1483
3000
4489 7.68.45
5285.00
4483.45
211410107601

Fluid mechanics is that branch of science which de with the behaviour of the fluids (liquids or gases) at rest as w as in motion. (4) Cooling of Microchips through fluid flow & phase change

(5)

Fluid mechanics may be divided into three divisions : (1)

Statics :- Fluid statics is the branch of science which deals with the behaviour of fluids at rest.

Kinematics :- Fluid kinematics is the branch of science which deals with the behaviour of fluids in motion, where pressure forces are not considered.

Dynamics :- Fluid dynamics is the branch of science which deals with the fluids in motion, where pressure forces are considered.

(Tornado, River, Raindrops

DIMENSIONS & UNITS :-

A Dimension is a name which describes the measurable character of an object such as mass, length, time & temperature etc,

A unit is an accepted standard for measuring the dimension

Physical dimensions used in fluid mechanics are expressed in four fundamental dimensions namely, mass, length time & temperature.

Mass [M]	kilogram	kg
Length [L]	metre	m
Time [T]	second	s
Temperature [θ]	kelvin [or] °celsius	K, °c

Swing

Derived units from the fundamental units are

· Density = mass per unit volume (kg/m³).

· Newton = a unit of force Force = mass × acceleration = kg uniformly applied over an area of $1m^2$.

$$\text{pressure} = \text{force per unit area}$$
$$= N/m^2 = Pa \text{ or Pascal.}$$

$$1 \text{ bar} = 10^5 \text{ Pa} = 100 \text{ kPa} = 0.1 \text{MPa}$$

$$1 \text{ atmosphere} = 101.325 \text{ kpa} = 1.01325 \text{ bar.}$$

Area	$[L^2]$	m^2.	force	$[MLT^{-2}]$	N
volume	$[L^3]$	m^3.	pressure	$[ML^{-1}T^{-2}]$	N/m^2 or pascal.
velocity	$[LT^{-1}]$	m/s	energy	$[ML^2T^{-2}]$	N/m, Joule = Nm.
Acceleration	$[LT^{-2}]$	m/s^2	power	$[ML^2T^{-3}]$	J/s Watt (W).
Density	$[ML^{-3}]$	kg/m^3	viscosity	$[ML^{-1}T^{-1}]$	Ns/m^2.

<u>Properties of fluids</u>:-

<u>DENSITY</u> (or) <u>MASS DENSITY</u> :- Defined as the ratio of mass of a fluid to its volume. It is denoted by symbol "ρ" (rho). unit of mass density in SI unit is kg per cubic metre.

$$\rho = \frac{\text{mass of fluid}}{\text{volume of fluid}} \quad (kg/m^3).$$

Density of water is $1 gm/cm^3$ or $1000 kg/m^3$.

Density of liquids considered as constant while that of gases changes with in the variation of pressure & temperature.

<u>SPECIFIC WEIGHT</u> or <u>WEIGHT DENSITY</u> :- Defined as the ratio weight of a fluid to its volume. It is denoted by "w".

$$w = \frac{\text{weight of fluid}}{\text{volume of fluid}} = \frac{(\text{mass of fluid}) \times \text{Acceleration due gravity}}{\text{volume of fluid}}$$

$$= \frac{\text{mass of fluid} \times g}{\text{volume of fluid}}$$

$$= \rho \times g$$

$$\therefore w = \rho g$$

Specific weight or weight density for water is 9.81 x 1000 N/m³ in SI units.

SPECIFIC VOLUME :- Defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid.

(2)

$$\text{specific volume} = \frac{\text{volume of fluid}}{\text{mass of fluid}} = \frac{1}{\frac{\text{mass of fluid}}{\text{volume}}} = \frac{1}{\rho}$$

specific volume is the reciprocal of mass density. units m^3/kg.

SPECIFIC GRAVITY :- Defined as the ratio of weight density of a fluid to the weight density of a standard fluid.

specific gravity is also called relative density. It is denoted by 'S'

$$S \text{ (for liquids)} = \frac{\text{weight density of liquid}}{\text{weight density of water}}$$

$$S \text{ (for gases)} = \frac{\text{weight density of gas}}{\text{weight density of air}}$$

weight density of a liquid = S x weight density of water

$= S \times 1000 \times 9.81 \; N/m^3$.

Density of a liquid = S x Density of water

$= S \times 1000 \; kg/m^3$.

prob :- calculate the specific weight, density and specific gravity one litre of a liquid which weighs 7 N.

sol :- Given Data : volume = 1 litre = $\frac{1}{1000} m^3$ [1 m³ = 1000 L]

weight = 7 N

i) $\text{specific weight } (w) = \frac{\text{weight}}{\text{volume}} = \frac{7\,N}{\left(\frac{1}{1000}\right) m^3} = 7000 \; N/m^3$.

ii) $\text{Density } (\rho) = \frac{w}{g} = \frac{7000}{9.81} \; kg/m^3 = 713.5 \; kg/m^3$.

iii) $\text{specific gravity} = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} = 0.7135$.

calculate the density, specific weight and weight of one litre of petrol of specific gravity = 0.7.

Sol:-

Given Data volume = 1 litre = $1 \times 1000\ cm^3$ = ~~1000~~ $0.001\ m^3$.

Specific gravity $S = 0.7$.

i) Density (ρ) = $S \times 1000\ kg/m^3 = 0.7 \times 1000 = 700\ kg/m^3$.

ii) Specific weight $w = \rho \times g = 700 \times 9.81 = 6867\ N/m^3$.

iii) Weight (W)

$$\text{specific weight}(w) = \frac{\text{Weight}}{\text{Volume}} = \frac{W}{V}.$$

$$\Rightarrow W = w \times V = 6867 \times 0.001 = 6.86\ N.$$

VISCOSITY :-

Defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

When two layers of a fluid, a distance 'dy' apart, move one over other at different velocities, say u and u+du, the viscosity together with relative velocity causes a shear stress ~~on the adjacent lower layer~~ acting between the fluid layers.

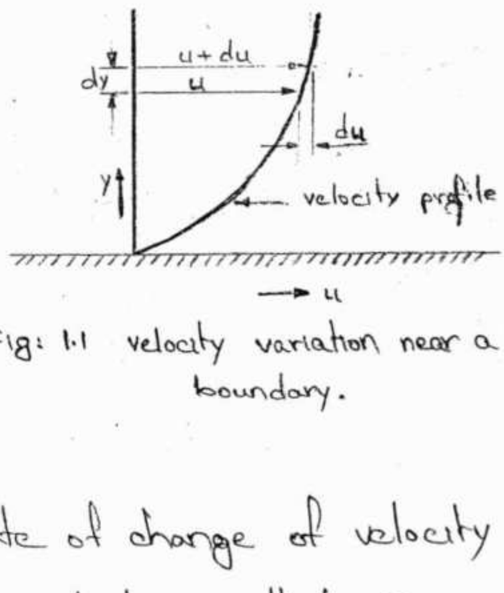


Fig: 1.1 velocity variation near a solid boundary.

shear stress is proportional to the rate of change of velocity with respect to y. It is denoted by symbol τ called Tau.

$$\text{i.e.,} \quad \tau \propto \frac{du}{dy}.$$

$$\tau = \mu \frac{du}{dy}.$$

where μ - constant of proportionality (coefficient of dynamic viscosity

$\frac{du}{dy} \rightarrow$ rate of shear strain or velocity gradient

$$\Rightarrow \mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$$

cohesion :- The force of attraction b/w molecules of the same substance is termed as cohesion

viscosity is also defined as the shear stress required to produ unit rate of shear strain.

<u>units of viscosity</u> :-

In S.I units : $N \cdot s/m^2$.

In M.K.S. units : $kg_f\ sec/m^2$.

$$\therefore \mu = \frac{\text{shear stress}}{\left(\frac{\text{change of velocity}}{\text{change of distance}}\right)} = \frac{\text{force/area}}{\left(\frac{\text{length}}{\text{time}} \times \frac{1}{\text{length}}\right)} = \frac{\text{force} \times \text{time}}{(\text{length})^2}$$

the unit of viscosity in C.G.S. is also called poise = $\frac{\text{dyne - sec}}{cm^2}$.

one poise = $\frac{1}{10}\ N \cdot s/m^2$.

<u>Note</u> :- viscosity of water at 20°c is 0.01 poise or 1.0 centipoise

<u>KINEMATIC VISCOSITY</u> :-

Defined as the ratio of between the dynamic viscosity and dens of fluid. It is denoted by the Greek symbol ν and called n

$$\nu = \frac{\text{viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

units of kinematic viscosity : S.I unit or M.K.S unit — m^2/sec

C.G.S unit — cm^2/s or stoke

$\therefore$ one stoke = cm^2/s = $10^{-4}\ m^2/sec$.

<u>EFFECT OF TEMPERATURE ON VISCOSITY</u> :-

viscosity is effected by temperature. The visco of liquids decreases with the increase of temperature but the viscosity of gases increases with increase of temperature.

This is due to the reason that viscous forces in a fluid are due to <u>cohesive forces</u> & molecular momentum transfer.

the relation between viscosity and temperature for liquids cases are :

Adhesion :- The force of attraction b/n molecules of one substance and molecules of another substance

i) for liquids, $\mu = \mu_0 \left(\frac{1}{1+\alpha t+\beta t^2}\right)$. where μ = viscosity of liquid at $t°C$ in poise

μ_0 = viscosity of liquid at $4°C$ in poise

α, β = constants for the liquid

for water, $\mu_0 = 1.79 \times 10^{-3}$ poise.

$\alpha = 0.03368$ $\beta = 0.000221$.

ii) for a gas, $\mu = \mu_0 + \alpha t - \beta t^2$.

for air $\mu_0 = 0.000017$, $\alpha = 0.000000056$, $\beta = 0.1189 \times 10^{-9}$.

EFFECT OF PRESSURE ON VISCOSITY :-

The viscosity under ordinary conditions is not appreciably affec by the changes in pressure. However, the viscosity of some o has been found to increase with increase in pressure.

NEWTON'S LAW OF VISCOSITY :-

Newton's law states that the shear stress (τ) on a flu element layer is directly proportional to the rate of shear strain

$$\tau = \mu \cdot \frac{du}{dy}. \quad \tau \propto \frac{du}{dy}$$

TYPES OF FLUIDS :-

Fluids may be classified into five types:

1. Ideal fluid 2. Real fluid
3. Newtonian fluid 4. Non-newtonian fluid
5. Ideal plastic fluid.

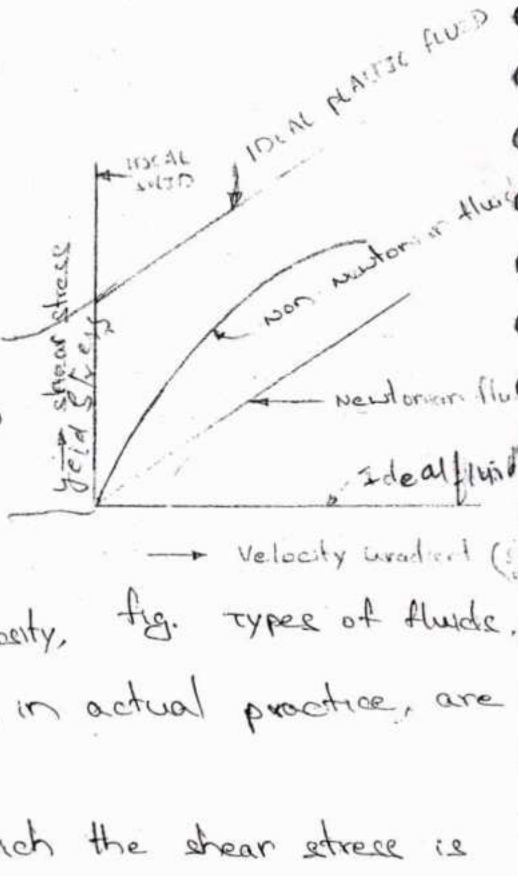


fig. Types of fluids.

IDEAL FLUID :- An ideal fluid is one which is incompressible and has zero viscosity is known as an ideal fluid. (Butter, printing ink)

REAL FLUID :- A fluid which possesses viscosity, is known as real fluid. All the fluids, in actual practice, are real fluids.

NEWTONIAN FLUID :- A Real fluid, in which the shear stress is directly proportional to the rate of shear strain is known as Newtonian fluid. Water, kerosene, air etc

Fluids which obeys newton's law of viscosity are known as newtonian fluids.

4

Ex:- water, kerosene, air etc,

4. **NON - NEWTONIAN FLUID** :- A real fluid, in which the shear stress not proportional to the rate of shear strain.

Fluids which do not obey the newton's law of viscosity are known as non-newtonian fluids.

Ex:- slurries, blood, mud flows, polymer solutions etc.,

5. **IDEAL PLASTIC FLUID** :- A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate shear strain. Sewage [eg :- Sewage Sludge, drilling Muds etc

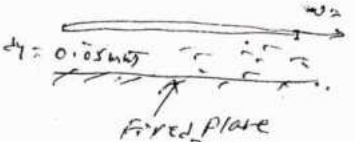
Relation between shear stress (τ) and rate of shear deformation for various fluids :

i) Ideal fluids : $\tau = 0$ ii) Newtonian fluids $\tau = \mu . \frac{du}{dy}$.

iii) Ideal plastic fluids : $\tau = \text{const} + \mu . \frac{du}{dy}$.

iv) non - newtonian fluids : $\tau = \left(\frac{du}{dy}\right)^n$.

prob :- A plate 0.05 mm distant from a fixed plate moves at 1.2 and requires a force of 2.2 N/m^2 to maintain this speed. find viscosity of the fluid between the plates.



sol:- Given Data

Distance between the plates, $dy = 0.05 \text{ mm} = 0.05 \times 10^{-3} \text{ m}$.

velocity of the moving plate, $u = 1.2 \text{ m/s}$.

Force on moving plate, $\tau = \frac{F}{A} = 2.2 \text{ N/m}^2$.

viscosity of the fluid $\mu = \frac{\tau}{\left(\frac{du}{dy}\right)} = \frac{2.2 \times 0.05 \times 10^{-3}}{1.2}$

$= 9.16 \times 10^{-4}$ poise.

... velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ in which u is the velocity in metre per second at a distance 'y' metre above the plate, determine the shear stress at $y=0$ & $y = 0.15$ m. take dynamic viscosity of fluid as 8.63 poise.

sol:- Given $u = \frac{2}{3}y - y^2$ $\therefore \frac{du}{dy} = \frac{2}{3} - 2y$

At $y = 0 \Rightarrow \left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = \frac{2}{3}$.

$y = 0.15 \rightarrow \left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2(0.15) = 0.367$

i.e, $\mu = 8.63$ poise $= 0.863$ Ns/m^2.

shear stress $\tau = \mu \cdot \frac{du}{dy}$.

$\Rightarrow y = 0 \Rightarrow \tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.863 \times 0.667 = 0.575$

$\tau_{0.15} = 0.863 \times 0.367 = 0.3167$ N/m^2.

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with air (gas) (or) b/n two immissible liquids

SURFACE TENSION :-

Defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two ~~immiscible~~ Immiscible liquids such that the contact surface behaves like a membrance under tension.

It is denoted by Greek letter "σ" (called sigma).

SI units — N/m.

consider three molecules A, B, C of a liquid in a mass of liquid. The molecule A is attracted in all directions equally by the surrounding molecules of the liquid. Thus the resultant force acting on the molecule A is zero.

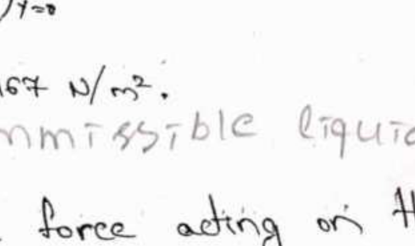


Fig: surface tension.

But the molecule B, which is situated near the free surface, is acted upon by upward and downward forces which are unbalanced.

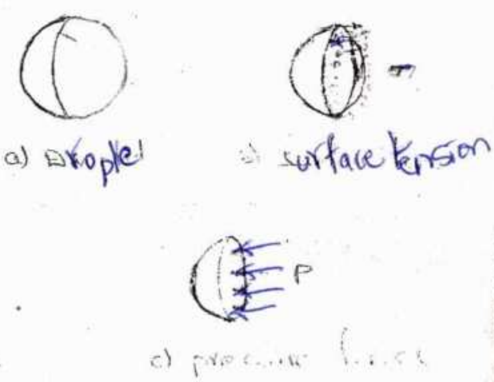
Thus a net resultant force on molecule B is acting in the down direction. The molecule C, situated on the free surface of liquid does experience a resultant downward force. All the molecules on the free surface experience a downward force.

SURFACE TENSION ON LIQUID DROPLET :-

Consider a small spherical droplet of liquid of diameter d and let it be cut into two halves. The forces acting on one-half will be

1) tensile force due to surface tension acting around the circumference of the cut portion as shown in fig.(b) & is equal to

$$= \sigma \times \text{circumference}$$
$$= \sigma \times \pi d.$$



ii) pressure force on the area $\frac{\pi}{4} d^2 \times p$

$p \times \frac{\pi}{4} d^2$ as shown in fig (c).

These two forces will be equal & opposite under equilibrium conditions, i.e.,

Forces on droplet

$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$
$$\Rightarrow \quad p = \frac{4\sigma}{d}$$

Thus the pressure intensity inside the droplet increases with the decrease of diameter of the droplet.

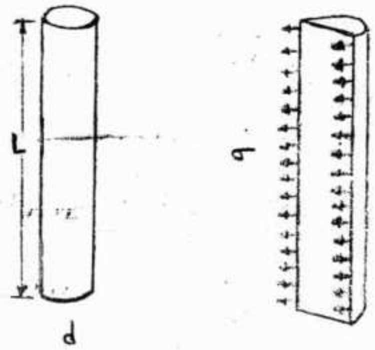
SURFACE TENSION ON A HOLLOW BUBBLE :-

A soap bubble i.e., Hollow bubble has two surfaces in contact with air; one inside and other outside. Surface tension force will act on both the surfaces and accordingly

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$
$$p = \frac{8\sigma}{d}.$$

Surface Tension on a Liquid Jet :-

consider a liquid jet of diameter "d" & length 'L' as shown in fig :



consider the equilibrium of the semi jet.

Force due to pressure $= p \times$ area of semi jet
$= p \times L \times d$

Force due to surface tension $= \sigma \times 2L$

$$\Rightarrow \quad p = \frac{\sigma \times 2\cancel{L}}{\cancel{L} \times d} \Rightarrow p = \frac{2\sigma}{d}$$

prob :- A soap bubble 62.5 mm diameter has an internal pressure in excess of the outside pressure of $20\,N/m^2$. What is tension in the soap film?

Sol :- Given Data. Diameter of bubble $d = 62.5$ mm $= 0.0625$ m

Internal pressure in excess of the outside pressure $p = 20\,N/m^2$.

surface tension on soap bubble

$$p = \frac{8\sigma}{d} \Rightarrow \quad \sigma = \frac{p \times d}{8} = \frac{20 \times 0.0625}{8}$$

$$= 0.156\ N/m.$$

prob :- The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm^2. calculate the pressure with in the droplet surface tension is given as 0.0725 N/m of water.

0.04mm
1243
0.052

Sol :- Given Data. Dia. of droplet $d = 0.04$ mm $= 0.04 \times 10^{-3}$ m

pressure outside the droplet $= 10.32\ N/cm^2 = 10.32 \times$

surface tension, $\sigma = 0.0725\ N/m$.

pressure inside the droplet, in excess of outside pressure

$$p = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{0.04 \times 10^{-3}} = 0.725\ N/cm^2.$$

$\therefore$ pressure inside the droplet $= p +$ pressure outside the droplet

$$= 0.725 + 10.32$$
$$= 11.045\ N/cm^2.$$

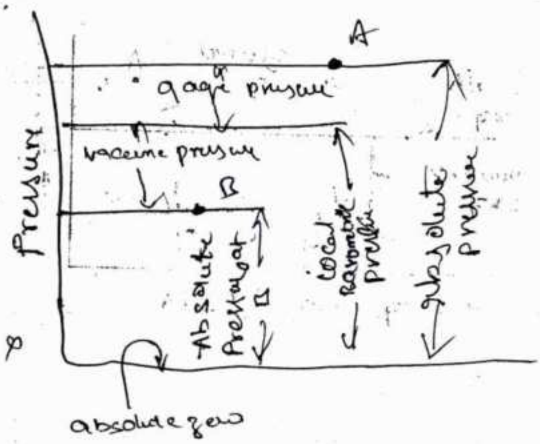
[illegible], [illegible], ATMOSPHERIC AND VACUUM PRESSURES :

The pressure on a fluid is measured in two different systems. one is Absolute & the other is Atmospheric. 6

Absolute pressure is defined as the pressure which is measured with reference to absolute zero or complete vacuum.

Gauge pressure is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum.

The atmospheric pressure on the scale is marked as zero.



Vacuum pressure is defined as the pressure below the atmospheric pressure.

The relationship between absolute, gauge & vacuum pressures are mathematically

i) Absolute pressure = Atm pressure + Gauge pressure

ii) vacuum pressure = Atm pressure − Absolute press

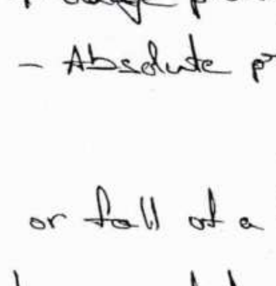
Capillarity :-

capillarity is defined as a phenomenon of rise or fall of a liq surface in a small tube relative to the adjacent general level liquid when the tube is held vertically in the liquid.

The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression. its value depends on specific weight of liquid, diameter of tube & surface tension of liquid. it is expressed in terms of cm o

Expression for capillary rise :

consider a glass tube of small diameter 'd' opened at both ends and is inserted in a liquid, say water. the liquid level rise in the tube above the level of the liquid.



Capillary rise

h = height of the liquid in the tube.

σ = surface tension of liquid.

θ = Angle of contact between liquid & glass tube.

weight of liquid of height h in the tube = (Area of tube × h) × ρ × g

$= \frac{\pi}{4} d^2 \times h \times \rho \times g$.

vertical component of the surface tensile force = (σ × circumference) × $\cos\theta$

$= \sigma \times \pi d \times \cos\theta$.

for equilibrium, we get

$$h = \frac{4\sigma\cos\theta}{\rho \times g \times d}$$

. θ value is taken as zero.

$\therefore$ rise of water is given by $h = \frac{4\sigma}{\rho g d}$.

Expression for capillary fall :-

If the glass tube is dipped in mercury, the level of mercury in the tube will be lower than the general level of the outside liquid.

let H = Height of depression in tube.

Two forces are acting on mercury inside the tube.

① surface tension acting in the downward direction $= \sigma \times \pi d \times \cos\theta$

② Hydrostatic force acting upward $= p \times \frac{\pi}{4} d^2 = \rho g h \times \frac{\pi}{4} d^2$

equating two. we get $h = \frac{4\sigma\cos\theta}{\rho g d}$.

$\therefore$ θ for mercury glass tube is 128°

11.05

452

452

487

10.5

452

11:0 11:5 5

MEASUREMENT OF PRESSURE :-

The pressure of a fluid is measured by the following devices.

1. Manometers. 2. Mechanical Gauges. 7 (7)

MANOMETERS :- Defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid the same or another column of the fluid. these are classifi as :

a) simple Manometers. b) Differential Manometers.

MECHANICAL GAUGES :- Defined as the devices used for measuring pressure by balancing the fluid column by the spring or dead wei

Commonly used mechanical pressure gauges are.

a) Diaphragm pressure gauge. b) Bourdon pressure gauge.
c) Dead - weight pressure gauge. d) Bellows pressure gauge.

SIMPLE MANOMETERS :-

A simple manometer consists of a glass tube whose one end is connected to a point where pressure is to be measured and the other end remains open to atmosphere.

common types of simple manometers are.

1. piezometer. 2. U-tube Manometer 3. single colum manome

PIEZOMETER :-

Piezometer is the simplest form of manometer used for measuring gauge pressure.

It consists of a glass tube, one end is connected to the point where pressure is to be measured and the other end is kept open to the atmosphere, such that liquid can rise freely in it upto certain height without overflowin

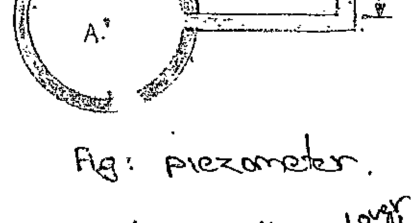


Fig: piezometer.

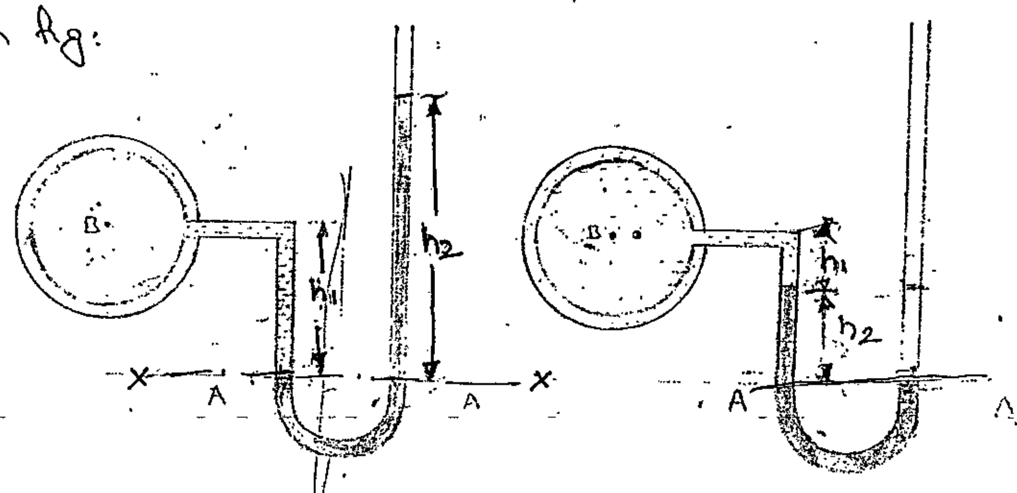
the rise of liquid gives the pressure head at that point.

Thus pressure at point A $= \rho \times g \times h$. (N/m^2)

where ρ = Density of fluid.
g = gravity
h = height of fluid.

U-TUBE MANOMETER :-

A U-tube Manometer consists of a glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in fig:



a) for gauge pressure. b) for vacuum pressure.
U-tube Manometer.

The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of liquid whose pressure is to be measured.

For Gauge pressure :

let B be the point at which pressure is to be measured, whose value is p and A-A is the datum line as shown in fig (a).

let h_1 = Height of the light liquid in the left limb above the datum line
h_2 = Height of the heavy liquid in the right limb above the datum line
S_1 = specific gravity of light liquid. & ρ_1 = Density of light liquid $= 1000 \times S_1$
S_2 = sp. gravity of heavy liquid. & ρ_2 = Density of heavy liquid $= 1000 \times S_2$.

As the pressure is the same for the horizontal surface. The pressure in the left limb and right limb above the datum line A-A are equal.

8 (8)

pressure head A-A in the left limb $= p + \rho_1 \times g \times h_1$

pressure head A-A in the right limb $= \rho_2 \times g \times h_2$.

Equating these two pressures, we get

$$p = (\rho_2 g h_2 - \rho_1 g h_1).$$

b) For vacuum Pressure:

Refer to the fig (b).

pressure head above A-A in the left limb $= \rho_2 g h_2 + \rho_1 g h_1 + p$.

pressure head above A-A in the right limb $= 0$.

Equating these two pressure,

$$p = -(\rho_2 g h_2 + \rho_1 g h_1).$$

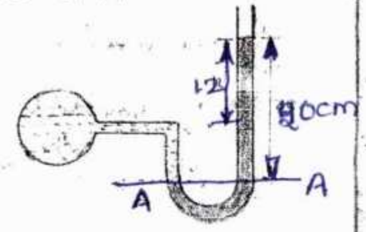
prob:- The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gr. of 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.

Sol:- Given sp. gr. of fluid $\rightarrow s_1 = 0.9 \Rightarrow \rho_1 = s_1 \times 1000 = 900\ kg/m^3$.

sp. gr. of Mercury $s_2 = 13.6 \Rightarrow \rho_2 = s_2 \times 1000 = 13600\ kg/m^3$

Difference of mercury level $h_2 = 20\ cm = 0.2m$.

Height of fluid from A-A $h_1 = 20 - 12 = 8\ cm = 0.08m$.



p = pressure of fluid in pipe.

$\Rightarrow p + \rho_1 g h_1 = \rho_2 g h_2$.

$\Rightarrow p = 13600 \times 9.81 \times 0.2 - 900 \times 9.81 \times 0.08$

$= 25977\ N/m^2$.

$= 2.597\ N/cm^2$.

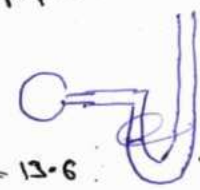
$1m^2 = 100cm$

304, 301, 305

$p = \rho_2 g h_2 - \rho_1 g h_1$

prob:- U-tube manometer containing mercury was used to find the negative or vacuum pressure in the pipe, containing water. The right limb was open to the atmosphere. Find the vacuum pressure on the pipe, if the difference of mercury level in the two limbs was 100 mm and height of water in the left limb from the centre of the pipe was found to 40 mm below.

sol:- Given Data: specific gravity of water, $s_1 = 1$

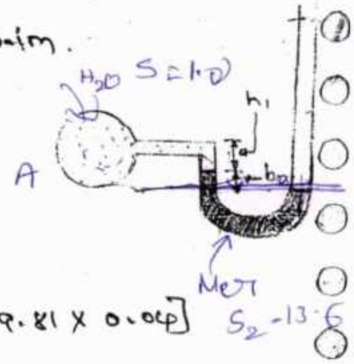


specific gravity of mercury $s_2 = 13.6$

Height of the water in the left limb, $h_1 = 40\ mm = 0.04 m$

Height of the water in the right limb, $h_2 = 100 mm = 0.1 m$.

let p = pressure in the pipe.



Equating the pressure above A - A datum line

$$\rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$$\Rightarrow p = -[13.6 \times 1000 \times 0.1 \times 9.81 - 1 \times 1000 \times 9.81 \times 0.04]$$

$$= -13.73 \text{ kpa}$$

$$= 13.73 \text{ kpa (vacuum)}.$$

SINGLE COLUMN MANOMETER :-

Single column manometer is a modified form of a U-tube manometer in which a reservoir, having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs (say left limb) of the manometer. ~~as shown in fig.~~ For any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure given by the height of liquid in the other limb. The other limb may be vertical or inclined.

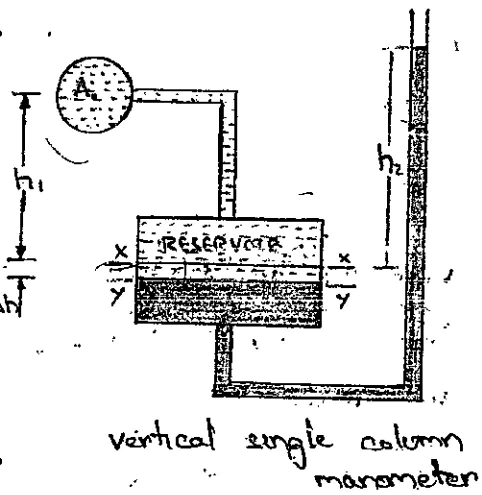
commonly single column manometers are classified into two types

1. vertical single column manometer.
2. inclined single column manometer.

fig. shows the vertical single column manometer.

let x-x be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe.

when the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.



vertical single column manometer.

let Δh = fall of heavy liquid in reservoir.

h_2 = Rise of heavy liquid in right limb.

h_1 = Height of centre of pipe above x-x.

P_A = Pressure at A, which is to be measured.

A = cross-sectional area of the reservoir.

a = cross-sectional area of the right limb.

S_1 = sp. gr. of liquid in pipe.

S_2 = sp. gr. of heavy liquid in reservoir and right limb.

ρ_1 = Density of liquid in pipe.

ρ_2 = Density of liquid in reservoir.

$A \times \Delta h = a \times h_2$

$\Delta h = \frac{a\, h_2}{A}$

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$$A \times \Delta h = a \times h_2 .$$

$$\Delta h = \frac{a \times h_2}{A}$$

let us now consider the pressure heads above datum line y-y

pressure in the left limb $= \rho_1 g \times (\Delta h + h_1) + P_A$.

pressure in the right limb $= \rho_2 g \times (\Delta h + h_2)$.

$\Rightarrow \rho_1 g \Delta h + \rho_1 g h_1$

$\rho_2 g \Delta h + \rho_2 g h_2$

Equating these pressures, we get

$$\Rightarrow P_A = \Delta h(\rho_2 g - \rho_1 g) + h_2 \rho_2 g - h_1 \rho_1 g$$

$P_A = \Delta h[\rho_2 g - \rho_1 g] + h_2\rho_2 - h_1$

$$\therefore P_A = \frac{a \times h_2}{A}(\rho_2 g - \rho_1 g) + h_2 \rho_2 g - h_1 \rho_1 g .$$

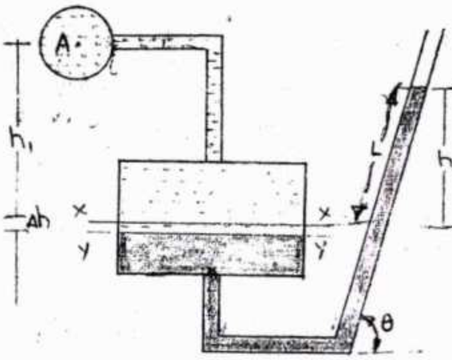
As the area A is very large as compared to a, hence a/A becomes very small & can be neglected.

$$\therefore \; P_A = h_2 \rho_2 g - h_1 \rho_1 g.$$

2) INCLINED SINGLE COLUMN MANOMETER :

- fig: shows the inclined single column manometer.

Due to inclination the distance moved by the heavy liquid in the right limb will be more.



INCLINED SINGLE COLUMN MANOMETER

Let L = length of heavy liquid moved in the right limb from X-X

θ = inclination of right limb with horizontal.

h_2 = vertical rise of heavy liquid in right limb from X-X

$= L \times \sin\theta.$

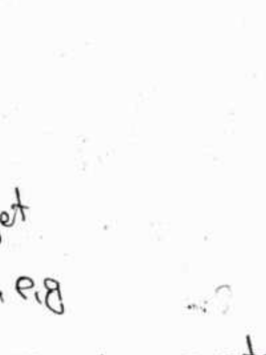
$\therefore$ pressure at A is $P_A = h_2 \rho_2 g - h_1 \rho_1 g$

$\therefore$ substituting the value of h_2, we get

$$P_A = L \times \sin\theta \times \rho_2 g - h_1 \rho_1 g$$

prob:- A single column manometer is connected to a pipe containing liquid of sp. gr. 0.9 as shown in fig.(1). Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube for the manometer reading as shown in fig (1). sp. gr. of mercury is 13.6

sol:- Given Data $S_1 = 0.9$, $S_2 = 13.6$. $h_1 = 20$ cm, $= 0.2$m. $\frac{A}{a} = 100$.
$h_2 = 40$ cm $= 0.4$m.



Let P_A = pressure in pipe.

$$\Rightarrow P_A = \frac{a}{A} h_2 [\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

$$= \frac{1}{100} \times 0.4 [13.6 \times 1000 \times 9.81 - 900 \times 9.81] + 0.4 \times 13.6 \times 1000 \times 9.81 - 0.2 \times 900 \times 9.81.$$

$$= 0.4/100 [133416 - 8829] + 53366.4 - 1765.8$$

$$= 52134 \; N/m^2$$

$$= 5.21 \; N/cm^2.$$

DIFFERENTIAL MANOMETERS:

Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, (11 containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. 10

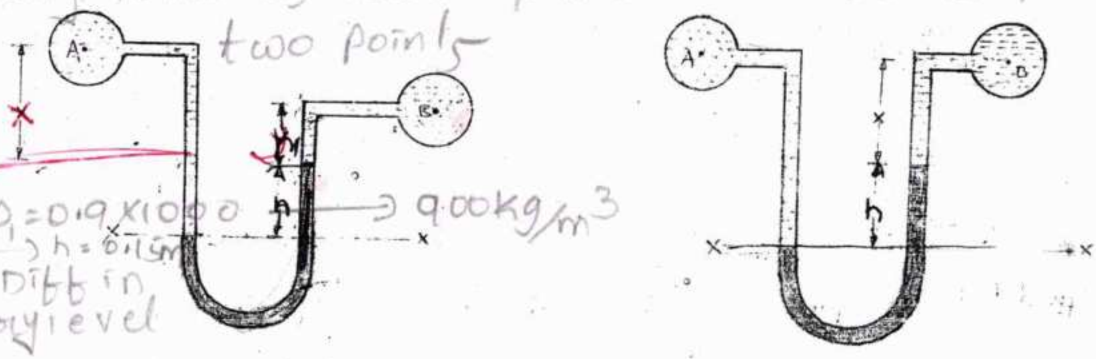
common types of Differential manometers are:

1. U-tube differential manometers.
2. Inverted U-tube differential manometer.

A pipe containing an oil of sp. gravity 0.9 A diff manometer connected at the two points A & B show a difference in mercury level as 15cm. Find the diff of pressure at the two points

U-TUBE DIFFERENTIAL MANOMETER:-

fig. shows the U-tube differential manometer



Sol:

$S_1 = 0.9$, $\rho_1 = 0.9 \times 1000 \Rightarrow 900\ kg/m^3$

$h = 15cm$ Diff in mercury level, $h = 0.15m$

a) Two pipes at different levels. b) Two pipes at same level

fig. U-tube differential manometers.

sp. gr of mercury $S_g = 13.6$ $\rho_g = 13.6 \times 1000\ kg/m^3$

a) let shows a differential manometer whose two ends are connected with two different points A and B at the same level and containing same liquid.

Diff in pres $\Rightarrow P_A - P_B = g \times h(\rho_g - \rho_1)$ $\Rightarrow 18688\ N/m^2$

let, h = Difference of mercury level in the U-tube

x = Distance of the centre of A, from the mercury level in left limb.

y = Distance of the centre of B, from the mercury level in the right limb.

ρ_1 = Density of liquid at A, ρ_2 = Density of liquid at B.

ρ_g = Density of heavy liquid or mercury.

pressure at A and B are P_A & P_B.

pressures in the left limb and right limb, above the datum are equal.

pressure above x-x in the left limb = $\rho_1 g(h+x) + P_A$.

pressure above x-x in the right limb = $\rho_g \times g \times h + \rho_2 \times g \times y + P_B$.

equating these pressures $\quad P_A + \rho_A g[x+h] = P_B + \rho_B g y + \rho_m g h$

$P_A - P_B = [\rho_B g h + \rho_m g h] - \rho_A g[x+h]$

$$\rho_1 g(h+x) + P_A = \rho_g \times g \times h + \rho_2 \times g \times y + P_B$$

$$\rho_1 g h + \rho_1 g x + P_A = \rho_g g h + \rho_2 g y + P_B \qquad P_A - P_B = gh[\rho_B - \rho_m] - \rho_A g x - \rho_A g h$$

$$\Rightarrow P_A - P_B = h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x \,..$$

$\therefore$ Difference of pressure at A and B = $h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$

b) Two pipes are at same level.

pressure above x-x in right limb = $\rho_g \times g \times h + \rho_1 \times g \times x + P_B$.

pressure above x-x in left limb = $\rho_1 \times g \times (h+x) + P_A$.

equating two pressures

$$\therefore P_A - P_B = \rho_g \times g \times h + \rho_1 g x - \rho_1 g(h+x)$$

$$\rho_g \times g \times h + \cancel{\rho_1 g x} - \rho_1 g h - \cancel{\rho_1 g x}$$

$$= g \times h(\rho_g - \rho_1).$$

probl :- A pipe contains an oil of sp. gr. 0.9. A differential manometer connected at the two points A and B shows a difference in mercury level as 15cm. find the difference of pressure at the two points.

sol :- Given Data $\quad$ sp. gr. of oil $S_1 = 0.9 \Rightarrow \rho_1 = 900\ kg/m^3$.

sp. gr. of mercury $S_g = 13.6 \Rightarrow \rho_g = 13600\ kg/m^3$.

Difference in mercury level $h = 15cm = 0.15m$.

$\Rightarrow$ Difference of pressure is $P_A - P_B = g \times h(\rho_g - \rho_1)$

$= 9.81 \times 0.15\,(13600 - 900)$

86,55,27

$= 18688\ N/m^2$.

probl :- A differential manometer is connected at the two points A and B of two pipes as shown in fig. the pipe A contains a liquid of sp. gr. 1.5 while pipe B contains a liquid of sp. gr. = 0.9. the pressures at A & B are 1kgf/cm² & 1.80 kgf/cm² respectively. find the difference in mercury level in the differential manometer.

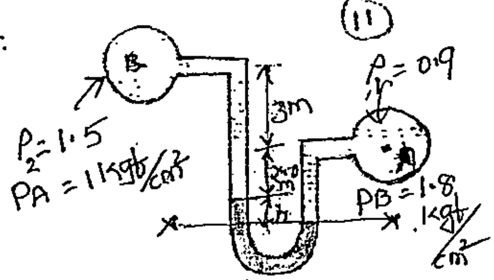
Given Data

$S_1 = 0.9$, $\rho_1 = 900$ $\qquad P_A = 1.80$ kgf/cm² $= 1.8\times10^4\times9.81$ N/m² $\qquad$ 1 kgf = 9

$S_2 = 1.5$, $\rho_2 = 1500$ $\qquad P_B = 1$ kgf/cm² $= 9.81\times10^4$ N/m²

$x = 2$, $y = 5$:

Density of mercury $\rho_g = 13600$ kg/m³.



difference of pressure at A & B.

$$P_A - P_B = h\times g\,(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$

$$\Rightarrow -(1.8-1)\times9.81\times10^4 = h\times9.81\,(13600-900) + 1500\times9.81\times5 - 900\times9.81\times2$$

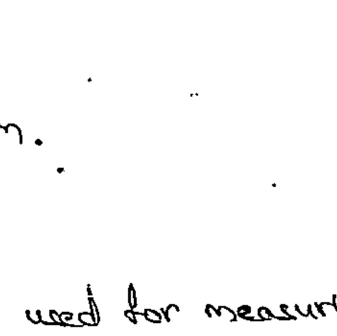
$$\rightarrow -78480 = h\times124587 + 73575 - 17658.$$

$$h = -22563/124587 = -0.181\text{ m.} = -18.1\text{ cm.}$$

INVERTED U-TUBE DIFFERENTIAL MANOMETER :-

Inverted U-tube differential manometer is used for measuring difference of two pressures where accuracy is the major consideration. It consists of an inverted U-tube, containing light liquid. the two ends of the tube are connected to the points, whose difference of pressure is to be measured.

let the pressure at A is more than the pressure at B.



let h_1 = Height of liquid in left limb below the datum line x-x.
h_2 = Height of liquid in right limb.
h = difference of light liquid.
ρ_1 = density of liquid at A.
ρ_2 = density of liquid at B.
ρ_s = density of light liquid
P_A = pressure at A. P_B = pressure at B.

pressures in the left limb and right limb below the datum line are equal.

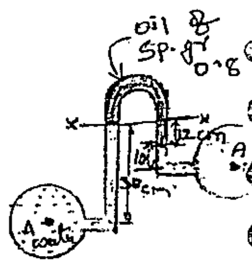
pressure in the left limb $= P_A - \rho_1\times g\times h_1$.
pressure in the right limb $= P_B - \rho_2 g h_2 - \rho_s\times g\times h$
Equating two pressures.

$$P_A - P_B = \rho_1\times g h_1 - \rho_2\times g\times h_2 - \rho_s\times g\times h.$$

065255

14900
1257.0
2430

Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp. gr. 0.8 is connected. the pressure head in the pipe A is 2m of water, find the pressure in the pipe B for the manometer readings as shown in fig.



Sol:- Given Data pressure head at A = $\frac{P_A}{\rho g}$ = 2m of water.

$\Rightarrow P_A = \rho \times g \times 2 = 19620\ N/m^2$

pressure in the left limb = $P_A - \rho_1 g h_1$

pressure in the right limb = $P_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$.

$\Rightarrow$ equating two pressures.

$19620 - 1000 \times 9.81 \times 0.3 = P_B - 1922.36$

$\Rightarrow P_B = 1.8599\ N/cm^2$.

probl :- An inverted differential manometer is connected to two pipes A and B carrying water under pressure as shown in fig. the fluid in manometer is oil of specific gravity 0.75. Determine the pressure diff between A and B.

Sol:- sp. gravity of oil $s = 0.75 \Rightarrow \rho_s = 750\ kg/m^3$.

difference of oil in two limbs = $(450 + 200) - 450$

$= 200\ mm$

pressure heads on the left & right limbs below the datum line x-x are equal

pressure in the left limb = $P_A - 1000 \times 9.81 \times 0.45$

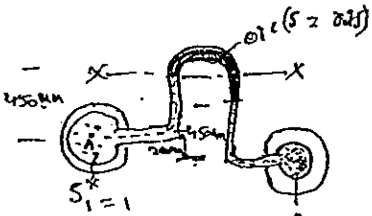
pressure in the right limb = $P_B - 1000 \times 9.81 \times 0.45 - 1000 \times 9.81 \times 0.75$

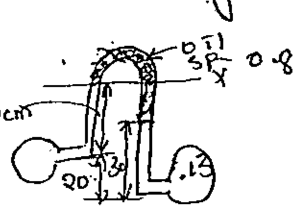
equating two pressures heads,

we get

$P_B - P_A = 4.4145 - 2.943$

$= 1.47\ kN/m^2$.





-> An inverted differential manometer is connected to two pipes A and B which convey water. the fluid in manometer is oil of sp. gr. 0.8 for the manometer readings shown in the fig. find the pressure diff b/n A and B.

Sol: Sp. gr of oil = 0.8 $\therefore \rho_s = 800\ kg/m^3$

Diff of oil in the two limbs

$= (30 + 20) - 30 = 20cm$

Taking datum line at X-X

pressure in the left limb below X-X

$= P_A - 1000 \times 9.81 \times 0.3$

Press. in the right below

$= P_B - 1000 \times 9.81 \times 0.3 - 800 \times 9.81 \times 0.2$

$= P_B - 2943 - 1569.6$

$= P_B - 4512.6$

equal -

$P_A - 2943 = P_B - 4512.6$

$P_A - P_B = 1569.6\ N/m^2$ //

(12)

FLUID KINEMATICS :

fluid kinematics is a branch of 'fluid mechanics' w deals with the study of velocity and acceleration of the partic of fluids in motion and their distribution in space without considering any force or energy involved.

STREAM LINE :-

A stream line may be defined as an imaginary line with in the flow so that the tangent at any point on i indicates the velocity at that point.

Equation of a stream line in a three dimensional flow is given as:

$$\frac{dx}{du} = \frac{dy}{v} = \frac{dz}{w}$$

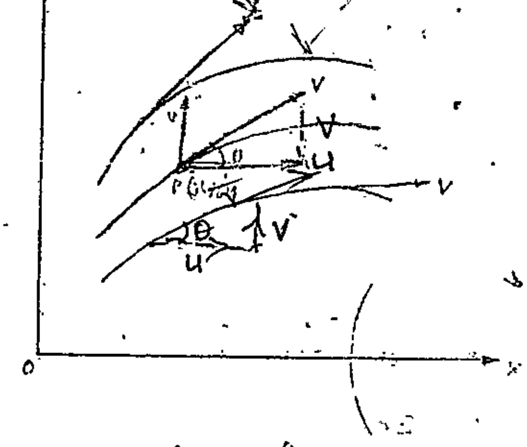
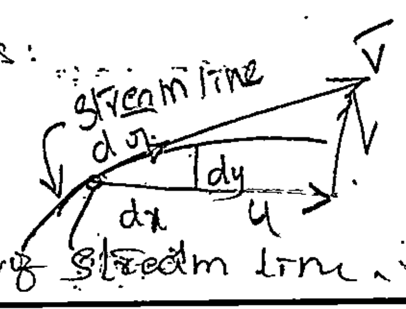


fig. shows some of the streamlines for a flow pattern in the xy plane in which a streamline passing through a point P(x,y) is tangential to the velocity vector v at P. If u and v are the components of V along x and y directions, then

$$\frac{opp}{adj} \quad \frac{v}{u} = \tan\theta = \frac{dy}{dx} = \frac{\text{change in height}}{\text{change in horizontal distance}}$$

From this, differential equation for stream-line in a three dimensional flow is given as:

$$\left\{\frac{dx}{u} = \frac{dy}{v}\right\} = \frac{dz}{w}.$$



$v\,dx = u\,dy \Rightarrow -v\,dx + u\,dy = 0 \rightarrow$ eq of stream line

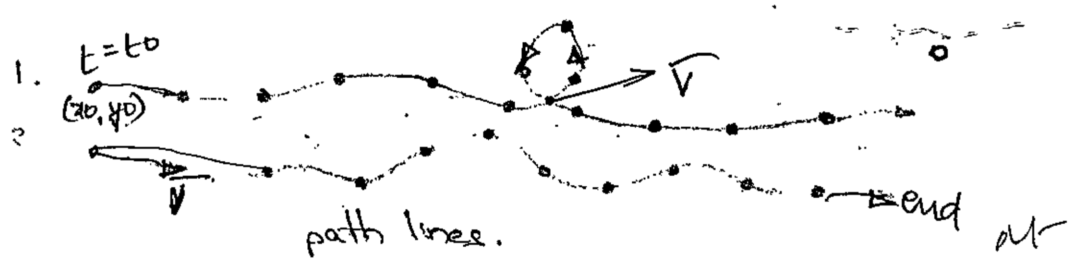
Some important points about streamlines

1. A streamline cannot intersect itself nor two streamlines can cross.
2. There cannot be any movement of the fluid mass across streamli
3. The series of streamlines represent the flow pattern at an ins
4. Stream line spacing varies inversely as the velocity: converging stream lines in any particular direction shows accelerated flo in that direction.

- In steady flow, the pattern of streamlines remains same a different times.
- In unsteady flow the pattern of streamlines may change fro time to time.

PATH LINE: A path line is the path followed by a fluid partic in motion. A path line shows the direction of particular parti as it moves ahead.

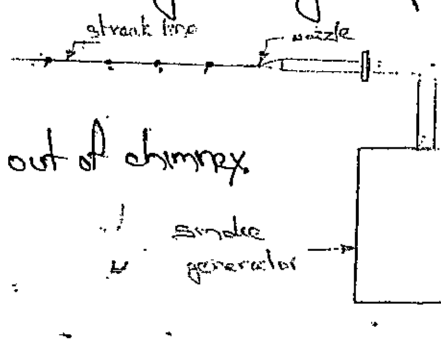
A path line intersect itself at different times. (x_0, y_0)



path lines.

- In steady flow the path lines and stream lines are identical.
- In unsteady flow, path lines and stream lines are different.

STREAK LINES:- A streak line is the ~~instantaneous~~ a curve which gives an instantaneous picture of the location of the fluid particles, which have passed through a given point i.e fixed point in a flow field.



Ex:- 1. path taken by smoke coming out of chimney.

STREAM TUBE :- A stream tube is a tube imagined to be formed by a group of stream lines passing through a small closed curve, which may or may not be a circular. (13)

or

simply, it is a fluid mass bounded by a group of streamlines.

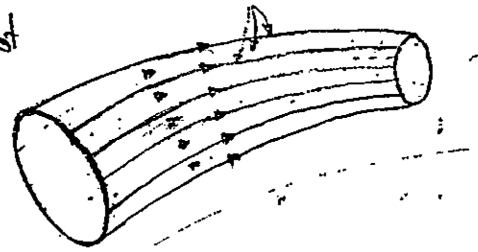


fig: stream tube.

Ex:- pipes & nozzles.

important points about stream tube are

1. The stream tube has finite dimensions.
2. The shape of a stream tube changes from ~~one instant to~~ another because of change in the position of stream lines.

TYPES OF FLUID FLOW :-

Fluid flow is classified as:

1. steady & unsteady flows. (T)
2. uniform & non uniform flows. (S)
3. Laminar & Turbulent flows.
4. compressible & incompressible flows. [density fluid change $f_{\,(T,P)}$]
5. Rotational & irrotational flows.
6. one, two & three - dimensional flows.

STEADY AND UNSTEADY FLOWS :-

Steady flow is defined as that type of flow in which fluid characteristics like, velocity, pressure, density etc, at a point do not change with time. Thus for steady flow mathematically, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0,y_0,z_0} = 0, \quad \left(\frac{\partial p}{\partial t}\right)_{x_0,y_0,z_0} = 0 ; \quad \left(\frac{\partial \rho}{\partial t}\right)_{x_0,y_0,z_0} = 0.$$

where (x_0, y_0, z_0) is a fixed point in a fluid field.

Ex:- Flow through a prismatic or non-prismatic conduit at a const. flow rate $Q\ m^3/sec$ is steady.

Unsteady flow is that type of flow, in which the velocity, pressure or density at a point changes with respect to time. Mathematically we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \; ; \left(\frac{\partial P}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \; ; \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} \neq 0.$$

$$\left(\frac{\partial P}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \quad \text{etc.,}$$

Ex:- Flow in a pipe whose valve is opened or closed gradually.

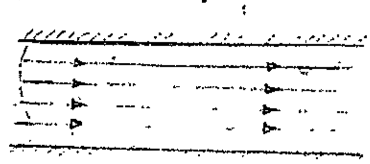
UNIFORM AND NON-UNIFORM FLOWS:-

Uniform flow is defined as that type of flow in which the vel at any given time does not change with respect to space. Mathematically we have,

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} = 0.$$

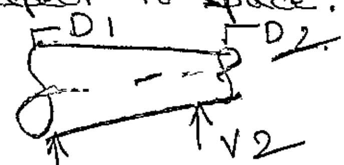
where
∂V = change in velocity
∂s = displacement in any d

Ex:- 1. Flow through a straight pipe of constant diameter.
2. Flow between parallel plates. →



Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Mathematically,

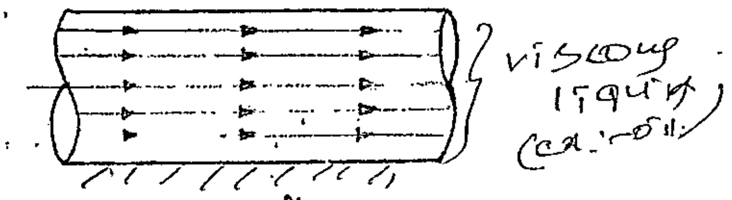
$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} \neq 0.$$



Ex:- 1. Flow through a non-prismatic conduit
2. Flow around a uniform diameter pipe-bend or a canal bend.

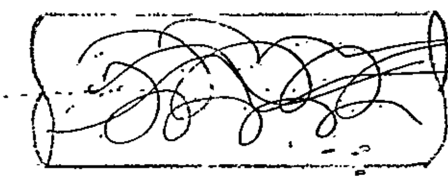
LAMINAR AND TURBULENT FLOWS:

Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream lines are straight & parallel. This type of flow is also called stream-line flow & viscous flow.



Laminar flow

fixed surface



Turbulent flow.

Ex:- 1. Ground water flow. 2. Flow through a capillary tube.

A turbulent flow is that type of flow in which the fluid particles move in a zig-zag way.

Ex:- High velocity flow in a conduit of large size.

Laminar & turbulent flows are characterized on the basis of Reynolds number $\left(\frac{VD}{\nu}\right)$. where V = mean velocity of flow in pipe

RNO :- ratio of inertia force to viscous force => $Re = \frac{I.F}{V.F}$ (mean vel)

D = diameter of pipe

ν = kinematic viscosity of fluid

$Re = \rho VL/\mu$ (flat) (μ dynamic) $Re = \frac{\rho V D}{\mu}$ (circular pipe)

Reynolds number (Re) < 2000 ..... flow is laminar.

Reynolds number (Re) > 4000 ..... flow is turbulent.

Reynolds number (Re) between 2000 & 4000 ..... flow may be laminar or turbulent.

Compressible & incompressible flows:-

Compressible flow is that type of flow in which the density of fluid changes from point to point. mathematically,

$\rho \neq$ constant. Ex:- Flow of gases through nozzle, orifices etc,

Incompressible flow is that type of flow in which the density of fluid is constant for the fluid flow. Mathematically,

ρ = constant. Ex:- subsonic aerodynamics.

ROTATIONAL AND IRROTATIONAL FLOWS:

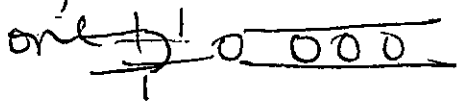
Rotational flow is that type of flow in which the fluid particles while flowing along stream lines, also rotate about their own axis.

Ex:- Motion of liquid in a rotating tank.

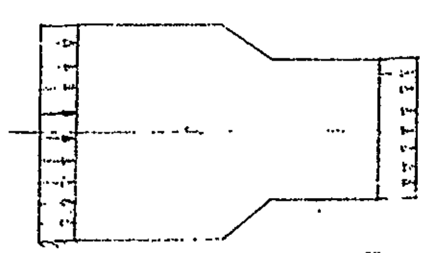
Irrotational flow is that type of flow in which the fluid particles while flowing along stream lines, do not rotate about their own axis.

Ex:- Flow above a drain hole of a stationary tank or a wash basin.

ONE, TWO AND THREE - DIMENSIONAL FLOWS:-

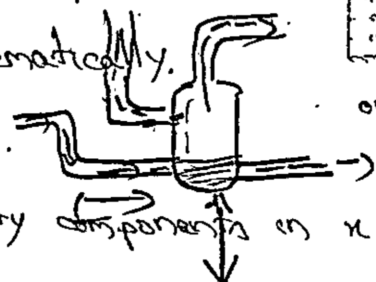


one - dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and one - space coordinate only, say x. mathematically.



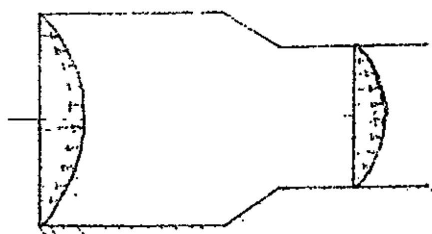
one - dimensional flow

$$u = f(x), \quad v = 0 \;\&\; w = 0.$$



where u, v, & w are the velocity components in x, y, & z. directions.

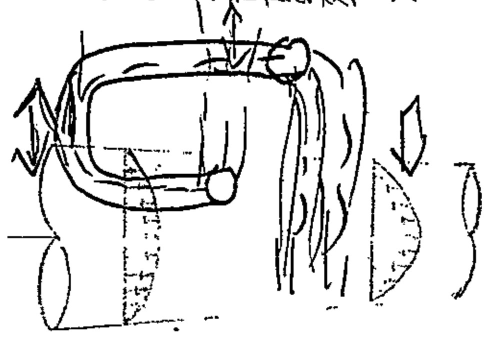
Two dimensional flow is that type of flow in which the velocity is a function of time and two rectangular space coordinates say x and y. mathematically.



Two dimensional flow.

$$u = f_1(x,y), \quad v = f_2(x,y) \;\&\; w = 0.$$

Three dimensional flow is that type of flow in which the velocity is a function of time & three mutually perpendicular directions. mathematically,



Three dimensional flow

$$u = f_1(x,y,z), \quad v = f_2(x,y,z), \quad w = f_3(x,y,z).$$

15

RATE OF FLOW (OR) DISCHARGE :-

Rate of flow (or) Discharge is defined as the quantity of a fluid flowing per second through a section of pipe or a channel. It is generally denoted by Q.

Let us consider a liquid flowing through a pipe.

Let, A = Area of cross-section of the pipe. &

V = Average velocity of the liquid across the section.

Then ∴ Discharge = Area × Average velocity

$$Q = A \times V \quad (m^3/sec).$$

CONTINUITY EQUATION :-

The continuity equation is based on the principle of conservation of mass. It states that

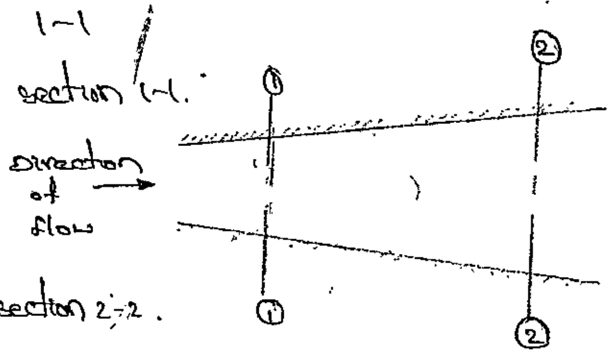
"If no fluid is added or removed from the pipe in any length then the mass passing across different sections shall be same."

Consider two cross-sections of a pipe as shown in fig.

- Let A_1 = Area of pipe at section 1-1

V_1 = velocity of the fluid at section 1-1.

ρ_1 = Density at section 1-1.



A_2, V_2, ρ_2 are corresponding values at section 2-2.

Rate of flow at section 1-1 = $\rho_1 A_1 V_1$

Rate of flow at section 2-2 = $\rho_2 A_2 V_2$

According to law of conservation of mass,

Rate of flow at 1-1 = Rate of flow at 2-2

$\rho = m/\text{volume}$

$m = \rho \times vol.$

$m/s = \rho \; vol/s$

$= \rho \times (av) \Rightarrow \rho Q.$

$m_1 = m_2$

$\rho_1 a_1 v_1 = \rho_2 a_2 v_2$

$$\therefore \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \longrightarrow \text{eqn. } ①.$$

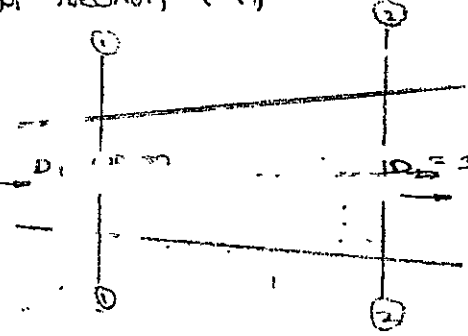
equation (1) is applicable to the compressible fluids as well as incompressible fluids, it is called continuity equation.
If the fluid is incompressible, then $\rho_1 = \rho_2$ ⇒ continuity equation

is $A_1V_1 = A_2V_2$.

prob:- The diameters of a pipe at the sections 1-1 and 2-2 are 200 mm and 300 mm respectively. If the velocity of water flowing through the pipe at section 1-1 is 4 m/s. find 1) Discharge through the pipe 2). velocity of water at section 2-2.

sol:- Given Data Diameter of the pipe at section 1-1,

$D_1 = 200\,mm = 0.2\,m$.



∴ Area $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314\,m^2$

velocity $v_1 = 4\,m/s$.

Diameter of the pipe at section 2-2 is

$D_2 = 300\,mm$.

∴ Area $A_2 = \frac{\pi}{4} \times (0.3)^2 = 0.07\,m^2$

1) Discharge through the pipe, Q:

$Q_1 = A_1 \times V_1 \Rightarrow Q_1 = 0.0314 \times 4 = 0.1256\ m^3/sec$.

2) velocity of water at section 2-2

we know that

$A_1V_1 = A_2V_2$

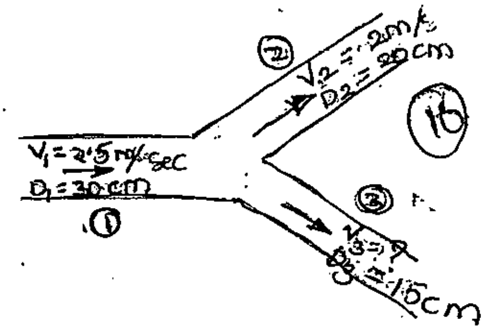
$\Rightarrow V_2 = \frac{A_1V_1}{A_2} = \frac{0.0314 \times 4}{0.07} = 1.77\ m/sec.$

prob:- A 30 cm diameter pipe, conveying water, branches into two pipes of diameters, 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm if the average velocity 20 cm diameter pipe is 2 m/s.

$D_1 = 30$ cm , $A_1 = \frac{\pi}{4}(0.3)^2 = 0.07\ m^2$, $V_1 = 2.5$ m/s

$D_2 = 20$ cm , $A_2 = \frac{\pi}{4}(0.2)^2 = 0.031\ m^2$, $V_2 = 2$ m/s.

$D_3 = 15$ cm , $A_3 = \frac{\pi}{4}(0.15)^2 = 0.017\ m^2$, $V_3 = ?$



According to continuity equation

$$Q_1 = Q_2 + Q_3.$$

Discharge Q_1 in pipe 1 is given by $Q_1 = A_1 \times V_1 = 0.07 \times 2.5 = 0.176\ m^3$

$$Q_2 = A_2 V_2 = 0.031 \times 2.0 = 0.0628\ m^3/s.$$

$$\Rightarrow \quad Q_3 = Q_1 - Q_2 = 0.1767 - 0.0628 = 0.1139\ m^3/s.$$

But $Q_3 = A_3 \times V_3 \Rightarrow V_3 = \frac{Q_3}{A_3} = \frac{0.113}{0.017} = 6.44$ m/s.

FLUID DYNAMICS :

Fluid dynamics is the study of fluid motion with the forces causing flow. The dynamic behaviour of the fluid flow is analysed by the newton's second law of motion, which relates the acceleration with forces.

According to newton's second law of motion, the net force f_x act on a fluid element in the direction of x is equal to mass m of fluid element multiplied by the acceleration a_x in the x-direct thus mathematically,

$$\boxed{f_x = m \cdot a_x.}$$

forces present in fluid flow are gravity force, pressure force, force due to viscosity, force due to turbulence, force due to compres

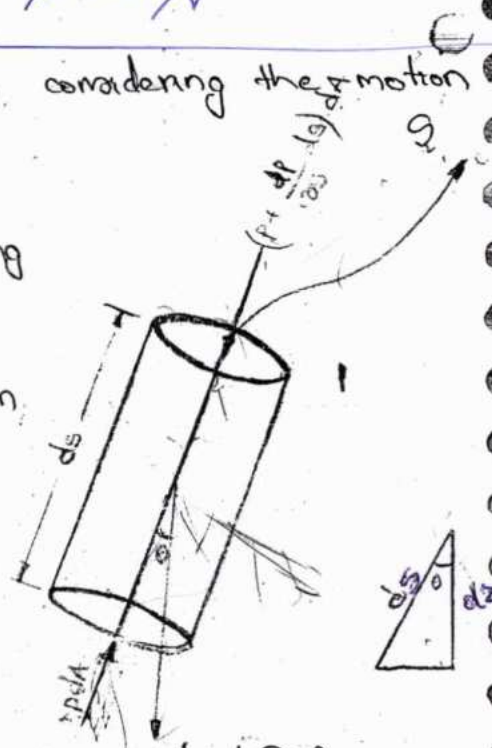
$\therefore$ net force $\boxed{f_x = (f_g)_x + (f_p)_x + (f_v)_x + (f_t)_x + (f_c)_x.}$

if the force is assumed to be ideal, viscous force (f_v) is zero and equation of motions are known as euler's equation of moti

$P = F/A \Rightarrow F = PA$
$\rho = m/V$

Euler's Equation of Motion :

Euler's equation of motion is derived by considering the motion of a fluid element along a stream line.



consider a stream line in which flow is taking place in s-direction as shown in fig. 6.1.

consider a cylindrical element of cross-section dA and length ds.

The forces acting on the cylindrical element are:

1. pressure force $p\,dA$ in the direction of flow.
2. pressure force $\left(p + \frac{\partial p}{\partial s}\right) dA$ opposite to the direction of flow.
3. weight of element $\rho g\,dA\,ds$.

$\rho g\,dA\,ds \cos\theta$

$W = mg$
$= \rho V g$
$= \rho g\,dA\,ds$

Let θ is the angle between the direction of flow and the line of action of the weight of element.

$F = m \cdot a$

Resultant force on the fluid element in the direction of s must be equal to the mass of fluid element × acceleration in the direction

$$\therefore\ p\,dA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g\,dA\,ds\cos\theta = \rho\,dA\,ds \times a_s$$

Now $a_s = \frac{dv}{dt}$, where v is a function of "s" and "t".

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\,\partial v}{\partial s} + \frac{\partial v}{\partial t} \qquad \left\{\because \frac{ds}{dt} = v\right\}$$

If the fluid is steady, $\frac{\partial v}{\partial t} = 0$

$$\therefore\ a_s = \frac{v\,\partial v}{\partial s}$$

substituting the value of a_s in equation ①. & simplifying, we get

$$-\frac{\partial p}{\partial s} ds\,dA - \rho g\,dA\,ds\cos\theta = \rho\,dA\,ds \times \frac{v\,\partial v}{\partial s}$$

Dividing by $\boxed{\rho\,ds\,dA}$, we get

$$-\frac{\partial P}{\rho \partial s} - g\cos\theta = \frac{v\partial v}{\partial s}$$

(or) $$\frac{\partial P}{\rho \partial s} + g\cos\theta + \frac{v\partial v}{\partial s} = 0.$$

17

But, we have $\underline{\cos\theta = \frac{dz}{ds}}$.

$$\frac{1}{\rho}\cdot\frac{\partial P}{\partial s} + g\cdot\frac{dz}{ds} + v\frac{\partial v}{\partial s} = 0 \quad (\text{or}) \quad \frac{\partial P}{\rho} + g\,dz + v\,dv = 0$$

(or) $$\boxed{\frac{\partial P}{\rho} + g\,dz + v\,dv = 0}$$

This is known as euler's equation of motion. $\frac{\partial P}{\rho} + g\,dz + v\,dv$

BERNOULLI'S EQUATION of MOTION :-

Bernoulli's equation of motion is obtained by integrating the Euler's equation of motion as

$$\int \frac{dP}{\rho} + \int g\,dz + \int v\,dv = \text{constant}.$$

If flow is incompressible, ρ is constant &

$$\frac{P}{\rho} + gz + \frac{v^2}{2} = \text{Constant} \quad (\text{or})$$

$$\boxed{\frac{P}{\rho g} + \frac{v^2}{2g} + z = \text{constant}.}$$ is a Bernoulli's equation

where $\frac{P}{\rho g}$ = pressure energy per unit weight of fl
or Pressure head

$v^2/2g$ = kinetic energy per unit weight

z = potential energy per unit weight.

$\frac{P}{\rho g} + \frac{V^2}{2g} + Z$

Assumptions :

Assumptions considered in the derivation of Bernoulli's equation :

1) fluid is ideal i.e., viscosity is zero.
2) flow is steady.
3) Flow is incompressible.
4) flow is irrotational

prob:- The water is flowing through a tapering pipe having diameter 300 mm and 150 mm at sections 1 & 2 respectively. The discharge through the pipe is 40 litres/sec. The section 1 is 10 m above datum & sec 2 is 6 m above datum. Find the intensity of pressure at section 2 if that at section 1 is 400 kN/m².

Sol: Given Data

$D_1 = 300\ mm = 0.3 m.$ $\quad A_1 = \frac{\pi}{4}(0.3)^2$

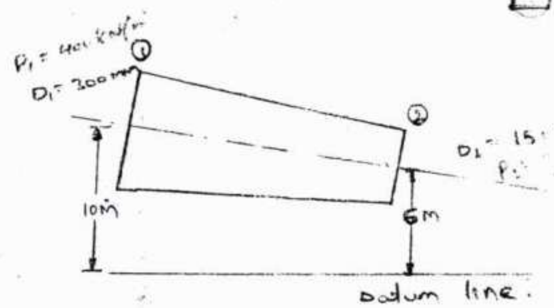
$P_1 = 400\ kN/m^2.$ $\quad = 0.07\ m^2.$

$Z_1 = 10\ m.$

$D_2 = 150\ mm = 0.15 m.$

$P_2 = ?$

$Z_2 = 6\ m.$



Discharge or rate of flow $Q = 40$ litres/sec $= \frac{40 \times 10^3}{10^6}$

$= 0.04\ m^3/sec.$

We know that

$Q = A_1 V_1 = A_2 V_2 \Rightarrow V_1 = \frac{Q}{A_1} = \frac{0.04}{0.07} = 0.56\ m/s.$

$V_2 = \frac{Q}{A_2} = \frac{0.04}{0.017} = 2.264\ m/s.$

Applying Bernoulli's equation at sections 1 & 2 we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2.$$

$$\Rightarrow \frac{P_2}{\rho g} = \frac{400 \times 10^3}{1000 \times 9.81} + \frac{(0.56)^2}{2 \times 9.81} + 10 - \frac{(2.264)^2}{2 \times 9.81} - 6$$

$$= 40.7 + 0.015 + 10 - 0.261 - 6$$

$$\frac{P_2}{\rho g} = 44.54$$

$$\Rightarrow P_2 = 44.54 \times 1000 \times 9.81 = 436.8\ kN/m^2.$$

Prob: A pipe line carrying oil of specific gravity 0.87, changes in diameter from 200 mm diameter at a position A to 500mm diameter at a position B which is 4m at a higher level. If the pressures at A and B are 9.81 N/cm² & 5.88 N/cm² respectively & the discharge is 200 litres/s. Determine the loss of head & direction of flow.

Sol: Given Data

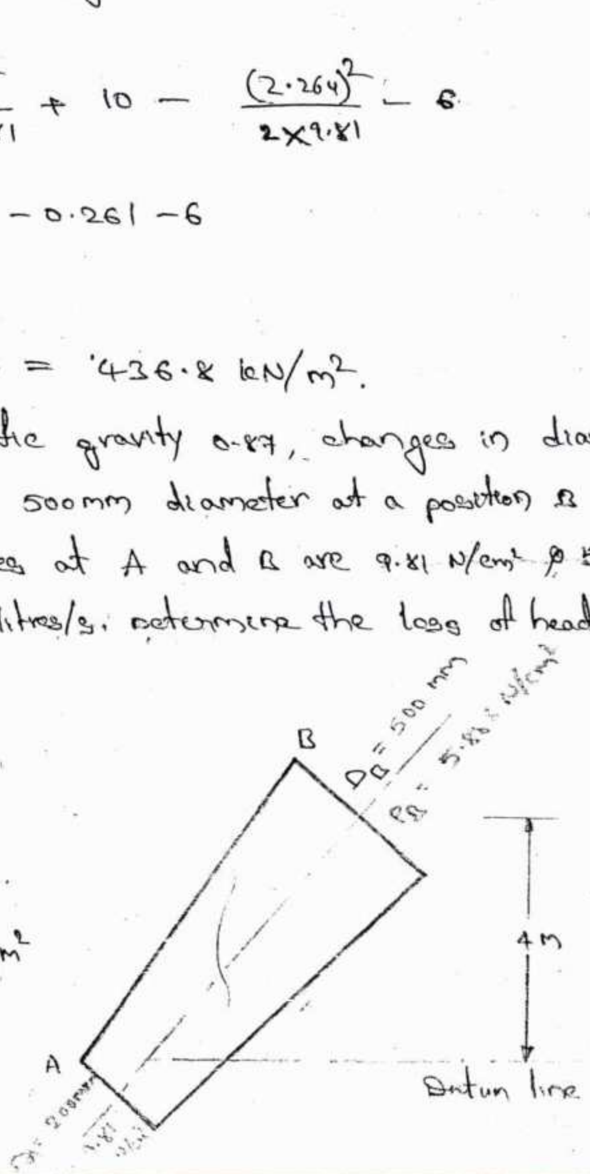
Discharge, $Q = 200\ lit/s = 0.2\ m^3/s.$

Sp. gr. of oil $= 0.87 \Rightarrow \rho = 870\ kg/m^3.$

$D_A = 200 mm \Rightarrow A_A = \frac{\pi}{4}(0.5)^2 = 0.031\ m^2$

$P_A = 9.81\ N/cm^2$

$= 9.81 \times 10^4 N/m^2.$



... the datum line is passing through A, then $Z_A = 0$.

$$V_A = \frac{Q}{A_A} = \frac{0.2}{0.0314} = 6.36 \text{ m/s.}$$ (18)

At section B,

$D_B = 500$ mm, $A_B = \frac{\pi}{4}(0.5)^2 = 0.196 \text{ m}^2$. $P_B = 5.886 \text{ N/cm}^2 = 5.886\times10^4$ N

$Z_B = 4.0$ m. $\quad V_B = \frac{Q}{A_B} = \frac{0.2}{0.196} = 1.018$ m/s.

Total energy at A $= E_A = \frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A$

$$= \frac{9.81\times10^4}{870\times9.81} + \frac{(6.36)^2}{2\times9.81} + 0$$

$$= 13.557 \text{ m}.$$

Total energy at B $= E_B = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B$

$$= \frac{5.886\times10^4}{870\times9.81} + \frac{(1.018)^2}{2\times9.81} + 4.0 = 10.948 \text{ m}$$

i) Direction of flow. $\quad E_A > E_B \therefore$ flow is from A to B.

ii) Loss of head $= h_L = E_A - E_B = 13.557 - 10.948 = 2.609$ m.

MOMENTUM EQUATION:

Momentum equation is based on the momentum principle, which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction.

The force acting on a fluid mass 'm' is given by the newton's second law of motion,

$$f = m\times a.$$

But $\quad a = \frac{dv}{dt}$

$$f = m.\frac{dv}{dt}$$

$$f = \frac{d(mv)}{dt} \rightarrow \text{equation. (a)}$$

Equ. (a) is known as the momentum principle & also can be written as $f.dt = d(mv)$ which is known as impulse-momentum equation.

Impulse momentum equations states that the impulse of a force F acting on a fluid of mass m in a short interval of time dt is equal to the change of momentum $d(mv)$ in the direction of force.

force exerted by a flowing fluid on a pipe-bend:

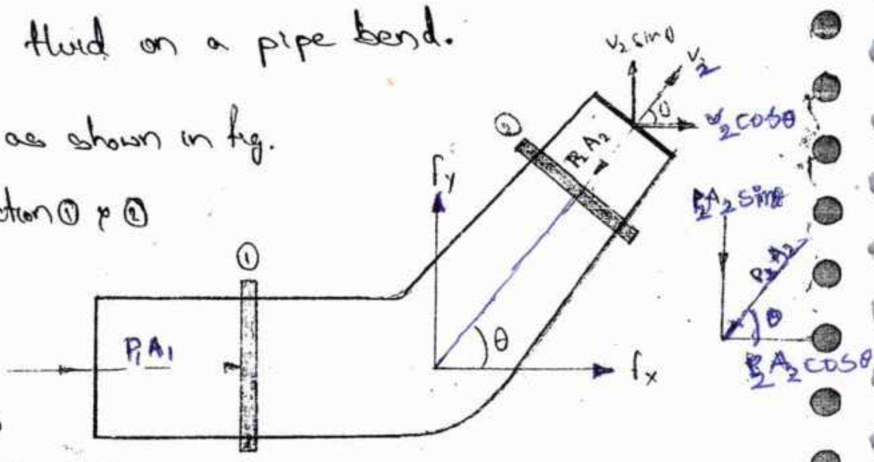
The impulse momentum equation is used to determine the resultant force exerted by a flowing fluid on a pipe bend.

Consider two sections ① & ② as shown in fig.

let V_1 & V_2 = velocity of flow at section ① & ②

P_1 & P_2 = pressure intensity at section ① & ②.

A_1 & A_2 = Area of cross-section of pipe at section ① & ②.



forces on bend.

Let f_x & f_y be the components of the forces exerted by the flowing fluid on the bend in x & y directions.

force exerted by the bend on the fluid in the direction of x and y will be equal to f_x & f_y but in opposite directions.

momentum equation in x-direction is given by

net force acting on fluid in the direction of x = Rate of change of momentum in x-

$$\therefore P_1A_1 - P_2A_2\cos\theta - f_x = (\text{mass per sec})(\text{change of velocity})$$

$$= \rho Q(\text{final velocity in the direction of } x - \text{initial velocity in the direction})$$

$$= \rho Q(V_2\cos\theta - V_1)$$

$\rho Q(V_1 - V_2\cos\theta) + P_1A_1 - P_2A_2$

$$f_x = \rho Q(V_1 - V_2\cos\theta) + P_1A_1 - P_2A_2\cos\theta.$$

similarly momentum equation in y-direction gives

$$\therefore f_y = \rho Q(-V_2\sin\theta) - P_2A_2\sin\theta.$$

Now the resultant force F_R acting on the bend $= \sqrt{f_x^2 + f_y^2}$

Angle made by the resultant force with horizontal direction $\boxed{\tan\theta = \frac{f_y}{f_x}}$

Prob: A 45° reducing bend is connected in a pipe line, the diameters at the inlet & outlet of the bend being 600 mm & 300 mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet to bend is 8.829 N/cm² & rate of flow of water is 600 litres/sec. (19)

Sol: Given Data,

Angle of bend, $\theta = 45°$

Dia. at inlet, $D_1 = 600mm = 0.6m$

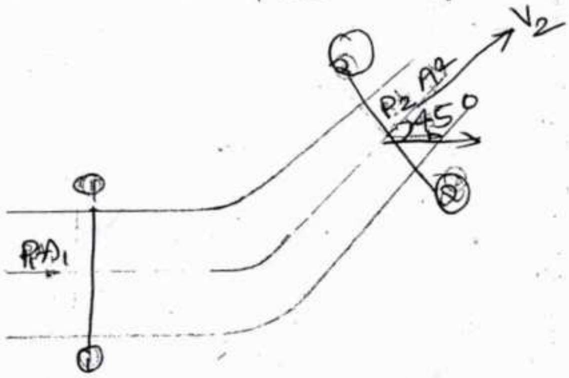
$\therefore$ Area, $A_1 = \frac{\pi}{4}(D_1)^2 = 0.2827 \, m^2$.

Dia. at outlet, $D_2 = 300 \, mm = 0.30m$

$\therefore$ Area. $A_2 = \frac{\pi}{4}(3)^2 = 0.07068 \, m^2$.

Pressure at inlet, $P_1 = 8.829 \times 10^4 \, N/m^2$.

Discharge $Q = 600 \, lit/s = 0.6 \, m^3/s$.



$V_1 = \frac{Q}{A_1} = \frac{0.6}{0.2827} = 2.122 \, m/s$. $V_2 = \frac{Q}{A_2} = 8.488 \, m/s$.

Applying Bernoulli's equation at ① & ②, we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

But

$$\therefore \frac{8.829 \times 10^4}{1000 \times 9.81} + \frac{(2.122)^2}{2 \times 9.81} = \frac{P_2}{\rho g} + \frac{(8.488)^2}{2 \times 9.81}$$

$$9 + 0.2295 = P_2/\rho g + 3.672$$

$$\Rightarrow \quad P_2 = 5.55 \times 1000 \times 9.81 = 5.45 \times 10^4 \, N/m^2.$$

Forces on the bend in x-direction is given as

$$F_x = \rho Q \, [V_1 - V_2 \cos\theta] + P_1 A_1 - P_2 A_2 \cos\theta$$

$$= 1000 \times 0.6 \, (2.122 - 8.48 \cos 45°) + 8.829 \times 10^4 \times 0.2827 - 5.45 \times 10^4 \times 0.0706 \times \cos 45°$$

$$= 19911.4 \, N.$$

Force on the bend in y-direction, $F_y = \rho Q \, (-V_2 \sin\theta) - P_2 A_2 \sin\theta$

$= -6322.2 \, N$. (F_y is in ↓ direction)

$\therefore$ Resultant force $F_R = \sqrt{F_x^2 + F_y^2}$

$= 20890.9 \, N$.

Angle made by the resultant force with x-axis $\tan\theta = \frac{F_y}{F_x} \Rightarrow \theta = 17° 36'$.

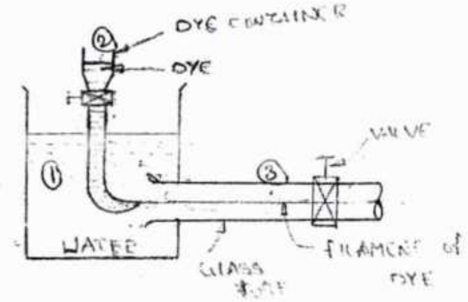
REYNOLDS EXPERIMENT :-

The apparatus consists of :

A tank containing water at constant head,

A small tank containing some dye.

A glass tube having a bell-mouthed entrance at one end & a regulating valve at other ends.

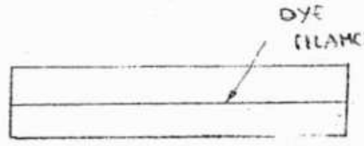


The water from the tank was allowed to flow through the glass tube. The velocity of flow was varied by the regulating valve.

A liquid dye having same specific weight as water was introduced into the glass tube as shown in fig.

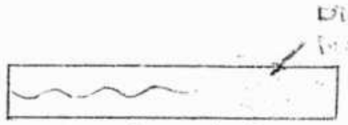
The following observations were made by Reynold :

1) when the velocity of flow was low, the dye filament in the glass tube was in the form of a straight line. This straight line of dye filament was parallel to the glass tube, which was the case of laminar flow.



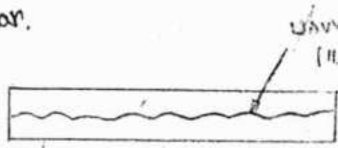
a) Laminar flow

2) with the increase of velocity of flow, the dye filament was no longer a straight-line but it became a wavy one. this shows that flow is no longer laminar.



b) Turbulent flow.

3) With further increase of velocity of flow, the wavy dye-filament broke-up and finally diffused in water.



b) Transition flow.

This means that the fluid particles of the dye at this higher velocity are moving in random fashion, which shows the case of turbulent flow.

In case of laminar flow, the loss of pressure head was found to be proportional to the velocity but in case of turbulent flow, Reynold observ that loss of head is approximately proportional to the square of velo more exactly the loss of head, $h_f \propto V^n$, where n varies from 1.75 to 2.0.

... EQUATION :-

Darcy - Weisbach equation is used for finding loss of head due to f[...] pipes.

EXPRESSION FOR LOSS OF HEAD DUE TO FRICTION IN PIPES :-

consider a uniform Horizontal pipe, having steady flow as shown in [...]

let 1-1 & 2-2 are two sections of pipe.

let P_1 = pressure intensity at section 1-1

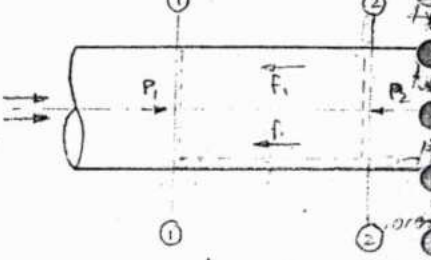
V_1 = velocity of flow at section 1-1.

L = length of the pipe between sections 1-1 & 2-2.

d = diameter of pipe.

f' = frictional resistance per unit wetted area per unit velocity.

h_f = loss of head due to friction.



uniform horizontal

& P_2, V_2 = are values of pressure intensity & velocity at section 2-2.

Applying Bernoulli's equations between sections 1-1 & 2-2,

Total head at 1-1 = Total head at 2-2 + loss of head due to friction 1-1 & 2-2

or $\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$

But $z_1 = z_2$ as pipe is horizontal. & $V_1 = V_2$ as diameter of pipe

$$\therefore \quad h_f = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} \longrightarrow \text{equati. ①}$$

But "h_f" is the head lost due to friction & hence intensity of p[...] will be reduced in the direction of flow by frictional resistance.

Now frictional resistance = frictional resistance per unit wetted area per unit velocity × wetted area × $(\text{velocity})^2$

$\therefore \quad F_1 = f' \times \pi d L \times V^2$ [$\because$ wetted area = $\pi d \times L$, velocity $V = V_1 = V_2$, $\therefore \pi d$ = perimeter

the forces acting on the fluid between sections 1-1 & 2-2 are:

1. pressure force at section 1-1 = $P_1 \times A$ $\quad \therefore A$ = Area of

2. pressure force at section 2-2 = $P_2 \times A$

3. frictional force F_1 as shown in fig.

Resolving all forces in the horizontal direction.

$P_1 A - P_2 A - F_1 = 0$ (or) $(P_1 - P_2)A = F_1 = f' \times P \times L \times V^2$

(or) $$P_1 - P_2 = \frac{f' \times P \times L \times V^2}{A}$$ (2)

But from equ. (1), $P_1 - P_2 = \rho g h_f$

Equating the value of $(P_1 - P_2)$, we get

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \longrightarrow \text{equation (2)}$$

In equation (2), $\frac{P}{A} = \frac{\text{wetted perimeter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{4}{d}$.

$$\therefore h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times V^2 = \frac{f'}{\rho g} \times \frac{4LV^2}{d} \rightarrow (3)$$

placing $\frac{f'}{\rho g} = \frac{f}{2}$, where f is known as co-efficient of friction

Equ (3) becomes $$h_f = \frac{4 \cdot f}{2g} \cdot \frac{LV^2}{d} = \frac{4 f L V^2}{2g \times d} \longrightarrow (4)$$

equation (4) is known as Darcy-Weisbach equation.

This equation is commonly used for finding loss of head due to friction in pipes.

Sometimes equation (4) is written as

$$\boxed{h_f = \frac{f_1 L V^2}{2g d}}$$

f_1 is known as friction fa
($f_1 = 4 f$)

Prob:- An oil of sp. gr. ~~0.7~~ 0.9 is flowing through a pipe of diameter ~~300 mm~~ 500 mm at the rate of ~~500~~ 700 lit/s. Find the head lost due to fric & power required to maintain the flow for a length of ~~1000 m~~ 1900 m. Take ν = ~~.29~~ stokes. $\nu = 0.5$ stokes

Sol: Given Data sp. gr. of oil, $S = 0.7$.

Dia. of pipe, $d = 300$ mm $= 0.3$ m.

Discharge, $Q = 500$ lit/s $= 0.5$ m^3/s.

Length of pipe, $L = 1000$ m.

Velocity, $V = \frac{Q}{Area} = \frac{0.5}{\frac{\pi}{4}d^2} = \frac{0.5\times 4}{\pi \times 0.3^2} = 7.073\ m/s$.

$\therefore$ Reynold number, $R_e = \frac{V\times d}{\nu} = \frac{7.073\times 0.3}{0.29\times 10^{-4}} = 7.316\times (10)^4$

$\therefore$ Coefficient of friction, $f = \frac{0.079}{R_e^{1/4}} = \frac{0.79}{(7.316\times 10^4)^{1/4}} = 0.0048$

$\therefore$ Head lost due to friction, $h_f = \frac{4\times f\times L\times V^2}{2g\times d} = \frac{4\times 0.0048\times 1000\times 7.073^2}{0.3\times 2\times 9.81}$

$= 163.18\ m$.

power required $= \frac{\rho g \cdot Q \cdot h_f}{1000}$ kw

where ρ = density of oil $= 0.7\times 1000$
$= 700$

$\therefore$ power required $= \frac{700\times 9.81\times 0.5\times 163.18}{1000}$

$= 560.28$ kw.

[for laminar flow
$f = \frac{16}{Re}$ $Re < 2000$
for turbulent
$Re = 4000$ to 10^6

MINOR ENERGY LOSSES :-

The loss of head or energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of flowing fluid in magnitude or direction is called minor loss of energy.

1) Loss of head due to sudden enlargement.
2) " " " " sudden contraction.
3) " " " at the entrance to a pipe.
4) " " " at the exit of a pipe.
5) " " " due to an obstruction in a pipe.
6) " " " due to bend in the pipe.
7) " " " in various pipe fittings.

Loss of Head Due to Sudden Enlargement :-

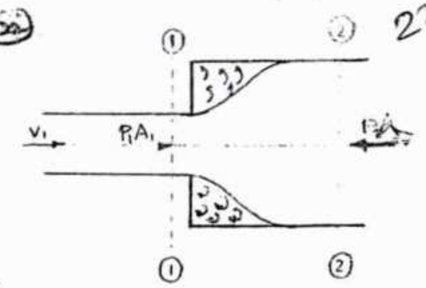
Consider a liquid flowing through a pipe which has sudden enlargement as shown in fig. consider two sections 1-1 & 2-2 before & after the enlargement.

Let P_1 = Pressure intensity at section 1-1

V_1 = velocity of flow at section 1-1,

A_1 = Area of pipe at section 1-1,

P_2, V_2 & A_2 = corresponding values at section 2-2.



sudden enlargement

Due to sudden change of diameter of the pipe from D_1 to D_2, the liquid flowing from the smaller pipe is not able to follow the abrupt change of the bounda Thus the flow separates from the boundary & turbulent eddies a formed.

Let p' = Pressure intensity of the liquid eddies on the area (A_2-A_1)

h_e = loss of head due to sudden enlargement.

Applying Bernoulli's equation to sections 1-1 & 2-2

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+\text{loss of head due to sudden enlargement}$$

But $z_1=z_2$ as pipe is horizontal.

$$\Rightarrow \quad h_e=\left(\frac{P_1-P_2}{\rho g}\right)+\left(\frac{V_1^2-V_2^2}{2g}\right) \longrightarrow ①$$

Consider the control volume of liquid between sections 1-1 & 2-2 Then the force acting on the liquid in the control volume in th direction of flow is given by

$$F_x = P_1A_1 + p'(A_2-A_1) - P_2A_2$$

But experimentally $p' = P_1$

$$F_x = P_1A_1 + P_1(A_2-A_1) - P_2A_2 \Rightarrow \cancel{P_1A_1} + \underline{P_1A_2} - \cancel{P_1A_1} - \underline{P_2A_2}$$

$$= (P_1-P_2)A_2 \longrightarrow ② \qquad (P_1-P_2)A_2$$

momentum of liquid/sec at section 1-1 = mass × velocity

$$= \rho A_1V_1 \times V_1$$

$$= \rho A_1V_1^2$$

Momentum of liquid/sec at section 2-2 = $\rho A_2 V_2 \times V_2 = \rho A_2 V_2^2$

$\therefore$ change of momentum/sec $= \rho A_2 V_2^2 - \rho A_1 V_1^2$

But from continuity equation, we have

$$A_1 V_1 = A_2 V_2 \quad \text{or} \quad A_1 = \frac{A_2 V_2}{V_1}$$

$\therefore$ change of momentum/sec $= \rho A_2 V_2^2 - \rho \times \frac{A_2 V_2}{V_1} \times V_1^2$

$$= \rho A_2 \left[V_2^2 - V_1 V_2\right]$$

Now net force acting on the control volume in the direction of flow must be equal to the rate of change of momentum.

$$(P_1 - P_2)\cancel{A_2} = \rho \cancel{A_2} \left[V_2^2 - V_1 V_2\right]$$

(or)

$$\frac{P_1 - P_2}{\rho} = V_2^2 - V_1 V_2$$

Dividing by 'g' to both sides, we have

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1 V_2}{g}$$

$$\Rightarrow \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g}$$

$$\Rightarrow \frac{V_1^2 + V_2^2 - 2V_1V_2}{2g}$$

$$\Rightarrow \frac{(V_1 - V_2)^2}{2g}$$

Substituting the value of $\left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right)$ in equation ①, we have

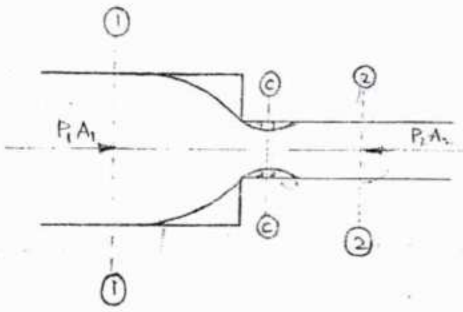
$$h_e = \left(\frac{V_2^2}{g} - \frac{V_1 V_2}{g}\right) + \left(\frac{V_1^2 - V_2^2}{2g}\right) \Rightarrow \frac{2V_2^2 - 2V_2V_1 + V_1^2 - V_2^2}{2g}$$

$$\therefore \boxed{h_e = \cancel{\frac{(V_1 - V_2)^2}{2g}}} = \frac{(V_1 - V_2)^2}{2g} \Rightarrow \frac{V_1^2 + V_2^2 - 2V_1V_2}{2g}$$

$$\Rightarrow \frac{(V_1 - V_2)^2}{2g}$$

Loss of Head Due to sudden contraction :-

Consider a liquid flowing in a pipe which has sudden contraction in area as shown in fig.



Consider two sections 1-1 & 2-2 before & after contraction.

As the liquid flows from large pipe to smaller pipe, the area of flow goes on decreasing & becomes minimum at section c-c as shown in fig.

This section c-c is called vena-contracta. After section c-c, a sudd enlargement of the area takes place. The loss of head due to sudd contraction is actually due to sudden enlargement from vena-contrac to smaller pipe. (23) 23

Let A_c = Area of flow at section c-c.

V_c = Velocity of flow at section c-c.

A_2 = Area of flow at section 2-2.

V_2 = Velocity of flow at section 2-2.

h_c = Loss of head due to sudden contraction.

Now h_c = actually loss of head due to enlargement from section c-c to section 2-2 & is given by $h_e = \frac{(V_1-V_2)^2}{2g}$

$$= \frac{(V_c-V_2)^2}{2g} = \frac{V_2^2}{2g}\left[\frac{V_c}{V_2}-1\right]^2 \quad \text{------} \ (1).$$

From continuity equation, we have

$$A_cV_c = A_2V_2 \quad \text{or} \quad \frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{(A_c/A_2)} = \frac{1}{C_c} \qquad \left[\because V_c = \frac{A_2}{A_c}\right.$$

substituting the value of $\frac{V_c}{V_2}$ in (1), we get

$$h_e = \frac{V_2^2}{2g}\left[\frac{1}{C_c}-1\right]^2$$

$$= \frac{kV_2^2}{2g}, \quad \text{where } k = \left[\frac{1}{C_c}-1\right]^2$$

If the value of C_c is assumed to be equal to 0.62, then

$$k = \left[\frac{1}{0.62}-1\right]^2 = 0.375$$

then $h_c = \frac{kV_2^2}{2g} = 0.375\frac{V_2^2}{2g}$.

If the value of C_c is not given then the head loss due to contraction is taken as $\boxed{h_c = 0.5\frac{V_2^2}{2g}}$

Loss of Head at the entrance of a pipe :

This is the loss of energy which occurs when a liquid enters a pipe which is connected to a large tank or reservoir. This loss is similar to the loss of head due to sudden contraction. This loss depends on form of entrance.

In practice the value of loss of head at the entrance (or inlet) of pipe with sharp cornered entrance is taken as

$$h_i = 0.5 \frac{V^2}{2g}$$ where, V = velocity of liquid in

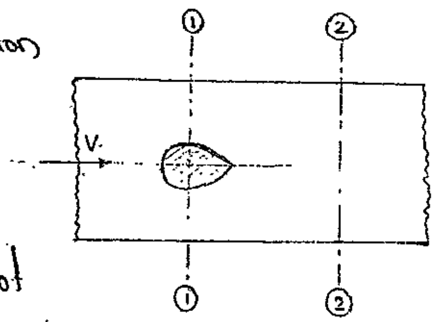
Loss of Head at the Exit of a pipe :

This is the loss of head due to the velocity of liquid at outlet of the pipe which is dissipated either in the form of a free jet or it lost in the tank or reservoir. This loss is equal to $\frac{V^2}{2g}$, where V is the velocity of liquid at the outlet of pipe.

$$\therefore h_o = \frac{V^2}{2g}$$ where V = velocity at outlet of pipe

Loss of Head Due to an obstruction in a pipe :

Whenever there is an obstruction in a pipe, the loss of energy takes place due to reduction of the area of the cross-section of the pipe at the place where obstruction is present. There is a sudden enlargement of the area of flow beyond the obstruction due to which loss of head takes place as shown in fig.



An obstruction in a pipe

Consider a pipe of area of cross-section A having an obst

Let a = Maximum area of obstruction.

A = Area of pipe.

V = velocity of liquid in pipe.

Then $(A-a)$ = Area of flow of liquid at section 1-1.

As the liquid flows and passes through section 1-1, a vena-con is formed beyond section 1-1, after which the stream of liquid widens again & velocity of flow at sections 2-2 becomes uniform equal to velocity, v in the pipe. This situation is similar to the f of liquid through sudden enlargement.

Let V_c = velocity of liquid at vena-contracta.

Loss of head due to obstruction = loss of head due to enlargemen from vena-contracta to section 2-2.

$$= \frac{(V_c-V)^2}{2g} \longrightarrow ①$$

From continuity, we have $a_c \times V_c = A \times V \longrightarrow ②$ where a_c = area of cross-se at vena-contr

-at C_c = coefficient of contraction.

then $$C_c = \frac{\text{area at vena-contracta}}{(A-a)} = \frac{a_c}{(A-a)}$$

$$\therefore \quad a_c = C_c \times (A-a) \longrightarrow ③$$

substituting ③ in ②, we get

$$C_c \times (A-a) \times V_c = A \times V$$

$$\therefore \quad V_c = \frac{A \times V}{C_c(A-a)}$$

substituting the value of V_c in equation ③, we get

$$\text{Head loss due to obstruction} = \frac{(V_c-V_2)^2}{2g} = \frac{v^2}{2g}\left(\frac{A}{C_c(A-a)}-1\right)^2$$

Loss of head due to Bend in a pipe:-

When there is any bend in a pipe, the velocity of flow changes, due to which the separation of the flow from the boundary & als formation of eddies takes place. Thus the energy is lost.

Loss of head in pipe due to bend is given as

$$h_b = \frac{kv^2}{2g}$$ where h_b = loss of head due to ben
v = velocity of flow,
k = coefficient of bend.

Loss of Head in various Pipe fittings:

loss of head in various pipe fittings such as valves, couplings etc is expressed as $= \frac{kv^2}{2g}$

k = coefficient of pipe fitt

M-I
prob: A horizontal pipe line 40 m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. for the first 25 m of its length from the tank, the pipe is 150 mm diameter & its diameter is suddenly enlarged to 300 mm. the height of water level in the tank is 8 m above the centre of pipe. considering all losses of head which occur, determine the rate of flow. Take $f = 0.01$ for both sections of pipe.

Sol: Given:

Total length of pipe, $L = 40$ m.

Length of 1st pipe, $L_1 = 25$ m.

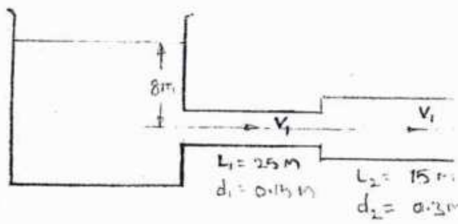
Dia. of 1st pipe, $d_1 = 150$ mm $= 0.15$ m.

Length of 2nd pipe, $L_2 = 40 - 25 = 15$ m

Dia. of 2nd pipe, $d_2 = 300$ mm $= 0.3$ m.

Height of water, $H = 8$ m.

Co-efficient of friction, $f = 0.01$



Applying Bernoulli's theorem to the free surface of water in the tank & outlet of pipe as shown in fig.

Taking the reference line passing through the centre of pipe

$$0 + 0 + 8 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + 0 + \text{all losses}$$

or. $8.0 = 0 + \frac{V_2^2}{2g} + h_i + h_{f_1} + h_e + h_{f_2}$ → equ. ①

But from continuity equation, we have

$$A_1V_1 = A_2V_2$$

$$\therefore \quad V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4} \times (0.3)^2 \times V_2}{\frac{\pi}{4} \times (0.15)^2} = 4 V_2$$

substituting the value of V_1 in different head losses, we have

h_{f_1} = head lost due to friction in pipe 1 = $\frac{4 \times f \times L_1 \times V_1^2}{d_1 \times 2g} = \frac{4 \times 0.01 \times 25}{0.15 \times 2 \times}$

$$= 106.67 \frac{V_2^2}{2g}.$$

h_i = loss of head at entrance = $0.5 \frac{V_1^2}{2g} = 0.5 \times \frac{(4V_2)^2}{2g} = \frac{8V_2^2}{2g}$

h_e = loss of head due to sudden enlargement = $\frac{(V_1 - V_2)^2}{2g} = \frac{(4V_2 - V_2)^2}{2g} = \frac{9}{2}$

h_{f_2} = Head lost due to friction in pipe 2 = $\frac{4 f_2 L V^2}{2g d_2} = \frac{4 \times 0.01 \times 15 \times V_2^2}{0.3 \times 2 \times 9.}$

$$= 2 \frac{V_2^2}{2g}.$$

substituting the values of these losses in equ. ①, we get

$$8.0 = \frac{V_2^2}{2g} + \frac{8V_2^2}{2g} + 106.67 \frac{V_2^2}{2g} + \frac{9V_2^2}{2g} + 2 \times \frac{V_2^2}{2g}$$

$$= V^2/2g \, [1 + 8 + 106.67 + 9 + 2] \Rightarrow 126.67 \frac{V_2^2}{2g}$$

$$8.0 = 126.67 \frac{V_2^2}{2g} \Rightarrow V_2^2 = \frac{2g(8.0)}{126.67} \Rightarrow V_2 = \sqrt{\frac{2 \times 9.81 \times 8}{126.67}}$$

$$\Rightarrow \quad V_2 = \sqrt{\frac{8 \times 2 \times 9.81}{126.67}} = 1.113 \text{ m/s.} \qquad = 1.113 \text{ m/sec}$$

$\therefore$ Rate of flow, $Q = A_2 \times V_2 = \frac{\pi}{4} \times (0.3)^2 \times 1.113 = 78.67$ litres/s

HYDRAULIC GRADIENT AND TOTAL ENERGY LINE :-

The concept of hydraulic gradient line & total energy line is very useful in the study of flow of fluids through pipes.

Hydraulic gradient line :- It is defined as the line which gives the [sum] of pressure head $\left(\frac{p}{w}\right)$ & datum head (z) of a flowing fluid in a pipe with respect to some reference line.

(or)

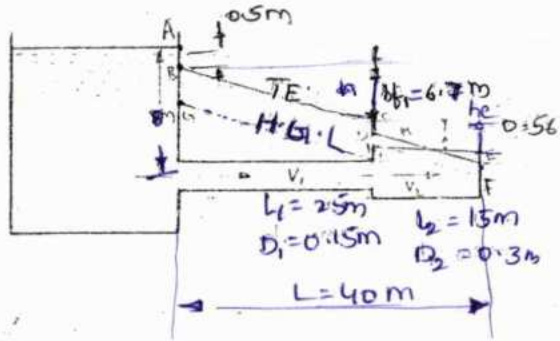
It is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head (p/w) of a flowing fluid in a pipe from the centre of pipe. (It is written as H.G.L).

Total Energy line :- It is defined as the line which gives the sum of pressure head, datum head & kinetic head of a flowing fluid in a pipe with respect to some reference line. It is briefly written as T.E.L.

prob :- Refer to the problem III. Hi-I, draw the hydraulic gradient line

sol :- The various head losses are :

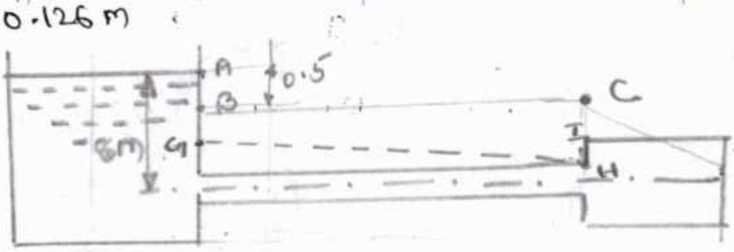
$$h_1 = \frac{8 \cdot V_2^2}{2g} = \frac{8 \times (1.11)^2}{2 \times 9.81} = 0.5\,m$$



$$h_{f_1} = 106.6 \frac{V_2^2}{2g} = \frac{106.6 \times (1.11)^2}{2 \times 9.81} = 6.7\,m$$

$$h_e = \frac{9V_2^2}{2g} = \frac{9 \times (1.11)^2}{2 \times 9.81} = 0.56\,m$$

$$h_{f_2} = \frac{2\,V_2^2}{2g} = \frac{2 \times (1.11)^2}{2 \times 9.81} = 0.126\,m$$



To draw T.E.G.L & H.G.L

Total energy line

1) point A lies on free surface of water
2) Take $AB = h_1 = 0.5\,m$.
3) from B, draw a horizontal line. Take BL equal to the length pipe, i.e, L_1. from L draw a vertical line downward.
4) cut the line $LC = h_{f_1} = 6.73\,m$
5) join the point B to C. from C, take a line CD vertically downward equal to $h_e = 0.568\,m$.

5) from D, draw DM horizontal and from point F which is lying on the centre of the pipe, draw a vertical line in the upward direc meeting at M. from M, take a distance ME = h_{f_2} = 0.126. Join DE.

(26)

Line ABCDE represents T.E.L.

2,2000

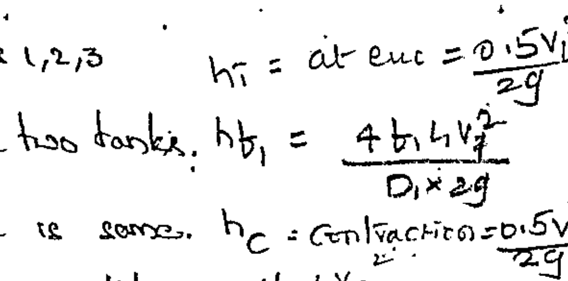
Hydraulic gradient line :

1) from B, take $BG = \frac{V_1^2}{2g} = 1.0m$.
2) draw the line GH parallel to the line BC.
3) from F, draw a line FI parallel to the line ED.
4) Join the point H & I.

Line GHIF represents H.G.L.

FLOW THROUGH PIPES IN SERIES:(or) COMPOUND PIPES:-

Pipes in series or compound pipes is defined as the pipes of different lengths & different diameters connected end to end (in s to form a pipe line.



let, L_1, L_2, L_3 = length of pipes 1, 2 & 3 respectively.

d_1, d_2, d_3 = diameter of pipes 1, 2 & 3 respectively.

V_1, V_2, V_3 = Velocity of flow through pipes 1, 2, 3

f_1, f_2, f_3 = coefficient of frictions for pipes 1, 2, 3

H = difference of water level in the two tanks.

h_i = at enc $= \frac{0.5V_1^2}{2g}$

$h_{f_1} = \frac{4f_1L_1V_1^2}{D_1 \times 2g}$

The discharge passing through each pipe is same.

h_c = contraction $= \frac{0.5V_2^2}{2g}$

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

$h_{f_2} = 4f_2LV_2$

h_e = enlarge $= \frac{(V_2-V_3)^2}{2g}$

The difference in liquid surface levels is equal to the sum of the total head loss in the pipes

$h_{f_3} = \frac{4f_3L_3V_3^2}{D_3 \times 2g}$

$$H = h_i + h_{f_1} + h_c + h_{f_2} + h_e + h_{f_3} + \frac{V_3^2}{2g}$$

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2g \times d_1} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2g \times d_2} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{2g \times d_3} + \frac{V_3^2}{2g}$$

If minor losses are neglected, then the above equation becomes as

$$H = \frac{4f_1L_1V_1^2}{2g\times d_1} + \frac{4f_2L_2V_2^2}{2g\times d_2} + \frac{4f_3L_3V_3^2}{2g\times d_3} \rightarrow \text{eqn (1)}$$

If the coefficient of friction is same for all pipes

i.e., $f_1 = f_2 = f_3 = f$, then eqn (1) becomes

$$H = \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right]$$

Prob:- Three pipes of diameters 300mm, 200mm & 400mm & lengths 450m, 255m & 315m respectively are connected in series. The difference of water surface levels in two tanks is 18m. Determine the rate of flow of water if co-efficient of friction are 0.0075, 0.0078 & 0.0072 respectively considering: 1) minor losses also & and 2) neglecting minor losses

Sol:- Given Data

Difference of water level, $H = 18m$

Length & dia. of pipe 1 : $L_1 = 450m$, $D_1 = 0.3m$, $f_1 = 0.0075$.

Length, dia. & co. of friction for pipe 2 : $L_2 = 255m$, $D_2 = 0.2m$, $f_2 = 0.0078$.

" " " " " 3 : $L_3 = 315m$, $D_3 = 0.4m$, $f_3 = 0.0072$.

1) Rate of flow of water considering minor losses:

Let V_1, V_2 & V_3 be the velocities in 1st, 2nd & 3rd pipes respectively

from continuity, we have $A_1V_1 = A_2V_2 = A_3V_3$.

$$\therefore V_2 = \frac{A_1V_1}{A_2} = \frac{\pi/4\times D_1^2\times V_1}{\pi/4\times D_2^2} = \frac{(0.3)^2}{(0.2)^2}\times V_1 = 2.25V_1$$

$$\& \quad V_3 = \frac{A_1V_1}{A_3} = \frac{D_1^2}{D_3^2}\times V_1 = \frac{(0.3)^2}{(0.4)^2}\times V_1 = 0.5625V_1$$

We know that
$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2g\times D_1} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2g\times D_2} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{2g\times D_3} + \frac{V_3^2}{2g} \ \ldots\ldots$$

$$\Rightarrow 18 = \frac{0.5 V_1^2}{2g} + \frac{4 \times 0.0075 \times 450 \times V_1^2}{0.3 \times 2g} + \frac{0.5 \times (2.25 V_1)^2}{2g} + \frac{4 \times 0.0078 \times 255 \times (2.25 V_1)^2}{0.2 \times 2g}$$

$$+ \frac{(2.25V_1 - 0.5625 V_1)^2}{2g} + \frac{4 \times 0.0072 \times 315 \times (0.5625 V_1)^2}{0.4 \times 2g} + \frac{(0.5625 V_1)^2}{2g}$$

27

$$18 = \frac{V_1^2}{2g}\left(0.5 + 45 + 2.53 + 201.4 + 2.847 + 7.176 + 0.316\right)$$

$$= 259.77 \frac{V_1^2}{2g}$$

$$\Rightarrow \quad V_1 = \sqrt{\frac{18 \times 2 \times 9.81}{259.77}} = 1.166 \text{ m/s}.$$

$\therefore$ Rate of flow, $Q = A_1 V_1 = (\pi/4) \times (0.3)^2 \times 1.166 = 0.0824 \text{ m}^3/\text{s}.$

II) Rate of flow neglecting minor losses:

we know that, $H = \frac{4 f_1 L_1 V_1^2}{2g \times D_1} + \frac{4 f_2 L_2 V_2^2}{2g \times D_2} + \frac{4 f_3 L_3 V_3^2}{2g \times D_3}$

$$\rightarrow \quad 18 = \frac{V_1^2}{2g}\left[\frac{4 \times 0.0075 \times 450}{0.3} + \frac{4 \times 0.0078 \times 255 \times 2.5^2}{0.2} + \frac{4 \times 0.0072 \times 315 \times (0.5}{0.4}\right.$$

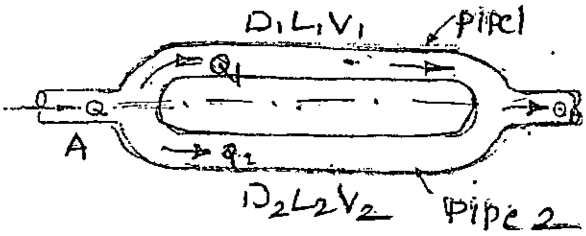
$$\rightarrow \quad V_1 = \sqrt{\frac{18 \times 2 \times 9.81}{253.57}} = 1.18 \text{ m}.$$

$\therefore$ Discharge $Q = A_1 V_1 = (\pi/4) \times (0.3)^2 \times 1.18 = 0.0834 \text{ m}^3/\text{s}.$

Flow Through parallel pipes :-

consider a main pipe which divides into two or more branches as shown in fig. & again join together downstream to form a single pipe, then the branches are said to be connected in parallel.

The rate of flow in the main pipe is equal to the sum of flow through branch pipes.



Thus $Q = Q_1 + Q_2$.

when the pipes are arranged in parallel, the loss of head in each pipe is same.

∴ Loss of head in pipe 1 = Loss of head in pipe 2.

$$\therefore \quad h_f = \frac{4 f_1 L_1 V_1^2}{2g \times D_1} = \frac{4 f_2 L_2 V_2^2}{2g \times D_2}$$

when $f_1 = f_2$, then

$$\frac{L_1 V_1^2}{d_1 \times 2g} = \frac{L_2 V_2^2}{2g \times d_2}$$

prob:- A main pipe divides into two parallel pipes which again forms one pipe. The length & diameter for the first parallel pipe are 2000 m & 1.0 m respectively. While the length & diameter of 2nd parallel pipe are 2000 m & 0.8 m. Find the rate of flow in each parallel pipe, if total flow is in the main is 3.0 m³/s. The coeff. of friction for each parallel pipe is same & equal to .005.

sol:- Given Data

Length of pipe 1, $L_1 = 2000$ m   pipe 2: $L_2 = 2000$ m.

Dia. of pipe 1, $D_1 = 1$ m   pipe 2: $D_2 = 0.8$ m

Discharge $Q = 3.0$ m³/sec

Let $f_1 = f_2 = f = .005$

Let Q_1 = Discharge in pipe 1.

Q_2 = Discharge in pipe 2.

$\Rightarrow \quad Q = Q_1 + Q_2 = 3.0$

Also we have

$$\frac{4 f_1 L_1 V_1^2}{2g \times D_1} = \frac{4 f_2 L_2 V_2^2}{2g \times D_2}$$

$f_1 = f_2 = 0.005$, $L_1 = L_2 = 2000$ m.

$$\Rightarrow \quad \frac{4 \times 0.005 \times 2000 \times V_1^2}{2g \times 1} = \frac{4 \times 0.005 \times 2000 \times V_2^2}{2g \times 0.8}$$

$$\Rightarrow \quad V_1^2 = 1 \times \frac{V_2^2}{0.8}$$

$$\Rightarrow \quad V_1 = \frac{V_2}{.894}$$

Now $Q_1 = \frac{\pi}{4} \times D_1^2 \times V_1 = \frac{\pi}{4} \times (1)^2 \times \frac{V_2}{0.894}$

$Q_2 = \frac{\pi}{4} \times 0.8^2 \times V_2 = \frac{\pi}{4} \times 0.64 \times V_2$

substituting Q_1 & Q_2 in $Q = Q_1 + Q_2$

$\Rightarrow \quad 3.0 = \frac{\pi}{4} \times 1 \times \frac{V_2}{0.894} + \frac{\pi}{4} \times 0.64 \times V_2$

$\Rightarrow \quad V_2 = \frac{3.0}{1.3841} = 2.17 \text{ m/s}.$

substituting the value V_2 in $V_1 = \frac{V_2}{0.894}$

$\Rightarrow \quad V_1 = 2.427 \text{ m/s}.$

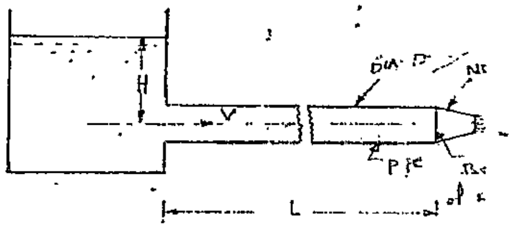
Hence $Q_1 = \frac{\pi}{4} D_1^2 \times V_1 = \frac{\pi}{4} \times 1^2 \times 2.427 = 1.906 \text{ m}^3/\text{sec}$

$Q_2 = \frac{\pi}{4} D_2^2 \times V_2$ or,

$Q = Q_1 + Q_2 \Rightarrow Q_2 = Q - Q_1 = 1.094 \text{ m}^3/\text{sec}$

Flow Through Nozzles :-

A Nozzle is a tapering mouthpiece, which is fitted to the outlet end of a pipe. The total energy at the end of a pipe consists of pressure energy & kinetic energy. By fitting the nozzle at the end of a pipe, the total energy is converted into kinetic energy. Thus nozzles are used where higher velocities of flow are requir[ed]. Examples are:



Nozzle fitted to a pipe

1. In case of pelton turbine, the nozzle is fitted at the end of the pipe to increase velocity.
2. In case of the extinguishing fire, a nozzle is fitted at the end of the hose to increase velocity.

Let D = diameter of the pipe, L = length of pipe,

A = area of pipe $= \frac{\pi}{4} D^2$, V = Velocity of flow in a pipe.

H = total head at the inlet of the pipe.

d = diameter of nozzle at ~~rest~~ outlet.

v = velocity of flow at outlet of nozzle,

a = area of nozzle at outlet $= \frac{\pi}{4} d^2$, f = coefficient of friction for pipe

Head lost due to friction in pipe, $h_f = \frac{4fLV^2}{2g \times D}$

∴ Head available at the end of a pipe or at the base of a N

= Head at inlet of pipe − Head lost due to fric

$= H - h_f = H - \left[\frac{4fLV^2}{2gD}\right]$.

Assuming minor losses and losses in the nozzle to be negligible we have

Total head at the nozzle outlet $= \frac{v^2}{2g}$

$$H = h_f + \frac{v^2}{2g} = \frac{4fLV^2}{2g \times D} + \frac{v^2}{2g} \quad \text{------ (1)}$$

from continuity equation, we have

$$AV = av$$

$$\Rightarrow V = \frac{av}{A} \longrightarrow (2)$$

substituting (2) in (1), we get

$$H = \frac{v^2}{2g} + \frac{4fL}{2gD} \times \left(\frac{av}{A}\right)^2 = \frac{v^2}{2g} + \frac{4fLa^2v^2}{2g \times D \times A^2}$$

$$v = \sqrt{\frac{2gH}{\left[1 + \frac{4fL}{D} \times \frac{a^2}{A^2}\right]}} \quad \text{------ (3)}$$

∴ Discharge through nozzle = $a \times v$.

Power Transmitted through Nozzle :-

(29)

The kinetic energy of the jet at the outlet of nozzle $= \frac{1}{2} mv^2$

Now mass of liquid at the outlet of the nozzle per second $= \rho a v$

$\therefore$ kinetic energy of the jet at the outlet per sec. $= \frac{1}{2} \rho a v \times v^2$

$= \frac{1}{2} \rho a v^3$.

$\therefore$ power in kW at the outlet of nozzle $= (\text{k.e/sec}) \times \frac{1}{1000}$

$= \dfrac{\frac{1}{2} \rho a v^3}{1000}$

$\therefore$ Efficiency of power transmission through nozzle,

$$\eta = \frac{\text{power at outlet of nozzle}}{\text{power at inlet of pipe}} = \frac{\frac{1}{2} \rho a v^3/1000}{\frac{\rho g \cdot Q \cdot H}{1000}}$$

$$= \frac{\frac{1}{2} \rho a v \cdot v^2}{\rho g \cdot a v \cdot H} \qquad [\because Q = av]$$

$$= \frac{v^2}{2gH} = \left[\frac{1}{1 + \frac{4fL}{D} \times \frac{a^2}{A^2}}\right]$$

Condition for maximum power transmitted through Nozzle :-

The total head at inlet of pipe = total head at the outlet of nozzle + losses.

i.e, $H = \frac{v^2}{2g} + h_f$

$$\rightarrow \frac{v^2}{2g} = H - \frac{4fLV^2}{2g \times D}$$

But power transmitted through nozzle $= \dfrac{\frac{1}{2} \rho a v^3}{1000}$

$$= \frac{\frac{1}{2} \rho a v}{1000} \times v^2$$

$$= \frac{\frac{1}{2} \rho a v}{1000} \left[2g \cdot \left[H - \frac{4fLV^2}{2g \times D}\right]\right]$$

$$= \frac{\rho g a v}{1000} \left[H - \frac{4fLV^2}{D \times 2g} \right]$$

Now from continuity equation, we have

$$AV = av$$

$$\rightarrow \quad V = \frac{av}{A}$$

substituting the value of V in equation, we get

power transmitted through nozzle $= \frac{\rho g a v}{1000} * \left[H - \frac{4fLa^2}{D \times 2g} \cdot \frac{v^2}{A^2} \right]$

power (P) will be maximum, when $\frac{d(P)}{dv} = 0$

$$\therefore \quad \frac{d}{dv} \left[\frac{\rho g a v}{1000} \left[H - \frac{4fL}{2g \times D} \cdot \frac{a^2 v^2}{A^2} \right] \right] = 0$$

$$\rightarrow \quad \frac{\rho g a}{1000} \left[H - 3 \cdot \frac{4fL}{D \times 2g} \cdot \frac{a^2 v^2}{A^2} \right] = 0.$$

$$\rightarrow \quad H - 3 \cdot \frac{4fL}{D \times 2g} \cdot V^2 = 0 \qquad \because V = \frac{av}{A}$$

$$H - 3h_f = 0. \qquad \left[h_f = \frac{4fLV^2}{2g \times D} \right] \therefore \text{head in P}$$

$$\rightarrow \quad \boxed{h_f = \frac{H}{3}}$$

gives the condition for maximum power transmitted through nozzle.

It states that power transmitted through nozzle is maximum when the head lost due to friction is one-third of the total head supplied at the inlet of pipe.

Prob:- The rate of flow of water through a pipe of length 2000 m and diameter 1 m is 2 m³/s. At the end of the pipe a nozzle of outside diameter 300 mm is fitted. Find the power transmitted through the nozzle if the head of water at inlet of the pipe is 200 m & co-efficient of friction for pipe is 0.01. (30)

Sol:- Given Data Length of pipe, L = 2000 m.

$D = 1m$, $Q = 2\ m^3/s$. $d = 300mm = 0.3m$. $H = 200m$. $f = 0.01$

$A = \frac{\pi}{4} D^2 = 0.7854\ m^2$

velocity of water through pipe, $V = \frac{Q}{A} = \frac{2.0}{0.7854} = 2.546\ m/s$

power transmitted through nozzle is given as

$$P = \frac{\rho g Q V}{1000} \cdot \left[H - \frac{4fLV^2}{D \times 2g}\right] \quad [\because Q \cdot V =$$

$$= \frac{1000 \times 9.81 \times 2.0}{1000} \cdot \left[200 - \frac{4 \times 0.01 \times 2000 \times (2.546)^2}{1 \times 2 \times 9.81}\right]$$

$$= 3405.43\ kw.$$

Measurement of flow :-

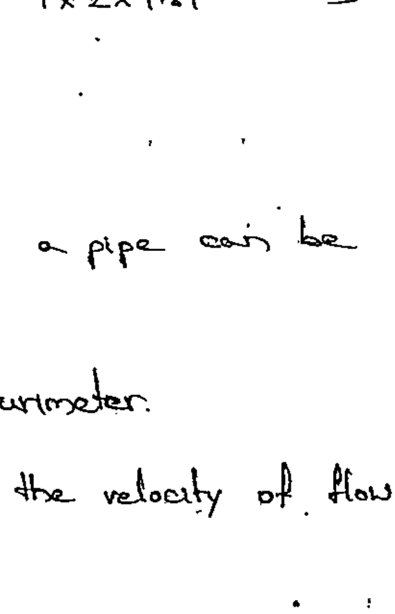
Rate of flow of a fluid flowing through a pipe can be measured by using the devices

1) pitot tube   2) orificemeter   3) Venturimeter

pitot tube :- It is a device used for measuring the velocity of flow at any point in a pipe or a channel.

pitot - tube consists of a glass tube, bent at right angles as shown in f

It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the k.E into pressure energy.



PITOT TUBE

The lower end is directed in the up-stream direction. The liquid rises up in the tube due to the conversion of k.E into pressure energy.

The velocity is determined by measuring the rise of liquid in the tube.
consider two points (1) & (2) at the same level in such a way that point (2) is just as the inlet of the pitot-tube & point (1) is far away from the tube.

let P_1 = intensity of pressure at point (1), P_2 = pressure at point (2)
v_1 = velocity of flow at (1) v_2 = velocity at (2), which is
H = depth of tube in the liquid.
h = rise of liquid in the tube above the free surface.

Applying Bernoulli's equations at points (1) & (2), we get

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 .$$

$\therefore$ $z_1 = z_2$ (points 1 & 2 are on same line & $v_2 = 0$).

$$\frac{P_1}{\rho g} = \text{pressure head at (1)} = H$$

$$\frac{P_2}{\rho g} = \text{pressure head at (2)} = (h+H).$$

$$\Rightarrow \quad H + \frac{v_1^2}{2g} = h + H \quad \therefore \quad h = \frac{v_1^2}{2g} \text{ or } v_1 = \sqrt{2gh}$$

this is theoretical velocity.

- Actual velocity is given by

$$(V)_{act} = C_v \cdot \sqrt{2gh} \qquad C_v = \text{co-efficient of pitot tube.}$$

$\therefore$ velocity at any point $v = C_v\sqrt{2gh}$.

prob:- A pitot static tube placed in the centre of a 300 mm pipe line has an orifice pointing upstream & other perpendicular to it. the mean velocity in the pipe is 0.80 of the central velocity. find the discharge through the pipe if the pressure difference between the two orifices 60 mm of water. Take the co-efficient of pitot tube as $C_v = 0.98$.

sol:- Given Data dia. of pipe, $d = 300$ mm $= 0.30$ m
diff of pressure head, $h = 60$ mm of water $= .06$ m of water
$C_v = 0.98$
mean velocity $\bar{V} = 0.80 \times$ central velocity.

central velocity is given by equation = $C_v \cdot \sqrt{2gh}$

$= 0.98 \times \sqrt{2 \times 9.81 \times 0.06} = 1.063$ m/s.

(31)

$\therefore \bar{V} = 0.80 \times 1.063 = 0.8504$ m/s.

Discharge Q = Area of pipe $\times \bar{V} = \frac{\pi}{4} \times d^2 \times \bar{V} = \frac{\pi}{4}(0.30)^2 \times 0.8504$

$= 0.06$ m³/s.

prob:- A pitot tube is inserted in a pipe of 300 mm diameter. The stat pressure in pipe is 100 mm of mercury (vacuum). The stagnation pressu at the centre of the pipe, recorded by the pitot-tube is 0.981 N/cm². Calculate the rate of flow of water through pipe, if the mean veloci of flow is 0.85 times the central velocity. Take $C_v = 0.98$.

sol:- Given Data dia. of pipe d = 300 mm = 0.3 m.

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.3)^2 = 0.07068$ m².

Static pressure head = 100 mm of mercury (vacuum)

$= -\frac{100}{1000} \times 13.6 = -1.36$ m of water.

Stagnation pressure = 0.981 N/cm² = $.981 \times 10^4$ N/m²

$\therefore$ stagnation pressure head $= \frac{.981 \times 10^4}{\rho g} = \frac{.981 \times 10^4}{1000 \times 9.81} = 1$ m.

h = stagnation pressure head − static pressure head.

$= 1.0 - (-13.6) = 2.36$ m of water.

$\therefore$ velocity at centre $= C_v \sqrt{2gh}$

$= 0.98 \times \sqrt{2 \times 9.81 \times 2.36} = 6.668$ m/s.

mean velocity, $\bar{V} = 0.85 \times 6.668 = 5.6678$ m/s.

$\therefore$ Rate of flow of water $= \bar{V} \times$ area of pipe

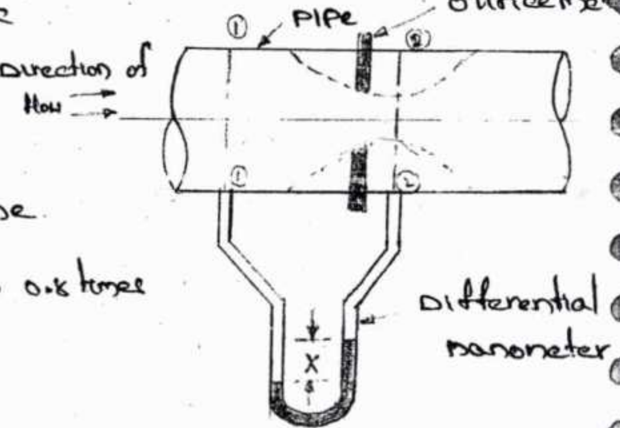
$= 5.6678 \times 0.07068$ m³/s

$= 0.40$ m³/s.

Orificemeter - Orificemeter is a device used for measuring the rate of flow of a fluid through a pipe.

It consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with the pipe.

The orifice diameter is kept generally 0.5 times the diameter of the pipe, though it may vary from 0.4 to 0.8 times the pipe diameter.



A differential manometer is connected at section (1), which is at distance of about 1.5 to 2.0 times the pipe diameter upstream from the orifice plate, & at section (2), which is at a distance of about half the diameter of the orifice on the down stream side from the orifice plate.

Let P_1 & P_2 = pressure at sections (1) & (2).

v_1 & v_2 = velocity at section 1 & 2.

a_1 & a_2 = area of pipe at section 1 & 2.

Applying Bernoulli's equation at sections 1 & 2, we get

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\frac{P_1}{\rho g} + z_1 - \frac{P_2}{\rho g} - z_2 = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $\left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = h$ = differential head.

$$\therefore \quad h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{or} \quad 2gh = v_2^2 - v_1^2$$

$$\therefore \quad v_2 = \sqrt{2gh + v_1^2} \quad \text{—} \ ①$$

Now section (2) is at the vena contracta & a_2 represents the area at the vena contracta.

If a_0 is the area of orifice then, we have $C_c = \frac{a_2}{a_0}$

C_c - coefficient of contraction

$$\therefore \quad a_2 = a_0 \times C_c \quad \text{—— (2)}$$

By continuity equation, we have

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2}{a_1} \cdot v_2 = \frac{a_0 C_c}{a_1} \cdot v_2 \quad \text{—— (3)}$$

substituting the value of v_1 in equation (1), we get

$$v_2 = \sqrt{2gh + \frac{a_0^2 C_c^2 v_2^2}{a_1^2}}$$

$$\therefore \quad v_2^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 C_c^2 v_2^2$$

$$\therefore \quad v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$\therefore$ Discharge $Q = a_2 \times v_2 = v_2 \times a_0 C_c$ $\quad \because a_2 = a_0 C_c$.

$$= \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 \times C_c^2}} \quad \text{—— (4)}$$

The above expression can be simplified by using

$$C_d = C_c \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 \cdot C_c^2}}$$

$$C_c = C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}$$

substituting the value of C_c in (4), we get

$$Q = a_0 \times C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 \cdot C_c^2}}$$

$$Q = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}.$$

where C_d - coefficient, discharge for orifice

The C_d for orificemeter is much smaller than that of a venturimeter.

An orifice meter dia 10cm is inserted in a pipe of 20 cm diameter, the pressure gauges fitted upstream & downstream of orificemeter gives readings of 19.62 N/cm² & 9.81 N/cm² respectively, co-ed of discharge for the meter is given as 0.6. find the discharge of water through pipe.

sol:- Given Data Dia. of orifice, $d_0 = 10cm$ ∴ area, $a_0 = \frac{\pi}{4}(10)^2$
$= 78.54\ cm^2$

Dia. of pipe. $d_1 = 20\ cm$. Area $a_1 = \frac{\pi}{4}(20)^2 = 314.16\ cm^2$.

$P_1 = 19.62\ N/cm^2 = 19.62\times10^4\ N/m^2$. $P_2 = 9.81\times10^4\ N/m^2$.

$h = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{19.62\times10^4}{1000\times9.81} - \frac{9.81\times10^4}{1000\times9.81} = 20.0 - 10.0 = 10\ m$ of w

$C_d = 0.6$.

Discharge $Q = C_d\cdot\frac{a_0 a_1}{\sqrt{a_1^2-a_0^2}}\cdot\sqrt{2gh}$

$= 0.6\times\frac{78.54\times314.16}{\sqrt{(314.16)^2-(78.54)^2}}\times\sqrt{2\times981\times1000}$

$= \frac{20736838.09}{304} = 68213.28\ cm^3/s = 68.21$ litres/s

prob:- An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. the pressure difference measured by a mercury oil differential manometer on the two sides of the orificemeter gives a reading of 50 cm of mercury. find the rate of flow of oil of sp. gr. 0.9 when the coefficient of discharge of the meter = 0.64

sol:- Given Data $d_0 = 15\ cm$; $a_0 = \frac{\pi}{4}(15)^2 = 176.7\ cm^2$

$d_1 = 30\ cm$; $a_1 = \frac{\pi}{4}(30)^2 = 706.85\ cm^2$

sp. gr. of oil, $S_0 = 0.9$

Reading of diff. manometer, $x = 50$ cm of mercury.

∴ Differential head, $h = x\left[\frac{S_g}{S_0}-1\right] = 50\left[\frac{13.6}{0.9}-1\right]$ cm

$= 705.5$ cm of oil.

$C_d = 0.64$

∴ Rate of flow $Q = 0.64\times\frac{176.7\times706.85}{\sqrt{(706.5)^2-(176.7)^2}}\times\sqrt{2\times981\times705.5}$

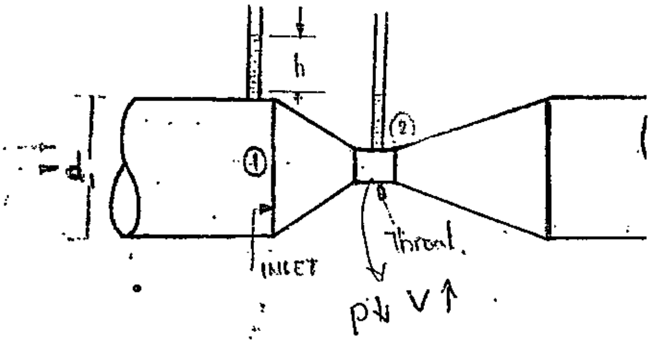
$= 137414.25\ cm^3/s = 137.414$ litres/s.

Venturimeter :- A venturimeter is a device used for measuring the rate of flow of fluid flowing through a pipe. It consists of three parts :

i) converging part ii) throat & iii) diverging part. (33)

It is based on the principle of Bernoulli's equation:

consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing as shown in fig.



Let d_1 = diameter at section (1).

P_1 = pressure at section (1).

v_1 = velocity of fluid at section (1).

a_1 = area at section (1) $= \frac{\pi}{4} d_1^2$

& d_2, P_2, v_2 & a_2 are corresponding values at section (2).

Applying Bernoulli's equation at sections (1) & (2), we get

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$.

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} \quad (\text{or}) \quad \frac{P_1 - P_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$\frac{P_1 - P_2}{\rho g}$ is the difference of pressure heads at sections 1 & 2 & it is equal to h or $\frac{P_1 - P_2}{\rho g} = h$.

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

Now applying continuity equation at sections (1) & (2).

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2 v_2}{a_1}$$

substituting v_1 in above equation.

$$h = \frac{v_2^2}{2g} - \frac{a_2^2 v_2^2}{2g \times a_1^2}$$

$$= \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right]$$

or $v_2 = \frac{a_1}{\sqrt{}}$

$$V_2 = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \cdot \sqrt{2gh}$$

$\therefore$ Discharge $Q = a_2 V_2' = a_2 \cdot \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

Above equation gives the discharge under ideal conditions & is called, theoretical discharge.

Actual discharge $Q_{act} = C_d \times Q_{the}$.

C_d is coefficient of venturi & is < 1.

value of 'h' given by differential U-tube manometer

case 1 :- manometer contains a liquid which is heavier than the liquid flowing through the pipe.

$$h = x\left[\frac{S_h}{S_0} - 1\right].$$

x - difference of heavier liq in u-tube.

S_h - sp. gr. of heavier liq.

S_0 - sp. gr. of liquid flowing in pipe.

case 2 :

$$h = x\left[1 - \frac{S_l}{S_0}\right].$$

case 3 : inclined venturimeter with differential U-tube manom

$$h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_0} - 1\right].$$

case 4 :

$$h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_l}{S_0}\right].$$

prob :- A horizontal venturimeter with inlet & throat diameters 30 cm & 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet & throat is 20 cm of mercury. Determine the rate of flow. Take $C_d = 0.98$.

sol :- Given Data $d_1 = 30$ cm , $a_1 = 706.85$ cm^2

$d_2 = 15$ cm , $a_2 = 176.7$ cm^2

$C_d = 0.98$

Difference of pressure head $h = x\left[\frac{S_h}{S_0} - 1\right]$.

$= 20\left[\frac{13.6}{1} - 1\right] = 252.0$ cm of

Discharge Through venturimeter

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \cdot \sqrt{2gh}$$ (34)

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$= 125756 \ cm^3/sec$

$= 125.756 \ lit/sec.$

Vent-

→ An oil of Sp. gr 0.8 is flowing through a venturimeter having Inlet dia 20cm & throat dia 10cm. The oil mercury differential Manometer shows a reading of 25cm. calculate the discharge of oil through the horizontal venturimeter Take $C_d = 0.98$

Sol: $S_o = 0.8$

$S_h = 13.6$

$x = 25cm$

$h = x\left[\frac{S_h}{S_o} - 1\right]$

$= 400$ cm of oil

$d_1 = 20$ cm

$a_1 = \pi/4 (20)^2 = 314.16$

$d_2 = 10cm$

$a_2 = 78.54$

$C_d = 0.98$

$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$

$= 70465 \ cm^3/sec$

$= 70.465 \ lit$

horizontal Venturimeter with inlet dia 20cm & throat dia 10cm is used to measure the flow of oil Sp. gra 0.8. The discharge of oil through venturimeter is 60 lit/sec find the reading of the oil mercury differential Manometer. Take $C_d = 0.98$

$d_1 = 20$ cm $a_1 = 314.16$

$d_2 = 10$ cm $a_2 = 78.54$

$C_d = 0.98$

$Q = 60 \ lit/sec = 60 \times 1000 \ cm^3/sec$

$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$\sqrt{h} = 17.029 \Rightarrow h^2 = 289.98$

$h = x\left[S_h/S_o - 1\right] \Rightarrow \left[\frac{S_h = 13.6}{S_o = 0.8}\right]$

$289.98 = x\left[\frac{13.6}{0.8} - 1\right] = 16x$

$x = 18.12 cm.$

FORCE Exerted by the Jet on the stationary plate:-

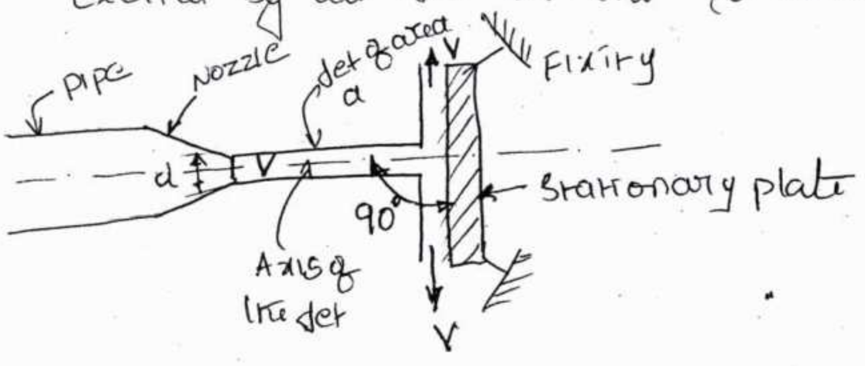


Fig shows a fluid jet striking a stationary flat plate held perpendicular to the flow direction.

Let a and V be cross-sectional area and velocity of the Jet respectively. The jet after striking the plate will get its direction changed through 90°; but, it will move on and off the plate with velocity V, if we neglect the friction b/n the Jet and the plate as is possible when the plate is smooth. If the friction is considered, the velocity of liquid coming off the plate will be slightly less than V.

The force exerted by the Jet on the plate (assuming it smooth) in the direction of Jet (x-direction)

F_x = Rate of change of momentum (in the direction of force)

= (initial momentum − final momentum) → Impulse-momentum principle

= (mass/sec) × [velocity of Jet before striking the plate − velocity of Jet after striking the plate]

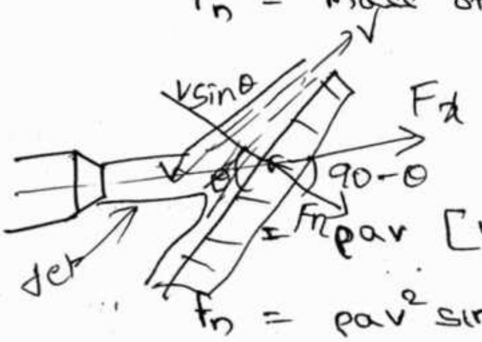
$= \rho a V(V - 0) \Rightarrow F_x = \rho a V^2$

After striking the plate (assuming it smooth), the jet leaves the plate with a velocity equal to initial velocity (v).

Let us apply the impulse - momentum equation in the direction normal to the plate.

Force exerted by the jet on the plate in the direction normal to the plate & is represented by F_n.

F_n = mass of jet striking per second × [Initial velocity of jet before striking in the direction of n — final velocity of jet after striking in the direction of n]



$= \rho a v\ [v\sin\theta - 0]$

$F_n = \rho a v^2 \sin\theta$.

This normal force can be resolved into two components.

F_x component in the direction of jet.
F_y component, normal to the direction of jet.

$$F_x = F_n \cos(90-\theta) = F_n \sin\theta = \rho a v^2 \sin^2\theta$$
$$F_y = F_n \sin(90-\theta) = F_n \cos\theta = \rho a v^2 \sin\theta \cos\theta$$

prob :- A jet of water, 75 mm in diameter, issues with a velocity of 30 m/s and impinges on a stationary flat plate which destroys its forward motion. find the force exerted by the jet on the plate & work done.

sol :- Given Data Diameter of jet, $d = 75\,mm = 0.075\,m$
Velocity of jet, $v = 30\,m/s$.

The force exerted by the jet on a stationary vertical plate is given by $F = \rho a v^2 = 1000 \times \frac{\pi}{4} \times 0.075^2 \times 30^2 = 3976.2\ N$

prob :- A jet of water of diameter 75 mm moving with a velocity 25 strikes a fixed plate in such a way that the angle between the jet & plate is 60°. find the force exerted by the jet on the plate i) in the direction normal to the plate &
ii) in the direction of the jet.

Sol:- Given data Diameter of jet, $d = 75mm = 0.075 m$.

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = 0.004417 m^2$

Velocity of jet, $V = 25 m/s$.

Angle between jet & plate $\theta = 60°$

i) force exerted by the jet of water in the direction normal to the plate

$$f_n = \rho a V^2 \sin\theta = 1000 \times 0.004417 \times 25^2 \times \sin 60°$$

$$= 2390.7 N$$

ii) force in the direction of the jet

$$f_x = \rho a V^2 \sin^2\theta$$

$$= 1000 \times 0.004417 \times 25^2 \sin^2 60° = 2070.4 N.$$

prob:- A jet of water of diameter 50 mm strikes a fixed plate in such a way that the angle b'n the plate & the jet is 30°. The force exerted in the direction of the jet is 1471.5 N. Determine the rate of flow of water.

Sol:- Given Data

1652.7

Diameter of jet $d = 50 mm = 0.050 m$

$\therefore$ Area $a = \frac{\pi}{4} \times (0.05)^2 = .00196 m^2$

Angle $\theta = 30°$

$f_x = 1471.5 N$

force in the direction of jet

$$f_x = \rho a V^2 \sin^2\theta$$

$$1471.5 = 1000 \times 0.00196 \times V^2 \sin^2 30$$

$$V = 54.77 m/s.$$

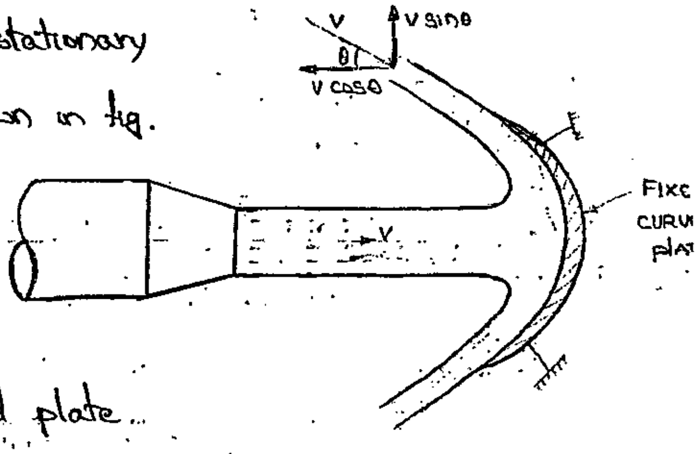
$\therefore$ Discharge Q = Area x velocity

= 107.5 liters/s.

Force exerted by a jet on stationary curved plate :-

Case I :- Jet strikes the curved plate at the centre :-

Consider a fluid jet striking a stationary curved plate at the centre as shown in fig. The jet after striking the plate, comes out with the same velocity if the plate is smooth, in the tangential direction of the curved plate.



The velocity at the outlet of the plate can be resolved into two components,

one in the direction of jet $= -V\cos\theta$.

other perpendicular to the direction of jet $= V\sin\theta$.

Applying impulse momentum equation, we have

Force exerted by the jet in the direction of jet,

$$F_x = \text{Mass per sec} \times [\text{I.V} - \text{F.V}]$$

$$= \rho a V \times [V - (-V\cos\theta)]$$

$$= \rho a V^2 [1 + \cos\theta]$$

III^{ly}

$$F_y = \text{mass per sec} \, [\text{I.V.} - \text{F.V}]$$

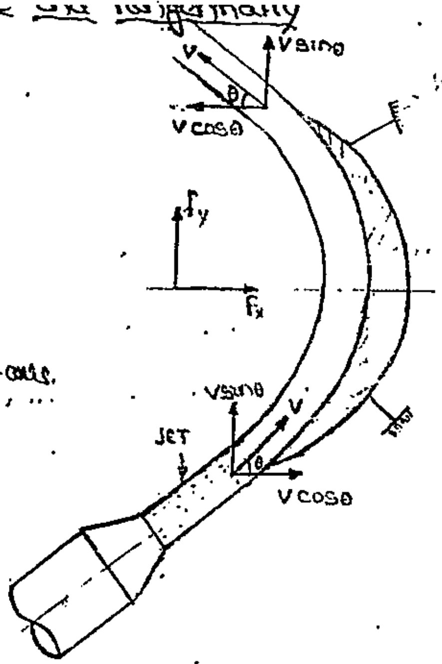
$$= \rho a V \, [0 - V\sin\theta]$$

$$= -\rho a V^2 \sin\theta$$

-ve sign indicates that force is acting in the downward direction.

Note : Angle of deflection of the jet $= (180^\circ - \theta)$.

a) when the plate is symmetrical.



Consider a fluid jet striking a stationary symmetrical curved plate at one ~~jet~~ end tangentially as shown in fig.

let the curved plate is symmetrical about x-axis.

let V = Velocity of the jet

θ = Angle made by the jet with x-axis at inlet tip of the curved plate.

The angle made by the tangents at the two ends of the plate will be same.

If the plate is smooth, the velocity of water at the outlet is equal 'V'.

force exerted by the jet of water in the directions of x & y are

$$f_x = \text{mass/sec} \times [\text{I.V} - \text{F.V}]$$
$$= \rho a v\,[V\cos\theta - (-V\cos\theta)]$$
$$= 2\rho a v^2 \cos\theta.$$

$$f_y = \rho a v\,[V\sin\theta - V\sin\theta] = 0.$$

b) when the plate is unsymmetrical.

when the plate is unsymmetrical about x-axis, then angle made the tangents drawn at the inlet & outlet tips of the plate with x will be different.

Let θ = angle made by the tangent at inlet tip with x-axis

ϕ = angle made " " " " outlet tip "

forces exerted by the jet of water in the directions of x & y

$$f_x = \rho a v\,[V\cos\theta - (-V\cos\phi)] = \rho a v^2\,[\cos\theta + \cos\phi]$$

$$f_y = \rho a v\,[V\sin\theta - V\sin\phi] = \rho a v^2\,[\sin\theta - \sin\phi].$$

$d = 150mm \Rightarrow 0.15 \Rightarrow 0.0176$

$V = \cancel{50}\ 30 m/sec$

$u = 15 m/sec$

leaves the vane at 60°

i) F_x ii) $F_x \times u$.

$\alpha = 0$

$\beta = 180^\circ - 60^\circ$

$= 120^\circ$

$u_1 = u_2 = u = 15 m/s$

$V_{r_1} = AB - AC$

$= V_1 - u_1$

$= 30 - 15 \Rightarrow 15$

$V_{w_1} = V_1 = 30 m/sec$

$V_{r_2} = V_{r_1} = 15$

$V_{r_2} = 15 m/sec$

$u_2 = 15 m/sec$

$\angle GEF = 180^\circ - (60^\circ + \phi)$

$= (120^\circ - \phi)$

89100 W

$\frac{EG}{\sin 60^\circ} = \frac{GF}{\sin(120^\circ - \phi)}$ (or)

$\frac{15}{\sin 60^\circ} = \frac{10}{\sin[120^\circ - \phi]}$

$\sin 60^\circ = \sin[120^\circ - \phi]$

$60^\circ = 120^\circ - \phi$

(or)

$120^\circ - 60^\circ = 60^\circ$

$V_{w_2} = u_2 - V_2 \cos\phi$

$= 15 - 15\cos 60^\circ$

$= 15 - 7.5$

$\Rightarrow 7.5 m/sec$

$F_x = \rho a V_{r_1} [V_{w_1} - V_{w_2}]$

$1000 \times 0.0176 \times 15$

$[30 - 7.5]$

5940 N

407, 432, 413, 439,
416, 458, 459, 442, 401

Prob: A jet of water of diameter 50 mm moving with a velocity of 40 strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet of water in the direction of the jet, if the jet is deflected through an angle of 120° at the outlet of the curved plat

Sol:- Given Data $d = 0.5m$ (38)

$$\text{Area} = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times 0.5^2 = 0.001963\ m^2$$

Velocity of jet, $V = 40\ m/s$

Angle of deflection $= 120°$

angle of deflection $= 180° - \theta$

$$\Rightarrow \theta = 180 - 120 = 60°$$

$$\therefore f_x = \rho a V^2 [1 + \cos\theta] = 1000 \times 0.001963 \times 40^2 [1 + \cos 60°]$$
$$= 4711.15\ N.$$

37

Prob:- A Jet of water of diameter 60 mm moving with a velocity of 40 strikes a curved fixed plate tangentially at one end at an angle of 30° to horizontal. The jet leaves the plate at an angle of 20° to the horizontal. Find the force exerted by the jet on the plate in the horizontal & vertical directions.

Sol:- Given Data Diameter of jet, $d = 0.06\ m$

$$\therefore \text{Area},\ a = \frac{\pi}{4} \times 0.06^2 = 0.002827\ m^2$$

Velocity of the jet, $V = 40\ m/s$.

Angle made by the jet at inlet tip with horizontal, $\theta = 30°$

" " " " " outlet " " " , $\phi = 20°$

Force exerted by the jet in x-direction,

$$\therefore F_x = \rho a v^2 (\cos\theta + \cos\phi) = 8167.6\ N$$

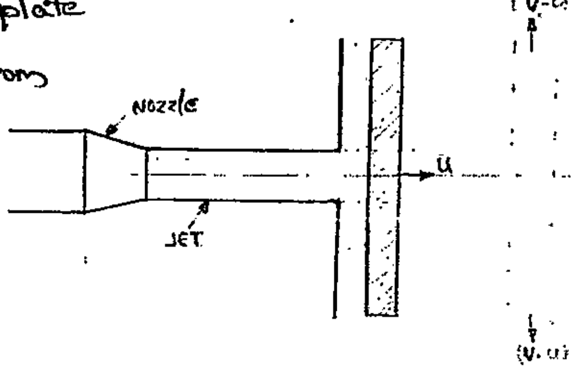
Force exerted by the jet in vertical direction,

$$F_y = \rho a v^2 (\sin\theta - \sin\phi) = 714.57\ N$$

FORCE EXERTED BY JET ON MOVING PLATES :-

i) Force on flat moving plate in the direction of jet :-

A fluid jet striking a flat vertical plate moving with a uniform velocity away from the jet.



Let V = Velocity of jet (absolute).

a = cross - sectional area of jet

u = Velocity of flat plate.

Jet does not strike the plate with a velocity 'V', but it strikes the plate with a relative velocity, which is equal to the absolute velocity of jet minus the velocity of the plate.

The relative velocity which the jet strikes the plate is $(V-u)$

Mass of water striking the plate per second = $\rho \times a \times (V-u)$.

∴ force exerted by the jet on the moving plate in the direction of the jet,

F_x = mass of water striking per sec × [I.V − F.V].

$= \rho a(V-u)\,[(V-u)-0]$

$= \rho a(V-u)^2$

∴ work done by the jet on the plate per second =

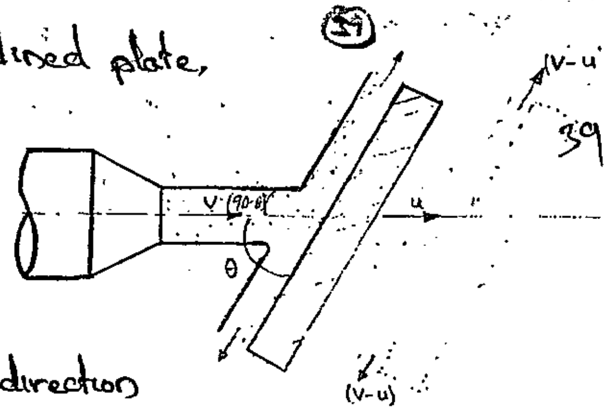
force × $\frac{\text{distance moved by the flat plate}}{\text{time}}$

∴ work done/sec $= F_x \times u$

$= \rho a\,(V-u)^2 \times u$.

Force exerted on a moving plate held inclined to the direction of :

Let a jet of water striking an inclined plate, which is moving with a uniform velocity in the direction of jet.



let V - absolute velocity of jet

u - velocity of plate in the direction of jet

a - cross-sectional area of jet

θ - Angle between jet and plate.

Relative velocity of jet of water = (V-u).

∴ the velocity with which jet strikes = (V-u).

force exerted by the jet on the plate in the direction normal to the plate is given as:

$$F_n = \rho a (V-u) [(V-u) \sin\theta - 0]$$

$$= \rho a (V-u)^2 \sin\theta.$$

components of this force in the direction of jet

$$F_x = F_n \sin\theta = \rho a (V-u)^2 \sin^2\theta.$$

$$F_y = F_n \cos\theta = \rho a (V-u)^2 \sin\theta \cos\theta$$

∴ work done $= F_x \times u = \rho a (V-u)^2 \sin^2\theta \times u$. Nm/s.

prob:- A 75 mm diameter jet having a velocity of 30 m/sec strike a plate, the normal of which is inclined at 45° to the axis of : find the normal pressure on the plate,

i) when the plate is stationary ii) when the plate is moving with a velocity of 15 m/sec in the direction of jet, away from the jet.

Also determine the power & efficiency of the jet when the plate is moving.

Sol:- Given Data diameter of jet, $d = 0.075$ m.

$\therefore$ Area $a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (0.075)^2 = 0.0044$ m^2

Angle between the plate & jet, $\theta = 90° - 45° = 45°$

Velocity of jet $V = 30$ m/sec.

1) When the plate is stationary, the normal force on the plate is:

$$F_n = \rho a V^2 \sin\theta = 1000 \times 0.0044 \times 30^2 \times \sin 45°$$

$$= 2811.6 \text{ N}$$

2) When the plate is moving with a velocity of 15m/s & moving away from the jet.

$$F_n = \rho a (V-u)^2 \sin\theta = 1000 \times 0.0044 (30-15)^2 \cdot \sin 45°$$

$$= 702.9 \text{ N}$$

Work done per second by the jet $= F_x \times u$

$= F_n \sin\theta \times u$

$= 702.9 \sin 45° \times 15$

$= 7455$ Nm/s

$\therefore$ power of jet $= 7.455$ kW.

Efficiency of jet $= \dfrac{\text{Work done/sec}}{\text{K.E. by jet}}$

$$= \frac{7455}{\frac{1}{2}(\rho a V) \cdot V^2}$$

$$= \frac{7455}{\frac{1}{2} \times 1000 \times 0.0044 \times 30 \times 30^2}$$

$= 0.125$ or 12.5%

prob:- A nozzle of 50 mm diameter delivers a stream of water at 20 m/s perpendicular to a plate that moves away from the jet at 5m/s. find force on the plate. work done.

Sol:- Given Data $d = 0.05$ m $a = \frac{\pi}{4} \times 0.05^2 = 0.0019$ m^2

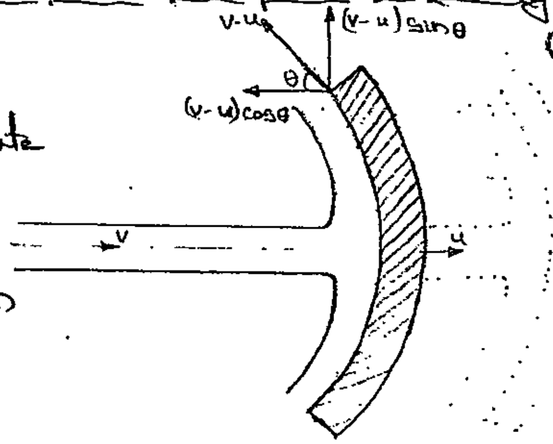
1) force on the plate $F_x = \rho a (V-u)^2 = 1000 \times 0.0019 \times (20-5)^2$

$= 441.78$ N

2) Work done by the jet $= F_x \times u = 2208.9$ Nm/s.

Force exerted on a curved plate when the plate is moving in the direction of jet :-

A jet of water strikes a curved plate at the centre which is moving with a uniform velocity in the direction of jet.



Let V = absolute velocity of jet

a = Area of jet

u = Velocity of plate in the direction of jet.

Relative velocity of jet of water, with which jet of water strikes the plate is $(V-u)$.

The component of this velocity in the direction of jet will be $= -(V-u)\cos\theta$.

Mass of water striking the plate $= \rho a(V-u)$

$\therefore$ Force exerted by the jet on the vane,

F_x = mass/sec × [I.V. with which the jet strikes the vane in the direction of jet − F.V].

$$= \rho a(V-u) \times [(V-u) - (-(V-u)\cos\theta)]$$

$$F_x = \rho a (V-u)^2 [1+\cos\theta].$$

Work done on the vane per second $= F_x \times u$

$$= \rho a (V-u)^2 [1+\cos\theta] \times u$$

Efficiency $\eta = \dfrac{\text{work done / sec}}{\text{K.E supplied by jet}}$

$$= \frac{\rho a (V-u)^2 [1+\cos\theta] \times u}{\frac{1}{2}\rho a V^3}$$

//

prob:- A jet of water of 60 mm diameter strikes a curved vane at its centre with a velocity of 18 m/s. The curved vane is moving with a velocity of 6 m/s in the direction of jet. The jet is deflected through an angle of 165°. Assuming the plate to be smooth find 1) thrust on the plate in the direction of jet 2) power of the jet 3) efficiency of the jet.

sol:- Given Data Diameter of jet $d = 60\,mm = 0.06\,m$

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = 0.0028\, m^2$

$V = 18\, m/s$ $\quad u = 6\, m/sec$. Angle of deflection of jet $= 165°$

$\theta = 180 - 165 = 15°$.

1) Thrust on the plate in the direction of the jet,

$$F_x = \rho a (v-u)^2 [1 + \cos\theta]$$

$$= 1000 \times 0.0028 \times (18-6)^2 [1 + \cos 15°] = 800.3\, N$$

2) power of jet $= F_x \times u = 800.3 \times 6 = 4801.8\, W = 4.8\, kW$

3) efficiency of jet $\eta_{jet} = \dfrac{\text{work done by jet}}{\text{k.E. of the jet}}$

$$= \frac{4801.8}{\frac{1}{2} \times \rho a v^3} = 58.25\,\%$$

prob:- A jet of water of diameter 50mm moving with a velocity 25 m/s impinges on a fixed curved plate tangentially at one end at an angle of 30° to the horizontal. Calculate the resultant force of the jet on the plate if the jet is deflected through an angle of 50°. Take $g = 10\, m/s^2$.

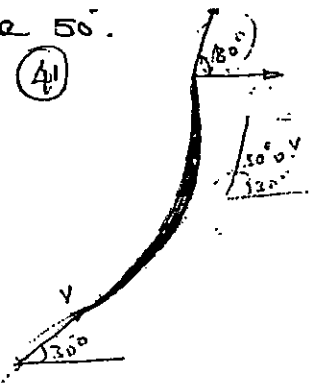
sol:- Given Data Dia. of jet $d = 0.05\, m$, $a = \frac{\pi}{4} \times d^2$

$= 0.00196\, m^2$

Angle made by the jet with horizontal at inlet $\theta = 30°$.

After striking it is deflected through an angle 50°.

$\Rightarrow \quad \phi = \theta + 50^\circ = 80^\circ$..

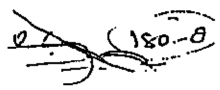


force exerted by the jet of water on plate

$f_x = \rho a V^2 (\cos\theta \mp \cos\phi)$

$= 1000 \times 0.00196 \times 25^2 \ (\cos 30^\circ \mp \cos 80^\circ)$

$= 849.7 \ N$



$f_y = 1000 \times 0.00196 \times 25^2 \ [\sin 30^\circ - \sin 80^\circ]$

$= -594.9 \ N$

Resultant force $f_R = \sqrt{f_x^2 + f_y^2} = 1037 \ N$.

Angle made by the resultant force with horizontal is

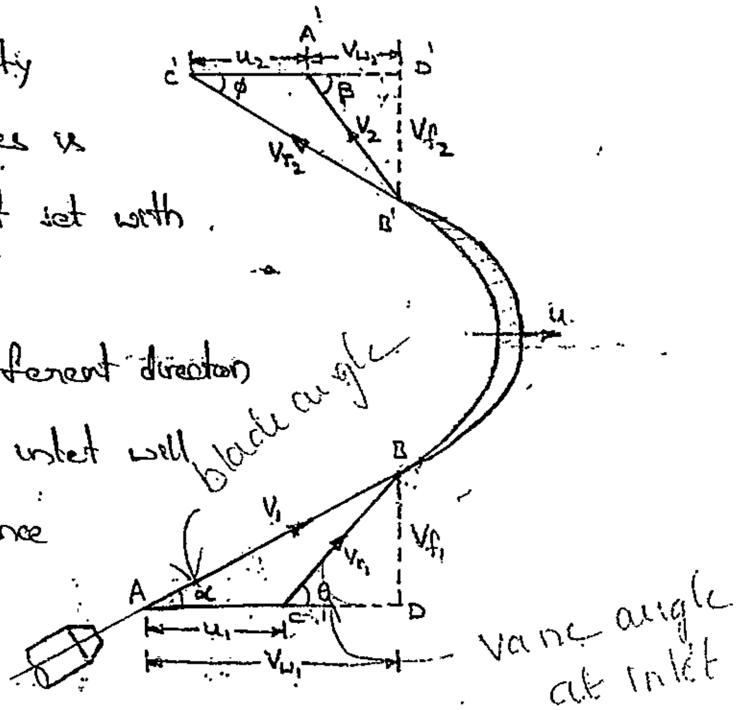
$$\alpha = \tan^{-1}\left(\frac{f_y}{f_x}\right) = 35^\circ.$$

__Force exerted by a jet of water on an un-symmetrical Moving curved plate when jet strikes tangentially at one of the Tip.__

A jet of water striking a curved vane which is moving, tangentially at one tip and leaving at the other.

As plate is moving, the velocity with which jet of water strikes is equal to the relative velocity of jet with respect to plate.

As the plate is moving in different direction of jet, the relative velocity at inlet will be equal to the vector difference of velocity of jet & velocity of plate at inlet.



The relative velocity V_{r_1} may be obtained by drawing the velocity triangle at inlet. the absolute velocity (V_2) at the exit may be obtained by drawing the velocity triangle at the outlet.

Let V_1, V_2 = Absolute velocities of the jet at inlet & outlet.

u_1, u_2 = Velocities of the plate at inlet & outlet.

V_{r_1}, V_{r_2} = Relative velocities of the vane & jet at inlet & outl

V_{f_1}, V_{f_2} = components of absolute velocities perpendicular to the direction of motion at inlet & outlet.

V_{w_1}, V_{w_2} = components of absolute velocities in the direction motion of vane at inlet & outlet.

α = Angle between the direction of jet & direction of motion of plate, also called Guide blade angle.

θ = Angle made by the relative velocity with direction of motion at inlet, also called vane angle at inlet.

β & ϕ be Angles at outlet.

It may be noted that all angles are measured with the direct of motion of vane.

Triangles ABD & A'C'D' are called inlet & outlet velocity triangles and are drawn as follows.

__Inlet Velocity triangle__ :-

- Take any point A & draw a line $AB = V_1$, making an angle with the horizontal line AD.
- Draw a line $AC = u_1$ & join C to B, CB then represents relative velocity of jet at inlet
- From B draw a perpendicular BD meeting the horizontal line produced at D. Then BD represents V_{f_1} & AD represents V_{w_1}

$\angle BCD = \theta$ = vane angle at inlet.

OUTLET VELOCITY TRIANGLE :-

- Draw $B'C'$ in the tangential direction of the vane at outlet & $B'C' = V_{r_2}$.
- from C' draw a line $C'A'$ in the direction of vane at outlet & equal to u_2. Join $B'A'$. $B'A'$ represents V_2 at outlet in magnit & direction.
- from B' draw a perpendicular $B'D'$ to meet the line $C'A'$ produce at D'. then $B'D'$ & $A'D'$ represent V_{f_2} & V_{w_2} at outlet.
- ϕ = angle of vane at outlet, β = angle made by V_2 at outle

If vane is smooth & is having velocity in the direction of moti at inlet & outlet equal then we have

$$u_1 = u_2 = u = \text{velocity of vane in the direction of motion}$$

$$V_{r_1} = V_{r_2}$$

Mass of water striking vane per sec $= \rho a V_{r_1}$

$\therefore$ force exerted by the jet in the direction of motion

$$F_x = \text{mass of water striking the vane} \times (I.V - F.V)$$

$$= \rho a V_{r_1} \left[V_{r_1} \cos\theta - (-V_{r_2} \cos\phi)\right] \quad -(-V\omega_2 + u_2)$$

$V_{r_1} \cos\theta = V_{w_1} - u_1$

$V_{r_2} \cos\phi = V_{w_2} + u_2$

$$= \rho a V_{r_1} \left[V_{w_1} - u_1 + u_2 + V_{w_2}\right] \quad V\omega_1 - u_1 + V\omega_2 + u_2$$

$[\because u_1 = u_2]$

$$F_x = \rho a V_{r_1} (V_{w_1} + V_{w_2})$$

The above equation is true only when β is an acute angl.

when $\beta = 90°$, $V_{w_2} = 0$ then $F_x = \rho a V_{r_1} (V_{w_1})$.

If β is an obtuse angle then $F_x = \rho a V_{r_1} [V_{w_1} - u_1 - (-u_2 + V$

$$F_x = \rho a V_{r_1} \left[V\omega_1 - u_1 - \left[-u_2 + V\omega_2\right]\right] = \rho a V_{r_1} \left[V_{w_1} - u_1 + u_2 - V_{w_2}\right]$$

$$= \rho a V_{r_1} \left[V\omega_1 - \cancel{u_1} + \cancel{u_2} - V\omega_2\right] \quad F_x = \rho a V_{r_1} \left[V_{w_1} - V_{w_2}\right].$$

$$= \rho a V_{r_1} \left[V\omega_1 - V\omega_2\right]$$

from the above equations, general equation can be written as

$$F_x = \rho a V_{r1} [V_{w1} \pm V_{w2}]$$

work done per second by the jet on the vane

$$= F_x \times u$$

$$= \rho a V_{r1} [V_{w1} \pm V_{w2}] \times u$$

work done per second per unit weight of fluid striking

$$= \frac{\rho a V_{r1} (V_{w1} \pm V_{w2}) \times u}{\rho a V_{r1} \times g}$$

$$= \frac{1}{g} (V_{w1} \pm V_{w2}) \times u.$$

Efficiency of jet :- $\eta = \frac{o/p}{i/p}$

$$= \frac{\rho a V_{r1} [V_{w1} \pm V_{w2}] \times u}{\frac{1}{2}(\rho a v_1) \times v_1^2}$$

K.E Supplied

prob:- A jet of water having a velocity of 40 m/s strikes a curved vane, which is moving with a velocity of 20 m/s. The jet makes an angle of 30° with the direction of motion of vane at inlet & leaves at an angle of 90° to the direction of motion of vane at outlet. Draw the velocity triangles at inlet & outlet & determine the vane angles at inlet & outlet so that the water enters & leaves the vane without shock.

sol:- Given Data Velocity of jet, $V_1 = 40$ m/s.

Velocity of vane $u_1 = 20$ m/sec.

Angle made by the jet at inlet, $\alpha = 30°$

Angle made by the jet at outlet $= 90°$

$\therefore \beta = 180° - 90° = 90°$ $\quad u_1 = u_2 = u = 20$ m/s

Vane angles at inlet & outlet:

Vane angles at inlet & outlet are θ & ϕ respectively.

From Δ BCD, we have

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} \qquad V_{f_1} = V_1 \sin\alpha = 40 \times \sin 30^\circ = 20\,m/s$$
$$V_{w_1} = V_1 \cos\alpha = 40 \times \cos 30^\circ = 34.64$$

$$\theta = \tan^{-1}\left(\frac{20}{34.6 - 20}\right)$$
$$= 53.79^\circ$$

From Δ BCD, we have $V_{r_1} = \frac{V_{f_1}}{\sin\theta} = \frac{20}{\sin 53.79^\circ} = 24.78\,m/s.$

$$\cos\phi = \frac{u_2}{V_{r_2}} = \frac{20}{24.78} = 0.8071$$

$$\phi = \cos^{-1}(0.8071) = 36.18^\circ.$$

Prob:- A jet of water having a velocity of 20 m/s strikes a curved vane, which is moving with a velocity of 10 m/s. The jet makes an angle of 20° with the direction of motion of vane at inlet & leaves at an angle of 130° to the direction of motion of va at outlet. Calculate:

1) vane angles, so that the water enters & leaves the vane with shock.

2) work done per second per unit weight of water striking the va per second.

Sol:- Given Data

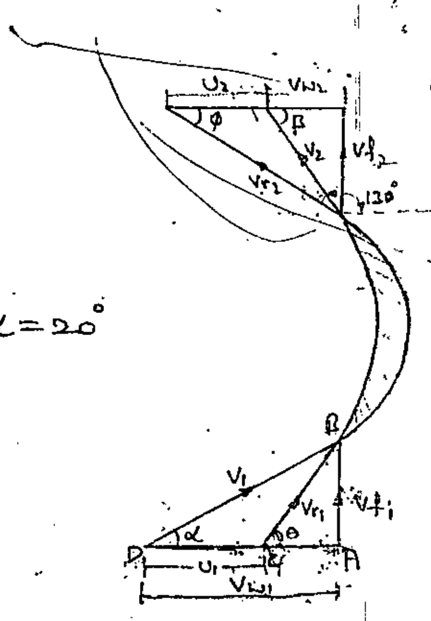
Velocity of jet $V_1 = 20\,m/s$

Velocity of vane $u_1 = 10\,m/s$

Angle made by the jet at inlet, $\alpha = 20^\circ$

Angle made by the leaving jet $= 130^\circ$

$$\beta = 180^\circ - 130^\circ$$
$$= 50^\circ$$



$u_1 = u_2 = 10\,m/s.$

$V_{r_1} = V_{r_2}$

i) vane angles at inlet & outlet i.e., θ & ϕ

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1}$$

$V_{f_1} = V_1 \sin\alpha$

$= 20 \sin 20^\circ = 6.84 \text{ m/s}$

$V_{w_1} = V_1 \cos\alpha$

$= 20 \cos 20^\circ = 18.79 \text{ m/s}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{6.84}{18.79 - 10}\right)$$

$$= 37^\circ \, 52.5'$$

from ΔABC $\sin\theta = \frac{V_{f_1}}{V_{r_1}} \Rightarrow V_{r_1} = \frac{6.84}{\sin 37.5} = 11.14 \text{ m/sec}$

$$V_{r_1} = V_{r_2} = 11.14 \text{ m/s.}$$

Applying sine rule

$$\frac{V_{r_2}}{\sin(180^\circ - \beta)} = \frac{u_2}{\sin(\beta - \phi)}$$

$$\Rightarrow \frac{11.14}{\sin\beta} = \frac{10}{\sin(50 - \phi)}$$

$\because \beta = 50$

$$\Rightarrow \phi = 6^\circ.33$$

$V_{w_2} = V_{r_2}\cos\phi - u_2 = 1.069$

ii) Work done per second per unit weight of water ~~of~~ striking the vane per second is

$$= \frac{1}{g}\left(V_{w_1} \pm V_{w_2}\right) \times u$$

$\because \beta$ is an acute a

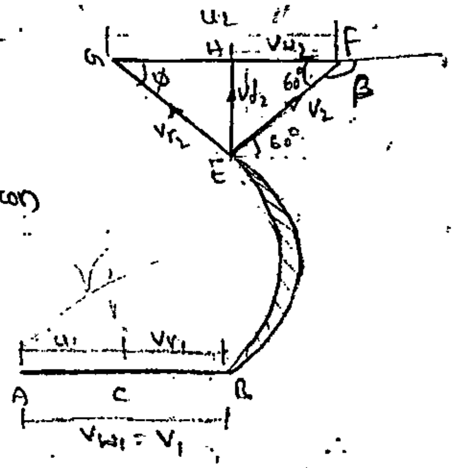
$$= \frac{1}{g}\left(V_{w_1} + V_{w_2}\right) \times u.$$

$$= \frac{1}{9.81}\left[18.79 + 1.06\right] \times 10 = 20.24 \text{ Nm/N}$$

Imp prob :- A jet of water of diameter 50 mm, having a velocity of 20 m/s strikes a curved vane which is moving with a ve of 10 m/s in the direction of jet. The jet leaves the vane c an angle of 60° to the direction of motion of vane at outl. Determine: i) force exerted by the jet on the vane in the direction of ~~set~~ motion.

2) work done per second by the set.

Sol:- Given Data Diameter of set $d = 50\,mm = 0.05\,m$



$\therefore$ Area $a = \frac{\pi}{4}(.05)^2 = .00196\,m^2$

$V_1 = 20\,m/s$, $u_1 = 10\,m/s$

As set & vane are moving in same direction

$\alpha = 0$, $\theta = 0$

Angle made by the set at outlet

$= 60°$

$\beta = 180° - 60° = 120°$

$u_1 = u_2 = u = 10\,m/s$, $V_{r_1} = V_{r_2}$

$V_{r_1} = V_1 - u_1 = 20 - 10 = 10\,m/s$

$V_{w_1} = V_1 = 20\,m/s$

$V_{r_2} = V_{r_1} = 10\,m/s$

ΔEFG ; $EG = V_{r_2} = 10\,m/s$, $GF = u_2 = 10\,m/s$

$\angle GEF = 180° - (60° + \phi) = 120° - \phi$

sine rule, we have

$$\frac{EG}{\sin 60°} = \frac{GF}{\sin(120° - \phi)}$$

$\Rightarrow \phi = 60°$

$V_{w_2} = u_2 - V_{r_2}\cos\phi = 5\,m/s$

1) forced exerted by the set on the vane

$F_x = \rho a V_{r_1}[V_{w_1} - V_{w_2}]$

$= 294.45\,N$

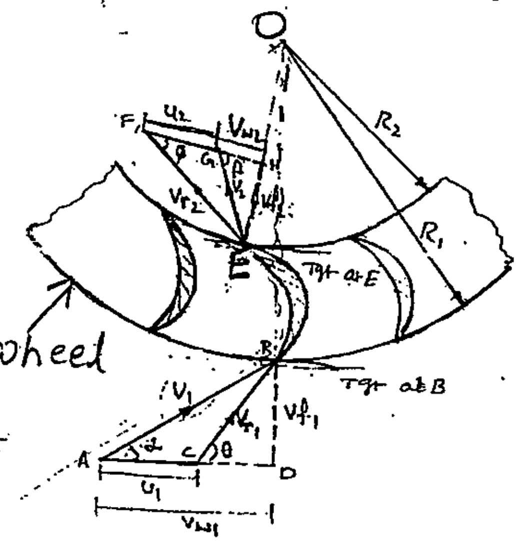
2) work done per second by the set $= F_x \times u$

$= 2944.5\,Nm/sec$

Force exerted on a series of Radial curved Vanes :-

Consider a series of radial curved vanes mounted on a wheel as shown in figure.

The jet of water strikes the vanes & the wheel starts rotating at a constant angular speed.



Let R_1 = Radius of wheel at inlet of vane.

R_2 = Radius of wheel at the outlet of vane

ω = Angular speed of the wheel.

Then $u_1 = \omega R_1$ & $u_2 = \omega R_2$

velocity triangles at inlet & outlet are drawn as shown in fig.

mass of water striking per second for a series of vanes.

$= \rho a V_1$

momentum of water striking the vanes in the tangential direction per sec at inlet $= \rho a V_1 \times V_{w_1}$ $[\because V_1 \cos\alpha = V_{w_1}]$

Angular momentum/s. at inlet = momentum at inlet × Radius at in

$= \rho a V_1 \times V_{w_1} \times R_1$

$|||^{ly}$ Angular momentum per second at outlet $= -\rho a V_1 \times V_{w_2} \times R_2$

Torque exerted by the water on the wheel.

T = Rate of change of Angular momentum.

$= \rho a V_1 V_{w_1} \times R_1 - (-\rho a V_1 V_{w_2} \times R_2)$

$= \rho a V_1 \left(V_{w_1} R_1 + V_{w_2} R_2\right)$

work done per second on the wheel = Torque × Angular vel

$= T \times \omega$

$= \rho a v_1 (V_{w_1} R_1 + V_{w_2} R_2) \times \omega$

$= \rho a v_1 (V_{w_1} \omega R_1 + V_{w_2} \omega R_2)$

$= \rho a v_1 (V_{w_1} U_1 + V_{w_2} U_2)$ [$\because$ β is acute angle]

General expression for work done per second on the wheel

$$= \rho a v_1 (V_{w_1} U_1 \pm V_{w_2} U_2).$$

Efficiency of radial curved vane

$$\eta = \frac{\text{work done per second}}{\text{kinetic energy per second}}$$

$$= \frac{\rho a v_1 (V_{w_1} U_1 \pm V_{w_2} U_2)}{\frac{1}{2}(\rho a v_1) \times v_1^2}$$

$$= \frac{2 [V_{w_1} U_1 \pm V_{w_2} U_2]}{V_1^2}$$

probl A jet of water having a velocity of 30 m/s strikes a seri of radial curved vanes mounted on a wheel which is rotating 200 r.p.m. the jet makes an angle of 20° with the tangent to the wh at inlet & leaves the wheel with a velocity of 5 m/s at an angle 130° to the tangent to the wheel at outlet. water is flowing fr outward in a radial direction. the outer & inner radii of the wheel are 0.5 m & 0.25 m respectively. Determine:
1) vane angles at inlet & outlet, 2) work done per unit weight of w 3) efficiency of the wheel.

sol:- Given Data $V_1 = 30$ m/s $N = 200$ r.p.m $\omega = \frac{2\pi N}{60} = 20.9$

$\alpha = 20^\circ$, $V_2 = 5$ m/s. $\beta = 180 - 130^\circ = 50^\circ$ $R_1 = 0.5$ m $R_2 = 0.25$ m

$u_1 = \omega R_1$ $\quad$ $u_2 = \omega R_2$

$= 20.94 \times 0.5$ $\quad$ $= 20.94 \times 0.25$

$= 10.47$ m/s $\quad$ $= 5.235$ m/s.

1) vane angles at inlet & outlet

$\theta = ?$ $\quad$ $\phi = ?$

from the inlet triangles

$$V_{w_1} = V_1 \cos\alpha = 30 \times \cos 20^\circ = 28.19 \text{ m/s}$$

$$V_{f_1} = V_1 \sin\alpha = 30 \times \sin 20 = 10.26 \text{ m/s}$$

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - U_1} \Rightarrow \theta = \tan^{-1} \frac{10.26}{28.19 - 10.47} = 30.07$$

$$V_{w_2} = V_2 \cos\beta = 5 \times \cos 50^\circ = 3.214 \text{ m/s}.$$

$$V_{f_2} = V_2 \sin\beta = 5 \sin 50^\circ = 3.83 \text{ m/s}.$$

$$\tan\phi = \frac{V_{f_2}}{U_2 + V_{w_2}} \Rightarrow \tan\phi = \frac{3.83}{5.2 + 3.2} \Rightarrow \phi = 24.385$$

2) work done per second on the wheel

$$= \rho a V_1 (V_{w_1} U_1 + V_{w_2} U_2) \qquad \because \beta < 90^\circ$$

work done per second per unit weight of water per second

$$= \frac{1}{g} [V_{w_1} U_1 + V_{w_2} U_2]$$

$$= \frac{1}{9.81} [28.19 \times 10.47 + 3.21 \times 5.23]$$

$$= 31.8 \text{ Nm/N}.$$

3) ~~efficiency~~ $\eta = \frac{2[V_{w_1} U_1 \pm V_{w_2} U_2]}{V_1^2} = \frac{2[28.19 \times 10.47 + 3.21 \times 5.}{30^2}$

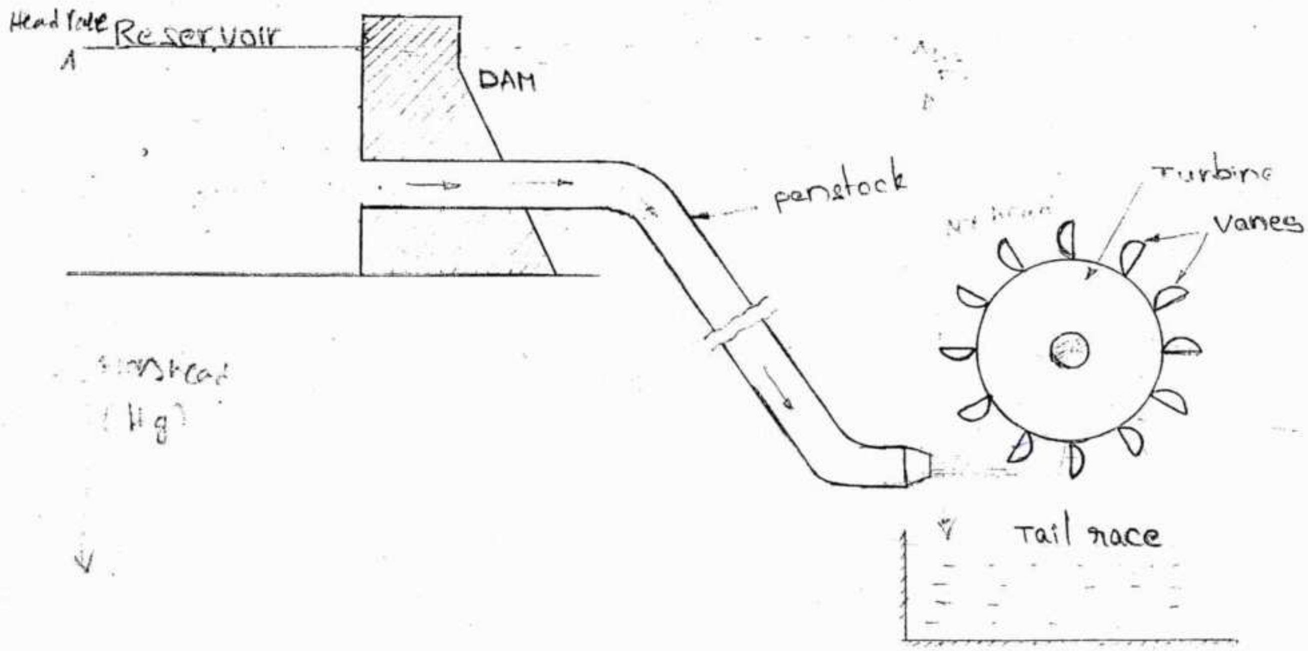
$$= 69.32\ \%$$

INTRODUCTION: The electric power which is obtained from the hyd energy is known as Hydro electric power.

A plant is a place where the generation of power is done.

Hydro-electric power plant is a type of plant which makes use energy of water to run the turbines which in turn run the electric generators. the energy of water utilised for power gene may be kinetic or potential.

GENERAL LAYOUT OF A HYDRO-ELECTRIC POWER PLANT :-



General layout of a hydro electric power plant

A Hydro-electric power plant consists of:

1) A dam constructed across a river to store water.
2) Pipes of large diameters called penstocks, which carry water under pressure from the storage reservoir, to the turbines.
3) Turbines having different types of vanes fitted to the wheels.
4) Tail race, which is a channel which carries water away from th

turbines after the water has worked on the turbines.

SELECTION OF SITE FOR A HYDRO-ELECTRIC PLANT :-

The following factors should be considered while selecting the site for a hydro-electric plant :

1) Availability of water 2) Water storage 3) Water head

4) Accessibility of site 5) Distance from load centre 6) Type of the land of site

ESSENTIAL FEATURES / ELEMENTS OF HYDRO-ELECTRIC POWER PLANT :-

The following are the essential elements of Hydro-electric power plant

1) Catchment area 2) Reservoir 3) Dam 4) Spill ways

5) Conduits 6) Surge tanks 7) Prime movers 8) Draft tubes.

9) power house & equipment.

C
R
D
S
C
S
P
D
P
C

6

Hydro-Electric Power Plant

6.1. Introduction. 6.2. Application of hydro-electric plants. 6.3. Advantages and disadvantages of hydro-electric plants. 6.4. Selection of site for a hydro-electric plant. 6.5. Essential features/elements of hydro-electric power plant—Catchment area—Reservoir—Dam—Spillways—Conduits—Surge tanks—Primemovers—Draft tubes—Power house and equipment. 6.6. Classification of hydro-electric power plants—High head power plants—Medium head power plants—Low head power plants—Base load plants—Peak load plants—Run-of-river plants without pondage—Run-of-river plant with pondage—Storage type plants—Pumped storage plants—Mini and microhydel plants. 6.7. Hydraulic turbines—Classification of hydraulic turbines— Description of various types of turbines—Specific speed of a turbine—Efficiencies of a turbine—Cavitation—Performance of hydraulic turbines—Governing of hydraulic turbines—Selection of turbines. 6.8. Plant layout. 6.9. Hydro-plant auxiliaries. 6.10. Cost of hydro-plant. 6.11. Average life of hydro-plant-components. 6.12. Hydro-plant controls. 6.13. Electrical and mechanical equipment in a hydro-plant. 6.14. Combined hydro and steam power plants. 6.15. Comparison of hydro-power stations with thermal power stations. 6.16. Underground hydro-plants. 6.17. Automatic and remote control of hydro-station. 6.18. Safety measures in hydro-electric power plants. 6.19. Preventive maintenance of hydro-plant. 6.20. Calculation of available hydro-power. 6.21. Cost of hydro-power. 6.22. Hydrology—Introduction—The hydrologic cycle—Measurement of run off—Hydrograph—Flow duration curve—Mass curve—6.23. Hydro power development in India. Worked Examples—Highlights—Theoretical Questions—Unsolved Examples—Competitive Examinations Questions.

6.1. INTRODUCTION

In hydro-electric plants energy of water is utilised to move the turbines which in turn run the electric generators. The energy of water utilised for power generation may be kinetic or potential. The *kinetic energy* of water is its energy in motion and is a function of mass and velocity, while the *potential energy* is a function of the difference in level/head of water between two points. In either case continuous availability of a water is a basic necessity ; to ensure this, water collected in natural lakes and reservoirs at high altitudes may be utilised or water may be artificially stored by constructing dams across flowing streams. The ideal site is one in which a good system of natural lakes with substantial catchment area, exists at a high altitude. *Rainfall is the primary source of water* and depends upon such factors as temperature, humidity, cloudiness, wind etc. The usefulness of rainfall for power purposes further depends upon several complex factors which include its intensity, time distribution, topography of land etc. However it has been observed that only a small part of the rainfall can actually be utilised for power generation. A significant part is accounted for by *direct evaporation,* while another similar quantity *seeps* into the soil and forms the underground storage. Some water is also absorbed by vegetation. Thus, only a part of water falling as rain actually flows over the ground surface as direct run off and forms the streams which can be utilised for hydro-schemes.

First hydro-electric station was probably started in America in 1882 and thereafter development took place very rapidly. In India the first major hydro-electric development of 4.5 MW capacity named as Sivasamudram Scheme in Mysore was commissioned in 1902. In 1914 a hydro-power plant named Khopoli project of 50 MW capacity was commissioned in Maharashtra. The hydro-power capacity, upto 1947, was nearly 500 MW.

Hydro (water) power is a conventional renewable source of energy which is clear, free from pollution and generally has a good environmental effect. However the following factors are major obstacles in the utilisation of hydro-power resources :

(*i*) Large investments (*ii*) Long gestation period

(*iii*) Increased cost of power transmission.

Next to thermal power, hydro-power is important in regard to power generation. The hydro-electric power plants provide 30 per cent of the total power of the world. The total hydro-potential of the world is about 5000 GW. In some countries (like Norway) almost total power generation is hydrobased.

6.2. APPLICATION OF HYDRO-ELECTRIC PLANTS

Earlier hydro-electric plants have been used as exclusive source of power, but the trend is towards use of hydropower in an *inter connected system with thermal stations*. As a self-contained and independent power source, a hydro-plant is most effective with adequate storage capacity otherwise the maximum load capacity of the station has to be based on minimum flow of stream and there is a great wastage of water over the dam for greater part of the year. This increases the per unit cost of installation. By inter connecting hydro-power with steam, a great deal of saving in cost can be effected due to :

(*i*) reduction in necessary reserve capacity,

(*ii*) diversity in construction programmes,

(*iii*) higher utilisation factors on hydroplants, and

(*iv*) higher capacity factors on efficient steam plants.

In an inter connected system the base load is supplied by hydropower when the maximum flow demand is less than the stream flow while steam supplies the peak. When stream flow is lower than the maximum demand the hydroplant supplies the peak load and steam plant the base load.

6.3. ADVANTAGES AND DISADVANTAGES OF HYDRO-ELECTRIC PLANTS

Advantages of hydro-electric plant :

1. No fuel charges.
2. An hydro-electric plant is highly reliable.
3. Maintenance and operation charges are very low.
4. Running cost of the plant is low.
5. The plant has no stand by losses.
6. The plant efficiency does not change with age.
7. It takes a few minutes to run and synchronise the plant.
8. Less supervising staff is required.
9. No fuel transportation problem.
10. No ash problem and atmosphere is not polluted since no smoke is produced in the plant.
11. In addition to power generation these plants are also used for flood control and irrigation purposes.
12. Such a plant has comparatively a long life (100-125 years as against 20-45 years of a thermal plant).
13. The number of operations required is considerably small compared with thermal power plants.
14. The machines used in hydro-electric plants are more robust and generally run at low speeds at 300 to 400 r.p.m. where the machines used in thermal plants run at a speed 3000 to 4000 r.p.m. Therefore, there are no specialised mechanical problems or special alloys required for construction.

(23)

47

15. The cost of land is not a major problem since the hydro-electric stations are situated away from the developed areas.

Disadvantages :

1. The initial cost of the plant is very high.
2. It takes considerable long time for the erection of such plants.
3. Such plants are usually located in hilly areas far away from the load centre and as such they require long transmission lines to deliver power, subsequently the cost of transmission lines and losses in them will be more.
4. Power generation by the hydro-electric plant is only dependent on the quantity of water available which in turn depends on the natural phenomenon of rain. So if the rainfall is in time and proper and the required amount of can be collected, the plant will function satisfactorily otherwise not.

6.4. SELECTION OF SITE FOR A HYDRO-ELECTRIC PLANT

The following factors should be considered while selecting the site for a hydro-electric plant :

1. Availability of water	2. Water storage
3. Water head	4. Accessibility of the site
5. Distance from load centre	6. Type of the land of site.

1. Availability of water :

The most important aspect of hydro-electric plant is the availability of water at the site since all other designs are based on it. Therefore the run-off data at the proposed site must be available before hand. It may not be possible to have run-off data at the proposed site but data concerning the rainfall over the large catchment area is always available. Estimate should be made about the average quantity of water available throughout the year and also about maximum and minimum quantity of water available during the year. These details are necessary to :

(*i*) decide the capacity of the hydro-electric plant,

(*ii*) setting up of peak load plant such as steam, diesel or gas turbine plant and to,

(*iii*) provide adequate spillways or gate relief during the flood period.

2. Water storage :

Since there is a wide variation in rainfall during the year, therefore, it is always necessary to store the water for continuous generation of power. The storage capacity can be calculated with the help of mass curve. Maximum storage should justify the expenditure on the project.

The two types of storages in use are :

(*i*) The storage is so constructed that it can make water available for power generation of one year only. In this case storage becomes full in the beginning of the year and becomes empty at the end of each year.

(*ii*) The storage is so constructed that water is available in sufficient quantity even during the worst dry periods.

3. Water head :

In order to generate a requisite quantity of power it is necessary that a large quantity of water at a *sufficient head* should be available. An increase in effective head, for a given output, reduces the quantity of water required to be supplied to the turbines.

4. Accessibility of the site :

The site where hydro-electric plant is to be constructed should be easily accessible. This is important if the electric power generated is to be utilised at or near the plant site. The site selected should have transportation facilities of rail and road.

5. Distance from the load centre :

It is of paramount importance that the power plant should be set up *near the load centre* ; this will *reduce the cost of erection and maintenance of transmission line.*

6. Type of the land of the site :

The land to be selected for the site should be cheap and rocky. The ideal site will be one where the dam will have largest catchment area to store water at high head and will be economical in construction.

The necessary requirements of the foundation rocks for a masonry dam are as follows :

(*i*) The rock should be strong enough to withstand the stresses transmitted from the dam structure as well as the thrust of the water when the reservoir is full.

(*ii*) The rock in the foundation of the dam should be reasonably impervious. (does not effect)

(*iii*) The rock should remain stable under all conditions.

6.5. ESSENTIAL FEATURES/ELEMENTS OF HYDRO-ELECTRIC POWER PLANT

The following are the *essential elements of hydro-electric power plant* :

1. Catchment area	2. Reservoir
3. Dam	4. Spillways
5. Conduits	6. Surge tanks
7. Primemovers	8. Draft tubes.
9. Powerhouse and equipment.	

Fig. 6.1 shows the flow sheet of hydro-electric power plant.

The description of various elements of hydro-electric plants is as follows :

6.5.1. Catchment Area

The whole area behind the dam draining into a stream or river across which the dam has been built at a suitable place, is called *catchment area.*

6.5.2. Reservoir

The water reservoir is the primary requirement of hydro-electric plant. A reservoir is employed to store water which is further utilised to generate power by running the hydraulic turbines.

A reservoir may be of the following two types :

1. Natural	2. Artificial

A *natural reservoir* is a lake in high mountains.

An artificial reservoir is built by erecting a dam across the river.

Water held in upstream reservoir is called *storage* whereas water behind the dam at the plant is called *pondage.*

6.5.3. Dam

A dam is a barrier to confine or raise water for storage or diversion to create a hydraulic head. An hydro-electric dam diverts the flow from the river to the turbines and usually increases the head. A reservoir dam stores water by raising its level.

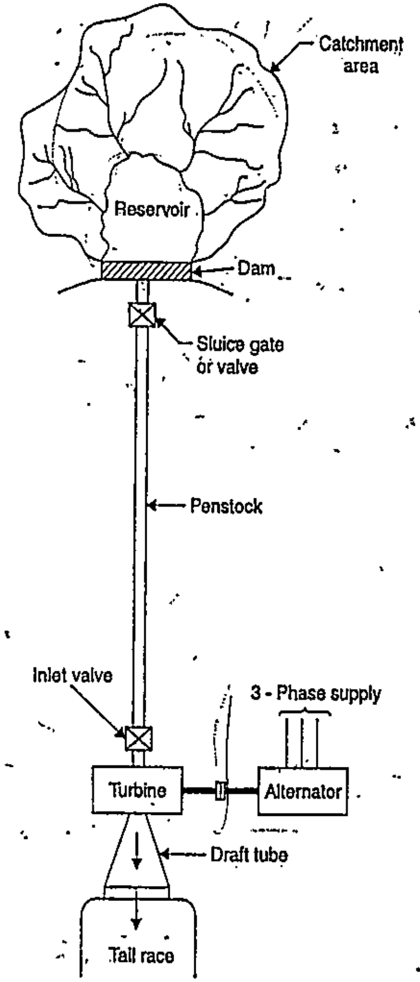


Fig. 6.1. Flow sheet of hydro-electric power plant.

Dams are built of *concrete* or *stone masonry*, *earth* or *rock fill*, or *timber*. Masonry dams may be the solid-gravity, buttress or arch type. A *barrage* is a diversion dam, especially at a tidal power project. A *weir* is a low overflow dam across a stream for measuring flow or maintain water level, as at a lake outlet. A *dike* is an embankment to confine water; a *levee* is a dike near the bank of a river to keep low land from being overflowed.

6.5.3.1. Types of dams

The different types of dams are as follows :

A. Fill dams

1. Earth dams
2. Rock-fill dams.

B. Masonry dams

1. Solid gravity dams
2. Buttress dams
3. Arch dams.

C. Timber dams

Selection of site for dams

The following points should be taken into consideration while selecting the site for a dam.

1. For achieving economy the water storage should be largest for the minimum possible height and length. Naturally site should be located in a narrow valley.

2. For safe and cheap construction good foundation should be available at moderate depth.

3. Good and suitable basin should be available.

4. Material for construction should be available at a dam site or near by. As huge quantities of construction materials are required for construction of the dam, the distance at which the material is available affects the total cost of the project.

5. For passing the surplus water, after the reservoir has been filled upto its maximum capacity, a spillway is to be provided. There should be good and suitable site available for spillway construction. It may be in dam itself or near the dam on the periphery of the basin.

6. The value of the property and the land likely to be submerged by the proposed dam should be sufficiently low in comparison with the benefits expected from the project.

7. The site of the dam should be easily accessible in all the seasons. It should be feasible to connect the site with good lines of communication.

8. There should be a good catchment on the upstream side of the site, that is the catchment should contribute good and sufficient water to the basin. The catchment area should not be easily erodable otherwise excessive silt will come in the reservoir basin.

9. There should be suitable site available for providing living accommodation to the labourers and engineering staff. It is very essential to see that the climate of the site is healthy.

10. Overall cost of construction and maintenance of the dam should be taken into consideration.

Selection of type of a dam

The selection of a type of dam is affected by the following topographical and geological factors :

1. Nature of foundation

Sound rock formation in the foundation	Any type of dam can be adopted
Poor rock and earth foundation	Earth dam

2. Nature of valley

Narrow valleys (with good rock abutments)	Arch dam
If george with rocky bed available	Solid gravity dam

48

If valley is wide and foundation is weak	Buttress dam
For any width of valley with good foundations	Steel dam
For any width of valley with any foundation and low height of water to be stored	Timber dam
For wide valley with gentle side slopes	Earth dam or rock fill dam.

3. Permeability of foundation material

When uplift pressure exerted on the base of the dam is excessive	Arch dam
When the foundations are pervious	Earth dam

In addition to these factors the following points should also be given consideration.

(*i*) Suitable site for locating spillways sometimes affects the selection of the type of a dam.

(*ii*) The availability of construction material may sometimes dictate the choice.

It may be mentioned that in general the most permanent and safe dam will be found to be most economical one.

Description of dams

A. *Fill dams :*

1. Earth dams. For small projects, in particular, dam constructed of earth fill or embankment are commonly used. Because of the great volume of material required, it is imperative that the fill be obtainable in the vicinity of dam site.

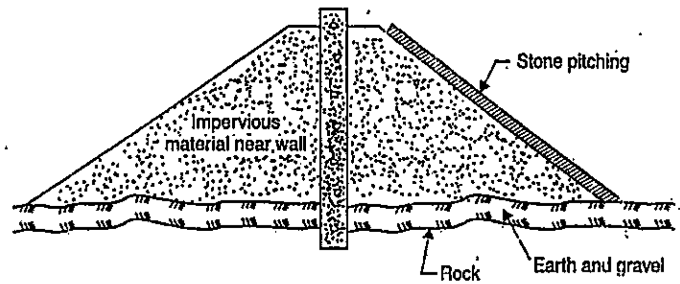


Fig. 6.2. Earth dam.

The earth dams have the following **advantages** :

(*i*) It is usually cheaper than a masonry dam.

(*ii*) It is suitable for relatively previous foundation.

(*iii*) It blends best with the natural surroundings.

(*iv*) It is the most permanent type of construction if protected from corrosion.

Disadvantages of the earth dam are :

(*i*) It has greater seepage losses than most other types of dams.

(*ii*) Since this type of dam is not suitable for a spillway structure, therefore it requires a supplementary spillway.

(*iii*) It is subject to possible destruction or serious damage from erosion by water either overtopping the dam or seeping through it.

Following are the *causes of failure* of earth dams :

1. Overtopping caused by insufficient spillway capacity.

2. Seepage along conduits through the dam.

3. Piping through the dam or its foundation.

2. Rock-fill dams. A rock-fill dam consists of loose rock of all sizes and has a trapezoidal shape with a wide base, with a watertight section to reduce seepage. It is *used in mountaneous locations* where rock rather than earth is available. A rock-fill dam may be destroyed if overtopped to any great extent, and so it needs a supplementary spillway of adequate capacity.

B. *Masonry dams :*

1. Solid gravity dams. This type of dam is more massive and bulky than the other types since it depends on its weight for stability. Because of its weight it requires a sound rock foundation. It may be used as a spillway section for a dam of another type on sand or gravel foundations if the stresses are limited and a suitable cut-off is provided. On a rock foundation the base of a solid gravity dam which is 0.7 of the head usually results in a satisfactory and economical section for either a bulkhead or spillway section. On an earth foundation the base generally equals the head.

2. Buttress dam. A buttress dam has an inclined upstream face, so that water pressure creates a large downward force which provides stability against overturning or sliding. The forces on the upstream face are transmitted to a row of buttresses or piers. This type of dam requires only about *one-third* the concrete needed for a solid gravity dam, but extra cost of reinforcing steel and framework and the skilled labour needed for the thinner sections may largely offset the saving in concrete. The relatively thin sections of concrete in a buttress dam are susceptible to damage from frost and temperature and may require protection or precautionary measures.

Fig. 6.3 and Fig. 6.4 show typical buttress dams.

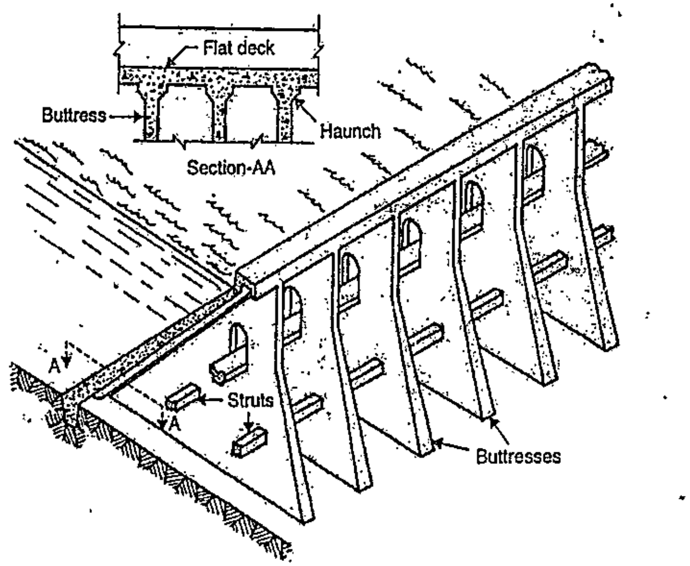


Fig. 6.3. Slab-and-buttress type.

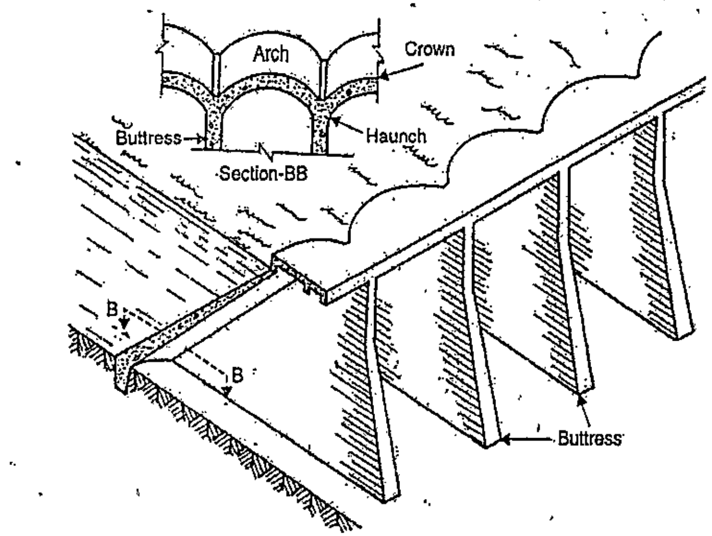


Fig. 6.4. Multiple-arch type.

3. Arch dam. Refer Fig. 6.5. This type of dam resists the water force by being braced against the canyon sides because of its curved shape. Few sites are suitable for the this type of dam, which requires a fairly narrow valley with steep slopes of solid rock to support the outward thrust of the structure. An arch dam is not ordinarily used for a spillway as the downstream face is too steep for the over- flowing water except for low discharges. It is generally necessary to provide a separate spillway for an arch dam, either a tunnel or conduit type, a side-channel wasteway at the end of the dam, or a spillway in another location.

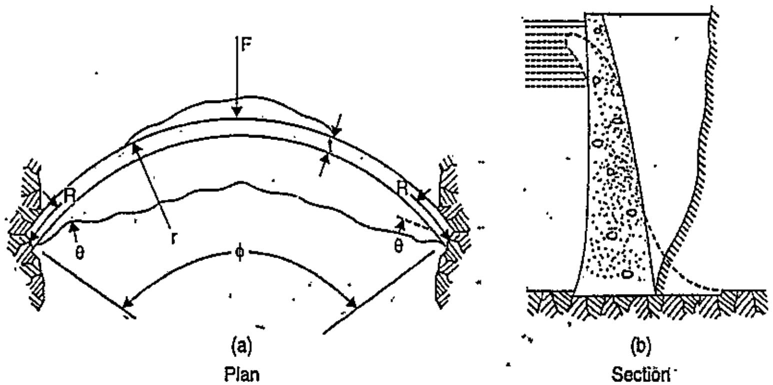


Fig. 6.5. Typical arch dam : (*a*) Plan (*b*) Section.

C. *Timber dams :*

When wood is plentiful and more durable materials are not accessible timber is sometimes used for low dams upto 12 m. In the early days, timber could be had for the cutting, but most of the original timber dams have been superseded by masonry or fill dams. Now-a-days wooden dams are uncommon.

6.5.4. Spillways

When the water enters the reservoir basin, the level of water in basin rises. This rise is arranged to be of temporary nature because excess accumulation of water endangers the stability of dam structure. To relieve reservoir of this excess water contribution, a structure is provided in the body of a dam or near the dam or on the periphery of a basin. This *safeguarding structure is called a spillway.*

A spillway should fulfill the following requirements :

1. It should provide structural stability to the dam under all conditions of floods.
2. It should be able to pass the designed flood without raising reservoir level above H.F.L. (high flood level).
3. It should have an efficient operation.
4. It should have an economical section.

Types of spillways

Following are some types of spillways :

(*i*) Overfall spillway or solid gravity spillway

(*ii*) Chute or trough spillway	(*iii*) Side channel spillway
(*iv*) Saddle spillway	(*v*) Emergency spillway
(*vi*) Shaft or glory hole spillway	(*vii*) Siphon spillway.

In the types from (*i*) to (*v*) *water spills and flows over the body of the spillway whereas* in the types (*vi*) and (*vii*) *water spills over the crest and then flows through the body of the spillway.*

The *selection* of type of spillway is generally based *on the type of the dam and the quantity of flood water to be discharged below* ; it also depends on the *site conditions.*

6.5.4.1. Overfall spillway or solid gravity spillway

Refer Fig. 6.6. This type of spillway is provided in case of concrete and masonry dams. It is situated in the body of the dam, generally in the centre. As it is provided in the dam itself the length of dam should be sufficient to accommodate the designed spillway crest.

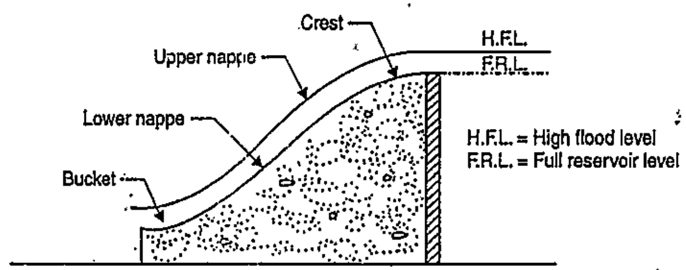


Fig. 6.6. Overfall spillway.

This spillway consists of an *ogee crest* and a *bucket*. Water spills and flows over the crest in the form of a rolling sheet of water. The bucket provided at the lower end of the spillway *changes the*

(85)

49

direction of the fast moving water. In this process the excess energy of fast moving water is *destroyed*. The portion between the front vertical face and the lower nappe of sheet of water is filled with concrete to conform the profile of the spillway to the lower nappe. This type of construction practically avoids the development of *negative pressures*. The section is always designed for maximum head of water over the crest of spillway.

6.5.4.2. Chute or trough spillway

This type of spillway is most suited under the situation when the *valley is too narrow* to accommodate the solid gravity spillway in the body of the dam or when the non-rigid type of dam is adopted. It is called chute spillway because after crossing over the crest of the spillway the water flow *shoots down a channel or a trough* to meet the river channel downstream of the dam.

In this type the crest of the spillway is at right angles to the centre line of the trough or the chute. The crest is isolated from the dam axis. The trough is taken straight from the crest to the river and it is generally lined with concrete.

6.5.4.3. Side channel spillway

A side channel spillway is employed when the *valley is too narrow in case of a solid gravity dams and when non-rigid dams are adopted*. In non-rigid dams it is undesirable to pass the flood water over the dam. When there is no room for the provision of chute spillway this type is adopted as it requires comparatively limited space. Thus the situations where chute and side channel spillways are required are mostly the same. *The side channel spillway differs from the chute spillway in the sense that after crossing over the spillway crest, water flows parallel to the crest length in former, whereas the flow is normal to the crest in the latter* (Fig. 6.7).

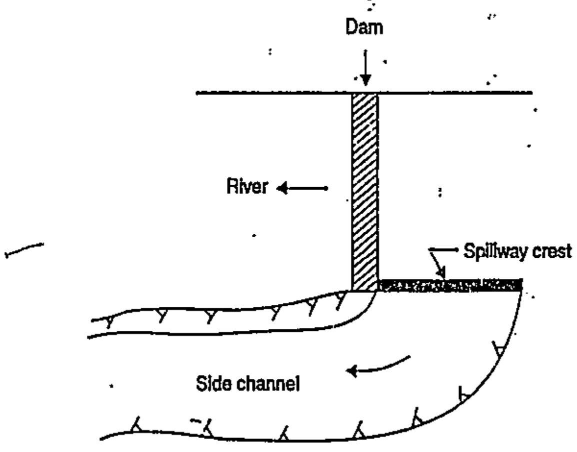


Fig. 6.7. Side channel spillway.

To maintain satisfactory flow conditions a sufficient longitudinal slope is given to the side channel.

6.5.4.4. Saddle spillway

A saddle spillway may be constructed when conditions are not favourable for any of the types mentioned above. There may be some natural depression or saddle on the periphery of the reservoir basin away from the dam as shown in Fig. 6.8. The depression may be used as a spillway. It is

essential that *the bottom of the depression should be at full reservoir level*. For ideal conditions there should be good rock formation at the site of a spillway.

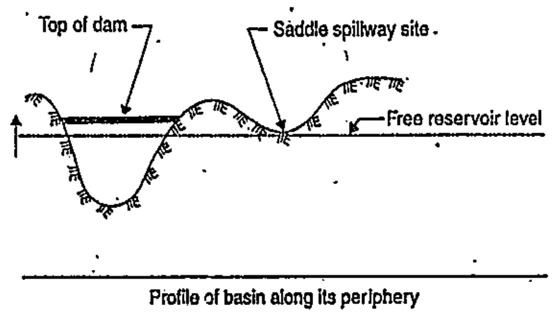


Fig. 6.8. Saddle spillway.

6.5.4.5. Emergency spillway

As the name suggests this type of spillway is very rarely put into action. Naturally it is not necessary to protect the structure, its foundation or its discharge channel from serious damage.

An emergency spillway comes into action when the occurring flood discharge *exceeds* the designed flood discharge.

6.5.4.6. Shaft or glory hole spillway

The shape of shaft spillway is just like a funnel. The lower end of the funnel is turned at right angles and then taken out below the dam horizontally. Water spills over the crest, which is circular, and then enters the vertical shaft and is taken out below the dam through a horizontal tunnel. Sometimes the flow is guided by means of radial piers on the crest of the spillway. It avoids creation of spiral flow in the shaft. The piers may be used to support a bridge around the crest. The bridge may be used to connect the spillway to the dam.

6.5.4.7. Siphon spillway

A siphon spillway, as the name suggests, is designed on the *principle of a siphon*.

Fig. 6.9 shows a saddle siphon spillway. The crest is fixed at full reservoir level (F.R.L.). When the water level in the reservoir rises above F.R.L. water starts spilling over the crest. The step or a joggle deflects the sheet of water and consequently the lower end is sealed. As the lower end is sealed the air gets entrapped in the lower limb. This air is driven out by incoming water completely. This process of evacuating and filling the lower limb by water is known as *priming*. Once the siphon is primed water starts flowing out till the level of the water in the reservoir falls below the level of the upper limb. Usually the lower end of the upper limb is kept below full reservoir level. It prevents blocking of the entrance due to the floating matter such as ice etc. Naturally if the water is emptied till the lower end of the upper limb emerges out the useful live storage is lost. To break the siphon action at proper time, that is when the water level falls to F.R.L., an air vent is provided on the crown as shown in Fig. 6.9. Thus when the water level falls to F.R.L. air enters in the lower limb through air vent and siphonic action is broken or stopped.

OTPR
AP1000295094
DOB :- 19/7/1981

GRP1110034 5373

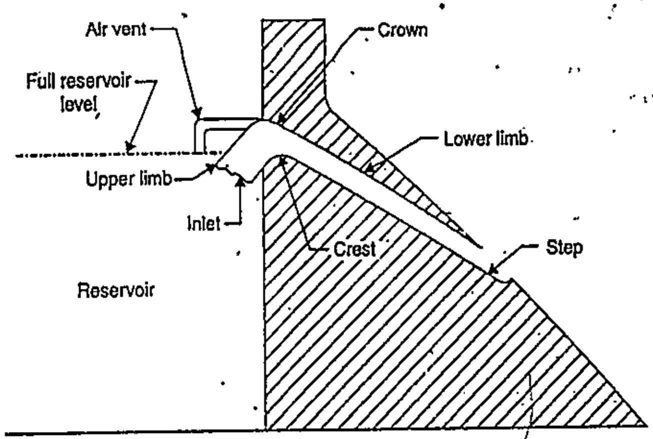


Fig. 6.9. Saddle siphon spillway.

6.5.5. Conduits

A headrace is a channel which leads water to a turbine and a tailrace is a channel which conducts water from the wheels. The conduit may be open or close.

Open conduits ...... Canals and flumes

Close conduits ...... Tunnels, pipelines and penstocks

Canal. A canal is an open waterway excavated in natural ground. It has to follow the contour of the ground, with perhaps a slight gradient corresponding to the head loss.

Flume. A flume is an open channel erected on the surface of supported above ground on a trestle. A flume might be used with a canal to cross a ravine or where the slope of the ground is greater than the hydraulic gradient.

Tunnel. It is a closed channel excavated through a natural obstruction such as a ridge of higher land between the dam and the powerhouse. A tunnel across a bend in the river might be cheaper than a conduit that goes around. Tunnels are also commonly used in diverting water from one drainage area to another, where the divide between watersheds is higher than the reservoir.

Pipeline. A pipeline is a closed conduit usually supported on or above the surface of the land. When a pipeline is laid on the hydraulic gradient, it is called a *flow line.*

Penstock. It is a closed conduit for supplying *water under pressure* to a turbine.

Advantages and limitations of different types of conduits :

Open channels are generally the *least expensive,* but the cost of a flume increases with the height of the trestle.

Where the land is fairly level at head water elevation between the dam and powerhouse sites, a *canal* would be feasible, but not many sites fit this requirement.

Tunnels are generally the *most costly type* of conduit for a given length but are justified if their use results in considerable saving in distance. While ordinarily tailraces are open channels, tunnels are used for the discharge from an underground hydro-station.

Penstocks are used where the *slope is too great for a canal,* especially for the final stretch of the diversion system where the land pitches steeply to the powerhouse. Surge tanks or other measures are necessary to prevent damage in closed conduits due to abnormal pressures.

Fig. 6.9 (*a*) shows the combination of tunnel, flume, and penstocks at a high-head development. The tunnel intake regulates the flow between the reservoirs, while the sluice gates at the entrance to the flume control the discharge to that needed by the turbines. The regulating forebay has a small storage capacity to care for minor flow fluctuations. It has an automatic spillway to discharge overflow when turbines shut down suddenly.

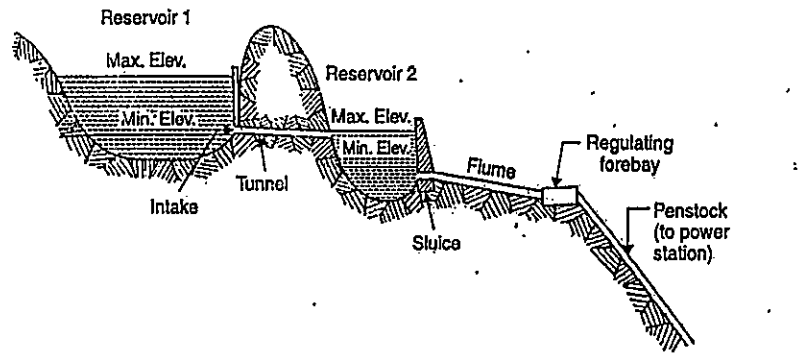


Fig. 6.9. (*a*) Combination of tunnel, flume and penstocks at a high-head development.

Penstocks ;

(*i*) *How to calculate penstock thickness ?*

The thickness of steel penstock which depends on the water head hoop/circumferential stress allowed in the material can be calculated by using the following relation :

$$t = \frac{pd}{2f\eta}$$

where t = Thickness of the penstock,

p = Pressure due to water including water hammer $= wH$, w and H being specific weight of water and head of water respectively,

d = Diameter (internal) of the penstock,

f = Permissible hoop/circumferential stress, and

η = Joint efficiency.

(*ii*) *Number of penstocks to be used :*

To supply water to a number of turbines penstocks needed may be decided from the *following alternatives :*

1. To provide *one penstock for each turbine* separately. In such a case water is supplied independently to each turbine from a separate penstock.

2. To use a *single penstock* for the entire plant. In this case the penstock should have as *many branches* as the number of hydraulic turbines.

3. To provide *multiple penstocks* but each penstock should supply water to atleast two hydraulic turbines.

86

50

While selecting the number of penstocks to be used for supplying water to the turbines, the following points need be considered :

(*a*) **Operational safety.** As far as possible a single penstock should not be used for supplying to different turbines for generating power because any damage to this penstock would mean shut down of the entire set of turbines.

(*b*) **Economy.** From view point of economy, if the length of penstock required is short then one penstock each may be provided to the turbines, however for longer penstocks a single penstock or as few penstocks as possible may be used.

(*c*) **Transportation facilities.** The penstock size should be so selected that it is easily transported from one place to another.

(*iii*) *Penstock materials and their suitability :*

— *Reinforced concrete penstocks* are suitable upto 18 m head as greater pressures cause rapid deterioration of concrete exposed to frost action.

— *Wood-stave penstocks* are used for heads upto about 75 m and consist of treated timbers laid side by side to form a cylinder held together by the steel hoops. The size and spacing of the hoops on a wood-stave pipe and the reinforcing steel in concrete increase with the head.

— *Steel penstocks* can be designed for any head, with the thickness varying with the pressure and diameter. The minimum thickness of steel plate is used for heads upto 45 m ; for lower heads it may be more economical to use wood or concrete pipe rather than steel.

The strength of a penstock can be expressed as the horse power it can carry. Since the size of a pipe depends on the flow, the product of head and flow determines both maximum stresses and power.

— *High pressure penstocks* are fabricated in 6 to 8 m lengths for mountaneous regions where transportation is difficult. The manholes give access to the interior of the penstocks for inspection and maintenance. *Welded joints are preferable to riveted ones because of the higher friction losses in the latter. Steel penstocks* are usually given protective coatings in the shop and after erection.

— Penstocks are generally supported by *concrete piers* or *cadles,* although they may be laid on or in the ground. A bridge or trestle is used to carry a penstock across a narrow defile. *Anchors* on steep grades support the weight of the fill penstock and also take the thrusts from water pressure acting at angles in the pipe. It may be cheaper to bury small pipes, while penstocks are sometimes covered to protect them from rock or snow slides, prevent freezing, or eliminate expansion joints. *Buried penstocks* are subject to corrosion, which can be eliminated or atleast greatly reduced by *cathodic protection* which prevents electrolysis from attacking the metal. *Exposed penstocks last longer and are more accessible for inspection and maintenance.*

6.5.6. Surge Tanks

A surge tank is a small reservoir or tank in which the water level rises or falls to reduce the pressure swings so that they are not transmitted in full to a closed circuit. In general a surge tank serves the following purposes :

1. To reduce the distance between the free water surface and turbine thereby reducing the water-hammer effect (the *water hammer* is defined as the change in pressure rapidly above or below normal pressure caused by sudden changes in the rate of water flow through the pipe according to the demand of prime mover) on penstock and also protect upstream tunnel from high pressure rises.

2. To serve as a *supply tank* to the turbine when the water in the pipe is accelerating during increased load conditions and as a *storage tank* when the water is decelerating during reduced load conditions.

Types of surge tanks

The different types of surge tanks in use are :

1. Simple surge tank
2. Inclined surge tank
3. The expansion chamber and gallery type surge tank
4. Restricted orifice surge tank
5. Differential surge tank.

1. Simple surge tank. A simple surge tank is a vertical stand pipe connected to the penstock as shown in Fig. 6.10. In the surge tank if the overflow is allowed, the rise in pressure can be eliminated but *overflow surge tank is seldom satisfactory and usually uneconomical.* Surge tanks are built high enough so that water cannot overflow even with a full load change on the turbine. It is always desirable to place the surge tank on grond surface, above the penstock line, at the point where the latter drops rapidly to the power house as shown in Fig. 6.10. Under the circumstances when suitable site for its location is not available the height of the tank should be increased with the help of a support.

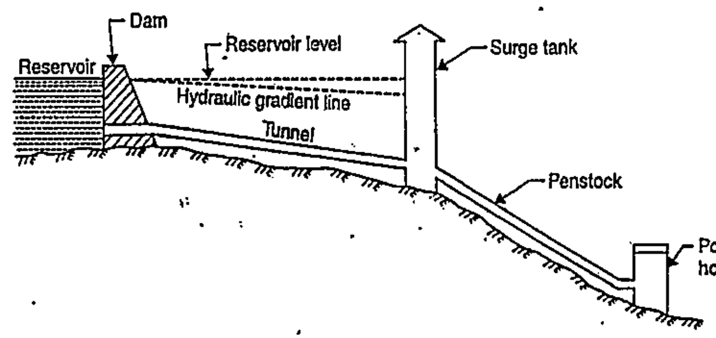


Fig. 6.10. Surge tank on ground level.

2. Inclined surge tank. When a surge tank is inclined (Fig. 6.11) to the horizontal its effective water surface increases and therefore, lesser height surge tank is required of the same diameter if it is inclined or lesser diameter tank is required for the same height. But this type of surge tank is *more costlier than ordinary type as construction is* difficult and is rarely used unless the topographical conditions are in favour.

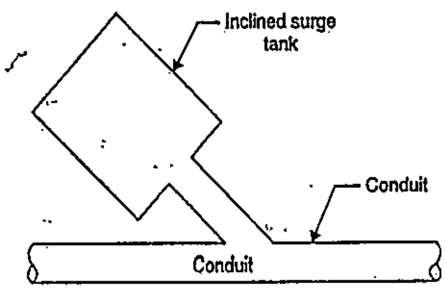


Fig. 6.11. Inclined surge tank.

3. **Expansion chamber surge tank.** Refer Fig. 6.12. This type of a surge tank has an expansion tank at top and expansion gallery at the bottom ; these expansions *limit the extreme surges*. The 'upper expansion chamber' must be *above the maximum reservoir level* and 'bottom gallery' must be *below the lowest steady running level in the surge tank*. Besides this the intermediate shaft should have a stable minimum diameter.

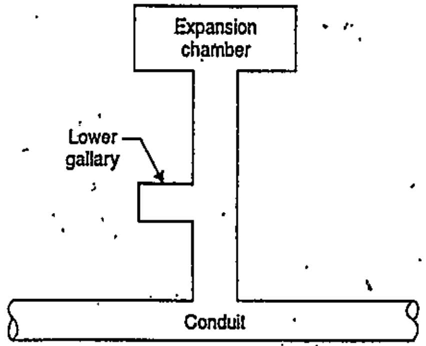


Fig. 6.12. The expansion chamber surge tank.

4. **Restricted orifice surge tank.** Refer Fig. 6.13. It is also called *throttled surge tank*. The main object of providing a throttle or restricted orifice is *to create an appreciable friction loss when the water is flowing to or from the tank*. When the load on the turbine is reduced, the surplus water passes through the throttle and a retarding head equal to the loss due to throttle is built up in the conduit. The size of the throttle can be designed for any designed retarding head. *The size of the throttle adopted is usually such as the initial retarding head is equal to the rise of water surface in the tank when the full load is rejected by the turbine* (a case when there is closure of the gate valve).

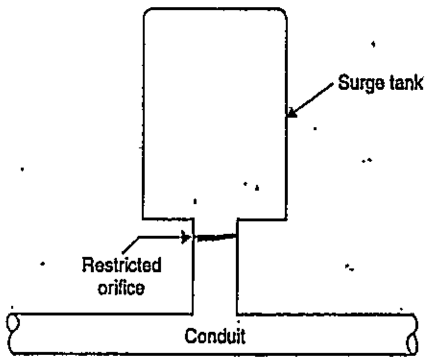


Fig. 6.13. Restricted orifice surge tank.

Advantage. Storage function of the tank can be separated from accelerating and retarding functions.

Disadvantage. Considerable portion of water hammer pressure is transmitted directly into the low pressure conduit.

In comparison to other types of surge tanks these are *less popular*.

5. **Differential surge tank.** Refer Fig. 6.14. A differential surge tank has a riser with a small hole at its lower end through which water enters in it. The function of the surge tank depends upon the area of hole.

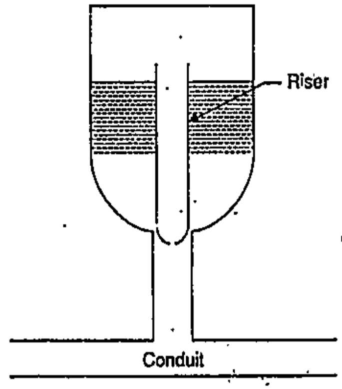


Fig. 6.14. Differential surge tank.

6.5.7. Primemovers

In an hydraulic power plant the primemover converts the energy of water into mechanical energy and further into electrical energy. These machines are classified on the basis of the action of water on moving blades. As per action of water on the primemover, they are classified as :

1. **Impulse turbine.** Here the *pressure energy of water is converted into kinetic energy* when passed through the nozzle and forms the high velocity jet of water. The formed water jet is used for driving the wheel.

2. **Reaction turbine.** In this case the water pressure combined with the velocity works on the runner. The power in this turbine is developed from the *combined action of pressure and velocity of water* that completely fills the runner and water passage.

For more details on hydraulic turbines refer article 6.7.

6.5.8. Draft Tubes

The draft tube serves the following two purposes :

1. *It allows the turbine to be set above tail-water level, without loss of head, to facilitate inspection and maintenance.*

2. *It regains, by diffuser action, the major portion of the kinetic energy delivered to it from the runner.*

At rated load the velocity at the upstream end of the tube for modern units ranges from 7 to 9 m/s, representing from 2.7 to 4.8 m head. As the specific speed (it is the speed of a geometrically similar turbine running under a unit head and producing unit power) is increased and the head reduced, it becomes increasingly important to have an efficient draft tube. Good practice limits the velocity at the discharge end of the tube to 1.5 to 2.1 m/s, representing less than 0.3 m velocity head loss.

Types of draft tubes. The following two types of draft tubes are commonly used :

(*i*) The straight conical or concentric tube (*ii*) The elbow type.

(87)

57

Properly designed, the two types are about equally efficient, over 85%.

(*i*) **Conical type.** The conical type is generally used on low-powered units for all specific speeds and, frequently, on large high-head units. The side angle of flare ranges from 4 to 6°, the length from 3 to 4 times the diameter and the discharge area from four to five times the throat area. Fig. 6.15 shows a straight conical draft tube.

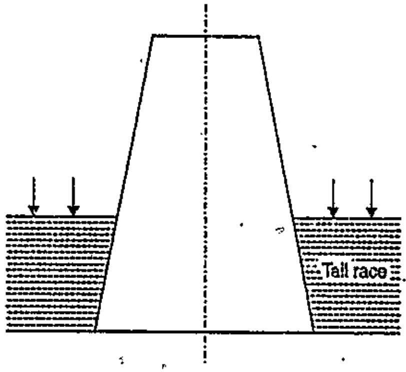


Fig. 6.15. Straight conical draft tube.

(*ii*) **Elbow type.** The elbow type of tube is now used with most turbine installation. With this type the vertical portion begins with a conical section which gradually flattens in the elbow section and then discharges horizontally through substantially regular sections to the tailrace. Most of the regain of energy takes place in the vertical portion, very little in the elbow section, which is shaped to deliver the water to the horizontal portion so that the regain may be efficiently completed. Fig. 6.16 shows an elbow type draft tube. One or two vertical piers are placed in the horizontal portion of the tube, for structural and hydraulic reasons.

Small conical tubes are sometimes made entirely of steel plate. Most tubes are made of concrete with a steel-plate lining extending from the upper end to a point where the velocity has been sufficiently reduced (say 5 m/s) to prevent erosion of the concrete. Sometimes the liner is carried around the elbow. Pier noses are also lined where necessary to prevent erosion for structural reasons.

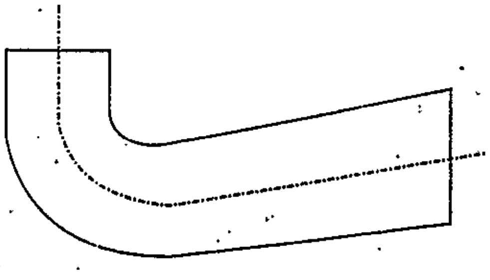


Fig. 6.16. Elbow type draft tube.

6.5.9. Power House and Equipment

A power house should have a stable structure and its layout should be such that adequate space is provided around the equipment (such as turbines, generators, valves, pumps, governors etc.), so that the dismantling and repairing may be easily carried out.

A power house, mostly, comprises of the following sub-divisons :

1. **The substructure.** This part of the powerhouse extends from top of generator to the soil or rock and houses most of the generating equipment. In case of Francis and Kaplan turbines the substructure not only accommodates various equipment but draft tube as well.

2. **Intermediate structure.** It is that part of the structure which extends from the top of the draft tube to top of generator foundation.

3. **The superstructure.** This part of the structure lies above the generator level. It houses mostly the cranes which handle the heavy equipment in the substructures.

Following important equipment may be provided in a power house :

(*i*) Hydraulic turbines	(*ii*) Electric generators
(*iii*) Governors	(*iv*) Gate valves
(*v*) Relief valves	(*vi*) Water circulating pumps
(*vii*) Flow measuring equipment	(*viii*) Air duct
(*ix*) Water circulating pumps	(*x*) Switch board equipment and instruments
(*xi*) Oil circuit breakers	(*xii*) Reactors
(*xiii*) Low tension and high tension bar	(*xiv*) Storage batteries
(*xv*) Cranes.	

Besides the above important equipment shops and offices are also provided in the power house.

6.6. CLASSIFICATION OF HYDRO-ELECTRIC POWER PLANTS

Hydro-electric power stations may be classified as follows :

A. According to availability of head

1. High head power plants
2. Medium head power plants
3. Low head power plants

B. According to the nature of load

1. Base load plants
2. Peak load plants

C. Accordingly to the quantity of water available

1. Run-of-river plant without pondage
2. Run-of-river plant with pondage
3. Storage type plants
4. Pump storage plants
5. Mini and micro-hydel plants

A. ***According to availability of head***

The following figures give a rough idea of the heads under which the various types of plants work :

(*i*) High head power plants	100 m and above
(*ii*) Medium head power plants	30 to 100 m
(*iii*) Low head power plants	25 to 80 m.

Note. It may be noted that figures given above overlap each other. Therefore it is difficult to classify the plants directly on the basis of head alone. The basis, therefore, technically adopted is the *specific speed* of the turbine used for a particular plant.

6.6.1. High Head Power Plants

These types of plants work under heads 100 m and above. Water is usually stored up in lakes on high mountains during the rainy season or during the reason when the snow melts. The rate of flow should be such that water can last throughout the year.

Fig. 6.17 shows high head power plant layout. Surplus water discharged by the spillway cannot endanger the stability of the main dam by erosion because they are separated. The tunnel through the mountain has a surge chamber excavated near the exit. Flow is controlled by head gates at the tunnel intake, butterfly valves at the top of the penstocks, and gate valves at the turbines. This type of site might also be suitable for an underground station.

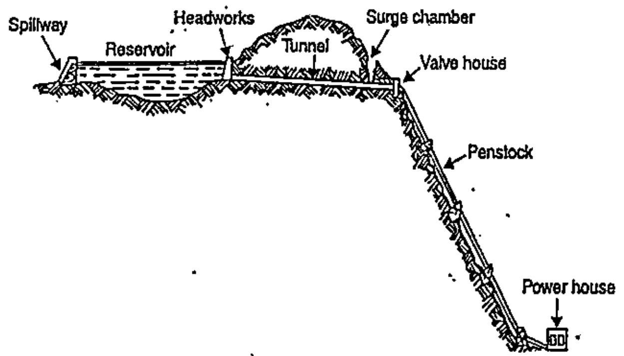


Fig. 6.17. High head power plant layout. The main dam, spillway, and powerhouse stand at widely separated locations. Water flows from the reservoir through a tunnel and penstock to the turbines.

The *Pelton wheel* is the common primemover used in high *head power plants*.

6.6.2. Medium Head Power Plants

Refer Fig. 6.18. When the operating head of water lies between 30 to 100 metres, the power plant is known as medium head power plant. This type of plant commonly uses *Francis turbines*. The forebay provided at the beginning of the penstock serves as water reservoir. In such plants, the water is generally carried in open canals from main reservoir to the forebay and then to the power-house through the penstock. The forebay itself works as a surge tank in this plant.

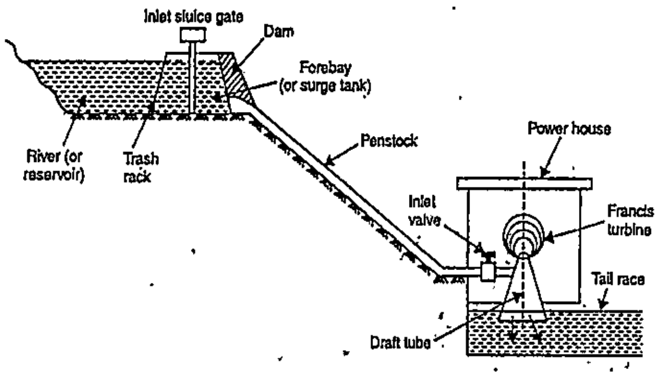


Fig. 6.18. Medium head power plant layout.

6.6.3. Low Head Power Plants

Refer Fig. 6.19. These plants usually consist of a dam across a river. A sideway stream diverges from the river at the dam. Over this stream the power house is constructed. Later this channel joins the river further downstream. This type of plant uses vertical shaft Francis turbine or Kaplan turbine.

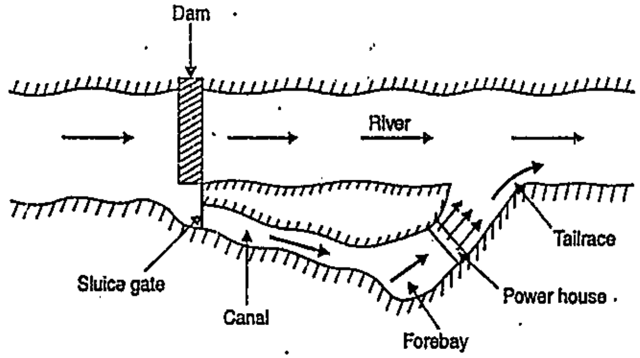


Fig. 6.19. Low head power plant layout.

B. *According to the nature of load*

6.6.4. Base Load Plants

The plants which cater for the base load of the system are called *base load plants*. These plants are required to *supply a constant power when connected to the grid*. Thus they *run without stop* and are often remote-controlled with which *least staff is required for such plants*. Run-of-river plants without pondage may sometimes work as baseload plant, but the firm capacity in such cases, will be muchless.

6.6.5. Peak Load Plants

The plants which can supply the power during peak loads are known as *peak load plants*. Some of such plants supply the power during average load but also supply peak load as and when it is there; whereas other peak load plants are required to work during peak load hours only. The run-of-river plants may be made for the peak load by providing pondage.

C. *According to the quantity of water available*

6.6.6. Run-of-river Plants without Pondage

A run-of-river plant without pondage, as the name indicates, does not store water and uses the water as it comes. There is *no control on flow of water* so that during high floods or low loads water is wasted while during low run-off the plant capacity is considerably reduced. *Due to non-uniformity of supply and lack of assistance from a firm capacity the utility of these plants is much less* than those of other types. The head on which these plants work *varies considerably*. Such a plant can be made a great deal more useful by providing sufficient storage at the plant to take care of the hourly fluctuations in load. This lends some *firm capacity* to the plant. During good flow conditions

88

52

these plants may cater to base load of the system, when flow reduces they may supply the peak demands. *Head water elevation for plant fluctuates with the flow conditions.* These plants without storage may sometimes be made to supply the base load, but the firm capacity depends on the minimum flow of river. The run-of-river plant may be made for load service with pondage, though storage is usually seasonal.

6.6.7. Run-of-river Plant with Pondage

Pondage usually refers to the collection of water behind a dam at the plant and increases the stream capacity for a short period, say a week. *Storage* means collection of water in up stream reservoirs and this increases the capacity of the stream over an extended period of several months. Storage plants may work satisfactorily as base load and peak load plants.

This type of plant, as compared to that without pondage, is *more reliable* and its generating capacity *is less dependent on the flow rates of water available.*

6.6.8. Storage Type Plants

A storage type plant is one with a reservoir of sufficiently large size to permit carry-over storage from the wet reason to the dry reason, and thus to supply firm flow substantially more than the minimum natural flow. This plant can be used as base load plant as well as peak load plant as water is available with control as required. The majority of hydro-electric plants are of this type.

6.6.9. Pumped Storage Plants

Refer Fig. 6.20.

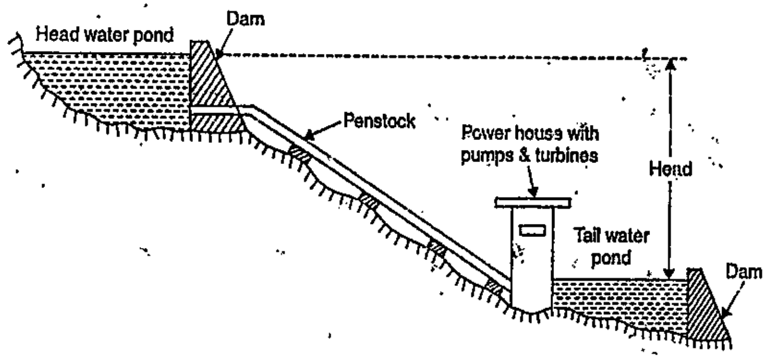


Fig. 6.20. Pumped storage plant.

Pumped storage plants are employed at the places where the quantity of water available for power generation is *inadequate*. Here the water passing through the turbines is stored in *'tail race pond'*. During low load periods this water is pumped back to the head reservoir using the extra energy available. This water can be again used for generating power during peak load periods. Pumping of water may be done seasonally or daily depending upon the conditions of the site and the nature of the load on the plant.

Such plants are *usually interconnected* with steam or diesel engine plants so that off peak capacity of interconnecting stations is used in pumping water and the same is used during peak load periods. Of course, the energy available from the quantity of water pumped by the plant is *less* than the energy input during pumped operation. Again while using pumped water the *power available is reduced* on account of losses occuring in primemovers.

Advantages. The pump storage plants entail the following *advantages* :

1. There is substantial increase in peak load capacity of the plant at comparatively low capital cost.
2. Due to load comparable to rated load on the plant, the operating efficiency of the plant is high.
3. There is an improvement in the load factor of the plant.
4. The energy available during peak load periods is higher than that of during off peak periods so that inspite of losses incurred in pumping there is *over-all gain.*
5. Load on the hydro-electric plant remains uniform.
6. The hydro-electric plant becomes partly independent of the stream flow conditions.

Under pump storage projects almost 70 percent power used in pumping the water can be recovered. In this field the use of *"Reversible Turbine Pump"* units is also worth noting. These units can be used as turbine while generating power and as pump while pumping water to storage. The generator in this case works as motor during reverse operation. The efficiency in such case is high and almost the same in both the operations. With the use of reversible turbine pump sets, additional capital investment on pump and its motor can be saved and the scheme can be worked more economically.

6.6.10. Mini and Microhydel Plants

In order to meet with the present energy crisis partly, a solution is to develop *mini* (5 m to 20 m head) and *micro* (less than 5 m head) hydel potential in our country. The low head hydro-potential is scattered in this country and estimated potential from such sites could be as much as 20,000 MW.

By proper planning and implementation, it is possible to commission a small hydro-generating set up of 5 MW with a period of one and half year against the period of a decade or two for large capacity power plants. Several such sets upto 1000 kW each have been already installed in Himachal Pradesh, U.P., Arunachal Pradesh, West Bengal and Bhutan.

To reduce the cost of micro-hydel stations than that of the cost of conventional installation the following considerations are kept in view :

1. The civil engineering work needs to be kept to a *minimum* and designed to fit in with already existing structures *e.g.* irrigation, channels, locks, small dams etc.
2. The machines need to be manufactured in a small range of sizes of simplified design, allowing the use of unified tools and aimed at reducing the cost of manufacture.
3. These installations must be automatically controlled to reduce attending personnel.
4. The equipment must be simple and robust, with easy accessibility to essential parts for maintenance.
5. The units must be light and adequately subassembled for ease of handling and transport and to keep down erection and dismantling costs.

Micro-hydel plants (micro-stations) *make use of standardised bulb sets with unit output ranging from 100 to 1000 kW working under heads between 1.5 to 10 metres.*

6.7. HYDRAULIC TURBINES

A ***hydraulic turbine*** *converts the potential energy of water into mechanical energy which in turn is utilised to run an electric generator to get electric energy.*

6.7.1. Classification of Hydraulic Turbines

The hydraulic turbines are classified as follows :

(*i*) According to the head and quantity of water available.

INTRODUCTION :-

Hydraulic machines are those machines which convert either hydraulic energy into mechanical energy (or) mechanical energy into hydraulic energy.

Hydraulic machines are classified into two types. They are

1) Turbines 2) pumps.

Turbines are defined as the hydraulic machines which convert hyd energy into mechanical energy. This mechanical energy is used in running an electric generator which is directly coupled to the sha of the turbine. The electric power which is obtained from the hydro energy is known as Hydro-electric power.

pumps are defined as the hydraulic machines which convert th mechanical energy into hydraulic energy.

Definitions :-

Gross Head :- The difference between the head race level & tail rac level when no water is flowing is known as Gross Head. It i denoted by "Hg".

Net Head :- Defined as the head available at the inlet of the tu when water is flowing from head race to the turbine, a loss of hea due to friction between the water & penstock occurs.

Net head on turbine is $H = Hg - h_f$

Efficiencies of a Turbine :

Important efficiencies of a turbine are

a) Hydraulic efficiency , η_h b) Mechanical efficiency , η_m

c) volumetric efficiency , η_v d) overall efficiency , η_o

a) Hydraulic efficiency (η_h) :- It is defined as the ratio of power given by water to the runner of a turbine to the power supplied by the water at the inlet of turbine.

~~Hmathematically~~ mathematically

$$\eta_h = \frac{\text{power delivered to runner}}{\text{power supplied at inlet}}$$

$$= \frac{\text{Runner Power}}{\text{water power}} = \frac{R.P}{W.P}$$

b) Mechanical efficiency (η_m) :- It is defined as the ratio of power available at the shaft of the turbine to the power delivered to the runner.

mathematically, $$\eta_m = \frac{\text{power at the shaft of turbine}}{\text{power delivered by water to runner.}} = \frac{S.P}{R.P}$$

c) Volumetric efficiency (η_v) :- It is defined as the ratio of volume of the water actually striking the runner to the volume of water supplied to the turbine.

$$\eta_V = \frac{\text{volume of water actually striking the runner}}{\text{volume of water supplied to the turbine}}$$

d) overall efficiency (η_o) :- It is defined as the ratio of power available at the shaft of the turbine to the power supplied by the water at the inlet of the turbine.

$$\eta_o = \frac{\text{~~volume~~ power available at the shaft of turbine}}{\text{power supplied at the inlet of turbine}}$$

$$= \frac{S.P}{W.P}$$

$$= \frac{S.P}{W.P} \times \frac{R.P}{R.P}$$

$$= \frac{S.P}{R.P} \times \frac{R.P}{W.P}$$

$$\boxed{\eta_o = \eta_m \times \eta_h}$$

Classification of Hydraulic Turbines :-

(i) The Hydraulic turbines are classified as

1) According to the type of energy at inlet.
   a) Impulse turbine b) Reaction turbine
2. According to the direction of flow through runner.
   a) Tangential flow turbine. b) Radial flow turbine
   c) Axial flow turbine d) mixed flow turbine.
3. According to the head at the inlet of turbine.
   a) High head turbine. b) medium head turbine.
   c) Low head turbine.
4. According to the specific speed of the turbine.
   a) Low specific speed turbine b) medium specific speed turbi
   c) High specific speed turbine.

The energy available at the inlet of the turbine is only kinetic energy. the turbine is known as impulse turbine.

The water possesses kinetic energy as well as pressure ener the turbine is known as reaction turbine.

If the water flows along the tangent of the runner, the turbine is known as tangential flow turbine.

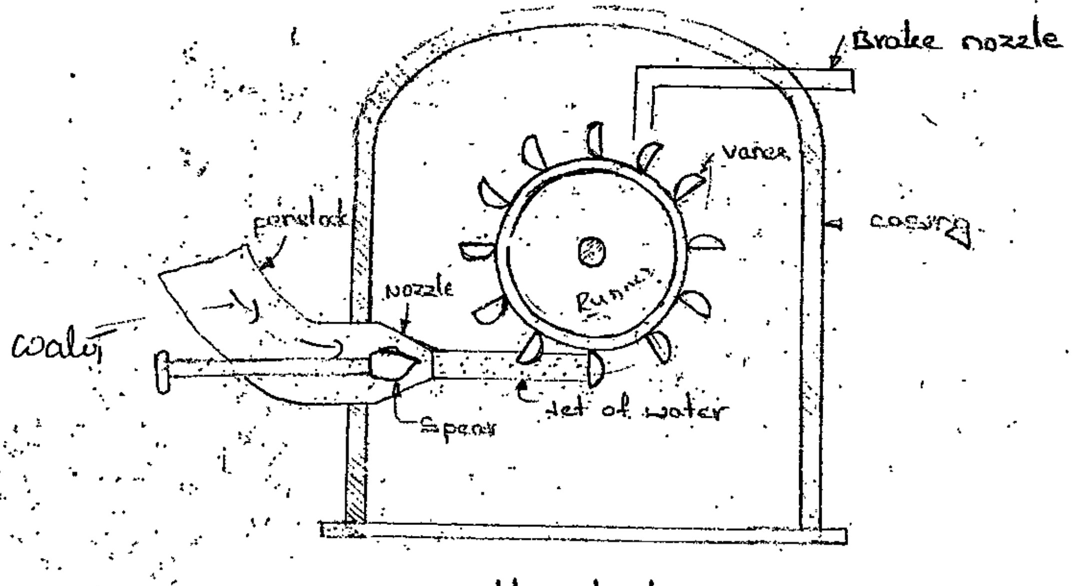
If the water flows in the radial direction through the runn the turbine is called radial flow turbine.

If the water flows from outward to inward radially, the turbine is called inward radial flow turbine.

If the water flows from inwards to outwards radially, the turbine is called outward radial flow turbine.

Impulse turbine :- I. turbine the P.E of H_2O is converted into K.E when passed through the nozzle & forms the high velocity jet of H_2O.

[illegible] :- pelton turbine is a tangential flow impulse turbine. The water strikes the bucket along the tangent to the runner. This turbine is used for high heads.
The main parts of the pelton turbine are:
1. Nozzle & spear 2. Runner & buckets 3. Casing 4. Breaking Jet.



pelton turbine

1) NOZZLE AND SPEAR :- The amount of water striking the bucket of runner is controlled by providing a spear in the nozzle. Nozzle is used to convert total energy of water into useful kinetic energy. spear is a conical shaped needle which is operated either by a hand wheel or automatically.

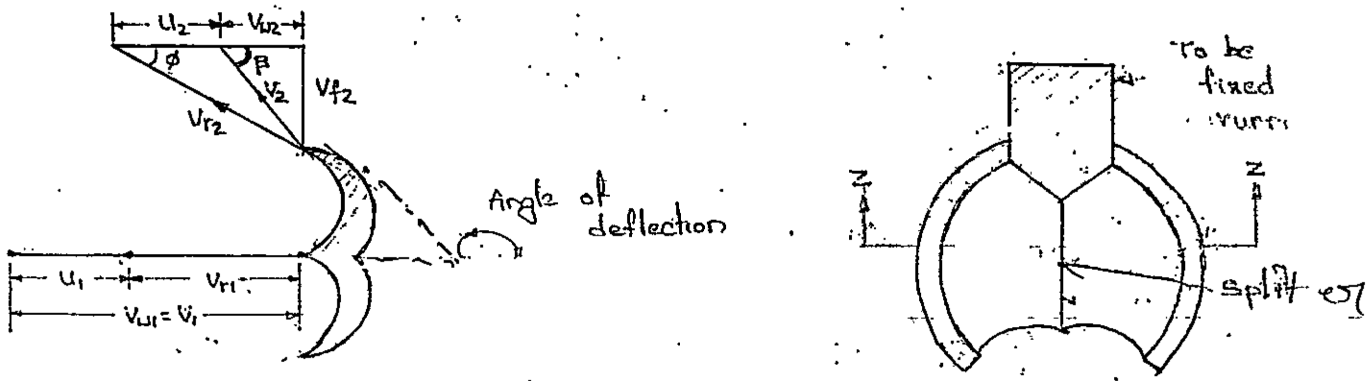
2) RUNNER WITH BUCKETS :- fig. shows the runner of a pelton wheel. It consists of a circular disc on the periphery of which a number of buckets evenly spaced are fixed. The shape of the buckets is a double hemispherical cup bowl. each bucket is divided into two symmetrical parts by a dividing wall which is known as splitter.

3) CASING :- The function of the casing is to prevent the splashing of water & to discharge water to tail race.

4) Breaking Jet :- In order to stop the runner in a short time a small nozzle is provided which directs the jet of water on the back of the vanes. This jet of water is called breaking. 53

Velocity Triangles & work done for Pelton wheel :- (48)

The jet of water from the nozzle strikes the bucket at the splitter, which splits up the jet into two parts. These parts of the jet, glides over the inner surfaces & comes out at the outer edge. The inlet velocity triangle is drawn at the splitter & outlet velocity triangle is drawn at the outer edge of the bucket.



shape of bucket.

Let H = Net head acting on the Pelton wheel
$= H_g - h_f$

N = speed of the wheel in r.p.m.

d = Diameter of jet

D = Diameter of the wheel

V_1 = velocity of jet at inlet $= \sqrt{2gH}$.

$u = u_1 = u_2 = \frac{\pi DN}{60}$.

The velocity triangle at inlet will be a straight line

$$V_{r1} = V_1 - u_1 = V_1 - u \quad , \quad V_{w1} = V_1$$

$$\alpha = 0^\circ \;\; \& \;\; \theta = 0^\circ$$

~~From~~ from the velocity triangle at outlet, we have

$$V_{r2} = V_{r1} \;\; \& \;\; V_{w2} = V_{r2}\cos\phi - u_2 .$$

The force exerted by the jet of water in the direction of motion is given as

$$\boxed{F_x = \rho a V_1 (V_{w1} + V_{w2})}$$

work done by the jet on the runner per second

$$= F_x \times u = \rho a V_1 (V_{w1} + V_{w2}) \times u \quad Nm/s$$

power given to the runner by the jet

$$= \frac{\rho a V_1 (V_{w1} + V_{w2}) \times u}{1000} \quad kW.$$

work done/s per unit weight of water striking/s

$$= \frac{\rho a V_1 (V_{w1} + V_{w2}) \times u}{\rho a V_1 \times g}$$

$$= \boxed{\frac{1}{g}\left[V_{w1} + V_{w2}\right] \times u}$$

The energy supplied to the jet at inlet is in the form of k.e & is equal to $\frac{1}{2} m V_1^2$

$\therefore$ k.e of jet per second $= \frac{1}{2}(\rho a V_1) \times V_1^2$

$\therefore$ Hydraulic efficiency, $\eta_h = \dfrac{\text{work done per second}}{\text{k.e of jet per second}}$

$$= \frac{\rho a V_1 \left[V_{w1} + V_{w2}\right] \times u}{\frac{1}{2} \rho a V_1 \times V_1^2}$$

$$\boxed{\eta_h = \frac{2\left[V_{w1} + V_{w2}\right] \times u}{V_1^2}}$$

Now $V_{w1} = V_1$, $V_{r1} = (V_1 - u)$

$V_{w2} = V_{r2} \cos\phi - u = (V_1 - u)\cos\phi - u$

substituting V_{w1} & V_{w2}.

$$\eta_h = \frac{2(V_1 - u)\left[1 + \cos\phi\right] \times u}{V_1^2}$$

The efficiency will be maximum for a given value of V_1 when

$$\frac{d}{du}(\eta_h) = 0 \Rightarrow \frac{d}{du}\left[\frac{2(V_1-u)[1+\cos\phi]u}{V_1^2}\right] = 0 \quad \text{(or)}$$

$$\Rightarrow \frac{2[1+K\cos\phi]}{V_1^2} \times \frac{d}{du}(V_1u-u^2) = 0$$

$$\Rightarrow \frac{2[1+K\cos\phi]}{V_1^2} \neq 0$$

$$\Rightarrow \frac{d}{du}[V_1u-u^2] = 0 \Rightarrow V_1 - 2u = 0 \Rightarrow u = \frac{V_1}{2}$$

$$\Rightarrow 2V_1 - 4u = 0$$

$$\boxed{u = \frac{V_1}{2}}$$

The above equation states that hydraulic efficiency of a Pelton will be maximum when the velocity of the wheel is half the velocity of the jet of water at inlet.

$$\therefore \text{maximum efficiency } \eta_h = \frac{2\left(V_1 - \frac{V_1}{2}\right)(1+\cos\phi) \times \frac{V_1}{2}}{V_1^2}$$

$$\boxed{\max.\eta_h = \frac{1+\cos\phi}{2}}$$

* Points to be Remembered for Pelton wheel :-

i) Velocity of jet at inlet is given by $V_1 = C_v\sqrt{2gH}$

where C_v = coefficient of velocity
= 0.98 or 0.99
H = net head on turbine.

ii) Velocity of wheel (u) is given by $u = \phi\sqrt{2gH}$

where ϕ = speed ratio = 0.43 to 0.48

iii) The angle of deflection of the jet through buckets is taken as 165°, if no angle of deflection is given.

iv) The mean diameter or the pitch diameter D of the pelton wheel is given by $u = \frac{\pi DN}{60}$ or $D = \frac{60u}{\pi N}$.

v) Jet Ratio :- It is defined as the ratio of pitch diameter D of the pelton wheel ~~is given by~~ to the diameter of jet (d). It is denoted by "m".

$$m = \frac{D}{d} \quad (= 12 \text{ for most cases})$$

(in general varies from 11 to 16)

vi) Number of Buckets on a runner is given by

$$Z = 15 + \frac{D}{2d} = 15 + 0.5m$$

vii) Number of Jets. It is obtained by dividing the total rate flow through the turbine by the rate of flow of water through a single jet.

$$\therefore \text{Number of Jets} = \frac{\text{Total discharge}}{\text{Discharge of single jet}}$$

Design of Pelton wheel :- Design of pelton wheel means the following data is to be determined.

1. Diameter of jet (d) 2. Diameter of wheel (D).
3. Width of buckets = 5xd   4. Depth of buckets = 1.2xd
5. Number of buckets on the wheel.

prob:- The water available for a pelton wheel is 4 m^3/sec (cumec) & the total head from the reservoir to the nozzle is 250 meters. The turbine has two runners with two jets per runner. All the four jets have the same diameters. The pipe line is 3000 metres long. The efficiency of power transmission through the pipe line & the nozzle is 91% & efficiency of each runner is 90%. The velocity coefficient of each nozzle is 0.975 & coefficient of friction '4f' for the pipe is 0.0045. Determine 1) power developed by the turbine 2) Diameter of the jet. 3) Diameter of the pipe line.

Sol:- Given Data Total Discharge $Q = 4 \ m^3/sec$

Total head or Gross head $H_g = 250$ m.

Total number of jets = 4

Length of the pipe $L = 3000$ m.

Efficiency of the pipeline & nozzle = 91% or 0.91

Efficiency of runner $\eta_h = 90\%$ or 0.90.

$C_v = 0.975$ $4f = 0.0045$.

Efficiency of power transmission through pipe lines & nozzles
given by $\eta = \frac{H_g - h_f}{H_g} \Rightarrow 0.91 = \frac{250 - h_f}{250}$ (10)

$\Rightarrow h_f = 22.5$ m.

$\therefore$ Net head $H = H_g - h_f = 250 - 22.5 = 227.5$ m

Velocity of jet $V_1 = C_v\sqrt{2gH} = 0.975 \times \sqrt{2 \times 9.81 \times 227.5}$

$= 65.14$ m/s.

1) power at inlet of the turbine is given as $W.P = \text{K.E/s}/1000$

$$\eta_h = \frac{\text{power developed by turbine}}{W.P}$$

$$\Rightarrow 0.90 = \frac{\text{power developed by turbine}}{W.P}$$

$\Rightarrow$ power developed by turbine $= 0.90 \times \frac{K.E/s}{1000}$

$$= 0.90 \times \frac{\frac{1}{2} m V_1^2}{1000}$$

$$= 0.90 \times \frac{1}{2} \times \frac{\rho Q V_1^2}{1000}$$

$= 7637.8$ kW $\frac{\rho Q \sqrt{2g}.H}{2}$

2) Discharge per jet $= \frac{\text{Total discharge}}{\text{no. of jets}}$

$$= \frac{4.0}{4.0} = 1.0 \text{ m}^3/\text{s}$$

$q_1 =$ Area of one jet $\times$ velocity of jet

$1.0 = \frac{\pi}{4} \times d^2 \times 65.14$

$\Rightarrow d = 0.14$ m

3) $h_f = \frac{4fLV^{*2}}{2g \times D}$ $V^* =$ velocity through the pipe $= \frac{Q}{\frac{\pi}{4}D^2}$

$$\Rightarrow h_f = \frac{4f \times L \times \left[\frac{4Q}{\pi D^2}\right]^2}{2g \times D}$$

$\Rightarrow D = 0.955$ m

Prob:- Three-jet Pelton turbine is required to generate 10,000 kW under a net head of 400m. The blade angle at outlet is 15° & the reduction in the relative velocity while passing over the blade 5%. If the overall efficiency of the wheel is 80%, $C_v = 0.98$ & speed ratio = 0.46, then find: 1) Diameter of jet 2) total flow in m^3/s & 3) force exerted by a jet on the buckets. If the jet ratio is not be less than 10, find the speed of the wheel for a frequency of 50 hertz/sec & the corresponding wheel diameter.

Sol:- Given No. of jets = 3 Total power P = 10,000 kW.

Net head, H = 400m Blade angle at outlet, $\phi = 15°$.

Relative velocity at outlet = 0.95 of relative velocity at inlet

$$V_{r2} = 0.95\, V_{r1}$$

overall efficiency, $\eta_0 = 0.80$. $C_v = 0.98$ speed ratio = 0.46

frequency f = 50 hertz/sec.

$$\eta_0 = \frac{P}{\rho g Q H/1000}$$

$$\Rightarrow Q = \frac{10,000}{0.8 \times 9.81 \times 400} = 3.18\ m^3/s.$$

Discharge from one nozzle = $\frac{3.18}{3} = 1.06\ m^3/s$.

i) Diameter of jet (d)

Discharge from one nozzle = Area of jet × velocity.

$$\Rightarrow 1.06 = \frac{\pi}{4} \times d^2 \times C_v\sqrt{2gH}$$

$$\Rightarrow d = 125\ mm.$$

ii) Total flow = 3.18 m^3/s

iii) force exerted by the jet on the wheel

$$\text{Speed ratio} = \frac{u_1}{\sqrt{2gH}}$$

$$u_1 = 0.46 \times \sqrt{2 \times 9.81 \times 400} = \cancel{84}\ m/s.$$

$$= 40.75\ m/s$$

now $V_{r1} = V_1 - u_1 = 87 - 40.75 = 46.25$ m/s

$V_{r2} = 0.95 V_{r1} = 44$ m/s.

$V_{w1} = V_1 = 87$ m/s.

$V_{w2} = V_{r2} \cos\phi - u_2 = 44 \cos 15^\circ - 40.75 = 1.75$ m/s.

force exerted by the jet on the buckets

$= \rho \times$ discharge through one jet $\times (V_{w1} + V_{w2})$

$= 1000 \times 1.06\ (87 + 1.75) = 94.075$ kN.

Jet ratio $\frac{D}{d} = 10$

$\therefore$ Diameter of wheel $D = 10 \times d = 1.25$ m

W.K.T $u_1 = \frac{\pi D N}{60} \Rightarrow N = \frac{60 \times 40.75}{\pi \times 1.25} = 620$ r.p.m.

using the relation $N = \frac{60 \times f}{P}$ $f \to$ frequency, $P \to$ pairs of poles.

$$P = \frac{60 \times f}{N}$$

$$= \frac{60 \times 50}{620} = 4.85 \cong 5 \text{ poles.}$$

corresponding to 5 pairs of poles, speed of the turbine

$$N = \frac{60 \times f}{P} = \frac{60 \times 50}{5} = 600 \text{ r.p.m}$$

But $u = \frac{\pi D N}{60}$.

As the peripheral velocity is constant. Hence with the change of speed, diameter of wheel will change.

$$D = \frac{60 \times u}{\pi \times N} = \frac{60 \times 40.75}{\pi \times 600} = 1.3 \text{ m}$$

$\therefore$ Jet ratio becomes $= \frac{D}{d} = \frac{1.3}{0.125} > 10$

$\therefore$ Hence the given condition is satisfied.

RADIAL FLOW REACTION TURBINES :-

Radial flow turbines are those turbines in which water flows in the radial direction.

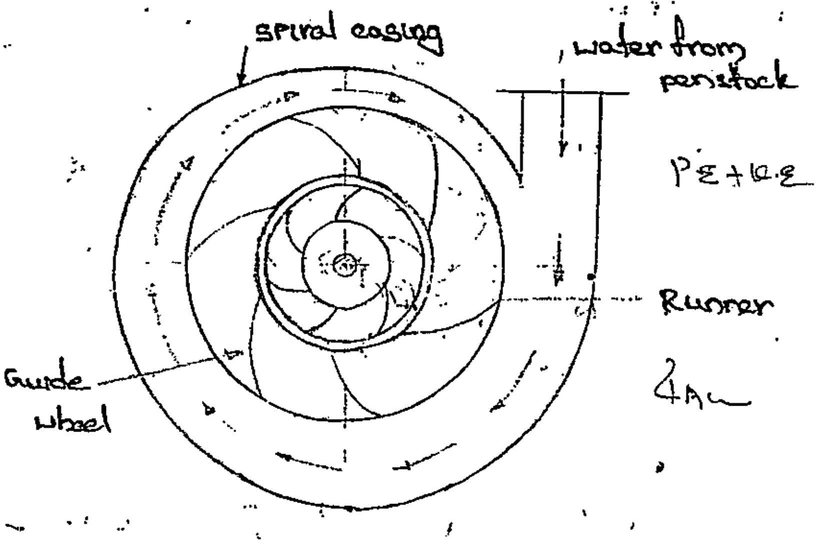
If the water flows from outwards to inwards through the runner the turbine is known as inward radial flow turbine.

If the water flows from inwards to outwards, the turbine is known as outward radial flow turbine. Ex: Francis, Kaplan & Propeller.

Main Parts of a Radial Flow Reaction Turbine :-

The main parts of a radial flow reaction turbine are:

1. Casing 2. Guide mechanism 3. Runner 4. Draft tube.



Casing :- The casing is made of concrete, cast steel or plate steel. casing and runner are always full of water. The casing completely surrounds the runner of the turbine.

Guide Mechanism :- It consists of a stationary circular wheel all round the runner of the turbine. stationary guide vanes are fixed on the guide mechanism. These guide vanes allow the water to strike the vanes fixed on the runner without shock at inlet.

Runner :- It is a circular wheel on which a series of radial curved vanes are fixed. The runner is made of cast steel, cast iron or stainless steel. The number of runner blades varies between 16 to 24.

Draft-tube :- The pressure at the exit of the turbine is generally less than atmospheric pressure. A tube of gradually increasing area is for discharging water from the exit of the turbine to tail race. tube of increasing area is called draft tube. (52)

Inward Radial Flow Turbine :- The water flows from outwards to inwards through the runner, the turbine is known as inward flow Turbine. The water flows over the moving vanes in the inward radial direction & is discharged at the inner diameter of the runner. Ex: Francis turbine.

Velocity Triangles & work done for francis turbine :-

As the turbine is a type of Radial flow turbine, the velocity triangles at inlet & outlet will be same as that of moving rad curved vanes. from the velocity triangles, work done by the wat on the runners & efficiency of the turbine can be obtained.

The work done per second on the runner by water is given a

$$= \rho a V_1 [V_{w1} u_1 \pm V_{w2} u_2].$$

$$\boxed{W.D/s = \rho Q [V_{w1} u_1 \pm V_{w2} u_2]}$$

where V_{w1} = Velocity of whi at inl

V_{w2} = Velocity of whi at out

u_1 = Tangential velocity o

$= \frac{\pi D_1 N}{60}$ D_1 = outer d

$u_2 = \frac{\pi D_2 N}{60}$ D_2 = ou

The work done per second per unit weight of water per second

$$= \frac{\rho Q [V_{w1} u_1 \pm V_{w2} u_2]}{\rho Q \times g} = \frac{1}{g} [V_{w1} U_1 \pm V_{w2}$$

This eqn is known as fundamental eqn of Hydrodynamic Machines also the eqn is called by other name as Euler's eqn of.

Hydraulic efficiency :- $\boxed{\eta_h = \frac{R.P}{W.P} = \frac{(V_{w1}u_1 \pm V_{w2}u_2)}{gH}}$

The hydraulic efficiency of the francis turbine varies from 85 % to 90 %.

points to be Remembered for francis Turbine :-

1. Ratio of width to diameter $\left(\frac{B}{D}\right)$ at inlet is represented by 'n'.

$$n = \frac{B_1}{D_1}$$ 'n' varies from 0.1 to 0.45

2. speed ratio (k_u) : It is the ratio of the peripheral speed at inlet to the theoretical jet velocity. Thus,

$$\boxed{k_u = \frac{u_1}{\sqrt{2gH}}}$$

k_u ranges from 0.6 to 0.9

u_1 Tangential velocity of wheel inlet

3. flow ratio (k_f) : It is the ratio of velocity of flow at inlet to the theoretical jet velocity. thus,

$$\boxed{k_f = \frac{V_{f1}}{\sqrt{2gH}}}$$

k_f varies 0.15 to 0.30

Design of francis turbine :-

1. Discharge of the turbine. $Q = \pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2}$

of the thickness of the vanes are considered, then the area through which the flow takes place is given by

$$Q = (\pi D_1 - n \times t) \times B_1 \times V_{f1}$$

2. Head on the turbine is given by $H = \frac{P_1}{\rho g} + \frac{V_1^2}{2g}$

3. Tangential velocity of wheel at inlet $u_1 = \frac{\pi D_1 N}{60}$

4. Hydraulic efficiency $\eta_h = \frac{V_{w1} u_1}{gH}$

5. obtain the guide vane angle (α) & the runner vane angle (θ) fro the following relations obtained from inlet velocity triangle. (53)

16, 17, 18, 19, 20, 21, 22

$$\tan\alpha = \frac{V_{f1}}{V_{w1}} \quad \& \quad \tan\theta = \frac{V_{f1}}{V_{w1}-u_1}.$$

6. Assume runner diameter D_2 at outlet to be approximately one-h the diameter at inlet.

$$D_2 = \frac{D_1}{2} \quad \& \quad u_2 = \frac{u_1}{2}.$$

7. Velocity of flow at the exit (V_{f2}) is obtained as follows:

$$Q = k_{f1}\pi D_1 B_1 V_{f1} = k_{f2}\pi D_2 B_2 V_{f2}$$

Thus $\dfrac{V_{f1}}{V_{f2}} = \dfrac{k_{f2}\pi D_2 B_2}{k_{f1}\pi D_1 B_1}$ $\quad \because V_{f1} = V_{f2}$ & $k_{f1} = k_{f2}$, $B_2 =$

K_{f1}, K_{f2} Blade thickness coefficient

8. Assuming the discharge at the runner exit to be radial ($\beta = 90°$), the runner vane angle at exit (ϕ) from the outlet triangle

$$\tan\phi = \frac{V_{f2}}{u_2}.$$

9. If there is no loss of energy when water flows through the vanes we have $H - \frac{V_2^2}{2g} = \frac{1}{g}[V_{w}$

prob:- An inward flow reaction turbine has external & internal diame as 1.08 m & 0.54 m. The turbine is running at 200 r.p.m. The width of the turbine at inlet is 240 mm & velocity of flow through the runn is constant & is equal to 2.16 m/s. The guide blades make an an of 10° to the tangent of the wheel & discharge at the outlet of th turbine is radial. Draw the inlet & outlet velocity triangles & determine: 1) The absolute velocity of water at inlet of the runne 2) The velocity of whirl at inlet 3) The relative velocity at inlet, 4) The runner blade angles. 5) width of runner at outlet, 6) weight of water flowing through the runner per second. 7) Head at inlet of the turbine 8) power developed 9) Hydraulic efficiency of the turbine.

8

Sol :- External diameter $D_1 = 1.08\ m$, Internal diameter $D_2 = 0.54\ m$

Speed $N = 200$ r.p.m , width at inlet $B_1 = 240\ mm = 0.24\ m$.

Velocity of flow, $V_{f_1} = V_{f_2} = 2.16\ m/s$ Guide blade angle, $\alpha = 10^\circ$.

Discharge at outlet is radial $\beta = 90^\circ$ & $V_{w_2} = 0$

Tangential velocity of wheel at inlet & outlet

$$u_1 = \frac{\pi D_1 N}{60} \quad , \quad u_2 = \frac{\pi D_2 N}{60}$$

$$= 11.31\ m/s \qquad\qquad = 5.65\ m/s$$

1) Absolute velocity of water at inlet of the runner, V_1 :

from inlet velocity triangle, we have

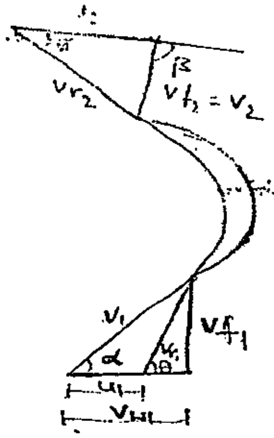
$$V_1 \sin\alpha = V_{f_1}$$

$$\therefore \quad V_1 = \frac{V_{f_1}}{\sin\alpha} = \frac{2.16}{\sin 10^\circ} = 12.44\ m/s$$

2) Velocity of whirl at inlet, V_{w_1} :

$$V_{w_1} = V_1 \cos\alpha = 12.44 \cos 10^\circ$$

$$= 12.25\ m/s$$



3) Relative velocity at inlet, V_{r_1} :

$$V_{r_1} = \sqrt{(V_{w_1} - u_1)^2 + V_{f_1}^2}$$

$$= 2.35\ m/s$$

4) Runner blade angles, θ, ϕ :

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} \Rightarrow \theta = 66.48^\circ$$

$$\tan\phi = \frac{V_{f_2}}{u_2} \Rightarrow \phi = 20.9^\circ$$

5) width of runner at outlet, B_2 :

$$\pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$$

$$\Rightarrow B_2 = \frac{D_1 B_1}{D_2} = 0.48\ m$$

6) weight of water flowing per second $= w \times Q = w \times \pi D_1 B_1 V_{f_1}$

$= 17.25\ kN/s$

(54) 62

7) Head at inlet of turbine, H:

$$H = \frac{1}{g}(V_{w_1}u_1 \pm V_{w_2}u_2) + \frac{V_2^2}{2g}$$ $[\because V_{w_2}=0]$, $[V_2 = V_{f_2}]$

$= 14.36 m$

8) power developed $= \rho Q \times V_{w_1}U_1 = 243.6$ kW

9) Hydraulic efficiency, $\eta_h = \frac{V_{w_1}U_1}{gH} = 98.35\%$.

prob:- The following data is given for a francis Turbine. Net Head H = speed N = 700 r.p.m ; shaft power = 294.3 kW ; $\eta_o = 84\%$; $\eta_h = 93\%$; flow ratio = 0.20 ; breadth ratio n = 0.1 ; outer diameter of the runner 2x inner diameter of runner. The thickness of vanes occupy 5% circumfe area of the runner, velocity of flow is constant at inlet & outlet & discharge is radial at outlet. Determine :

i) Guide blade angle, ii) Runner vane angles at inlet & outlet

iii) Diameters of runner at inlet & outlet, and iv) width of wheel at i

v) Degree of reaction.

sol:- Given Data Net Head H = 60 m speed N = 700 r.p.m

shaft power = 294.3 kW $\eta_o = 84\% = 0.84$ $\eta_h = 0.93$.

Flow ratio, $\frac{V_{f_1}}{\sqrt{2gH}} = 0.20$ $\Rightarrow$ $V_{f_1} = 0.20 \times \sqrt{2\times9.81\times60} = 6.86$ m/s

Breadth ratio, $\frac{B_1}{D_1} = 0.1$, outer dia $D_1 = 2\times D_2$, $V_{f_1} = V_{f_2} = 6.8$

Thickness of vanes = 5% of circumferential area of runner

$\therefore$ Actual area of flow $= 0.95\,\pi D_1 B_1$

Discharge at outlet is Radial $\therefore V_{w_2} = 0$ & $V_{f_2} = V_2$

using relation $\eta_o = \frac{S.P}{W.P}$

$$0.84 = \frac{294.3}{W.P} \Rightarrow W.P = 350.35 \text{ kW}$$

But $W.P = \frac{WH}{1000} = \frac{\rho g Q H}{1000} \Rightarrow Q = 0.59 \text{ m}^3/\text{s}$

Discharge Q = Actual area of flow × velocity of flow

$$= 0.95 \times \pi D_1 B_1 \times V_{f1}$$

$$0.59 = 0.95 \times \pi D_1 \times 0.1 D_1 \times 6.86$$

$$\Rightarrow D_1 = 0.54\ m.$$

$$\Rightarrow B_1 = 0.1 \times 0.54 = 0.054\ m$$

Tangential speed of the runner at inlet, $u_1 = \dfrac{\pi D_1 N}{60}$

$$= \frac{\pi \times 0.54 \times 700}{60}$$

$$= 19.79\ m/s$$

W.K.T $\eta_h = \dfrac{V_{w1} U_1}{gH}$

$$\Rightarrow V_{w1} = \frac{0.93 \times 9.81 \times 60}{19.79} = 27.66\ m/s.$$

i) Guide blade angle (α)

from inlet velocity Δ^{le}, $\tan\alpha = \dfrac{V_{f1}}{V_{w1}}$

$$\Rightarrow \alpha = 13^\circ 55$$

ii) Runner vane angles at inlet & outlet (θ & ϕ)

$$\tan\theta = \frac{V_{f1}}{V_{w1} - U_1} \Rightarrow \theta = 41^\circ 5.4'$$

$$\tan\phi = \frac{V_{f2}}{u_2} = \frac{6.86}{9.89} \Rightarrow \phi = 34^\circ 44.4' \qquad u_2 = \frac{\pi D_2 N}{60} = 9.89\ m/s$$

iii) Diameters of runner at inlet & outlet

$$D_1 = 0.54 \qquad D_2 = 0.27\ m$$

iv) width of wheel at inlet $B_1 = 54\ mm$

v) Degree of reaction $R = 1 - \dfrac{\cot\alpha}{2(\cot\alpha - \cot\theta)}$ $\alpha = 13.92$, $\theta = 41.09^\circ$

$$= 0.3$$

for francis Turbine, the degree of reaction varies from

0 to 1 i.e. $0 \le R \le 1$

AXIAL FLOW REACTION TURBINES :-

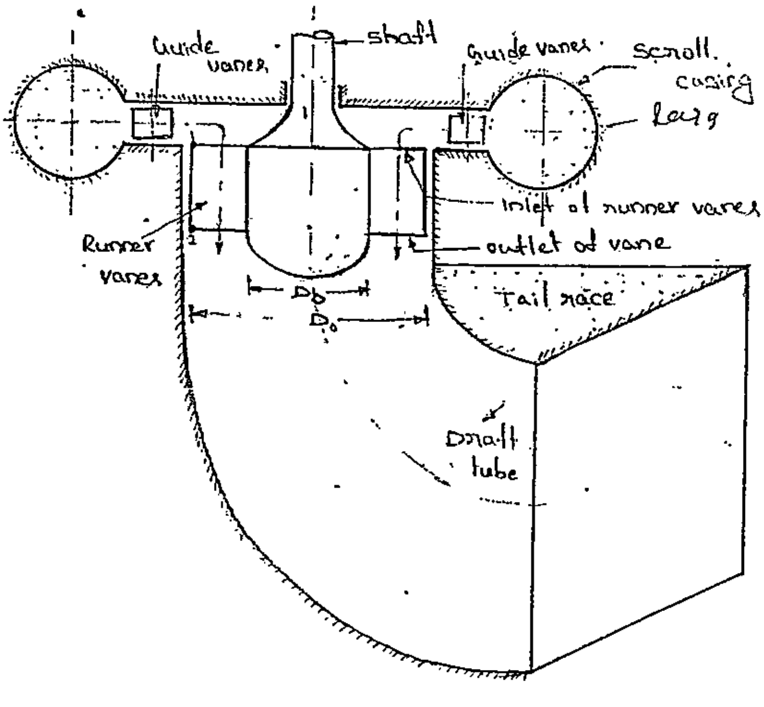
If the water flows parallel to the axis of the rotation of the sha the turbine is known as axial flow turbine. (55)

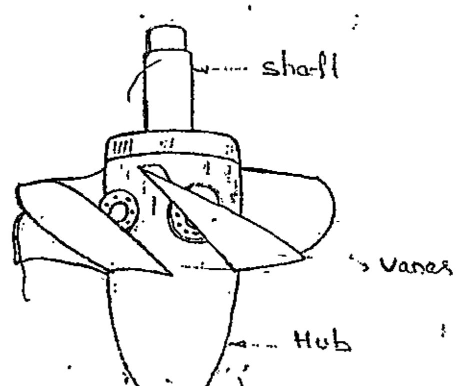
For axial flow turbine, the shaft of the turbine is vertical, the lower end of the shaft is made larger which is known as 'hub' or 'boss' the vanes are fixed on the hub & hence hub acts as a runner. propeller turbine & kaplan turbine are the important types of axial flow reaction turbines.

When the vanes are fixed to the hub & they are not adjustable the turbine is called as propeller turbine.

When the vanes on the hub are adjustable, the turbine is known as kaplan turbine.

Kaplan turbine :- kaplan turbine is suitable where a large quan of water at low heads is available. The number of vanes on t runner is varies from 3 to 8.





Kaplan turbine runner

The main components of a kaplan turbine are :

1. Scroll casing , 2) Guide vane mechanism
3. Hub with vanes or runner of the turbine 4. Draft tube.

The water from the penstock enters the scroll casing and then moves to the guide vanes. from the guide vanes the water turns through 90° and flows axially through the runner.

Discharge through the runner is

$$\boxed{Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}}$$

runner

D_o — outer diameter of runner

D_b — diameter of hub.

Important points for kaplan turbine :-

1. Discharge Q = Area of flow × velocity of flow

$$= \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_f$$

$$= \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times k_f\sqrt{2gH}$$ k_f = flow ratio

$$Q = \frac{\pi}{4} D_o^2 \left(1 - n^2\right) \times k_f \cdot \sqrt{2gH}$$ $\left[\because n = \frac{D_b}{D_o}\right]$.

value 'n' varies from 0.35 to 0.60.

n = no. of buckets.

2. peripheral velocity at inlet & outlet are equal

$$u_1 = u_2 = \frac{\pi D_o N}{60}$$

3. Velocity of flow is constant. $V_{f_1} = V_{f_2}$.

prob :- A kaplan turbine develops 22000 kW at an average head of 35m. Assuming a speed ratio of 2, flow ratio of 0.6, diameter of the boss equal to 0.35 times the diameter of the runner & an overall efficiency of 88 percent, calculate the diameter, speed and specific speed of the turbine?

Sol :- Given Data shaft power $P = 22000$ kW ; Head $H = 35$m;
speed ratio, $k_u = 2.0$, flow ratio $k_f = 0.6$; $D_b = 0.35\, D_o$;
$\eta_o = 88\%$

from $k_u = \frac{u_1}{\sqrt{2gH}} \Rightarrow u_1 = 2.0 \times \sqrt{2 \times 9.81 \times 35} = 52.4$ m/s.

$k_f = \frac{V_{f_1}}{\sqrt{2gH}} \Rightarrow V_{f_1} = 15.7$ m/s

$\eta_0 = \frac{S.P}{W.P} \Rightarrow W.P = \frac{22000}{0.88} =$

$$Q = \frac{22000}{0.88 \times 9.81 \times 35} = 72.8 \ m^3/s.$$

1) Diameter of runner D_o ;

W.K.T Q = Area of flow x velocity of flow

$$72.8 = \frac{\pi}{4} (D_o^2 - D_b^2) \times V_{f_1}.$$

$$72.8 = \frac{\pi}{4} (D_o^2 - (0.35 D_o)^2) \times V_{f_1}$$

$$\Rightarrow D_o = 2.6 \ m.$$

2) speed of the turbine, N ;

$$u_1 = \frac{\pi D_o N}{60} \Rightarrow N = 384.6 \ rpm$$

3) specific speed of the turbine, N_s :

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{384.9 \times \sqrt{22000}}{(35)^{5/4}}$$

$$= 670.6 .$$

prob : A propeller reaction turbine of runner diameter 4.5m is running at 40 r.p.m. the guide blade angle at inlet is 145° & runner blade angle at outlet is 25° to the direction of vane. The axial flow area of water through runner is 25 m². If the runner blade angle at inlet is radial determine: 1) Hydraulic efficiency of turbine. 2) Discharge through turbine 3) power developed by the runner & 4) specific speed of the turbine.

Sol:- Given Data Runner diameter $D_o = 4.5$m , speed N = 40 rpm Guide blade angle, $\alpha = 145°$, Runner blade angle at outlet, $\phi = 25°$

Flow area, $a = 25\ m$

Runner blade angle at inlet is radial $\therefore \theta = 90^\circ$, $V_{r_1} = V_{f_1}$ & $u_1 = V_{w_1}$.

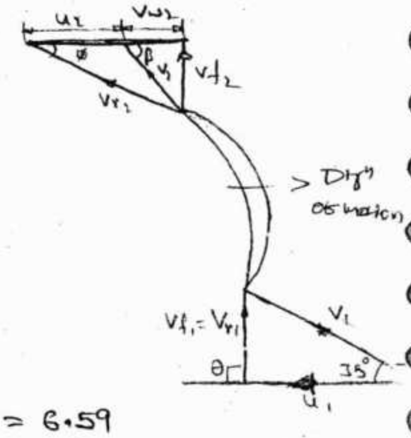
As area of flow is constant & $V_{f_1} = V_{f_2}$.

$\because Q =$ area of flow $\times V_{f_1}$ = area of flow $\times V_{f_2}$

Tangential velocity of turbine at inlet

$$u_1 = \frac{\pi D_0 N}{60} = \frac{\pi \times 4.5 \times 40}{60}$$

$$= 9.42\ m/s.$$



$$\therefore u_2 = u_1 = 9.42\ m/s.$$

from inlet velocity Δ^{le},

$$\tan(180 - \alpha) = \frac{V_{f1}}{u_1}$$

$$\tan(180 - 145) = \frac{V_{f_1}}{u_1}$$

$$\Rightarrow V_{f_1} = V_{w_1} \tan(35) = 9.42 \tan 35 = 6.59$$

$$V_{w_1} = u_1 = 9.42\ m/s.$$

from outlet velocity Δ^{le}, $\tan\phi = \frac{V_{f_2}}{u_2 + V_{w_2}} \Rightarrow \tan 25^\circ = \frac{6.59}{9.42 + V_{w_2}}$

$$\therefore V_{w_2} = 4.71\ m/s.$$

$$V_2 = \sqrt{V_{f_2}^2 + V_{w_2}^2} = 8.1\ m/s$$

Head at inlet, $H - \frac{V_2^2}{2g} = \frac{1}{g}\left[V_{w_1}U_1 - V_{w_2}u_2\right]$.

'−ve' V_1 & V_2 are in same direct.

$$H - \frac{8.1^2}{2 \times 9.81} = \frac{1}{9.81}\left[9.42 \times 9.42 - 4.71 \times 9.42\right]$$

$$\Rightarrow H = 7.866\ m$$

1) Hydraulic efficiency $\eta_h = \frac{V_{w_1}u_1 - V_{w_2}u_2}{gH} = 57.5\%$.

2) Discharge through turbine, $Q = 25 \times V_{f_1} = 164.75\ m^3/s$.

3) power developed by turbine, $P = \rho Q(V_{w_1}U_1 - V_{w_2}u_2)/1000$

$$= 6867\ kW$$

4) specific speed $N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{40 \times \sqrt{6867}}{7.866^{5/4}} = 251.6$ r.p.m.

DRAFT TUBE :- The draft tube is a pipe of gradually increasing area which connects the outlet of the runner to the tail race. (57) one end of the draft tube is connected to the outlet of the ru while the other end is sub-merged below the level of water in tail race.

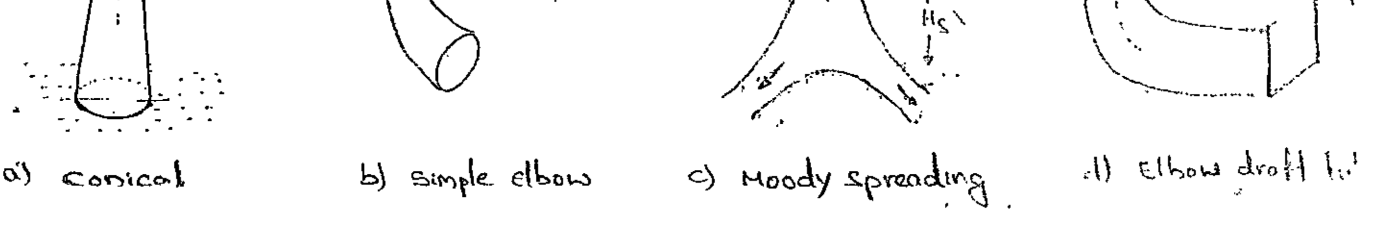
purpose of Draft tube :-

1. It converts a large portion of kinetic energy rejected at the outlet of the turbine into useful pressure energy.
2. It permits a negative head to be established at the outlet of the runner & there by increase the net head on the turbine. The turbine may be placed above the tail race with out any loss of net head &, hence turbine may be inspected properly.

Types of Draft tubes :-

commonly used important types of draft tubes are:

1) conical draft tubes
2) simple elbow tubes
3) Moody spreading tube
4) Elbow draft tube with circular inlet & rectangular outlet.



a) conical   b) simple elbow   c) Moody spreading   d) Elbow draft tu

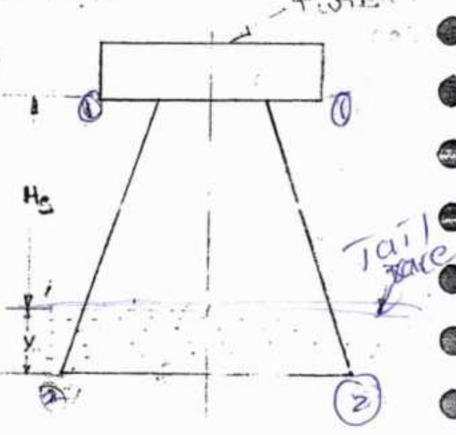
The conical draft tubes & Moody spreading draft-tubes are most efficient than other draft tubes.

Draft - tube theory :-

Consider a conical draft tube.

Let H_s = vertical height of draft tube above tail race

y = distance of bottom of draft tube from tail race.



Applying Bernoulli's equation at inlet 1-1 & outlet 2-2 of the draft tube.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + (H_s + y) = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + 0 + h_f$$

But $\frac{P_2}{\rho g} = \frac{P_a}{\rho g} + y$

$$(or)\ \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + H_s = \frac{P_a}{\rho g} + \frac{V_2^2}{2g} + h_f$$

$$P_1/\rho g = \frac{P_a}{\rho g} + \frac{V_2^2}{2g} + h_f - V_1^2/2g - H_s$$

$$\Rightarrow \frac{P_1}{\rho g} = \frac{P_a}{\rho g} - H_s - \left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_f\right] \quad \Rightarrow \frac{P_a}{\rho g} - H_s - \left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_f\right]$$

$\frac{P_1}{\rho g}$ is less than Atmospheric pressure

Efficiency of draft tube :- efficiency of draft tube is defined as the ratio of actual conversion of kinetic head into pressure head in the draft - tube to the kinetic head at inlet of the draft tube.

$$\eta_d = \frac{\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - h_f}{\frac{V_1^2}{2g}}$$

prob :- A conical draft tube having inlet & outlet diameters 1m & 1.5m discharges water at outlet with a velocity of 2.5 m/s. The total length of the draft tube is 6m & 1.20m of the length of draft - tube is immersed in water. If the atmospheric pressure head is 10.3m of water & loss of head due to friction in

, the draft-tube is equal to 0.2 x velocity head at outlet of the tube, find: 1) pressure head at inlet 2) Efficiency of draft tube.

(58)

Sol:- Given Data: Diameter at inlet $D_1 = 1.0m$

Diameter at outlet $D_2 = 1.5m$, velocity at outlet $V_2 = 2.5 m/s$.

Total length of tube $H_s + y = 6.0m$

Length of tube in water, $y = 1.20m$

$$H_s = 6.0 - 1.20 = 4.80m$$

Atmospheric pressure head $\frac{P_a}{\rho g} = 10.3m$

$$h_f = 0.2 \times \frac{V_2^2}{2g}$$

Discharge through tube, $Q = A_2 \times V_2 = \frac{\pi}{4} \times (1.5)^2 \times 2.5 = 4.41 m^3/$

velocity at inlet $V_1 = \frac{Q}{A_1} = \frac{4.4}{\frac{\pi}{4} \times 1^2} = 5.625 m^3/sec$

1) pressure head at inlet $\left(\frac{P_1}{\rho g}\right)$.

$$\frac{P_1}{\rho g} = \frac{P_a}{\rho g} - H_s - \left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_f\right]$$

$$= 10.3 - 4.8 - \left[\frac{5.625^2}{2 \times 9.81} - \frac{2.5^2}{2 \times 9.81} - 0.2 \times \frac{V_2}{2}\right.$$

$$\cong 4.27m.$$

2) efficiency of Draft-tube (η_d)

$$\eta_d = \frac{\left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right] - h_f}{\frac{V_1^2}{2g}} = \frac{V_1^2 - 1.2V_2^2}{V_1^2}$$

$$= 76.3\%$$

prepared by
Raja.k

6

Example

(19)

GEOMETRIC SIMILARITY :- Geometric similarity is the similarity of form shape. Two systems — the model & prototype are said to be geometrically similar if the ratios of all corresponding linear dim in the model & prototype are equal.

For geometric similarity: $\frac{l_m}{l_p} = \frac{b_m}{b_p} = \frac{d_m}{d_p}$, where l, b, d are len breadth & depth respec m – model, p – protot

The model is the small scale replica of the actual structure. The actual structure is called prototype.

SPECIFIC SPEED :- The specific speed of a turbine is defined as the speed of a turbine which is identical in shape, geometrical dimensio blade angles, gate opening, etc, which would develop unit power i working under a unit head. It is denoted by the symbol N_s.

Significance of Specific Speed :- specific speed plays an important ro for selecting the type of turbine. Also the performance of a turbine can be predicted by knowing the specific speed of the turbine.

S.No.	Specific speed (S.I units)	Type of turbine.
1.	8.5 to 30	pelton wheel with single
2.	30 to 51	pelton wheel with two or m
3.	51 to 225	Francis turbine
4.	225 to 860	kaplan or propeller turb

In M.K.S unit, unit power is taken as one horse power & unit head as one metre, But in S.I units, unit power is taken a one kilowatt & unit head as one metre.

[illegible]

The overall efficiency of any turbine is given by,

$$\eta_0 = \frac{\text{Shaft Power}}{\text{Water power}} = \frac{P}{\rho g Q H / 1000}.$$

where H = Head, Q = discharge & P = shaft power

$$\Rightarrow \quad P = \eta_0 \times \frac{\rho g Q H}{1000}$$

$$P \propto Q \times H$$

Let D = diameter of actual turbine, N = speed of actual turbine,
u = Tangential velocity of turbine, N_s = specific speed of turbine
V = Absolute velocity of water.

The absolute velocity, tangential velocity & head on the turbine are related as,

$$u \propto V, \quad V \propto \sqrt{H}$$

$$\Rightarrow \quad u \propto \sqrt{H}$$

But $\quad u = \frac{\pi D N}{60}, \quad u \propto DN$

$$\Rightarrow \quad DN \propto \sqrt{H}$$

$$D \propto \frac{\sqrt{H}}{N}.$$

Discharge through the turbine is given by

$$Q = \text{Area} \times \text{velocity}$$

$$\text{Area} \propto B \times D$$

$$\propto D^2$$

$$\therefore \quad Q \propto D^2 \times \sqrt{H}$$

$$\propto \left(\frac{\sqrt{H}}{N}\right)^2 \times \sqrt{H}$$

$$\propto \frac{H^{3/2}}{N^2}.$$

$$\Rightarrow \quad P \propto \frac{H^{3/2}}{N^2} \times H \Rightarrow P \propto \frac{H^{5/2}}{N^2}.$$

$\therefore \quad P = K \cdot \frac{\cdots}{N^2}$

If $P=1$, $H=1$, the speed N = specific speed, N_s.

we get $\quad 1 = \frac{k \times (1)^{5/2}}{N_s^2}$ or $N_s^2 = k$.

$\therefore \quad P = N_s^2 \cdot \frac{H^{5/2}}{N^2}$

$\Rightarrow \quad \boxed{N_s = \frac{N\sqrt{P}}{H^{5/4}}}$

Prob:- A turbine is to operate under a head of 25m at 200 r.p.m. The discharge is 9 m^3/s. If the overall efficiency is 90%, determine i) power generated; ii) specific speed of turbine; iii) Type of turb

Sol:- Given Data Head $H = 25m$, speed $N = 200$ r.p.m.

Discharge $Q = 9\ m^3/s$. $\eta_o = 90\%$.

i) Power generated, P: $\quad P = \eta_0 \times wQH = 0.9 \times 9.81 \times 9 \times 25 = 1986.5$ kW

ii) Specific speed, N_s: $\quad N_s = \frac{N\sqrt{P}}{H^{5/4}} = 159.4$ rpm.

iii) Type of turbine. As the specific speed lies between 51 to 225 the turbine is a francis turbine.

Prob:- A turbine develops 7225 kW power under a head of 25 metre at 135 rpm. calculate the specific speed of the turbine & state the type of turbine.

Sol:- Given Data power developed, $P = 7225$ kW

Head, $H = 25m$, speed $N = 135$ r.p.m.

Specific speed $N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{135 \times \sqrt{7225}}{25^{5/4}} = 205.28$ rpm.

As the N_s lies between 51 & 255 the type of turbine is francis turbine.

A pelton wheel develops 8000 kW under a net head of 130m and speed of 200 r.p.m. Assuming the co-efficient of velocity for the nozzle 0.98, hydraulic efficiency 87%, speed ratio 0.46 & jet diameter to wheel diameter ratio $\frac{1}{9}$, determine:

1) The discharge required; 2) the diameter of the wheel, 3) specific speed 4) The diameter & no of jets required, mechanical efficiency is 75%.

Sol:- Given Data power developed $P = 8000$ kW.

Net head, $H = 130$ m. speed, $N = 200$ r.p.m. $C_v = 0.98$,

$\eta_h = 0.87$. Speed ratio $\frac{u_1}{\sqrt{2gH}} = 0.46$

$\Rightarrow u_1 = 0.46 \cdot \sqrt{2 \times 9.81 \times 130} = 23.23$ m/s.

$\frac{d}{D} = \frac{1}{9}$, $\eta_m = 0.75$.

W.K.T $\eta_o = \eta_m \times \eta_h = 0.87 \times 0.75 = 0.652$

$\therefore \eta_o = \frac{S.P}{W.P} \Rightarrow 0.652 = \frac{8000}{W.P \text{ in kW}}$

$\Rightarrow$ W.P in kW $= \frac{\rho g Q H}{1000}$

$12260.536 = g Q H$

$\Rightarrow Q = 9.614 \ m^3/s$

1) Discharge required $Q = 9.614 \ m^3/s$.

2) Diameter of wheel (D), $u_1 = \frac{\pi D N}{60} \Rightarrow D = 2.218$ m

3) Diameter of jet & No. of jets

$\frac{d}{D} = \frac{1}{9} \Rightarrow d = 246.4$ mm

no. of jets $= \frac{Q}{q} = \frac{9.614}{\frac{\pi}{4} d^2 \times V_1}$ $\quad \therefore V_1 = C_v \sqrt{2gH}$

$= 4$ jets

4) Specific speed, $N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{200 \times \sqrt{8000}}{130^{5/4}}$

$= 40.75$ ~~r.p.m.~~

UNIT QUANTITIES :- In order to predict the behaviour of a turbine working und varying conditions of head, speed, output & gate opening, the (61) results are expressed in terms of quantities which may be obtained when th head on the turbine is reduced to unity. the conditions of the tur under unit head are such that the efficiency of the turbine rem unaffected.

The three important unit quantities which must be studied under unit head are

1) Unit speed 2) Unit power 3) Unit discharge.

UNIT SPEED :- Unit speed is defined as the speed of a turbine working under unit head (i.e., under a head of 1m). It is denot by 'N_u'.

Let, N = speed of a turbine under a head H,
H = Head, u = Tangential velocity.

The tangential velocity, absolute velocity & head on the turbine are related as

$$u \propto V \qquad \therefore V \propto \sqrt{H}$$
$$\propto \sqrt{H}$$

But $u = \frac{\pi D N}{60}$

For a given turbine, Diameter (D) is constant

$$u \propto N \quad \text{or} \quad N \propto \sqrt{H}$$

$$N = k_1 \sqrt{H}$$

If the head on the turbine is unity, the speed becomes unit sp

$$H = 1, \quad N = N_u.$$

$$N_u = k_1 \cdot \sqrt{1} = k_1$$

$$\therefore N = N_u \sqrt{H} \quad \text{or} \quad \boxed{N_u = \frac{N}{\sqrt{H}}}$$

UNIT DISCHARGE :- Unit discharge is defined as the discharge passing through a turbine, which is working under unit head.
It is denoted by 'Q_u'.

Let: H = Head of water on the turbine.

Q = Discharge., a = Area of flow of water.

Q = Area of flow × Velocity

$Q \propto$ Velocity $\propto \sqrt{H}$ ∵ Area of flow is constant for a turbine.

$Q = k_2 \sqrt{H}$.

If $H=1$, $Q = Q_u$ then $Q_u = k_2$

$$\therefore \boxed{Q_u = \frac{Q}{\sqrt{H}}}$$

UNIT POWER :- Unit power is defined as the power developed by a turbine, working under a unit head. It is denoted by 'P_u'.

Let H = Head of water on the turbine.

P = power developed under a Head, H.

Q = Discharge through turbine under a Head H.

w.k.t $\eta_o = \dfrac{P}{\frac{\rho g Q H}{1000}}$

$$\Rightarrow P = \eta_o \times \frac{\rho g Q H}{1000}$$

$$P \propto QH \qquad [\because Q \propto \sqrt{H}]$$

$$\propto \sqrt{H} \cdot H$$

$$P = k_3 \cdot H^{3/2}$$

when $H=1$, $P = P_u$, $P_u = k_3 (1)^{3/2} = k_3$

$$\therefore \boxed{P_u = \frac{P}{H^{3/2}}}$$

prob:- A turbine is to operate under a head of 25m at 200 r.p.m. the discharge is 9 m^3/sec. If the efficiency is 90% determine the performance of turbine under a head of 20m. (62) 71

sol:- Given Data Head under which turbine works, $H_1 = 25m$.

Speed of the turbine, $N_1 = 200$ r.p.m.

Discharge through the turbine, $Q_1 = 9\ m^3/s$.

$\eta_0 = 90\%$

performance of turbine under a head of 20m; N_2; Q_2; P_2:

$$\eta_0 = \frac{P_1}{\frac{wQ_1H_1}{1000}} \Rightarrow P_1 = 0.90 \times 9.81 \times 9 \times 25$$

$$= 1986.5\ kW.$$

W.K.T
$$\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}} \Rightarrow N_2 = \frac{N_1\sqrt{H_2}}{\sqrt{H_1}} = 178.88\ rpm.$$

$$\frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}} \Rightarrow Q_2 = \frac{Q_1\sqrt{H_2}}{\sqrt{H_1}} = \frac{9 \times \sqrt{20}}{\sqrt{25}} = 8.05\ m^3/s.$$

$$\frac{P_1}{\sqrt{H_1^3}} = \frac{P_2}{\sqrt{H_2^3}} \Rightarrow P_2 = \frac{P_1 \times H_2^{3/2}}{H_1^{3/2}} = 1421.4\ kW.$$

prob:- A turbine develops 500 kW power under a head of 100 metres at 200 r.p.m. what would be its normal speed & output under a head of 81 metres?

sol:- Given Data power. $P_1 = 500\ kW$, Head. $H_1 = 100m$

Speed, $N_1 = 200$ r.p.m, $H_2 = 81m$, $N_2 = ?$, $P_2 = ?$

W.K.T.
$$\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}} \Rightarrow N_2 = 180\ r.p.m$$

$$\frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

$$\Rightarrow P_2 = H_2^{3/2} \times \frac{P_1}{H_1^{3/2}}$$

$$= \frac{729}{1000} \times 500$$

$$= 364.5\ kW.$$

CHARACTERISTIC CURVES OF HYDRAULIC TURBINES:-

In general the turbines are designed for specific values of speed, discharge, power & efficiency. But the turbines may be required to operate under conditions different from those for which these have been designed. To know the exact behaviour of turbines under varying conditions, it is necessary to conduct tests. The results so obtained are usually represented graphically & the curves obtained are known as "characteristic curves".

The important parameters which are varied during a test on turbine are:

1) Speed (N) 2) Head (H) 3) Discharge (Q) 4) Power (P)
5) overall efficiency (η_o) 6) Gate opening.

The important characteristic curves of a turbine are:

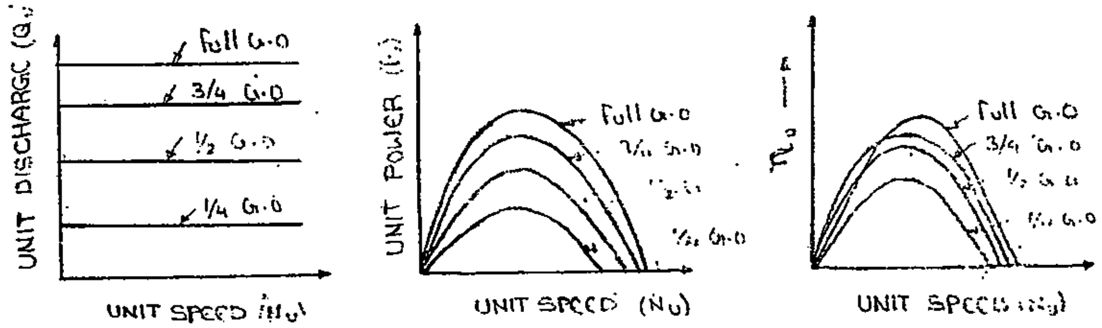
1) Main characteristic curves or constant Head curve.
2) operating characteristic curves or constant speed curve.
3) Muschel curves or constant efficiency curves.

MAIN CHARACTERISTIC CURVES or CONSTANT HEAD CURVES:-

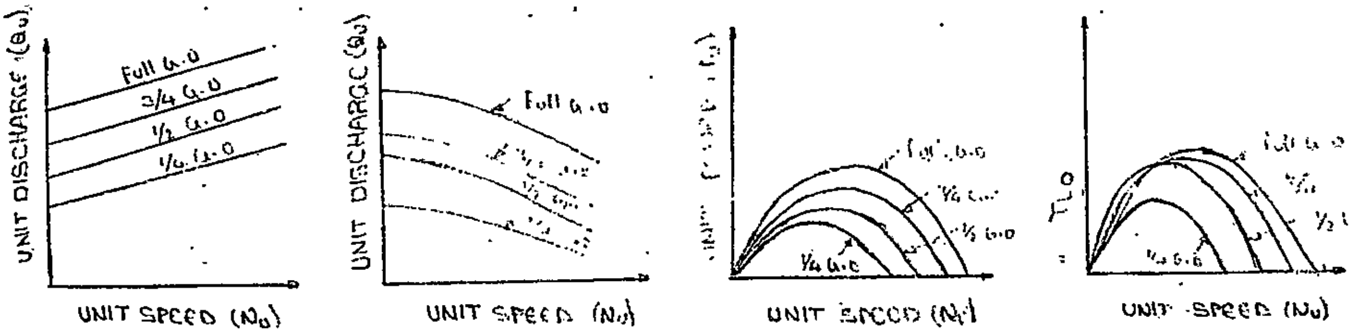
Main characteristic curves are obtained by maintaining a constant head & a constant gate opening (G.O), on the turbine.

Speed is varied by allowing a variable quantity of water to flow through the inlet opening. For each value of the speed, the corresponding values of the power (P) & discharge (Q) are obtained. Then the overall efficiency (η_o) for each value of the speed is calculated. From these readings the values of unit speed (N_u), unit power (P_u) & unit discharge (Q_u) are determined.

Taking N_u as abscissa, the values of Q_u, P_u & η_o are plotted for different gate openings.



Main characteristic curves for pelton wheel.



a) for kaplan turbine b) for francis turbine

Main characteristic curves for Reaction turbine.

main characteristic curves yield the following information:

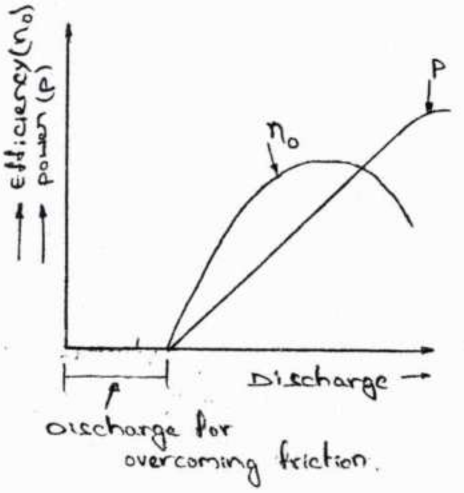
1) The discharge Q_u for a pelton wheel depends only upon the gate opening & is independent of N_u; the curves for Q_u are horizontal.

2) The curves between Q_u & N_u for a francis turbine are falling curv

3) The curves between Q_u & N_u for kaplan turbine are rising curves. the discharge increases with increase in speed.

4) <u>operating characteristic curves or constant speed curves</u>:-

operating characteristic curves are plotted when the speed on the turbine is constant. For operating characteristics N & H are cons and hence the variation of power & efficiency with respect to dischar Q are plotted. The power curve for turbines shall not pass through the origin because certain amount of discharge is needed to produc power to overcome initial friction. Hence the power & efficiency curves will be slightly away from the origin.

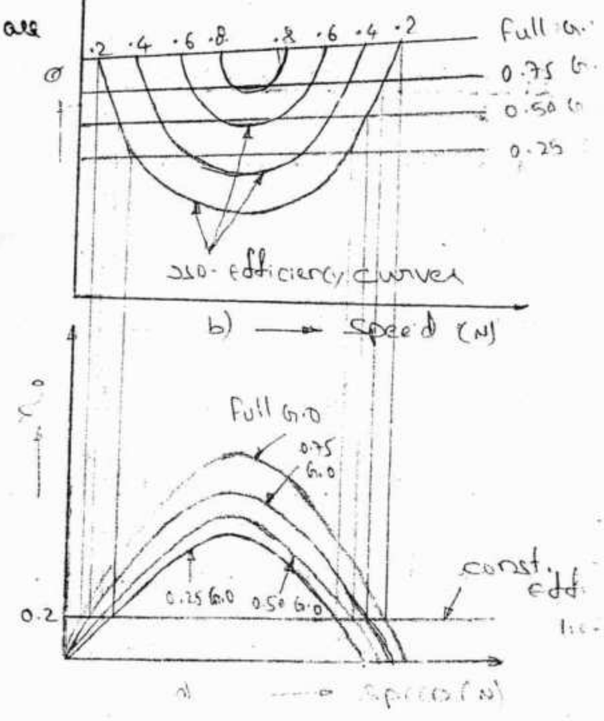


operating characteristic curves.

<u>constant Efficiency curves</u> or <u>MUSCHEL curves</u> (or) <u>iso - efficiency curves</u>

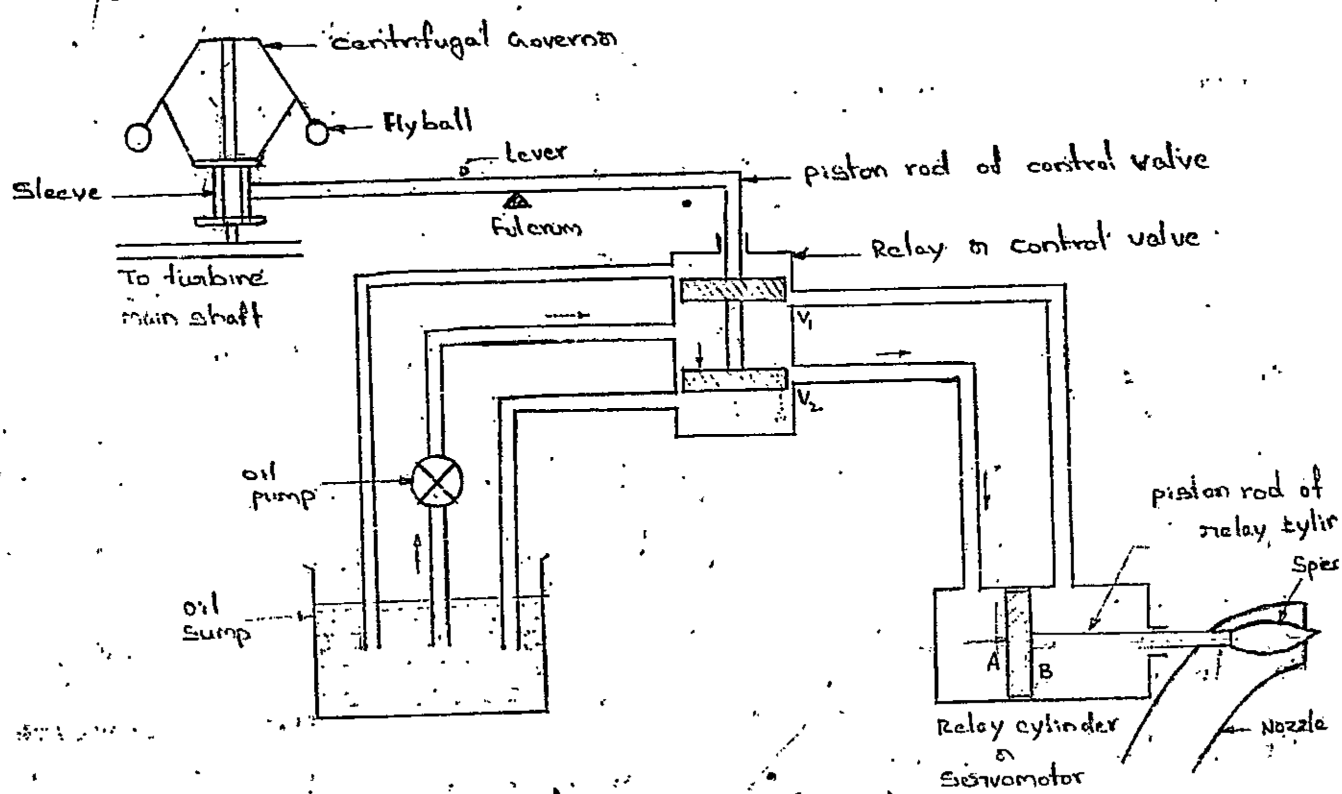
constant efficiency curves are obtained from the speed vs efficiency and speed vs. discharge curves for different openings. As η_o vs N curve is of parabolic nature, there exists two speeds for one value of η_o. at the η_o is maximum there is only one speed. corresponding to these values of speeds there are two values of discharge for each value of efficiency. These two values of speed two values of discharge corresponding to a particular gate openin are plotted. The procedure is repeated for different gate openings the curves Q vs. N are plotted. The points having the same efficienc are joined. The curves having same η_o are called iso - efficiency curves.

For plotting the iso - efficiency curves, horizontal lines representing the same η_o are drawn on η_o - N curves. The points at which these lines cut the η_o curves at various gate openings are transferred to corresponding Q ~ speed curves. The points having the same η_o are joined by smooth curves, these are called iso-efficiency curves



GOVERNING OF TURBINES :

Governing of a turbine is defined as the operation by which speed of the turbine is kept constant under all conditions of working.



Governing of turbine. (Pelton)

It is done automatically by means of a governor, which regulates the rate of flow through the turbines according to the changing load conditions on turbine.

GOVERNING OF PELTON TURBINE :-

Governing of pelton turbine is done by means of oil press governor, which consists of the following parts:

1) oil sump. 2) Gear pump also called oil pump, which is driven by the power obtained from turbine shaft.
3) The servomotor also called relay cylinder.
4) The control valve or the distribution valve or relay valve.

5) The centrifugal governor or pendulum which is driven by belt gear from the turbine shaft.
6) pipes connecting the oil sump & control valve & servo motor.
7) The spear rod or needle.

Fig. shows the position of the piston in the relay cylinder, position control or relay valve & fly-balls of the centrifugal governor, wh the turbine is running at the normal speed.

When the load on the generator decreases, the speed of the generator increases. This increases the speed of the turbine bey the normal speed. the centrifugal governor, which is connected the turbine main shaft, will be rotating at an increased spe Due to increase in the speed of the centrifugal governor, the fly-balls move upward due to the increased centrifugal fo on them. Due to the upward movement of the fly ~~wheels~~ ba the sleeve will also move upward. A horizontal lever, supporte over a fulcrum, connects the sleeve & the piston rod of the control valve. As the sleeve moves up, the lever turns abou the fulcrum & the piston rod of the control valve moves down This closes the valve V_1 & opens the valve V_2.

The oil, pumped from the oil sump to the control valve, un pressure will flow through the valve V_2 to the servomotor & w exert force on the face A of the piston of relay cylinder. Th piston along with piston rod & spear will move towards right this will decrease the area of flow of water to the turbine which consequently reduces the speed of the turbine. when the speed of the turbine becomes normal, the fly balls, sleeve lever & piston rod come to its normal position.

When the load on the generator increases, the speed of the gene & hence the speed of the turbine decreases. The speed of the centrifugal governor also decreases & hence centrifugal force ac on the fly - balls also reduces. This brings fly balls in the downward direction. Due to this, the sleeve moves downward & the lever turns about the fulcrum, moving the piston rod of cont valve in the upward direction. This closes the valve V_2 & opens the valve V_1. The oil under pressure from the control valve, will move through valve V_1 to the servomotor & will exert a force on the face B of the piston. This will move the piston along with th piston rod & spear towards left, increasing the area of flow of water at outlet of the nozzle. This will increase the rate of flow of water to the turbine & consequently, the speed of the turbine will also increase, till the speed of the turbine becomes normal.

CAVITATION :-

Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below its vapour pressure & the sudden collapsing of these vapour bubbles in a region of high pressure.

When the vapour bubbles collapse, a very high pressure is created. The metallic surfaces, above which these vapour bubl collapse, is subjected to these high pressures, which causing pitting action on the surface. Thus cavities are formed on th metallic surface & also considerable noise & vibrations are produced.

Precaution against Cavitation :-

The following precautions should be taken against cavitation :

1) The pressure of the flowing liquid in any part of the hydraulic system should not be allowed to fall below its vapour pressure. If the flowing liquid is water, then the absolute pressure head should not be below 2.5 m of water.

2) The special materials or coatings such as aluminium - bronze & stainless steel, which are cavitation resistant materials, should used.

Effects of Cavitation :-

The following are the effects of cavitation:

1) The metallic surfaces are damaged & cavities are formed on the surfaces.

2) Due to sudden collapse of vapour bubble, considerable noise vibrations are produced.

3) The efficiency of a turbine decreases due to cavitation.

Thoma's Cavitation factor for Reaction Turbines:-

Prof. D. Thoma suggested a dimensionless number, called after his name Thoma's cavitation factor, which can be used f determining the region where cavitation takes place in reaction turbines.

Thoma's cavitation factor is given by

$$\sigma = \frac{H_b - H_s}{H} = \frac{(H_{atm} - H_v) - H_s}{H}$$

where H_b - Barometric pressure head
H_{atm} - Atmospheric " "
H_v - Vapour " "
H_s - Suction pressure head.
H - Net head.

Selection of Type of Turbine :-

The following points should be considered while selecting right t of turbine for Hydroelectric power plant.

1. Specific Speed :- High specific speed is essential where head is low and output is large, because otherwise the rotational speed wi be low which means cost of turbo-generator & power house will be high. on the other hand, there is practically no need of che a high value of specific speed for high installations, because even with low specific speed high rotational speed can be attaine with medium capacity plants.

2. Rotational speed :- Rotational speed depends on specific speed. Also the rotational speed of an electric generator with which the turbine is to be directly coupled, depends on the frequency no. of pair of poles. The value of specific speed adopted should be such that it will give the synchronous speed of the generator.

3. Efficiency :- The turbine selected should be such that it gi the highest overall efficiency for various operating conditions.

4. part load operations :- In general, the efficiency at partloads overloads is less than normal. for the sake of economy the turbine should always run with maximum possible efficiency to more revenue.

5. Cavitation :- The installation of water turbines of reaction type over the tail race is affected by cavitation.

6 Disposition of turbine shaft :- vertical shaft arrangement is better for large-sized reaction turbines. on case of large si size impulse turbines, horizontal shaft arrangement is mostly empl

7) HEAD :- very high heads (350 m & above) → pelton turbine.

High heads (150 m to 350 m) → pelton or francis turbine.

Medium heads (60 m to 150 M) → francis turbine.

Low heads (below 60 m) → kaplan turbine.

Surge tank :- A surge tank is a small reservoir or tank in whic the water level rises or falls to reduce the pressure swings so that they are not transmitted in full to a closed circuit.

In general a surge tank is employed to serve the following purp

1) To reduce the distance between free water surface & turbi thereby reducing the water hammer effect.

2) To serve as supply tank to the turbine when the water in the pipe is accelerating during increased load conditions & storage tank when the water is decelerating during reduc load conditions.

Water hammer :- In a long pipe, when the ~~water~~ is flowing wa is suddenly brought to rest by closing the valve or by any similar cause, there will be a sudden rise in pressure due t the momentum of water being destroyed. A pressure wave is transmitted along the pipe. A sudden rise in pressure has th effect of hammering action on the walls of the pipe. This phenomenon of sudden rise in pressure is known as wate hammer. The magnitude of pressure rise depends on:

1) speed at which valve is closed.
2) velocity of flow.
3) length of pipe.
4) The elastic properties of the pipe material.

The hydraulic machines which convert the mechanical energy into hydraulic energy are called pumps. The hydraulic energy is the form of pressure energy.

CLASSIFICATION OF PUMPS :-

on the basis of transfer of mechanical energy the pumps can be broadly classified as follows:

- pumps
  - Rotodynamic
    - Radial flow
      - volute
      - Turbine
        - single stage
        - multi stage
    - Axial flow
    - mixed flow
  - positive displacement pumps
    - Reciprocating
      - piston
      - plunger
      - Diaphragm
    - Rotary
      - Gear
      - vane
      - screw
      - Axial

The radial flow type pumps are commonly called centrifugal pumps

CLASSIFICATION OF CENTRIFUGAL PUMPS :-

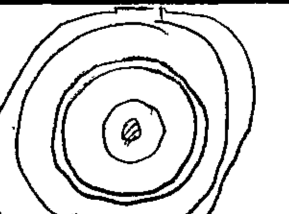
on the basis of characteristics, the centrifugal pumps are classified as follows :

1. Type of casing :-

   i) volute pumps ii) Turbine pump or diffusion pump.

2. Working head :

   i) Low lift centrifugal pump
   ii) medium lift centrifugal pump
   iii) High lift centrifugal pump.



3. Liquid handled :

i) closed impeller pump   ii) semi - open impeller pump.

iii) open impeller pump.

4. Number of impeller per shaft :

i) single stage centrifugal pump   ii) multi - stage centrifugal pump.

5. Number of entrances to the impeller :

i) single entry or single suction pump.

ii) Double entry or double suction pump.

6. Direction of flow through impeller :

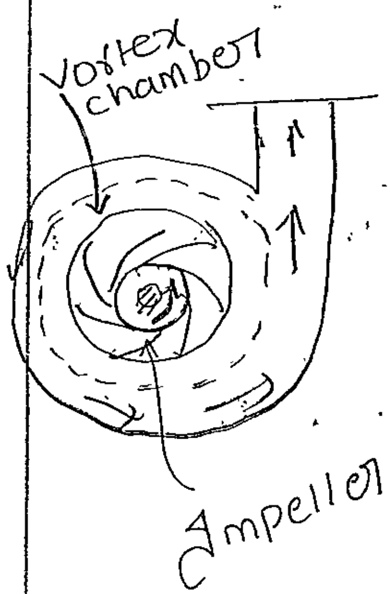
i) Radial flow pump   ii) Axial flow pump   iii) mixed flow pump.

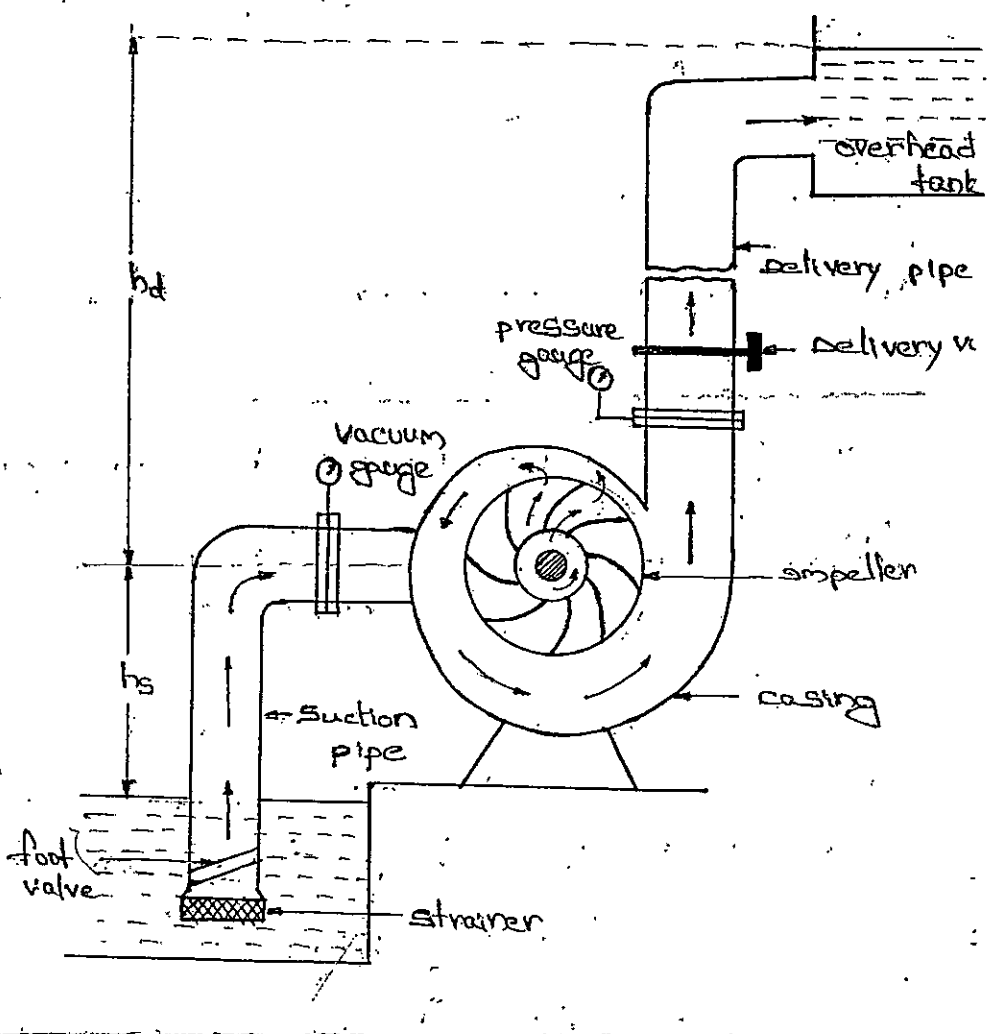
MAIN PARTS OF A CENTRIFUGAL PUMP :-

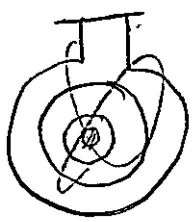
The main parts of a centrifugal pump are :

1. Impeller   2. casing   3. suction pipe with a foot valve & a str

4. Delivery pipe.







IMPELLER:- The rotating part of a centrifugal pump is called impelle. It consists of a series of backward curved vanes. The impeller is mounted on a shaft which is connected to the shaft of an electric motor.

CASING:- The casing of a centrifugal pump is an air-tight passag surrounding the impeller & is designed in such a way that the kinetic energy of water discharged at the outlet of the impeller is converted to pressure energy before the water leaves the casing & enters the delivery pipe.

In general three types of casings are commonly adopted:

1) volute casing 2) vortex casing 3) casing with guide blades.

SUCTION PIPE:- A pipe whose one end is connected to the inlet of the pump & other end dips into water in a sump is known as suction pipe.

A foot valve is a non-return valve fitted at the lower end of suc pipe. the foot valve opens only in the upward direction.

A strainer is also fitted at the lower end of the suction pipe.

DELIVERY PIPE:- A pipe whose one end is connected to the outlet the Pump & other end delivers the water at a required height is known as delivery pipe.

WORK DONE BY THE CENTRIFUGAL PUMP (IMPELLER) ON WATER:-

Working:-

The working of a centrifugal pump is explained below:

The delivery valve is closed & the pump is primed so that no air pocket is left.

keeping the delivery valve still closed the electric motor is started to rotate the impeller. the rotation of the impeller causes strong suction or vacuum just at the eye of the casing.

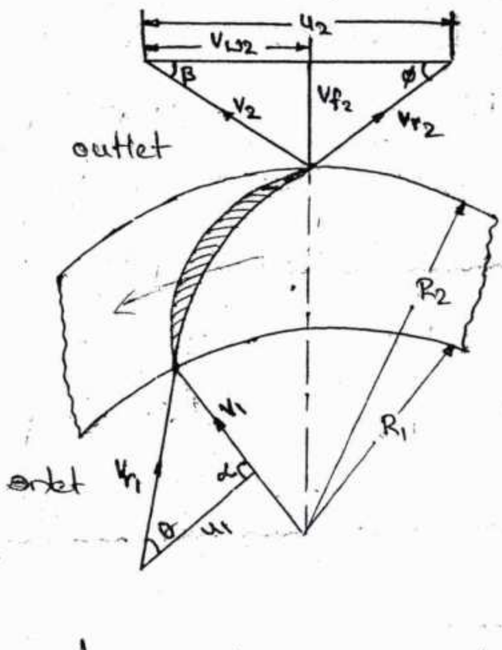
The speed of the impeller is gradually increased till the impeller rotates at its normal speed & develops normal energy required for pumping the liquid. After the impeller attains the normal speed the delivery valve is opened, when the liquid is continuously sucked In the suction pipe, it passes through the eye of casing & enters the impeller at its centre. This liquid is sent out at the outlet tips of vanes into casing. From casing, the liquid passes into pipe & is lifted to the required height.

Work done by impeller :-

In case of centrifugal pump, work is done by the impeller on the water. The expression for the work done by the impeller on the water is obtained by drawing velocity triangles at inlet & outlet of the impeller.

The water enters the impeller radially at inlet for best efficiency of the pump, which means the absolute velocity of water at inlet makes an angle of 90° with the direction of motion of the impeller at inlet.

$\therefore \alpha = 90^\circ$ & $V_{w1} = 0$.



A centrifugal pump is the reverse of a radially inward flow reaction turbine. But in case of a radially inward flow reaction turbine, work done by the water on the runner per second per unit weight of water striking per second is given by

$$= \frac{1}{g}\left[V_{w1}u_1 - V_{w2}u_2\right].$$

∴ Work done by the impeller on the water per second per unit weigh of water striking per second = − [work done in case of turbine] (6)

$$= -\frac{1}{g}\left[V_{w1}u_1 - V_{w2}u_2\right]$$

$$= \frac{1}{g}\left[V_{w2}u_2 - V_{w1}u_1\right] \qquad [\because V_{w1}=0]$$

$$= \frac{1}{g}V_{w2}u_2 \qquad \text{Per unit wt. per s}$$

This gives the head imparted to water by impeller (or) Energy given by impeller to water

Work done by impeller on water per second $= \frac{W}{g} \cdot V_{w2}u_2$.

$W = \rho g Q$.

$Q = \pi D_1 B_1 V_{f1}$

$= \pi D_2 B_2 V_{f2}$

DEFINITIONS OF HEADS AND EFFICIENCIES OF A CENTRIFUGAL PUMP:-

1) **SUCTION HEAD (h_s)**:- It is the vertical height of the centre line of centrifugal pump above the water surface in the tank or pump from which water is to be lifted. It is denoted by "h_s".

2) **DELIVERY HEAD (h_d)**:- The vertical distance between the centre line of the pump & the water surface in the tank to which water is delivered is known as delivery head. It is denoted by "h_d".

3) **STATIC HEAD (H_m)**:- The sum of suction head & delivery head is known as static head. This is represented by "H_s".

$$H_s = h_s + h_d.$$

4) **MANOMETRIC HEAD (H_m)**:- It is defined as the head against which centrifugal pump has to work. It is denoted by 'H_m'.

a) H_m = Head imparted by the impeller to the water − Loss head in the pump.

$$= \frac{V_{w2}u_2}{g} - \text{loss of head in impeller \& casing.}$$

$$= \frac{V_{w2}u_2}{g} \qquad \text{if loss of pump is zero.}$$

b) H_m = Total head at outlet of the pump − Total head at the inlet of the pump.

$$= \left(\frac{p_o}{\rho g} + \frac{V_o^2}{2g} + z_o\right) - \left(\frac{p_i}{\rho g} + \frac{V_i^2}{2g} + z_i\right)$$

c) $H_m = h_s + h_d + h_{f_s} + h_{f_d} + \frac{V_d^2}{2g}$.

where h_s − suction head

h_d − delivery head

h_{f_s} − frictional head loss in suction pipe

h_{f_d} − frictional head loss in delivery pipe

V_d − velocity of water in delivery pi

Manometric head
Head imp by ...

pw giv to H_2O at outlet $= \frac{\omega H_m}{1000}$ KW

power at the imp $= \frac{wd/sec}{1000}$ KW

$= \frac{\omega}{g} \times \frac{V_{\omega 2} U_2}{1000}$ KW

$\eta_{man} = \frac{\omega H_m / 1000}{\frac{\omega}{g} \times \frac{V_{\omega 2} U_2}{1000}}$

η_{max} H_m

79

EFFICIENCIES OF A CENTRIFUGAL PUMP :- In case of a centrifugal pump the power is transmitted from the shaft of the electric motor to the shaft of the pump & then to the impeller. from the impeller, the power is given to the water.

The following are the important efficiencies of a centrifugal pump:

a) Manometric efficiency η_{man} b) Mechanical efficiency c) overall efficiency.

Manometric efficiency (η_{man}) : The ratio of the manometric head to the head imparted by the impeller to the water is known as manometric efficiency.

$$\eta_{man} = \frac{\text{Manometric head}}{\text{Head imparted by impeller to water}}$$

$$\boxed{\eta_{man} = \frac{g H_m}{V_{w2} u_2}}$$

The power at the impeller of the pump is more than the power given to the water at outlet of the pump.

The ratio of the pump given to water at outlet of the pump to the power available at the impeller, is known as manometric efficiency.

power given to water at outlet of the pump $= \frac{W H_m}{1000}$ kw

power at the impeller $= \frac{\text{W.D by the impeller/sec}}{1000}$ kw

$$= \frac{W}{g} \times \frac{V_{w2} u_2}{1000}$$

$$\therefore \boxed{\eta_{man} = \frac{W H_m / 1000}{\frac{W}{g} \times \frac{V_{w2} u_2}{1000}} = \frac{g H_m}{V_{w2} u_2}}$$

MECHANICAL EFFICIENCY (η_m) :- The ratio of the power available at the impeller to the power at the shaft of the centrifugal pump is known as mechanical efficiency.

$$\eta_m = \frac{\text{power at the impeller}}{\text{power at the shaft}}$$

$$\eta_m = \frac{W/g\ (V_{w_2}u_2/1000)}{S.P}$$

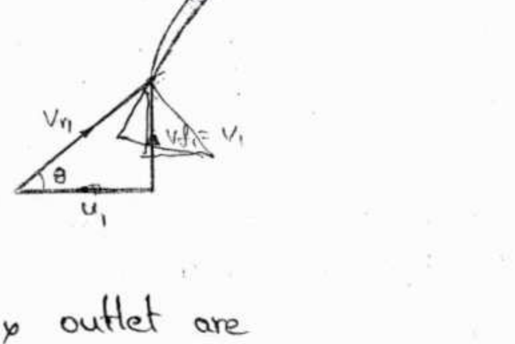
$$\boxed{\eta_m = \frac{\frac{W}{g}\left(\frac{V_{w_2}u_2}{1000}\right)}{S.P}}$$

<u>OVERALL EFFICIENCY (η_o)</u> :- It is defined as the ratio of power outp of the pump to the power input to the pump.

$$\eta_o = \frac{(WH_m/1000)}{S.P}.$$

$$\therefore \boxed{\eta_o = \eta_{man} \times \eta_m}$$

prob :- A centrifugal pump having outer diameter equal to two times th inner diameter & running at 1000 r.p.m works against a total head of 40m The velocity of flow through the impeller is constant & equal to 2.5 m/sec the vanes are set back at an angle of 40° at outlet. of the outer diameter of the impeller is 500 mm & width at outlet is 50mm. determine: 1) vane angle at inlet, ii) work done by impeller on wat per second, & 3) Manometric efficiency.



Sol:- Given Data

speed $N = 1000$ r.p.m, Head $H_m = 40$ m

velocity of flow, $V_{f_1} = V_{f_2} = 2.5$ m/s

vane angle at outlet, $\phi = 40°$

outer diameter of impeller $D_2 = 0.5$m

Inner dia. of impeller $D_1 = \frac{0.5}{2} = 0.25$ m

width at outlet $B_2 = 50$ mm

Tangential velocity of impeller at inlet & outlet are

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = 13.09 \text{ m/s} \qquad u_2 = \frac{\pi D_2 N}{60} = 26.18 \text{ m/s}.$$

Discharge $Q = \pi D_2 B_2 V_{f_2} = \pi \times 0.5 \times 0.05 \times 2.5 = 0.196\ m^3/sec$. (71)

i) Vane anglet at inlet (θ), $\tan\theta = \frac{V_{f_1}}{U_1} \Rightarrow \theta = 10^\circ 48'$.

ii) Work done by impeller on water per second $= \frac{\rho g Q}{g} \times V_{w_2} u_2$

$= 119227.9\ Nm/s$.

$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} \Rightarrow V_{w_2} = 23.2\ m/s$.

iii) Manometric efficiency (η_{man}) $\eta_{man} = \frac{g H_m}{V_{w_2} u_2} = \frac{9.81 \times 40}{23.2 \times 26.18} = 64.4$

prob:- A centrifugal pump is running at 1000 r.p.m. the outlet vane angle of the impeller is 45° & velocity of flow at outlet is 2.5 m/s. The discharge through the pump is 200 litres/s when the pump is wor against a total head of 20 m. & the manometric efficiency of the p is 80%. determine the width of the impeller at outlet.

i) determine the diameter of the impeller.

sol:- Given Data

$N = 1000$ r.p.m, $\phi = 45^\circ$, $V_{f_2} = 2.5\ m/s$, $Q = 0.2\ m^3/s$, $H_m = 20\ m$

$\eta_{man} = 0.80$,

$$\boxed{ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$ $\qquad \eta_{man} = \frac{g H_m}{V_{w_2} u_2}$

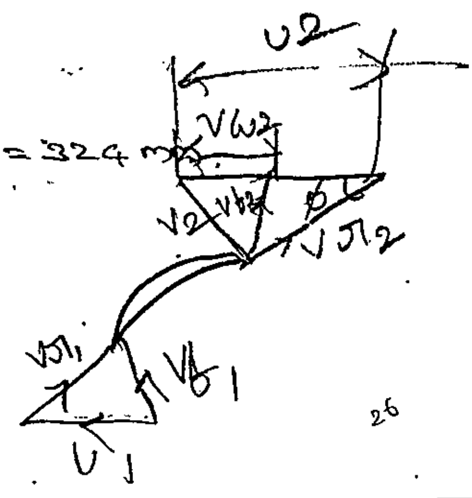
$V_{w_2} = u_2 - 2.5$ $\qquad V_{w_2} u_2 = 245.25$

$u_2 = \frac{2.5 \pm \sqrt{(2.5)^2 - 4 \times 245.25}}{2}$

$= 16.96$.

Diameter of impeller (D_2)

$u_2 = \frac{\pi D_2 N}{60} \Rightarrow D_2 = 324\ mm$.



width of impeller at outlet (B_2).

$Q = \pi D_2 B_2 \times V_{f_2}$

$\Rightarrow B_2 = 78.6\ mm$.

MINIMUM SPEED FOR STARTING A CENTRIFUGAL PUMP :-

If the pressure rise in the impeller is more than or equal to manometric Head (H_m). The centrifugal pump will start delivering water otherwise, the pump will not discharge any water, though the impeller rotating. When the impeller is rotating, the water in contact with the impeller is also rotating. This is the case of forced vortex.

In case of forced vortex, the centrifugal head or head due to pressure rise in the impeller

$$= \frac{\omega^2 r_2^2}{2g} - \frac{\omega^2 r_1^2}{2g}$$

$\therefore$ Head due to pressure rise in impeller $= \frac{u_2^2}{2g} - \frac{u_1^2}{2g}$

Flow of water is possible only if

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} \geq H_m$$

For minimum speed we must have $\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m$

W.K.T $\quad \eta_{man} = \frac{g H_m}{V_{w_2} u_2}$

$$H_m = \eta_{man} \times \frac{V_{w_2} u_2}{g}$$

$$\therefore \frac{u_2^2}{2g} - \frac{u_1^2}{2g} = \eta_{man} \times \frac{V_{w_2} u_2}{g}$$

$\because u_2 = \frac{\pi D_2 N}{60}$, $u_1 = \frac{\pi D_1 N}{60}$

$$\frac{1}{2g}\left(\frac{\pi D_2 N}{60}\right)^2 - \frac{1}{2g}\left(\frac{\pi D_1 N}{60}\right)^2 = \eta_{man} \times \frac{V_{w_2} u_2}{g}$$

Dividing by $\frac{\pi N}{g \times 60}$,

$$\frac{\pi N}{120}\left(D_2^2 - D_1^2\right) = \eta_{man} \times V_{w_2} D_2$$

$$\boxed{N = \frac{120 \times \eta_{man} \times V_{w_2} \times D_2}{\pi \left(D_2^2 - D_1^2\right)}}$$

MULTISTAGE CENTRIFUGAL PUMPS :-

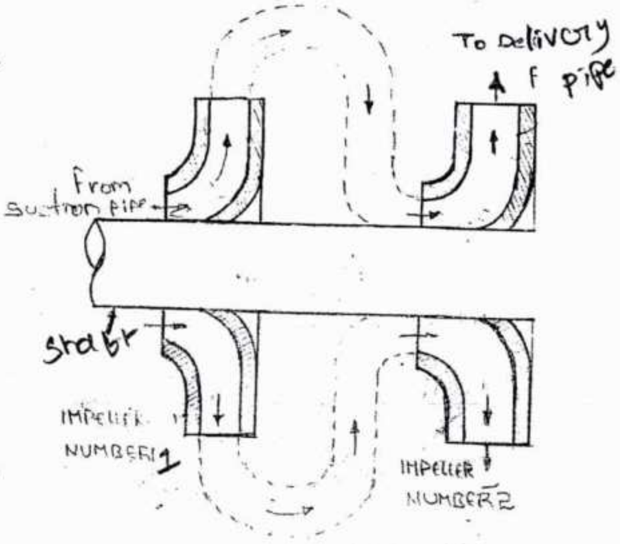
If a centrifugal pump consists of two or more impellers, the pump is called a multistage centrifugal pump. the impellers may be mounted on the same shaft or on different shafts.,

Functions :- 1) To produce a high head

2) To discharge a large quantity of water.

Multistage pumps for High Heads :-

For developing a high head, a number of impellers are mounted in series or on the same shaft.



Pipe connecting outlet of 1st IMPELLER to inlet of 2nd Impeller.

Impellers in series.

The water from suction pipe enters the 1st impeller at inlet & is discharged at outlet with increased pressure.

The water with increased pressure from the outlet of the 1st impeller is taken to the inlet of the 2nd impeller with the help of a connecting pipe.

At the outlet of the 2nd impeller, the pressure of water will be more than the pressure of water at the outlet of the 1st impeller. Thus, if more impellers are mounted on the same shaft, the pressure at the outlet will be increased further.

Let n = no of identical impellers mounted on the same shaft.

H_m = Head developed by each impeller.

Total head developed $= n \times H_m$.

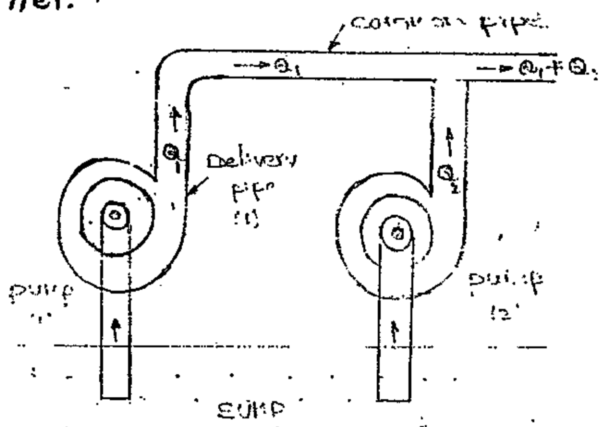
Multistage Centrifugal Pumps for High Discharge :-

For obtaining high discharge, the pumps should be connected in paral
Each of the pumps lifts the water from a common pump & discharges
water to a common pipe to which the delivery pipes of each pump i
connected. Each of the pump is working against the same head.

Let n = no of identical pumps arranged in //el.

Q = Discharge from one pump.

∴ Total discharge = n×Q



prob :- A four-stage centrifugal pump has four identical impellers, keye
to the same shaft. The shaft is running at 400 r.p.m & the total manom
head developed by the multistage pump is 40m. The discharge throu
the pump is 0.2 m³/s. The vanes of each impeller are having outlet
angle as 45°. If the width & diameter of each impeller at outlet is 5 cm
60cm respectively. Find the manometric efficiency.

Sol :- Given Number of stage, n = 4.

Speed $N = 400$ r.p.m Total manometric head = 40m

Manometric head for each stage, $H_m = \frac{40}{4} = 10.0$ m

Discharge $Q = 0.2\ m^3/sec$. outlet vane angle, $\phi = 45°$

Width at outlet, $B_2 = 5$ cm $= 0.05$ m

Dia. at outlet, $D_2 = 60$ cm $= 0.6$ m.

Tangential velocity of impeller at outlet, $u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 4}{60}$

$= 12.56$ m/s

Velocity of flow at outlet, $V_{f_2} = \frac{Q}{\pi D_2 B_2} = \frac{0.20}{\pi \times 0.6 \times 0.5} = 2.122$

W.K.T $\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}}$ ⇒ $V_{w2} = u_2 - 2.122 = 10.438$ m/sec (73) 82

$\eta_{man} = \frac{g H_m}{V_{w2} u_2} = \frac{9.81 \times 10}{10.43 \times 12.56} = 0.7482$ or 74.8 %

SPECIFIC SPEED OF A CENTRIFUGAL PUMP (N_s) :-

The specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump which would deliver one cubic metre of liquid per second against a head of one metre. It is denoted by 'N_s'.

Expression for Specific Speed for a Pump :-

The discharge Q, for a centrifugal pump is given by the relation

$$Q = \text{Area} \times \text{Velocity of flow}$$

$$= \pi D_1 B_1 V_{f1} \quad \text{or}$$

$$Q \propto D B V_f$$

W.K.T $B \propto D$

$$Q \propto D^2 V_f$$

But W.K.T $u = \frac{\pi D N}{60}$

$$u \propto DN$$

Now the tangential velocity (u) & velocity of flow (V_f) are related to the manometric head (H_m) as

$$u \propto V_f \propto \sqrt{H_m}$$

$$\therefore \sqrt{H_m} \propto DN$$

$$D \propto \frac{\sqrt{H_m}}{N}$$

$$\therefore Q \propto \frac{H_m}{N^2} \times V_f$$

$$\propto \frac{H_m^{3/2}}{N^2}$$

$$Q = K \cdot \frac{H^{3/2}}{N^2}$$

If $H_m = 1m$ & $Q = 1 m^3/s$. N becomes $= N_s$.

$$1 = \frac{k}{N_s^2}$$

$$\Rightarrow N_s^2 = k$$

$$\therefore Q = N_s^2 \cdot \frac{H_m^{3/2}}{N^2}$$

$$\Rightarrow \boxed{N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}}$$

prob:- find the number of pumps required to take water from a deep well under a total head of 89m. All the pumps are identical and running at 800 Rpm. The specific speed of each pump is given as 25 while the rated capacity of each pump is $0.16 m^3/sec$.

Sol:- Given Data Total head = 89m

speed $N = 800$ r.p.m, $N_s = 25$, $Q = 0.16 m^3/sec$.

H_m = Head developed by each pump.

W.k.T $N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}$

$$\Rightarrow H_m^{3/4} = \frac{800 \times \sqrt{0.16}}{25}$$

$$H_m = 29.94 \text{ m}$$

$\therefore$ No. of pumps $= \frac{\text{Total head}}{\text{Head from one pump}}$

$$= \frac{89}{29.94} \simeq 3.$$

PRIMING OF A CENTRIFUGAL PUMP :-

priming of a centrifugal pump is defined as the operation in which the suction pipe, casing of the pump and a portion of the deliver pipe upto the delivery valve is completely filled up from outside source with the liquid to be raised by the pump before starting the pump. thus the air from these parts of the pump is remov & these parts are filled with the liquid to be pumped.

CHARACTERISTIC CURVES OF A CENTRIFUGAL PUMP :-

characteristic curves of centrifugal pumps are defined as the curves which are plotted from the results of a number o tests on the centrifugal pump.

These curves predict the behaviour & performance of the pump when the pump is working under different flow rate, head & sp the important characteristic curves for pumps are :

1. Main characteristic curves.
2. operating characteristic curves
3. constant efficiency or Muschel curves.

Main characteristic curves :-

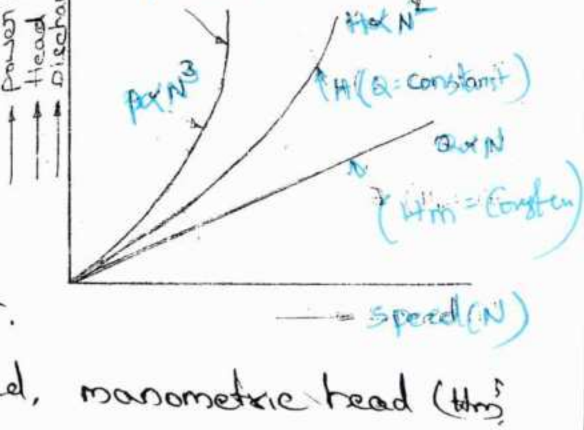
$\sqrt{H_m} \propto 1$

$\frac{Q}{D^3 N} = ca$

$Q \propto$

$P \propto$

The main characteristic curves of a centrifugal pump consists of a variation of head (Hm), power & discharge with respected to speed.



For plotting curves of manometric head versus speed, discharge is kept constant.

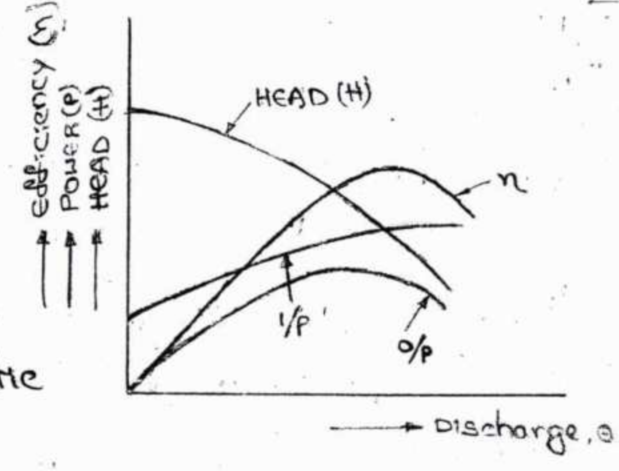
For plotting curves of discharge vs speed, manometric head (Hm) is kept constant.

For plotting curves of power vs speed, manometric head (Hm) & discharge are kept constant. fig. shows main characterist curves.

Operating characteristic curves :-

1. If the speed is kept constant,
the variation of manometric head,
power & efficiency with respect to
discharge gives the operating
characteristics of the pump.

fig. shows the operating characteristic
curves of a pump.



Constant efficiency curves :-

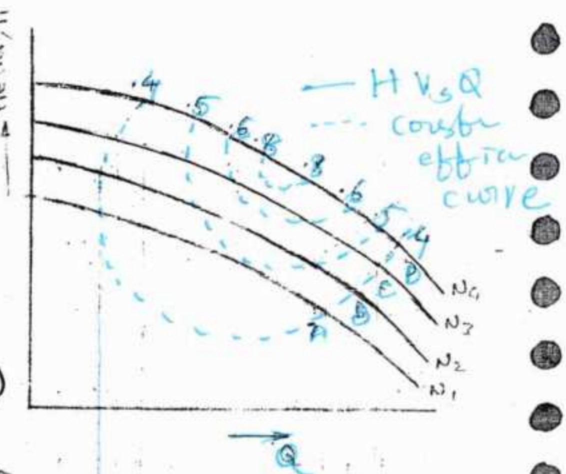
for obtaining constant efficiency
curves for a pump, the head vs
discharges curves & efficiency vs
discharge curves for different speed
are used.

fig. (a) shows the head vs discharge
curves for different speeds.

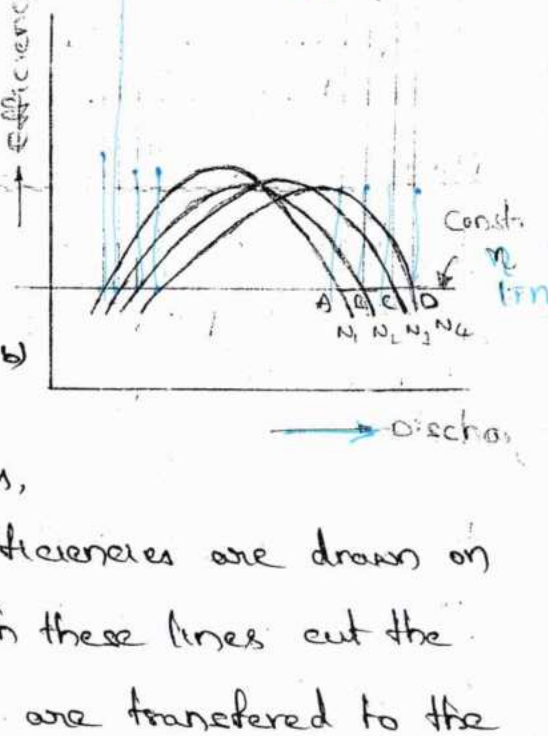
fig (b) shows efficiency vs discharge
curves for different speeds.

By combining these curves, constant
efficiency curves are obtained.

a)



b)



for plotting the constant efficiency curves,
horizontal line represents constant efficiencies are drawn on
the η ~ Q curves. the points, at which these lines cut the
efficiency curves at various speeds, are transfered to the
corresponding H ~ Q curves. the points having the same efficie
are then joined by smooth curves. these smooth curves represe
iso efficiency curves.

Net Positive Suction Head (NPSH) :

The net positive suction head (NPSH) is defined as the absolute pressure head at the inlet to the pump, minus the vapour pressure head plus the velocity head.

$\therefore$ NPSH = Absolute pressure head at inlet of the pump − vapour pressure head (absolute units) + velocity head

$$= \frac{P_1}{\rho g} - \frac{P_v}{\rho g} + \frac{V_s^2}{2g}$$ ($\because$ Absolute pressure at in of pump = P_1)

$\therefore$ Absolute pressure head at inlet of pump is given as

$$\frac{P_1}{\rho g} = \frac{P_a}{\rho g} - \left(\frac{V_s^2}{2g} + h_s + h_{fs}\right)$$

$$\therefore \text{NPSH} = \left[\frac{P_a}{\rho g} - \left(\frac{V_s^2}{2g} + h_s + h_{fs}\right)\right] - \frac{P_v}{\rho g} + \frac{V_s^2}{2g}$$

$$= \frac{P_a}{\rho g} - \frac{P_v}{\rho g} - h_s - h_{fs}$$

$$= \left[(H_a - h_s - h_{fs}) - H_v\right]$$

$\because \frac{P_a}{\rho g} = H_a$

$\frac{P_v}{\rho g} = H_v$.

The right hand side of the equation is the total suction h

$\therefore$ NPSH is defined as the total head required to make the liquid flow through the suction pipe to the pump impeller.

prob:- A centrifugal pump rotating at 1000 r.p.m delivers 160 lit of water against a head of 30 m. The pump is installed at a pl where atmospheric pressure is 1×10^5 Pa (abs) & vapour pressure of water is 3 kPa (abs). The head loss in suction pipe is equivale to 0.2 m of water. Calculate i) Minimum NPSH. ii) maximum allowable height of pump from free surface of water in the sump.

Given, $N = 1000$ r.p.m, $Q = 0.15\ m^3/s$. $H_m = 30\ m$, $P_a = 1 \times 10^5\ N/m^2$

$P_v = 3 \times 10^3\ N/m^2$. $h_{fs} = 0.2\ m$.

Minimum NPSH

$$\sigma = \frac{NPSH}{H_m}$$

NPSH is $\propto$ proportional to thoma's cavitation factor.

$$\therefore \sigma_c = \frac{(NPSH)_{min}}{H_m}$$

$\Rightarrow$ critical value of $\sigma_c = 1.03 \times 10^{-3} \times N_s^{4/3}$.

$$\therefore N_s = \frac{N\sqrt{Q}}{H_m^{3/4}} \Rightarrow N_s =$$

$$\sigma_c = 0.1012.$$

$$\therefore (NPSH)_{min} = H_m \times 0.1012 = 3.036\ m.$$

(ii) Maximum allowable height of the pump from free surface of water in the sump. (h_s)

$$\therefore NPSH = H_a - H_v - h_s - h_{fs}.$$

$$\Rightarrow (NPSH)_{min} = H_a - H_v - (h_s)_{max} - h_{fs}$$

$$\Rightarrow (h_s)_{max} = 10.193 - 0.305 - 0.2 - 3.036$$

$$= 6.652\ m$$

$H_a = \frac{P_a}{\rho g} = 10.193$ m of water

$H_v = \frac{P_v}{\rho g} = 0.305$ m of water

RECIPROCATING PUMPS :- The hydraulic machine which converts the (76) mechanical energy into hydraulic energy which is mainly in the form of pressure energy. If the mechanical energy is converted into hydraulic energy by sucking the liquid into a cylinder in which piston is reciprocating (moves front & back), which exerts the thr on the liquid & increases its hydraulic energy, the pump is known as reciprocating pump.

CLASSIFICATION OF RECIPROCATING PUMPS :-

Reciprocating pumps may be classified as:

1) According to the water being in contact with one side or both sides of the ~~pump~~ piston.
2) According to the number of cylinders provided.

According to the contact of water

1) single - acting pump — Water is in contact with one side of piston
2) Double acting pump — water is in contact with both sides of pis

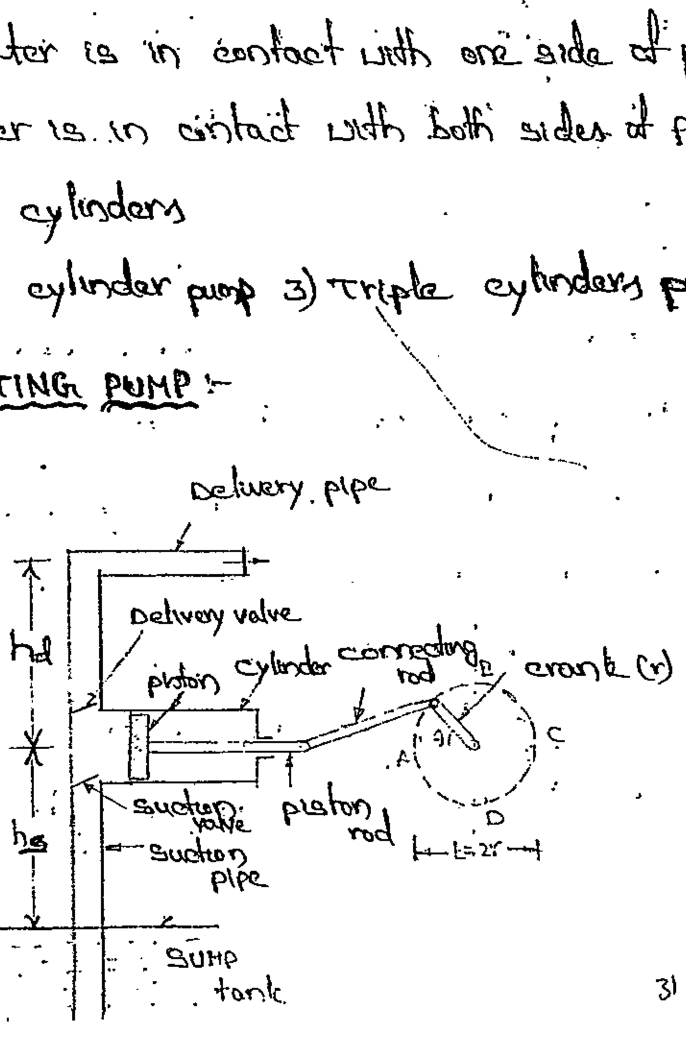
According to the number of cylinders

1) single cylinder pump 2) Double cylinder pump 3) Triple cylinders pum

MAIN PARTS OF A RECIPROCATING PUMP :-

Main parts of a reciprocating pump are.

1. A cylinder with a piston, piston rod, connecting rod & a crank.
2. suction pipe.
3. Delivery pipe.
4. Delivery valve
5. suction valve.



WORKING OF A RECIPROCATING PUMP :-

A single acting reciprocating pump, which consists of a pisto which moves forwards & backwards in a close fitting cylinder, shown in fig.

The movement of the piston is obtained by connecting the pis rod to crank by means of a connecting rod. The crank is rot by means of an electric motor. suction & delivery pipes with suction valve & delivery valve are connected to the cylinder.

when crank starts rotating, the piston moves to & fro in the cylinder. when crank is at A., the piston is at the extreme left position in the cylinder. As the crank is rotating from A to C, (ie., from $\theta = 0$ to $\theta = 180°$), the piston is moving towar right in the cylinder. the movement of the piston towards right creates a partial vacuum in the cylinder. But on the surface ~~the cylinder~~ liquid in the sump atmospheric pressure is acting which is more than the pressure inside the cylinder. thus the liquid is forced in the suction pipe from the sump. this liqu opens the suction valve & enters the cylinder.

when crank is rotating from C to A (ie from $\theta = 180°$ to $\theta = 3$ the piston from its extreme right position starts moving tow left in the cylinder. the movement of the piston towards left increases the pressure of the liquid inside the cylinder more than atmospheric pressure. Hence suction valve closes & deliv valve opens. The liquid is forced into the delivery pipe & is raised to a required height.

DISCHARGE THROUGH A RECIPROCATING PUMP :-

Consider a single reciprocating pump.

Let D = diameter of the cylinder

A = cross-sectional area of the piston or cylinder $= \frac{\pi}{4} D^2$

r = radius of crank, N = speed of crank.

L = Length of stroke $= 2 \times r$

h_s = Height of axis of cylinder from water surface in sump

h_d = Height of delivery outlet above the cylinder axis.

Discharge of water in one revolution = Area × stroke length

$= A \times L$

∴ Discharge of water from pump per second

Q = Discharge in one revolution × no. of revolutions per sec

$= A \times L \times \frac{N}{60}$

$$\boxed{Q = \frac{ALN}{60}}$$

Weight of water delivered per second $W = \rho \times g \times \frac{ALN}{60}$

WORK DONE BY RECIPROCATING PUMP :-

Work done by the reciprocating pump per second is given by

Work done/sec = Weight of water lifted/sec × Total height through which water is lifted.

$= \rho g \frac{ALN}{60} \times (h_s + h_d)$

∴ power required to drive the pump, in kW

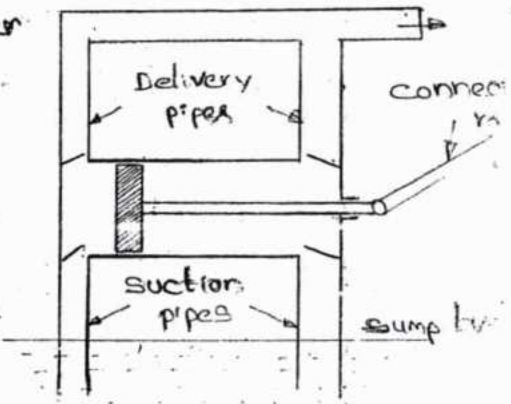
$$P = \frac{W.D/sec}{1000}$$

$$\boxed{P = \frac{\rho g ALN (h_s + h_d)}{60{,}000} \text{ kW}}$$

Amma. Nagulavari Jyothi Yojula

DISCHARGE, WORK DONE AND POWER REQUIRED TO DRIVE A DOUBLE [illegible]TING PUMP :-

In case of double acting pump, the water is acting on both sides of the piston.
thus we require two suction pipes & two delivery pipes for double acting pump.
When there is a suction stroke on one side of the piston, there is a delivery strobe on the other side of the piston at same time.



thus for one complete revolution of the crank there are two delivery strokes & water is delivered to the pipes by the pump dur these two delivery strokes.

let D = Diameter of piston.
d = diameter of piston rod.

$\therefore$ Area on one side of piston $A = \frac{\pi}{4} D^2$.

Area on other side of piston $A_1 = \frac{\pi}{4}(D^2 - d^2)$

Discharge for one revolution of crank $= AL + A_1L$

$$= \frac{\pi}{4} D^2 \cdot L + \frac{\pi}{4}(D^2 - d^2) \times L$$

Discharge per second $= \left[\frac{\pi}{4} D^2 + \frac{\pi}{4}(D^2 - d^2)\right] \times L \times \frac{N}{60}$

at 'd' $<< D$, it can be neglected.

$$\therefore Q = \left[\frac{\pi}{4} D^2 + \frac{\pi}{4} D^2\right] \times L \times \frac{N}{60}$$

$$= 2 \times \frac{\pi}{4} D^2 \times L \times \frac{N}{60}$$

$$\boxed{Q = \frac{2\,ALN}{60}}$$

Work done by double acting reciprocating pump

$$\text{Work done per second} = \rho g \times \frac{2ALN}{60} \times (h_s + h_d)$$

power required to drive the double-acting pump in kW

$$\boxed{P = \frac{2\rho g ALN (h_s + h_d)}{60{,}000}}$$

SLIP OF RECIPROCATING PUMP :-

Slip of a pump is defined as the difference between the theoretical discharge & actual discharge of the pump.

The actual discharge of a pump is less than the theoretical discharge due to leakage.

$$\text{slip} = Q_{th} - Q_{act}$$

But slip is mostly expressed as percentage slip

$$\%\ \text{slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$$

$$= (1 - C_d) \times 100 \qquad \because \frac{Q_{act}}{Q_{th}} = C_d.$$

NEGATIVE SLIP :- If actual discharge of a pump is more than the theoretical discharge, the slip of the pump will become '-ve'. In that case, slip of pump is known as negative slip.

prob :- A single acting reciprocating pump, running at 50 r.p.m & delivers $0.01\ m^3/s$ of water. The diameter of the piston is 200 mm & stroke length 400 mm. Determine: 1) theoretical discharge of the pump 2) coefficient of discharge 3) slip & the % slip of the pump.

Sol:- Given speed of the pump, N = 50 r.p.m.

Actual discharge $Q_{act} = 0.01\ m^3/sec$. Dia of piston $D = 200\ mm = 0.2\ m$

Area $A = \frac{\pi}{4}(D)^2 = 0.0314\ m^2$

stroke $L = 0.4\ m$

theoretical discharge for single acting pump.

$$Q_{the} = \frac{ALN}{60} = \frac{0.0314 \times 0.4 \times 50}{60} = 0.01047\ m^3/s$$

2) Coefficient of discharge, $C_d = \frac{Q_{act}}{Q_{the}} = \frac{0.01}{0.01047} = 0.955$

3) slip $= Q_{the} - Q_{act} = 0.01047 - 0.01 = 0.00047\ m^3/sec.$

% slip $= (1 - C_d) \times 100 = 4.48\%$.

prob:- A double acting reciprocating pump, running at 40 r.p.m. is discharging 1.0 m^3 of water per minute. The pump has a stroke 400 mm. The diameter of the piston is 200 mm. The delivery & suc head are 20 m & 5 m respectively. Find the slip of the pump & pow required to drive the pump.

sol:- Given Data $N = 40$ r.p.m, $Q_{act} = 1.0\ m^3/min = 0.016\ m^3/sec$.

$L = 0.4\ m$, $D = 0.2\ m$, $A = \frac{\pi}{4} D^2 = 0.0314\ m^2$, $h_s = 5\ m$, $h_d = 20$

$$Q_{the} = \frac{2ALN}{60} = \frac{2 \times 0.0314 \times 0.4 \times 0.2}{60} = 0.01675\ m^3/sec.$$

slip $= Q_{the} - Q_{act} = 0.00009\ m^3/s$.

power required to drive the pump

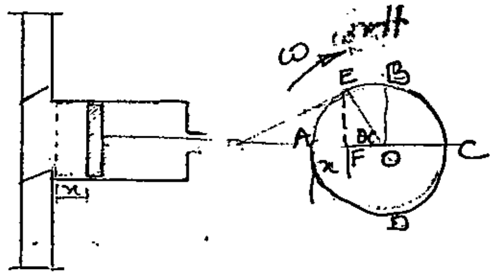
$$P = \frac{2\ \rho g\ ALN\ (h_s + h_d)}{60 \times 1000}$$

$$= 4.109\ kW$$

VARIATION OF VELOCITY AND ACCELERATION IN THE SUCTION AND DELIVER
DUE TO ACCELERATION OF THE PISTON :-

(79)

When crank starts rotating, the piston moves forwards & backwards in the cylinder. At the extreme left position & Right position of the piston in the cylinder, the velocity of piston is zero. The velocity of piston is maximum at the centre of the cylinder.



Velocity & Acceleration of piston

This means that the start of a suction stroke the velocity of the piston is zero & this velocity becomes maximum at the centre of each stroke & again becomes zero the end of each stroke. Thus at the beginning of each stroke, the piston will be having an acceleration & at the end of each st the piston will be having a retardation. The water in the cylinder i in contact with the piston & hence the water, flowing from suctio pipe on to the delivery pipe will have an acceleration at the begi of each stroke & a retardation at the end of each stroke. This m the velocity of flow of water in the suction & delivery pipe will n be uniform. Hence, an accelerative or retarding head will be acting on the water flowing through the suction or delivery pip This accelerative or retarding head will change the pressure inside the cylinder.

Let the crank is rotating at a constant angular speed.

Let ω = Angular speed of the crank in rad/s.

A = Area of cylinder l = length of pipe.

a = area of pipe. r = radius of the crank.

In the beginning, the crank is at A and the piston in the cylr is at a position shown by dotted lines. The crank is rotating with an angular velocity 'ω' & let in time 't' seconds, the crank turn through an angle θ from A.

let θ = Angle turned by crank in radians in time 't'.

$= \omega t$

The distance x travelled by the piston is given as

$$x = \text{Distance } AF$$
$$= r - r\cos\theta$$
$$= r - r\cos(\omega t)$$

Velocity of piston is obtained by differentiating the above equation

$$\therefore \; V = \frac{dx}{dt} = \frac{d}{dt}\left[r - r\cos\omega t\right]$$
$$= \omega r \sin\omega t$$

Now, from continuity equation, the volume of water flowing out cylinder per second is equal to the volume of water flowing from the pipe per second.

$$V \times A = v \times a$$

v = velocity of water in p

a = area of pipe

$$v = \frac{A}{a} \times V$$
$$= \frac{A}{a}\,\omega r \sin\omega t$$

Acceleration of water in pipe is obtained by differentiating the above equation.

$$\therefore \text{ Acceleration of water in pipe} = \frac{dv}{dt}$$
$$= \frac{A}{a}\,\omega^2 r \cos\omega t$$

Mass of water in pipe = $\rho \times$ Volume of water in pipe

$$= \rho \times a \times l$$

$\therefore$ force required to accelerate the water in the pipe

= mass of water in pipe $\times$ Acceleration of water in

$$= \rho a l \times \frac{A}{a} \omega^2 r \cos \omega t.$$

$\therefore$ intensity of pressure due to acceleration

$$= \frac{\text{force required to accelerate water}}{\text{Area of pipe}}$$

$$= \frac{\rho a l \times \frac{A}{a} \omega^2 r \cos \omega t}{a}$$

$$= \rho l . \frac{A}{a} \omega^2 r \cos \omega t$$

$\therefore$ pressure head (h_a) due to acceleration

$$h_a = \frac{\text{intensity of pressure due to acceleration}}{\text{weight density of liquid}}$$

$$h_a = \frac{\rho l . \frac{A}{a} \omega^2 r \cos \theta}{\rho g}$$

$$\boxed{h_a = \frac{l}{g} \times \frac{A}{a} \omega^2 r \cos \theta}$$

The pressure head due to acceleration in the suction & delivery pipes is obtained as

$$\boxed{h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \cos \theta}$$

$$\boxed{h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \cos \theta}$$

The pressure head (h_a) due to acceleration, varies with θ. The values for different values of θ are:

1. when $\theta = 0°$, $h_a = \frac{l}{g} \times \frac{A}{a} \omega^2 r$ as $\cos 0° = 1$
2. when $\theta = 90°$, $h_a = 0$ as $\cos 90° = 0$
3. when $\theta = 180°$, $h_a = -\frac{l}{g} \times \frac{A}{a} \omega^2 r$ as $\cos 180° = -1$

$\therefore$ Maximum pressure head due to acceleration

$$\boxed{(h_a)_{max} = \frac{l}{g} \times \frac{A}{a} \omega^2 r}$$

Effect of variation of velocity on friction in the suction and delivery pipes:-

The velocity of water in suction or delivery pipe is given by

$$v = \frac{A}{a} \omega r \sin\theta$$

Loss of head due to friction in pipes is given by

$$h_f = \frac{4fLv^2}{2gd}$$

$$\Rightarrow \quad h_f = \frac{4fL}{2gd} \left(\frac{A}{a} \omega r \sin\theta\right)^2$$

The variation of h_f with 'θ' is parabolic.

1) when $\theta = 0°$ $\sin\theta = 0$, $h_f = \frac{4fL}{2g \times d} \times 0 = 0$
2) when $\theta = 90°$. $\sin 90° = 1$, $h_f = \frac{4fL}{d \times 2g} \times \left[\frac{A}{a} \omega r\right]^2$
3) when $\theta = 180°$, $\sin 180° = 0$, $h_f = 0$.

$\therefore$ maximum value of loss of head due to friction;

$$(h_f)_{max} = \frac{4fL}{d \times 2g} \times \left[\frac{A}{a} \omega r\right]^2$$

DIFFERENCES BETWEEN CENTRIFUGAL PUMPS AND RECIPROCATING PUMPS:

Centrifugal pumps	Reciprocating pumps
1. The discharge is continuous & smooth.	1. The discharge is fluctuating & pulsating.
2. It can handle large quantity of liquid.	2. It handles small quantity of liquid only.
3. It can be used for lifting highly viscous liquids.	3. It is used only for lifting pure water or less viscous liq
4. It is used for large discharge through smaller heads.	4. It is meant for small discharge & high heads.
5. cost of centrifugal pump is less compared to reciprocating pump.	5. cost of reciprocating pump is approximately four times the cost of centrifugal pump.
6. centrifugal pump runs at high speed.	6. reciprocating pump runs at low speeds.
7. Maintainance cost is low.	7. Maintainance cost is high.
8. operation of centrifugal pump is smooth & without much noise.	8. operation of reciprocating pump is complicated & with much noise.
9. centrifugal pump needs smaller floor area & installation cost is low.	9. Reciprocating pump requi large floor area & installati cost is high.
10. efficiency is high	10. efficiency is low.
11. No. of moving parts are less	11. more no of moving parts.

Mon :- 9:05 to 9:25

wed :- 2:45 to 3:05

18 - 211, 204, 228, 213, 218

9A - 208, 205, 206

Mon 14, Tu 15, wed 16, th 17, Fri 18, 19

May 3 → -2000

Ma

Bushanpini → 2000
3000

Madam → 2200

Ravi → 3000

car → 340

→ 85

panipuri → 40