DEPARTMENT OF MATHEMATICS

Course Structure and Syllabi for Pre Ph.D Programme

SUBJECT – 1

S.No	Course Code	Title of the Course
1	24CMGT01T	Research Methodology

SUBJECT - 2

S.No	Course Code	Title of the Course
1	24CMGT02T	Research and Publication Ethics

CORE SUBJECTS

Choose any Two subjects from the following list

S.No	Course Code	Title of the Course
1	24CMAT01T	Topics in Analysis
2	24CMAT02T	Mathematical Methods
3	24CMAT03T	Discrete Mathematics and Graph Theory
4	24CMAT04T	Advance Statistical Inference
5	24CMAT05T	Theory of Differential Equations
6	24CMAT06T	Topics In Algebra
7	24CMAT07T	Fluid Mechanics
8	24CMAT08T	Operations Research

Topics in Analysis (24CMAT01T)

Unit 1: Abstract Integration

The concept of measurability - Simple functions – Properties of measure, Integration of positive functions and complex functions – Set of measure zero.

Unit 2: Positive Borel Measures

Vector spaces – Review of topological preliminaries leading to locally compact Hausdorff spaces – Riesz representation theorem – Regularity properties of Borel measures – Lebesgue measures – Continuity property of measureable functions.

Unit 3: L^p-Spaces

Convex functions and inequalities – The L^p-spaces – Approximation by continuous functions.

Banach Space Techniques: Banach spaces – Consequences of Baire's theorem – Fourier coefficients of L^1 functions – Hahn Banach theorem.

Unit 4: Integration on product spaces

Measurability on Cartesian products – Product measure and its completion – Fubini's theorem – Convolution – Distribution functions.

Unit 5: Harmonic Functions

Laplacian of a harmonic function – Poisson integral of L^1 function – Mean value property – Boundary behavior of Poisson Integrals – Representation theorems.

Analytic continuation: Regular and Singular Points – Continuation along curves natural boundaries – Monodromy theorem.

Reference Books:

1. Walter Rudin, Real & Complex Analysis, Tata McGraw-Hill, Third Edition (Year of publication/Reprint: 1987).
2. H.L. Royden, Real Analysis, Collier Macmillan (Year of publication/Reprint: 1988)
3. P.R.Halmos, Measure theory – Graduate Text in Mathematics, Springer Verlag, New York (Year of Publication/Reprint 1974).
4. M. Thamban Nair, Functional Analysis – Prentice Hall, India (Year of Publication/Reprint 2003).
5. E.Kreyszig, Introductory Functional Analysis with Applications, John Wiley and sons. (Year of Publication/Reprint 1989).

6. L.V. Ahlfors, Complex Analysis, Mc Graw Hill. (Year of Publication/Reprint 1988)
7. J.B.Conway, Functions of one complex Variables I, Narosa Publishing House. (Year of Publication/Reprint 2000)
8. S.Lang, Complex Analysis, Springerm Verlag (Year of Publication/Reprint 2003).

Mathematical Methods (24CMAT02T)

Unit 1: Harmonic Functions

Harmonic Functions: Basic properties of harmonic functions-harmonic functions on a disk- sub harmonics and super harmonic functions-the Dirichlet problem-Green's functions.

Entire Functions: Jensen's formula-the genus and order of an entire function-Hadamard factorization theorem.

Steady temperatures: Steady temperatures- Steady temperatures in a half plane-a related problem.

Unit 2: Laplace Transforms

Temperatures in a Quadrant with part of One Boundary Insulated. Electrostatic Potential. Potential in a Cylindrical space. Two- dimensional Fluid Flow. The stream Function. Flow around a Corner. Flow around a Cylinder.

Laplace Transforms – Inverse Laplace Transforms – Error functions – Application to boundary value problems (Heat equation- Laplace equation) – Fourier transform – Fourier integral formula – Finite &infinite Fourier sine and cosine transforms – Application to integral equations and Boundary Value problems.

Unit 3: Special function: Bessel functions

Bessel functions: recurrence relations for the Bessel co-efficients – Series expansion for Bessel co-efficients – Integral expression for the Bessel co-efficients. The additions formula for the Bessel co-efficients.

Numerical solution of partial differential equations – Introduction – Finite difference approximation to derivatives – Finite difference methods – Laplace's equation - parabolic equations – Cranice – Nicholson Method – Jacobi Method - Gauss Siedel method.

Unit 4: Finite Element Methods

Finite Element Methods - Integral formulation and Variational Methods: Need for Weighted-Integral forms – Some mathematical concepts and formulas – Boundary, Initial and Eigen value problems – Integral relations – Functionals – The Variational Symbol – Weak formulation of Boundary Value problems – Weighted – Integral and Weak formulations – Linear and Bilinear forms and Quadratic Functional – examples. Variational methods of approximation – The Rayleigh – Ritz Method – Petrov – Galerkin method.

Unit 5: Differential and Integral Equations

Maxima and Minima- The Simplest Case, Illustrative Examples, Natural Boundary Conditions and transition conditions, The Variational notation, The more general case, Constraints.

Lagrange Multipliers, Variable end points, Sturm-Liouville problems. integral equations: Introduction, Relations between Differential and Integral Equations, The Green's function, Alternative Definition of the Green's function, Linear Equations in Cause and Effect-The influence function.

Reference Books:

1. Functions of one complex variable, Second edition, John B. Conway, Springer International Student Edition. (Chapter X and XI)
2. Integral Transforms Goyal and Gupta
3. Introductory Methods of Numerical Analysis by S. S. Sastry, Printis Hall Publication.
4. Standard and treatment "AN INTRODUCTION TO THE FINITE ELEMENT METHOD" G.N.REDDY McGraw-Hill Inc. (Second Edition) (Chapters 1 and 2).
5. "Methods of Applied Mathematics", FRANCIS B. HILDEBRAND, Second Edition, PHI Ltd, New Delhi.

"Special functions of Mathematical physic and Chemistry" I.N. Sneddon, of Longman Publications. (Chapter 4).

Discrete Mathematics and Graph Theory (24CMAT03T)

Unit 1: Mathematical Logic

Statements & Notation – Connectives - Well-formed formulas – Duality law – Functionally complete sets of connectives – Normal forms and principal normal forms.

Theory of Inference for the statement calculus – Rule of inference – Automatic theorem proving – Predicate calculus.

Unit 2 : Relations and Algebraic Structure

Representation of Discrete Structures – Relations and ordering – Functions – Composition of functions and inverse functions – Recursive functions, sets and predicates.

Lattices and Boolean algebra – Lattices as a partially ordered sets – Some properties of lattices, lattices as algebraic systems – Sublattices – Direct product and Homomorphism – special Lattices – Boolean algebra – Boolean functions, - Representation and minimization of Boolean functions.

Unit 3: Graph theory-1

Graphs – Isomorphism – Sub graphs – Euler Graphs – Hamiltonian paths and Circuits – Travelling salesman problem – Trees – Properties of trees – Spanning trees – Minimal spanning trees – Kruzkal's algorithm – Prim's algorithm – Dijkstra's algorithm.

Unit 4: Graph theory-2

Cut-sets and cut-vertices – Planar graph duality in planner graphs – Matrix representation of graphs – incidence matrix – Adjacent matrix path matrix – Circuit matrix – Cut set matrix – Transitive closur or a graph – Warshall's algorithm.

Coloring covering and partitioning – Chromatic number – Chromatic partitioning – Chromatic polynomial – Matching – Coverings the four-color problem.

Unit 5: Graph theory and Trees

Directed graphs – Digraphs – Types of Digraphs – Directed paths and connectedness – Euler digraphs – Trees with directed edges – Fundamental circuits in digraphs – Adjacency matrix of a digraphs – Acyclic digraphs & decyclization

Reference Books:

1. Discrete Mathematical structures with Applications to Computer Science, J.P. Tremblay & R. Manohar, TATA McGraw-Hill Edition (Chapter 1.1 to 1.5, 2.2 to 2.6 and 4.1 to 4.4).
2. Graph Theory with applications to Engineering and Computer Science. Narsingh Deo,PHI Prentice-Hall India.

Advance Statistical Inference (24CMAT04T)

Unit 1: Principle of Data Reduction

Principle of Data Reduction: Sufficiency principle - Factorization criterion – minimal sufficiency – completeness and bounded completeness – likelihood principle – Equivariance principle.

Unit 2: Theory of estimation

Theory of estimation: Basic concepts of estimation – Point estimation – methods of estimation- method of moments - method of maximum likelihood – Unbiasedness – Minimum variance estimation – Cramer – Rao bound and its generalization – Rao Blackwell theorem – Existence of U.M.V.U.E estimators.

Unit 3: General decision problem

General decision problem: Loss function, Risk function – Non-randomized and Randomized decision rules – Admissibility – Bayes' and Minimax decision rules – Sequential decision rules.

Unit 4: Testing of Hypothesis

Testing of Hypothesis: Critical region and power of the test – Neyman-Pearson lemma – Likelihood ratio principle – Uniformly most powerful tests – Unbiased test – Sequential probability ratio test.

Unit 5 : Analysis of variance

Analysis of variance: one way classification and its extension – Simple linear regression analysis with normal distribution.

Reference Books:

1. Rao. C.R Linear statisica inference and its Application (Wiley Easterm Ltd) 2nd Edition (Year of Publication/Reprint: 2001).
2. Ferguson, T: Mathematical Statistics – A Decision Theoreitc Approach (wiley & Sons) (Year of Publication/Reprint: 1967)
3. Berger, J.O: Statistical Decision Theory and Bayesian Analysis (springer – Verlag) (Year of Publication/Reprint: 1985)
4. Lehman, E.L: Testing of Statistical Hypothesis , Wiley Eastern Ltd. (Year of Publication/Reprint: 1959)
5. Lehman, E.L: Point Estimation, John Wiley & Sons (Year of Publication/Reprint: 1984).
6. G.Casella, R.L Berger: Statistical Inference 2nd Edition, Duxbury Press (Year of Publication/Reprint: 2002).

Theory of Differential Equations (24CMAT05T)

Unit 1: System of differential equations

System of differential equations: System of first order equations – existence and uniqueness of solution – Gronwall's inequality – continuous dependence on initial conditions and parameters

Unit 2: Linear systems

Linear systems: Autonomous systems – Transition matrix – Phase- space of two-dimensional systems – time varying systems – fundamental matrix and its properties – linear systems with periodic coefficients.

Unit 3: Stability of differential systems

Stability of differential systems: Stability of linear systems – almost linear systems – stability of periodic solutions – Lyapunov stability theorems for non-linear system – limit cycles – Poincare – Bendixon theorem – Lienard System – Construction of Lyapunov function – Bifurcations (Transcritical, Saddle-node, Pitchfork, Hopf, Sotomayor theorem)

Unit 4: Review of first order PDE

Review of first order PDE: classification – solution method for quasi-linear and nonlinear PDE – discontinuous solution – conservations laws and shocks.

Four important linear PDE's (transport, Laplace, heat and wave equations): fundamental solution – mean value formulae – properties of harmonic functions – Green's function and energy method.

Unit 5: Sobolev spaces and Elliptic Equations

Sobolev spaces: Definition – approximations – sobolev inequalities – extensions –traces – compactness – dual spaces.

Elliptic Equations: Definitions - Existence of Weak solutions – Regularity – Maximum principles – Eigne values and eigen –function. Linear evolution equations: Parabolic equation – hyperbolic equations – semigroup theory.

Reference Books:

1. G.F. Simmons, Differential Equations with Applications and Historical Notes, 2nd Ed., McGraw – Hill (Year of Publication/Reprint: 1991).
2. R.P Agarwal, D.O's Regan, An Introduction to Ordinary Differential Equations, Springer ((Year of Publication/Reprint: 2008).

3. K.S Bhamra – Ordinary Differential Equations – Narosa Publications ((Year of Publication/Reprint: 2015).
4. I.N. Sneddon, Elements of Partial Differential Equations – McGraw –Hill ((Year of Publication/Reprint: 1957).
5. L.C. Evans, Partial Differential Equations, 2nd Ed., American Mathematics Society. (Year of Publication/Reprint: 2015).
9. M. Renardy, R.C Rogers, An Introduction to Partial Differential Equations, 2nd Ed., Springer (Year of Publication/Reprint: 2010).
10. S. Kesavan, Topics In innctional Analysis and its Applications, New Age International (P) Ltd. (Year of Publication/Reprint: 2012).

Topics in Algebra (24CMAT06T)

Unit 1: Groups

Review of group actions and Sylow's theorems – Free groups and relations – normal series – nilpotent and solvable groups.

Unit 2: Rings and Ideals

Review of rings and ideals – PID – Euclidean domains and UFD. Modules – direct sums of modules – free modules – exact sequences – finitely generated modules over a PID – Structure of finitely generated abelian groups – rational and Jordan canonical forms.

Unit 3: L^p-Spaces

Review of algebraic extensions of fields – algebraic closure and splitting fields. Normal extensions and separable extensions – finite fields- Galois theory – The fundamental theorem of Galois Theory – roots of unity – cyclotomic extensions – cyclic extensions Galois group of a polynomial – solvable and radical extension – insolvability of the quintic.

Unit 4 : Integration on product spaces

Artinian and Noetherian modules and rings- modules of finite lengths – simple and semi simple modules and rings – Wedderburn –Artin theorem – nil radical and Jacobson radical of an Artinian ring.

Unit 5: Harmonic Functions

Commutative rings: Primary decompositions of Ideals and modules.

Reference Books:

1. N. Herstein University of Chicago Topics in Algebra 2nd Edition John Wiley & Son's New York .
2. Dummit D.S and Foote R.M., "Abstract Algebra" John Wiley and Sons (3rd Edition) (Year of Publication/Reprint: 2003).
3. Hungerford T.W., "Algebra", Springer. (Year of Publication/Reprint: 1980).
4. Bhattacharya P.B., Jain S.K and Nagpaul S.R., "Basic Abstract Algebra", Cambridge University Press (2nd Edition) (Year of Publication/Reprint: 1995).
5. LangS., "Algebra", Springer (3rd Edition) (Year of Publication/Reprint: 2005).
6. Jacobson N., "Basic Algebra Vol. I & Vol .II " Dover Publications (2nd Edition) (Year of Publication/Reprint: 2009).
7. Musuli C., "Introduction to Rings and Modules", Narosa Publishing House (2nd Edition) (Year of Publication/Reprint: 1997)

Fluid Mechanics (24CMAT07T)

Unit 1: Kinematics of fluids in motion

Kinematics of fluids in motion: Real fluid and ideal fluids - Velocity of fluid at a point - Stream lines and path lines - Steady flow and unsteady flow, velocity potential, velocity vector, local and particle of fluid, conditions at a rigid boundary, general analysis of fluid motion.

Equations of motion of a fluid: Pressure at a point in a fluid at rest - Pressure at a point in a moving fluid - Conditions at boundary of two inviscid in compressive fluids - Euler's equation of motion - Bernoulli's equation.

Unit 2: Three dimensional flows

Three dimensional flows: Sources – Sinks – Doublets - Images in a rigid infinite plane - Images in solid spheres - Axisymmetric flows - Stokes stream function for axisymmetrical irrotational motions.

Two dimensional flows: Meaning of two-dimensional flow - Use of cylindrical polar coordinates - stream function, complex potential for two dimensional irrotational incompressible flow - Complex velocity potentials for standard two-dimensional flow - Uniform stream line sources and line sinks - Line doublets line vortices.

Unit 3: Stress and Strain Analysis

Milne Thompson circle theorem - application s of circle theorems extensions of circle theorem - theorem of Blasius.

Stress components in a real fluid – Relations between Cartesian components of stress – Translational motion of fluid element - The rate of strain quadric and principal stresses – Some further properties of the rate of strain quadric – Stress analysis in fluid motion – Relations between stress and rate of strain – The coefficient of viscosity and laminar flow.

Unit 4 : Viscous Flow and Solutions of the Navier-Stokes Equations

The Navier-Stokes equations of motion of a viscous fluid – Some solvable problems in viscous flow – Steady motion between parallel planes – Steady flow through tube of uniform circular cross-section – steady flow between concentric rotating cylinders – Steady viscous flow in tubes of uniform cross-section – Tube having equilateral triangular cross-section.

Unit 5: Harmonic Functions

Diffusion of vorticity – Energy dissipation due to viscosity – steady flow past a fixed sphere – Dimensional analysis; Reynolds number Prandtl's boundary layer.

Reference Books:

1. F. Chorlton, Textbook of Fluid Dynamics, CBS Publishers & Distributors.

Operations Research (24CMAT08T)

Unit 1: Linear Programming and Solution Methods

Introduction to Linear program problem method -Simplex Method-big M-method and Dual Simplex methods.

Unit 2: Transportation, Assignment, and Routing Problems

Transportation problems – Assignment models and The traveling salesman (Routing) Problem.

Unit 3: Markov Processes and Stochastic Analysis

Markov Analysis: Introduction – Stochastic (Random) process – Markov process – Transition probability – Transition probability matrix – First order and higher order Markov process – n-Step transition probabilities – Markov chain – Steady state (Equilibrium) condition – Markov Analysis.

Unit 4: Game Theory and Job Sequencing Optimization

Game Theory in job sequencing Minimax (Maximin) Criterion and optimal strategy – Saddle point, optimal strategies and the value of game – Solution of games with saddle point(s) – Illustrative examples – Rectangular games without saddle point – Minimax-Maximin principle for mixed strategy games – Equivalence of Rectangular game and Linear programming – Minimax Theorem (Fundamental theorem of game theory) – solution of m x n games by linear programming – Two by-two (2 x 2) games – Principle of dominance to reduce the size of the game – Graphical method for (2 x n) and (m x 2) games – Matrix method for m x n games

Job sequencing: Introduction – Terminology and notations – Principal assumptions – Solution of sequencing problem – Processing n jobs through 2 machines – Processing n jobs through 3 machines – Processing 2 jobs through m machines – Processing n jobs through m machines

Unit 5: Inventory Models and Queuing Theory

Deterministic Elementary inventory models: Concept of average inventory – Concept of economic ordering quantity (EOQ) – the EOQ model without shortage – The EOQ model with shortages – multi-item deterministic models (The EOQ with constraints).

Solution of queueing models and limitations for the applications – Model (M|M|1) : FCFS) : Birth and Death model – Model ii (A) General Erlang queueing model (Birth-Death process) – Model III, (M|M|1) : (N|FCFS) – Model IV (A), (M|M|s) : (¥|FCFS) – Non-Poisson queueing model.

Reference Books:

1.Operations Research, S. D. Sharma, Kedar Nath Ram Nath & Co. Publishers