

UNIT-I: WAVE OPTICS

INTERFERENCE

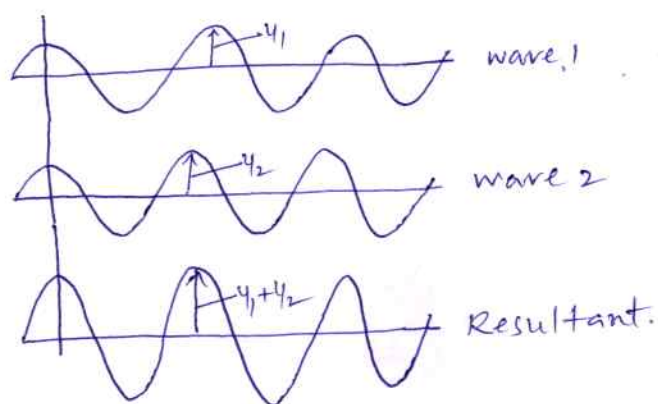
principle of superposition:

The principle of superposition states that when two or more number of light waves are combined at a point then resultant displacement is algebraic sum of the individual displacements of light waves.

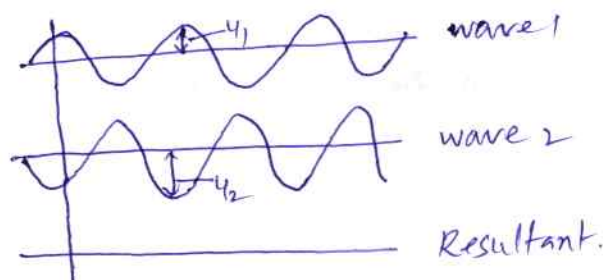
Explanation: Consider two waves travelling in medium with displacements y_1 & y_2 and combined at a point to produce resultant displacement y . A/c to superposition principle the resultant displacement y is the sum of two displacements when they travel in same direction and difference when travel opposite.

$$y = y_1 \pm y_2$$

Case(1): $y = y_1 + y_2$, when travel in same direction.



Case(2): $y = y_1 - y_2$ when travel in opposite direction.



Case(3): If y_1 and y_2 are in phase difference θ then

$$y^2 = y_1^2 + y_2^2 + 2y_1y_2 \cos \theta.$$

Interference :

Modification of intensity due to superposition of two or more number of light waves at a point in medium is called Interference.

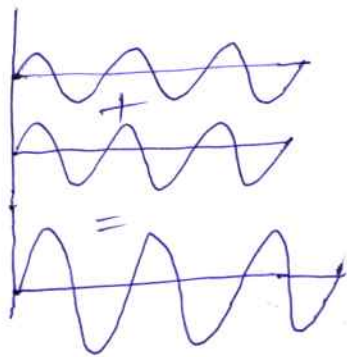
Examples : Rainbow, colours of soap bubble, colours on surface of oil films

Experimental example : Newton rings, wedge method, Young's double slit etc.

Types of Interference :

1. **Constructive interference :** Constructive interference is when two waves travelling in the same direction overlap and their crests and troughs combine to produce a larger wave.

In this interference a bright fringe is obtained on the screen.



Conditions of Constructive interference :

Path difference $\Delta = 0, \lambda, 2\lambda, 3\lambda, \dots, n\lambda$

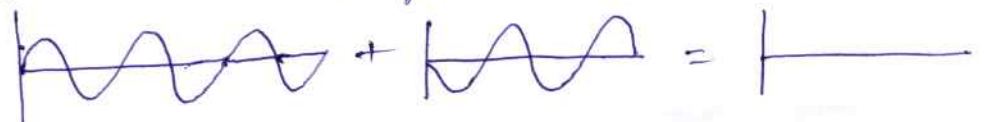
Phase difference $\phi = 0, 2\pi, 4\pi, 6\pi, \dots, 2n\pi$

Intensity of Interference $I = 4a^2 \cos^2\left(\frac{\phi}{2}\right)$

Bright fringe intensity $I = 4a^2$

2. **Destructive Interference :** Destructive Interference is when two waves travelling in the same direction are aligned at the crest of one wave and the trough of the other. The waves cancel out and the amplitude of resultant wave becomes zero.

In this interference a dark fringe is obtained on the screen.



Conditions of destructive interference:

path difference $\Delta = \lambda/2, 3\lambda/2, 5\lambda/2, \dots (2n+1)\lambda/2$

Phase difference $\phi = \pi, 3\pi, 5\pi, \dots (2n+1)\pi$

Dark fringe Intensity $I = 0$.

Conditions for Interference:

1. The two sources should be coherent
2. The two sources should be monochromatic
3. The two sources should be narrow
4. The two sources should emit continuous light of same wavelength λ
5. The distance b/w the two sources should be small
6. The distance b/w the two sources and screen should be large
7. To view interference fringes, the background should be dark
8. The amplitude of the light waves should be equal

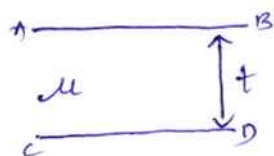
Thin films: A medium formed b/w two surfaces with a refractive index and micrometer thickness is called thin film.

Examples: oil film on surface of water, soap bubble, air film at the time of rainbow.

Types of thin films:

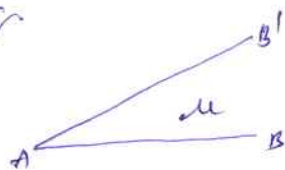
1. parallel thin films:

A thin film formed with uniform or constant thickness is called parallel thin film. A parallel thin film formed with thickness t and refractive index μ is shown in fig.



2. Non parallel thin film:

A Thin film formed with varying or non-uniform thickness is called non parallel thin film. A non parallel thin film formed with refractive index μ is shown in fig.

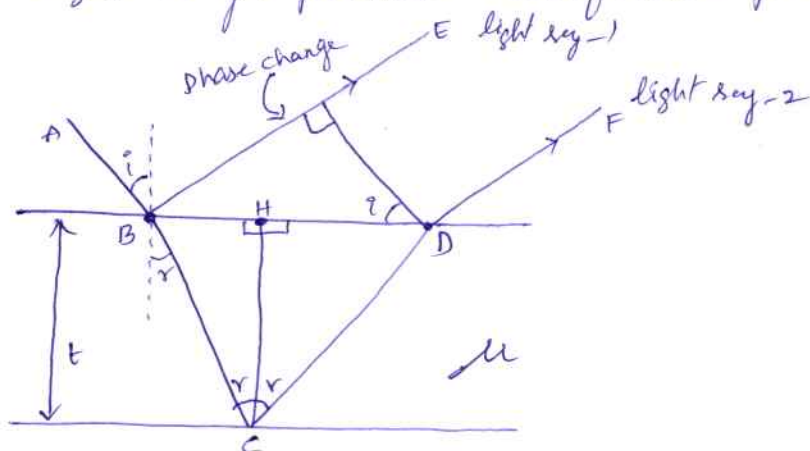


Applications of thin films:

- Thin films are used commercially in anti-reflection coatings, mirrors, and optical filters.
- These films are created through deposition processes in which material is added to a substrate in a controlled manner. Methods include chemical vapour deposition and various physical vapour deposition techniques.

Interference in Thin film by Reflection:

When a white light is reflected from thin films like soap bubbles, they produce variety of colours due to interference. When the light is reflected from the upper and lower boundaries of a thin film the reflected light rays produce interference pattern.



- consider a thin film of thickness 't' with refractive index ' μ ' &
- Let the light AB is incident on the thin film at 'B'
- BE is reflected light called as light ray-1
- BC is refracted light and reflected at 'C' and passes as CF called as light ray 2.
- Be Light rays 1 & 2 produce interference.

The path difference b/w light rays 1 & 2 is

$$= \mu (BC + CD) \text{ in film} - BG \text{ in air} \quad \text{--- (1)}$$

To calculate BC & CD :-

$$\text{From (i) } \Delta BCH; \cos r = \frac{HC}{BC} = \frac{t}{BC}$$

$$BC = \frac{t}{\cos r} \quad \text{--- (2)}$$

$$\text{(ii) } \Delta DCH; \cos r = \frac{HC}{CD} = \frac{t}{CD}$$

$$CD = \frac{t}{\cos r} \quad \text{--- (3)}$$

$$\therefore BC + CD = \frac{t}{\cos r} + \frac{t}{\cos r} = \frac{2t}{\cos r} \quad \text{--- (4)}$$

To calculate BG :-

From the figure, we have

$$BD = BH + HD \quad \text{--- (5)}$$

$$\text{From (i) } \Delta BHC; \tan r = \frac{BH}{HC} = \frac{BH}{t}$$

$$BH = t \tan r$$

$$\text{(ii) } \Delta DHC; \tan r = \frac{HD}{HC} = \frac{HD}{t}$$

$$HD = t \tan r$$

$$\therefore BD = t \tan r + t \tan r = 2t \tan r \quad \text{--- (6)}$$

From ΔBGD : we have

$$\sin i = \frac{BG}{BD}$$

$$BG = BD \sin i$$

$$BG = 2t \tan r \sin i \quad \text{--- (7)}$$

$$\text{From Snell's law: } \mu = \frac{\sin i}{\sin r}$$

$$\sin i = \mu \sin r$$

Substituting the values of $\sin i$ in eq (7)

$$\therefore BG = 2t \tan r \mu \sin r$$

$$BG = 2\mu t \tan r \sin r \quad \text{--- (8)}$$

$\therefore$ From eq (1) we have

$$\text{path difference} = \mu \left(\frac{2t}{\cos r} \right) - 2\mu t \tan r \sin r$$

$$= \frac{2\mu t}{\cos r} - 2\mu t \frac{\sin r}{\cos r} \sin r$$

$$= \frac{2\mu t}{\cos r} [1 - \sin^2 r]$$

$$= \frac{2\mu t}{\cos r} \cos^2 r$$

$$= 2\mu t \cos r \quad \text{--- (9)}$$

This is calculated the path difference in the case of reflected.
When light ray BE undergoes an additional phase change of π (or)
an additional path difference of $\lambda/2$.

$$r^2 = 2Rt - t^2$$

As 't' is small, t^2 will be negligible

$$r^2 = 2Rt$$

$$t = \frac{r^2}{2R}$$

(i) condition for bright rings:

$$2t = (2n-1)\lambda/2$$

$$2\left(\frac{r^2}{2R}\right) = (2n-1)\lambda/2$$

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

Replacing r by $D/2$, the diameter of n^{th} bright rings will be

$$\frac{D^2}{4} = \frac{(2n-1)\lambda R}{2}$$

$$D^2 = (2n-1) 2\lambda R$$

$$D = \sqrt{2n-1} \sqrt{2\lambda R}$$

$$D \propto \sqrt{2n-1}$$

$$D \propto \sqrt{\text{odd natural numbers.}}$$

So, the diameter of bright rings are proportional to the square root of odd natural numbers.

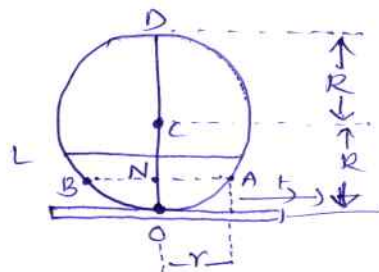
(ii) condition for dark ring:

$$2t = n\lambda$$

$$2\left(\frac{r^2}{2R}\right) = n\lambda$$

$$r^2 = n\lambda R$$

Replacing by $D/2$, the diameter of n^{th} dark ring will be



The path difference b/w two rays $= 2\mu t \cos r + \lambda/2$

For air film $\mu=1$ and for normal incidence $r=0$,

the path difference $= 2t + \lambda/2$

At the point of contact $t=0$,

the path difference is $\lambda/2$.

i.e. The incident and reflected lights are out of phase and destructive interference occurs. The centre spot is always dark.

(i) Condition for bright ring:-

$$2t + \lambda/2 = n\lambda$$

$$2t = n\lambda - \lambda/2$$

$$2t = (2n-1)\lambda/2 \quad \text{where } n = 1, 2, 3, \dots$$

(ii) Condition for dark ring:-

$$2t + \lambda/2 = (2n+1)\lambda/2$$

$$2t = (2n+1)\lambda/2 - \lambda/2$$

$$2t = n\lambda \quad \text{where } n = 0, 1, 2, 3, \dots$$

Theory of Newton's rings:

To find the diameters of dark and bright rings, let 'L' be a lens placed on a glass plate P. The convex surface of the lens is the part of spherical surface with centre at 'C'. Let R be the radius of curvature and r be the radius of Newton's rings corresponding to the film thickness 't'.

From the property of the circle, $NA \times NB = ND \times ND$

$$r \times r = t \times (2R - t)$$

Now, the total path difference = $2\mu t \cos r + \lambda/2$

(i) condition for n^{th} bright fringe:-

$$2\mu t \cos r + \lambda/2 = n\lambda$$

$$2\mu t \cos r = n\lambda - \lambda/2$$

$$= (2n-1)\lambda/2$$

where $n = 0, 1, 2, 3, \dots$

(ii) condition for n^{th} dark fringe:-

$$2\mu t \cos r + \lambda/2 = (2n+1)\lambda/2$$

$$2\mu t \cos r = (2n+1)\lambda/2 - \lambda/2$$

$$= n\lambda$$

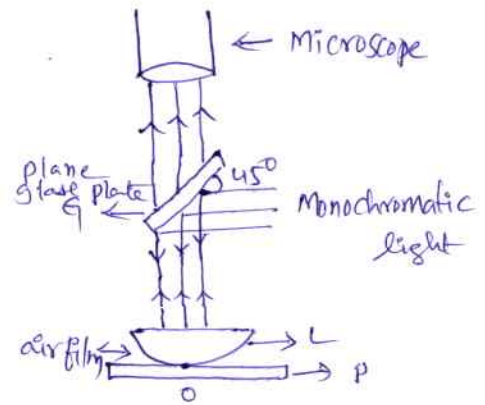
where $n = 0, 1, 2, 3, \dots$

Newton's rings:

Newton's rings are one of the best examples for the interference in a non uniform thin film. When a planoconvex lens with its convex surface is placed on a plane glass plate, an air film of increasing thickness is formed b/w the two. At the point of contact, the thickness of the film is zero. If monochromatic light is allowed to fall normally and the film is viewed in the reflected light, we observe alternate bright and dark rings around the point of contact. These rings are known as Newton's rings.

Experimental arrangement:

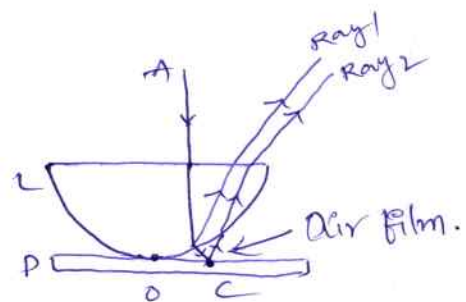
- The experimental arrangement of Newton's rings is shown in fig
- The planoconvex lens (L) of large radius of curvature is placed with its convex surface on a plane glass plate (P). The planoconvex lens touches the glass plate at O.



- The monochromatic light falls on a glass plate G held at an angle of 45° with the vertical. The glass plate G reflects normally a part of the incident light towards the air film enclosed by the lens L and the glass plate P.
- The light rays reflected from the front and back surface of the film. These reflected rays interfere and give rise to an interference pattern in the form of circular rings. These rings are seen through a travelling microscope.

Explanation of Newton's rings:

A part of the incident monochromatic light AB is reflected at B in the form of ray (1) with an additional phase change. The other part of the light is reflected refracted through BC. Then at C, it is again reflected in the form of ray (2) with additional phase change of π or path change of $\lambda/2$.



$$\frac{D^2}{4} = n\lambda R$$

$$D^2 = 4n\lambda R$$

$$D = 2\sqrt{n\lambda R}$$

$$D \propto \sqrt{n}$$

$$D \propto \sqrt{\text{natural numbers.}}$$

So, the diameter of dark rings are proportional to the square root of natural numbers.

With increase in the order (n), the rings get closer and the fringe width decreases. and $\rightarrow$

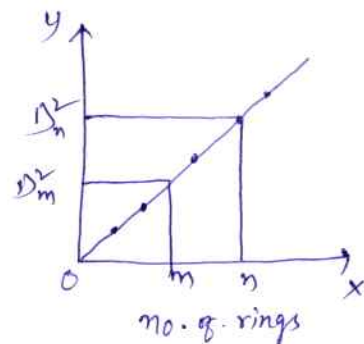
Determination of wavelength of light source:

Let R be the radius of curvature of a planoconvex lens, λ be the wavelength of light used. Let D_m and D_n are the diameters of m^{th} and n^{th} dark rings respectively. Then.

$$D_m^2 = 4m\lambda R \text{ and } D_n^2 = 4n\lambda R$$

$$D_n^2 - D_m^2 = 4(n-m)\lambda R$$

$$\lambda = \frac{D_n^2 - D_m^2}{4(n-m)R}$$



Newton's rings are formed with suitable experimental setup. With the help of a travelling microscope, the readings for different orders of dark rings were noted from one edge of the rings to the other edge. The diameters of different orders of the rings can be known. A plot b/w D^2 and the no. of rings gives a straight line as shown in fig.

From the graph,

$$\frac{D_n^2 - D_m^2}{n - m} = \frac{AB}{CD}$$

The radius R of the planoconvex lens can be obtained with the help of a spherometer. using these values in the formula λ can be calculated.

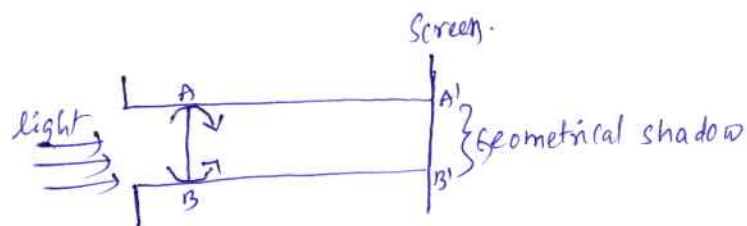
Engineering applications of Interference:

1. Determination of wavelength of light source, radius of curvature
2. Determination of refractive index of a transparent liquid
3. Determination of diameter of thin wire
4. Finding the thickness of thin transparent glass plate.
5. Data read and writing of CD/DVD
6. Construction of hologram in holography.
7. Signaling and image processing
8. To check the flatness of surface.

DIFFRACTION

Diffraction: Bending of light at the edges of obstacles to form bright and dark fringes in geometric shadow region is called diffraction.

Explanation:



The light from source is allowed to pass through the slit AB . as shown in ~~below~~ fig. when the slit width (e) is approximately equal to wavelength (λ) then the light bends at the edges of slit and diffracted. Due to diffraction alternate bright and dark fringes formed on the screen in geometric shadow.

Classification of diffraction:

The diffraction phenomenon is classified into general classes.

1. Fresnel Diffraction: Both source of light and screen on which the diffraction pattern is observed are at finite distance from the obstacle is Fresnel diffraction
2. Fraunhofer Diffraction: Both, source of light and screen on which the diffraction pattern is observed are at finite distance from the obstacle is Fraunhofer diffraction.

Fresnel's Diffraction

1. For diffraction to occur the light source and screen are at finite distance from the obstacle.
2. To study the diffraction, lenses are not necessary
3. Study of diffraction is complicated.
4. Diffraction can be studied only in the direction of propagation of light
5. The incident wavefronts are either spherical or cylindrical.

Fraunhofer diffraction

1. For diffraction to occur, the light source and screen are at finite distance from obstacle.
2. To study the diffraction, lenses are necessary
3. Study of diffraction is easy.
4. Diffraction can be studied in any direction of propagation.
5. The incident wavefronts are plane.

Difference b/w Interference and Diffraction.

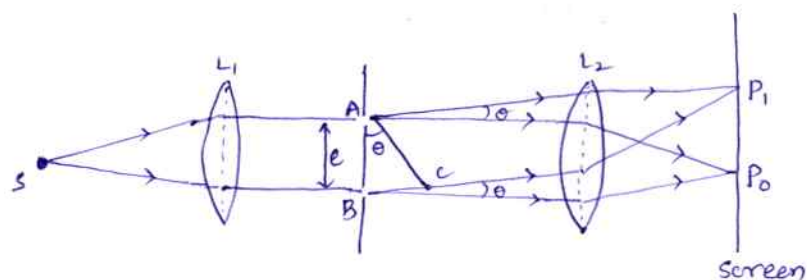
Interference

1. Interference is the superposition of primary wave fronts.
2. Intensity of all bright fringes are same
3. All the dark fringes intensity is zero
4. Interference fringe width is constant.

Diffraction.

1. Diffraction is the superposition of Secondary wave fronts
2. Intensity of all the diffracted bright fringes are not equal
3. All the dark fringes intensity is not completely zero
4. Diffraction fringe width is not constant.

Fraunhofer Diffraction by a narrow single slit:



- Consider a single slit AB and size is 'e'
- Consider a source 's' and placed L_1 & L_2
- When light is incident on lens L_1 and from here light rays travelled parallelly. So the light rays travelling without deviation, will focus on the screen at P_0 . Here we will get maximum intensity
- Now travelled at deviation of ' θ ' and these light rays are focused at point P_1 on the screen.
- When P_1 is bright or dark will depends on path difference b/w two light rays
- In order to calculate path difference

$$\text{path difference, } \Delta ABC \Rightarrow \sin \theta = \frac{BC}{AB}$$

$$\sin \theta = \frac{BC}{e}$$

$$\therefore BC = e \sin \theta \quad \text{--- (1)}$$

- The phase difference b/w two light rays

$$\phi = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$\phi = \frac{2\pi}{\lambda} e \sin \theta \quad \text{--- (2)}$$

Let the width of the slit divided into 'n' equal parts and the amplitude of the wave from each part is 'a'.

So the phase difference b/w two consecutive points

$$= \frac{1}{n} (\text{total phase})$$

$$= \frac{1}{n} \left[\frac{2\pi}{\lambda} \cdot e \sin \theta \right] = d \quad \text{say} \quad \text{--- (3)}$$

Then the resultant amplitude R is calculated by using the method of vector addition of amplitudes.

$$R = a \frac{\sin\left(\frac{n\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$$

$$R = a \frac{\sin\left(\frac{n}{2} \pm \frac{2\pi}{\lambda} e \sin \theta\right)}{\sin\left(\frac{1}{2} \pm \frac{2\pi}{\lambda} e \sin \theta\right)}$$

$$R = a \frac{\sin\left(\frac{\pi}{\lambda} e \sin \theta\right)}{\sin\left(\frac{\pi}{n\lambda} e \sin \theta\right)}$$

$$R = a \frac{\sin \alpha}{\sin \frac{\alpha}{n}} \quad \text{where } \alpha = \frac{\pi}{\lambda} \cdot e \sin \theta.$$

$$R = a \frac{\sin \alpha}{\frac{\alpha}{n}} \quad (\alpha/n \text{ is very small, } \sin \frac{\alpha}{n} = \frac{\alpha}{n})$$

$$R = na \frac{\sin \alpha}{\alpha}$$

$$R = A \frac{\sin \alpha}{\alpha} \quad \text{--- (4)} \quad (na = A)$$

$\therefore$ The intensity is given by

$$I = R^2 = A^2 \frac{\sin^2 \alpha}{\alpha^2} \quad \text{--- (5)}$$

Case (1): principal maximum;

$$\text{From eq (4) } R = A \frac{\sin \alpha}{\alpha}$$

Takes max value for $\alpha = 0$

$$\Rightarrow \alpha = \frac{\pi}{\lambda} e \sin \theta = 0$$

$$\sin \theta = 0$$

$$\theta = 0$$

$\therefore \theta = 0$ means, the max intensity is formed at θ_0 and is known as principal maximum.

Intensity of principal maximum.

$$R_{\max} = \lim_{\alpha \rightarrow 0} A \frac{\sin \alpha}{\alpha}$$

$$R_{\max} = A \lim_{\alpha \rightarrow 0} \frac{\sin \alpha}{\alpha}$$

$$R_{\max} = A \cdot 1$$

$$R_{\max} = A$$

$$I_{\max} = R_{\max}^2 = A^2 \quad \text{--- (6)}$$

Case (ii): Minimum intensity position:

Intensity (I) will be minimum, when $\sin \alpha = 0$

$$\therefore \alpha = \pm \pi, \pm 2\pi, \pm 3\pi, \dots \pm m\pi$$

$$\alpha = \frac{\pi}{\lambda} e \sin \theta = \pm m\pi$$

$$e \sin \theta = \pm m\pi \quad \text{--- (7)} \quad (m = 1, 2, 3, \dots)$$

In the above equation (7) $m=0$ is not applicable because corresponds to principal max. Therefore the positions A/c to eq (7) are on either side of the principal max.

Case (iii): Secondary maxima:

In between these minima, we get secondary maxima

From eq (5)

$$I = A^2 \frac{\sin^2 \alpha}{\alpha^2} = 0$$

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha} A^2 \left(\frac{\sin^2 \alpha}{\alpha^2} \right) = 0$$

$$\frac{dI}{d\alpha} = A^2 \frac{2 \sin \alpha}{\alpha} \cdot \left(\frac{\alpha (\cos \alpha - \sin \alpha)}{\alpha^2} \right) = 0$$

$$A^2 \neq 0 : \sin \alpha \neq 0.$$

Because $\sin \alpha = 0$ corresponds to minima positions.

$$\therefore \alpha \cos \alpha - \sin \alpha = 0$$

$$\alpha \cos \alpha = \sin \alpha$$

$$\alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\alpha = \tan \alpha \quad \text{--- (8)}$$

The values of α satisfying the above equation are obtained graphically by plotting the curves $y = \alpha$ and $y = \tan \alpha$ on the same graph

The points of intersections are $\alpha = 0, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \dots$

$$\therefore I = A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2$$

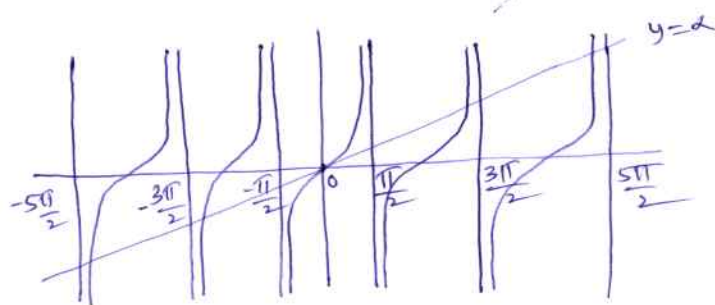
When $\alpha = 0$, $I_0 = A^2$ (principle max)

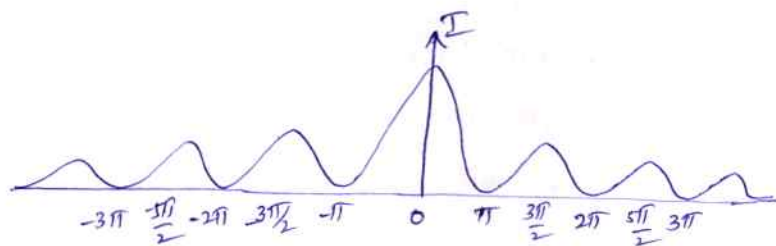
$$\alpha = \pm \frac{3\pi}{2} \Rightarrow I_1 = A^2 \left(\frac{\sin \frac{3\pi}{2}}{\frac{3\pi}{2}} \right)^2$$

$$\alpha = \pm \frac{5\pi}{2} \Rightarrow I_2 = A^2 \left(\frac{\sin \frac{5\pi}{2}}{\frac{5\pi}{2}} \right)^2 \text{ and so on}$$

So, the most of the incident light is concentrated in the principal max and the rest is redistributed into the secondary max.

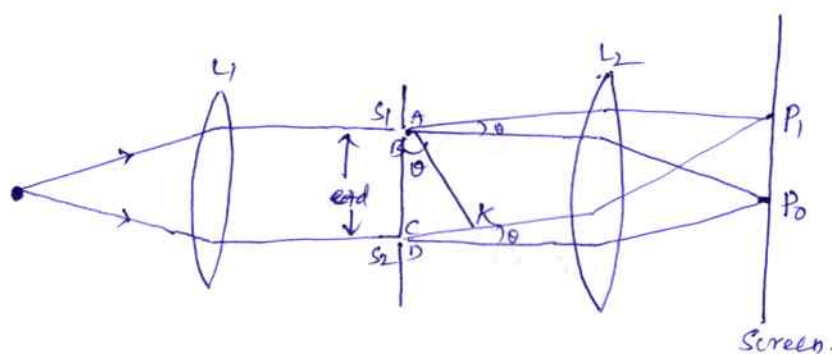
Case (iv): Intensity distribution;





- The diffraction pattern consists of a central principle max for $\alpha = 0$
- There are secondary maxima of decreasing intensity on either of it at positions $\alpha = \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2} \dots$
- Between secondary maxima, there are minima at positions $\alpha = \pm \pi, \pm 2\pi, \pm 3\pi$.

Fraunhofer diffraction due to double slit:



- In this diffraction, two narrow slits are separated by a narrow gap. The narrow slits are AB and CD .
- AB is the first slit and CD is second slit. Size e
- The gap b/w two slits, let us BC has ' d '

$$AB = CD = e$$

$$BC = d$$

- So, the centre of the two slits called as S_1 and S_2 . The gap b/w S_1 and S_2 is $e+d$.
- A monochromatic light rays incident normally on the two slits

→ when light is incident on lens L, and here light rays travelled parallelly. so the light rays travelling without deviation, will focus on the screen at P_0 . Here we will get maximum intensity.

→ Now travelled at deviation of θ and these light rays are focused at point P_1 on the screen.

→ when P_1 is bright or dark, will depends on path difference b/w two light rays.

→ path difference

$$\Delta S_1 S_2 K = \sin \theta = \frac{S_2 K}{S_1 S_2}$$

$$\sin \theta = \frac{S_2 K}{e d}$$

$$S_2 K = (e d) \sin \theta \quad \text{--- (1)}$$

phase difference

$$\phi = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$\phi = \frac{2\pi}{\lambda} (e d) \sin \theta$$

$$\phi = 2\beta$$

→ From figure, the resultant intensity or resultant amplitude is due to two single slits.

→ we know, the study of diffraction due to single slit, the resultant amplitude is

$$R = A \frac{\sin \alpha}{\alpha}$$

→ From figure, we have two slits. each slit has contribution intensity on the screen.

→ Resultant amplitudes $R_1 = A \frac{\sin \alpha}{\alpha}$; $R_2 = A \frac{\sin \alpha}{\alpha}$

The resultant amplitude can be calculated with the help of parallelogram law of Vectors.

$$R = \sqrt{R_1^2 + R_2^2 + 2R_1R_2 \cos \phi}$$

$$R = \sqrt{A^2 \frac{\sin^2 \alpha}{\alpha^2} + A^2 \frac{\sin^2 \alpha}{\alpha^2} + 2A \frac{\sin \alpha}{\alpha} A \frac{\sin \alpha}{\alpha} \cos \phi}$$

$$R = A \frac{\sin \alpha}{\alpha} \sqrt{1+1+2 \cos \phi}$$

$$R = A \frac{\sin \alpha}{\alpha} \sqrt{2+2 \cos \phi}$$

$$R = A \frac{\sin \alpha}{\alpha} \sqrt{2(1+\cos \phi)}$$

$$R = A \frac{\sin \alpha}{\alpha} \sqrt{2 \cdot 2 \cos^2 \phi/2}$$

$$R = A \frac{\sin \alpha}{\alpha} \sqrt{4 \cos^2 \phi/2}$$

$$R = A \frac{\sin \alpha}{\alpha} 2 \cos \frac{\phi}{2}$$

$$R = A \frac{\sin \alpha}{\alpha} 2 \cos \frac{2\beta}{2} \quad (\phi = 2\beta :)$$

$$R = A \frac{\sin \alpha}{\alpha} 2 \cos \frac{2\beta}{2}$$

$$R = 2A \frac{\sin \alpha}{\alpha} \cos \beta.$$

$$\therefore I = R^2 = 4 A^2 \frac{\sin^2 \alpha}{\alpha^2} \cos^2 \beta \longrightarrow (3)$$

From the above expression it is clear that the resultant intensity is the product of two factors.

1. $A^2 \frac{\sin^2 \alpha}{\alpha^2}$ which represents the diffraction pattern due to single slit.
2. $\cos^2 \beta$ which gives the interference pattern due to single slit.

(i) Diffraction effect: The diffraction term $A^2 \frac{\sin^2 \alpha}{\alpha^2}$ gives the principal maxima at the centre of the screen with alternate minima and -

Secondary maxima of decreasing intensity.

we get principal maximum for $\theta = 0$

we get minima for $\sin \alpha = 0$

$$\therefore \alpha = \pm \pi, \pm 2\pi, \pm 3\pi, \dots, \pm m\pi.$$

$$\alpha = \frac{\pi}{2} e \sin \theta = \pm m\pi$$

$$e \sin \theta = \pm m\pi \quad \text{where } m = 1, 2, 3, \dots$$

The positions of Secondary maxima occurs for $\alpha = \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \dots$

(ii) Interference effect:

The interference term $\cos^2 \beta$ gives the equidistant bright and dark fringes.

Maximum: If $\cos^2 \beta = 1$

$$\beta = \pm n\pi \quad n = 0, 1, 2, 3, \dots$$

$$\frac{\pi}{\lambda} (e+d) \sin \theta = \pm n\pi$$

$$(e+d) \sin \theta = \pm n\lambda$$

$$\therefore \text{path difference} = \pm n\lambda$$

Minimum: If $\cos^2 \beta = 0$

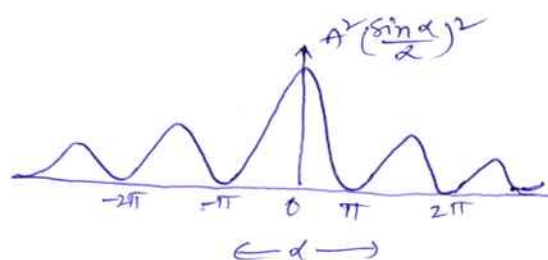
$$\beta = \pm (2n+1) \frac{\pi}{2} \quad n = 0, 1, 2, 3, \dots$$

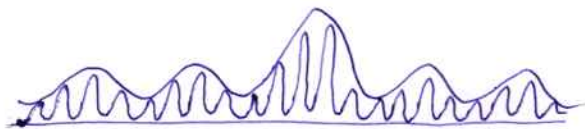
$$\frac{\pi}{\lambda} (e+d) \sin \theta = \pm (2n+1) \frac{\pi}{2}$$

$$(e+d) \sin \theta = \pm (2n+1) \frac{\lambda}{2}$$

$$\therefore \text{path difference} = \pm (2n+1) \frac{\lambda}{2}$$

→



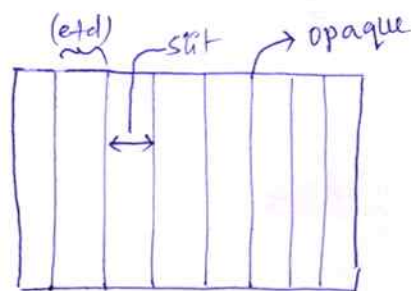


→ From fig it is clear that the resultant minima are not equal to zero, still they have some minimum intensity due to interference.

Diffraction Grating:

A transparent glass plate drawn with equal distance line to form diffraction is called "Diffraction grating."

When light falls on the grating, the light gets diffracted through each slit. As a result, both diffracted and interference of diffracted light gets enhanced and forms a diffraction pattern. This pattern is known as Diffraction spectrum.



Let 'e' be the width of the line and 'd' be the width of the slit.

Then (e+d) is known as grating element. If 'N' is the number of lines per inch on the grating then

$$N(e+d) = 1'' = 2.54 \text{ cm}$$

$$e+d = \frac{1}{N} \text{ (or)} \frac{2.54}{N} \text{ cm}$$

The relation between grating element and the angles of diffracted beams of light is called grating equation and is given by

$$(e+d) \sin \theta = \pm n\lambda$$

$$\frac{1}{N} \sin \theta = \pm n\lambda \quad n = 0, 1, 2, 3, \dots$$

$$\lambda = \frac{\sin \theta}{Nn}$$

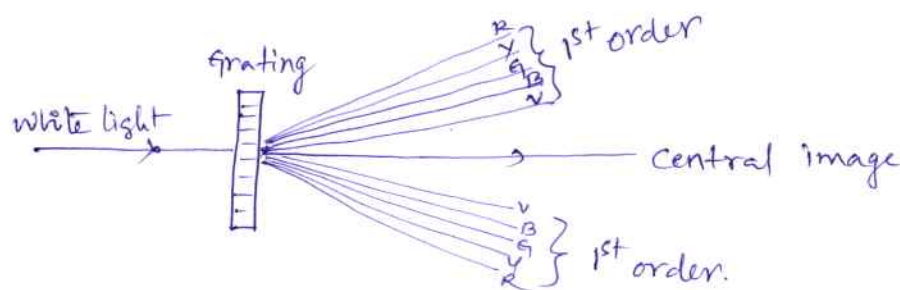
by using the above equation, the wavelength of light can be determined.

Grating Spectrum:

The condition to form the principal maxima in a grating is given by

$$(e+d) \sin \theta = \pm n\lambda$$

where $(e+d)$ is the grating element and the equation is known as grating equation.



- The spectral lines are almost straight and sharp
- most of the incident light intensity goes to zero order image. It will appear as bright
- Spectra of different orders are situated symmetrically on both sides of zero order image
- In each spectrum, spectral colours are in the order from violet to red.
- The spectral lines are more and more dispersed as order increases i.e. $n \propto \Delta \lambda$

→ Maximum number of orders available with a grating is

$$n = \frac{(e+d) \sin \theta}{\lambda}$$

Here n — order of the spectrum,

e — width of each slit

d — width of each opaque part

$(e+d)$ — grating element

θ — angle of diffraction

λ — wavelength of light source.

If $\theta = 90^\circ$ it gives max number of possible orders.

$$\therefore n = (e+d)/\lambda$$

Dispersive power of grating:

Dispersive power of grating is defined as the ratio of the difference in the angle of diffraction of any two neighbouring spectral lines to the difference in wavelength b/w the two spectral lines.

It can also be defined as the difference in the angle of diffraction per unit change wavelength.

The order of the n th order principal maximum for a wavelength λ is given by the equation

$$(e+d) \sin \theta = n\lambda$$

Differentiating the equation with respect to θ & λ

$$(e+d) \cos \theta d\theta = n d\lambda$$

$$\frac{d\theta}{d\lambda} = \frac{n}{(e+d) \cos \theta}$$

$$\frac{d\theta}{d\lambda} = \frac{nN}{\cos \theta} \quad \text{where } (e+d) = \frac{1}{N}$$

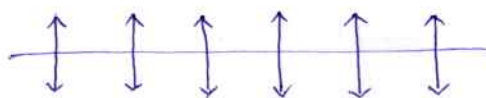
Engineering applications of Diffraction:

- wavelength of light source by using diffraction method
- Crystal Structural Study by x-ray diffraction
- The study of reconstruction of hologram
- study design of 3D images as molecular structures
- Diffraction is used in monochromators, spectrometers, lasers, division multiplexing devices. and many other optical instruments.
- Diffraction is used in the study of DNA, the building-blocks of human genetics.

POLARIZATION

Polarization of light: The light rays that are vibrating in only one direction are known as polarization of light.

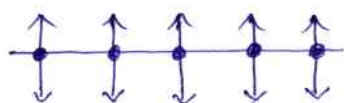
Representation of polarized and unpolarized light:



plane polarized light - parallel to plane of the paper



plane polarized light - perpendicular to the plane of paper.



or



unpolarized light.

Types of polarizations:

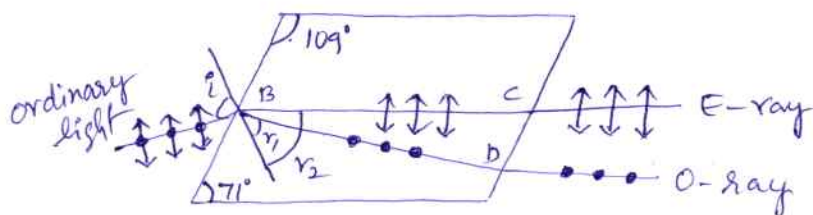
- (i) plane polarized light:— The resultant polarized light after passing through certain crystals, the incident light is confined to a particular plane only and it is known as plane polarized light or linearly polarized light.
- (ii) circularly polarized light:— The polarized light after passing through certain crystals, traces circular paths and is known as circularly polarized light.
- (iii) Elliptically polarized light:— The resultant polarized light ^{after} passing through certain crystals, traces an elliptical path and is known as elliptically polarized light.

Methods of polarizations:

1. Reflection
2. Refraction
3. Double refraction
4. Scattering
5. Selective absorption.

Double Refraction:

"Bending of light ray into two refracted rays instead of one refracted ray is called 'Double refraction'."



- when unpolarized light passes through a calcite crystal, it is split into two refracted rays.
- The one which always obeys the laws of refraction and having vibrations perpendicular to the principal section is known as ordinary ray. The other which does not obey the laws of refraction and has vibrations in the principal section is called extraordinary ray. This phenomenon is known as double refraction and was discovered by Bartholinus.
- Inside the crystal the rays break up into ordinary and extraordinary rays.
- The ordinary ray travelling along BD and makes an angle of refraction r_1 , while extraordinary ray travelling along BC and makes an angle of refraction r_2 .
- The refractive indices of ordinary and extraordinary rays can be expressed as

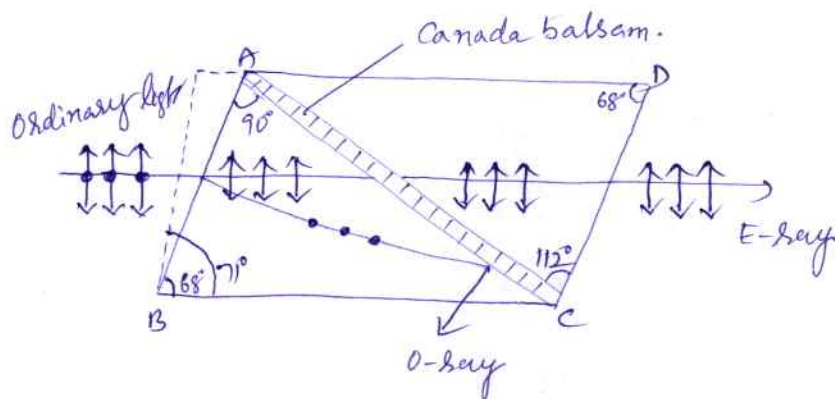
$$\mu_o = \frac{\sin i}{\sin r_1} \quad \text{and} \quad \mu_e = \frac{\sin i}{\sin r_2}$$

→ In case of calcite $n_1 < n_2$, $\mu_o > \mu_e$.

i.e. the velocity of light for ordinary ray inside the crystal will be less than extraordinary ray. It is observed that μ_o is same for all angles of incidence. While μ_e varies with angle of incidence. Therefore, the ordinary ray travels with the same speed in all directions while extraordinary ray has different speeds in different directions.

Nicol prism:

- It is an optical device made from calcite and is used to produce and analyse plane polarized light is called Nicol prism.
- It was invented by William Nicol, a Scotch physicist in 1828
- The Nicol prism is designed in such a way that it can transmit the extraordinary rays and cut off the ordinary rays.
- The emergent beam from a Nicol prism is a plane polarized light.



- The Nicol prism is constructed using a calcite crystal having three times of its width. Grounded angles of principal section of calcite crystal is 68° and 112° instead of 71° & 109° .
- The crystal cut into two parts along AC corners and well polish the faces. Now the two parts are pasted with Canada balsam as a single crystal.

- The Canada Canada balsam has refractive index ($\mu_{cb} = 1.55$) lies b/w ordinary ($\mu_o = 1.658$) and extra ordinary rays ($\mu_{eo} = 1.486$). So it works as rarer and denser medium to fulfill total internal reflection.
- When unpolarized light enter the crystal, it splits into ordinary and extraordinary rays.
- The Canada balsam layer is rarer medium for the ordinary ray. Therefore the ordinary ray travelling from a denser to rarer medium. Hence it undergoes total internal reflection. If the angle of incidence made by the ordinary at the Canada balsam layer is greater than the critical angle.
- The critical angle for the ordinary ray for calcite to Canada balsam layer is

$$\theta_c = \sin^{-1} \left[\frac{1.55}{1.658} \right] = 69.2^\circ.$$
- Under these conditions, the ordinary ray is completely reflected at calcite - balsam surface and is absorbed by the tube containing the Nicol prism.
- The extraordinary ray is not totally reflected because it is travelling from rarer (calcite) to denser (Canada balsam), and hence it travels in a straight line and emerges out from the Nicol prism. The emergent beam is plane polarized light.

Wave plates (Retardation plates):

These plates are made from uniaxial crystal to produce phase difference b/w extraordinary and ordinary waves. These are two types.

1. Quarter wave plate
2. Half wave plate.

1. Quarter wave plate:

- Consider the case of a calcite plate cut with optic axis parallel to the surface. When plane polarized light is incident on calcite crystal, the light split in to ordinary ray and extraordinary ray.
- They travel along the same path with different velocities. velocity of extraordinary ray is greater than the velocity of ordinary ray. As a result a phase difference is introduced b/w them. The thickness of this device is cut in such a way that it can produce a path difference of $\lambda/4$ b/w ordinary and extraordinary rays.

- let μ_o and μ_e be the refractive indices of the extraordinary ray and ordinary rays respectively.

Let 't' be the thickness of the crystal.

Then for a negative crystal ($\mu_o > \mu_e$) and positive crystal ($\mu_o < \mu_e$).

The path difference between extraordinary ray and ordinary ray is

$$\Delta = \mu_o t - \mu_e t = (\mu_o - \mu_e) t \quad \text{--- (1)}$$

If this crystal act as quarter wave plate, then path difference between extraordinary ray and ordinary ray is

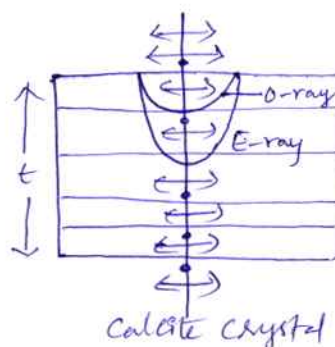
$$\Delta = \lambda/4 \quad \text{--- (2)}$$

From eq (1) & (2), $(\mu_o - \mu_e) t = \lambda/4$

$$t = \frac{\lambda}{4(\mu_o - \mu_e)} \quad \text{--- (3)}$$

Thickness of the quarter wave plate,

$$t = \frac{\lambda}{4(\mu_o - \mu_e)}.$$



2. Half wave plate:

- If the thickness of calcite crystal plate, cut with its faces parallel to optic axis, is such that it produces a path difference $\lambda/2$ b/w ordinary and extraordinary waves. then it is called half wave plate.
- Let μ_o and μ_e be the refractive indices of the extraordinary ray and ordinary rays respectively. let 't' be the thickness of the crystal. then, for a negative crystal ($\mu_o > \mu_e$) and positive crystal ($\mu_o < \mu_e$) the path difference b/w extraordinary ray and ordinary ray is

$$\Delta = \mu_o t - \mu_e t = (\mu_o - \mu_e) t \quad \text{--- (1)}$$

If this crystal act as half wave plate, then path difference b/w extraordinary ray and ordinary rays is

$$\Delta = \lambda/2 \quad \text{--- (2)}$$

From eq (1) & (2), $(\mu_o - \mu_e) t = \lambda/2$

$$t = \frac{\lambda}{2(\mu_o - \mu_e)} \quad \text{--- (3)}$$

Thickness of the half wave plate, $t = \frac{\lambda}{2(\mu_o - \mu_e)}$

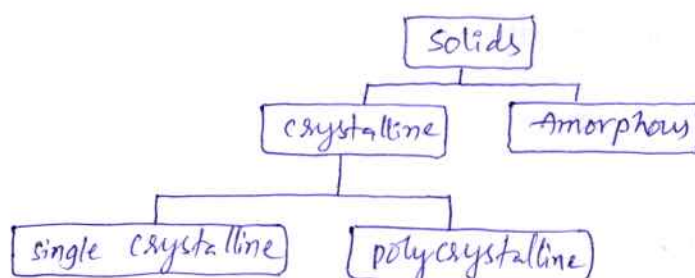
Engineering applications of polarization:

1. polarization is used to differentiate longitudinal and transverse waves.
2. As sunglasses to cut off unwanted reflected light
3. polarization is used to produce and show 3D movies.
4. polarization is used in infrared spectroscopy

5. used in the measurement of photo elasticity
6. polarization is utilized for mirror used in automobile industries
7. Frequently exploited using polaroid microscope to identify minerals.
8. All radio transmitting and receiving antennas and RADARs are intrinsically polarized.
9. In Engineering, the character of polarization is useful to study differences b/w stress and strain
10. polarization of cosmic microwave background is being used to study the physical physics of early univers.

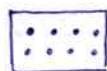
Introduction: The branch which is studied about the geometry and structural properties of crystal substances is known as Crystallography. Matter exists in three different states - gas, liquid, and solid. In gaseous and liquid states the atoms or molecules of the substance move from one place to other, and there is no fixed position of atoms in the substance. In solids, the positions of the atoms or molecules are fixed and may or may not be present periodically at regular intervals of distances.

Solids are classified into two categories based on the arrangement of atoms or molecules. They are crystalline and amorphous solids.



Crystalline solids:

* atoms are arranged regularly



* True solids.

* Have sharp melting point

* They exhibit different magnitudes of physical properties in different directions.

* These are anisotropic in nature

* If a crystalline solid breaks the broken pieces also have regular shape.
eg: Salt, Sugar.

Amorphous solid

* Atoms are arranged randomly.



* Pseudo solids (super cooled liquids).

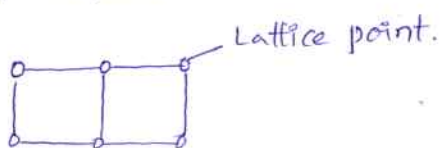
* Doesn't have sharp melting point

* They exhibit same magnitudes of physical properties in different direction.

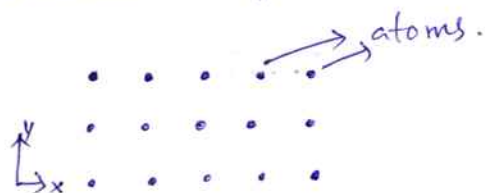
* These are isotropic nature.

* If an amorphous solid breaks, the broken pieces have irregular shape. ex: Glass, plastic.

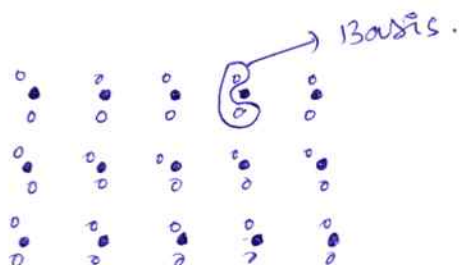
Lattice:- The regular and orderly arrangement of atoms or molecules in a row is called lattice.



Space Lattice or Crystal Lattice:- A space lattice is an array of points showing how particles are arranged at different sites in two or three dimensional spaces.



Basis:- A group of identical atoms or molecules in composition is called basis or pattern.



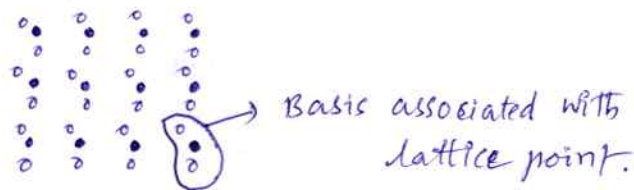
Crystal structure:

→ Atomic arrangement and symmetry is most important part in Crystal structure.

→ Crystal structure is a combination of lattice and basis.

$$\text{Crystal structure} = \text{lattice} + \text{Basis}$$

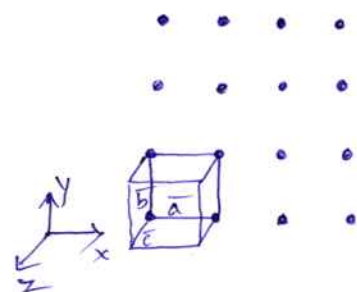
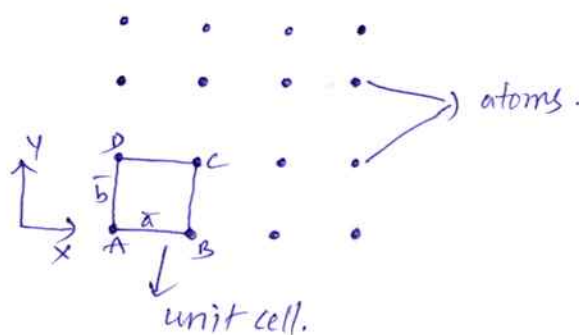
→ Depending upon the crystal structure, the basis may be mono-atomic, diatomic etc.



unit cell:-

→ The crystal structure of a material can be described in terms of its unit cell. It is the smallest unit.

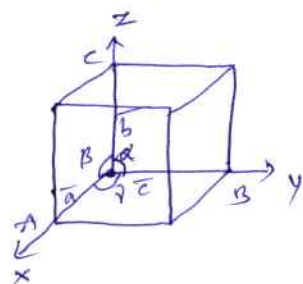
→ A unit cell is the smallest geometric structure, which repeated regularly gives the crystal structure is known as unit cell.



Lattice parameters:

→ A unit cell has 6 lattice parameters to represent the structure and shape, i.e. a, b, c and α, β, γ . These quantities are known as lattice parameters.

→ The lattice parameters a, b, c may or may not be equal and α, β, γ may or may not be right angles.



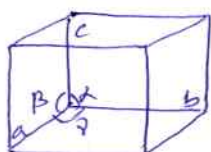
Crystal systems:

→ crystal systems are classified into seven systems on the basis of the shape of the unit cell or lattice parameters.

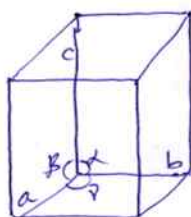
→ These are classified in terms of length of unit cells and the angle of inclination b/w them.

Seven Crystal systems

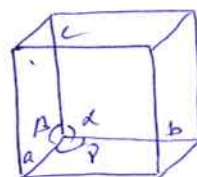
1. No.	<u>Crystal system</u>	<u>Axes</u>	<u>Angles</u>	<u>Examples</u>
1.	Cubic	$a=b=c$	$\alpha=\beta=\gamma=90^\circ$	Fe, Cu
2.	Tetragonal	$a=b \neq c$	$\alpha=\beta=\gamma=90^\circ$	Sn, MnO_2
3.	Orthorhombic	$a \neq b \neq c$	$\alpha=\beta=\gamma=90^\circ$	Iodine, KNO_3
4.	Monoclinic	$a \neq b \neq c$	$\alpha=\beta=90^\circ, \gamma \neq 90^\circ$	Sugar
5.	Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90^\circ$	$\text{K}_2\text{Cr}_2\text{O}_7$
6.	Hexagonal	$a=b \neq c$	$\alpha=\beta=90^\circ, \gamma=120^\circ$	Quartz
7.	Trigonal	$a=b=c$	$\alpha=\beta=90^\circ, \gamma \neq 90^\circ$	As, Bi.



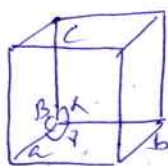
Cubic



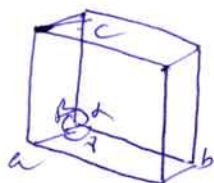
Tetragonal.



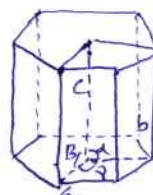
Orthorhombic



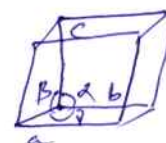
Monoclinic



Triclinic



Hexagonal

Rhombohedral
(Trigonal)

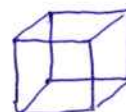
Bravais Lattices:

- If all atoms are identical at lattice points then that lattice knowns as Bravais lattice.
- Bravais showed that there are 14 different types of unit cells under the seven crystal systems.
- The classification of Bravais lattice is based on the following crystal lattices.

1. Primitive Lattice (P)
2. Body Centred Lattice (I)
3. Face Centred Lattice (F)
4. Base Centred Lattice (C)

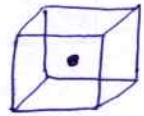
1. primitive Lattice (P)

In this lattice the unit cell consists of eight corners atoms and all these corners atoms contribute only one effective atom for the lattice.



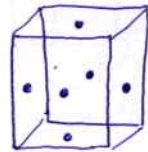
2. Body Centered Lattice (F):

In addition to the eight corner atoms it consists of one complete atom at the center.



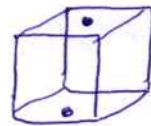
3. Face centered Lattice (F):

Along with corner atoms, each face will have one center atom.



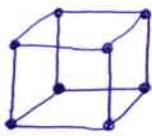
4. Base Centered Lattice (C):

The base and opposite face will have center atoms along with corner atoms.

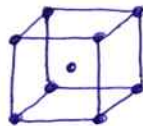


→ Bravais lattices for cubic

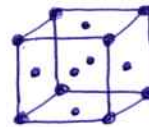
b)



(P)

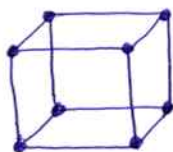


(I)

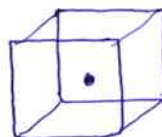


(F)

→ Bravais lattice: Tetragonal

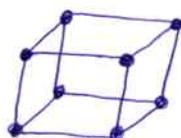


(P)

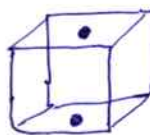


(I)

→ Bravais lattice: monoclinic

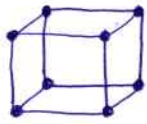


(P)

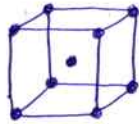


(C)

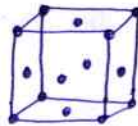
→ Bravais lattice: orthorhombic



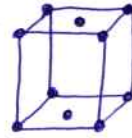
(P)



(I)



(F)



(C)

→ Triclinic:



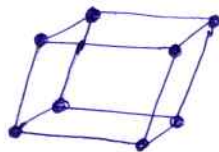
(P)

→ Hexagonal:



(P)

→ Rhombohedral (Trigonal)



(P)

Basic definitions of cubic crystal:

1) Number of atoms per unit cell (n):

The total no. of atoms per unit cell by considering the contribution of corner atoms, center atoms and face centered atoms. for SC, BCC and FCC are 1, 2, and 4.

2. Atomic Radius ($r = \frac{a}{2}$):

It is defined as half of the distance b/w two atoms. $r = \frac{a}{2}$

3. Coordination number (N)

It is number of nearest neighbours that an atom has in given structure. The co-ordination number for SC, BCC and FCC are 6, 8, and 12.

4. Atomic packing Factor (APF):

It is the ratio of volume of atoms in a unit cell to volume of unit cell. The APF for SC, BCC and FCC are 0.52, 0.68, and 0.74 respectively.

$$\text{APF} = \frac{\text{Volume of atoms in unit cell}}{\text{Volume of unit cell}} = \frac{N \times \text{Volume of each atom}}{\text{Volume of unit cell}}$$

Packing fractions of SC, BCC, and FCC

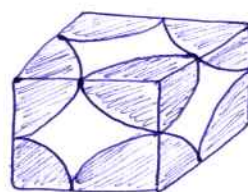
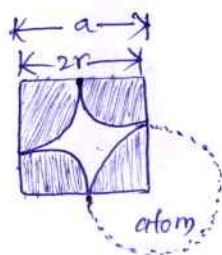
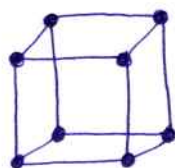
(1) Simple cubic (SC):

→ There are 8 atoms at 8 corners of the cube. The corner atoms touch with each other

→ The 8 corner atoms are shared by 8 unit cells, so that the contribution of a corner atom to a unit cell is $\frac{1}{8}$. Hence the no. of atoms in the unit cells is given as

$$n = 8 \text{ corner atoms} \times \frac{1}{8}$$

$$n = 1 \text{ atom.}$$



$$\text{Volume of atom} = \frac{4}{3} \pi r^3$$

$$\text{Volume of cube} = a^3 = (2r)^3 = 8r^3$$

$$\text{Atomic Packing Factor (APF)} = \frac{\text{No. of atoms} \times \text{Volume of each atom}}{\text{Volume of unit cell}}$$

$$= \frac{1 \times \frac{4}{3} \pi r^3}{8r^3}$$

$$= \frac{4}{3} \pi r^3 \times \frac{1}{8r^3} = \frac{\pi}{6} = 0.52.$$

Thus, the packing fraction of a simple cubic lattice is 0.52.

or 52%. This means that only 52% of the volume is occupied by atoms and remaining 48% is empty. This empty space inside a crystal lattice is known as 'void'.

Examples: Barium and polonium.

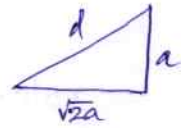
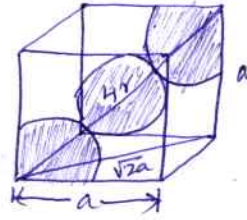
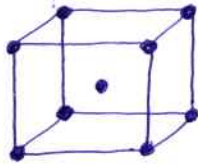
2. Body Centered Cubic (BCC):

→ In BCC, there are 8 atoms at 8 corners and 1 atom at the body center.

→ The 8 corner atoms are shared by 8 unit cell, so that contribution of a corner atom to a unit cell is $\frac{1}{8}$, and the center atom is entirely within the unit cell. It is not shared by any surrounding unit cell, Hence, the number of atoms in the unit cell is given as

$$n = (1 \text{ body centered atom} \times 1) + (8 \text{ corner atoms} \times \frac{1}{8})$$

$$n = 1 + 1 = 2 \text{ atoms.}$$



If the sides of the unit cell are 'a' then the length of the face diagonal will be $\sqrt{2}a$ - and the length of the body diagonal will be $\sqrt{3}a$. using Pythagoras Theorem.

$$d^2 = a^2 + (\sqrt{2}a)^2$$

$$d^2 = a^2 + 2a^2$$

$$d^2 = 3a^2$$

$$d = \sqrt{3}a.$$

Three atoms lie along the body diagonal with the total length being equal to $4r$. Thus, we have

$$4r = \sqrt{3}a$$

$$a = \frac{4r}{\sqrt{3}}$$

$$\therefore \text{Volume of cube} = a^3 = \left(\frac{4r}{\sqrt{3}}\right)^3$$

$$\text{Volume of atom (sphere)} = \frac{4}{3}\pi r^3$$

$$\text{Atomic Packing Factor (APF)} = \frac{\text{No. of atoms} \times \text{Volume of each atom}}{\text{Volume of unit cell}}$$

$$= \frac{2 \times \frac{4}{3}\pi r^3}{\left(\frac{4r}{\sqrt{3}}\right)^3}$$

$$= \frac{8}{3}\pi r^3 \frac{(\sqrt{3})^3}{4^3 r^3}$$

$$= \frac{\sqrt{3}\pi}{8} = 0.68.$$

Thus the packing fraction of a body centered cubic lattice is 0.68 or 68%. i.e. only 68% of the volume is occupied by atoms and remaining 32% is empty (void).

Examples: Sodium, potassium, ---

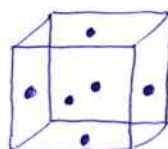
3. Face centered cubic (FCC):

→ In FCC, there are 8 atoms at 8 corners of the unit cell and each face (6 faces) has a center atom.

→ The 8 corner atoms are shared by 8 unit cells, so that the contribution of a corner atom to a unit cell is $\frac{1}{8}$. The face centered atom is shared by 2 unit cells and only $\frac{1}{2}$ of each atom belongs to an individual cell. Hence, the no. of atoms in the unit cells is given as.

$$n = (6 \text{ face centred atom} \times \frac{1}{2}) + (8 \text{ corner atoms} \times \frac{1}{8})$$

$$n = 3 + 1 = 4 \text{ atoms.}$$



If the sides of the unit cell are 'a' then the length of the face diagonal will be $\sqrt{2}a$.

$$\therefore 4r = \sqrt{2}a$$

$$a = \frac{4r}{\sqrt{2}} = \frac{\sqrt{16} r}{\sqrt{2}}$$

$$= \sqrt{\frac{16}{2}} r$$

$$= \sqrt{8} r$$



$$a^3 = (\sqrt{8} r)^3$$

$$\text{volume of atom (sphere)} = \frac{4}{3} \pi r^3$$

$$\text{APF} = \frac{\text{No. of atoms} \times \text{volume of each atom}}{\text{volume of unit cell.}}$$

$$= \frac{4 \times \frac{4}{3} \pi r^3}{(\sqrt{8} r)^3}$$

$$= 4 \times \frac{4}{3} \pi r^3 \times \frac{1}{8 \sqrt{8} r^3}$$

$$= \frac{2\pi}{3\sqrt{8}} = \frac{\pi}{3\sqrt{2}} = 0.74.$$

i.e. The packing fraction of a face centered cubic lattice is 0.74 or 74%. That is only 74% of the volume is occupied by atoms and remaining 26% is empty (Void)

examples: Cu, Ag, Al, ..

Miller Indices:

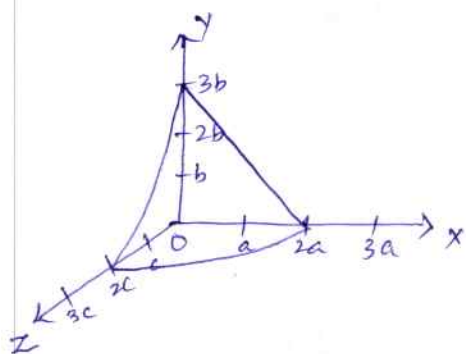
The crystal plane is represented by a set of three smallest integers called "Miller indices". These are denoted as (hkl).

Procedure to find Miller Indices:

1. The intercepts made by the crystal plane along x, y, z axis in terms of lattice parameters a, b, c are noted.
2. Express the intercepts as multiple of a, b, c.
3. Take the reciprocals of the intercepts.

4 → Convert the reciprocals into integers by multiplying each of them with their LCM to get smallest number as (hkl) miller indices. This represents the miller indices of the crystal plane.

Example:



→ ABC is the crystal plane which makes intercepts $2a$, $3b$ and $2c$ along the x , y , z axes respectively. Where a , b , c are the lattice parameters.

→ The intercepts of the multiples of a , b , c are $2, 3, 2$

→ reciprocals of the intercept are $\frac{1}{2}, \frac{1}{3}, \frac{1}{2}$. Their LCM is 6. multiply the reciprocals by 6, so that they become integers $3, 2, 3$.

→ The integers are written within the parenthesis as (323) . (323) represents the miller indices of the crystal plane ABC.

Important features of miller indices of crystal planes:

→ miller indices represents the orientation of crystal planes in a crystal lattice.

→ All equally spaced parallel planes have the miller indices

→ miller indices does not define particular plane but a set of parallel planes.

→ If a plane cuts an axis on the -ve side of the origin, then the corresponding index is -ve. and is indicating by placing a minus sign above the index. i.e. ($\bar{1}00$) for negative x-axis, ($0\bar{1}0$) for negative y axis, and ($00\bar{1}$) for negative z axis.

→ When the integers used in the miller indices contain more than one digit, they must be separated by commas.

example (3, 2, 4).

→ If (hkl) is the miller indices of a crystal plane, then the intercepts made by the plane with the coordinate axes are $\frac{a}{h}, \frac{b}{k}, \frac{c}{l}$ where a, b, c are the lattice parameters.

→ For cubic crystal $h:k:l = \frac{a}{p} : \frac{a}{q} : \frac{a}{r}$

For any crystal, $h:k:l = \frac{a}{p} : \frac{b}{q} : \frac{c}{r}$.

→ For a cubic crystal, the interplanar spacing b/w the adjacent plane is given by

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

where a is the lattice constant, and (hkl) is the indices of the crystal plane.

Separation b/w successive (hkl) planes :

→ Let us consider a rectangular co-ordinate system with origin 'O' at any one of the lattice points.

→ Let (hkl) be the miller indices of a plane ABC, which makes intercepts OA, OB, OC on x, y, z axes, respectively.

→ Since the plane segment 'a' into 'h' equal parts 'b' into 'k' equal parts, and 'c' into 'l' equal parts, then the intercepts OA, OB and OC are such that

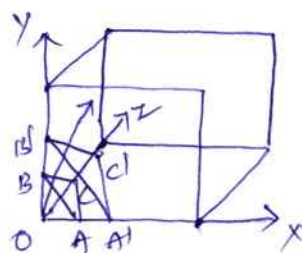
$$OA = \frac{a}{h}; OB = \frac{b}{k}, \text{ and } OC = \frac{c}{l}$$

From figure:

$$\Delta ONA, \cos \alpha = \frac{ON}{OA} = \frac{d}{\frac{a}{h}} = d \left(\frac{h}{a} \right)$$

$$\Delta ONB, \cos \beta = \frac{ON}{OB} = \frac{d}{\frac{b}{k}} = d \left(\frac{k}{b} \right)$$

$$\Delta ONC, \cos \gamma = \frac{ON}{OC} = \frac{d}{\frac{c}{l}} = d \left(\frac{l}{c} \right)$$



Use cosine law of directions,

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\left[d \left(\frac{h}{a} \right) \right]^2 + \left[d \left(\frac{k}{b} \right) \right]^2 + \left[d \left(\frac{l}{c} \right) \right]^2 = 1$$

$$\frac{d^2}{a^2} (h^2) + \frac{d^2}{b^2} (k^2) + \frac{d^2}{c^2} (l^2) = 1$$

For a cubic system: $a=b=c$ then the above eq.

$$\frac{d^2}{a^2} (h^2 + k^2 + l^2) = 1$$

$$d^2 = \frac{a^2}{h^2 + k^2 + l^2}$$

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

The above equation represents the interplanar spacing b/w two parallel planes.

X-RAY DIFFRACTION

X-ray diffraction:

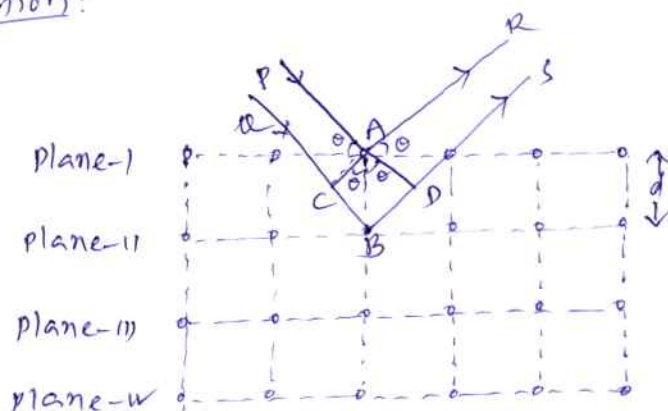
- The diffraction occurs when the size of obstacle must be comparable with wavelength of the incident light. In crystals the separation b/w the planes or atomic arrangement is nearly $1-2\text{\AA}$.
- If light is allowed will not be diffracted because light has wavelength $4000-7000\text{\AA}$. Thus X-rays are allowed to pass through crystals whose wavelength $1-10\text{\AA}$. Diffraction through crystal is observed. This is called X-ray diffraction.

Bragg's Law:

- Bragg's law was introduced by Sir W.H. Bragg and his son Sir W.L. Bragg.

Statement: Bragg's law states that when X-ray is incident onto a crystal surface, its angle of incidence ' θ ' will reflect back with a same angle of scattering ' θ '. (or) Bragg's law states that the diffraction of reflected X-rays occurs at condition of constructive interference.

Derivation:



- Consider a crystal have a planar separation as 'd' (i.e. $AB = d$) and is allowed to incident with x-rays as ray 1 (PA) and ray 2 (QB).
- The ray 1 reflected by plane 1 as AR and ray 2 as BG
- These diffracted (reflected) rays x-rays will interference constructively or destructively depending on the path difference b/w the x-rays.
- To calculate the path difference, two normal AC and AD are drawn from A to QB and BG

$$\text{Path difference} = CB + BD$$

$$\Delta ACB, \sin \theta = \frac{CB}{AB}$$

$$CB = AB \sin \theta = d \sin \theta \quad \text{--- (1)}$$

$$\text{Ily. } \Delta ADB, \sin \theta = \frac{BD}{AB}$$

$$BD = AB \sin \theta = d \sin \theta$$

From eq (1) & (2)

$$\begin{aligned} \text{Path difference} &= CB + CD = d \sin \theta + d \sin \theta \\ &= 2d \sin \theta \end{aligned}$$

To get maximum intensity, path difference should be equal to integral multiple of λ

$$\text{Path difference} = n\lambda$$

$$2d \sin \theta = n\lambda \quad \text{where } n = 1, 2, 3, \dots$$

The above expression represents Bragg's law.

X-ray Diffractometer:

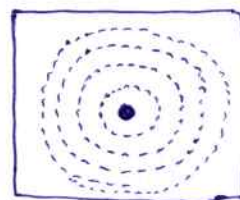
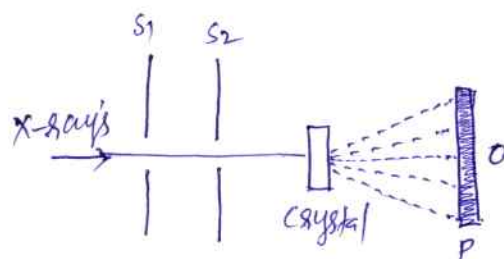
The instrument used to estimate parameters of crystals by applying Bragg's is called X-ray Bragg's spectrometer.

There are 3 diffractometers.

1. Laue method
2. powder method
3. Rotating crystal method

Laue's method:

Construction:-



Laue spot.

- The laue method is the oldest of the x-ray diffraction method.
- In this method white x-rays are used to analyse the crystal structure.
- The x-rays are converted to parallel beam of x-ray by using parallel slit arrangement S_1 and S_2
- In the path of x-rays a single crystal is arranged at fixed position.
- A photographic film is used to record x-ray diffraction pattern at certain distance from crystal as shown in fig.

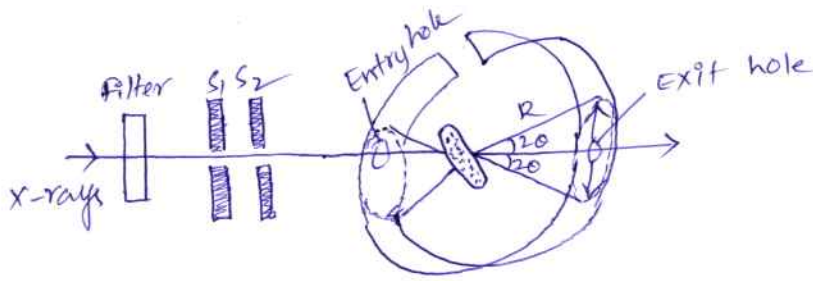
Working:-

- A collimated beam of continuous (white x-rays) falls on the crystal. The crystal plane in the crystal diffract the x-rays satisfying Bragg's law.
- The diffracted x-rays are allowed to fall on a photographic plate.
- The diffraction pattern consists of a series of bright spots corresponding to interference maximum for a set of crystal planes satisfying the Bragg's equation $2d \sin \theta = n\lambda$. for a particular wavelength of the incident beam.
- The distribution of spots depends on the symmetry of the crystal and its orientation with respect to x-ray beam.
- Hence various diffraction spots are found in photographic plate as shown in fig.

Powder method:

- The powder method is an x-ray diffraction technique used to study the structure of tiny crystalline in the form of powder.
- This method developed by Debye and Scherrer in Germany also known as Debye and Scherrer method.

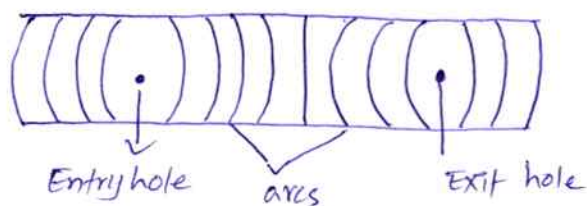
Construction:-



- The experimental arrangement is shown in fig. It consists of a cylindrical camera, called the Debye-Scherrer camera, consisting of a film in the inner portion.
- The finely powdered sample is filled in a thin capillary tube and mounted at the centre of the camera.

Working:-

- The x-rays from the source are made approximately monochromatic by filter.
- These are collimated by two lead slits S_1, S_2 falls on the powder specimen.
- Since the powder consists of randomly oriented crystallites, all possible α and d values are available for diffraction of incident x-rays, which satisfy the Bragg's relation $2d \sin \theta = n\lambda$ where λ is constant for monochromatic x-rays.
- For the values of θ , the beam appears at the corresponding 2θ deviation.
- The transmitted x-rays come out the exit hole. The diffracted x-ray cones make impression on the film in the form of arcs on either side of exit and entry holes with their center coinciding with hole.



→ If the distance of pair of arcs is 's' and radius of the camera is 'R', then for a crystalline size we have

$$4\theta = \frac{s}{R}$$

From the above expression, θ can be calculated.

Applications:

1. Powder method is useful for chemical Analysis - phase identification.
2. Determination of unit cell dimensions
3. Measurement of Sample purity
4. most useful for cubic crystal
5. used for determining the complex structure of material and Alloys.

DIELECTRIC MATERIALSElectric Dipole :-

A dipole is defined as a pair of two equal and opposite charges q and $-q$ separated by distance r .

Dipole moment :-

Dipole moment is equal to the product of one of charges and the separation between them

$$\mu = q \cdot r$$

S.I Unit - coulomb-meter (or) Debye

$$1 \text{ Debye} = 3.3 \times 10^{-30} \text{ C-m}$$

Permittivity :-

Permittivity represents the easily polarizable nature of dielectric medium. Permittivity of dielectric is denoted by ϵ and Permittivity of free space is denoted by ϵ_0 .

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

Dielectric Constant :-

The ratio of permittivity of medium to permittivity of free space is called dielectric constant

$$\epsilon_r = \epsilon / \epsilon_0$$

Consider a capacitor with air as medium having charge Q on its plates are separated by a distance d . Let area of cross section of plates is A . V_0 is potential difference and E_0 is the electric field across plates.

Capacity of capacitor is

$$C_0 = Q/V_0 \quad \text{and} \quad C_0 = A\epsilon_0/d$$

When the dielectric is introduced into the capacitor opposite charges are induced into the two dielectric faces which are close to the charge plates. These induced positive and negative charges act as dipoles and it has net dipole moment. This phenomenon is dielectric polarization. Due to the dipoles, the resultant potential difference decreases, so that capacity increases. Due to polarization, the polarization electric field E_p is induced in opposite direction to applied field E_0 . Resultant electric field is

$$E = E_0 - E_p$$

Let V_d is potential difference across capacitor's plates. Then capacity given by.

$$C_d = Q/V_d \rightarrow (1)$$

$$\epsilon_r = \epsilon/\epsilon_0$$

The ratio of capacity of capacitor with dielectric to capacity of capacitor without dielectric is called dielectric constant

$$\epsilon_r = C_d/C_0$$

$$\epsilon_r = V_0/V_d$$

Dielectric constant has no units

Dielectric polarization (P) :

Polarization is defined as the dipole moment per unit volume. Let N is the no. of molecules per unit volume and μ is dipole moment of molecule.

$$P = N \cdot \mu$$

$$\text{S.I unit} = \text{Coulomb/meter}^2 \text{ (Cm}^2\text{)}$$

Dielectric Susceptibility (χ)

It measures the amount of polarization is given electric field produced in a dielectric polarization. Polarization is proportional to the product of permittivity of free space ϵ_0 and electric field E .

$$P \propto \epsilon_0 E$$

$$P = \chi \epsilon_0 E$$

$$\boxed{\chi = P / \epsilon_0 E}$$

Electric flux density

The total number of lines passing through unit area of cross section of dielectric is called electric displacement (or) electric flux density.

Consider a dielectric in an electric field E the electric lines of force passing through dielectric and it can be polarized. Electric displacement D is proportional to the applied electric field E .

$$D \propto E$$

$$\boxed{D = \epsilon E}$$

→ ①

Relation between dielectric constant & Susceptibility :-

Electric displacement D is also related to electric field E by.

$$D = \epsilon_0 E + P \quad \text{--- ②}$$

From eq. ① & ②

$$\epsilon E = \epsilon_0 E + P$$

$$\epsilon_0 \epsilon_r E = \epsilon_0 E + P$$

$$P = \epsilon_0 \epsilon_r E - \epsilon_0 E$$

$$P = \epsilon_0 E (\epsilon_r - 1)$$

$$(\epsilon_r - 1) = P / \epsilon_0 E$$

From definition, $\chi = P / \epsilon_0 E$

$$\boxed{\chi = \epsilon_r - 1}$$

This is the relation for susceptibility and relative Permittivity ϵ_r (or) dielectric constant.

Dielectric polarizability (α)

Dielectric polarizability is defined as the net dipole moment per unit applied electric field

Net dipole moment is proportional to the applied electric field

$$\mu \propto E$$

$$\boxed{\mu = \alpha E}$$

S.I unit - $F \cdot m^2$

$$\text{Polarization } P = N \mu$$

$$P = N \alpha E$$

$$\Rightarrow \alpha = \frac{P}{NE}$$

$$\therefore \alpha = \mu / E$$

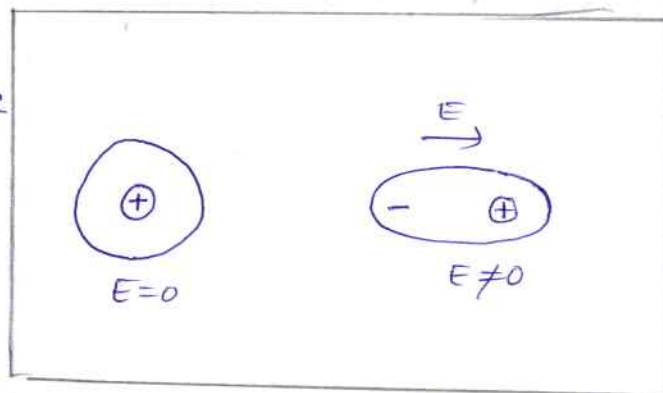
Types of Polarizations :-

Polarization occurs due to several atomic mechanisms. When specimen is placed in dc electric field. 4 types of polarizations occur in dielectric material.

- i> Electronic
- ii> Ionic
- iii> Dipolar / orientation
- iv> Space charge polarization

Electronic polarization :

Displacement of nucleus and electron cloud in opposite direction on application of electric field is called electronic polarization.



When electric field is applied on dielectric atom, electron cloud is displaced with respect to the nucleus in a direction opposite to the electric field. This phenomenon is called electronic polarization.

Consider an atom of dielectric material such that its atomic number is z and atomic radius is R . The centers of electron cloud and positive charged nucleus are coincided hence there is no dipole moment. $\therefore$

charge density of atom $\rho = \frac{-ze}{\frac{4}{3}\pi R^3} \longrightarrow \text{①}$

When this atom is placed in electric field of strength E , due to Lorentz force, the electron

cloud will move in opposite to the electric field. The centers of nucleus and electron cloud do not coincide, columb force develop between them. When columb force and Lorentz force are equal and opposite, then there will be equilibrium between nucleus and electron cloud. Hence dipole moment will be greater than zero. The atom get polarized.

Let the distance between centers of nucleus and electron cloud is 'x'

Total negative charge in the sphere.

$$Q_e = \frac{4}{3} \pi x^3 \rho \quad \text{--- (2)}$$

$$Q_e = \frac{4}{3} \pi x^3 \left(\frac{-ze}{\frac{4}{3} \pi R^3} \right) \quad \text{--- (3)}$$

$$Q_e = \frac{-ze x^3}{R^3} \quad \text{--- (4)}$$

The total positive charge of the atom

$$Q_p = +ze \quad \text{--- (5)}$$

$$\text{The columb force } F_c = \frac{1}{4\pi\epsilon_0} \frac{Q_e Q_p}{x^2}$$

Substitute eq (3) & (4) in equation (5)

$$F_c = \frac{1}{4\pi\epsilon_0 x^2} \left[\frac{-ze x^3}{R^3} \right] [ze]$$

$$= \frac{-z^2 e^2 x}{4\pi\epsilon_0 R^3} \quad \text{--- (6)}$$

$$\text{Lorentz force } F_L = -zeE \quad \text{--- (7)}$$

At equilibrium, Lorentz force = columb force

$$-zeE = \frac{-z^2 e^2 x}{4\pi\epsilon_0 R^3}$$

$$E = \frac{-ze\alpha}{4\pi\epsilon_0 R^3}$$

$$\alpha = \frac{4\pi\epsilon_0 R^3 E}{ze} \longrightarrow \textcircled{8}$$

Induced dipole moment $M_e = ze\alpha \longrightarrow \textcircled{9}$

Sub. eq $\textcircled{8}$ in eq $\textcircled{9}$

$$M_e = ze \frac{4\pi\epsilon_0 R^3 E}{ze}$$

$$\Rightarrow M_e = 4\pi\epsilon_0 R^3 E \longrightarrow \textcircled{10}$$

From definition $M_e = \alpha_e E \longrightarrow \textcircled{11}$

From eq. $\textcircled{10}$ & $\textcircled{11}$,

Electronic polarizability $\boxed{\alpha_e = 4\pi\epsilon_0 R^3}$

The induced electronic dipole moment per unit volume is called electronic polarization

$$P = N\alpha_e E$$

$$\epsilon_0 E(\epsilon_r - 1) = N\alpha_e E$$

$$\alpha_e = \frac{\epsilon_0 E(\epsilon_r - 1)}{N}$$

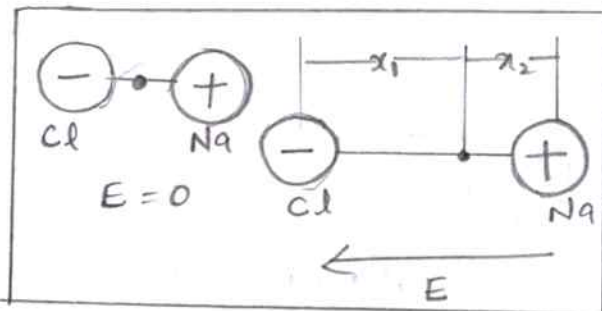
Ionic polarization;

The ionic polarization is due to the displacement of cations and anions in opposite directions when an electric field is applied in the +ve x direction

the positive ions move to the right by x_1 distance and negative ions move to left by x_2 distance.

The resultant dipole moment per unit cell is

$$M_i = e(x_1 + x_2) \longrightarrow \textcircled{1}$$



If K_1 and K_2 are restoring force constants of cation and anion, F_R is the force due to applied field

$$F_R = K_1 x_1 = K_2 x_2$$

$$\therefore x_1 = \frac{F_R}{K_1}, \quad x_2 = \frac{F_R}{K_2}$$

Where $K_1 = m\omega_0^2$ and $K_2 = M\omega_0^2$ where m is mass of positive ion, M is mass of negative ion and ω_0 is the angular frequency

At equilibrium $F_R = F_L$ (Lorentz force)

$$F_L = eE \quad \text{then}$$

$$x_1 = \frac{eE}{m\omega_0^2} \quad \text{and} \quad x_2 = \frac{eE}{M\omega_0^2}$$

$$x_1 + x_2 = \frac{eE}{\omega_0^2} \left(\frac{1}{m} + \frac{1}{M} \right) \quad \text{--- (2)}$$

Ionic dipole moment $\mu_i = e(x_1 + x_2)$

$$\mu = \frac{e^2 E}{\omega_0^2} \left(\frac{1}{m} + \frac{1}{M} \right)$$

ionic polarizability $\alpha_i = \frac{\mu_i}{E}$

$$\boxed{\alpha_i = \frac{e^2}{\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right)}$$

Dipolar (or) Orientational Polarization:-

When an electric field is applied on polar dielectric material, all the dipoles are aligned in the direction of applied field. The polarization produced in polar dielectric material due to the application of electric field is called orientation polarization.

5
 Orientational polarization depends on temperature.
 Thermal energy randomizing the dipoles.

Orientational polarization $P_0 = N \mu_0 = N \alpha_0 E$

Therefore, $P_0 = \frac{NM^2 E}{3KT} = N \alpha_0 E$

Orientational polarizability $\alpha_0 = \frac{P_0}{NE}$

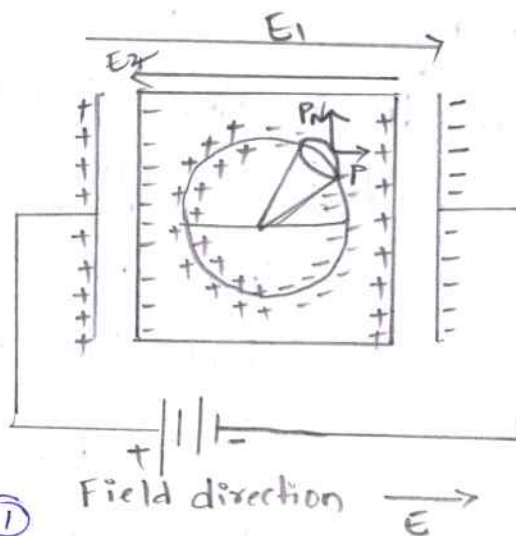
$\alpha_0 = \frac{M^2}{3KT}$

Therefore, the resultant polarizability due to all the polarizations is given by

$\alpha = \alpha_e + \alpha_i + \alpha_0 = 4\pi\epsilon_0 R^3 + \frac{e^2}{m\omega_0^2} \left(\frac{1}{M} + \frac{1}{m} \right) + \frac{M^2}{3KT}$

Internal field (or) Local field (Lorentz method) :-

Let a dielectric is placed between the plates of a parallel plate capacitor. Let there is an imaginary spherical cavity around an atom A inside the dielectric. The internal field at the atom A is made up of four components.



$E_i = E_1 + E_2 + E_3 + E_4 \longrightarrow \textcircled{1}$

Field E_1 :

E_1 is the electric field intensity at A due to the charge density on the plates. From field theory

$E_1 = D/\epsilon_0$

$D = \epsilon_0 E + P$

$E_1 = \frac{\epsilon_0 E + P}{\epsilon_0} \Rightarrow E_1 = E + \frac{P}{\epsilon_0} \text{---} \textcircled{2}$

Field E_2 :-

E_2 is the field intensity at A due to the charge density induced on the two sides of the dielectric.

$$E_2 = -\frac{P}{\epsilon_0} \longrightarrow (3)$$

Field E_3 :-

E_3 is the field intensity at A due to the other atoms contained in the cavity. we are assuming a spherical structure,

$$E_3 = 0$$

Field E_4 :-

E_4 is the field intensity due to polarization charges on the surface of the cavity.

if dA is the surface area of the spherical shell of radius r lying between θ and $d\theta$

$$\text{Then } dA = 2\pi(P_2)(r \sin\theta) d\theta$$

$$\text{but } \sin\theta = P_2/r$$

$$\Rightarrow P_2 = r \sin\theta$$

$$\text{And, } d\theta = r/r \Rightarrow r = r d\theta$$

$$\text{Hence, } dA = 2\pi r \sin\theta r d\theta$$

The charge dq on the surface dA is equal to the normal component of the polarization multiplied by the surface.

$$\begin{aligned} dq &= \text{normal component of } P \times \text{area of charge} \\ &= P \cos\theta dA \end{aligned}$$

Substitute eq (5) in eq (6)

$$dq = P \cos \theta (2\pi r \sin \theta r d\theta)$$

$$dq = P (2\pi r^2 \sin \theta \cos \theta d\theta) \longrightarrow (7)$$

The field due to the charge on A is denoted by dE_f in the direction $\theta = 0$

$$dE_f = \frac{dq \times 1 \times \cos \theta}{4\pi \epsilon_0 r^2}$$

$$= \frac{P \cos \theta 2\pi r^2 \sin \theta d\theta \cos \theta}{4\pi \epsilon_0 r^2}$$

$$= \frac{P}{2\epsilon_0} \cos^2 \theta \sin \theta d\theta$$

Thus the total field due to surface of entire cavity is

$$\int dE_f = \frac{P}{2\epsilon_0} \int_0^\pi \cos^2 \theta \sin \theta d\theta$$

$$= \frac{P}{2\epsilon_0} \int_0^\pi \cos^2 \theta d(-\cos \theta)$$

$$= \frac{-P}{2\epsilon_0} \left[\frac{\cos^3 \theta}{3} \right]_0^\pi$$

$$= \frac{-P}{2\epsilon_0} [-1 - 1]$$

$$E_f = P/3\epsilon_0 \longrightarrow (8)$$

The internal field / Lorentz field

$$E_i = E_1 + E_2 + E_3 + E_4$$

$$= E + \frac{P}{\epsilon_0} - \frac{P}{\epsilon_0} + 0 + \frac{P}{3\epsilon_0}$$

$$\therefore E_i = E + \frac{P}{3\epsilon_0} \longrightarrow (9)$$

The above equation is also known as Lorentz relation

Clausius - Mosotti relation :-

Clausius Mosotti relation makes the relationship between microscopic and macroscopic quantities of polarization. The dipole moment is proportional to the internal field (or) Local field.

$$P \propto N \alpha E_i \longrightarrow (1)$$

$$P = N \alpha \left(E + \frac{P}{3 \epsilon_0} \right)$$

$$P = N \alpha E + \frac{N \alpha P}{3 \epsilon_0}$$

$$\Rightarrow P - \frac{N \alpha P}{3 \epsilon_0} = N \alpha E$$

$$P \left(1 - \frac{N \alpha}{3 \epsilon_0} \right) = N \alpha E$$

$$P = \frac{N \alpha E}{1 - \frac{N \alpha}{3 \epsilon_0}} \longrightarrow (2)$$

We know that $D = \epsilon_0 E + P$

$$\epsilon E = \epsilon_0 E + P$$

$$\epsilon_0 \epsilon_r E = \epsilon_0 E + P$$

$$P = \epsilon_0 \epsilon_r E - \epsilon_0 E$$

$$P = \epsilon_0 E (\epsilon_r - 1) \longrightarrow (3)$$

Substitute eq (3) in eq (2)

$$\epsilon_0 E (\epsilon_r - 1) = \frac{N \alpha E}{1 - \frac{N \alpha}{3 \epsilon_0}}$$

$$1 - \frac{N \alpha}{3 \epsilon_0} = \frac{N \alpha}{\epsilon_0 (\epsilon_r - 1)} \Rightarrow 1 = \frac{N \alpha}{3 \epsilon_0} + \frac{N \alpha}{\epsilon_0 (\epsilon_r - 1)}$$

$$\Rightarrow \frac{N \alpha}{3 \epsilon_0} = \frac{1}{\left(1 + \frac{3}{\epsilon_r - 1} \right)}$$

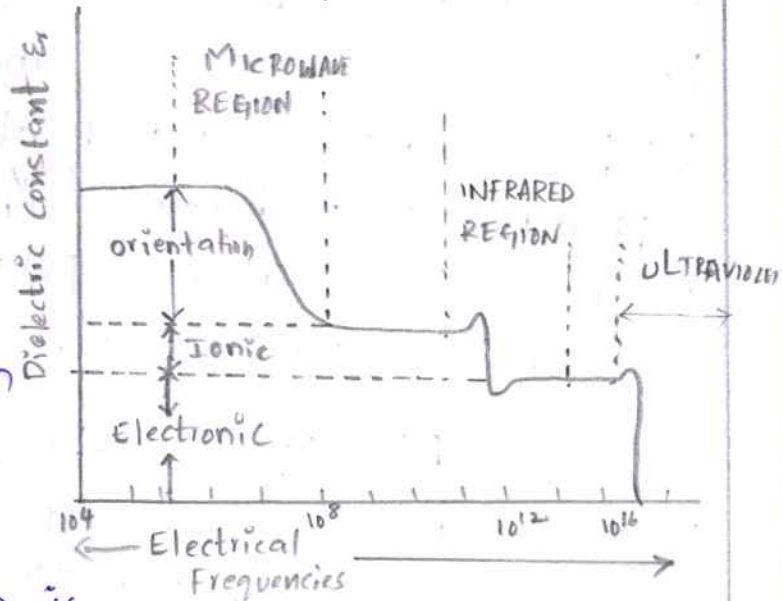
$$\boxed{\frac{N \alpha}{3 \epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}}$$

Frequency Dependence of the Polarizability :-

On application of electric field, polarization process occurs as the function of time

$$P(t) = P[1 - \exp(-t/\tau_r)]$$

where P is maximum polarization and τ_r is the relaxation time for particular polarization.



Electronic polarization is extremely rapid. Even when the frequency of the applied voltage is very high in the optical range ($\sim 10^{15}$ Hz), electronic polarization occurs during every cycle of applied voltage.

Ionic polarization is due to displacement of ions over small distances due to applied field. Since ions are heavier than the electron cloud, the time taken for displacement is larger. The frequency with which ions are displaced is in the same order of lattice vibration frequency ($\sim 10^{13}$ Hz). At optical frequencies ions do not respond. If frequency of applied voltage is less than 10^{13} Hz, the ions respond. Hence at 10^{13} Hz, we have both electronic & ionic polarizations.

Orientation polarization is even slower than ionic polarization. The relaxation time for orientation polarization in liquid is less than that in a solid. Orientation polarization occurs, when the frequency of applied voltage is in the order of audio frequency.

Space charge polarization is the slowest process, as it involves the diffusion of ions over several

inter atomic distances. The relaxation time for this process is related to the frequency of successful jumps of ions under the influences of the applied field having frequency in the order of 10^2 Hz. Space charge polarization occurs at power frequencies (50-60 Hz).

At optical frequencies (10^{15} Hz) only electronic polarization occurs. At 10^{13} Hz, ionic polarization occurs in addition to electronic polarization. At 10^6 Hz range, dipolar polarization occurs in addition to electronic and ionic polarizations. At 10^2 Hz, Space charge polarization also contributes.

Applications :-

- 1) Capacitors with vacuum, air or inert gas as dielectric these are used in radio frequency circuits and in low frequency measuring circuits.
- 2) capacitors with mineral oils as dielectrics: these are used in high voltage applications.
- 3) capacitors with solid as dielectric: these are used standard capacitors because dielectric constant does not vary with temperature.
ex: glass, mica etc.
- 4) capacitors with both solid and liquid dielectrics: these are used in power distribution system for power factor correction
Ex: combinations of solids such as papers, glass, mica etc.

UNIT-III Magnetic Materials

Magnetic Dipole moment :-

When an electric current of I ampere flows through a circular wire of one turn having an area of cross section $A \text{ m}^2$, it is said to have a magnetic moment of

$$\mu_m = I \times A$$

It is expressed in Am^2 . It is a vector quantity pointing from South to North pole

Magnetization :-

The magnetization M is defined as the magnetic moment per unit volume. It is expressed in Am^{-1}

Magnetic Susceptibility :-

The magnetic susceptibility χ is defined as the ratio of the magnetization produced in a sample to the magnetic field strength

$$\chi = M/H$$

χ has no units.

Magnetic permeability :-

The magnetic permeability μ is measure of the amount of magnetic lines of forces penetrating through a material.

$$\mu = B/H \Rightarrow B = \mu H$$

Magnetic flux density :-

Number of magnetic lines per unit area of cross section is called magnetic flux density.

$$B = \phi / A$$

S.I unit = web/m^2 or Tesla

Magnetic field intensity :-

Force experienced by unit ^{North} Pole is called magnetic field intensity.

$$H = F/m$$

Relative permeability :-

The ratio of permeability of free space and permeability of material

$$\mu_r = \mu / \mu_0$$

It has no units.

Relation between B, H & M :-

$$B = \mu H$$

$$B = \mu_0 \mu_r H$$

$$B = \mu_0 \mu_r H + \mu_0 H - \mu_0 H$$

$$\text{But, } M = H(\mu_r - 1)$$

$$B = \mu_0 (H + \mu_0 M)$$

$$B = \mu_0 (H + M)$$

Relation between χ and μ_r :-

$$B = \mu_0 (H + M)$$

$$\mu_r = \mu / \mu_0$$

$$\mu_r = \frac{B/H}{B/(H+M)}$$

$$M_r = \frac{H+M}{H}$$

$$M_r = 1 + M/H$$

$$M_r = 1 + \chi$$

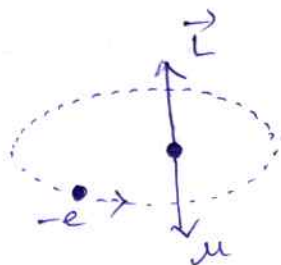
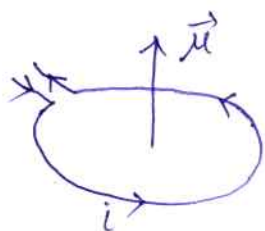
Origin of magnetic moment - Bohr magneton :-

Magnetic moment arises to the atom due to

1. Orbital motion of electron
2. Spin motion of electron
3. Spin motion of nucleus

Orbital magnetic moment :-

The electron of charge ' $-e$ ' moves in circular orbit around the nucleus.



The magnitude of the orbital moment dipole moment for a current loop is

$$M_m = I \times A$$

where I is the current and A is the area of loop.

$$I = -e/T$$

where e is the magnitude of the electron charge and T is its time period

If we assume that the electron travels in a perfectly circular orbit, the time period is

$$T = 2\pi/\omega \Rightarrow I = -e\omega/2\pi$$

where r is the radius of the orbit and v is the speed of the electron in its orbit. Given that the area of a circle is πr^2 , the absolute magnetic moment is

$$M_m = -\frac{evr^2}{2}$$

$$M_m = -\frac{emvr^2}{2m}$$

$$M_m = -\frac{eL}{2m}$$

where -ve sign indicates that magnetic moment and orbital angular momentum are in opposite directions.

Orbital angular momentum $L = \frac{lh}{2\pi}$

where l = orbital quantum number $1, 2, 3, \dots$

$$M_m = -\frac{elh}{4\pi m}$$

where, Bohr magneton $M_B = \frac{eh}{4\pi m}$

$$M_m = -M_B l$$

$$M_m = -M_B, -2M_B, -3M_B, \dots \quad [\because l = 1, 2, 3]$$

Spin magnetic moment :-

The spinning of electron about its own axis establishes the magnetic moment called spin magnetic moment.

$$\text{Spin magnetic moment } M_{\text{spin}} = \gamma \frac{e}{2m} S$$

where γ is the spin gyro magnetic ratio.

For an electron $S = \frac{h}{4\pi}$

$$\mu_{\text{spin}} = v \left(\frac{e}{2m} \right) \left(\frac{h}{4\pi} \right)$$

Magnetic moment due to nuclear spin :-

Spin motion of nucleus contributes magnetic moment to atom.

$$\text{Nuclear Spin magnetic moment } \mu_{\text{nuclear}} = \frac{eh}{4\pi M_p}$$

where M_p is the mass of proton.

Types of Magnetic Materials :-

Depending upon the magnetic moment of materials, they can be classified into the following 5 types.

1. Diamagnetic Materials
2. Paramagnetic materials
3. Ferro magnetic material
4. Ferri magnetic Material
5. Anti Ferro magnetic Materials.

Dia magnetic material :-

1. Diamagnetism is a weak magnetism and is the fundamental property of all matter.
2. Diamagnetism is mainly due to the non-cooperative behaviour of the orbital electrons under the application of external magnetic field.
3. In diamagnetism substances, all the atoms have paired electrons and there are no

Unpaired electrons in the shells. Thus the net magnetic moment of the atom of a diamagnetic substance is zero.

4) However, when an external magnetic field is applied on these substances, these materials are magnetized opposite to the field direction. Thus they have negative magnetization.

5) Relative permeability is small and less than one.

$$\mu_r < 1$$

6) Susceptibility is negative and susceptibility is independent of temperature (χ)

Example

Organic materials.

Paramagnetic materials :-

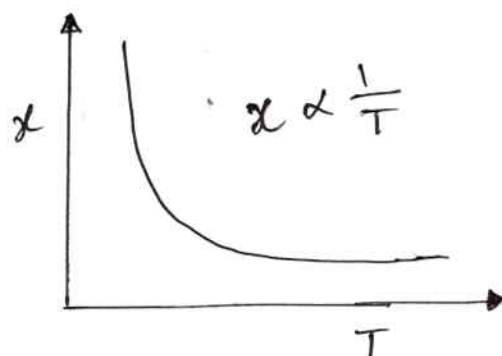
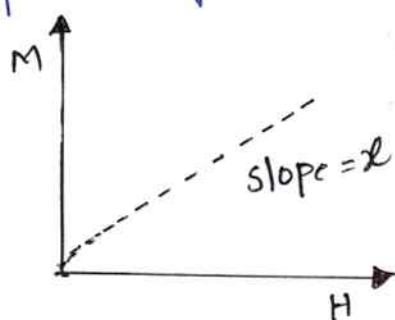
1. In these materials, the atoms or ions have unpaired electrons in partially filled orbitals.
2. That means each atom in a paramagnetic substance has a small net magnetic moment.
3. But, there is no interaction between these atomic magnets.
4. In the presence of an external magnetic field there will be a partial alignment of these atomic magnetic moments in the direction of applied magnetic field resulting in a net positive magnetization.

5. Relative permeability μ_r and susceptibility χ is small and positive.

6. When the applied field is zero, the magnetization also becomes zero

7. The susceptibility is inversely proportional to the absolute temperature.

Susceptibility $\chi = \frac{M}{H}$



Paramagnetism

Example :

Alkali metals, transition metals, rare earth metals.

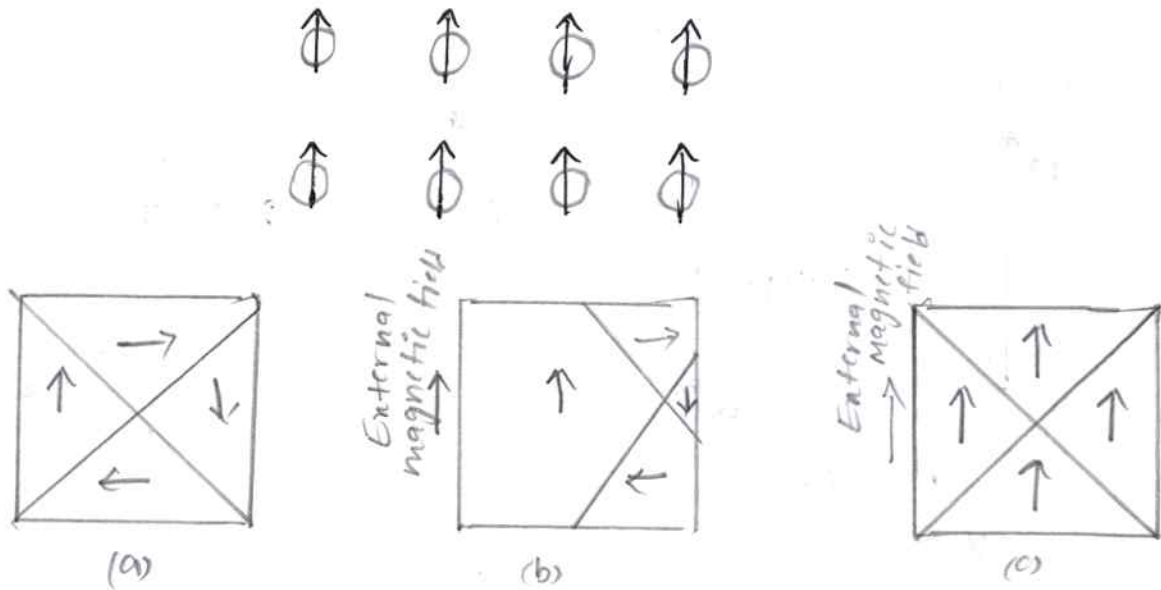
Ferro magnetic materials :-

In these substances

1. The materials that are affected strongly with external magnetic field and have permanent dipoles
2. A spinning electron behaves as a tiny magnet. There exists a strong interaction between the atomic magnets. These interaction forces are exchange type of forces. The interaction force between the atoms is due to exchange of electrons.

3. Atomic magnets are aligned parallel to each other. Under the influence of these exchange forces even in the absence of external magnetic field. These atomic magnets are aligned parallel to each other.

Parallel alignment



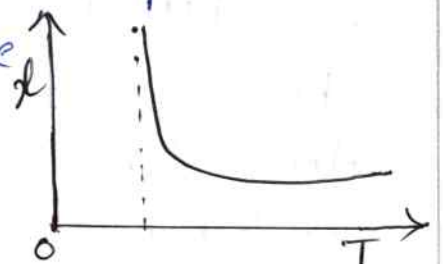
4. Ferromagnetic susceptibility is high and positive.

5. In ferromagnetic substances at a particular temperature called Curie temperature and above Curie temperature, the ferromagnetic is disordered and works as para magnet. But below Curie temperature, it is ordered and has ferromagnetic nature. The saturation magnetization becomes zero at the Curie temperature.

6. Relative permeability is very high and positive.

7. Susceptibility depends on temperature as shown below.

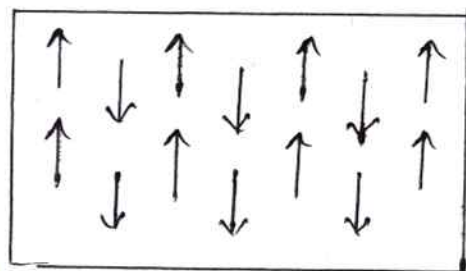
Susceptibility $\chi =$



Example :- Iron, Nickel & Cobalt

Antiferro-magnetic material :-

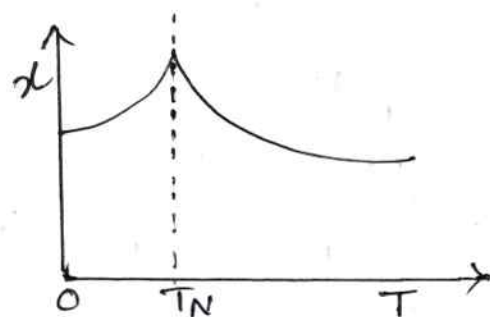
1. Spin dipoles are equal and oriented in opposite direction as shown
2. The net magnetic moment is small
3. Net magnetic moment is zero in the absence of external magnetic field
4. Net magnetization is induced in the direction of external magnetic field
5. It allows magnetic lines of forces to through it
6. Intensity of magnetization is positive and moderate
7. Relative permeability μ_r is greater than one ($\mu_r > 1$)
8. Susceptibility is low and positive
9. Susceptibility depends on temperature



Susceptibility $\chi =$

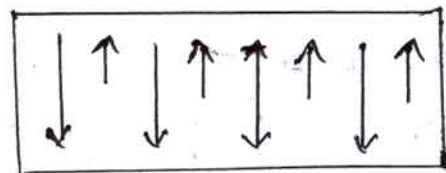
10. When $T < T_N$, the material is a antiferromagnetic. when $T > T_N$ the material is paramagnetic

Ex: Salts of Transition metals



Ferri magnetic materials :-

1. Ferri magnetic substance also exhibits spontaneous magnetization
2. Spin dipoles are unequal and oriented in opposite direction due to Super exchange interactions
3. There is a net magnetic moment even in the absence of external magnetic field.

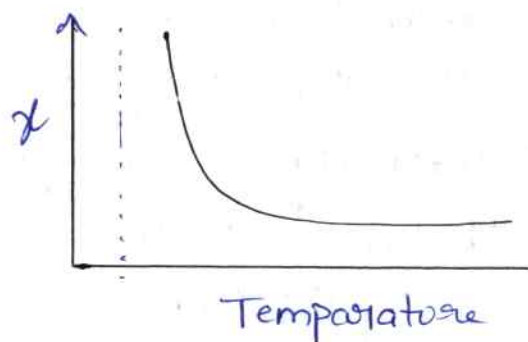


5. It allows magnetic lines of forces to pass through it
6. Intensity of magnetization is positive and high
7. Relative permeability is small and greater than one
8. Susceptibility is high and positive
9. Susceptibility depends on temperature.

Susceptibility $\chi =$

10. When $T < T_N$, the material is ferromagnetic, when $T > T_N$, the material is paramagnetic

Ex: Ferrites.



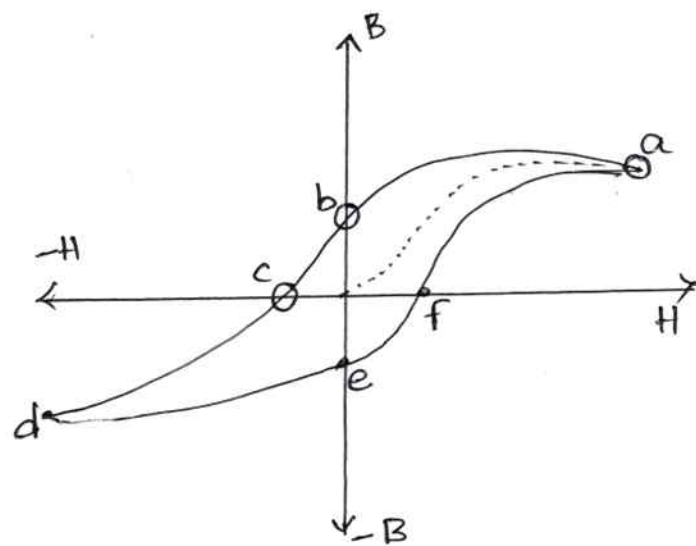
Hysteresis :-

When a specimen of ferromagnetic material is placed in a magnetic field, the specimen is magnetized by induction. As the magnetic intensity H changes, the flux density does not change linearly with H . The variation of B with H is shown in the fig. The point O represents initially unmagnetized specimen and a zero magnetic intensity. As H is increased, the magnetic flux density B increases nonlinearly and reaches saturation point 'a'. If H is decreased, B also decreases but following the path 'ab'. Thus B lags behind H . This property is called hysteresis.

When H becomes zero, B still has a value ' O_b '. The magnetic flux density remaining in the specimen in the absence of external magnetic intensity is called remanence magnetism or residual magnetism B_r . This property is called retentivity. If the magnetic intensity is increased in opposite direction, the value of B decreases and becomes 0 when H has the value equal to ' O_c '. This magnetic intensity is called coercive force. The coercivity (H_c) is measure of the magnetic intensity required to destroy the residual magnetism of the specimen. As H is increased beyond O_c , the specimen is increasingly magnetized in opposite direction and reached the Saturation point 'd'.

By taking H back from its negative maximum value to its original positive value through 0, a similar curve will be obtained.

The closed curve 'abcdefa' which represents a cycle of magnetization of the specimen is known as hysteresis curve (loop) of the specimen.



Hysteresis Curve

Soft and Hard magnetic materials :-

(x The magnetic materials are heated to required temperature and cool)

The magnetic materials are classified based on hysteresis loop into two types

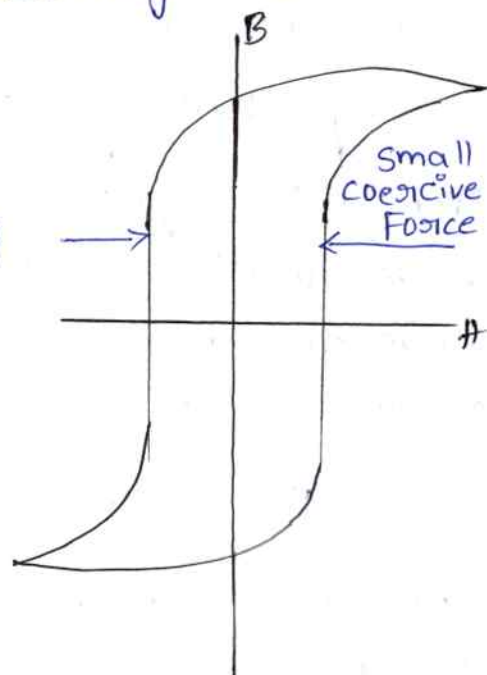
1. Soft magnetic materials.
2. Hard magnetic materials.

Soft magnetic materials :-

The magnetic materials are heated to required temperature and cooled slowly them to prepare soft ferromagnetic materials.

Properties :-

1. They are easily magnetized and demagnetized
2. Their hysteresis loop is 'steep'
3. The area of hysteresis loop is
4. They have high magnetic susceptibility and permeability
5. They exhibit lowest retentivity and coercivity
6. They exhibit high resistivity
7. They require low magnetic field to magnetize



Example ;

1. Iron - Silicon alloys
2. Nickel - Iron alloys
3. Iron - cobalt alloys.

Applications :-

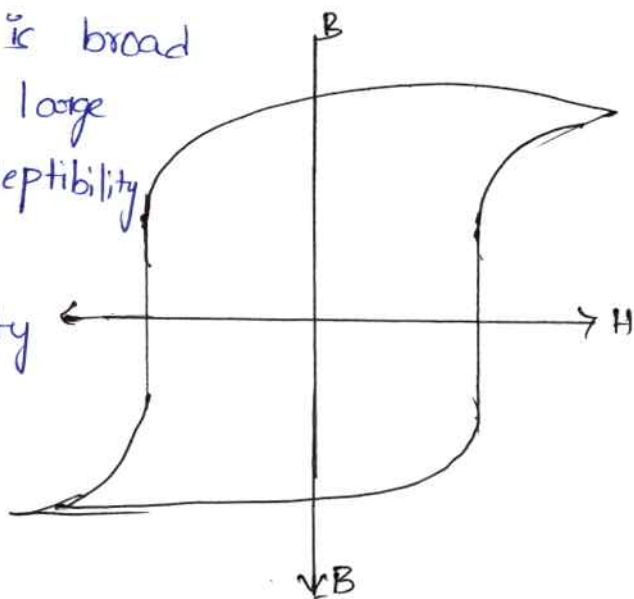
1. They are used in electric equipment and magnetic cores of transformers.
2. They are used in motors, relays & sensors.
3. They are used in microwave isolators.

Hard magnetic materials :-

The magnetic materials are heated to required temperature and suddenly cooled by dipping into cold solution to prepare hard magnetic material.

Properties :-

1. They are hard to magnetize and demagnetize.
2. Their hysteresis loop is broad
3. The area of hysteresis loop is broad large. hence hysteresis loss is large
4. They have low magnetic susceptibility and permeability
5. They exhibit high retentivity and coercivity
6. They exhibit low resistivity
7. They require high magnetic field to magnetize



example :-

1. Carbon steel alloys
2. Tungsten steel alloys etc.

Applications :-

1. They are used to make permanent magnets
2. They are used in speedometers and sensors in automobiles, motors etc.
3. They are used in meters and measuring devices.

LASERS

Lasers :-

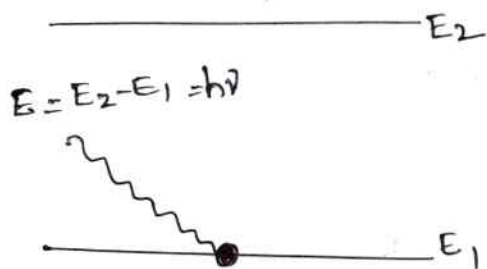
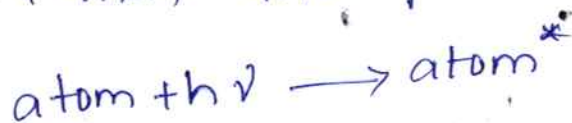
Laser means light amplification by stimulated emission of radiation

Speciality of Lasers :

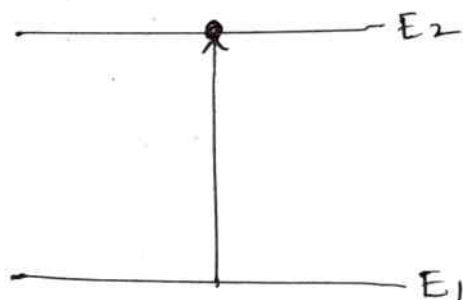
Any light beam is composed of photons, two photons are said to be coherent if they have same energy, phase and direction. Laser beam is composed of such coherent photons whereas ordinary light contains incoherent photons. So the property of coherence adds special characteristics to LASER beam such as high directionality, high intensity, monochromaticity and penetration.

Spontaneous Absorption :-

When an atom absorbs amount of energy $h\nu$ in the form of photons from the external agency and excited into the higher energy level from ground state, then this process is known as absorption



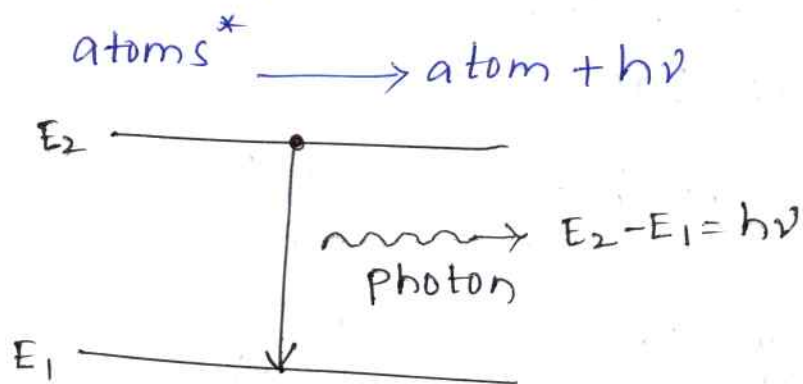
Before Absorption



After Absorption

Spontaneous Emission :-

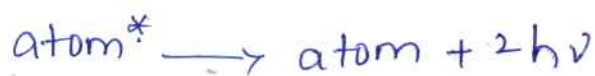
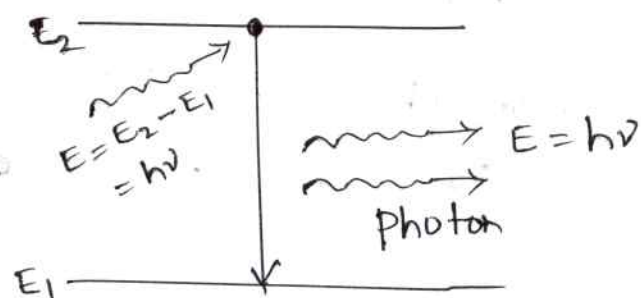
When an atom in the excited state emits a photon of energy $h\nu$ coming down to ground state by itself without any external agency, such an emission is called Spontaneous emission.



Photons released in Spontaneous emission are not coherent. Hence spontaneous emission is not useful for producing Laser.

Stimulated Emission :-

When an atom in the excited state, emits two photons of same energy $h\nu$ while coming to down to ground state with the influence of an external agency, such an emission is called Stimulated emission.



In the two photons one photon induces Stimulated emission and the second one is released

16
by the transition of atom from highest energy level to lowest energy level. Both the photons are strictly coherent. Hence Stimulated emission is responsible for Laser production.

Characteristics of LASER Beam

Some of the special properties which distinguish Lasers light from ordinary light sources are characterised by

1. Directionality
2. High intensity

3. Monochromaticity
4. Coherence

Directionality :-

Laser emits radiation only in one direction. The directionality of Laser beam is expressed in terms of angle of divergence i.e., it is twice the angle that the outer edge of the beam makes with the axis of the beam with reference to emitting source. The outer edge of the beam is defined a point at which the intensity of beam falls to $1/37$ times to that of value at the centre.

Divergence or angular spread is given by

$$\theta = \frac{a_2 - a_1}{d_2 - d_1}$$

Where d_1, d_2 are any two distances from the Laser source emitted.

a_1, a_2 are the radii of the beam spots at a distance d_1 and d_2 respectively.

High Intensity :

Generally light from conventional source spread uniformly in all directions. For ex. take 100 watt bulb and look at a distance of 30 cm, the power enter into the eyes is less than thousand of a watt. This is due to uniform distribution of light in all directions. But in case of Lasers light is a narrow beam and its energy is concentrated within the small region. The concentration of energy accounts for greater intensity of Lasers.

Monochromaticity :-

The light emitted by Laser is highly monochromatic than any of the other conventional monochromatic light. A comparison b/w normal light and laser beam ordinary sodium (Na) light emits radiation at wavelength of $5893 \text{ \AA}$ with the line width of $1 \text{ \AA}$. But He-Ne Laser of wavelength $6328 \text{ \AA}$ with a narrow width of only $10^{-7} \text{ \AA}$ i.e., monochromaticity of Laser is 10 million times better than normal light. The degree of monochromaticity of the light is estimated by line of width.

Coherence :-

If any wave appears as pure sine wave is perfectly for long time and infinite space, then it is said to be perfectly coherent. partially

no wave is perfectly coherent including Lasers. But compared to other light sources, Lasers have high degree of coherence because all the energy is concentrated within the small region. There are two independent concepts of coherence

- 1) Temporal coherence
- 2) Spatial coherence

Laser Production Principle :-

Two coherent photons produced in stimulated emission, interacts with other two excited atoms resulting in four coherent photons. Thus, coherent photons are multiplied in a Lasing medium. The continuous successive emission of photons results for the production of Laser beam.

Laser Action :-

Laser production is the laser action which involves the following 4 steps

1. Absorption (or) pumping process
2. population inversion
3. Stimulated Emission
4. Amplification

1. Pumping Process :- This process is required to achieve population inversion. pumping process is defined as the process which excites the atoms from ground state to excited state to achieve

Population inversion". Pumping can be done by number of ways.

- i) Optical pumping $\rightarrow$ excitation by strong source of light
- ii) Electrical pumping $\rightarrow$ Excitation by electron impact
- iii) Chemical pumping $\rightarrow$ Excitation by chemical reactions
- iv) Direct conversion $\rightarrow$ Electrical energy is directly converted into radiant.

Energy in devices like LED's, population inversion is achieved in forward bias.

Population Inversion:-

Generally number of atoms in the ground state is greater than the number of atoms in highest energy state. But in order to produce a laser beam the minimum requirement is stimulated emission. Stimulated emission takes place only if the number of atoms in the higher energy level is greater than the number of atoms in the lower energy level. Simply population inversion is nothing but number of atoms in higher energy level is greater than the number of atom in lower energy level.

If N_1 represents number of atoms in the ground state and N_2 represents number of atoms in the excited state. then the amplification of light takes place only when $N_2 > N_1$.

If $N_2 > N_1$ there will be population inversion so induced emission and beam are in the same directions and strictly coherent then the resultant laser is said to be amplified.

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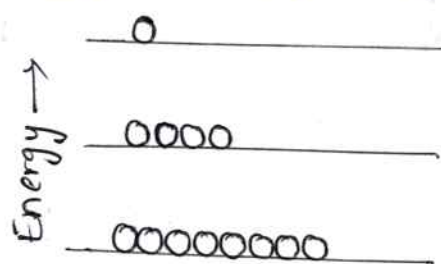
Boltzmann's principle gives the information about the fraction of atom found on average in any particular energy state at equilibrium temperature as

$$\frac{N_1}{N_2} = \exp(E_2 - E_1) / kT$$

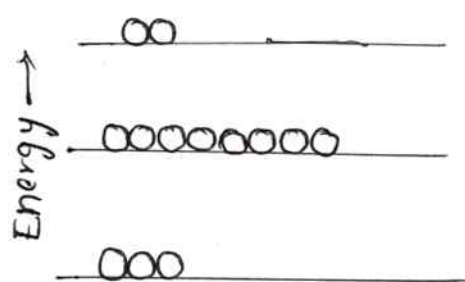
$$= \exp \Delta E / kT$$

where $N_1 = N_0 \exp(-h\nu / kT)$ and

$$N_2 = N_0 \exp(-h\nu / kT)$$



Normal
Distribution



population
inversion

3. Stimulated Emission :-

If majority of atoms are present in highest energy state than the lowest energy state then the process becomes very easy. So, if there is a population inversion there by only stimulated emission will be able to produce Laser beam. Emission of radiation by transition of electron from highest to lowest with supplying photon is called stimulated emission.

4. Amplification :-

The process of multiplying the energy is called amplification

Principle of Spontaneous and Stimulated Emission - Einstein's Co-efficients :-

In 1917, Einstein predicted the existence of two different kinds of processes by which an atom emits radiation. Transition b/w the atomic energy states is statistical process. It is not possible to predict which particular atom will make a transition from one state to another state at a particular instant. For an assembly of very large number of atoms it is impossible to calculate the rate of transitions b/w two states.

Einstein was the first to calculate the probability of such transition, assuming the atomic system to be in equilibrium with electromagnetic radiation.

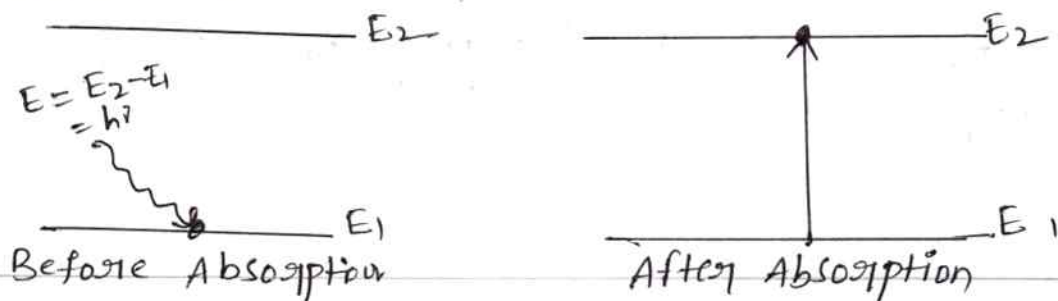
Let us consider an atom exposed to light photons of energy $h\nu$. three distinct processes take place

a) Absorption

b) Spontaneous emission

c) Stimulated emission

An atom in the lower energy level or ground state energy level E_1 absorbs the incident photon radiation of energy and goes to the higher energy level (or) excited level E_2 as shown in figure



Where N_1 = Number of atoms in state E_1 , $H\nu(Q)$ = Energy density of induced beam B_{12} = Probability of an absorption coefficient. The number of spontaneous transitions N_{sp} taking place in time ' Δt ' depends on only no. of atoms N_2 lying in excited state.

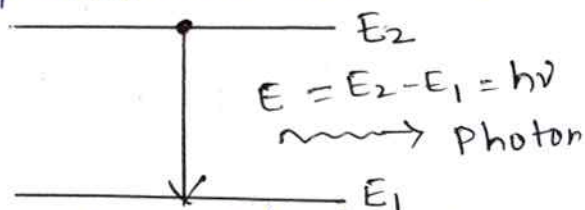
The rate of absorption (R_{12}) is proportional to the following factors. i.e. N_1 and Q

$$R_{12} \propto B_{12} N_1 Q \text{ ----- 1}$$

Where B_{12} = Probability of spontaneous transition.

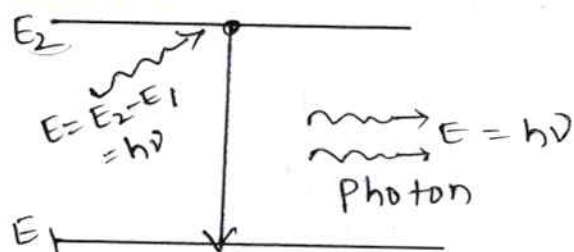
The number of Spontaneous transitions N_{st} occurring during the time Δt may be written as

$$R_{sp} = A_{21} N_2 \text{ ----- 2}$$



The number of stimulated transitions N_{st} occurring during the time Δt may be written as:

$$R_{st} = B_{21} N_2 Q \text{ ----- 3}$$



Where B_{21} = Probability of stimulated emission

Under the thermal equilibrium number of upward transitions = number of downward transitions per unit volume per second.

So, We can write:

$$A_{21} N_2 + B_{21} N_2 Q = B_{12} N_1 Q \rightarrow 4$$

The co-efficients A_{21} B_{12} B_{21} are known as Einstein coefficients

Number of atoms excited into higher energy level
= Number of atoms that made.

Transition into lowest energy level

$B_{12} = B_{21}$ i.e. absorption = stimulated emission

$$B_{12} N_1 Q - B_{21} N_2 Q = A_{21} N_2$$

Rearranging the eq. (4), we have

$$B_{12} N_1 Q - B_{21} N_2 Q = A_{21} N_2$$

$$Q(B_{12} N_1 - B_{21} N_2) = A_{21} N_2$$

$$Q = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2} \longrightarrow (6)$$

Dividing numerator and denominator by $B_{12} N_2$

we have,

$$Q = \frac{\frac{A_{21} N_2}{B_{21} N_2}}{\frac{B_{12} N_1}{B_{21} N_2} - \frac{B_{21} N_2}{B_{21} N_2}}$$

$$Q = \frac{A_{21}}{B_{21}} \cdot \frac{1}{\left(\frac{B_{12}}{B_{21}}\right) \frac{N_1}{N_2} - 1} \longrightarrow (7)$$

On substituting $\frac{N_1}{N_2} = e^{h\nu/KT}$ ($\because$ Boltzmann distribution)

$$Q = \frac{A_{21}}{B_{21}} \cdot \frac{1}{\left(\frac{B_{12}}{B_{21}}\right) e^{h\nu/KT} - 1} \longrightarrow (8)$$

Planks radiation formula for energy distribution

$$Q = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1} \longrightarrow \textcircled{9}$$

Comparing the eq (8) & eq (9) we have

$$\frac{B_{21}}{B_{12}} = 1 \longrightarrow \textcircled{10}$$

$$\text{and } \frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3} \quad (\text{or}) \quad \frac{8\pi h}{\lambda^3} \longrightarrow \textcircled{11}$$

Since $B_{12} = B_{21}$,

Einstein's coefficients are termed as A & B coefficient

Conclusion:-

- The Spontaneous emission is more predominant than the stimulated emission. The Laser light is due to stimulated emission. Therefore, stimulated emission should be greater than spontaneous emission. To achieve this population inversion is required.
- The equation (11) gives the relation between spontaneous emission and stimulated emission coefficients. Since this ratio is proportional to ν^3 , the probability of spontaneous emission increases with the energy differences between the two states.

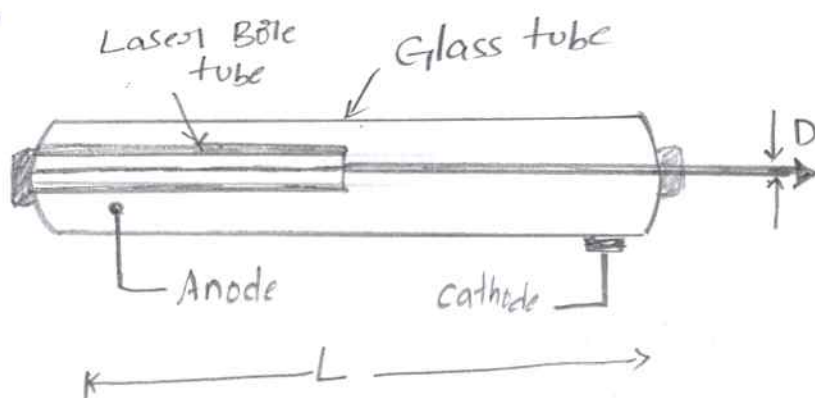
He-Ne Laser :-

The first gas laser to be operated successfully was the He-Ne laser in 1961 by the Scientist A Tawan. In this method, two gases helium & Neon were mixed in the ratio 10:1 in a discharge tube made of quartz crystal. The dimensions of the discharge tube are nearly 80 cm length and 1.5 cm diameter, with its windows substances. The purpose of placing Brewster windows on either side of the discharge tube is to get plane polarized Laser output. Windows slanted at Brewster's angle. $\theta = \tan^{-1}(n)$ where n = refractive index of the window substance. Two concave mirrors M_1 & M_2 are made of dielectric material arranged on both sides of the discharge tube so that their foci lines within the interior of discharge tube. One of the two concave mirrors M_1 is thick so that all the incident photons are reflected back into lasing medium. The thin mirror M_2 allows part of the incident radiation to be transmitted to get laser output.

Working :-

The discharge tube is filled with Helium at a pressure of 1 mm of Hg and Neon at 0.1 mm of Hg. The gas discharge is established by the application of large potential difference between the two electrodes of the discharge tube. The atoms are easily excited to higher energy levels by colliding with each other.

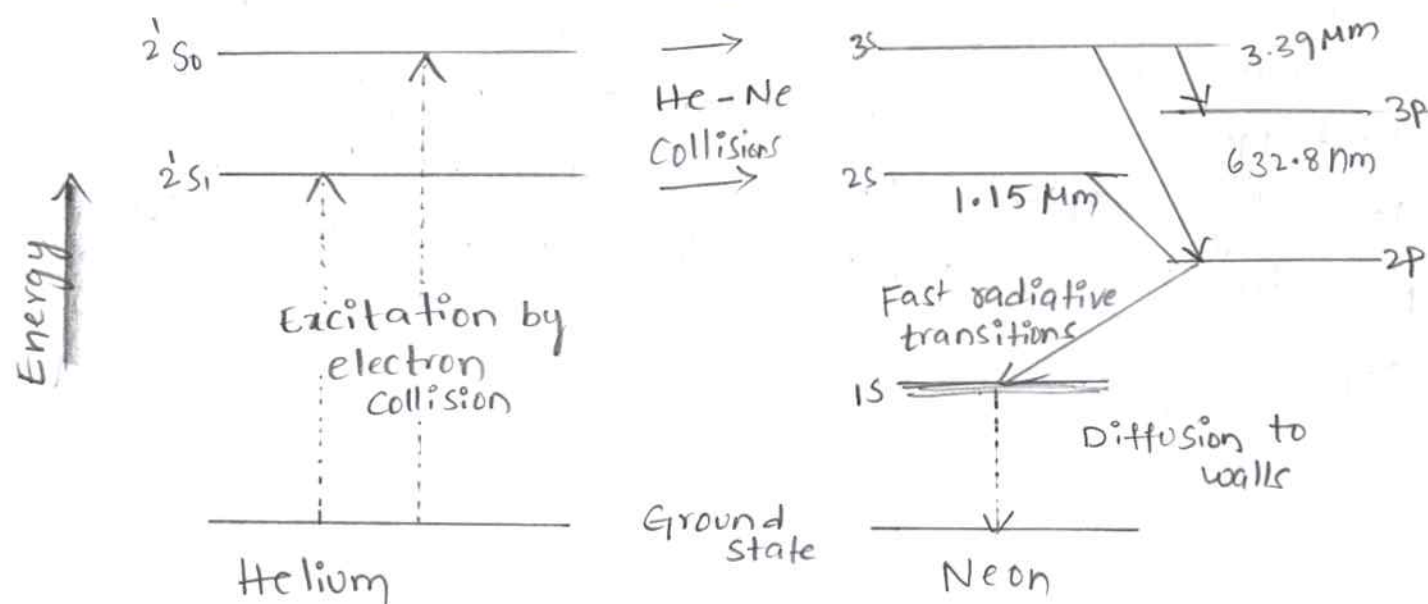
Neon atoms present in less concentration, to excite the Ne atoms pumping process must be required. The excited He atoms collide with Ne atoms and transfer the excitation energy to the Ne atoms. Because both the atoms having highest energy levels very close to each other i.e., 20.61 eV & 20.66 eV for He and Ne respectively. Here He is the pumping medium and Ne is the Lasing medium. Neon atoms undergo a radioactive transfer through 18.7 eV energy level by emitting photons of wavelength $6328 \text{ \AA}$. Ne atoms are pumped into highest energy levels continuously.



Description of Energy Levels :-

When electric discharge is set up in the tube, collisions b/w the atoms raises the amount of He & Ne atoms to the highest energy levels F_1 , F_2 and E_4 , E_6 respectively. Ne has two energy levels E_4 , E_5 at nearly same energy level of F_2 to He. The Ne atoms drop down to the levels E_3 , E_5 a radiative transfer and reaches to the level E_2 through no radiative transfer. From level E_2 Ne atoms are brought back to the ground state through collisions.

with walls of the discharge tube. The transition from $E_6 \rightarrow E_5$, $E_4 \rightarrow E_3$ and $E_6 \rightarrow E_3$ emit radiations corresponding to the wavelength $3390 \text{ \AA}$, $1150 \text{ \AA}$ and $6328 \text{ \AA}$ respectively. $6328 \text{ \AA}$ is the well known red light of He-Ne laser because other two wavelengths are not in the region of electromagnetic radiation (visible region)



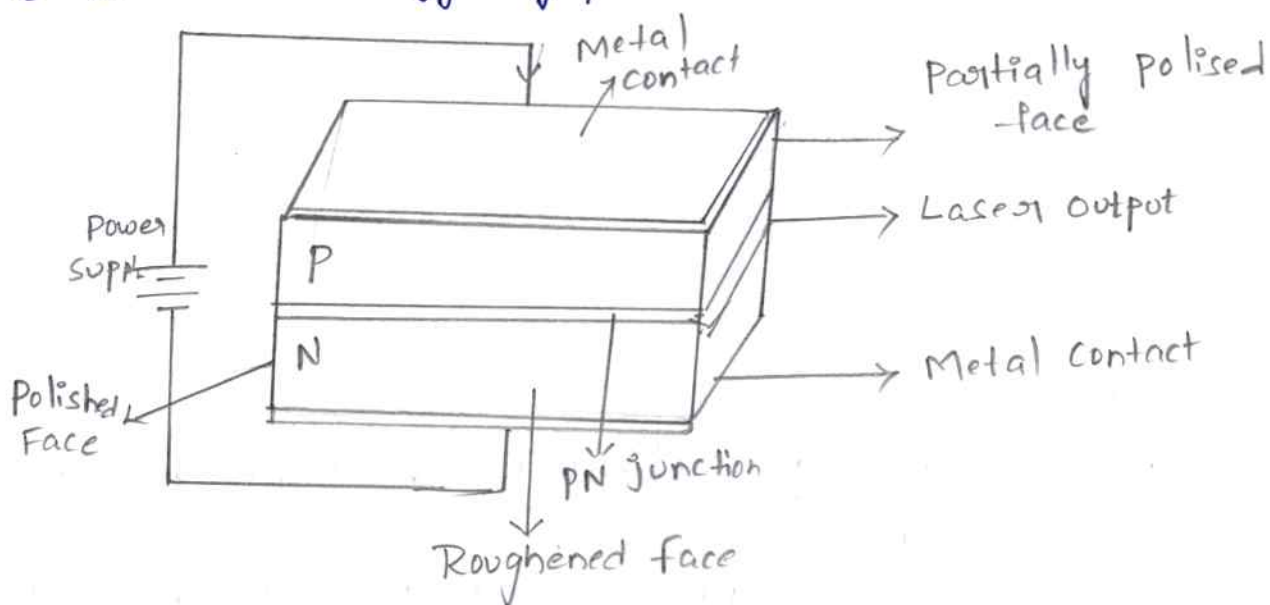
Semiconductor Laser

Principle and working:

After the invention of He-Ne Laser in 1961, Lasers have become at common use. In conventional Lasers, Lasers are generated due to transition of e^- s from highest to lowest energy level. But in Semiconductor laser transitions takes place from conduction band to valency band. The basic mechanism responsible for light emission from a semiconductor laser is the recombination of electrons and holes at PN junction when current is passed

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through the diode. Stimulated emission can occur when the incident radiation stimulates an electron in conduction band to make a transition into valency band in that process radiation will be emitted. As the current is further increased at some threshold value of current the amplification will take place and lasers begin to emit coherent radiation. The properties of semiconductor laser depends upon the energy gap i.e., the induced radiation must have a frequency greater than the value of the energy gap.

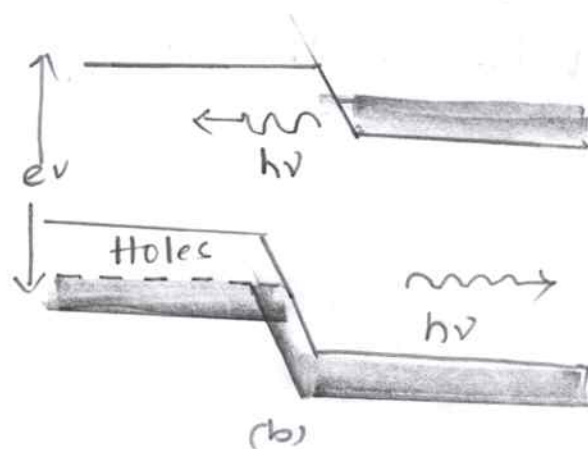
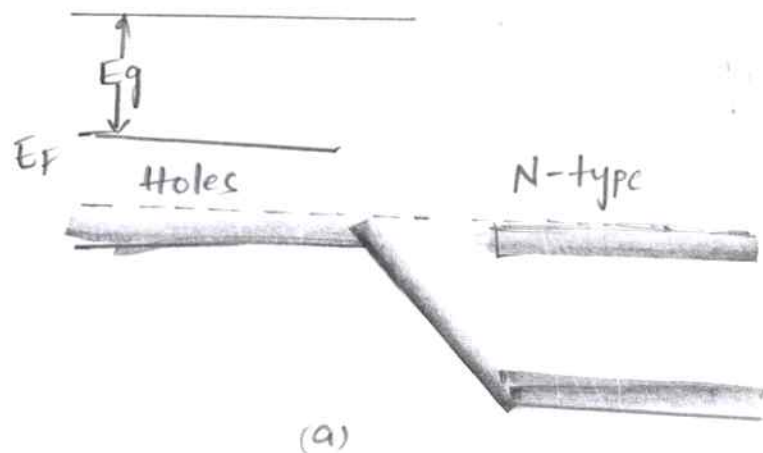


CONSTRUCTION OF SEMICONDUCTOR LASER

Working :-

Ga-As is heavily doped impurities in both P & N regions. N region is doped with tellurium & P region by Zn. When a forward bias with the source is applied to a semiconductor, electrons from N-region and holes from P-region move to cross the junction in opposite directions. In neutral region the electrons and holes recombination is possible due to transition of

electron from CB to VB. For low currents the population inversion does not take place hence only Spontaneous emission takes place and photon released are not coherent. When forward current is further increased beyond the certain threshold value population inversion takes place and coherence photons are released. The energy gap of Gallium Arsenide (Ga-As) is 1.487 eV and corresponding wavelength of radiation is $6435 \text{ \AA}$ which is responsible for Laser medium.



Applications :-

Lasers have wide applications in different branches of Science and engineering.

1. Lasers are used in optical communications, due to narrow band width.
2. The laser beam can be used for communication b/w earth and moon (or) other satellites.
3. By the use of Lasers, the storage capacity of information in computers is greatly improved.
4. Lasers have industrial applications. They can make holes in diamond & hard steels.
5. Lasers have wide medical applications, military purposes.
6. These are used to find the size, shape biological cells such as erythrocytes.

UNIT-IV FIBER OPTICS

Optical fiber :-

A thin flexible and transparent cylindrical waveguide prepared for light Propagation is called optical fiber and also called a light pipe. It is made up from the dielectric materials like glass and plastic.

Construction of an optical Fiber :-

Structure of an Optical fiber consists of three parts : core, cladding and outer jacket. Core diameters range from 5 to 100 μm while cladding diameters vary from 125 to 750 μm .

Core :

- The core is a cylindrical rod of dielectric material
- Light Propagates mainly along the core of the fiber
- The core is generally made of glass or plastic
- The core is described as having an index of refraction,
- Refractive Index : The refractive index of a substance is a measure of the speed of light in that substance. It is expressed as a ratio of speed of light in vacuum relative to that in the considered medium ($n = c/v$)

Cladding :

- * The core is surrounded by a layer of material called the cladding which is generally made of glass or plastic.

* The cladding layer is made of a dielectric material with an index of refraction n_2 .

* The index of refraction of the cladding material is less than that of the core material.

* The cladding performs the following functions.

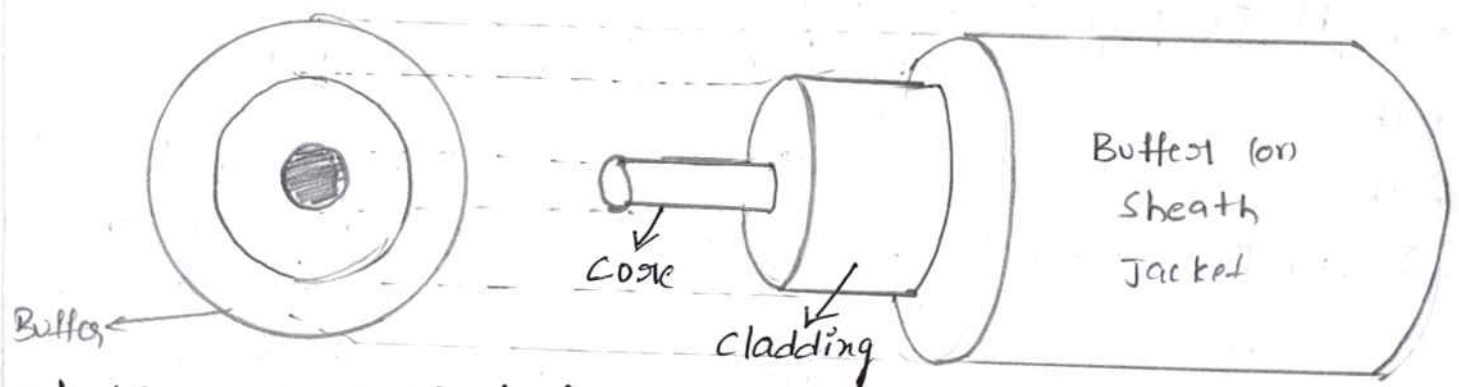
1. Reduces loss of light from the core into air.
2. Reduces scattering loss at the surface of the core.
3. protects the fiber from absorbing surface contaminants.
4. Adds mechanical strength.

Buffer :-

* The coating or buffer is a layer of material used to protect an optical fiber from physical damage.

* The material used for a buffer is a type of plastic.

* The buffer is elastic in nature and prevents abrasions.



Working of Optical fiber

The light propagates through optical fiber by the principle "Total internal reflection"

Total internal reflection appears due to following reasons. When light travel from optically denser

rarer medium to denser medium the refracted ray moves towards normal drawn on the interface of media as in Snell's law. Conversely, if light traverse from denser to optically rarer medium, the refracted ray moves away from the normal drawn on the interface of the medium. If the angle of incidence increases as shown in fig (b) to certain value for which the refracted ray happen to be on the interface of medium with $r=90^\circ$. This angle of incidence is known as Critical angle (θ_c) if the incident angle $i > c$ as shown in fig c.

Conditions for total internal reflections:

1. The ray of light should be traverse from denser to rarer medium
2. The incident angle should be more than the Critical angle ($\theta_i > \theta_c$)

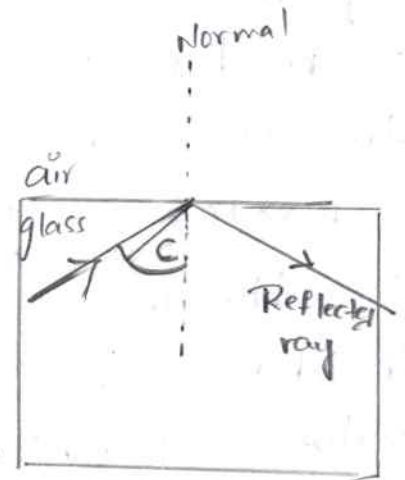
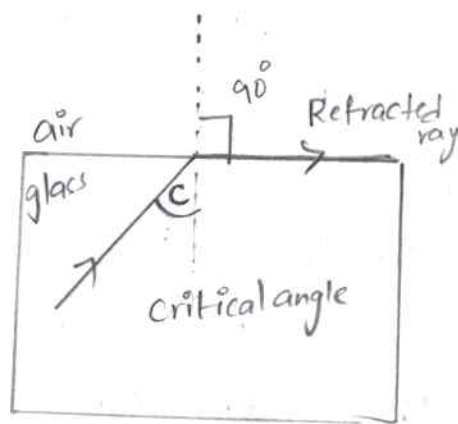
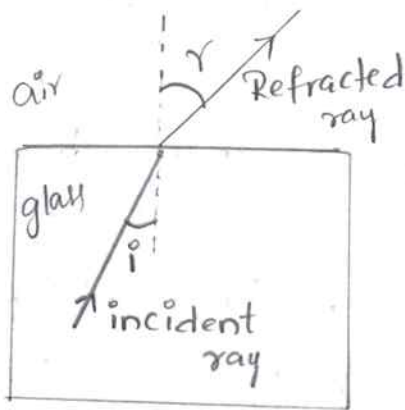
Total internal reflection :-

When light travels from denser to rarer medium if the incident ray exceeds the critical angle then refraction would be turned into reflection is called total internal reflection. If ' i ' is the angle of incidence less than critical angle and reflection will not takes place.

From figure

- a. when $i < c$, it is refracted into rarer medium
- b. when $i = c$, it traverses along the interface so that angle of refraction is $r = 90^\circ$

c. when $i > c$, it is totally reflected back into the denser medium itself.



(a) when $i < c$
no reflection

(b) $i = c$
critical angle

(c) $i > c$
Total internal reflection

When $i = c$ then at critical angle, using Snell's law

According to law of refraction

$$n_1 \sin \theta = n_2 \sin r$$

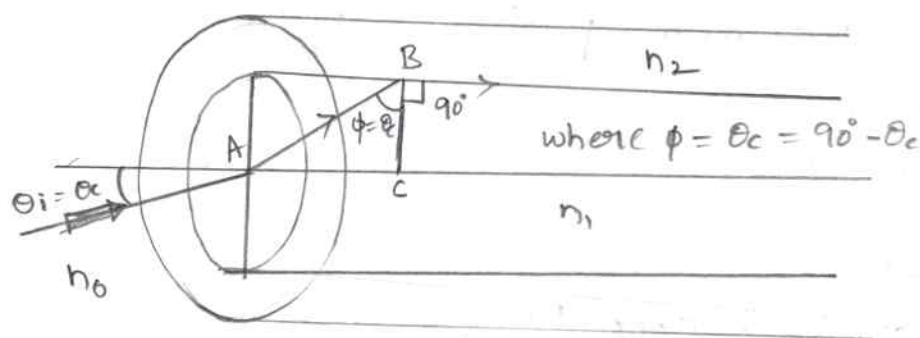
$$\theta = \theta_c \rightarrow r = 90^\circ$$

$$\sin \theta_c = \frac{n_2}{n_1} \sin 90^\circ$$

$$\sin \theta_c = \frac{n_2}{n_1}$$

Expressions for Acceptance angle (θ_m) & Numerical Aperture (NA) :-

All light rays falling on the optical fiber are not transmitted through the fiber. Only those light rays making $\theta_i > \theta_c$ at the core-cladding interface are transmitted through the fiber by undergoing TIR. For which angle of incidence, the refraction angle is greater than 90° will be propagated through TIR. **Acceptance Angle (AA) :** The maximum angle of incidence to the axis of optical fiber at which the light ray may enter the fiber so that it can be propagated through interface of core & cladding.



Consider an optical fiber through which the light is being sent. The end at which light enters is called launching end. Let the refractive indices of the core and cladding be n_1 and n_2 respectively, $n_1 > n_2$. Let refractive index of the medium from which the light is launched be n_0 . Let the light ray enter at an angle i to the axis of the fiber. The ray refracts at an angle r . The ray strikes the core-cladding interface at an angle θ . If θ is greater than the critical angle θ_c , the ray undergoes TIR.

at the surface. Let us now find out up to what maximum value of i at A, the TIR at B is possible

In triangle ABC, $90 - r = \theta \longrightarrow (1)$

From Snell's law

$$\frac{\sin i}{\sin r} = \frac{n_1}{n_0}$$

$$\frac{\sin i}{\sin(90 - \theta)} = \frac{n_1}{n_0}$$

$$\sin i = \cos \theta \left(\frac{n_1}{n_0} \right) \longrightarrow (2)$$

If θ is less than the critical angle θ_c , the ray will be lost by refraction. Therefore, limiting values for containing the beam inside the core by TIR is

θ_c .

When $i = i_m$ the maximum possible angle of incidence at the fiber end.

face A, then $\theta = \theta_c$.

From equation (2)

$$\sin i_m = \cos \theta \frac{n_1}{n_0} \longrightarrow (3)$$

But from the interface of core & cladding at critical angle,

$$\sin \theta_c = n_2 / n_1$$

$$\cos \theta_c = \sqrt{(n_1^2 - n_2^2)} / n_1$$

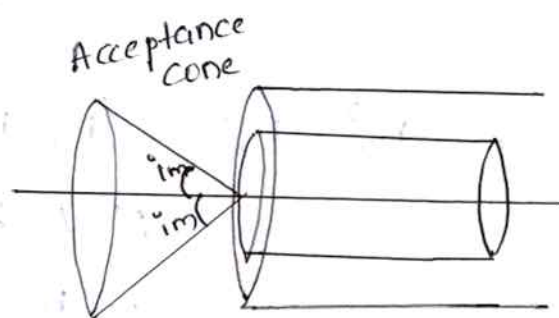
by sub. this in eq (3) one can get

$$i_m = \sin^{-1} \sqrt{(n_1^2 - n_2^2)} / n_0$$

This angle i_m is the maximum acceptance angle.

Maximum Acceptance angle :-

Acceptance angle is defined as the maximum angle that a light ray can have relative to the axis of the fiber and propagate down the fiber. or the maximum angle or below which the light can suffer TIR is called acceptance angle.



Acceptance Cone :-

An optical fiber accepts only those rays which are incident within a cone having a semi full angle $2\theta_{im}$ are accepted and transmitted along the fiber. Therefore the cone is called the acceptance cone. Light incident angle beyond θ_{im} refracts through the cladding and the corresponding optical energy is lost.

Numerical Aperture :-

The numerical aperture NA is defined as the sine of the acceptance angle.

$$NA = \sin(\theta_{im}) = \sqrt{(n_1^2 - n_2^2)} n_0$$

NA determines the light gathering capability of the fiber. It is a measure of amount of light that can be accepted by a fiber.

Fractional Index Change :-

It is the ratio of refractive index difference in core and cladding to the refractive index.

$$\text{Fractional index change } \Delta = \frac{n_1 - n_2}{n_1}$$

$$\frac{n_1 - n_2}{n_1} = \Delta \cdot n_1$$

$$n_1^2 - n_2^2 = (n_1 - n_2)(n_1 + n_2) \approx (n_1 - n_2)(2n_1)$$

$$\approx \Delta \cdot 2n_1^2 NA$$

$$= n_1 \sqrt{2\Delta}$$

Classification of optical fibers:-

Based on variation in the core refractive index (n_1) optical fibers are divided into two types.

1. Step index fiber
2. Graded index fiber

Based on mode of propagation, fibers are further classified into 2 types as.

1. Single mode propagation and multimode propagation

Step index fiber propagations.

2. Whereas graded index fibers have multimode propagation only.

All together in total 3 types of fibers

1. Single mode step index fiber
2. Multimode step index fiber
3. Multimode graded index fiber

Transmission of signal in optical fibers

Step index fiber:-

Refractive index of core material is uniform throughout and undergoes a sudden change in the form of step at the core-clad interface.

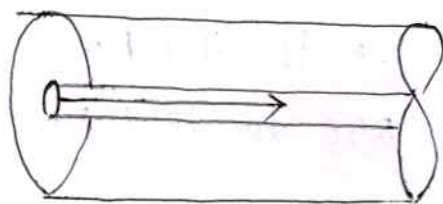
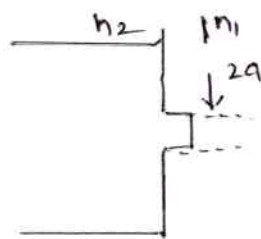
Single Mode Step index fiber :-

It has longitudinal cross section and the variation of the refractive index in step manner is called step index optical fiber. The refractive index of this optical fiber as a function of distance can be mathematically represented as refractive index profile in single mode step index fiber

$$n(r) = n_1 \quad \text{when } r < a$$
$$= n_2 \quad \text{when } r > a$$

where r is radial distance and a is radius of core for glass fiber

- * Core diameter : 8 to 12 μm , usually 8.5 μm
- * Cladding Diameter : Around 125 μm
- * Sheath Diameter : 250 to 1000 μm
- * NA : 0.15 to 0.25 usually 0.22.



— 125 μm cladding

— 8-12 μm (Core)

For single mode step index fiber, it might appear that for $V = 2.405$, i.e., $N \approx 2$. This is not a mistake. It simply reflects the fact that the single mode contains two polarizations, which are indistinguishable from each other because of circular symmetry of the optical fiber. In other words, a single mode fiber contains two modes that are together.

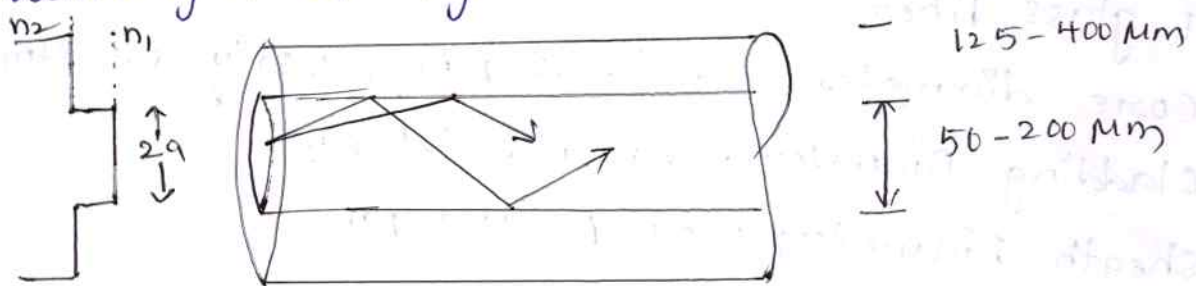
$$V \text{ number} = \frac{2\pi}{\lambda} \cdot NA$$

Number of mode propagation in S.I Single mode

$$N_{SI} = V^2/2$$

Multi Mode step Index fibers :-

These fibers reasonably large core diameters and large NA to facilitate efficient transmission to coherent or coherent sources. These fibers allow finite number of modes. Normalized frequency is the cut-off frequency, below which a particular mode cannot exist. This is related to NA, Radius the core and wavelength of light



- i) Core diameter : 50 to 200 μm
- ii) Cladding Diameter : 125 to 400 μm
- iii) Sheath Diameter : 250 to 1000 μm
- iv) NA : 0.16 to 0.5

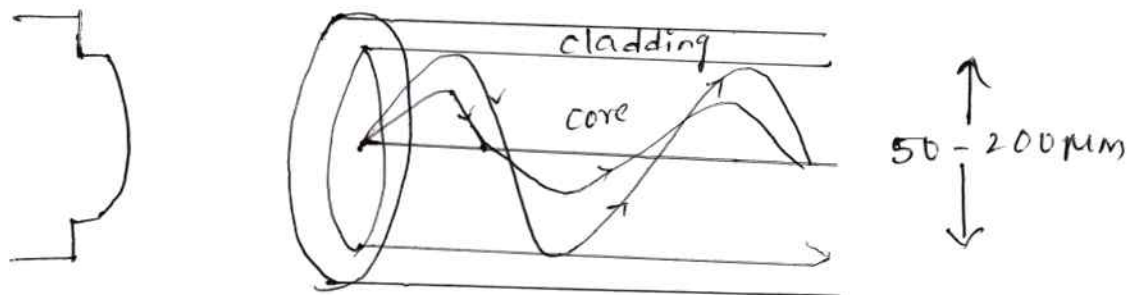
Multimode Graded Index fibers :-

In case of graded index fiber, the refractive index of core is made to vary as a function of radial distance from the center of the optical fiber. Refractive index increases from one end of core diameter to centre and attains maximum value at the center. Again refractive index decreases as

as moving away from center to towards the other end of the core diameter. The refractive index variation is represented as refractive index profile in multimode graded index fiber

$$n(r) = n_1 \sqrt{1 - 2(r/a)^2 \Delta} \quad \text{when } r < a$$

$$= n_2 \quad \text{when } r > a$$



$$V\text{-number} = (2\pi/\lambda) \cdot NA$$

Number of mode propagation in G.I multimode

$$N_{GI} = N_{SI}/2$$

- i) Core Diameter : 30 to 100 μm
- ii) Cladding Diameter : 105 to 150 μm
- iii) Sheath Diameter : 250 to 1000 μm
- iv) NA : 0.2 to 0.3

Attenuation and Losses in Fibers :-

When the light signal propagates in the optical fiber, losses arise due to different factors and these losses are referred to as attenuation in optical fibers. The various factors causing attenuation in optical fibers are :

1. Material dispersion (or) impurity losses

2. Scattering losses

3. Absorption losses

4. Bending losses

5. Radiation induced losses

6. Inherent defect losses

7. Inverse square law losses

8. Transmission losses

9. Core and cladding losses.

Losses are expressed in decibels per kilometer (dB/km)

The attenuation is given by

$$P_{out} = P_{in} 10^{-\alpha L / 10}$$

where P_{out} = Power at a distance L from the input P_{in}
amount of power coupled in to fiber α = fiber attenuation
factor in dB/km and L = Length of the optical fiber.

1. Material (or) impurity losses :-

The doped impurities present in the fabrication of an optical fiber in order to vary the refractive index causes, losses

2. Scattering losses :-

In a glass fiber, the glass contains many microscopic inhomogeneities and material content. Due to this a portion of the light signal passing through the glass fiber gets scattered. This scattering losses vary inversely with the fourth power of wavelength.

3. Absorption losses :-

It is caused by the nature of the core material and varies inversely to the transparency of the material. For glass fibers, ion-resonance absorption, ultraviolet absorption and infrared absorption are the three separate mechanisms which contribute to total absorption losses.

4. Bending losses :-

Whenever a fiber deviates from a straight line path, radiate losses occur. These losses are prominent in improperly installed single mode optical cable.

5. Radiation induced losses :-

When the glass molecular matrix interacts with electrons, neutrons, x-rays and gamma rays, the structure of the glass molecules is altered and the fiber darkens.

6. Inherent defect losses :-

The inherent defect present in the core-cladding causes losses of the propagating light signal through it. The surface defect in the core causes losses.

7. Transmission losses :-

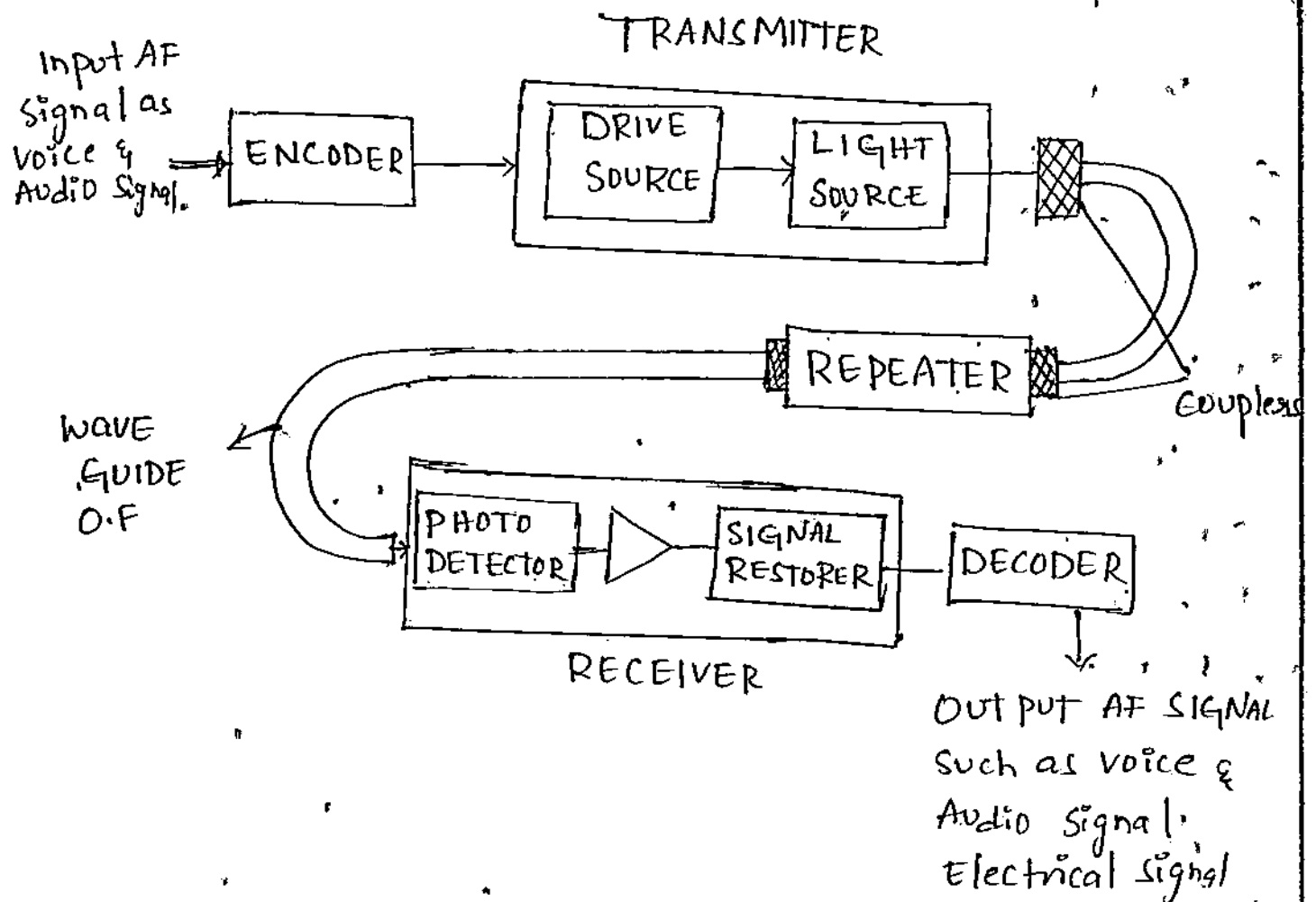
The losses are caused by light which is caught in the cladding material of the optical fibers. This light is either lost or trapped in the cladding layer.

Optical Fiber Communication System :-

Optical fiber communication system essentially consists of three parts namely

- a) Transmitter
- b) Optical fiber
- c) Receiver

The transmitter includes modulator, encoder, light source, drive circuit and couplers. Basically the fiber optic system simply converts an electrical signal to binary data by an encoder.



Transmitter :-

The transmitter consists of an analog to digital A/D converter called a decoder, and a light source. The A/D converter is used to convert continuous analog signals as voice or video (TV) signals into a series of digital pulses. This binary data comes out as a stream of electrical pulses and these pulses are converted into pulses of optical power, by modulating light source is either a light emitting diode LED or an injection laser diode (ILD). Then the drive circuit directly modulates the intensity of the light with the encoded digital signal. This digital optical signal is launched into the optical fiber cable. The light beam pulses are then fed into a fiber-optic cable using couplers, where they are transmitted over long distances.

Optical fiber :-

The optical fiber consists of a glass or plastic fiber core, a cladding and a protective jacket. To transmit signals to long distances, repeaters are used after certain lengths in the optical fiber. Finally at the end of the optical fiber, the signal is fed to the receiver.

Receiver :-

The receiver includes a light detector or photocell and a decoder. The light detector is very often either a PN diode or an APD. The light detector, acting as

the receiving element, converts the received light pulses back to pulses of electrical current. The electrical pulses are amplified and reshaped.

Medical applications of optical fibers:-

Fiber optic Scope in endoscope is one of the widely used optical technique to view the internal parts of the disease affected body.

The basic principle is the light from the source is transmitted through the outer fibers which falls on the inner portions of the body. The reflected light from the inside is carried by the inner fiber to the observer's eye to observe the image of the illuminated portion.

Optical fibers in Sensors:-

Temperature Sensor: The light from the source passes through the upper fiber, and then through the silicon layer having reflective coating at the end placed in the temperature field to be sensed.

These reflected light after passing through the bottom fiber is detected by the photo detector.

When light passes through the silicon layer it absorbs certain light and hence, received intensity varies. The amount of light absorbed by the silicon varies with temperature.

Semiconductors

Origin of Energy bands in Solids :-

A Solid is nothing else than a collection of interacting atoms. According to the energy band theory, from each atomic level a huge number of different energy levels is generated due to interaction between atoms. A schematic splitting of atomic levels by decreasing the distance between the atoms is presented in fig. From the point of view of Solid State physics there are two atomic levels which are of considerable importance; The last-occupied and the first unoccupied by electrons. These two atomic levels give rise to two different bands in the solid as shown in fig.

Valance Band : The band resulting from the last occupied level is called the valency band.

Conduction Band : The band resulting from the first unoccupied level is called C.B.

Forbidden Energy gap :- The energy band diagram has a gap between V.B & C.B named as forbidden gap

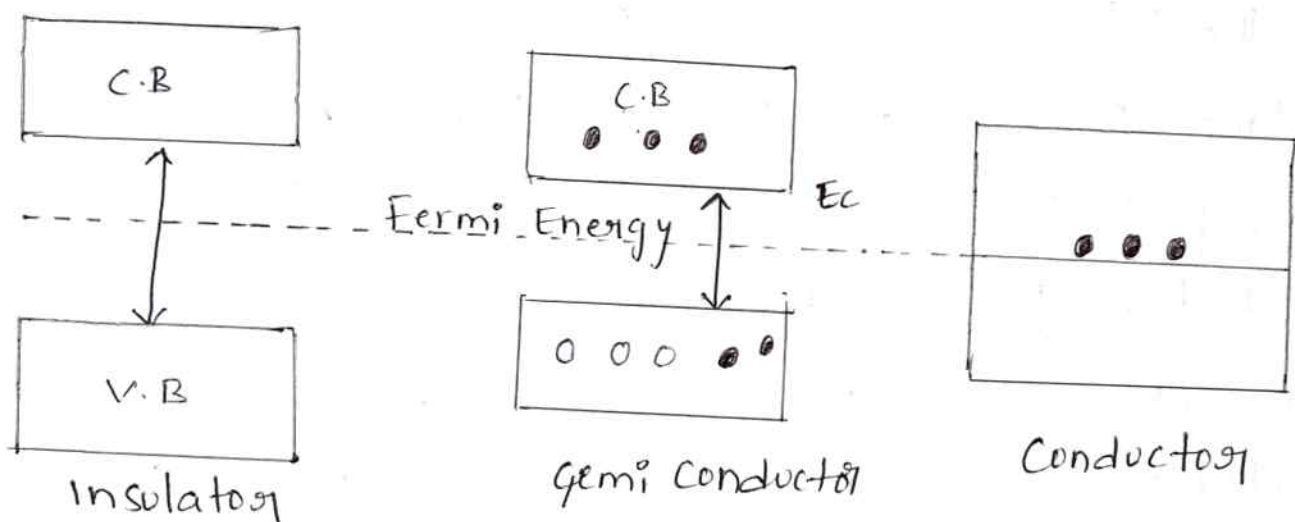
Thus the valency band is totally occupied by electrons, whereas the C.B is partially or totally free of electrons at 0K temp. The metallic

semiconductor or dielectric properties of a solid are determined by the fact how the two bands are filled by electrons.

classification of solids based on Energy bands:-

Based on the energy gap between the V.B and C.B the materials classified as conductor, insulator and semiconductor as shown in fig.

Conductor: The valency band is completely filled and the conduction band is empty, but they overlap each other. So the energy gap is almost zero $E_g \approx 0 \text{ eV}$. Because of overlap, it is considered as single band. Hence conduction band is assumed as partially filled. Therefore, on applying a small electric field, the metals conduct electricity.



Insulator: In insulator like diamond, the forbidden energy gap is quite large. The forbidden energy gap value around $E_g > 6 \text{ eV}$, so minimum of 6 eV energy is required for electron to move from V.B to C.B.

32

Normally C.B is empty and V.B is full for insulators. Since there is no electron for C.B, these type of materials behave as an insulators.

Semiconductor :-

The energy band structure of the semiconductor is similar to the insulator, but the forbidden gap is very narrow. The forbidden energy gap ranges $E_g = 0.1 - 3 \text{ eV}$ for silicon 1.1 eV and for Ge 0.7 eV . The semiconductor acts as an insulator at 0°K and at room temperature the thermal energy absorbed from the surrounding is enough for the electron to jump from the valence band to C.B. So a semiconductor starts conducting at room temperature.

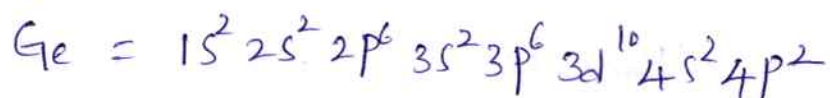
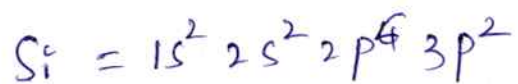
Types of Semiconductors :-

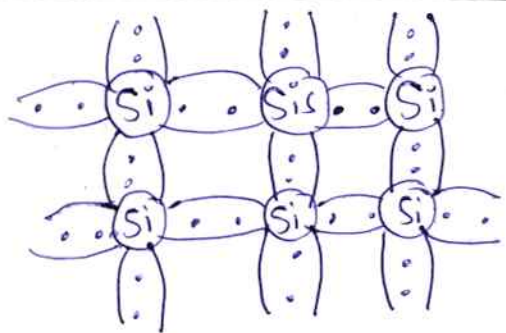
Semiconductors are of two types. They are

- pure or intrinsic
- impure or extrinsic

Intrinsic semiconductor

A pure semiconductor is also called as semiconductor. Most of the fourth group elements are semiconductor. Germanium (Ge) and Silicon (Si) are important among them. Carbon is sometimes used as a semiconductor.





At absolute zero

Germanium has 32 electrons and silicon has 14 electrons in their atomic structure. Since each of them have 4 valence electrons, they are called tetravalent atoms. The neighbouring atoms form covalent bonds by sharing four electrons with each other. This covalent bond is shown in fig. Since all the 4 valence electrons are very tightly bound to their atoms an intrinsic semiconductor acts as a perfect insulator at 0°K. It does not have free charge carriers at 0°K. The diagram shows the energy level diagram of an intrinsic semiconductor. The width of the forbidden band is 0.72 eV for Ge and 1.1 eV for Si. Thus semiconductor works as insulator at 0°K.

The covalent bonds can be broken when the temperature of semiconductor is increased. When a covalent bond is broken, an electron jumps to the C.B. leaving behind an empty space in the valency band called a hole which behaves like a +ve charge with a mass equal to the mass of electron.

Thus intrinsic semiconductor works as conductor at room temperature. The bond diagram and energy band diagram of pure semiconductor is shown in fig.

Extrinsic Semiconductor :-

Pure semiconductors unable to produce large number of charge carriers at normal temperature. Hence their use is very limited. Whenever a very small amount of impurity is added to the pure semiconductor, its conductivity increases drastically. owing to creation of large no. of charge carriers.

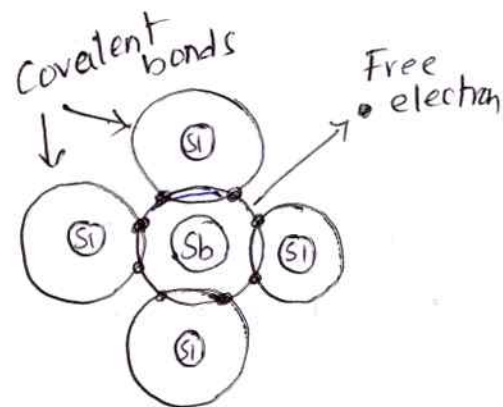
The process is called doping and the resulting s.c is called extrinsic semiconductor.

(i) P-type semiconductor

(ii) N-type semiconductor

N-type Semiconductor :-

When a very small amount of pentavalent impurity element (group V) is added to a pure semiconductor we can get an N-type semiconductor.



eg.

Arsenic or Antimony or phosphorous is added to Ge or Si

In a Si crystal the 4 valance electrons bound b.. when a phosphorous impurity is added to Si the phosphorous atom with 5 valance electrons is connected by covalent bonds on the four sides with the 5th electron left free, as shown in fig. This acts as a conduction band on the four sides with the fifth electron left free. A small electric field can make

electrons move. Now semiconductor has majority of negatively charged electron.

Hence it is called N-type semiconductor.

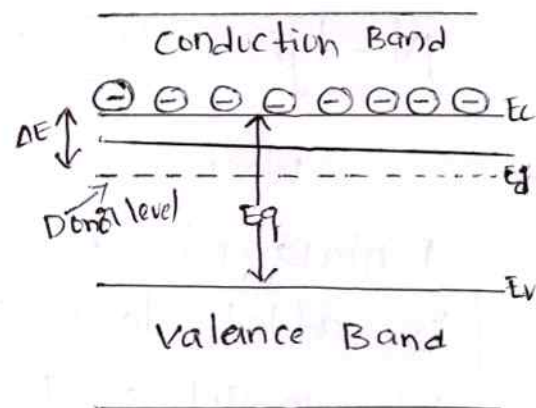
Each impurity atom contributes a free electron, thus increasing the no. of charge carriers. When temperature increases above 0°C , the heat energy is used to break the covalent bond & release electrons & an equal no. of holes. Thus the number of charge carriers is increased due to doping. Since a large number of electrons & a small no. of holes are created due to this type of doping, electrons are majority charge carriers and holes are minority charge carriers.

From the charge neutrality condition, the total negative charge of mobile electrons is equal to the total positive charge created in the crystal

$$\text{Thus } n_n = N_d + P_n$$

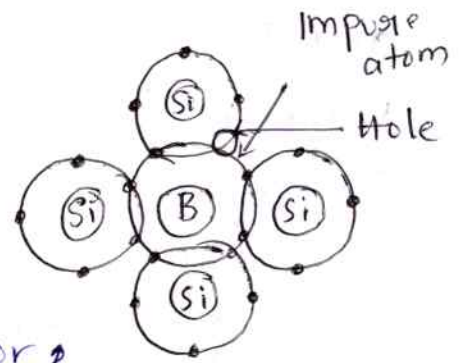
where N_d is the donor concentration

n_n & P_n are the electron and hole concentration in n-type semiconductor.



P-type Semiconductor :-

When a very small amount of trivalent impurity element is added to a pure semiconductor, it becomes a P-type semiconductor. Eg. Ga or In or B is added to Ge or Si.



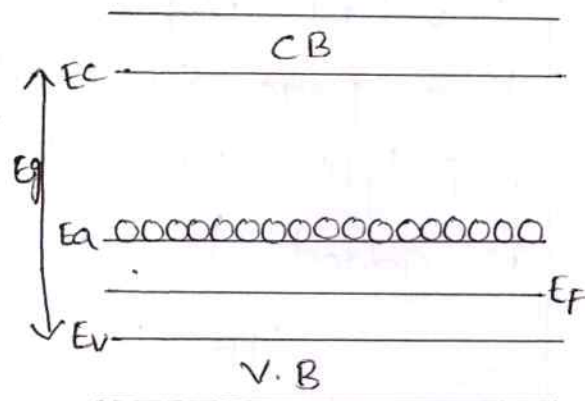
In a Si crystal, valence electrons are bound by covalent bonds. When a boron impurity is added to Si, the boron atoms with 3 valence electrons are connected by covalent bonds on three sides and a vacancy on the fourth side, as shown. A vacancy means absence of a negative charge and hence is equivalent to the presence of a positive charge or a hole. With charge the neutral Si now becomes a majority of positively charged holes. Hence it is called P-type.

Each impurity atom contributes a hole, thus increasing the no. of charge carriers. When temperature increases above 0°K , the heat energy is used to break the covalent bonds and release electrons and equal no. of holes. Thus the no. of charge carriers is increased due to doping. Since a large number of holes and a small no. of electrons are created due to doping.

Even at relatively low temperature the acceptor atoms get ionized taking electrons from valence band and thus giving rise to holes in valence band for conduction. Due to ionization of acceptor atoms only holes

and no electrons are created. Thus holes are more in number than electrons and hence holes are majority carriers and electrons are minority carriers in P-type Semiconductors.

From the charge neutrality condition, the total -ve charge of mobile electrons is equal to the total positive charge created in the crystal.



Where N_A is the acceptor concentration, n_p and p_p are the electron & hole concentration in p-type semiconductor. Thus $p_p = N_A + n_p$

Differences between p-type and N-type Semiconductors

N - type	P - type
1. It is obtained by doping the impurity atoms of pentavalent atoms like P, Bi, As into Si and Ge atom.	1. It is obtained by doping the impurity atoms of trivalent atoms like boron, indium, aluminium into Si & Ge.
2. It is also known as donor type semiconductor.	2. It is also known as acceptor type semiconductor.
3. Electrons are majority carriers and holes are minority carriers.	3. Holes are majority carriers and electrons are minority carriers.
4. The electron density n_e is much greater than the hole density n_h i.e., $n_e > n_h$.	4. The hole density n_h is much greater than the electron density n_e . i.e., $n_h > n_e$.

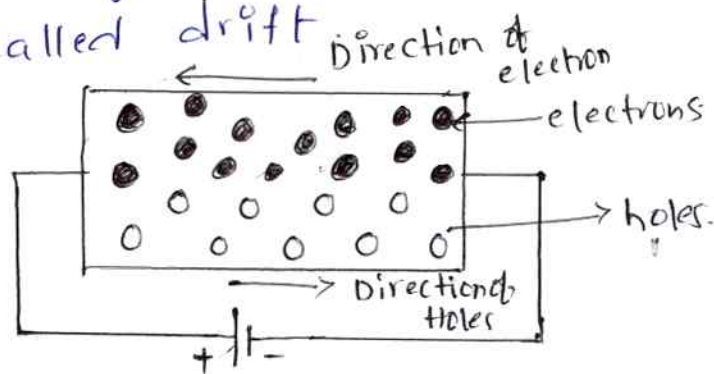
Drift and Diffusion Semiconductor :-

In semiconductor electrical conduction takes place by means of two processes they are

1. Drift
2. Diffusion

Drift :-

When a voltage applied across the semiconductor material each charge carrier experience a force attracting the electrons to the positive potential and holes to the negative potential. This net movement in charge is called drift.



In the absence of external electrical field, the charge in semiconductor move randomly. Hence the net charge flowing the S.C is zero.

When the external electric field is applied the electrons experience a force equal to

$$F = -qE$$

But the velocity of charge carrier does not increase continuously due to collisions with atomic nuclei. Hence the charge carriers will attain a constant velocity or steady state velocity is called drift velocity V_d . The drift velocity is directly proportional to the strength of the applied electric field.

$$V_d \propto E$$

$$V_d = -\mu_n E$$

Hence μ_n is the mobility of electrons and -ve sign indicates the mobility is against drift velocity. The drift of electrons against electric field develops current density J_n and is given by

$$J_n = -neV_d$$

$$J_n = -ne\mu_n E \rightarrow (1)$$

Here n is the electron concentration and e is the electron charge

Also we know that the current density as

$$J_n = \sigma_n E \rightarrow (2)$$

Here σ is conductivity of electrons

From (1) & (2) equations

$$\sigma_n E = ne\mu_n E$$

$$\sigma_n = ne\mu_n$$

$$\sigma_p = pe\mu_p$$

By for conductivity of holes is
Since the S.C both electrons & holes, the net conductivity is given by $\sigma = \sigma_n + \sigma_p$

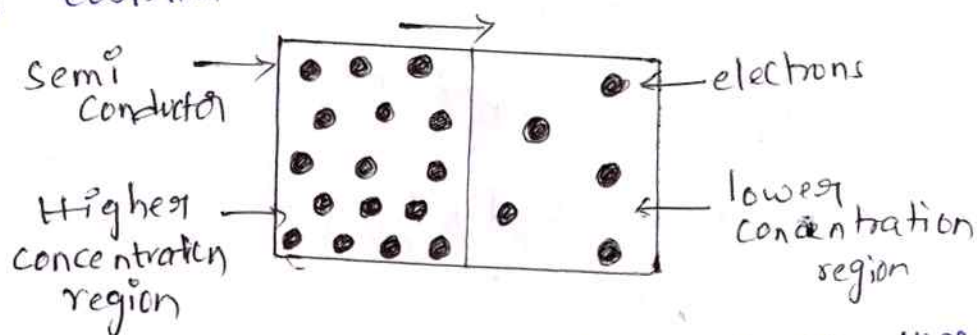
$$\sigma = ne\mu_n + pe\mu_p$$

Drift Current density $J_{drift} = J_n + J_p$

$$J = neV_d + peV_d$$

Diffusion :-

When there is a non uniform charge carriers in s.c diffusion takes place i.e., the electron and hole concentration vary with the position of the sample. Any such special variation of charge carriers can cause net motion of charge carriers from the regions of highest concentration to the regions of lowest concentration. This type of net motion of charge carriers is called diffusion. This diffusion constitutes a current called diffusion current.



According to Fick's law the rate of diffusion of electrons is directly proportional to dn/dx and it takes place in the direction of negative gradient.

Therefore the current density in n-type semiconductor

$$J_n = e D_n (dn/dx)$$

The current density in p-type semiconductor

$$J_p = -e D_p (dp/dx)$$

Where D_n & D_p are diffusion constants or diffusion coefficient of electrons & holes.

Total diffusion current density is given by

$$J = J_n + J_p$$

$$J = e D_n (dn/dx) - e D_p (dp/dx)$$

Einstein's relation :-

It gives the relation between drift and diffusion current. In the absence of external electric field the net current due to the motion of charge carriers is equal to zero.

When the external electric field is applied the diffusion current flows and internal electric fields produced. Due to this field a drift current flows balancing diffusion current

$$J_n(\text{Drift}) = J_n(\text{diffusion})$$

$$ne\mu_n E = eD_n \left(\frac{dn}{dx} \right)$$

$$neE = eD_n/\mu_n \left(\frac{dn}{dx} \right)$$

Force due to this field

$$F = neE = e \frac{D_n}{\mu_n} \left(\frac{dn}{dx} \right) \longrightarrow (1)$$

Comparing the charge carriers in S.C gas molecules at low pressure then the force corresponding to the pressure gradient is given by

$$F = KT \frac{dn}{dx} \longrightarrow (2)$$

From eq (1) & (2) we have

$$e \frac{D_n}{\mu_n} \left(\frac{dn}{dx} \right) = KT \frac{dn}{dx}$$

$$\frac{D_n}{\mu_n} = KT/e$$

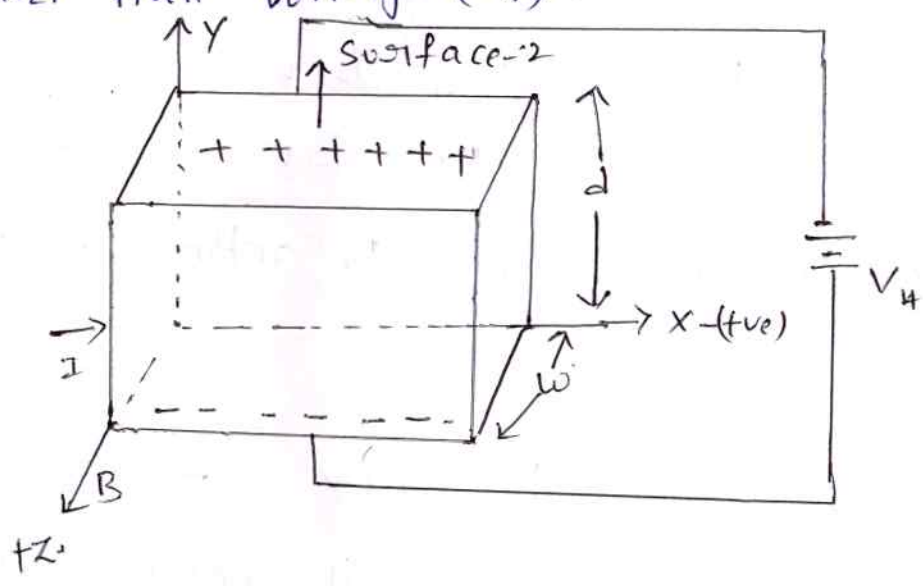
||y

$$D_p/\mu_p = KT/e$$

Therefore we get $\frac{D_n}{\mu_n} = D_p/\mu_p = KT/e$ is called Einstein relation.

Hall Effect and Hall Voltage :-

When a current carrying semiconductor placed in transverse magnetic field then perpendicular to both a voltage is developed is called Hall effect. This voltage is called Hall voltage (V_h).



Consider a semiconductor width ' w ' and a thickness ' d ' which carrying current, I along x -direction. The semiconductor now subjected to magnetic field of inductor B along z -direction. Thus an electric field and magnetic field forces are developed in perpendicular to each other and a resultant Lorentz force is developed along y -direction.

Electric force, $F_e = E \cdot q$

Magnetic force $F_m = B q v_d$ where v_d is drift velocity

$F_L = F_e + F_m$

The force experienced by the positive charge given by Fleming left hand rule is in the upward direction as shown. Thus the top face will get positive charges and the bottom face will get -ve charge due to

drifting of charges. The separation of charges shall continue till the electrical field of attraction between them due to hall field E_H developed between the two faces balances the force due to magnetic field B .

Thus

At equilibrium, the two forces are equal

$$\text{Hence, } eE_H = Bev$$

$$E_H = Bv \longrightarrow (1)$$

The current density J_x acting along x -axis is

$$v = \frac{J_x}{ne}$$

$$J_x = nev \longrightarrow (2)$$

where ' n ' is the carrier concentration of holes or electrons
Sub eq (2) in eq (1) we get.

$$E_H = \frac{J_x B}{ne}$$

$$\frac{1}{ne} = \frac{E_H}{J_x B}$$

$$R_H = \frac{1}{ne} = \frac{E_H}{J_x B} \longrightarrow (3)$$

The positive sign in R_H that the material is a p-type semiconductor and

For negative sign in R_H shows that the material is n-type semiconductor

$$R_H = -\frac{1}{ne} = \frac{E_H}{J_x B} \longrightarrow (4)$$

Hall coefficient :

If 'd' is the thickness of the specimen & V_H is the voltage developed then

$$V_H = E_H d \longrightarrow (4)$$

Sub. eq (3) in eq (4) we get

$$V_H = R_H I_x B d \longrightarrow (5)$$

Area of the specimen $A = \text{width } (w) \times \text{thickness } (d)$

$$A = wd \longrightarrow (6)$$

$$\text{Current density } I_x = \frac{I_x}{\text{Area of Specimen } (A)} \longrightarrow (7)$$

Sub eq (7) in eq (5) we get

$$\begin{aligned} V_H &= R_H I_x B d / wd \\ &= R_H I_x B / w \end{aligned}$$

$$\boxed{R_H = \frac{V_H d}{I_x B}} \text{ is called Hall coefficient}$$

Applications:

1. Determination of Semiconductor type: Hall coefficient R_H is positive for p-type and -ve for n-type semiconductor. The sign of R_H indicates the type of semiconductor.
2. Calculation of carrier Concentration: Carrier concentration in a semiconductor is calculated using the value R_H

$$n = \frac{1}{e R_H}$$
3. Calculation of mobility: mobility of charge carriers is calculated from the values of hall coefficient and electrical conductivity.
4. Measurement of magnetic flux density
5. Measurement of power flow in an electromagnetic wave.

$$V_H = \left(\frac{R_H I_x}{b} \right) B \quad (63)$$

$V_H \propto B$ when I_x is constant

Hence Hall voltage is used to measure magnetic field

6> Measurement of power flow in an electromagnetic wave
Hall effect is also used to measure power flow in an electromagnetic wave. In an electromagnetic wave B & E are at right angles. When semiconductor is placed parallel to E , a current I passes through it.

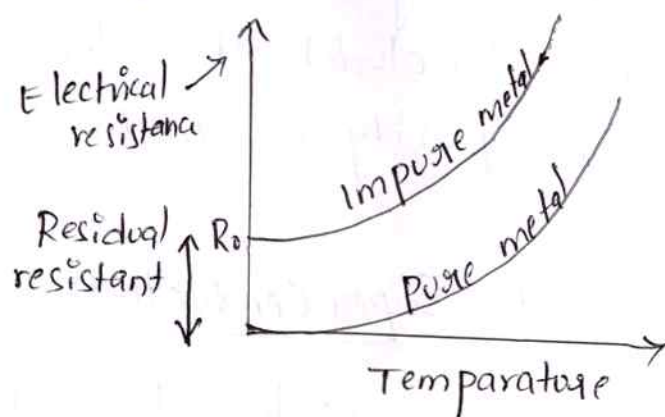
7. Hall effect multiplier :

When a current I is passed through a coil, an axial magnetic field B is produced. B is proportional to I .

UNIT-V Super Conductivity

Introduction :-

Super Conductivity is one of the important and interesting properties of Solids at low temperature. We know that the resistivity of metals (ρ) depends on temperature. In case of impure metals the resistivity exists due to impurities and also due to phonon vibration as shown



But for fully pure metals, the resistivity decreases with decreases in temperature. And approaches to '0' at $T = 0K$

Def: The phenomenon of attaining Zero resistivity (or) infinite conductivity at low temperature is known as Super conductivity. The material becomes 'Super conductor' (or)

Certain metals alloys exhibit almost zero resistivity when they cooled to low temperature is known as 'Super conductivity'

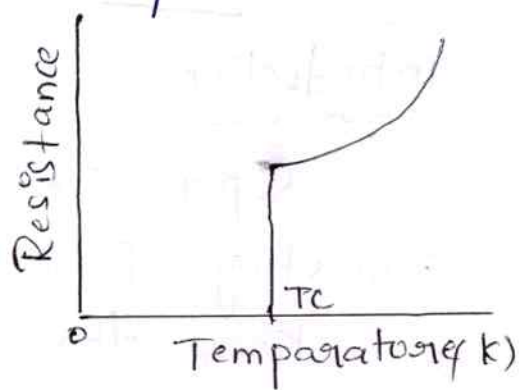
Transition / critical temperature

The temperature at which the materials undergoes a transition from normal state to Super

conducting state is known as Critical temperature (T_c).

Since perfect crystals are not exists in nature. This phenomenon was thought as an ideal one until 1911. But in the year 1911. H.K. Onnes first observed this phenomenon during his experiment. In his experiment, he

observed that the resistivity of mercury suddenly to zero at 4.2 K and he concluded that the super conductivity is not depends on purity of metals. It is a different phenomenon.



General properties of Super Conductors :-

1. Super Conductivity is a low temperature phenomenon
2. The transition from normal state to super conducting state occurs below the critical temperature
3. Different metals will show different critical temp.
4. The current once set up in a super conductor persists for a long time due to zero resistivity
5. Super conductors do not allow magnetic lines through them and behave as a diamagnetic nature. This property of expulsion is known as Meissner effect
6. The magnetic field at which a super conductor loses its super conductivity & becomes a normal conductor is known as the critical magnetic field

$$\text{Critical magnetic field } H_c = H_0 [1 - (T/T_c)^2]$$

7. Critical current $I_c = 2\pi r H_c$
8. Critical current density $J_c = I_c / A$
9. Super conductivity occurs in a metallic elements in which the number of valence electrons lies between 2 & 8
10. Super conductors are not good conductors at room temperature

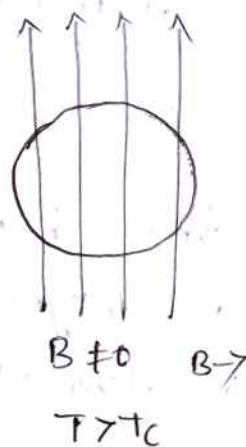
Meissner Effect :

In 1933 the Scientist Meissner suggested that, when a weak magnetic field is applied to a super conducting material at a temperature below transition temperature T_c

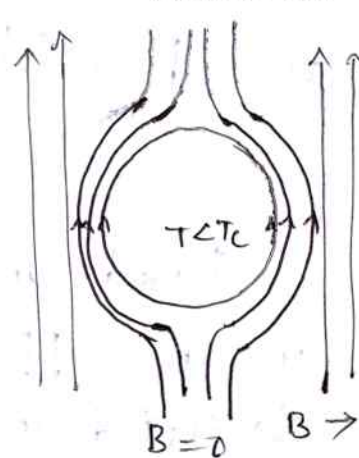
the magnetic flux lines are repelled by the material and

it acts as a perfect diamagnetic material. This effect is known as Meissner Effect

Normal
Conductor



Super
conductor



This process is reversible i.e. if the temperature field reaches its critical value, then magnetic flux lines suddenly enters into material and is converted into normal material.

Under normal condition, the magnetic flux density is given by $B = \mu_0(H + M) \rightarrow (1)$

But under super conducting state $B = 0$

$$\text{from (1)} \Rightarrow 0 = \mu_0(H + M)$$

$$H + M = 0$$

$$H = -M \rightarrow (2)$$

The induced magnetization is in direction opp. to applied magnetic field.

For Superconductors

$$\chi_m = \frac{M}{H}$$

$$\chi = -1$$

From ② & ③ it is clear that Superconductors are perfect diamagnetic material.

Classification of Superconductors

Based on the behaviour of Superconducting material in an applied magnetic field, Superconductors are classified into two types.

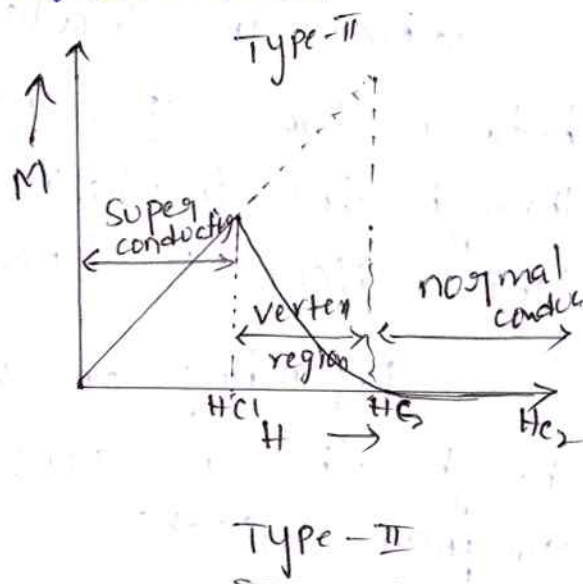
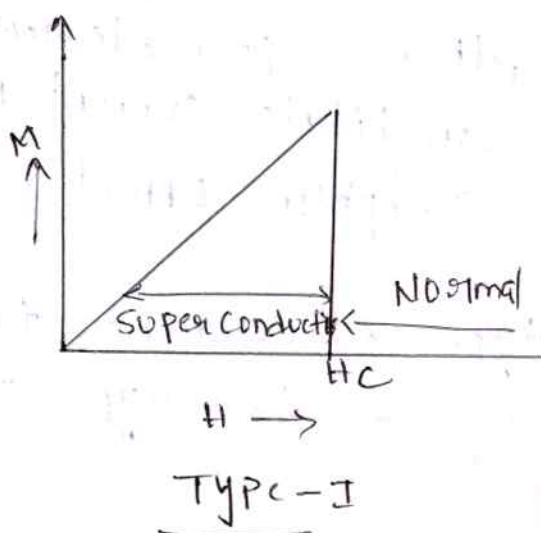
1. Type-I Superconductors
2. Type-II Superconductors

Type-I Superconductors:

For type-I Superconductors as magnetic field increases, then magnetization is also increases in the opposite direction, so they exhibit complete Meissner effect. They are completely diamagnetic materials as long as $H < H_c$ as shown. At $H = H_c$ these materials losses their property abruptly and they become normal material as shown.

These materials only one critical magnetic field H_c that is also very small. So they are used to carry very small currents. Their critical

Their critical fields are nearly 0.1 T . These materials are also known as Soft Superconductors.



Type-II Superconductors :-

For this type of material, as the applied magnetic field increases, the magnetization is also increases as shown up to lower critical field H_C , up to this field H_{C1} , it acts as a perfect diamagnetic material. After this magnetic flux lines are start to penetrate and increase up to H_{C2} . After all H_{C2} all the flux enters into material and is converted into normal material. Here H_{C2} is called upper critical field.

Type-II Superconductors has two critical fields H_{C1} and H_{C2} . Below H_{C1} they are Superconductors. Above they are normal materials. But in between H_{C1} & H_{C2} they are in mixed state and this region is called vertex region.

The value of H_{C2} is 100 times greater than H_{C1} in type-II superconductors. So these materials are used to very large currents & used in all applications.

Josephson Effect :-

When an insulating material is placed in between two superconductors, then Super electrons tunnel through the insulator and constitute current in it. This effect is known as Josephson Effect.

Depending on what type of current flowing through the insulator, there are two types of Josephson effects

1. DC Josephson effect
2. AC Josephson effect.

DC Josephson Effect :-

A DC current flow through the insulator due to tunneling of Super electron in the absence of external voltage is known as 'DC Josephson effect'.

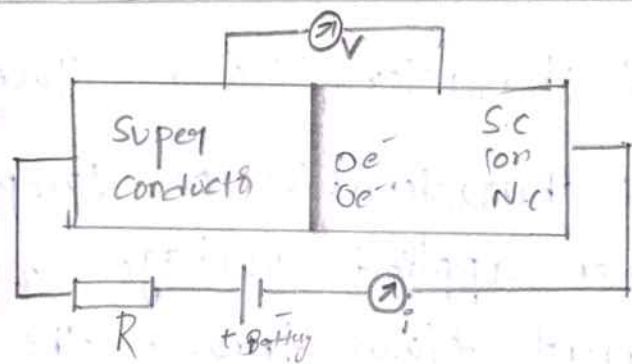
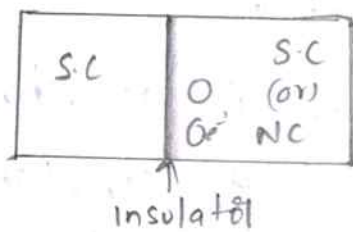
According to Josephson when tunneling occurs through the insulator then there is a phase difference ϕ_0 between the two parts of the wave function of electrons before and after the insulator Josephson junction

The tunneling current is given by

$$I = I_0 \sin \phi_0$$

where ϕ_0 phase difference in wave function.

I_0 is maximum current that can flow through the Josephson



AC Josephson Effect

An AC current flows through the junction due to tunneling of super electrons with the application of external dc voltage known as AC Josephson effect.

Now let us assume, a static potential V_0 is applied to Josephson junction as shown. This results in the additional phase difference introduced by the super electrons during tunneling across the junction. This additional phase difference $\Delta\phi$ at any time 't' can be written by quantum mechanics as

$$\Delta\phi = \frac{Et}{\hbar} \rightarrow (1)$$

where E - total energy of the system

$E = 2e(V_0)$ since Cooper pair contains 2 electrons

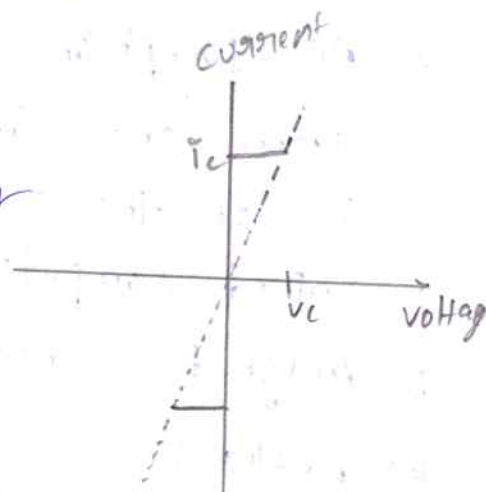
$$\text{Hence } \Delta\phi = \frac{2eV_0 t}{\hbar}$$

Now the tunneling current is given by

$$\begin{aligned} I &= I_0 \sin(\phi_0 + \Delta\phi) \\ &= I_0 \sin\left[\phi_0 + \frac{2eV_0 t}{\hbar}\right] \\ &= I_0 \sin(\omega t + \phi_0) \end{aligned}$$

$$\text{where } \omega = \frac{2eV_0}{\hbar}$$

This represents an AC current with angular frequency ω .



1-V characteristics of Josephson Effect :-

1-V characteristics of Josephson effect is as shown here
When applied voltage $V_0 = 0$ there is a constant current flows through the Josephson effect called superconducting current. This effect is called DC Josephson effect.

2) As long as $V_0 < V_c$, a constant current I_0 flows through the junction

3) When $V_0 < V_c$ the junction has a finite resistance, and the current oscillates with frequency $\omega = \frac{2eV}{\hbar}$ this is AC current and effect is called AC Josephson effect.

Applications :-

1) Josephson effect is used to generate microwaves of frequency $\omega = \frac{2eV_0}{\hbar}$

2) AC Josephson effect is used to define Standard Volt.

3) AC Josephson effect is used to measure very low temp.

4) AC Josephson effect is used for switching of signals from one circuit to another. The switching time is of the order of 10^{-12} s hence very useful in high speed computers

5) Magnetic Sensors

6) Decoders

7) Microprocessors

BCS Theory

The microscopic theory of superconductivity was put forward by Bardeen Cooper and Schrieffer in 1957. The basic features on which the theory stands are.

1) Electron-phonon-electron interaction

2) Formation of Cooper pair

3) Existence of energy gap

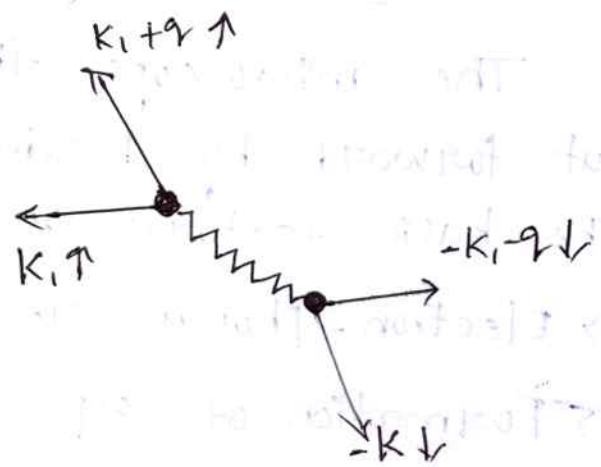
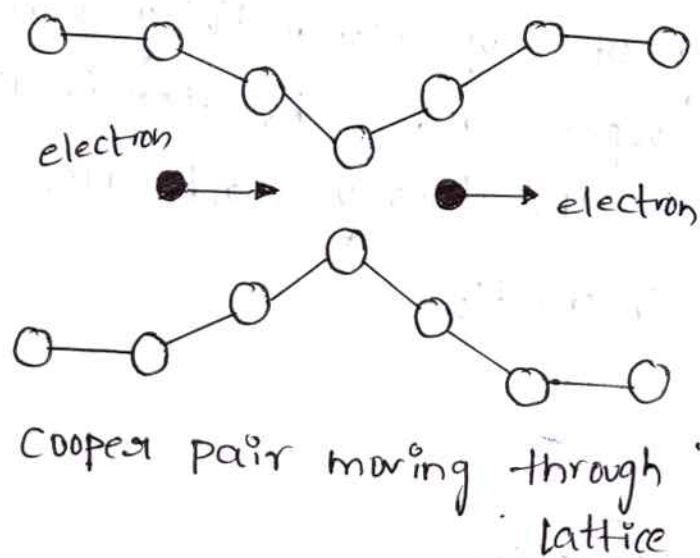
Electron-phonon-electron interaction :-

When an electron with a wave vector k_1 encounters a lattice ion, it suffers a coulomb interaction, as a result of this interaction, the momentum of the electron decreases by an amount proportional to $k_1 - q$ and the lattice is set into motion.

Another electron with a wave vector k_2 may enter this volume and this electron interacts with the distorted lattice and gains momentum. Therefore it can be thought that the electron with wave vector k_1 emits a virtual photon ' q ' which is absorbed by the second electron with wave vector k_2 and the two electrons are scattered with wave vectors $k_1 - q$ and $k_1 + q$. The process is virtual one and the energy is not conserved. The short lived phonon is called virtual phonon.

The two electrons are interacting via the distorted lattice or virtual phonon this type of interaction

is called 'electron-phonon-electron' interaction



The attraction of two electrons via phonon field is observed and is called Cooper pair. A Cooper pair is a system of two electrons having equal and opposite momentum and spin. A Cooper pair is denoted by (k, k)

Energy gap :-

The energy of the Cooper pair of the electrons in the bound state is less than the energy of the pair in free state. The difference between the two states is the binding energy of the Cooper pair. To break the Cooper pair into two separate electrons the energy equivalent to binding energy of the Cooper pair should be supplied. The energy differences between the free state of the electron and the paired state appears the energy gap.

Merits

- 1) BCS theory explains the thermal properties
- 2) The BCS theory explains the penetration depth, coherent length, flux.

Applications of Superconductors :-

1> Energy production and transport

- Alternators
- Transformers
- Cables
- Energy Storage (SMES)
- Current limiters (FCL)

2> Transports

- Magnetic levitation
- Motors
- MHD propulsion

3> Medicine : MRI , biomagnetic measurements

4> Research : Laboratory magnets

NMR Spectrometers
 Magnets for nuclear fusion
 Superconducting detectors
 SQUID

5> Fast electrical switching

6> Electric power transmission lines