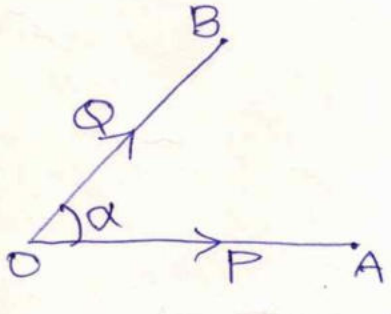
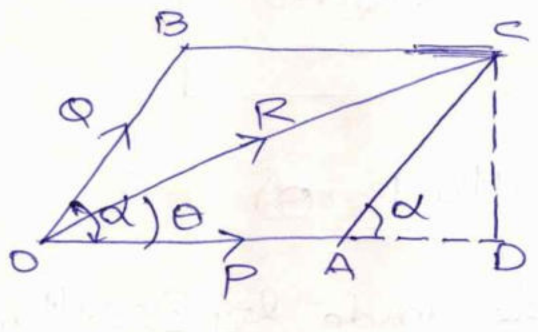
represented in magnitude and direction by the diagonal of the parallelogram passing through that point".

Let two forces P and Q act at a point 'O' as shown in fig. The force P is represented in magnitude and direction by OA whereas the force Q is presented in magnitude and direction by OB. Let the angle between both the two forces be 'α'.

The resultant of these two forces will be obtained in magnitude and direction by the diagonal (passing through O) of the parallelogram of which OA and OB are two adjacent sides.





Magnitude of Resultant: (R)

From 'C' draw CD perpendicular to OA produced.

Let α = angle between two forces P and Q

$= \angle AOB$

Now $\angle DAC = \angle AOB = \alpha$

In parallelogram OACB, AC is parallel and equal to OB.

$AC = Q$

In triangle ACD,

$$AD = AC\cos\alpha = Q\cos\alpha$$

$$CD = AC\sin\alpha = Q\sin\alpha$$

In ΔOCD,

$$OC^2 = OD^2 + CD^2$$

$$OC = R,\ OD = P + AD,\ CD = Q\sin\alpha$$

$$OD = P + Q\cos\alpha$$

$$R^2 = (P + Q\cos\alpha)^2 + (Q\sin\alpha)^2$$

$$R^2 = P^2 + Q^2\cos^2\alpha + 2PQ\cos\alpha + Q^2\sin^2\alpha$$

$$= P^2 + Q^2(\sin^2\alpha + \cos^2\alpha) + 2PQ\cos\alpha$$

$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}}$$

Direction of Resultant:- (θ)

Let θ = angle made by resultant with OA

from Δ^{le} OCD,

$$\tan\theta = \frac{CD}{OD} = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)}$$

Case 1: If the two forces are at right angles, then

$$\alpha = 90^\circ$$

Resultant $R = \sqrt{P^2+Q^2+2PQ\cos 90}$ ②

$$\boxed{\therefore R = \sqrt{P^2+Q^2}}$$

Direction angle $\theta = \tan^{-1}\left(\frac{Q\sin 90}{P+Q\cos 90}\right)$

$$\theta = \tan^{-1}\left(\frac{Q}{P}\right)$$

case: 2: If the two forces P and Q are equal and are acting at an angle α between them, then the magnitude and direction of Resultant is given as

$$R = \sqrt{P^2+Q^2+2PQ\cos\alpha}$$

$$P = Q$$

$$= \sqrt{P^2+P^2+2P^2\cos\alpha} = \sqrt{2P^2+2P^2\cos\alpha}$$

$$= \sqrt{2P^2(1+\cos\alpha)} = \sqrt{2P^2 \cdot 2\cos^2\frac{\alpha}{2}} = \sqrt{4P^2\cos^2\frac{\alpha}{2}}$$

Resultant magnitude $\boxed{R = 2P\cos\frac{\alpha}{2}}$

Resultant direction: $\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P+Q\cos\alpha}\right)$

$$= \tan^{-1}\left(\frac{\cancel{P}\sin\alpha}{P+P\cos\alpha}\right) = \tan^{-1}\left(\frac{\sin\alpha}{1+\cos\alpha}\right)$$

$$= \tan^{-1}\frac{\cancel{2}\sin\frac{\alpha}{2}\cancel{\cos\frac{\alpha}{2}}}{\cancel{2}\cos^2\frac{\alpha}{2}} = \tan^{-1}\left(\tan\left(\frac{\alpha}{2}\right)\right) = \frac{\alpha}{2}$$

$\therefore \boxed{\theta = \frac{\alpha}{2}}$

Law of Triangle of forces:- It states that "if three forces acting at a point be represented in magnitude and direction by the three sides of a triangle, taken in order, they will be in equilibrium".

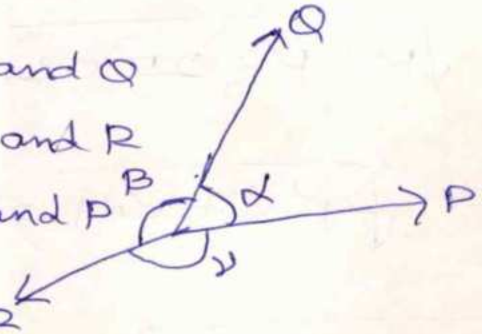
Lami's Theorem:- It states that " If three forces acting at a point are in equilibrium, each force will be proportional to the sine of the angle between the other two forces".

Suppose the three forces P, Q and R are acting at point 'O' and they are in equilibrium as shown in fig.

Let α = angle between P and Q

β = " " Q and R

γ = " " R and P

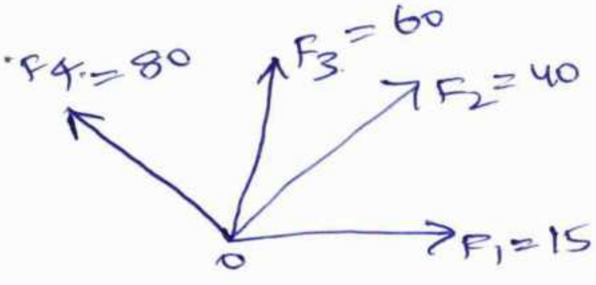


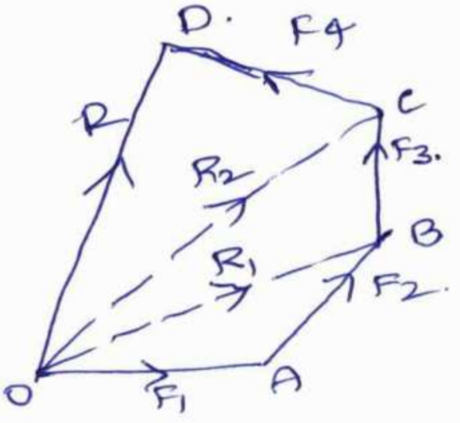
Then according to lami's theorem.

$$\boxed{\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha}}$$

Triangle law of forces:- It may be stated as " If two forces acting on a body are represented one after another by the sides of a triangle, their resultant is represented by the closing side of the triangle taken from first point to the last point".

Polygon law of forces:- It may be stated as "If a number of concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon, taken in a order, then the resultant is represented in magnitude and direction by the closing side of the polygon, taken from first point to last point."

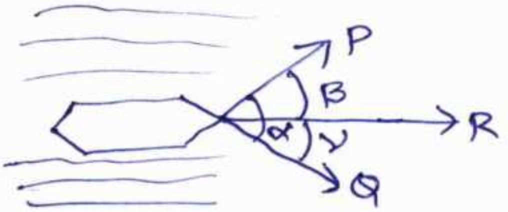




Q) A man of weight W = 712N holds one end of a rope that passes over a pulley vertically above his head and to the other end of which is attached a weight Q = 534N. Find the force with which the man's feet press against the floor. [Ans: 178N]

Q) A boat is moved uniformly along a canal by two horses pulling with forces P = 890N and Q = 1068N acting under an angle α = 60° (Fig A) Determine the magnitude of the resultant pull on the boat and the angles β and γ as shown in the fig. [Ans: R = 1698N, β = 33°; γ = 27°]

[Use parallelogram Law of forces].



Q) What force Q combined with a vertical pull P = 27N will give a horizontal resultant force R = 36N?

[Ans: 45N Inclined by 36°.86]

Q) To move a boat uniformly along a canal at a given speed requires a resultant force R = 1780N. This is accomplished by two horses pulling with forces P and Q on two ropes, as shown in fig A. If the angles that the two ropes make with the axis of the canal are β = 35° and γ = 25°, what are the corresponding tensions in the ropes?

[Use Lami's Theorem]

[Ans: P = 868N; Q = 1179N]

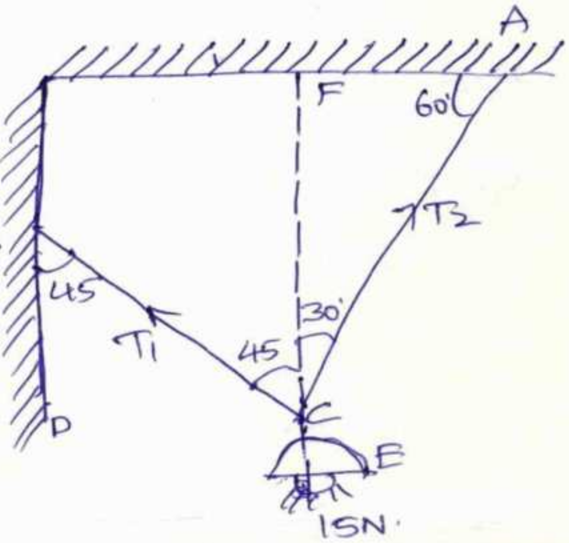
Q) If, in fig. the horses pull with the forces P = 1068N and Q = 890N, what must be the angles β and γ to give the resultant $R = 1780N$?

[Ans: $\beta = 22°39'$; $\nu = 27.07$]

(7)

a) An electric light fixture weighing 15N hangs from a point C, by two strings AC and BC. AC is inclined at 60° to the horizontal and BC at 45° to the vertical as shown in fig. Using lami's theorem or otherwise determine the forces in the strings AC and BC.

[Ans: $T_1 = 7.76N$ $T_2 = 10.98N$]



Resolution of a force:-

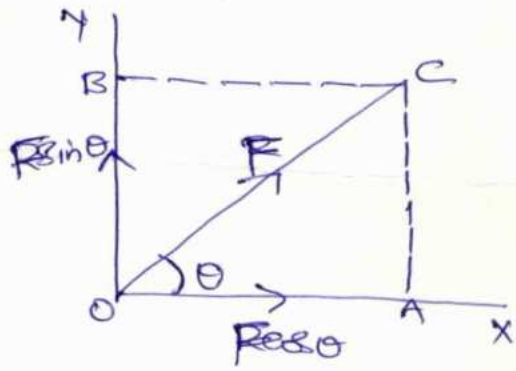
Resolution of a force means "finding the components of a given force in two given directions".

Let a given force be R which makes an angle θ with X-axis as shown in fig. It is required to find the components of the force R along X-axis and Y-axis.

components of R along X-axis = $R\cos\theta$

" " " Y-axis = $R\sin\theta$

Hence, the resolution of forces is the process of finding components of forces in specified directions.



Resolution of a number of coplanar forces:

Let a number of coplanar forces (forces acting in one plane are called co-planar forces) $R_1, R_2, R_3 \ldots$ are acting at a point as shown in fig.

Let θ_1 = Angle made by R_1 with X-axis

θ_2 = " " " R_2 " "

θ_3 = " " " R_3 " "

H = Resultant component of all forces along X-axis

V = " " " " " Y-axis

Each force can be resolved into two components, one along X-axis and other along Y-axis. (8)

component of R_1 along X-axis $= R_1 \cos\theta_1$

" " " Y-axis $= R_1 \sin\theta_1$

Similarly, the components of R_2 and R_3 along X-axis and Y-axis are $(R_1 \cos\theta_2, R_2 \sin\theta_2)$ and $(R_3 \cos\theta_3, R_3 \sin\theta_3)$ respectively.

Resultant component along X-axis

= sum of components of all forces along X-axis

$H = R_1 \cos\theta_1 + R_2 \cos\theta_2 + R_3 \cos\theta_3 + \ldots\ldots$

Resultant component along Y-axis

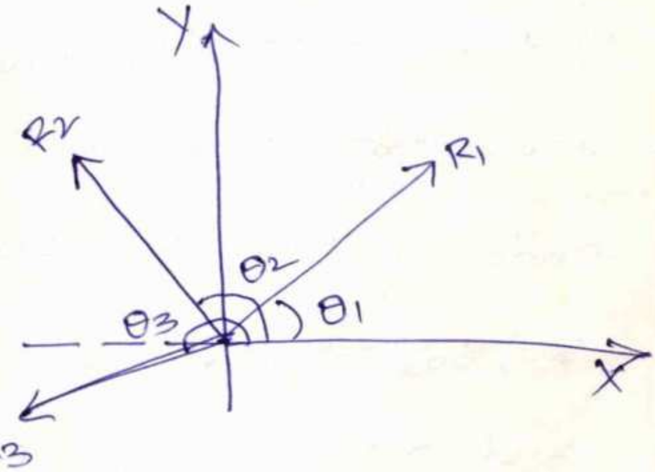
= sum of components of all forces along Y-axis

$V = R_1 \sin\theta_1 + R_2 \sin\theta_2 + R_3 \sin\theta_3$

Then resultant of all the forces, $R = \sqrt{H^2 + V^2}$

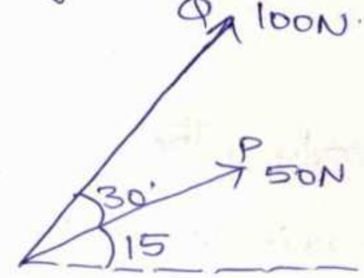
The angle made by R with X-axis is given by,

$$\boxed{\tan\theta = \frac{V}{H}}$$



Problems

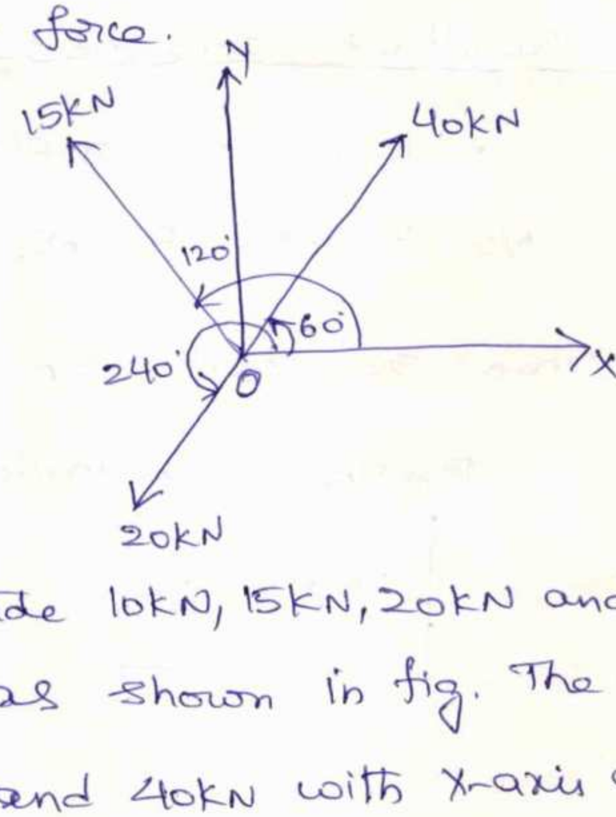
1) Two forces are acting at a point 'O' as shown in fig. Determine the resultant in magnitude and ~~resultant~~ direction.



[Ans: R = 145.46N θ = 35.10°]

2) Three forces of magnitude 40kN, 15kN and 20kN are acting at a point O as shown in fig. The angles made by 40kN, 15kN and 20kN forces with X-axis are 60°, 120° and 240° respectively. Determine the magnitude and direction of the resultant force.

[Ans: R = 30.41kN
θ = 85.28°]



3) Four forces of magnitude 10kN, 15kN, 20kN and 40kN are acting at a point 'O' as shown in fig. The angles made by 10kN, 15kN, 20kN and 40kN with X-axis are 30°, 60°, 90° and 120° respectively. Find the magnitude and direction of the resultant force.

[Ans: R = 72.73kN θ = 93.03°

<u>Moment of a force</u>: The product of a force and the perpendicular distance of the line of action of the force from a point is known as moment of the force about that point.

P = A force acting on a body

r = $\perp^r$ dist between the point 'O' and line of action of the force P.

The moment of the force P about 'O' $= P \times r$

The tendency of the moment $P \times r$ is to rotate the body in the clockwise direction about 'O'.

Hence this moment is called clockwise moment. If the tendency of rotation is anti-clockwise, the moment is called anti-clockwise moments.

Units: Nm or Nmm

<u>Effect of force and Moment on a body</u>:-

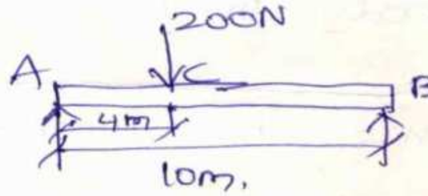
The force acting on a body causes linear displacement while moment causes an angular displacement. Hence a body when acted by a number of coplanar forces will be in equilibrium if:

(i) Resultant component of forces along any direction is zero i.e, resultant component of forces in the direction of x, in the direction of y and in the direction of ~~z~~ z are zero.

(ii) Resultant moments of the forces about any point in the plane of the forces is zero or clockwise moment is equal to anti-clockwise moment

Problems

1) A beam of span 10m is carrying a point load of 200N at a dist 4m from A. Determine the beam reactions.



clockwise moment = anticlockwise moment

$R_A + R_B = 200$

A

$R_B \times 10 = 200 \times 4 = 800$

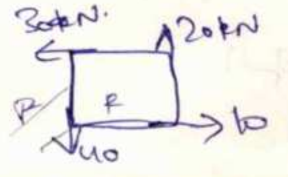
$R_B = 80$

$R_A = 200 - R_B$

$R_A = 120$

2) Four forces of magnitudes 10N, 20N, 30N and 40N are acting respectively along the four sides of a square ABCD as shown in fig. Determine the magnitude, direction and position of the resultant force.

[Ans: $R = 20\sqrt{2}$ N $L = \frac{5a}{2\sqrt{2}}$
$\theta = 225^\circ$]



Law of Mechanics:-

1) Newton's first law and second law of motion
2) Newton's third law
3) Gravitational law of attraction
4) The parallelogram law
5) The principle of transmissibility of forces.

Newton's first and second laws of motion:-

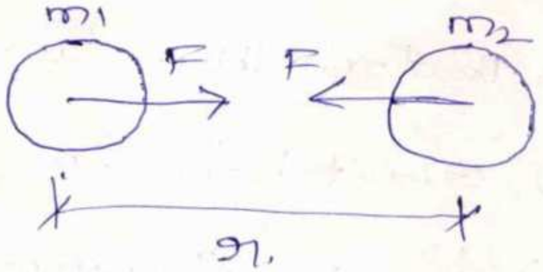
Newton's first law states that "Every body continues in a state of rest or in motion in a straight line unless it is compelled ~~by~~ to change that state by some external force acting on it."

Newton's second law states that "The net external force acting on a body in a direction is directly proportional to the rate of change of momentum in that direction.

Newton's Third law:- Newton's Third law states, "To every action there is always equal and opposite reaction".

The Gravitational law of attraction: It states that two bodies will be attracted towards each other along their connecting line with a force which is direct proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

$$F \propto \cancel{G}\,\frac{m_1 m_2}{r^2}$$



$$F = G\,\frac{m_1 m_2}{r^2}$$

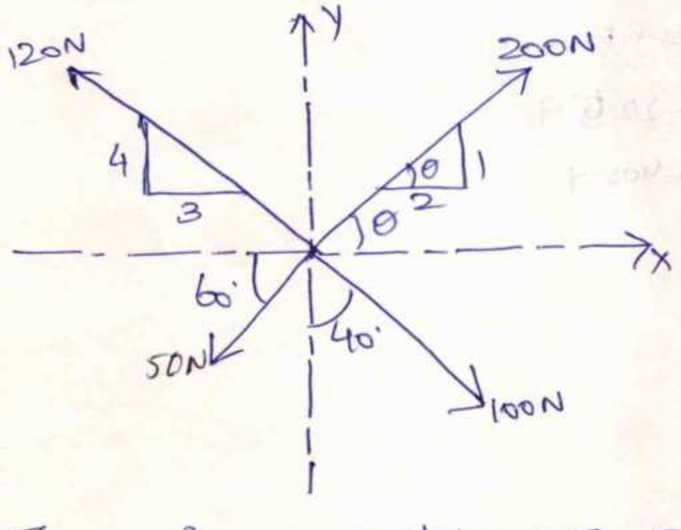
where G = Universal gravitational constant of proportionality

Principle of transmissibility of forces: It states that "if a force, acting at a point on a rigid body, is shifted to any other point which is on the line of action of the force, the external effect of the force on the body remains unchanged.

1) The resultant of two forces, one of which is double the other is 260N. If the direction of the larger force is reversed and the other remain unaltered, the resultant reduced to 180N. Determine the magnitude of the forces and the angle between the forces:

Ans P = 200, θ = 63°

2) A system of ~~bar~~ four forces acting at a point on a body is shown in fig. Determine the resultant.

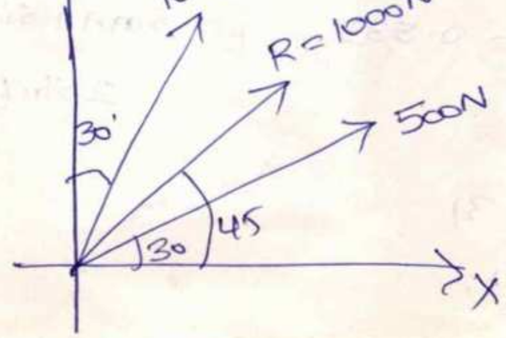


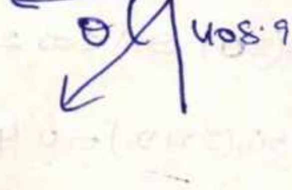
$R = 160.18\ N$

$\alpha = 24.15'$

3) Two forces acting on a body are 500N and 1000N as shown in fig. Determine the third force F such that the resultant of all the three forces is 1000N directed at 45° to x-axis

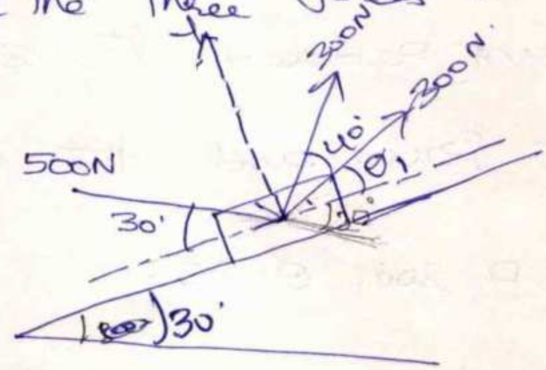
[Ans: θ = 61.08, F = 467.2 N]





4) Three forces acting at C.G of a block are shown in fig. The direction of 300N forces may vary, but the angle between them is always 40°. Determine the value of θ_1 for which the resultant of the three forces is directed parallel to the plane.

[Ans: $\theta_1 = 6.31°$]



3) $R\cos\theta = \Sigma F_x = 500\cos30 + 1000\sin30 + F\cos\theta_1$

$R\sin\theta = \Sigma F_y = 500\sin30 + 1000\cos30 + F\sin\theta_1$

$\tan 45 = \dfrac{933 + F\cos\theta_1}{1116 + F\sin\theta_1}$ $\quad F\cos\theta_1 = -225.9$

$\quad F\sin\theta_1 = -408.9$

$1116 + F\sin\theta_1 = 933 + F\cos\theta_1$

$F(\sin\theta_1 - \cos\theta_1) = -183.$

$\tan\theta_1 = 1.81$

$\theta_1 = 61.08$

$F = -467.145$

4) $300\cos\theta_1 + 300\cos(40+\theta_1) + 500\cos30 = \Sigma F_x$

$300\sin\theta_1 + 300\sin(40+\theta_1) - 500\sin30 = 0.$

$\sin\theta_1 + \sin(40+\theta_1) = 0.833$

$2\sin(20+\theta_1)\cos 20 = 0.833.$ $\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$\sin(20+\theta_1) = 0.44.$

$20 + \theta_1 = 26.31$

$\theta_1 = 6.31.$

The forces, which are having their line of actions
parallel to each other, are known Parallel forces. The two parallel forces will not inter-sect at ~~each~~ a point.

Classification of a force system:-

When several forces act on a body, then they are called force system or a system of forces. In a system in which all the forces lie in the same plane, it is known as coplanar force system.

Force system

- 1. coplanar
  - Collinear
  - Concurrent
  - parallel
  - Non-concurrent Non-parallel
- 2. Non-coplanar
  - concurrent
  - parallel
  - Non-concurrent Non-parallel

Resultant of several forces:-

When a number of coplanar forces are acting on a rigid body then these forces can be replaced by a single force which has the same effect on the rigid body as that of ~~the~~ all the forces acting together then this single force is known as resultant of several forces.

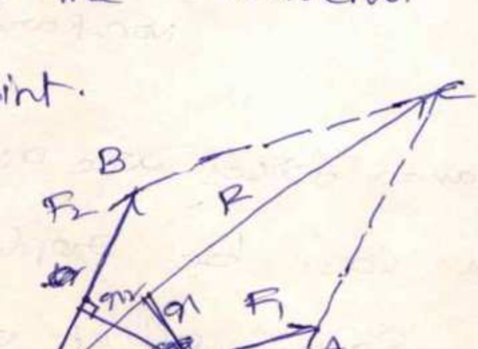
Moment of a force:- The Product of a force and the ⊥r dist ~~bet~~ of the line of action of the force ~~and~~ from a point is known as moment of the force about that point.

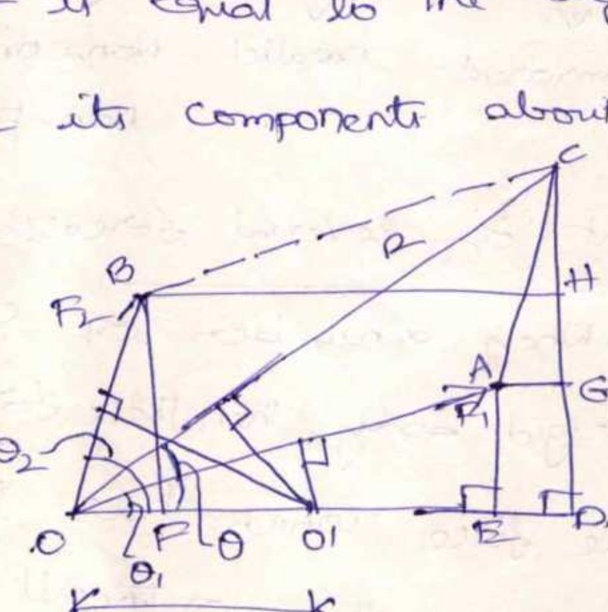
$M = F \times r$.

Principle of moments (OR Varignon's Principle):-

Principle of moments states that the moment of the resultant of a number of forces about any point is equal to ~~an~~ the algebraic sum of the moments of all the forces of the system about the same point.

According to Varignon's principle, the moment of a force about any point is equal to the algebraic sum of the moments of its components about that point.





Problems

1) Two forces of magnitude 10N and 8N are acting at a point. If the angle between the two forces is 60° determine the magnitude of the resultant force.

[Ans: R = 15.62N]

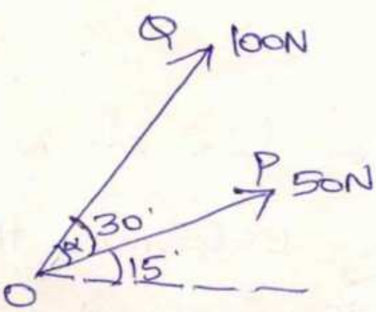
2) Two equal forces are acting at a point with an angle of 60° between them. If the resultant force is equal to $20 \times \sqrt{3}$ N, find magnitude of each force.

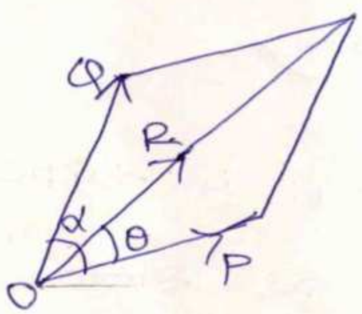
[Ans: R = 20N]

3) The resultant of the two forces, when they act at an angle of 60° is 14N. If the same forces are acting at right angles, their resultant is $\sqrt{136}$ N. Determine the magnitude of the two forces.

[Ans: P = 10N Q = 6N]

4) Two forces are acting at a point 'O' as shown in fig. Determine the resultant in magnitude and direction





[Ans: R = 145.46N θ = 35.10°

5) The resultant of two concurrent forces is 150
the angle between the forces is 90°. The resultant

angle of 36° with one of the force. Find the magnitude of each force.

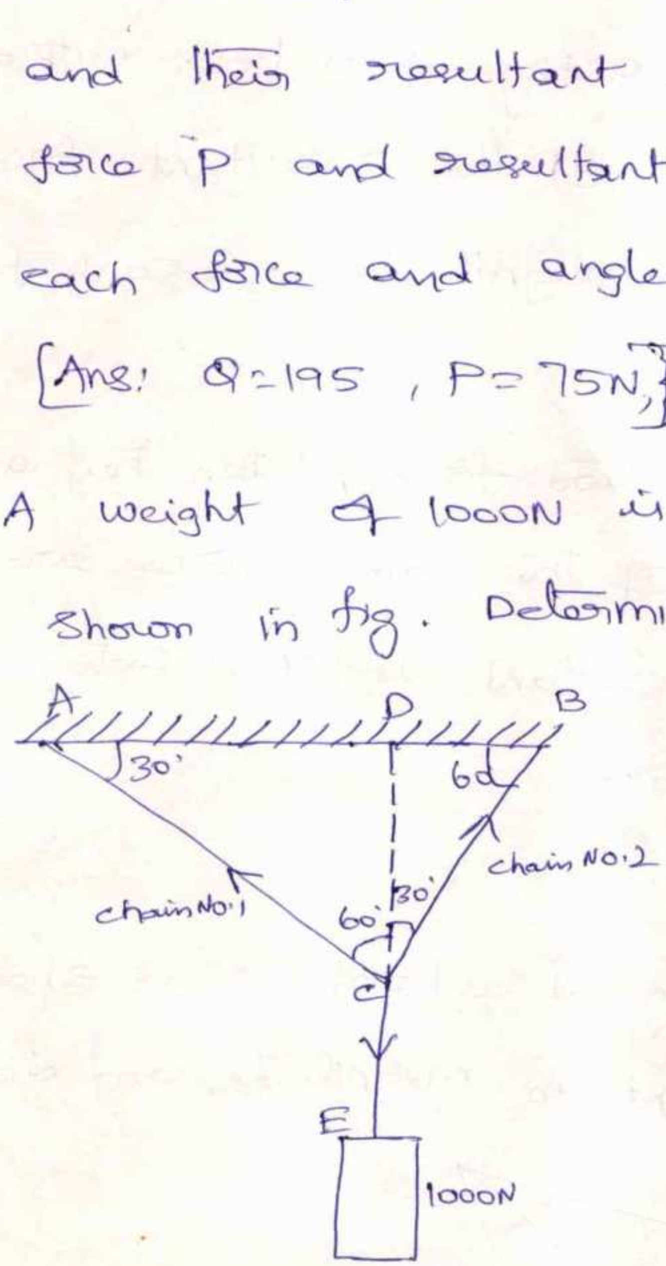
[Ans: P = 1213.86N   Q = 881.~~86~~ 67N]

6) The sum of two concurrent forces P and Q is 270N and their resultant is 180N. The angle between the force P and resultant R is 90°. Find the magnitude of each force and angle between them.

[Ans: Q = 195, P = 75N, } α = 112.618°]

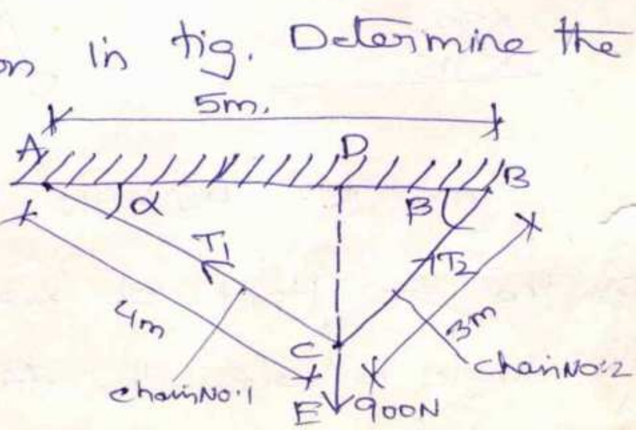
7) A weight of 1000N is supported by two chains as shown in fig. Determine the tension in each chain.

[Ans: T_1 = 500N
T_2 = 866N]



8) A weight of 900N is supported by two chains of length 4m and 3m as shown in fig. Determine the tension in each chain.

[Ans: T_1 = ~~776N~~ 537.44N
T_2 = ~~10.98N~~ 720N]



... of Varignon's Principle!

Fig shows two forces F_1 and F_2 acting at point O. These forces are represented in magnitude and direction by OA and OB. Their resultant R is represented in magnitude and direction by OC which is the diagonal of parallelogram OACB.

Let O' is the point in the plane about which moments of F_1, F_2 and R are to be determined.

From point O' draw ⊥'s on OA, OC and OB.

Let r_1 = ⊥r dist between F_1 and O'

r = " " " R and O'

r_2 = " " " F_2 and O'.

Then according to Varignon's Principle

Moment of R about O' must be equal to algebraic sum of moments of F_1 and F_2 about O'.

$$R \times r = F_1 \times r_1 + F_2 \times r_2$$

Now refer to fig. Join Od and produce it to D.

From points C, A and B draw ⊥'s on OD meeting at D, E and F respectively.

From A and B also draw ⊥'s on CD meeting the line CD at G and H respectively.

Let θ_1 = angle made by F_1 with OD

θ = " " " R with OD

θ_2 = " " " F_2 with OD.

In fig OA = BC , OB = AC & GD = CH

Then from fig

$F_1 \sin\theta_1 = AE = GD = CH$

$F_1 \cos\theta_1 = OE$

$F_2 \sin\theta_2 = BF = HD.$

$F_2 \cos\theta_2 = OF = ED.$

$R \sin\theta = CD.$

$R \cos\theta = OD.$

Let the length $od = x$

Then $x \sin\theta_1 = r_1$, $x \sin\theta = r$, $x \sin\theta_2 = r_2$

Now moment of R about O^1

$= R \times d$ (⊥r dist bet O' and R) $= R \times r$

$= R \times x \sin\theta$

$= R \sin\theta \times x$

$= CD \times x$

$= (CH + HD) \times x$

$= (F_1 \sin\theta_1 + F_2 \sin\theta_2) \times x$

$= F_1 \times x \sin\theta_1 + F_2 \times x \times \sin\theta_2$

$= F_1 \times r_1 + F_2 \times r_2$

$= F_1 \times r_1 + F_2 \times r_2$

= Moment of F_1 about O^1 + Moment of F_2 about O^1.

Hence moment of R about any point is the algebraic sum of moments of its components about the same point. Hence varignon's principle is proved.

Types of parallel forces: The following are the important types of parallel forces:

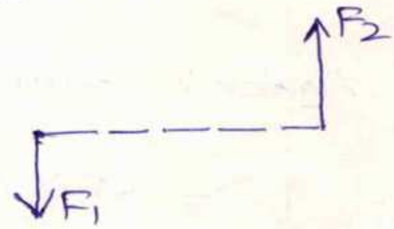
1) Like parallel forces

2) Unlike "

1) like parallel forces:-

The parallel forces which are acting in the same direction are known as like parallel forces. These forces may be equal or unequal in magnitude.

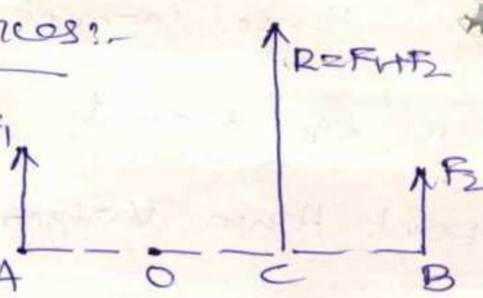
Unlike parallel forces:- The parallel forces which are acting in the opposite direction are known as unlike parallel forces. These forces may be equal or unequal in magnitude.



Resultant of two parallel forces:-

1. Two parallel forces are like
2. Two " " are unlike and are unequal in magnitude
3. " " " " " but equal in magnitude

1. Resultant of two like parallel forces:-



For the two parallel forces F_1, F_2 which are acting in the same direction, Resultant R is given by

$$R = F_1 + F_2$$

In order to find the point at which the resultant is acting Varignon's principle (or principle of moments) is used.

The algebraic sum of moments of F_1 and F_2 about any point should be equal to the moment of the resultant (R) about that point.

choose any point 'O' along line AB and moments of all forces about this point.

Moment of F_1 about 'O' = $F_1 \times AO$ (c.w)

" F_2 " " = $F_2 \times OB$ (A.C.W)

Algebraic sum of moments of F_1 and F_2 about 'O'

$$= -F_1 \times AO + F_2 \times OB$$

Moment of Resultant about 'O' = $R \times OC$ (a.c.w)

$$-F_1 \times AO + F_2 \times OB = R \times OC$$

$$-F_1 \times AO + F_2 \times OB = (F_1 + F_2) OC$$

$$F_2 (OB - OC) = F_1 (OC + AO)$$

$$F_2 \times CB = F_1 \times AC$$

$= F_2 \times AB$ (C.W)(−)

~~R×~~ Now the moment of resultant 'R' about A.

$= R \times AC$ (A.C.W)

$R \times AC = F_2 \times AB$

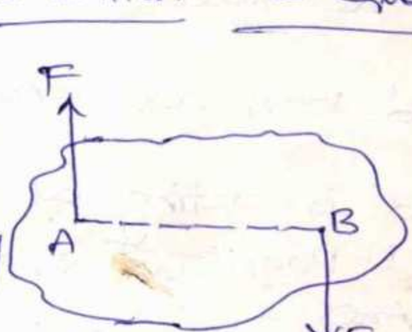
$(F_1 - F_2) \times AC = F_2 \times AB$

$F_1 \times AC = F_2 (AB + AC)$ ~~Hence F_1~~

$F_1 \times AC = F_2 \times CB$

As F_1 and F_2 and AB are known, the AC can be calculated, the location of point C is known.

<u>Resultant of two unlike parallel forces which are equal in magnitude</u>:- When two equal and opposite parallel forces act on a body at some distance apart, the two forces ~~form~~ form a couple which has a tendency to rotate the body. The ⊥r dist bet the parallel forces is known as arm of the couple.



let F = force at A and at B.

a = ⊥r distance (or arm of the couple)

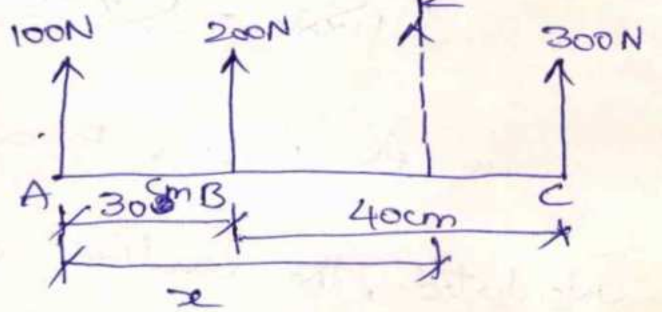
Moment of the couple ⇒ $M = F \times a$

units: Nm.

Resolution of a force into a force and a couple

Problems

1) Three like parallel forces 100N, 200N and 300N are acting at points A, B and C respectively on a st. line ABC as shown in fig. The distances are AB = 30cm and BC = 40cm find the resultant and also the dist of the resultant from point A on line ABC.

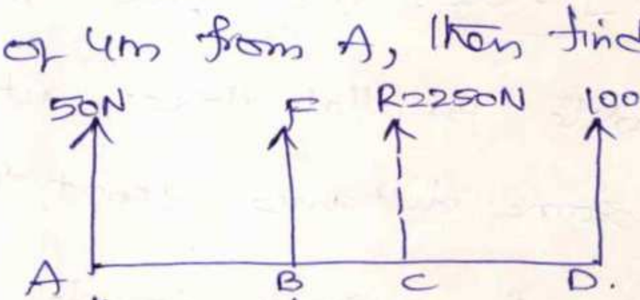


[Ans: R = 600N x = 45cm]

2) The three like parallel forces of magnitude 50N, P and 100N as shown in fig. If the resultant R = 250N and is acting at a dist of 4m from A, then find.

(i) Magnitude of force P

(ii) Distance of P from A



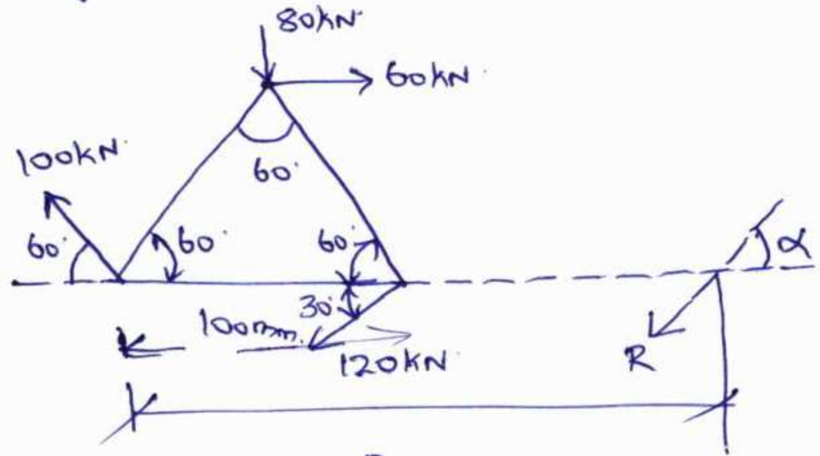
[Ans: P = 100, x = 3m]

3) Four parallel forces of magnitudes 100N, 150N, 25N and 200N are shown in fig. Determine the magnitude of the resultant and also the distance of the resultant from point A.

[Ans: R = 125N x = 3.06m

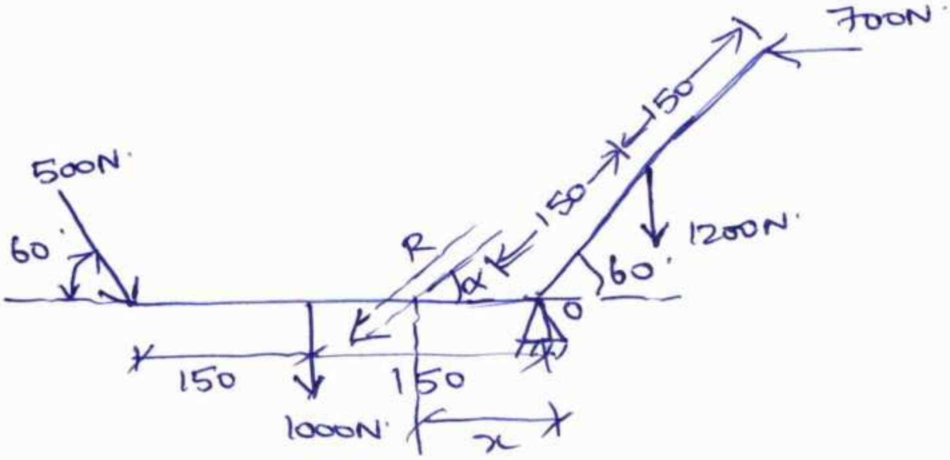
Q) Find the resultant of the force system shown in fig acting on a lamina of equilateral triangular shape.

[Ans: R = 108 kN
α = 29.62°
x = 284.6 mm] (16)



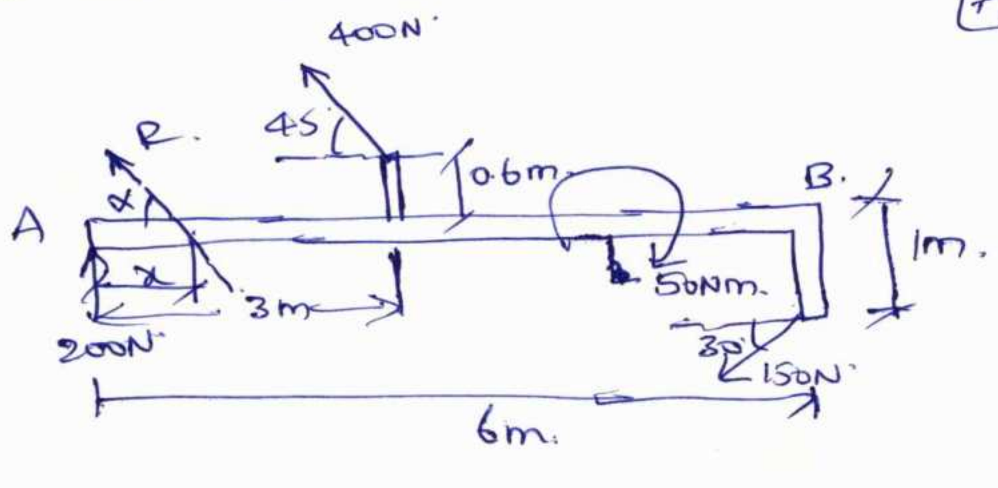
Q) The system of forces acting on a bell crank is shown in fig. Determine the magnitude, direction and the point of application of the resultant.

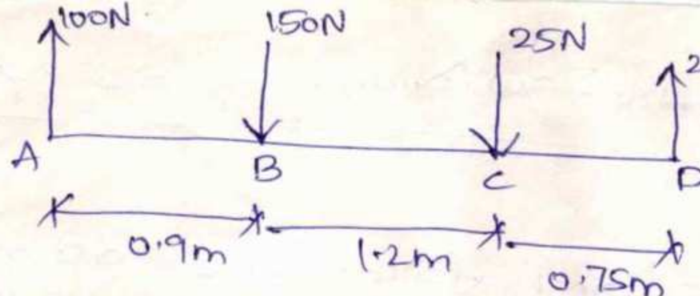
[Ans: R = 2671.2N
α = 80.30'
x = 141.2mm]



Q) A bracket is subjected to three forces and a couple as shown in fig. Determine magnitude, direction and the line of action of the resultant.

[Ans: R = 580.2N
α = 44.76'
x = 0.952m].

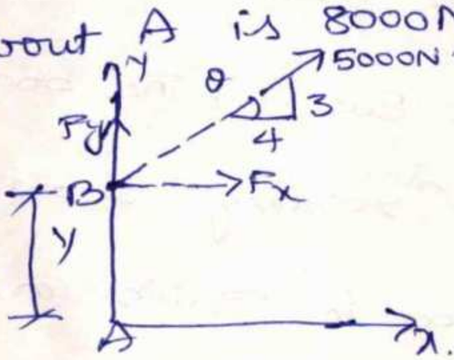




Chapter: coplanar Parallel forces.
Problems: 1 to 8 in Bansal

10/11	8,10
11/11	10

Q) what will be Y-intercept of 5000N force shown in fig. if its moment about A is 8000Nm.



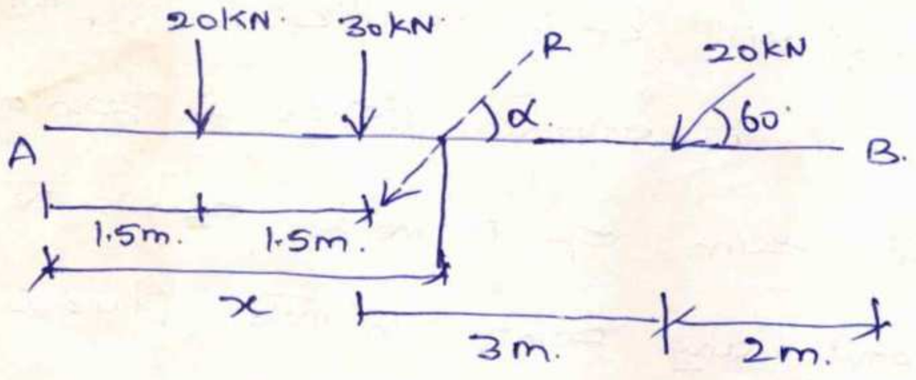
[Ans:

$(F_x \times y) + (F_y \times 0) = 8000$

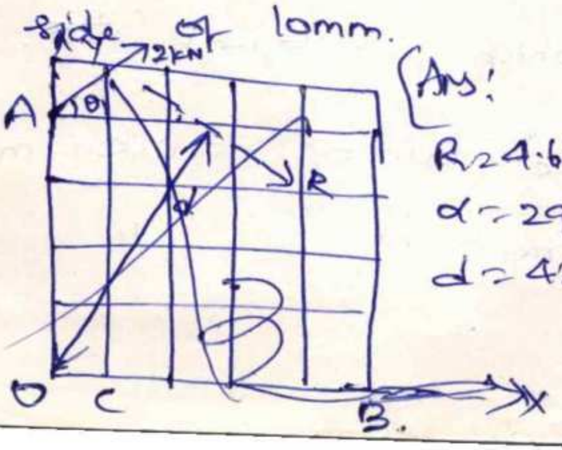
$(5000\cos 36.86 \times y) = 8000.$

$y = 2m.$]

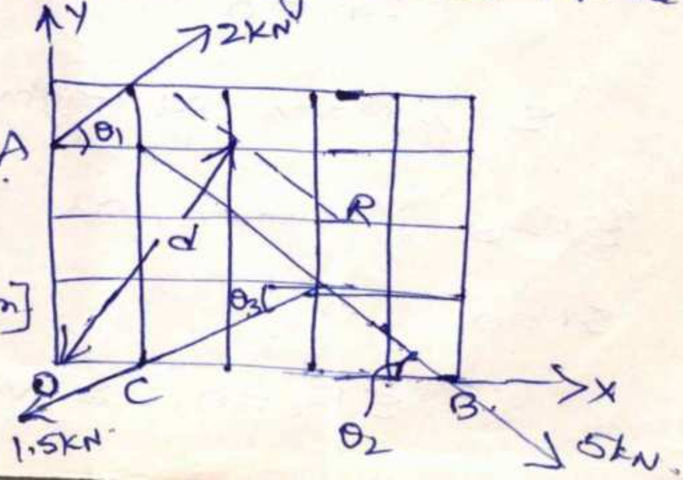
Q) Determine the resultant of the ~~forces~~ system of forces acting on a beam as shown in fig. [Ans: R = 68.06kN
α = 81.55
x = 3.326m.]



Q) Find the resultant of the system of coplanar forces acting on a lamina as shown in fig. Each square has a side of 10mm.



[Ans:
R = 4.655kN
α = 29
d = 42.8mm]



~~Conditi~~ ~~Conditi~~ Equilibrium of force systems

Introduction:

When some external forces are acting on a stationary body (concurrent or parallel forces) the body may start moving or may start ~~moving~~ rotating about any point. But if the body does not start moving and also does not start rotating about any point, then the body is said to be in equilibrium.

Principle of Equilibrium:-

The principle of Equilibrium states that, a stationary body which is subjected to coplanar forces (concurrent or parallel) will be in equilibrium if the algebraic sum of all the external forces is zero and also the algebraic sum of moments of all the external forces about any point in their plane is zero.

$$\Sigma F = 0$$
$$\Sigma M = 0$$

'Σ' is known as Sigma which is a Greek letter.

$\Sigma F = 0$ is known as Force law of Equilibrium

$\Sigma M = 0$ " " Moment " " "

$$\Sigma F_x = 0, \quad \Sigma F_y = 0$$

$\frac{120^2 \times 12}{1440}$

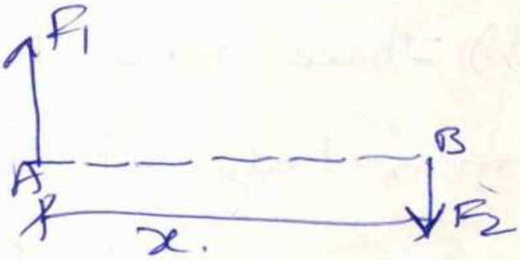
Force law of Equilibrium:- ~~G. Ananth~~

(18)

(i) Two force system
(ii) Three " "
(iii) Four " "

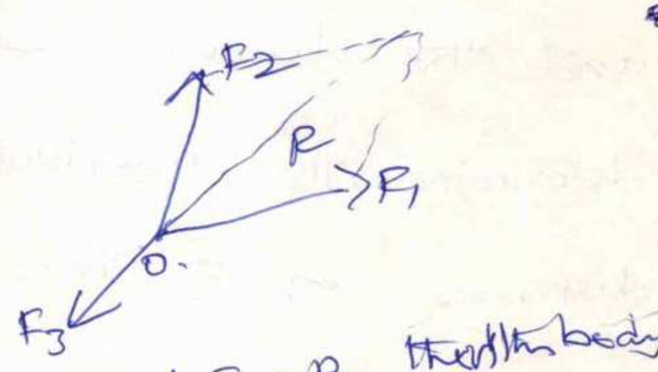
1) Two force system:-

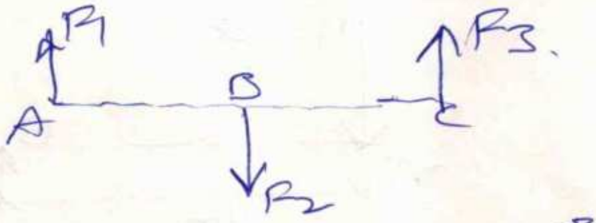
When a body is subjected to two forces, then the body will be in equilibrium if the two forces are collinear, equal and opposite



2) ~~Force~~

2) Three force system:-





if $F_3 = R$ then the body is in equm

$\Sigma F = 0 \Rightarrow F_1 + F_3 = F_2$

$\Sigma M = 0 \Rightarrow$ Solution

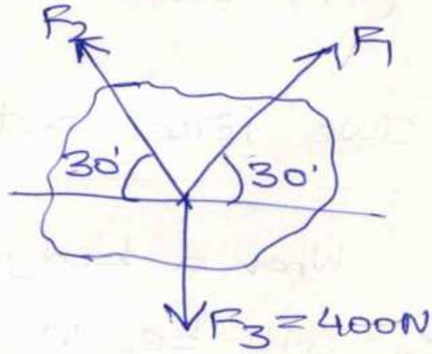
3) Four force system:-

$\Sigma F = 0 \Rightarrow \Sigma F_x = 0, \Sigma F_y = 0$

$\Sigma M = 0$

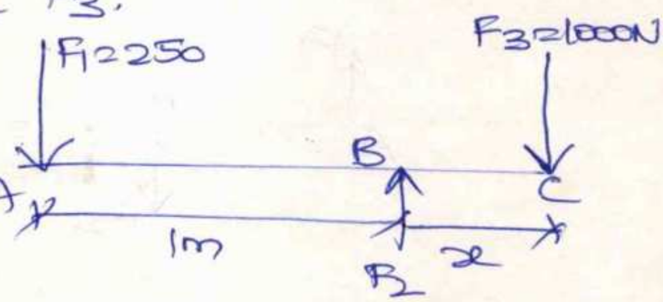
1) Three forces F_1, F_2 and F_3 are acting on a body as shown in fig and the body is in equilibrium. If the magnitude of force F_3 is 400 N, find the magnitudes of force F_1 and F_2.

[Ans: $F_1 = F_2 = 400N$]



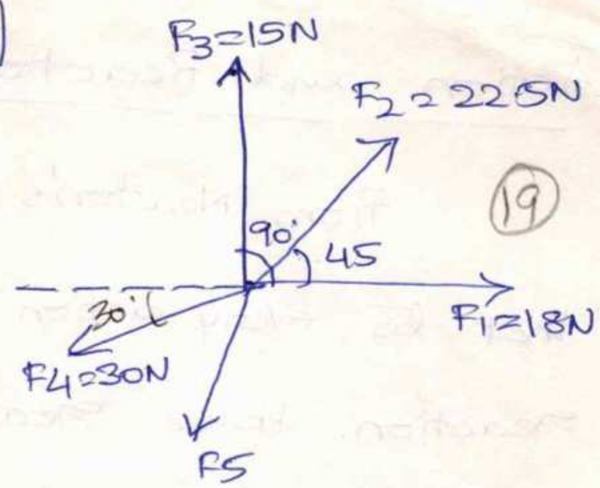
2) Three parallel forces F_1, F_2 and F_3 are acting on a body as shown in fig and the body is in equilibrium. If force $F_1 = 250N$ and $F_3 = 1000N$ and the distance between F_1 and $F_2 = 1m$ then determine the magnitude of force F_2 and the distance of F_2 from force F_3.

[Ans: $x = 0.25m$]



3) The five forces F_1, F_2, F_3, F_4 and F_5 are acting at a point on a body as shown in fig. and the body is in equilibrium. If $F_1 = 18N$, $F_2 = 22.5N$, $F_3 = 15N$ and $F_4 = 30N$, find the force F_5 in magnitude and direction.

[Ans: $\theta = 63.52$, $F_5 = 17.76N$]



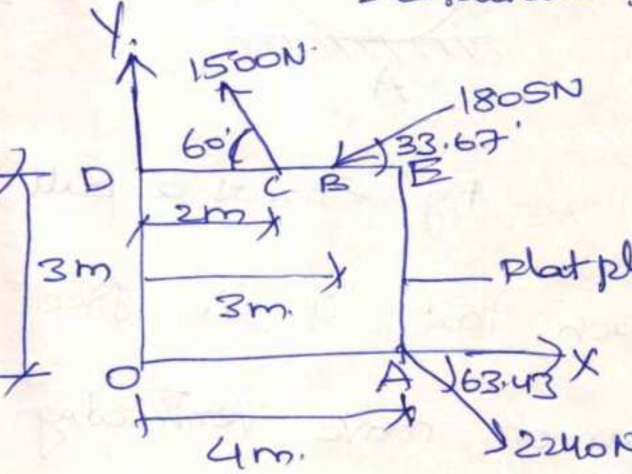
4) Fig shows the coplanar system of forces acting on a flat plate. Determine

(i) the resultant and (ii) x and y intercepts of the resultant.

Ans: $R = 2114.4N$

$x = 0.97\,m$

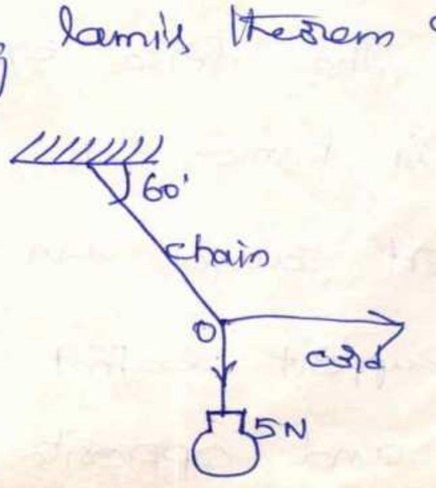
$y = 1.32\,m$



5) A light weighing 5N is suspended from the ceiling by a chain. It is pulled aside by a horizontal cord untill the chain makes an angle of 60° with the ceiling as shown in fig. Find the tensions in chain and the cord by applying lamis theorem and also by graphical method.

[Ans: $T_1 = 2.886N$

$T_2 = 5.773N$]



Action and Reaction:-

From Newton's third law of motion, we know that to every action there is equal and opposite reaction. Hence reaction is always equal and opposite to the action.

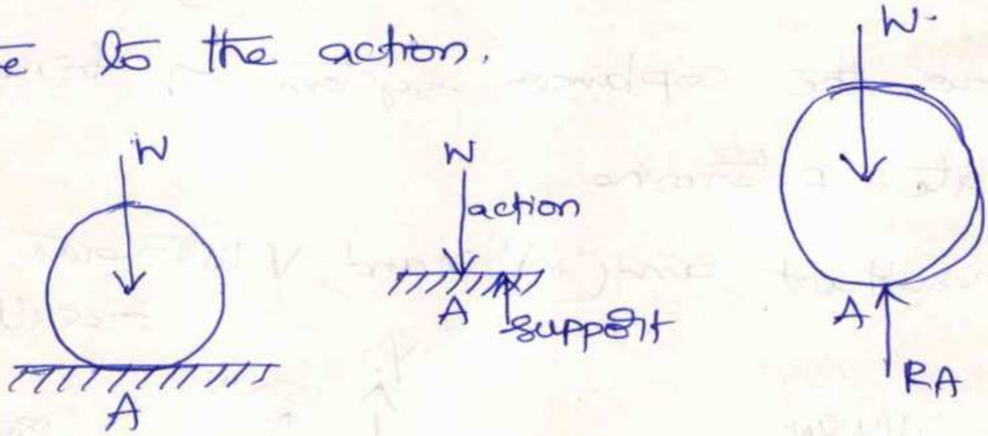


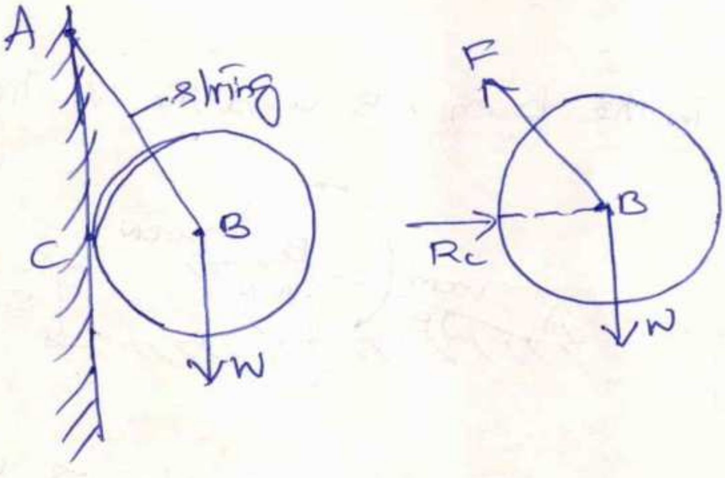
Fig shows a ball placed on a horizontal surface such that it is free to move along the plane but cannot move vertically downward. Hence the ball will exert a force vertically downwards at the support as shown in fig. This force is known as action. The support will exert an equal force vertically upwards on the ball at the point of contact.

The force exerted by the support on the ball is known as Reaction. Hence "any force on a support causes an equal and opposite force from the support so that action and reaction are two equal and opposite forces".

Free body diagram:-

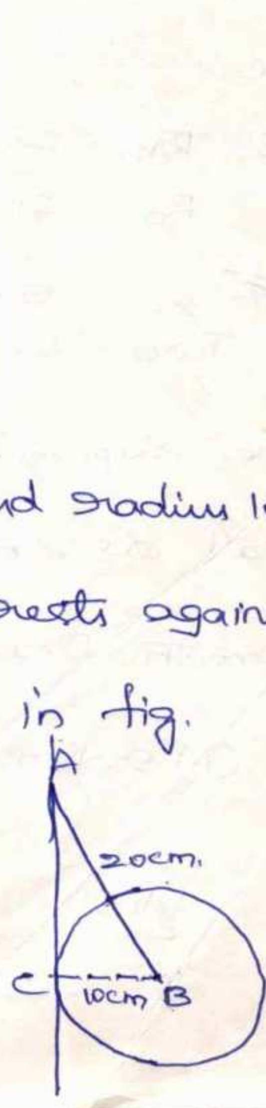
If we remove the supporting surface and replace it by the reaction RA that the surface exerts on the ball as shown in fig.

A fig in which the ball is completely isolated from its support and in which all forces acting on the ball are shown by vectors, is known a free-body diagram.



Problems

1) A circular roller of weight 100N and radius 10cm hangs by a tie rod AB= 20cm and rests against a smooth vertical wall at C as shown in fig.



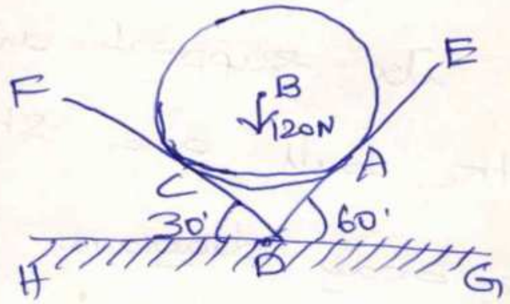
Determine :(i) the force F in the tie rod,

(ii) the Reaction R_c at point C,

[Ans: F= 115.47N, R_c= 57.73N]

2) A ball of weight 120N rests in a right-angled groove, as shown in fig. The sides of the groove are inclined at an angle of 30° and 60° to the horizontal. If all the surfaces are smooth, then determine the reactions RA and Rc at the point of contact.

[Ans: RA = 60N, Rc = 103.92 N]



3) Find the tension in the bar AB and the vertical reaction at C.

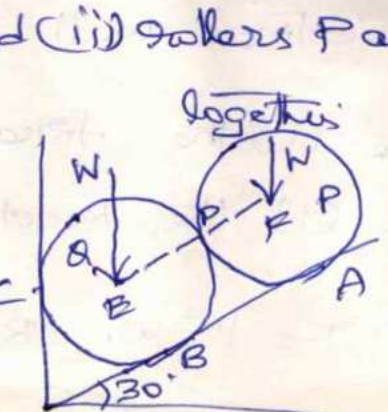
[Ans: T_{AB} = 230.94
Rc = 215.47N]

θ=60°

4) Two identical rollers P and Q, each of weight W, are supported by an inclined plane and a vertical wall as shown in fig. Assume all the surfaces to be smooth. Draw the F.B.D of:

(i) roller Q (ii) roller P and (iii) rollers P and Q taken together

[Ans:



Weight of each roller = W

Radius of each roller = R (21)

Let R_A = Reaction at point A

R_B = " " " B

R_C = " " " C

Two rollers are also in contact at point D Hence there will be a reaction R_D at the point D.

(i) Free body diagram of roller Q: To draw the FBD of roller Q, isolate the roller completely and find the forces acting on the roller Q. The roller Q has points of contact at B, C and D. The forces acting on the roller 'Q' will be:

(i) weight of roller W

(ii) Reaction R_B at point B.

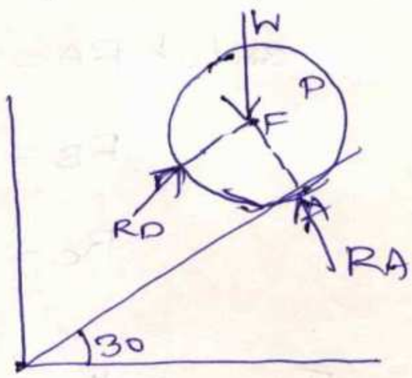
This will be normal to the surface BA at point B

(iii) Reaction R_C at point C. This will be normal to the vertical surface at point C.

(iv) Reaction R_D at point D. This will be normal to the tangent at point D.

The reactions R_B, R_C and R_D will pass through the centre E of the roller Q. ~~These~~

(ii) FBD of roller P:- Free body diagram of roller P is shown in fig. The roller P has points of contacts at A and D. The forces acting on the roller P are:

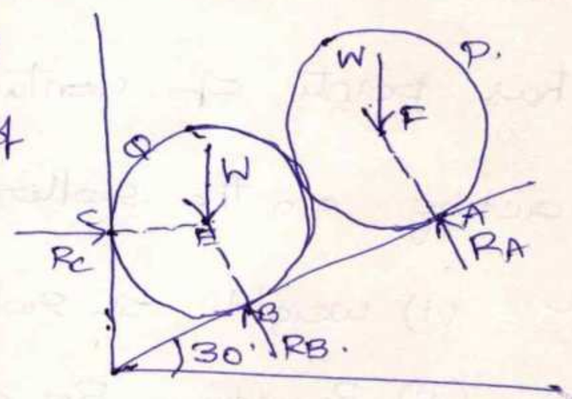


1) weight 'W'
2) Reaction RA at point A
3) " RD " " D.

The reactions RA and RD will pass through point F. i.e centre of roller 'P'.

(iii) FBD of rollers P and Q taken together:-

When the rollers P and Q are taken together, then points of contact are A, B and C.

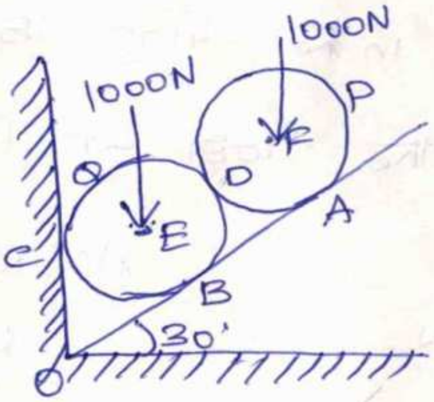


The forces acting are:

(1) Weight 'W' on each roller
(2) Reaction RA at point A
(3) Reaction RB " " B
(4) " Rc " " C

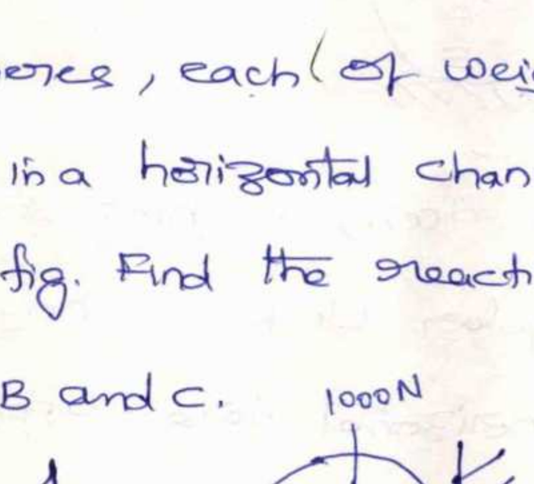
13|4|
10

1) Two identical rollers, each of weight W=1000N, are supported by an inclined plane and a vertical wall as shown in fig. Find the reactions at the points of supports A, B and C. Assume all the surfaces to be smooth



[Ans: $R_A = 866.17N$, $R_B = 1443.3N$, $R_C = 1154.45N$
$R_D = 499.78$]

2) Two spheres, each of weight 1000N and of radius 25cm rest in a horizontal channel of width 90cm as shown in fig. Find the reactions on the points of contact A, B and C.



[Ans! $R_A = 1333.33N$, $R_B = 2000N$, $R_C = 1333.33N$]
$R_D = 1667.05N$

3) Two smooth circular cylinders, each of weight W=1000N and radius 15cm, are connected at their centres by a string AB of length =40cm and rest upon a horizontal plane, supporting above them a third cylinder of weight = 2000N and rad 15cm as shown in fig. Find the force S in the string AB and the pressure produced on the floor at the point of contact D and E.

Ans $R_E = 2000N$

$R_D = 2000N$

$R_F = \cancel{1341.84N}$ 1414.21N

$R_G = \cancel{1341.84N}$ 1414.21N

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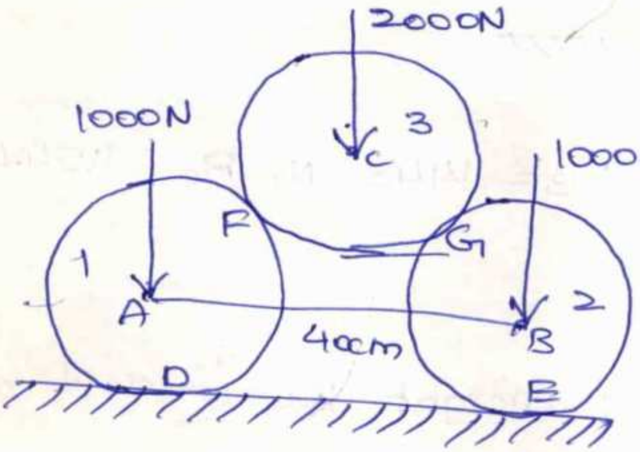
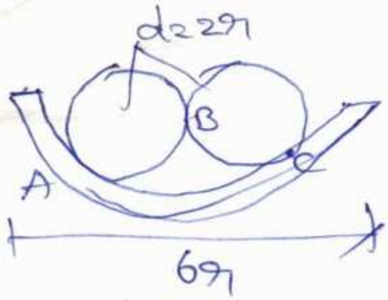
18

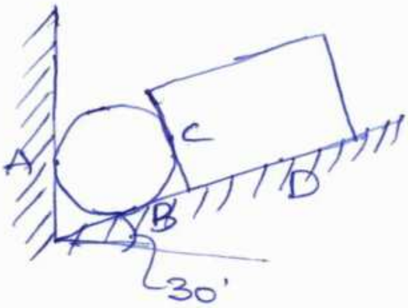


4) A roller of radius 40cm, weighing 3000N is to be pulled over a rectangular block of height 20cm as shown in fig. by a horizontal force applied at the end of a string wound round the circumference of the roller. Find the magnitude of the horizontal force which will just turn the roller over the corner of the rectangular block. Also determine the magnitude and direction of reactions at A and B.



$R_A = R_C = 2 \cdot \frac{W}{\sqrt{3}}$ $R_B = \frac{W}{\sqrt{3}}$

10)

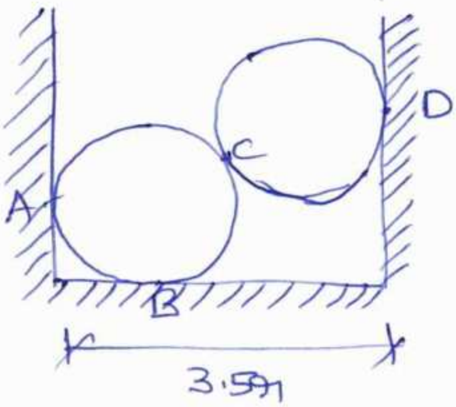


Weight of sphere = 50N

Weight of block = 150N

[Ans: $R_A = 115.5N$, $R_B = 101.04N$, $R_C = 75N$, $R_D = 129.9N$]

11)

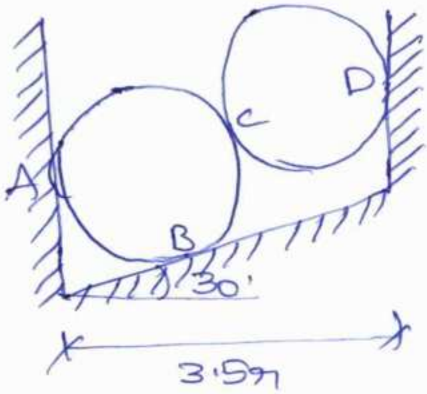


radius of cylinders = r

Weight " = W

[Ans: $R_A = R_D = 1.134W$, $R_B = 2W$, $R_C = 1.512W$]

12)

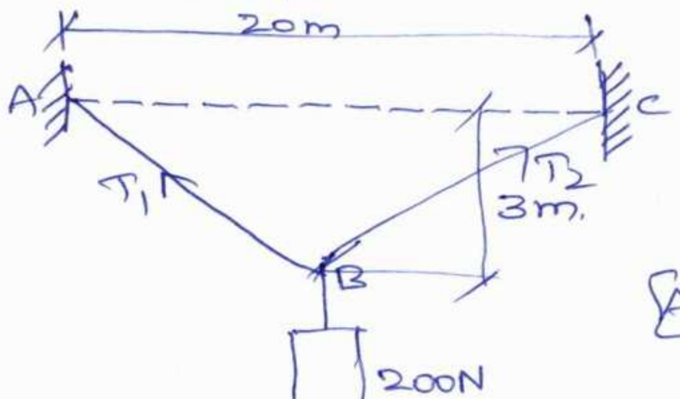


radius = r

Weight = W

[Ans: $R_A = 2.29W$, $R_B = 2.31W$, $R_C = 1.512W$, $R_D = 1.134W$]

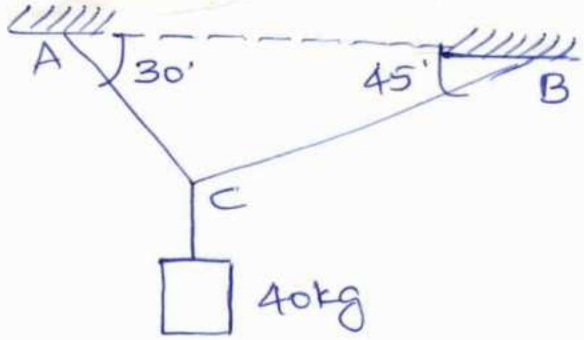
13)



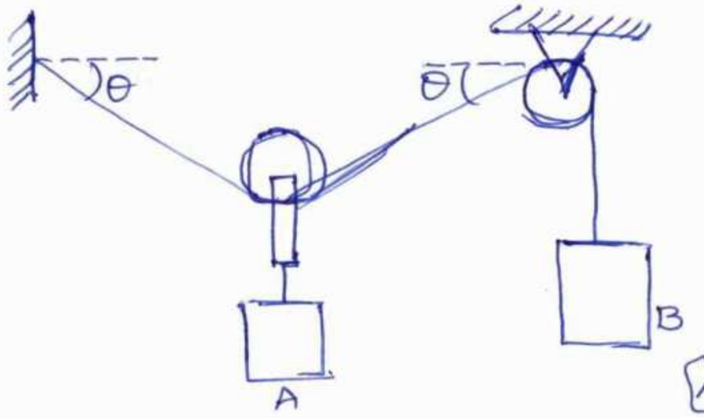
$T_1 = 3 \cdot W$

$T_2 = 3 \cdot W$

[Ans: $T_{AB} = T_{BC} = 717.1N$]

14) 

[Ans: $T_{AC} = 287.3\,N$
$T_{BC} = 351.8\,N$]

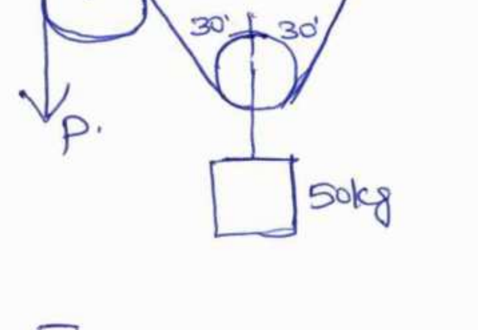
15) 

Det the angle θ

a) $W_A = W_B = W$

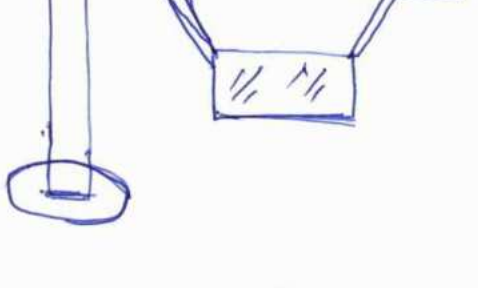
b) $W_A = W_B = 2W$

[Ans: a) 30° b) 14.5°]

16) 

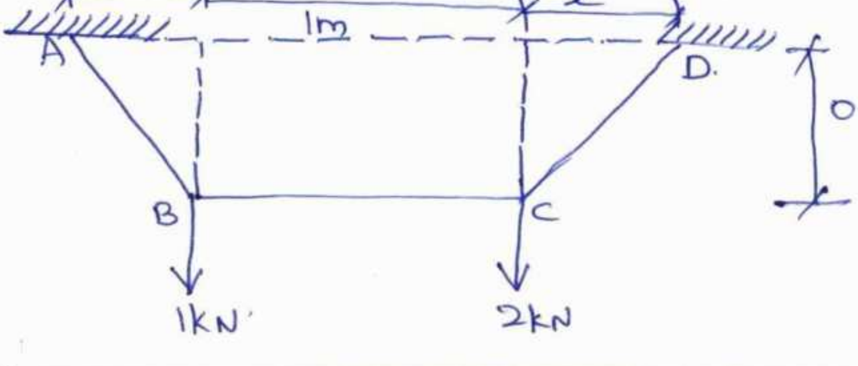
Det the force P

[Ans $P = 283.2\,N$]

17) 

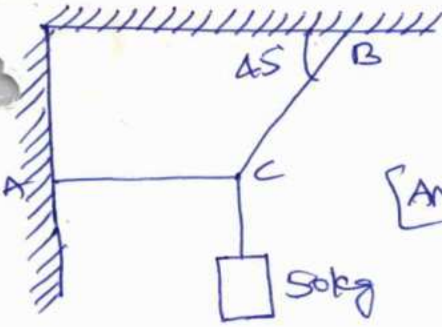
If the two ropes make the same angle with the vertical, det the angle when the tension in each rope is equal to the weight of the signboard.

[Ans: $\theta = 60°$]

18) 

[Ans: $T_{AB} = 1.25\,kN$
$T_{BC} = 0.75\,kN$
$T_{CD} = 2.14\,kN$
$x = 0.15\,m$]

19)



Tensions in AC and BC

[Ans: $T_{AC} = 490.5N$
$T_{CB} = 693.7N$]

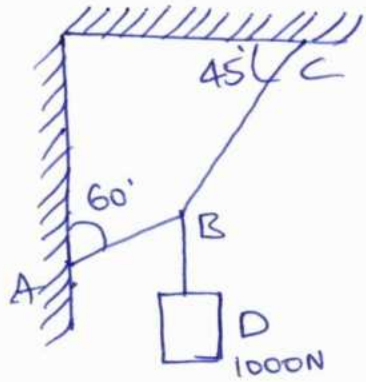
(24)

Bhavikatti

Page No: 79 – 3.1
: 80 – 3.4
80 – 3.6.

: 42 } 2.1 to
43 } 2.8

20)



~~A~~ Det Tensions in each cable

[Ans: $T_{AB} = 2732.05N$
$T_{BC} = 3346.06N$]

~~FBD~~

Timoshenko

Page No. 2.1 to 2.5

Page No: 54 – 2.34

2.31, 2.32, 2.33.
2.28 2.29, 2.30

Equilibrium of coplanar Non-concurrent forces

Beam: Beam is a structural member that is designed to resist forces transverse to its axis.

Types of Beams:

Based on supports, It is divided into five beams

1) cantilever beams
2) simply supported beams
3) overhanging beams
4) Fixed beams
5) continuous beams.

06/12
02/12
09/12

Types of loads:

1) concentrated load (or) point load
2) Uniformly distributed load (UDL)
3) " Varying load (UVL)

Types of supports:-

1) Fixed support
2) Roller "
3) Hinged "
4) Simply support

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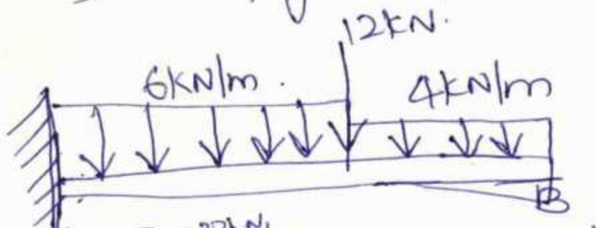
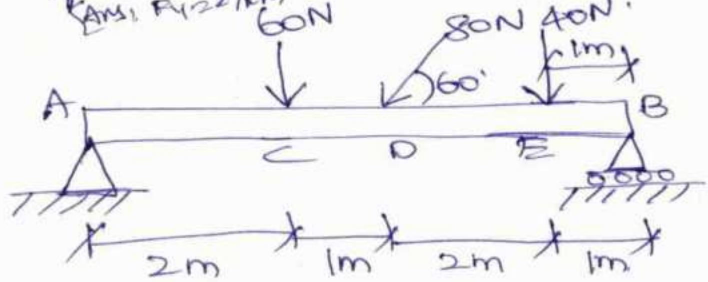
1(a)

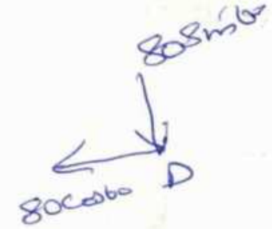


3)

[Ans: Fy=27kN,

2)

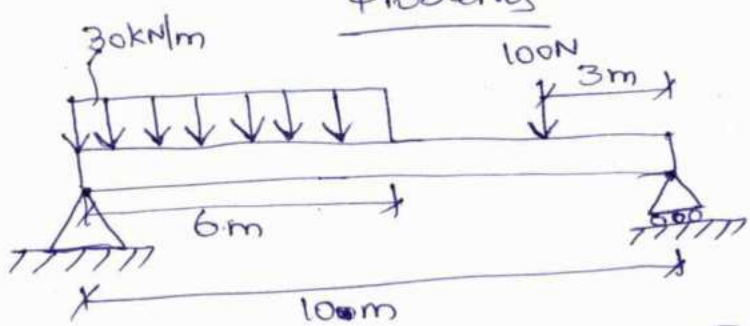




[Ans Fx = 40N, Fy = 81.3N, By = 88N]

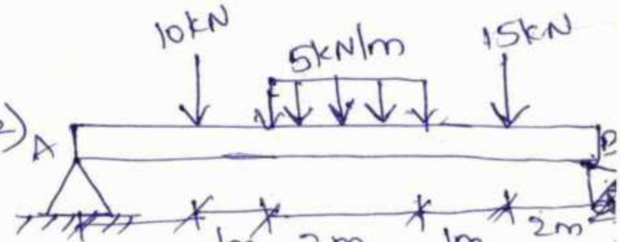
Problems

1)



[Ans: Ay = 156N, By = 124N]

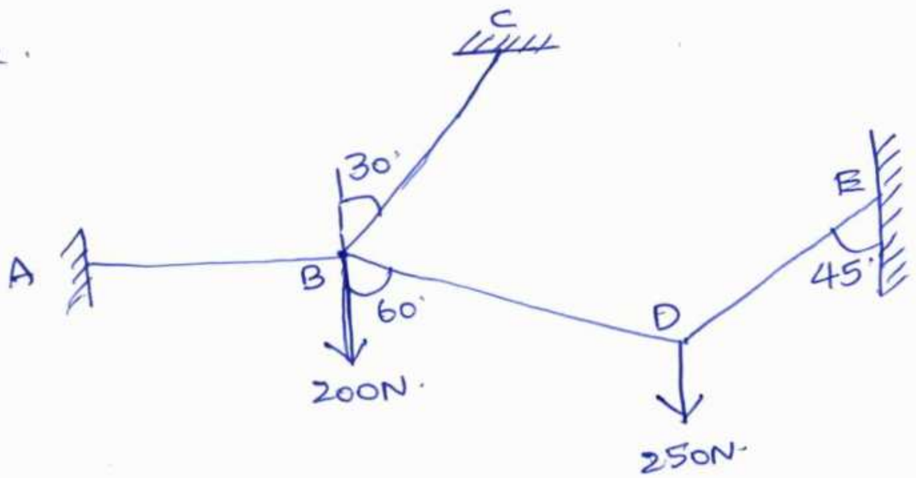
2)



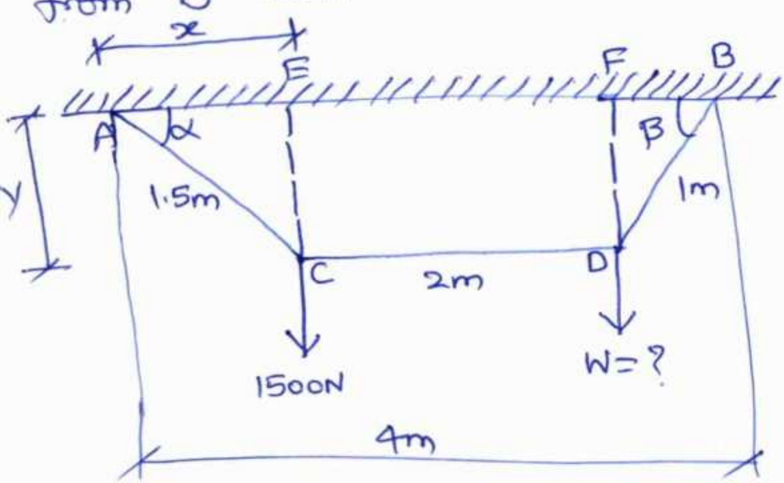
[Ans: Ax = 18.75kN,
Ay = 16.25kN,
RB = 26.52 k

Q) A system of connected flexible cables as shown in fig is supporting two vertical forces 200N and 250N at points B and D. Determine the forces in various segments of the cable.

[Ans: $F_{AB} = 326.66N$
$F_{BC} = 336.53N$
$F_{BD} = 182.9N$
$F_{DE} = 224.14N$]



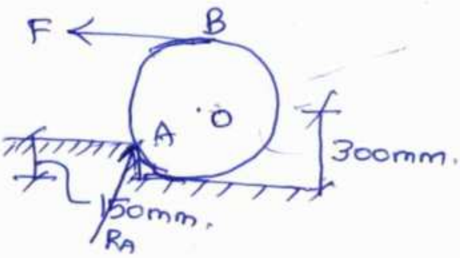
Q) Rope AB shown in fig is 4.5m long and is connected at two points A and B at the same level 4m apart. A load of 1500N is suspended from a point 'c' on the rope at 1.5m from A. What load connected at point D on the rope, 1m from B will be necessary to keep the position CD level?



[Ans: $F_{AC} = 3097.9N$
$F_{CD} = 2710.66N$
$F_{BD} = 3942.23N$
$W = 2862.43N$]

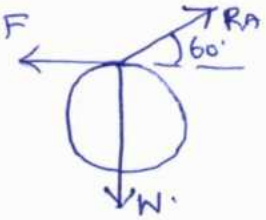
equate $y^2 = 1^2 - (2-x)^2$. $\alpha = 29.12$
$y^2 = 1.5^2 - x^2$ $\beta = 46.6$

Q) A roller of radius $r = 300mm$ and weighing 2000N is to be pulled over a curb of height 150mm, as shown in fig by applying a horizontal force F applied to the end of a string wound around the circumference of the roller. Find the magnitude of force F required to start the roller move over the curb. What [illegible] the least pull F through the centre of the wheel to just turn the roller over the curb?.

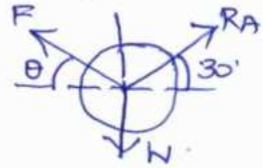


[Ans: (i) F = 1154.7N (ii) F = 1732N.]

Sol: (i)



(ii)



$F\cos\theta = R_A\cos30 \Rightarrow R_A = \frac{F\cos\theta}{\cos30}$

$F\sin\theta + R_A\sin30 = W \Rightarrow F\sin\theta + F\cos\theta\tan30 = W$

$\sin\theta + \cos\theta\tan30 = \frac{W}{F}$

In order to be max, $\frac{d}{d\theta}\left(\frac{W}{F}\right) = 0 \Rightarrow \cos\theta - \sin\theta\tan30 = 0$

$\cot\theta = \tan30$

$\sin60 + \cos60\tan30 = \frac{2000}{F}$ $\quad \theta = 60°$

$F = 1732.05N.$

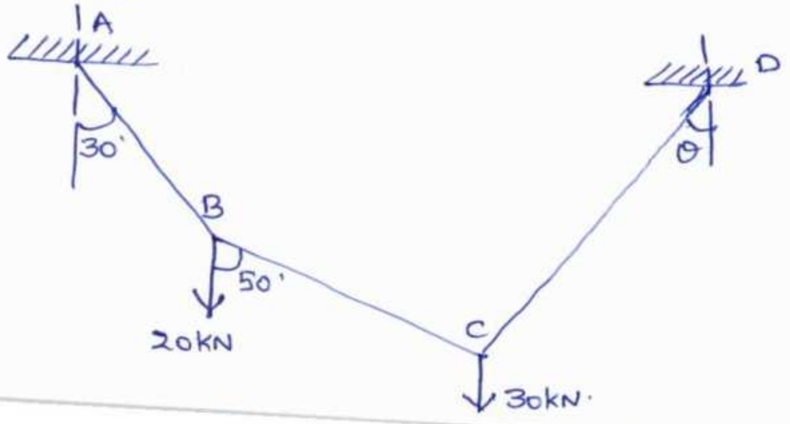
Q) A wire rope is fixed at two points A and D as shown in fig. weights 20kN and 30kN are attached to it at B and C respectively. The weights rest with portions AB and BC inclined at 30° and 50° respectively, to the vertical as shown in the fig. Find the tension in segments AB, BC and CD of the wire. Determine the inclination of the segment CD to vertical.

[Ans: F_{AB} = 44.79 kN

F_{BC} = 29.23 kN

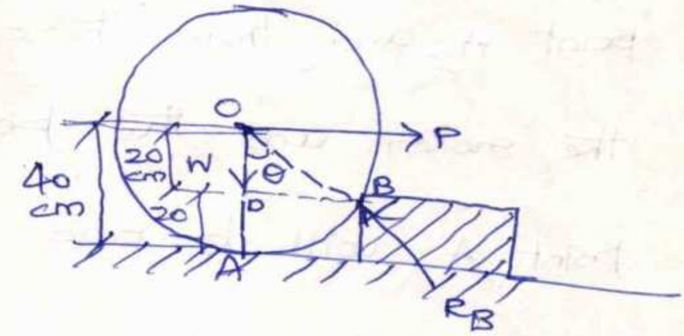
F_{CD} = 29.03 kN

θ = 63.4°]

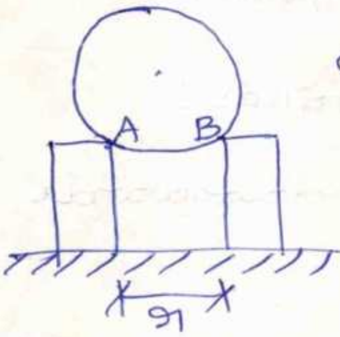


5) If in the above problem, the force P is applied horizontally at the center of the roller, what would be magnitude of this force? Also determine the least force and its line of action at the roller centre, for turning the roller over the rectangular block.

[Ans: P=



6)



diameter = 2r

weight = W

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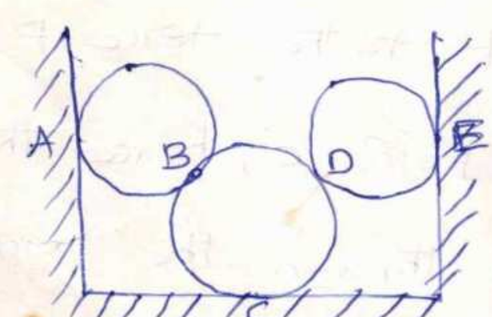
[Ans: $R_A = R_B = \frac{W}{\sqrt{3}}$]

7)

~~RA~~ Weight = W

[Ans: $R_A = \frac{W}{2}$, $R_B = \sqrt{3} \cdot \frac{W}{2}$]

8)

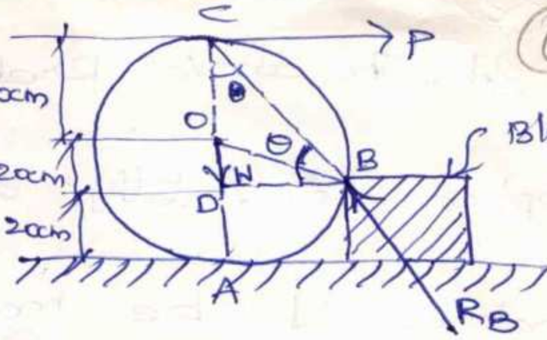


Weight of small cylinders = W, radii = r

large " = 2W, " = 2r

[Ans: $R_A = R_E = 0.895W$

$R_B = R_D = 1.342W$, $R_C = 4W$]



Sol: Find horizontal force P, reaction R_A and reaction R_B when the roller just turns over the block.

When the roller is about to turn over the corner of the rectangular block, the roller lifts at the point A and then there will be no contact between the roller and the point A. [Hence reaction R_A at point A will become zero.]

Now the roller will be in equilibrium under the action of the following three forces:

(i) its weight W acting vertically downwards

(ii) horizontal force P.

(iii) reaction R_B at point B. The direction of R_B is unknown.

$P \times CD =$
$W \times DB.$
$60 = 3000 \times$
$P =$

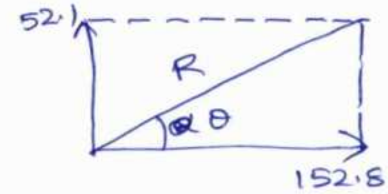
For the equilibrium, these three forces should pass through a common point. As the force P and weight W is passing through point 'C', hence the reaction R_B must also pass through the point C. Therefore, the line BC gives the direction of the reaction R_B.

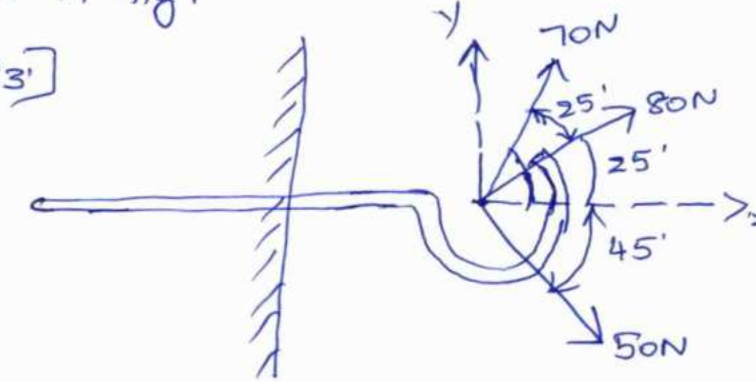
[Ans: $R_B = 3464.2N$, $P = 1732.1N$]

~~RAB~~

Q) Determine the resultant of the three forces acting on a hook as shown in fig.

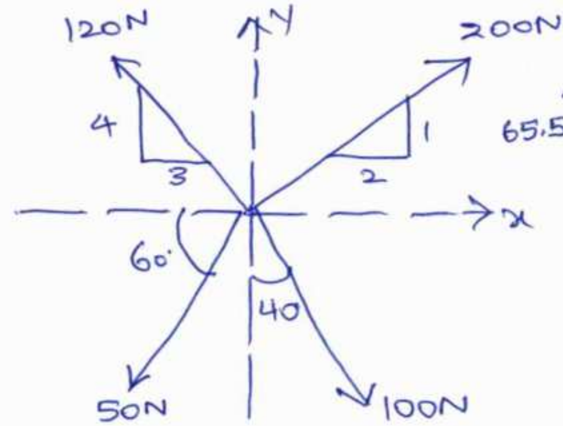
Sol. [Ans: $R = 161.5N$, $\theta = 18.83°$]

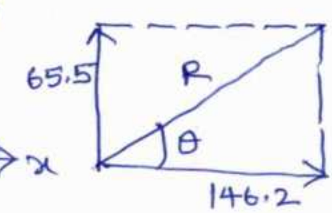




Q) A system of four forces acting at a point on a body is as shown in fig. Determine the resultant and direction.

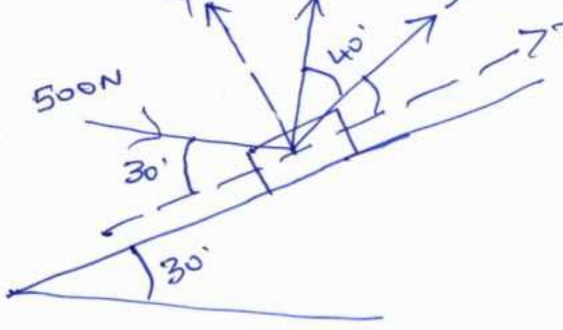
[Ans: $R = 160.2N$, $\theta = 24.1$]



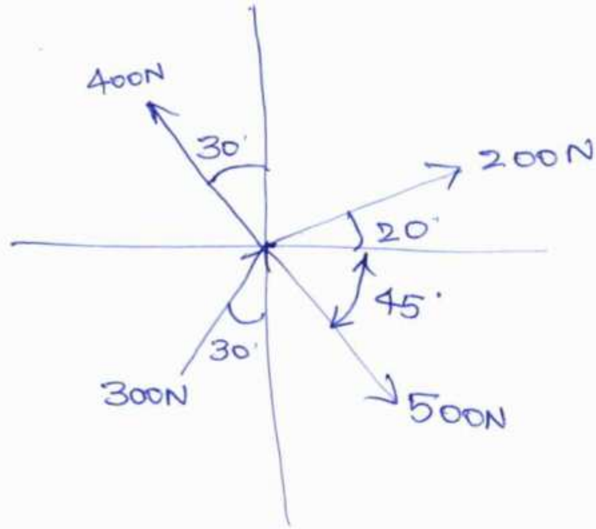


Q) Three forces acting at C.G of a block are shown in fig. The direction of 300N forces may vary, but the angle between them is always 40°. Determine the value of θ for which the resultant of the three forces is directed parallel to the plane.

[Ans: $\theta = 6.31$]



Q) A system of four forces is acting at a point as shown in fig. Find out the resultant of the force system by scalar resolution method and also vector method

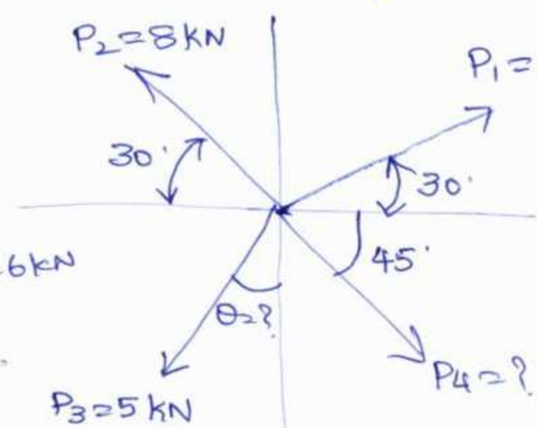


[Ans: $R = 587.08N$
$\phi = 33.15°$]

$\Sigma F_x = 491.5N$

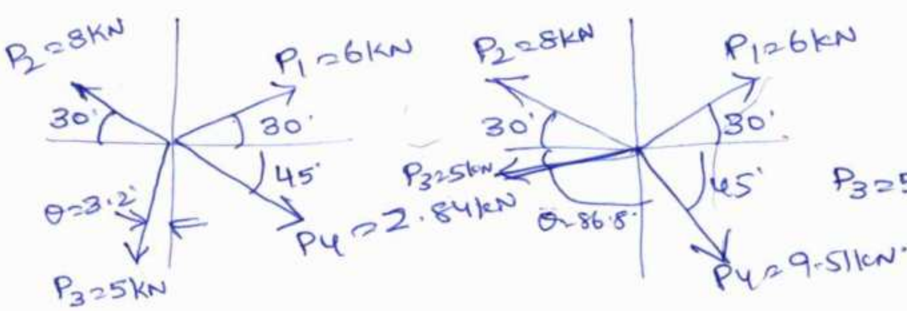
$\Sigma F_y = 321.07N$

Q) Find out the magnitude of the force P_4 and the direction of force P_3 if the resultant of four coplanar concurrent forces P_1, P_2, P_3 and P_4 shown in fig. is zero.

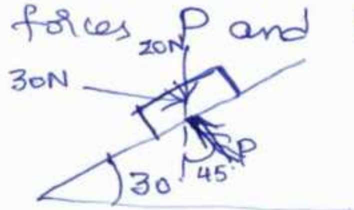


[Ans: $P_4 = 2.84kN$, for $\theta = 3.2°$
$P_4 = 9.51kN$, for $\theta = 86.8°$]

Two Answers came and are indicated



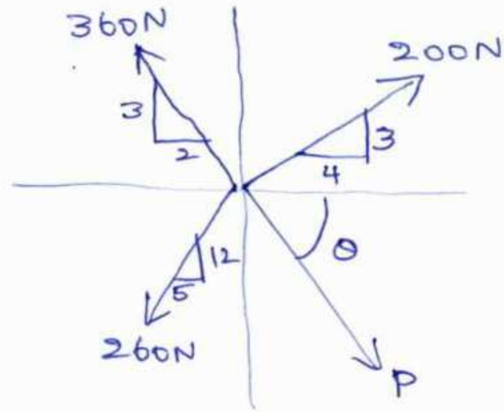
Q) The resultant R of the forces acting on a block on an incline as shown in fig. is parallel to the incline. Determine the forces P and R. [Ans: $P = 33.46N$, $R = 15.98N$ acting up the incline]



27

Q) The resultant of the force system shown in fig is 520N along the +ve direction of Y axis. Determine P and θ.

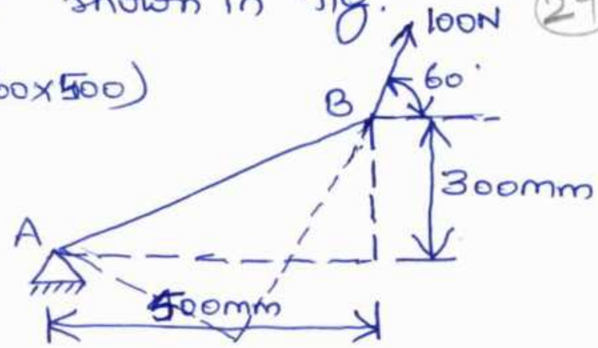
[Ans: P = 368N, θ = 67.64°]



Couple :- Two parallel forces equal in magnitude and opposite in direction and separated by a definite distance are said to form a couple. (28)

Q)

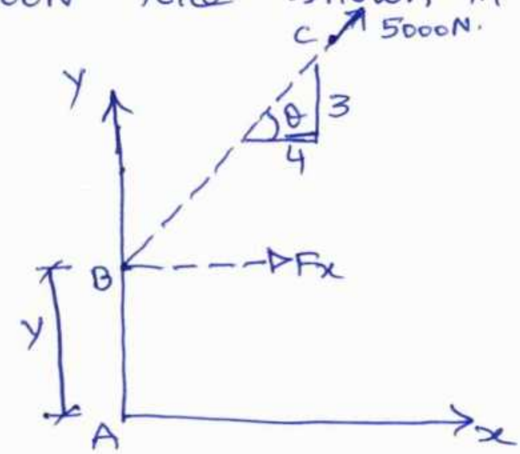
Q) Determine the moment of 100N force acting at B' about moment centre A as shown in fig. (29)



[Ans: $M = (100\cos 60 \times 300) - (100 \times \sin 60 \times 500)$

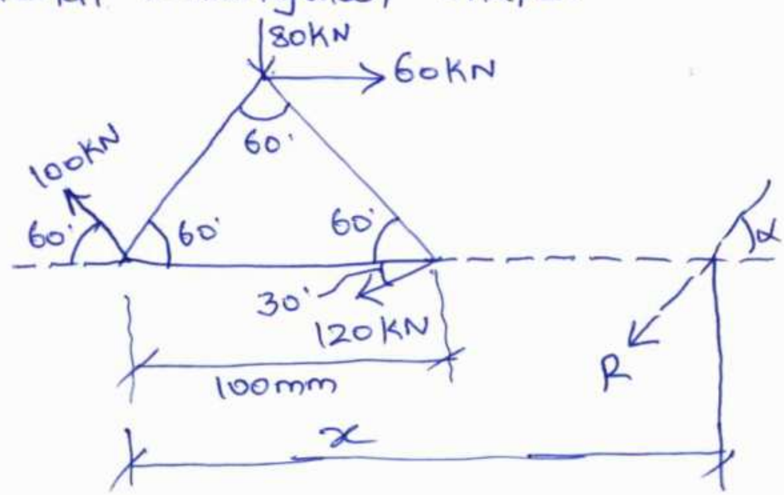
$= -28301$ Nmm. (A.C.W)

Q) What will be Y-intercept of 5000N force shown in fig, if its moment about A is 8000N-m?



[Ans: Y = 2m]

Q) Determine the resultant of the force system shown in fig acting on a lamina of equilateral triangular shape.



[Ans: R = 108 N

x = 284.6mm]

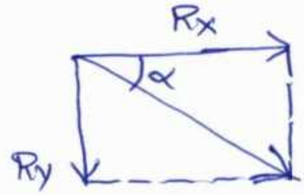
Q) Find the resultant of the system of coplanar forces acting on a lamina as shown in fig. Each square has a side of 100mm.

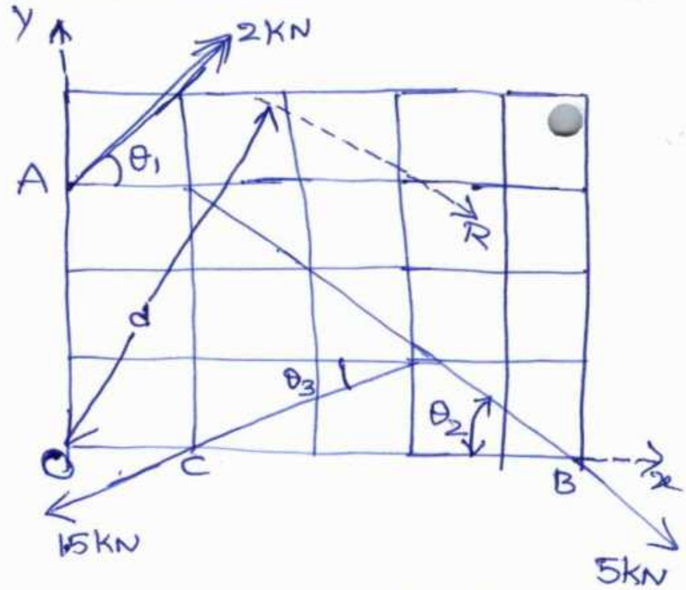
[Ans: $R = 4.655\,kN$

$\Sigma M_o = 200\,kNmm$

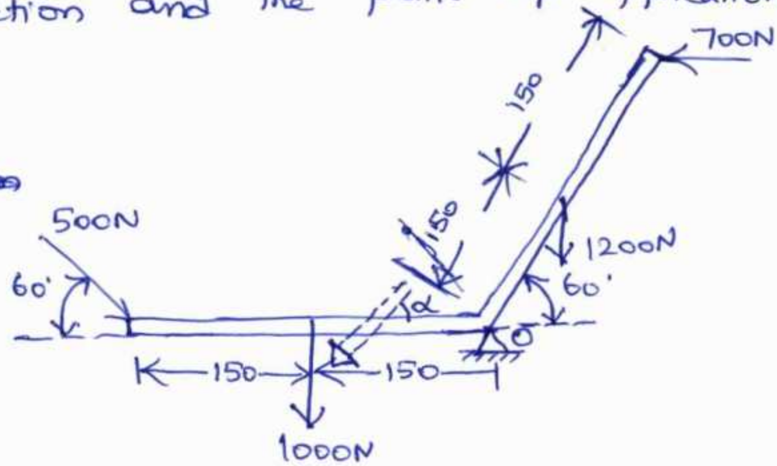
$\alpha = 29°$

$d = 42.8\,mm$]





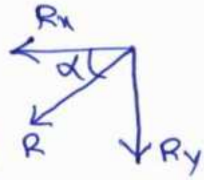
Q) The system of forces acting on a bell crank is shown in fig. Determine the magnitude, direction and the point of application of the resultant.



[Ans: $R_x = -450\,N$, $R_y = -2633\,N$

$R = 2671.2\,N$

$\alpha = 80.3°$

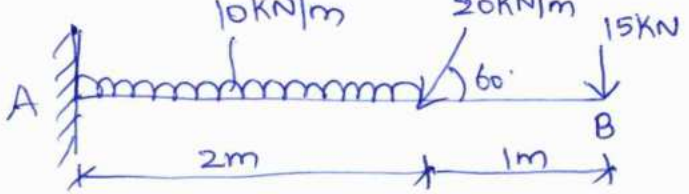


$\Sigma M_o = -371769.14\,Nmm$ (A.C.W)

$R \sin\alpha \times x = \Sigma M_o$

$$x = \frac{-371769.14}{(2671.2 \times \sin 80.3)} = 141.2\,mm$$]

Q) Determine the reactions developed in the cantilever beam shown in fig.

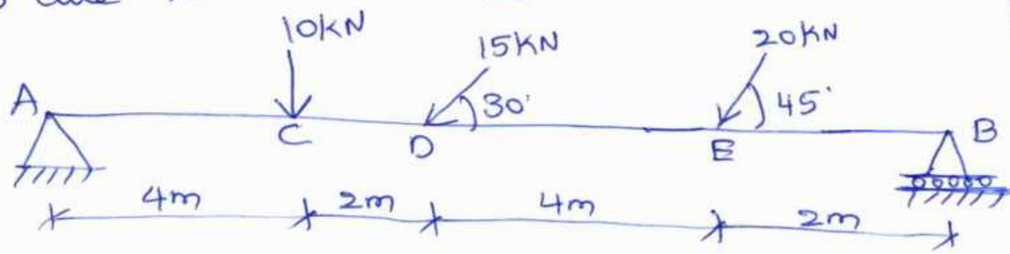


[Ans: $(R_Y)_A = 52.32$ KN

$(R_X)_A = 10$ KN

$M_A = 99.64$ kN m]

Q) The beam AB of span 12m shown in fig is hinged at A and is on rollers at B. Determine the reactions at A and B due to the loading shown in the figure.

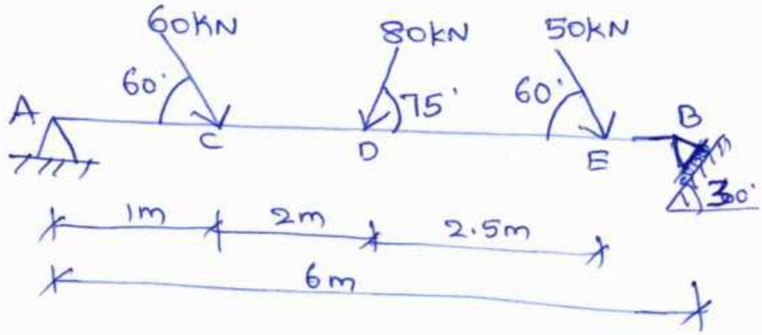


[Ans: $(R_A)_Y = 12.77$ kN (↑)

$R_B = 18.86$ KN (↑)

$(R_A)_x = 27.13$ kN (→)

Q) Find the magnitude and direction of reactions at supports A and B in the beam AB shown in fig.



[Ans: $(R_A)_x = 15.93$ kN.

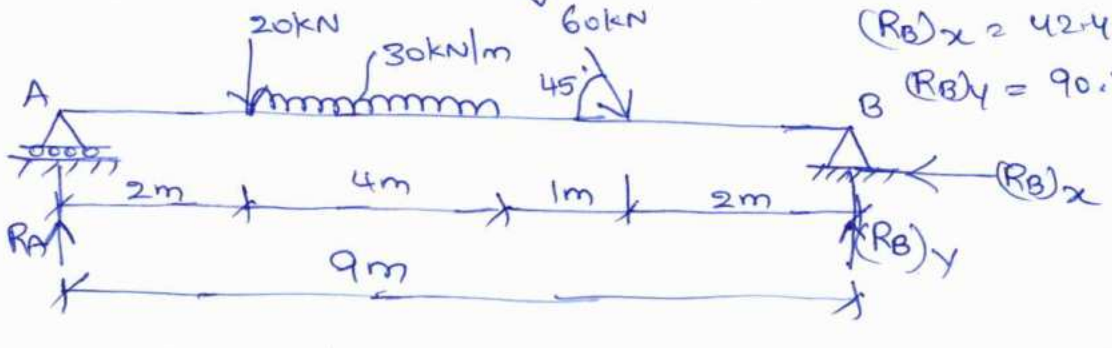
$(R_A)_Y = 85.54$ kN

$R_B = 100.45$ kN.

$R_A = \sqrt{(R_A)_x^2 + (R_A)_y^2} = 87.02$ kN.

$\alpha_A = \tan^{-1}\frac{85.54}{15.93} = 79.45°$

Q) Find the reactions developed at supports A and B of the loaded beam shown in fig.

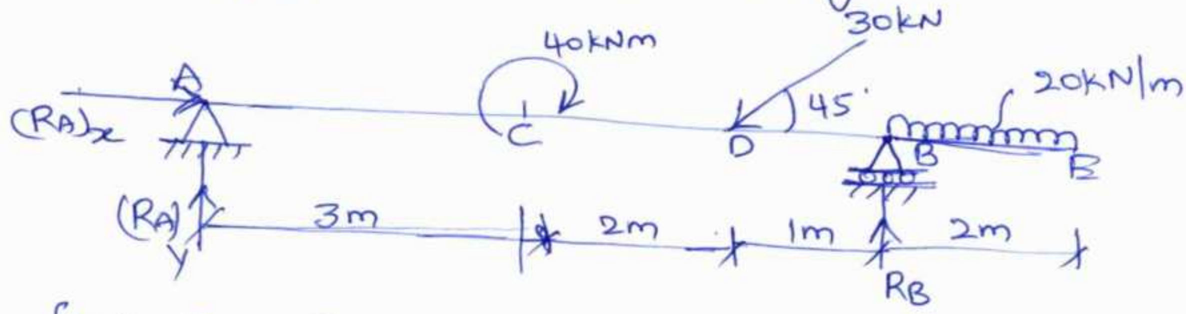


[Ans: $R_A = 91.64$ KN

$(R_B)_x = 42.42$ KN (←)

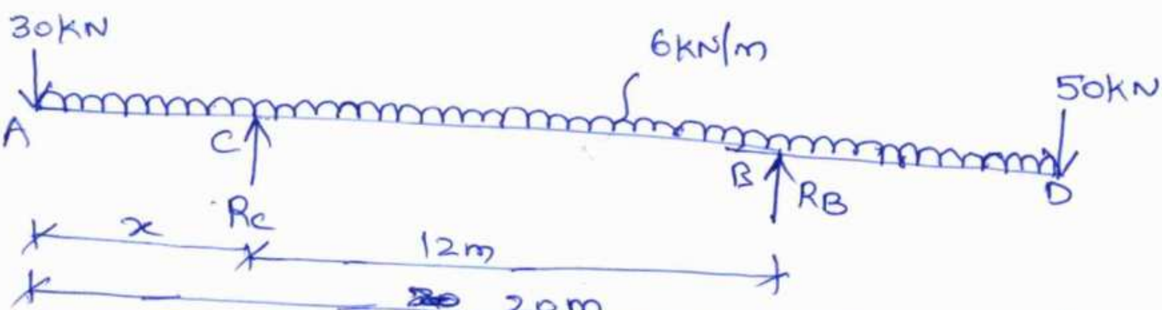
$(R_B)_Y = 90.77$ kN]

Q) Determine the reactions developed at supports A and B of overhanging beam shown in fig.



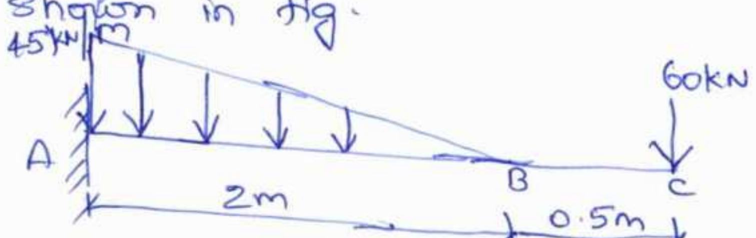
[Ans: $(R_A)_y = -9.81 (\downarrow)$ $(R_A)_x = 21.21$ $(R_B) = 71.01$ kN]

Q) A beam 20m long supported on two intermediate supports, 12m apart, carries a UDL of 6kN/m and two concentrated loads of 30kN at left end A and 50kN at the right end D as shown in fig. How far away should the first support 'c' be located from the end A so that the reactions at both the supports are equal.



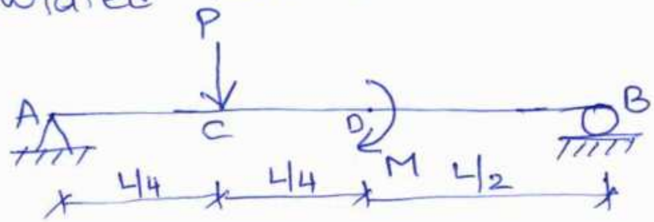
[Ans: $R_C = 100$ kN ; $R_B = 100$ kN , $x = 5$ m]

Q) Determine the reactions developed in the cantilever beam shown in fig.



[Ans: $(R_A)_y = 105$ kN
$(R_A)_x = 0$
$(M_A) = 180$ kNm]

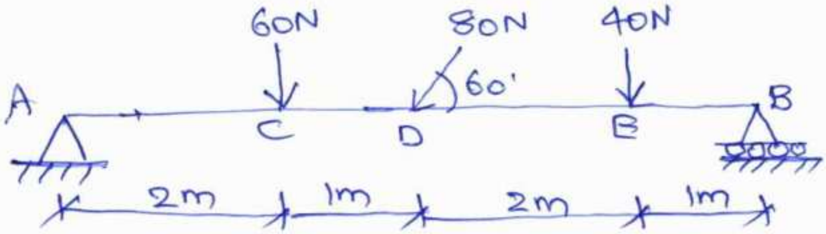
Q) A beam AB of span L is subjected to a concentrated load P and a couple M as shown in fig.



[Ans: $R_A = \frac{3PL - \cancel{4}M}{4L} = \frac{3P}{4} - \frac{M}{L}$

$R_B = \frac{4M + PL}{4} = M + \frac{PL}{4}$

Q) Find the support reactions for the simply supported beam AB loaded as shown in fig.

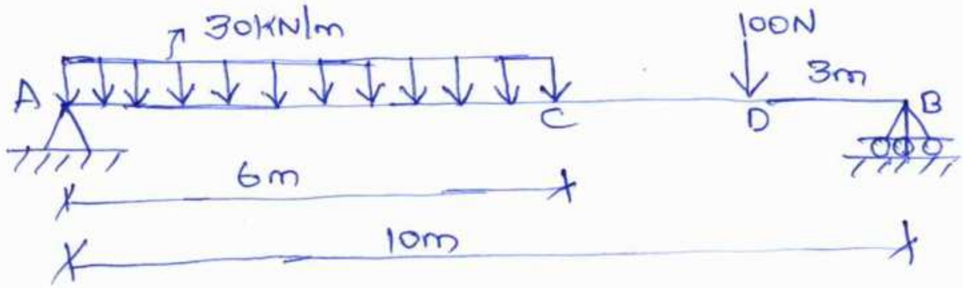


[Ans: $(R_A)_x = 40N$

$(R_A)_y = 81.3N$

$R_B = 88N$]

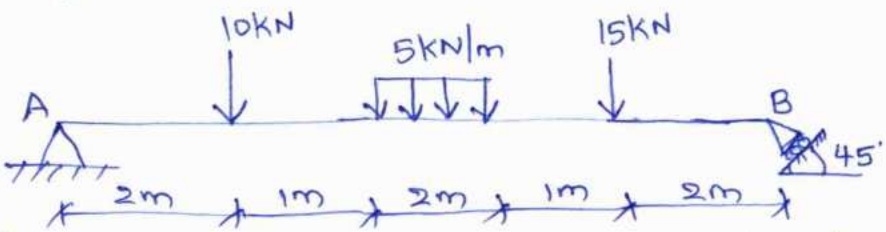
Q)



[Ans: $R_A = 156N$

$R_B = 124N$]

Q)



[Ans: $(R_A)_x = 18.75KN$; $(R_A)_y = 16.25$ KN

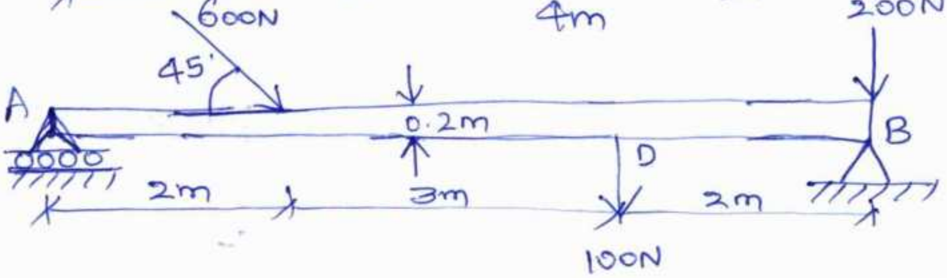
$R_B = 26.52KN$

Q)

[Ans: $R_A = 7.17$ KN

$R_B = 14.82$ KN]

Q)



[Ans: $R_A =$

$(R_B)_x =$

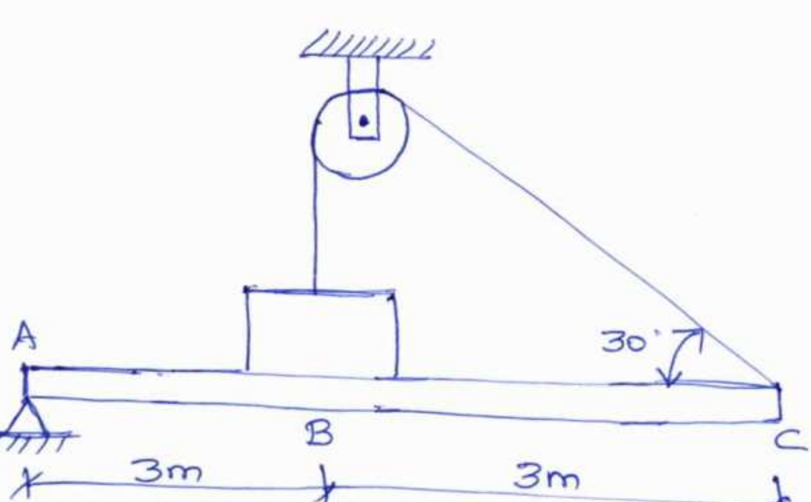
$(R_B)_y =$

Q) A beam AC hinged at A is held in a horizontal position by a cable attached at end 'c' and passing over a smooth pulley as shown in fig. The free end of the cable is connected to a weight 2000N that rests on the beam. Determine the reaction at A and tension in the cable. Neglect the weight of the beam.

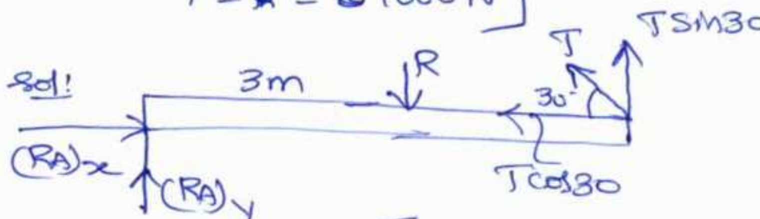
[Ans: $(R_A)_x = 866.03N$

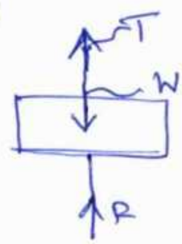
$(R_A)_y = 500N$

$T = R = 1000 N$]



Sol!





$\Sigma F_y = 0$

$T + R = W$

$T + R = 2000$

$T + T = 2000$

$2T = 2000$

$T = 1000$

$R = 1000$

$\Sigma F_x = 0$ $\qquad$ $\Sigma F_y = 0$

$(R_A)_x = T\cos30$; $(R_A)_y + T\sin30 = R$

$\Sigma M_A = 0 \Rightarrow T\sin30 \times 6 = R \times 3$

$T = R$

$(R_A)_x = 1000\cos30 = 866.02N$

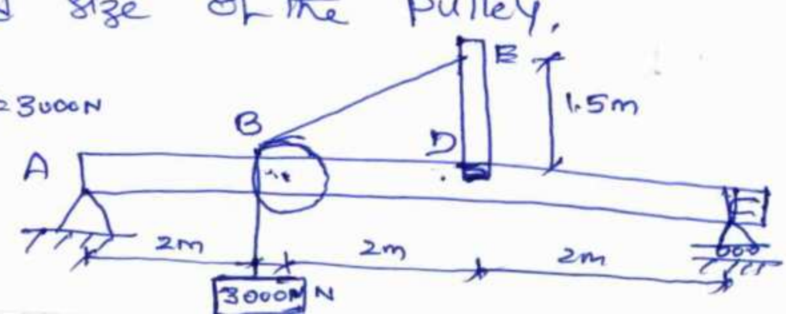
$(R_A)_y = 500N.$

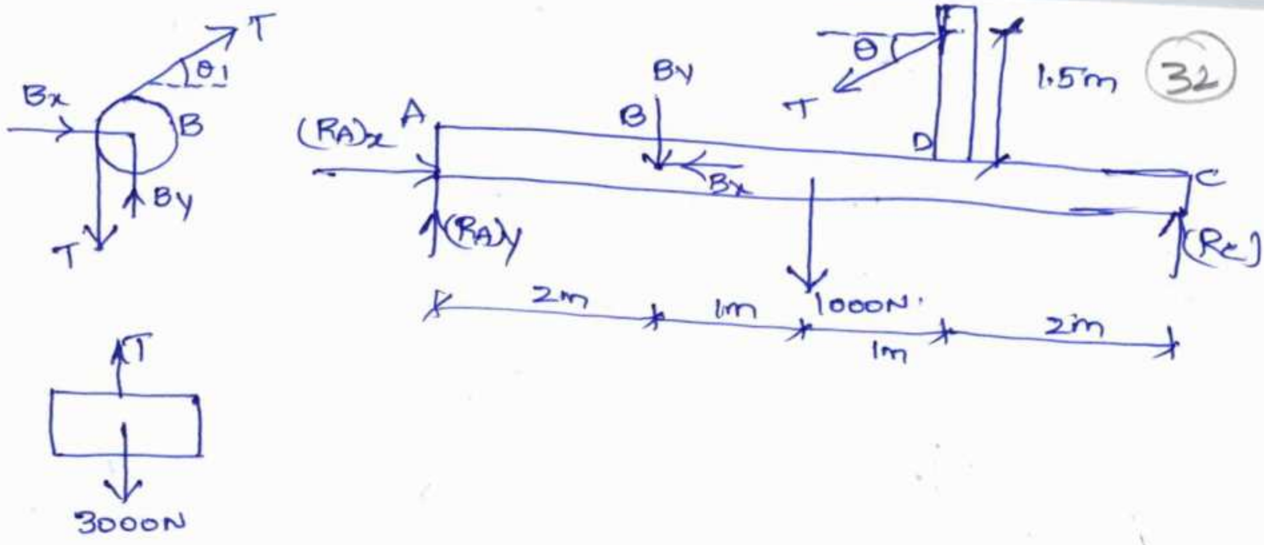
Q) A smooth pulley supporting a load of 3000N is mounted at B on a horizontal beam AC as shown in fig. If the beam weighs 1000N, find the support reactions at A and C. Neglect the weight and size of the pulley.

[Ans: $(R_C) = 1500N$; $(R_A)_x = 0$; $T = 3000N$

$(R_A)_y = 2500N$

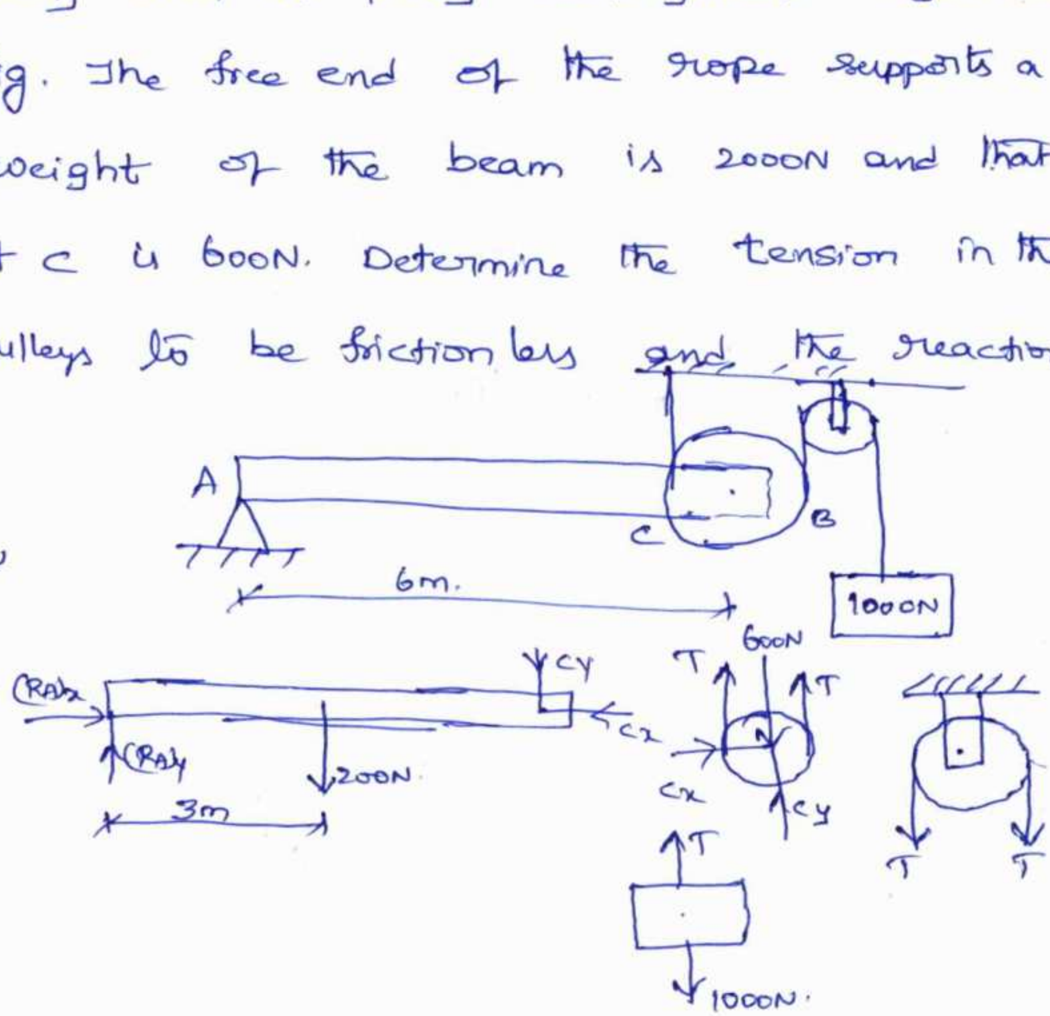
$B_y = 1200N$; $B_x = -2400N$.



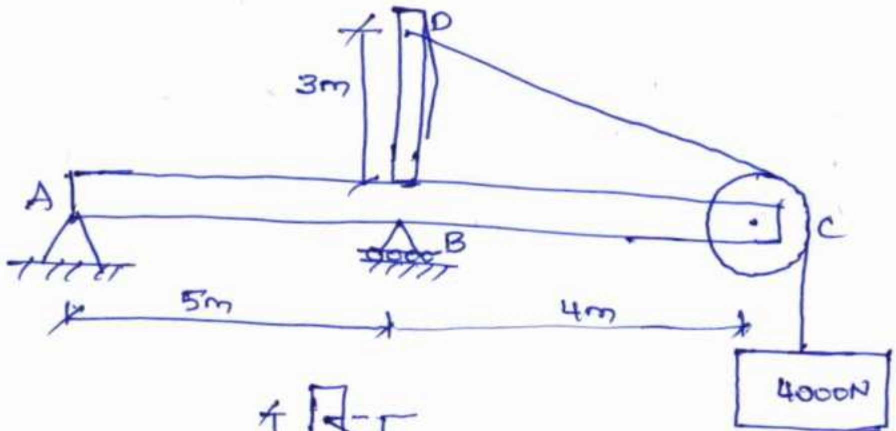


Q) A beam AB hinged at A is supported in a horizontal position by a rope passing over a pulley arrangement hinged at C as shown in fig. The free end of the rope supports a load of 1000N. The weight of the beam is 2000N and that of the pulley hinged at C is 600N. Determine the tension in the rope assuming the pulleys to be frictionless and the reaction at A.

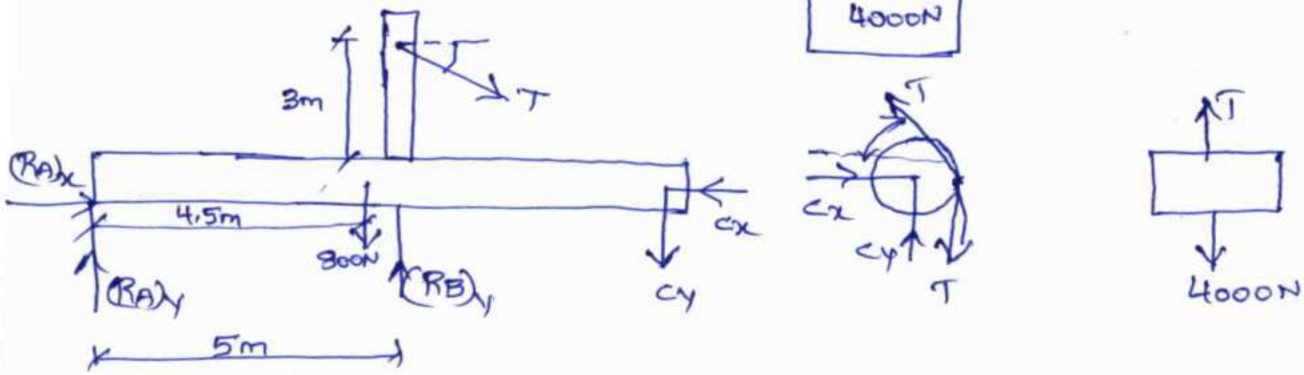
[Ans: T = 1000N,
$C_x = 0$
$C_y = -1400N$
$(R_A)_x = 0$
$(R_A)_y = 600N$



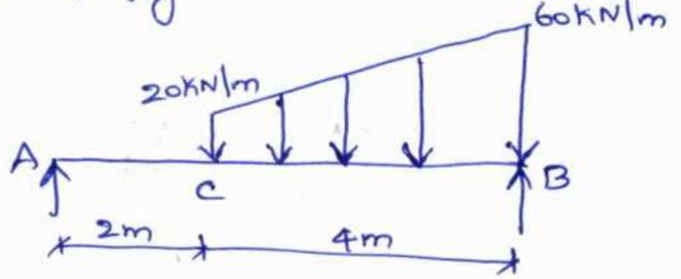
Q) A pulley supporting a load of 4000N is mounted at 'C' on a horizontal beam as shown in fig. If the beam weighs 800N, find the support reaction at A and B. Neglect the weight of the pulley.



[Ans: $(R_A)_x = 0$
$(R_A)_y = -3120N$
$(R_B)_y = 7920N$.
$T = 4000N$]

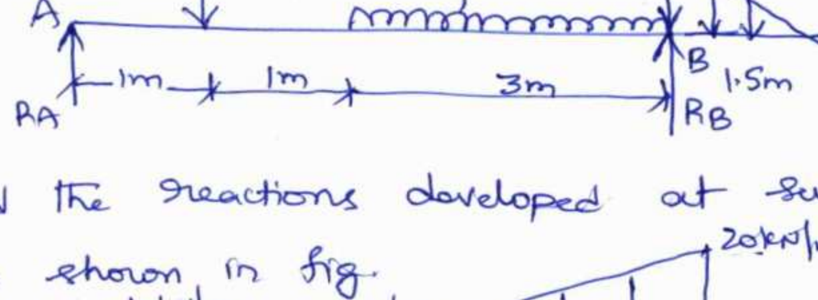


Q) Determine the reaction developed in the simply supported beam shown in fig.



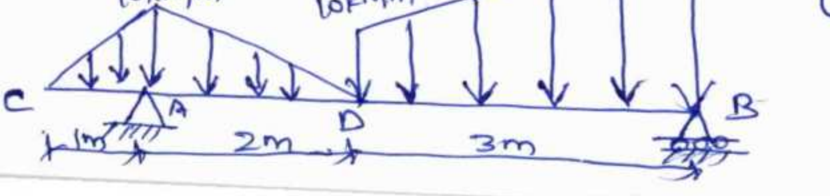
[Ans: $R_A = 44.44kN$
$R_B = 115.56\ kN$]

Q) Determine the reactions at supports A and B of the overhanging beam shown in fig.



[Ans:
$R_A = 42.6kN$
$R_B = 89.4kN$]

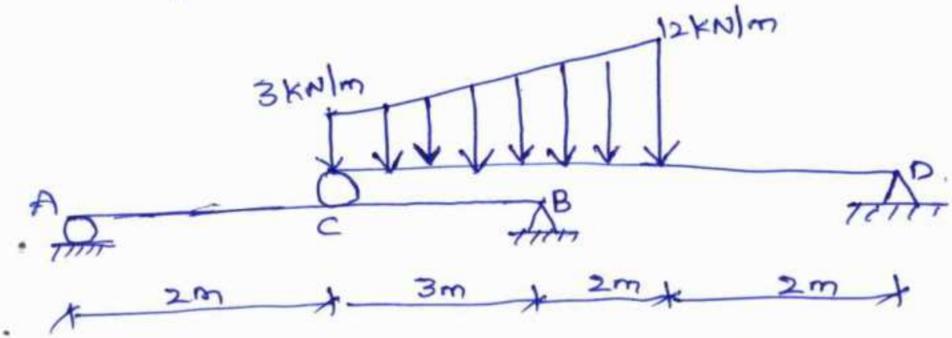
Q) Find the reactions developed at supports A and B of the loaded beam shown in fig.

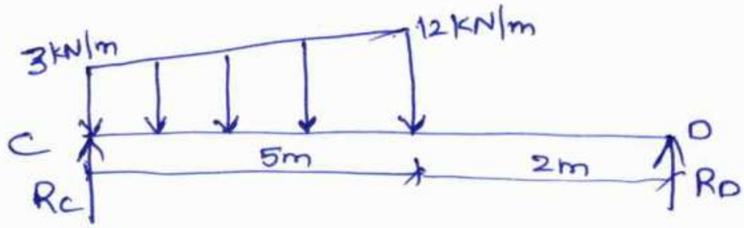


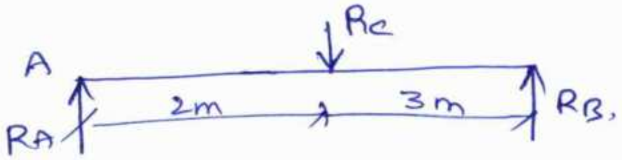
[Ans: $R_A = 26kN$
$R_B = 34kN$]

Q) Determine the reactions at A, B and D of the compound beam shown in fig. Neglect the self-weight of the members.

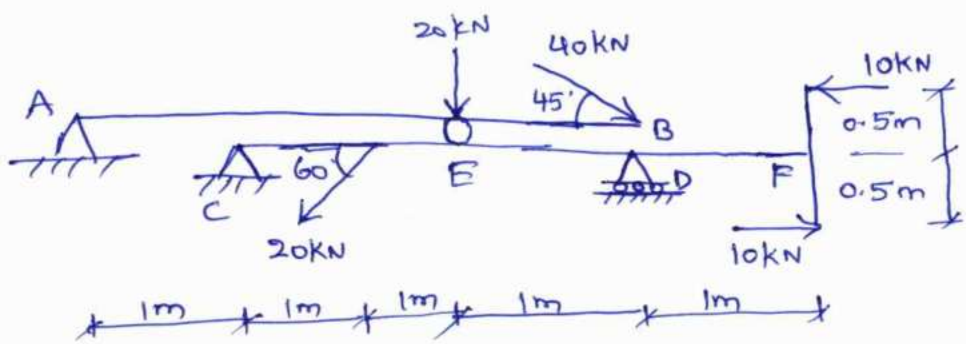
[Ans: $R_C = 21.43$ kN
$R_D = 16.07$ kN
$R_B = 8.57$ kN
$R_A = 12.86$ kN]



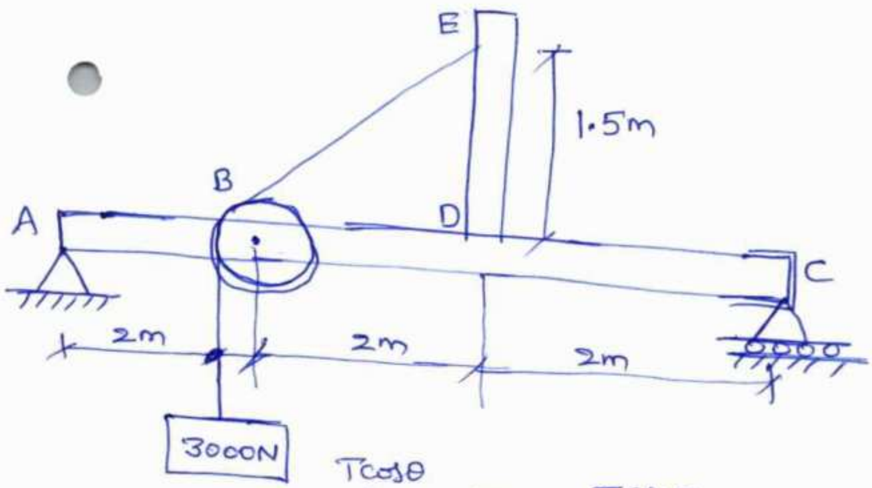




Q) The beam AB and CF are arranged as shown in fig. Determine the reactions at A, C and D due to the loads acting on the beam as shown in the fig.



[Ans: $(R_A)_x = 28.28$ kN
$(R_A)_y = 9.43$ kN (downward)
$R_E = 57.71$ kN.
$(R_C)_x = 10$ kN
$(R_C)_y = 34.12$ kN.
$R_D = 40.91$ kN.



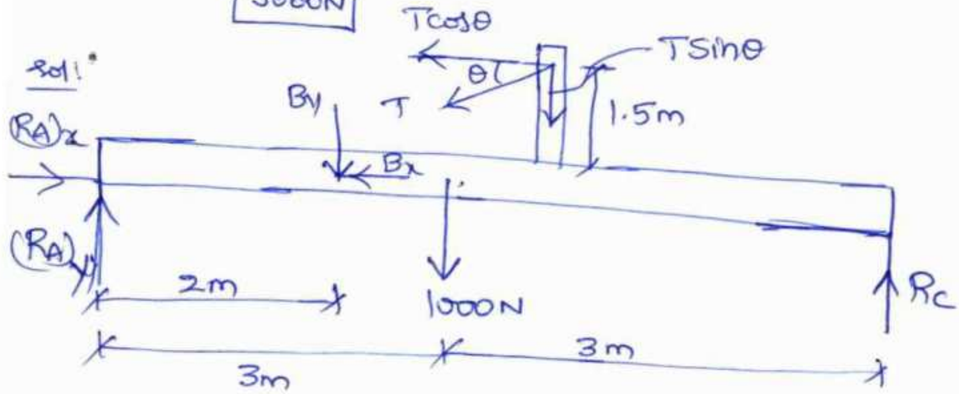
[Ans: $(R_A)_x = 0$

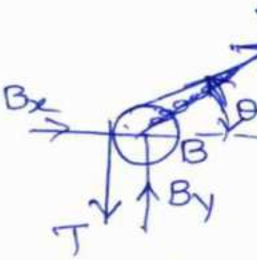
$(R_A)_y = 2500N$

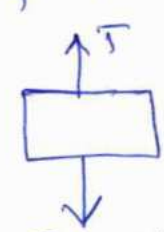
$R_C = 1500N$

$T = 3000N$]

Sol:







$\Sigma F_x = 0$

$B_x + T\cos\theta = 0$

$B_x = -T\cos\theta$

$\Sigma F_y = 0$

$T\sin\theta + B_y = T$

$B_y = 1200.41$ N

$\Sigma F_y = 0$

$T = 3000N$

$\Sigma F_x = 0 \Rightarrow (R_A)_x = B_x + T\cos\theta \Rightarrow (R_A)_x = 0$

$\Sigma F_y = 0 \Rightarrow (R_A)_y + R_c = B_y + 1000 + T\sin\theta \Rightarrow$

Here $\theta = 36.86$ $\qquad (R_A)_y = 2500N$

$\Sigma M_A = 0 \Rightarrow R_C \times 6 = (1000 \times 3) + (B_y \times 2) + (T\sin\theta \times 4) - (T\cos\theta \times 1.5)$

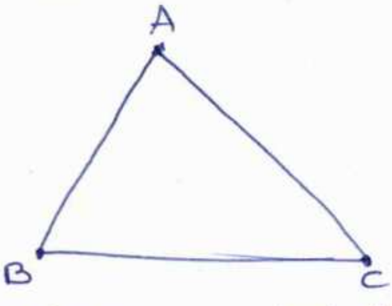
$R_C = 1500N,$

Analysis of Framed structures

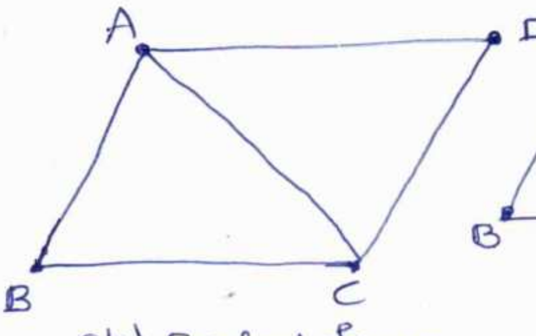
(Pin-Jointed plane)

A framed structure consists of a number of members connected to each other so as to form a frame to support an external load system.

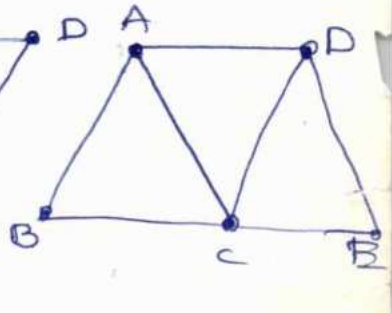
The simplest frame is a triangle, consisting of three members pin-jointed to each other. This can be easily analysed by the conditions of equilibrium. This frame is called the basic perfect frame. It has three members AB, BC and CA and three joints A, B and C



(a) Basic perfect frame



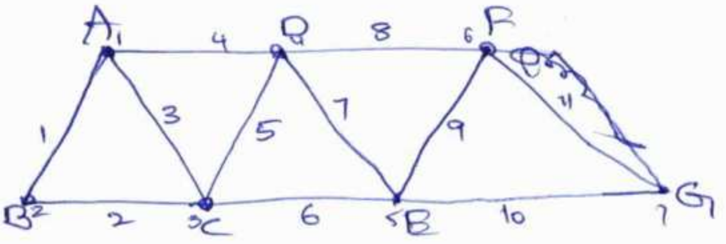
(b) Perfect frame



Suppose we add to this basic perfect frame two members AD and CD and a joint D, we get a frame (fig b) which can also be analysed by the conditions of equilibrium. This frame is called a perfect frame.

Relation between the no. of joints and the no. of members in a perfect frame:- let there are 'n' no. of members and j no. of joints in a perfect frame.

$3 = 2(3) - 3$
$= 6 - 3$
$3 = 3$

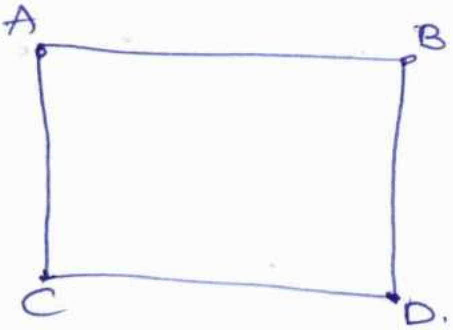


$11 = 2(7) - 3$
$= 14 - 3$
$11 = 11$

$n = 2j - 3$ n = no of members
j = no. of joints

$n = 2(4) - 3$
$4 = 5$

Hence for a stable frame the minimum no. of members required = twice the no. of joints minus three. If the number provided is less than the above requirements the frame will not be stable.



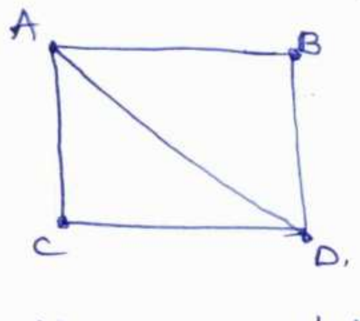
$n = 4$ $j = 4$

$4 = 2(4) - 3 = 5$

$4 < 5 \Rightarrow$ deficient frame

$\therefore$ Frame is unstable.

If we add one member the frame becomes stable and perfect.



$n = 5,\ j = 4$

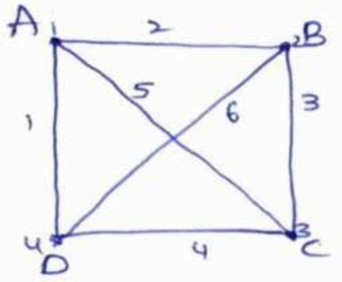
$5 = 2(4) - 3$

$= 8 - 3 = 5$

$5 = 5 \Rightarrow$ Perfect frame.

Hence, a deficient frame has less number of members than what is req for a perfect frame.

If the frame has more no. of members than what is reqd for a perfect frame. Such a frame is called redundant frame.



$n=6 \quad j=4$

$6 = 2(4)-3$

$6 = 8-3 = 5$

$6 > 5 \Rightarrow$ Redundant frame

In general let a frame have j joints and n members

If $n = 2j-3$, the frame is a perfect frame

If $n < 2j-3$, " " " deficient frame

If $n > 2j-3$ " " redundant frame.

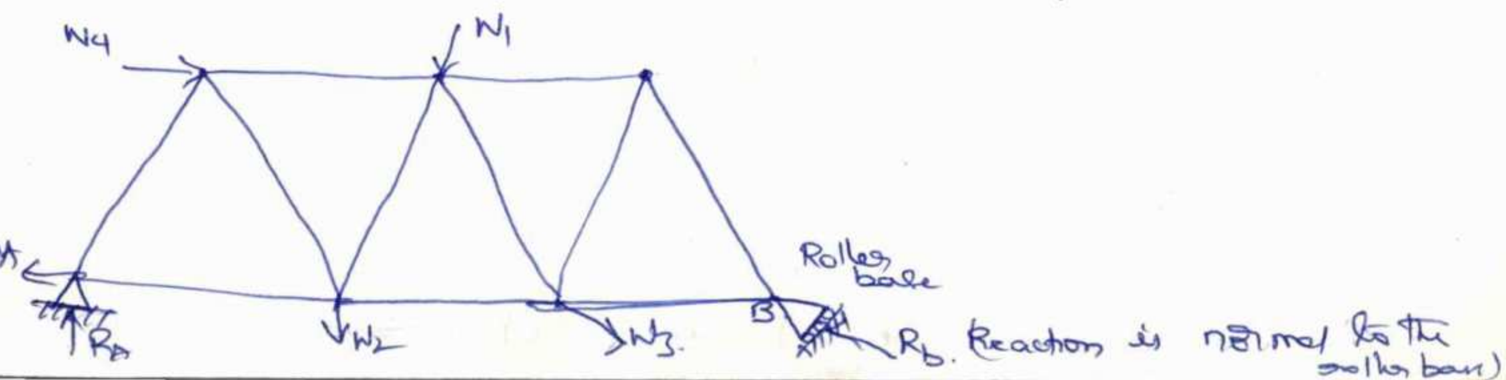
A perfect frame can always be analysed by conditions of equilibrium.

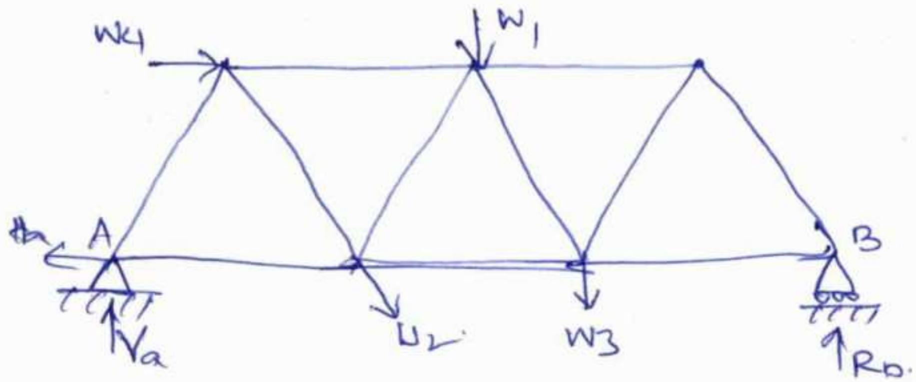
In this chapter we will discuss ~~only~~ analysis of perfect frames. only.

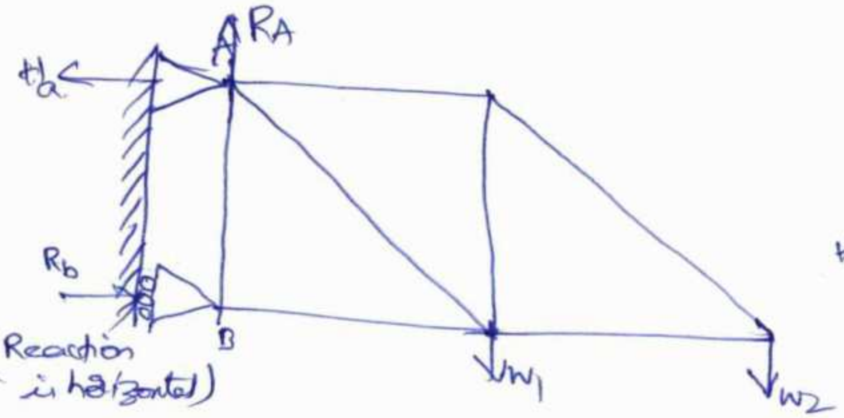
Reactions at supports:-

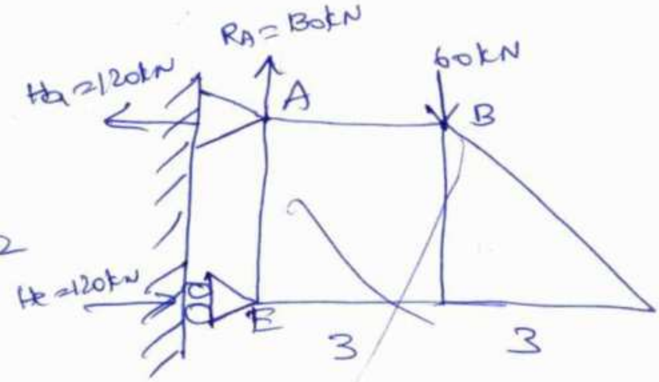
Frames are usually provided with either

(i) Roller or free supports (ii) hinged support.









To determine the reactions:-

Reactions at the supports can be determined by the conditions of equilibrium.

Consider the cantilever truss shown in fig. The truss is provided with a hinged support at A and a roller support at E.

The roller base at E being vertical the reaction at E is horizontal. Hence there will be no vertical reaction at E.

Taking moments about A.

$$H_e \times 4 = (60 \times 3) + (40 \times 3) + (30 \times 6)$$

$$= 180 + 120 + 180$$

$$H_e \times 4 = 480 \Rightarrow H_e = 120\,kN\ (\rightarrow)$$

∴ Total applied Vertical force = 60+40+30 = 130 kN (↓)

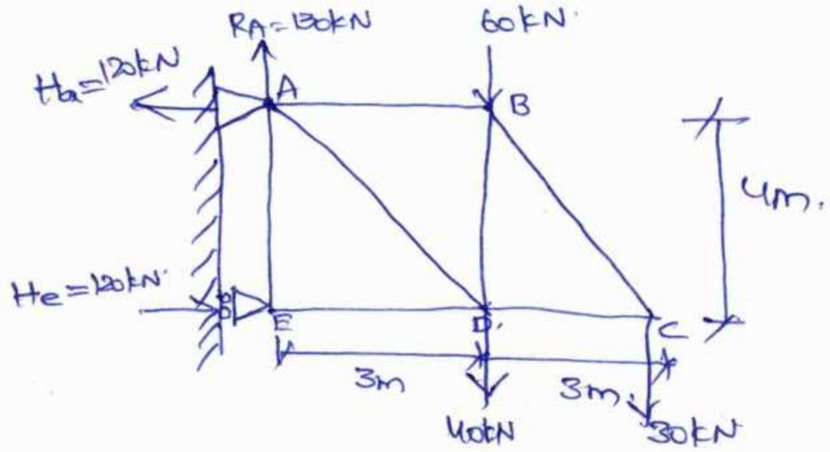
∴ Vertical reaction at A = R_A = 130 kN (↑)

Resolving the forces horizontally we get

H_a = 120 kN (←).

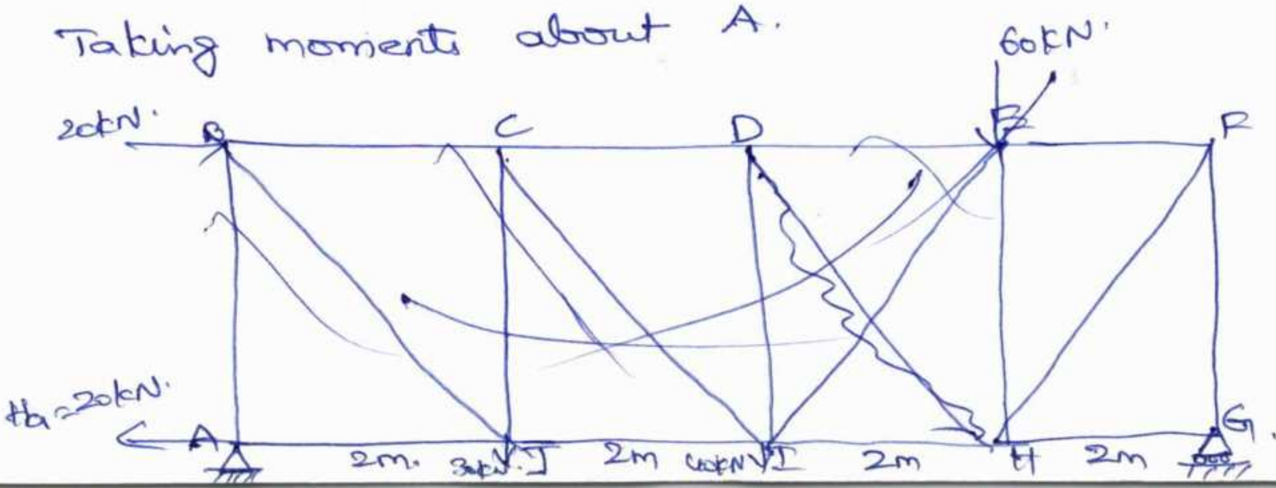
Thus the reaction at A consists of a Vertical component R_A = 130 kN (↑) and a horizontal component

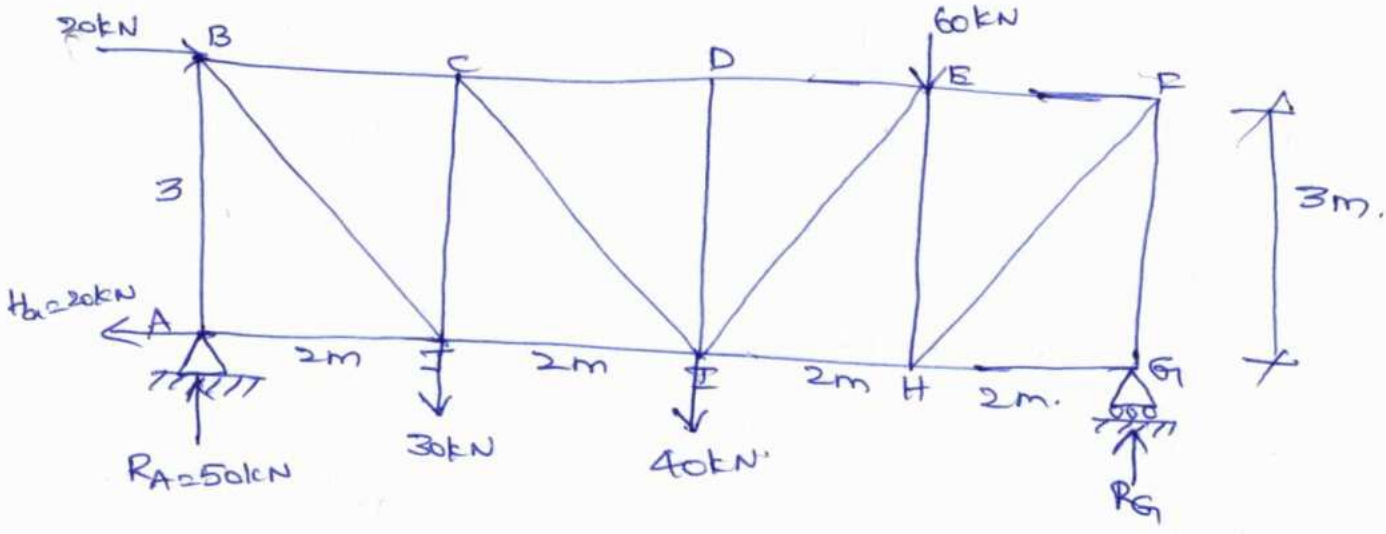
H_a = 120 kN (←)



Now consider the truss shown in fig provided with a hinged support at A and a roller support at G. The roller base at G is horizontal and hence the reaction at G is entirely Vertical. There will be no horizontal reaction at G.

Taking moments about A.





$R_G \times 8 = (60 \times 6) + (30 \times 2) + (40 \times 4) + (20 \times 3)$

$= 360 + 60 + 160 + 60$

$R_G \times 8 = 580 + 60 = 640$

$R_G = \frac{640}{8} = 80$ kN. (↑)

Total applied Vertical force = $60 + 30 + 40 = 130$ kN (↓).

∴ Vertical reaction at A = $R_A = 130 - 80 = 50$ kN (↑)

Total applied horizontal force = 20kN (→)

∴ Horizontal reaction at A = H_a = 20kN (←)

<u>Assumptions made in finding out the forces in a frame:</u>

(1) The frame is a perfect frame

(2) The frame carries load at the joints

(3) All the members are pin jointed.

Analysis of a ~~truss~~ frame:-

The analysis of a ~~truss~~ frame consists of the following

(i) Det of the reactions at the supports

(ii) Det of the forces in the members of the frame.

The reactions are determined by the condition that the applied load system and the induced reaction at the supports form a system in equilibrium.

The forces in the members of the frame are determined by the condition that every joint should be in equilibrium and so, the forces acting at every joint should form a system in equilibrium.

A framed structure can be analysed by the following methods

(1) Method of Joints or Method of Resolution

(2) Method of Sections

(3) Graphical analysis.

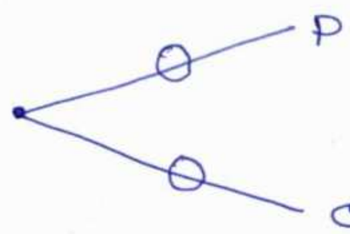
To determine which members of a frame do not carry forces:- In a frame carrying a load system some members may not carry forces. such members can be identified by using the following Principles.

(a) A single force cannot form a system in equilibrium. Hence if there is only one force acting at a joint, then for the equilibrium of the joint, this force equals zero.

$P = 0$

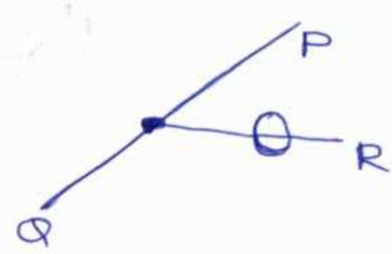
(b) If two forces act at a joint, then for the equilibrium of the joint these two forces should act along the same st line. The two forces will be equal and opposite. If the only two forces acting at a joint are not along the same st line, then for the equilibrium of the joint each force = 0.

$P = 0$ and $Q = 0$.

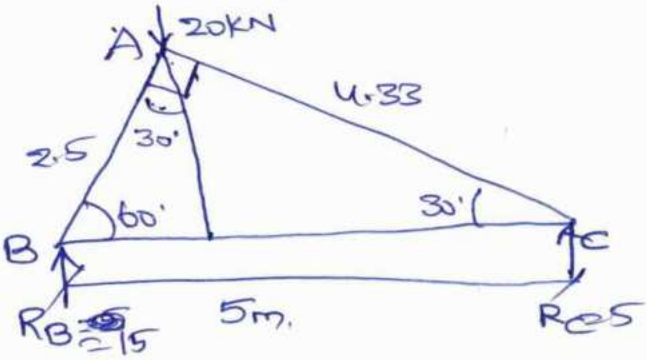


(c) If three forces act at a joint and two of them are along the same st line then for the equilibrium of the joint, the third force should be equal to zero.

$R = 0$



1) Find the forces in the members AB, AC and BC of the truss shown in fig.



$R_B + R_C = 20$

$R_C \times 5 - (20 \times 2.5\cos 60) = 0$

$R_C = 5$

$R_B = 15$

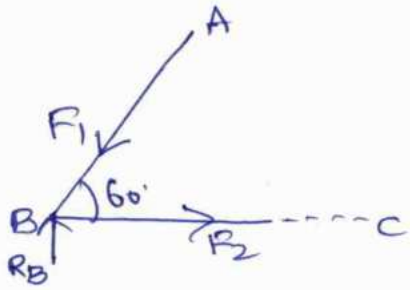
$\sin 60 = \frac{AC}{BC} = \frac{AC}{5}$

$AC = 5\sin 60 = 4.33$

$BC^2 = AC^2 + AB^2$

$AB^2 = BC^2 - AC^2$

$AB = 2.5m.$



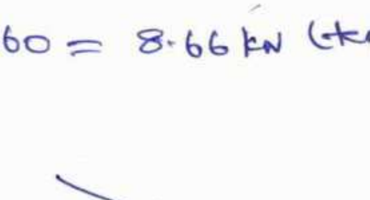
Joint B: let the force F_1 is acting towards the joint B and the force F_2 is acting away from the joint B.

Resolving the forces acting on the joint B, Vertically

$R_B = F_1 \sin 60 \Rightarrow F_1 = \frac{R_B}{\sin 60} = \frac{15}{\sin 60} = 17.32$ KN (compressive)

$F_2 = F_1 \cos 60 = 8.66$ kN (tensile)

Joint C:



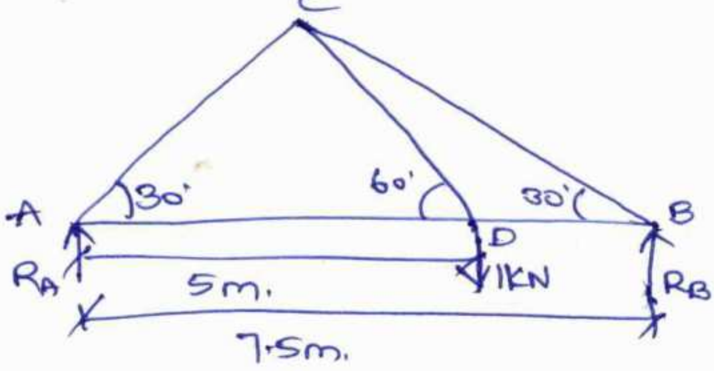
$F_3 \sin 30 = R_C$

$F_3 = \frac{R_C}{\sin 30} = 10$ kN (compressive)

~~$F_3 \cos 30 = F_2$~~

~~$F_2 = 4.54$~~

2) A truss of span 7.5m carries a point load of 1kN at joint D as shown in fig. Find the reactions and forces in the member of the truss.



Sol:

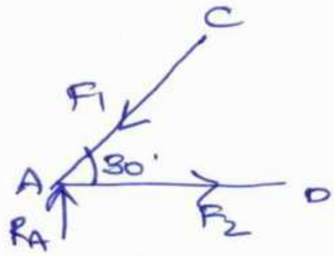
$R_A + R_B = 1$

$R_B \times 7.5 = 1 \times 5$

$R_B = \frac{5}{7.5} = 0.66\,kN$

$R_A = 1 - 0.66 = 0.33\,kN$

Joint A:

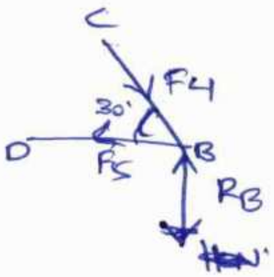


$F_1 \sin 30 = R_A$

$F_1 = \frac{R_A}{\sin 30} = \frac{0.33}{\sin 30} = 0.66$ (compressive)

$F_2 = F_1 \cos 30 = 0.57$ (tensile).

Joint B:

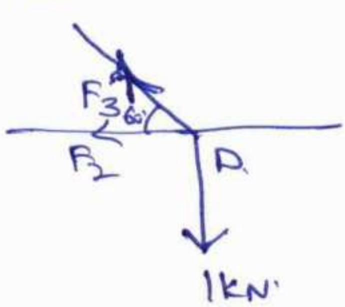


$F_4 \sin 30 = R_B$

$F_4 = \frac{R_B}{\sin 30} = 1.32\,kN$ (Compressive)

$F_5 = F_4 \cos 30 = 1.14\,kN$. (Tensile)

Joint D:-



$F_3 \sin 60 = 1$

$F_3 = 1.154\,kN$ (tensile).

~~F_2~~

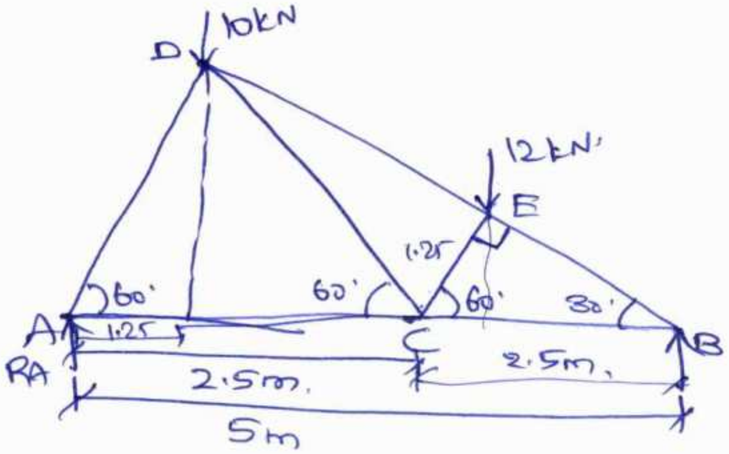
Hence forces in the member are:

$F_1 = 0.66\,kN$ (compr) $F_2 = 0.57$ (tensile)

$F_3 = 1.154\,kN$ (tensile) $F_4 = 1.32\,kN$ (compr)

$F_5 = 1.14\,kN$ (tensile).

3) A truss of span 5m is loaded as shown in fig find the reactions and forces in the members of the truss.



Sol.

$R_A + R_B = 22$

$R_B \times 5 - 10 \times 1.25 - 12 \times 3.12 = 0$

$R_B = 9.98\,kN$

$R_A = 12.02\,kN$

$\sin 60 = \frac{BE}{BC}$

$BE = 2.5 \times \sin 60$

$BE = 2.16$

$\cos 60 = \frac{CX}{CB}$

$CX = 0.625$

$\sin 60 = \frac{DB}{5}$

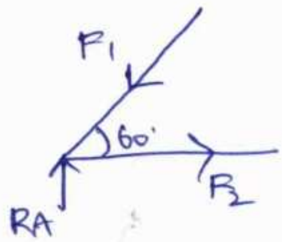
$DB = 5 \sin 60$

$DB = 4.33$

$DA = 2.5$

Joint A:

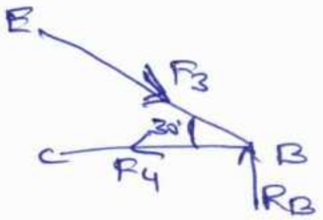
$R_A = F_1 \sin 60$



$F_1 = \frac{R_A}{\sin 60} = 13.87\,kN$ (compr)

$F_2 = F_1 \cos 60 = 6.93\,kN$ (tensile)

Joint B:



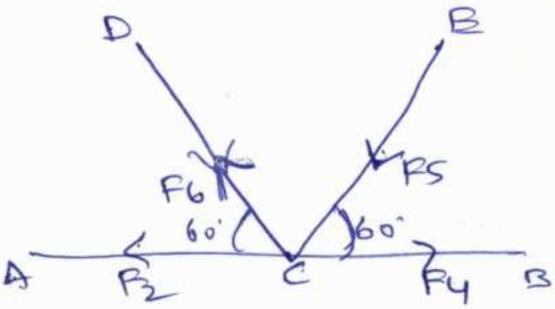
$F_3 \sin 30 = R_B$

$F_3 = 19.96\,kN$ (compr)

$F_4 = F_3 \cos 30 = 17.28$ (tensile)

Joint C:

$F_5 \sin 60 + F_6 \sin 60 = 0$



$F_5 + F_6 = 0$

$F_5 = -F_6$

$F_2 - F_6 \cos 60 = F_4 - F_5 \cos 60$

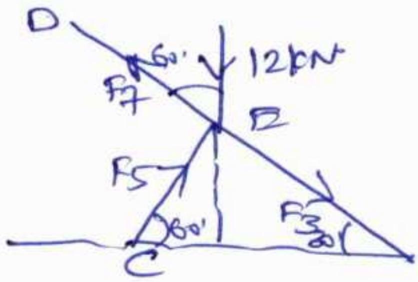
$6.93 - F_6 \cos 60 = 17.28 + F_6 \cos 60$

$-10.35 = F_6 (2 \times \cos 60)$

$F_6 = +10.35$ kN (tensile)

$F_5 = 10.35$ kN (compressive)

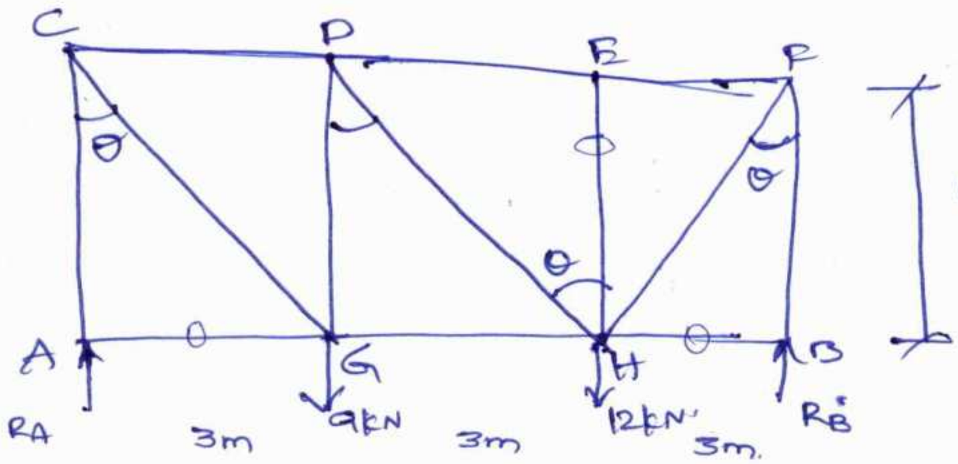
Joint E:-



$F_7 + 12 \cos 60 = F_3$

$F_7 = 13.96$ kN (compr.)

4)

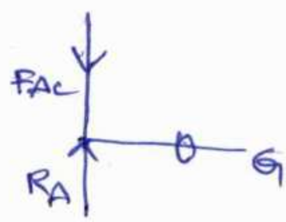


Find the forces and Reactions in the members of the frame.

$R_A + R_B = 21$ kN $R_B \times 9 - (12 \times 6) - (9 \times 3) = 0$

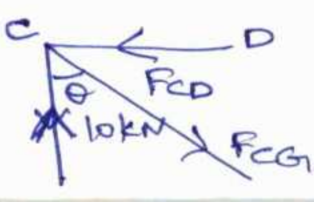
$R_A = 10$ kN $R_B = \frac{72 + 27}{9} = \frac{99}{9} = 11$ kN

Joint A:



$F_{AC} = R_A = 10$ kN (compr).

Joint C:-



$\cos\theta = \frac{3}{4} \Rightarrow \theta = \cos^{-1}\left(\frac{3}{4}\right) = 41.4$

$\cos\theta = \frac{F_{CA}}{F_{CG}} \Rightarrow F_{CG} = \frac{F_{CA}}{\cos\theta} = \frac{10}{(3/4)} = \frac{40}{3} = 13.33\,kN.$

$F_{CG} \cos\theta = F_{AC}$ $\qquad \tan\theta = \frac{3}{4} = 0.75$

$\theta = 36.86$

$F_{CG} = 12.5\,kN$ (tensile)

$F_{CG} \sin\theta = F_{CD}$

$F_{CD} = 7.49\,kN$ (compr)

Joint G:

$F_{CG} \sin\theta = F_{GH}$

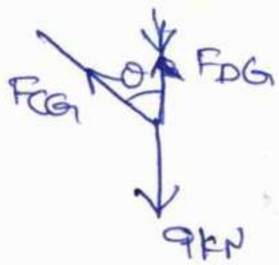
$F_{GH} = 7.49\,kN$ (tensile)

$F_{DG} + F_{CG} \cos\theta = 9$

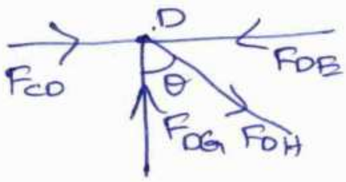
$F_{DG} = -1\,kN$

assumed direction wrong

$F_{DG} = 1\,kN$ (~~Tensile~~, Compr)



Joint D:



$F_{CD} = F_{DE} - F_{DH} \sin\theta$ $\qquad F_{DG} = F_{DH} \cos\theta$

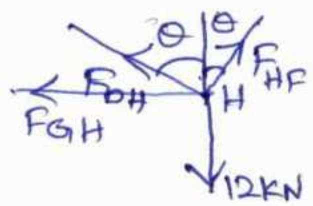
$F_{DE} = 8.24\,kN$ (compr) $\qquad F_{DH} = 1.24\,kN$ (Tensile)

Joint E:

$F_{EF} = F_{DE} = 8.24\,kN$ (compr)

$\therefore F_{EF} = 8.24\,kN$ (compr).

Joint H:-



~~$F_{DH}\sin\theta = F_{HF}\sin\theta$~~

~~$F_{DH} = F_{HF}$~~

$F_{DH}\cos\theta + F_{HF}\cos\theta = 12$

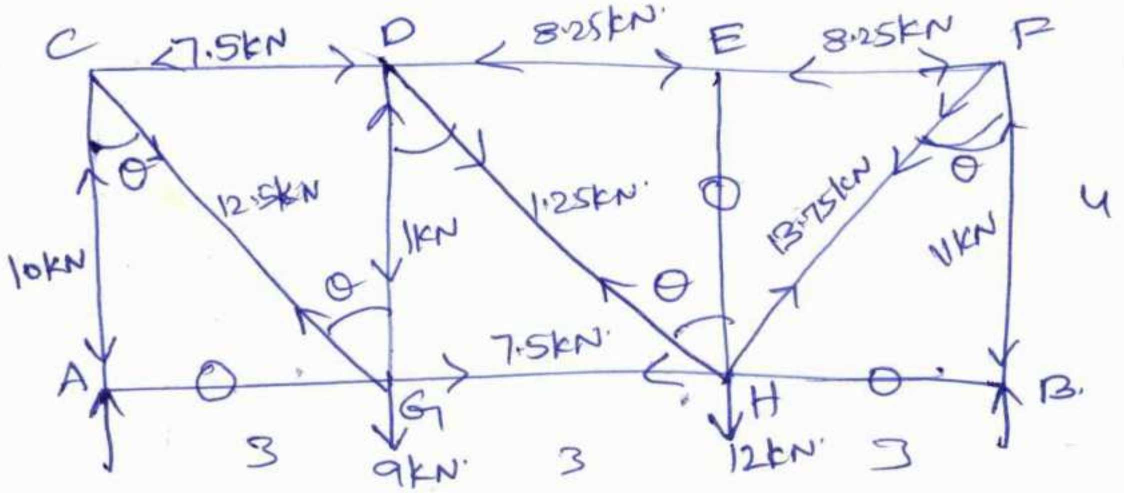
$F_{GH} + F_{DH}\sin\theta = F_{HF}\sin\theta$

$F_{HF} = 13.75\,kN$ (Tensile)

$F_{GH} = 7.5\,kN$ (Tensile).

Joint B:

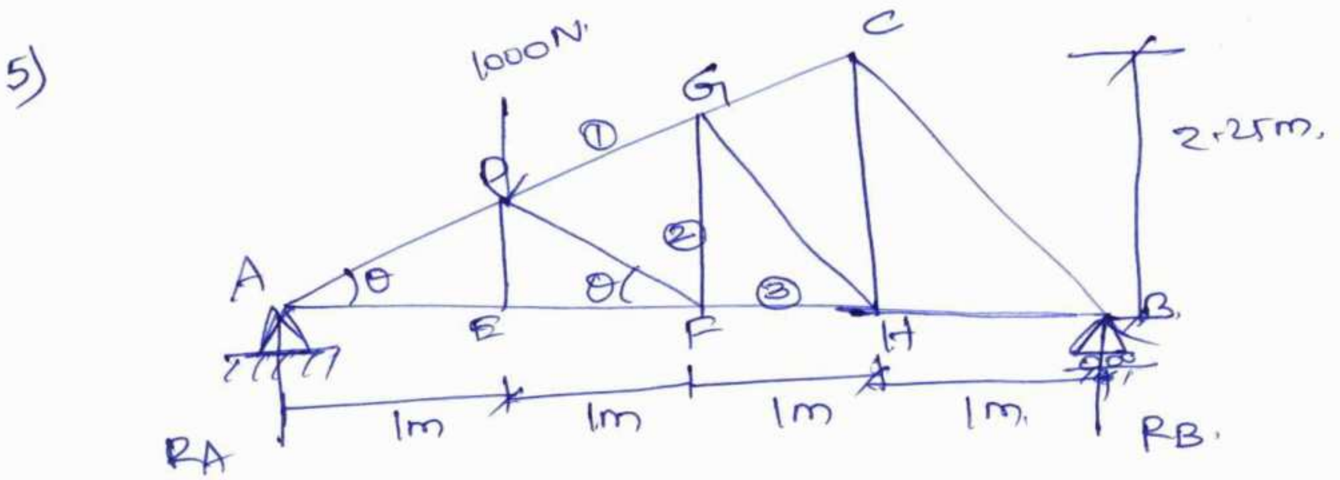
$F_{FB} = R_B = 11\,kN$ (compr)



Member.	Force in member
AC	10kN (compr)
AG	0
CG	12.5kN (Tens)
CD	7.5kN (Compr)
DG	10kN (compr)
DE	8.25kN (Comp)
DH	1.25kN (Tens)
GH	7.5kN (Tens)

EH	0
EF	8.25 kN (comp)
HB	0
HF	13.75 kN (Tens).
BF	11 kN (Comp).

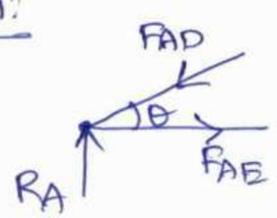
5)



$R_A + R_B = 1000$ $\quad R_B \times 4 - 1000 \times 1 = 0$

$R_B = 250$ kN $\quad R_A = 750$ kN

Joint A:



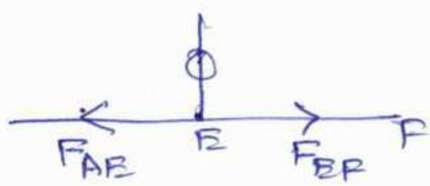
$\tan\theta = \frac{2.25}{3} =$

$\theta = 36.86°$

$F_{AD}\sin\theta = R_A$ $\quad F_{AD}\cos\theta = F_{AE}$

$F_{AD} = 1250$ kN (compr) $\quad F_{AE} = 1000$ kN (Tensile)

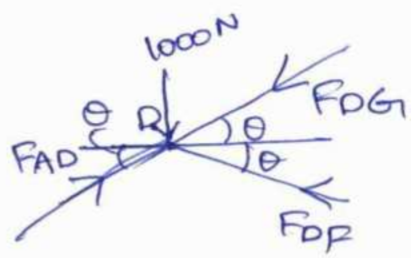
Joint E:-



$F_{AE} = F_{EF} = 1000$ kN

$F_{EF} = 1000$ kN (Tensile)

Joint D:-



$1000 + F_{DG}\sin\theta = F_{AD}\sin\theta + F_{DF}\sin\theta$

$\cancel{F_{DG} - F_{DF} = 417N}$ $F_{DF} - F_{DG} = 417N$

$F_{AD}\cancel{\cos}\theta = F_{DG}\cancel{\cos}\theta + F_{DF}\cancel{\cos}\theta$

$F_{DG} + F_{DF} = 1250N$

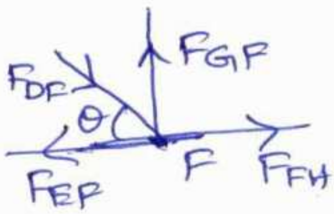
$2F_{DF} = 1667N$

$F_{DF} = 833.5\text{ N (compr)}$

$F_{DG} = 416.5\text{ N (compr)}$

$F_1 = F_{DG} = 416.5N\text{ (compr)}$

Joint F:



$F_{FH} + F_{DF}\cos\theta = F_{EF}$

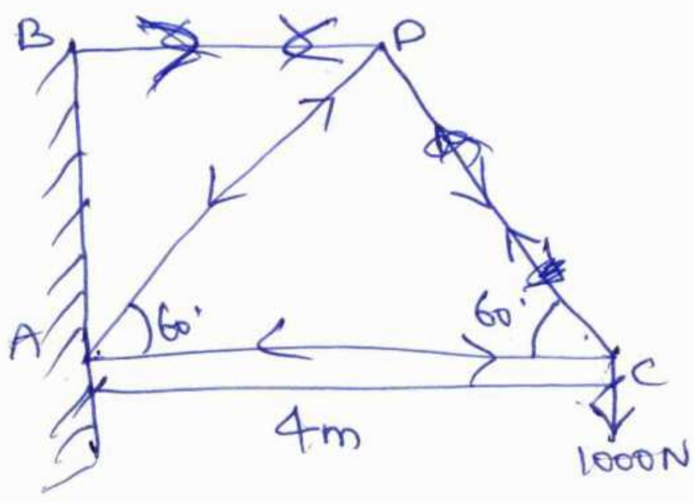
$F_3 = F_{FH} = 333.11\text{ N (Tensile)}$

$F_{GF} = F_{DF}\sin\theta$

$F_2 = F_{GF} = 500\text{N (Tensile)}$

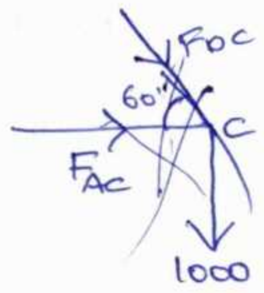
$\therefore F_1 = 416.5N\text{(compr)}$ $F_3 = 333.11N\text{(Tensile)}$ $F_2 = 500N\text{(Tensile)}$

6)



Apply conditions of Equilibrium:

Joint 'C'

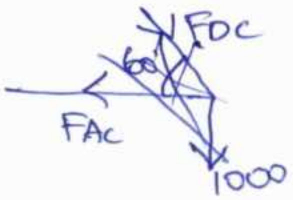


$F_{DC}\cos 60 + F_{AC} = 0 \Rightarrow F_{AC} = -577.3N$

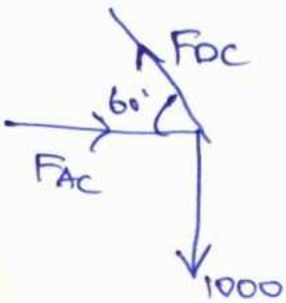
$1000 = F_{DC}\sin 60$

$F_{DC} = 1154$ (compressive)

Assumed direction is zero for F_{AC}



$\therefore F_{AC} = 577.3N$ (Tensile)

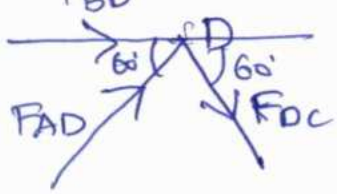


$F_{DC}\sin 60 = 1000 \Rightarrow F_{DC} = 1154.7N$ (Tensile)

$F_{DC}\cos 60 = F_{AC}$

$F_{AC} = 577.35N$ (compressive)

Joint D:-



$F_{BD} + F_{AD}\cos 60 + F_{DC}\cos 60 = 0$

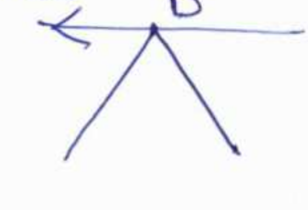
$F_{AD}\cancel{\sin 60} = F_{DC}\cancel{\sin 60}$

$F_{AD} = F_{DC} = 1154.7N$ (compression)

$F_{BD} + (2\cos 60)F_{DC} = 0$

$F_{BD} = -1154.7N$

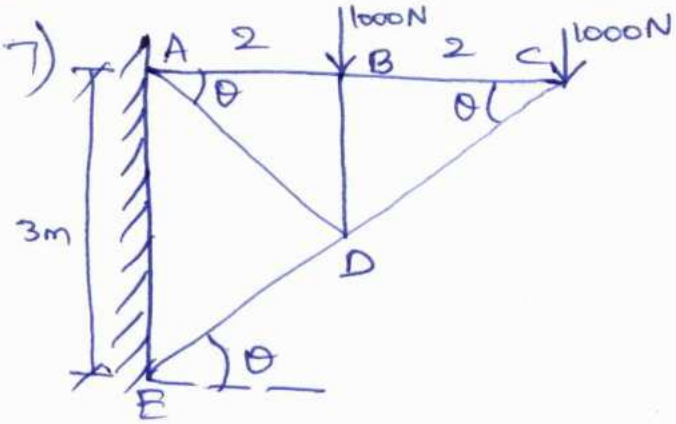
Assumed direction of F_{BD} is wrong



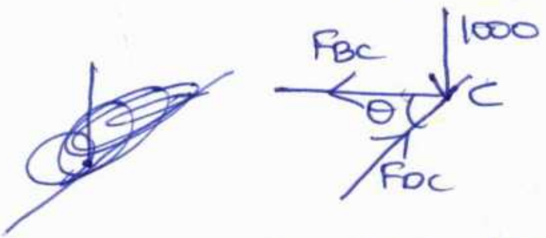
$\therefore F_{BD} = 1154.7N$ (Tensile).

$\therefore F_{AD} = 1154.7N$ (compr)

7)



Joint C:-



$\theta = 36.86$

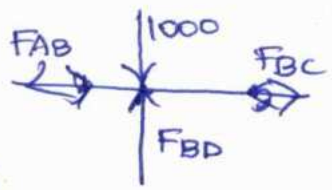
$F_{DC} \sin\theta = 1000$

$F_{DC} = 1666.66N$ (compr)

$F_{DC} \cos 36.86 = F_{BC}$

$F_{BC} = 1333.5N$ (Tensile)

Joint 'B':-

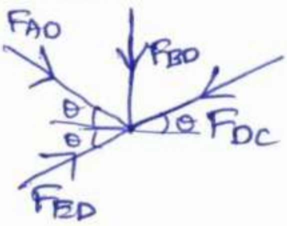


$F_{BD} = 1000$ (compr)

$F_{AB} = F_{BC}$

$F_{AB} = 1333.5N$ (Tensile)

Joint 'D':



~~$F_{BD} = F_{ED}\sin\theta$~~

$F_{AD}\cos\theta + F_{ED}\cos\theta = F_{DC}\cos\theta$

$F_{AD} + F_{ED} = 1666.66N$

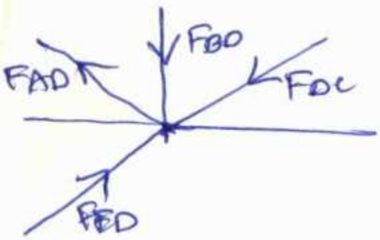
$F_{BD} + F_{AD}\sin\theta + F_{DC}\sin\theta = F_{ED}\sin\theta$

$3333.71 = F_{ED} - F_{AD}$

$F_{ED} = 2500N$ (compr)

$F_{AD} = -833.52$

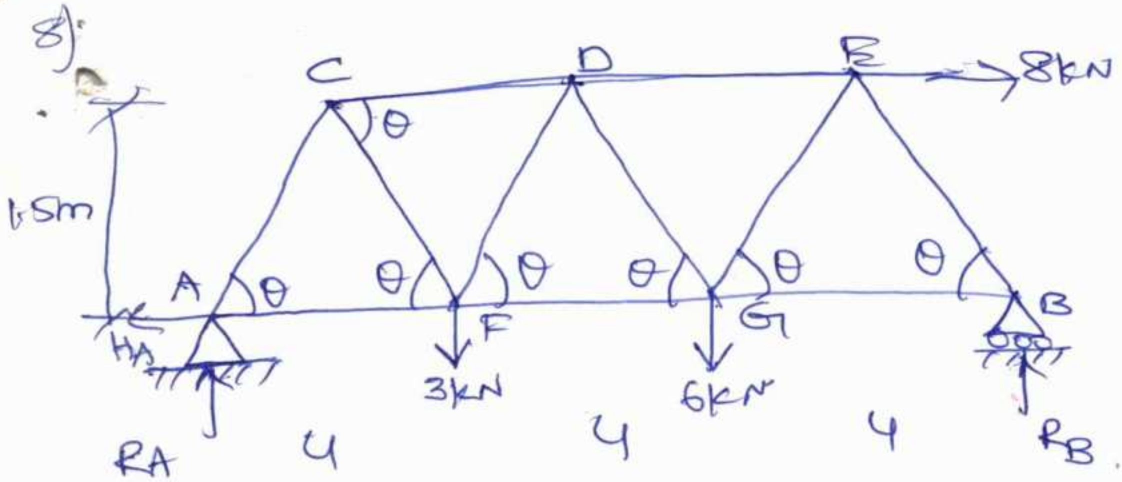
Assumed direction of F_{AD} is wrong



$\therefore F_{AD} = 833.52N$ (Tensile)

AB	1333.5N (Tensile)
BC	1333.5N (Tensile)
AD	833.52N (Tensile)
BD	1000 (compr)
DC	1666.66N (compr)
ED	2500N (compr)

8)



$\theta = 36.86$

$R_A + R_B = 9$

$R_B \times 12 = (6\times8) + (3\times4) + (8\times1.5)$

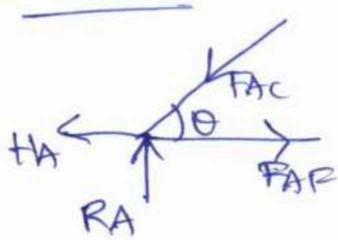
$R_B \times 12 = 72$

$R_B = 6$

$R_A = 3$

$H_A = 8kN$

Joint A:



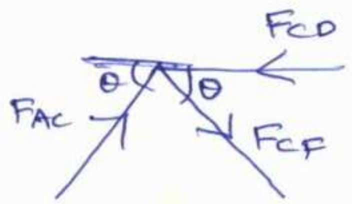
$R_A = F_{AC} \sin 36.86$

$F_{AC} = 5kN$ (compr) $\rightarrow$ ①

$H_A + F_{AC}\cos\theta = F_{AF}$

$F_{AF} = 12kN$ (Tensile) $\rightarrow$ ②

Joint C:-



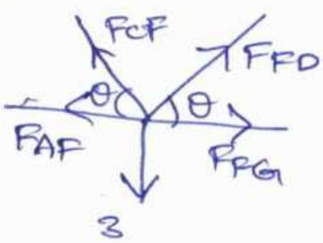
$F_{AC}\cos\theta + F_{CF}\cos\theta = F_{CD}$

$F_{AC}\sin\theta = F_{CF}\sin\theta$

$F_{CF} = 5kN$ (Tensile) $\rightarrow$ ③

$F_{CD} = 8kN$ (compr) $\rightarrow$ ④

Joint F:-



$F_{CF}\cos\theta + F_{AF} = F_{FD}\cos\theta + F_{FG}$

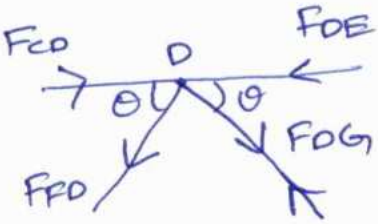
$F_{CF}\sin\theta + F_{FD}\sin\theta = 3 \Rightarrow$

$F_{FD}\cos\theta + F_{FG} = 16$

$F_{FD} = 0 \rightarrow$ ⑤

$F_{FG} = 16kN$ (Tensile) $-$ ⑥

Joint D:-



$$F_{CD} + F_{DG}\cos\theta = F_{DE} + F_{FD}\cos\theta$$

$$F_{FD}\sin\theta + F_{DG}\sin\theta = 0$$

$$F_{FD} = -F_{DG}$$

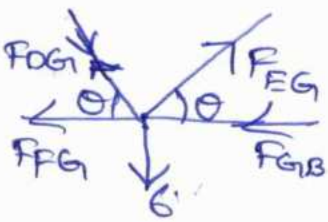
$$F_{CD} = F_{DE} - F_{DG}\cos\theta$$

$$8 = F_{DE} - F_{DG}\cos\theta$$

$$F_{DE} = 8\,kN\,(compr) \rightarrow (7)$$

$$\therefore F_{DG} = 0 \rightarrow (8)$$

Joint G:



$$F_{DG}\cos\theta + F_{EG}\cos\theta = F_{GB} + F_{FG}$$

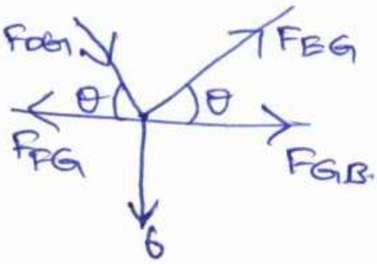
$$F_{DG}\sin\theta + 6 = F_{EG}\sin\theta$$

$$F_{EG} = 10\,kN\,(Tensile) \rightarrow (9)$$

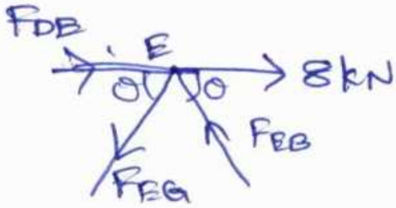
$$F_{GB} = -8\,kN\,(Compr)$$

Assumed direction is wrong.

$$\therefore F_{GB} = 8\,kN\,(Tensile) \rightarrow (10)$$



Joint E:-

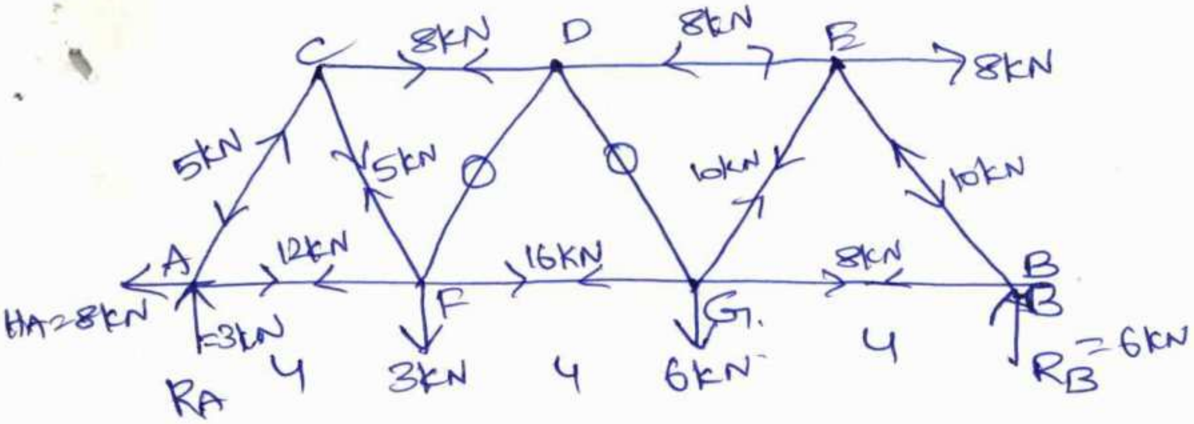


$$F_{DE} + 8 = F_{EG}\cos\theta + F_{EB}\cos\theta$$

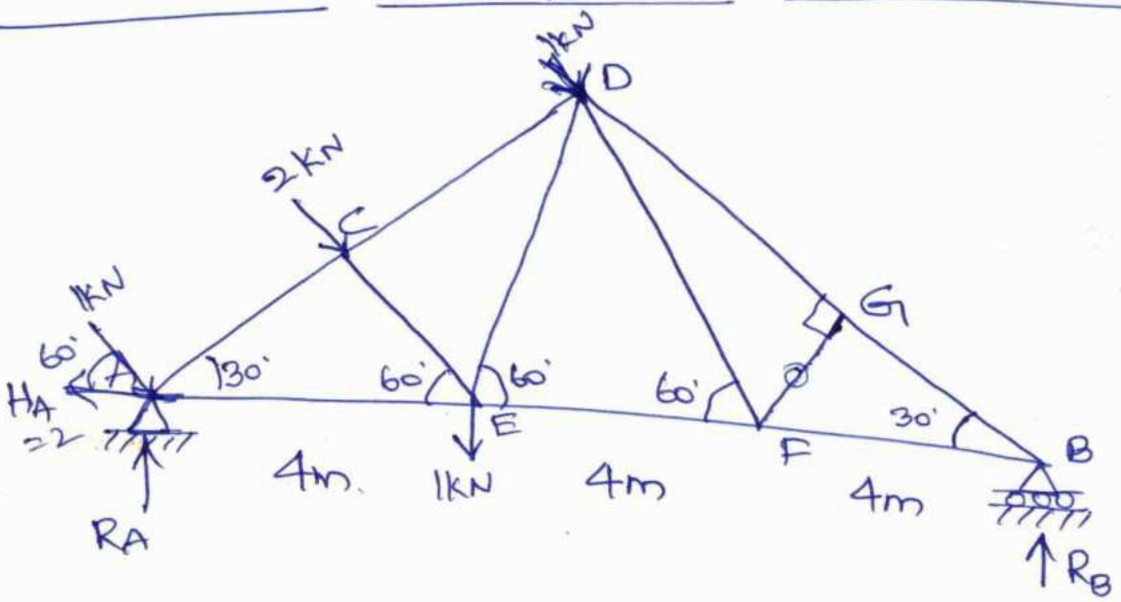
$$F_{EG}\sin\theta = F_{EB}\sin\theta$$

$$F_{EG} = F_{EB}$$

$$\therefore F_{EB} = 10\,kN\,(compr)$$



Method of Joints applied to Trusses carrying inclined loads



$R_A + R_B = 1 + (1\sin 60) + (2\sin 60) + (1\sin 60)$

$AC = 4\cos 30$

$AD = 8\cos 30$

$= 1 + 0.866 + 1.73 + 0.866$

$= 4.46\,kN$

$R_B \times 12 = (1 \times AD) + (2 \times AC) + (1 \times 4)$

$= (1 \times 8\cos 30) + (2 \times 4\cos 30) + 4.$

$= 17.85$

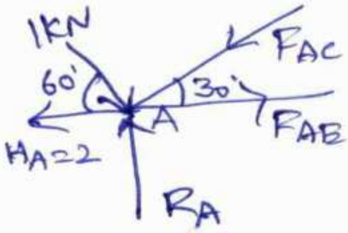
$R_B = 1.48\,kN$

$R_A = 2.97\,kN$

$H_A = (1\cos 60 + 2\cos 60 + 1\cos 60)$

$H_A = 2$

Joint A:-



~~$F_{AC}\sin 30 = R_A$~~

~~$F_{AC} = \frac{R_A}{\sin 30} = 5.94\,kN$~~

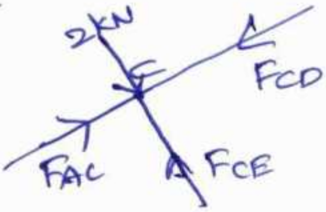
$F_{AC}\sin 30 + 1\sin 60 = R_A$

$F_{AC} = 4.2\,kN$ (compr)

$1\cos 60 + F_{AB} = H_A + F_{AC}\cos 30$

$F_{AB} = 5.13\,kN$ (Tensile)

Joint C:-

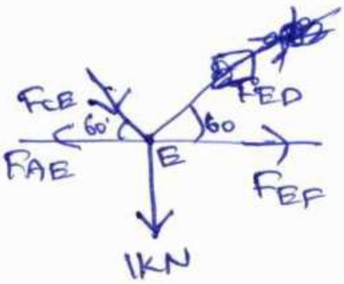


$F_{CE} = 2$ (compr)

$F_{AC} = F_{CD}$

$F_{CD} = 4.2\,kN$ (compr)

Joint E:-



$F_{CE}\cos 60 + F_{EF} + F_{ED}\cos 60 = F_{AE}$

$F_{EF} + F_{ED}\cos 60 = 4.913$

$F_{CE}\sin 60 + 1 = F_{ED}\sin 60$

~~$F_{ED} = 0.84\,kN$~~

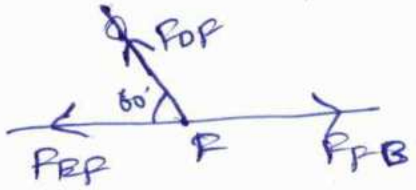
3.15 kN (Tens)

$F_{ED} =$ [illegible]

$F_{EF} = 2.55\,kN$ (Tensile)

Joint 'D':-

Joint 'F':-



$F_{DF}\sin 60 = 0$

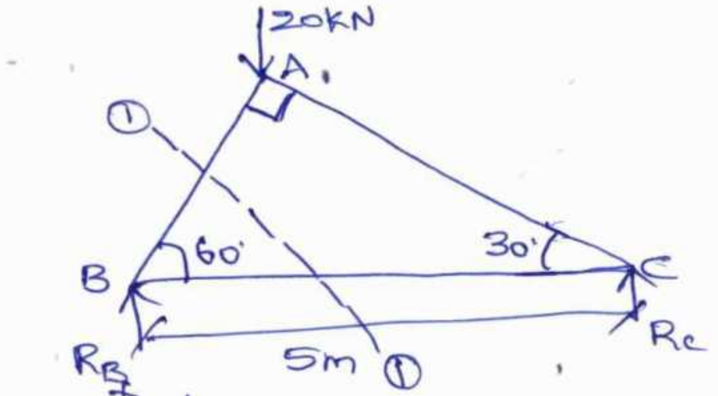
$F_{DF} = 0$

$F_{DF}\cos 60 + F_{EF} = F_{FB}$

$F_{EF} = F_{FB} = 2.55\,kN$ (Tensile)

If the magnitude of the forces, in the members cut by a section line, is +ve then the assumed direction is correct. If magnitude of a force is -ve, then reverse the direction of that force.

P1) Find the forces in the members AB and AC of the truss shown in fig. using method of section.



Sol!

$AB = BC \cdot \cos 60 = 2.5m$

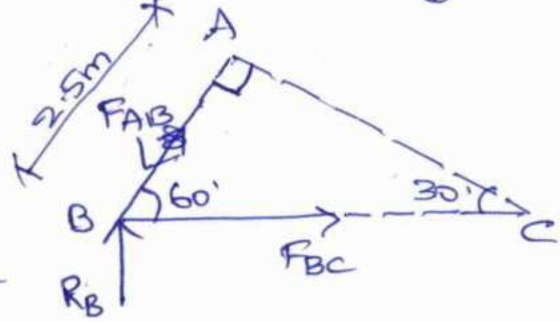
$R_B + R_C = 20$

$R_C \times 5 = 20 \times (2.5\cos 60)$

$R_C \times 5 = 25$

$R_C = 5KN$

$R_B = 15KN$



Now consider the left part (equilibrium) of the section.

Now taking the moments of all the forces acting on the left part about point c, we get

$R_B \times 5 + F_{AB} \times (BC\sin 60) = 0$

$F_{AB} = -17.32kN$

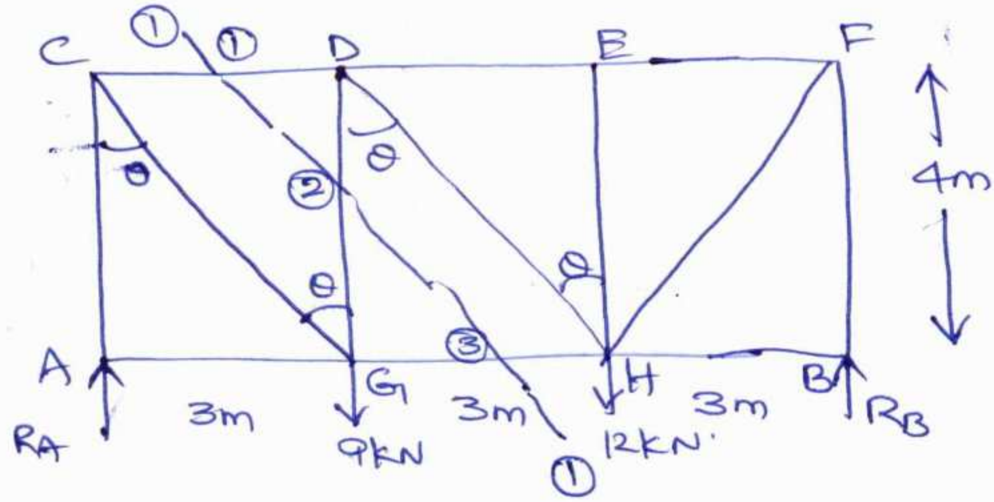
$\therefore F_{AB} = 17.32kN$ (compressive)

Now take the moments of the forces about point A

$R_B \times (\cancel{A}B \sin 30) = (F_{BC} \times A\cancel{B} \sin 60)$.

$\frac{R_B \times \sin 30}{\sin 60} = F_{BC} \Rightarrow F_{BC} = 8.66\,kN$ (Tensile)

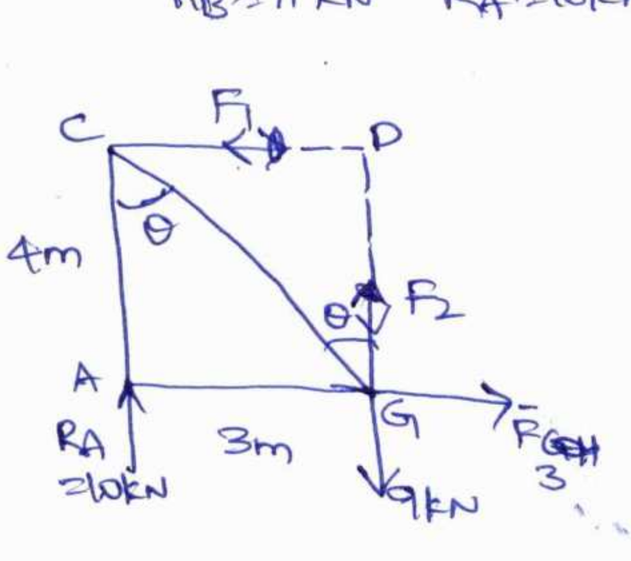
2) A truss of span 9m is loaded as shown in fig. Find the reactions and forces in the members marked 1, 2 and 3.



Sol: $R_A + R_B = 21$

$R_B \times 9 = (12 \times 6) + (9 \times 3) = 72 + 27 = 99$

$R_B = 11\,kN \quad R_A = 10\,kN$



Taking moments of all forces about point D

$R_A \times 3 = F_3 \times 4$

$30 = F_3 \times 4$

$F_3 = 7.5\,kN$ (Tensile)

Taking moments of all forces about G

$(R_A \times 3) + (F_1 \times 4) = 0$

~~30/09/11~~
~~10, 12, 16, 19~~,
~~28, 29~~, 33,
~~34~~

$F_1 = \frac{-30}{4} = -7.5\,kN$

Assumed direction is wrong

$\therefore F_1 = 7.5\,kN$ (compression)

Take moments about point C

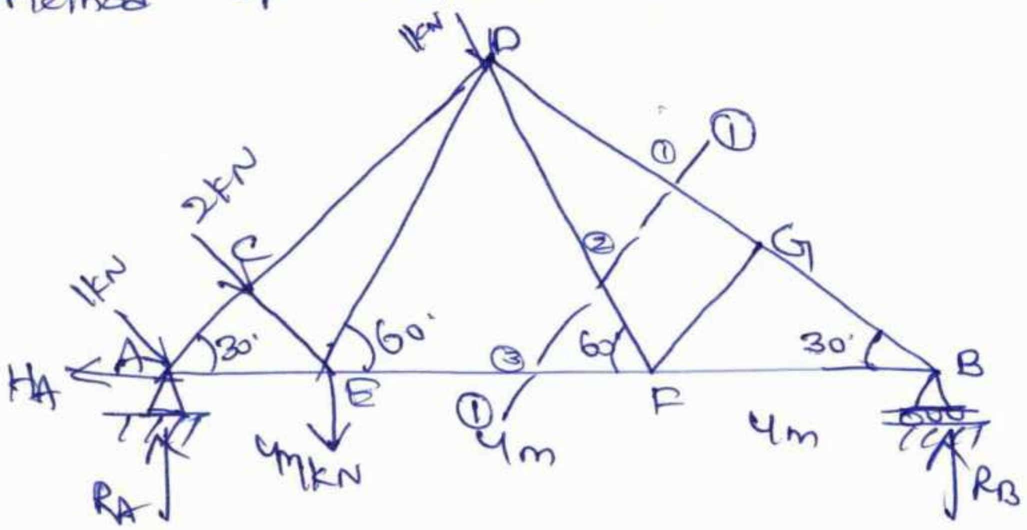
$F_3 \times 4 + F_2 \times 3 = (9 \times 3)$

$(7.5 \times 4) + (F_2 \times 3) = 27$

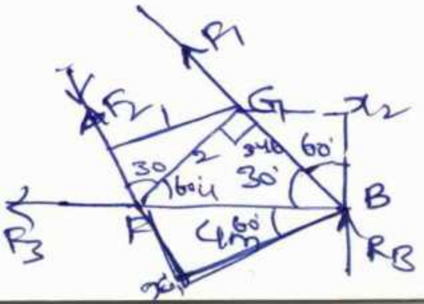
$F_2 = \frac{27 - 30}{3} = \frac{-3}{3} = -1\,kN$

$\therefore F_2 = 1\,kN$ (compr).

3) A truss of 12m span is loaded as shown in fig. Det the forces in the members DG, DF and EF, using method of sections.



Sol:



GB = 3.46
$x_2G = GB \sin 60$
$x_2G = 3$

$x_1B = FB \cos 60$
$= 4 \times \cos 60$
$= 2$

consider Equilibrium of the right part of the truss

Take moments about point B

$F_2 \times 2 = 0$

$\therefore F_2 = 0$

Taking moment about point G

$R_B \times 3 = (F_2 \times 1) + (F_3 \times 1.73)$

$F_3 = \frac{R_B \times 3}{1.73} = 2.56 \text{ kN}$ (Tension) $\quad (\because F_2 = 0)$

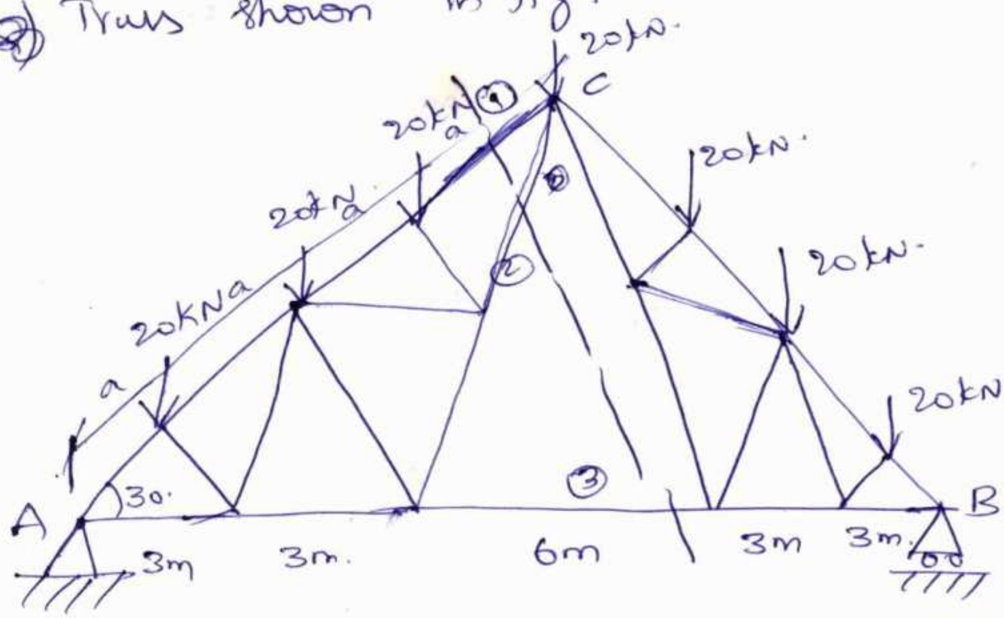
Taking moments about point F

$F_1 \times 2 + R_B \times 4 = 0$

$F_1 = -2.96 \text{ kN}$

$\therefore F_1 = 2.96 \text{ kN}$ (compression)

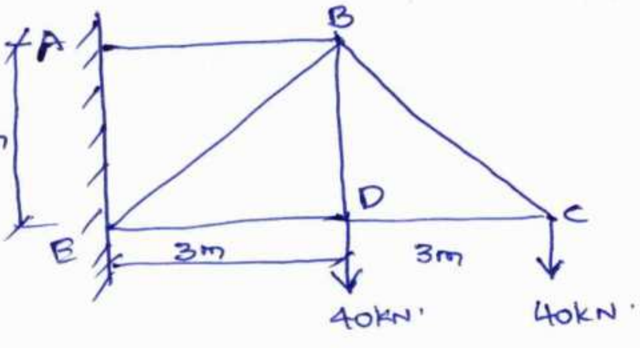
Q) Find the forces in the members ①, ② & ③ of Truss shown in fig.



Ans: $F_1 = 110kN$ (compr) $F_2 = 51.96kN$ (Tens) $F_3 = 69.28kN$ (Tens)

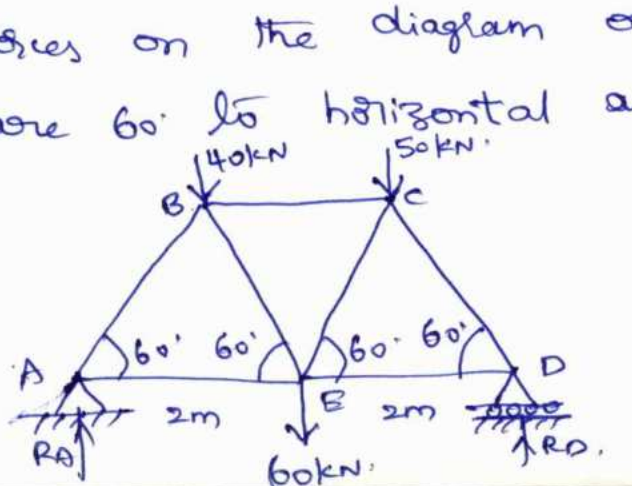
Method of Joints Problems:-

Q) Find the forces in all the members of the truss shown in fig. Tabulate the results.



Member	Magnitude of force in kN	Nature
AB	120	Tension
BC	56.57	"
CD	40	compression
DE	40	"
BE	113.14	"
BD	40	Tension.

Q) Determine the forces in all the members of the truss shown in fig and indicate the magnitude and nature of the forces on the diagram of the truss. All inclined members are 60° to horizontal and length of each member is 2m.



Ans: $R_A = 72.5kN$, $R_D = 77.5kN$

$F_{AB} = 83.72kN$ (compr) | $F_{BC} = 31.76kN$ (Tens)

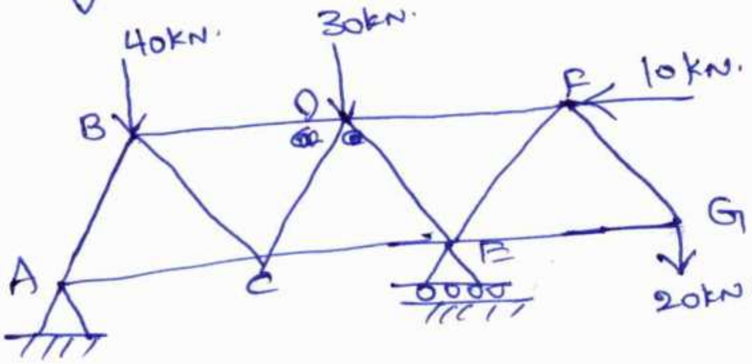
$F_{AE} = 41.86kN$ (Tens) | $F_{EC} = 60.62kN$ (compr)

$F_{DC} = 89.5kN$ (compr)

$F_{ED} = 44.75kN$ (Tens)

$F_{BE} = 37.53kN$ (Tens).

Q) Analyse the truss shown in fig. All members are 3m long.



$R_A = 31.83kN$, $R_E = 58.16kN$.

$F_{AB} = 36.75 KN$ (compr).

$F_{AC} = 8.38kN$ (Tens).

$F_{BC} = 9.44kN$ (compr).

$F_{CE} = 1.06kN$ (compr).

$F_{CD} = 9.44kN$ (Tens)

$F_{BD} = 13.66kN$ (compr)

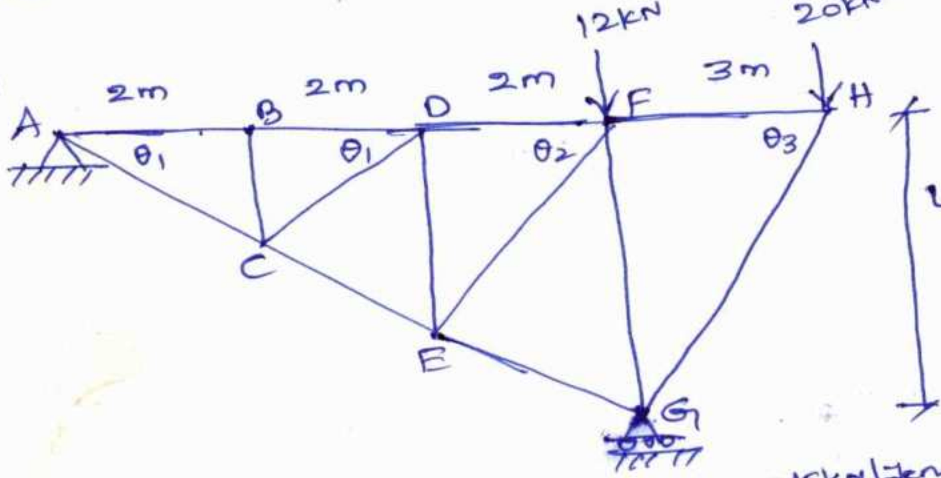
$F_{DE} = 44.08kN$ (compr)

$F_{DF} = 13.1kN$ (Tens)

$F_{FG} = 23.1kN$ (Tens).

$F_{EG} = 11.55kN$ (compr).

Q) Find the forces in all the members of the truss shown in fig.



[Ans: $\theta_1 = 33.69$

$\theta_2 = \theta_3 = 53.13'$

$R_G = 42kN$

$R_A = -10kN$ (downwards)

$F_{AB} = 15kN$ (Tens)

$F_{AC} = 18.03kN$ (comps)

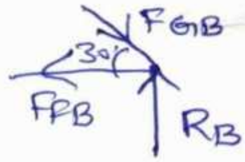
$F_{BC} = 0$

$F_{BD} = 15kN$ (Tens)

$F_{CD} = 0$, $F_{DE} = 0$, $F_{EF} = 0$, $F_{DF} = 15kN$ (Tens).

$F_{FG} = 12kN$. (comps) $F_{FH} = 15kN$ (Tens), $F_{GH} = 25kN$ (comps)

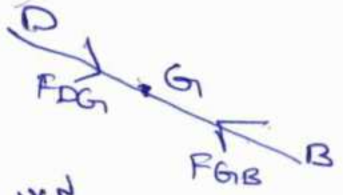
Joint 'B':-



$F_{GB} \sin 30 = R_B \qquad F_{GB} \cos 30 = F_{FB}$

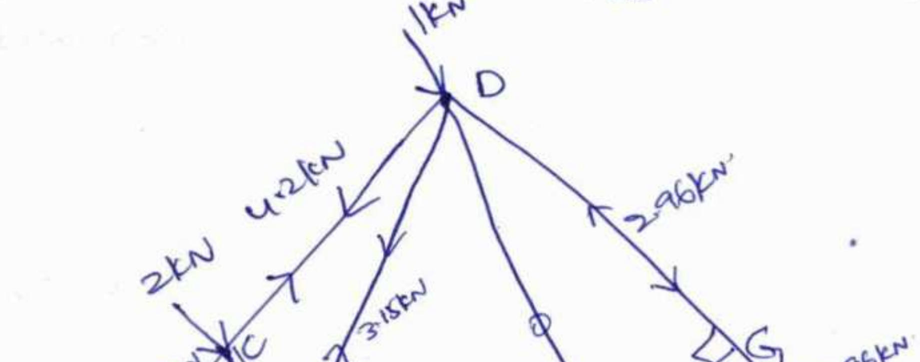
$F_{GB} = 2.96\,kN$ (comps) $\qquad$ ~~$F_{FB} = 2.56\,kN$ (Tens)~~

Joint 'G':-



$F_{DG} = F_{GB} = 2.96\,kN$

$\therefore F_{DG} = 2.96\,kN$ (comp).



Method of Sections:-

When the forces in a few members of a truss are to be determined, then the method of section is mostly used.

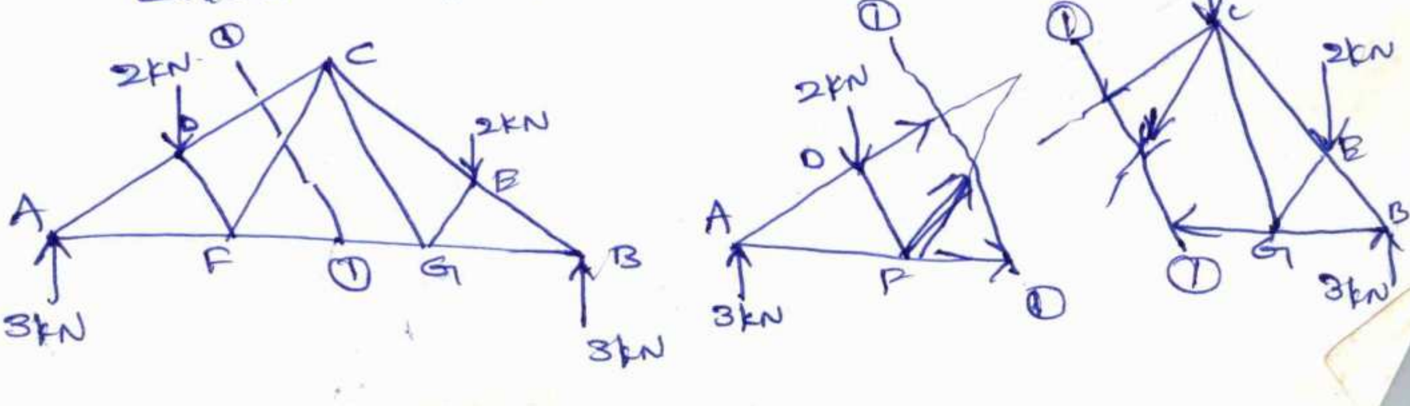
In this method, a section line is passed through the members, in which forces are to be determined as shown in fig.

The section line should be drawn in such a way that it does not cut more than three members in which the forces are unknown.

The part of the truss, on any one side of the ~~truss~~ section line, is treated as a free body in equilibrium under the action of external forces on that part and forces in the members cut by the section line.

The unknown forces in the members are thus determined by using equations of equilibrium

$\Sigma F_x = 0 \quad \Sigma F_y = 0, \quad \Sigma M = 0$



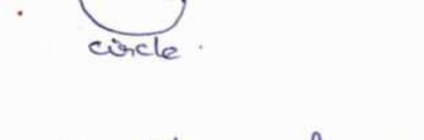
UNIT-IV

CENTRE OF GRAVITY & CENTROID

Centre of Gravity:- C.G of a body is the point through which total weight of the body acts. A body is having only one C.G for all positions of the body. It is represented by C.G or 'G'

Centroid:- The point at which the total area of a plane figure is assumed to be concentrated, is known as centroid of that area. The centroid is also represented by C.G or simply 'G'. The centroid and C.G are at the same point.

Centroid or Centre of Gravity of a simple plane figures:

(i) 

(ii)

Triangle

(iii) 

Rectangle (or) parallelogram

(iv) 

circle

C.G of plane figures by the method of moments:-

Fig shows a plane figure of total area A whose C.G is to be determined. Let this area A is composed of a number of small areas $a_1, a_2, a_3, a_4, \ldots$ etc.

$\therefore A = a_1 + a_2 + a_3 + a_4 + \dots$

Let x_1 = The dist of the C.G of area a_1 from axis OY

let x_2 = " " " " a_2 from axis OY

let x_3 = " " " " a_3 from "

let x_4 = " " " " a_4 from "

and so on

The moments of all small areas about the axis OY

$$= a_1x_1 + a_2x_2 + a_3x_3 + \dots \rightarrow (1)$$

Let G = C.G of total area A whose dist from the axis OY is $\bar{x}$.

Then moment of total area about OY = $A.\bar{x}$ $\rightarrow (2)$

The moment of all ~~forces~~ small areas about the axis OY must be equal to the moment of total area about the same axis.

$\because$ Hence equating eq(1) = eq(2).

$$a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \dots = A.\bar{x}$$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \dots}{A} \rightarrow (3)$$

where $A = a_1 + a_2 + a_3 + a_4 + \dots$

If we take the moments of the small areas about OX axis and also the moment of total area about the axis OX, we will get.,

3

$$\bar{Y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + \dots}{A} \rightarrow (4)$$

where $\bar{Y}$ = dist of C.G from axis OX

y_1 = dist of C.G of the area a_1 from OX.

y_2 = " " " a_2 from "

y_3 = " " " a_3 " "

⋮

C.G of plane figures by Integration method:-

Eq(3) & Eq(4) can be written as.

$$\bar{x} = \frac{\Sigma a_i x_i}{\Sigma a_i} \text{ and } \bar{Y} = \frac{\Sigma a_i y_i}{\Sigma a_i}$$

where $i = 1, 2, 3, 4, \dots$

The Value of i depends upon the no. of small areas.

If the no. of small areas are larger in number then the summations in the above equations can be replaced by integration.

$$\bar{x} = \frac{\int x^* dA}{\int dA} \qquad \text{where } \int x^* dA = \Sigma x_i a_i$$

$$\int dA = \Sigma a_i$$

$$\bar{Y} = \frac{\int y^* dA}{\int dA} \qquad \int y^* dA = \Sigma y_i a_i$$

where x^* = dist of C.G of area dA from axis OY

y^* = " " " " " " OX

C.G of a line:- The C.G of a line which may be straight or curve, is obtained by dividing the given line into a large number of small lengths as shown in fig. (4)

The C.G of a line is obtained by replacing ~~by~~ dL instead of dA in the above eq.

$$\bar{x} = \frac{\int x^* dL}{\int dL} \qquad \bar{Y} = \frac{\int y^* . dL}{\int dL}$$

where x^* = Dist of C.G of length 'dL' from Y-axis

y^* = " " " 'dL' " X-axis.

If the lines are straight, the the above eq are written as

$$\bar{x} = \frac{L_1 x_1 + L_2 x_2 + L_3 x_3 + \ldots}{L_1 + L_2 + L_3 + \ldots}$$

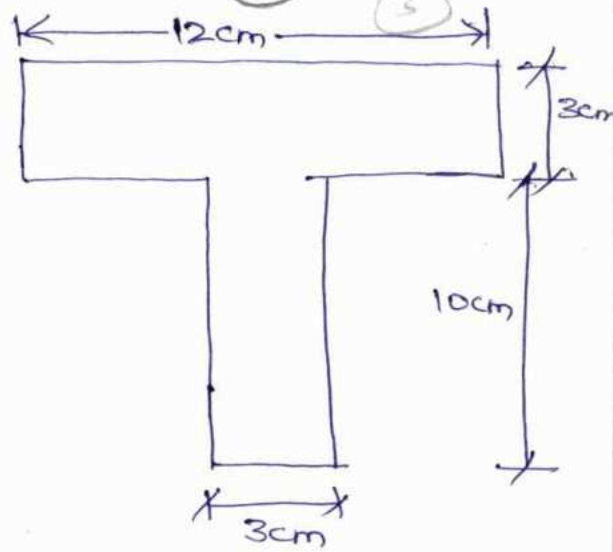
$$\bar{Y} = \frac{L_1 Y_1 + L_2 Y_2 + L_3 Y_3 + \ldots}{L_1 + L_2 + L_3 + \ldots}$$

Important points:-

(i) The axis OX and OY are reference axis

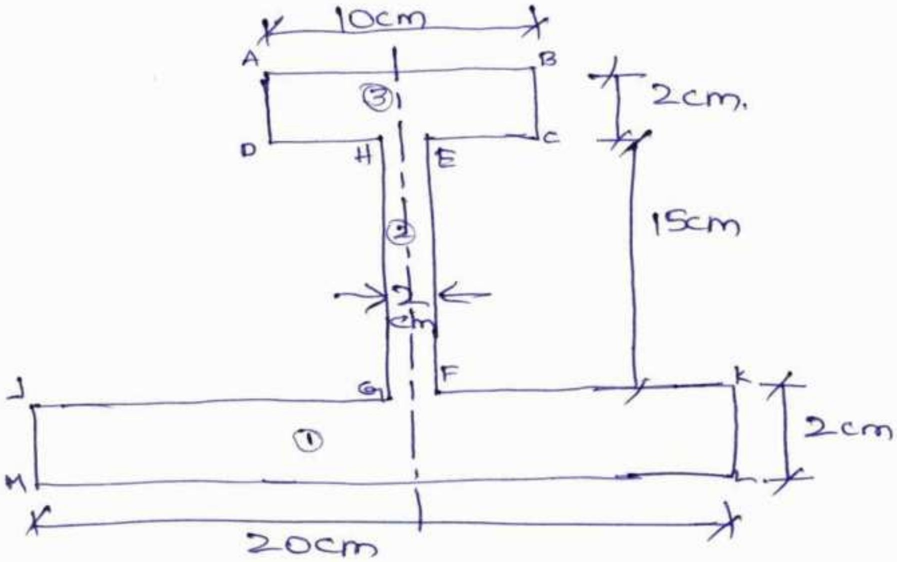
Problems

1) Find the C.G of the T-Section



[Ans: $\bar{y} = 8.545$ cm]

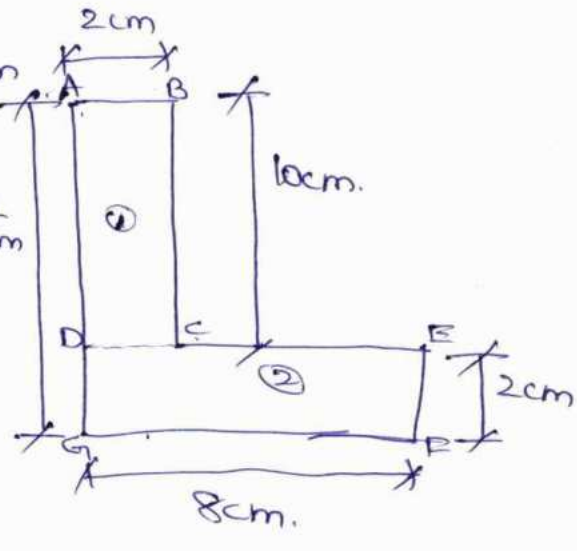
2) Find the C.G of the I-Section



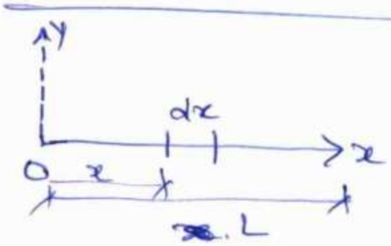
[Ans: $\bar{y} = 7.611$ cm]

3) Find the C.G of the L-Section

[Ans: $\bar{x} = 2.33$ cm
$\bar{y} = 4.33$ cm]



Centroid of a straight line:-



Consider a small length 'dx' at a dist x from 'o', then its first moment about the Y-axis is

$$dM_y = x \cdot dx.$$

∴ The first moment of the entire length about the Y-axis is

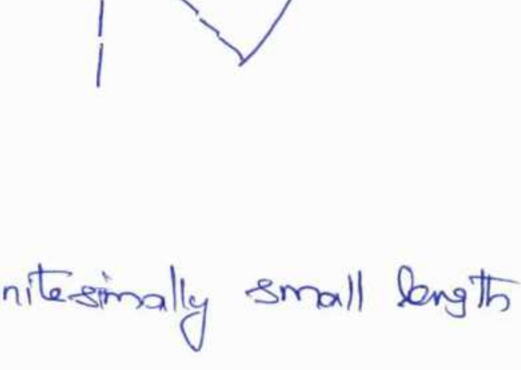
$$M_y = \int_0^L x\,dx = \frac{L^2}{2}$$

∴ x-coordinate of the centroid is given as

$$\bar{x} = \frac{M_y}{L} = \frac{L^2/2}{L} = \frac{L}{2}$$

∴ We can conclude that the centroid of a st line lies at the mid-point of the line.

Centroid of an arc of a circle:-



$$dL = R \cdot d\theta$$

∴ Total length of the arc is

$$L = \int_{-\alpha}^{\alpha} R\,d\theta = 2R\alpha$$

The first moment of the infinitesimally small length about the Y-axis is

$$dM_y = x \cdot dL = (R\cos\theta) \cdot R\,d\theta = R^2\cos\theta \cdot d\theta$$

Hence, first moment of the entire arc about the Y-axis is given by

(7) (w)

$$M_y = \int_{-\alpha}^{\alpha} R^2 \cos\theta \, d\theta$$

$$= +R^2 [\sin\theta]_{-\alpha}^{\alpha} = +R^2 . \sin\alpha . 2$$

$$= 2R^2 \sin\alpha$$

$\therefore$ x- coordinate of the centroid of the arc is given as

$$\bar{x} = \frac{M_y}{L} = \frac{2R^2 \sin\alpha}{L} = \frac{\cancel{2} R^{\cancel{2}} \sin\alpha}{\cancel{2} \cancel{R} \alpha} = \frac{R \sin\alpha}{\alpha}$$

Due to the symmetric of the arc about X-axis

$$\bar{Y} = 0$$

$$\therefore \boxed{\bar{x} = \frac{R\sin\alpha}{\alpha}}$$

For a semi circular arc, θ Varies from $-\pi/2$ to $\pi/2$ hence the location of ~~the~~ its centroid is obtained by substituting $\alpha = \pi/2$ in the above eq.

$$\bar{x} = \frac{R \sin \pi/2}{\pi/2} = \frac{2R}{\pi}$$

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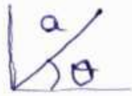
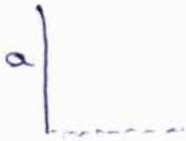
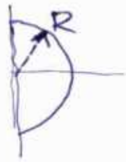
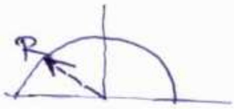
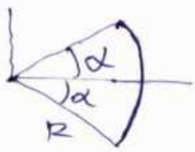
19,22

$$\therefore \boxed{\bar{x} = \frac{2R}{\pi}}, \quad \bar{y} = 0$$

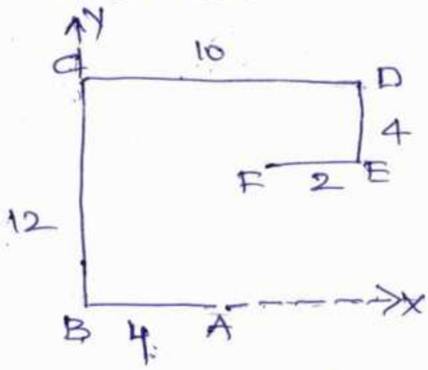
Note: The centroid of a circular arc due to symmetry about the X-axis and Y-axis must lie at the center of the circle.

Centroids of a simple curves:-

S.NO	Shape	Figure	length	$\bar{x}$	$\bar{Y}$
1.	straight line (inclined)		a	$\frac{a}{2}\cos\theta$	$\frac{a}{2}\sin\theta$
2.	straigh line (horizontal)		a	$\frac{a}{2}$	0
3.	straight line (Vertical)		a	0	$\frac{a}{2}$
4.	Semicircular arc		πR	$\frac{2R}{\pi}$	0
5.	Semicircular arc		πR	0	$\frac{2R}{\pi}$
6.	Quarter circular arc		$\frac{\pi R}{2}$	$\frac{2R}{\pi}$	$\frac{2R}{\pi}$
7.	Arc of a circle		$2R\alpha$	$\frac{R\sin\alpha}{\alpha}$	0

Problems

1) Find the centroid of a wire bent as shown in fig.



S.No	Elements	L_i	$\bar{x}_i$	$\bar{y}_i$	$L_i\bar{x}_i$	$L_i\bar{y}_i$
1	AB	4	2	0	8	0
2	BC	12	0	6	0	72
3	CD	10	5	12	50	120
4	DE	4	10	$12-(4/2)=10$	40	40
5	EF	2	$10-(\frac{2}{2})=9$	8	18	16

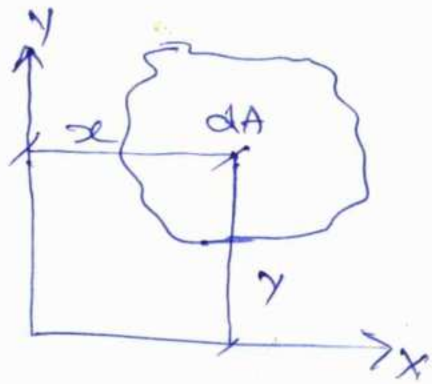
$$\bar{x} = \frac{\Sigma L_i x_i}{\Sigma L_i} = \frac{8+0+50+40+18}{4+12+10+4+2} = \frac{116}{32} = 3.625 \text{ cm.}$$

$$\bar{y} = \frac{\Sigma L_i y_i}{\Sigma L_i} = \frac{0+72+120+40+16}{4+12+10+4+2} = \frac{248}{32} = 7.75\text{cm}$$

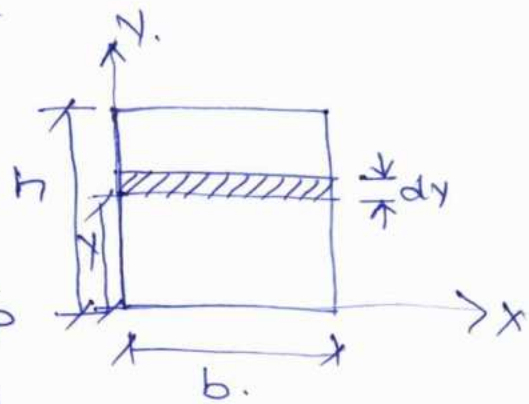
Centroid of an Area:-

$$\bar{x} = \frac{\int x^* \, dA}{\int dA}$$

$$\bar{y} = \frac{\int y^* \, dA}{\int dA}$$



Centroid of a Rectangle:-



consider a rectangle of base 'b' and height 'h'.

let us consider a small strip parallel to x-axis and $\perp^r$ to y-axis and of infinitesimally small thickness dy then its area is given as $dA = b.dy$.

Hence the area of the rectangle is

$$A = \int_0^h dA = \int_0^h b \cdot dy = b \cdot h$$

Moment of small area about the OX-axis

$$dM_x = y\,dA = y \cdot b \cdot dy$$

Moment of total area about the x-axis is

$$M_x = \int_0^h y\,dA = \int_0^h y(b \cdot dy) = \frac{b \cdot h^2}{2}$$

Hence the y-coordinate of the centroid of the rectangle is given as

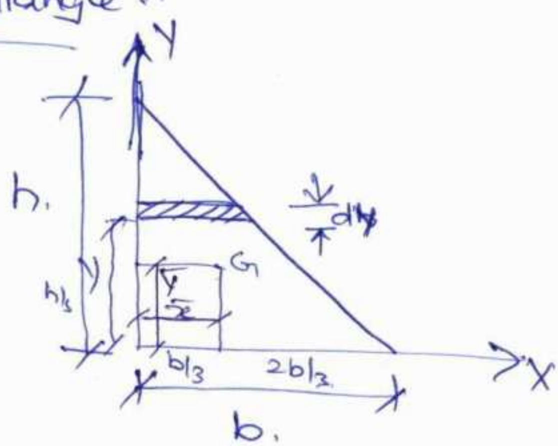
$$\bar{y} = \frac{M_x}{A} = \frac{bh^2}{2 \cdot b \cdot h} = \frac{h}{2}$$

In the same way

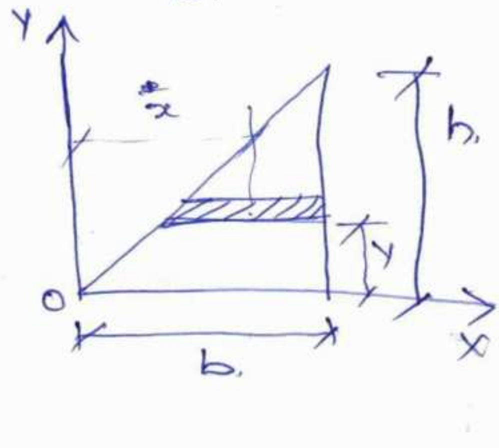
$$\bar{x} = \frac{b}{2}$$

Centroid of a Right-Angled triangle :-

$$\bar{x} = \frac{b}{3} \quad \bar{y} = \frac{h}{3}$$



$$\bar{x} = \frac{2b}{3} \quad \bar{y} = \frac{h}{3}$$

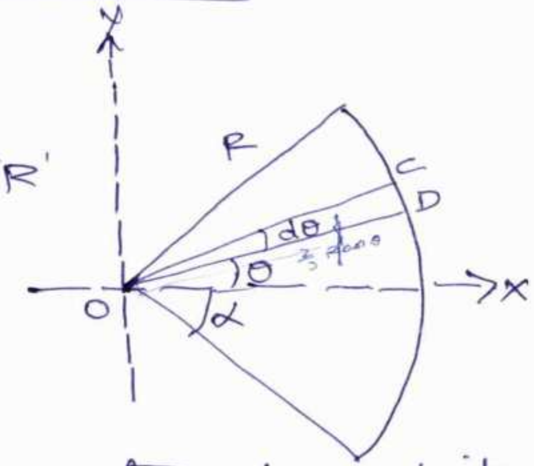


Controid of a triangle in General:- (11)

a) Controid of area of a circular sector:-

Q) " " " Parabola

Controid of area of a circular sector:-



Let us consider a circular sector of a circle of radius 'R' and angle 2α and symmetric about X-axis.

OCD can be considered as a triangle and its area is then given as

$$dA = \frac{1}{2} \cdot R \cdot R d\theta = \frac{1}{2} R^2 \cdot d\theta$$

The controid of this $\triangle^{le}$ lies at a distance of $\frac{2}{3}R$ from O. Hence, the x and Y coordinates of the controid are

$$x = \frac{2}{3} R\cos\theta \qquad Y = \frac{2}{3} R\sin\theta$$

Area of the entire clar sector is obtained by integrating the expression for dA between limits

$$A = \int_{-\alpha}^{\alpha} \frac{R^2}{2} d\theta = R^2\alpha$$

Taking the first moment of the $\triangle^{le}$ OCD about OY axis

$$dM_Y = x \cdot dA = \frac{2}{3} R\cos\theta \cdot \frac{R^2}{2} \cdot d\theta$$

$dM_y = \frac{R^3}{3} \cancel{R^3} \cos\theta \, d\theta$

∴ First moment of the entire area about the Y-axis

$$M_y = \int_{-\alpha}^{\alpha} x \, dA = \int_{-\alpha}^{\alpha} \frac{2}{3} R\cos\theta \, \frac{R^2}{2} \, d\theta$$

$$= \int_{-\alpha}^{\alpha} \frac{R^3}{3} \cos\theta \, d\theta$$

$$= \frac{1}{3} R^3 \cdot \left[\sin\theta\right]_{-\alpha}^{\alpha}$$

$$= \frac{R^3}{3} \cdot 2\sin\alpha$$

$$= \frac{2}{3} R^3 \sin\alpha$$

∴, the x-coordinate of the centroid is

$$\bar{x} = \frac{M_y}{A} = \frac{\frac{2}{3} R^3 \sin\alpha}{R^2 \alpha} = \frac{2}{3} \frac{R \sin\alpha}{\alpha}$$

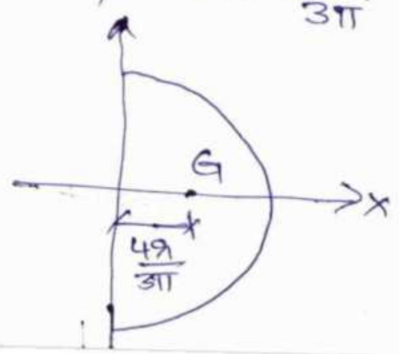
$$\bar{y} = 0$$

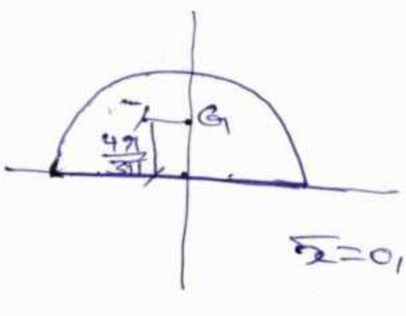
For a semicircular area, we know that θ varies from $-\pi/2$ to $\pi/2$. Hence, its centroid is obtained by substituting $\alpha = \pi/2$ in the above expression for $\bar{x}$.

$$\bar{x} = \frac{4R}{3\pi} \text{ and } \bar{y} = 0$$

$\bar{x} = \frac{2}{3} \cdot \frac{R \cdot \sin \pi/2}{\pi/2}$

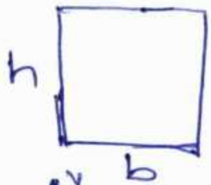
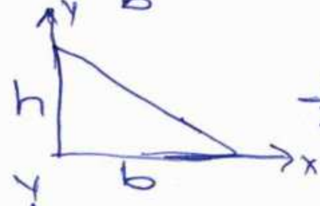
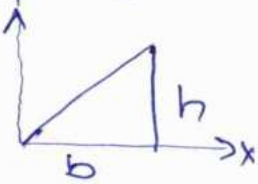
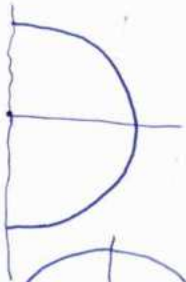
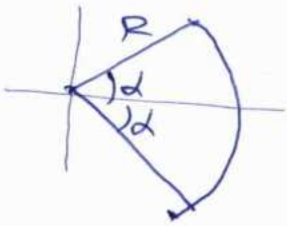
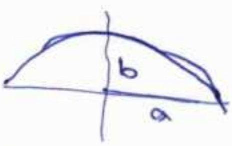
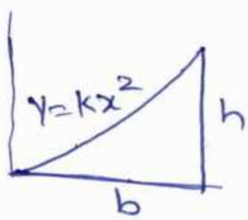
$= \frac{4}{3} \cdot \frac{R}{\pi}$





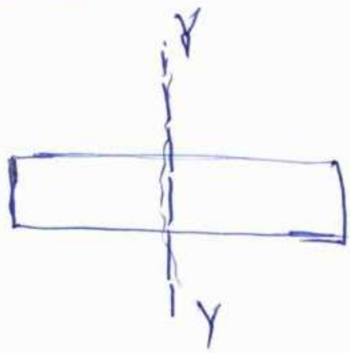
$\bar{x} = 0, \; \bar{y} = \frac{4R}{3\pi}$

Centroid ~~of a parabola~~:-

S.No	Shape	Figure	Area	$\bar{x}$	$\bar{y}$
1.	Rectangle		$b.h$	$\frac{b}{2}$	$\frac{h}{2}$
2.	Right-angled $\triangle$le		$\frac{1}{2}.b.h$	$\frac{b}{3}$	$\frac{h}{3}$
3.	Right-angled $\triangle$le		$\frac{1}{2}.b.h$	$\frac{2b}{3}$	$\frac{h}{3}$
4.	Semicircle		$\frac{\pi r^2}{2}$	$\frac{4r}{3\pi}$	0
5.	Semicircle		$\frac{\pi R^2}{2}$	0	$\frac{4r}{3\pi}$
6.	Quadrant		$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
7.	Circular sector		αR^2	$\frac{2R\sin\alpha}{3\alpha}$	0
8.	Semi-Elliptical area		$\frac{\pi ab}{2}$	0	$\frac{4b}{3\pi}$
9.	Parabola		$\frac{1}{3}.b.h$	$\frac{3b}{4}$	$\frac{3h}{10}$

Axis of symmetry:-

If an area has an axis of symmetry (Say Y-Y axis) then its centroid will lie on that axis.

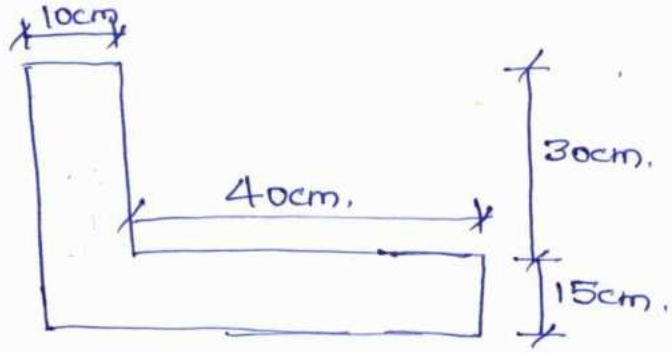


Problems

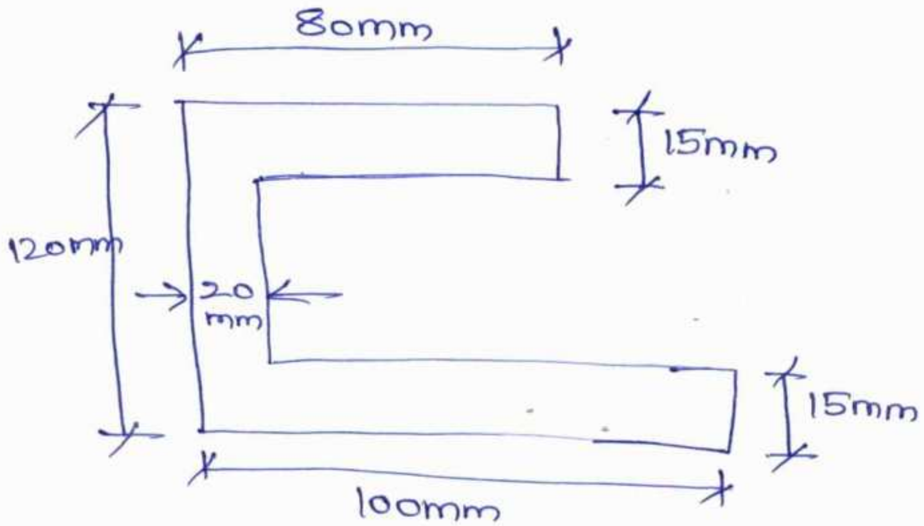
1) Find the centroid of the plain lamina

$\bar{x} = 19.29\,cm.$

$\bar{Y} = 13.93\,cm.$

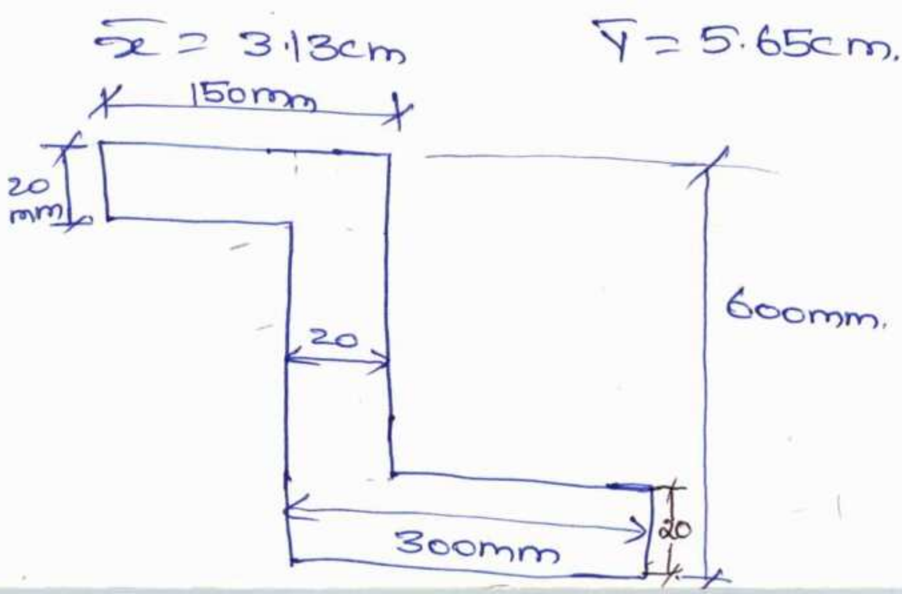


2).



$\bar{x} = 3.13\,cm$ $\bar{Y} = 5.65\,cm.$

3).



$\bar{x} = 17.19\,cm.$

$\bar{Y} = 25.69\,cm.$

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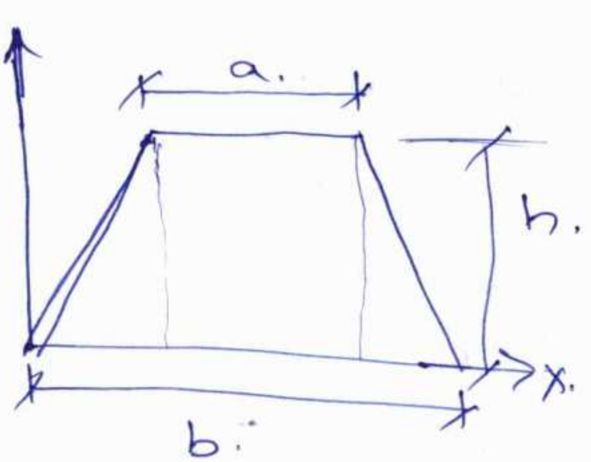
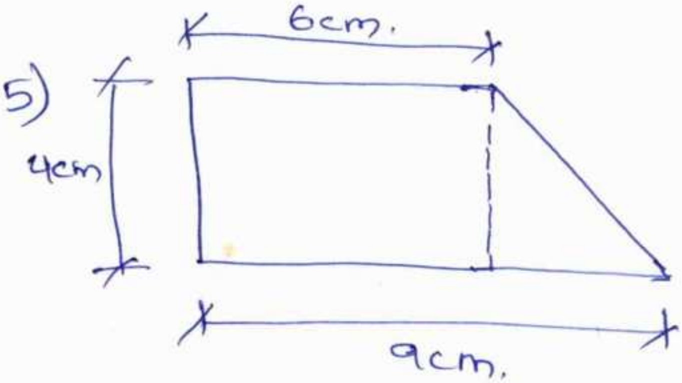
4)



$\bar{x} = \frac{b}{2}$

$\bar{Y} = \frac{h}{3}\left[\frac{b+2a}{b+a}\right]$

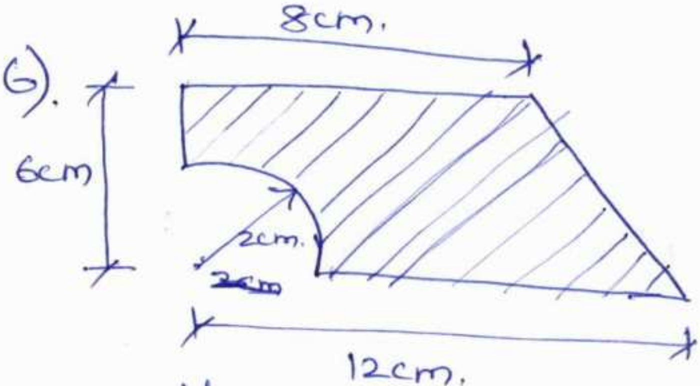
5)



$\bar{x} = 3.8\text{cm}$

$\bar{Y} = 1.87\text{cm}$

6)



$\bar{x} = 5.3\text{cm}$

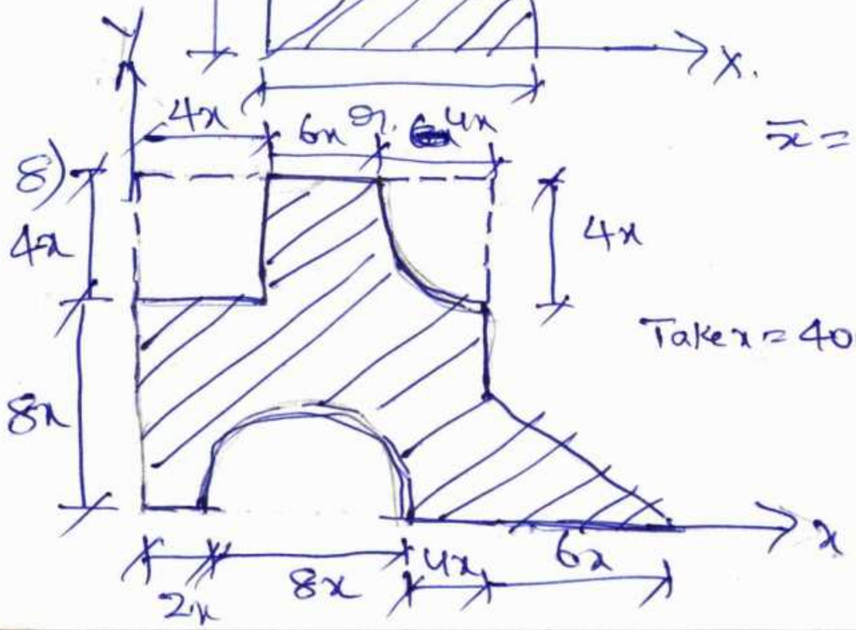
$\bar{Y} = 2.91\text{cm}$

7)

$\bar{x} = \frac{2r}{\pi} = 0.637r$

$\bar{Y} = 0.349r$

8)



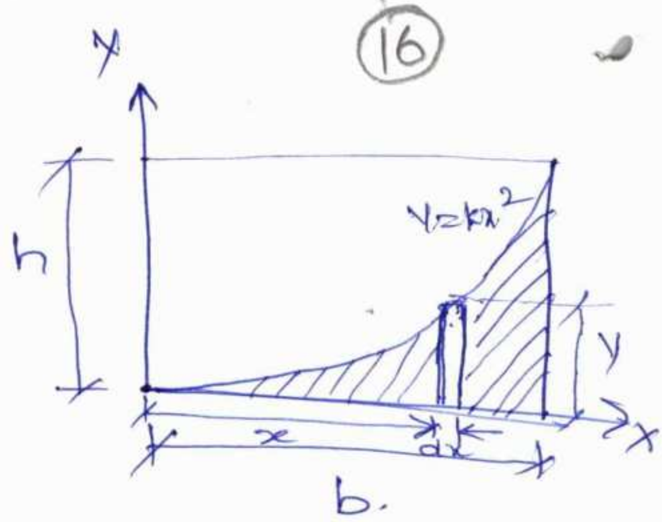
$\bar{x} = 8.159x$, $\bar{Y} = 5.47x$

$\bar{x} = 326\text{mm}$

Take $x = 40\text{mm}$ $\bar{Y} = 219.12$

Centroid of a parabola:-

Consider a shaded area bounded by a parabola of equation $y = kx^2$, [X-axis and line $x = b$].



$$dA = y \cdot dx$$

$$y = mx^2$$

$$m = \frac{y}{x^2} = \frac{h}{b^2} \rightarrow (1).$$

$$\therefore m = \frac{h}{b^2}$$

$$y = \frac{h}{b^2} \cdot x^2$$

$$dA = \frac{h}{b^2} \cdot x^2 dx$$

$$A = \int_0^b dA = \int_0^b \frac{h}{b^2} \cdot x^2 \cdot dx = \frac{h}{b^2} \left[\frac{x^3}{3}\right]_0^b$$

$$= \frac{h}{b^2} \cdot \frac{b^3}{3} = \frac{hb}{3}$$

$$\therefore A = \frac{hb}{3}$$

$$dM_y = \int dA \cdot x = \int y \cdot x \cdot dx$$

$$M_y = \int_0^b x \frac{h}{b^2} \cdot x^2 dx.$$

$$= \int_0^b \frac{h}{b^2} x^3 dx$$

$$= \frac{h}{b^2} \left[\frac{x^4}{4}\right]_0^b$$

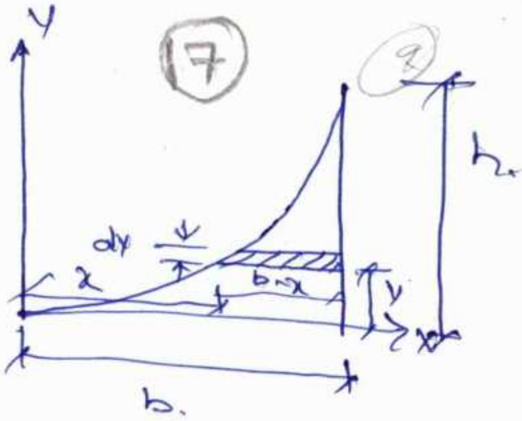
$$= \frac{h}{b^2} \cdot \frac{b^4}{4}$$

$$M_y = \frac{hb^2}{4}$$

$$\bar{x} = \frac{M_y}{A} = \frac{hb^2/4}{bh/3}$$

$$\bar{x} = \frac{3b}{4}$$

17



$dA = (b-x)\,dy$

$A = \int_0^h (b-x)\,dy$

$\cancel{A = (b-x)h}$

$y = \frac{h}{b^2} \cdot x^2$

$x = \sqrt{\frac{yb^2}{h}}$

$dM_x = \int dA \cdot y$

$M_x = \int_0^h dA \cdot y = \cancel{\int_0^h \frac{h}{b^2} \cdot x^2 (b-x)\,dy}$

$\cancel{= \frac{h}{b^2} \int_0^h \left(\frac{yb^2}{h}\right)\left(b - \sqrt{\frac{yb^2}{h}}\right)dy}$ $= \int_0^h y \cdot (b-x)\,dy$

$\cancel{= \frac{h}{b^2} \int_0^h \left[\frac{b^3 \cdot y}{h} - \left(\frac{yb^2}{h}\right)^{3/2}\right]dy}$ $= \int_0^h y \cdot \left(b - \sqrt{\frac{yb^2}{h}}\right)dy$

$= \left[\frac{by^2}{2} - \frac{b}{h^{1/2}} \frac{y^{5/2}}{5/2}\right]_0^h$

$\cancel{= \frac{h}{b^2} \left[\frac{b^3}{h}\frac{y^2}{2} - \frac{b^3}{h^{3/2}} \cdot \frac{y^{5/2}}{5/2}\right]_0^h}$ $= \left[\frac{bh^2}{2} - \frac{2b \cdot h^2}{5}\right]$

$\cancel{= \frac{h}{b^2}\left[\frac{b^3 \cdot h}{2} - \frac{2b^3 \cdot h^2}{5}\right] = \frac{h}{b^2} b^3 h \left[\frac{1}{2} - \frac{2h}{5}\right]}$

$\cancel{M_x = h^2 b \left(\frac{1}{2} - \frac{2h}{5}\right)}$ $= \frac{5bh^2 - 4bh^2}{10}$

$\cancel{\bar{y} = \frac{M_x}{A} = \frac{bh^2 \left[\frac{1}{2} - \frac{2h}{5}\right]}{\left(b - \sqrt{\frac{yb^2}{h}}\right) h}}$ $M_x = \frac{bh^2}{10}$

$\bar{y} = \frac{M_x}{A} = \frac{bh^2/10}{bh/3}$

$$\bar{y} = \frac{bh^2/10}{bh/3} = \frac{3h}{10}$$

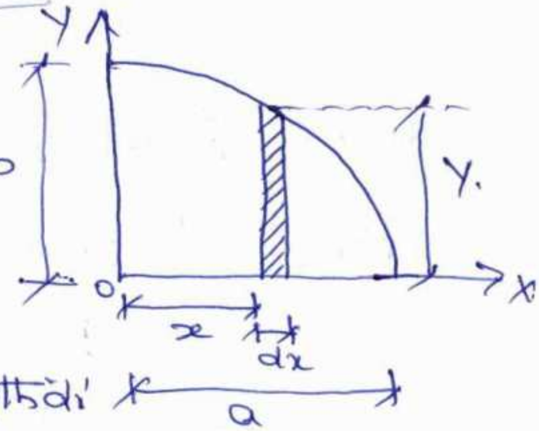
(18)

$$\therefore \boxed{\bar{x} = \frac{3b}{4} \qquad \bar{y} = \frac{3h}{10}}$$

Centroid of quadrant of an Ellipse :-

Eq of ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$y = \sqrt{b^2\left[1 - \frac{x^2}{a^2}\right]}$$



consider a small strip of length 'dx' and height y. at a distance x from 'O'

$$dA = y \cdot dx$$

$$A = \int_0^a dA = \int_0^a y \cdot dx = \int_0^a \frac{b}{a}\sqrt{a^2 - x^2} \cdot dx$$

$$= \cancel{\frac{b}{a}\sqrt{a^2-x^2}} = \frac{b}{a}\left[\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= \frac{\pi ab}{4}$$

$$dM_y = x \cdot dA = x \cdot y \cdot dx = x \cdot \frac{b}{a}\sqrt{(a^2-x^2)}\, dx$$

$$= \frac{-b}{a}\left[\frac{(a^2-x^2)^{3/2}}{3}\right]_0^a = \frac{a^2 b}{3}$$

$\therefore$ x-coordinate of the centroid of the quadrant of the ellipse is given as

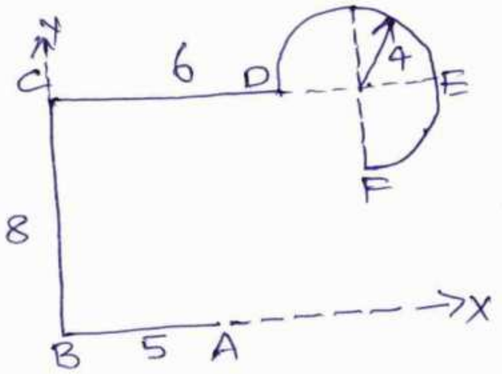
$$\bar{x} = \frac{M_y}{A}$$

$$= \frac{a^2 b/3}{\pi a b/4} = \frac{4a}{3\pi}$$

Similarly $\bar{Y} = \frac{4b}{3\pi}$

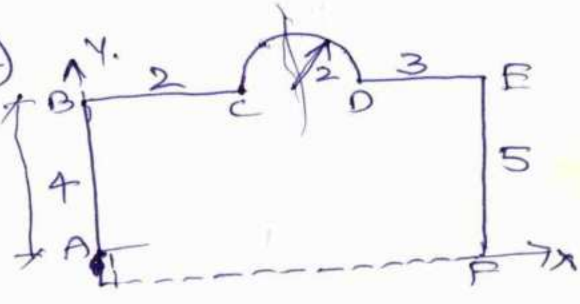
Problems

1)



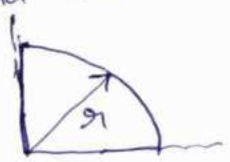
$\bar{x} = 6.2\text{cm}$ $\bar{Y} = 6.5\text{cm}$

2)

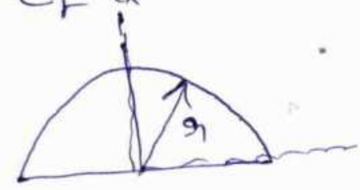


$\bar{x} = 4.7\text{cm}$ $\bar{Y} = 4.4\text{cm}$

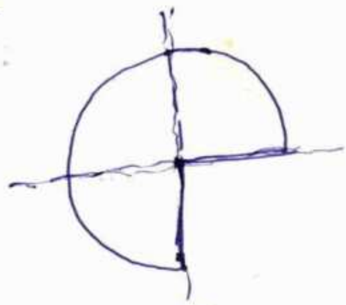
3) Find the centroid of a bent wire



$\bar{x} = \bar{Y} = \frac{3r}{4+\pi}$



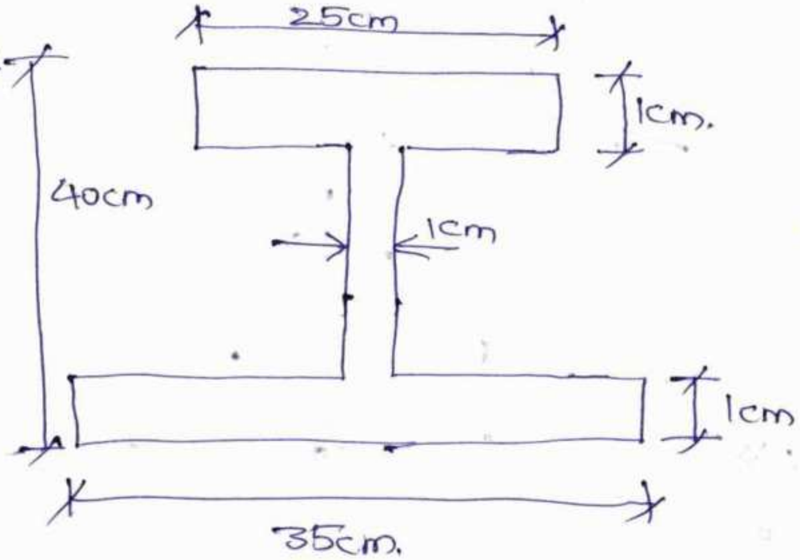
$\bar{x} = 0, \bar{Y} = \frac{2r}{r+\pi}$



$\bar{x} = \frac{-r}{3\pi+4}$

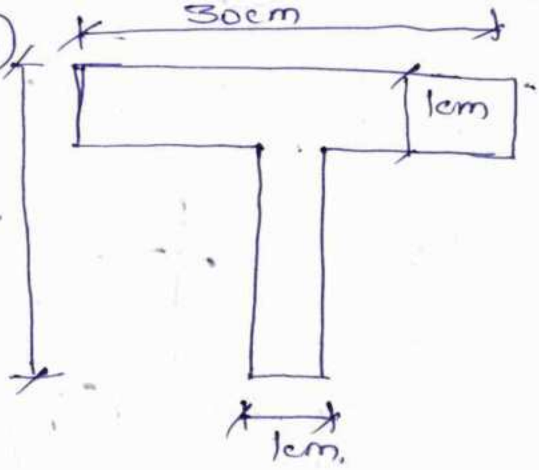
$\bar{Y} = \frac{r}{3\pi+4}$

4)



$\bar{x} = 0$ $\bar{Y} = 18\text{cm}.$

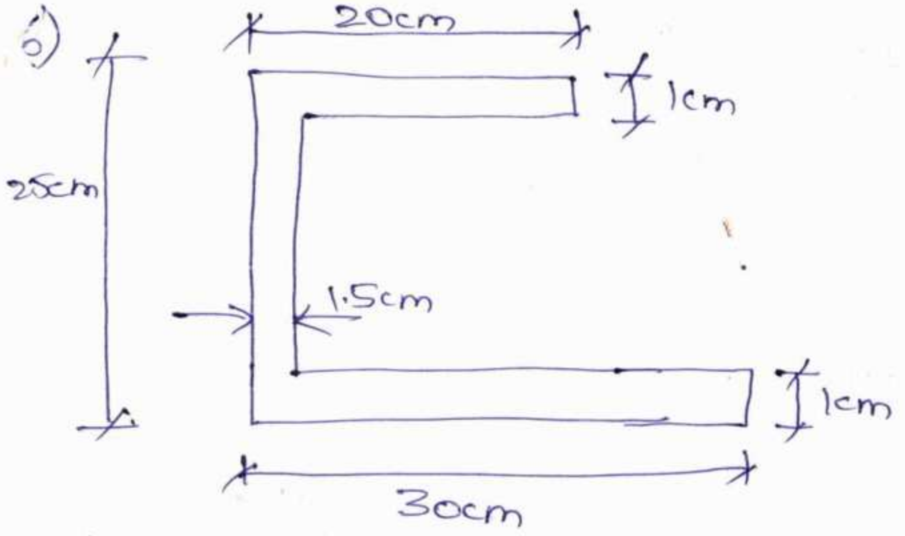
5)



$\bar{x} = 0$

$\bar{Y} = 28.2\text{cm}$

6)

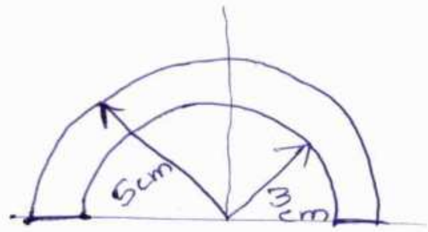


20

$\bar{x} = 8cm$

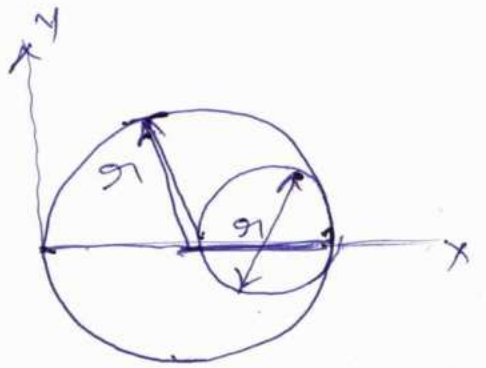
$\bar{Y} = 11.1cm.$

7)



$\bar{x} = 0, \bar{Y} = 2.6cm.$

8)

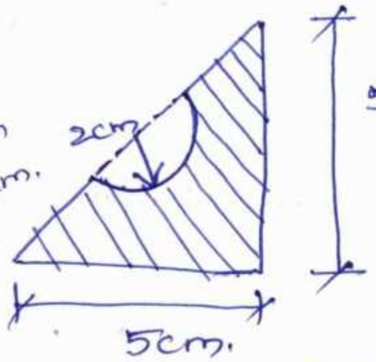


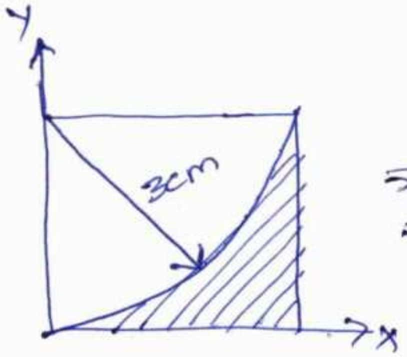
$\bar{x} = 0.839$
$\bar{Y} = 0.$

$\bar{x} = 4.17cm \quad \bar{Y} = 0cm.$

9)

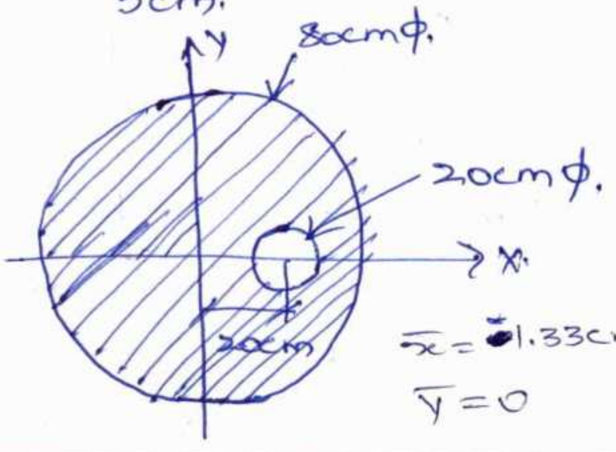
$\bar{x} = 3.6cm$
$\bar{Y} = 1.4cm.$





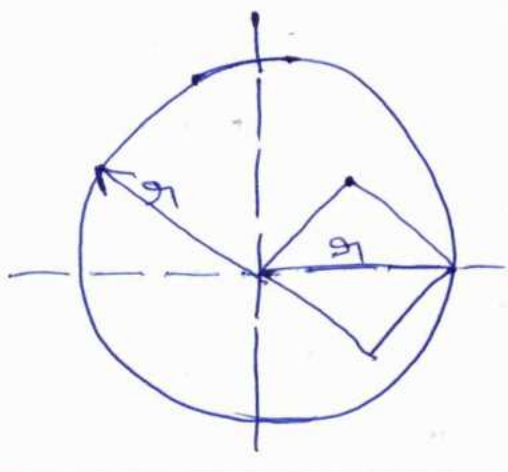
$\bar{x} = 2.33cm$
$\bar{Y} = 0.66cm.$

10.



$\bar{x} = 1.33cm$

$\bar{Y} = 0$



Theorems of Pappus and Guldinus

Pappus and Guldinus are two mathematicians developed two theorems.

Theorem: 1:- "The area of a surface generated by revolving a plane curve about a non-intersecting axis in the plane of the curve is equal to the product of the length of the curve and distance travelled by the centroid G of the curve during revolution."

proof:- consider a curve of length L and let it be revolved about the ox axis through 2π radians. Then an infinitesimally small element of length dL will generate a hoop of area $2\pi y\,dL$.

$\therefore$, the total surface area generated by the curve is given as

$$A = \int 2\pi\, y \cdot dL$$

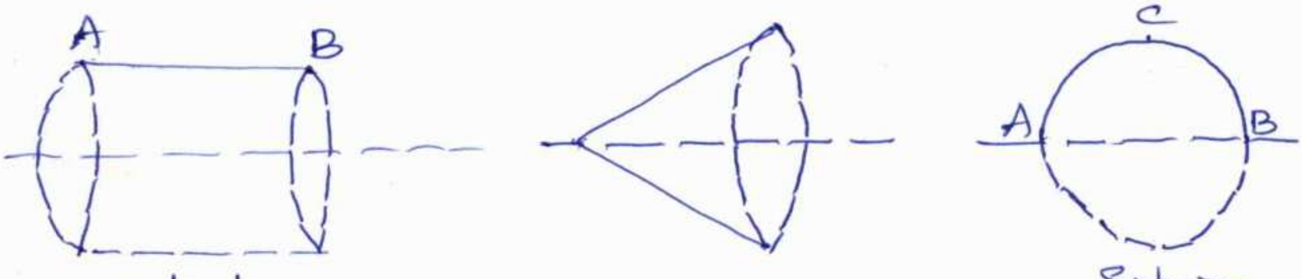
$$= 2\pi \int y \cdot dL = 2\pi\, \bar{y} \cdot L \quad \left(\because \bar{y} = \frac{\int y\,dL}{\int dL}\right)$$

Depending upon the generating curve, the surface area generated are differented as shown below

A st line parallel to the axis of revolution generates surface area of a cylinder;

An inclined ~~plane~~ line with one end touching the axis of revolution generates surface area of a cone

A semicircular arc with the ends touching the axis of revolution generates surface area of a sphere and a ~~cos~~



cylinder   cone   Sphere

Theorem 2:- The volume of a solid generated by revolving a plane area about a non-intersecting axis in its plane is equal to the product of the area and length of the path travelled by centroid 'G' of the area during the revolution about the axis.

$$V = \int 2\pi \cdot y \cdot dA$$

$$V = 2\pi \int y \cdot dA \qquad \left[\bar{y} = \frac{\int y\,dA}{A} \right]$$

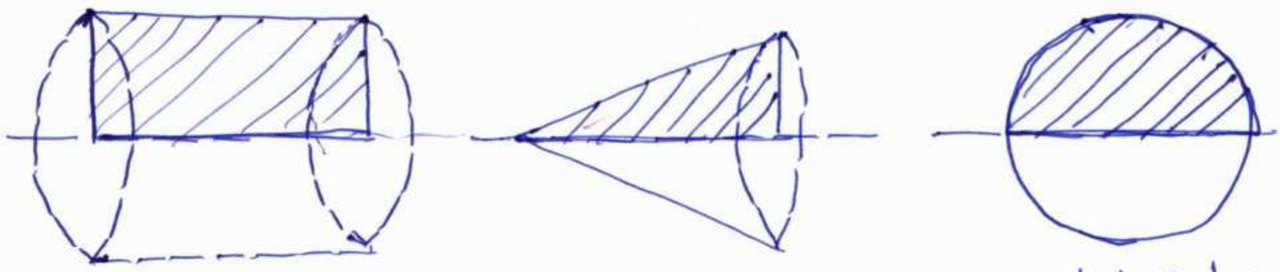
$$V = 2\pi \cdot \bar{y} \cdot A$$

Depending upon the generating area, the volumes generated are differented as shown below.

A rectangular area when rotated about one of its sides generates volume of a cylinder.

A right-angled triangle when rotated about a side other than the hypotenuse generates volume of a cone.

A semicircular area when rotated about its diameter generates volume of a sphere.



Solid cylinder Solid Cone solid sphere

Problems

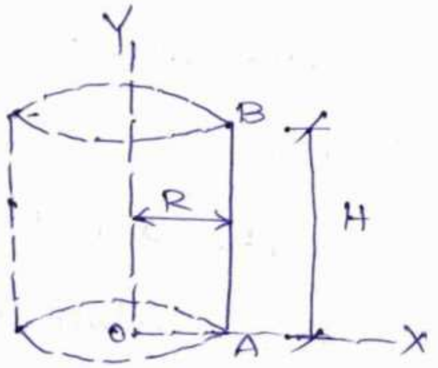
1) Determine the surface area and volume of a cylinder using the Pappus and Guldinus Theorems

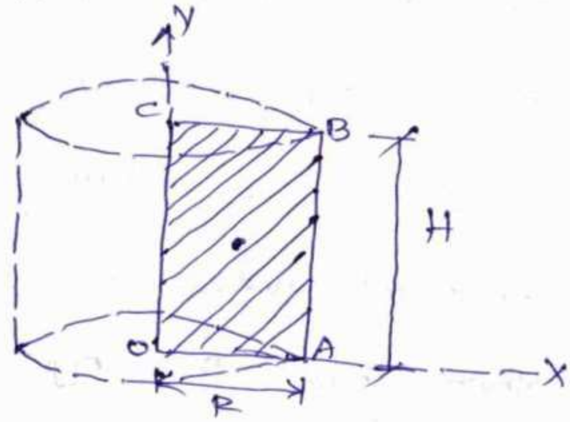
Sol: consider a st line AB of length H, parallel to the Y-axis at a distance R from the Y-axis

Revolving this line about the Y-axis through 360° will generate surface area of a cylinder of radius R and Height H.

$A = (\text{length of curve})\, \bar{x} \cdot \theta$

$= H \cdot R \cdot 2\pi = 2\pi \cdot R \cdot H$





Similarly considering a rectange OABC and rotating it about the Y-axis will generate a solid cylinder. Its volume can be determined by using Theorem II as

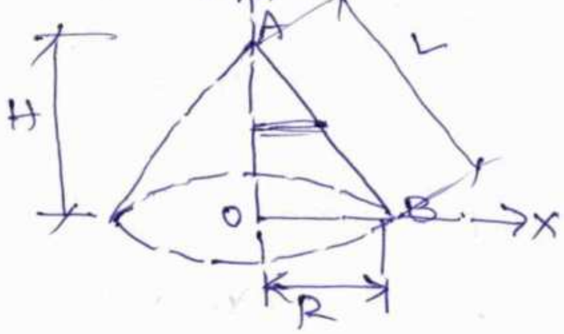
$V = (\text{area of the plane}) \cdot \bar{x} \cdot \theta$

$= HR \cdot \frac{R}{\cancel{2}} \cdot \cancel{2}\pi$

$= HR^2\pi$

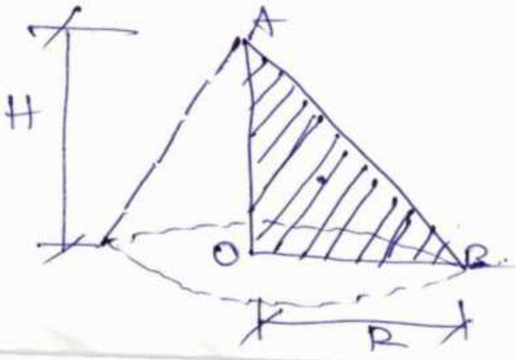
2) Determine the surface area and Volume of a cone using Pappus and Guldinus Theorem

$\frac{R}{H} = \bar{x}$



$A = L \cdot \frac{R}{2} \cdot 2\pi$

$A = \pi R \cdot L$



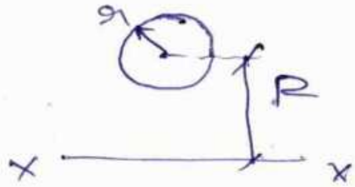
$V = \left(\frac{1}{\cancel{2}} \times R \times H\right) \cdot \frac{R}{3} \cdot \cancel{2}\pi$

$V = \frac{R^2 \cdot H \cdot \pi}{3} = \frac{1}{3}\pi R^2 H.$

$\theta_y = \cos^{-1}\left(\frac{R_y}{R}\right) = \cos^{-1}\left(\frac{-215.64}{4337.38}\right) = 92.85^\circ$ 2b (13)

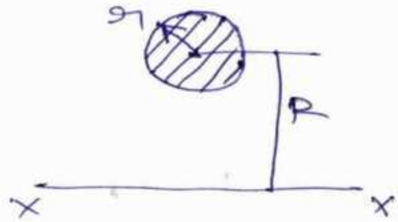
$\theta_z = \cos^{-1}\left(\frac{R_z}{R}\right) = \cos^{-1}\left(\frac{-4326.37}{4337.38}\right) = 175.91^\circ$

3) Determine the surface area and Volume of a torus of radius ~~R~~ r. obtained by rotating a circle of radius R about an axis away from the circle



$A = (\text{length of the curve}) \cdot \bar{y} \cdot 2\pi$

$= 2\pi r \cdot R \cdot 2\pi = 4\pi^2 \cdot r \cdot R$

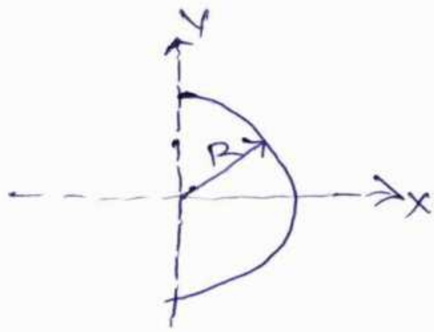


$V = (\text{Area of the curve}) \times \bar{y} \times 2\pi$

$= \pi r^2 \times R \times 2\pi$

$= 2R \cdot r^2 \cdot \pi^2$

4) Determine the surface area and volume of a sphere using the Pappus and Guldinus theorems

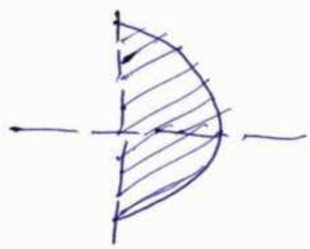


$A = (\pi R) \times \left(\frac{2R}{\pi}\right) \times 2\pi$

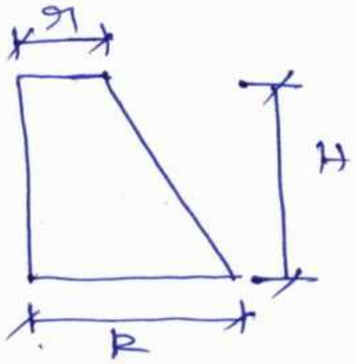
$A = 4\pi R^2$

$V = \frac{\pi R^2}{2} \times \frac{4R}{3\pi} \times 2\pi$

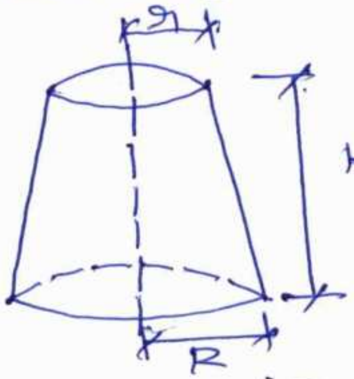
$V = \frac{4}{3}\pi R^3.$



5) Determine the volume of a frustum of a cone of base radius R, top radius r and height H.

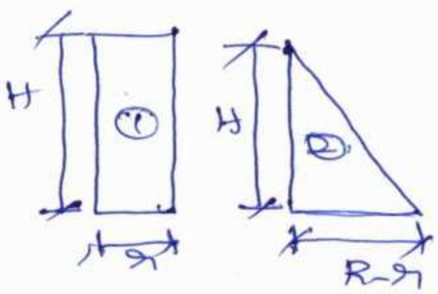


sub R = 5cm
$r = 3cm$.
$H = 8cm$.



$V = \left[Rr + \frac{1}{2}(R-r)H\right]$
$= \left(\frac{r}{2} + \frac{R-r}{3}\right)$
2π

V = (area of the plane) $\times \bar{x} \times 2\pi$



$$A = a_1 + a_2 = (rH) + (R-r)\cdot\frac{1}{2}\cdot H =$$
$$= Hr + \frac{HR}{2} - \frac{Hr}{2}$$
$$= \frac{Hr + HR}{2} = \frac{H(R+r)}{2}$$

$A_1 = rH$

$A_2 = \frac{(R-r)}{2}\cdot H$

$x_1 = \frac{r}{2}$

$x_2 = r + \left(\frac{R-r}{3}\right) = \frac{(R+2r)}{3}$

$$\bar{x} = x_2 = \frac{r}{2} + \left(\frac{R-r}{3} + r\right)$$
$$= \frac{r}{2} + \frac{R}{3} + \frac{2r}{3} = \frac{7}{6}r + \frac{R}{3}$$
$$= \frac{7r + 2R}{6} =$$

$\frac{2}{3} + \frac{1}{2}$

$\frac{4+3}{6} = \frac{7}{6}$

$$V = \cancel{\frac{H}{2}(R+r)\cdot\frac{(7r+2R)}{6} \times 2\pi}$$

$x_1 = \frac{r}{2}$ $\quad x_2 = \frac{R-r}{3} + r = \frac{R}{3} - \frac{r}{3} + r = \frac{R}{3} + \frac{2r}{3}$

$= \frac{1}{3}(R+2r)$

$$\bar{x} = \frac{rH\cdot\frac{r}{2} + \frac{(R-r)}{2}\cdot H\cdot\left(\frac{R+r}{3}\right)}{rH + (R-r)\frac{1}{2}H}$$

$$= \frac{H}{2}\left\{r^2 + \frac{(R-r)^2}{3}\right\} \Big/ H\left[r + \frac{R}{2} - \frac{r}{2}\right]$$

$$= \frac{H}{2}\left[3r^2 + R^2 + r^2 - 2Rr\right] \Big/ \frac{H}{2}\left[R + r\right]$$

$$= \frac{r^2 + R^2 - 2Rr}{R+r} \cdot \frac{1}{3}$$

$$\bar{x} = \frac{rH \cdot \left(\frac{r}{2}\right) + \left(\frac{R-r}{2}\right) \cdot H \cdot \left(\frac{R+2r}{3}\right)}{rH + \left(\frac{R-r}{2}\right) H}$$

$$= \left[\frac{Hr^2}{2} + \frac{H}{6}\left(R^2 + 2Rr - Rr - 2r^2\right)\right] \Big/ \left[Hr + \frac{HR}{2} - \frac{Hr}{2}\right]$$

$$= \left[\frac{Hr^2}{2} + \frac{HR^2}{6} + \frac{HRr}{6} - \frac{Hr^2}{3}\right] \Big/ \frac{H(R+r)}{2}$$

$$= \left[\frac{Hr^2}{6} + \frac{HR^2}{6} + \frac{HRr}{6}\right] \Big/ \frac{H(R+r)}{2}$$

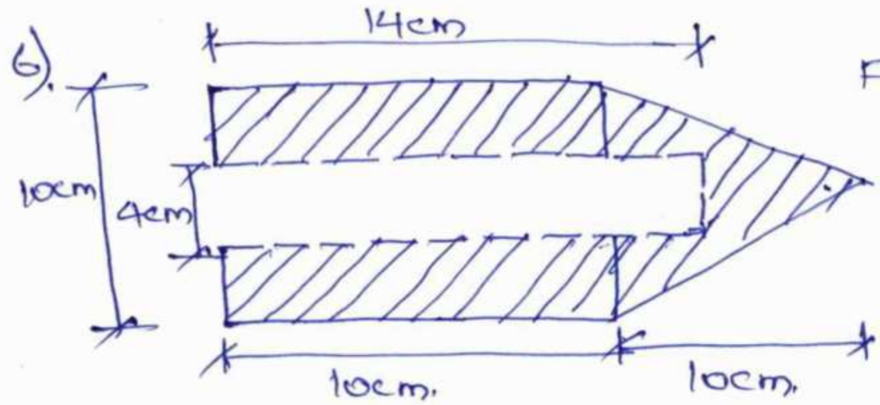
$$= \frac{H}{2}\left[\frac{r^2}{3} + \frac{R^2}{3} + Rr\right] \Big/ \frac{H(R+r)}{2}$$

$$\bar{x} = \frac{\left[R^2 + r^2 + Rr\right]}{3(R+r)}$$

$$V = 2\pi \cdot \frac{H(R+r)}{2} \cdot \frac{\left[R^2 + r^2 + Rr\right]}{3(R+r)}$$

$$V = \frac{\pi H}{3}\left(R^2 + r^2 + Rr\right)$$

6).

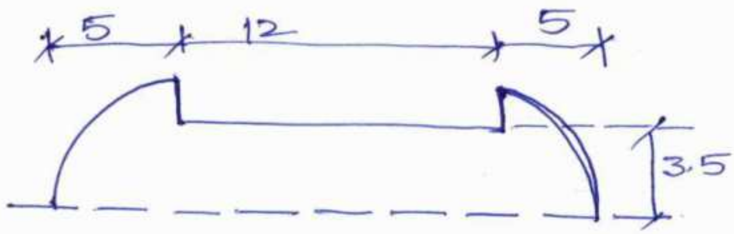


Find the volume of the body

Ans: $V = 871.16\ cm^3$

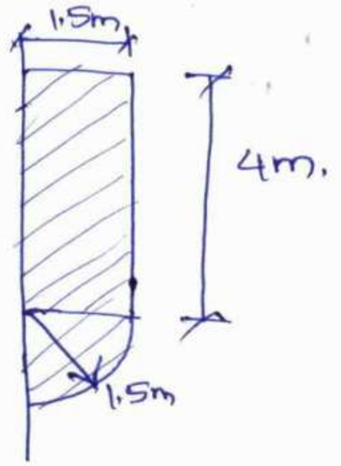
7) Find the surface area and Volume of the body ~~formed~~

(28)



[Ans: $V = 985.5 cm^3$, $A = 658 cm^2$]

8) Find the Volume of the body



[Ans: $V = 35.35 m^3$].

Centroids of Volumes:-

$$\bar{x} = \frac{\int x\,dv}{V} \quad \bar{y} = \frac{\int y\,dv}{V} \quad \text{and} \quad \bar{z} = \frac{\int z\,dv}{dV}$$

Centre of mass:-

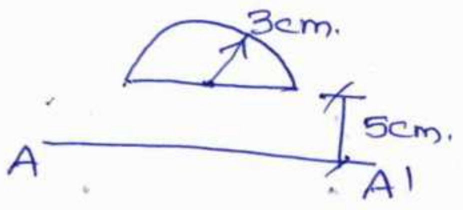
$$\bar{x} = \frac{\int x\,dm}{\int dm} \quad \bar{y} = \frac{\int y\,dm}{\int dm} \quad \bar{z} = \frac{\int z\,dm}{\int dm}$$

Problems

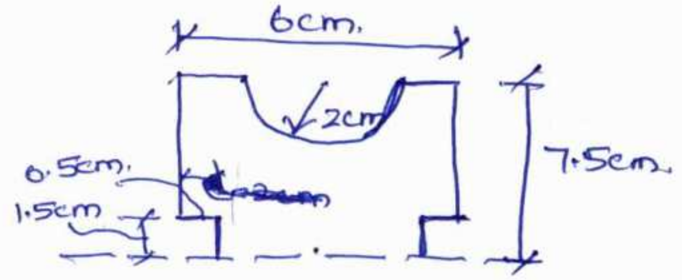
1) Determine the volume generated by the revolution of a square of side 'a' about one of its diagonals

[Ans: $V = \frac{\pi a^3}{\sqrt{18}}$]

2)

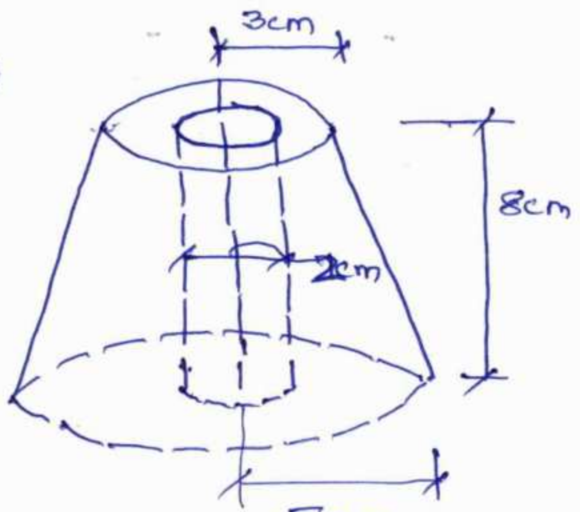


[Ans: S.A = 597.7 cm^2
V = 557.2 cm^3]

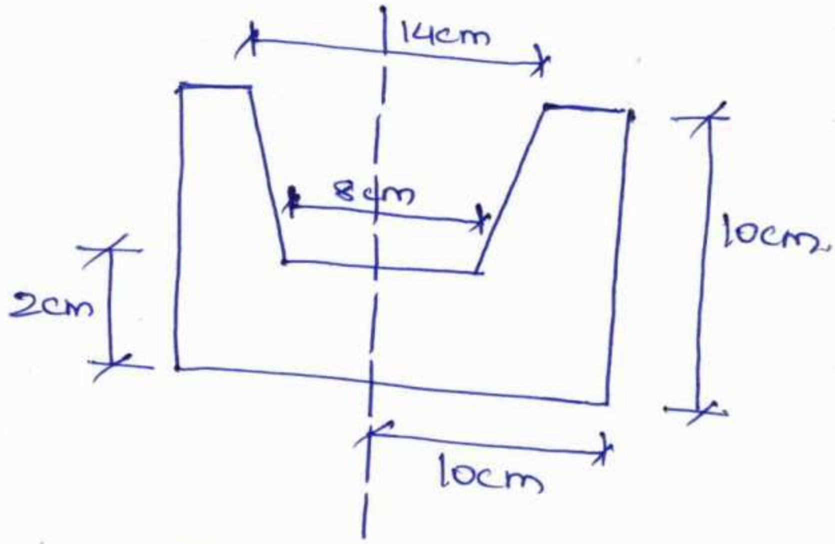


[Ans: V = 794.4 cm^3]

3)

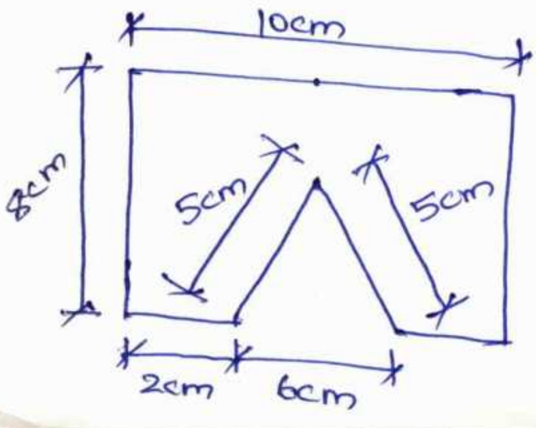


[Ans: 386 cm^3]



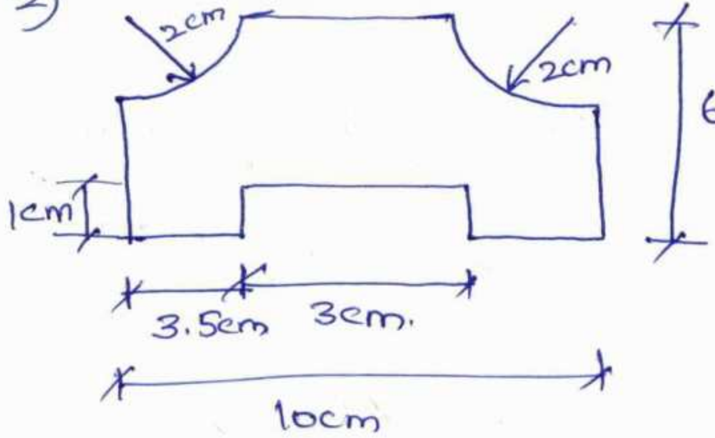
[Ans: A = 1448.1 cm^2
V = 2362.5 cm^3]

1). Determine the centroid of the composite sections

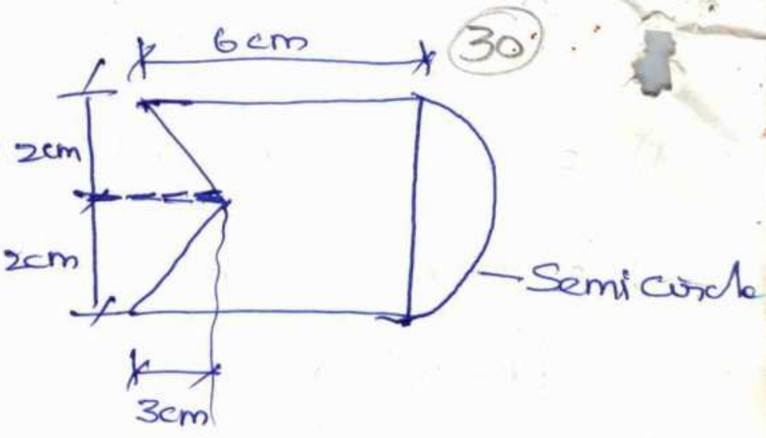


[Ans: $\bar{Y}$ = 4.5cm from base]

2)

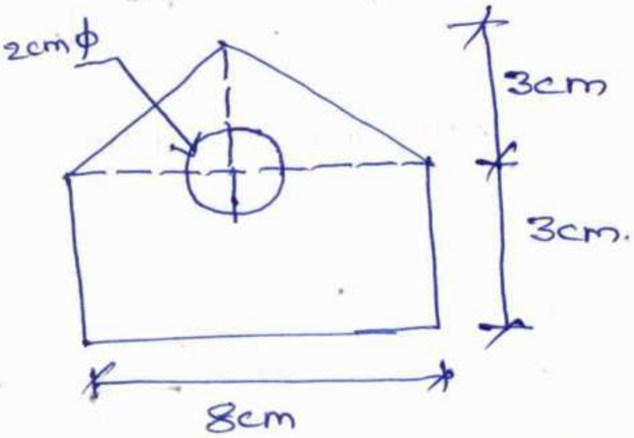


[Ans: $\bar{y} = 2.9$cm from base]



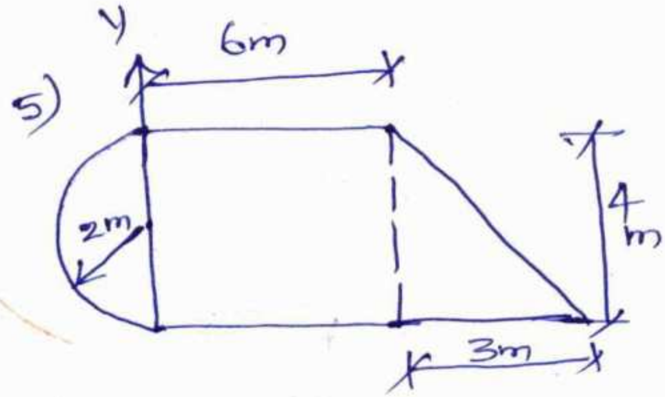
[Ans: $\bar{x} = 4.5$cm, $\bar{y} = 0$.]

3)



[Ans: $\bar{x} = 0$, $\bar{y} = 2.3$cm].

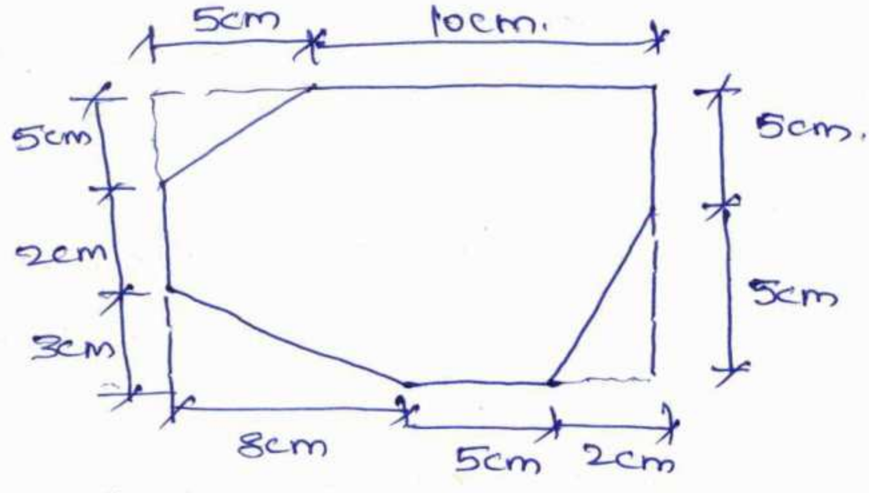
5)



$\bar{x} = 2.995$m,

$\bar{y} = 1.89$m.

4)



[Ans: $\bar{x} = 8.3$cm, $\bar{y} = 5.2$cm]