Mechanical Properties

The following are the most important mechanical properties of engineering materials:

(i) Elasticity (ii) Plasticity
(iii) Ductility (iv) Brittleness
(V) Malleability (Vi) Toughness
(Vii) Hardness (Viii) Strength.

(i) Elasticity:-

When external forces are applied on a body, made of engineering materials, the external forces tends to deform the body while the molecular forces acting between the molecules offer resistance against deformation. The deformation or displacement of the particles continues till full resistance to the external forces is setup. If the forces are now gradually removed, the body will return, wholly or partly to its original shape.

"Elasticity is the property by virtue of which a material deformed under the load and is enabled to return to its original dimension when the load is removed."

If a body regains completely its original shape, it is said to be perfectly elastic. For any particular material, a limited or critical value of the load at which the body deforms and regain its original ~~deformation~~ shape after removal of the load, up to that value of the load is known as Elastic limit. The value of the load beyond the Elastic limit tends to deform the body and after removal of the load it does not regain its original shape results in a permanent deformation or permanent set.

Ex: Steel, aluminium, copper, stone, Concrete etc may be considered to be perfectly elastic, within certain limits.

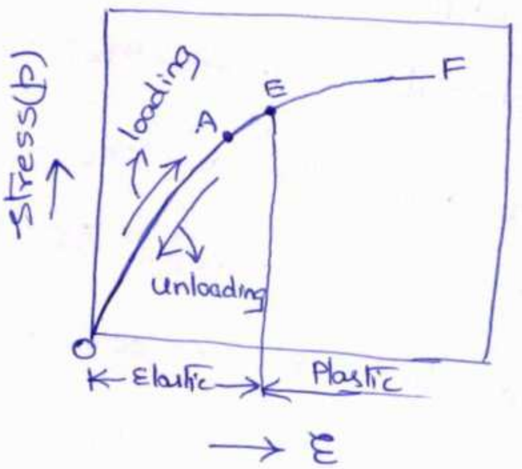


Fig: (a)

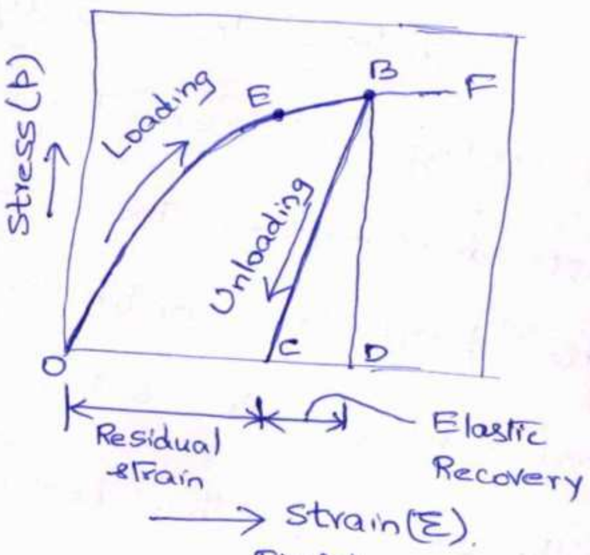


Fig: (b)

In Fig (a), the specimen is loaded only up to point A, i.e within the Elastic limit E. Due to the applied load upto point 'A', the path traces the curve 'OA'. During unloading, again it traces the curve OA. So, finally loading and unloading curves are same when the load is applied upto point 'A'.

In fig(b), the specimen is loaded up to point 'B', beyond the elastic limit E. When the path traced by the curve during loading is curve OB and during unloading it traces the curves (or path) BC, resulting in a residual strain (OC) or permanent strain.

Homogeneity and Isotropy:- A material is homogenous if it has same composition throughout the body.

If a material is equally elastic in all the directions, it is said to be isotropic.

If, it is having different elastic properties in different directions, it is called anisotropic.

(ii) Plasticity:- It is the converse of Elasticity.

"It is the characteristic of the material by which it undergoes deformation permanently beyond the elastic limit and is known as plasticity"

In the plastic region, i.e the body is loaded beyond the elastic limit and if it undergoes large deformations, the material is said to undergo plastic flow.

This property is very useful in pressing and forging operations and it is also useful for the design of structural members, utilising its ultimate strength.

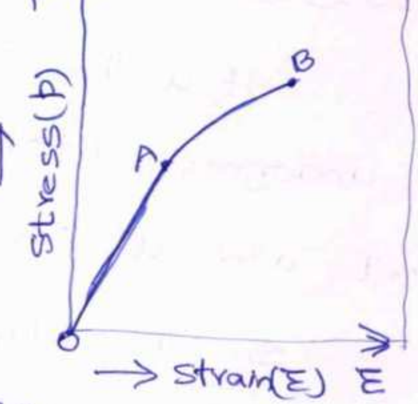
(iii) Ductility:- It is the characteristic, which permits a material to be drawn out longitudinally to a reduced section, under the action of a tensile force.

In a ductile material, large deformation is possible before absolute failure or rupture takes place. It possess a high degree of plasticity and strength. In a ductile material it shows a certain degree of Elasticity, together with a considerable degree of plasticity.

Ductility is measured in the tensile test of specimen of the material, either in terms of % elongation or in terms of % reduction in the cross-sectional area of the test specimen. The property of ductility is utilised in wire drawing.

(iv) Brittleness:-

A material is said to be brittle when it is not possible to drawnout smaller section by applying tensile load.



In Brittle material, fracture takes place without warning and it does not deform significantly before failure takes place under the load. This property of the material is highly undesirable.

Examples of brittle materials are: cast Iron, high carbon steel, concrete, stone, glass, ceramic materials.

(V) Malleability:- Malleability is a property of a material which permits the materials to be extended in all directions without rupture.

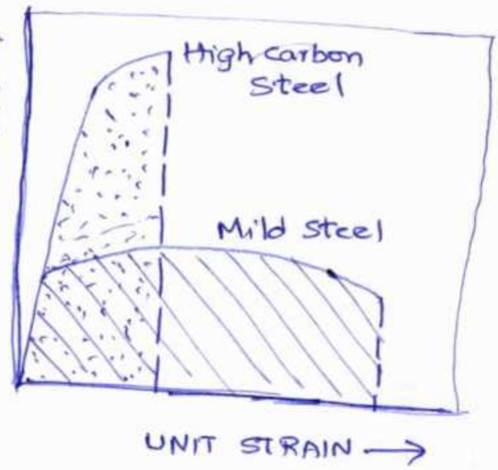
A malleable material possesses a high degree of plasticity, but not necessarily great strength.

This property is utilised in many operations such as forging, hot rolling, drop-stamping etc.

(Vi) Toughness:-

Toughness is the property of a material which enables it to absorb energy without ~~failure~~ fracture

Toughness is measured in terms of energy required per unit volume of the material, to cause rupture under the action of gradually increasing tensile load.



This property is very desirable in components subject to cyclic or shock loading.

(Vii) Hardness:- Hardness is the ability of a material to resist permanent indentation or impression.

(viii) Strength:- The strength of a material enables it to resist fracture under load.

Chapter - 1

Simple Stresses and Strains

Statics
Dynamics } Study of external effects on Rigid bodies

S.M → Study of Internal effects and deformation that are caused by the applied loads.

Introduction:- Materials which we come across may be classified into three types.

1. Elastic 2. Plastic 3. Rigid

1. Elastic material:- An elastic material undergoes a deformation when subjected to an external loading and regain its original position after removal of the load. Ex: Rubber band

2. Plastic material:- A plastic material undergoes a continuous deformation during the period of loading and the deformation is permanent and the material does not regain its original dimensions on the removal

Rigid Material: A Rigid material does not undergo any deformation. Ex: Iron Rod.,

In practice no material is absolutely elastic nor plastic nor rigid. These properties are applicable when the deformations are certain limit.

Resistance to Deformation:-

The resistance offered by the material as long as the member is forced to remain in the deformed state is called strength of the material.

[Stress:- The force of resistance offered by a body against the deformation is called the stress.

(or)

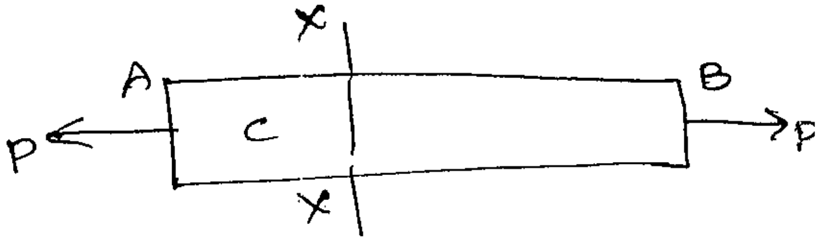
Resistance offered by the body against the deformation per unit area is called stress.

(or)

Force of resistance offering per unit area is called stress.]

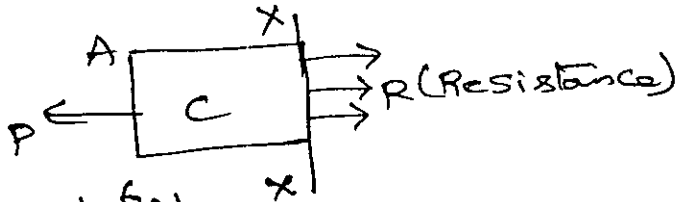
Load: The external force acting on the body is called stress | ... |

Stress is denoted by 'σ' (Sigma) (or) P



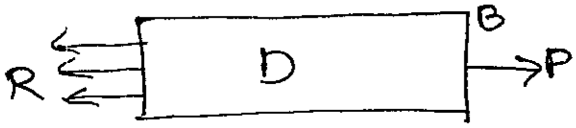
$$\sigma = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

$F \rightarrow N$, $A \rightarrow m^2$



$N/m^2 \Rightarrow 1$ pascal

$1kN = 10^3 \times N$, $1MN = 10^6 N$



$1 N/mm^2 = 10^6 N/m^2$

$= 10^6$ pa

$$\boxed{1 N/mm^2 = Mpa}$$

~~1,3,4,~~5,7,8,10, ~~232,7,28,~~29,31, ~~34,25,35,38,~~39,42

$$\boxed{\sigma = \frac{F}{A}}$$

σ is expressed in terms of Mpa, Gpa

Strain:- It is denoted by epsilon (ε) or e

It is defined as ratio of change in length (δl) to the original length (l) of the body.

$$\varepsilon = \frac{\delta l}{l} = \frac{m}{m}$$ units: No units

Types of stresses:-

1) Tensile 2) compressive 3) shear

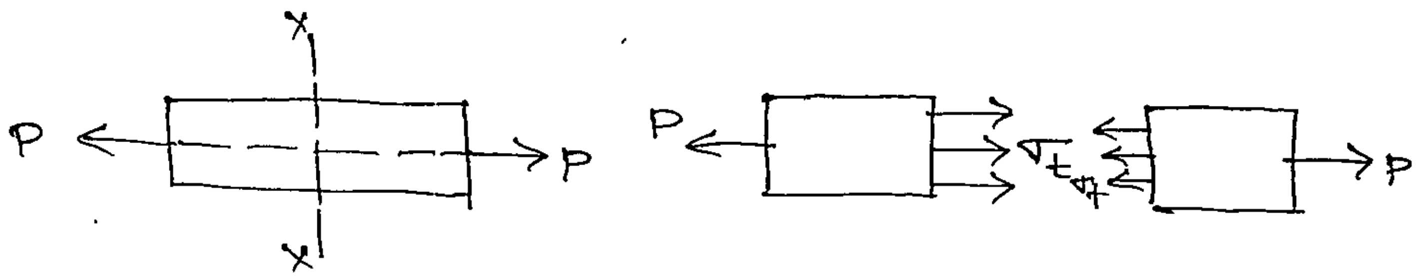
Stress:- When some external system of forces or loads act on a body, the internal forces (equal and opposite) are set up at various sections of the body, which resist the external forces. This internal force per unit area at any section of the body is known as unit stress or simply a stress. It is denoted by a Greek Letter sigma (σ), Mathematically

$$\text{Stress, } \sigma = P/A$$

P = Force or load acting on a body

A = c/s Area of the body.

Tensile stress and strain:-



When a body is subjected to two equal and opposite axial pulls P (also called tensile load), then the stress induced at any section of the body is known as tensile stress.

Due to the tensile load, there will be decrease in c/s area and an increase in length of the body. The ratio of increase in length to the original length is known as tensile strain.

Let P = Axial tensile force acting on the body

A = c/s Area of the body

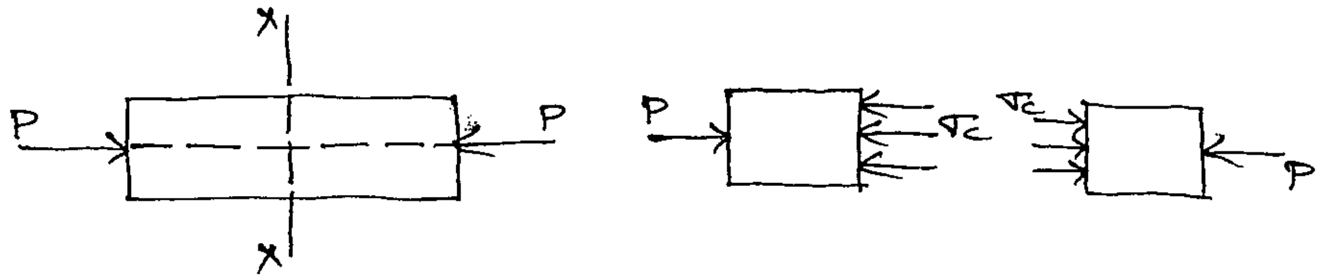
l = original length

δl = Increase in length

$\therefore$ Tensile stress, $\sigma_t = \frac{P}{A}$

tensile strain, $\varepsilon_t = \frac{\delta l}{l}$

Compressive stress and strain:-



When a body is subjected to two equal and opposite axial pushes P (also called compressive load), then the stress induced at any section of the body is known as compressive stress.

Due to the compressive load, there will be an increase in c/s area and a decrease in length of the body. The ratio of decrease in length to the let P = Axial compressive force

A = C/s area of the body

l = original length

δl = decrease in length

∴ Compressive stress, $\sigma_c = \frac{P}{A}$

" strain, $\varepsilon_c = \frac{\delta l}{l}$

<u>Note</u>:- These two stresses Tensile and compressive acts Normal to the c/s area of the body, so these stresses are called Normal stresses.

Shear stress and Strain:-

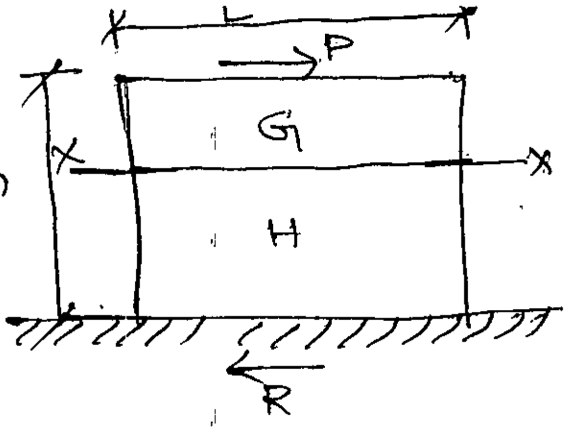
When a body is subjected to two equal and opposite forces, acting tangentially across the resisting section, as a result of which the body tends to shear off the section, then the stress induced is called shear stress. The corresponding strain is known as shear strain.

Shear stress is denoted by Tau (τ) (or) 'q' and shear strain is denoted by Phi (ϕ).

Let us consider a rectangular block of height 'h' and length L and width unity.

Let the bottom face of the block be fixed to a surface.

Let a force P be applied tangentially along the top face of the block. Such a force acting tangentially along a surface is called shear force.



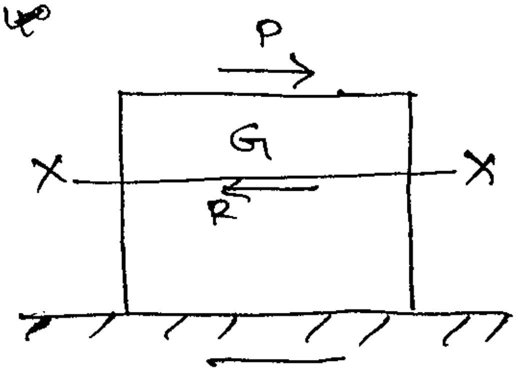
For the Equilibrium of the block, Tangential reaction R equal and opposite to the applied tangential force P.

The resistance R is called Shear Resistance

~~4~~,6,7,9, 10,~~21~~,23, 28,~~31~~,35, 40

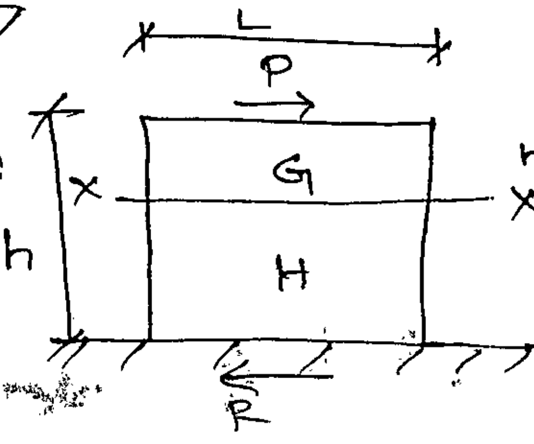
$$\text{Shear stress} = \tau = \frac{R}{A} = \frac{P}{A} = \frac{\text{Shear Force}}{\text{Shear Area}}$$

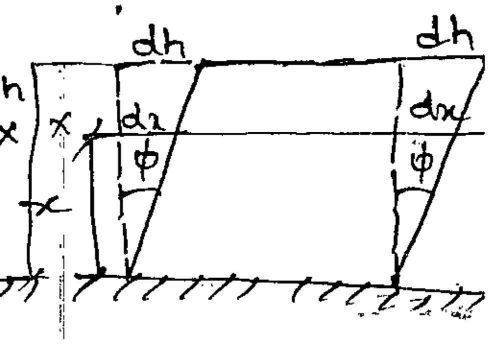
$$= \frac{R}{L \times h} = \frac{P}{L \times h}$$



If $R < P \Rightarrow$ Shear failure takes place

Shear Deformation:





$\delta \frac{dl}{l} = \frac{\text{change in length}}{\text{original length}}$

$\tan\phi = \frac{dh}{h}$

∵ Since ϕ is very small $\tan\phi = \phi$

$\phi = \frac{dh}{h}$ = shear strain.

angular deformation ϕ in radians.

<u>Elastic Limit</u>:- For every material the property of assuming or regaining its previous shape and size is exhibited on the removal of the loading, when the intensity of stress is within a certain limit called the Elastic limit.

<u>Hooke's law</u>:- Hooke's law states that when a material is loaded within elastic limit, the stress is directly proportional to strain

$\sigma \propto \varepsilon$

<u>For Normal stresses</u>:-

For Normal stresses

$\sigma \propto \varepsilon \Rightarrow \sigma = E\varepsilon$

$E = \frac{\sigma}{}$

where E is constant of proportionality and is called Young's Modulus or Modulus of Elasticity.

$$E = \frac{\sigma}{\varepsilon} = \frac{N/m^2}{-} = N/m^2$$

~~E~~ units: N/m^2

It can ~~be~~ also be expressed in terms of KPa, MPa, GPa.

<u>For Tangential Stress:-</u>

stress $\propto$ strain

$$\tau \propto \phi$$

$$\tau = G\phi$$

$$G = \frac{\tau}{\phi}$$

where G is ~~called~~ constant of proportionality and is called Rigidity Modulus or Modulus of Rigidity.

$$G = \frac{N/m^2}{-} = N/m^2$$

units: N/m^2

It is also denoted by 'C' or 'N' or 'G'

Problems

1) An elastic rod 25mm in diameter, 200mm long exceeds by 0.25mm under a tensile load of 40kN. Find the intensity of stress, the strain and the elastic modulus for the material of the rod.

[Ans: $\sigma = 81.52\ N/mm^2$, $\varepsilon = 0.00125$, $E = 65216\ N/mm^2$]

2) A C.I column has an external diameter of 300mm and 20mm thick. Find the safe compressive load on the column with a factor of safety of 5 if the crushing strength of the material is $550\ N/mm^2$.

[Ans: P = 1934240 N or 1934.24kN] 1935.22kN.

3) A 30mm diameter steel rod when subjected to an axial tensile force was, subjected to a strain of 0.6×10^{-3}, find the tensile force that caused the above strain. Take $E = 200\ kN/mm^2$.

[Ans: P = 84780N or 84.78kN]

Stress - Strain curves for mild steel and Typical Engineering Materials :-

Stress strain curve for mild steel :-

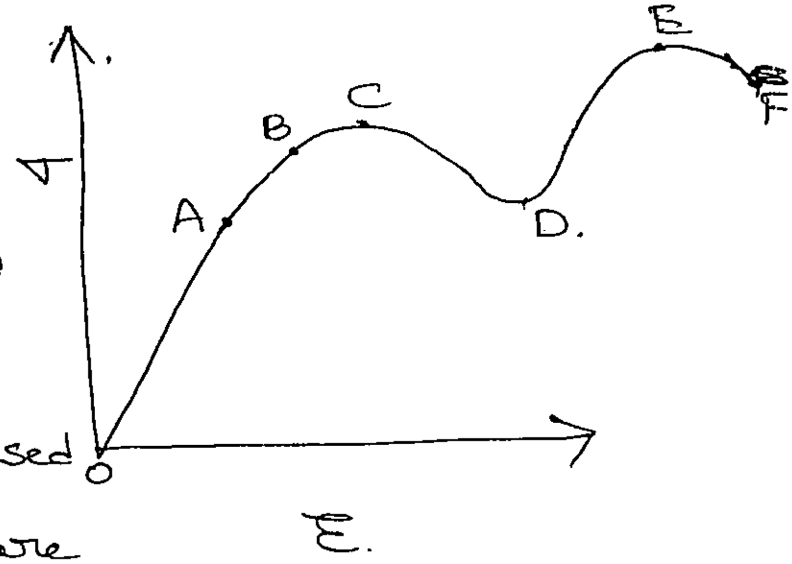


Fig show the stress - strain diagram obtained for a mild steel specimen subjected to a tensile test. The mechanical properties mostly used in mechanical Engg Practice are commonly determined from a tensile test.

The plot from O to A is a straight line. The stress corresponding to the point A is called the limit of proportionality. In this range stress is proportional to strain (ε) (or) the load is proportional to extension i.e Hooke's law is applicable.

In the range A to B the relation between stress and strain is not linear. The stress at B is called the Elastic limit.

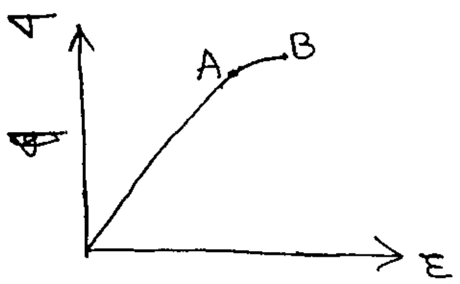
If the specimen is extended beyond the elastic limit, the plastic deformation takes place. In the range B to C the strain increases with almost constant stress. The stress at C is called upper yield point and the stress

As the load is increased, the extension increases and at the condition shown at E a waist or necking of the specimen is developed. The stress corresponding to E is called ultimate tensile stress.

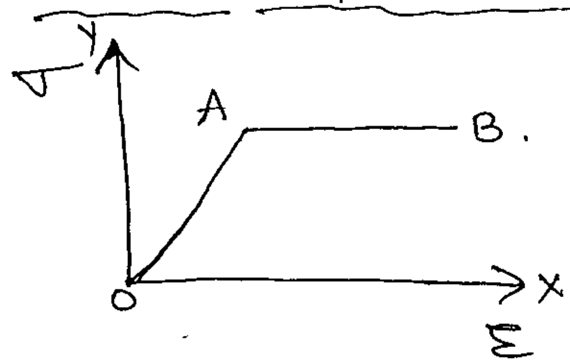
The stress at F is called Failure stress.

σ-ε curves for typical Engg materials :-

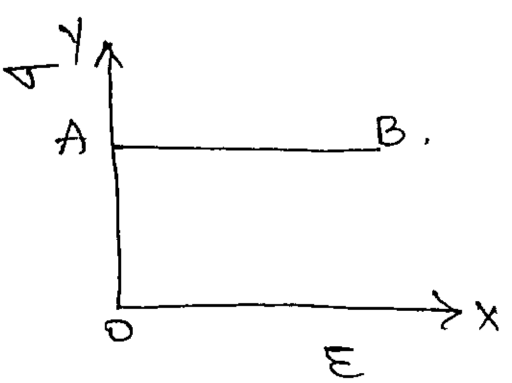
For Elastic material :



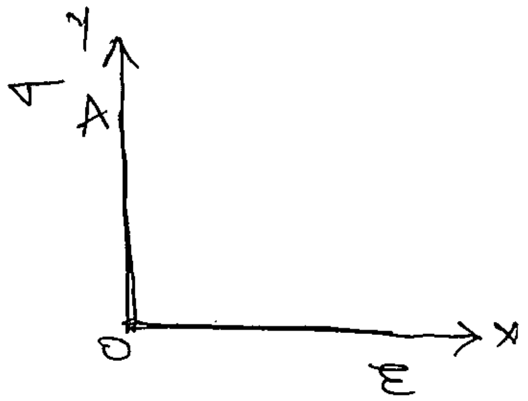
For Elastic - plastic material :



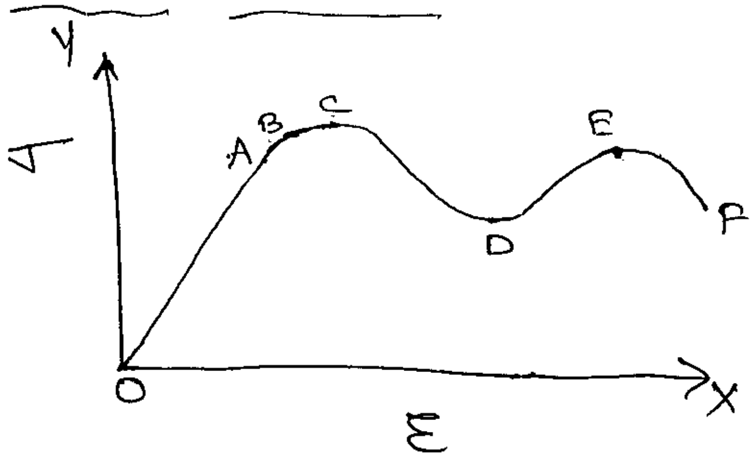
For purely plastic material



For rigid materials

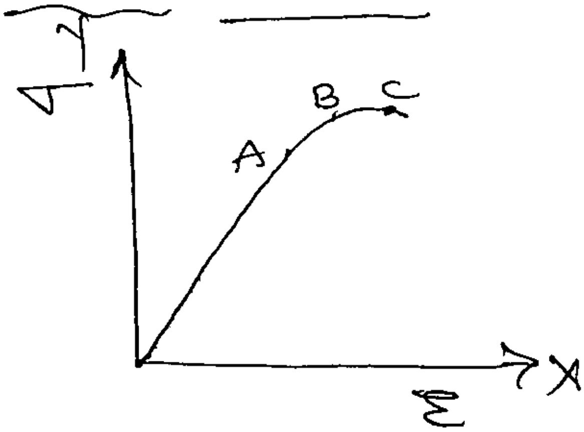


Ductile materials :-



Maximum stress = Ultimate stress

Brittle materials



Max stress = Yield point

Working stress:- When designing a machine part, it is desirable to keep the stress lower than the maximum or ultimate stress at which the failure of the material takes place. This stress is known as Design stress or working stress. It is also known as safe or allowable stress.

Factor of Safety: (F.S)

It is defined in general as the ratio of max stress to the working stress.

$$F.S = \frac{\text{Maximum stress}}{\text{Working (or) design stress}}$$

In case of Brittle materials, where the yield point is clearly defined, the Factor of Safety is based upon the yield point stress.

$$F.S = \frac{\text{Yield point stress}}{\text{Working or design stress}}$$

For Ductile materials

$$F.S = \frac{\text{Ultimate stress}}{\text{Working (or) Design stress}}$$

Note:- The above relations are valied for static loading only

F.S

For timber: 4 to 6 ; For concrete = 3.

For steel: 1.85

1) A copper rod 3mm in diameter when subjected to a pull of 495N Exceeds by 0.07mm over a gauge length of 100mm. Calculate the Young's Modulus for copper. [Ans: 100.04 KN/mm^2]

2) A wooden tie is 75mm wide, 150mm deep and 1.5m long. It is subjected to an axial pull of 45000N. The stretch of the member is found to be 0.638mm. Find the Young's modulus for the material.

[Ans: 9404.38 N/mm^2]

3) A load of 4000N has to be raised at the end of a steel wire. If the unit stress in the wire must not Exceed 80N/mm^2. What is the minimum diameter req? What will be extension of 3.5m length of wire? Take $E = 2\times10^5$ N/mm^2. [Ans: d = 7.98mm ; δl = 1.4mm]

4) Find the minimum diameter of a steel wire, which is used to raise a load of 4000N if the stress in the rod is not to Exceed 95MN/m^2. [Ans: d = 7.32mm]

5). The safe stress for a hollow steel column which carries an axial load of 2.1×10^3 kN is 125 MN/m^2. If the external diameter of the column is 30cm. Determine the internal diameter. [Ans: di = 261.93mm]

6) The ultimate stress for a hollow steel column which carries an axial load of 1.9MN is 480 N/mm^2. If the

External diameter of the column is 200mm, det the internal diameter. Take F.S = 4. [Ans: $d_i = 140.85mm$]

7) A square steel rod 20mm x 20mm in section is to carry an axial load (compressive) of 100kN. Calculate the shortening in a length of 50mm. $E = 2.14 \times 10^8 \, kN/m^2$.

[Ans: $\delta l = 0.058mm$].

8). The following dimensions were made during a tensile test on a mild steel specimen 40mm in diameter and 200mm long.

Elongation with 40kN load, $\delta l = 0.0304mm$.

Yield load = 161kN   Max load = 242kN.

Length of specimen at fracture = 249mm.

Determine 1) E   2) Yield point stress   3) ultimate stress

4) Percentage elongation.

[Ans 1) $E = 209414.39 \, N/mm^2$ 2) $128.119 \, N/mm^2$ 3) $192.57 \, N/mm^2$

4) 24.5%.]

9) A steel wire 2m long and 3mm in diameter is extended by 0.75mm when a weight W is suspended from the wire. If the same weight is suspended from a brass wire, 2.5m long and 2mm in diameter, it is elongated by 4.64mm. Determine the modulus of elasticity of brass if that of steel be $2 \times 10^5 \, N/mm^2$.

Ans: $E_b = \cancel{9092} \; 90921.33 \, N/mm^2$

Longitudinal strain:- When a body is subjected to an axial tensile or compressive load, there is an axial deformation in the length of the body.

The ratio of axial deformation to the original length of the body is known as longitudinal or linear strain.

$\therefore$ longitudinal strain, $\varepsilon_l = \dfrac{\text{change in length}}{\text{original length}}$

lateral strain:- The strain at right angles to the direction of applied load is known as lateral strain.

Consider a
let a Rectangular bar of length 'L', breadth 'b' and depth 'd' is subjected to an axial tensile load P as shown in fig.

The length of the bar will increase while the breadth and depth will decrease.

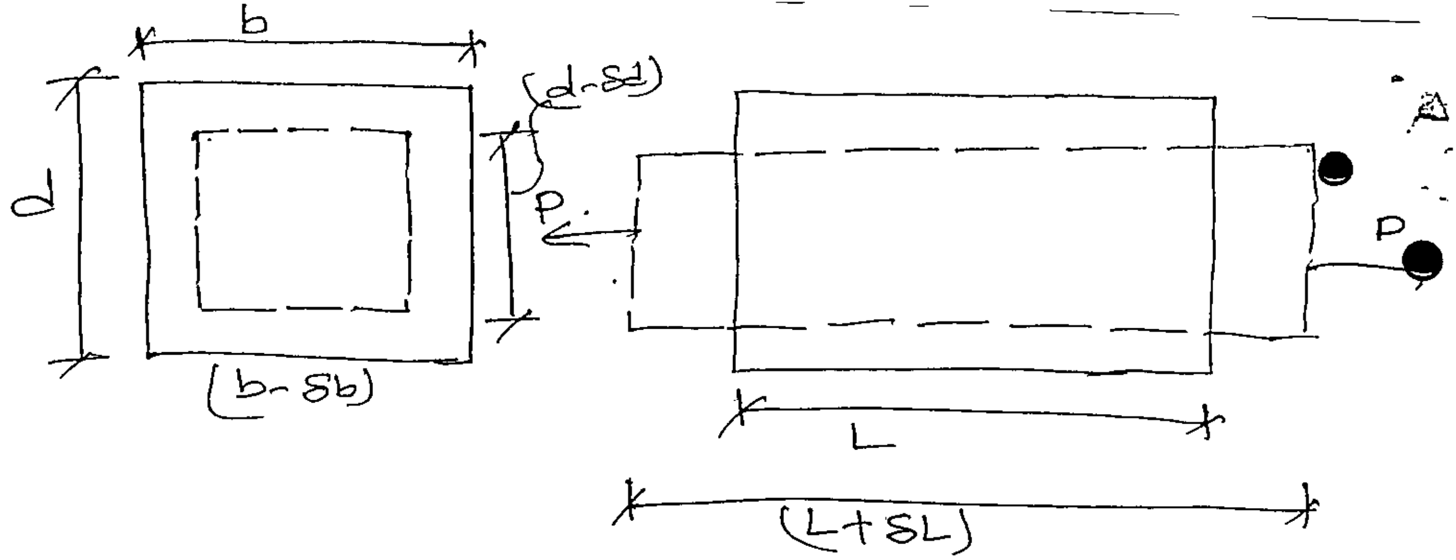
let δL = Increase in length

δb = Decrease in breadth

δd = " depth.

Then longitudinal strain = $\dfrac{\delta L}{L}$

and lateral strain = $\dfrac{\delta b}{b}$ or $\dfrac{\delta d}{d}$



Note:-

(i) If longitudinal strain is tensile, the lateral strains will be compressive.

(ii) If longitudinal strain is compressive, then lateral strain will be tensile.

(iii) Hence every longitudinal strain in the direction of load is accompanied by lateral strains of the opposite kind in all directions perpendicular to the load.

Poisson's Ratio:- It is denoted by 'μ' (or) $\frac{1}{m}$.

The ratio of lateral strain to the longitudinal strain is a constant, when the deformation of the member is within the elastic limit. This ratio is called 'Poisson's Ratio.'

$$\text{Poisson's Ratio } (\mu) \text{ or } \left(\frac{1}{m}\right) = \frac{\text{Lateral strain}}{\text{longitudinal strain}}$$

The Value of poisson's ratio Varies from 0.25 to 0.33. For rubber, its value ranges from 0.45 to 0.5

lateral strain = $\mu \times$ longitudinal strain

As lateral strain is opposite in sign to longitudinal strain, hence algebrically

lateral strain = $-\mu \times$ longitudinal strain.

Volumetric strain:- The ratio of change in Volume to the original volume of a body is called Volumetric strain. It is denoted by ε_v.

$$\varepsilon_v = \frac{\delta V}{V}$$

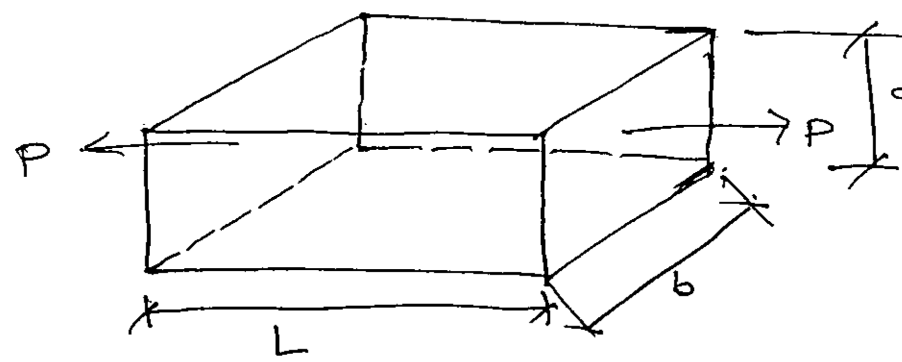
, where δV = change in volume
V = original Volume.

Bulk Modulus:- It is denoted by 'k'

When a body is subjected to three mutually perpendicular stresses of equal intensity, then the ratio of direct stress to the corresponding volumetric strain is known as Bulk modulus.

$$k = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\delta V/V}$$

Volumetric strain of a Rectangular bar which is subjected to an axial load P in the direction of its length :-



Consider a rectangular bar of length L, width b and depth d which is subjected to an axial load P in the direction of its length as shown in fig.

Let δL = change in length

δb = " breadth

δd = " depth.

$\therefore$ Final length of the bar $= L + \delta L$

" width " " $= b + \delta b$

" depth " " $= d + \delta d$

Now original Volume of the body $= Lbd$.

Final Volume of the body $= (L+\delta L)(b+\delta b)(d+\delta d)$

$$= Lbd + Lb\cdot\delta d + L\cdot\delta b\cdot d + \cancel{L\cdot\delta b\cdot\delta d}^{\,0} + \delta L\cdot b\cdot d + \cancel{\delta L\cdot b\cdot\delta d}^{\,0}$$

$$+ \cancel{\delta L\cdot\delta b\cdot d}^{\,0} + \cancel{\delta L\cdot\delta b\cdot\delta d}^{\,0}$$

$$= Lbd + Lb\cdot\delta d + L\cdot d\cdot\delta b + b\cdot d\cdot\delta L$$

Change in Volume = Final Volume − Original Volume

$= \cancel{Lbd} + Lb \cdot \delta d + L \cdot d \cdot \delta b + b \cdot d \cdot \delta L - \cancel{Lbd}$

$\delta V = L \cdot b \cdot \delta d + L \cdot d \cdot \delta b + b \cdot d \cdot \delta L$

∴ Volumetric strain, $\varepsilon_V = \dfrac{\delta V}{V}$

$$= \frac{Lb\delta d + L \cdot d \cdot \delta b + b \cdot d \cdot \delta L}{Lbd}$$

$$= \frac{\delta d}{d} + \frac{\delta b}{b} + \frac{\delta L}{L}$$

But $\frac{\delta L}{L}$ = longitudinal strain

$\frac{\delta b}{b}$ or $\frac{\delta d}{d}$ = lateral strain

ε_V = longitudinal strain + (2 × lateral strain)

But lateral strain = $-\mu \times$ longitudinal strain

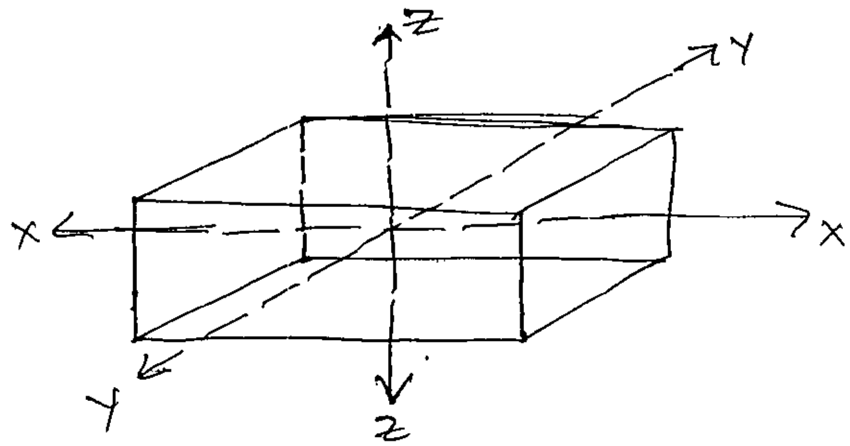
ε_V = longitudinal strain − 2μ longitudinal strain

= longitudinal strain $(1 - 2\mu)$.

$$\boxed{\varepsilon_V = \frac{\delta L}{L}(1 - 2\mu)}$$

or $\boxed{\varepsilon_V = \frac{\sigma}{E}(1 - 2\mu)}$

Volumetric strain of a Rectangular bar subjected to three forces which are mutually perpendicular:-



Consider a rectangular block of dimensions x, y and z subjected to three direct tensile stresses along three mutually perpendicular axis as shown in fig.

Then volume of block $V = xyz$

Taking logarithms on both sides

$$\log V = \log x + \log y + \log z$$

Differentiating on both sides we get

$$\frac{1}{V} dV = \frac{1}{x} dx + \frac{1}{y} dy + \frac{1}{z} dz$$

But $\frac{dV}{V} = \frac{\text{change in volume}}{\text{original vol}}$ = Volumetric strain

$||^{ly}$ $\frac{dx}{x} = \frac{\text{change in dimensions } x}{\text{original dimension } x}$

= strain in the x-direction $= \varepsilon_x$.

$\frac{dy}{y} = \frac{\text{change in dimensions } y}{\text{original dimensions } y}$ = strain in the y-direction

$= \varepsilon_y$.

$\therefore \frac{dz}{z} = \frac{\text{change in dimensions } z}{\text{original dimensions } z}$

$= \text{strain in the } z\text{-direction} = \varepsilon_z$

$$\therefore \boxed{\frac{dV}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z}$$

Now,

Let σ_x = Tensile stress in x-x direction

σ_y = " " y-y "

σ_z = " " z-z "

E = Young's modulus

μ = poisson's Ratio.

Now σ_x will produce a tensile strain equal to $\frac{\sigma_x}{E}$ in the direction of x and a compressive strain equal to $\mu \frac{\sigma_x}{E}$ in the direction of y and z.

Similarly, σ_y will produce a tensile strain equal to $\frac{\sigma_y}{E}$ in the direction of y and a compressive strain equal to $\mu \frac{\sigma_y}{E}$ in the direction of x and z.

Similarly, σ_z produce a tensile strain equal to $\frac{\sigma_z}{E}$ in the direction of z and a compressive strain equal to $\mu \frac{\sigma_z}{E}$ in the direction of x and y.

Hence σ_y and σ_z will produce compressive strains equal to $\mu\frac{\sigma_y}{E}$ and $\mu\frac{\sigma_z}{E}$ in the direction of x.

$\therefore$ Net tensile strain along x-direction is given by

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \mu(\sigma_y+\sigma_z)\right) \qquad \text{or } \varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu}{E}(\sigma_y+\sigma_z)$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\mu}{E}(\sigma_x+\sigma_z)$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\mu}{E}(\sigma_x+\sigma_y)$$

Adding all the strains

$$\varepsilon_x+\varepsilon_y+\varepsilon_z = \frac{\sigma_x+\sigma_y+\sigma_z}{E} - \frac{\mu}{E}(2\sigma_x+2\sigma_y+2\sigma_z)$$

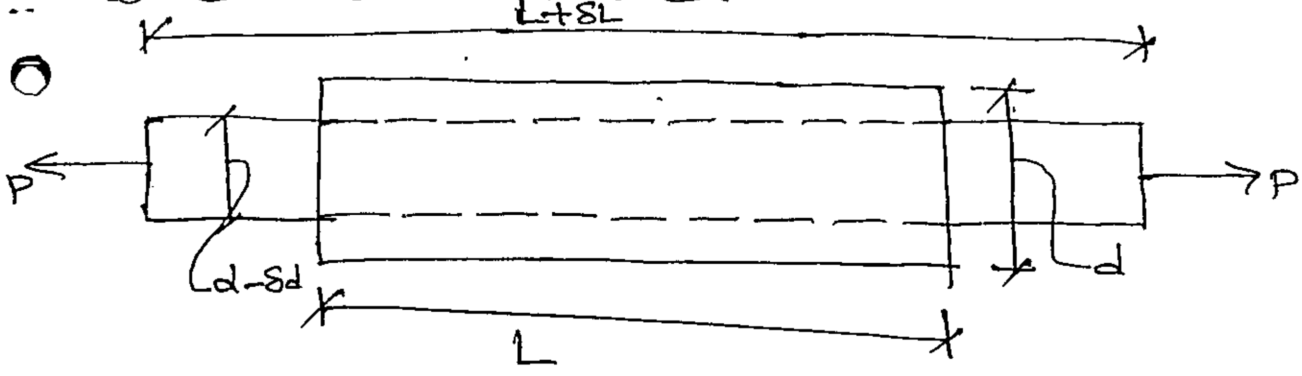
$$= \frac{\sigma_x+\sigma_y+\sigma_z}{E} - \frac{2\mu}{E}\left[\sigma_x+\sigma_y+\sigma_z\right]$$

$$\boxed{\frac{\delta V}{V} = \frac{(\sigma_x+\sigma_y+\sigma_z)}{E}(1-2\mu)}$$

In this equation the stresses σ_x, σ_y and σ_z are all tensile. If any of the stresses is compressive, it may be regarded as -ve, and the above eq holds good.

If the value of $\frac{\delta V}{V}$ is +ve, it represents increase in volume whereas the negative value of $\frac{\delta V}{V}$ represents a decrease in volume.

Volumetric strain of a cylindrical Rod:-



consider a cylindrical rod which is subjected to an axial tensile load 'p'.

let d = diameter of the rod

L = length of the rod.

Due to tensile load P, there will be an increase in the length of the rod, but the diameter of the rod will decrease.

∴ Final length = $L + \delta L$

∴ Final diameter = $d - \delta d$

Now original Volume of the rod

$$V = \frac{\pi}{4} d^2 L.$$

Final Volume $= \frac{\pi}{4}(d+\delta d)^2 (L+\delta L)$

$$= \frac{\pi}{4} d^2 L + \frac{\pi}{4} d^2 \delta L + \frac{\pi}{4} \cancelto{0}{(\delta d)^2} . L + \frac{\pi}{4} \cancelto{0}{(\delta d)^2 (\delta L)}$$

$$+ \frac{\pi}{4} . 2 . d . \delta d L + \frac{\pi}{4} 2 . \cancelto{0}{d \delta d \div \delta L}$$

$$= \frac{\pi}{4} d^2 L + \frac{\pi}{4} d^2 \delta L + 2 d . L \frac{\pi}{4} \delta d .$$

change in volume = Final Volume − Initial vol

$$= \frac{\pi}{4}\cancel{d^2 L} + \frac{\pi}{4}d^2 \delta L + 2dL\cdot\frac{\pi}{4}\delta d - \frac{\pi}{4}\cancel{d^2 L}$$

$$= \frac{\pi}{4}d^2 \delta L + 2dL\cdot \delta d \cdot \frac{\pi}{4}$$

$$\delta V = \frac{\pi}{4}\left(d^2 \delta L + 2dL\cdot \delta d\right)$$

Volumetric strain, $\varepsilon_v = \frac{\text{Change in Volume}}{\text{Original volume}}$

$$\varepsilon_v = \frac{\delta V}{V} = \frac{\frac{\pi}{4}\left(d^2 \delta L + 2dL\cdot \delta d\right)}{\frac{\pi}{4}d^2 L}$$

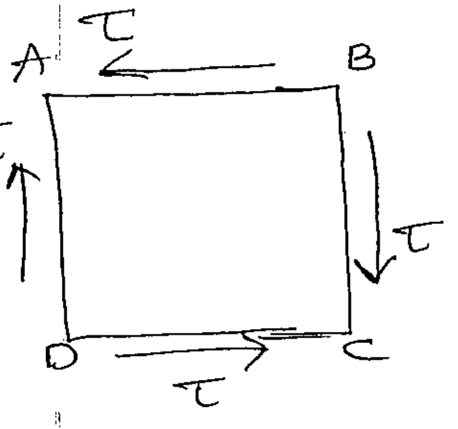
$$\boxed{\varepsilon_v = \frac{\delta L}{L} + 2\frac{\delta d}{d}}$$

∴ Volumetric strain = strain in length + twice the strain of diameter.

Note:

Volumetric strain of a sphere $= 3.\varepsilon_d = 3.\frac{\delta d}{d}$.

Relation between the modulus of Elasticity and the modulus of rigidity :-



Consider a square block ABCD of side a and of thickness unity perpendicular to the plane of the drawing.

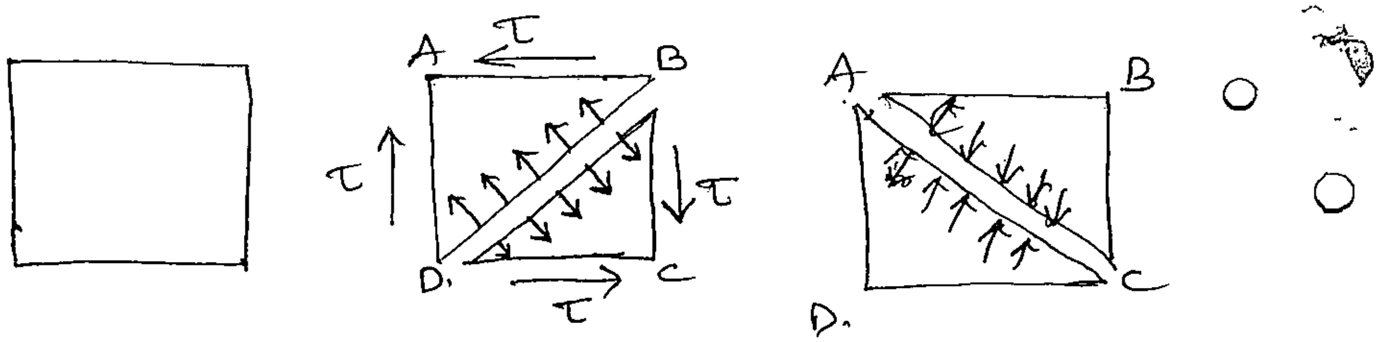
Let the block be subjected to shear stresses of intensity τ. Due to these stresses the block will be subjected to a deformation such that the diagonal AC is elongated and the diagonal BD is shortened. Consider the diagonal AC.

The increase in length of the diagonal can be computed by considering the effect of the diagonal tensile and diagonal compressive stresses.

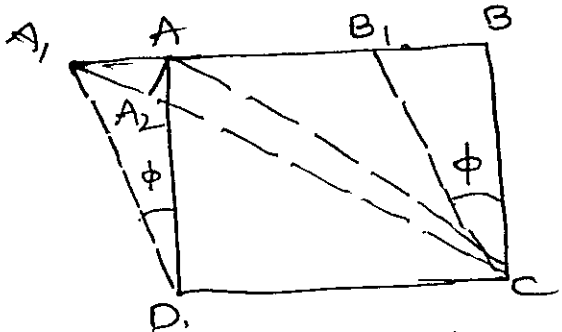
Strain in the length of the diagonal AC = strain in the length of AC due to diagonal tensile stresses on the plane BD + strain in the length of AC due to diagonal compressive stresses on the plane AC.

$$\text{strain of } AC = \frac{\tau}{E} + \frac{\tau}{E}\cdot\mu = (1+\mu)\cdot\frac{\tau}{E}$$

$$\therefore \text{strain of the diagonal } AC = (1+\mu)\cdot\frac{\tau}{E} \rightarrow (1)$$



strain of the diagonal AC may also be determined from the geometry of the distorted shape of the block



Let the block ABCD deform to the position A_1B_1CD through the angle ϕ.

Increase in the length of the diagonal AC $= A_1C - AC$

Let AA_2 be $\perp^r$ to A_1A_2. Since the $\angle ACA_2$ is very small

$$AC = A_2C.$$

$\therefore$ Increase in the length of the diagonal AC

$$= A_1C - A_2C$$

$$= A_1A_2 = AA_1 \cos AA_1A_2$$

But the angle $\angle AA_1A_2$ is nearly equal to $\angle BAC = 45°$

$\therefore$ Increase in length of the diagonal $AC = AA_1 \cos 45°$

$$AA_2 = AA_1 \cos 45° = \frac{AA_1}{\sqrt{2}}$$

$$\text{shear strain} = \phi = \frac{AA_1}{AD} = \frac{AA_1}{a}$$

$$AA_1 = a\phi$$

$\therefore$ Increase in the length of diagonal $AC = AA_2 = \frac{a\phi}{\sqrt{2}}$

But length of the diagonal $AC = \sqrt{2}a$

$\therefore$ Strain of the diagonal $AC = \frac{\text{Increase in length}}{\text{original length}}$

$$= \frac{a\phi}{\sqrt{2}} \Big/ \sqrt{2}a = \frac{\phi}{2} \rightarrow (2)$$

But eq(1) = eq(2).

$$\frac{\phi}{2} = \frac{\tau}{E}(1+\mu)$$

$$\frac{E}{2} = \frac{\tau}{\phi}(1+\mu)$$

$$\frac{E}{2} = G(1+\mu)$$

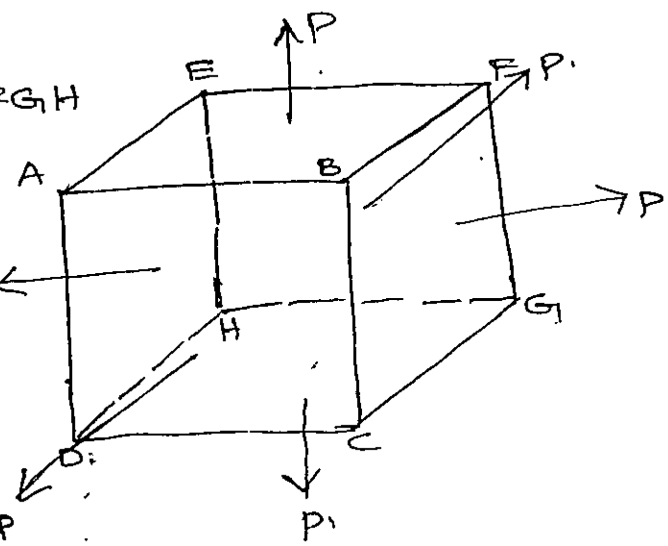
$$\boxed{\therefore E = 2G(1+\mu)}$$

where E = Young's modulus

G = Modulus of rigidity

Relation between Young's Modulus and Bulk modulus

Fig shows a cube ABCDEFGH of side a.



Let the faces of the cube be subjected to a direct stress of intensity σ.

Let E = Young's modulus

μ = poisson's ratio.

Let us now consider the strain of one of the edge

Say AB.

strain of AB due to stresses on the faces AEHD, BFGC.

$$= \frac{\sigma}{E}$$

strain of the ~~face~~ AB due to stresses on the faces ABFE and DHGC

$$= -\frac{\mu\sigma}{E}$$

strain of AB due to stresses on the faces ABCD and EFGH

$$= -\frac{\mu\sigma}{E}$$

$\therefore$ Total strain of AB $= \frac{\sigma}{E} - \frac{\mu\sigma}{E} - \frac{\mu\sigma}{E}$

$$\therefore \quad = \frac{\sigma}{E}(1-2\mu).$$

original volume of the cube $= V = a^3$

$$\delta V = 3a^2 . \delta a$$

Volumetric strain $= \frac{\delta V}{V} = \frac{3a^2 . \delta a}{a^3} = 3 . \frac{\delta a}{a}$

$\therefore$ Vol strain $= 3 \times$ strain of AB $= 3 . \frac{\sigma}{E}(1-2\mu)$

Bulk modulus, $k = \frac{\text{stress}}{\text{Vol. strain}}$

$$k = \frac{\sigma}{3\frac{\sigma}{E}(1-2\mu)} = \frac{E}{3(1-2\mu)}$$

$$\boxed{\therefore E = 3k(1-2\mu)}$$

Relation between E, C and K:-

$$\boxed{E = 2G(1+\mu)} \quad , \quad \boxed{E = 3K(1-2\mu)}$$

$$2G(1+\mu) = 3K(1-2\mu)$$

$$(2G+6K)\mu = 3K-2C$$

$$\boxed{\mu = \frac{3K-2C}{6K+2C}}$$

$$1+\mu = \frac{E}{2G} \quad , \quad 1-2\mu = \frac{E}{3K}$$

$$2+2\mu = \frac{E}{G} \rightarrow (1) \quad , \quad 1-2\mu = \frac{E}{3K} \rightarrow (2)$$

Adding (1) and (2)

$$3 = \frac{E}{G} + \frac{E}{3K} \Rightarrow 3 = E\left[\frac{1}{G}+\frac{1}{3K}\right]$$

$$\boxed{E = \frac{9KG}{3K+G}} \quad \text{or} \quad \boxed{E = \frac{9KC}{3K+C}}$$

Problems

1) $E = 1.25\times10^5\ N/mm^2$, $\mu = 0.25$, $G=?$, $K=?$

[Ans: $G = 0.5\times10^5\ N/mm^2$, $K = 0.83\times10^5\ N/mm^2$.]

[2) $\delta v = ?$, $K = 2.5\times10^5$, Find the change in volume of a cubical block of steel of side 250mm when placed at a depth of 5km in sea water.]

2) A bar 30mm × 30mm × 250mm long is subjected to a pull of 90kN in the direction of its length. The extension of the bar was found to be 0.125mm, while the decrease in each lateral dimension is found to be 0.00375mm. Find the Young's modulus, poisson's ratio, Modulus of rigidity and Bulk modulus for the material of the bar.

[Ans: $E = 2\times10^5 N/mm^2$, $\mu = 0.25$, $G = 8\times10^4 N/mm^2$, $K = 1.33\times10^5 N/mm^2$

3) C or $G = 0.8\times10^5 N/mm^2$. When a 6mm × 6mm rod of this material was subjected to an axial pull of 3600N it was found that the lateral dimension of the rod changed to 5.9991 mm × 5.9991mm. Find the μ, E

[Ans: $\mu = 0.315$, $E = 2.1\times10^5 N/mm^2$]

4) $E = 1.1\times10^5 N/mm^2$, $C = 0.43\times10^5 N/mm^2$, $K = ?$, $\delta d = ?$, d = 40mm

$l = 2.5m$, $\delta l = 2.5mm$.

[Ans: $K = 8.29\times10^4 N/mm^2$, $\delta d = 0.0112mm$]

5) A concrete cylinder of diameter 150mm and length 300mm when subjected to an axial compressive load of 240kN resulted in an increase of diameter by 0.127mm and a decrease in length of 0.28mm. Compute the value of poisson's ratio and modulus of elasticity E.

($\mu = 0.907$, $E = 14.55 GN/m^2$),

6) For a given material, $E = 110\ GN/m^2$ and $G = 42\ GN/m^2$. Find the Bulk Modulus $K = ?$, and lateral contraction of a round bar of 37.5mm diameter and 2.4m length when stretched 2.5mm.

[Ans: $K = 96.77\ GN/m^2$, $\delta d = 0.0121\ mm$]

7) The following data relate to a bar subjected to a tensile test: Diameter of the bar, $d = 30mm$, Tensile load $P = 54kN$, Gauge length $l = 300mm$, Extension of the bar, $\delta l = 0.112mm$, change in diameter $\delta d = 0.00366m$. Calculate: (i) Poisson's Ratio (ii) The value of three moduli

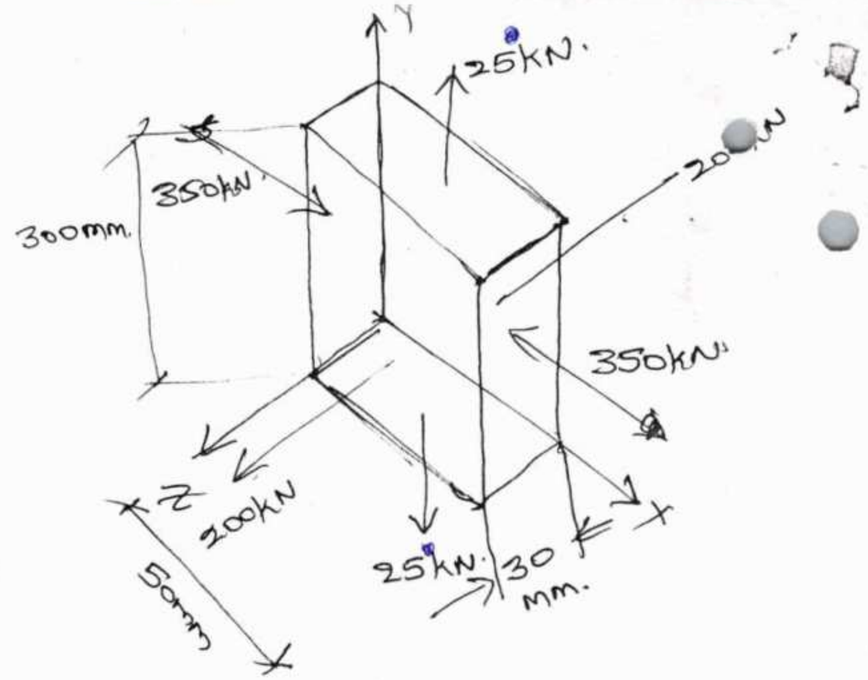
[Ans: $\mu = 0.327$, $E = 2.05 \times 10^5\ MN/m^2$, $C = 0.77 \times 10^5\ MN/m^2$

$K = 1.97 \times 10^5\ MN/m^2$].

8) A C.I flat, 300mm long and of 30mm × 50mm uniform section, is acted upon by the following forces uniformly distributed over the respective cross-section: 25kN in the direction of length (tensile): 350kN in the direction of width (compressive) and 200kN in the direction of thickness (tensile). Determine the change in volume of the flat.

Take $E = 140\ GN/m^2$ and $m = 4$.

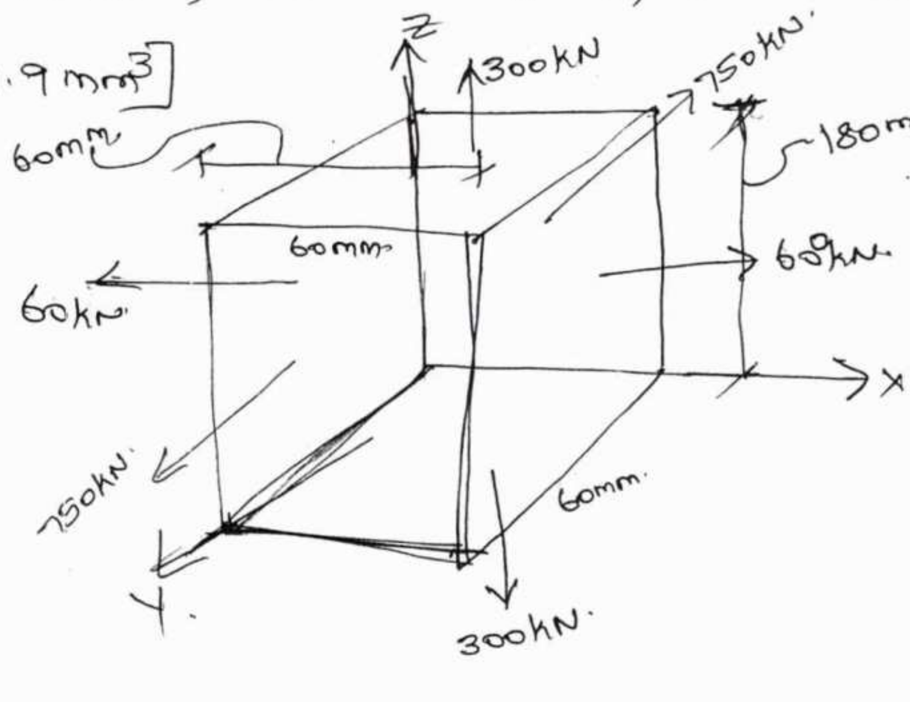
[Ans: $\delta V = 14.13\ mm^3$]



9) A bar of steel is 60mm x 60mm in section and 180mm long. It is subjected to a tensile load of 300kN along the longitudinal axis and tensile loads of 750kN and 600kN on the lateral faces. Find the change in the dimensions of the bar and the change in volume.

Take $E = 200 GN/m^2$ and $\frac{1}{m} = 0.3$

[Ans: $\delta L = 0.0412 mm$, $\delta b = 0.00291 mm$, $\delta d = 0.00833 mm$,
$\delta V = 269.9 mm^3$]



Deformation of bars under axial loading:-

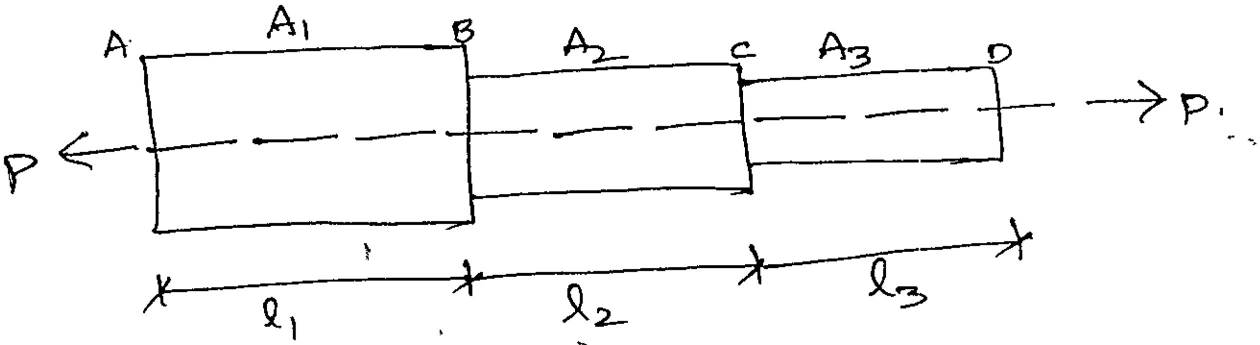


Fig shows a bar which consists of three lengths l_1, l_2 and l_3 with c/s area of A_1, A_2 and A_3 and subjected to an axial load 'P'.

Total force on each section is same but the intensity of stress will be different for the three sections.

Intensity of stress for the portion AB

$$= \sigma_1 = \frac{P}{A_1}$$

stress for the portion BC, stress for the portion CD

$$\sigma_2 = \frac{P}{A_2} \qquad \sigma_3 = \frac{P}{A_3}$$

let E be the Young's modulus

$\therefore$ strain of the part AB, $e_1 = \frac{\sigma_1}{E_1} = \frac{P}{A_1 E_1}$

$$e_2 = \frac{\sigma_2}{E_2} = \frac{P}{A_2 E_2}, \qquad e_3 = \frac{\sigma_3}{E_3} = \frac{P}{A_3 E_3}$$

$\therefore$ change in length of the part AB =

$$\delta l_1 = l_1 \cdot \varepsilon_1 = \frac{Pl_1}{A_1 E_1}$$

$\therefore$ change in length of the part BC

$$\delta l_2 = l_2 \cdot \varepsilon_2 = \frac{Pl_2}{A_2 E_2}$$

change in length of the part CD

$$\delta l_3 = l_3 \cdot \varepsilon_3 = \frac{Pl_3}{A_3 E_3}$$

$\therefore$ Total change in length of the bar

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$= \frac{Pl_1}{A_1 E_1} + \frac{Pl_2}{A_2 E_2} + \frac{Pl_3}{A_3 E_3}$$

If $E_1 = E_2 = E_3 = E$

05/08
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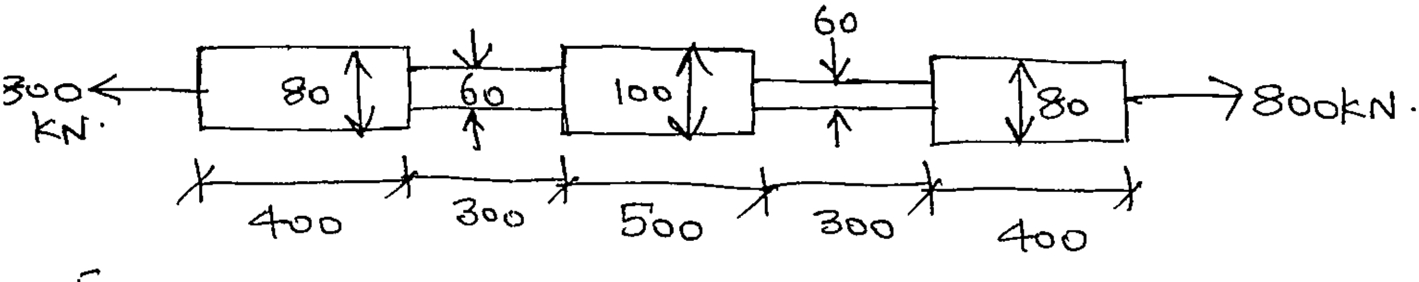
$$= \frac{Pl_1}{A_1 E} + \frac{Pl_2}{A_2 E} + \frac{Pl_3}{A_3 E}$$

$$\boxed{\delta l = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right]}$$

1. A circular steel bar of various c/s is subjected to a pull of 800 kN as shown in fig. Det the Extension of the bar. E = 204 GPa.



[Ans: $\delta L = 1.708$ mm].

Problems

1. A hollow steel tube is to be used to carry a axial compressive load of 140 kN. The yield stress for steel is 250 N/mm². A Factor of safety of 1.75 is to be used in the design. The following three classes tubes of External diameter 101.6 mm are available.

Class	Thickness
Light	3.65 mm
Medium	4.05 mm
Heavy	4.85 mm.

which section do you recommend?.

[Ans: t = 3.169 mm, use of light section is recommended]

2. A Specimen of Steel 25 mm diameter with a gauge length of 200 mm is Tested to destruction. It has an Experiment of 0.16 mm under a load of 80 kN and the load at elastic limit is 160 kN. The max load is 180 kN.

The total Extension at fracture is 56 mm and diameter at neck is 18 mm. Find

(i) the stress at elastic limit

(ii) Young's Modulus

(iii) Percentage elongation

(iv) Percentage reduction in area

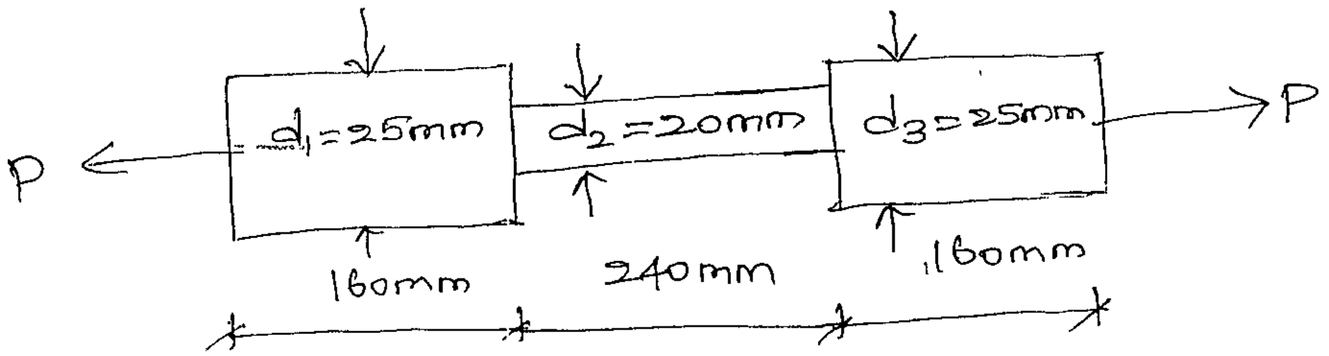
(v) Ultimate tensile stress.

[Ans: $\sigma = 325.949\ N/mm^2$, $E = 203718\ N/mm^2$

% elongation = 28%   % reduction in area = 48.16%

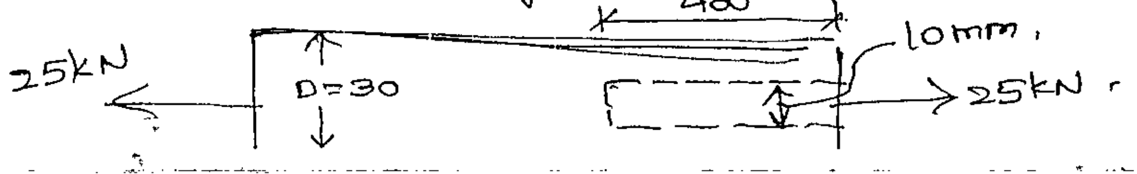
$\sigma_{ult} = 366.693\ N/mm^2$]

3. The bar shown in fig is tested in Universal Testing machine. It is observed that at a load of 40kN the total extension of the bar is 0.285 of the material. Determine the Young's Modulus of the material.



[Ans: $E = 198714.72\ N/mm^2$]

4. A bar of length 1000mm and diameter 30mm is centrally bored for 400mm, the bore diameter being 10mm as shown in fig. Under a load of 25kN, if the extension of the bar is 0.183 mm, what is the modulus of elasticity of the bar? [Ans: $E = 200736\ N/mm^2$]



$202929.8\ N/mm^2$

Principle of superposition:-

When a number of loads are acting on a body, the resulting strain, will be the algebraic sum of strains caused by individual loads is called the Principle of superposition.

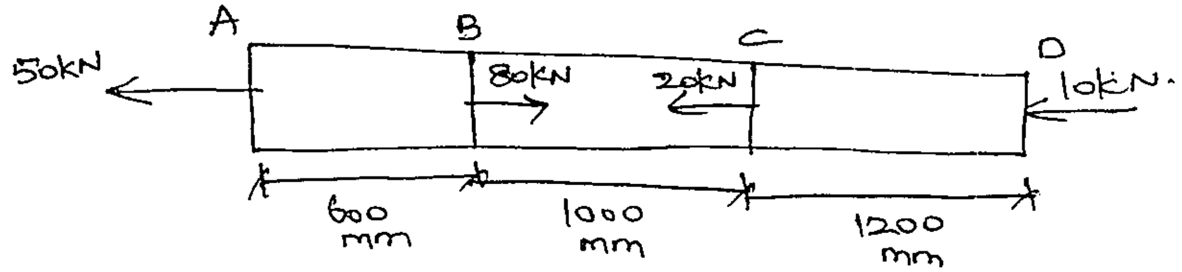
steps to solve: 1) Draw the free body diagram of individual section

2) Deformation of each section is obtained.

3) Total deformation of the body will be then equal to the algebraic sum of deformations of the individual sections.

Problems

1) A brass bar having a c/s area of $1000 mm^2$ is subjected to axial forces as shown in fig. Find the total change in length of the bar. Take $E = 1.05 \times 10^5 N/mm^2$.



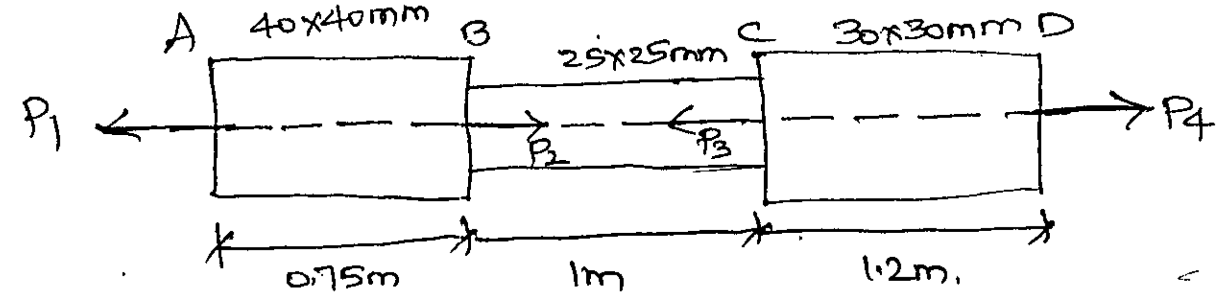
[Ans: $\delta L_1 = 0.285$ (Extension), $\delta L_2 = -0.2857$ (contraction)

$\delta L_3 = -0.114$ (contraction)., $\delta L = -0.1143$ mm (contraction).

A member ABCD is subjected to point loads P_1, P_2, and P_4 as shown in fig. Calculate the force P_3

necessary for Equilibrium if $P_1 = 120kN$, $P_2 = 220$

and $P_4 = 160kN$. Det also the net change in length of the member. Take $E = 2 \times 10^5 N/mm^2$.

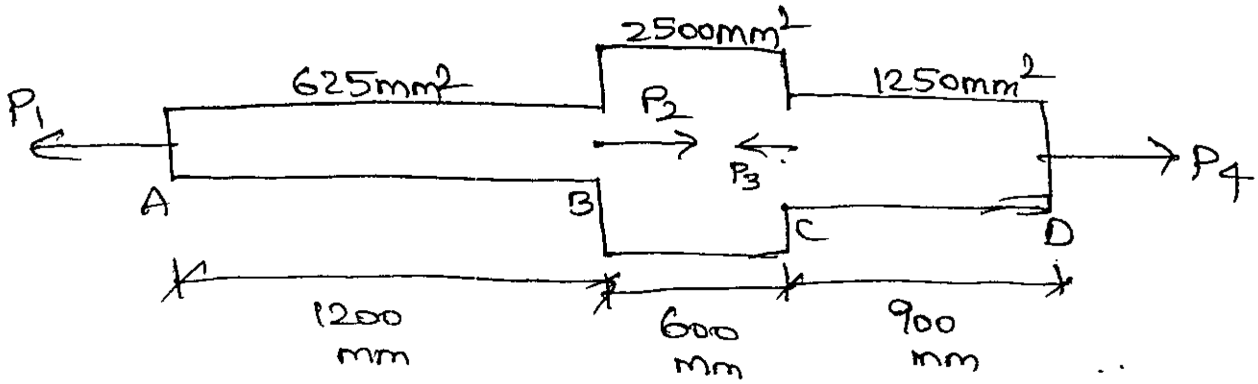


[Ans: $P_3 = 260kN$, $\delta L_1 = +0.28mm$ (Extension),

$\delta L_2 = -0.8mm$ (contraction), $\delta L_3 = +1.07mm$ (Extension),

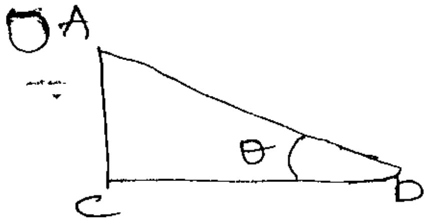
$\delta L = +0.55mm$ (Extension).

3) A member ABCD is subjected to point loads, P_1, P_2, P_3 and P_4 as shown in fig. Calculate the force P_2 necessary for equilibrium if $P_1 = 45kN$, $P_3 = 450kN$, $P_4 = 130kN$. Det the total elongation of the member, assuming the modulus of Elasticity to be $2.1 \times 10^5 N/mm^2$.

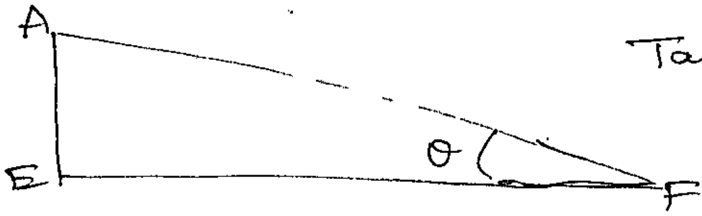


[Ans: $P_2 = 365kN$, $\delta L_1 = 0.411$ (Extension) $\delta L_2 = -0.365$ (contr $\delta L_3 = 0.445$ (Exten

Applying ~~similar~~ law of similar triangles



$$\tan\theta = \frac{AC}{CD} = \frac{r_1 - r_x}{x} \rightarrow (1)$$



$$\tan\theta = \frac{AE}{EF} = \frac{r_1 - r_2}{L} \rightarrow (2)$$

eq(1) = eq(2)

$$\frac{r_1 - r_x}{x} = \frac{r_1 - r_2}{L}$$

$$r_1 - r_x = (r_1 - r_2)\cdot\frac{x}{L} \Rightarrow r_x = r_1 - \frac{x}{L}(r_1 - r_2)$$

||ly

$$D_x = D_1 - \frac{x}{L}(D_1 - D_2) \rightarrow (3)$$

Area of section $A_x = \frac{\pi}{4} D_x^2$

$$= \frac{\pi}{4}\left(D_1 - \frac{x}{L}(D_1 - D_2)\right)^2$$

$$A_x = \frac{\pi}{4}(D_1 - kx)^2 \qquad \left[\text{where } k = \frac{D_1 - D_2}{L}\right]$$

stress in the section: $\sigma_x = \frac{P}{A_x} = \frac{P}{\frac{\pi}{4}(D_1 - kx)^2}$

$$\sigma_x = \frac{4P}{\pi(D_1 - kx)^2}$$

strain in section $= \varepsilon_x = \frac{\sigma_x}{E} = \frac{4P}{\pi E (D_1 - kx)^2}$

→ Deformation produced in the section

$$\varepsilon_x = \frac{\text{deformation}}{\text{original length}} = \frac{\text{deformation}}{dx}$$

$$\text{deformation} = \varepsilon_x \cdot (dx)$$

$$= \frac{4P}{\pi E (D_1 - kx)^2} \cdot dx$$

Deformation produced for the total body

$$\delta L = \int_0^L \frac{4P}{\pi E (D_1 - kx)^2} \cdot dx$$

$$\delta L = \frac{4P}{\pi E}\left[\frac{(D_1-kx)^{-2+1}}{-2+1} \cdot \frac{1}{-k}\right]_0^L = \frac{4P}{\pi E}\left[\frac{1}{k} \cdot \frac{1}{(D_1-kx)}\right]_0^L$$

$$= \frac{4P}{\pi E}\left[\frac{1}{k} \cdot \frac{1}{(D_1-kL)} - \frac{1}{k \cdot D_1}\right]$$

$$= \frac{4P}{\pi E k}\left[\frac{1}{\left(D_1 - \left(\frac{(D_1-D_2)}{L} \cdot L\right)\right)} - \frac{1}{D_1}\right]$$

$$\delta L = \frac{4P}{\pi E k}\left[\frac{1}{D_2} - \frac{1}{D_1}\right] = \frac{4P(D_1-D_2)}{\pi E k \cdot D_1 D_2} = \frac{4P(D_1-D_2)}{\pi E\left(\frac{D_1-D_2}{L}\right) D_1 \cdot D_2}$$

$$\boxed{\delta L = \frac{4PL}{\pi E D_1 \cdot D_2}}$$

where P = Load acting on the body
L = length of the body
D_1 = Dia of the body at one end
D_2 = " the other end

Thermal Effect:- When a material undergoes a change in temperature, it either elongates or contracts depending upon whether heat is added to or removed from the material.

When a member is free to expand or contrac due to the rise or fall of temperature, no stresses will be induced in the member, but it undergoes a strain.

The strain due to temperature change is called thermal strain and is expressed as.

$$\varepsilon_T = \alpha \cdot \Delta T$$

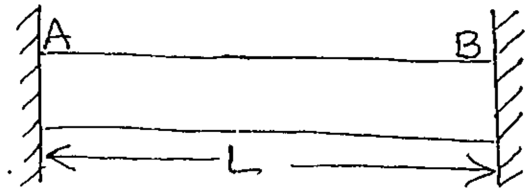
Where α is a material property known as coefficient of thermal Expansion

ΔT indicates the change in Temperature

Since strain is a dimensionless quantity and ΔT is expressed in k or °c. α has a unit that is reciprocal of k or °c.

The free Expansion or Contraction of materials When restrained ~~produces~~ induces stresses in the material and it is referred to as thermal stress

○ Consider a rod ~~of~~ AB of length L which is fixed at both ends as shown in fig.



Let the Temperature of the rod be raised by ΔT and as the Expansion is restricted, the material develops a compressive stress.

In this Problem, static equilibrium equations alone are not sufficient to solve for unknowns and hence is called statically indeterminate problem.

To determine the stress due to ΔT, assume that the support at the end B is removed and the material is allowed to Expand freely.

Increase in the length of the rod δ_T due to free Expansion can be found out using Equation

$$\delta_T = \varepsilon_T \cdot L = \alpha \cdot (\Delta T) \cdot L$$

Now apply a compressive load P at the end B to bring it back to its initial position and the deflection due to mechanical load

$$\delta_T = \frac{PL}{AE}$$

Problems

1. A steel rod is stretched between two rigid walls and carries a tensile load of 5000N at 20°c. If the allowable stress is not to exceed 130MPa at -20°c, what is the minimum diameter of the rod?

Assume: $\alpha = 11.7 \mu m/m\text{-}^\circ c$ and $E = 200$ Gpa.

[Ans: $d = 13.22mm$]

2. A steel rod 15m long is at a temperature of 15°c. Find the free expansion of the length when the temp is raised to 65°c. Find the temp stress produced when:

   1. The expansion of the rod is prevented
   2. The rod is permitted to expand by 6mm.

   Take, $\alpha = 12 \times 10^{-6}/^\circ c$ and $E = 200 GN/m^2$.

   (Ans: (i) $120MN/m^2$ (ii) $40MN/m^2$]

3. Calculate the values of the stress and strain in portions AC and CB of the steel bar shown in fig. A close fit exists at both of the rigid supports at room temp and the temp is raised by 75°c. Take $E = 200$ Gpa and $\alpha = 12 \times 10^{-6}/^\circ c$ for steel. Area of c/s of AC is $400mm^2$ and of BC is $800mm^2$.

   (Ans: $\varepsilon_{AC} = 900 \times 10^{-6}$; $\varepsilon_{CB} = 900 \times 10^{-6}$; $\sigma_{CB} = 180MN/m^2$]

As the magnitude of δ_T and δ are equal and their signs differ,

$$\delta_T = -\delta$$

$$\alpha \cdot \Delta T \cdot L = -\frac{PL}{AE}$$

Thermal stress, $\sigma_T = \frac{P}{A} = -\alpha \cdot \Delta T \cdot E$

Minus sign in the equation indicates a compressive stress in the material.

With the decrease in temperature, the stress developed is tensile stress as ΔT becomes negative.

Thermal stress produces the same effect in the material similar to that of mechanical stress. ~~and~~

~~It can be de~~

The S.E stored in a body due to impact load

$$\sigma = \frac{P}{A}\left(1+\sqrt{1+\frac{2AEh}{PL}}\right)$$

Toughness:

Toughness of a material can be defined as

"The ability of a material to absorb sudden Shock without breaking or Shattering".

(or)

Toughness in materials science & metallurgy, is the resistance to fracture of a material when stressed. It is defined as the amount of Energy per Volume that a material can absorb before rupturing.

Hardness:-

"Hardness is the measure of how resistant Solid matter is to Various kinds of Permanent Shape change when a force is applied".

Strength:- ~~"Strength is a measure of the Extent of a material's elastic range, or Elastic and plastic ranges together".~~

The ability to withstand an applied stress without failure.

So hardness is about Permanent change,

~~Strength is about Elastic (non-Permanent) change;~~ and

toughness is about Energy (rather than force).

UNIT – II

SHEAR FORCES AND BENDING MOMENTS

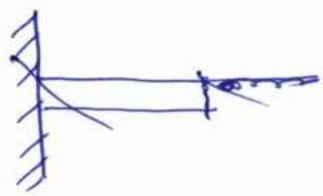
Definitions:-

Beam: A Beam is a structural member subjected to a system of External forces at right angles to its axis. (transverse loads).

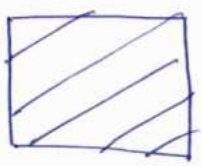
Types of supports:-

1) Fixed Support:-

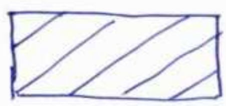
An Fixed support there is no displacement



Beams may be of the following Type Section

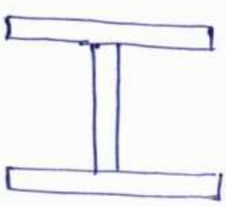


Square section

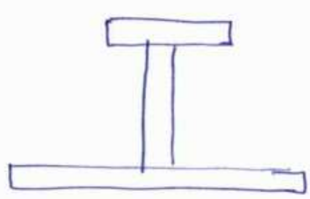


Rectangular section

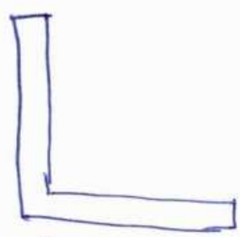
Circular Section



Equal I-Section

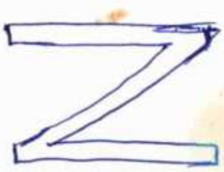


Unequal I-Section



L-Section

T-Section



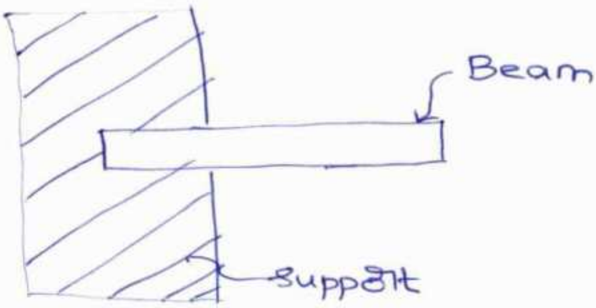
Z-Section

The type of Section depends upon the requirement of strength, stability etc.

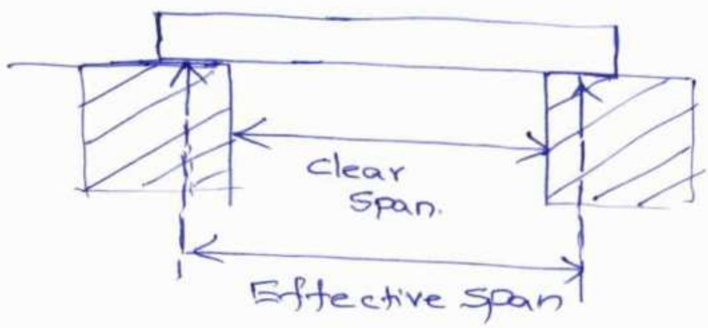
Classification of Beams:-

Depending upon the type of supports beams are classified as follows.

1. Cantilever: A cantilever is a beam whose one end is fixed (or built-in) and the other end is free.

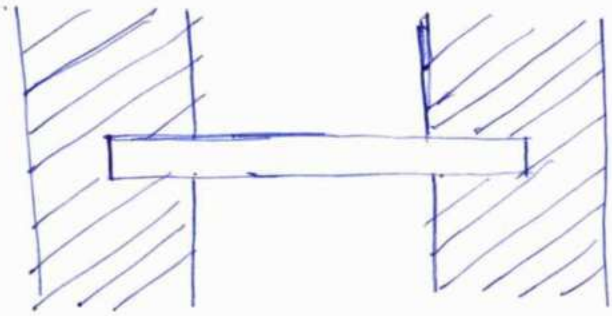


2. Simply supported Beam:- If the ends of a Beam are made to freely rest on supports the beam is called a freely or simply supported beam.

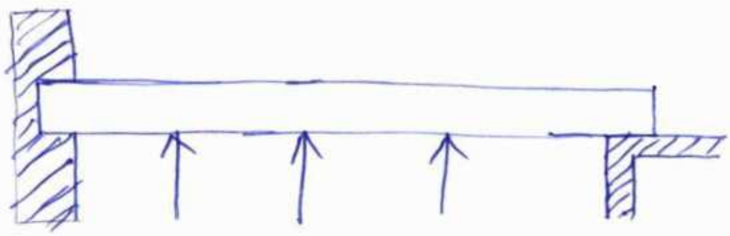


The clear horizontal distance between the walls is called the clear span of the beam. The horizontal distance between the centres of the end bearings is called the Effective span of the beam

3) Fixed Beam:- If the Beam is fixed at both its ends, it is called a built-in, encastered or fixed Beam.

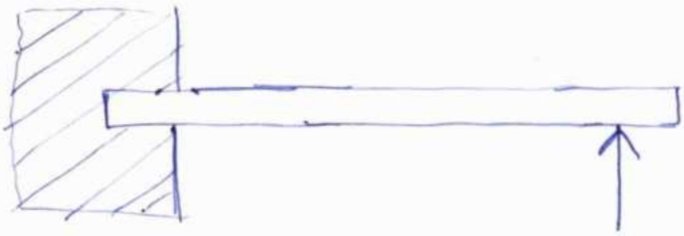


4. Continuous Beam:- A Beam which is provided with more than two supports is called a continuous Beam.



5. overhanging Beam:- If the Beam is Extended beyond the supports, it is called a overhanging Beams.

6. Propped Cantilever Beam:- When a support is provided at some suitable point of a cantilever beam in order to resist the deflection of the beam, it is known as Propped Cantilever beam.

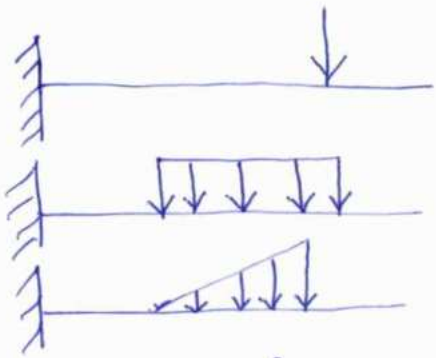


Types of loads:- Beams are commonly loaded with the following three types of loads.

i) Concentrated or point load

ii) Uniformly distributed load

iii) uniformly varying load.



i) Concentrated or point Load:- A concentrated load (also called a point load) is a load applied over a very small area and is regarded as a single load.

ii) Uniformly distributed load:-

A distributed load is a load which is spread on some length of a beam.

A distributed load is expressed by its intensity (Ex: Newton/metre).

If the intensity of the distributed load is constant the load is called uniformly distributed load.

iii) Uniformly Varying Load:- A Varying Load has an intensity that Varies according to some law along the length of the beam.

The Beams Cantilever, simply supported beam and overhanging beams are known as Statically Determinate Beams as the reactions of these beam at their supports can be determined by the use of equations of static equilibrium and the reactions are independent of the deformation of Beams.

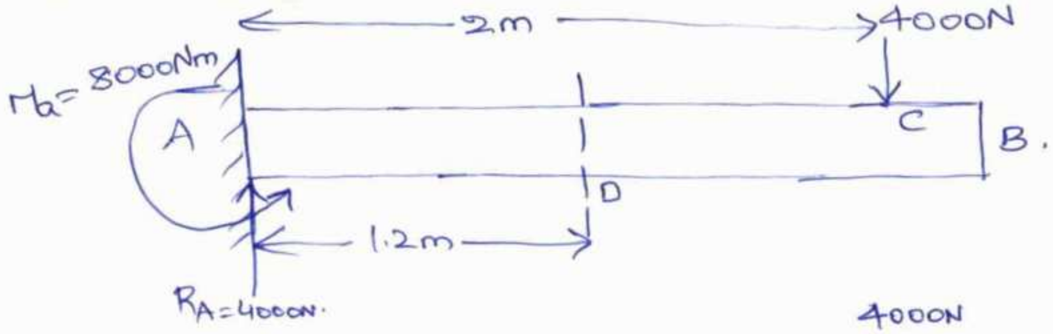
Fixed Beams and continuous Beams are known as statically Indeterminate beams as their reactions at supports cannot be determined by the use of equations of static equilibrium.

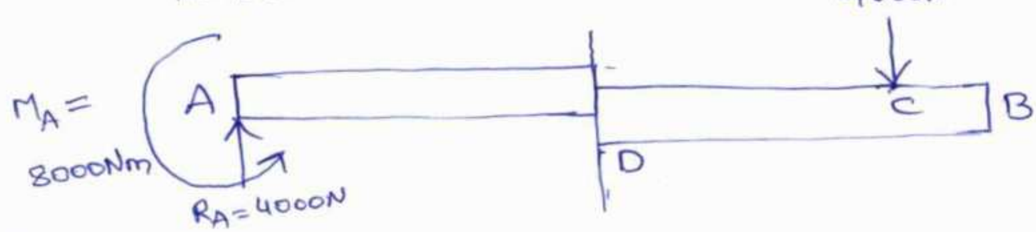
Conception of Shear force and Bending Moment:-

Fig shows a Cantilever AB whose end A is fixed. Let the cantilever carry a vertical load of 4000N at C. For the equilibrium of the Cantilever the fixed support at A will provide a vertical reaction vertically upwards of magnitude $R_A = 4000N$.

Taking moments about A, we have a clockwise moment of $4000 \times 2 = 8000 Nm$.

Hence for the equilibrium of the Cantilever the fixed support at A must also provide a reacting moment of 8000Nm of an Anticlockwise order.





SHEAR FORCE:

Now consider a Section D. At this Section there is a possibility of failure by shear as shown in fig.

If such a failure will occur at Section D, the cantilever is liable to be sheared off into two parts.

It is clear that the force acting normal to the centre line of the member on each part equals F$\cancel{S}$= 4000 N.

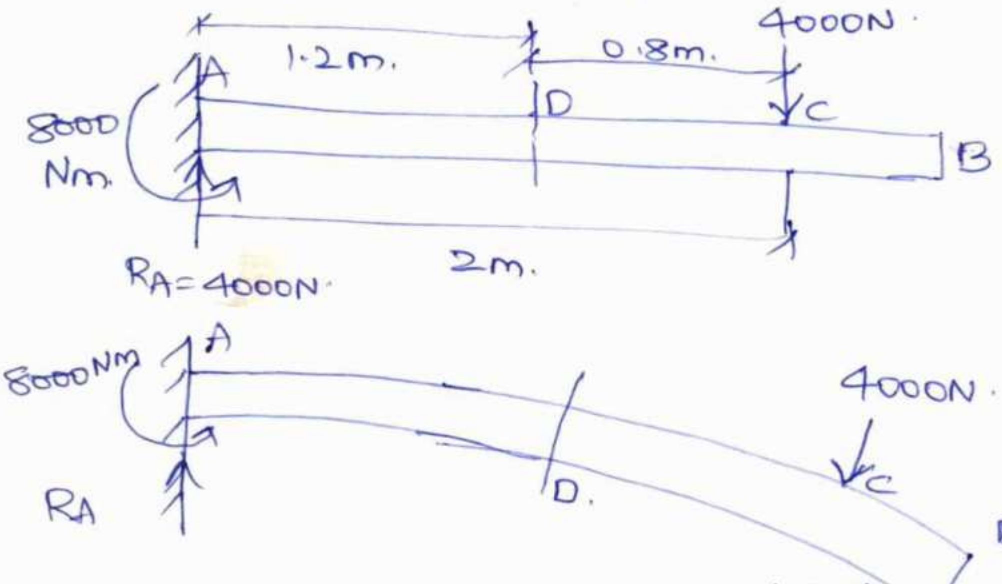
The force acting on the right part of the Section D is downward. The resultant force acting on the left part is upward.

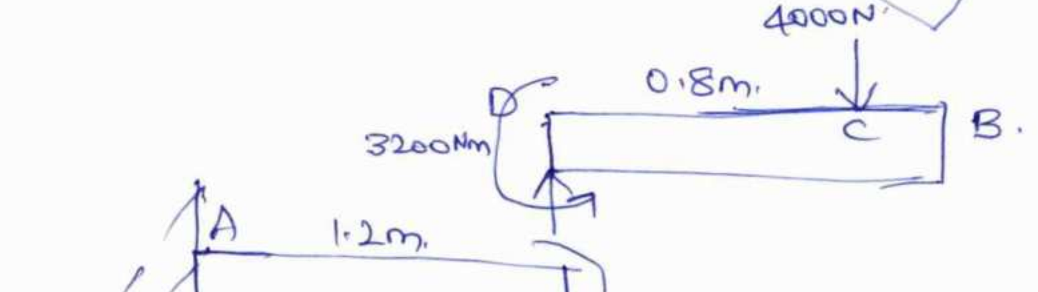
"The resultant force acting on the left part is upward. any one of the parts normal to the axis of the member is called the shear force at the Section D."

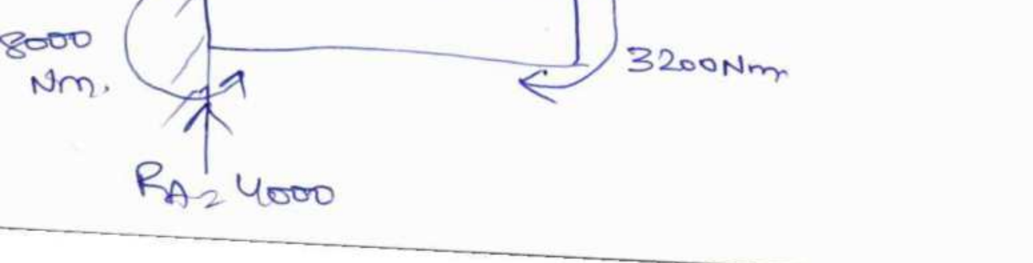
Bending Moment:-

For the Equilibrium of the cantilever, the fixed support at A will provide a reacting or resisting anticlockwise moment of 8000Nm. If the support A is not able to provide such a resisting moment the cantilever will not be in equilibrium and will therefore rotate about A in the clockwise order.

The magnitude of the reacting moment at A depends on (i) the magnitude of the load and (ii) the position of the load.







Now consider for instance, the section D.

Suppose the part DB was free to rotate about D, obviously the load on the part DB would cause the part DB to rotate in a clockwise order about D.

Considering the part DB taking moments about D, we find there is a clockwise moment of $4000 \times 0.8 = 3200\,Nm$ about D. Hence for the equilibrium of the part DB it is necessary that the part DA of the cantilever should provide a reacting or restoring anti-clockwise moment of 3200Nm about D.

Let us now discuss the equilibrium of the part AD. Taking moments about D, we have following moments about D.

(i) $R_A \times 1.2 = 4000 \times 1.2 = 4800\,Nm$. (clockwise)

(ii) Couple $= 8000\,Nm$ (A.C.W).

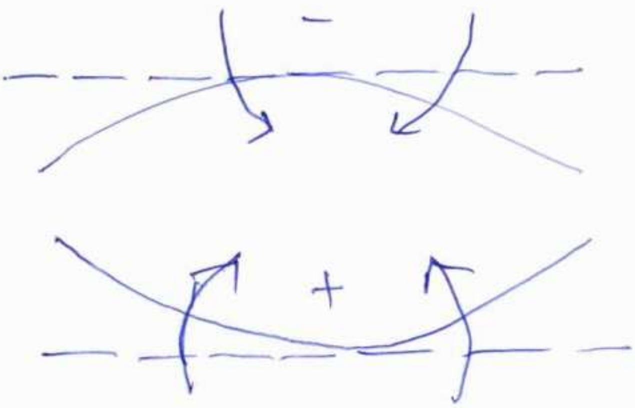
$\therefore$ Net moment about $D = 8000 - 4800 = 3200\,Nm$ (A.C.W)

Hence for the equilibrium of the part AD, the part DB should provide a clockwise moment of 3200Nm.

Hence, we find that at the section D, the part DB provides a C.W moment of 3200Nm and the part DA provides an A.C.W moment of 3200Nm.

we say at the section D there is a bending moment of 3200Nm.

The Bending moment at the section D is the algebraic sum of the moment of forces and reactions acting on one side of the section about the section.



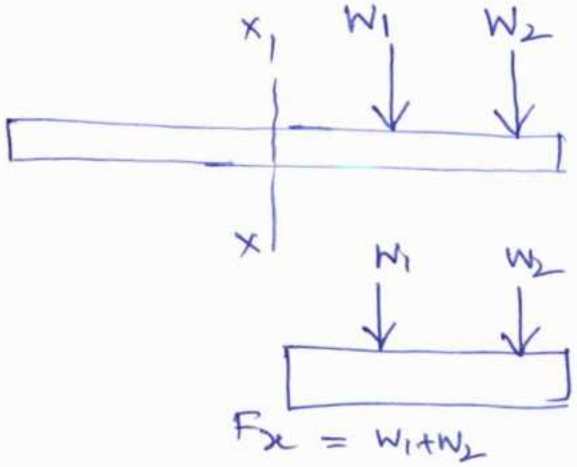
Effect of hogging B.M.

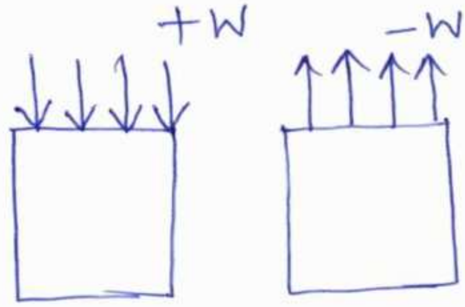
Effect of sagging B.M.

SHEAR FORCE:- "Shear force at a Section in a beam is the force that is trying to shear off the section and is obtained as the algebraic sum of all the forces including the reactions acting normal to the axis of the beam either to the left or to the right of the section".

Bending Moment:- Bending Moment at a Section in a beam is the moment that is trying to bend it and is obtained as the algebraic sum of the moments about the section of all the forces (including the reactions) acting on the beam either to the left or to the right of the section".

Sign conventions:-



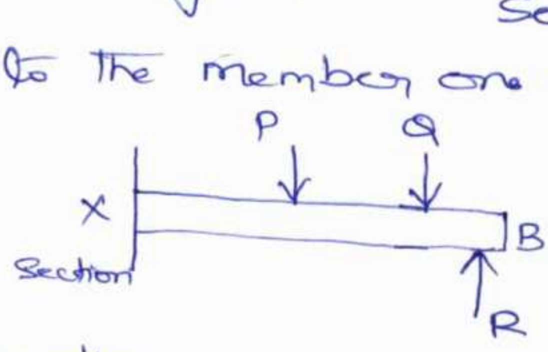


1) ~~If the for~~ Some Important Hints to be noted

Shear force:-

(i) Consider the left or the right part of the Section.

(ii) Add the forces normal to the member on one of the parts.



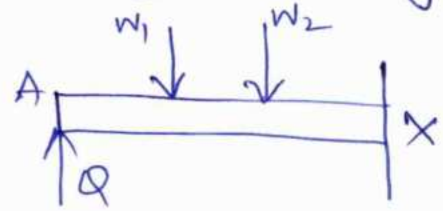
If the right part of the section is chosen, a force on the right part acting downwards is +ve while a force on the right part acting upwards is negative.

For instance, if the SF at a section X of a beam is required and if the right part XB be considered the forces P and Q are +ve while The force R is -ve.

$\therefore$ S.F at $X = P + Q - R$

If the left part of the section be chosen, a force on the left part acting upwards is +ve and a force on the left part acting downwards is negative. For instance, if the S.F at X of a beam is required and if XA is the left part, force Q is +ve while the forces W_1 and W_2 are negative.

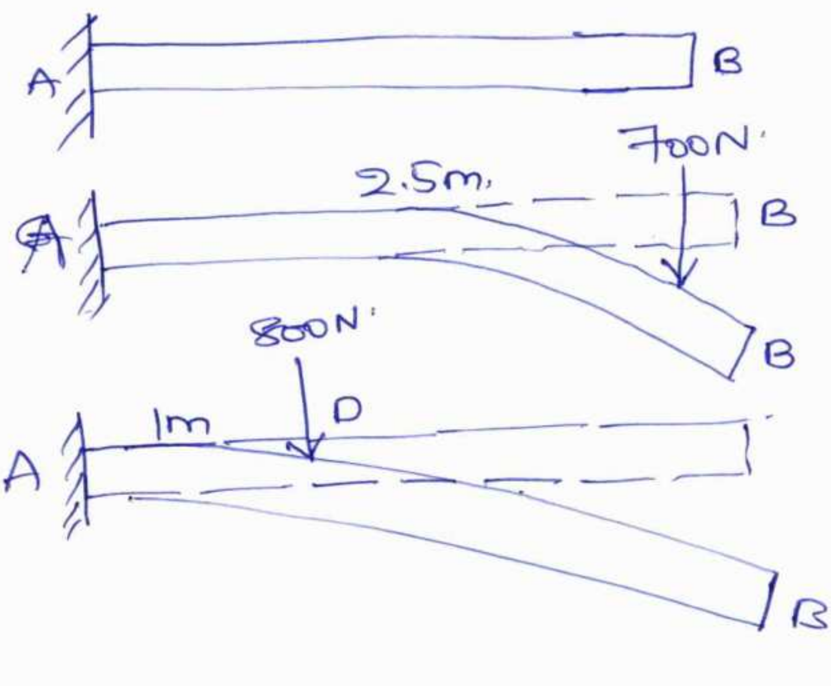
S.F at $X = Q - W_1 - W_2$



Bending Moment:-

1) consider the left or right part of the section

2) Remove all restriants and all forces on the part selected.



(iii) Now Introduce each force or reacting element one at a time and find its effect at the section Treat sagging moments as positive and hogging moments as negative.

Note that the moment due to every downward force is negative and the moment due to every upward force is positive.

→ If the right part of the section be selected

→ Remove the restraints on the part GB.

→ Introduce the load of 700N at E.

→ The independent effect of the load is to produce a hogging moment of -700×2.5
$= -1750 \, Nm$

Now consider the independent effect of the 800N load, at D.

obviously this will also produce a hogging moment of $-800 \times 1 = -800 \, Nm$.

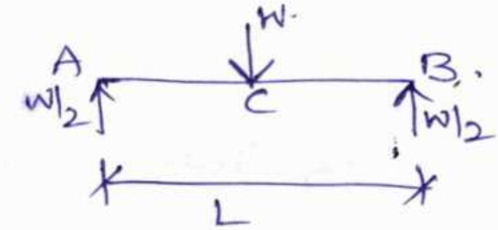
∴ Resultant B.M at $G = -1750 - 800 = -2550 \, Nm$ (hogging)

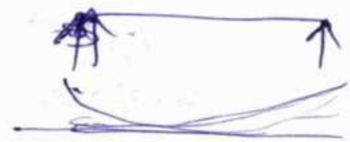
Shear Force Diagram:- A shear force for a structural member is a diagram [illegible] the values of shear forces at various sections of the member.

Bending Moment diagram:- A Bending Moment diagram for a member is a diagram which shows the values of Bending Moment at various sections of the member.

B) Beams freely supported at the two ends:
(Simply Supported Beams):

1) Simply supported beam of span l carrying a concentrated load at mid span.



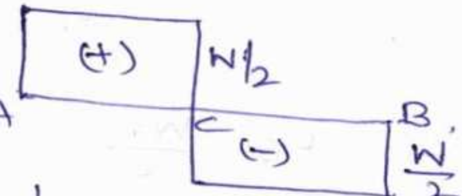


~~S.F.B at~~ Reaction at each support $= \frac{W}{2}$

$$R_A = R_B = \frac{W}{2}$$

for any section between A and C

$$S.F = S_x = \frac{W}{2}$$



$\frac{w}{2}x -$
$w(\ldots)$

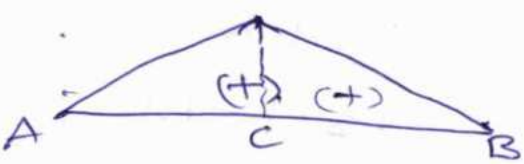
C and B.

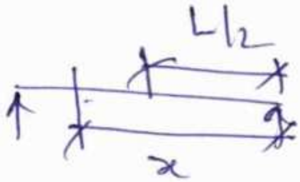
$$M_x = \frac{W}{2}x = \frac{W}{2}\frac{L}{2} = \frac{WL}{4} \quad \text{at } x = \frac{L}{2}$$

$\frac{WL}{2} + \frac{WL}{2}$

~~$M_x = \frac{W}{2}x$~~

~~at $x = L$~~





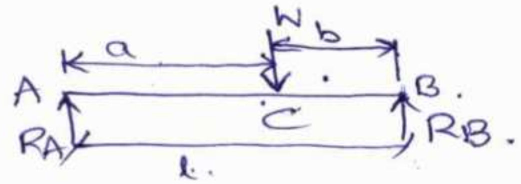
A and C

$(x - \frac{L}{2})$

$$M_x = \left[\frac{W}{2}(x) - W\left(\frac{L}{2}\right)\right] \text{ (or) } \left[\frac{W}{2}x - W\left(x - \frac{L}{2}\right)\right]$$

at $x = L$ $\quad M = \frac{WL}{2} - \frac{WL}{2} = 0.$

(ii) Simply supported beam carrying a concentrated load placed eccentrically on the Span.



S.F at a section X from B ~~at~~ at a dist of x.

~~consider~~ For ~~S.F~~ the static equilibrium of the beam Take the moments about any point equal to zero.

~~MRA~~ $M_A = 0$

$R_B \times L - Wa = 0$

$R_B = \frac{Wa}{L}$

$R_A + R_B = W.$

$R_A = W - R_B$

$= W - \frac{Wa}{L}$

$= W\left(\frac{L-a}{L}\right)$

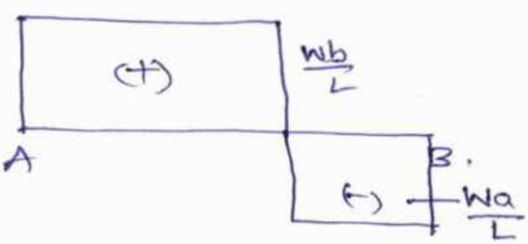
$R_A = \frac{Wb}{L}$

$\therefore R_A = \frac{Wb}{L} \quad R_B = \frac{Wa}{L}$

~~S.F at B~~ $S_x = -R_B. = -\frac{Wa}{L}$

at $C = -R_B + W = -\frac{Wa}{L} + W = \frac{Wb}{L}$

at $A = -R_B + W - R_A = 0$



M_x at ~~x~~ a dist x from ~~a~~ 'B'

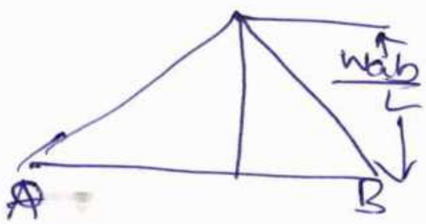
$M_C = R_B \times b$

$M_A = R_B \times L - W \times a$

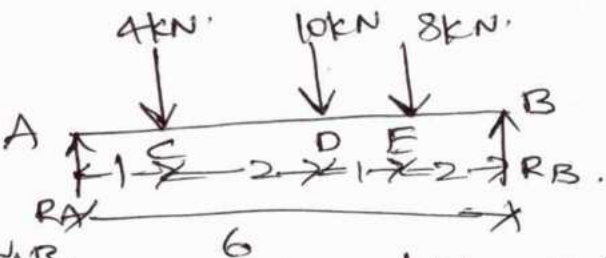
$= \frac{Wa}{L} \times L - Wa$

$M_A = 0$ $\qquad M_D = \frac{Wa}{L} \times b = \frac{Wab}{L}$

$M_B = 0$



(iii) simply supported beam: Several point loads.



Let us consider section E and B.
At any section bet E and B at a dist of x from B.

$M_x = R_B \times x$.

Taking Moments about A is equal to zero

$M_A = 0$. $\qquad R_A + R_B = 22$

$R_B \times 6 - 8 \times 4 - 10 \times 3 - 4 \times 1 = 0$. $\qquad R_A = 22 - 11$

$R_B = \frac{32 + 30 + 4}{6} = \frac{66}{6} = 11$ $\qquad R_A = 11$

$R_B = 11 kN$.

$M_x = \cancel{R_B \times x} \; 11x$

At $x = 2$

$M_x = 22$ at E

At any section bet D and E at a dist of x from B.

$M_D = R_B \times x - F_E \times (x-2)$

at $x = 3$

$= 11 \times 3 - 8 \times 1$

$= 33 - 8$

$M_D = 25$

At any section bet C and D at a dist of x from B.

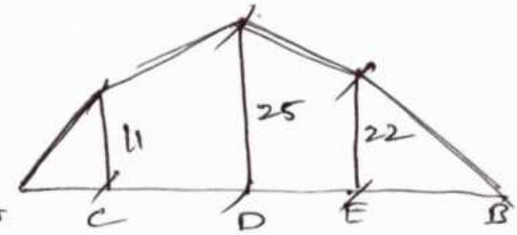
$M_C = R_B \times x - F_E \times (x-2) - F_D\,(x-3)$,

at $x = 5$

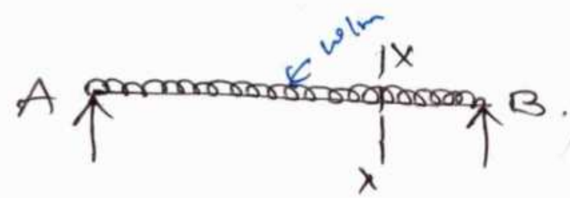
$= 11 \times 5 - 8 \times (3) - 10 \times 2$

$= 55 - 24 - 20$

$M_C = 11$



(iv) simply supported beam: UDL over the whole span.



$R_A = R_B = \frac{wL}{2}$

S.F.D $F_x = -R_B + wx$

At $x=0$

$F_x = -R_B$

$F_B = -\frac{WL}{2}$

At $x = \frac{L}{2}$

$F = -R_B + \frac{WL}{2}$

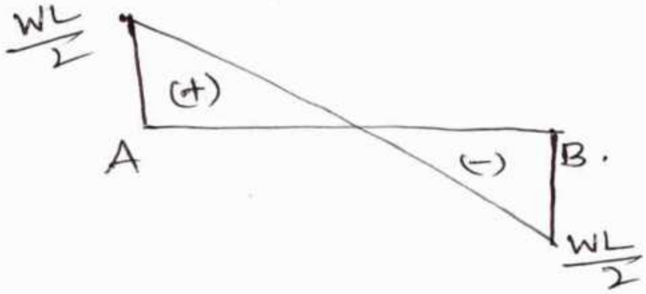
$F = 0$

At $x = L$

$F_A = -R_B + WL$

$= -\frac{WL}{2} + WL$

$= \frac{WL}{2}$



B.M.D?

at any

$M_a = +\frac{WL}{2} \times x - wx \times \frac{x}{2}$

at $x=0$

$M_A = 0.$

at $x = \frac{L}{2}$

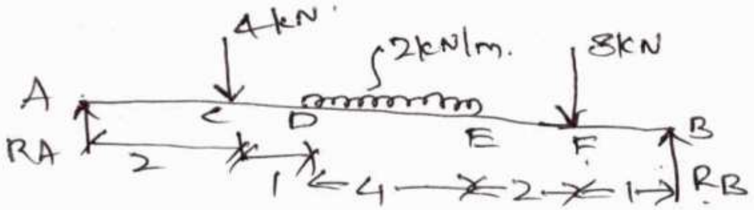
$M = \frac{WL^2}{4} - \frac{WL^2}{8}$

$M = \frac{WL^2}{8}$

at $x = L$

$M = \frac{WL^2}{2} - \frac{WL^2}{2} = 0.$

(v) Simply supported beam: combination of loads:



At ~~any~~ section bet F and B at a dist of x from B.

S.F $F_x = -R_B$

$\Sigma M_A = 0.$

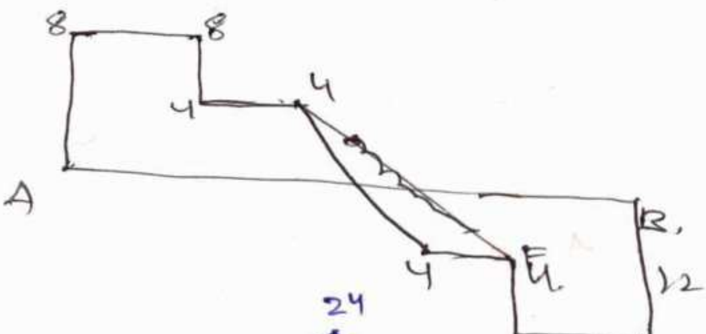
$R_B \times 10 - F_F \times 9 - F_{DE} \times 5 - F_C \times 2 = 0$

$R_B = \frac{72 + 8\times5 + 8}{10} = \frac{72+40+8}{10}$

$= \frac{120}{10} = 12$

$R_A + R_B = 14$

$R_A = 14 - 12 = 2$



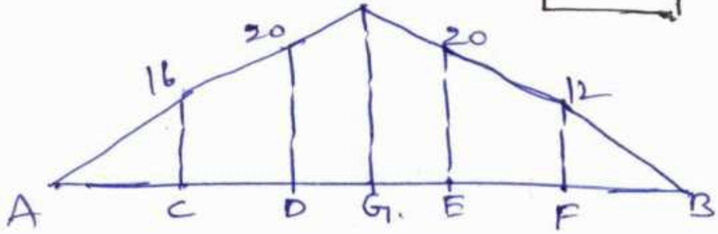
S.F @ $F_B = -12$

S.F @ $F_F = -12$

S.F at $E = -12 + 8 = -4.$

S.F at $D = -4 + 8 = 4.$

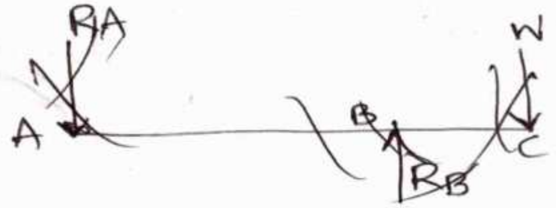
S.F at $A = 4 + 4 = 8$

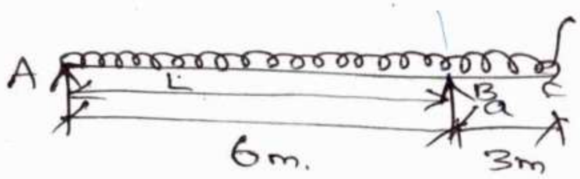


9/9
25/08
6:24,3829

(B) simply supported beam with one side overhanging!

~~point load~~ UDL





$\Sigma M_A = 0$

$$R_B \times L = W \times \frac{(L+a)^2}{2}$$

$$R_B = \frac{w(L+a)^2}{2L} \uparrow$$

Hence

$$R_A = w(L+a) - \frac{w(L+a)^2}{2L}$$

$$= w(L+a)\left[1 - \frac{L+a}{2L}\right]$$

$$R_A = w(L+a)\left[\frac{L-a}{2L}\right].$$

$\Sigma M_A = 0.$

$$\cancel{R_B \times 6 - (2 \times 9 \times 4.5) = 0}$$

$$\cancel{R_B = \frac{81}{6} = 1}$$

$$\therefore R_B = \frac{w(L+a)^2}{2L} \uparrow$$

$$R_A = \frac{w(L+a)(L-a)}{2L} \uparrow.$$

27/08-3
1, 2, 12, 13,
14, 15, 21, 30,
6.x 38, 39

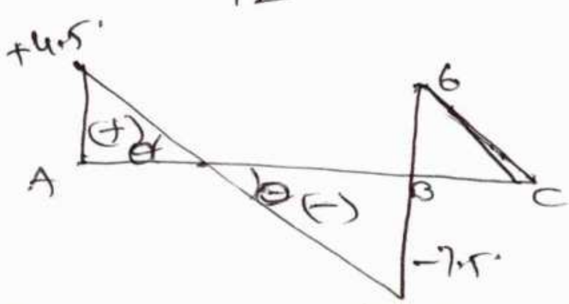
∴ If $L > a$, RA will act upwards.

If $L < a$, RA will act downwards.

If $L = a$ $RA = 0$

##Warranty##

Sol $R_A = \frac{2(9)(3)}{12} = 4.5 \uparrow$ $R_B = \frac{2(9)^2}{12} = \frac{2 \times 81}{12} = 13.5$



$6 - 13.5 = -7.5 + 12$
$= +4.5$

B.M:

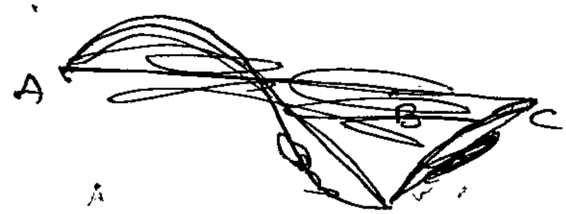
$M_A = 0$

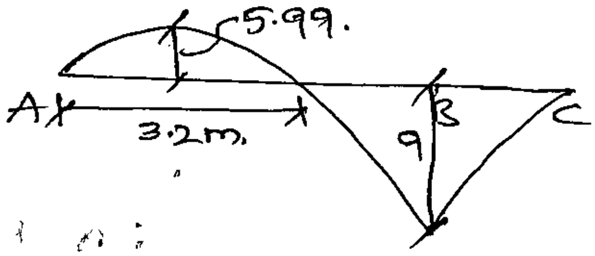
$M_B = -(6\times1.5) = -9$

$M_A = -(9\times2\times4.5) + (13.5\times6)$

$= -81 + 81$

$\therefore M_A = 0.$





$\frac{6.8}{L-x} = \frac{4.5}{x}$

$6.8x = 4.5L - 4.5x.$

$11.3x = 4.5\times6$

$x = 2.38.$

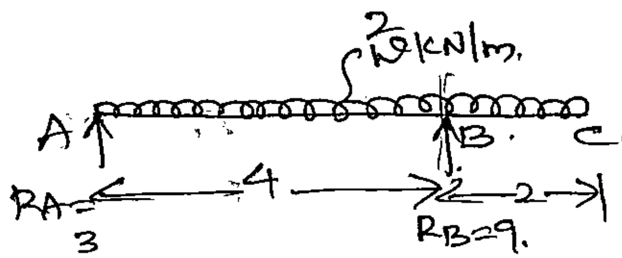
$M_x = R_B\times(6-2.38) - (2\times6.62\times3.31)$

$= 13.5\times3.69 - 43.82$

$M_x = +5.99$

Point of contraflexure (or) point of inflexion: It is the point where the B.M is zero after changing its sign from positive to Negative or vice-versa.

Overhanging beams:



$\Sigma M_A = 0$.

$R_B \times 4 - (2\times6\times3) = 0$

$R_B = \frac{36}{4} = 9.$

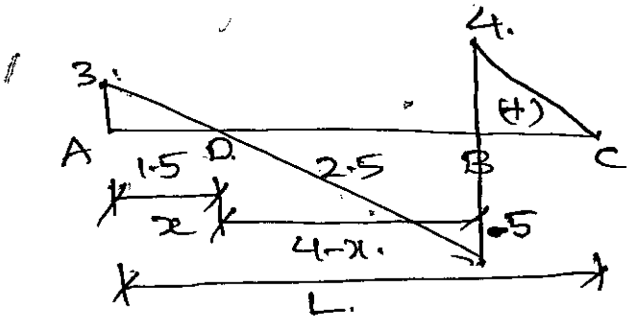
$R_A = 12 - 9 = 3.$

S.F at point C, $F_C = 0$

" " B, $F_B = (2\times2) - 9 = 4 - 9 = -5.$

" " A, $R_A = 12 - 9 = 3.$

$R_A = 3 - 3 = 0.$



$\frac{5}{4-x} = \frac{3}{x}$

$5x = 12 - 3x$

$8x = 12$

$x = \frac{12}{8} = 1.5$

B.M at point C, $M_C = 0$

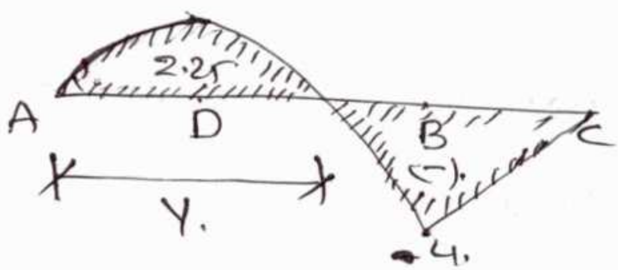
" " B, $M_B = -(2\times2\times1) = -4.$

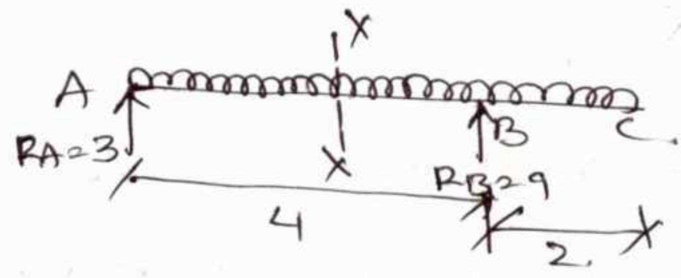
" " D, $M_D = -\left(\frac{2\times(4.5)^2}{2}\right) + (9\times2.5)$

$= -20.25 + 22.5$

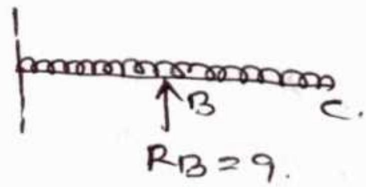
$M_D = 2.25$

$M_A = -(2\times6\times3) + (9\times4) = -36+36 = 0$





At any section bet A and B is given by



$M_x = 0$

$= -2 \times \frac{x^2}{2} + 9 \times (x-2)$

$\Rightarrow -x^2 + 9x - 18 = 0$

$x^2 - 9x + 18 = 0$

$x^2 - 6x - 3x + 18 = 0$

$x(x-6) - 3(x-6) = 0$

$x = \cancel{6}, 3$

$\therefore x = \cancel{6}\ 3$

$\therefore Y = 3.$

$-W(L+a) + R_C L + Wa = 0$

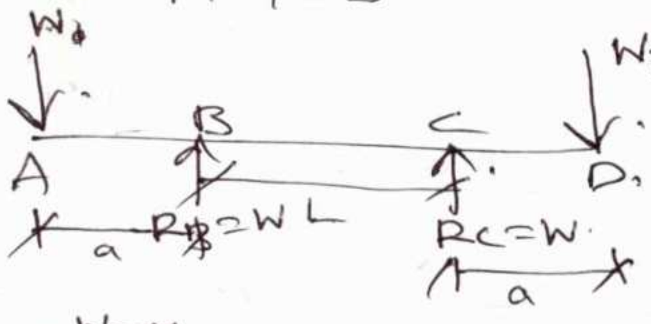
$R_C = \frac{W(L+a)}{L}$

$R_C L + Wa = W(L+a)$

$= WL$ +ve

$R_C = W.$

15)

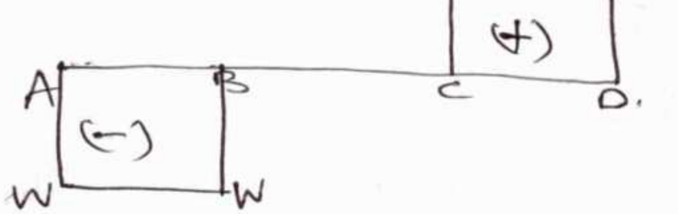


S.F at D $F_D = +W$

at C $F_C = W - W = 0.$

at B $F_B = 0 - W = -W.$

at A $F_A = -W + W = 0$



3.11 at D $M_D = 0$

at C $\quad M_C = -W \times a = -Wa$

at B $\quad M_B = -W(L+a) + WL$

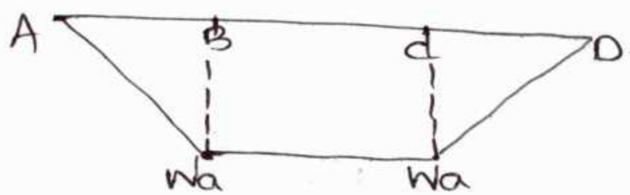
$M_B = -Wa$

at A $\quad M_A = -W(L+2a) + W(L+a) + Wa$

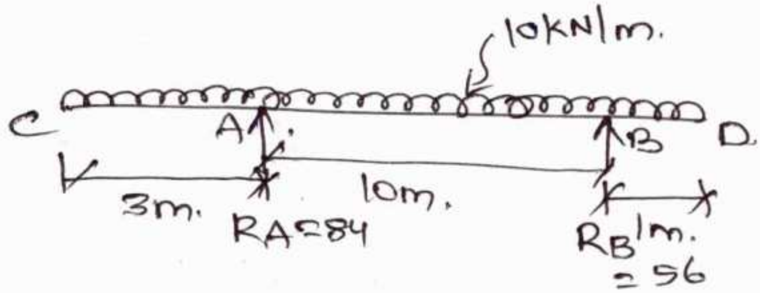
$= -\cancel{WL} - 2aW + \cancel{WL} + \cancel{Wa} + Wa$

$= 0$

Good 13/10/11 9, 26, 42, 43.



16).



$\Sigma V = 0$: $\quad R_A + R_B = 10 \times 14 = 140$

$\Sigma M_A = 0 \quad R_B \times 10 - (10 \times 14 \times (7-3)) = 0$

$R_B = 14 \times 4 = 56$

$R_B = 56$

$R_A = 84$.

SF at D : $F_D = 0$

at B: $F_B = 10 \times 1 - R_B$

$= 10 - R_B$

$= 10 - 56$

$F_B = -46$

at A $\quad F_A = -46 + 100$

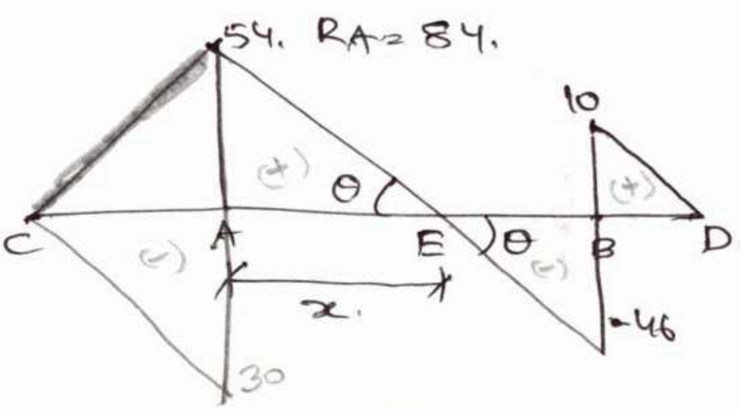
$= 54$.

$F_A = 54 - 84 = -30$

at C $\quad$ ~~$F_C = 54$~~

$F_C = -30 + 30$

$F_C = 0$



~~MB~~ B.M at D $M_D = 0$

at ~~B~~ $M_B = -10 \times 1 \times 0.5 = -5$ kNm.

at A $M_A = (-10 \times 11 \times 5.5) + (56 \times 10)$

$= -605 + 560$

$M_A = -45$

$M_C = (-10 \times 14 \times 7) + (56 \times 13) + (84 \times 3)$

$= -980 + 728 + 252$

$M_C = 0$

$\frac{54}{x} = \frac{46}{10-x}$

$54(10-x) = 46x$

$540 - 54x = 46x$

$540 = 100x$

$x = 5.4$

$M_E = -10 \times \frac{(1+4.6)^2}{2} + (56 \times 4.6)$

$= -156.8 + 257.6 = 100.8$ kNm.

$M_x = 0$

$= 10 \times \frac{(1+(10-x))^2}{2} - (56 \times (10-x)) = 0$

$\frac{10(11-x)^2}{2} = 56(10-x)$

$10(11-x)^2 = \cancel{56}\,112(10-x)$

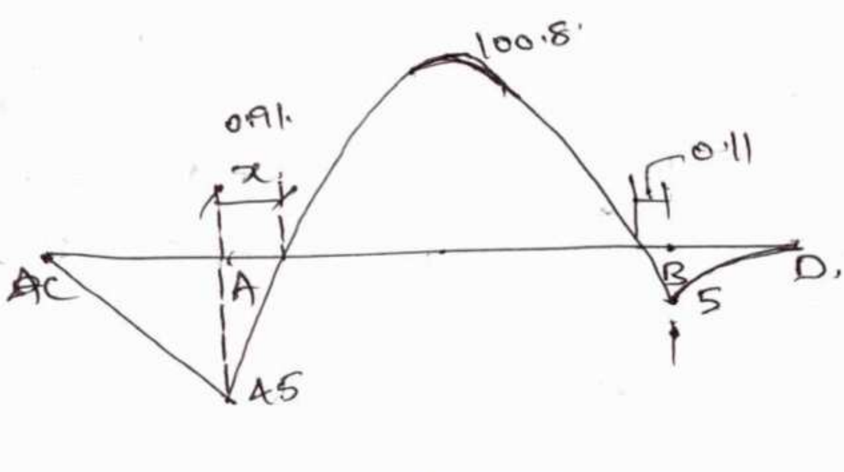
$(11-x)^2 = 11.2(10-x)$

$121 + x^2 - 22x = 112 - 11.2x$

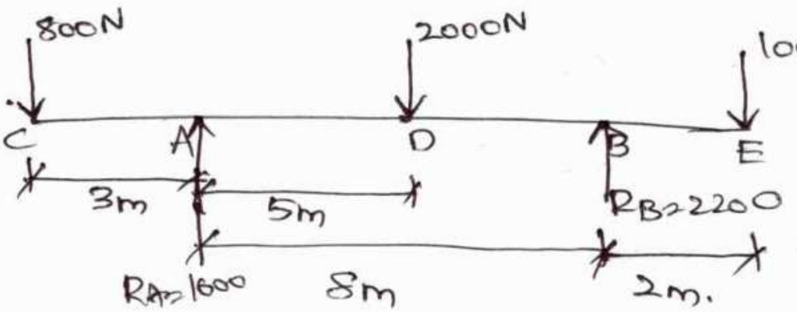
$x^2 - 10.8x + 9 = 0$

$x = 0.91$ m.

$x = 9.89$ m



17)



$\Sigma V = 0 \quad R_A + R_B = 800 + 2000 + 1000$

$= 3800$

$\Sigma M_A = 0$

$R_B \times 8 - 1000 \times 10 - 2000 \times 5 + (800 \times 3) = 0$

$R_B \times 8 + 2400 = 10000 + 10000$

$= 20000$

$= \frac{20000 - 2400}{8} = 2200$

$R_B = 2200$

$R_A = 1600$

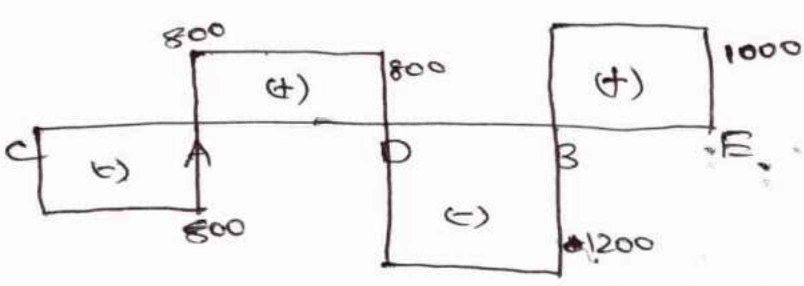
S.F at E $\quad F_E = 1000$

at B $\quad F_B = 1000 - 2200 = -1200$

at D $\quad F_D = 1000 - 2200 + 2000 = 800$

at A $\quad F_A = 800 - 1600 = -800$

at C $\quad F_C = -800 + 800 = 0.$



~~21, 26, 32~~,
43.

B.M at point E : $M_E = 0$

at " B : $M_B = -1000 \times 2 = -2000$

" D : $M_D = (-1000 \times 5) + (2200 \times 3)$

$= -5000 + 6600$

$M_D = 1600$

" A $M_A = (-1000 \times 10) + (2200 \times 8) - (2000 \times 5)$

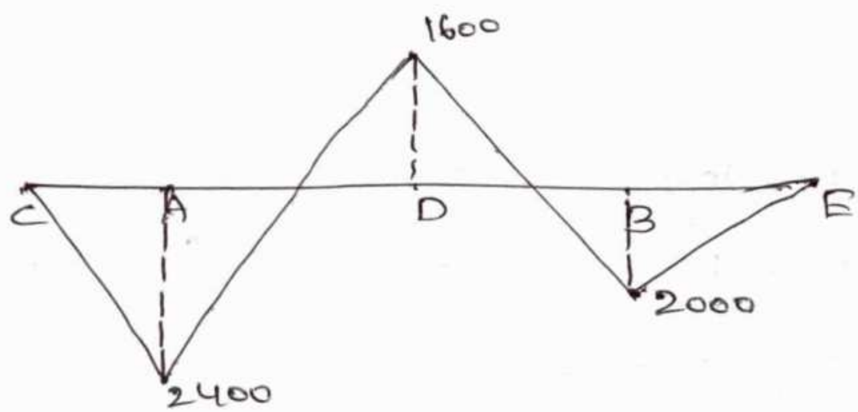
$= -10000 + 17600 - 10000$

$M_A = -2400$

" c $M_C = (-1000 \times 13) + (2200 \times 11) - (2000 \times 8) + (1600 \times 3)$

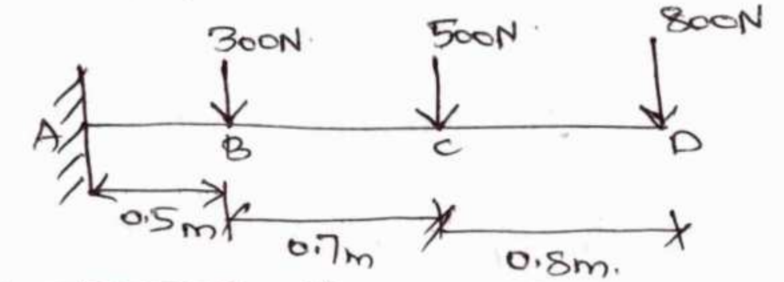
$= -13000 + 24200 - 16000 + 4800$

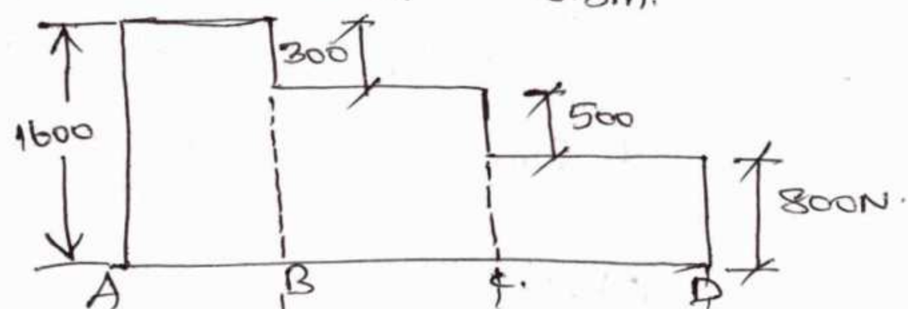
$M_C = 0$

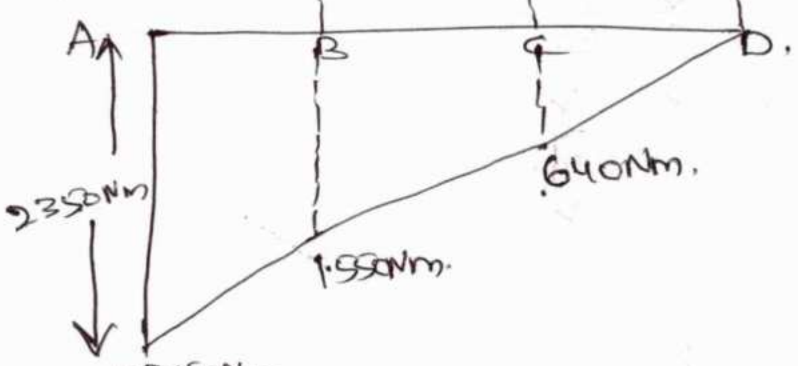


Problems:

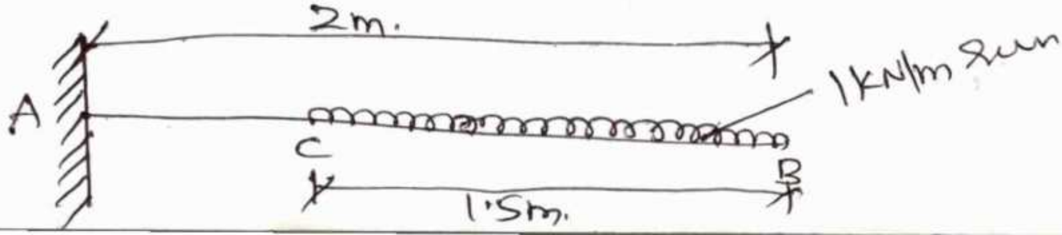
1) A cantilever beam of length 2m carries the point loads as shown in fig. Draw the S.F & B.M. diagrams for the cantilever beam.

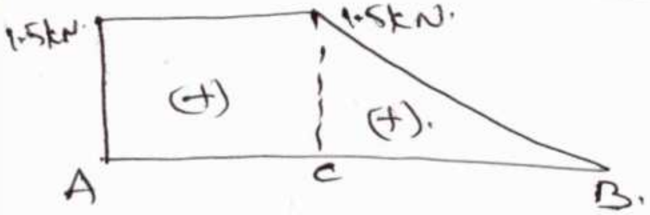


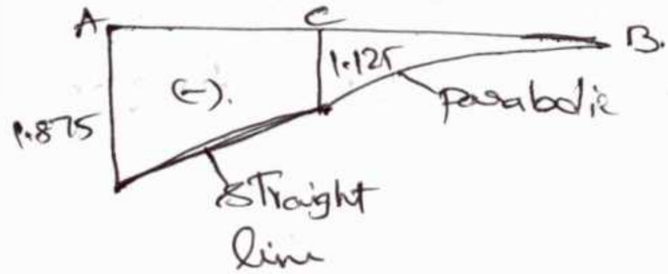




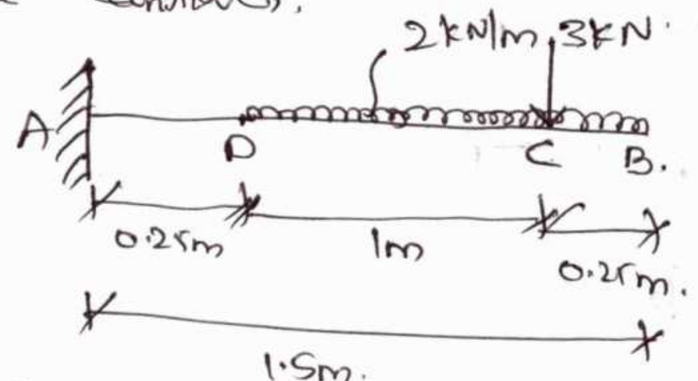
2). A cantilever of length 2m carries a UDL of 1kN/m run over a length of 1.5m from the free end. Draw the S.F & B.M diagram for the cantilever.

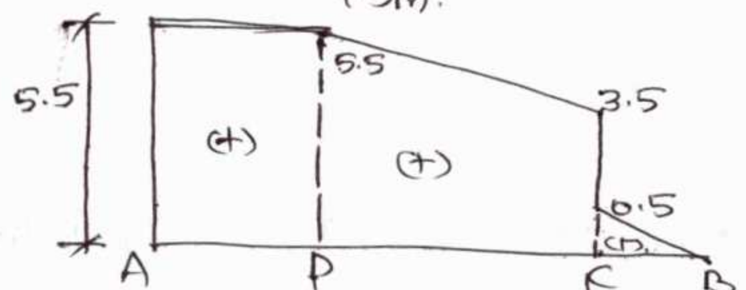


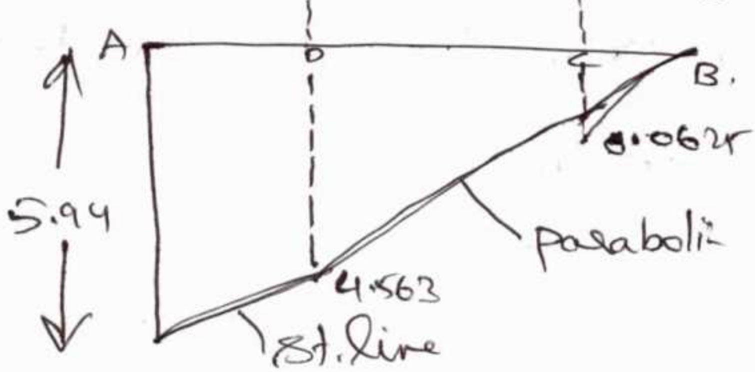




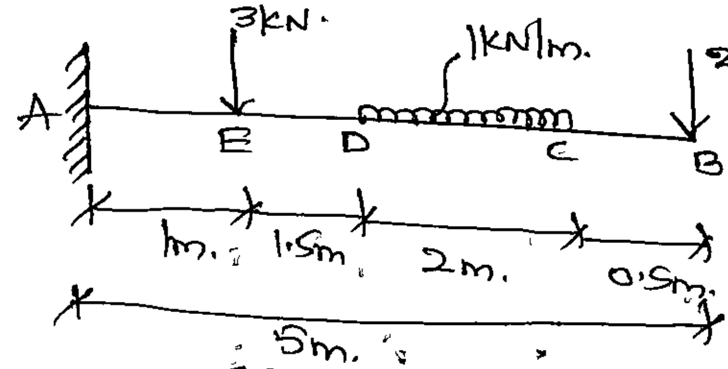
3) A cantilever 1.5m long is loaded with a UDL of 2kN/m run over a length of 1.25m from the free end. It also carries a point load of 3kN at a dist of 0.25m from the free end. Draw the S.F & B.M. diagrams for the cantilever.

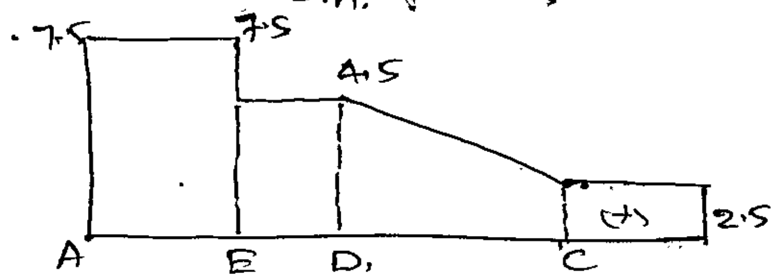


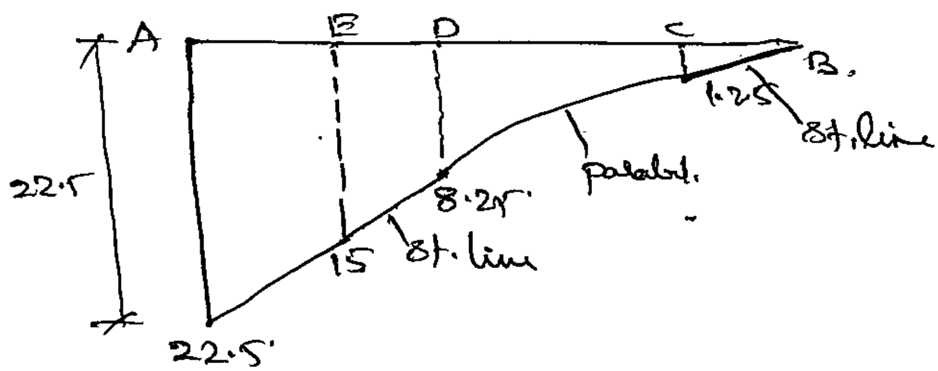




4) A cantilever of length 5m is loaded as shown in fig. Draw the S.F and B.M. diagrams for the cantilever.

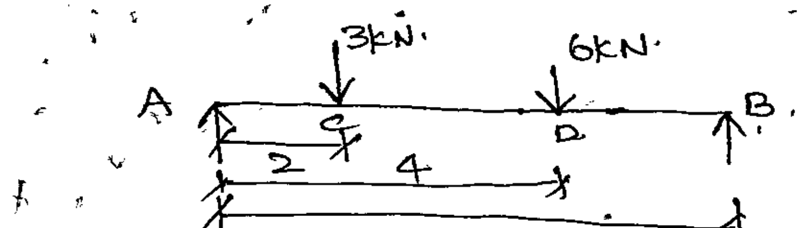


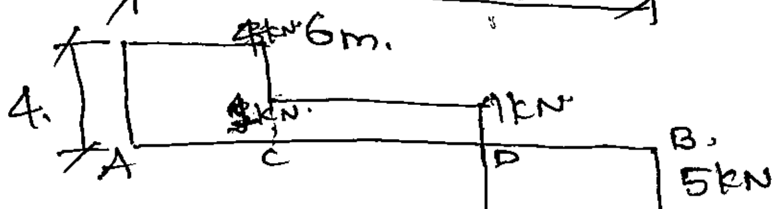


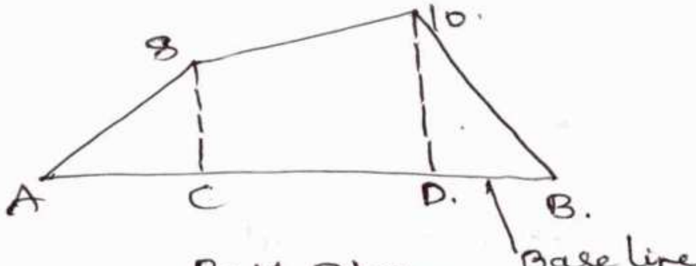


S.S.B:

5) A Simply supported beam of length 6m, carries point load of 3kN and 6kN at dist of 2m and 4m from the left end. Draw the S.F & B.M diagrams for the beam.

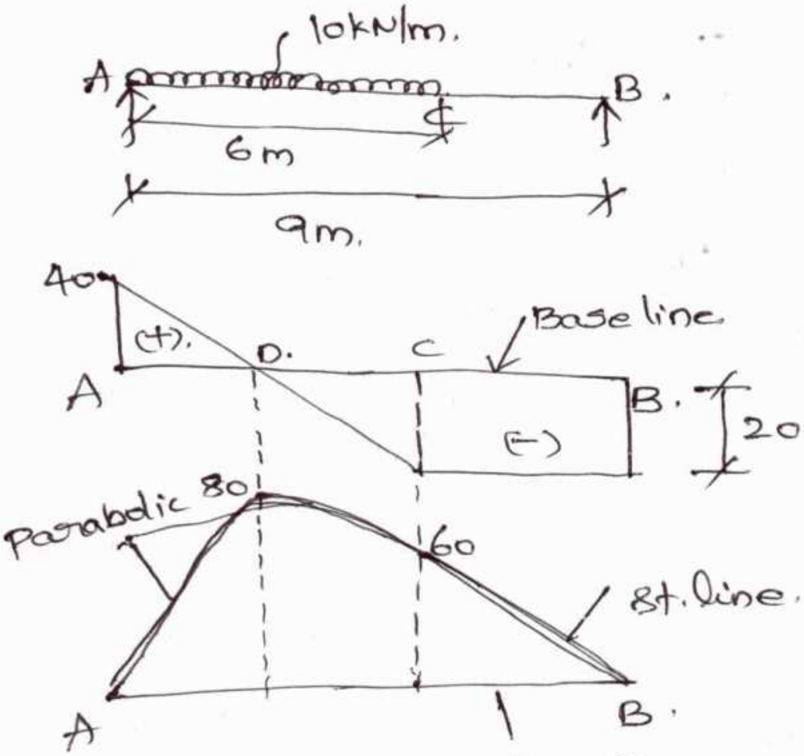




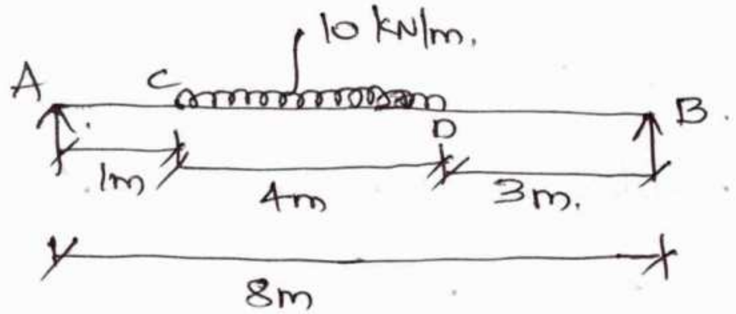


B.M. Diagram.

6). Draw the S.F & B.M diagram for a simply supported beam of length 9m and carrying a UDL of 10kN/m for a distance of 6m from the left end. Also calculate the Max B.M on the section.



7). Draw the S.F and B.M diagrams for a simply supported beam of length 8m and carrying a UDL of 10kN/m for a dist of 4m as shown in fig.



$$\frac{15}{4-x} = \frac{25}{x}$$

$$15x = 100 - 25x$$

$$40x = 100$$

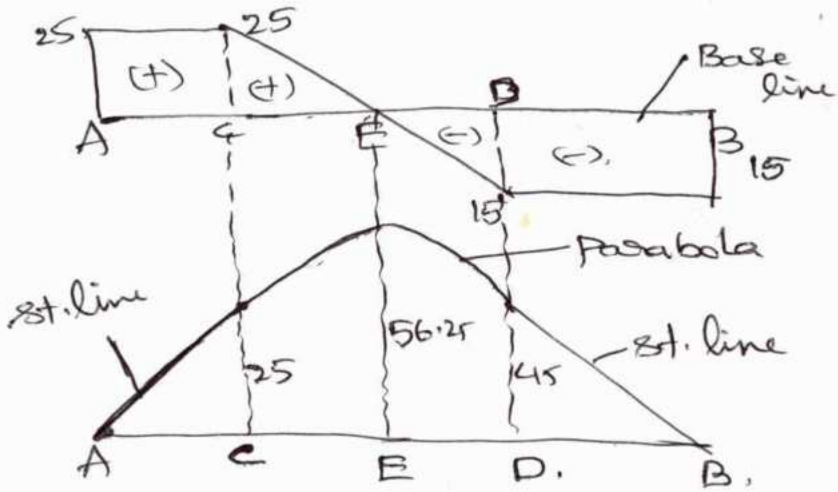
$$x = \frac{100}{40} = 2.5$$

$4-x$

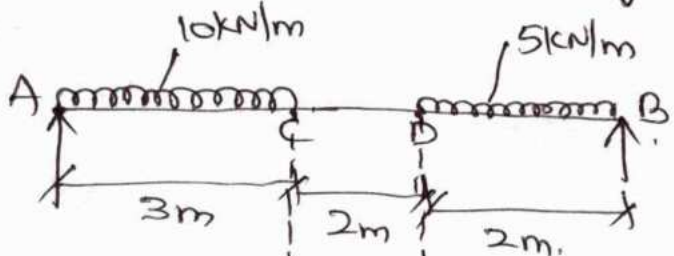
$$15 \times 4.5 - \left(15 \times 2.5 \times \frac{2.5}{2}\right)$$

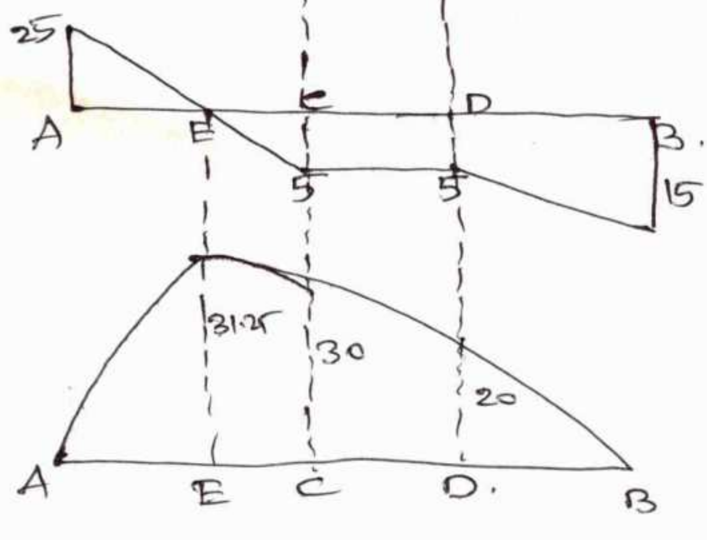
$$= 67.5 - 22.5 \times 5$$

$$= 62.5$$

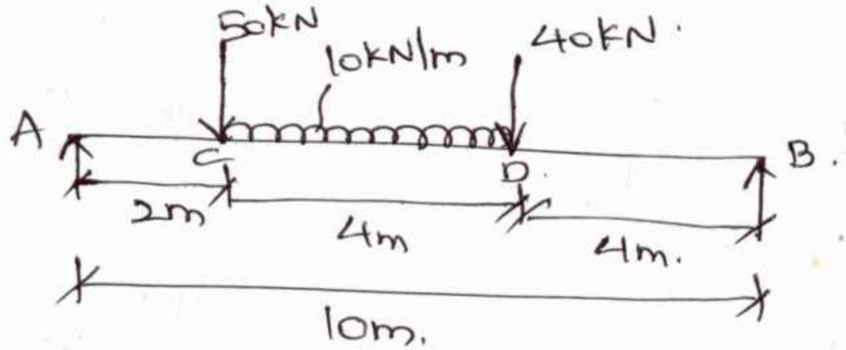


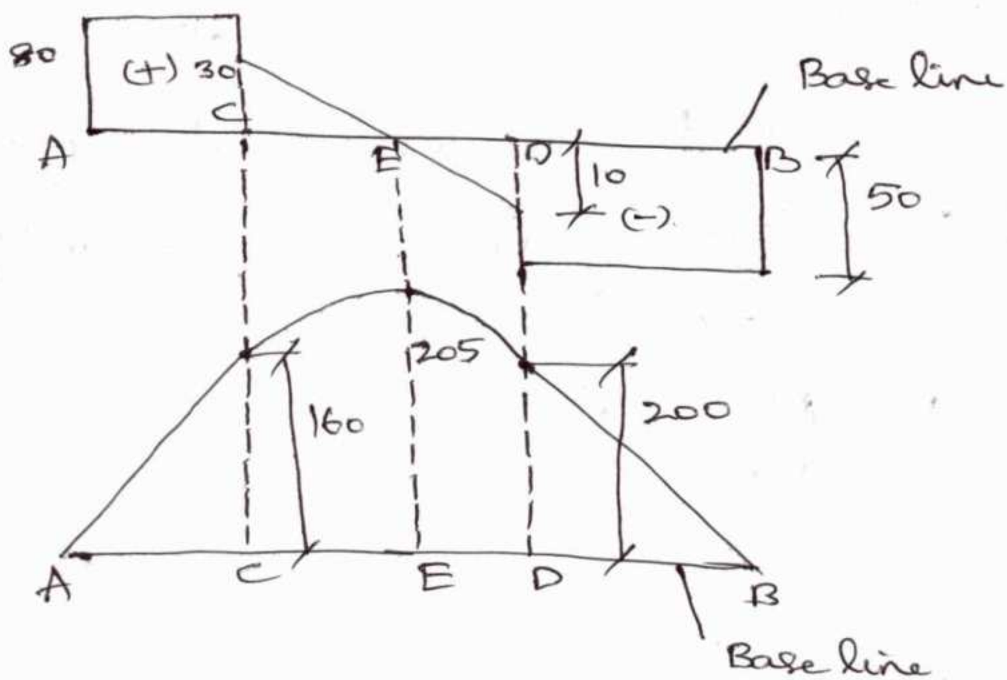
8) Draw the S.F and B.M diagrams of a simply supported beam of length 7m carrying UDL as shown in fig.





9). A simply supported beam of length 10m, carries the UDL and two point loads as shown in fig. Draw the S.F & B.M diagram for the beam. Also calculate the B.M.





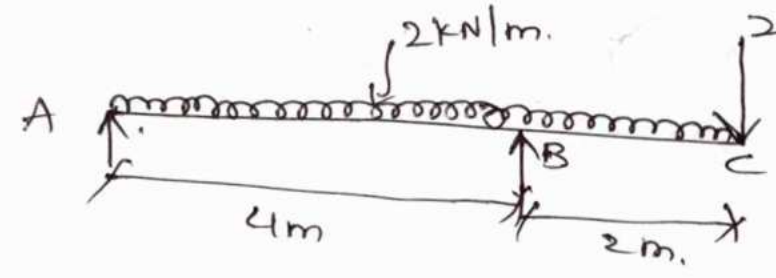
$\frac{30}{x} = \frac{10}{4-x}$

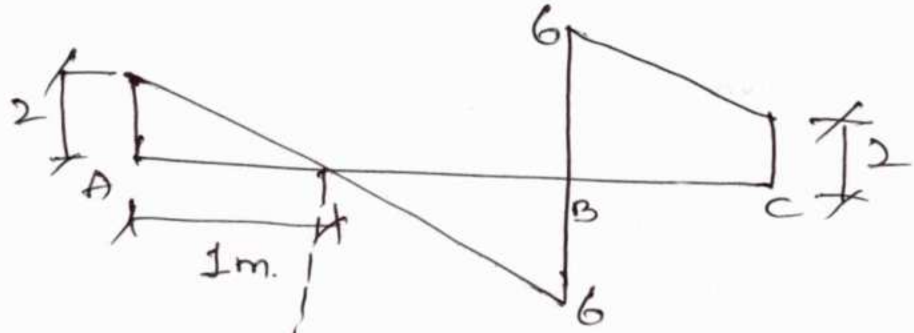
$120 - 30x = 10x$

$120 = 40x$

$x = 3$

10) Draw the S.F & B.M diagrams for the following overhanging beam: 13



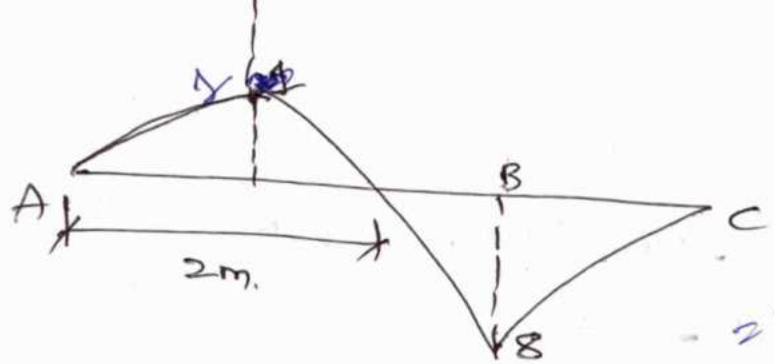


$\frac{2}{x} = \frac{6}{4-x}$

$8 - 2x = 6x$

$8x = 8$

$x = 1.$



$= -(2\times5).$

$(2\times5) + (12\times3) - (2\times25)$

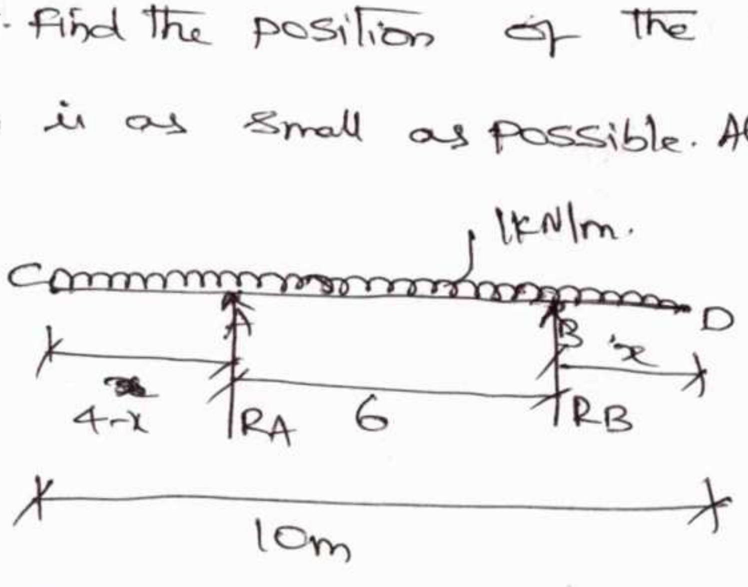
$= -10 + 36 - (25)$

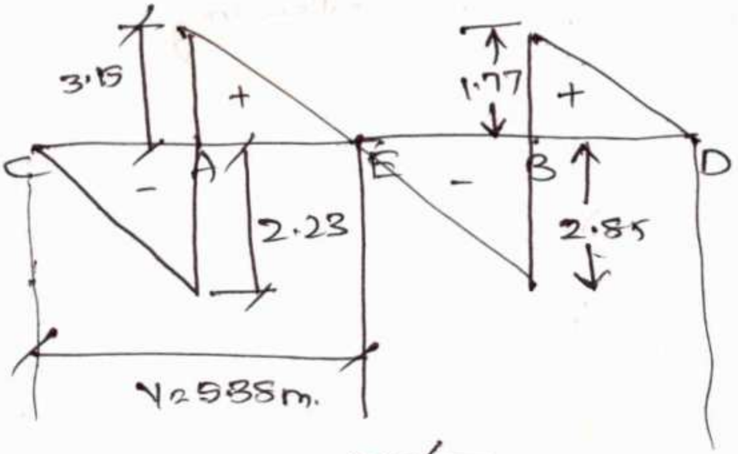
$= 26 - (25)$

$= 1.$

11). A horizontal beam 10m long is carrying a UDL of 1KN/m. The beam is supported on two supports 6m apart. Find the position of the supports, so that B.M on the beam is as small as possible. Also draw the S.F & B.M diagram.

$x = 1.77$





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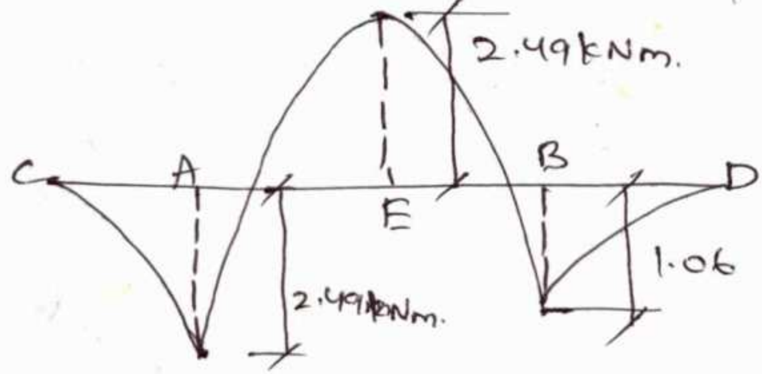
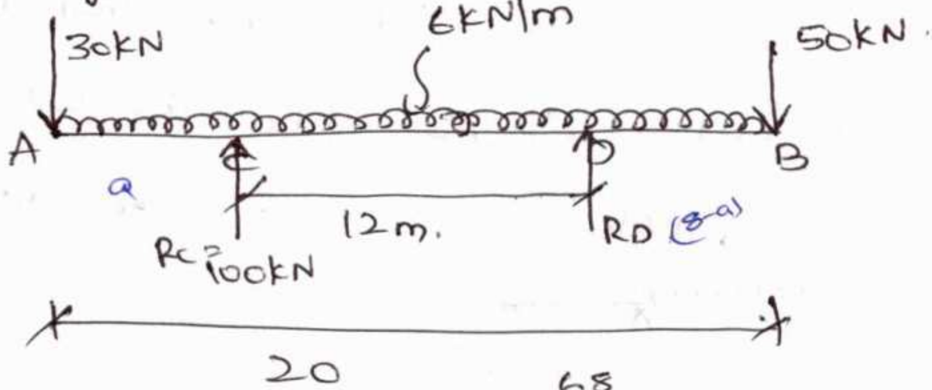
2, 11, 22, 25, 26, 27,



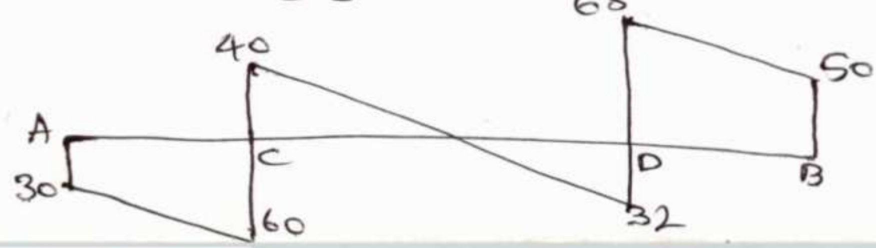
28/08/12

2, 21, 38

12). A beam AB, 20m long supported one two intermediate props 12m apart carries a UDL 6KN/m together with concentrated loads of 30kN at the left end A and 50kN at the right end B. The props are so located that the reaction is the same of each support. Det the position of props and draw B.M & S.F diagrams.



a = 5m.



B.M at a Section due to a couple:

Let cantilever (AB) of length 'l' subjected to couple in anticlockwise direction M = Pp be applied at a section distance 'a' from A.

couple here consists of the equal and parallel force P with a lever arm 'p' between them.

Hence at every section X, in AC

B.M = Moment of the individual forces 'P' of the couple.

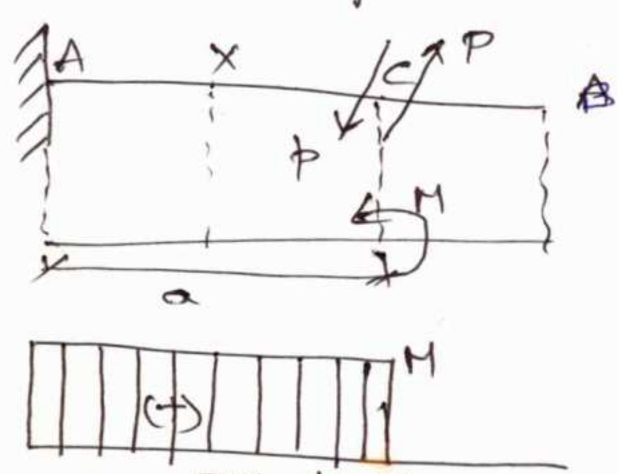
= anticlockwise moment Pp.

= Sagging Moment Pp = M

Hence at every section between A and C there will be a sagging moment M.

Due to the couple alone there will be no shear force.

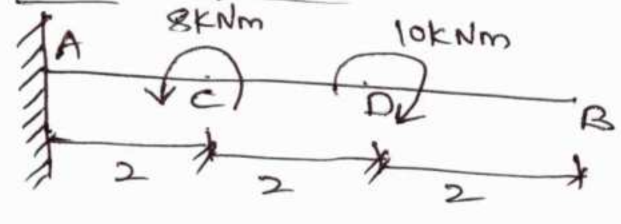
fig shows the B.M for the diagram.

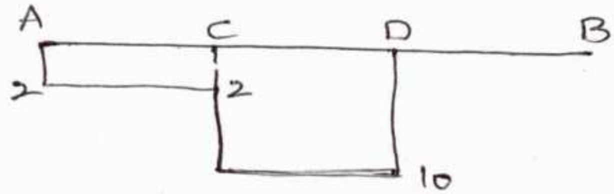


B.M diagram.

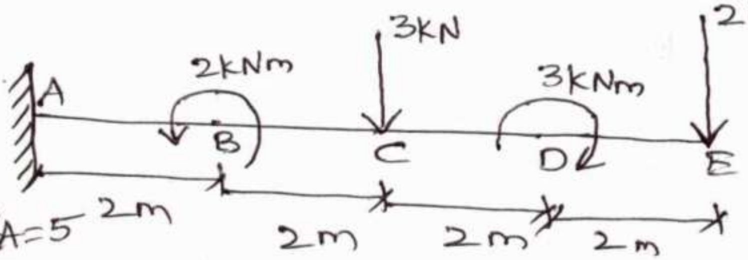
Cantilever beams:

1)

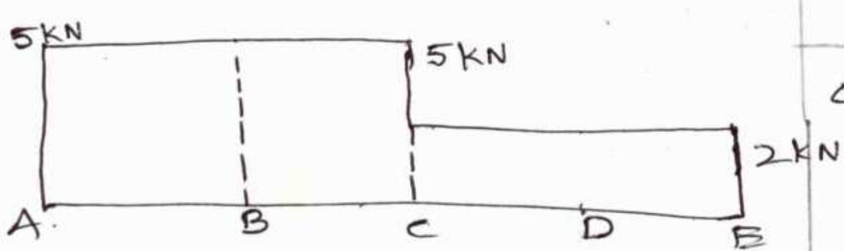


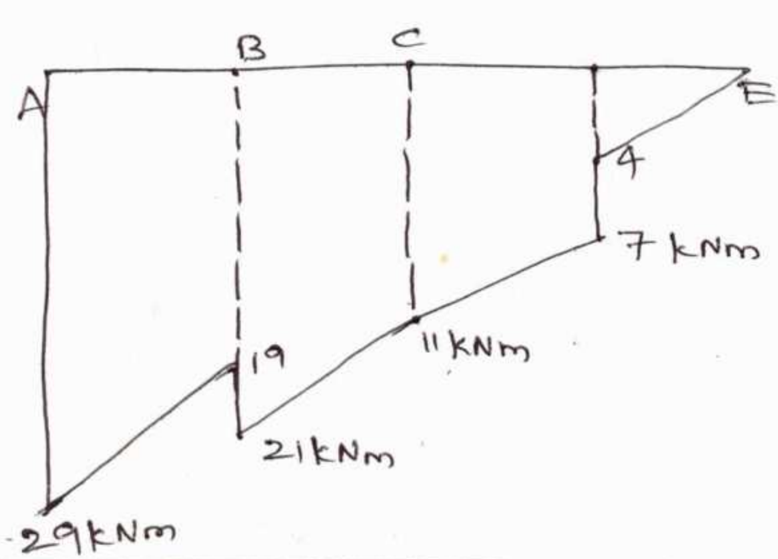


2)

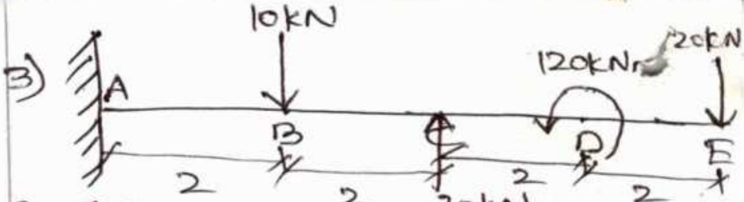


$R_A = 5$

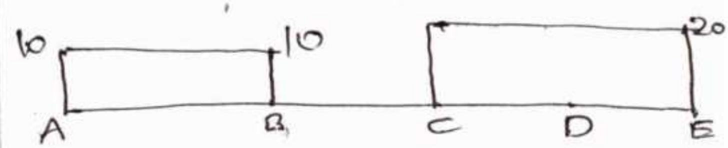


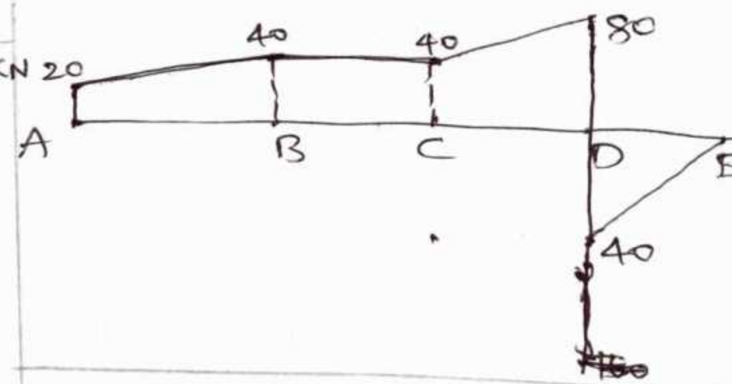


3)

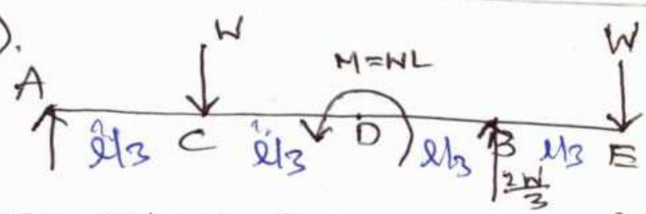


$R_A = 10kN$



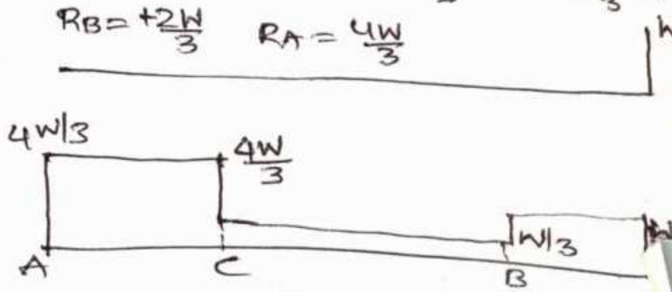


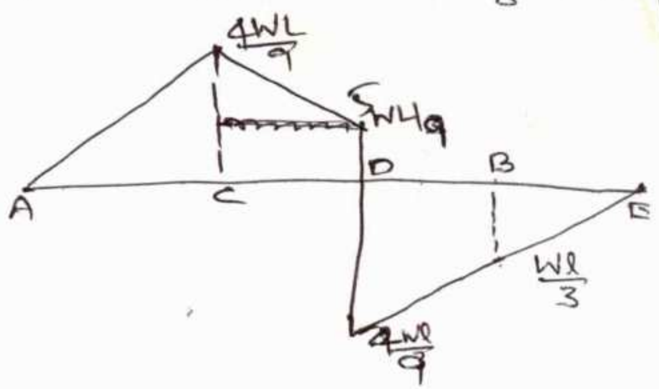
4).



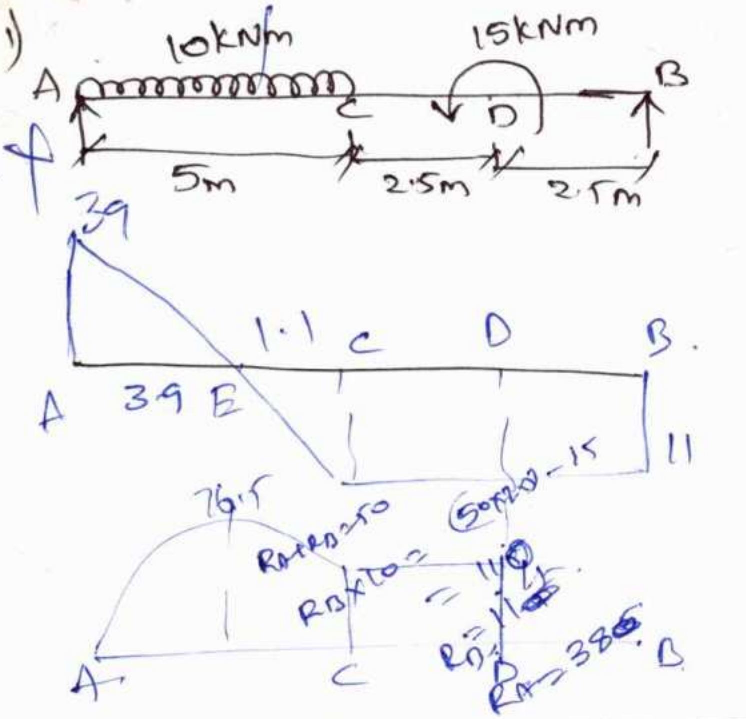
$R_A + R_B = 2W$; $R_B \times l - W \times \frac{4l}{3} + WL - \frac{Wl}{3} = 0$

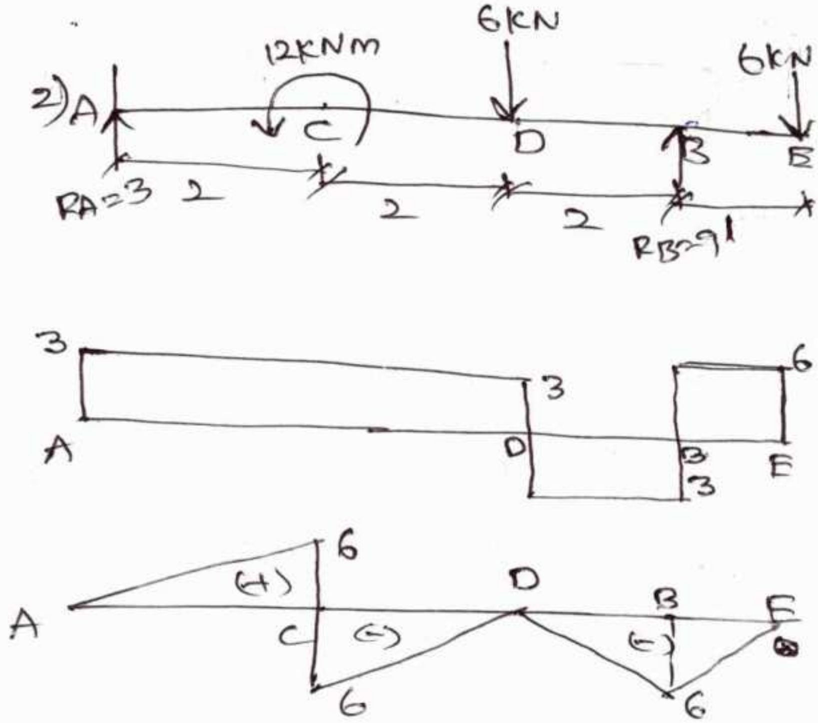
$R_B = +\frac{2W}{3}$ $R_A = \frac{4W}{3}$

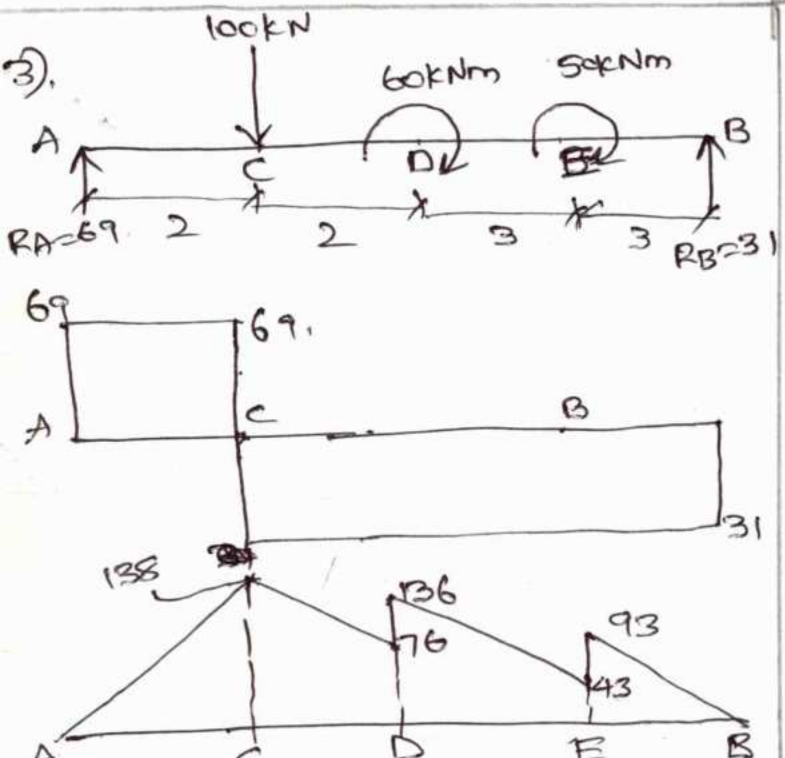


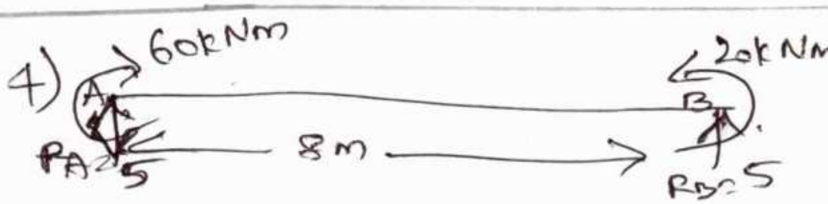


Simply supported beams:



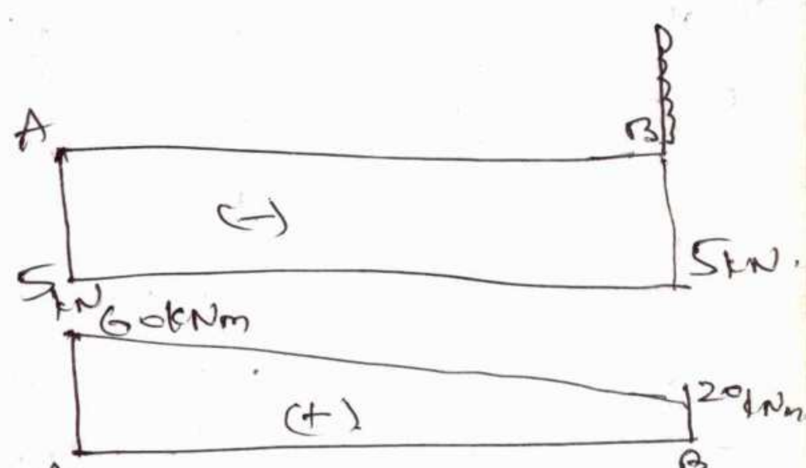






$RB \times 8 + 20 = 60$ | $RA \times 8 + 60 = 20$

$RB = \frac{40}{8} = 5$ | $RA = -\frac{40}{8} = -5$

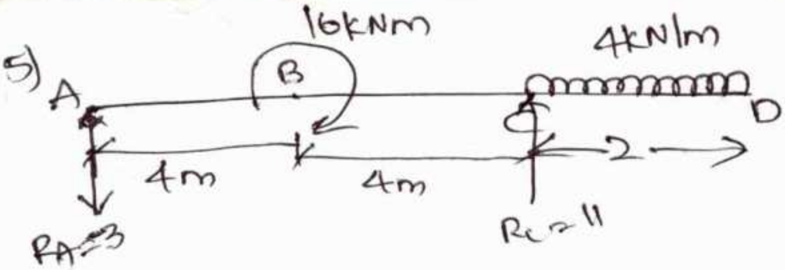


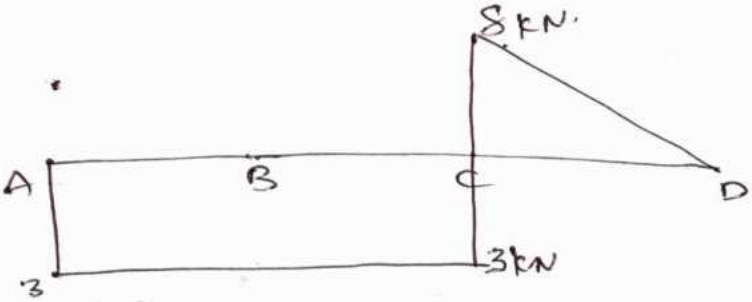
$M_D = 27.5$

$M_C = 70$ $\frac{39}{x} = \frac{11}{5-x}$

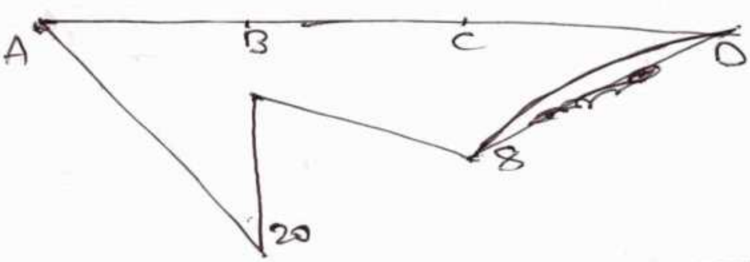
$x = 3.9$

5)

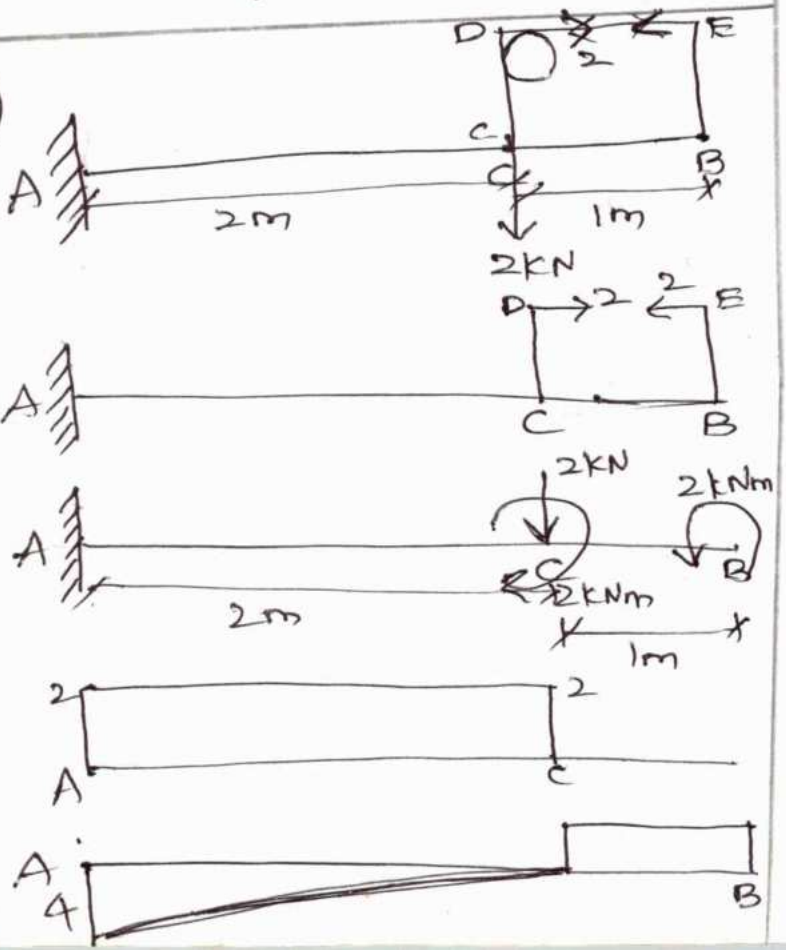


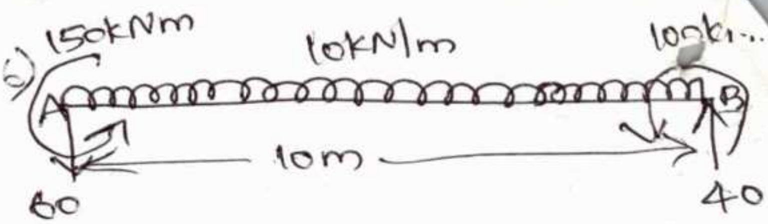


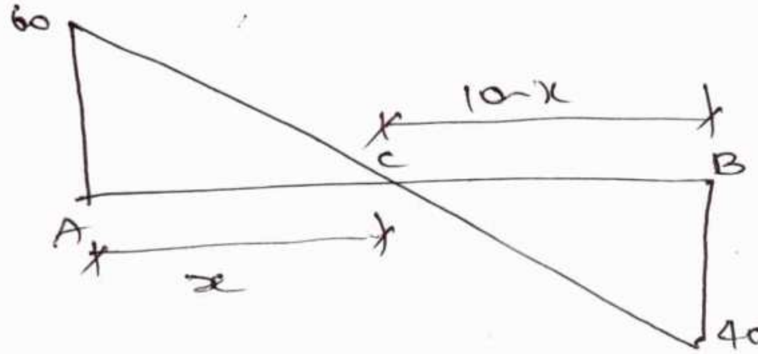
09 - 2/ 11, 12, 14, 15
11, 17, 31, 32, 37,
38



6)







$$\frac{60}{x} = \frac{40}{10-x}$$

$$60(10-x) = 40x$$

$$600 - 60x = 40x$$

$$600 = 100x$$

$$x = 6$$

6, 8, 17, 19,
24, 33,
44

$M_B = 100$

$M_C = (40\times4) + 100 - (10\times4\times2)$

$= 160 + 100 - 80$

$M_C = 180$

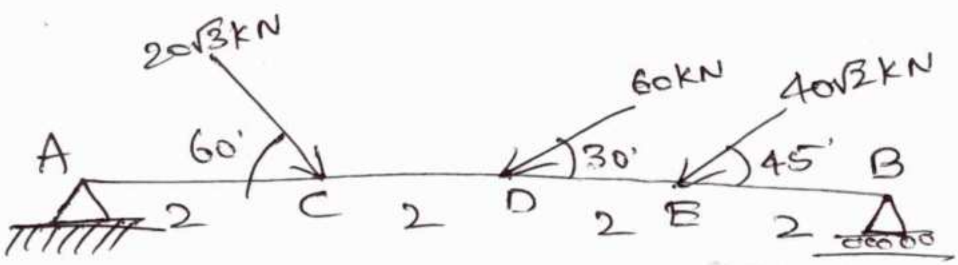
$M_B = 2$

$M_C = 2 - 2 = 0$

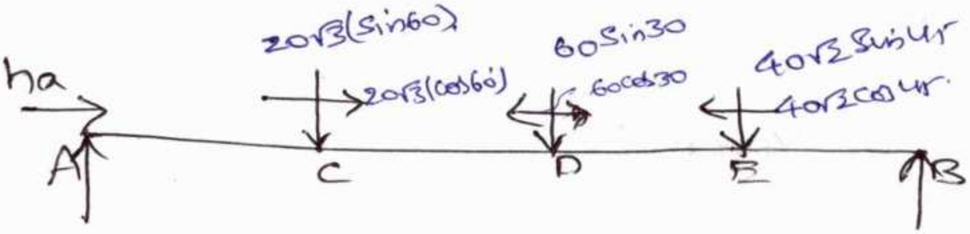
$M_A = 2 - 2 - (2\times2)$

$M_A = -4.$

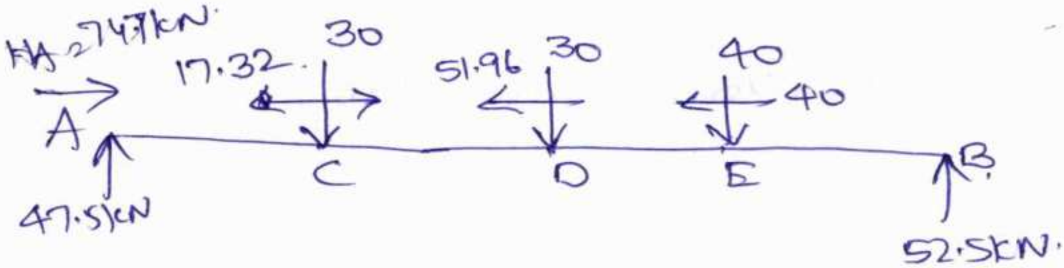
Members with oblique loading:-



1) Hinged support
2) Roller support
3) Fixed support



2) $\uparrow$ Y
$V_y \neq 0$



$R_A + R_B = 100$

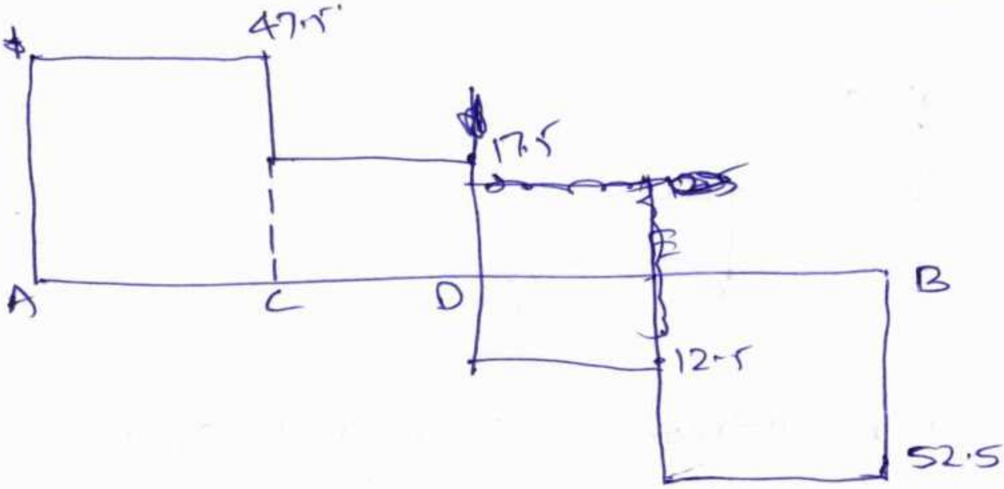
$H_A = 40 + 52 - 17.32$

$H_A = 74.7$ kN $(\leftarrow)$

$R_B \times 8 - (40 \times 6) - (30 \times 4) - (30 \times 2) = 0$

$R_B = 52.5$ kN

$R_A = 47.5$ kN



$F_B = -52.5$

$F_E = -52.5 + 40$

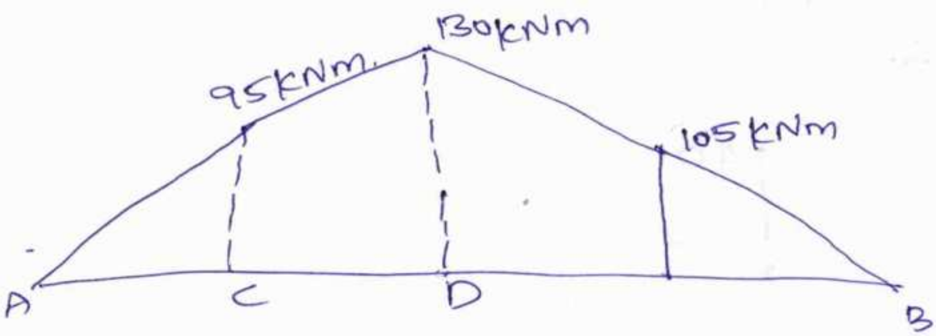
$= -12.5$

$F_D = -12.5 + 30$

$= 17.5$

$F_C = 17.5 + 30$

$F_C = 47.5$



B.M at B: $M_B = 0$

" " E: $M_E = 52.5 \times 2$

$= 105$ kNm

" " D: $M_D = 52.5 \times 4 - 40 \times 2$

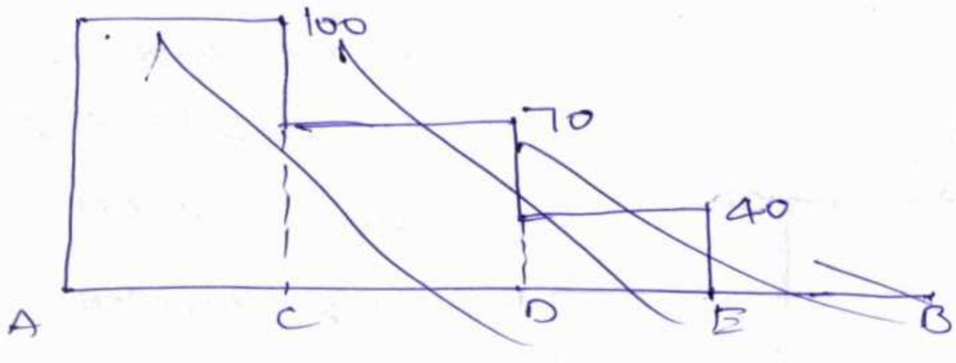
$= 130$ kNm

" " C: $M_C = (52.5 \times 6) - (40 \times 4) - (30 \times 2)$

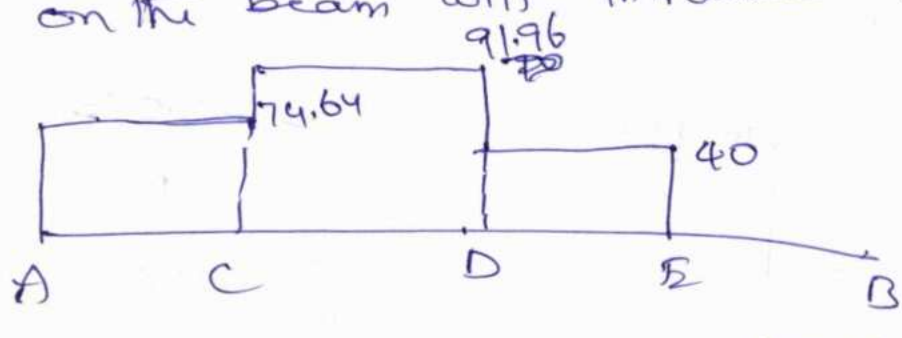
$= 95$ kNm

" " A: $M_A = (52.5 \times 8) - (40 \times 6) - (30 \times 4) - (30 \times 2)$

$= 0$

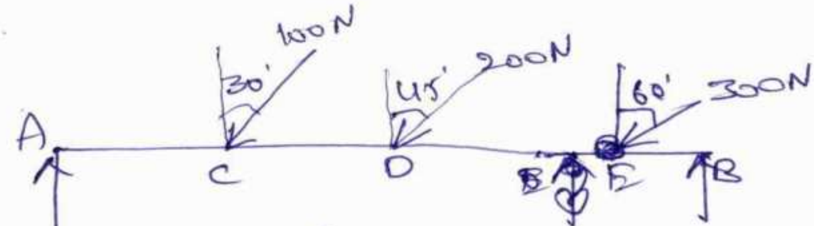


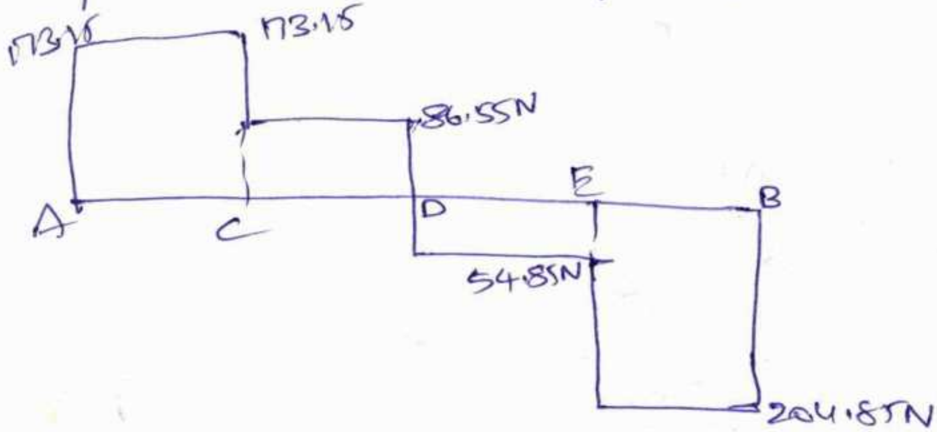
Thrust diagram: The horizontal components of the loads on the beam will introduce axial force (or) thrust in the members.

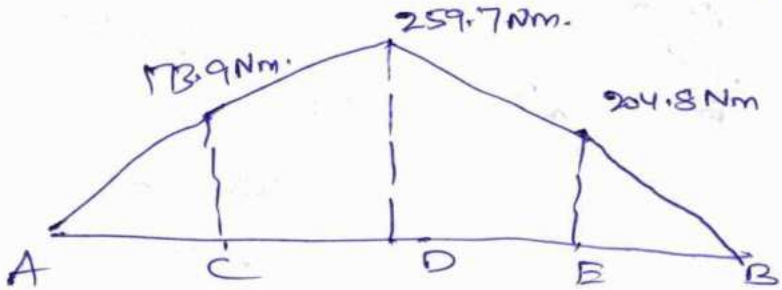


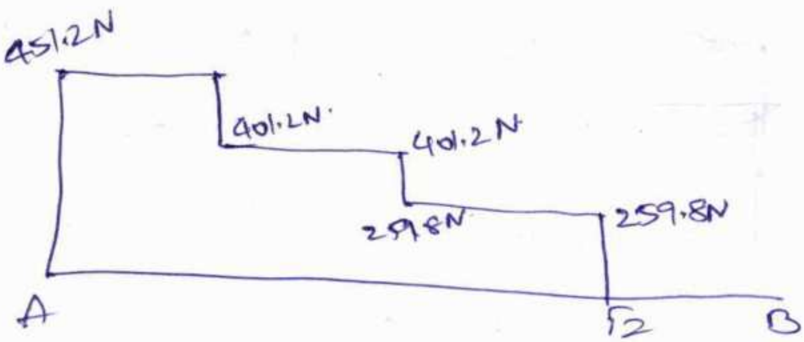
A diagram which shows the variation of the axial force for all sections of the span is called thrust diagram

Ass 1)



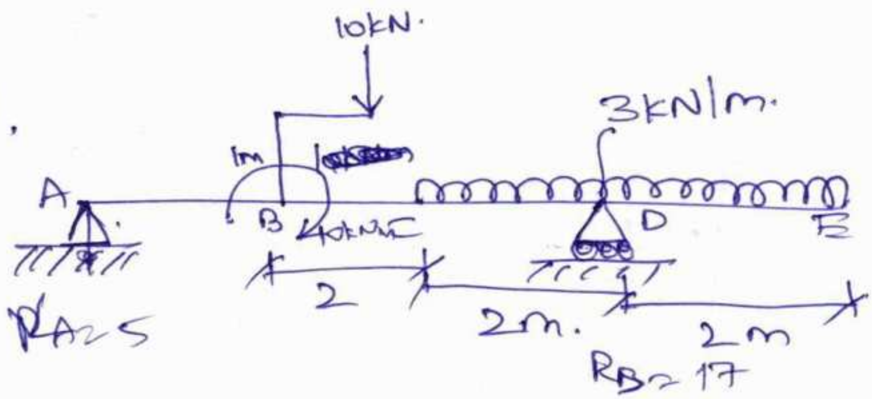


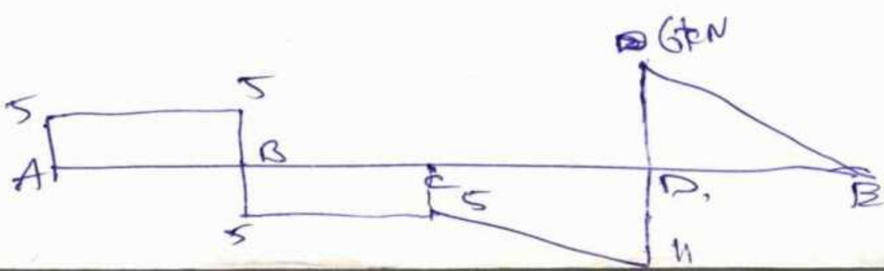




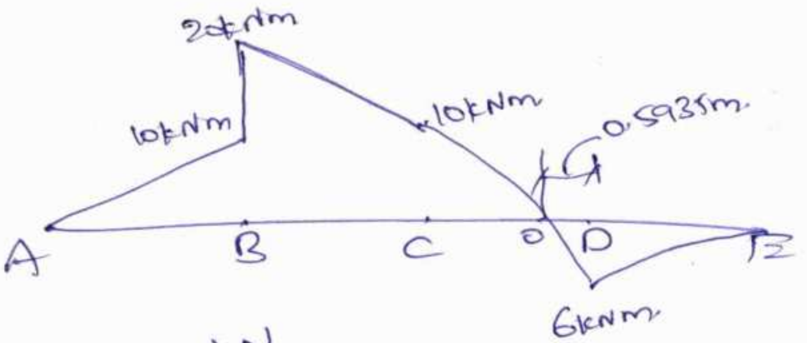
03/09/
7, 10, 13, 15, 18, 22,
27,

Ass 2).

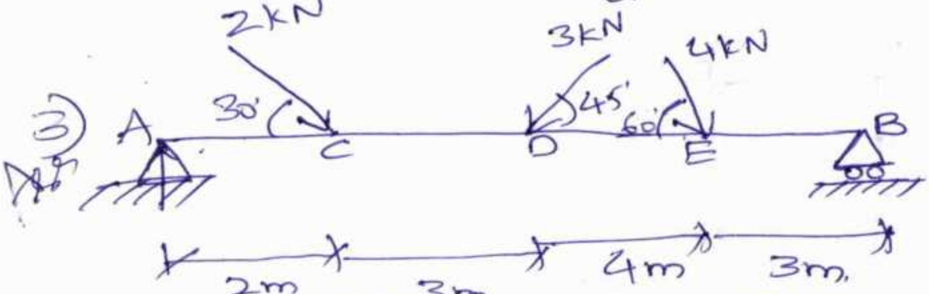


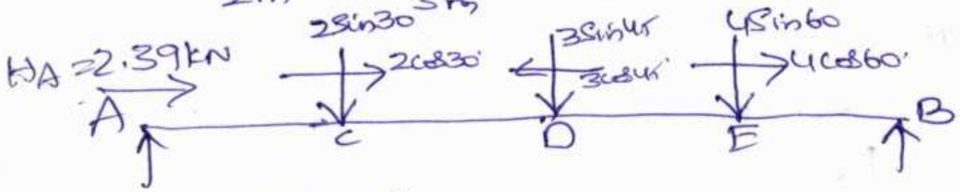


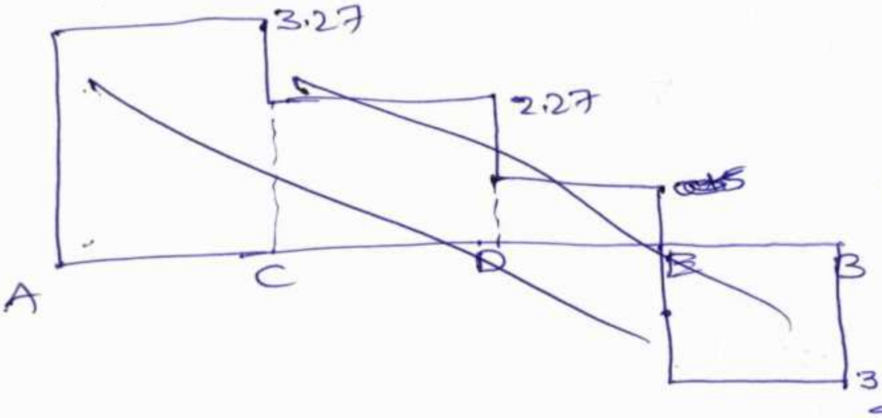
YSRBECYVU
4EC

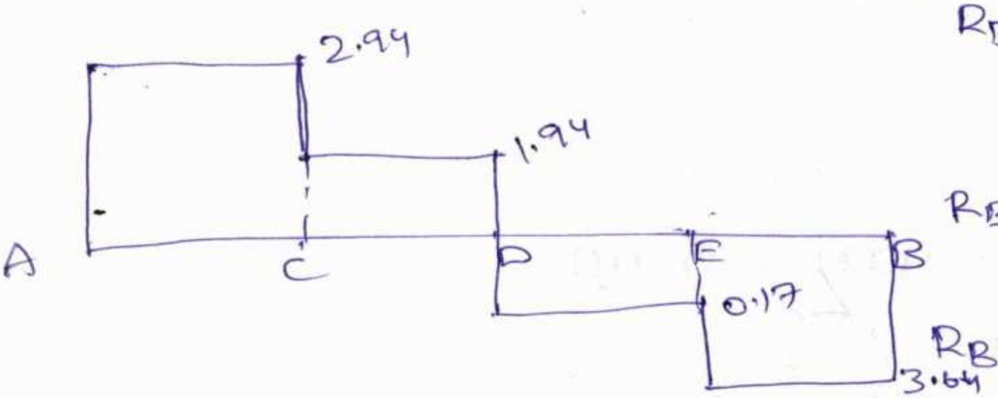


3)









$\Sigma V = 0$

$R_A + R_B = (2\sin 30 + 3\sin 45 + 4\sin 60)$

$= 6.585$

$\Sigma H = 0$

$(-2\cos 30 + 3\cos 45 + 4\cos 60) = 0$

$\Sigma H = 1 + 2.12 + 3.464$

$= -1.73 + 2.12 + 2$

$= 2.39$ kN (←)

$\Sigma M = 0$

$R_B \times 12 - (4\sin 60 \times 9) - (3\sin 45 \times 5) - (2\sin 30 \times 2) = 0$

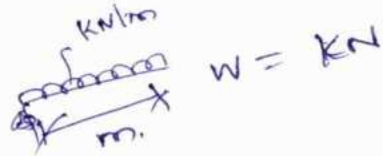
$R_B = \frac{31.17 + 10.6 - 2}{12}$

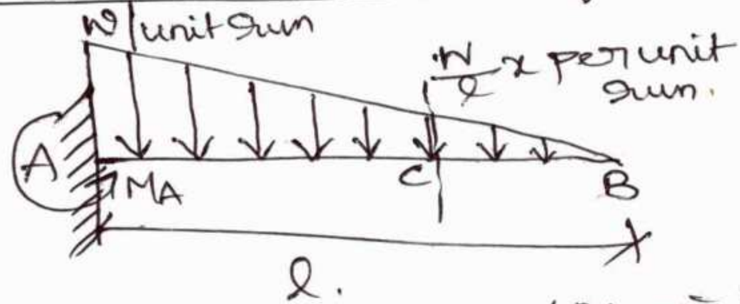
$R_B = \frac{39.77}{12} = 3.314$

$R_A = 3.27$

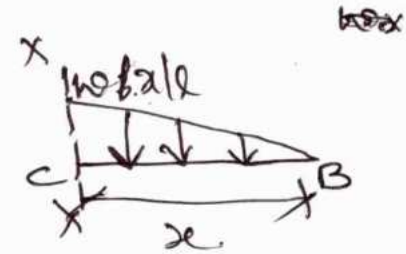
$H_A =$ ← (1.61) kN

Beams subjected to uniformly varying load:-





Let us consider a section x-x at a dist of 'x' from 'B'



R_A = Weight of UVL load.

= Area of the Triangle

$R_A = \frac{1}{2} \times w \times l.$

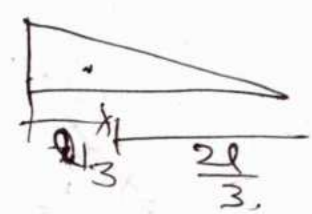
$A_x = \frac{1}{2} \times x \times \frac{wx}{l}.$

$w : l.$

$y : x$

$A_x = \frac{wx^2}{2l}.$

$F_x = +\frac{wx^2}{2l}.$



at $x=0$ i.e at B $F_B = 0$

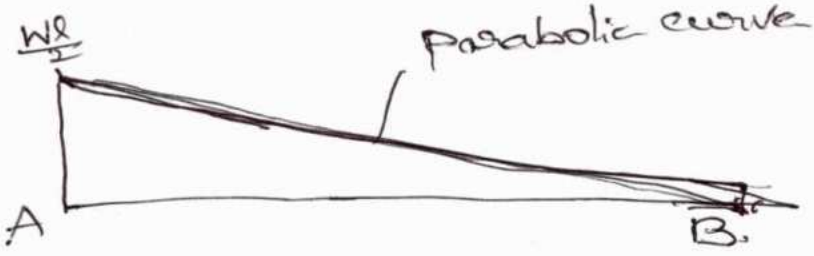
at $x = L$ i.e at A $F_A = \frac{wl^2}{2l} = \frac{wl}{2}$

$M_x = -\left(\frac{wx^2}{2l} \times \frac{\cancel{x}}{3}\right) = -\frac{wx^3}{6l}.$

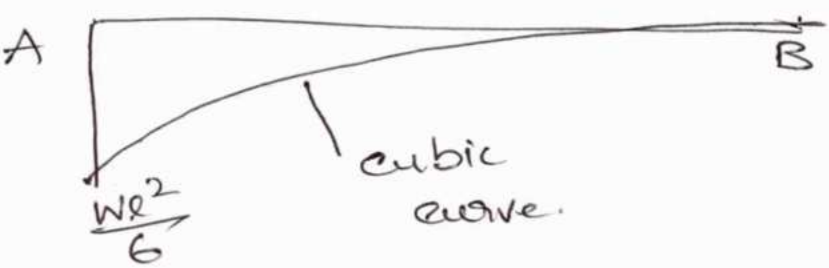
at $x=0$ i.e at B $M_B = 0$

at $x = L$ i.e at A $M_A = -\frac{wl^3}{6l} = \frac{-wl^2}{6}$

S.F Diagram:



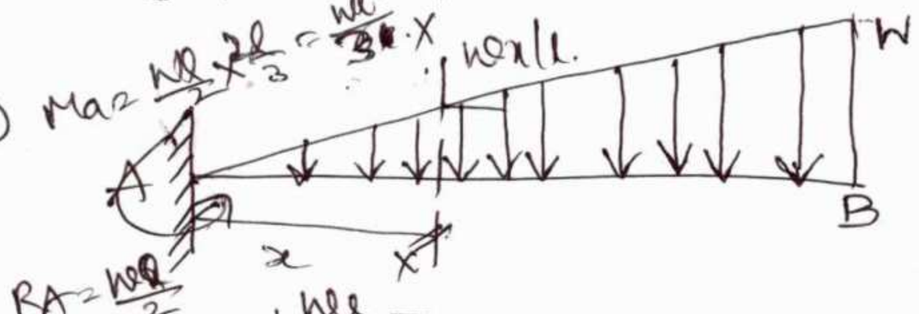
B.M Diagram:



S.F varies following a parabolic ~~curve~~ law and
B.M " cubic ~~curve~~ law.

2) $M_A = \frac{wl}{2} \times \frac{2l}{3} = \frac{wl^2}{3} \cdot x$ $wx/l.$ W

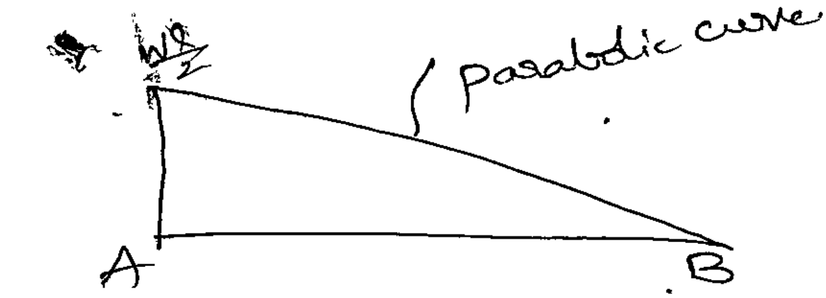
$R_A = \frac{wl}{2}$



$F_x = +\frac{wl}{2} - \frac{1}{2} \times x \times \frac{wx}{l} = \frac{wl}{2} - \frac{wx^2}{2l}.$

at $x=0$ i.e at A , at $x=L$

$F_A = 0 + \frac{wl}{2}$ $F_B = \frac{wl}{2} - \frac{wl}{2} = 0.$



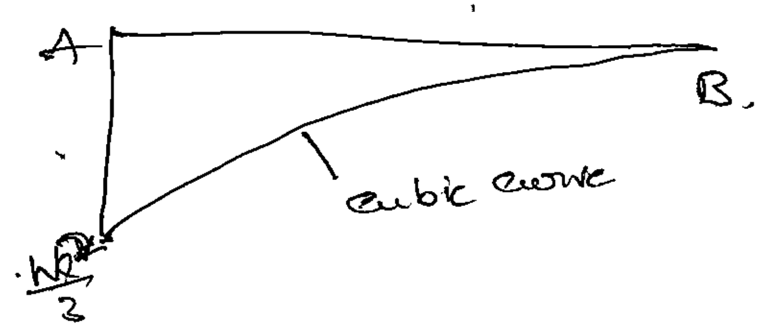
$M_x = +\frac{wl}{2} \times x - \left(\frac{wx}{l} \times \frac{1}{2} \times x \times \frac{x}{3}\right)$

at $x = 0$ $\quad \frac{wl^2}{3}$

$M_A = -\frac{wl^2}{3}$

at $x = l$

$M_B = +\frac{wl^2}{2} - \frac{wl^3}{6l} -$

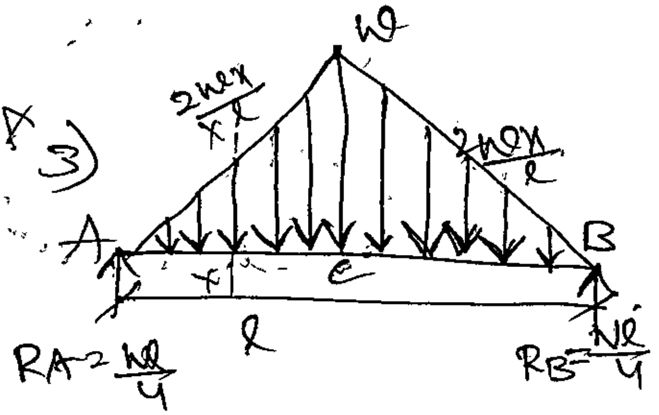


$= +\frac{wl^2}{2} - \frac{wl^2}{6} - \frac{wl^2}{3}$

$M_B = \cancel{\frac{2wl^2}{6}} = \cancel{\frac{wl^2}{3}} + \frac{wl^2}{3}$

$M_B = 0$

3)

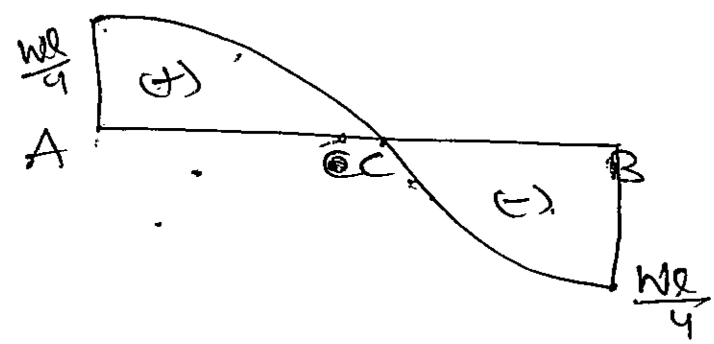


$M_B = \frac{2wl^2}{6} - \frac{wl^2}{3} = \frac{wl^2}{3} - \frac{wl^2}{3}$

$= 0$

$\cancel{A_x = \frac{wx}{8} \frac{1}{2} \times w \times l}$

$F_x = -\left(\frac{2wx}{l} \times \frac{1}{2} \times x\right) + \frac{wl}{4}$



at $x = 0$ $\quad$ at $x = \frac{l}{2}$

$F_A = \frac{wl}{4}$ $\quad$ $F_B = -\frac{wl^2}{8l} + \frac{wl}{4}$

at $x = l$

$F_B = -\frac{wl^2}{2l} + \frac{wl}{4} = \frac{wl}{4}.$

$= -\frac{wl}{4}$

$$F_x = +\frac{wl}{4} - \left(\frac{2wx}{l} \times \frac{1}{2} \times x\right).$$

at $x = 0$ at $x = \frac{l}{2}$

$$F_A = \frac{wl}{4} \qquad F_C = \frac{wl}{4} - \left(\cancel{2} \times \frac{w \times l}{\cancel{2}\cancel{l}} \times \frac{1}{2} \times \frac{l}{2}\right)$$

$$F_C = \frac{wl}{4} - \frac{wl}{4} = 0$$

at $x = l$ i.e at B

$$F_B = -\frac{wl}{4}$$

$$M_x = +\frac{wl}{4}(x) \mp \left(\frac{1}{\cancel{2}} \times \frac{\cancel{2}wx}{l} \times x \times \frac{x}{3}\right)$$

at $x = 0$ at $x = \frac{l}{2}$

$$M_A = 0 \qquad M_C = +\frac{wl}{4}\left(\frac{l}{2}\right) \mp \left(\frac{w}{3l}\left(\frac{l}{2}\right)^3\right)$$

at $x = l$

$$= \pm\frac{wl^2}{8} \mp \frac{wl^2}{24}$$

$$= \frac{(+3 \mp 1)\, wl^2}{24} = +\frac{wl^2}{12}$$

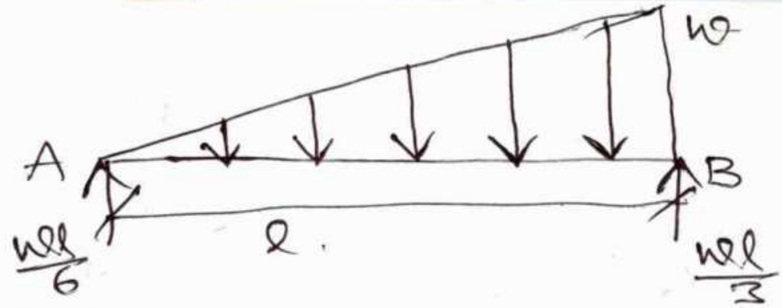
$$M_B = \frac{wl^2}{4} - \frac{w}{3l} l^2 \quad 0$$

$$= \frac{wl^2}{12}$$

$\frac{1}{4} - \frac{1}{3}$

$3 - 4$
-1

4)



$R_A + R_B = \frac{1}{2} \times w \times l$

~~For~~ $\cancel{R_A \times l - \left(\frac{wx}{l} \times \frac{1}{2} \times x \times \frac{x}{3}\right) = 0}$

$\cancel{R_A = \frac{wx^3}{6l^2}}$

$\cancel{R_B = \frac{wl}{2} - \frac{wx^3}{6l^2}}$

$R_A \times l - \left(\frac{1}{2} \times w \times l \times \frac{l}{3}\right) = 0$

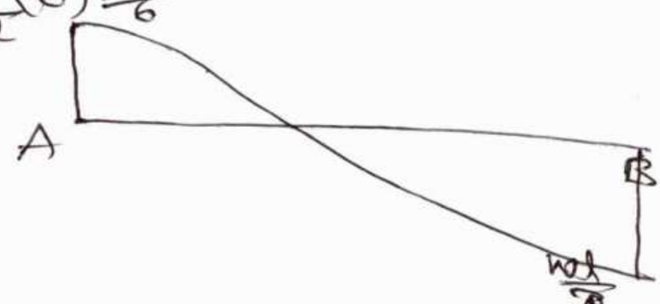
$R_A = \frac{wl}{6} \qquad R_B = \frac{wl}{2} - \frac{wl}{6} = \frac{2wl}{6} = \frac{wl}{3}$

S.F at X $\quad F_x = +R_A - \left(\frac{wx}{l} \times \frac{1}{2} \times x\right)$

at $x = 0$

$F_A = \frac{wl}{6} - \frac{w}{2l}(0)$

$F_A = \frac{wl}{6}$



at $x = l$

$F_B = \frac{wl}{6} - \frac{w}{2l} l^2 = \frac{wl}{6} - \frac{wl}{2} = -\frac{\cancel{2}wl}{\cancel{6}\,3}$

$F_B = -\frac{wl}{3}$

$\because F_x = 0$

$\frac{\cancel{w}l}{\cancel{6}\,3} = \frac{\cancel{w}x^2}{\cancel{2}l}$

$x^2 = \frac{l^2}{3}$

$x = \frac{l}{\sqrt{3}}$

$M_x = \frac{wl}{6}(x) - \left(\frac{wx}{l} \times \frac{x}{2} \times \frac{x}{3}\right)$

at $x = 0$

$M_A = 0$

at $x = l$

$M_B = \frac{wl^2}{6} - \left(\frac{wx^3}{6l}\right)$

$\frac{2(4.6}{2.13}$

~~F_C~~ $= R_A - \frac{w}{2l}\left(\frac{l}{\sqrt{3}}\right)^2$

$= \frac{wl^2}{6} - \frac{wl^2}{6}$

$= 0$

$\cancel{= \frac{wl}{6} - \frac{wl^2}{6l} = 0}$

$M_B = \cancel{\frac{3wl^2}{12} - \frac{2wl^2}{12} - \frac{wl^2}{12}}$

at $x = \frac{l}{\sqrt{3}}$

$M_C = \frac{wl}{6}\left(\frac{l}{\sqrt{3}}\right) - \left(\frac{w}{6l}\left(\frac{l}{\sqrt{3}}\right)^3\right)$

$= \frac{wl^2}{6\sqrt{3}} - \frac{wl^2}{6\times3\times\sqrt{3}}$

$= \frac{wl^2}{6\sqrt{3}}\left[1 - \frac{1}{3}\right] = \frac{2wl^2}{18\sqrt{3}} = \frac{wl^2}{9\sqrt{3}}$

Thin cylinders

The vessels such as boilers, compressed air receivers etc are of cylindrical and spherical forms.

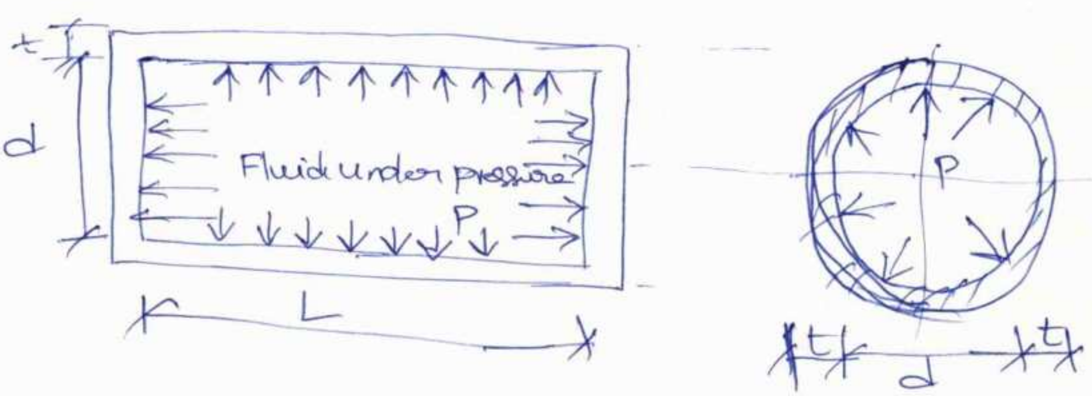
These vessels are generally used for storing fluids (liquids or gas) under pressure.

The walls of such vessels are thin as compared to their diameters.

If the thickness of the wall of the cylindrical vessel is less than $\frac{1}{15}$ to $\frac{1}{20}$ of its internal diameter, the cylindrical vessel is known as thin cylinder.

In case of thin cylinders, stress distribution is assumed uniform over the thickness of the wall.

<u>Thin cylindrical vessel subjected to internal pressure:-</u>



Let d = Internal diameter of the thin cylinder

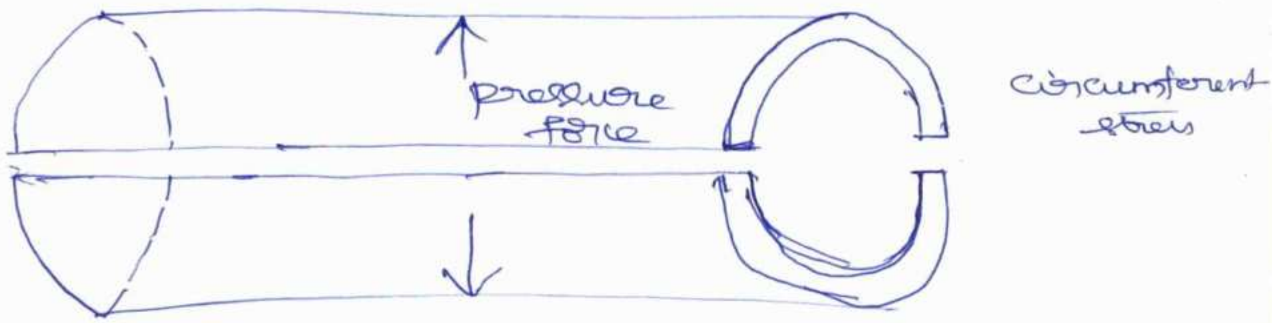
t = thickness of the wall of the cylinder

p = Internal pressure of the fluid

L = Length of the cylinder.

On account of the internal pressure p, the cylindrical vessel may fail by splitting up in any one of the two ways as shown in fig.

The force due to pressure of the fluid acting vertically upwards and downwards on the thin cylinder, tend to burst the cylinder as shown in fig.



The forces, due to pressure of the fluid, acting at the ends of the thin cylinder, tend to burst the thin cylinder as shown in fig.

longitudinal stress.

Stresses in a thin cylindrical Vessel subjected to internal pressure:-

When a thin cylindrical vessel is subjected to internal fluid pressure, the stresses in the wall of the cylinder on the c/s along the along the axis and on the c/s ⊥r to the axis are set up. These stresses are tensile and are known as

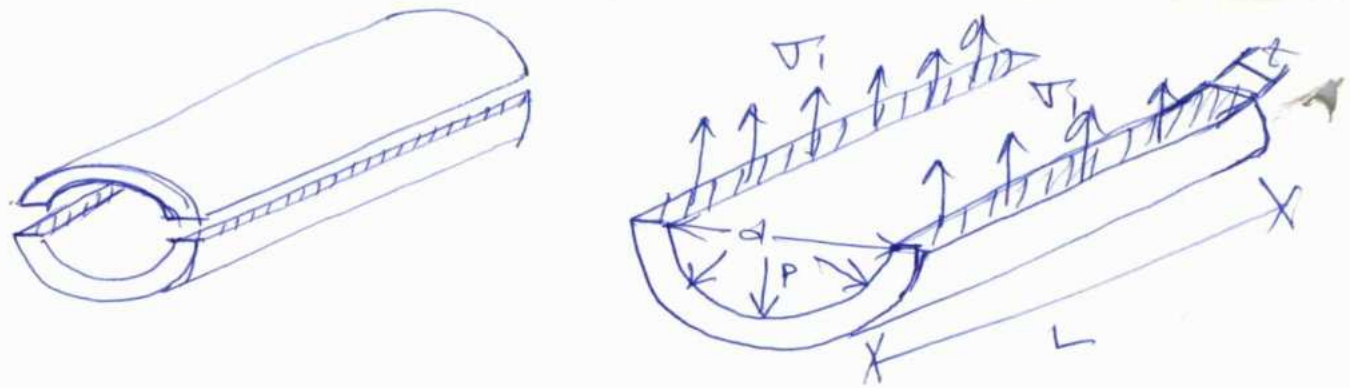
1. circumferential stress (or hoop stress) and
2. Longitudinal stress

The stress acting along the circumference of the cylinder is called circumferential stress (hoop stress) fig 1

The stress acting along the length of the cylinder is known as longitudinal stress. fig 2

Expression for circumferential stress (or hoop stress)

Consider a thin cylindrical vessel subjected to an internal fluid pressure. The circumferential stress will be setup in the material of the cylinder, if the bursting of the cylinder takes place as shown in fig.



The expression for hoop stress or circumferential stress (σ_1) is obtained as given below.

Let p = Internal pressure of fluid

d = " diameter of the cylinder

t = Thickness of the wall of the cylinder

σ_1 = circumferential or hoop stress in the material

The bursting will take place if the force due to fluid pressure is more than the resisting force due to circumferential stress set up in the material.

In the limiting case, the two forces should be equal.

$\therefore$ Force due to fluid pressure = $p \times$ Area on which p is acting

$$= p \times (d \times L) \longrightarrow (1)$$

($\because$ p is acting on projected area $d \times L$)

Force due to circumferential ~~pressure~~ stress

$$= \sigma_1 \times \text{Area on which } \sigma_1 \text{ is acting}$$

$$= \sigma_1 \times (L \times t + L \times t) = \sigma_1 \times 2Lt = 2\sigma_1 \times L \times t \longrightarrow (2)$$

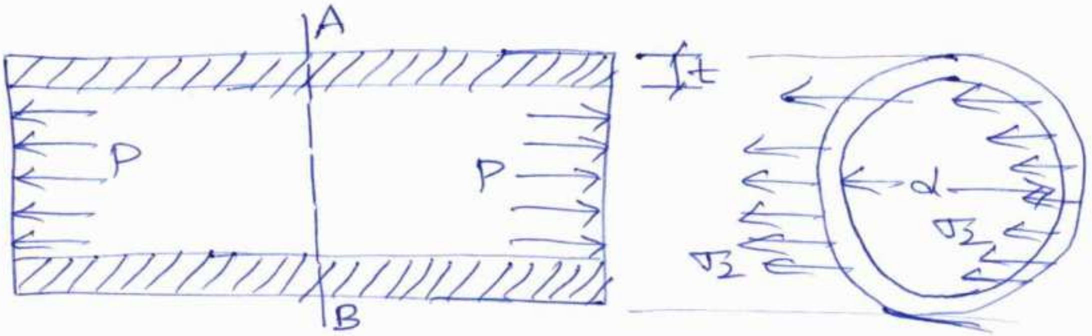
Equating (1) = (2)

$$p \times d \times \cancel{L} = 2\sigma_1 \times t \times \cancel{L}$$

$$\boxed{\sigma_1 = \frac{pd}{2t}}$$

<u>Expression for Longitudinal stress:-</u>

Consider a thin cylindrical vessel subjected to internal fluid pressure. The longitudinal stress will be set up in the material of the cylinder, if the bursting of the cylinder takes place along the section AB of fig.



The longitudinal stress (σ_2) developed in the material is obtained as:

Let p = Internal pressure of fluid stored in thin cylinder

d = Internal diameter of cylinder

t = thickness of the cylinder

σ_2 = longitudinal stress in the material.

The bursting will take place if the force due to fluid pressure acting on the ends of the cylinder is more than the resisting force due to longitudinal force (σ_2) developed in the material as shown in fig. In the limiting case, both the forces should be equal

Force due to fluid pressure = $p \times$ Area on which p is acting

$$= p \times \frac{\pi}{4} d^2$$

Resisting force = $\sigma_2 \times$ Area ~~of~~ on which σ_2 is acting

$$= \sigma_2 \times \pi d \times t$$

Force due to fluid pressure = Resisting force

$$p \times \frac{\cancel{\pi}}{4} d^{\cancel{2}} = \sigma_2 \times \cancel{\pi} \cancel{d} \times t$$

$$\boxed{\sigma_2 = \frac{pd}{4t}}$$

σ_2 is also tensile

$$\sigma_2 = \frac{1}{2} \cdot \frac{pd}{2t} = \frac{1}{2} \cdot \sigma_1$$

$\therefore$ Longitudinal stress = Half of circumferential stress

This also means that circumferential stress (σ_1) is two times the longitudinal stress (σ_2).

Maximum shear stress:- At any point in the material of the cylindrical shell, there are two principal stresses, namely a circumferential stress of magnitude $\sigma_1 = \frac{pd}{2t}$ acting circumferentially and a longitudinal stress of magnitude $\sigma_2 = \frac{pd}{4t}$ acting parallel to the axis of the shell. These two stresses are tensile and $\perp^r$ to each other.

$\therefore$ Maximum shear stress

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2} = \frac{pd}{8t}$$

$$\therefore \boxed{\tau_{max} = \frac{pd}{8t}}$$

~~18, 19, 21, 22, 27~~

Efficiency of a Joint:-

The cylindrical shells such as boilers are having two types of joints namely longitudinal joint and circumferential joint.

~~A~~ If the efficiency of the longitudinal joint and circumferential joint are given then the circumferential and longitudinal stresses are obtained as:

Let η_l = Efficiency of a longitudinal joint

η_c = " " circumferential "

Then the circumferential stress (σ_1) is given as

$$\sigma_1 = \frac{pd}{2t \cdot \eta_l}$$

and the longitudinal stress (σ_2) is given as

$$\sigma_2 = \frac{pd}{4 \cdot t \cdot \eta_c}$$

Problems

1) A cylindrical pipe of diameter 1.5m and thickness 1.5cm is subjected to an internal fluid pressure of $1.2 N/mm^2$. Det

(1) longitudinal stress developed in the pipe

(2) circumferential " " "

[Ans: $\sigma_2 = 30 N/mm^2$ $\sigma_1 = 60 N/mm^2$]

2) A cylinder of internal diameter 2.5m and of thickness 5cm contains a gas. If the tensile stress in the material is not to exceed $80 N/mm^2$, Determine the internal pressure of the gas.

[Ans: $p = 3.2 N/mm^2$]

3) A thin cylinder of internal diameter 1.25m contains a fluid at an internal pressure of $2 N/mm^2$. Det the maximum thickness of the cylinder if:

(i) The longitudinal stress is not to exceed $30 N/mm^2$

(ii) The circumferential stress is not to exceed $45 N/mm^2$

{Ans: (i) $t = 2.77 cm$, (ii) $t = 2.08 cm$

Effect of internal pressure on the dimensions of a thin cylindrical shell:-

When a fluid having internal pressure (P) is stored in a thin cylindrical shell, due to internal pressure of the fluid the stresses set up at any point of the material of the shell are:

(i) Hoop or circumferential stress (σ_1), acting on longitudinal section.

(ii) longitudinal stress (σ_2) acting on the circumferential section.

These stresses are principal stresses, as they are acting on principal planes. The stress in the third principal plane is zero as the thickness (t) of the cylinder is very small.

Actually the stress in the third principal plane is radial stress which is very small for thin cylinders and can be neglected.

Let p = internal pressure of fluid

L = Length of cylindrical Shell

d = diameter of the " "

t = thickness " " "

E = Modulus of Elasticity for the material of the Shell

σ_1 = Hoop stress in the material

σ_2 = Longitudinal stress in the material.

μ = poisson's ratio

δd = change in diameter

δL = " " Length

δV = " " Volume.

The Values of σ_1 and σ_2 are given by

$$\sigma_1 = \frac{Pd}{2t} \qquad \sigma_2 = \frac{Pd}{4t}$$

circumferential strain

$$e_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E}$$

$$e_1 = \frac{Pd}{2tE} - \mu \cdot \frac{Pd}{4tE} = \frac{Pd}{2tE}\left[1 - \frac{\mu}{2}\right]$$

$$\boxed{e_1 = \frac{Pd}{2tE}\left[1 - \frac{\mu}{2}\right]} \longrightarrow (1)$$

longitudinal strain

$$e_2 = \frac{\sigma_2}{E} - \frac{\mu \sigma_1}{E}$$

$$= \frac{Pd}{4tE} - \mu\frac{Pd}{2tE} = \frac{Pd}{2tE}\left[\frac{1}{2} - \mu\right]$$

$$\boxed{e_2 = \frac{Pd}{2tE}\left\{\frac{1}{2} - \mu\right]} \longrightarrow (2)$$

But circumferential strain is also given as

$$e_1 = \frac{\text{change in circumference due to pressure}}{\text{original circumference.}}$$

$$= \frac{\text{Final circumference - original circumf}}{\text{original circumference}}$$

$$= \frac{\pi(d+\delta d) - \pi d}{\pi d} = \frac{\cancel{\pi}\cdot \delta d}{\cancel{\pi} d} = \frac{\delta d}{d}$$

$$e_1 = \frac{\delta d}{d} \rightarrow (3)$$

equating eq (1) and eq (3)

$$\frac{P\cancel{d}d}{2tE}\left[1 - \frac{\mu}{2}\right] = \frac{\delta d}{d}$$

$$\boxed{\delta d = \frac{Pd^2}{2tE}\left[1 - \frac{\mu}{2}\right]}$$

similarly longitudinal strain is also given

$$e_2 = \frac{\text{change in length due to pressure}}{\text{original length}}$$

$$e_2 = \cancel{\delta L} \frac{\delta L}{L} \longrightarrow (4)$$

Equating eq(2) and eq(4)

$$\frac{Pd}{2tE}\left[\frac{1}{2} - \mu\right] = \frac{\delta L}{L}$$

$$\boxed{\delta L = \frac{PdL}{2tE}\left[\frac{1}{2} - \mu\right]}$$

Volumetric strain:- It is defined as change in volume divided by original volume.

$$\therefore \text{Volumetric strain} = \frac{\delta V}{V}$$

But change in volume = Final volume − original volume

original volume = Area of cylindrical shell × length

$$= \frac{\pi}{4} d^2 L$$

$$\text{Final volume} = \frac{\pi}{4}((d + \delta d)^2 \cdot (L + \delta L))$$

$$= \frac{\pi}{4}\left[(d^2 + \cancel{\delta d^2}^{\,0} + 2d \cdot \delta d)(L + \delta L)\right]$$

$$= \frac{\pi}{4}\left[d^2 L + d^2 \delta L + 2d\,\delta d \cdot L + 2d\,\cancel{\delta d}^{\,0} \cdot \delta L\right]$$

$$= \frac{\pi}{4}\left[d^2L + d^2\delta L + 2d.\delta d.L\right]$$

$\therefore$ change in volume (δV) = Final vol − original vol

$$= \frac{\pi}{4}\left[d^2L + 2.d.L.\delta d + d^2\delta L\right] - \frac{\pi}{4}d^2L$$

$$= \frac{\pi}{4}\left[2dL\,\delta d + d^2\delta L\right]$$

$\therefore$ Volumetric strain $= \frac{\delta V}{V}$

$$= \frac{\cancel{\pi}}{\cancel{4}}\left[2d.L.\delta d + d^2\delta L\right] \Big/ \frac{\cancel{\pi}}{\cancel{4}}d^2L$$

$$= \frac{2.\delta d}{d} + \frac{\delta L}{L}$$

$$= 2e_1 + e_2$$

$\therefore$ Also change in volume $(\delta V) = V \times (2e_1 + e_2)$

~~14/01/11 – 2~~

~~2, 6, 15, 18, 28,~~

Formulas

1) circumferential stress $\sigma_1 = \frac{pd}{2t}$

2) longitudinal stress $\sigma_2 = \frac{pd}{4t}$

3) Maximum shear stress $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{pd}{8t}$

4) change in dimensions

$$\delta d = \frac{pd^2}{2tE}\left[1 - \frac{\mu}{2}\right]$$

$$\delta L = \frac{pdL}{2tE}\left[\frac{1}{2} - \mu\right]$$

5) Change in volume

$$\delta v = V(2e_1 + e_2) \quad \text{where } e_1 = \frac{pd}{2t}\left[1 - \frac{\mu}{2}\right]$$

$$e_2 = \frac{pd}{2t}\left[\frac{1}{2} - \mu\right]$$

6) Major principal stress $= \cancel{\frac{\sigma_1 + \sigma_2}{2} = \left[\frac{pd}{2t} + \frac{pd}{4t}\right]\frac{1}{2}}$

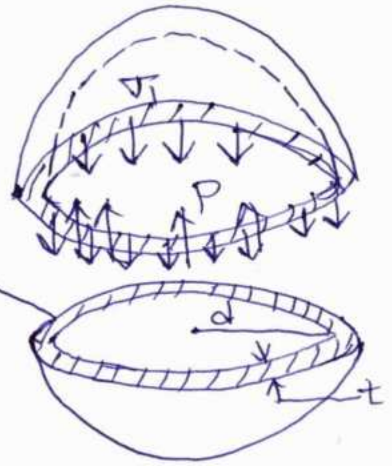
$$= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{1}{2}(\sigma_1 + \sigma_2) + \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

7) Minor principal stress

$$= \frac{1}{2}(\sigma_1 + \sigma_2) - \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

Thin spherical shells:-



Resisting area πdt

Fig shows a thin spherical shell of internal diameter 'd' and thickness 't' subjected to an internal fluid pressure 'p'. The fluid inside the shell has a tendency to split the shell into two hemispheres along x-x axis.

The force (P) which has a tendency to split the shell

$$= p \times \frac{\pi}{4} d^2$$

The area resisting this force $= \pi . d . t$

$\therefore$ Hoop or circumferential stress (σ_1) induced in the material of the shell is given by

$$\sigma_1 = \frac{\text{Force } P}{\text{Area resisting the force } P} = \frac{p \times \frac{\cancel{\pi}}{4} d^{\cancel{2}}}{\cancel{\pi} . \cancel{d} . t}$$

$$\boxed{\sigma_1 = \frac{pd}{4t}}$$

$\sigma_1 \times \pi d t$
$= p \times \frac{\pi}{4} d^2$
$\sigma_1 = \frac{pd}{4t}$

The stress σ_1 is tensile in nature

The fluid inside the shell is also having tendency to split the shell into two hemispheres along Y-Y axis. Then it can be shown that the tensile hoop stress will also be equal to $\frac{pd}{4t}$.

let this stress is σ_2.

$$\boxed{\sigma_2 = \frac{pd}{4t}}$$

The stress σ_2 will be at right angles to σ_1.

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{pd}{4t} - \frac{pd}{4t} = 0$$

$\boxed{\tau_{max} = 0}$ for thin spherical shells

<u>change in dimensions of a thin spherical shell</u>
<u>due to an internal pressure</u> :-

$[\because \sigma_1 = \sigma_2]$

$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} = \frac{\sigma_1}{E}[1-\mu]$$

$$\boxed{\varepsilon_1 = \frac{pd}{4tE}[1-\mu]} \rightarrow (1)$$

$$\varepsilon_2 = \frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E} = \frac{\sigma_2}{E}[1-\mu] = \frac{pd}{4tE}[1-\mu] \rightarrow (2)$$

$$\varepsilon_1 = \text{change } \frac{\delta d}{d} \rightarrow (3)$$

eq(1) = eq(3)

$$\frac{\delta d}{d} = \frac{pd}{4tE}[1-\mu]$$

$$\boxed{\delta d = \frac{pd^2}{4tE}[1-\mu]}$$

Volumetric strain :-

[For a sphere volume

$V = \frac{4}{3}\pi r^3$

$$\frac{\delta v}{V}$$

$$V = \frac{\pi}{6} d^3$$

$$\delta v = \frac{\pi}{6}(d+\delta d)^3 - \frac{\pi}{6}d^3 = \frac{\pi}{6}\left[d^3 + \cancel{(\delta d)^3} + 3d^2(\delta d) + 3d\cancel{(\delta d)^2}\right] - \frac{\pi}{6}d^3$$

$$\delta v = \frac{\pi}{6}\left[d^3 + 3d^2(\delta d)\right] - \frac{\pi}{6}d^3 = \frac{\pi}{6}(3d^2 \delta d)$$

$$\therefore \varepsilon_v = \frac{\delta v}{V} = \frac{\pi}{6}\left[3d^2 \delta d\right] \Big/ \frac{\pi}{6} d^3$$

$$\varepsilon_v = 3\frac{\delta d}{d}$$

$$\frac{\delta V}{V} = 3\frac{\delta d}{d}$$

$$\boxed{\frac{\delta V}{V} = 3 \cdot \frac{pd}{4tE}(1-\mu)}$$

Problems

1) Calculate: (i) the change in diameter (ii) change in length (iii) change in volume of a thin cylindrical shell 100cm diameter, 1cm thick and 5m long when subjected to internal pressure of $3 N/mm^2$. $E = 2\times10^5 N/mm^2$ $\mu = 0.3$

[Ans: $\delta d = 0.6375 mm$, $\delta L = 0.75 mm$, $\delta V = 5595.96\times10^3 mm^3$]

2) A cylindrical thin drum 80cm in diameter and 3m long has a shell thickness of 1cm. If the drum is

subjected to an internal pressure of $2.5 N/mm^2$. determine (i) change in diameter (ii) change in length (iii) change in volume, Take $E = 2 \times 10^5 N/mm^2$ $\mu = 0.25$

[Ans: $\delta d = 0.35$, $\delta l = 0.375$, $\delta V = 1507964.47 mm^3$]

~~20/1/24~~ ~~All present~~

3)

A thin cylindrical vessel subjected to internal fluid pressure and a Torque:-

When a thin cylindrical vessel is subjected to internal fluid pressure (p), the stresses set up in the material of the vessel are circumferential stress (σ_1) and longitudinal stress (σ_2). These two stresses are tensile and are acting ⊥r to each other. If the cylindrical vessel is subjected to a torque, shear stresses will also be setup in the material of the vessel.

Hence at any point in the material of the cylinder, there will be two tensile stresses mutually ⊥r to each other accompanied by a shear stress.

Let σ_1 = circumferential stress

σ_2 = longitudinal stress

τ = shear stress due to torque.

The major principal stress $= \frac{\sigma_1+\sigma_2}{2} + \frac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}$

minor " " $= \frac{\sigma_1+\sigma_2}{2} - \frac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}$

Maximum shear stress $= \frac{\sigma_1-\sigma_2}{2}$

Problems

1) A thin cylindrical tube 80mm internal diameter and 5mm thick, is closed at the ends and is subjected to an internal pressure of 6N/mm². A torque of 2009600 Nmm is also applied to the tube. Find the hoop stress, longitudinal stress, maximum and minimum principal stress and the maximum shear stress.

[Ans: $\sigma_1 = 48 N/mm^2$, $\sigma_2 = 24 N/mm^2$, Max principal stress $= 59.32 N/mm^2$, Min principal stress $= 12.68 N/mm^2$, $\tau_{max} = 23.32 N/mm^2$]

2) A spherical shell of internal diameter 0.9m and of thickness 10mm is subjected to an internal pressure of 1.4N/mm². Det the increase in diameter and increase in volume. Take $E = 2 \times 10^5 N/mm^2$ $\mu = \frac{1}{3}$

[Ans: $\delta d = 0.0945mm$ $\delta v = 120285 mm^3$]

Summary

1) If the thickness of the wall cylinder is less than or equal to $\frac{1}{15}$ to $\frac{1}{20}$ of its internal diameter the cylinder is known as thin cylinder

If it is greater than $\frac{1}{20}$ of its internal diameter the cylinder is known as thick cylinder.

If $\frac{t}{d} < \frac{1}{20} \Rightarrow$ Thin cylinder

$\frac{t}{d} > \frac{1}{20} \Rightarrow$ Thick cylinder.

2) In case of thin cylinders stress distribution is uniform over the thickness of the wall.

3) Due to internal pressure of the fluid in thin cylinder may two types of stresses will be developed

1) circumferential stress or hoop stress

2) Longitudinal stress.

The stress acting along the circumference of the cylinder is known as circumferential or hoop stress

The stress acting along the length of the cylinder is known as longitudinal stress.

circumferential or hoop stress $\sigma_1 = \frac{Pd}{2t}$

longitudinal stress $\sigma_2 = \frac{Pd}{4t}$

where p = pressure of the fluid

d = internal dia of the cylinder

t = thickness of the cylinder

Maximum shear stress!

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{Pd}{8t}$$

$$\tau = \frac{Pd}{8t}$$

Effect of internal pressure on the dimensions of a thin cylindrical shell :-

$$\varepsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$\frac{\delta d}{d} = \frac{Pd}{2tE} - \mu\frac{Pd}{4tE} = \frac{Pd}{2tE}\left[1 - \frac{\mu}{2}\right]$$

$$\delta d = \frac{Pd^2}{2tE}\left[1 - \frac{\mu}{2}\right]$$

$$\therefore \varepsilon_1 = \frac{Pd}{2tE}\left[1 - \frac{\mu}{2}\right] \quad \text{or} \quad \delta d = \frac{Pd^2}{2tE}\left[1 - \frac{\mu}{2}\right]$$

$$\varepsilon_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} = \frac{Pd}{4tE} - \mu\frac{Pd}{2tE}$$

$$\varepsilon_2 = \frac{Pd}{2tE}\left[\frac{1}{2} - \mu\right] \quad \text{or} \quad \delta L = \frac{PdL}{2tE}\left[\frac{1}{2} - \mu\right]$$

Volumetric strain:

$$\frac{\delta v}{V} = 2e_1 + e_2$$

where $e_1 = \frac{Pd}{2tE}\left[1 - \frac{\mu}{2}\right]$

$e_2 = \frac{Pd}{2tE}\left[\frac{1}{2} - \mu\right]$

$\therefore$ change in volume

$$\delta v = V\left[2e_1 + e_2\right]$$

Thin spherical shell:-

$$\sigma_1 = \sigma_2 = \frac{Pd}{4tE}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = 0$$

change in dimensions:

$$e_1 = \frac{\delta d}{d} = \frac{Pd}{4tE}\left[1-\mu\right] \Rightarrow \delta d = \frac{Pd^2}{4tE}\left[1-\mu\right]$$

$$e_2 = \frac{\delta L}{L} = \frac{Pd}{4tE}\left[1-\mu\right] \Rightarrow \delta L = \frac{PdL}{4tE}\left[1-\mu\right]$$

Volumetric strain.

$$\frac{\delta V}{V} = 3e_1$$

$$e_1 = \frac{Pd}{4tE}\left[1-\mu\right]$$

$$\delta V = V.3.e_1$$

Problems

1. Calculate the bursting pressure for a cold drawn seamless steel tubing of 60mm inside diameter with 2mm wall thickness. The ultimate strength of steel is $380 MN/m^2$.

   [Ans: P = 25.33 MPa]

2. Calculate the thickness of the metal required for a cast-iron main 800mm in diameter for water at a pressure head of 100m if the max permissible tensile stress is $20 MN/m^2$ and weight of water is $10 kN/m^3$.

   [Ans: 19.62 mm (or) 20mm]

3. A cylindrical water tank of height 25m, inside diameter 2.2m, having vertical axis is open at the top. The tank is made of steel having yield stress of $210 MN/m^2$. Determine the thickness of steel used when the tank is full of water.

   Given: Efficiency of the longitudinal joint = 70%, F.S = 3.

   [Ans: t = 5.6mm say t = 6mm]

4. A boiler shell is to be made of 15mm thick plat having tensile stress of $120 MN/m^2$. If the Efficiences of the longitudinal and circumferential joints are 70% and 30% respectively. determine!

(i) Maximum Permissible diameter of the shell for an internal pressure of $2 MN/m^2$.

(ii) Permissible intensity of internal pressure when the shell diameter is 1.5m.

[Ans: (i) d = 1.26m (1260mm) (ii) P = 1.44 MPa]

5. A cylindrical air drum is 2.25m in diameter with plates 1.2cm thick. The Efficiencies of the longitudinal and circumferential joints are respectively 75% and 40%. If the tensile stress in the plating is to be limited to $120 MN/m^2$. find the maximum safe air pressure.

[Ans: P = 0.96 MPa]

6. A cylindrical vessel whose ends are closed by means of rigid flange plates is made of steel plate 3mm thick. The internal length and diameter of vessel are 50cm and 25cm respectively.

Determine the longitudinal and circumferential stresses in the cylindrical shell due to an internal fluid pressure of $3 MN/m^2$. Also calculate increase in length, diameter and volume of the vessel.

Take $E = 200 GN/m^2$ and $\mu = 0.3$

[Ans: $\sigma_1 = 125 MN/m^2$; $\sigma_2 = 62.5 MPa$; $\delta d = 0.01328 m$ (increase)

$\delta l = 0.0625 mm$ (increase) $\delta V = 29150 mm^3$ (increase)]

7. A built up cylindrical shell of 300mm diameter, 3m long and 6mm thick is subjected to an internal pressure of $2 MN/m^2$. Calculate the change in length, diameter and volume of the cylinder under that pressure if the efficiencies of the longitudinal and circumferential joints are 80% and 50% respectively. $E = 200 GN/m^2$; $m = 3.5$.

[Ans: $\sigma_c = 62.5 MPa$; $\sigma_l = 50 MPa$, $\delta d = 0.0723 mm$ (increase)

$\delta l = 0.483 mm$ (increase), $\delta V = 136300 mm^3$ (increase)]

8. A copper cylinder 90cm long, 40cm external diameter and wall thickness of 6mm has its both ends closed by rigid blank flanges. It is initially

fully of oil at atmospheric pressure. Calculate the additional volume of oil which must be pumped into it in order to raise the oil pressure $5 MN/m^2$ above atmospheric pressure. For copper assume $E = 100 GN/m^2$ and poisson's ratio $= \frac{1}{3}$. Take Bulk modulus of oil as $2.6 GN/m^2$.

[Ans: $\delta V_1 = 301298.56 mm^3$, $\delta V_2 = 204640.92 mm^3$,

$\delta V = 505939.5 mm^3$]

9. A cylindrical shell 90cm long and 20cm internal diameter having thickness of metal as 8mm is filled with fluid at atmospheric pressure. If an additional $20 cm^3$ of fluid ~~at~~ is pumped into the cylinder, find (i) the pressure exerted by the fluid on the cylinder. and (ii) the hoop stress induced.

$E = 2\times10^5 N/mm^2$

74.45

[Ans: $P = 5.958$ Mpa ; $\sigma_1 = \cancel{78.59}$ MPa] $\mu = 0.3$

([illegible])

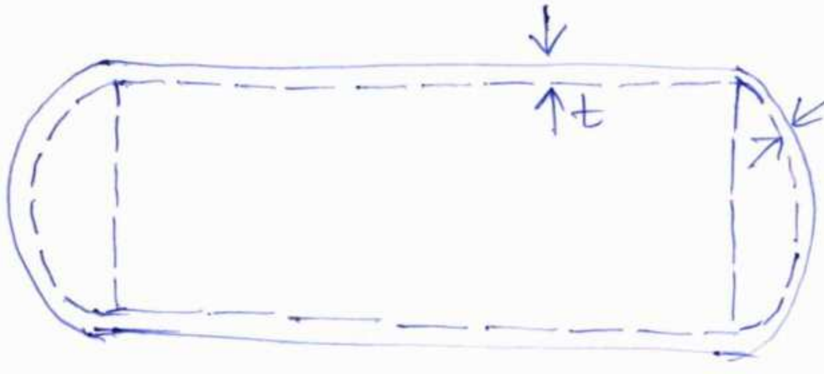
74.45

10. Calculate the increase in volume of a spherical shell 1m in diameter and 1cm thick when it is subjected to an internal pressure of $1.6 MN/m^2$. Take $E = 200 GN/m^2$ and $\frac{1}{m} = 0.3$. [Ans: $\delta V = 219911.48 mm^3$]

11. A thin spherical shell 1m in diameter with its wall of 1.2cm thickness is filled with a fluid at atmospheric pressure. What intensity of pressure will be developed in it if $175 cm^3$ more of fluid is pumped in to it? Also, calculate the circumferential stress at that pressure and the increase in diameter. Take $E = 200 GN/m^2$ and $\frac{1}{m} = 0.3$

[Ans: $P = 1.522 MN/m^2$; $\delta d = 0.1109 mm$; $\sigma_c = 31.714 MPa$]

Thin cylinder with spherical ends:-



Let 't' be the thickness of the cylindrical portion and 't'' of the hemispheric portion of the shell. The internal diameter may be taken as 'd' both for the cylinder and for the spherical ends.

Stresses in the cylinder:-

Hoop stress, $\sigma_c = \frac{pd}{2t}$ and longitudinal stress $\sigma_l = \frac{pd}{4t}$

Hoop strain, $\varepsilon_c = \frac{\sigma_c}{E} - \mu \frac{\sigma_l}{E}$

$$\varepsilon_c = \frac{pd}{2tE}\left[1 - \frac{\mu}{2}\right]$$

Stresses in the Hemispherical portion:-

Hoop stress, $\sigma_c = \frac{pd}{4t'}$

Hoop strain, $\varepsilon = \frac{pd}{4t'E}(1-\mu)$

If there is no distortion of the junction under pressure

$$\varepsilon_c = \varepsilon$$

$$\frac{\cancel{pd}}{\cancel{2}t\cancel{E}}\left[1 - \frac{\mu}{2}\right] = \frac{\cancel{pd}}{\underset{2}{\cancel{4}}t'\cancel{E}}[1-\mu]$$

$$\frac{1}{t} - \frac{\mu}{2t} = \frac{1}{2t'} - \frac{\mu}{2t'} \Rightarrow \boxed{\frac{t'}{t} = \frac{[1-\mu]}{2-\mu}}$$

If μ is taken as 0.3, $\frac{t'}{t} = \frac{7}{17}$

Then $\sigma = \frac{pd}{4t'} = \frac{pd}{4\left(\frac{7}{17}\right)t} = \frac{17pd}{28t} = \frac{17}{14}\left(\frac{pd}{2t}\right)$

which is greater than the hoop stress in the cylinder, $\frac{pd}{2t}$

For maximum stress to be equal

$$\boxed{\frac{t'}{t} = \frac{1}{2}}$$

1) A cylindrical boiler drum has hemispherical ends. The cylinder portion is 1.6m long, 800mm in diameter and 20mm thick. After filling it with water at atmospheric pressure, it is put on a hydraulic test and the pressure is raised to 12MPa. Find the additional volume of water required to be filled in the drum at this pressure. Assume the hoop strain at the junction of cylinder and the hemisphere to be the same for both.

$E = 205$ GPa, $k = 2080$ MPa and $\mu = 0.3$

[Ans: Increase in capacity ~~to~~ in cylinder $(\delta V_1) = 1.789 \times 10^6$ mm^3

" " " sphere $(\delta V_2) = 0.8 \times 10^6$ mm^3

Increase in Volume of water $(\delta V_3) = 6.187 \times 10^6$ mm^3

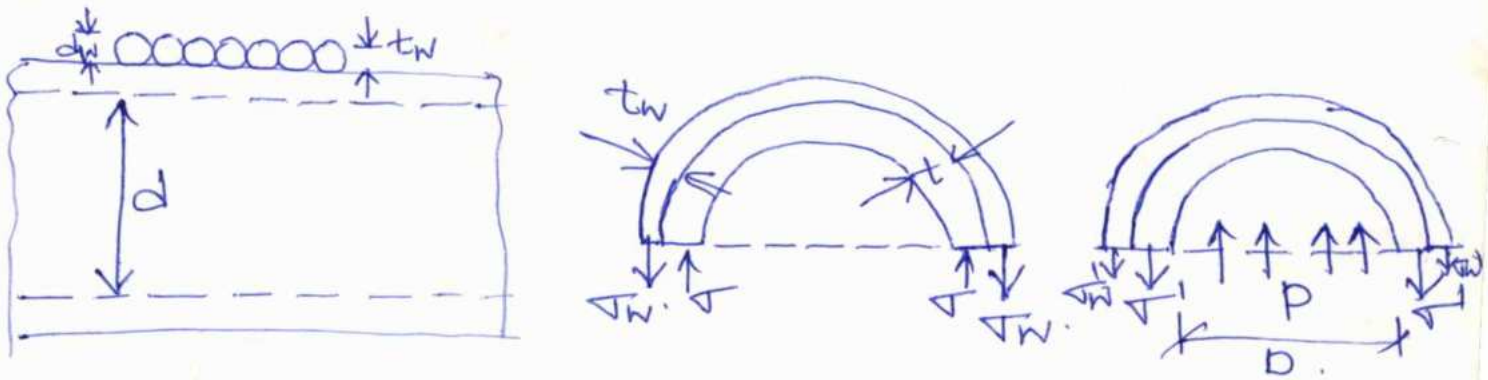
$\therefore$ Total Increase $(\delta V) = 8.776 \times 10^6$ mm^3.]

Wire Winding of thin cylinders

A tube can be strengthened against the internal pressure by winding it with wire under tension and putting the tube wall in compression.

As the pressure is applied, the resultant hoop stress produced is much less as it would have been in the absence of wire.

The analysis of wire wound cylinders is made on the assumption that one layer of wire of diameter d_w is closely wound on the tube with an initial tension T. The procedure is as follows:



1. Initial Tensile stress in the wire, $\sigma_w = \dfrac{T}{\frac{\pi}{4}d_w^2} = \dfrac{4T}{\pi d_w^2}$

2. Replace the wire by a wire of Rectangular c/s of thickness t_w and width d_w having the same c/s area of circular wire. Thus.

$$t_w \cdot \cancel{d_w} = \frac{\pi}{4} d_w^{\cancel{2}}$$

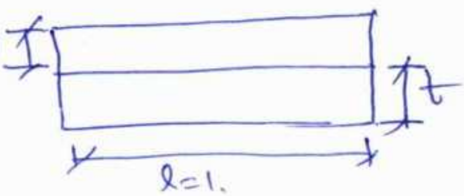
$$t_w = \frac{\pi d_w}{4} \rightarrow (1)$$

Thus now the cylinder is assumed to be wound with a rectangular wire of width d_w and thickness t_w.

2. For unit axial length of the cylinder

The initial compressive hoop stress σ in the cylinder can be found by equating the compressive circumferential force in the cylinder to tensile force in the wire for a unit axial length i.e.

$$(t \cdot 1) \cdot \sigma = (t_w \cdot 1) \cdot \sigma_w$$



$$\sigma = \frac{t_w \cdot \sigma_w}{t} \rightarrow (2)$$

3. Stresses due to fluid pressure alone

On applying an internal pressure p, let the stresses be σ' tensile (hoop) in the cylinder and σ_w' tensile in the wire due to fluid pressure alone. Then for Equilibrium

Resisting force in the cylinder and wire = Fluid force on projected area.

$$(2t \cdot 1)\sigma' + (2t_w \cdot 1) \cdot \sigma_w' = p \times d$$

$$2t\sigma' + 2t_w \sigma_w' = p \cdot d \rightarrow (3)$$

4. Equating the circumferential strains of the wire and the cylinder.

$$\frac{\sigma'}{E} - \mu\frac{\sigma_l'}{E} = \frac{\sigma_w'}{E_w} \longrightarrow (4)$$

On solving eq(3) and eq(4), σ' and σ_w' can be determined.

Final stresses ~~can be~~ are calculated by taking the algebraic sum of the initial stresses and stresses due to fluid pressure.

Final stresses: In the pipe $= \sigma' - \sigma$

In the wire $= \sigma_w' + \sigma_w$.