ANNAMACHARYA UNIVERSITY

EXCELLENCE IN EDUCATION; SERVICE TO SOCIETY

(ESTD, UNDER AP PRIVATE UNIVERSITIES (ESTABLISHMENT AND REGULATION) ACT, 2016)

Rajampet, Annamayya District, A.P – 516126, INDIA

CIVIL ENGINEERING

Lecture Notes

on

Engineering Mechanics

Written by

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Civil Engineering

Unit- I

Introduction to Engg mechanics

Basic concepts- System of forces- Resultant of a force system, moment of forces and its applications & couples, Spatial forces- Compo[nents] in space, resultant equilibrium of system forces, free body diag[ram]

Unit - II

Types of supports

Support reactions for beams with different types of loading Concentrated, UDL, UVL & couple.

Analysis of frames

Types of frames- Assumptions for forces in members of a perfect frame, method of joints & method of sections, Cantilever and simply supported trusses.

Unit- III

Friction

Types of friction - Static and dynamic frictions, laws of friction- limiting friction and impending ~~friction~~ motions - Cone of limiting friction - Motion of bodies - Wedge friction- Ladder friction.

Unit- IV

Centroid & Centre of gravity

> Centroids of simple figures - Centroids of composite figures - Centre of gravity of bodies - Theorem of Pappus and Guldinus centre of gravity of Composite figures. (simple problems only)

UNIT- V

Moment of Inertia

Area moment of Inertia - Parallel axis and perpendicular axis theorems - Moment of Inertia of composite figures

Mass moment of Inertia;

Moment of Inertia of simple solids, moment of Inertia of composite masses, simple masses

Fundamentals - Basic units - Independent, Unit adopted for measure of a base quantity

Mass - kilograms - kg

length - meters - m

Time - Seconds - S

Temperature - K

Electric current - A

luminous intensity - cd

Amount of substance - moles

Physical Quantities

Acceleration - m/s^2 velocity/time

Angular acceleration - Rad/s^2 Rate of change of angular velocity

Angular displacement - Rad angle b/n initial & final positions

Angular momentum - $kg\text{-}m/s^2$ I x angular velocity

Angular velocity - Rad/s.

Area - m^2 l x b

Couple / moment - N-m equal forces act in opp directions

Density - kg/m^3 mass/volume

Displacement - m change in dimension

Energy - J / N-m Quantity property that must transfer an object in order to perform work

Force - Newton Push or pull, Action that tends to motion / alter motion of a body

Frequency - per second S^{-1} No of cycles / second

Momentum - kg-m/S mass x velocity

Moment of Inertia of mass - kgm^2

Plane angle - Radian

Power - Watt

Pressure - N/m^2 (or) Pascal

Speed - m/s distance/time

Time - S

Newton - $kg\,m/s^2$	force	
Joule	N-m	Energy
Watt	J/S	Power
Pascal	N/m^2	Pressure, Stress
Hertz	S^{-1}	Frequency

10 power representation

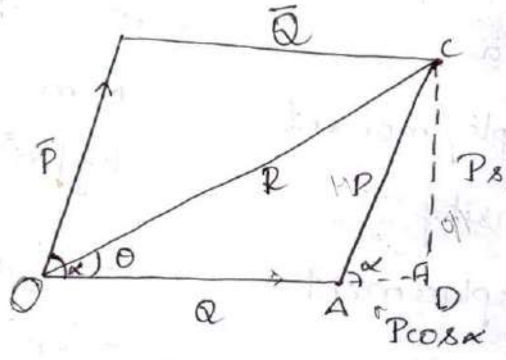
Milli - 10^{-3}
Micro - 10^{-6}
Nano - 10^{-9}
Pico - 10^{-12}
Tera

Tera - 10^{12}
Giga - 10^{9}
Mega - 10^{6}
Kilo - 10^{3}

10^{2} - hecto
10^{1} - deca
10^{-1} - deci
10^{-2} - centi

Parallelogram law:

let $\bar{P}$ & $\bar{Q}$ be two vectors



magnitude

from △ ACD

$$\sin\alpha = \frac{CD}{AC} = \frac{CD}{P}$$

$$CD = P\sin\alpha$$

$$\cos\alpha = \frac{AD}{AC} = \frac{AD}{P} \quad \therefore AD = P\cos\alpha$$

A/c to Δ_{le}

$$R^2 = CD^2 + OD^2$$

$$R^2 = CD^2 + (OA + AD)^2$$

$$= CD^2 + (Q + P\cos\alpha)^2$$

$$P^2\sin^2\alpha + Q^2 + P^2\cos^2\alpha + 2PQ\cos\alpha$$

$$= P^2(\sin^2\alpha + \cos^2\alpha) + Q^2 + 2PQ\cos\alpha$$

$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

Direction;

$$\tan\theta = \frac{CD}{OD} = \frac{CD}{OA+OD} = \frac{P\sin\alpha}{Q+P\cos\alpha}$$

$$\theta = \tan^{-1}\left[\frac{P\sin\alpha}{Q+P\cos\alpha}\right]$$

Direction

$$\theta = \tan^{-1}\left[\frac{P\sin\alpha}{Q+P\cos\alpha}\right]$$

$$Q = \tan^{-1}\left[\frac{P\sin\alpha}{P+P\cos\alpha}\right]$$

$$\tan^{-1}\left[\frac{P\sin\alpha}{P(1+\cos\alpha)}\right]$$

$$\tan^{-1}\left[\frac{2\sin\alpha/2\cos\alpha/2}{\frac{1+\cos\alpha/2}{2\cos^2\alpha/2}}\right]$$

$$\theta = \tan^{-1}\left[\frac{\sin\alpha/2}{\cos\alpha/2}\right]$$

$$\theta = \tan^{-1}(\tan\alpha/2)$$

$$\theta = \alpha/2$$

$$2\theta = \alpha$$

If $\alpha = 90^\circ$

$R^2 = P^2 + Q^2 + 2PQ\cos 90^\circ$

$= P^2 + Q^2 + 2PQ(0)$

$R^2 = \sqrt{P^2 + Q^2}$

$R = \sqrt{P^2 + Q^2}$

$\theta = \tan^{-1}\left[\dfrac{P\sin 90^\circ}{Q + P\cos 90^\circ}\right]$

$\theta = \tan^{-1}\left[\dfrac{P}{Q + P(0)}\right]$

$\theta = \tan^{-1}\left[P/Q\right]$

If two forces are equal

$P = Q$

$R^2 = P^2 + P^2 + 2P^2\cos\alpha$

$R^2 = 2P^2 + 2P^2\cos\alpha$

$= 2P^2(1 + \cos\alpha)$

$= 2P^2(2\cos^2\alpha/2)$

$R^2 = 4P^2(2\cos^2\alpha/2) \quad 8P^2\cos^2\alpha/2 \quad = \quad R = \sqrt{8P^2\cos^2\alpha/2}$

$R' = 2P\cos\alpha/2$

Trigonometric formulae

$\sin\theta = \dfrac{Opp}{Hyp} \quad \cos\theta = \dfrac{Adj}{Hyp} \quad \tan\theta = \dfrac{Opp}{Adj}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

$\sin 2A = 2\sin A\cos A$

$\sin^2\theta + \cos^2\theta = 1$

$\dfrac{\partial}{\partial x} U \cdot V = U\dfrac{\partial V}{\partial x} + V \cdot \dfrac{\partial U}{\partial x}$

$\int x^n\,dx = \dfrac{x^{n+1}}{n+1}$

$\int 4\,dx = 4x$

$\dfrac{\partial}{\partial x}(4) = 0$

$\dfrac{\partial}{\partial x}\sin x = \cos x, \quad \dfrac{\partial}{\partial x}\cos x = -\sin x, \quad \dfrac{\partial}{\partial x}(\tan x) = \sec^2 x$

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CIVIL ENGINEERING

Engineering Mechanics

UNIT-1

Introduction to Engineering mechanics;

Basic concepts;

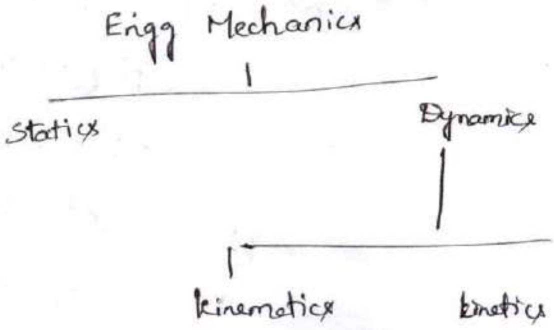
Mechanics;

Branch of physics concerned with the state of rest or state of motion of bodies subjected to action of forces

(or)

Study of forces acting on body when it is at rest or motion.

classification



Mechanics of rigid bodies

" " Deformed bodies

Fluid

Mechanics of rigid bodies;

<u>Statics</u>; A branch of Mechanics, deals with the study of forces acting on body in equilibrium. (Either at rest or Uniform motion)

<u>Dynamics</u>; A branch of Mechanics, deals with the study of forces acting on body in motion. Further divided in to Kinetics & Kinematics

<u>Kinetics</u>; A branch of dynamics, deals with study of body in motion under the influence of forces.

<u>Kinematics</u>; A branch of dynamics, deals with study of body in motion with out considering forces.

(5) Force;

A push or pull, which creat's motion or tends to create motion, destroys or tends to destroy the motion.

→ In Engg mechanics - force is the action of one body on another body

→ A force tends to move a body in the direction of its action.

→ A force is characterised by its point of application, magnitude and direction i.e., force is a vector quantity.

Units of force; CGS FPS

$1 N = kg\text{-}m/s^2$ $1 Dyne - g\text{-}cm/s^2$ $1 P = 4.448 N$

$1 N = 10^5$ dynes

Particle - Body has mass but no size (neglected)

Rigid body - A body in which relative positions of any two particles do not change under the action of forces means the distance between two points or particles remain same before and after applying external forces.

③ Basic quantities; length, mass & time.

length; To locate the position of particles and to describe the size of the physical system. measurement or extent of something from end to end.

1 cm = 10 mm   1 m = 3.2808 feet

1 m = 100 cm   1 m = 39.36 inch

1 m = 1000 mm   1 mile = 1.609 km

Mass; Property of matter by which we can compare, action of one body over the other.

Property of a physical body and a measure of its resistance to acceleration when net force is applied. Also determines the strength of its mutual gravitational attraction to other bodies. kg

Time; Indefinite continued progress of existence and events in the past, present and future regarded as a whole.

Basic quantity in dynamics.

④ Internal System of units (SI)

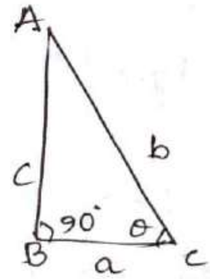
length - m, Mass - kg, Time - seconds

CGS System;

length - cm, Mass - gm, Time - seconds

Trigonometric formulae;

Δ^{le} ABC

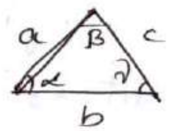


$\sin\theta = \frac{Opp}{Hyp} = \frac{AB}{AC} = \frac{c}{b}$

$\cos\theta = \frac{adj}{Hyp} = \frac{BC}{AC} = \frac{a}{b}$

$\tan\theta = \frac{AB}{BC} = \frac{c}{b}$

Sine rule



$$\frac{a}{\sin\partial} = \frac{b}{\sin\beta} = \frac{c}{\sin\alpha}$$

Cosine rule

$$a = \sqrt{b^2 + c^2 - 2bc\cos\partial}$$

$$b = \sqrt{c^2 + a^2 - 2ac\cos\beta}$$

$$c = \sqrt{a^2 + b^2 - 2ab\cos\alpha}$$

(2) Scalar and vector quantities

Scalar quantity; Only magnitude

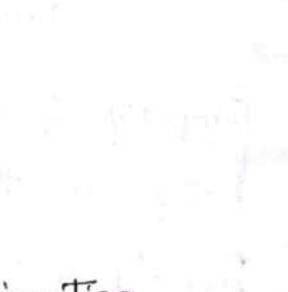
Ex; mass, length, Time, distance etc.,

Vector quantity; Both magnitude and direction

Ex; velocity, acceleration, force, displacement etc.,

Vector quantity is represented by straight line, length indicates magnitude and arrow mark indicates direction. →

A crow files north-wards from pole A to pole B and covers a distance of 8km it then flies east-wards to pole C and covers 6km. Find the net displacement and its direction?



$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$

$\frac{6}{\sin\alpha} = \frac{10}{\sin 90^\circ} \Rightarrow \frac{\cancel{6}^{3}}{\sin\alpha} = \frac{\cancel{10}^{5}}{1}$

$\sin\alpha = 3/5$

$\alpha = \sin^{-1}\left[3/5\right] = 37^\circ$

$90^\circ - 37^\circ = 53^\circ$

A Traveller travels 10km east, 20km north, 15km west and 80 km south. Find the displacement of traveller from starting point.

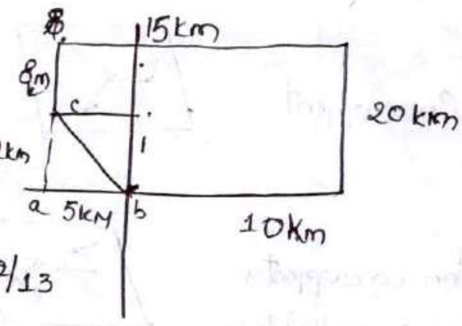
$c^2 = a^2 + b^2$

$c^2 = 144 + 25 = 169$

$c = \sqrt{169} = 13$

$\frac{a}{\sin\alpha} = \frac{13}{\sin\gamma} \Rightarrow \frac{12}{\sin\alpha} = \frac{13}{\sin 90^\circ}$ $\sin\alpha = 12/13$

$\alpha = \sin^{-1}\left[12/13\right]$



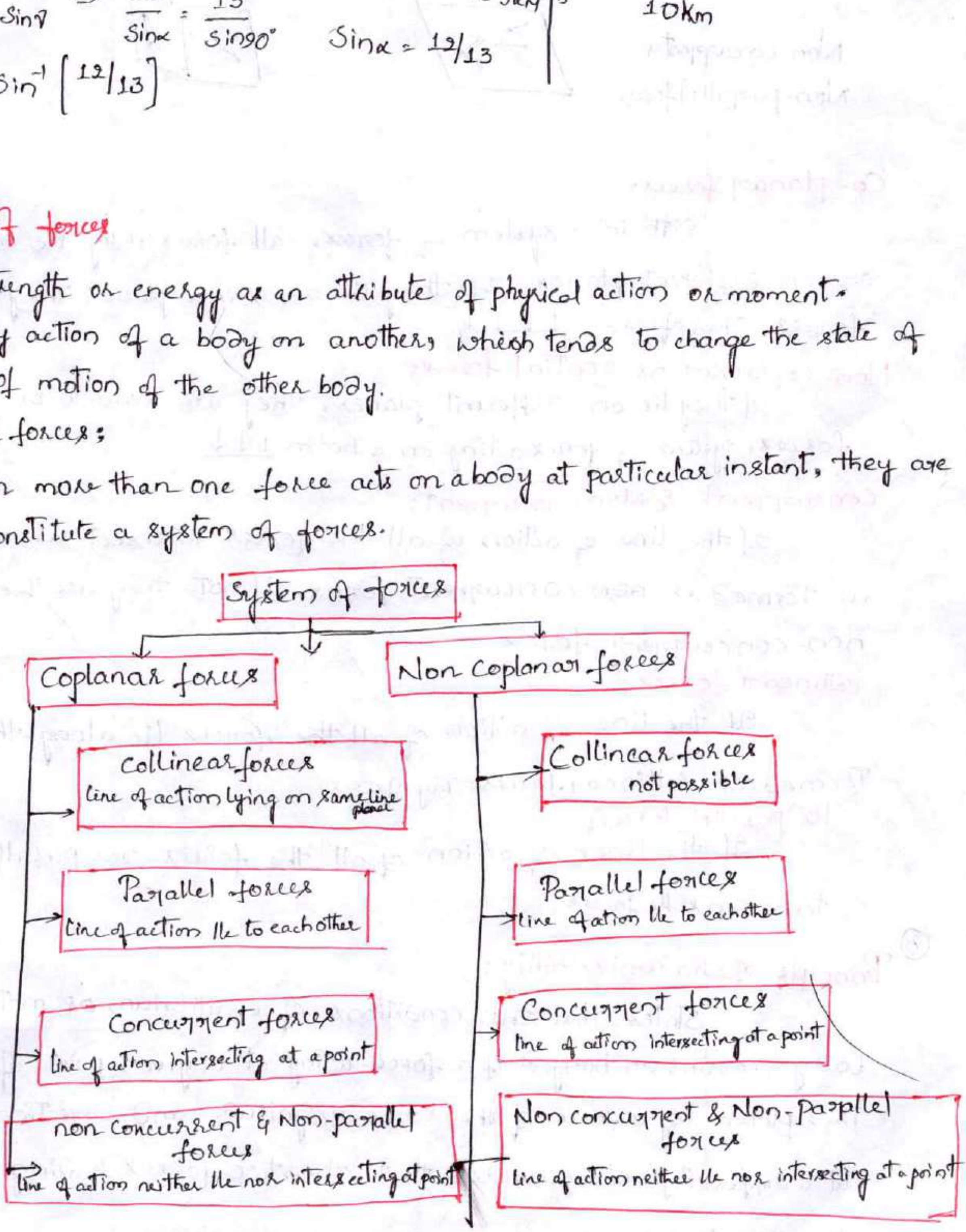
⑥ System of forces

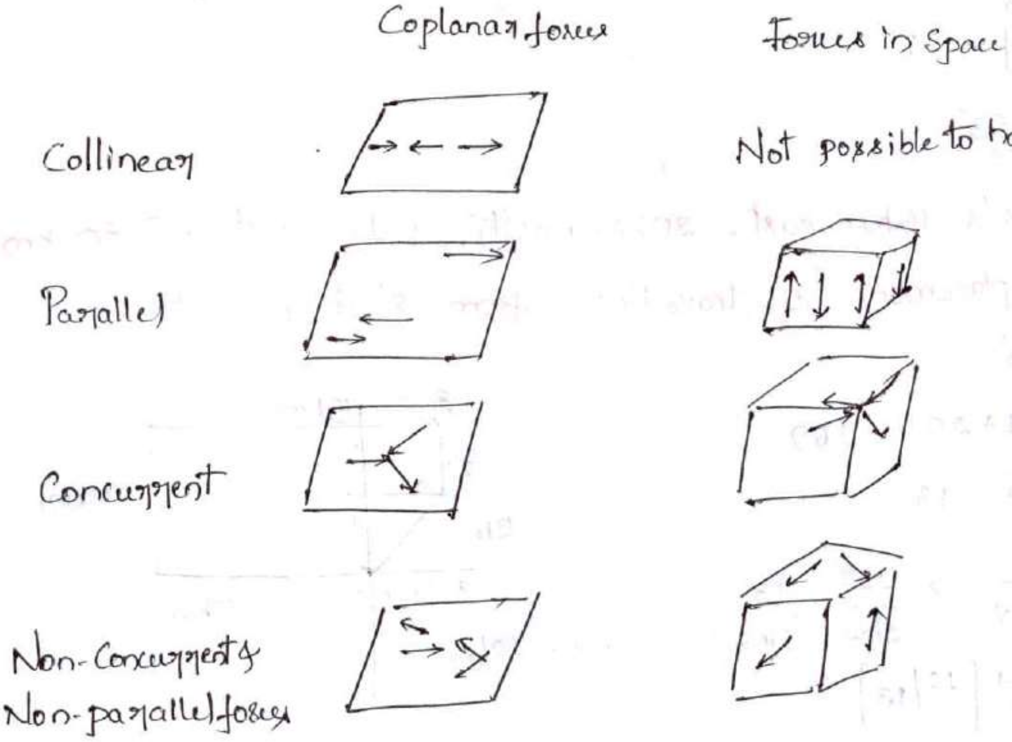
force - Strength or energy as an attribute of physical action or moment.

Any action of a body on another, which tends to change the state of rest or of motion of the other body.

System of forces:

When more than one force acts on a body at particular instant, they are said to constitute a system of forces.





Co-planar forces;

With in a system of forces, all forces may lie on the same plane or on different planes. If they lie on same plane, they are coplanar forces. Tug of war

Non-coplanar or spatial forces;

If they lie on different planes, they are said to be non coplanar forces. system of forces acting on a beam

Concurrent & Non concurrent;

If the line of action of all the forces intersect at a point, they are termed as non concurrent forces. If not, they are termed as non-concurrent forces

Collinear forces;

If the line of action of all the forces lie along the same plane. Termed as Collinear forces. Tug of war

|| parallel forces;

If the lines of action of all the forces are parallel to each other termed as || forces

⑧ Principle of Transmissibility;

states that the conditions of equilibrium or motion of a rigid body remains unchanged if a force acting at a given point of the rigid body is replaced by a force of the same magnitude and direction, but acting at a different point, provided that the two forces having the same line of action.

~~effect~~ $I_{zz} = I_{xx} + I_{yy}$

states that if a force, acting at a point on a rigid body, is shifted to any other point which is on the line of action of the force, the external effect of the force on the body remains unchanged

(a) Resultant of a force system;

state of rest or motion of the rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude & direction but acting anywhere on the body along the line of action of the replaced force.

A single force which will have the same effect as that of a number of forces acting on a body. Such single force is called resultant force and the process of finding the resultant force is called Composition of forces.

(7) Parallelogram law of forces;

If two forces are acting simultaneously on a particles, be represented by two adjacent sides of a parallelogram then their resultant ~~may be represented by a~~ will be equal to the diagonal of parallelogram which passes through the same point of concurrency

Take $F_1 = 100N$, $F_2 = 150N$ & $\theta = 45°$ find the resultant & direction

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$$

$$= \sqrt{100^2 + 150^2 + 2 \times 100 \times 150 \times \cos 45°} = 250N$$

$$\text{Direction } \theta = \tan^{-1}\left[\frac{F_1 \sin\alpha}{F_2 + F_1\cos\alpha}\right] = \tan^{-1}\left[\frac{100\sin 45°}{150 + 100\cos 45°}\right] =$$

Determine the magnitude and resultant direction of 2 forces 7kN & 8kN, acting a point with an included angle of 60° in b/n them The force of 7kN being horizontal.

⑩ Lami's theorem;

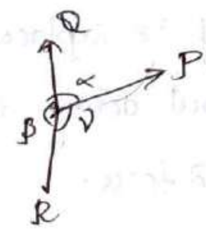
If there are forces acting at a point are in equilibrium, each force will be proportional to the sine angle b/w the other two forces.

Suppose three forces P, Q & R are acting at a point O and they are in equilibrium.

let α = Angle b/n forces P & Q

β = Angle b/n forces Q & R

γ = Angle b/n forces R & P



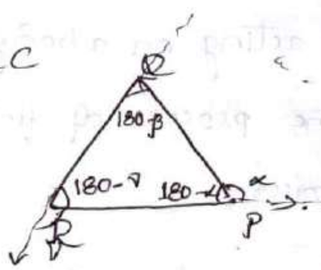
A/c to Lami's theorem $P \propto \sin\beta$

$Q \propto \sin\gamma$

$R \propto \sin\alpha$

$$\frac{P}{\sin\beta} = \text{constant}, \quad \frac{Q}{\sin\gamma} = C \quad \& \quad \frac{R}{\sin\alpha} = C$$

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha}$$



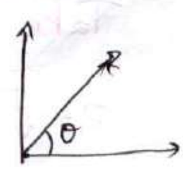
Proof
$$\frac{P}{\sin(180-\beta)} = \frac{Q}{\sin(180-\gamma)} = \frac{R}{\sin(180-\alpha)}$$

Three forces acting on a point, are in equilibrium and hence they can be represented by three sides of the Triangle taken in the same order.

⑪ Resolution of forces;

Finding the components in two directions of a given force.

let R be the force acting and making an angle θ with the axis.



The component along x-Axis is $R\cos\theta$

Y-axis $R\sin\theta$

Note; Parallelogram law

$$\theta = \tan^{-1}\left[\frac{Q\sin\alpha}{P + Q\cos\alpha}\right]$$

Case (i)

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$ If $\alpha = 90°$ then $R = \sqrt{P^2 + Q^2 + 2PQ\cos 90°}$ $\cos 90 = 0$

If $\alpha = 90°$
$$R = \sqrt{P^2 + Q^2}$$

$$\theta = \tan^{-1}\frac{Q}{P}$$

Case 2;

If $P = Q$

$$R = \sqrt{P^2+Q^2+2PQ\cos\alpha} = \sqrt{P^2+P^2+2P\times P\cos\alpha} = \sqrt{2P^2+2P^2\cos\alpha}$$

$$= \sqrt{2P^2(1+\cos\alpha)}$$

$$= \sqrt{2P^2 \times 2\cos^2\alpha/2}$$

$$= \sqrt{4P^2\cos^2\alpha/2} = 2P\cos\alpha/2$$

$$R = 2P\cos\alpha/2$$

$$\theta = \tan^{-1}\left[\frac{Q\sin\alpha}{P+Q\cos\alpha}\right] = \tan^{-1}\left[\frac{P\sin\alpha}{P+P\cos\alpha}\right] = \tan^{-1}\left[\frac{P\sin\alpha}{P(1+\cos\alpha)}\right]$$

$$\tan^{-1}\frac{2\sin\alpha/2\cos\alpha/2}{2\cos^2\alpha/2} = \tan^{-1}\frac{\sin\alpha/2}{\cos\alpha/2} = \tan^{-1}(\tan\alpha/2) = \alpha/2$$

$$\theta = \alpha/2$$

Problem 2;

Two equal forces are acting at a point with an angle of 60° b/n them. If the resultant force is equal to $20\sqrt{3}$, find the magnitude of each force.

Angle b/n the force $\alpha = 60°$

Resultant $R = 20\sqrt{3}$

We know $R = 2P\cos\alpha/2 = 2\cdot P\cdot\cos\frac{60°}{2} = 2P\cos 30°$

$$= 2P\sqrt{3}/2 = P\sqrt{3}$$

$$20\sqrt{3} = P\sqrt{3}$$

Magnitude of force $P = 20N$

Problem 3: The resultant of two forces, when they act at an angle of 60° is 14N. If the same forces are acting at right angles, their resultant is $\sqrt{136}$ N. Determine the magnitude of two forces.

Case 1;

Resultant $R_1 = 14N$

Angle $\alpha = 60°$

Case 2;

Resultant $R_2 = \sqrt{136}$ N, $\alpha = 90$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos 60^\circ} = \sqrt{P^2 + Q^2 + 2PQ\,\frac{1}{2}}$

$14^2 = \sqrt{P^2 + Q^2 + PQ}$ Squaring on both sides

$196 = P^2 + Q^2 + PQ$

Using equation $R = \sqrt{P^2 + Q^2}$

$\sqrt{136} = \sqrt{P^2 + Q^2}$

Squaring on both sides

$136 = P^2 + Q^2$

Substracting equation (ii)

$196 - 136 = P^2 + Q^2 + PQ - (P^2 + Q^2)$

$60 = PQ$

Multiplying the above equation by two, we get

$60 \times 2 = 2PQ$

$120 = 2PQ$

$PQ = \frac{120}{2} = 60$

$136 + 120 = P^2 + Q^2 + 2PQ$

13

$256 = P^2 + Q^2 + 2PQ$

$(16)^2 = (P+Q)^2$

$P + Q = 16$

$P = 16 - Q$

Substituting the value of P in equation

$60 = (16 - Q) \times Q$

$60 = 16Q - Q^2$

$Q^2 - 16Q + 60 = 0$

Quadratic equation solving $Q = 10$ and 6

Hence forces are 10N & 6N.

The resultant of two concurrent forces is 1500N and the angle b/n the forces is 90°. The resultant makes an angle of 36° with one of the force. Find the magnitude of each force.

Resultant $R = 1500N$

Angle b/n the forces $\alpha = 90^\circ$

$$\frac{\sin 90^\circ}{R} = \frac{\sin 36^\circ}{Q} = \frac{\sin 54^\circ}{P}$$

$$\frac{\sin 90^\circ}{R} = \frac{\sin 36^\circ}{Q}$$

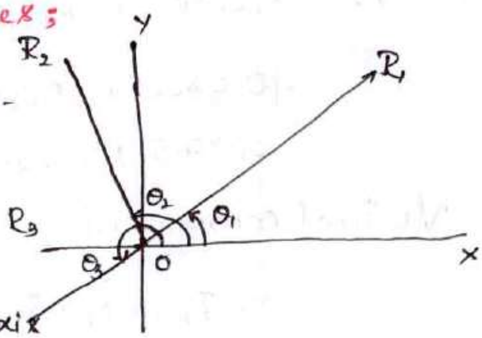
$$Q = \frac{R \sin 36^\circ}{\sin 90^\circ} = \frac{1500 \times 0.587}{1} = 881.6\ N$$

Also we have

$$\frac{\sin 90^\circ}{R} = \frac{\sin 54^\circ}{P}$$

$$P = \frac{R \sin 54^\circ}{\sin 90^\circ} = \frac{1500 \times 0.809}{1} = 1213.52\ N$$

Resolution of a number of coplanar forces:



Let a number of coplanar forces $R_1, R_2, R_3 \ldots$ are acting at a point

$\theta_1, \theta_2, \theta_3$ are the angles made by R_1, R_2 & R_3 with X-axis

H = Resultant component of all forces along X-axis

V = " " " " " Y-axis

R = Resultant of all forces

θ = Angle made by resultant - X-axis

Each force can be resolved into two components

R_1 along X-axis = $R_1 \cos \theta_1$

R_1 along Y-axis = $R_1 \sin \theta_1$

IIly the components of forces R_2 & R_3 along X-Y axis.

Along X-axis

$$H = R_1 \cos \theta_1 + R_2 \cos \theta_2 + R_3 \cos \theta_3$$

Along Y-axis

$$V = R_1 \sin \theta_1 + R_2 \sin \theta_2 + R_3 \sin \theta_3$$

Resultant $R = \sqrt{H^2 + V^2}$

Angle by R - $\tan \theta = \frac{V}{H}$

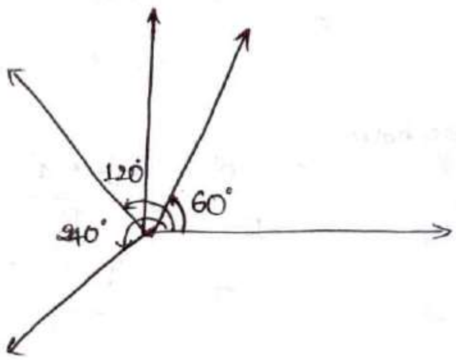
Prob 1:

Three forces of magnitude 40 kN, 15 kN and 20 kN are acting at a point O. The angles made by 40 kN, 15 kN and 20 kN forces with x-axis are 60°, 120° & 240° respectively. Determine the magnitude and direction of the resultant forces.

$R_1 = 40 kN \quad \theta_1 = 60°$

$R_2 = 15 kN \quad \theta_2 = 120°$

$R_3 = 20 kN \quad \theta_3 = 240°$



Horizontal component

$H = R_1 \cos\theta_1 + R_2 \cos\theta_2 + R_3 \cos\theta_3$

$= 40\cos 60° + 15\cos 120° + 20\cos 240°$

$= 20 - 7.5 - 10 = 2.5 kN$

Vertical component

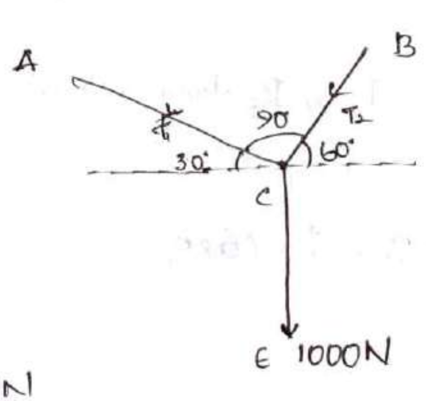
$V = R_1 \sin\theta_1 + R_2 \sin\theta_2 + R_3 \sin\theta_3$

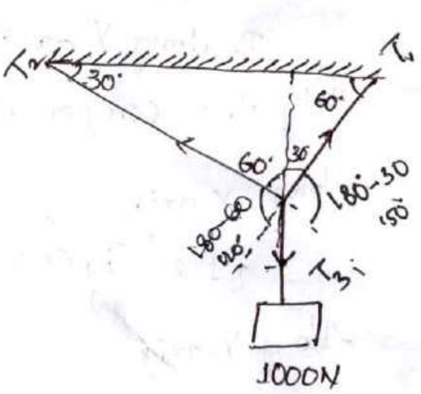
$= 40\sin 60° + 15 \cdot \sin 120° + 20\sin 240°$

$= 30.31 kN$

Resultant $R = \sqrt{H^2 + V^2} = \sqrt{2.5^2 + 30.31^2} = 30.41 kN$

A weight of 1000N is supported by two chains. Determine the tension in chain





Weight at C = 1000N

$\angle CAB = 30°$

$\angle CBA = 60°$

$\angle ACB = 90°$

In right angled triangle ADC

$\angle ADC = 90° - 30° = 60°$

In Δ BDC

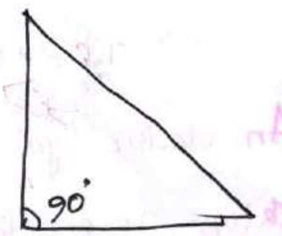
$\angle BDC = 90° - 60° = 30°$

T_1 = Tension in chain 1

T_2 = Tension in chain 2

$\angle ACE = 180° - 60° = 120°$

$\angle BCE = 180° - 30° = 150°$



Applying Lamis theorem

$$\frac{T_1}{\sin 120°} = \frac{T_2}{\sin 150°} = \frac{1000}{\sin 90°} = 1000$$

$T_2 = 1000 \sin 120° = 500N$

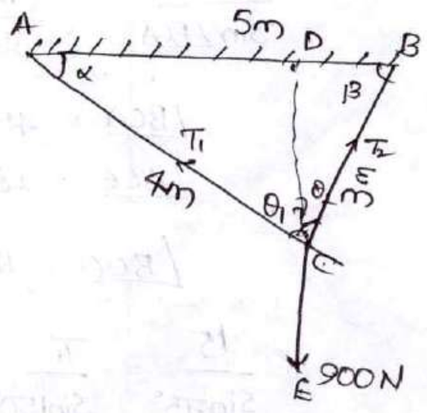
$T_1 = 1000 \sin 150° = 866N$

A weight of 900N is supported by two chains of lengths 4 & 3m as shown in fig. Determine the Tension in each chain

Weight at C = 900N

length AC = 4m

BC = 5m



Δ^{le} ABC $\quad AC^2 + BC^2 = 4^2 + 3^2 = 16 + 9 = 25$

$AB^2 = 5^2 = 25$

Triangle ABC

$$\sin\alpha = \frac{BC}{AB} = \frac{3}{5} = 0.6$$

$\alpha = 36°52'$ and $\alpha + \beta = 90°$

$\beta = 90° - 36°52' = 53°8'$

Δ^{le} ADC $\quad \theta_1 = 90° - \alpha = 90° - 36°52' = 53°8'$

Δ^{le} BDC $\quad \theta_1 = 90° - \beta = 90° - 53°8' = 36°52'$

$\angle ACE = 180° - \theta = 180 - 53°8' = 126°52'$

$\angle BCE = 180° - \theta_2 = 180° - 36°52' = 143°8'$

$\angle ACB = 90°$

Applying lami's theorem

$$\frac{T_1}{\sin \text{ of } \angle BCE} = \frac{T_2}{\sin \angle ACE} = \frac{900}{\sin 90°}$$

$$\frac{T_1}{\sin 143^\circ 8'} = \frac{T_2}{\sin 126^\circ 52'} = 900$$

$T_1 = 900 \sin 143^\circ 8' = 900 \times 0.597 = 537.44N$

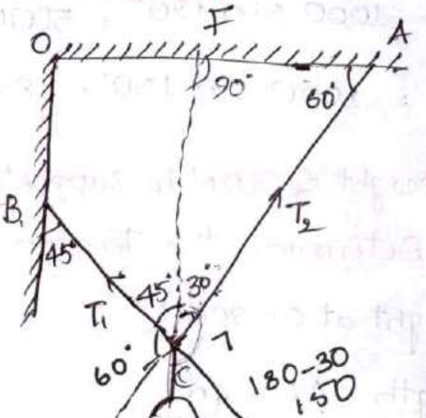
$T_2 = 900 \sin 126^\circ 52' = 720N.$

An electric light fixture weighing 15N hangs from a point C, by two strings AC and BC. AC is inclined at 60° to the horizontal and BC at 45° to the vertical as shown in fig. Using lami's theorem

Weight at C = 15N

$\angle OAC = 60^\circ$

$\angle CBD = 45^\circ$



Using lami's theorem

$$\frac{15}{\sin \angle BCA} = \frac{T_1}{\sin \angle ACE} = \frac{T_2}{\sin \angle ACE}$$

$\angle BCA = 45^\circ + 30^\circ = 75^\circ$ $\quad \angle BCA \quad 180-45 = 135$

$\angle ACE = 180 - 30^\circ = 150^\circ$

$\angle BCE = 180 - 45^\circ$

$$\frac{15}{\sin 75^\circ} = \frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 135^\circ}$$

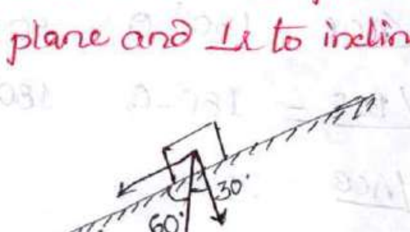
$$T_1 = \frac{15 \sin 150^\circ}{\sin 75} = 7.76N$$

$$T_2 = \frac{15 \cdot \sin 135^\circ}{\sin 75} = 10.98N$$

A small block of weight 100N is placed on an inclined plane which makes an angle θ = 30° with the horizontal. what is the component of this weight. (i) Parallel to the inclined plane and ⊥r to inclined plane

Weight of block, W = 100N

Inclined plane θ = 30°



weight of block is acting vertically downwards through the C.G of the block. Resolve these weight into two components i.e., one ⊥r to inclined plane and other ∥le to the inclined plane

Hence component of weight ⊥r to inclined plane $= W\cos 30°$

$= 100\cos 30° = 8.6N.$

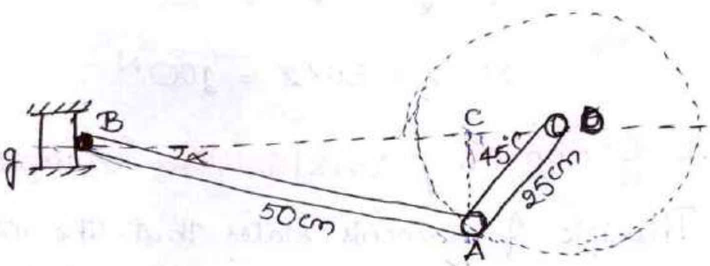
Component of weight ∥le to inclined plane $= W\sin 30°$

$= 100 \times 0.5 = 50N.$

Problem Figure shows a particular position of the connecting rod BA and crank AO. At this position, the connecting rod of the engine exerts a force 2500N on the crank pin at A. Resolve this force into horizontal and vertical components at A. Also resolve the given force at A along AO and along a direction ⊥r to AO.

BA = 50cm
AO = 25cm

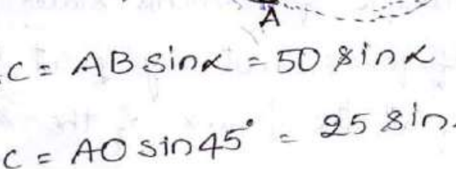
Force exerted by connecting rod BA at A = 2500N



In triangle ABC $\sin\alpha = \frac{AC}{AB}$ $AC = AB\sin\alpha = 50\sin\alpha$

Δ^{le} AOC $\sin 45° = \frac{AC}{AO}$ $AC = AO\sin 45° = 25\sin 45°$

Equating



$50\sin\alpha = 25\sin 45°$

$\sin\alpha = 0.3535$

$\alpha = 20.7°$

$H = 2500\cos 20°41' = 2338.86N$ $V = 2500\sin 20°41'$

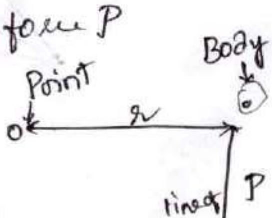
Moment of a force;

The product of a force and the ⊥r distance of the line of action of the force from a point is known as moment of force about a point.

P = A force acting on a body

r = ⊥r distance b/n the point O and line of action of force P

Moment of force P about O = P × r

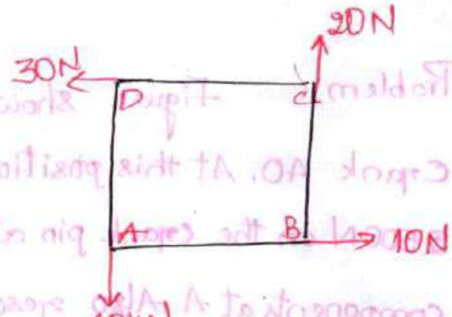


Tendency of the moment P×r is to rotate the body in clock wise direction about O.

Hence this moment is called clockwise moment. If the tendency of rotation is anticlockwise

In mks system the moment is expressed as kgf m where as in S.I system moment is expressed as N.m

Four forces of magnitude 10N, 20N, 30N & 40N are acting respectively along the four sides of a square ABCD as shown in fig. Determine the resultant moment about the point A. Each side of the square is given 2m.



Given

length AB = BC = CD = DA = 2m

force at B = 10N

C = 20N

D = 30N

A = 40N

Forces at A and B are in the direction of A

Hence the distance is zero

moment at A = $F_C \times L_C + F_D \times L_D$

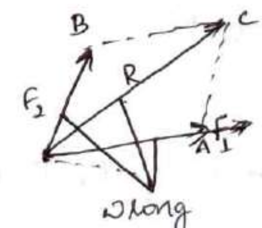
$= 20 \times 2 + 30 \times 2 = 100N$

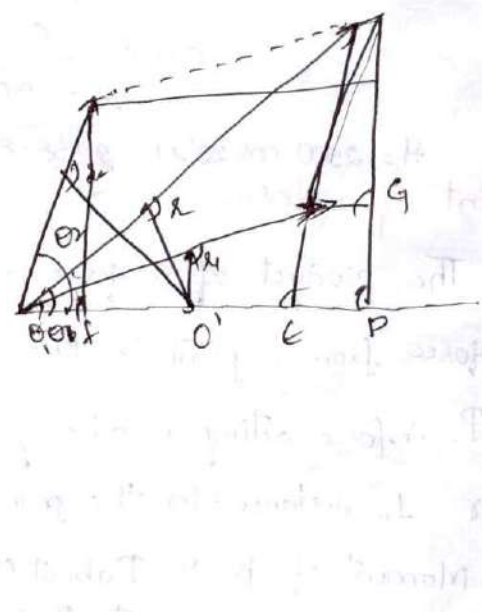
Principle of moments / VARIGNONS Principle ;

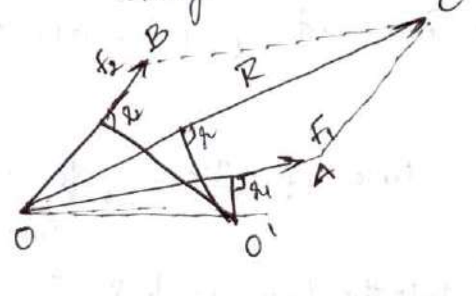
Principle of moments states that the moment of the resultant of a number of forces about any point is equal to the algebraic sum of the moments of all the forces of the system about the same point.

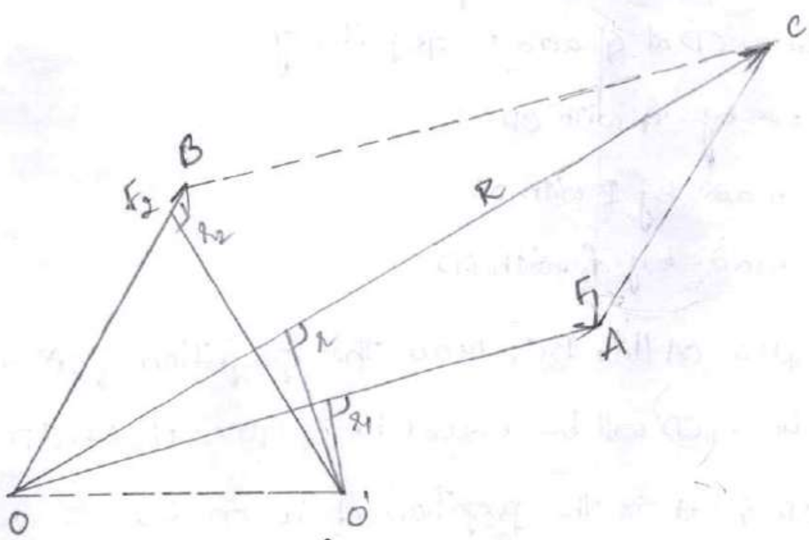
According to Varignon's principle, the moment of a force about any point is equal to the algebraic sum of the moments of its components about that point.

Proof

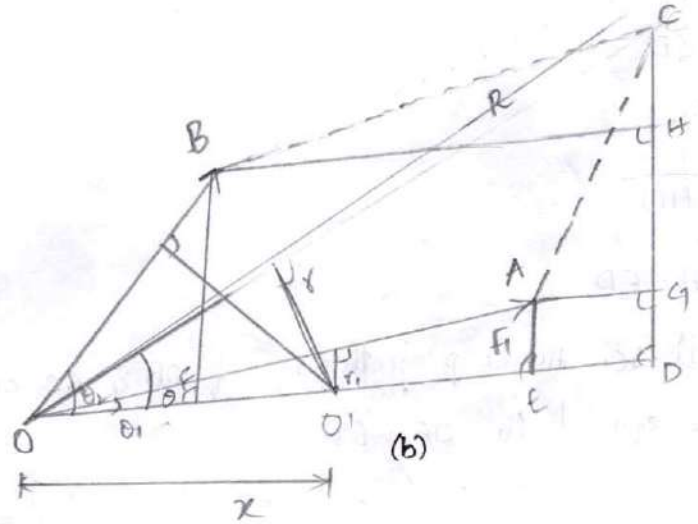








(a)



(b)

Proof ;

(a) fig shows two forces F_1 & F_2 acting at point O. These forces are represented in magnitude and direction by OA & OB. Their resultant R is represented in magnitude and direction by OC which is diagonal of parallelogram OACB. let O' is the point in the plane about which moments of F_1, F_2 and R are to be determined. From point O', draw ⊥r on OA, OC and OB

let r_1 = ⊥r distance b/n F_1 and O'

r = ⊥r distance b/n R & O'

r_2 = ⊥r distance b/n F_2 & O'

A/c to Varignons principle

Moment of R about O' must be equal to algebraic sum of moments of F_1 and F_2 about O'.

$$R \times r = F_1 r_1 + F_2 r_2$$

Now refer 3.4 (b) Join OO' and produce it to D. from points C, A and B draw

draw ⊥s on OD meeting at D, E and F respectively. From A & B also draw ⊥s on CD meeting the line CD at G and H respectively

θ_1 = Angle made by F_1 with OD

θ = Angle made by R with OD

θ_2 = Angle made by F_2 with OD

fig 34(b) OA = BC and also OA ∥ to BC, hence the projection of OA and BC on the same vertical line CD will be equal i.e., GD = CH as GD is the projection of OA on CD & CH is the projection of BC on CD

Then from fig 34(b)

$F_1 \sin\theta_1 = AE = GD = CH$

$F_1 \cos\theta_1 = OE$

$F_2 \sin\theta_2 = BF = HD$

$F_2 \cos\theta_2 = OF = ED$

∴ OB = AC and also OB ∥ AC. Hence projections of OB & AC on the same horizontal line OD will be equal to OF = ED

$R \sin\theta = CD$

$R \cos\theta = OD$

∴ let the length $OO' = x$

Then $x \sin\theta_1 = r_1$

$x \sin\theta = r$

$x \sin\theta_2 = r_2$

Now moment of R about O'

R × (⊥r distance b/n O' & R) = R × r

$R\, x \sin\theta$

$R \sin\theta\, x$

$CD\, x$

$(CH + HD)\, x$

$(F_1 \sin\theta_1 + F_2 \sin\theta_2)\, x$

$F_1 x \sin\theta_1 + F_2 x \sin\theta_2$ ∵ $x \sin\theta_1 = r_1$

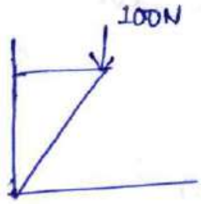
$F_1 r_1 + F_2 r_2$ $x \sin\theta_2 = r_2$

= Moment of F_1 about O' + Moment of F_2 about O'

Hence moment of R about any point in the algebraic sum of moments of its components (i.e., F_1 & F_2) about the same point. Hence Varignons principle is proved.

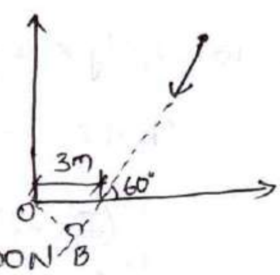
The principle moments is applicable to concurrent and coplanar. Concurrent or non-concurrent parallel force system.

A force of 100N is acting at a point A as shown in fig. Determine moment of this force about O.



Given

Force at A = 100N



Draw ⊥r from O on the line of action of force 100N. Hence OB is the ⊥r on the line of action of 100N

Δ^{le} OBC is a right-angled triangle

OBC = 60°

$\sin 60° = \frac{OB}{OC}$ ∴ $OB = OC \sin 60° = 3 \times 0.866 = 2.598$

Moment of the force 100N about O

$100 \times OB = 100 \times 2.598 = 259.8$ Nm

Types of ‖le forces;

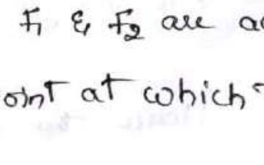
like parallel forces - The ‖le forces which are acting in the same direction are known as ‖le forces. Two ‖le forces F_1 & F_2 acting in the same direction. Hence they are called as like parallel forces. may be equal or unequal magnitudes.

Unlike parallel forces; ‖le forces which are acting in opposite direction

Unlike ‖le forces may be divided into

Unlike equal ‖le forces

Unlike unequal ‖le forces.



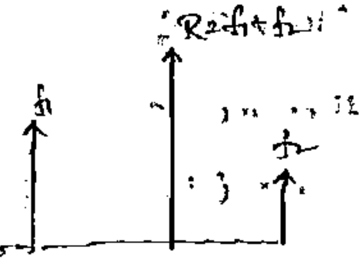
Resultant of 2 ‖le forces;

fig shows a body on which 2 like ‖le forces F_1 & F_2 are acting. It is required to determine the resultant (R) and also the point at which the

R is acting

Resultant $R = F_1 + F_2$

To find the point at which the R is acting.



let 'O' be any point on the line AB. Now calculating the moments about O.

Moment of F_1 about $O = F_1 \times OA$ -ve ↺

F_2 @ $O = F_2 \times OB$ +ve ↻

R @ $O = R \times OC$ +ve ↻

A/c to Varignon's principle

$$R \times OC = -F_1(OA) + F_2(OB)$$

$$(F_1 + F_2) OC = -F_1 \times OA + F_2 \times OB$$

$$F_1 OC + F_2 OA = F_2 OB - F_2 OC$$

$$F_1 (OC + OA) = F_2 (OB - OC)$$

$$\frac{F_1}{F_2} = \frac{(OB - OC)}{(OC + OA)}$$

$$\frac{F_1}{F_2} = \frac{BC}{AC}$$

Above relation shows that 'R' acts at point 'C', ll to the line of action of given forces F_1 & F_2 in such a way that the resultant divides the line AB in the ratio inversely proportional to the magnitude of F_1 & F_2, also the point 'C' lies in AB.

Can also calculate the point of 'R' acting on line 'AB' by taking moments WRT to point A

The moment of force F_1 and F_2 about 'A' will be 0 and

F_2 AB +ve

Moment of resultant WRT 'A' = $R \times AC$ +ve

A/c Varignon's principle, $R = F_1 + F_2$

$$F_2 AB = R \cdot AC$$

Hence the distance AB should be greater than AC

$F_2 AB = (F_1 + F_2) AC$

$F_2 AB = F_1 AC + F_2 AC \quad AB > AC$

$F_2 AB = R \times AC$

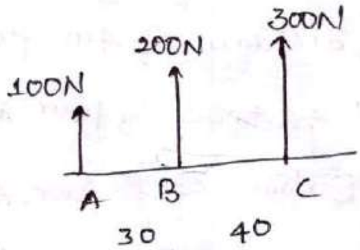
$R = (F_1 + F_2)$

Problem

Three like ll forces 100N, 200N & 300N are acting at points A, B & C respectively on a straight line ABC. The distances are AB = 30cm & BC = 40cm Find the resultant from Point A on line AB

Given 3 ll forces 100N, 200N & 300N

AB = 30cm & BC = 40cm



Resultant of forces = 100 + 200 + 300

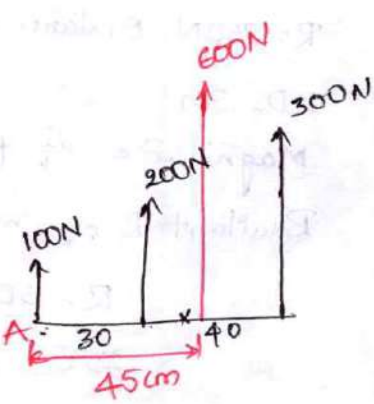
$R = 600N$

Taking moments with respect to point A

Moment of 100N @ A = 0

$200 @ A = 200 \times \frac{30}{100} = 60 N$

$300 @ A = 300 \times \frac{70}{100} = 210 N$



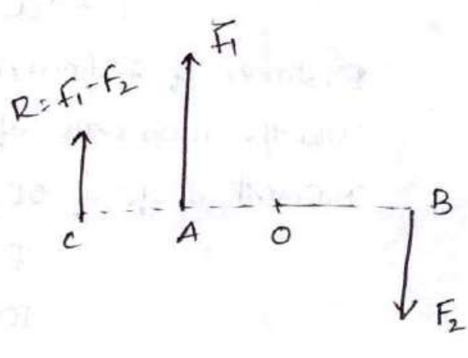
$R \times x = 60 + 210$

$600 x = 60 + 210$

$x = \frac{270}{600} = 0.45 m$

$x = 45cm$

Resultant of 2 unlike ll forces -



Resultant $R = F_1 - F_2$

Moment of F_1 about O $= F_1 \cdot OA$

$F_2 @ O = F_2 \cdot OB$

$\Sigma M = m_1 + m_2$

$= F_1 OA + F_2 OB$

Moment of Resultant forces R @ O

$= R \times CO$

$= (F_1 - F_2) CO$

$= F_1 \cdot CO - F_2 \cdot CO$

But A/c to varignons principle

$$F_1 \cdot OA + F_2 BO = F_1 \cdot CO - F_2 CO$$

$$F_2 (BO + CO) = F_1 (CO - OA)$$

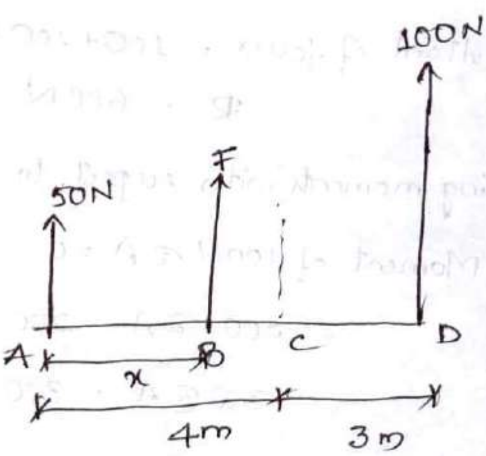
$$F_2 BC = F_1 AC$$

$$\frac{BC}{AC} = \frac{F_1}{F_2}$$

2) Three like parallel forces of magnitude 50N, F and 100N are shown in fig 3·13. If the resultant R = 250N and is acting at a distance of 4m from A, then find the resultant

Magnitude of force F.

Distance of F from A.



A = 50N at B = F and D = 100N

R = 250N, Distance AC = 4m,

CD = 3m

Magnitude of force F

Resultant R of three like forces is

$$R = 50 + F + 100$$

$$250 = 50 + F + 100$$

$$F = 100N$$

Distance of F from A

Take the moments of all forces about point A

Moment of force 50 N about A = 0

$$F @ A = F \times x$$

$$100 N @ A = 100 \times AD = 100 \times 7$$

$$= 700 Nm$$

$$\Sigma M = 0 + Fx + 700$$

$$F \cdot x + 700$$

Moment of resultant R about A = R × 4

$$= 250 \times 4$$

$$= 1000 Nm$$

ΣM of all forces about A must be equal to moment of resultant R @ A

$$Fx + 700 = 1000$$

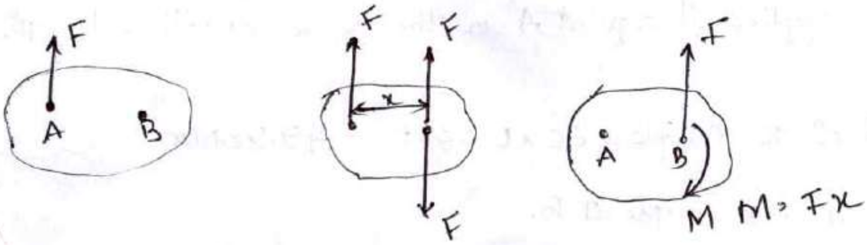
$$x = \frac{300}{F} = \frac{300}{100} = 3m$$

Assignment

Four ll forces of magnitude 100 N, 150N, 25N and 200N are shown in fig 3.14. Determine the magnitude of the resultant and distance from the of the resultant from point A. Pg 55 – problem 3.5

Resolution of a force into a force and a couple:

A given force F applied to a body at any point A can always be replaced by an equal force applied to another point B together with a couple which will be equivalent to the original force.



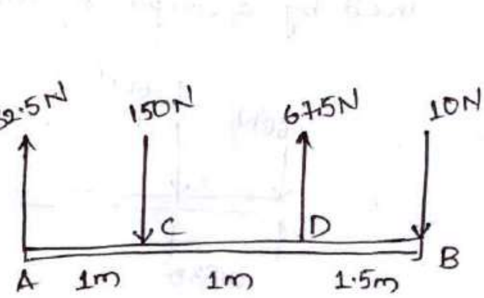
This force is to be replaced at the point B. Introduce 2 equal and Force acting at a point in a rigid body can be replaced by an equal and ll forces at any other point in the body, and a couple.

A system of ll forces are acting on a rigid bar as shown in fig. Reduce this system to

→ a single force

→ A single force and a couple at A

→ A single force and couple at B.



Given data

Forces at A, C, D & B are 32.5 N, 150N, 67.5 N and 10N

Distances AC = 1m, CD = 1m and BD = 1.5m

Single force system will consist only resultant force in magnitude and location. All the forces are acting in the vertical direction and hence their resultant R in magnitude is given by

$$R = 32.5 - 150 - 67.5 - 10 = -60N$$

A single force

$\Sigma M_A = 0$

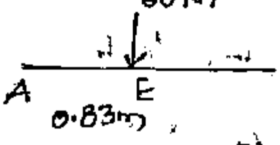
$$R \cdot x = -150 \times 1 + 67.5 \times 2 - 10 \times 3.5$$

$-60x$

$$-60x = -150 + 135 - 35 = -50$$

$$x = \frac{-50}{-60} = 0.83\,m$$

Hence the given force is equivalent to a single force



60N acting vertically downwards at point E at a distance of 0.833m from A

ii) A single force and a couple at A.

+ve sign shows that R is acting vertically upwards. To find the distance of R from point A, take the moments of all forces about point A.

let x = distance

Resultant force R acting at point E (as shown in fig) can be replaced by an equal force applied at a point A in the same direction together with a couple.

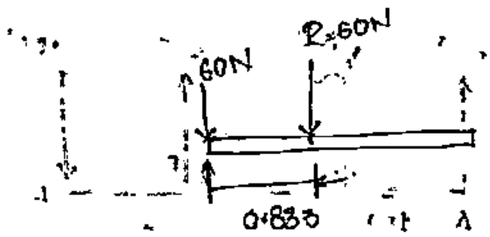
Moment of the couple = 60 × 0.83 = 49.98 Nm

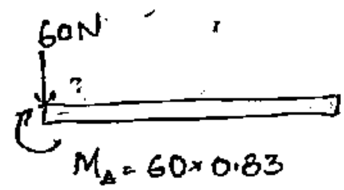
iii) A single force and a couple at B.

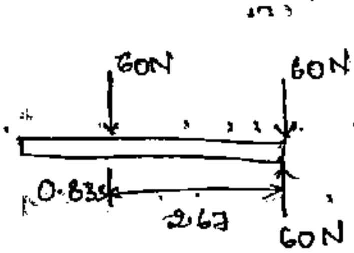
First find distance BE. But from fig 3.16 (b), the distance

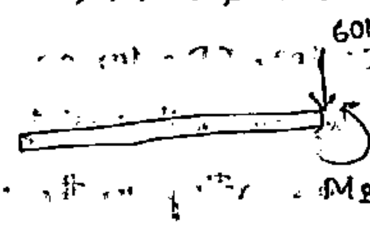
$$BE = AB - AE = 3.5 - 0.833$$
$$= 2.667\,m$$

Now if the force R = 60N is moved to the point B, it will be accompanied by a couple of moment 60 × BE or 60 × 2.67









Moment of couple = 60 × 2.67 = 160 Nm

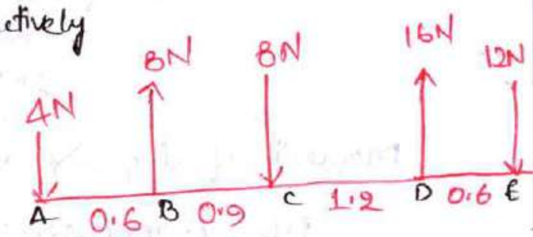
Determine the resultant of the parallel force system

Forces at A B C D & E are 4, 8, 8, 16 & 12 N respectively

Distance .

AB = 0.6 m   CD = 1.2 m

BC = 0.9 m   DE = 0.6 m



Resultant $R = -4+8-8+16-12 = 0$

moment @ A

ΣM_A

$M_A = -4\times 0 + 8\times 0.6 - 8\times(0.9+0.6) + 16\times(2.7) - 12\times(3.3)$

$\Sigma M = -3.6\,Nm$ clockwise couple

Equivalent System

An equivalent system for a given system of coplanar forces, is a combination of a force passing through a given point and a moment about that point. The force is the resultant of all forces acting on the body. The moment is the sum of all the moments about that point

Three external forces are acting on a L-shaped body as shown in fig Determine the equivalent system through point O.

Given

Force at A = 2000N $\theta = 30^\circ$

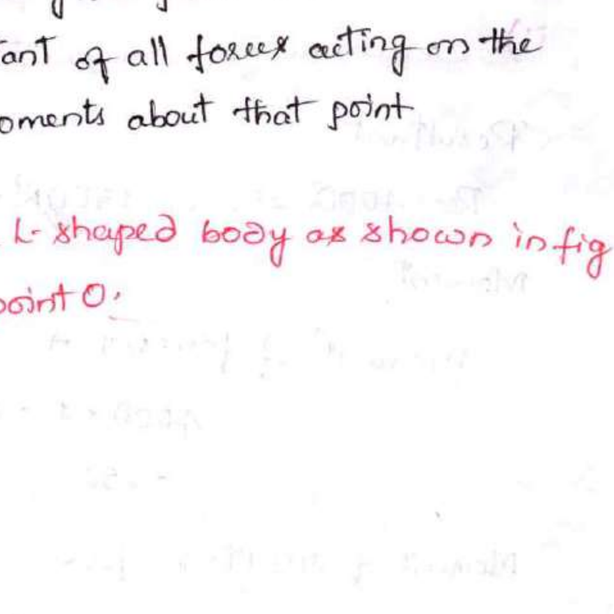
B = 1500 N

C = 1000N

Distance OA = 200 mm , OB = 100mm, BC = 200mm

Detum

Forces at A are resolved into two components



Component along x-axis = 2000 cos30°

= 1732N

Along y-axis = 2000 sin 30°

= 1000N

Resolving all forces along X-axis

$\Sigma F_x = 2000\cos 30^\circ - 1500 - 1000 = -768N$

$\Sigma F_y = -2000\sin 30^\circ = -1000$

$$R = \sqrt{\Sigma fx^2 + fy^2} = \sqrt{(-768)^2 + (-1000)^2}$$
$$= 1260.8 N$$

Moment of all forces about point O;

$$M_0 = (-2000 \sin 30) \times 2000 + 1500 \times 100 + 1000 \times 300$$
$$= -200000 + 150000 + 30.0000$$
$$= 2,50,000 \text{ Nmm} = 250 \text{ Nm}$$

Equivalent system through point O

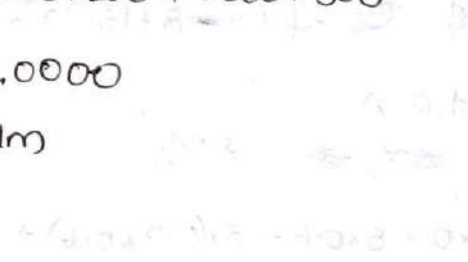
$R = 1260.8N$

$M = 250 Nm$

Two vertical forces and a couple of moment 2000Nm acting on a horizontal rod which is fixed at end A

(i) Determine the resultant of the system

ii) Determine an equivalent system through A



Resultant

$R = 4000 - 2500 = 1500N \downarrow$

Moment

Moment of forces at A

$= 4000 \times 1 - 2500 \times 2.5 + 2000$

$= -250$

Moment of resultant force $= 1500x$

$1500x = 250$

$x = \frac{250}{1500} = 0.16$

Equivalent system;

Resultant force - 1500N

Single moment $= 1500 \times 0.166 = 250 N m$

Conditions of equilibrium

When some external forces are acting on a stationary body, the body may start moving or rotating about any point. But if the body does not start moving or rotation about any point, then the body is said to be in equilibrium.

Principles of Equilibrium

A stationary body which is subjected to coplanar forces will be in equilibrium if the algebraic sum of all the external forces is zero and also the algebraic sum of moments of all the external forces about any point in their plane is zero. mathematically expressed as

$$\Sigma F = 0 \ , \ \Sigma M = 0$$

forces generally resolved in two directions i.e., horizontal and vertical

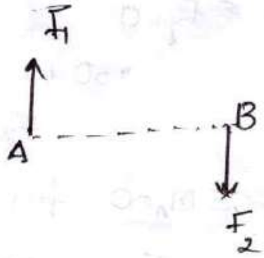
$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

Two force system;

$$\Sigma F_x = 0$$

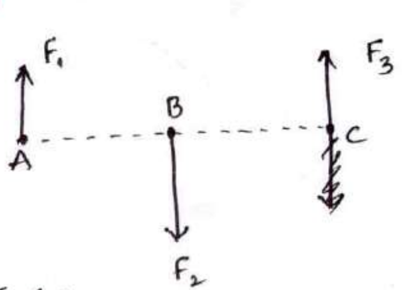
$$\Sigma F_y = 0$$

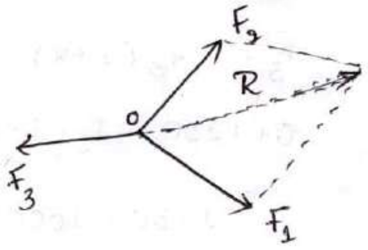


$$F_1 = F_2 \qquad M_A = -F_2 \times AB \ , \quad M_B = -F_1 \times AB$$

Three force system;

$$R = F_1 + F_2 + F_3$$





$$\Sigma F_y = F_1 + F_3 = F_2$$

$$\Sigma M_A = -F_2 \, AB + F_3 \, AC$$

Four force system;

$$\Sigma F_x = 0 \ , \quad \Sigma F_y = 0 \ , \quad \Sigma M = 0$$

Two forces F_1 & F_2 acting on a body is in equilibrium. If the magnitude of force F_1 is acting at O along x-axis as shown in fig

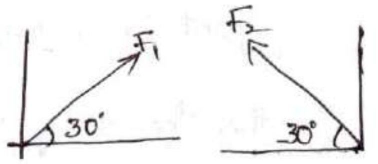
$$F_1 = F_2 = 100N$$

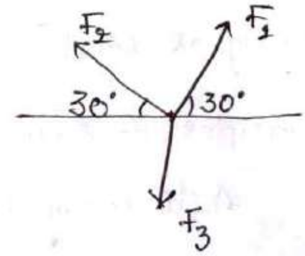
Three forces F_1, F_2 & F_3 are acting on a body. If the magnitude of force F_3 is 400N. Find the magnitude of forces F_1 & F_2.

$\Sigma f_x = 0$

$\Sigma f_y = 0$

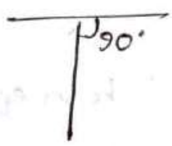
$\Sigma M = 0$





$F_{x_1} = F_1 \cos 30°$ $\quad F_{x_2} = F_2 \cos 30°$

$F_{y_1} = f_1 \sin 30°$ $\quad F_{y_2} = F_2 \sin 30°$



$F_{x_3} = F_3 \cos 90°$

$f_{y_3} = F_3 \sin 90° = 400 \times 1 = -400$

$\Sigma F_x = F_1 \cos 30° - F_2 \cos 30° + 0$

$F_1 \cos 30° = F_2 \cos 30°$

$F_1 = F_2$

$\Sigma F_y = 0$

$F_1 \sin 30° + F_2 \sin 30 - 400$

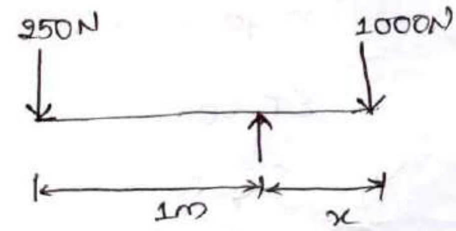
$2 F_1 \sin 30° = 400$

$F_1 \sin 30° = 200$

$F_1 = 400N$

$F_2 = 400N$

(3) 3lle forces F_1, F_2 and F_3 are acting on a body is in Equilibrium If force $F_1 = 250N$ & $F_3 = 1000N$ and the distance b/n F_1 & F_2 = 1m. Determine the magnitude of force F_2



$\Sigma F_y = 0$

$250 + 1000 - F_2 = 0$

$F_2 = 1250N$

$\Sigma M_A = 0$ for $F_1 = 0$

$F_2 = 1250 \times 1 = 1250$

$F_3 = -F_3(1+x)$

$0 + 1250 - F_3(1+x) = 0$

$1250 = 1000(1+x)$

$1250 - 1000 = 1000x$

$250 = 1000x$

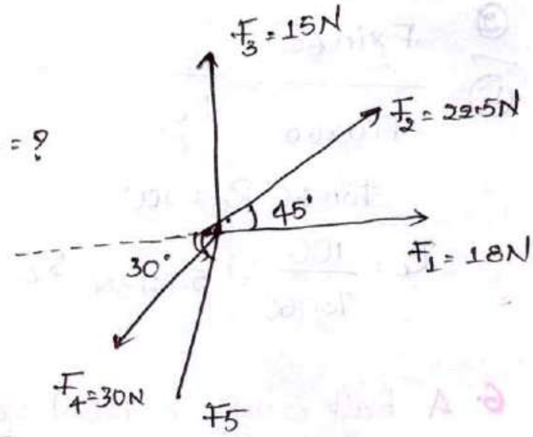
$0.25m = \frac{250}{1000} = x$

5 forces $F_1, F_2, F_3 \ldots F_5$ are acting at a point on a body as shown in fig and the body is in equilibrium. If $F_1 = 18N$, $F_2 = 22.5N$, $F_3 = 15N$ and $F_4 = 30N$. Find the force f_5 in magnitude and direction.

Given

$F_1 = 18N,\ F_2 = 22.5,\ F_3 = 15N,\ F_4 = 30N,\ F_5 = ?$

$\Sigma F_x = 0,\quad \Sigma F_y = 0,\quad \Sigma M = 0$



$F_{x_1} = 18\cos 0°$ | $f_{x_2} = 22.5\cos 45°$
$F_{y_1} = 18\sin 0°$ | $F_{y_2} = 22.5\sin 45°$

$F_{x_3} = 15\cos 90°$ | $F_{x_4} = -30\cos 30°$ | $F_{x_5} = -F_5\cos\theta$
$F_{y_3} = 15\sin 90$ | $F_{y_4} = -30\sin 30°$ | $F_{y_5} = -F_5\sin\theta$

$\Sigma F_x = 0$

$= 18\cos 0° + 22.5\cos 45° + 15\cos 90° - 30\cos 30° - F_5\cos\theta$

$\Rightarrow 18 + 15.9 + 0 - 25.98 - F_5\cos\theta = 0$

$F_5\cos\theta = 7.92$

$\Sigma F_y = 0$

$18\sin 0 + 22.5\sin 45° + 15\sin 90° - 30\sin 30° - F_5\sin\theta$

$F_5\sin\theta = 0 + 15 + 15.90 - 15$

$F_5\sin\theta = 15.90$

$$\frac{\sin\theta}{\cos\theta} = \frac{15.90}{7.92}$$

$\tan\theta = 2.0075$

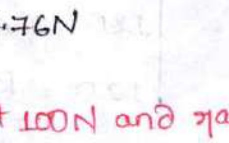
$\theta = \tan^{-1}(2.007)$

$\theta = 63°3'$

$$F_5 = \frac{15.90}{\sin(63°31')} = 17.76N$$

A circular roller of weight 100N and radius 10cm hangs by a tie rod AB equals to 20cm and rests against a smooth vertical wall at 'C' as shown in fig and body is in equilibrium. Determine the force 'F' in tie rod ii) Reaction 'R' at point c

$\theta = 30°$ $\Sigma F_x = 0$

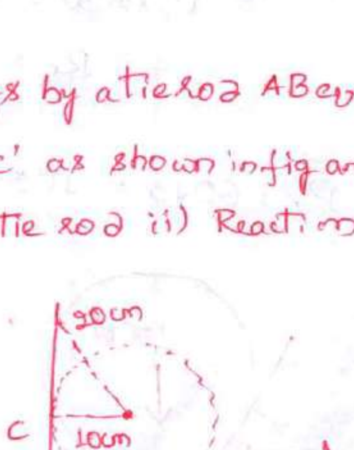


$\Sigma F_y = 0$

$F_{x_1} = -F\cos 60°$ $\quad W = F\sin 60°$

$F_{y_1} = F\sin 60$

$R_c - F\sin 60° = 0$



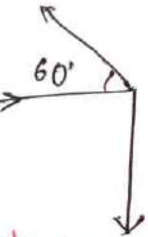
$R_c = F\cos 60° \rightarrow (1)$ $\qquad$ $W = F\sin 60° \rightarrow (2)$

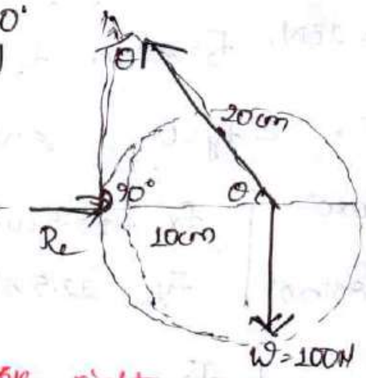
$\frac{(2)}{(1)}$ $\frac{F\sin 60°}{F\cos 60°} = \frac{W}{R_c}$ $\qquad$ $F = R_c / \cos 60°$

$F = 115.47N$

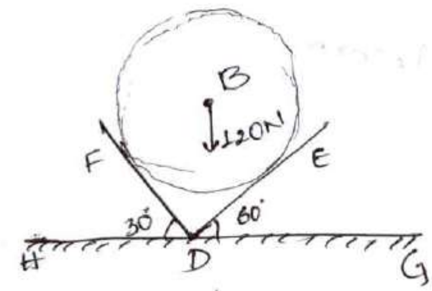
$\tan 60° R_c = 100$

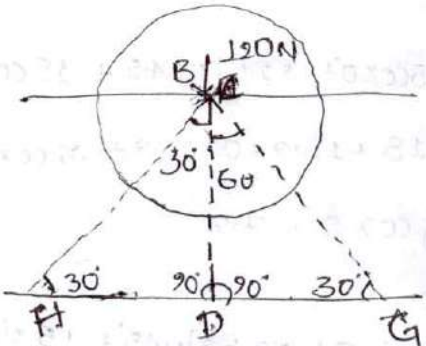
$R_c = \frac{100}{\tan 60°} = 57.73N$





6. A ball weight 120N rests in a right angled groove are inclined to an angle of 30° and 60° to horizontal. If all the surfaces are smooth then determine reactions R_A & R_c at points of contact





FBD

$\Sigma F_x = 0$

$\Sigma F_y = 0$

Given ball weight = 120N

Angle of groove = 90°

$\angle CDH = 30°$ $\quad$ $\angle DGA = 30°$

$\angle DCH = 90°$ $\quad$ $\angle HBL = 30°$

$\angle DHC = 60°$ $\quad$ $\angle LBG = 60°$

$\Sigma F_x = 0 \quad R_c \sin 30° - R_A \sin 60° = 0 \rightarrow (1)$

$R_c \sin 30° = R_A \sin 60°$

$R_c = R_A \frac{\sin 60°}{\sin 30°}$

$R_c = 60\sqrt{3} = 103.92N$

$\Sigma F_y = 0$

$120 - R_A\cos 60° - R_c\cos 30° = 0 \quad (2)$

$R_c = \sqrt{3} R_A \rightarrow (3)$

$120 = R_A\cos 60° + R_c\cos 30°$

$120 - R_A\cos 60° - \sqrt{3} R_A \cos 30° = 0$

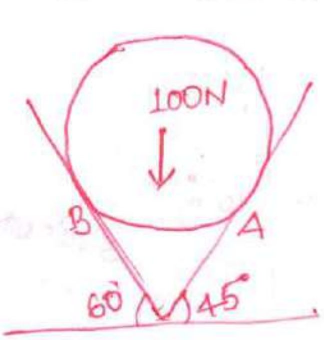
$120 - R_A/2 - \sqrt{3} R_A(\sqrt{3}/2) = 0$

$120 - R_A/2 - 3/2 R_A = 0$

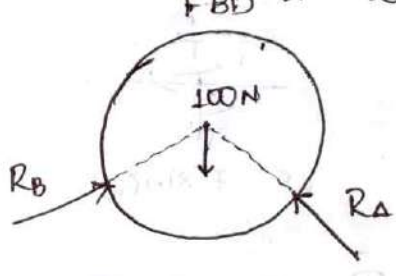
$120 = \frac{R_A}{2} + \frac{3}{2} R_A$

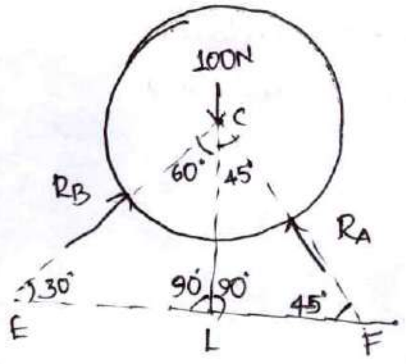
$120 = \frac{4 R_A}{2}$

$R_A = 60N$



FBD





$\Sigma F_x = 0$

$R_B \sin 60° - R_A \sin 45° = 0 \longrightarrow (1)$

$\Sigma F_y = 0$

$100 - R_A \cos 45° - R_B \cos 60° = 0 \longrightarrow (2)$

From (1)

$R_B \sin 60° = R_A \sin 45°$

$R_B(\sqrt{3}/2) = R_A(1/\sqrt{2})$

$R_A = R_B\left[\frac{\sqrt{3}\times\sqrt{2}}{2}\right] = \frac{\sqrt{6}}{2} R_B$

$R_B \sin 60 = R_A \sin 45°$

$R_B = R_A \frac{\sin 45°}{\sin 60°}$

$R_B = R_A \cdot \frac{1/\sqrt{2}}{\sqrt{3}/2}$

$R_B = R_A \cdot 2/\sqrt{6}$

From (2) eq

$100 - R_A\, 1/\sqrt{2} - R_A(2/\sqrt{6})(1/2) = 0$

$100 - R_A/\sqrt{2} - R_A/\sqrt{6} = 0$

$100 = \frac{R_A}{\sqrt{2}} + \frac{R_A}{\sqrt{6}} = \frac{R_A}{1.424} + \frac{R_A}{2.499}$

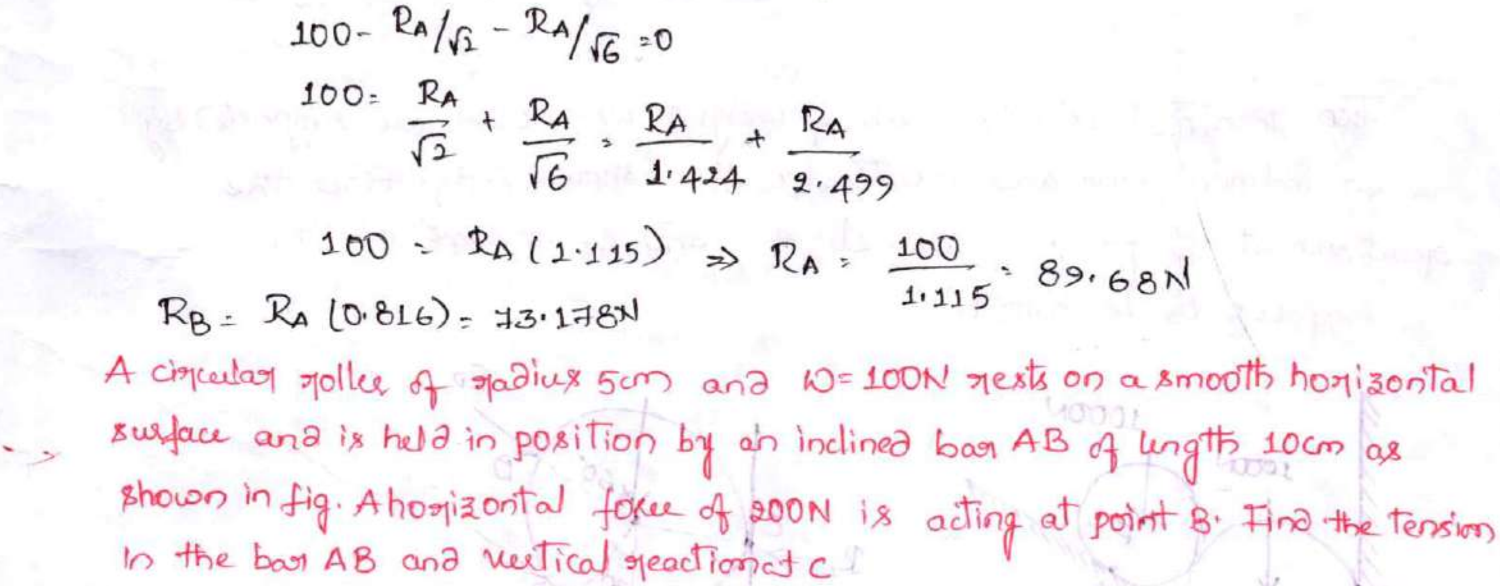
$100 = R_A(1.115) \Rightarrow R_A = \frac{100}{1.115} = 89.68N$

$R_B = R_A(0.816) = 73.178N$

A circular roller of radius 5cm and W = 100N rests on a smooth horizontal surface and is held in position by an inclined bar AB of length 10cm as shown in fig. A horizontal force of 200N is acting at point B. Find the tension in the bar AB and vertical reaction at C

Given a circular roller radius 5cm & W = 100N

- length AB = 10cm, Horizontal force = 200N



$\sin\theta = \frac{5}{10} = 1/2$

$\theta = 30°$

$\Sigma F_x = 0$

$F\cos 30° = 200N$

$F = \frac{200}{\cos 30°} = 230.94N$

$\Sigma F_y = 0$

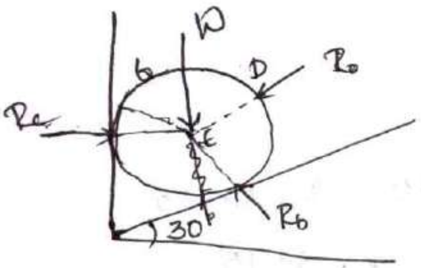
$R_c - 100 - F\sin 30° = 0$

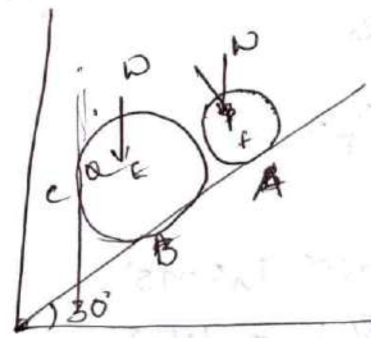
$R_c = 100 + (230.9)\sin 30°$

$R_c = 215.45N$

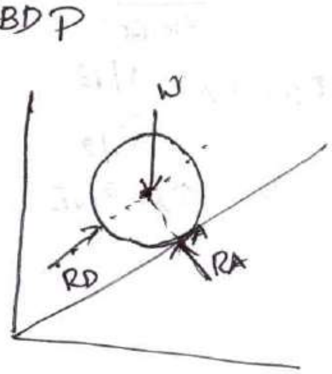
identical rollers P & Q each of weight W are supported by an [inclined] plane and a vertical wall as shown in fig. Assume all surfaces to be smooth. Draw the FBDs of Roller Q and Roller P, Roller P and Q taken together.

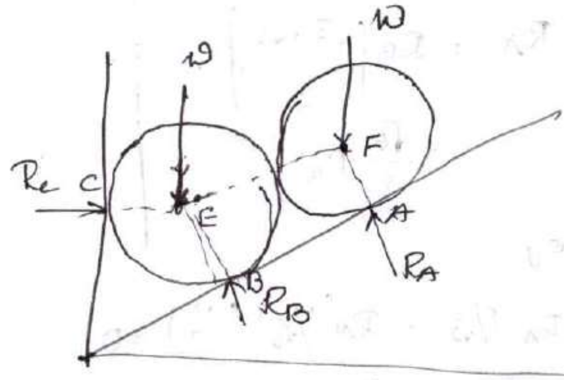
Free Body diagram of roller Q



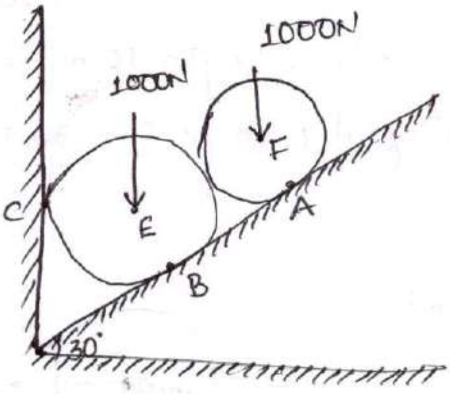


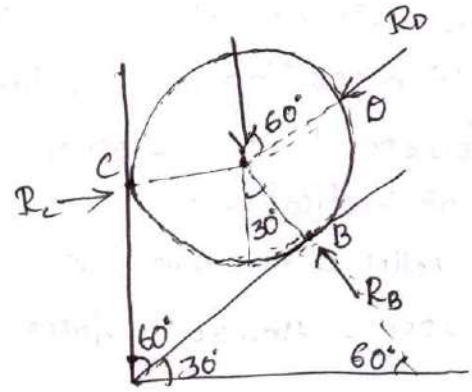
FBD P

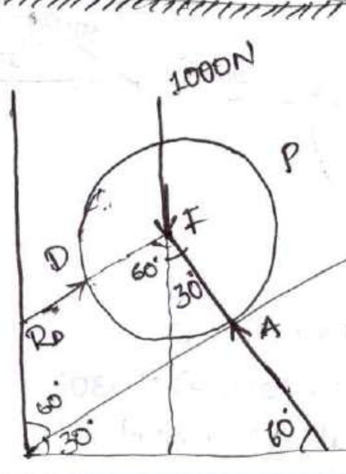




Two identical rollers, each of weight W = 1000N, are supported by an inclined plane and a vertical wall as shown in fig. Find the reactions at the points of supports A, B and C. Assume all the surfaces to be smooth.







Given
Weight of each roller = 1000N
Radius of each roller is same

Equilibrium of roller P

$\Sigma F_x = 0$

$R_D \sin 60^\circ - R_A \sin 30^\circ = 0$

$R_D = R_A \dfrac{\sin 30^\circ}{\sin 60^\circ} = 0.577 R_A \rightarrow (1)$

$\Sigma F_y = 0$

$R_D \cos 60^\circ + R_A \cos 30^\circ - 1000 = 0$

Substituting (1) in the above eq

$0.577 R_A \cos 60^\circ + R_A \cos 30^\circ = 1000$

$1.1545 R_A = 1000 \; ; \; R_A = \dfrac{1000}{1.1545} = 866.17N$

Equilibrium of Roller Q

$\Sigma F_x = 0$

$-R_c + R_B \sin 30^\circ + R_D \sin 60^\circ = 0$

$0.5 R_B + 449.8 \times 0.866 - R_c = 0$

$R_c = 0.5 R_B + 432.8$

$\Sigma F_y = 0$

$R_B \cos 30^\circ - 1000 - R_D \cos 60^\circ = 0$

$R_B \cdot 0.866 - 1000 - 449.78 \times 0.5 = 0$

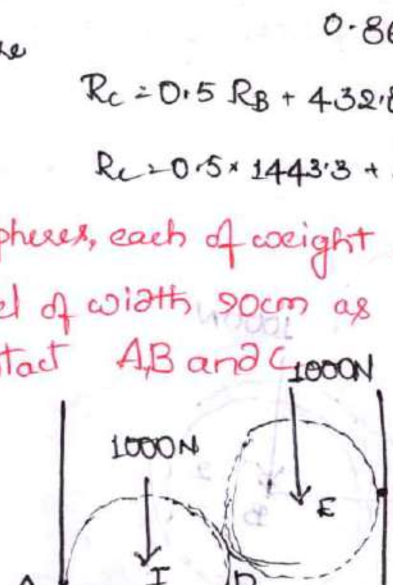
$0.866 R_B - 1249.89 = 0 \quad \therefore R_B = \dfrac{1249.89}{0.866} = 1443.3N$

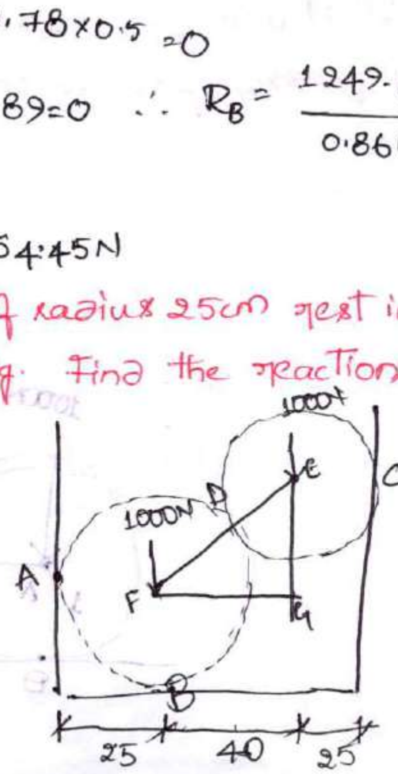
where

$R_c = 0.5 R_B + 432.8$

$R_c = 0.5 \times 1443.3 + 432.8 = 1154.45N$

Two spheres, each of weight 1000N and of radius 25cm rest in a horizontal channel of width 90cm as shown in fig. Find the reactions on the points of contact A, B and C





$\cos\theta = \frac{EG}{EF} = \frac{30}{50} = 3/5$ $\sin\theta = \frac{FG}{EF} = \frac{40}{50} = 4/5$

Equilibrium of Sphere (2)

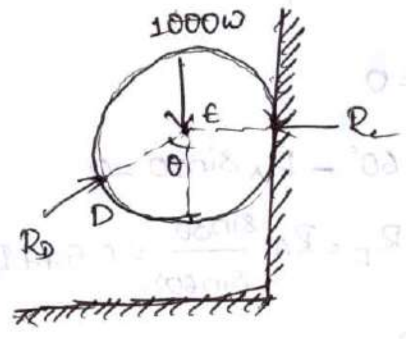
$\Sigma F_x = 0$

$R_D \sin\theta - R_c = 0$

$\Sigma F_y = 0$

$R_D \cos\theta - 1000$

$R_D = \frac{1000}{3/5} = \frac{5000}{3} N$



$\frac{5000}{3} \sin\theta = R_c$, $\frac{5000}{3} \cdot \frac{4}{5} = R_c$, $1333.3 = R_c$

Equilibrium of Sphere (1)

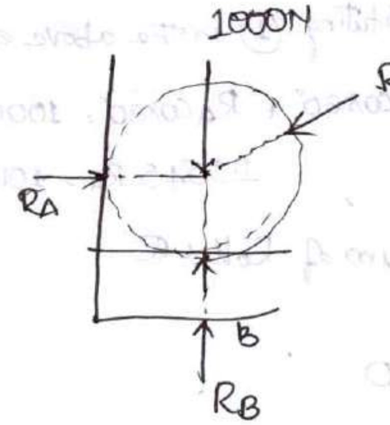
$\Sigma F_x = 0$

$R_A - R_D \sin\theta = 0$

$R_A = R_D \sin\theta$

$= \frac{5000}{3} \times \frac{4}{5}$

$= 1333.3 N$

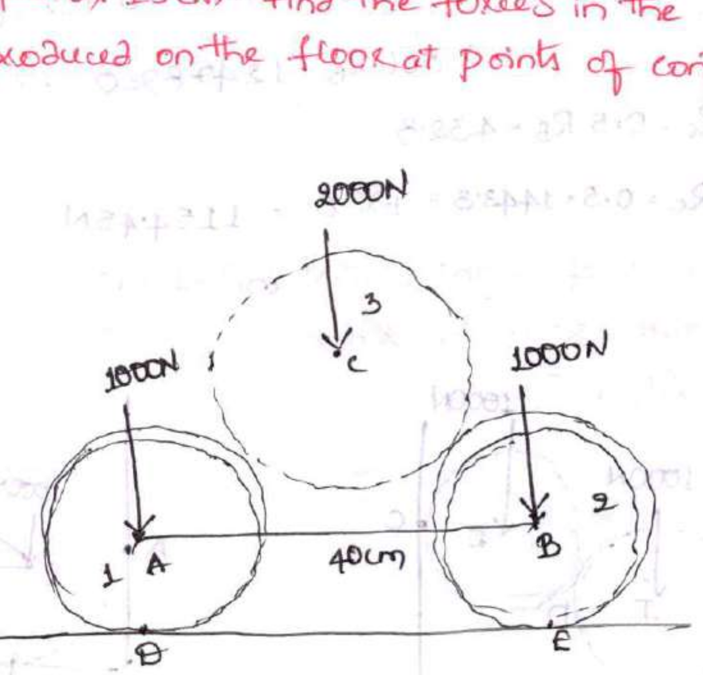


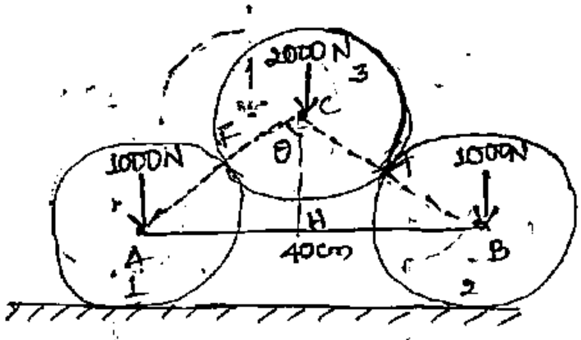
$\Sigma F_y = 0$

$R_B - 1000 - R_D \cos\theta = 0$

$R_B = 1000 + R_D \cos\theta$; $R_B = 2000 N$

Two smooth circular cylinders, each of weight W = 1000N and radius 15cm, are connected at their centres by a string AB of length = 40cm and rest up on a horizontal plane supporting above 3rd cylinder of weight 2000 N and radius 15cm. Find the forces in the string AB & the pressure produced on the floor at points of contact.





weight of cylinders 1 and 2 = 1000N

Weight of cylinder 3 = 2000N

Radius of each cylinder = 15cm

length of string AB = 40cm

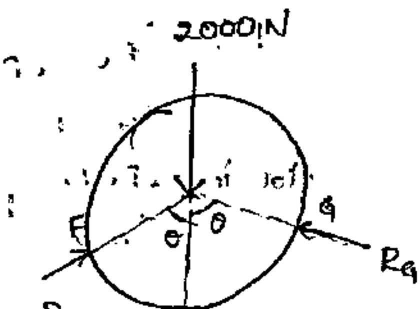
$AC = AF + FC = 15 + 15 = 30\,cm$

$AH = \frac{1}{2} AB = \frac{1}{2} \times 40 = 20\,cm$

From $\triangle ACH$ $\sin\theta = \dfrac{AH}{AC} = \dfrac{20}{30} = 0.667$

$\theta = \sin^{-1} 0.667 = 41.836$

Equilibrium of cylinder 3 ;



$\Sigma X = 0$

$R_f \sin\theta - R_g \sin\theta = 0$

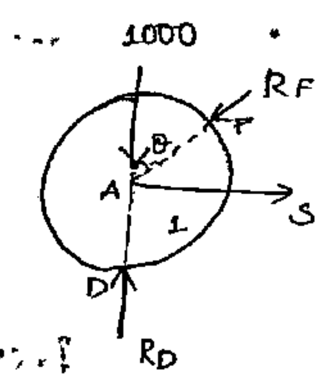
$R_f = R_g$

$\Sigma Y = 0$

$R_f \cos\theta + R_g \cos\theta = 2000$

$R_f = \dfrac{2000}{2\cos\theta} = \dfrac{1000}{\cos 41.836} = 1342.179$

Equilibrium of cylinder 1



$\Sigma F_x = 0$

$S - R_f \sin\theta = 0$

$S = R_f \sin\theta$

$= 1342.18 \sin 41.836°$

$= 895.2$

$\Sigma F_y = 0$

$R_D - 1000 - R_f \cos\theta = 0$

$R_D = 1000 + R_f \cos\theta$

$= 1000 + 1342.18 \cos 41.836$

$= 1999.9 \simeq 2000\,N$

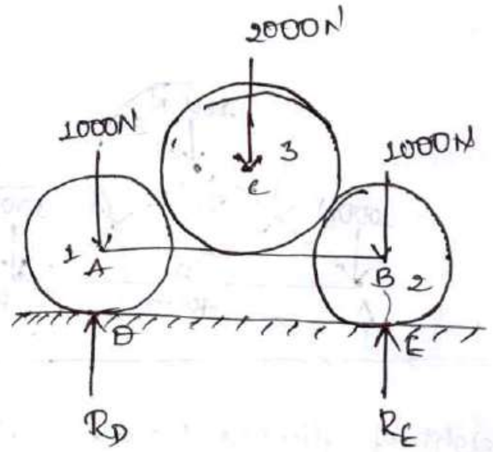
Equilibrium of cylinders 1, 2 and 3

$R_D + R_E - 1000 - 2000 - 1000 = 0$

$R_E = 1000 + 2000 + 1000 - R_D$

$R_E = 4000 - R_D$

$= 4000 - 2000 = 2000N$



A roller of radius 40cm, weighing 3000N is to be pulled over a rectangular block of height 20cm. by a horizontal force applied at the end of a string wound round the circumference of the roller. Find the magnitude of horizontal force which will just turn the roller over the corner, Determine the magnitude and direction of reaction A & B.

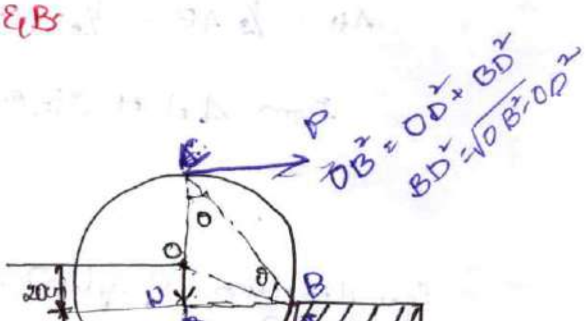
ΔBOD BO = 40cm - radius

OD = OA - AD

= 40 - 20 = 20cm

$BD = \sqrt{BO^2 - OD^2} = \sqrt{40^2 - 20^2} = \sqrt{1200}$

= 34.64



Now in ΔBCD

$\tan\theta = \frac{BD}{CD} = \frac{34.64}{CO + OD} = \frac{34.64}{40 + 20} = 0.5773$

$\theta = \tan^{-1} 0.5773 = 29.99° \simeq 30°$

Resolving forces horizontally $\Sigma X = 0$

$P = R_B \sin\theta = R_B \sin 30° = 0.5 R_B$

$\Sigma Y = 0$

$W - R_B \cos\theta$ $3000 - R_B \cos 30° = 0$

$R_B = \frac{3000}{\cos 30°} = 3464.2N$

$P = 1732.1 N$ ✓

&

$P \times CD = W \times BD$

$P \times 60 = 3000 \times 34.64$

$P = \frac{3000 \times 34.64}{60} = 1732.0 N$

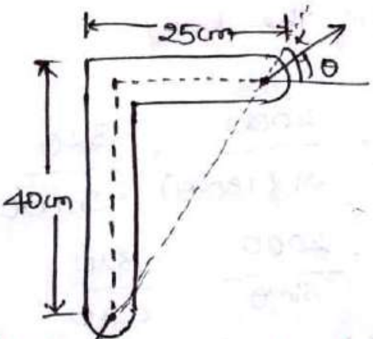
A L-shaped body ABC is hinged at A with a force F acting at its end C. Determine the angle θ which this force should make with the horizontal to keep the edge AB of the body vertical.

Δ^{le} ABC

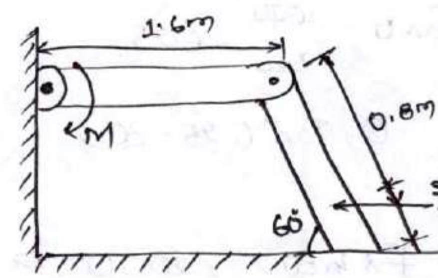
$$\tan\alpha = \frac{AB}{BC} = \frac{40}{25} = 1.6$$

$$\alpha = \tan^{-1} 1.6 = 57.99°$$

$$\theta = 57.99°$$



A horizontal force 200N is applied on sloping bar BCD whose bottom rests on a horizontal plane find reaction at B, equilibrium



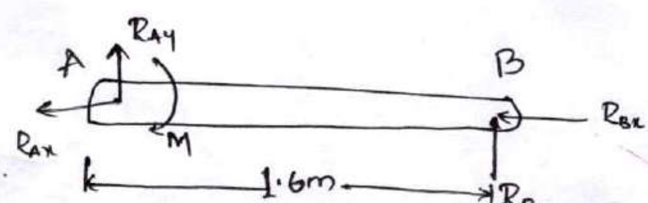
length AB = 1.6m

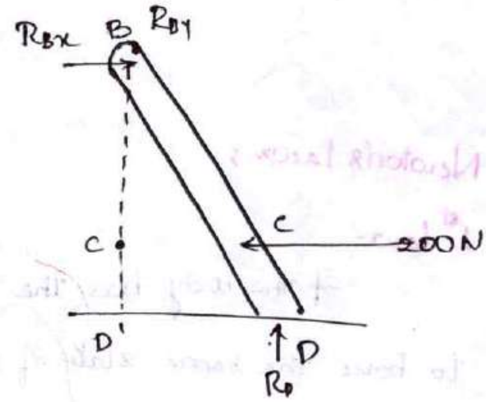
BD = 1.2m

BC = 0.8m

CD = 0.4m

horizontal Force at C = 200 N





Equilibrium of bar

$\Sigma f_x = 0$

$R_{Bx} = 200N$

$\Sigma f_y = 0$ then $R_{By} = R_D$

Taking moments of all forces at point B

$\Sigma M_B = 0$

$R_D \times BD' = 200 \times BC'$

$R_D \cdot BD\cos 60° = 200 \cdot BC\sin 60°$

$R_D \; 1.2\cos 60° = 200 \times 0.8\cos 60°$

$R_D = 230.93N$

$\Sigma M_A = 0$

$M = R_{By} \times 1.6$

$= 230.93 \times 1.6$

$= 369.44 Nm$

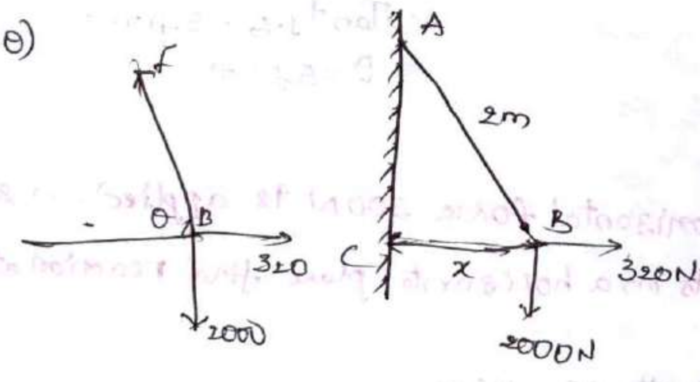
Reaction at B $\quad R_B = \sqrt{R_{Bx}^2 + R_{By}^2}$

$= \sqrt{200^2 + 230.93^2} = 305.4N$

A body weighing 2000 N is suspended with a chain AB 2m long. It is pulled by a horizontal force of 320N. Find the force in the chain and lateral displaced of the body

$$\frac{F}{\sin 90^\circ} = \frac{2000}{\sin(180-\theta)} = \frac{320}{\sin(90+\theta)}$$

$$\frac{F}{1} = \frac{2000}{\sin\theta} = \frac{320}{\cos\theta}$$



$F\sin\theta = 2000$

$F\cos\theta = 320$

$\tan\theta = \frac{2000}{320} = 6.25$

$\theta = \tan^{-1} 6.25 = 80.9^\circ$

$F\sin 80.9^\circ = 2000 \Rightarrow F = \frac{2000}{\sin 80.9} = 2025.5N$

$\cos\theta = \frac{x}{2}$ or $x = 2\cos\theta = 2\cos 80.9^\circ$

$= 0.3163 m$

Newton's laws;

1^{st} law-

If the body has the state of rest or uniform motion, then it will continue to have the same state of condition untill and unless an external force influences.

2^{nd} law-

When a body is under acceleration or deceleration, then the rate of change of momentum of the body in the direction of the motion is equal to the algebraic sum of the forces acting along the same direction of motion

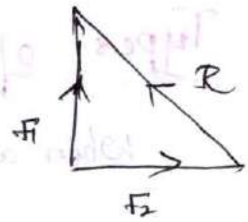
$$\Sigma F = \frac{\partial}{\partial t}(mv) = ma$$

3^{rd} law

For every action there is an equal and opposite reaction.

Triangular law

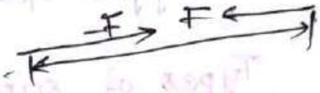
If F_1 and F_2 are two forces acting on a particle that can be represented by the two sides of a triangle in magnitude and direction taken one after the other, then the side that closes the triangle represents the resultant in opposite direction.



Newtons law of gravitation;

It states that the gravitational force of attraction between these bodies in proportional to the product of the masses of the two bodies and inversely proportional to the square of the distance between them.

$F \propto \frac{m_1 m_2}{r}$ $F = G\frac{m_1 m_2}{r^2}$



$G = 66.73 \times 10^{-12}\ m^3/kg \cdot s^2$

Acceleration due to gravity $g = 9.81\ m/s^2$

Resultant of 2 concurrent forces is 1500N and angle b/n the forces is 90°. The resultant makes an angle 36° with one force. Find magnitude

$\alpha = 90°$ $\theta = 36°$ $R = 1500k$

Ak ▱^{le} law $\theta = \tan^{-1}\left[\frac{Q\sin\alpha}{P+Q\cos\alpha}\right]$, $\tan 36° = Q/P$ $0.72 = Q/P$ $Q = 0.72P$

$R = \sqrt{P^2+Q^2+2PQ\cos\alpha}$, $1500 = \sqrt{P^2+(0.7P)^2+0}$, $1500 = \sqrt{P^2+0.49P^2}$, $1500 = \sqrt{1.49P^2}$

$1500 = 1.22P$; $P = \frac{1500}{1.22}$ $P = 1229.5\ N$ $Q = 0.72 \cdot 1229.5 = 885.24N$

Sum of two concurrent forces P & Q is 270N and their resultant is 180N. The angle b/n the force P and resultant R is 90°. Find the magnitude of each force and angle b/n them.

Given $P+Q = 270N$, $R = 180N$ & $\theta = 90°$

$\tan\theta = \frac{Q\sin\alpha}{P+Q\cos\alpha}$, $\tan 90° = \frac{Q\sin\alpha}{P+Q\cos\alpha} = \frac{1}{0}$, $P+Q\cos\alpha = 0$, $Q\cos\alpha = -P$

$R = \sqrt{P^2+Q^2+2PQ\cos\alpha}$, $180 = \sqrt{P^2+Q^2+2P(-P)}$, $180 = \sqrt{P^2+Q^2-2P^2}$, $180 = \sqrt{Q^2-P^2}$

$\sqrt{(Q-P)(Q+P)} = 180$, $\sqrt{(Q-P)(270)} = 180$, $(Q-P)270 = 180^2$, $(Q-P)270 = 32,400/-$

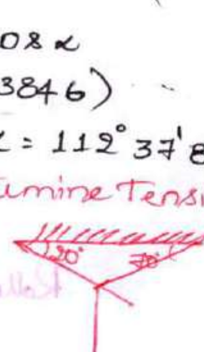
$(Q-P) = 120$, $P+Q = 270$

$Q-P = 120$
$Q+P = 270$
$2Q = 390$
$Q = 195N$

$P = 270 - 195 = 75N$

$R = \sqrt{75^2+195^2+2(75)(195)\cos\alpha}$
$180^2 = 5625 + 38025 + 29250\cos\alpha$
$\cos\alpha = -0.3845$, $\alpha = \cos^{-1}(-0.3846)$
$\alpha = 112°37'8.08''$

A weight of 800N is supported by 2 chains as shown. Determine Tension



$\frac{800}{\sin 90°} = \frac{T_1}{\sin 160°} = \frac{T_2}{\sin 110°}$

$\frac{T_1}{\sin 160°} = \frac{800}{\sin 90°}$

$T_1 = 800 \cdot \sin 90 = 273.61N$

$\frac{T_2}{\sin 110°} = \frac{800}{\sin 90°}$

$T_2 = 751.75N$

ANNAMACHARYA UNIVERSITY

EXCELLENCE IN EDUCATION; SERVICE TO SOCIETY

(ESTD, UNDER AP PRIVATE UNIVERSITIES (ESTABLISHMENT AND REGULATION) ACT, 2016)

Rajampet, Annamayya District, A.P – 516126, INDIA

CIVIL ENGINEERING

Engineering Mechanics

UNIT-2

Types of supports; Introduction.

When a number of forces are acting on a body, and the body is supported on another body, then the second body exerts a force known as reactions on the first body at the points of contact so that the first body is in equilibrium. The second body is known as support and the force, exerted by the second body on the first body, is known as support reactions.

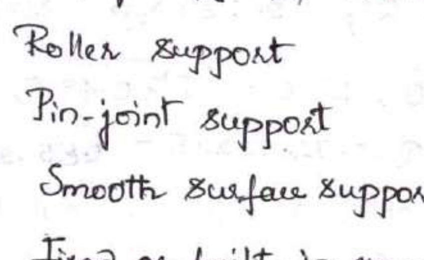
Types of supports:

Though there are many types of supports, yet the following are important from the subject point of view;

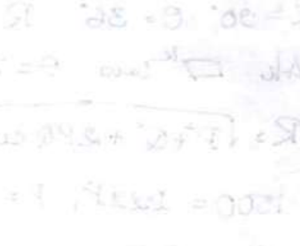
- Simply supported or knife edge supports
- Roller support
- Pin-joint support
- Smooth surface support
- Fixed or built-in supports

Simple support or knife edge support;

A beam supported on the knife edges. A and B is shown in fig 5.11(a). The reactions at A and B in case of knife edges support will be normal to the surface of the beam. The reactions R_A and R_B with free body diagram of the beam



Roller support



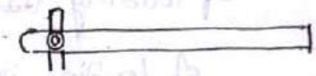
knife edge support

Roller Support;

A beam supported on the rollers at points A and B is shown in fig (5.2a. The reactions in case of roller supports will be normal to the surface on which rollers are placed.

Pin joint or Hinged support;

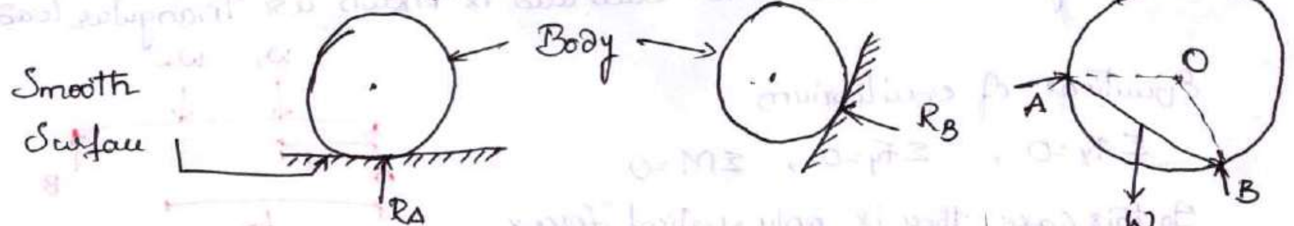
A beam is hinged at point A, is shown in fig



The reaction at the hinged end may be either vertical or inclined depending upon the type of loading. If the load is vertical, then the reaction will also be vertical. But if the load is inclined, then the reaction at hinged end will also be inclined.

Smooth surface Support;

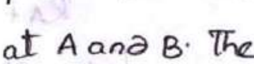
A body in contact with a smooth surface. The reaction will always act normal to the support



AB resting inside a sphere, whose surface are smooth. Here the rod becomes body and sphere becomes surface. The reactions on the ends of the rod will be normal to the sphere surface at A and B. The normal at any point on the surface of the sphere will always pass the centre of the sphere. Hence reactions R_A & R_B will have directions

Fixed or Built in support;

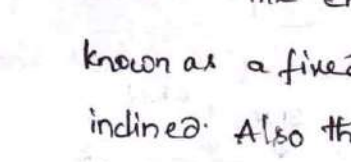
The end A of a beam, which is fixed. Hence the support at A is known as a fixed support. In case of fixed support, the reaction will be inclined. Also the fixed supports will provide a couple.

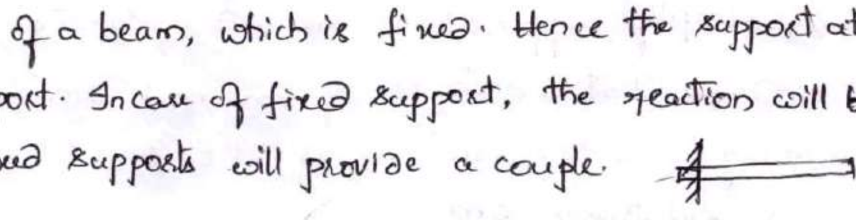


Types of loading

Concentrated/point load;

A Concentrated load is one which is considered to act at a single point, although in practice it must really be distributed over a small area.

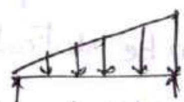




Uniformly distributed load;

load which spreads over a beam in such a manner that rate of loading is only ~~varies~~ from ~~point to point~~ along the length beam. The rate of loading is expressed as w N/m run.

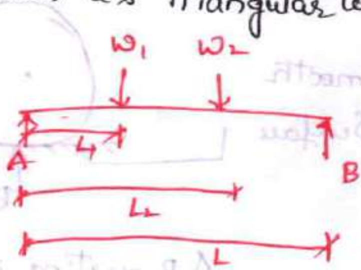
Uniformly varying load;



A uniformly varying load is one which is spread over a beam in such a manner that rate of loading varies from point to point along the beam in which load is zero at one end and increases uniformly to the other end. Such load is known as Triangular load.

Equations of equilibrium

$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$



In this case there is only vertical forces

$\therefore \Sigma F_y = 0 \quad \Sigma M = 0$

$R_A + R_B = W_1 + W_2$

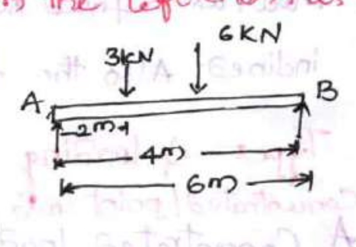
For $\Sigma M_A = 0$

$R_B \times L - W_1 L_1 - W_2 L_2 = 0$

$R_B = \dfrac{W_1 L_1 + W_2 L_2}{L}$

$R_A = (W_1 + W_2) - R_B$

A simply supported beam AB of span 6m carries point loads of 3kN and 6kN at a distance of 2m and 4m from the left end A as shown in fig. Find the reactions at A & B.



Span of Beam = 6m

Support reactions;

Taking moments of all forces about A, and equating the resultant moment at zero

$\Sigma M_A = 0$

$R_B \times 6 - 3 \times 2 - 6 \times 4 = 0$

$6 R_B = 6 + 24$

$R_B = 30/6 = 5kN$

Considering Equilibrium $\Sigma F_y = 0$

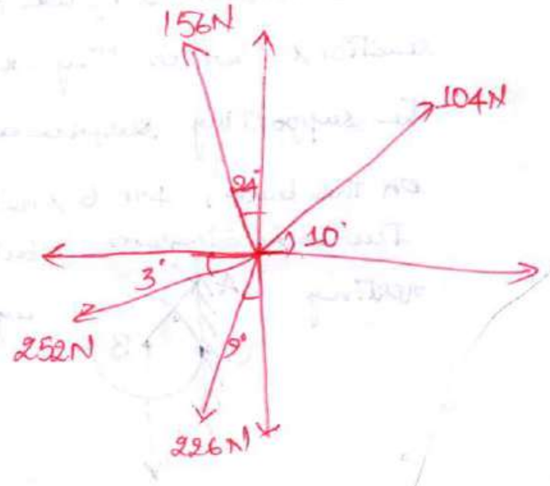
$R_A + R_B = 3 + 6 = 9$

$R_A = 9 - R_B$

$R_A = 9 - 5 = 4kN$

Problems 1st Unit;

Four Coplanar forces acting at a point as shown in figure. Determine the direction and magnitude



$\Sigma V = 0$

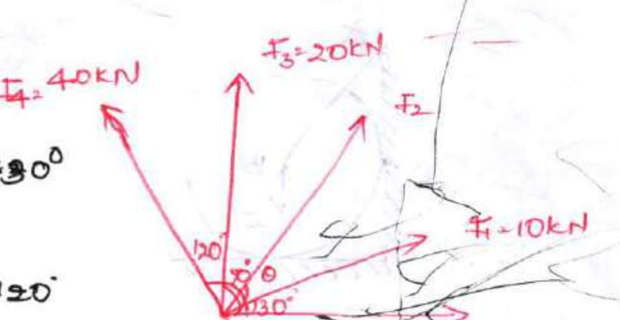
$= 104 \sin 10° - 252 \sin 3° - 226 \sin 81°$
$- 156 \sin 66°$
$= -248.05\,N$

$\Sigma H = 0$

$104 \cos 10° - 252 \cos 3° - 226 \cos 81°$
$- 156 \sin 66°$
$= -75.83$

$R = \sqrt{\Sigma H^2 + \Sigma V^2} = 259.38\,N$

$\tan\theta = \frac{\Sigma V}{\Sigma H} = 0.3057 \quad \theta = 16°59'$



$\Sigma H = 0$

$10 \cos 30° + F_2 \cos\theta + 20 \cos 30° - 40 \cos 30°$

$\Sigma V = 0$

$10 \sin 30° + F_2 \sin\theta + 20 \sin 30° + 40 \sin 120°$

The resultant is along Y-axis ∴ Sum of horizontal components is zero and the algebraic sum of vertical components = resultant

$R = \sqrt{\Sigma V^2 + \Sigma H^2} \quad \text{where } \Sigma H = 0$

$R = \sqrt{\Sigma V^2} \quad R = \Sigma V$

$R = F_2 \sin\theta + 59.64$

2/1 $\quad F_2 \sin\theta = 12.36\,kN$ — ② $, \quad F_2 \cos\theta = 11.34\,kN \rightarrow$ ①

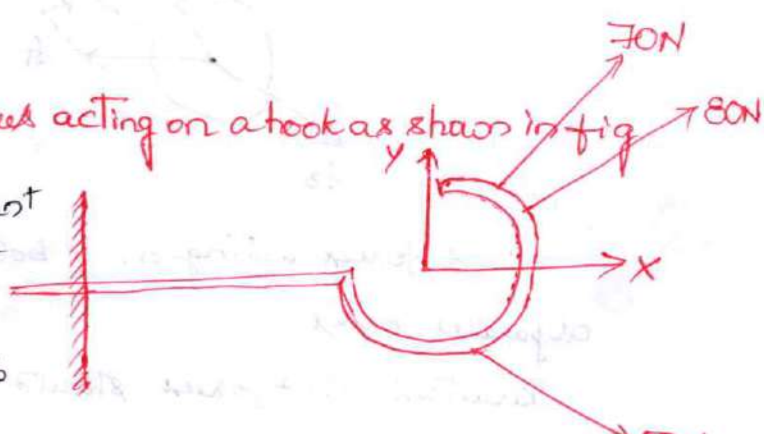
$\tan\theta = 1.08 \quad \theta = 47°12'$

$F_2 \cos\theta = 11.34$

$F_2 \cos(47°12') = 11.34$

$F_2 = 16.92\,kN$

Determine the resultant of 3 forces acting on a hook as shown in fig



Force	X-Component	Y-Component
70N	45.00	53.62
80N	72.50	33.81
50N	35.36	-35.36

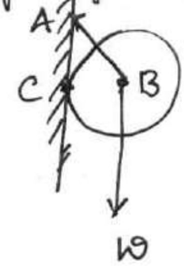
$R = \sqrt{152.86^2 + 52.07^2}$

$\theta = \tan^{-1} V/H \;\; \frac{\Sigma V}{\Sigma x}$

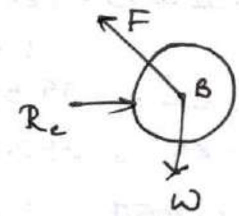
Free body diagram

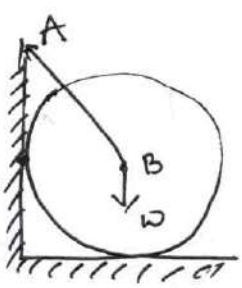
The equilibrium of the bodies which are placed on the supports can be considered if we remove the supports and replace them by reactions which they exert on the body. In fig 4.10 if we remove the supporting surface and it by reaction that the surface exerts on the balls, 4.10 b shall be free body diagram.

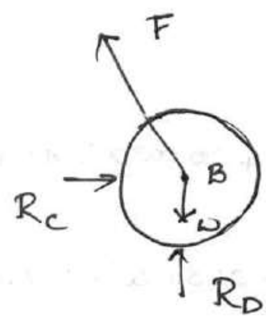
Free body diagram of ball of weight W supported by a string AB and resting



against a smoot vertical wall at C

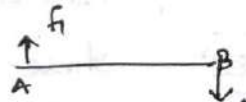






Force law of equilibrium

Two force system

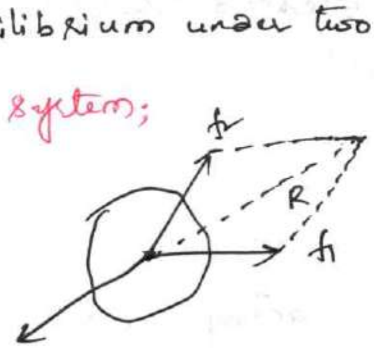


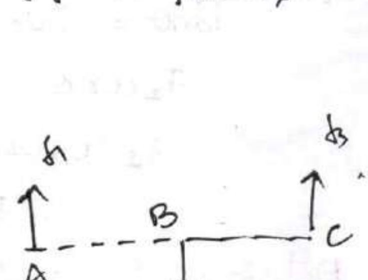
$M_A = f_1 \cdot AB$

$\Sigma f_y = 0 \qquad \Sigma f_x = 0 \qquad M_A \text{ is } \neq 0$

Hence 3rd condition is not satisfied. Hence body will not be in equilibrium under two equal and opp ‖le forces

Three force system;





3 forces acting on a body in equilibrium either concurrent or parallel forces

Resultant of 2 forces should be equal and opposite to 3rd force

If f_1 f_2 f_3 in same direction then $R = f_1 + f_2 + f_3$. The three forces are acting in opposite direction and their magnitude is 80 adjusted that there will be no resultant force and body is in equilibrium.

Apply
$\Sigma f_x = 0$

$\Sigma f_y = 0$
$\Sigma M = 0$ about any point

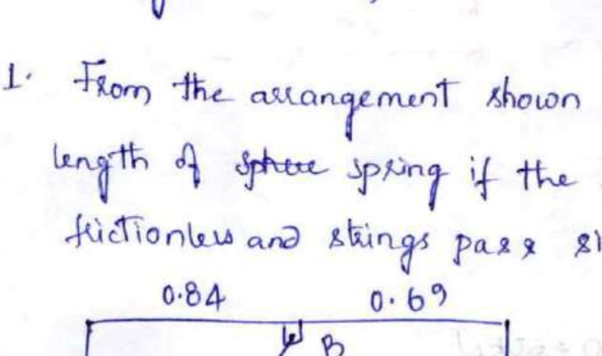
If $\Sigma M_A = 0$ then $-f_2 AB + f_3 AC = 0$

Force force system:

$\Sigma H = 0 \quad \Sigma V = 0 \quad \Sigma M = 0$

Assignment exam

1. From the arrangement shown in fig. Determine the value of W and Unstretched length of sphere spring if the spring constant is 800 N/m. Assume pulley as frictionless and strings pass simply over pulleys

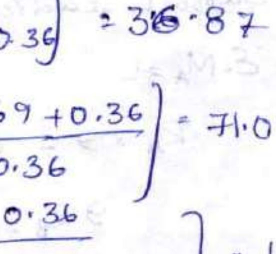


$\alpha = \tan^{-1}\left[\frac{0.36}{0.84-0.36}\right] = 36.87$

$\beta = \tan^{-1}\left[\frac{0.69+0.36}{0.36}\right] = 71.07$

$\theta = \tan^{-1}\left[\frac{0.36}{0.69+0.36}\right] = 18.92$

length $AB = \sqrt{0.36^2 + (0.69+0.36)^2} = 1.11m$



$F_{AB} = (1.11 - L) \times 800 N$

$F_{AB} = (1.11 - L) 800 \cos\beta) + (1.11 - L) 800 \sin\beta$

$F_{AD} = (F_{AD}\cos\theta + F_{AD}\sin\theta) = (0.946 f_{AD} + 0.324 f_{AD})j$

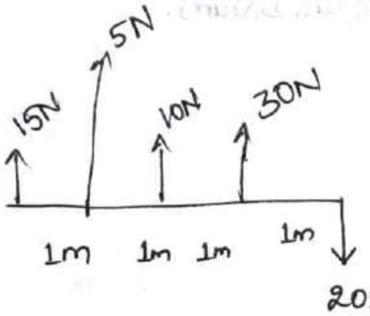
$F_{AC} = -F_{AC}\cos\alpha + F_{AC}\sin\alpha) = (-0.79 f_{AC})i + 0.6 f_{AC})j$

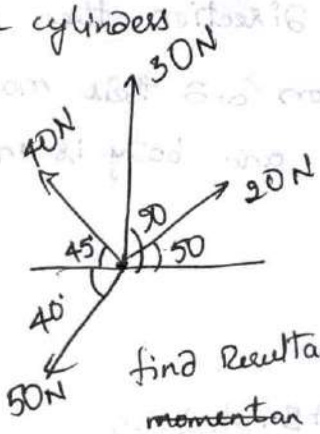
Resultant

$\Sigma H : 259.53(1.11 - L) + 0.946W - 0.79 \times 3W = 0$

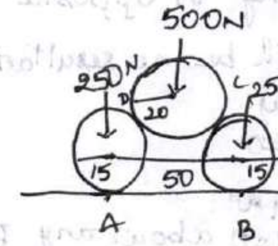
$\Sigma V \quad 756.73 (1.11 - L) + 0.324W + 0.6 \times 3W = 400$

Two smooth circular cylinders





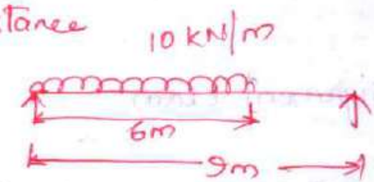
find Resultant momentum



Determine the magnitude and reaction at A B C & D

Resultant moment and distance

A SS beam carries UDL of 10kN/m for a distance of 6m from the left end. Calculate the reactions at A and B



Given

length of beam = 9m

Rate of UDL = 10 kN/m

length of UDL = 6m

Total load due to UDL : $6 \times 10 = 60kN$

Support reactions

$\Sigma M_A = 0$

$R_B \times 9 - 6 \times 10 \times 3 = 0$

$9 R_B = 180$

$R_B = \frac{180}{9} = 20 kN$

$\Sigma F_y = 0$

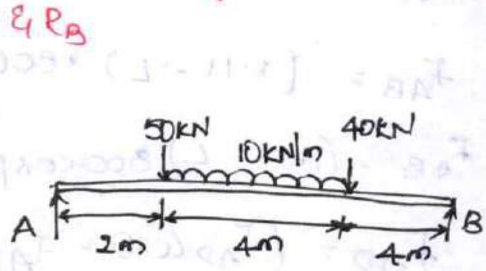
$R_A + R_B = 6 \times 10 = 60$

$R_A = 60 - R_B$

$= 60 - 20$

$= 40 kN$

A SS beam of length 10m, carries UDL and point load as shown in fig 5.15. Calculate the reactions at R_A & R_B



length of beam = 10m

length of UDL = 4m

Rate of UDL = 10kN/m

Total load due to UDL = $4 \times 10 = 40kN$

$\Sigma M_A = 0$

$R_B \times 10 - 50 \times 2 - 40 \cdot (2+4) - (10 \times 4)\left[2 + \frac{4}{2}\right] = 0$

$10 R_B - 100 - 240 - 160 = 0$

$10 R_B = 100 + 240 + 160 = 500$

$R_B = \frac{500}{10} = 50kN$

$R_A + R_B = 50 + (10\times4) + 40 = 130$

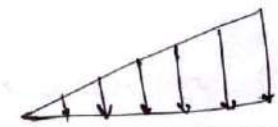
$R_A = 130 - R_B = 130 - 50 = 80\,kN$

A Simply Supported beam of Span 9m Carries a UVL from zero at end A to 900 N/m at end B. Calculate the reactions at the two ends of the support.

Span of beam = 9m

load at end A = 0

Load at end B = 900 N/m



Total Load on beam = Area of ABC = $\frac{1}{2} AB \times BC = 9\times900\times\frac{1}{2}$

$= 4050\,N$

C.G of Δ ABC i.e., $2/3 \times AB = 2/3 \times 9 = 6m$

Taking moments of all the forces about point A and equating the resultant moment at zero, we get

$R_B \times 9$ = Total Load on beam × Distance of Total Load from A

$= 4050\times6 \quad R_B = \frac{4050\times6}{9} = 2700\,N$

$R_A + R_B = 4050$

$R_A = 4050 - 2700 = 1350\,N$

A Simply Supported beam of length 5m Carries a UVL of 800 N/m at one end to 1600 N/m at the other end. Calculate the reactions at both ends.

Total load on beam = Area of load diagram ABDC

+ Area of Rectangle ABEC

+ Area of ΔCED

$= AB\times AC + \frac{CE \times ED}{2} = 5\times800 + 1/2\times5\times800$

$= 4000 + 2000 = 6000\,N$

C.G of rectangle ABEC will be at a distance of $5/2 = 2.5m$ from A, where as the CG of Δ^{le} CED will be at a distance of $2/3\,5 = 3.33m$

$R_B \times 5$ − (Load due to $\square^{le}$) Distance of CG of rectangle from A − load due to Δ^{le} × Distance of CG of Triangle from A

$5R_B - (5\times800)\cdot2.5 - 1/2\times5\times800\cdot 2/3\,5 = 0$

$5R_B - 1000 - 6666.66 = 0$

$R_B = \frac{16666.66}{5} = 3333.3\,N$

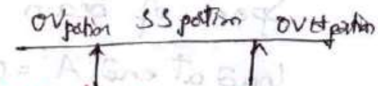
$R_A + R_B$ = Total Load $= 6000$

$R_A = 6000 - 3333.3$

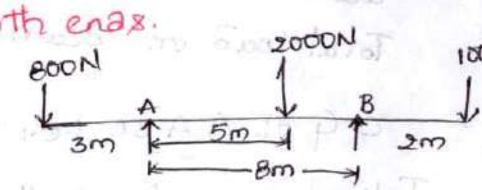
$= 2666.67\,N$

Overhanging beam

If the end portion of a beam is extended beyond the support, then the beam is known as overhanging beam. overhanging portion may be at one end of the beam or at either ends.



A beam AB of span 8m, overhanging on both sides, is loaded as shown in fig. Calculate the reactions at both ends.



Span of beam = 8m

Taking moments of all forces about point A

$\Sigma M_A = 0$

$R_B \times 8 + 800 \times 3 - 2000 \times 5 - 1000(8+2) = 0$

$8R_B + 2400 - 10{,}000 - 10000 = 0$

$-8R_B = 17600$

$R_B = \frac{17600}{8} = 2200N$

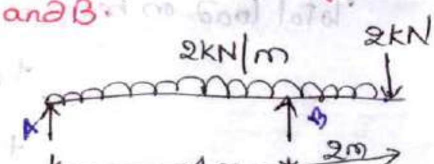
Also for the equilibrium of the beam.

$R_A + R_B = (800 + 2000 + 1000)$

$= 3800$

$R_A = 3800 - 2200 = 1600\,N$

A beam AB of span 4m, over hanging on one side up to a length of 2m, carries UDL of 2kN/m over the entire length of 6m and a point load of 2kN/m. Calculate the reactions at A and B.



Span of beam = 4m

Total length = 6m

Rate of UDL = 2kN/m

Total load due to UDL = $2 \times 6 = 12$ kN/m

Taking moments of all forces about point A and equating the resultant moment to zero, we get

$R_B \times 4 - 2 \times 6 \times 3 - 2 \times (4+2) = 0$

$4R_B - 36 - 12 = 0$

$R_B = 48/4 = 12$ kN

$\Sigma F_y = 0 \quad R_A + R_B = 12 + 2 = 14$

$R_A = 14 - R_B = 14 - 12 = 2$kN

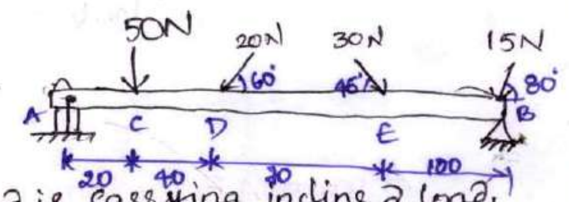
Problems on Roller and Hinged supported beams.

In case of roller supported beams, the reaction on the roller end is always normal to the support. All the steel trusses of the bridges is generally having one of their ends supported on rollers. The main advantage of such support is it an accomodate, Temperature change. Can move easily towards left or right, on account of expansion and contraction.

In case of hinge supported beam, the reaction on the hinged end may be either vertical or inclined, depending up on the type of loading. The main advantage of hinged end is that the beam remains stable Hence all steel trusses and bridges have one end roller and the other end as hinged.

A beam AB 1.7m long is loaded as shown in fig 5.9.1. Determine the reactions at A and B.

length of beam 1.7m



Since the beam is supported on rollers at B, therefore the reaction R_B will be vertical. The beam is hinged at A, and is carrying inclined load, therefore the reaction R_A will be inclined. This means reaction on R_A will have Two components i.e., vertical component and horizontal component

First resolve all the inclined loads in to the vertical and horizontal components

	Vertical components	Horizontal component
At D	$20\sin 60° = 20 \times 0.86 = 17.32N$	$20\cos 60° = 10N$
At E	$30\sin 45° = 21.21N$	$30\cos 45° = 21.21N$
At B	$15\sin 80° = 14.77N$	$15\cos 80° = 2.6N$

From conditions of equilibrium $\Sigma F_x = 0$

$$R_{Ax} - 10 + 21.21 - 2.6 = 0$$

$$R_{Ax} = 10 - 21.21 + 2.6 = -8.61N$$

-ve sign indicates direction of R_{Ax} is wrong. Correction direction will be opposite to the assumed direction. $R_{Ax} = 8.61N \leftarrow$

To find R_B Taking moments of all forces about A

for equilibrium $\Sigma M_A = 0$

$50 \times 20 + 20 \sin 60° \times (20+40) + 30 \sin 45° \times (20+40+70) +$
$15 \sin 80° \times 170 - 170 R_B = 0$

$1000 + 1039.2 + 2757.7 + 2511 - 170 R_B = 0$

$$R_B = \frac{7307.9}{170} = 42.98 N$$

To find R_{Ay}, applying conditions of equilibrium $\Sigma F_y = 0$

$-R_{Ay} + R_B = 50 + 20 \sin 60° + 30 \sin 45° + 15 \sin 80°$

$R_{Ay} + 42.98 = 50 + 17.32 + 21.21 + 14.77$

$= 103.3 N$

$$R_A = \sqrt{R_{Ax}^2 + R_{Ay}^2} = \sqrt{8.61^2 + 60.32^2} = 60.92 N$$

Angle made by R_A

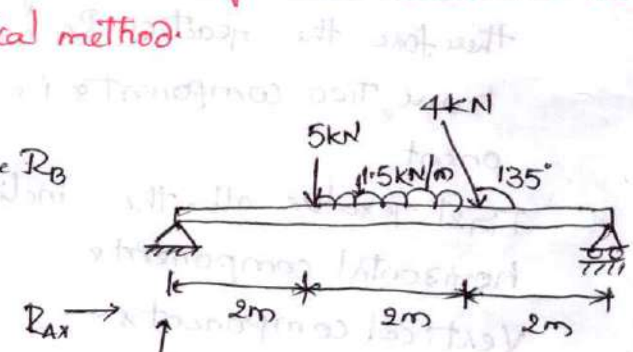
$$\tan\theta = \frac{R_{Ay}}{R_{Ax}} = \frac{60.32}{8.61} = 7.006$$

$\theta = 81.87°$

A beam AB 6m long is loaded as shown in fig: 5.22. Determine the reactions at A and B by analytical method.

length of beam = 6m

let Reaction at A = R_A & Reaction at B be R_B



Horizontal component of 4kN at D

$4 \cos 45° = 2.828 kN \rightarrow$

Vertical component

$4 \sin 45° = 2.828 kN \downarrow$

For equilibrium $\Sigma F_x = 0$

$-R_{Ax} + 2.828 = 0$

$R_{Ax} = 2.828 N$

To find R_B take the moments of all forces about point A

For equilibrium $\Sigma M_A = 0$

$R_B \times 6 - 5 \times 2 - 2 \times 1.5 \left[2 + \frac{2}{2}\right] - (4 \sin 45°)(2+2) = 0$

$6R_B - 10 - 9 - 11.312 = 30.312$

$$R_B = \frac{30.312}{6} = 5.052 kN$$

To find R_{AY}, Apply the Conditions of equilibrium $\Sigma F_y = 0$

$$R_{AY} + R_B - 5 - (1.5 \times 2) - 4 \sin 45^\circ = 0$$

$$R_{AY} + 5.052 - 5 - 3 - 2.828 = 0$$

$$R_{AY} = -5.052 + 5 + 3 + 2.82 = 5.776 \text{ kN}$$

Reaction at A

$$R_A = \sqrt{R_{AX}^2 + R_{AY}^2} = \sqrt{2.82^2 + 5.776^2} = \sqrt{41.36} = 6.43 \text{ kN}$$

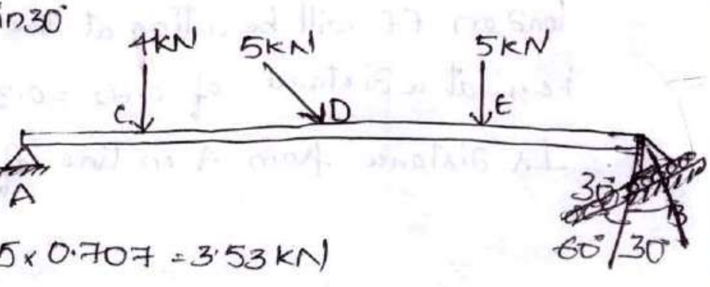
θ = Angle made by R_A with X - direction

$$\tan\theta = \frac{R_{AY}}{R_{AX}} = \frac{5.775}{2.828} = 2.0424 \quad \theta = 63.9^\circ$$

A Beam AB 10 m long is hinged at A and supported on rollers over a smooth surface inclined at 30° to the horizontal at B. The beam is loaded as shown in fig 5.24. Determine reactions at A and B.

Vertical Component of R_B = $R_B \cos 30^\circ$

Horizontal component of R_B = $R_B \sin 30^\circ$



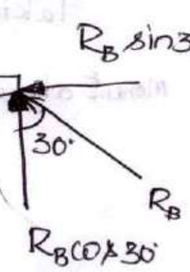
Resolving 5 kN load

Vertical component = $5 \sin 45^\circ = 3.53$ kN

Horizontal component = $5 \cos 45^\circ = 5 \times 0.707 = 3.53$ kN

For equilibrium of the beam, the moments of all forces about any point should be zero.

Taking moments about point A



$$(R_B \cos 30^\circ) \times 10 - 4 \times 2.5 - 5 \sin 45^\circ - 5 \times 8 = 0$$

$$8.66 R_B - 10 - 17.675 - 40 = 0$$

$$R_B = \frac{10 + 17.675 + 40}{8.66} = 7.81 \text{ kN}$$

For equilibrium $\Sigma F_x = 0$

$$R_{AH} + 5 \cos 45^\circ - R_B \sin 30^\circ = 0$$

$$R_{AH} + 3.535 - 7.81 \times 0.5 = 0$$

$$R_{AH} = 7.81 \times 0.5 - 3.535 = 0.37 \text{ kN}$$

For equilibrium $\Sigma F_y = 0$

$$R_{AV} + R_B \cos 30^\circ - 4 - 5 \sin 45^\circ - 5 = 0$$

$$R_{AV} + 6.763 - 12.535 = 0$$

$R_{AV} = 12.535 - 6.763 = 5.77 kN$

Reaction at A, $R_A = \sqrt{R_{AH}^2 + R_{AV}^2} = \sqrt{0.37^2 + 5.77^2} = 5.78 kN$

The angle made by R_A with x-direction is given by

$$\tan\theta = \frac{R_{AV}}{R_{AH}} = \frac{5.77}{0.37} = 15.59$$

$$\theta = \tan^{-1} 15.59 = 86.33'$$

Find reactions at supports of an L-bent shown in fig

Force at point D = 100 N at angle 30' with ____

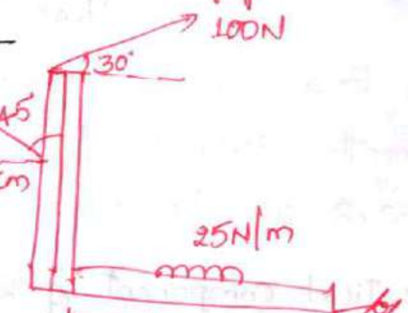
Force at point C = 70 N at angle 45' with ____

load on EF = 250 N/m

= 250 × length EF in metre

= 250 × 0.6

= 150 N



load on EF will be acting at the middle point of EF i.e., at a distance of 0.6/2 = 0.3m from E

⊥r distance from A on line of action of R_B = AO = AB cos 20°

= 180 × cos 20°

= 1.8 cos 20°

Taking moments of all forces about point A, we get of all forces @ point A moment about any point should be zero.

= (Horizontal component at D) × AD − (Horizontal component at C) × AC + load on EF × 90 − R_B × AO = 0

$$(100\cos 30°) \times 80 - (70\sin 45°) \times 40 + 150 \times 90 - R_B\, 180\cos 20° = 0$$

$$6928 - 1979.6 + 13{,}500 - 169.14 R_B = 0$$

$$18448.4 = 169.14 R_B$$

$$R_B = \frac{18448.4}{169.14} = 109.07 kN$$

The reaction at A can be resolved in two components i.e., R_{AX} & R_{AY}

For equilibrium $\Sigma F_x = 0$

$$R_{AX} + 100\cos 30° - 70\sin 45° - R_B \sin 20 = 0$$

$$R_{AX} = R_B \sin 20° + 70\sin 45° - 100\cos 30°$$

$$= 109.07 \times 0.342 + 70 \times 0.707 - 100 \times 0.866$$

$= 37.3 + 49.49 - 86.6 = 0.19 N$

For equilibrium $\Sigma F_y = 0$

$R_{AX} + 100\sin 30^\circ + 70\cos 45^\circ + R_B\cos 20^\circ = 150$

$R_{AY} = 150 - 100\sin 30^\circ - 70\cos 45^\circ - R_B\cos 20^\circ$

$= 150 - 50 - 49.49 - 109.07 \times 0.9396$

$= -51.98 N$

$R_A = \sqrt{R_{AX}^2 + R_{AY}^2} = \sqrt{(0.19)^2 + (-51.98)^2} = 51.98 kN$

Angle made by R_A with x-axis is given by

$\tan\theta = \frac{R_{AY}}{R_{AX}} = \frac{51.98}{0.19} = 273.57$

$\theta = \tan^{-1} 273.57 = 89.79^\circ$

Beams to couples

A simply supported beam AB of 7m span is subjected to

i) 4kNm clockwise couple at 2m from A

ii) 8kNm anti-clockwise couple at 5m from A and

iii) A triangular load with zero intensity at 2m from A increasing to 4kN/m at a point 5m from A. Determine the reactions at A and B.

Given

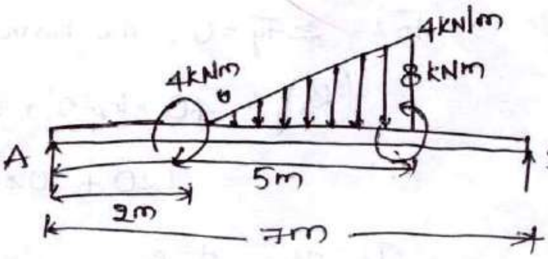
Span of beam = 7m

Couple at C = 4kNm

Couple at D = 8kNm

Triangular load at C = 0

Vertical load at D = 4kN/m



Total load on beam = Area of triangle CDE = ½ CD × DE = ½ × 3 × 4 = 6 kN

C.G of △CDE 2/3 CD = 2/3 × 3

Towards end A = 2 + 2 = 4m

Taking moments of all forces about point A and equating the resultant moment to zero $\Sigma M_A = 0$

$R_B \times 7 + 4 - 8 + (\frac{1}{2} \times 3 \times 4) \times 4 = 0$

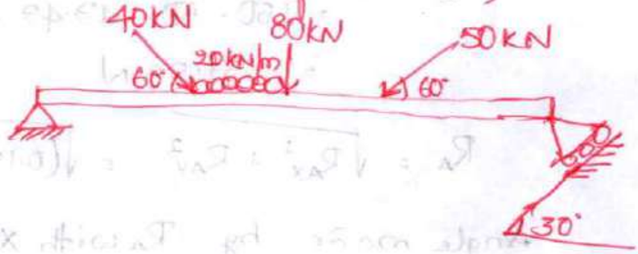
$R_B = 20/7 = 2.8 kN$

Also for the equilibrium of the beam $\Sigma F_y = 0$

$$R_A + R_B = 6\,kN$$

$$R_A = 6 - R_B = 6 - 20/7 = \frac{22}{7}\,kN$$

Find the reactions at the supports A and B of the beam



length of beam = 8m

R_{Ax} = Horizontal component of R_A

R_{Ay} = Vertical component of R_A

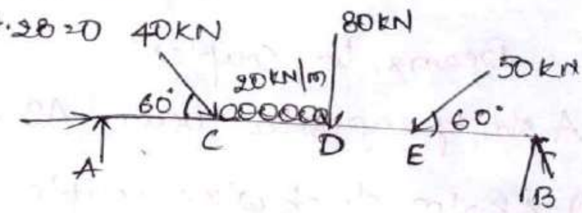
$\Sigma M_A = 0$

$$R_B \cos 30° \times 8 - 50 \sin 60° \times 6 - 80 \times 4 - 40 \times 3 - 40 \sin 60° \times 2 = 0$$

$$R_B \times 0.866 \times 8 - 50 \times 0.866 \times 6 - 320 - 120 - 40 \times 0.86 \times 2 = 0$$

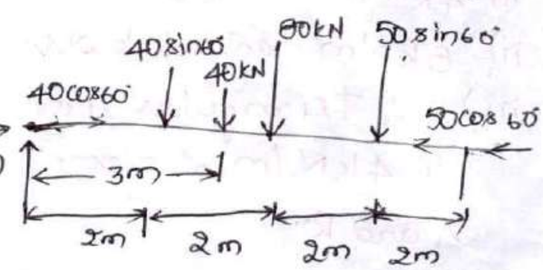
$$6.928\,R_B - 259.8 - 320 - 120 - 69.28 = 0$$

$$R_B = \frac{769.08}{6.928} = 111\,kN$$



$\Sigma F_x = 0$ & $\Sigma F_y = 0$

For $\Sigma F_x = 0$ we have



$$(R_A)_x + 40 \cos 60° - 50 \cos 60° - R_B \sin 30° = 0$$

$$R_{Ax} + 20 - 25 - 111 \times 0.5 = 0$$

$$R_{Ax} = -20 + 25 + 111 \times 0.5 = 60.5\,kN$$

For $\Sigma F_y = 0$, we have $(R_A)_y - 40 \sin 60° - 40 - 80 - 50 \sin 60° + R_B \cos 30° = 0$

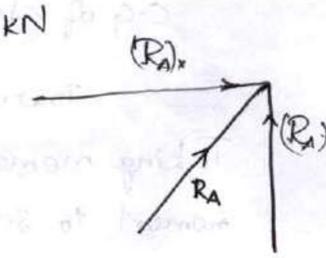
$$(R_A)_y = 40 \sin 60° + 120 + 50 \sin 60° - 111 \times 0.8666$$

$$= 120 + 90 \sin 60° - 111 \times 0.866 = 101.8\,kN$$

Reaction at A

$$R_A = \sqrt{(R_{Ax})^2 + (R_A)_y^2} = \sqrt{(60.5)^2 + (101.8)^2}$$

$$= \sqrt{3660.25 + 10363} = 118.42\,kN$$

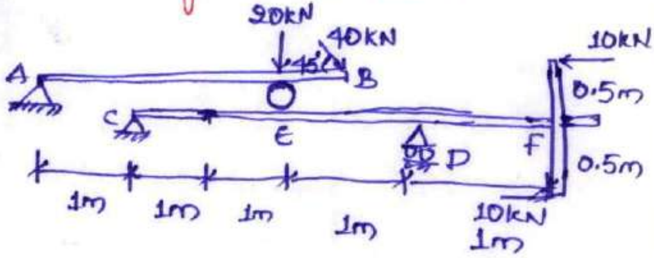


$$\tan\theta = \frac{(R_A)_y}{(R_A)_x} = \frac{101.8}{60.5} = 1.682$$

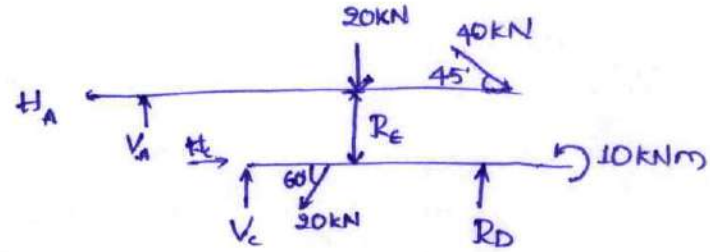
$$\theta = \tan^{-1}(1.682) = 59.27°$$

The beam AB and CF are arranged as shown in fig. Determine the reactions at A, C and D due to the forces acting on the beam

Freebody diagrams of beams AB and CF



The two ten kN forces can be replaced by a couple of $\{10(0.5+0.5)\} = 10KN$ Moment acting at F as shown. Now Consider the beam AB.



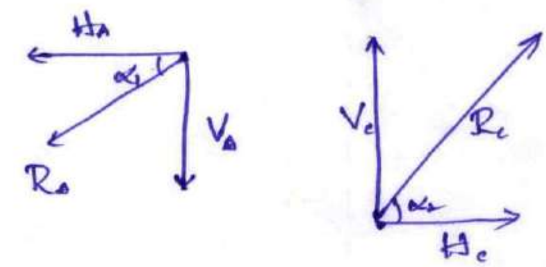
$\Sigma M_A = 0$

$R_E \times 3 - 20\times 3 - 40\sin 45^\circ \times 4 = 0$

$R_E = 57.712\ KN$

$\Sigma H = 0$

$H_A = 40\cos 45^\circ = 28.284KN$



$\Sigma V = 0$

$V_A + 57.712 - 20 - 40\sin 45^\circ = 0$

$V_A = -9.428$

$V_A = 9.428 \downarrow$

$R_A = \sqrt{H_A^2 + V_A^2} = 29.814KN$

$\alpha_1 = \tan^{-1}\dfrac{V_A}{H_A} = 18.43^\circ$

Consider beam CF

$\Sigma M_c = 0$

$R_D \times 3 - 20\sin 60^\circ \times 1 - 57.712\times 2 + 10 = 0$; $R_D = 40.915\ kN$

$\Sigma H = 0$

$H_c - 20\cos 60^\circ = 0$; $H_c = 10\ KN$

$\Sigma V = 0$

$V_c - 20\sin 60^\circ - 57.712 + 40.915 = 0$

$V_c = 34.117\ KN$

$R_c = \sqrt{V_c^2 + H_c^2} = 35.553KN$

$\alpha_2 = \tan^{-1}\dfrac{V_c}{H_c} = 73.6^\circ$

UNIT - II

Analysis of frames

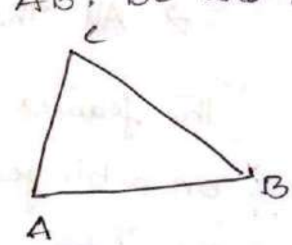
A structure made up of several bars (or members) riveted or welded together is known as frame. [If the frame is composed of such members which are just sufficient to keep the frame in equilibrium when the frame is supporting an external load, then the frame is known as perfect frame.]

Types of frames

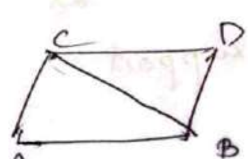
The different types of frames are

Perfect frame: The simplest perfect frame is a Triangle, which consists of 3 members and three joints. 3 members are AB, BC and AC where as the three joints are A, B and C.

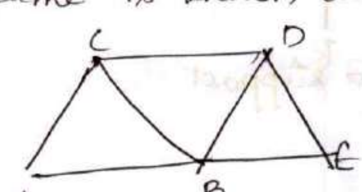
This frame can be easily analyzed by the conditions of equilibrium.



Let the two members CD and BD and a joint D are added to Triangular frame ABC. This frame can be analyzed by conditions of equilibrium. This frame is known as perfect frame



a



Suppose we add a set of two members and a joint again, we get a perfect frame. Hence for a perfect frame, the no of joints and no of members are given by

$n = 2j - 3$ n = number of members
j = no of joints.

Imperfect frame:

A frame in which number of members and number of joints are not given by $n = 2j - 3$

is known as imperfect frame. This means that number of members in an imperfect frame will be either more or less than (2j-3)

→ If the number of members in a frame less than (2j-3) then the frame is known as defecient frame.

→ If the number of members in a frame are more than (2j-3) Redundant frame.

Assumptions

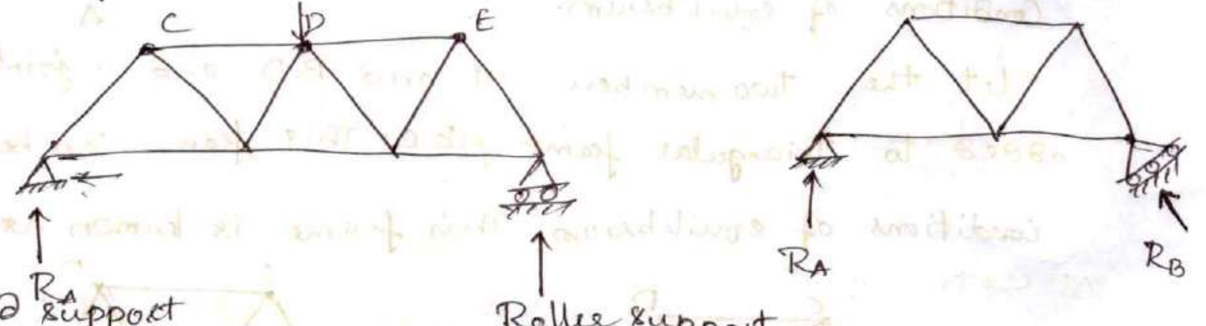
The assumptions made in finding out the forces in the frame are

→ The frame is a perfect frame

→ The frame carries load at the joints

→ All the members are pin-jointed.

The frames are generally supported on a roller support or on a hinged support.



Analysis of frame

→ Determination of reactions at the supports

→ Determination of forces in the members of the frame

Reactions are determined by the conditions that the applied load system and the induced reactions at the supports from a system in equilibrium

Forces - Conditions that every joint should be in equilibrium and so, the forces acting at every joint should form a system in equilibrium

By two methods
Method of joints and
Method of sections
Graphical method.

Method of joints;

After determining the reactions at the supports, the equilibrium of every point is considered. means sum of all vertical forces as well as the horizontal forces acting on a joint is equated to zero. The joint should be selected in such a way that at any time there are only two members, in which the forces are unknown. The forces in the member will be compressive if the member pushes the joint to which it is connected where as the force in the member will be Tensile if the member pulls the joint to which it is connected.

Determine the reactions R_B and R_C

line of action of load 20kN acting at A is vertical. This load is at a distance of AB cos 60° from the point B. Now let us find the distance AB

BA △ABC is a right angled triangle with BAC = 90°

Hence $AB = BC \cos 60°$

$AB = 5 \cos 60° = 5 \times 1/2 = 2.5\,m$

Distance of line of action of 20kN from B is $AB \cos 60° = 2.5 \cos 1/2 = 1.25\,m$

Taking moments about B, we get

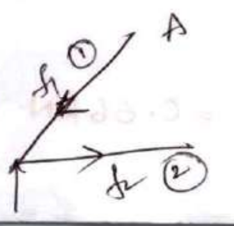
$R_C \times 5 - 20 \times 1.25 =$

$R_C = \frac{25}{5} = 5\,kN$

$R_B + R_C = 20\,kN$

$R_B = 20 - 5 = 15\,kN$

Joint B



$\Sigma V = 0$ Resolving the forces acting on joint B, Vertically

$F_1 \sin 60° = 15$

$F_1 = \frac{15}{\sin 60°} = 17.32\,kN$ Compressive

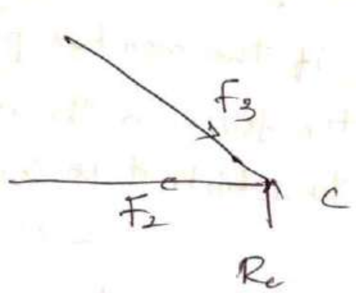
F_1 is pushing the joint B, this force is compressive

$\Sigma H = 0$ Resolving horizontally

$F_2 = F_1 \cos 60° = 17.32 \times 1/2 = 8.6\,kN$

As f_2 is pulling the joint B, hence force will be tensile.

Joint C



$\Sigma V = 0$

$F_3 \sin 30° = R_C$

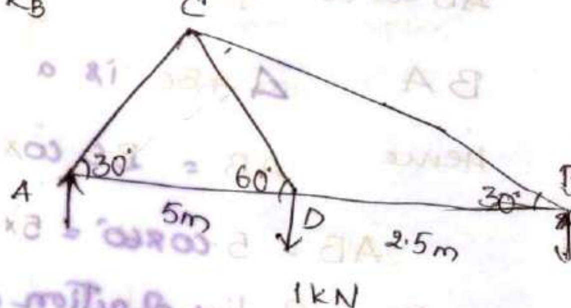
$F_3 \sin 30° = 5$

$F_3 = \frac{5}{\sin 30°} = 10\,kN$

Compressive.

A Truss of span 7.5m carries a point load of 1kN at joint D as shown in fig. Find the reactions and forces in the members of the Truss.

Let us determine the reactions R_A and R_B



$\Sigma V = 0$

$R_A + R_B = 1\,kN$

$R_B \times 7.5 = 5 \times 1$

$R_B = \frac{5}{7.5} = \frac{2}{3} = 0.667\,kN$

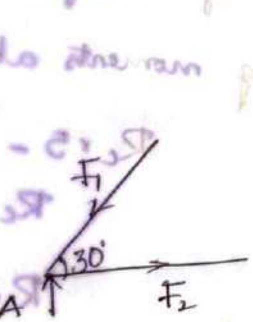
$R_A = \text{Total load} - R_B$

$= 1 - 0.667 = 0.33\,kN$

Joint A

F_1 = force in member AC

F_2 = Force in member AD



$\Sigma V = 0$

$F_1 \sin 30° = R_A$

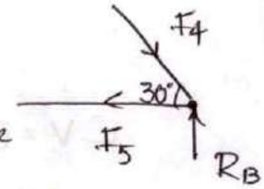
$F_1 = \frac{R_A}{\sin 30°} = \frac{0.33}{0.5} = 0.66\,kN$ compressive

Resolving the forces horizontally

$F_2 = F_1 \cos 30^\circ = 0.66 \cos 30^\circ = 0.66 \times 0.86 = 0.5767\,kN$

Joint B

$\Sigma V = 0$



$F_4 \sin 30^\circ = R_B = 0.667$

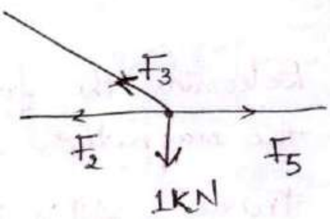
$F_4 = \dfrac{0.667}{\sin 30^\circ} = 1.334\,kN$ compressive

$\Sigma H = 0$

$F_5 = F_4 \cos 30^\circ$

$= 1.334 \cos 30^\circ = 1.15\,kN$ Tensile

Joint D ;



$F_3 \sin 60^\circ = 1$

$F_3 = 1 / \sin 60^\circ = 1.1547\,kN$

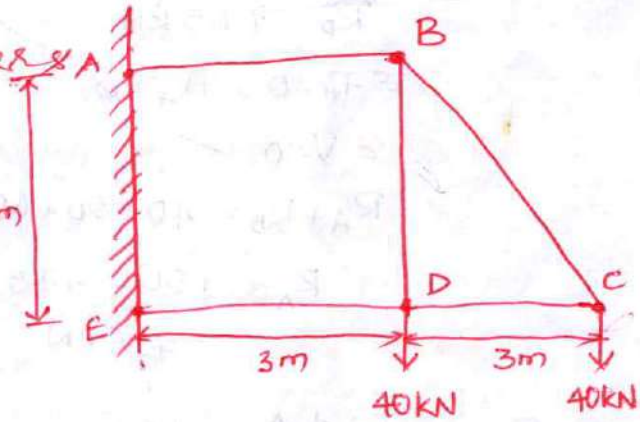
Hence the forces in the members are

$F_1 = 0.66\,kN$ C $\quad F_4 = 1.334\,kN$ C

$F_2 = 0.576\,kN$ T $\quad F_5 = 1.15\,kN$ T

$F_3 = 1.154\,kN$ T.

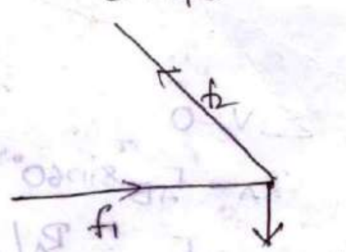
Find the forces in all the members



Suppor Inclination of inclined member $\tan\theta = 3/3$

$\theta = 45^\circ$

Joint C ;



$\Sigma V = 0$

$F_2 \sin 45^\circ - 40 = 0$

$F_2 = \dfrac{40}{\sin 45^\circ} = 56.57\,kN$

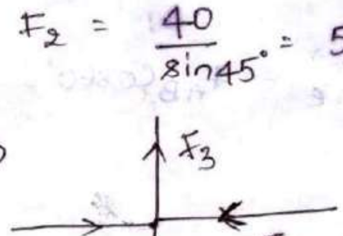
$\Sigma H = 0$

$F_2 \cos 45^\circ - F_1 = 0$

$F_1 = F_2 \cos 45^\circ$

$F_1 = 56.57 \cos 45^\circ$
$= 40$

Joint D



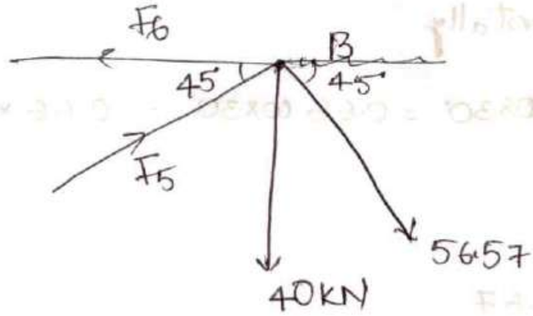
$\Sigma V = 0$

$F_3 = 40\,kN$

$\Sigma H = 0$

$F_4 = 40\,kN$

int B



$\Sigma H = 0$

$F_6 - F_5 \cos 45°$

$- 56.57 \cos 45$

113.14 kN

$F_6 = F_5 \cos 45° + 56.57 \cos 45°$

$= 120$ kN Tensile

$\Sigma V = 0$

$56.57 \sin 45° - F_5 \sin 45°$

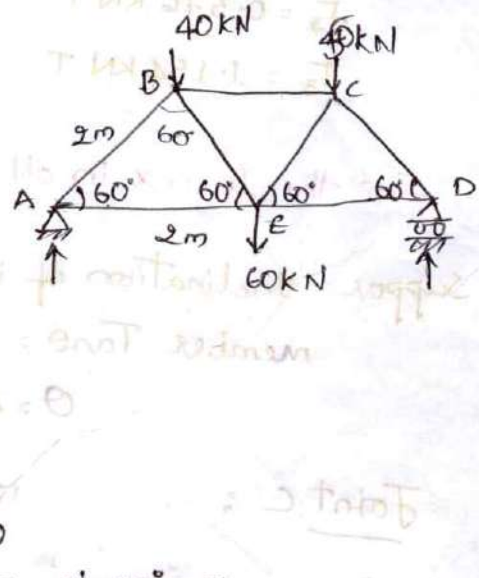
$40 + 56.57 \sin 45° = F_5 \sin 45°$

$F_5 = 113.14$ KN Compression

Determine the forces in all the members of the truss and indicates the magnitude and nature of the forces on the diagram of the truss. All inclined members are at 60° to horizontal and lengths of each member is 2m

line of action of load

$\Sigma M_A = 0 \quad AB \cos 60° = 1m$



$R_D \times 4 - 40 \times 1 - 50(2+1) - 60 \times 2 = 0$

$R_D = 77.5$ KN

$\Sigma F_H = 0 \quad H_A = 0$

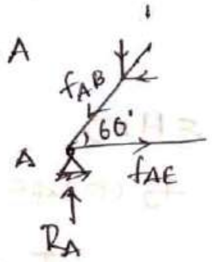
$\Sigma V = 0$

$R_A + R_B = 40 + 50 + 60 = 130$ KN

$R_A = 150 - 77.5$

$= 72.5$ kN

Joint-A



$\Sigma V = 0$

$R_A - F_{AB} \sin 60° = 0$

$F_{AB} = R_A / \sin 60° = \frac{72.5}{\sin 60°} = 83.7$ KN

$\Sigma M = 0$

$F_{AE} - F_{AB} \cos 60° = 0 \quad F_{AE} = F_{AB} \cos 60° = 83.7 \cos 60°$

$F_{AE} = 41.85$ kN

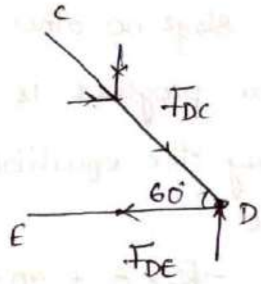
Joint D

$\Sigma V = 0$

$R_D - F_{DC} \sin 60° = 0$

$F_{DC} = R_D / \sin 60° = \frac{77.5}{\sin 60°}$

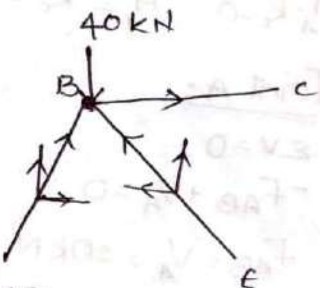
$F_{DC} = 89.5 \text{ kN } (C)$



$\Sigma H = 0$

$F_{DE} - F_{DC} \cos 60° = 0 \qquad F_{DE} = 89.5 \cos 60° = 44.8 \text{ kN } (T)$

Joint B



$\Sigma V = 0$

$F_{AB} \sin 60° - 40 \quad F_{BE} \sin 60° = 0$

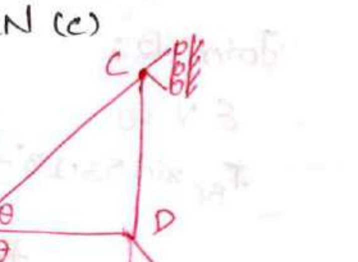
$F_{BE} = \frac{F_{AB} \sin 60° + 40}{\sin 60°} = \frac{83.7 \sin 60° + 40}{\sin 60°} = 129.9 \text{ kN } (C)$

$\Sigma H = 0$

$F_{AB} \cos 60° + F_{BC} - F_{BE} \cos 60° = 0$ | $F_{BC} - 23.1 = 0$

$83.7 \cos 60° + F_{BC} - 129.9 \cos 60° = 0$ | $F_{BC} = 23.1 \text{ kN } (T)$

Joint C



$\Sigma V = 0$

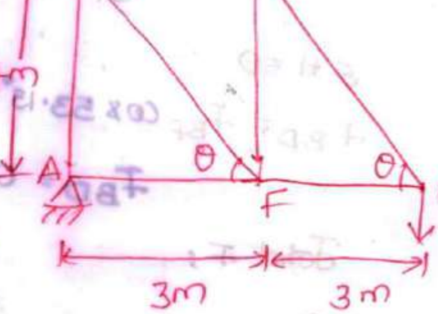
$50 - F_{CE} \sin 60° - F_{CD} \sin 60° = 0$

$50 - F_{CE} \sin 60° - 89.5 \sin 60° = 0$

$F_{CE} \sin 60° = 27.5$

$F_{CE} = 27.5 / \sin 60° = 31.76 \text{ kN } (C)$

Analyse the truss shown in fig

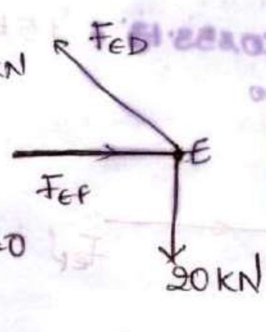


$\tan\theta = 4/3 = 53.13°$

As soon as a joint is analysed the forces on the joints are marked as members

Joint E $\quad \Sigma V = 0$

$F_{ED} = \frac{20}{\sin 53.13} = 25 \text{ kN}$



$\Sigma F_H = 0$

$F_{EF} - F_{ED} \cos\theta = 0$

$F_{EF} = 25 \cos 53^\circ 13' = 15\,kN$

At this stage no other joint is having only two unknown forces. Hence no further progress is possible. Let us find the reactions at supports. Considering the equilibrium of entire truss let the reactions be as shown

$\Sigma M_A = 0$

$-R_C \times 8 + 20 \times 6 = 0 \qquad \therefore R_C = 15\,kN$

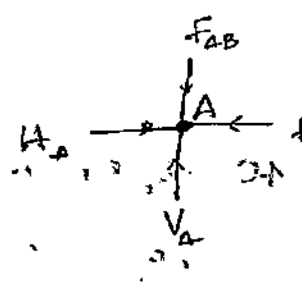
$\Sigma V = 0$

$V_A - 20 = 0 \qquad \therefore V_A = 20\,kN$

$\Sigma H = 0$

$H_A - R_C = 0 \qquad H_A = R_C = 15$

Joint A:



$\Sigma V = 0$

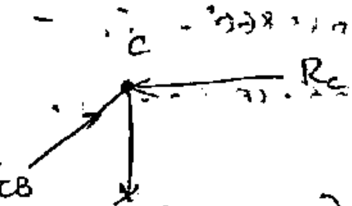
$-F_{AB} + V_A = 0$

$F_{AB} = V_A = 20\,kN$

$\Sigma H = 0$

$H_A - F_{AF} = 0 \qquad F_{AF} = H_A = 15\,kN$

Joint C;



$\Sigma H = 0$

$F_{CB} \cos\theta - R_C = 0$

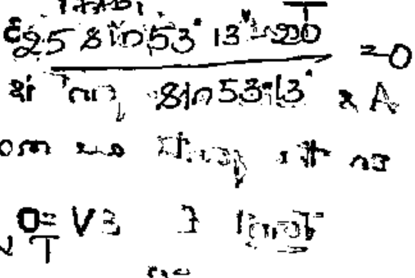
$F_{CB} = \frac{R_C}{\cos\theta} = \frac{15}{\cos 53^\circ 13'} = 25\,kN$ (Compression)

$\Sigma V = 0$

$F_{CB} \sin 53^\circ 13' - F_{CD} = 0$

$F_{CD} = 25 \sin 53^\circ 13' = 20\,kN$ Tension

Joint B;



$\Sigma V = 0$

$F_{BF} \sin 53^\circ 13' - F_{BC} \sin 53^\circ 13' + F_{AB} = 0$

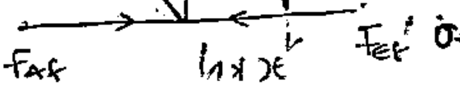
$F_{BF} = \frac{F_{BC} \sin 53^\circ 13' - F_{AB}}{\sin 53^\circ 13'} = \frac{25 \sin 53^\circ 13' - 20}{\sin 53^\circ 13'} = 0$

$\Sigma H = 0$

$F_{BD} - F_{BF} \cos 53^\circ 13' - F_{BC} \cos 53^\circ 13' = 0$

$F_{BD} = 0 + 25 \cos 53^\circ 13' = 15\,kN$

Joint F;

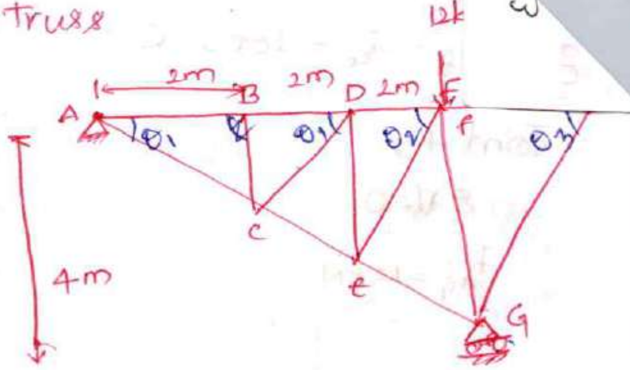


$\Sigma V = 0$

$F_{FD} = 0$

$F_{BF} = 0$

Exam find forces in all members of the truss



$\tan\theta_1 = 4/6 = 33.69°$

$\theta_2 = \tan^{-1}(8/3) \times 1/2 = 53.13°$

$\theta_3 = \tan^{-1} 4/3 = 53.13°$

Joint H;

$\Sigma V=0$

$F_{HG}\sin\theta_3 = 20 \quad F_{HG} = 25\,kN\ C$

$\Sigma H=0$

$F_{HF} - F_{HG}\cos\theta_3 = 0$

$F_{HF} = 25\cos 53.13° = 15\,kN\ T$

Now reactions at supports are to be found

$\Sigma M_A = 0$

$-R_G \times 6 + 20\times 9 + 12\times 6 = 0$

$R_G = 42\,kN$

$\Sigma Y=0 \quad V_A + R_G - 12 - 20 = 0$

$V_A = 32 - 42 = -10\,kN$

$\Sigma H = 0 \rightarrow H_A = 0$

Joint A

$F_{AC}\sin\theta_1 - 10 = 0$

$F_{AC} = \frac{10}{\sin 33.69} = 18.03\,kN$

$\Sigma F_H = 0$

$F_{AB} - F_{AC}\cos\theta_1 = 0$

Joint B $\quad F_{AB} = 18.03\cos 33.69° = 15\,kN$

$\Sigma V = 0 \quad F_{BC} = 0$

$\Sigma H = 0 \quad F_{CD} = F_{BA} = 15\,kN$

Joint C

$\Sigma Y=0 \quad F_{CD}=0 \quad F_{BC}=0$

$\Sigma H=0$

$F_{CE} = F_{AC} = 18.03\,kN$

Joint D;

$\Sigma F_V = 0 \quad F_{DE} = 0$

$\Sigma H = 0 \quad F_{DF} = F_{BD} = 15\,kN$

Joint E

$\Sigma V = 0 \quad F_{EF} = 0$

$\Sigma H = 0$

$F_{EG} = F_{CE} = 18.03$ C

Joint F:

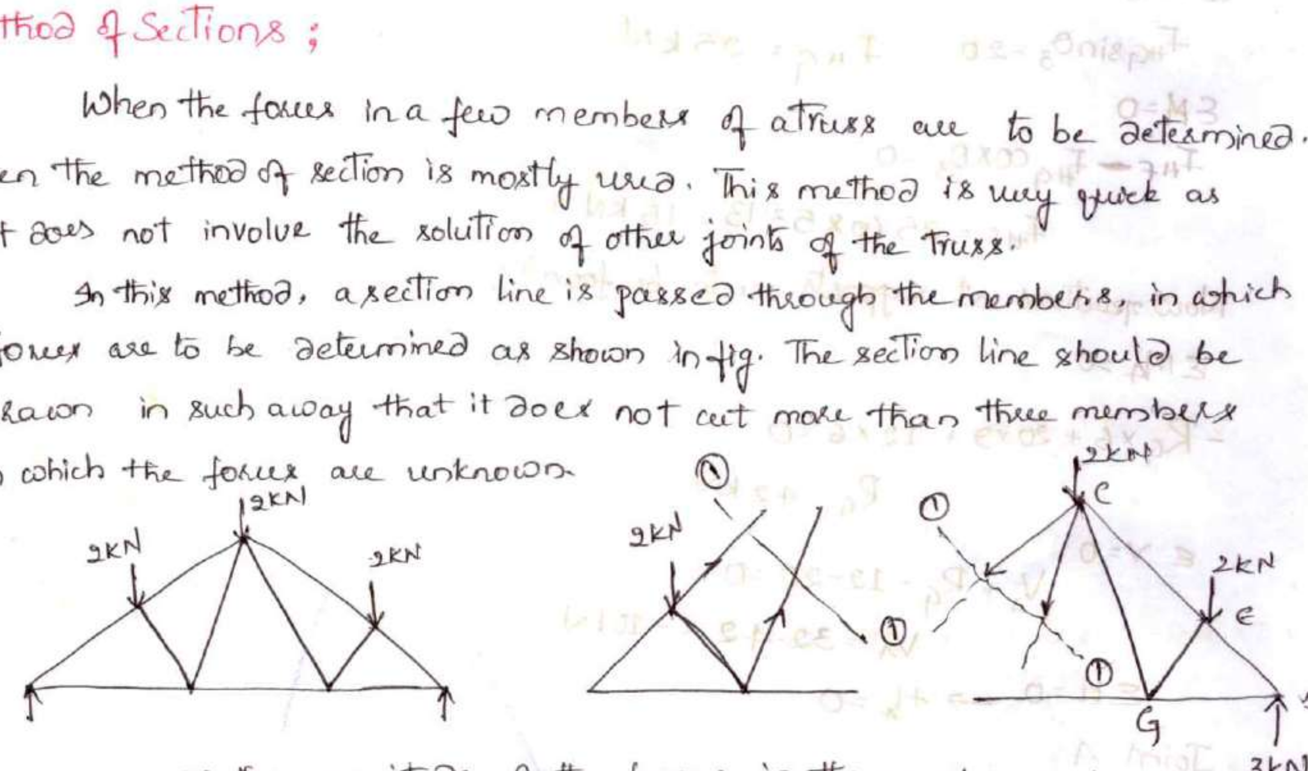
$\Sigma H = 0$

$F_{AG} = 12$ kN

Method of Sections;

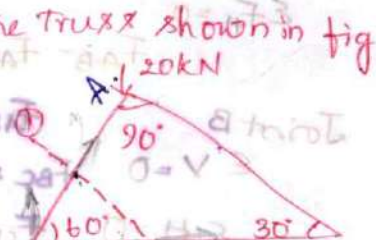
When the forces in a few members of a truss are to be determined. Then the method of section is mostly used. This method is very quick as it does not involve the solution of other joints of the truss.

In this method, a section line is passed through the members, in which forces are to be determined as shown in fig. The section line should be drawn in such a way that it does not cut more than three members in which the forces are unknown.



If the magnitude of the forces in the members cut by a section line, is positive then the assumed direction is correct. If magnitude of a force is -ve, then reverse the direction of that force.

Find the forces in the members AB and AC of the truss shown in fig.



Determining the reactions R_B and R_C

Distance of line of action $= AB\cos 60°$

$2.5 \times 1/2 = 1.25$ m

$\Sigma M_B = 0$

$R_C \times 5 = 20 \times 1.25 \quad R_C = \frac{20 \times 1.25}{5} = 5$ kN

$R_B = 20 - 5 = 15$ kN

Now draw a section ①-① cutting the members AB and BC in which forces are to be determined. Now consider the equilibrium of the left part as shown

let the direction of F_{BA} and F_{BC} are assumed as

Now taking moments of all the forces acting on left part about point C

$15 \times 5 + (F_{BA} \times AC) = 0$

$75 + F_{BA} \times 5 \times \cos 30° = 0$

$F_{BA} = \frac{-75}{5\cos 30°} = -17.32\,kN$

(17.32) compressive

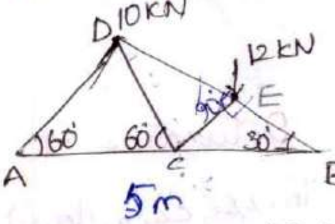
Again taking the moments of all forces acting on the left part about point A we get 15 × ⊥r distance between the line of action of 15 kN and point C

$F_{BC} \times$ ⊥r distance b/n F_{BC} and point A

$15 \times 2.5 \cos 60° = F_{BC}\, 2.5 \sin 60°$

$F_{BC} = 8.66\,kN$ (Te)

A Truss of span 5m loaded as shown in fig. find the reactions and forces in the members marked 4, 5 and 7 using method of sections



Support Reactions

$ADB = 90°$

$AD = AB \cos 60° = 5 \times 0.5 = 2.5\,m$

Distance of line of action of vertical load 10 KN from point A will be $AD \cos 60°$

$2.5 \times 0.5 = 1.25\,m$

from Triangle ACD, we have

$AC = AD = 2.5\,m$

$BC = 5 - 2.5 = 2.5$

In right angled Triangle CED, we have

$BE = BC \cos 30° = 2.5\,\sqrt{3}/2$

Distance of line of action of vertical load 12kN from point B will be BE

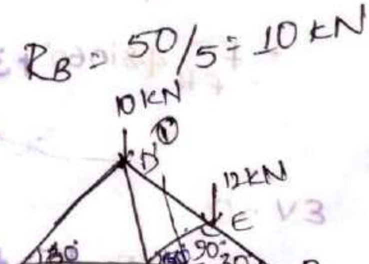
$BE \cos 30°$ $BE\,\sqrt{3}/2 = (2.5\sqrt{3}/2)\,\sqrt{3}/2 = 1.875\,m$

$5 - 1.875 = 3.125\,m$

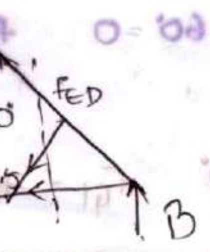
Taking moments @A, we get

$R_B \times 5 = 10 \times 1.25 + 12 \times 3.125 = 50$ $R_B = 50/5 = 10\,kN$

$R_A = (10+12) - 10 = 12\,kN$



$\Sigma M_E = 0$



$R_B \times BE\cos 30° = F_4\, BE \sin 30°$

$10 \times 2.5\sqrt{3}/2 \times \sqrt{3}/2 = F_4 \times 2.5 \times \sqrt{3}/2 \times 0.5$

$F_4 = 10 \times \sqrt{3}/2 \times 1/0.5 = 17.32$

Taking moments @ B $\Sigma M_B = 0$

$12\, BE \cos 30^\circ + F_5 \times BE = 0$

$12 \cos 30^\circ + F_5 = 0$

$F_5 = -12 \cos 30^\circ = -10.32\, kN$

Now taking moments @ point C all the forces acting on right part

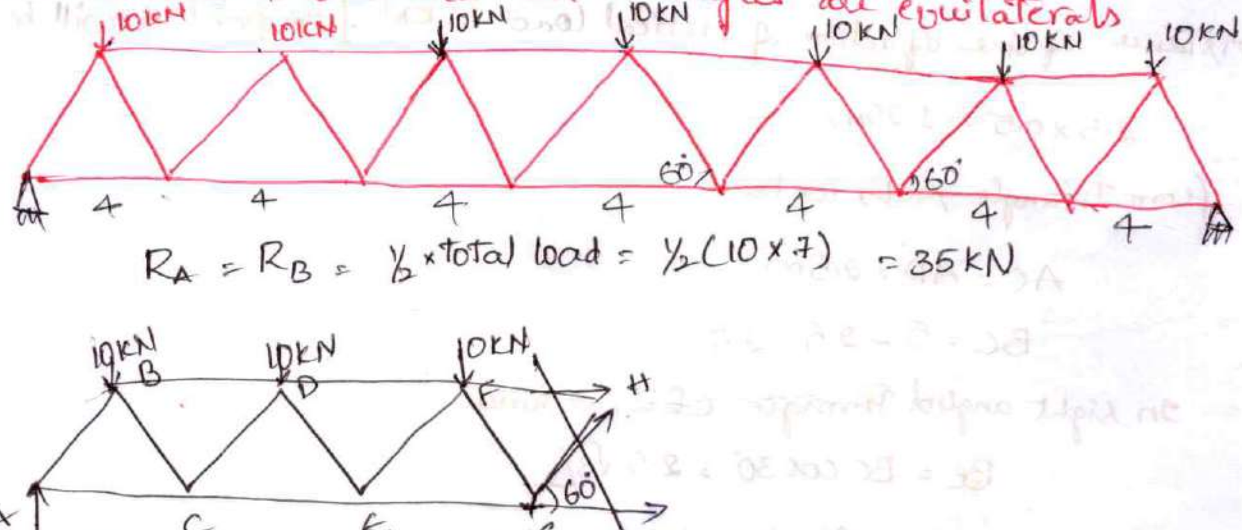
$12 \times (2.5 - BE \cos 30^\circ) = F_{ED} \times CE + R_B \times BC$

$12(2.5 - 2.5\sqrt{3}/2 \times \sqrt{3}/2] = F_{ED}\, 2.5 \sin 30^\circ + 10 \times 2.5$

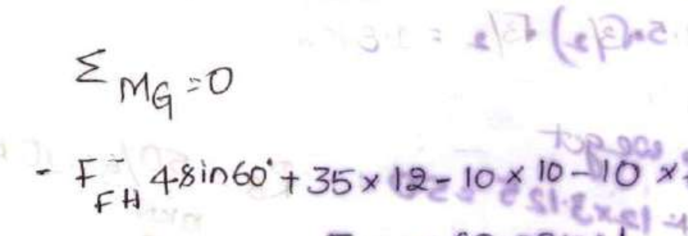
$12 \times (2.5 - 1.875) = F_{ED}\, 1.25 + 25$

$F_{ED} = \frac{7.5 - 25}{1.25} = -14\, kN$

Determine the forces in all the members FH, HG and GI in the Truss. Each load is 10 kN and all Triangles are equilaterals



$R_A = R_B = \frac{1}{2} \times \text{total load} = \frac{1}{2}(10 \times 7) = 35\, kN$



$\Sigma M_G = 0$

$-F_{FH}\, 4 \sin 60^\circ + 35 \times 12 - 10 \times 10 - 10 \times 6 - 10 \times 2 = 0$

$F_{FH} = 69.28\, kN$

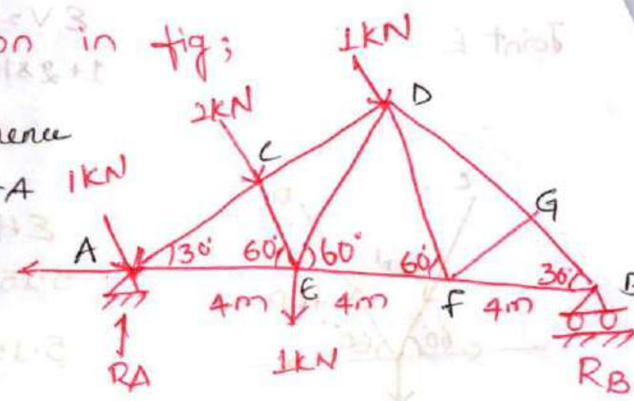
$\Sigma V = 0$

$35 - 10 - 10 - 10 - F_{GH} \sin 60^\circ = 0 \qquad F_{GH} = 5.77\, kN$

$\Sigma H = 0$

$F_{GI} - F_{FH} - F_{GH} \cos 60^\circ \qquad F_{GI} = 72.17\, kN$

Determine the forces in Truss as shown in fig;



Truss is supported on roller at B and hence R_B will be vertical. The Truss hinged at A hence the support reaction at A consists

H_A R_A

Now length

$AC = 4\cos 30° = 4 \times 0.86 = 3.46$

$AD = 2 \times AC = 2 \times 3.464 = 6.928$

Now taking moments @ A

$R_B \times 12 = 2 \times AC + 1 \times AD + 1 \times AE$

$R_B = \frac{17.86}{12}$ KN

$R_B = 1.49$ KN

Total vertical components of inclined loads $= (1+2+1)\sin 60° + 1$
$= 4.464$ KN

Horizontal components of inclined loads

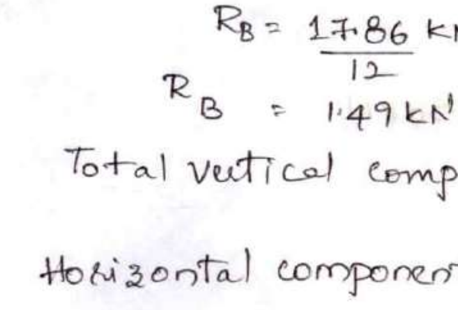
$(1+2+1)\cos 60° = 4 \times 0.5 = 2$ KN

$R_A =$ Vertical components $- R_B$

$= 4.464 - 1.49 = 2.974$

$H_A = 2$ KN

Joint A



$\Sigma V = 0$

$F_{AC} \sin 30° + 1 \sin 60° = 2.974$

$F_{AC} = 4.216$ KN C

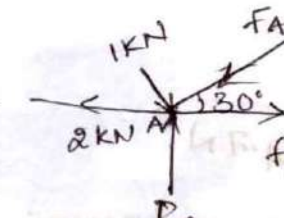
$\Sigma H = 0$

$F_{AE} = 2 + F_{AC} \cos 30° - 1 \cos 60°$

$= 2 + 4.216 \times 0.866 - 0.5$

$= 5.15$ KN T

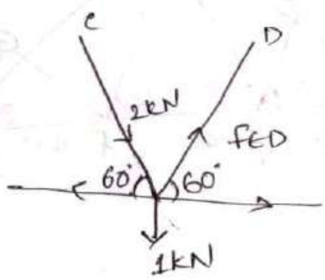
Joint C



$F_{CD} = F_{AC} = 4.216$

$F_{CE} = 2$ KN

Joint E



$\Sigma V = 0$

$1 + 2\sin 60° = F_{ED}\sin 60°$

$F_{ED} = 2 + \frac{1}{\sin 60} = 3.155\ T$

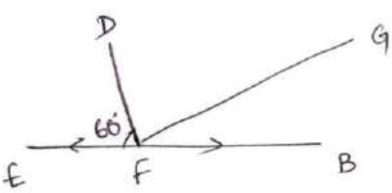
$\Sigma H = 0$

$5.15 - 2\cos 60° - F_{ED}\cos 60° - F_{EF} = 0$

$5.15 - 2\times 1/2 - 3.15\ 1/2 - F_{EF} = 0$

$F_{EF} = 2.58\,kN$

Join F



$\Sigma V = 0$

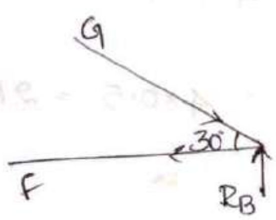
$F_{DF}\sin 60° = 0$

$F_{DF} = 0$

$\Sigma H = 0$

$F_{FB} = F_{EF} = 2.58\,kN$

Joint B



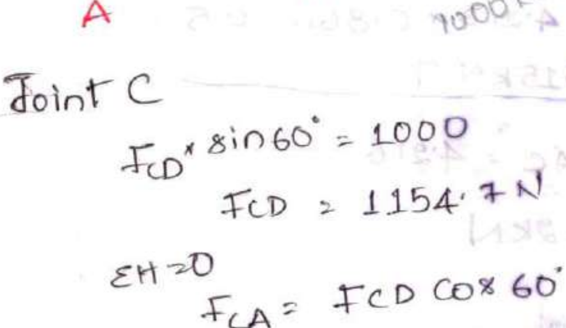
$\Sigma Y = 0$

$F_{BG}\sin 30° = 1.49$

$F_{BG} = 2.98\,kN$

Joint G

$F_{GD} = F_{BG} = 2.98\,kN$



Joint D

$F_{CD} = 1154.7\,N$

$F_{AD}\cos 30° = 1154.7\cos 30°$

$F_{AD} = 1154.7\,N$

Joint C

$F_{CD} \times \sin 60° = 1000$

$F_{CD} = 1154.7\,N$

$\Sigma H = 0$

$F_{CA} = F_{CD}\cos 60°$

$= 1154.7\cos 60°$

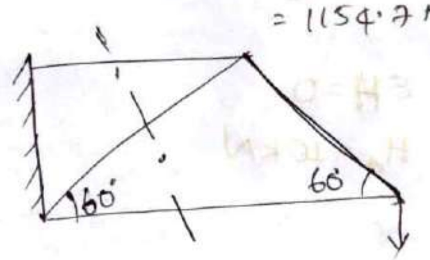
$= 577.35\,N$

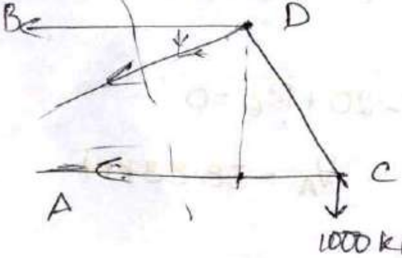
$\Sigma H = 0$

$F_{BD} = F_{AD} \sin 30^\circ + F_{DC} \sin 30^\circ$

$= 1154.7 \times 0.5 + 1154.7 \times 0.5$

$= 1154.7\,N\,(T)$





~~$\Sigma M_C = 0$~~

$\Sigma M_D = 0$

$1000[(AC \cos 60^\circ) \times \cos 60^\circ] + F_{AC}((AC \cos 60^\circ) \sin 60^\circ) = 0$

$1000 \times (4 \times 1/2)(1/2) + F_{AC}(2 \sin 60^\circ) = 0$

$F_{AC} = 577.35$

~~$\Sigma H = 0$~~ $\Sigma M_C = 0$

$F_{BD}(AC \cos 60^\circ) \sin 60^\circ + (F_{DA} \sin 60^\circ) \cdot AC \cos 60^\circ \times \cos 60^\circ = 0$

$\Sigma V = 0$

$1000 + F_{DA} \sin 60^\circ = 0$

$F_{DA} = 1154.7\,kN$

~~$F_{BD}(2 \sin 60^\circ) + F_{DA}\sqrt{3}/2 \cdot \ldots = 0$~~

~~1499~~

~~$F_{BD} = -866$~~

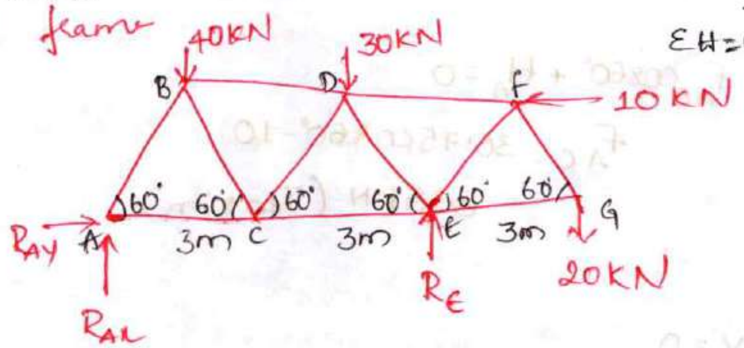
$\Sigma H = 0$

$F_{AC} + F_{BD} + F_{DA} \cos 60^\circ = 0$

$-577.35 + F_{BD} + -1154.7 \cos 60^\circ$

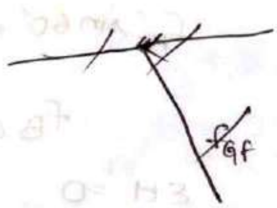
$F_{BD} = +1154.7\,kN$

Analyse the portal frame



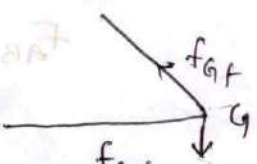
Joint G;

$\Sigma f_V = 0$



$F_{GF} \sin 60^\circ = 20$

$f_{GF} = \frac{20}{\sin 60^\circ} = 23.1$ (Tension)



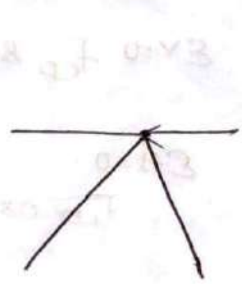
$\Sigma H = 0$

$f_{GE} - f_{GF} \cos 60^\circ = 0$

$f_{GE} = 23.1 \cos 60^\circ = 11.55\,kN$ (compression)

Joint F

$\Sigma f_V = 0$



$f_{EF} \sin 60^\circ - F_{GF} \sin 60^\circ = 0$

$F_{FE} = F_{GF} = 23.1\,kN$ (compression)

$\Sigma H = 0$

$F_{FD} + 10 - F_{GF} \cos 60^\circ - F_{FE} \cos 60^\circ = 0$

$F_{FD} = 13.1\,kN$ (Tension)

Support Reactions $\Sigma M_A = 0$

$R_E \times 6 - 20 \times 9 - 30 \times (3 + 3\cos 60°) - 10 \times 3\sin 60° - 40 \times 3\sin 60° = 0$

$R_E = 58.17\,kN$

$\Sigma V = 0$

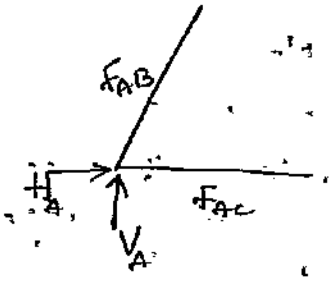
$V_A - 40 - 30 - 20 + R_E = 0$

$V_A = 38.83\,kN$

$\Sigma H = 0$

$H_A = 10\,kN$

Joint A



$\Sigma V = 0$ $F_{AB}\sin 60° - 31.83 = 0$

$F_{AB} = 36.75\,kN$

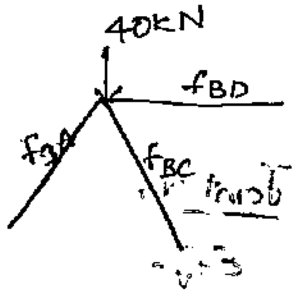
$\Sigma H = 0$

$F_{AC} - F_{AB}\cos 60° + H_A = 0$

$F_{AC} = 36.75\cos 60° - 10$

$= 8.38\,kN$ (Tension)

Joint B



$\Sigma V = 0$

$-F_{BC}\sin 60° + F_{AB}\sin 60° - 40 = 0$

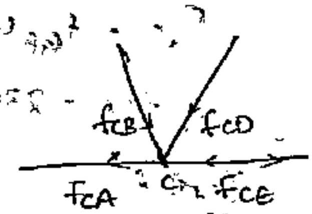
$F_{BC} = 9.44\,kN\ (C)$

$\Sigma H = 0$

$F_{AB}\cos 60° - F_{BC}\cos 60° - F_{BD} = 0$

$F_{BD} = 36.75\cos 60° - 9.44\cos 60° = 13.66\ (C)$

Joint C



$\Sigma V = 0$ $F_{CD}\sin 60° - F_{BC}\sin 60° = 0$

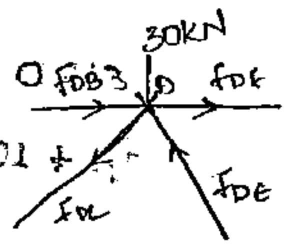
$F_{CD} = F_{BC} = 9.44\,kN$

$\Sigma H = 0$

$F_{CD}\cos 60° + F_{BC}\cos 60° - F_{AC} - F_{CE} = 0$

$F_{CE} = 1.06\,kN\ (C)$

Joint D;

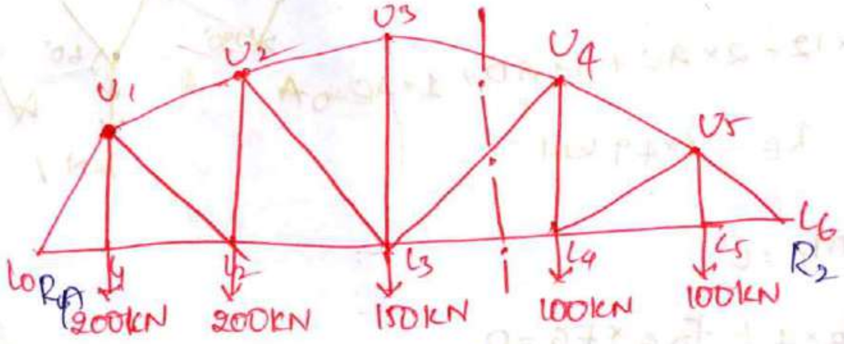


$\Sigma V = 0$ $F_{DE}\sin 60° - F_{CD}\sin 60° - 30 = 0$

$F_{DE} = 44.08\,kN\ (C)$

Sections problems

Find the magnitude and nature of force in the members U_3U_4 L_3L_4 and U_3L_4 of the loaded truss.



$6\times6=36$

$\Sigma M_{L_0}=0$

$200\times6+200\times12+150\times18+100\times24+100\times30-R_2\times36=0$

$R_2=325$

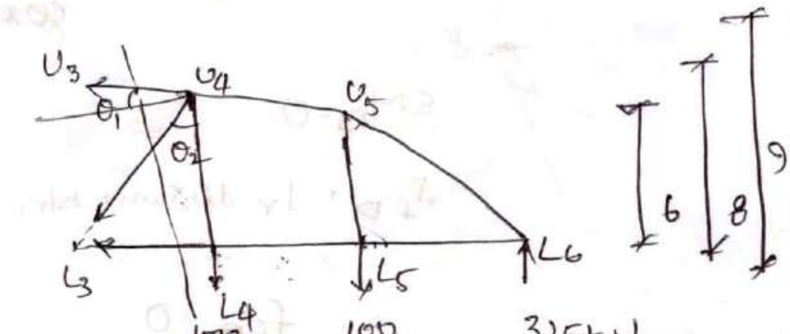
$\Sigma V=0$

$R_1+R_2-200-200-150-100-100=0$ $R_2=325$ kN

$R_1=425$ kN

$\theta_2=\tan^{-1}6/8 = 36.87°$

$\theta_1=\tan^{-1}1/6 = 9.46°$



$\Sigma M_{U_4}=0$

F_{L_3}

$325\times12-100\times6-F_{L_3}\times8=0$

$F_{L_3L_4}=412.5$ kN

$\Sigma M_{L_3}=0$

$F_{U_3U_4}\cos\theta_1\times8+F_{U_3U_4}\sin\theta_1\times6+100\times6+100\times12-325\times18$

$\Sigma M_{L3}=0$ → $F_{U_3U_4}=456.2$ kN (C)

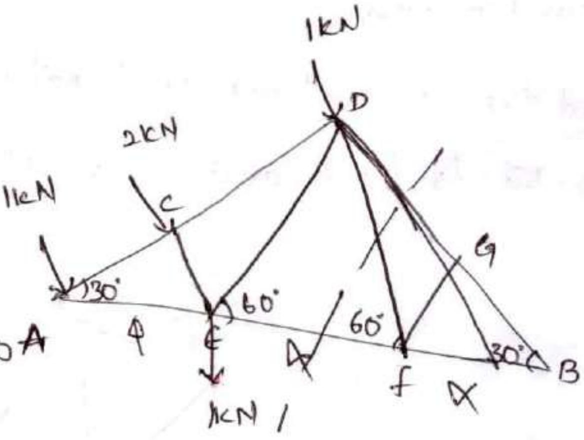
$\Sigma H=0$

$F_{U_3U_4}\cos\theta_1-F_{3}L_4-F_{U_4L_3}\sin\theta_2=0$

$F_{U_4L_3}=62.49$ kN (T)

$AC = AE \times \cos 30° = 3.464$

$AD = 2 \times AC = 2 \times 3.464$
$= 6.928$



$R_B \times 12 - 2 \times AC + 1 \times AD + 1 \times AE = 0$

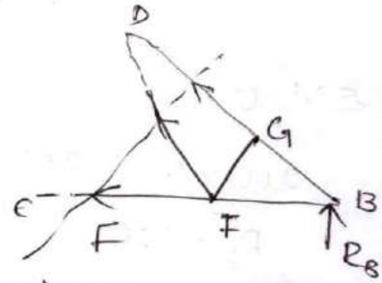
$R_B = 1.49$ kN

$\Sigma M_f = 0$

$R_B \times 4 + F_{DG} \times FG = 0$

$1.49 \times 4 + F_{DG}(4\sin 30°) = 0$

$F_{DG} = -2.98$ kN (com)



$\Sigma M_D = 0$

$F_G = 4\sin 30°$

$R_B \times BD\cos 30° - F_{FE}\ BD\sin 30°$

$F_{FE} = \frac{1.49\cos 30°}{\sin 30°} = \frac{1.49 \times 0.86}{0.5} = 2.58$ kN

$\Sigma M_B = 0$

$F_{FD} \times$ ⊥r distance b/n F_{FD} and B $= 0$

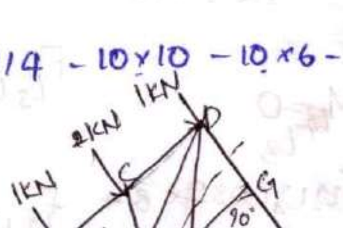
$F_{FD} = 0$

$R_0 \times 28 - 10 \times 26 - 10 \times 22 - 10 \times 18 - 10 \times 14 - 10 \times 10 - 10 \times 6 - 10 \times 2 = 0$

$R_0 = 35$

$AC = AE\cos 30° = 3.464$

$AD = 2 \times AC = 2 \times 3.46 = 6.928$ m



$\Sigma M_f = 0$

$R_B \times 4 + F_{GD} \times fG = 0$

$R_B \times 4 + F_{GD}\ 4\sin 30°$ $\quad F_{GD} = -2.98$

$\Sigma M_B = 0$

$\Sigma M_D = 0$

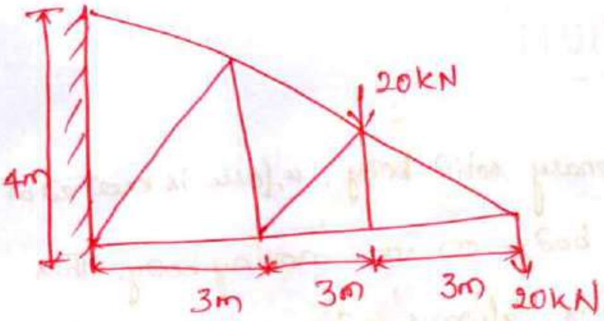
$-F_{FE} \times ED\sin 30° + R_B \times BD\cos 30° = 0$

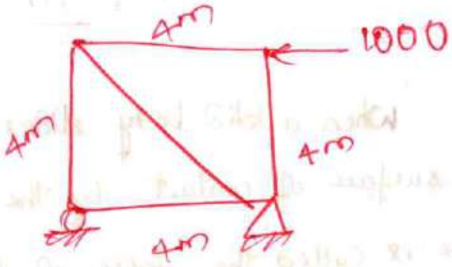
$F_{FE} = 2.58$ kN

$\Sigma V = 0$

$R_B + F_{FD}\sin 60° + F_{GD}\sin 30° = 0$

F_{FD}


20kN
4m
3m
3m
3m
20kN


4m
1000
4m
4m
4m

UNIT: III FRICTION

When a solid body slides over a stationary solid body, a force is exerted at the surface of contact by the stationary body on the moving body. This force is called the force of friction and is always acting in the direction opposite to the direction of motion. The property of the body by virtue of which a force is exerted by a stationary body to resist the motion of the moving body is called friction

Defination

Consider a solid body placed on a horizontal plane surface.

W = Weight of body acting through CG downward

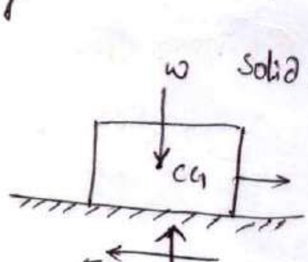
R = Normal reaction of body acting through CG upward

P = Force acting on the body through CG and parallel to horizontal surface

If P is small, the body will not move as the force of friction acting on the body in the stage comes, when the solid body is on the point of motion. At this stage, the force of friction acting on the body is called limiting state of friction. The limiting force of friction is denoted by F. Resolving the forces on the body vertically & horizontally

$R = W$

$F = P$.



If the magnitude of P is further increased the body will start moving. The force of friction acting on the body when the body is moving, is called kinetic friction.

Coefficient of friction;

Ratio of limiting force of friction to the normal reaction (R) between two bodies. It is denoted by the symbol μ.

$$\mu = \frac{\text{limiting force of friction}}{\text{Normal Reaction}} = \frac{F}{R} \qquad F = \mu R$$

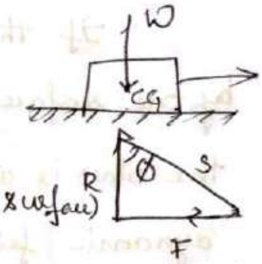
Angle of friction;

Angle made by the resultant of the normal reaction R and the limiting force of friction (F) with the normal reaction (R). It is denoted by ϕ. Solid body resting on horizontal plane.

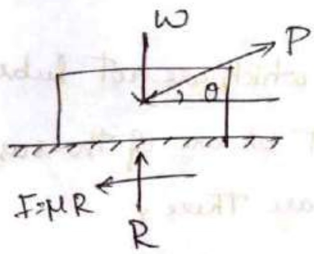
let S = Resultant of normal reaction R and limiting force of friction F

$$\tan\phi = \frac{F}{R} = \frac{\mu R}{R}$$

μ = coefficient of friction



A block of weight W is placed on a rough plane (horizontal surface) and a force P is applied at an angle θ with the horizontal such that block just tends to move



In this case the normal reaction R will not be equal to weight of body. The normal reaction is obtained by resolving the forces on the block horizontally and vertically. The force P is resolved in two components i.e., $P\cos\theta$ in the horizontal direction and $P\sin\theta$ in the vertical direction.

Resolving forces

$$F = P\cos\theta$$

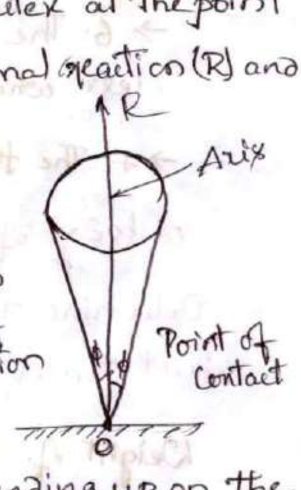
$$\mu R = P\cos\theta$$

Resolving forces on the block vertically, we get

$$R + P\sin\theta = W \;;\; R = W - P\sin\theta$$

Cone of friction;

It is defined as the right circular cone with vertex at the point of contact of the two bodies, axis in the direction of normal reaction (R) and semi vertical angle equal to angle of friction ϕ



O = Point of contact between two sides

R = Normal Reaction and also axis of the cone of friction

ϕ = Angle of friction

Types of friction;

The friction is divided in to following two types depending up on the nature of the two surfaces in contact.

Static friction

Dynamic friction

If the two surfaces, which are in contact, are at rest, the force experience by one surface is called static friction. But if one surface starts moving and the other is at rest, the force experienced by the moving surface is called dynamic friction. If between the two surfaces, no lubrication is used, the friction, that exist between two surface is called solid friction or Dry friction.

Laws of friction:

The friction that exists between two surfaces, which are not lubricated is called solid friction. The two surfaces may be at rest or one of the surface is moving and other surface is at rest. following are these:

→ 1 The force of friction is equal to the force applied to the surface, so long as the surface is at rest

→ 1. The force of friction acts in the opposite direction in which surface is having tendency to move.

→ 3. When the surface is on the point of motion, the force of friction is maximum and this maximum frictional force called limiting friction force.

→ 4. The limiting frictional force bears a constant ratio to the normal reaction between two surfaces.

→ 5. The limiting frictional force does not depend upon the shape and areas of the surfaces in contact.

→ 6. The ratio between limiting friction and normal reaction is slightly less when the two surfaces are in motion.

→ 7 The force of friction is independent of the velocity of sliding.

A body of weight 100N is applied on a rough horizontal plane Determine the coefficient of friction of a horizontal force of 60 N just causes the body to slide over the horizontal plane.

Weight of body $W = 100\,N$, Horizontal force applied $P = 60\,N$, Limiting force of friction $F = P = 60\,N$

Normal reaction of the body is given as $R = W = 100\,N$

$F = \mu R$

$\mu = F/R = 60/100 = 0.6$

~~A boa~~

The force required to pull a body of weight 50N on a rough horizontal plane is 15N. Determine the coefficient of friction if the force is applied at an angle of 15° with the horizontal.

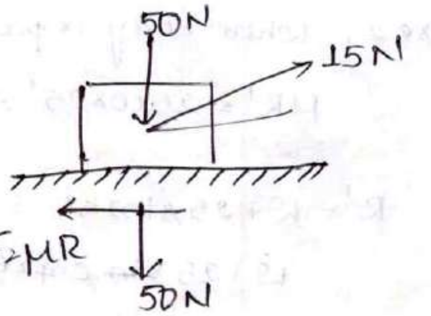
Given data

Weight of the body $W = 50N$

Force applied $P = 15N$

Angle made by force P $\theta = 15°$

Coefficient of friction $= \mu$



$F = \mu R$

$\Sigma H = 0$

$F = P\cos\theta$

$\mu R = 15\cos 15°$

$\Sigma V = 0$

$R + 15\sin 15° = 50$

$R = 46.12k$

$F = \mu R$

$15\cos 15° = \mu \times 46.12$

$\mu = 0.314N$

A Pull of 20N, inclined at 25° to the horizontal plane, is required just to move a body placed on a rough plane Horizontal. But the push required to move the body is 25N. If the push is inclined at 25° to the horizontal, find the weight of the body and coefficient of friction?

Given data

Pull required $P = 20N$

Inclination of pull $\theta = 25°$

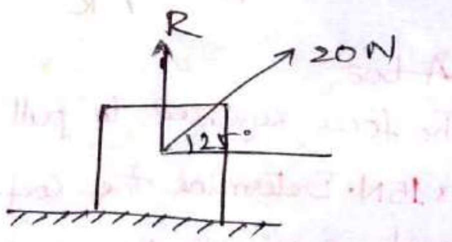
Push required $P' = 25N$

Inclination of push $\theta' = 25°$

Case 1; When body is pulled

$\mu R = 20\cos 25^\circ = 20 \times 0.9063$

$= 18.126$



$R + 20\sin 25^\circ = W$

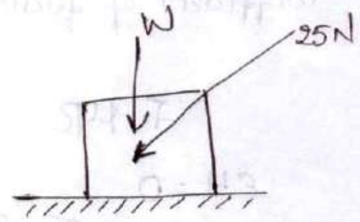
$R = W - 20\sin 25^\circ$

$= W - 8.452$

$\mu(W - 8.45) = 18.126 \longrightarrow ①$

Case 2; When body is pushed

$\mu R' = 25\cos 25^\circ = 25 \times 0.9063 = 22.657$



$R' = W + 25\sin 25^\circ$

$W + 25 \times 0.426 = W + 10.565$

$\mu(W + 10.565) = 22.657 \rightarrow ②$

Dividing ① ÷ ②

$$\frac{\mu(W - 8.45)}{\mu(W + 10.565)} = \frac{18.126}{22.657}$$

$22.657(W - 8.452) = 18.126(W + 10.565)$

$W = \frac{383}{4.53} = 84.547$

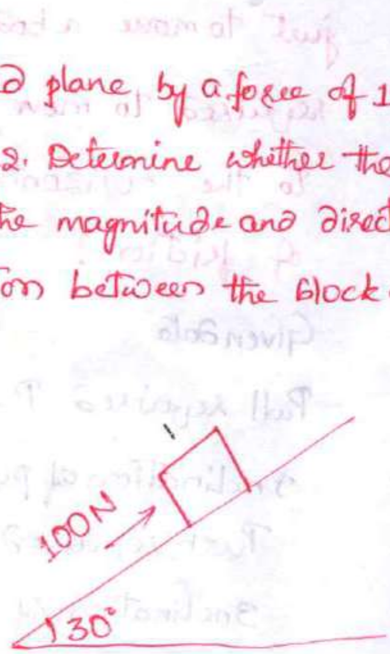
Substituting value of W in eq)

$\mu(84.547 - 8.452) = 18.126$

$\mu = 0.238$

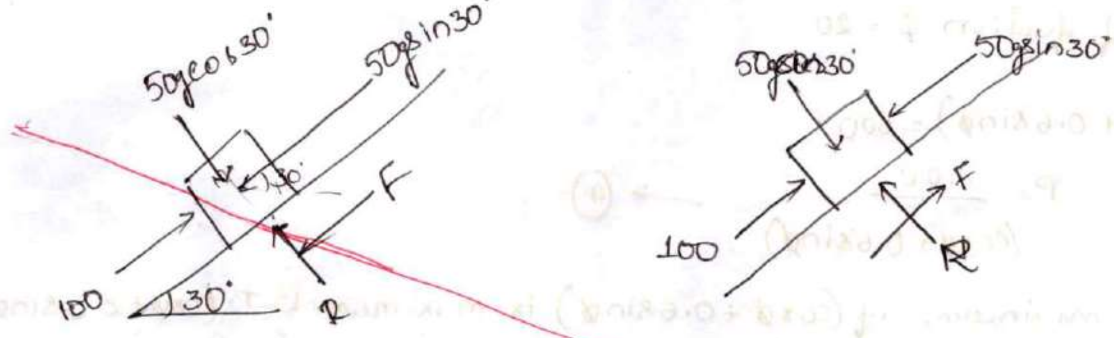
A block of 50 kg mass is pushed up an inclined plane, by a force of 100 N acting parallel to the incline as shown in fig 6.12. Determine whether the block shown in equilibrium or not. Also find the magnitude and direction of the frictional force. The coefficient of friction between the block and the plane are $\mu_s = 0.25$ and $\mu_k = 0.2$

$\Sigma V = 0$



$R - 50\cos 30^\circ = 0$

$R = 50 \times 9.81 \times \cos 30^\circ = 424.79$

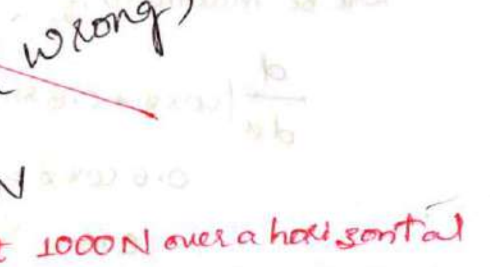


$F_{smax} = \mu_s R = 0.25 \times 424.79$

$\Sigma H = 0$

$100 - 50g\sin 30° - F = 0 \quad \therefore F = -145.25\,N$

A man wishing to slide a stone block of weight 1000N over a horizontal Concrete floor, ties a rope to the block and pulls it in a direction inclined upward at an angle of 20° to the horizontal. Calculate the min pull necessary to slide the block if the coefficient of friction $\mu = 0.6$. Calculate also the pull required if the inclination of the rope with the horizontal is equal to the angle of friction and prove that this is the least force required to slide the block



Weight $W = 1000N$

Angle with horizontal $\theta = 20°$

Coefficient of friction $\mu = 0.6$

Resolving forces horizontally

$$P\cos\theta = \mu R$$

$$P\cos 20° = 0.6\,R$$

Resolving force vertically $R + P\sin\theta = W$

$$R + P\sin 20° = 1000$$

$$R = 1000 - P\sin 20°$$

Substituting the value of R

$$P\cos 20° = 0.6(1000 - P\sin 20°)$$

$$= 600 - 0.6P\sin 20°$$

$$P\cos 20° + 0.6P\sin 20° = 600$$

$$P[\cos 20° + 0.6\sin 20°] = 600$$

$$P = 524N$$

Pull required if the inclination of the rope with horizontal is equal to angle of friction.

Angle of friction $\phi = 20^\circ$

$$P(\cos\phi + 0.6\sin\phi) = 600$$

$$P = \frac{600}{(\cos\phi + 0.6\sin\phi)} \longrightarrow (4)$$

P will be minimum, if $(\cos\phi + 0.6\sin\phi)$ is maximum. But $(\cos\phi + 0.6\sin\phi)$ will be maximum if

$$\frac{d}{d\phi}(\cos\phi + 0.6\sin\phi) = 0 \qquad -\sin\phi + 0.6\cos\phi = 0$$

$$0.6\cos\phi = \sin\phi$$

$$0.6 = \sin\phi / \cos\phi = \tan\phi$$

$$\tan\phi = \mu = 0.6$$

$$\tan\phi = 0.6$$

$$\phi = \tan^{-1} 0.6 = 30.96^\circ$$

Substituting this value of ϕ in eq (4)

$$P = \frac{600}{(\cos 30.96^\circ + 0.6\sin 30.96^\circ)} = 514.5\ N$$

Angle of Repose ;

Angle of repose is defined as the maximum inclination of a plane at which a body remains in equilibrium over the inclined plane by the assistance of friction only.

Consider a body of weight W, resting on a rough inclined plane

R = Normal Reaction

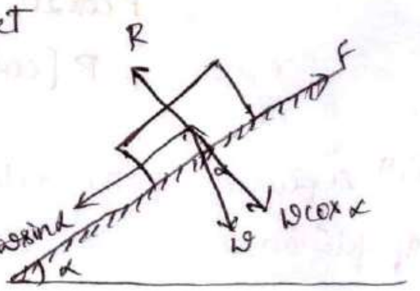
α = Inclination of plane

F = Frictional force acting upward along the plane

Let the angle of inclination be gradually increased, till the body just starts sliding down the plane. This angle of inclined plane, at which a body just begins to slide down the plane is called angle of repose.

Resolving the forces along the plane, we get

$$W\sin\alpha = F$$



$$\Sigma V = 0$$

$$W\cos\alpha = R$$

(1) $\div$ (2)

$$\frac{W\sin\alpha}{W\cos\alpha} = \frac{F}{R}$$

$\tan\alpha = F$

$\tan\phi = F/R$

ϕ = Angle of friction

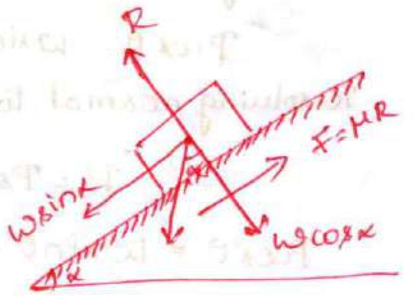
Hence from equations (iii) & (iv) we have

$\tan\alpha = \tan\phi$

$\alpha = \phi$

$\therefore$ Angle of repose = Angle of friction

Prove that the angle of friction (ϕ) is equal to the angle made by an inclined plane with the horizontal when a solid body, placed on the inclined plane is about to slide down.



$\Sigma H = 0$

$W\sin\alpha = F = \mu R$

$W\cos\alpha = R$

$$\frac{W\sin\alpha}{W\cos\alpha} = \frac{\mu R}{R} = \mu$$

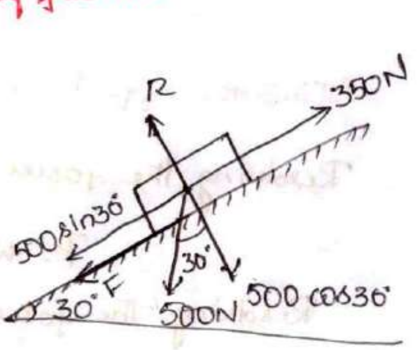
$\tan\alpha = \mu$

ϕ = Angle of friction

$\tan\alpha = \tan\phi = \mu$

$\alpha = \phi$

A body of weight 500N is pulled up an inclined plane, by a force of 350N. The inclination of the plane is 30° to the horizontal and the force is applied parallel to the plane. Determine coefficient of friction.



$W = 500N$

$P = 350N$

$\alpha = 30°$

The body is in equilib

Resolving the forces ~~along~~ Normal to the plane,

$R = 500\cos30° = 500 \times 0.866 = 433N$

Along the plane

$500\sin30° + F = 350$

$500\sin30° + \mu R = 350$

$500\sin30° + \mu \times 433 = 350$

$\mu = 0.23$

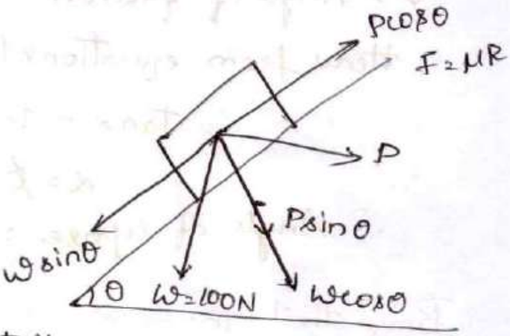
A rough inclined plane, coefficient of friction = μ, rises 1cm for every 5cm of its length. Calculate the effort required to drag a body weighing 100N up the plane

> the effort is applied horizontally and

en the effort is applied parallel to the plane.

Coefficient of friction = μ

Rise of plane is 1cm for every 5cm of its length



$\sin\theta = 1/5 = 0.2$

$\theta = \sin^{-1} 0.2 = 11.53^\circ$

$W = 100N$

(i) Find the effort when it is applied horizontally

The body is in equilibrium under the action of forces

Resolving the forces along the plane,

$$P\cos\theta = W\sin\theta + \mu R$$

Resolving normal to the plane

$$R = P\sin\theta + W\cos\theta$$

$$P\cos\theta = W\sin\theta + \mu(P\sin\theta + W\cos\theta)$$

$$= W\sin\theta + \mu P\sin\theta + \mu W\cos\theta$$

$$P\cos\theta - \mu P\sin\theta = W\sin\theta + \mu W\cos\theta$$

$$P(\cos\theta - \mu\sin\theta) = W(\sin\theta + \mu\cos\theta)$$

$$P = \frac{W(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}$$

$$= \frac{100(0.2 + 0.979\mu)}{(0.9798 - 0.2\mu)}$$

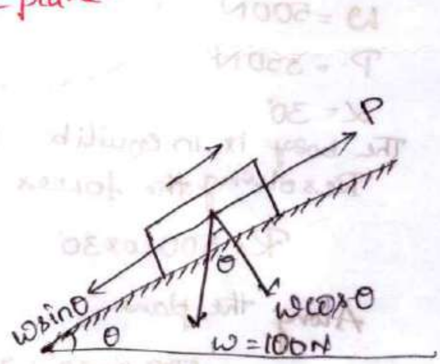
Find the effort when it is applied || to the plane.

Resolving the forces along the plane

$$P = W\sin\theta + \mu R$$

Resolving the forces normal to the plane

$$R = W\cos\theta$$



Substituting R

$$P = W\sin\theta + \mu W\cos\theta$$

$$= W(\sin\theta + \mu\cos\theta)$$

$$= 100(\sin 11.53^\circ + \mu\cos 11.53^\circ)$$

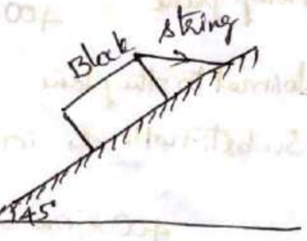
$$= 100(0.2 + \mu \times 0.9798)$$

Determine the coefficient of friction and limiting friction. Block A weighing 15 N is a rectangular prism resting on a rough inclined plane. The block is tied by a horizontal string which has a tension of 5N find μ, R & F

$W = 15N$

Tension in string = 5N

Inclination of plane $\alpha = 45°$



~~F = μR~~

~~ΣH = 0~~

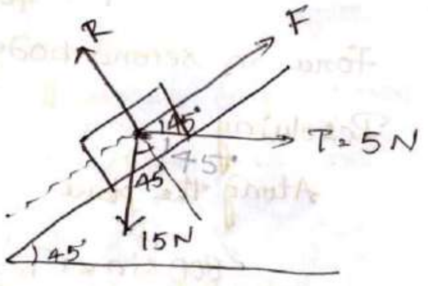
~~F = 15 sin45°~~ Resolving

~~ΣV = 0~~ Along the plane

~~R = F + 5cos45°~~ $15\sin 45° = F + 5\cos 45°$

$F = 15\sin 45° - 5\cos 45°$

$F = 7.07 N$



Resolving forces normal to inclined plane

$R = 15\cos 45° + T\cos 45°$

$= 15\cos 45° + 5\cos 45°$

$= 14.14 N$

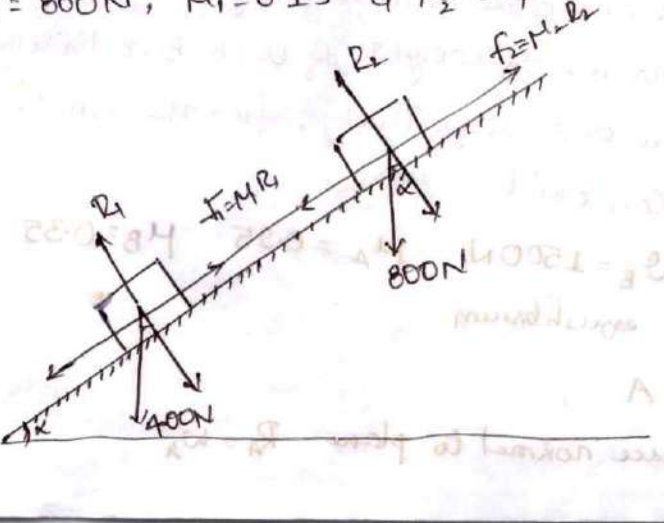
$F = \mu R$

$\mu = F/R = 7.07/14.14 = 0.5$

A cord connects two bodies of weights 300N and 800N. The two bodies are placed on an inclined plane and cord is parallel to inclined plane. The coefficient of friction for the weight of 400N is 0.15 and that for 800N is 0.4. Determine the inclination of the plane to the horizontal and tension in the cord when the motion is about to take place, down the inclined plane. The body weighing 400N is below the body weighing 800N.

$W_1 = 400N$, $W_2 = 800N$, $\mu_1 = 0.15$ & $\mu_2 = 0.4$

F



Force on the first body

Resolving

Along plane $400\sin\alpha = T + F_1 = T + \mu_1 R_1 = T + 0.15 R_1$

Normal to the plane $400\cos\alpha = R_1$

Substitutin R_1 in ① eq

$$400\sin\alpha = T + 0.15(400\cos\alpha)$$

$$T = 400\sin\alpha - 60\cos\alpha$$

Force on second body

Resolving

Along the plane

$$800\sin\alpha + T = F_2 = \mu_2 R_2 = 0.4 R_2$$

Normal to the plane

$$R_2 = 800\cos\alpha$$

Substituting value of R_2

$$800\sin\alpha + T = 0.4 \times 800\cos\alpha = 320\cos\alpha$$

$$T = 320\cos\alpha - 800\sin\alpha$$

Equating values of T

$$400\sin\alpha - 60\cos\alpha = 320\cos\alpha - 800\sin\alpha$$

$$400\sin\alpha + 800\sin\alpha = 320\cos\alpha + 60\cos\alpha$$

$$\tan\alpha = 0.316$$

$$\alpha = 17°56'$$

substing value of α in eq T

$$T = 400\sin 17°56' - 60\cos 17°56' = 63.48N$$

Two blocks A and B are connected by a horizontal rod and are supported on two rough planes. If weight of block B is 1500N and μ of block A and B are 0.25 and 0.35 respectively. find the smallest weight of block A for equilibrium can exist

Given $W_B = 1500N$ $\mu_A = 0.25$ $\mu_B = 0.35$ Let W_A be smallest weight of block for equilibrium

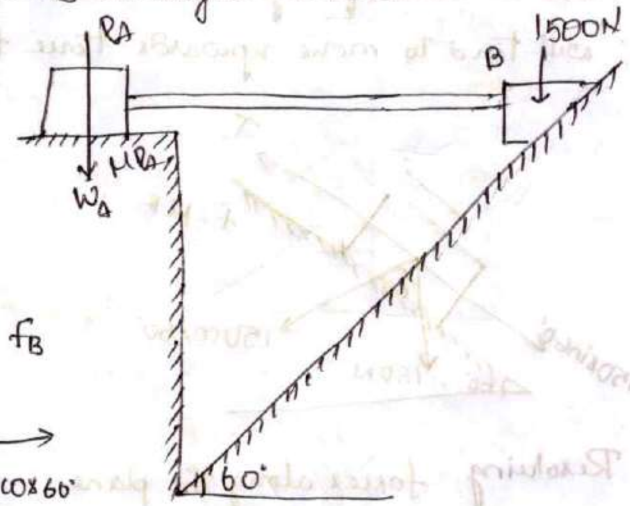
For Block A

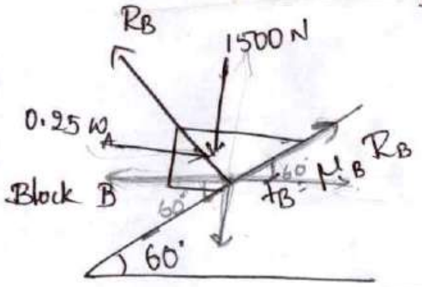
Resolving force normal to plane $R_A = W_A$

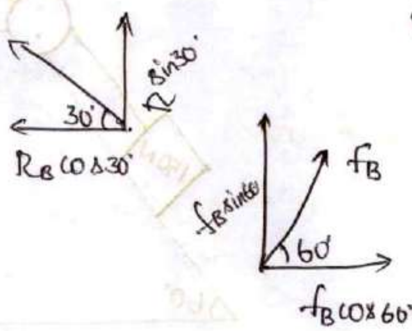
Force of friction $F_A = \mu_A R_A = \mu \times 0.25 \times W_A = 0.25 W_A$

This force will be transmitted to block B through rod AB

For block B







$W_B = 1500$ N

Horizontal force $= 0.25 W_A$, Force of friction $F_B = \mu_B R_B = 0.35 R_B$

Resolving forces Horizontally

$0.25 W_A + F_B \cos 60° = R_B \cos 30°$

~~$0.25 W_A + 0.35 \times 0.5 R_B$~~

$0.25 W_A + 0.35 R_B \cos 60° = R_B \cos 30°$

$0.25 W_A + 0.175 R_B = 0.866 R_B$

$0.25 W_A = 0.691 R_B$

Resolving vertically

$R_B \sin 30° + F_B \sin 60° = 1500$

$R_B \times 0.5 + 0.35 R_B \cdot 0.866 = 1500$

$0.5 R_B + 0.303 R_B = 1500$

$R_B = 1868$ N

Substituting this value of R_B

$0.25 W_A = 0.691 \times 1868$

$W_A = \frac{0.691 \times 1868}{0.25} = 5163$ N

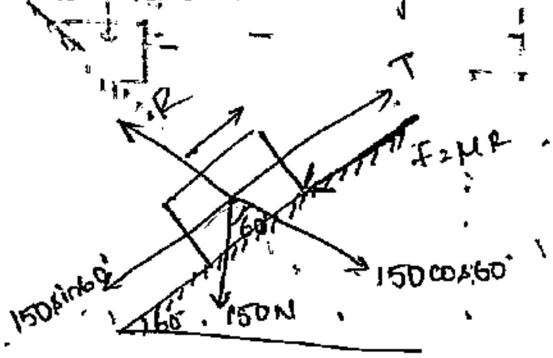
Refering to the fig 8.29 determine the least value of the force P to cause motion to impend rightwards. Assume the coefficient of friction under the blocks to be 0.2 and pulley to be frictionless

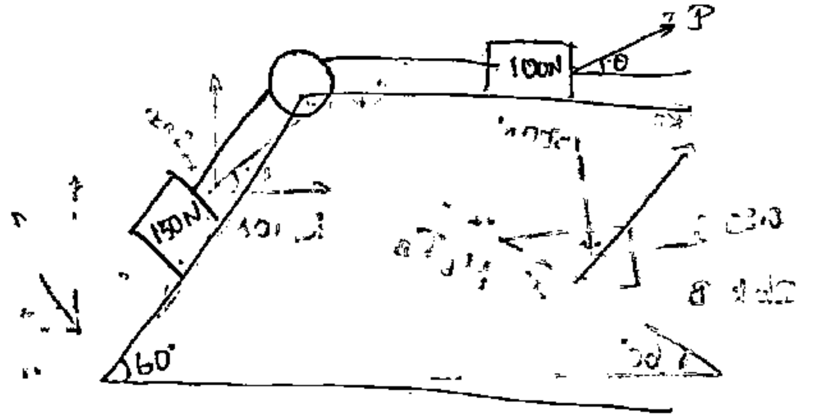
Given $\mu = 0.2$

Pulley is frictionless. Motion of block of weight 100N towards right

Case 1;

As the block of weight 100N tends to move rightwards, the block of weight 150N will tend to move upwards. Hence the force of friction





Resolving forces along the plane,

$T = 150 \sin 60° + \mu R$

Normal to inclined plane

$R = 150 \cos 60° = 75\ N$

Substituting

$T = 150 \sin 60° + 0.2 \times 75 = 144.9\ N$

Case 2;

Resolving forces along the plane

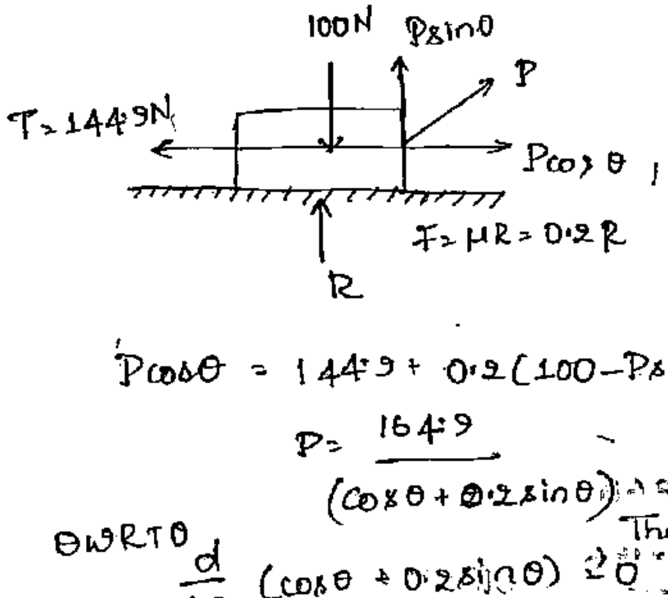
$P \cos\theta = T + 0.2R$

$= 144.9 + 0.2R$

Normal to the plane

$R + P\sin\theta = 100$

$R = 100 - P\sin\theta$



$P\cos\theta = 144.9 + 0.2(100 - P\sin\theta)$

$$P = \frac{164.9}{(\cos\theta + 0.2\sin\theta)}$$

The force P will be minimum if $\cos\theta + 0.2\sin\theta$ is max

θ WRT θ

$$\frac{d}{d\theta}(\cos\theta + 0.2\sin\theta) = 0$$

$-\sin\theta + 0.2\cos\theta = 0$

$0.2\cos\theta = \sin\theta$

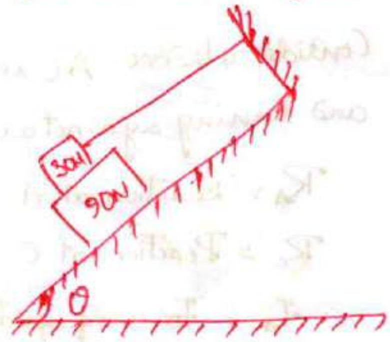
$0.2 = \tan\theta$

$\theta = 11.309°$

Substituting

$$P = \frac{164.9}{(\cos 11.309° + 0.2 \sin 11.309°)} = 161.88\ N$$

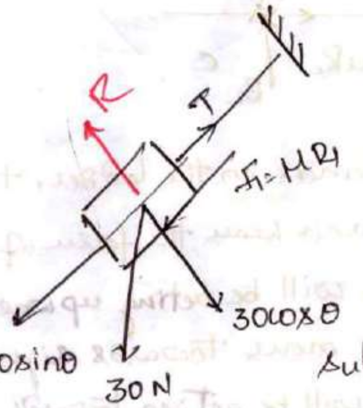
What should be the angle θ so that the motion of 90N block impends down the plane? The coefficient of friction μ for all the surfaces is 1/3



$\mu = 1/3$

Motion of weight impends down the plane

→ First consider the equilibrium of weight 30N.



Resolving

Along the plane $T = 30\sin\theta + \mu R_1$

$= 30\sin\theta + \frac{1}{3}R_1$

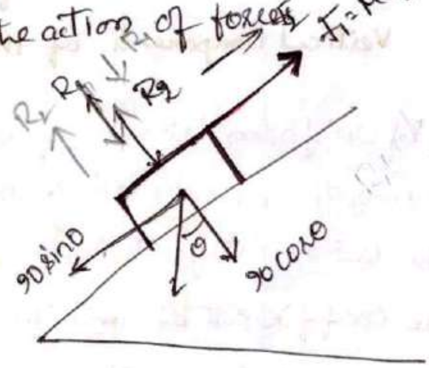
Normal to plane $R_1 = 30\cos\theta$

Substituting R_1 in T eq

$T = 30\sin\theta + \mu\, 30\cos\theta = 30\sin\theta + 10\cos\theta$

→ Secondly consider equilibrium of 90 N weight

weight of 90N will be in equilibrium under the action of forces



Along the plane

$90\sin\theta = \mu R_1 + \mu R_2$

$= \frac{1}{3}R_1 + \frac{1}{3}R_2$

$= \frac{1}{3} \times 30\cos\theta + \frac{1}{3}R_2$

$= 10\cos\theta + \frac{1}{3}R_2$

Resolving the forces normal to the plane

$R_2 = R_1 + 90\cos\theta$

$= 30\cos\theta + 90\cos\theta = 120\cos\theta$

Substituting the value of R_2

$90\sin\theta = 10\cos\theta + \frac{1}{3} \times 120\cos\theta$

$= 10\cos\theta + 40\cos\theta = 50\cos\theta$

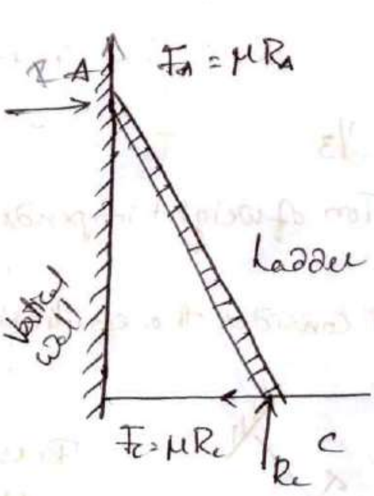
$\frac{\sin\theta}{\cos\theta} = \frac{50}{90} = 0.55$

$\tan\theta = 0.55$

$\theta = \tan^{-1} 0.55 = 29.05^{\circ}$

Analysis of ladder friction;

Consider a ladder AC resting on the ground and leaning against a wall.



R_A = Reaction at A

R_c = Reaction at C

F_A = Force of friction at A $= \mu R_A$

F_c = Force of friction at C μR_c

Due to self weight of ladder or when some man stands on the ladder, the upper end A of the ladder tends to slip downwards and hence the force of friction b/n the ladder and vertical wall $F_A = \mu R_A$ will be acting upwards similarly lower end C of the ladder will tend to move towards right and hence a force of friction b/n ladder and floor will be acting towards left.

$$F_c = \mu R_c$$

For the equilibrium of system, the algebraic sum of horizontal and vertical components of the forces must be zero

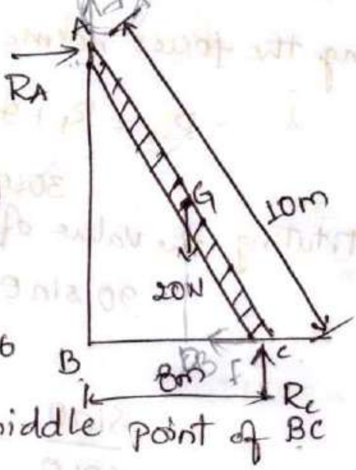
A uniform ladder of length 10m and weighing 20N is placed against a smooth vertical wall with its lower end 8m from the wall. In this position the ladder is just to slip. Determine
The coefficient of friction between the ladder and floor
Frictional force acting on the ladder at the point of contact b/n ladder & floor

Weight of ladder W = 20N

length of ladder AC = 10m

Distance of lower end of ladder from wall

BC = 8m



$AB = \sqrt{AC^2 - BC^2} = \sqrt{10^2 - 8^2} = \sqrt{100-64} = 6$

line of action of W will pass through the middle point of BC

$CD = \frac{1}{2} BC = 8/2 = 4m$

Resolving $\Sigma V = 0 \quad R_c = 20N$

$\Sigma H = 0$

$R_A = F_c = \mu \times R_c = \mu \times 20 = 20\mu$

$\Sigma M_c = 0$

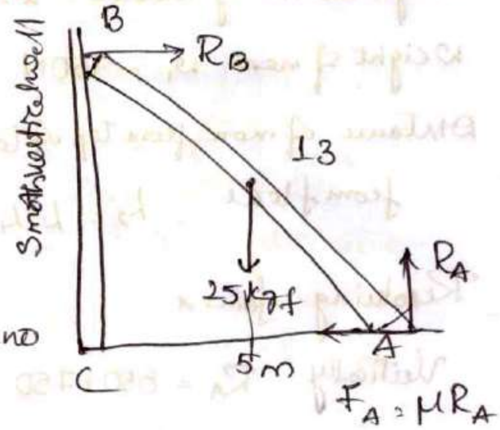
$R_A \cdot AB - 20 \times CD = 0$

$20\mu \times 6 = 20 \times 4$

$\mu = \frac{20\times4}{20\times6} = 0.67$

Frictional force $F_c = \mu R_c = 0.67 \times 20 = 13.4$

A uniform ladder of length 13 m and weighing 25 N is placed against a smooth vertical wall with its lower end 5 m from the wall. The coefficient of friction between the ladder and the floor is 0.3. Show that the ladder will remain in equilibrium in this position. What is the friction force acting on the ladder at the point of contact between the ladder and floor?



length of ladder $L = AB = 13m$

Weight $W = 25m$

$\mu = 0.3$

Vertical wall is smooth and hence there will be no force of friction between ladder and wall

$F_A = \mu R_A = 0.3 R_A$

~~Equating vertical forces~~

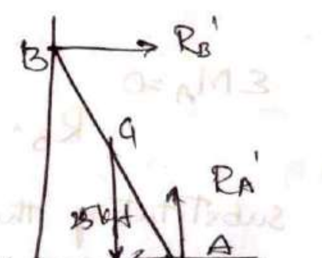
ladder AB placed. The weight of 25 N is acting at the middle point of AB. vertically downwards. If ladder is not in equilibrium. It will start moving at A towards right and force of friction F_A will act towards left. forces acting on the ladder

equating vertical forces $R_A = 25 N$

Eq Horizontal forces $R_B = F_A = \mu R_A = 0.3 R_A = 0.3 \times 25 = 7.5 N$

Max amount of force of friction available at A $F_A = 7.5 N$

To prove ladder in equilibrium



F_A' R_B' R_A'

$AD = CD = 2.5m$

$AB^2 = AC^2 + BC^2$

$BC = \sqrt{AB^2 - AC^2} = \sqrt{13^2 - 5^2} = 12m$

$\Sigma M_A = 0$

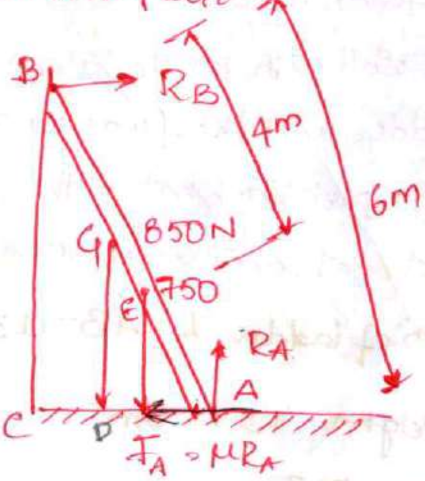
$25 \times AD = R_B' \times BC = R_B' \times 12$

$R_B' = 5.21 N$

$\Sigma H = 0$

$F_A' = R_B' = 5.21 N$ Hence proved

A uniform ladder of weight 850 N and of length 6m rests on a horizontal ground and leans against a smooth vertical wall. The angle made by the ladder from the top of ladder, the ladder is at the point of sliding. Determine the coefficient of friction b/n ladder and floor.



Weight of ladder $W = 850N$

Length of ladder $L = AB = 6m$

Angle made by ladder $\alpha = 65°$

Weight of man $W_1 = 750N$

Distance of man from top of ladder $L_1 = 4m$

from floor $L_2 = L - L_1 = 6 - 4 = 2m$

Resolving forces

Vertically $R_A = 850 + 750 = 1600 N$

$\Sigma H = 0$

$R_B = F_A = \mu R_A = \mu \times 1600 = 1600\mu N$

In Right angled ΔABC $BC = AB \sin 65° = 6 \sin 45° = 5.437m$

$AC = AB \cos 65° = 2.5357m$

As G is the middle point of AB and GD is normal to AC

$\therefore$ D is the middle point of AC

$$AD = \frac{AC}{2} = \frac{2.5357}{2} = 1.267$$

$$AH = AE \cos 65° = (AB - BE) \cos 65° = (6-4) \cos 65°$$

$$= 2 \cos 65° = 0.8452m$$

$\Sigma M_A = 0$

$$R_B \times BC = 850 \times AD + 750 \times AH$$

Substituting the values of BC, AD, AH and R_B from equation

$$1600\mu \times 5.437 = 850 \times 1.267 + 750 \cdot 0.8452$$

$$\mu = 0.199.$$

A uniform ladder of weight 200N of length 4.5m rests on a horizontal ground and leans against a rough vertical wall. The coefficient of friction between is placed on the ladder at a distance of 1.2m from the top of ladder

the ladder is at the point of sliding find Angle made by ladder with horizontal
Reaction at the foot of ladder
Reaction at the top of ladder.

$W = 200N$

$L = AB = 4.5m$

$\mu_A = 0.4$ ladder & floor

$\mu_B = 0.2$ ladder & wall

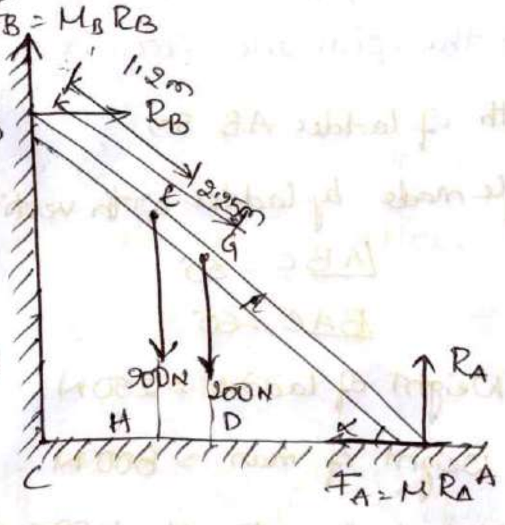
$W_1 = 900N$

$BE = 1.2m$

α = Angle made by ladder with horizontal

R_A $F_A = \mu_A R_A$

$F_B = \mu_B R_B$



From Δ^{le} ABC

$$BC = AB\sin\alpha = 4.5\sin\alpha \ ; \ AC = AB\cos\alpha = 4.5\cos\alpha$$

$$AD = AG\cos\alpha = \frac{AB}{2}\cos\alpha = \frac{4.5}{2}\cos\alpha = 2.25\cos\alpha$$

$$AH = AE\cos\alpha = (AB - BE)\cos\alpha = (4.5 - 1.2)\cos\alpha = 3.3\cos\alpha$$

Forces acting on ladder

$\Sigma V = 0$

$$R_A + F_B = 900 + 200 = 1100N$$

$$R_A + \mu_B R_B = 1100 \ ; \ R_A + 0.2R_B = 1100$$

$\Sigma H = 0$

$$R_B = F_A = \mu_A R_A = 0.4R_A$$

Substituting Value of R_B

$$R_A + 0.2 \times 0.4R_A = 1100$$

$$R_A + 0.08R_A = 1100$$

$$R_A = 1018.52$$

$$R_B = 0.4R_A = 407.41$$

$\Sigma M_A = 0$

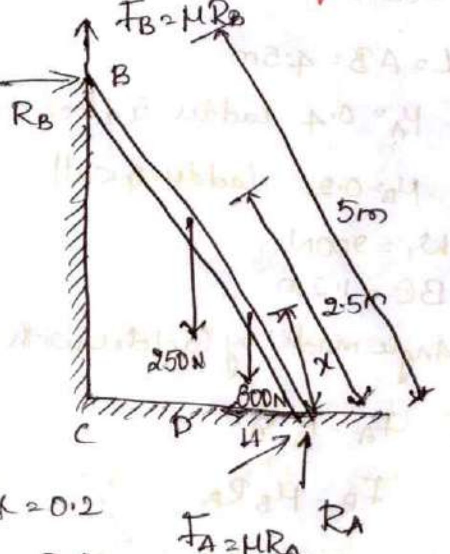
$$200 \times AD + 900 \times AH = R_B \times BC + F_B \times AC$$

$$200 \times 2.25\cos\alpha + 900 \times 3.3\cos\alpha = 407.41 \times 4.5\sin\alpha + \mu_B R_B \times 4.5\cos\alpha$$

$$\alpha = 59^\circ 65'$$

A ladder 5m long and 250N weight is placed against a vertical wall in a position where its inclination to the vertical is 30°. A man weighting

800N climbs the ladder at what position will he induce sliping? The coefficient of friction for both the contact surfaces of the ladder i.e. with the wall and floor is 0.2.



Length of ladder AB = 5m

Angle made by ladder with vertical = 30°

$\angle ABC = 30°$

$\angle BAC = 60°$

Weight of ladder = 250 N

Weight of man = 800 N

Coefficient of friction b/n ladder and floor = 0.2

ladder and wall = 0.2

Forces acting on the ladder

$\Sigma V = 0$

$R_A + \mu R_B = 250 + 800 = 1050$

$\Sigma H = 0$

$R_B = \mu R_A = 0.2 R_A$

Substituting the value of R_B in equation

$R_A + 0.2(0.2 R_A) = 1050$

$R_A + 0.04 R_A = 1050$

$1.04 R_A = 1050 \quad R_A = \frac{1050}{1.04} = 1009.6N$

Substituting the value of R_A

$R_B = 0.2 \times 1009.6 = 201.92N$

Δ^{le} AGD $AD = AG\cos 60° = 2.5 \times \frac{1}{2} = 1.25m$

Δ^{le} AEH $AH = x\cos 60° = x \times \frac{1}{2} = x/2$

$\Sigma M_A = 0$

$800\, AH + 250 \cdot AD = R_B\, BC + F_B\, AC$

$BC = AB\cos 30° = 5 \times 0.86 = 4.3m$

$AC = AB\cos 60° = 5 \times \frac{1}{2} = 2.5m$

$F_B = \mu R_B = 0.2 \times 201.92 = 40.384\ N$

Substituting the above values in eq (vii) we get

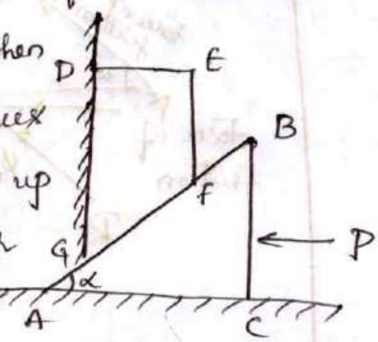
$$800\frac{x}{2} + 250\times1.25 = 201.92 \times 4.33 + 40.384 \times 2.5$$

$$x = \frac{66.276}{40} = 1.65 = 1.657\,m$$

Analysis of Wedge Friction;

A wedge ~~friction~~ is a piece of metal or wood which is usually of a triangular or trapezoidal in c/s. It is used for either lifting loads or used for slight adjustments in the position of a body i.e., for tightening fits or keys for shafts.

When lifting a heavy load the wedge is placed below the load and a horizontal force P is applied. If the force P is just sufficient to lift the load, the wedge will move towards left and load will move up. When the wedge moves towards left, the sliding of the surfaces AC and AB will take place. At the same time load moves up and sliding of the load takes place along GD. Thus for the wedge and load as fig sliding takes place along surfaces AB, AC and DG. Hence there will be three normal reactions at AB, AC and DG.



The problems

Equilibrium method;

In this method, the equilibrium of the load (or the body placed on the wedge) and the equilibrium of the wedge are considered.

Equilibrium of wedge;

Consider the equilibrium of the wedge. The force acting on the wedge are

(i) The force P applied horizontally on face BC

(ii) Reaction R_1 on the face AC. The reaction R_1 will be inclined at an angle ϕ_1 with normal.

(iii) Reaction R_2 on the face AB. The reaction R_2 will be inclined at an angle ϕ_2 with normal

When the force P is applied on the wedge, the surface CA will be moving towards left and hence force of friction on this surface will be acting towards right. Ully the force of friction on face AB will be acting from A to B. These forces

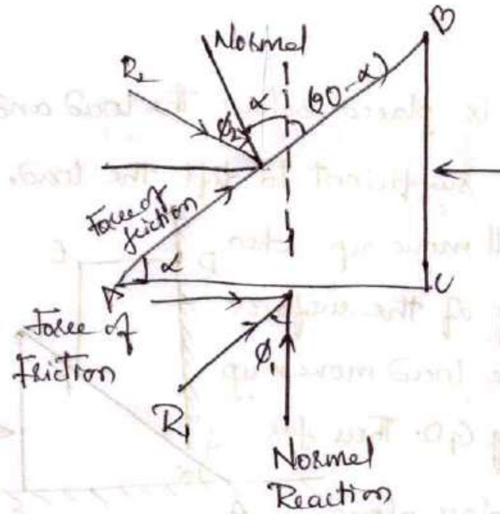
Resolving the forces horizontally

$$R_1 \sin\phi_1 + R_2 \sin(\phi_2 + \alpha) = P$$

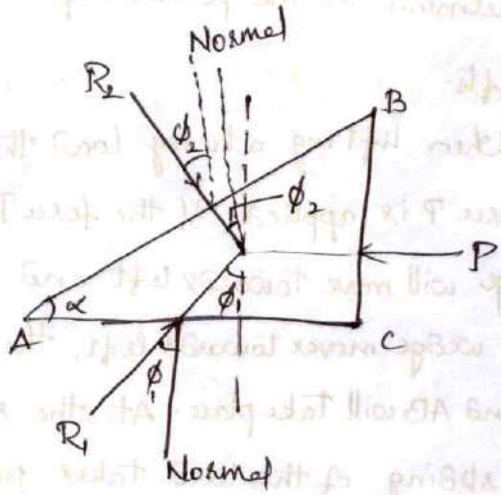
Resolving the forces vertically

$$R_1 \cos\phi_1 = R_2 \cos(\phi_2 + \alpha)$$

By lami's theorem;



Equilibrium of wedge



(2) fig

The wedge is in equilibrium under the action of three forces namely R_1, R_2 and P. These forces, when produced, will meet at a point. Apply lami's theorem, we get as shown in fig (2)

$$\frac{P}{\sin(180 - \phi_1 - \phi_2 - \alpha)} = \frac{R_1}{\sin(90 + \alpha + \phi_2)} = \frac{R_2}{\sin(90 + \phi_1)}$$

Equilibrium of body placed on wedge;

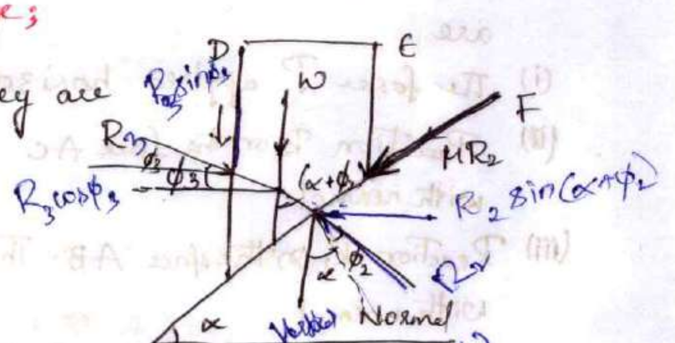
The forces acting on the body. They are

(i) The weight W of the body

(ii) Reaction R_3 on the face GD.

(iii) Reaction R_2 on the face GF.

These forces are as shown in fig



Resolving the forces $\Sigma H = 0$

$$R_3 \cos\phi_3 = R_2 \sin(\alpha + \phi_2)$$

$\Sigma V = 0$

$W + R_3 \sin\phi_3 = R_2 \cos(\alpha + \phi_2)$

By Lami's theorem

The forces R_3 R_2 and W are produced to meet at a point. The body in equilibrium under the action of these three forces.

Hence applying Lami's theorem, we get

$$\frac{W}{\sin(90 + \phi_3 + \alpha + \phi_2)} = \frac{R_2}{\sin(90 - \phi_3)} = \frac{R_3}{\sin[180 - (\alpha + \phi_2)]}$$

A block overlying a 10° wedge on a horizontal floor and leaning against a vertical wall and weighing 1500N is to be raised by applying a horizontal force to the wedge. Assuming coefficient of friction b/n all the surfaces in contact to be 0.3, determine the minimum horizontal force to be applied to raise the block.

Given

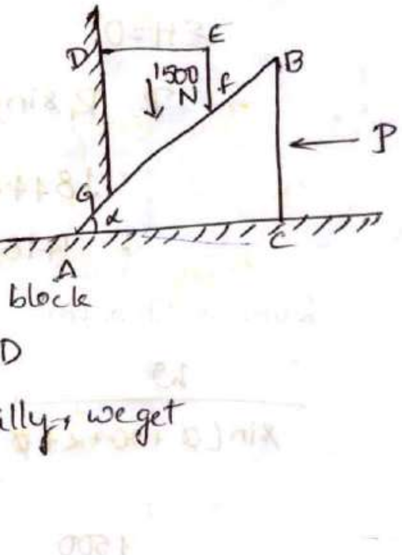
Angle of wedge $\alpha = 10°$

Weight of block $W = 1500N$

$\mu = 0.3$

$\mu = \tan\phi = 0.3$

$\phi = \tan^{-1} 0.3 = 16°42'$



1st method Considering the equilibrium of the block

R_2 = Reaction on face GF R_3 = Reaction on face GD

Resolving the forces acting on the block horizontally, we get

$R_3 \cos\phi = R_2 \sin(\alpha + \phi)$

$R_3 \cos 16°42' = R_2 \sin(10° + 16°42')$

$R_2 = 2.1317\ R_3$

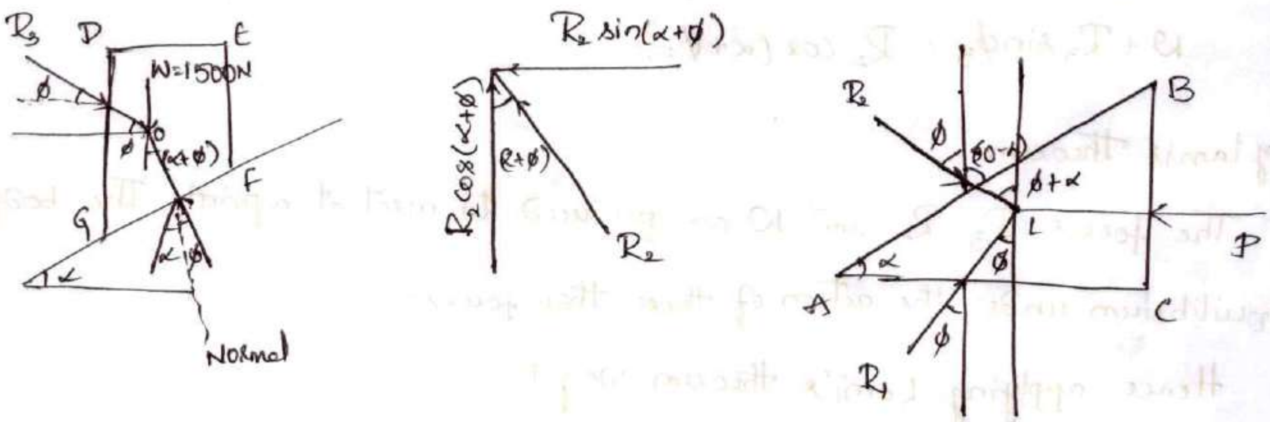
$\Sigma V = 0$

$R_3 \sin\phi + W = R_2 \cos(\alpha + \phi)$

$R_3 \sin 16°42' + 1500 = R_2 \cos(10 + 16°42')$

Substituting $R_2 = 2.1317\ R_3$

$R_3 = 927.54N$



$R_2 = 2.1317 R_3 = 1977.45N$

Now consider the equilibrium of the wedge

Resolving the forces vertically, we get

$R_2 \cos(\alpha+\phi) = R_1 \cos\phi$

$R_1 = \frac{R_2 \cos(\alpha+\phi)}{\cos\phi}$

$R_1 = 1977.45 \times \frac{\cos(10^\circ + 16^\circ 42')}{\cos 16^\circ 42'}$

$1977.45 \times \frac{0.8973}{0.9578} = 1844.49N$

$\Sigma H = 0$

$P = R_1 \sin\phi + R_2 \sin(\alpha+\phi)$

$= 1844.49 \sin(16^\circ 42') + 1977.45 \times \sin(10^\circ + 16^\circ 42')$

$= 1418.57N$

Lami's theorem

$$\frac{W}{\sin(\phi+90+\alpha+\phi)} = \frac{R_3}{\sin[180-(\alpha+\phi)]} = \frac{R_2}{\sin(90-\phi)}$$

$$\frac{1500}{\sin(133^\circ 24')} = \frac{R_3}{\sin(153^\circ 18')} = \frac{R_2}{\sin(72^\circ 18')}$$

$$\frac{1500}{\sin(133^\circ 24')} = \frac{R_3}{\sin(153^\circ 18')}$$

$R_3 = 927.66N$

$R_2 = 1500 \times \frac{0.9578}{0.7268} = 1977.56N$

Now consider the equilibrium of the wedge. Three forces R_1, R_2 and P when produced are meeting at the point L. Applying lami's theorem to the point L

$$\frac{R_1}{\sin(90+\alpha+\phi)} = \frac{R_2}{\sin(90+\phi)} = \frac{P}{\sin(180-\phi-(\phi+\alpha))}$$

$$P = \frac{R_2\left[\sin(180-\phi-(\phi+\alpha)\right]}{\sin(90+\phi)} = 1418.44$$

Impending motion; The moment where the body is on the verge of slipping. static friction force reaches the max value. For a given mating surfaces, (c) Motion the body starts moving in the direction of applied force

Unit- IV

Centroid & Center of gravity

Centroids of simple figures;

Centre of gravity;

Centre of gravity of a body is the point through which the whole weight of the body acts. A body is having only one centre of gravity for all positions of the body. It is represented by C.G or simply G.

Centroid;

The point at which the total area of a plane figure (like rectangle square, Triangle, quadrilateral, circle etc.) is assumed to be concentrated is known as the centroid of that area. The centroid and centre of gravity are at the same point.

Centroid or Centre of gravity of simple plane figures;

→ The C.G of a uniform rod lies at its middle point.

→ The C.G of Triangle lies at the point where the three medians of the Triangle meet.

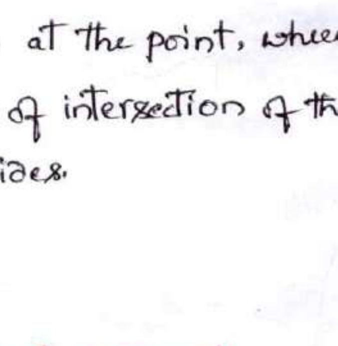
→ The C.G of rectangle or of a parallelogram at the point, where its diagonal meet each other. It is also the point of intersection of the lines joining the middle points of the opposite sides.

→ The C.G of a circle is at its centre.

Centre of gravity of plane figures by method of moments

Fig shows a plane figure of total area A whose CG is to be determined. Let this area A is composed of a number of small areas $a_1, a_2\ a_3\ a_4 \ldots$ etc

$A = a_1 + a_2 + a_3 + a_4 + \ldots$



Let x_1 = Distance of the C.G of the area a_1 from axis OY

x_2 = " " " " " " a_2 from "

x_3 = a_3 OY

x_4 = a_4 OY ....

The moment of small areas about the axis OY

$= a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \dots$

let G is the centre of gravity of the total area A whose distance from the axis OY is $\bar{x}$.

Then moment of Total area about OY = $A\bar{x}$

The moments of all small areas about the axis OY must be equal to the moment of total area about the same axis Hence equating the equation

$$a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \dots = A\bar{x}$$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \dots}{A}$$

Where $A = a_1 + a_2 + a_3 + a_4 + \dots$

If we take the moments of the small areas about the axis OX and also the moments of total area about the axis OX we will get

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4 + \dots}{A}$$

$\bar{y}$ = Distance of CG from axis OX

y_1 = Distance of CG of the area a_1 from axis OX

y_2 y_3 y_4 = Distance of CG of the area a_4, a_2, a_3 from axis OX respectively.

Centre of gravity of plane by integration method

$$\bar{x} = \frac{\Sigma a_i x_i}{\Sigma a_i} \qquad \bar{y} = \frac{\Sigma a_i y_i}{\Sigma a_i}$$

where $i = 1, 2, 3, 4, \dots$

x_i = Distance of CG of area a_i from axis OY and

y_i = Distance of CG of area a_i from axis OX

The value of i depends up on the number of small areas. If small areas are large in number, then the summations in the above equations can be replaced by integration. let small areas are represented by dA instead of 'a' then the above equation can be rewritten as

$$\bar{x} = \frac{\int x * dA}{\int dA} \qquad \bar{y} = \frac{\int y * dA}{\int dA}$$

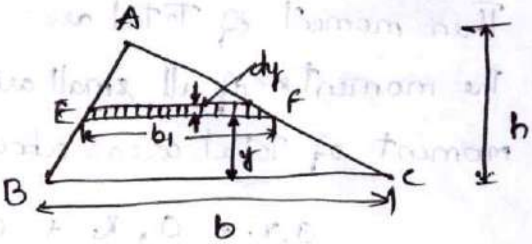
where $\int x^2 dA = \Sigma x_i a_i$ $\int y^2 dA = \Sigma y_i a_i$

$\int dA = \Sigma a_i$

Centroid of a Triangle

Consider a Δ^{le} ABC
Base width = b
Height = h

b_1 = width of elemental strip
dy = thickness at a distance y



Since ΔAEF & ΔABC are similar Δ^{le}

$$\frac{b_1}{b} = \frac{h-y}{h}$$

$$b_1 = \left[\frac{h-y}{h}\right] b = \left[1 - \frac{y}{h}\right] b$$ Area of element = $dA = b_1 dy$

$$dA = \left[1 - \frac{y}{h}\right] b\,dy$$ Area of triangle = $\frac{1}{2} bh$

$$\bar{y} = \frac{\int y\,dA}{A}$$

$$\int y\,dA = \int_0^h y\left[1 - y/h\right] b\,dy$$

$$= \int_0^h \left[y - y^2/h\right] b\,dy = b\left[\frac{y^2}{2} - \frac{y^3}{3h}\right]_0^h$$

$$= \frac{bh^2}{6}$$

$$\bar{y} = \frac{\int y\,dA}{A} = \frac{bh^2}{6} \times \frac{1}{\frac{1}{2} bh} = h/3$$

centroid from $h/3$ base   Apex $2h/3$

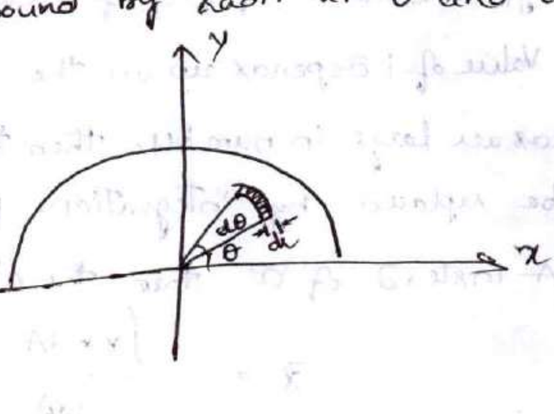
Semi Circle

Semicircle radius R, distance from diametral axis be $\bar{y}$. To find $\bar{y}$. Consider an element at a distance r from the centre O of the semicircle, radial distance being dr and bound by radii at θ and $\theta + d\theta$

Area of element $r\,d\theta\,dr$

Its moment about diametral axis x

$$r\,d\theta\,dr\,r\sin\theta = r^2 \sin\theta\,dr\,d\theta$$



Total moment of area about

diametral axis

$$\int_0^{\pi}\int_0^{R} r^2 \sin\theta \, dr \, d\theta = \int_0^{\pi} \left[\frac{r^3}{3}\right]_0^R \sin\theta \, d\theta$$

$$= \frac{R^3}{3}\left[1 - \cos\theta\right]_0^{\pi}$$

$$\frac{R^3}{3}(1+1) = \frac{2R^3}{3}$$

Area of semi. circle $A = \frac{1}{2}\pi R^2$

$$\bar{y} = \frac{\text{Moment of area}}{\text{Total area}} = \frac{2R^3/3}{\frac{1}{2}\pi R^2} = \frac{4R}{3\pi}$$

Centroid of Sector of a circle

Consider the sector of a circle of angle 2α as shown in fig. Due to symmetry, centroid lies on x-axis. To find its distance from the centre O,

Area of the element $= r\,d\theta\,dr$

Its moment about y axis $= r\,d\theta\,dr\; r\cos\theta = r^2\cos\theta\,dr\,d\theta$

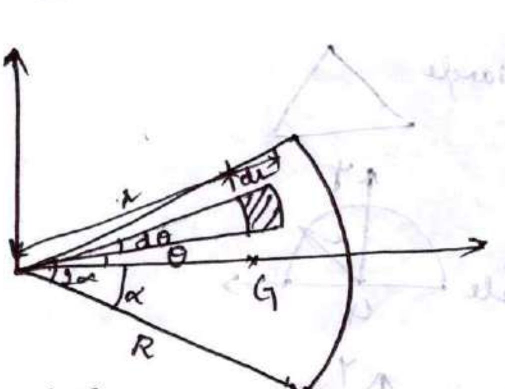
Total moment of area about Y-axis

$$\int_{-\alpha}^{\alpha}\int_0^{R} r^2\cos\theta\,dr\,d\theta = \left[\frac{r^3}{3}\right]_0^R \left[\sin\theta\right]_{-\alpha}^{\alpha} = \frac{R^3}{3}\,2\sin\alpha$$

Total area of the sector

$$= \int_{-\alpha}^{\alpha}\int_0^{R} r\,dr\,d\theta = \int_{-\alpha}^{\alpha}\left[\frac{r^2}{2}\right]_0^R d\theta$$

$$= \frac{R^2}{2}\left[\theta\right]_{-\alpha}^{\alpha} = R^2\alpha$$

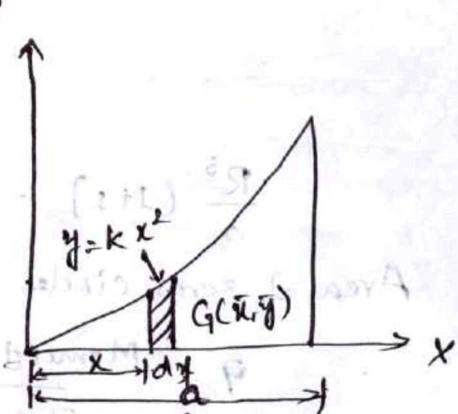


Distance of centroid from centre O $= \dfrac{\text{Moment of area about Y axis}}{\text{Area of the fig.}}$

$$\frac{\frac{2R^3}{3}\sin\alpha}{R^2\alpha} = \frac{2R}{3\alpha}\sin\alpha$$

Parabolic Spandrel

Consider the parabolic spandrel. Height of the element at a distance x from O is $y = kx^2$



Width of element $= dx$

Area of element $= kx^2\,dx$

Total area of spandrel $= \int_0^a kx^2\,dx$

$$\left[\frac{kx^3}{3}\right]_0^a = \frac{ka^3}{3}$$

Moment of area about y-axis $= \int_0^a kx^2\,dx\,x = \int_0^a kx^3\,dx$

$$\frac{ka^4}{4}$$

Moment of area about x-axis $= \int_0^a kx^2\,dx\,\frac{kx^2}{2} = \int_0^a \frac{k^2x^4}{2}\,dx$

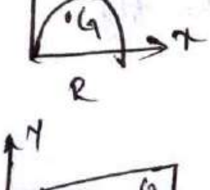
$$= \frac{k^2a^5}{10}$$

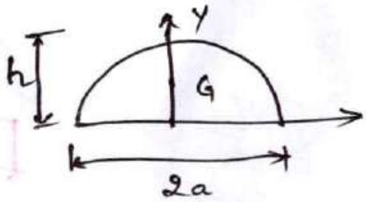
$$\bar{x} = \frac{ka^4}{4} \div \frac{ka^3}{3} = \frac{3a}{4} \quad \Bigg| \quad \bar{y} = \frac{k^2a^5}{10} \div \frac{ka^3}{3} = \frac{3}{10}ka^2$$

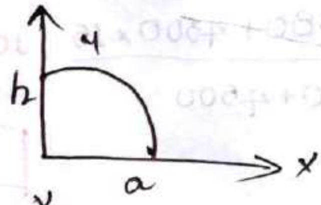
From the fig 5.9 at $x = a,\ y = h$

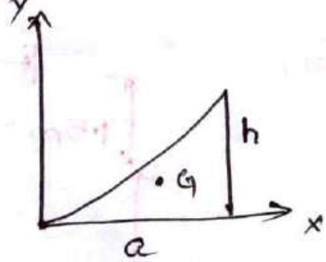
$$h = ka^2 \text{ or } k = \frac{h}{a^2}$$

$$\bar{y} = \frac{3}{10} \cdot \frac{h}{a^2}a^2 = \frac{3h}{10}$$

		$\bar{x}$	$\bar{y}$	Area
Triangle			$h/3$	$bh/2$
Semi circle		0	$4R/3\pi$	$\frac{\pi R^2}{2}$
Quarter		$4R/3\pi$	$4R/3\pi$	$\frac{\pi R^2}{4}$
Sector		$\frac{2R}{3\alpha}\sin\alpha$	0	αR^2

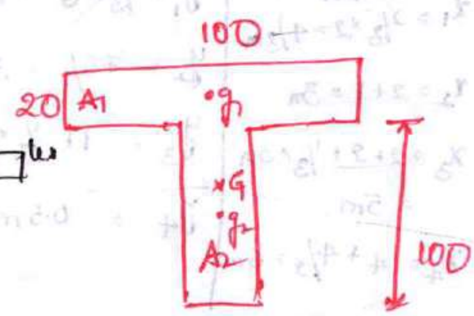
Parabola  0 $\frac{3h}{5}$ $\frac{4}{3}$

Semiparabola  $\frac{3}{8}a$ $\frac{3h}{5}$ $\frac{2ah}{3}$

Parabolic Spandrel  $\frac{3a}{4}$ $\frac{3h}{10}$ $\frac{ah}{3}$

Centroid of Composite figures

Locate the centroid of T-section



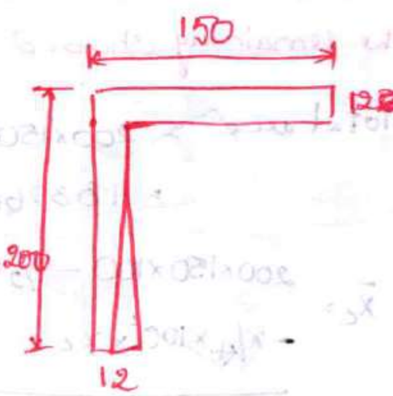
Given T-section is divided in to two parts □les

$A_1 = 100 \times 20$ $A_2 = 20 \times 100$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{100 \times 20 \times (10) + 20 \times 100\,(70)}{(100 \times 20) + (20 \times 100)} = 40\,mm$$

Centroid of T-section is on the symmetric axis at a distance 40 mm from top.

Centroid of unequal angle



$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$= \frac{1800 \times 75 + 2256 \times 6}{4056} = 36.62$$

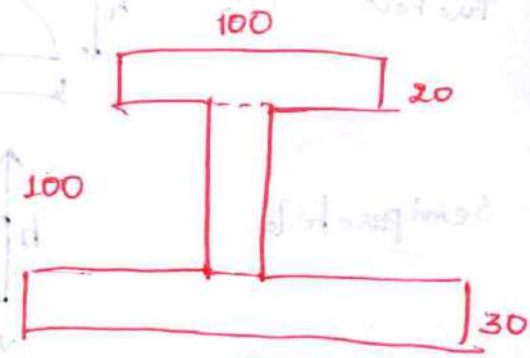
$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{1800 \times 6 + 2256 \times 106}{4056} = 61.62\,mm$$

$\bar{x} = 36.62\,mm$ & $\bar{y} = 61.62\,mm$

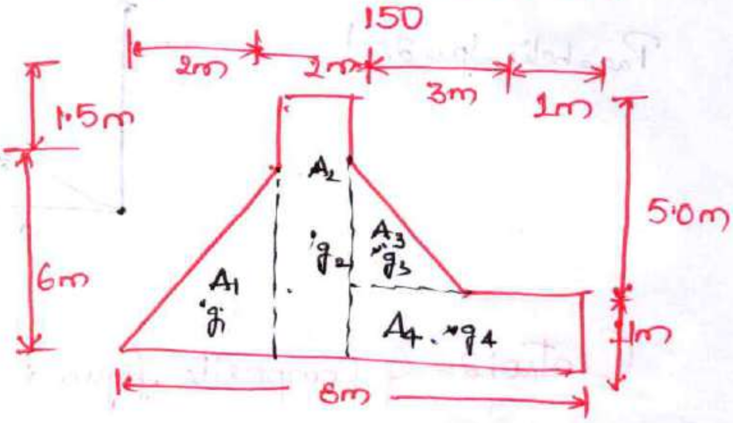
$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A}$$

$$= \frac{2000 \times 140 + 2000 \times 80 + 4500 \times 15}{2000 + 2000 + 4500}$$

$$= 59.71 mm$$



Centroid of the section



$A_1 = \frac{1}{2} \times 2 \times 6 = 6 m^2$

$A_2 = 2 \times 7.5 = 15 m^2$

$A_3 = \frac{1}{2} \times 3 \times 5 = 7.5 m^2$

$A_4 = 4 \times 1 = 4 m^2$

$\Sigma A_i =$

$x_1 = \frac{2}{3} \times 2 = \frac{4}{3} m$ | $y_1 = \frac{1}{3} \times 6 = 2m$

$x_2 = 2 + 1 = 3m$ | $y_2 = 7.5/2 = 3.75$

$x_3 = 2 + 2 + \frac{1}{3} \times 3m = 5m$ | $y_3 = 1 + \frac{1}{3} \times 5 = 8/3 m$

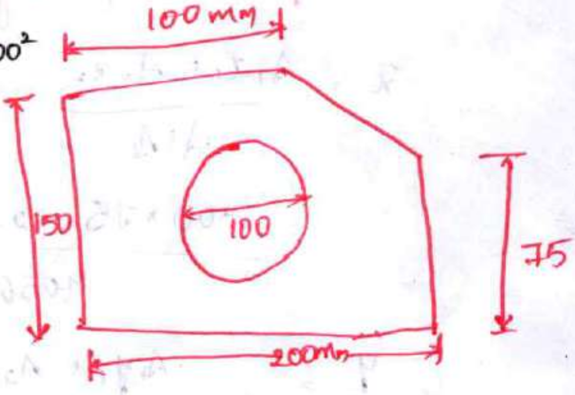
$x_4 = 4 + 4/2 = 6m$ | $y_4 = 0.5m$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 + A_4 x_4}{A_1 + A_2 + A_3 + A_4}$$

$$\bar{x} = \frac{6 \times 4/3 + 15 \times 3 + 7.5 \times 5 + 4 \times 6}{32.5} = 3.523 m$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 + A_4 y_4}{A_1 + A_2 + A_3 + A_4} = \frac{6 \times 2 + 15 \times 3.75 + 7.5 \times 8/3 + 4 \times 0.5}{32.5} = 2.77 m$$

Determine the coordinates x_c and y_c of the centre of a 100 mm diameter circular hole cut in a thin plate so that this point will be the centroid of the remaining shaded area.



Total area $= 200 \times 150 - (\frac{1}{2} \times 100 \times 75) - \frac{\pi}{4} \times 100^2$

$= 18396 mm^2$

$$\bar{x}_c = \frac{200 \times 150 \times 100 - \frac{1}{2} \times 100 \times 75 \times \left(200 - \left[\frac{100}{3}\right]\right) - \frac{\pi}{4} \times 100^2 \times x_c}{18396}$$

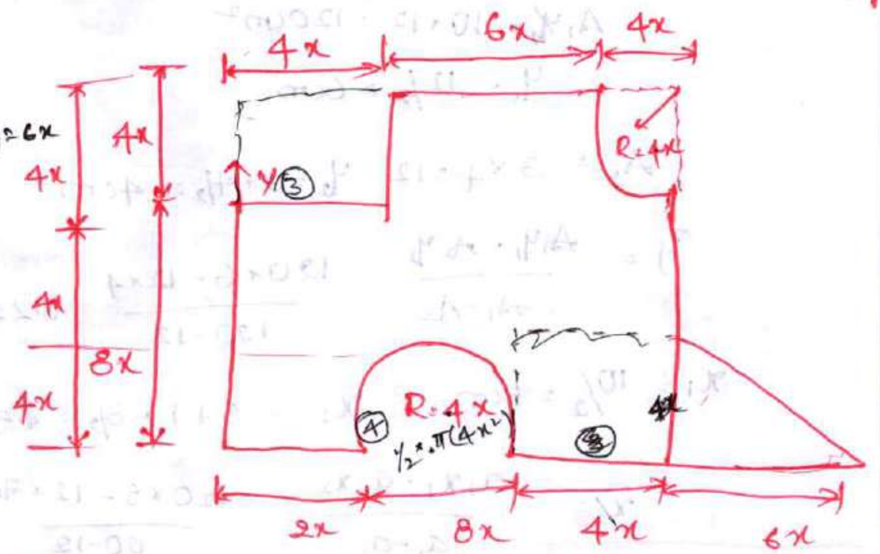
$$x_c \cdot 18396 = 200 \times 150 \times 100 - \frac{1}{2} \times 100 \times 75 \times 166.67 - \frac{\pi}{4} \times 100^2 x_c$$

$$x_c = 90.48 mm$$

$$18396 y_c = 200 \times 150 \times 75 - \frac{1}{2} \times 100 \times 75 \times (150 - 25) - \frac{\pi}{4} \times 100^2 y_c$$

$y_c = 67.86mm$

Determine the coordinates of the centroid of the plane area shown in fig. Take $x = 40mm$.



$A_1 = 14x \times 12x = 168x^2$, $x_1 = 7x$, $y_1 = 6x$

$A_2 = \frac{1}{2} 6x \times 4x = 12x^2$

$x_2 = 14x + 2x = 16x$

$y_2 = \frac{4x}{3}$

$A_3 = 4x \times 4x = 16x^2$

$x_3 = 2x \quad y_3 = 8x + 2x = 10x$

$A_4 = \frac{1}{2}\pi(4x)^2 = -8\pi x^2 \quad x_4 = 6x \quad y_4 = \frac{4R}{3\pi} = 4 \cdot \frac{4x}{3\pi} = \frac{16x}{3\pi}$

$A_5 = \frac{1}{4} \pi 4x^2 = -4\pi x^2 \quad x_5 = 14x - \frac{4R}{3\pi} = 14x - 4\left[\frac{4x}{3\pi}\right] = 12.3023x$

$y_5 = 12x - 4\left(\frac{4x}{3\pi}\right) = 10.3023x$

Total area $= 168x^2 + 12x^2 - 16x^2 - 8\pi x^2 - 4\pi x^2$

$= 126.3009x^2$

$\bar{x} = \frac{\Sigma Ax}{A}$

$\Sigma Ax = 168x^2 \times 7x + 12x^2 \times 16x - 16x^2 \times 2x - 8\pi x^2 \times 6x - 4\pi x^2 \times 12.3023x$

$= 1030.6083x^3$

$\bar{x} = \frac{\Sigma Ax}{A} = \frac{1030.6083x^3}{126.3009x^2} = 8.1599x$ Since $x = 40mm$

$= 326.4mm$

$\bar{y} = \frac{\Sigma Ay}{\Sigma A}$

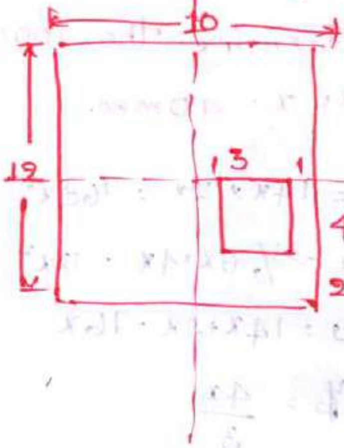
$\Sigma Ay = 168x^2 \times 6x + 12x^2 \times \frac{4x}{3} - 16x^2 \times 10x - 8\pi x^2 \cdot \frac{16x}{3\pi} - 4\pi x^2 \times 10.3023x$

$= 691.87x^3$

$\bar{y} = \frac{691.87x^3}{126.3009x^2} = 5.478x \quad [\because x = 40mm]$

$x = 219.12mm$

Find Centroid rectangular lamina ABCD 10cm×12cm a rectangular hole of 3m×4m is cut



$A_1 y_1 = 10\times12 = 120cm^2$

$y_1 = 12/2 = 6cm$

$A_2 = 3\times4 = 12 \quad y_2 = 2+4/2 = 4cm$

$\bar{y} = \dfrac{A_1y_1 - A_2y_2}{A_1 - A_2} = \dfrac{120\times6 - 12\times4}{120-12} = 6.22cm$

$x_1 = 10/2 = 5cm \quad ; \quad x_2 = 5+1+3/2 = 7.5$

$\bar{x} = \dfrac{a_1x_1 - a_2x_2}{a_1 - a_2} = \dfrac{120\times5 - 12\times7.5}{120-12} = 4.72cm$

$A_1 = 12\times6 = 72$

$A_2 = \frac{12\times6}{2} = 36$

$A_3 = \frac{\pi r^2}{2} = 18\pi$

$\bar{x}$

$x_1 = -6$

$x_2 = -4$

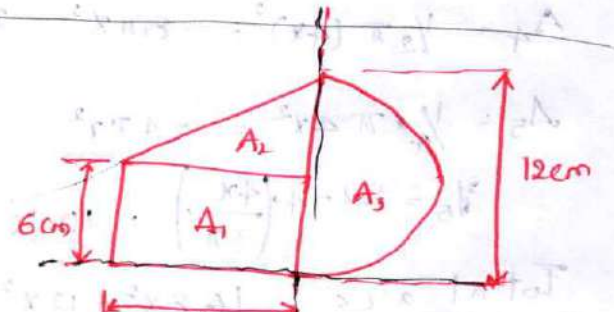
$x_3 = \frac{4\times6}{3\pi} = \frac{8}{\pi}$

$\bar{y}$

$y_1 = 3$

$y_2 = 8$

$y_3 = 6$



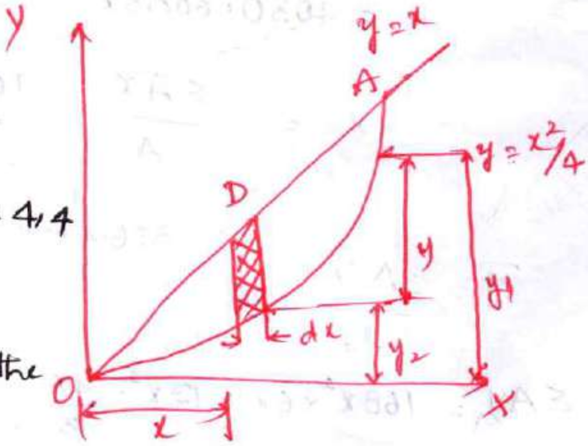
$\bar{x} = \dfrac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} = \dfrac{72(-6) + 36(-4) + 18\pi\times\frac{8}{\pi}}{72+36+18\pi} = -2.625 cm$

Determine the coordinates of C.G of shaded area b/n the Parabola $y = x^2/4$ and the straight line $y = x$

$x = x^2/4 \quad , \quad 4 = x^2/x = x$

$x = 4$ If $x = 4$ then $y = x$ $y = 4$

Hence the coordinates of point A are 4,4



let $dA = y\,dx = (y_1 - y_2)\,dx$

y_1 = Co-ordinate of point D which lies on the straight line OA

y_2 = Co-ordinate at point E

The values $y_1 = x$ and $y_2 = x^2/4$ $\quad dA = \left[x - \frac{x^2}{4}\right]dx \quad x^* = x$

$y^* = y_2 + \frac{y}{2} = y_2 + \frac{y_1 - y_2}{2} = \frac{2y_2 + y_1 - y_2}{2} = \frac{y_1 + y_2}{2} = \frac{x + x^2/4}{2} = \frac{1}{2}\left(x + x^2/4\right)$

Let $\bar{x}$ = Distance of CG of shaded area from y-axis

$\bar{y}$ = D " " from x-axis

$$\bar{x} = \frac{\int x^* \, dA}{\int dA} \qquad dA = \left[x - x^2/4\right] dx$$

$$\int x^* \, dA = \int_0^4 x\left[x - x^2/4\right] dx = \int_0^4 \left(x^2 - x^3/4\right) dx = \left[\frac{x^3}{3} - \frac{x^4}{16}\right]_0^4$$

$$= 16/3$$

$$\int dA = \int_0^4 \left[x - x^2/4\right] dx = \left[\frac{x^2}{2} - \frac{x^3}{12}\right]_0^4 = 16/6$$

$$\bar{x} = \frac{\int x^* \, dA}{\int dA} = \frac{16/3}{16/6} = 2$$

$$\bar{y} = \frac{\int y^* \, dA}{\int dA} \qquad y^* = 1/2\left[x + x^2/4\right] \qquad dA = \left[x - \frac{x^2}{4}\right] dx$$

$$\int y^* \, dA = \int_0^4 1/2\left[x + x^2/4\right]\left[x - x^2/4\right] dx$$

$$= 1/2 \int_0^4 \left(x^2 - x^4/16\right) dx = 1/2\left[\frac{x^3}{3} - \frac{x^5}{5 \times 16}\right]_0^4$$

$$= 64/15$$

$$\int dA = 16/6$$

$$\bar{y} = \frac{\int y^* \, dA}{\int dA} = \frac{64/15}{16/6} = 8/5$$

The Volume of a solid generated by revolving a plane area about a non intersecting axis in its plane, equal to the product of the area and length of the path travelled by centroid.

Theorems of Pappus & Guldins;

Pappus and Guldinus are two mathematicians developed theorems

Theorem 1;

Area of surface generated by revolving a plane curve about a non intersecting axis in the plane of the curve is equal to the product of length of curve and distance travelled by the centroid G of the curve during revolution

Proof;

Consider a curve of length l and let it be revolved about the OX-axis through 2π radians. Then an infinitesimally small element of length dL will generate a hoop of area $2\pi y dl$

Total surface area generated by the curve is given as

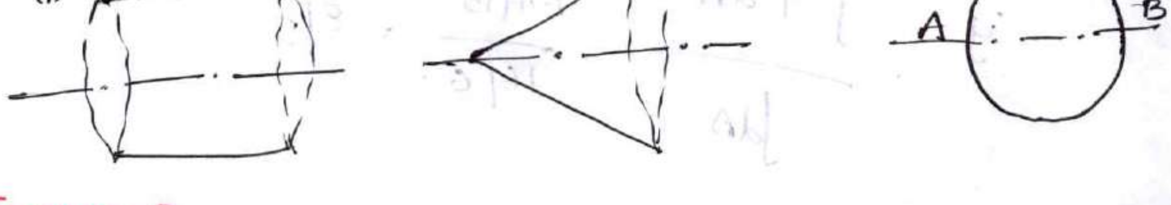
$$A = \int 2\pi y \, dL$$

$$2\pi \int y \cdot dL = 2\pi \bar{y} L \qquad \bar{y} = \frac{\int y \, dL}{\int dL}$$

Depending up on the generation of curve, the surface area generated are differen as shown. A straight line ∥le to the axis of revolution generates surface area of cylinder.

An inclined line with one end touching the axis of revolution generates surface area of a cone.

A semicircular arc with the ends touching the axis of revolution generates surface area of a sphere



Theorem-2

The volume of a solid generated by revolving a plane area about a non-intersecting axis in its plane is equal to the product of the area and length of the path travelled by centroid 'G' of the area about the axis

$$\bar{Y} = \frac{\int Y dA}{A}$$

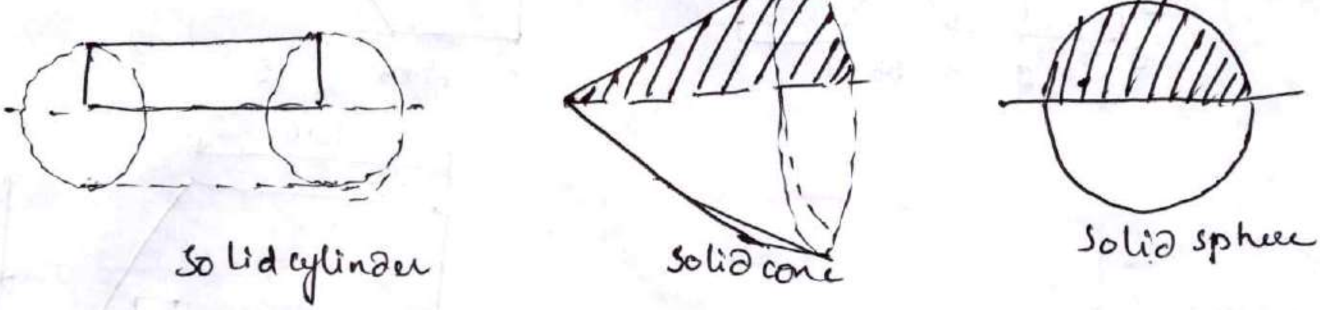
$$V = \int 2\pi y \, dA = 2\pi \int y \, dA = 2\pi \bar{y} A$$

Depending up on the generating area, the volumes generated are differentiated as shown.

A rectangular area when rotated about one of its sides generates volume of a cylinder.

A right angled triangle when rotated about a side other than the hypotenuse generates volume of cone.

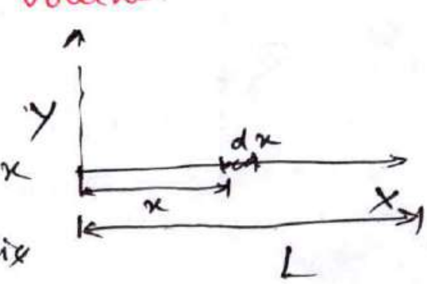
A semicircular area when rotated about its diameter generates volume of sphere.



Solid cylinder   Solid cone   Solid sphere

Determine the surface area and volume

Centroid of a straight line



Consider a small length 'dx' at a distance x from O then its first moment @ Y-axis

$$dM_y = x \times dx$$

First moment of entire length $\int_0^L x\,dx = L^2/2$

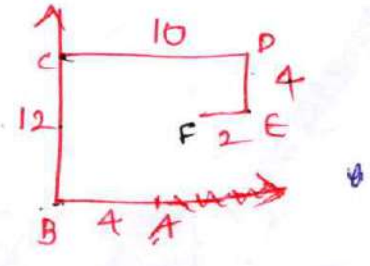
x-coordinate of the centroid is given as

$$\bar{x} = \frac{M_y}{L} = \frac{L^2/2}{L} = L/2$$

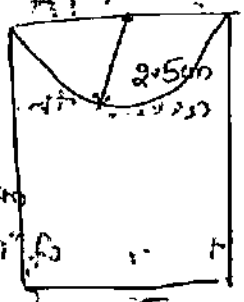
Can conclude that centroid of a straight line lies at the midpoint

Centroid of a wire bent

	x_i	y_i
AB = 4	2	0
BC = 12	0	6
CD = 10	5	12
DE = 4	10	12-(4/2) = 10
EF = 2	10-(2/2) = 9	8



of the area about the axis



Depending upon the [illegible]

area diff of the [illegible] as shown

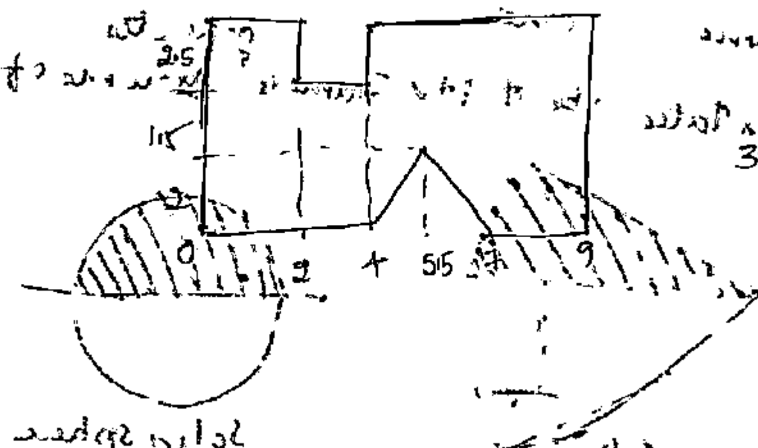
A rectangle [illegible]

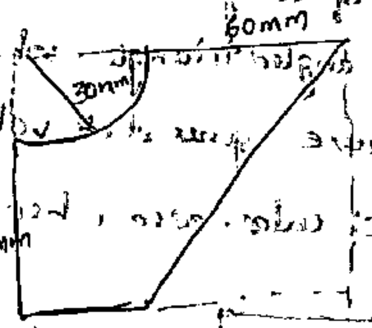
Volume of a cylinder

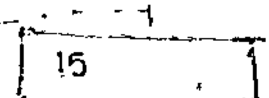
A right angled triangle

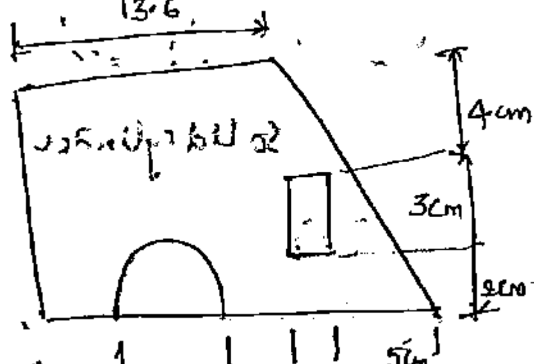
Hemisphere [illegible] volume of sphere

A semicircular [illegible]









Consider a small length dx at a distance x from the fixed end [illegible]

First moment of entire length

x - co-ordinate of the centroid is given

Can conclude that centroid [illegible] right lies at the midpoint

AB = 4

BC = 12

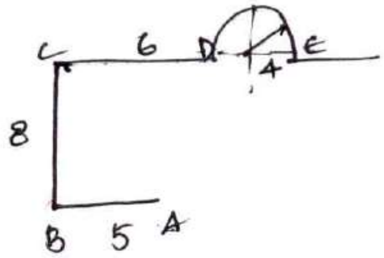
CD = 10

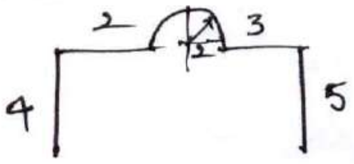
$$\bar{x} = \frac{L_1x_1 + L_2x_2 + L_3x_3 + L_4x_4 + L_5x_5}{L_1+L_2+L_3+L_4+L_5}$$

$$= \frac{(4\times 2) + (12\times 0) + (10\times 5) + (4\times 10) + 2(9)}{4+12+10+4+2} = 3.62\ cm$$

$$\bar{y} = \frac{L_1y_1 + L_2y_2 + L_3y_3 + L_4y_4 + L_5y_5}{L_1+L_2+L_3+L_4+L_5}$$

$$= \frac{(4\times 0) + (12\times 6) + (10\times 12) + (4\times 10) + 2\times 8)}{4+12+10+4+2} = 7.75\ cm$$





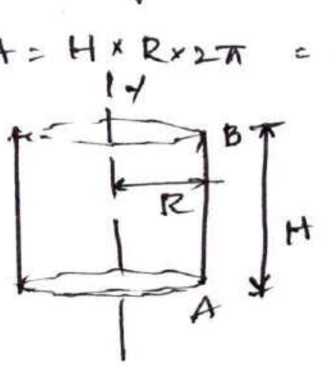
$\bar{x} = 4.7 \quad \bar{y} = 4.4$

$\bar{x} = 6.2 \quad \bar{y} = 6.5$

Determine the surface area and volume of a cylinder using Pappus and Guldinus theorem

Consider a straight line AB of length H parallel to Y-axis at a distance R from Y-axis. Resolving the line about the Y-axis through 360° will generate surface area of cylinder R

$A = H \times R \times 2\pi = 2\pi R \times H \qquad A = \text{length of curve} \times \bar{x} \times \theta$



Similarly considering a □^le ABCD and rotating about the Y-axis will generate a solid circular cylinder. Its volume can be

$V = \text{area of plane}\ \bar{x}\ \theta \quad = HR \times R/2\ 2\pi \quad = HR^2\pi$

UNIT- V

Moment of Inertia ;

Moment of force about any point is the product of force and the perpendicular b/n them. If this first moment is again multiplied by ⊥r distance between them, the product so obtained is called second moment of force. If instead of force the area of figure or mass of the body is considered, it is called second moment of area or second moment of mass. They are termed as moments of Inertia.

When mass moment of Inertia is used in conjunction with rotation of rigid bodies, it can be regarded as the measure of resistance of the body to rotation. Similarly area 'moment of Inertia', when used in conjunction with deflection or deformation of members in bending, can be regarded as the measure of resistance to bending.

Area of moment of Inertia ;

Consider a lamina of area A. let x = Distance of CG of area A from the axis OY.

y = Distance of CG of area A from the axis OX

Moment of area

= Area × ⊥r distance of C.G of area from axis OY

= A x

Eq is known as first moment of area about the axis OY.

This first moment of area is used to determine the centroid of gravity of the area.

If the moment of area given by eq is again multiplied by the ⊥r distance between the C.G of the area and axis OY then the quantity $(Ax)x = Ax^2$ is known as moment of the ~~moment~~ area ~~and~~ or second moment of area or area moment of Inertia about the axis OY. This second moment of area is used in

~~Second~~ moment of area about the axis OX = A Y

Second moment of area about OX axis $= AY \cdot Y = AY^2$

Moment of Inertia when mass is taken into consideration about OY axis $= mx^2$ and about OX axis $= my^2$

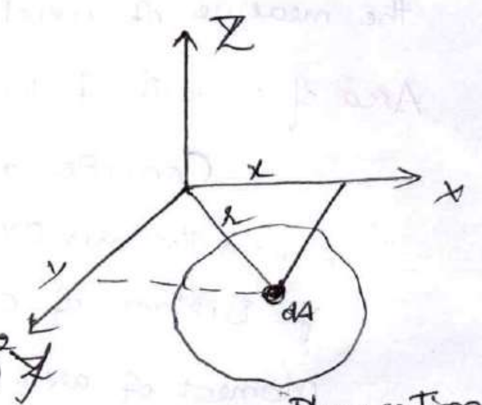
Theorem of Perpendicular axis ;

If I_{xx} and I_{yy} be the moment of inertia of a plane section about two mutually perpendicular axis X-X and Y-Y in the plane of the section. Then the moment of inertia of the section I_{zz} about the axis Z-Z ⊥r to the plane passing through the intersection of X-X and Y-Y is given by

$$I_{zz} = I_{xx} + I_{yy}$$

Proof ;

A plane section of area A and lying in plane x-y as shown. let OX and OY be the two mutually ⊥r axis, and OZ be the ⊥r axis. Consider a small area dA



Plane section of area A

x = Distance of dA from axis OY

y = Distance of dA from axis OX

r = Distance of dA from axis OZ

Then $r^2 = x^2 + y^2$

Now moment of inertia of dA about x-axis

$= dA \times (\text{Distance of } dA \text{ from } x\text{-axis})^2$

$= dA \times y^2$

Moment of inertia of total area A about x-axis $I_{xx} = \Sigma dA y^2$

Illy moment of inertia of total area A about Y-axis $I_{yy} = \Sigma dA x^2$

Moment of inertia of total area A about Z-axis $I_{zz} = \Sigma dA r^2$

$$\Sigma dA(x^2 + y^2)$$

$$\Sigma dA x^2 + \Sigma dA y^2$$

$$= I_{yy} + I_{xx}$$

$$I_{zz} = I_{xx} + I_{yy}$$

Above equation shows that the moment of inertia of an area about an axis at origin normal to x, y plane is the sum of moments of inertia about corresponding x and y axis. I_{zz} is known as polar moment of inertia.

Theorem of parallel axis:

If the moment of Inertia of a plane area about an axis in the plane of area through the C.G of the plane area be represented by I_G, then the moment of Inertia of the given plane area about a parallel axis AB in the plane of area at a distance h from C.G of the area is given by

$$I_{AB} = I_G + Ah^2$$

I_{AB} = Moment of Inertia of the given area about AB

I_G = M.I of " " " CG

A = Area of the section

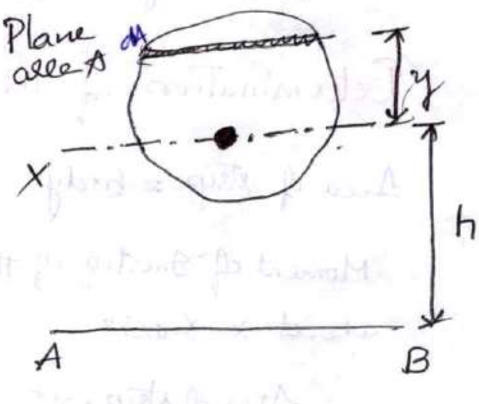
h = Distance b/n the C.G of the section and the axis AB.

Proof; A lamina of plane area A is shown in fig

let X-X = The axis in the plane area A and passing through the C.G of the area

AB = . " " " " and parallel to axis X-X

h = Distance b/n AB and X-X



Let the area of strip = dA

Moment of Inertia of area dA about X-X axis

$= dA\, y^2$

M.I of total area about X-X axis

$$I_{xx} \text{ or } I_G = \Sigma dA y^2$$

M.I of the area dA about AB

$$dA (h+y)^2$$

$$dA [h^2 + y^2 + 2hy]$$

M.I of total area A about AB

$$I_{AB} = \Sigma dA (h^2 + y^2 + 2hy)$$

$$= \Sigma dA h^2 + \Sigma dA y^2 + \Sigma dA 2hy$$

As h or h^2 is constant and hence they can be taken outside the summation sign. Hence the above equation becomes

$$I_{AB} = h^2 \Sigma dA + \Sigma dAy^2 + 2h \Sigma dAy$$

But $\Sigma dA = A$. Also from equation ① $\Sigma dAy^2 = I_G$ substituting these Values in the above eq, we get

$$I_{AB} = h^2 A + I_G + 2h \Sigma dAy$$

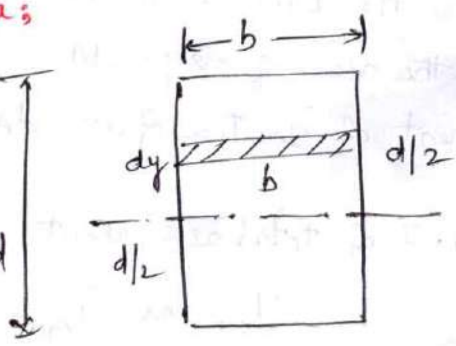
But $dA \times y$ represents the moment of area of strip about X-X axis. And ΣdAy represents the moments of the total area about X-X axis. But the moments of the total area about X-X axis is equal to the product of the total area from X-X axis is zero, hence ΣdAy will be equal to zero substituting this value in eq

$$I_{AB} = h^2 \times A + I_G + 0$$

$$I_{AB} = I_G + Ah^2$$

Thus if moment of Inertia of an area with respect to an axis in the plane of area is known, the M.I with respect to any ll axis in the plane may be determined by using the above eq.

Determination of Moment of Inertia:



Area of strip = $b \times dy$

Moment of Inertia of the area of strip about X-X axis

$$\text{Area of strip} \times y^2 = (b \times dy) \times y^2$$

$$= by^2 \times dy$$

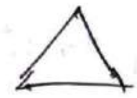
M.I obtained by Integration $-b/2$ to $b/2$

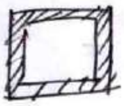
$$\int_{-d/2}^{d/2} by^2 dy = b\int_{-d/2}^{d/2} y^2 dy = b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2} = b/3\left[\left(\frac{d}{2}\right)^3 - \left[-\frac{d}{2}\right]^3\right]$$

$$b/3\left[\frac{d^3}{8} + \frac{d^3}{8}\right] = \frac{b}{3} \times \frac{2d^3}{8} = \frac{bd^3}{12}$$

$$I_{YY} = \frac{db^3}{12}$$

 $- \frac{bd^3}{12}$

 $= \frac{bh^3}{36}$

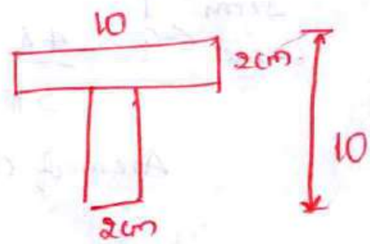
 $- \frac{BD^3}{12} - \frac{bd^3}{12}$

$= \frac{ML^2}{3}$

 $- \frac{\pi D^4}{64}$

 $- \frac{\pi}{64}(D^4 - d^4)$

T-section of dimensions 10 x 10 x 2. Determine the M.I of the section about the Horizontal and vertical axis passing through the C.G of see



$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(10 \times 2) \times 9 + (2 \times 8) \times 4}{20 + 16}$$

$$= \frac{180 + 64}{36} = 6.77 \text{ cm from bottom.}$$

$$I_{xx} = I_G + Ah^2$$

$$I_{G_1} = \frac{10 \times 2^3}{12} = 6.67 \qquad I_{G_2} = \frac{2 \times 8^3}{12} = 85.33$$

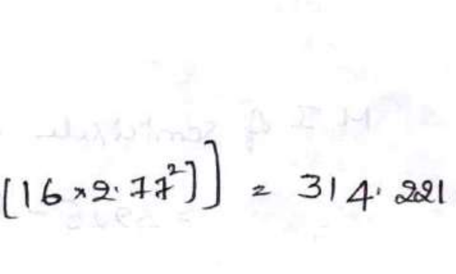
$$h_1 = y_1 - \bar{y} = 9.0 - 6.7 = 2.22$$

$$h_2 = \bar{y} - y_2 = 6.7 - 40 = 2.77$$

$$I_{xx} \left| I_{G_1} + Ah_1^2 \right] + \left[I_{G_2} + Ah_2^2 \right]$$

$$= \left[6.67 + (20 \times 2.22^2)\right] + \left[85.3 + [16 \times 2.77^2]\right] = 314.221 \text{ cm}^4$$

$$I_{yy} = \frac{2 \times 10^3}{12} + \frac{8 \times 2^3}{12} = 172 \text{ cm}^4$$

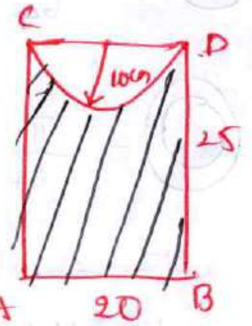


$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$\bar{y} = \frac{[(10 \times 2) \cdot (2 + 10 + 2/2)] + [10 \times 2)(2 + 10/2) + 20 \times 2 (2/2)}{20 + 20 + 40} = 5.5 \text{ cm}$$

$I_{xx} = \left(I_G + Ah^2\right)$

$$I_{xx} = \left[\frac{10 \times 2^3}{12} + (10 \times 2)(13-5.5)^2 + \frac{2 \times 10^3}{12} + (2 \times 10)(7.0-5.5)^2 + \frac{20 \times 2^3}{12} + (20 \times 2)(5.5-1)^2\right] = 2166.667 \text{ cm}^4$$



$$\text{M.I of rectangle} = \frac{bd^3}{12} = \frac{20 \times 25^3}{12} = 104.17 \text{ cm}^4$$

M.I of Semicircle = ½ (M.I of circle)

$$= \frac{1}{2}\frac{\pi d^4}{64} = \frac{1}{2} \times \frac{\pi}{64} \times 20^4 = 3.92 \text{ cm}^4$$

Semi circle CG $\bar{y} = \frac{4r}{3\pi} = \frac{4 \times 10}{3\pi} = 4.24 \text{ cm}$

Area of circle $A = \frac{\pi r^2}{2} = \frac{\pi \times 10^2}{2} = 157.1 \text{ cm}^2$

MOI of semicircle about AB = $1100.72 + 157.1 \times 20.76^2$

$= 68807.3$

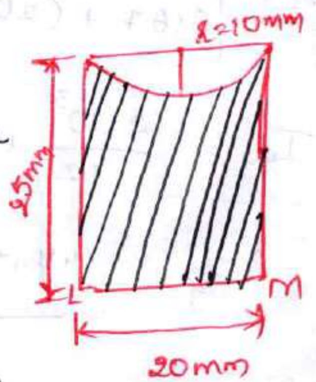
MOI of shaded portion about AB

$= 104.167 - 68807.3$

$= 35359.7 \text{ cm}^4$

MI of semicircle about CD

$= 3925 - 157.1 \times 4.24^2 = 1100.72 \text{ cm}^4$

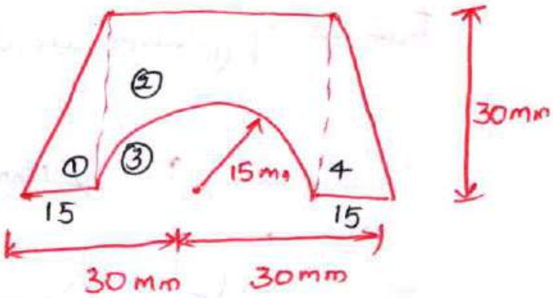


I_{LM} = MOI of ~~triangle~~ rectangle − MOI of Semicircle

$$= \left[\frac{20 \times 25^3}{12} + 20 \times 25 \times \left[\frac{25}{2}\right]^2\right] - \left[0.11 \times 10^4 + \frac{\pi \times 10^2}{2} \cdot \left[25 - \frac{4 \times 10}{3\pi}\right]^2\right] = 104166 - 68771$$

$= 35395 \, mm^4$

Find the centroidal moment of Inertia of the shaded area as shown in fig?



		'y' centroidal distance
Triangle 1	$\frac{1}{2}\times15\times30 = 225$	$30/3 = 10$
Rectangle 2	$30\times30 = 900$	$30/2 = 15$
Semicircle 3	$\pi/4\times15^2 = 353.4$	$4\times15/3\pi = 6.37$
Triangle 4	$\frac{1}{2}\times15\times30 = 225$	$30/3 = 10$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{15748.8}{996.6} = 15.8$$

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3} + I_{xx_4}$$

$$I_{xx} = \left[\frac{15\times30^3}{36} + \frac{1}{2}\times15\times30\left[15.8 - 30/2\right]^2\right] + \left[\frac{30\times30^3}{12} + 30\times30\times(15.8-15)^2\right]$$

$$-\left[0.11\times15^4 + \frac{\pi\times15^2}{12}\times\left(15.8 - \frac{4\times15}{3\pi}\right)^2\right] + \left[\frac{15\times30^3}{36} + \frac{1}{2}\times15\times30\left[15.8 - \frac{30}{3}\right]^2\right]$$

$$= 18819 + 68076 - 37023 + 18819 = 68691 \, mm^4$$

$$I_{yy} = I_{yy_1} + I_{yy_2} - I_{yy_3} + I_{yy_4}$$

$$= \left[\frac{30\times15^3}{36} + \frac{1}{2}\times30\times15\left[30 - 2/3\times15\right]^2 + \left[\frac{30\times30^3}{12}\right] - \left[\frac{\pi\times30^4}{64\times2}\right]\right]$$

$$+ \left[\frac{30\times15^3}{36} + \frac{1}{2}\times30\times15\times\left[30 - 2/3\times15\right]^2\right]$$

$$= 92812.5 + 67500 - 19800.4 + 92812.5 = 233244.6 \, mm^4$$

I_{xx}	I_{yy}	
$\frac{BD^3}{12}$	$\frac{DB^3}{12}$	Rectangle
$\frac{BD^3}{12} - \frac{bd^3}{12}$		Hallow Rectangle
$\frac{\pi D^4}{64}$		Circle
$\pi/64 \, (D^4 - d^4)$		Hallow circle

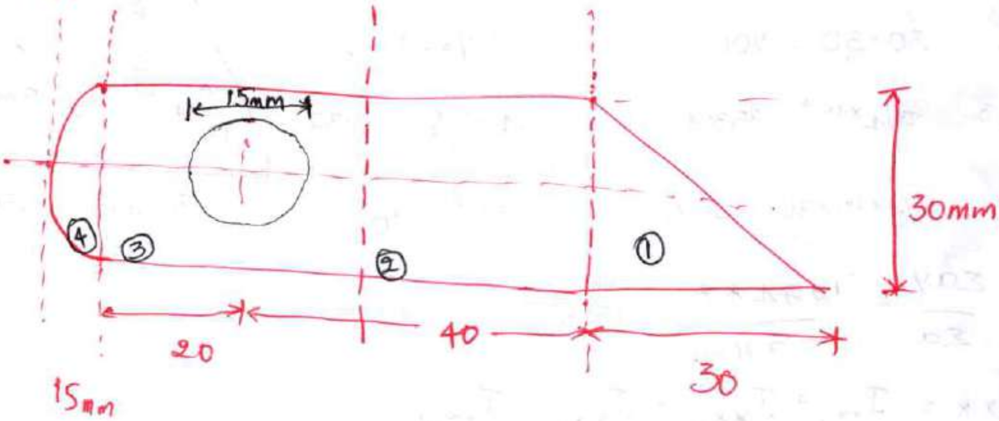
$bh^3/36$ - $h/3$ - Triangle

Find the following for shaded area

Position of the centroid

Second moment of area about the base

Radius of gyration about the base



	a	x	y	ax	ay
Triangle	$1/2 \times 30 \times 30$	$15+20+40+30/3 = 85$	$30/3 = 10$	38250	4500
Rectangle	60×30	$15 + \frac{60}{2} = 45$	$30/2 = 15$	81000	27000
circle	$\pi \times 7.5^2$	$15+20 = 35$	$30/2 = 15$	6184.8	2650.6
Semi circle	$\frac{\pi \times 15^2}{2} =$	$15 - \frac{4 \times 15}{3\pi} = 8.63$	$30/2 = 15$	3050.5	5301.4

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{116115.2}{2426.7} = 47.84 \text{ mm}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{34450.8}{2426.7} = 14.07 \text{ mm}$$

$$I_{LL} = I_G + Ah^2$$

= MOI of triangle (1) + MI of rectangle − MOI of circle + MOI of semicircle

$$= \frac{30 \times 30^3}{12} + \left[\frac{60 \times 30^3}{12} + 60 \times 30 \times 15^2\right] - \left[\frac{\pi \times 15^4}{64} + \frac{\pi \times 15^2}{4} \times 15^2\right] + \left(\frac{\pi \times 30^4}{64 \times 2} + \frac{\pi \times 30^2}{8} \times 15^2\right)$$

$$= 664656 \text{ mm}^4$$

Mass moment of Inertia;

Consider a body of mass M. let x = Distance of the centre of gravity of mass M from OY axis

y = Distance of the C-G of mass M from OX axis.

Then moment of the mass about the axis $OY = M \cdot x$

The above equation is known as first moment of mass about the OY axis

If the moment of mass given by the above equation is again multiplied by the ⊥r distance b/n the C-G of the mass and axis OY, then the quantity $(M \cdot x) x = M \cdot x^2$ is known as second moment of mass about the axis OY. This second moment of mass is known as mass moment of Inertia. Similarly, the second moment of mass or mass moment of Inertia about the axis OX $= My^2$

Hence the product of the mass and square of the distance of the C-G of the mass from an axis is known as the mass moment of Inertia about that axis. Mass moment of Inertia is represented by I_m. Hence

Mass moment of Inertia about the OX axis $= (I_m)_{xx}$

About the OY axis $= (I_m)_{yy}$

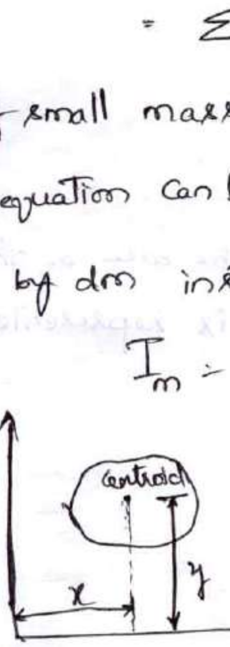
Consider a body which is split up into small masses $m_1, m_2, m_3, \ldots$ etc., let the C.G of the small areas from a given axis be at a distance of $r_1, r_2, r_3, \ldots$ etc. The mass moment of Inertia of the body about the given axis is given by

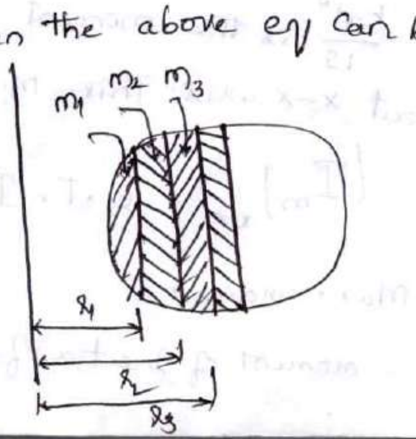
$$I_m = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots$$

$$= \Sigma m r^2$$

If small masses are large in number then the summation in the above equation can be replaced by integration. let the small masses are replaced by dm instead of 'm', then the above eq can be written as

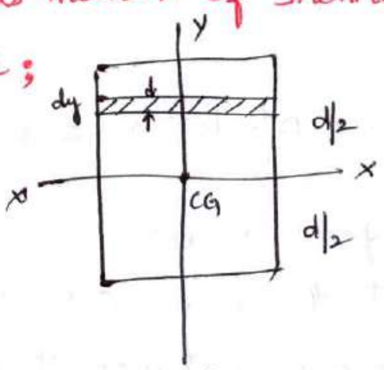
$$I_m = \int r^2 \, dm$$

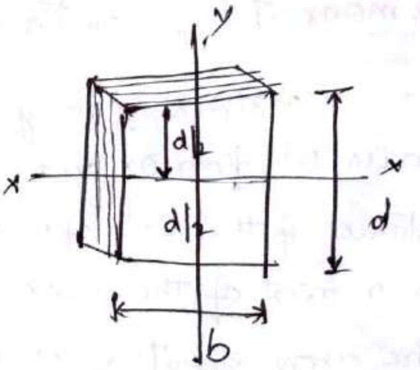




Derivation of mass moment of Inertia :

Rectangular plate ;





Rectangle plate of width b, depth d and uniform thickness t. Consider a small element of width b at a distance y from x-x axis

Here x - x axis the horizontal line passing through CG of the plate

Area of the element = b×dy

Mass of the element = Density × Volume of element

$= \rho \times (\text{Area} \times \text{Thickness})$

$= \rho \times (b \times dy \times t) = \rho\, bt\, dy$

Mass moment of Inertia of the element @ x-x axis

= Mass of element × y^2

$= M \times y^2$

$\rho\, b \times t \times dy \times y^2$

Mass moment of inertia of the plate will be obtained by integrating the above equation b/n the limits $-d/2$ to $d/2$

$$I_{m(xx)} = \int_{-d/2}^{d/2} \rho\, bt\, y^2\, dy = \rho\, bt \int_{-d/2}^{d/2} y^2\, dy = \rho\, bt \left[\frac{y^3}{3} \right]_{-d/2}^{d/2}$$

$$\frac{\rho bt}{3} \left[\left[\frac{d}{2}\right]^3 - \left[-\frac{d}{2}\right]^3 \right]^{-d/2} = \frac{\rho bt}{3} \left[\frac{d^3}{8} + \frac{d^3}{8} \right] = \frac{\rho bt}{3} \left[\frac{2d^3}{8} \right] = \frac{\rho bt}{3} d^3$$

$$= \rho \times t\, \frac{bd^3}{12}$$

But $\frac{bd^3}{12}$ is the moment of Inertia of the area of the rectangular section about x-x axis. This M.I of the area is represented by I_{xx}

$$(I_m)_{xx} = \rho \cdot t \cdot I_{xx}$$

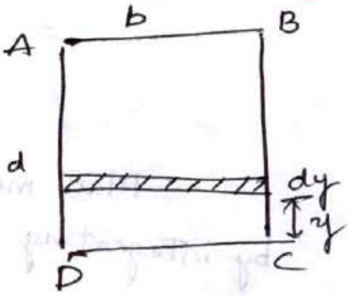
$(I_m)_{xx}$ = Mass moment.

I_{xx} = moment of Inertia of area

$$(I_m)_{xx} = \rho\frac{bd^3}{12} = \rho b\times d\times t\times\frac{d^2}{12} = M\times\frac{d^2}{12} = \frac{1}{12}Md^2$$

$$I_{myy} = \frac{1}{12}Mb^2$$

Mass moment of inertia of the rectangular plate about a line passing through the base;



Area of strip $dA = b\times dy$

Volume of strip $= dA\times t = b\times dy\times t = b\times t\times dy$

Mass of strip dm = Density × volume of strip

$= \rho(b\times t\times dy) = \rho bt\,dy$

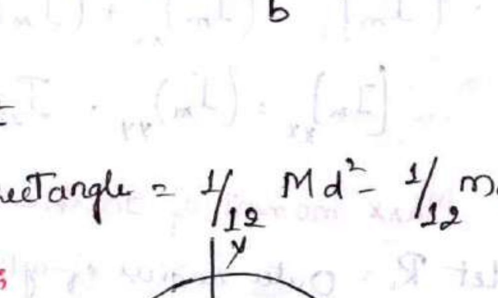
Mass moment of inertia of the strip $= M\times y^2 = dM\times y^2 = y^2 dm$

Mass moment of inertia of the whole rectangular plate about line CD is obtained by integrating the above eq limits 0 to d

$$\int_0^d y^2\times dm = \int_0^d y^2(\rho\times b\times t\times dy) = \rho bt\int_0^d y^2dy$$

$$= \rho bt\left[\frac{y^3}{3}\right]_0^d = \rho bt\times\frac{d^3}{3} = \rho btd\times d^2/3 = M\times d^2/3$$

Mass moment of inertia of a hallow rectangular plate



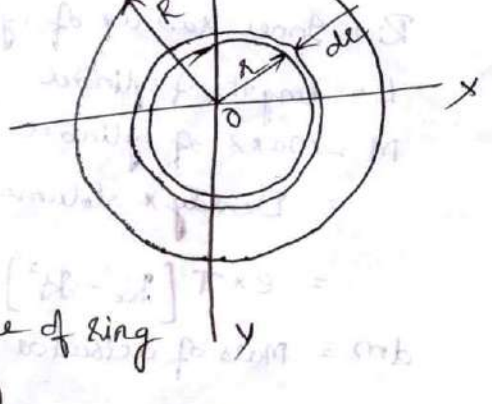
Mass moment of inertia of ABCD plate about X-X axis $= \frac{1}{12}Md^2$

Mass moment of inertia of EFGH about X-X axis $= \frac{1}{12}m d_1^2$

$M = \rho bdt \qquad m = \rho b_1 d_1 t$

Mass moment of inertia of a hallow rectangle $= \frac{1}{12}Md^2 - \frac{1}{12}md_1^2$

Moment of Inertia of a circular plate;



Area of ring $dA = 2\pi r\,dr$

Volume of ring = Area of ring × t

$= dA\times t$

$= 2\pi r\,dr\times t$

Mass of ring dm = Density × volume of ring

$= \rho\,(2\pi r\,dr\,t)$

In this case first the mass moment of Inertia about an axis passing through O and ⊥r to the plane of the paper about axis Z-Z

Mass moment of Inertia of the circular ring about axis Z-Z

$$= \text{Mass of ring} \times (\text{Radius of ring})^2$$

$$= dm \times r^2 = (\rho \times 2\pi r \times dr \times t)\, r^2$$

$$= \rho \times t \times 2\pi r^3 dr$$

Mass moment of Inertia of the whole circular plate will be obtained by integrating the above equation b/n 0 to R.

∴ Mass moment of Inertia of circular plate about Z-Z axis is given by

$$(I_m)_{zz} = \int_0^R \rho \times t \times 2\pi r^3 dr$$

$$= 2\pi \times \rho \times t \int_0^R r^3 dr = 2\pi \rho t \left[\frac{r^4}{4}\right]_0^R$$

$$= 2\pi \rho t \times \frac{R^4}{4} = \pi \times \rho \times t \times \frac{R^4}{2}$$

Now mass of circular plate

$$M = \rho \times \text{Volume of plate} \qquad \text{Volume} = \text{Area} \times t = \pi R^2 \times t$$

$$= \rho \times \pi R^2 \times t$$

Substituting this value in above eq

$$(I_m)_{zz} = \rho \times \pi R^2 \times t \times \frac{R^2}{2} = \frac{MR^2}{2}$$

But from the theorem of ⊥r axis $I_{zz} = I_{xx} + I_{yy}$

$$(I_m)_{zz} = (I_m)_{xx} + (I_m)_{yy} \qquad \text{Due to symmetry } I_{xx} = I_{yy}$$

$$(I_m)_{xx} = (I_m)_{yy} = I_{zz}/2 = \left[\frac{MR^2}{2}\right] \Big/ 2 = \frac{MR^2}{4}$$

Mass moment of Inertia of a Hollow circular cylinder

let R_o = Outer radius of cylinder

R_i = Inner radius of cylinder

L = length of cylinder

M = mass of cylinder

= Density × Volume of cylinder

$$= \rho \times \pi \left[R_o^2 - R_i^2\right] \times L$$

dm = Mass of a circular ring of radius 'r' width 'dr' and length L

$= \text{Density} \times \text{Volume of ring} = \rho \times \text{Area of ring} \times L$

$= \rho \times 2\pi r\,dr \times L$

Mass moment of Inertia of circular ring about Z-Z axis

$= \text{Mass of ring} \times \text{radius}^2$

$= (\rho \times 2\pi r\,dr \times L) \times r^2$

Mass moment of Inertia of a hollow circular cylinder will be obtained by integrating the above equation b/n the limits R_i to R_o

∴ Mass moment of Inertia of hollow circular cylinder about Z-Z axis

$$(I_m)_{zz} = \int_{R_i}^{R_o} (\rho \times 2\pi r\,dr \times L)\, r^2$$

$$\rho \times 2\pi \times L \int_{R_i}^{R_o} r^3\,dr = \rho \times 2\pi \times L \left[\frac{r^4}{4}\right]_{R_i}^{R_o} = \rho \times 2\pi \times L \left[\frac{R_o^4 - R_i^4}{4}\right]$$

$$R_o^4 - R_i^4 = (R_o^2 - R_i^2)(R_o^2 + R_i^2)$$

$$= \rho \times 2\pi \times L \left[\frac{R_o^2 - R_i^2}{4}\right](R_o^2 + R_i^2)$$

$$= \rho \times \pi\,[R_o^2 - R_i^2] \times L \times \frac{R_o^2 + R_i^2}{2} = \frac{M(R_o^2 + R_i^2)}{2}$$

$$I_{m_{xx}} = I_{m_{yy}} = \frac{(I_m)_{zz}}{2} = \frac{M(R_o^2 + R_i^2)}{4} \qquad [\rho \times \pi \times (R_o^2 - R_i^2) = M]$$

Mass moment of Inertia of a Right circular cone of Base radius R, Height H and mass M about its axis

let R = Radius of the base of the cone

H = Height of cone

M = Mass of cone

$= \text{Density} \times \text{Volume of cone} = \rho \times \frac{1}{3}\pi R^2 \times H$

Consider an elemental plate of thickness dy and of radius x at a distance y from vertex

$$\tan\alpha = \frac{x}{y} = \frac{R}{H}$$

$$x = \frac{R}{H} \times y$$

Mass of elemental plate

$dm = \rho \times \text{volume}$

$= \rho \times \pi x^2\,dy$

$$= \rho \left[\pi \frac{R^2 y^2}{H^2}\,dy\right]$$

The mass moment of Inertia of the circular elemental plate about the axis of the cone is given by eq

$$(I_m)_{zz} = \frac{\text{Mass of plate} \times \text{Radius}^2}{2} = \frac{(dm) \times r^2}{2} = \frac{(dm) \times x^2}{2}$$

$$= \left[\rho \times \frac{\pi R^2 y^2}{H^2} \times dy\right] \times \frac{x^2}{2}$$

$$= \left[\rho \cdot \frac{\pi R^2 y^2}{H^2} \times dy \times \left[\frac{R^2 y^2}{H^2}\right]\right] \cdot 1/2$$

$$= \frac{\rho \times \pi r^4 \times y^4}{2H^4} dy$$

Now the total mass moment of Inertia of circular cone will be obtained by integrating the above eq b/n the limits 0 to H.

$$(I_m)_{zz} = \int_0^H \frac{\rho \pi R^4 \times y^4}{2H^4} dy = \frac{\rho \pi R^4}{2H^4}\left[\frac{y^5}{5}\right]_0^H$$

$$\frac{\rho \pi R^4}{2H^4} \times \frac{H^5}{5} = \frac{\rho \pi R^4 \times H}{2 \times 5}$$

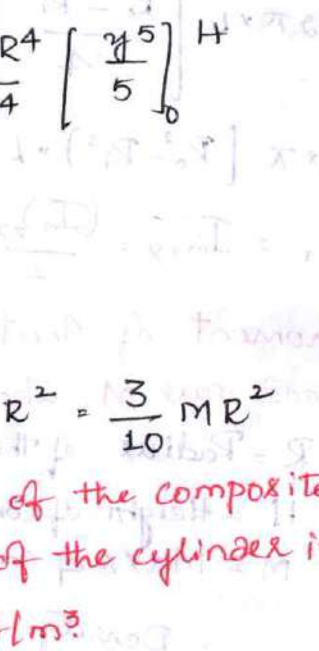
Mass of cone $M = \frac{\rho \pi R^4 \times H}{3}$

$$[I_m]_{zz} = \frac{\rho \pi R^2 \times H}{3} \times \frac{R^2 \times 3}{10} = M \times \frac{3}{10} \times R^2 = \frac{3}{10} MR^2$$

Determine the mass moment of Inertia of the composite body about Z-axis shown in fig. The mass density of the cylinder is 6000 kg/m³ and the rectangular prism is 7000 kg/m³.

mass density of cylinder $\rho_1 = 6000 \, kg/m^3$

mass density of rectangular prism $\rho_2 = 7000 \, kg/m^3$



Mass of cylinder $M_1 = \rho_1 \times A_1 \times L$

$= 6000 \pi r^2 \times 0.08$

$= 6000 \times \pi \times 0.02^2 \times 0.08$

$= 0.6 \, kg$

Mass of rectangular prism $M_2 = \rho_2 \times \text{Volume of prism}$

$= 7000 \times [0.08 \times 0.04 \times 0.12] = 2.69\,kg$

Mass moment of Inertia of the cylinder about Z-axis

$= \frac{1}{2} m r^2 = \frac{1}{2} M_1 \times 0.02^2 = \frac{1}{2} \times 0.6 \times 0.02^2 = 1.2 \times 10^{-4}\,kgm^2$

Mass moment of Inertia of $\square^{r}$ prism about centroidal axis ∥ to Z-axis

$$I_z = \frac{1}{12} m(a^2+b^2) = \frac{1}{12} \times M_2 \times (0.08^2 + 0.12^2)$$

$$= 4.66 \times 10^{-3}\,kg\,m^2$$

Mass moment of Inertia of rectangular prism about given Z-axis

$I_z + M_2 \times d^2$ $\qquad d = \frac{120}{2} - \frac{40}{2} = 60 - 20 = 40\,mm$

$= 4.66 \times 10^{-3} + 2.69 \times 0.04^2$

$= 8.96 \times 10^{-3}\,kg\,m^2$

Mass moment of Inertia of a composite body about Z-axis

= MOI of cylinder about Z-axis + MOI of prism about Z-axis

$= 0.12 \times 10^{-3} + 8.96 \times 10^{-3} = 9.08 \times 10^{-3}\,kg\,m^2$

Mass moment of Inertia

$I_{xx} = I_{yy}$ = Circle = $MR^2/4$ $\qquad I_{zz} = \frac{MR^2}{2}$

Prism circular

$I_{zz} = \frac{3}{10} MR^2$

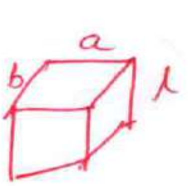
Rectangular section

$I_{xx} = I_{xy} = \frac{1}{12} Md^2 \qquad I_{yy} = \frac{1}{12} Mb^2$

Solid sphere

$I_{xx} = I_{yy} = \frac{2}{5} MR^2$

Parallelepiped $I_z = \frac{1}{12} M(a^2+b^2)$



Cylinder $= MR^2/2$

parallel axis theorem $I_A = I_g + Md^2$

Assignment marks I B.Tech

157 - 9

169 - 9

168 - 9

161 - 9

162 - 9

158 - 10

175 - 9

164 - 9

173 - 9

149 - 9

147 - 8

163 - 10

145 - 8

171 - 8

153 - 7