ANNAMACHARYA UNIVERSITY

EXCELLENCE IN EDUCATION; SERVICE TO SOCIETY

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Rajampet, Annamayya District, A.P – 516126, INDIA

CIVIL ENGINEERING

Lecture Notes
on
Fluid Mechanics, Hydraulics & Hydraulic Machinery

Written by

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Title of the Course: **Fluid Mechanics, Hydraulics and Hydraulic Machinery**
Category: PCC
Semester: III Semester
Couse Code: 24ACIV33T
Branch(s): CE

Lecture Hours	Tutorial Hours	Practice Hours	Credits
3	-	-	3

Course Objectives:

1. To introduce fundamental fluid properties, fluid statics concepts, and methods of pressure measurement.
2. To explain fluid kinematics and dynamics, including flow classification, governing equations, and their applications.
3. To develop understanding of flow through pipes, including energy losses, flow regimes, and pipe network analysis.
4. To study open channel flow characteristics, critical flow computation, and flow measurement techniques.
5. To familiarize students with hydraulic turbines, pumps, their working principles, design, and performance characteristics.

Course Outcomes:

At the end of the course, the student will be able to

1. Identify and explain physical properties of fluids, fluid statics principles, and analyze forces on submerged surfaces.
2. Apply fluid kinematics and dynamics principles to classify flow types and solve problems using continuity, Bernoulli, and momentum equations.
3. Analyze pipe flow systems by calculating major and minor losses, understanding laminar and turbulent flows, and solving pipe network problems.
4. Evaluate open channel flow parameters, classify flow regimes, and design hydraulic structures like weirs and notches.
5. Understand the operation and efficiency of hydraulic turbines and pumps, interpret characteristic curves, and address issues like cavitation and surge.

Unit 1 Basic Definitions and Fluid statics 12

Properties of fluids: mass density, specific weight, specific volume, specific gravity, viscosity, compressibility, vapor pressure, surface tension and capillarity. Pressure measurement devices - manometers, pressure gauges.

Fluid statics: Fluid Pressure, Pressure at a point, Hydrostatic law, Pascal's law, Atmospheric, Gauge and Absolute pressure, Equation of state, Hydrostatic forces on submerged plane and curved surfaces, Total pressure and Centre of Pressure, Practical applications-Dams and Gates, Buyoyancy, Buyoyant force, Centre of Buoyancy.

Unit 2 Fluid Kinematics and Fluid Dynamics 12

Fluid Kinematics : Classification of fluid flow-Steady and Unsteady flow, Uniform and Non-uniform flow, Laminar and Turbulent flow, Rotational and Irrotational flow, One, two and three dimensional flows; Stream line, Path line and stream tube, Velocity, Acceleration and rotation of fluid particles

Fluid Dynamics : Derivation of continuity equation-one-dimensional, three dimensional; stream function, velocity potential; Flow net-uses. Euler's Equation, Bernoulli's Equation, Applications of Bernoulli's equation-

Venturimeter, Orifice Meter and Pitot tube, Momentum equation, Momentum correction factor, Applications of momentum equation.

Unit 3 Flow through pipes **12**

Reynold's experiment, energy losses-major and minor losses; Laws of fluid friction; Darcy-Weisbach equation, Hydraulic Grade Line and Total Energy Line; Pipes in series and parallel; Equivalent pipe, Branched pipe, Siphon, Water Hammer in pipes,

Laminar flow-Laminar flow through circular pipes-Hazen poiseuille law; Laminar flow between parallel plates-Bothplates at rest, one plate moving and other at rest, Turbulent flow-Hydrodynamically smooth and rough boundaries, resistance to flow of fluid in smooth and rough pipes, Moody'sdiagram

Unit 4 Open Channel Flow **12**

Types of channels – Velocity distribution – Chezy's, Manning's and Bazin's formulae for uniform flow – Most Economical sections - Critical flow – Specific Energy - Critical depth – Computation of critical depth – Critical, subcritical and super critical flows – Velocity measuring instruments. Non uniform flow - Dynamic equation for gradually varied flow - Mild, critical, steep, horizontal and adverse slopes – Surface profiles - Rapidly varied flow - Hydraulic jump and its applications - Energy dissipation

Weirs and notches: Flow over Notches and Weirs: Types of Notches and Weirs; Flow over - Rectangular, Triangular, Trapezoidal Notches and Weirs.

Dimensional Analysis :methods of Dimension Analysis, types of Similarities and Similarity Laws

Unit 5 Hydraulic Turbines and Centrifugal Pumps **12**

Hydraulic Turbines : Layout of a typical hydropower installation-Heads and efficiencies-classification of turbines-Pelton wheel- Francis turbine-Kaplan turbine-working proportions- Velocity diagrams-Work done and efficiency- Hydraulic design, Surge tanks, Cavitation.

Centrifugal pumps: pump installation details-Heads-Losses and efficiencies-Limitation of suction lift-Work done-Minimum starting speed-Specific speed- Multistage pumps-Pumps in parallel-Performance of pumps-Characteristic curves-Net Positive Suction Head-Priming devices

Prescribed Textbooks:

1. P. M. Modi and S. M. Seth, Hydraulics and Fluid Mechanics, Standard Book House 22nd, 2019.
2.K. Subrahmanya, Theory and Applications of Fluid Mechanics, Tata McGraw Hill, 2nd edition 2018

Reference Textbooks:

1. R. K. Bansal, A text of Fluid mechanics and hydraulic machines, Laxmi Publications(P) Ltd., New Delhi 11th edition, 2024.
2. N. Narayana Pillai, Principles of Fluid Mechanics and Fluid Machines, Universities Press Pvt Ltd, Hyderabad. 3rd Edition 2009.
3. Fluid Mechanics by Frank M. White, Henry Xue, Tata McGraw Hill, 9th edition , 2022.
4. C. S. P. Ojha, R. Berndtsson and P. N. Chadramouli, Fluid Mechanics and Machinery, Oxford University Press, 2010.
5. Introduction to Fluid Mechanics & Fluid Machines by S K Som, Gautam Biswas, S Chakraborty Tata McGraw Hill, 3rd edition 2011.

CIVIL ENGINEERING

Fluid Mechanics, Hydraulics & Hydraulic Machinery

UNIT-1

UNIT- I

Fluid properties

Matter
- solid → Intermolecular attractive forces are very strong in solid so that they have definite shape & volume.
- fluid
  - liquids
  - gases

→ In liquid - comparitively less than solids. they donot have shape but it have definite volume.

→ In Gases - Extremely weak attractive force. neither definite shape nor volume

→ liquid at rest cannot resist shear, it flows.

A fluid is a substance that which deform continuously under the action of shear force & cannot regain its original position. In a continum approach the fluid is treated a continuous mass of substance a statical average distance travelled by two molecules in very small mean free path is very small kundsen number as $1/L$.

properties of fluids

1. Density (or) Mass Density : Density of a fluid is defined as the ratio of the mass of a fluid to its volume. It is denoted by the symbol ρ (rho). The unit of density in SI is kg/m^3. The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.

$$\rho = \frac{\text{Mass of fluid}}{\text{volume of fluid}}$$

The value of density of water is $1\ gm/cm^3$ (or) $1000\ kg/m^3$.

2. Specific weight (or) Weight Density :

Specific weight of a fluid is the ratio between the weight of a fluid to its volume. It is denoted by symbol(w).

$$w = \frac{\text{weight of fluid}}{\text{volume of fluid}} = \frac{\text{mass} \times g}{V} = \rho g.$$

The value of 'w' for water - 9810 N/m^3 in SI units.

3. Specific volume :

Specific volume of fluid is defined as the volume of a fluid occupied by a unit mass (or) volume per unit mass.

$$\text{specific volume} \quad v = \frac{\text{volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\rho}$$

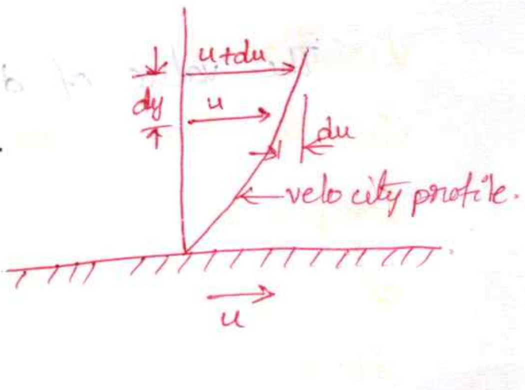
4. Specific Gravity :

It is defined as the ratio of the weight density of fluid to the weight density of standard fluid. For liquids, the standard fluid is taken water and for gases, the standard fluid is air.

$$S\,(or)\,G = \frac{\text{weight density [density] of liquid}}{\text{weight density [density] of std fluid}}.$$

Viscosity : Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid. When two layers of a fluid, a distance 'dy' apart, move one over the other at different velocities, say u and u+du as shown in fig., the viscosity together with relative velocity causes a shear stress acting between the fluid layers.

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity w.r.t y.



$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy} \Rightarrow \mu = \frac{\tau}{\left(\frac{du}{dy}\right)}.$$

Thus viscosity is also defined as the shear stress required to produce unit rate of shear strain.

CGS unit of viscosity $= \frac{dyne-sec}{cm^2}$

The unit of 'μ' in CGS is also called Poise which equal to $\frac{dyne-sec}{cm^2}$

SI unit of viscosity $= N\,s/m^2 = Pa \cdot s$.

"The viscosity of water at 20°C is 0.01 poise or 1 centipoise."

② **Kinematic viscosity:** It is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by ν [nu].

$$\nu = \frac{\mu}{\rho} = \left\{ \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \, \frac{\text{Mass}}{(\text{Length})^3}} = \frac{(\text{Length})^2}{\text{time}} \right\}$$

The unit of kinematic viscosity is m^2/sec. In CGS units, kinematic viscosity is also known as stoke.

One stoke $= 10^{-4}\, m^2/s$.

Newton's law of viscosity:

It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the coefficient of viscosity.

$$\tau = \mu \frac{du}{dy}$$

Fluids which obey the above relation are known as Newtonian fluids and ~~the~~ which donot obey the above relation are called Non-Newtonian fluids.

Variation of viscosity with temperature: Temperature affects the viscosity. The viscosity of ~~fluids~~ liquids decreases with increase of temperature while the viscosity of gases increases with the increase of temperature. This is due to reason that the viscous forces in a fluid are due to cohesive forces and molecular momentum transfer. In liquids, the cohesive forces predominates the molecular momentum transfer, due to closely packed molecules and with increase in temperature, the cohesive forces decreases with the result of decreasing viscosity. But in case of gases the cohesive forces are small and molecular momentum transfer predominates. With the increase in temparature, molecular momentum transfer increases and hence viscosity increases.

(i) For liquids, $\mu = \mu_0 \left[\frac{1}{1+\alpha t+\beta t^2}\right]$

where μ = viscosity of liquid at $t°c$, in poise

μ_0 = Viscosity of liquid at $0°c$, in poise

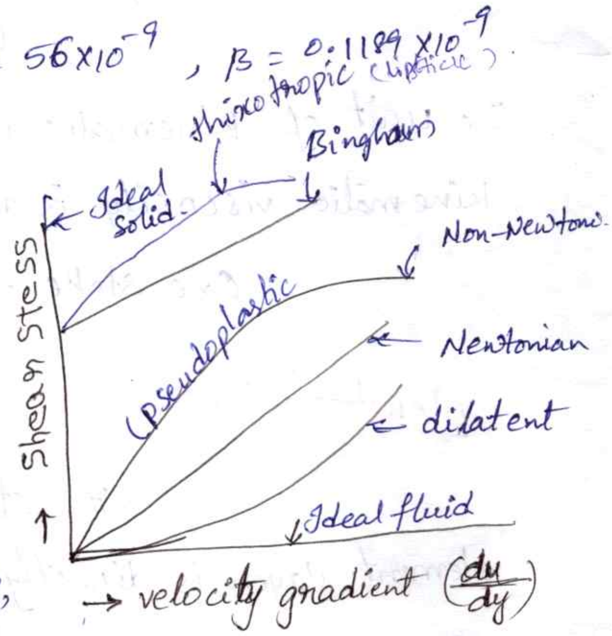
α, β = constants for the liquid.

For water, $\mu_0 = 1.79 \times 10^{-3}$ poise, $\alpha = 0.03368$ and $\beta = 0.000221$.

(ii) For a gas, $\mu = \mu_0 + \alpha t - \beta t^2$

where for air $\mu_0 = 0.000017$, $\alpha = 56 \times 10^{-9}$, $\beta = 0.1189 \times 10^{-9}$

Types of Fluids :



① Ideal fluid : A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid.

② Real Fluid : A fluid, which possesses viscosity is known as real fluid. All fluids, in actual practice are real fluids.

③ Newtonian Fluid : A real fluid, in which the shear stress is directly proportional to the rate of shear strain is known as Newtonian

④ Non-Newtonian fluid : A real fluid in which the shear stress is not proportional to the rate of shear strain, known as Non-Newtonian fluid.

⑤ Ideal plastic fluid [Binghum] : A fluid in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain, is known as ideal plastic fluid. [Sewage sludge, toothpaste.

⑥ Dilatent : Viscosity increases with increases rate of strain

Eg : Sugar solution, butter, quicksand.

for Non-Newtonian fluids $\tau = \mu \left(\frac{du}{dy}\right)^n$

(paper pulp, rubbers) Pseudo plastic — $n<1$
(quicksand, butter, printing inks) dilent — $n>1$ } Time Independent.

sewage sluge, drilling muds. Bingham = $\tau_0 + \mu \frac{du}{dy}$.

Time dependent Non-Newtonian fluids

$$\tau = \mu \frac{du}{dy} + f(t)$$

Thixotropic : τ = If f(t) is decreases then it is thixotropic.

Eg: paints., lipstick

Rheopectic : If f(t) is increases then it is Rheopectic.

Eg: solid - liquid combination. gypsum past

Thermodynamic properties :

Fluids consists of liquids or gases. But gases are compressible fluids and hence thermodynamic properties play an important role. with the change of pressure and temperature, the gases undergo large variation in density. The relationship between pressure (absolute), specific volume and temperature of a gas is given by the equation of state as

$\forall$ = specific volume = $\frac{1}{\rho}$

$$P\forall = RT \quad \text{or} \quad P = \rho RT$$

where P = absolute pressure of a gas: in N/m^2
R = Gas constant
T = Absolute temperature in °K, ρ = density of gas.

Dimension of 'R'. The gas constant R, depends upon the particular gas. The dimension of R is obtained from the eq $R = \frac{P}{\rho T}$

(i) In MKS units $R = \frac{kgf/m^2}{\left(\frac{kg}{m^3}\right) °K} = \frac{kgf - m}{kg \; °K}$.

ii) In SI units, P is in N/m^2 $R = \frac{N\text{-}m}{kg\text{-}K} = \frac{\text{Joule}}{kg\text{-}K}$

for air, R in MKS = 29.3 $\frac{kgf\text{-}m}{kg\text{-}K}$

R in SI = 287 J/kg-K

Isothermal Process : If the change in density occurs at constant temperature, then the process is called isothermal and relationship between pressure (P) and density (ρ) is given by

$$\frac{P}{\rho} = \text{constant}$$

Adiabatic Process : If the change in density occurs with no heat exchange to and from the gas, the process is called adiabatic. And if no heat is generated within the gas due to friction, the relationship between pressure and density is given by

$$\frac{P}{\rho^k} = \text{constant}$$

where k is ratio of specific heat of a gas at constant pressure and constant volume, k = 1.4 for air.

Universal Gas Constant :

let m = Mass of a gas in kg, ∀ = volume of gas of mass m, P = abs. press.

we have $P\forall = mRT$, R = gas constant.

The above equ can be made universal, i.e., applicable to all gases if it is expressed in mole-basis.

let n = no. of moles in volume of a gas.

∀ = Volume of the gas

$$M = \frac{\text{mass of the gas molecules}}{\text{Mass of a hydrogen atom}}$$

m = Mass of gas in kg.

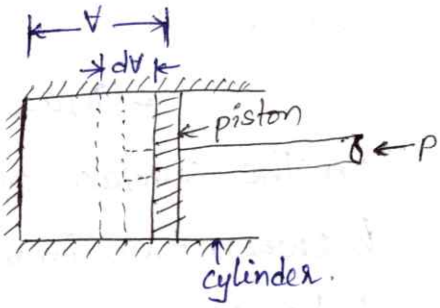
$nM = m$.

$$P\forall = n \times M \times RT$$

The product $M \times R$ is called universal gas constant and is equal to $848 \frac{kgf\text{-}m}{kg\text{-}mole\ ^\circ K}$ in MKS units, 8314 in SI units.

⊛ One kilogram mole is defined as the product of one kilogram mass of the gas and its molecular weight.

⊛ Bulk Modulus and Compressibility :



compressibility is the reciprocal of the bulk modulus of elasticity, k which is defined as the ratio of compressive stress to volumetric strain.

consider a cylinder fitted with a piston as shown in fig.

let $\forall$ = volume of a gas enclosed in the cylinder.

P = Pressure of gas when volume is $\forall$.

Let the pressure is increased to $p+dp$, the volume of gas decreases from $\forall$ to $\forall - d\forall$

The increase in pressure $= dp$ kgf/m², $K = 2\times10^{9}$ N/m²

$\therefore$ Volumetric strain $= -\dfrac{d\forall}{\forall}$ [∴ $d\forall$ = decrease in volume]

–ve sign means the volume decreases with increases of pressure.

$$\therefore \text{Bulk modulus} \quad K = \frac{\text{Increase of pressure}}{\text{Volumetric strain}} = \frac{dp}{\left(\frac{-d\forall}{\forall}\right)}$$

⊛ → As temperature ↑ for liquids k will ↓, for gas as temp ↑, k ↑.

compressibility $= \dfrac{1}{K}$.

Relationship between Bulk Modulus (K) and Pressure (P) of a Gas :

(i) For Isothermal Process :

$$\frac{P}{\rho} = \text{constant} = P\forall$$

Differentiating this equation, we get (P and $\forall$ both are variables)

$$Pd\forall + \forall dp = 0 \quad \text{(or)} \quad Pd\forall = -\forall dp, \quad \text{(or)} \quad P = \frac{-dp}{\left(\frac{d\forall}{\forall}\right)}$$

$$\boxed{K = P}$$

(ii) For Adiabatic process : $\dfrac{P}{\rho^k}$ = constant $= P\forall^k$

Differentiating, $P(d\forall^k) + \forall^k dp = 0$.

$$P k \forall^{k-1} d\forall + \forall^k dp = 0$$

$$Pk\, d\forall + \forall dp = 0 \quad \text{(or)} \quad Pk = \frac{dp}{-\frac{d\forall}{\forall}}$$

$$K = Pk$$

where K = Bulk modulus, k = Ratio of specific heats.

Surface Tension And Capillary :

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas (or) on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension. The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. S.T values drop with rise in temperature

→ Surface Tension on Liquid Droplet :

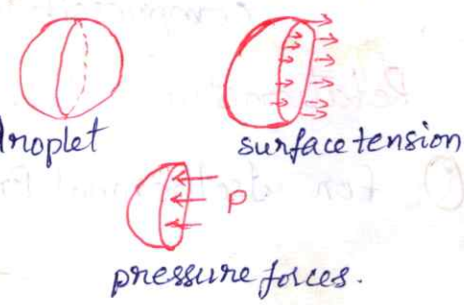
Consider a small droplet of a liquid of radius 'r'. On the entire surface of the droplet, the tensile force due to surface tension will be acting.

Let σ = Surface tension of the liquid.
p = pressure intensity inside the droplet
d = Dia. of droplet

Let the droplet is cut into two halves. The forces acting on one half will be

i) tensile force due to surface tension acting around the circumference of the cut portion as shown fig. and this is equal to $= \sigma \times$ Circumference $= \sigma \times \pi d$

ii) pressure force on the area $\frac{\pi}{4} d^2 = p \times \frac{\pi}{4} d^2$



These two forces will be equal and opposite under equilibrium conditions, i.e.

$$p \frac{\pi}{4} d^2 = \sigma \times \pi d \Rightarrow p = \frac{4\sigma}{d}$$

This equation shows that with the decrease of dia of droplet, pressure intensity inside the droplet increases.

⇒ Surface Tension on a Hollow Bubble :

A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces are subjected to surface tension. In such case, we have

$$p \times \frac{\pi}{4} d^2 = 2(\sigma \times \pi d)$$

$$p = \frac{8\sigma}{d}.$$

Surface Tension on a Liquid Jet :

Consider a liquid jet of dia 'd' and length 'L' as.

let P = pressure intensity inside the liquid jet above the outside pressure

σ = Surface tension of the liquid.

Consider the equilibrium of the semi Jet, we have

Force due to pressure = P × area of semijet = P × L × d

Force due to surface tension = $\sigma \times 2L$

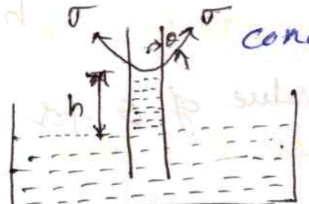
Equating the forces, we have $P \times L \times d = \sigma \times 2L$

$$P = \frac{\sigma \times 2L}{L \times d}$$

" σ = 0.073 N/m for air-water interface, σ = 0.48 N/m for air-mercury interface

Capillarity : Capillarity is defined as a phenomenon of rise or fall of liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression. It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, dia of the tube and surface tension.

Expression for Capillary Rise



Consider a glass tube of small dia 'd' opened at both ends and is inserted in a liquid, say water. The liquid will rise in the tube above the level of the liquid.

Let h = height of the liquid in the tube. Under a state of equilibrium, the weight of liquid of height h is balanced by force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.

let σ = Surface tension of liquid

θ = angle of contact between liquid and glass tube.

The weight of liquid of height h in the tube = (Area of tube × h) × ρ × g.

$= \frac{\pi}{4} d^2 \times h \times \rho \times g$.

$\frac{80 \times 73 \times 10^{-3}}{7.35 \times 10^{-3}}$ $\frac{10}{10}$

vertical component of the surface tensile force = (σ x Circumference) θ.

$= \sigma \times \pi d \times \cos\theta$.

For equilibrium, equating, we get,

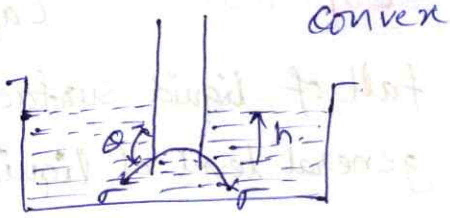
$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$$

$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

* The value of θ between water and clean glass tube is appr. 'zero' and hence $\cos\theta$ is equal to unity.

Then rise of water $h = \frac{4\sigma}{\rho g d}$

Expression for Capillary fall:



If the glass tube is dipped in mercury, the level of mercury in tube will be lower than the general level of the outside liquid as shown.

Let h = height of depression in tube.

$$\sigma \times \pi d \times \cos\theta = \rho g h \times \frac{\pi}{4} d^2$$

$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

value of θ for mercury and glass is 128°.

why the concept of surface tension is not applied to gases?

→ For gases, the inter-molecular distance among gas molecules is very large and consequently there is no appreciable force of cohesion and as such the characteristic property of surface tension is not exhibited.

→ **Applications of Surface tension:**

→ needle placed on water can be made to float due to S.T.
→ warm water is used for washing purpose as heating ↑ surface area
→ Mosquito eggs can float on water
→ Surface tension prevents water from passing through pores of umbrella

Vapour pressure and cavitation :

A change from the liquid state to gaseous state is known as vaporization. The vaporization occurs because of continuous escaping of the molecules through the free liquid surface.

consider a liquid [say water] which is confined in a closed vessel, let the temperature of liquid is 20°c and pressure is atmospheric. This liquid will vaporise at 100°c. when vaporization takes place, the molecules escapes from the free surface of the liquid. These vapour molecules get accumulated in the space between the free liquid surface and top of the vessel. These accumulated vapours exert a pressure on the liquid surface. This pressure is known as vapour pressure of the liquid or this is the pressure at which the liquid is converted into vapours.

Consider a flowing liquid in a system. If the pressure at any point in this flowing liquid becomes equal to or less than the vapour pressure, the vaporization of the liquid starts. The bubbles of these vapours are carried by the flowing liquid into the region of high pressure where they collapse, giving rise to high impact pressure. The pressure developed by the collapsing bubbles is so high that the material from the adjoining boundaries gets eroded and cavities are formed on them. This phenomenon is known as "Cavitation".

Hence the cavitation is the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below the vapour pressure and sudden collapsing of these vapour bubbles in a region of higher pressure.

Fluid pressure at a point :

Consider a small area dA in large mass of fluid. If the fluid is stationary, then the force exerted by the surrounding fluid on the area dA will always be perpendicular to the surface dA. Let dF is the force acting on the area dA in the normal direction. Then the ratio of $\frac{dF}{dA}$ is known as the intensity of pressure or simply pressure and this ratio is represented by 'P'. Hence the pressure at a point in a fluid at rest is

$$P = \frac{dF}{dA}$$

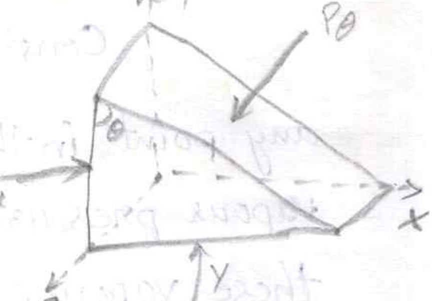
If the force (F) is uniformly distributed over the area (A), then pressure at any point is given by $P = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}$

Force (or) pressure force, $F = P \times A$

The units of pressure are (i) kgf/m^2 in MKS (ii) N/m^2 in SI units

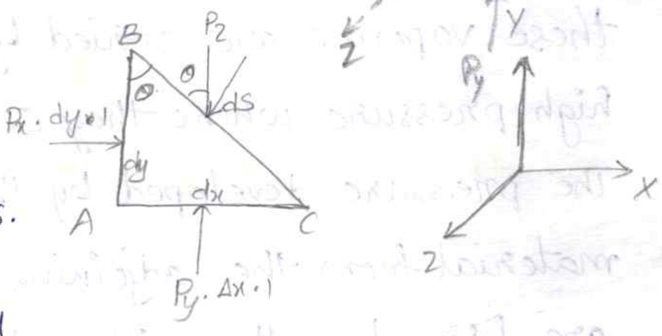
N/m^2 is known as Pascal

$$\text{one bar} = 100\ kPa = 10^5\ N/m^2$$



PASCAL'S LAW

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions.



The fluid element is of very small dimensions i.e., dx, dy, and ds.

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown in Fig. Let width of the element perpendicular to the plane of paper is unity and P_x, P_y and P_z are the pressures (or) intensity of pressure acting on the face AB, AC and BC respectively.

Let $\angle ABC = \theta$. Then the forces acting on the element are.

1. Pressure forces normal to the surfaces, and
2. weight of element in the vertical direction.

The forces on the faces are:

force on the face AB $= P_x \times$ Area of face AB

$= P_x \times dy \times 1$

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similarly force on the face AC $= P_y \times dx \times 1$

" " BC $= P_z \times ds \times 1$

weight of element = (Mass of element) × g

$= (\text{volume} \times \rho) \times g = \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g$

Resolving the forces in x-direction, we have

$$P_x \times dy \times 1 - P_z (ds \times 1) \sin(90° - \theta) = 0$$

$$P_x\,dy - P_z\,ds \cos\theta = 0$$

$$ds \cos\theta = AB = dy$$

$$\therefore P_x = P_z \quad \text{—— ①}$$

1ly In y-direction, we get

$$P_y \times dx \times 1 - P_z \times ds \times 1 \cdot \cos(90 - \theta) - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$

But $ds \sin\theta = dx$ and also the element is very small and hence weight is negligible.

$$P_y\,dx - P_z\,dx = 0$$

$$P_y = P_z \quad \text{—— ②}$$

from ① & ② equations, we have $P_x = P_y = P_z$

The above equation shows that the pressure at any point in x, y and z-directions is equal.

(Q) 10 ltrs of a liquid of Specific gravity 1.3 is mixed with 6 ltrs of a liquid of Specific gravity 0.8. If the bulk of the liquid shrinks by 1.5% on mixing, Calculate the specific gravity, density, volume & weight of the mixture.

Ans weight of 10 ltrs liquid of Specific gravity 1.3

$= 10 \times 10^{-3} \times 9810 \times 1.3\ N = 127.53\,N$

weight of 6 ltrs liquid of Specific gravity 0.8 $= 6 \times 10^{-3} \times 9810 \times 0.8$

$= 47.1\,N$

Total volume = 10 + 6 = 16 ltrs

bulk shrinks by 1.5%

Net total volume = 0.985 × 16 = 15.76 ltr

weight of equal volume of water $= 15.76 \times 10^{-3} \times 9810 = 154.6\,N$

weight of mixer = 127.53 + 47.1 = 174.63 N

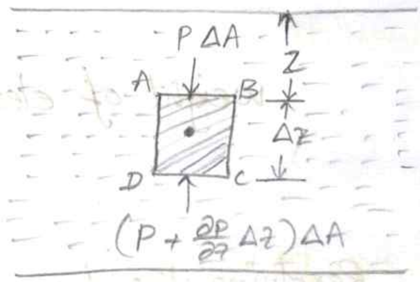
(i) Specific gravity $= \frac{174.63}{154.6}$

ii) Density $= \frac{M}{V} = 1128\ kg/m^3$

iii) volume = 15.76

iv) weight = 174.63 N

Pressure variation in a Fluid at rest : [Hydrostatic Law]



The pressure at any point in a fluid at rest is obtained by the hydrostatic law which states that the rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point.

Consider a small fluid element as shown in Fig.

let ΔA = Cross-sectional area of element

Δz = height of fluid element.

P = Pressure on face AB

z = Distance of fluid element from free surface.

forces acting on the fluid element are :

1. Pressure force on AB = $P \times \Delta A$ and acting ⊥ to face AB in downward.
2. Pressure force on CD = $(P \times \frac{\partial p}{\partial z}\Delta z)\Delta A$, acting ⊥ to CD, vertically upward.
3. weight of the fluid element = Density × g × Volume = $\rho \times g \times (\Delta A \times \Delta z)$
4. Pressure forces on surfaces BC and AD are equal and opposite.

for equilibrium of fluid element, we have

$$P\Delta A - \left(P + \frac{\partial p}{\partial z}\Delta z\right)\Delta A + \rho \times g \times (\Delta A \times \Delta z) = 0$$

$$-\frac{\partial p}{\partial z}\Delta z \Delta A + \rho g \Delta A \times \Delta z = 0$$

$$\frac{\partial p}{\partial z} = \rho \times g = w \quad — (3) \quad (\because \rho g = w)$$

Equation (3) states that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point. This is Hydrostatic Law.

By integrating the above eq (3) for liquids, we get

$$\int dp = \int \rho g \, dz$$

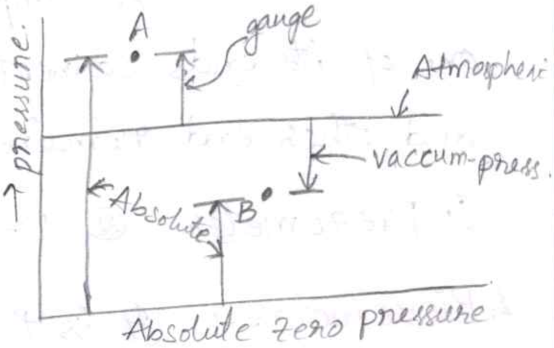
$$P = \rho g z$$

where P is the pressure above atmospheric pressure and z is the height of the point from free surfaces.

$$z = \frac{P}{\rho \times g}$$

Here z is called pressure head.

Absolute, Gauge, Atmospheric And Vaccum pressures.



1. Absolute pressure : It is defined as the pressure which is measured with reference to absolute vaccum pressure.

2. Gauge pressure : It is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric press is taken as datum. The atm. pressure on the scale is marked as zero.

3. Vaccum pressure : It is defined as the pressure below the atm press.

mathematically :

i) Absolute pressure = Atmospheric pressure + Gauge pressure.

$$P_{abs.} = P_{atm} + P_{gauge}.$$

ii) vacuum pressure = Atm. pressure − Abs. pressure.

NB The atm. pressure at sea level at 15°C is $101.3\ kN/m^2$

→ The atmospheric pressure head is 760 mm of mercury or 10.33 m of water.

→ Measurement of pressure :

The pressure of fluid is measured by following devices.

1. Manometers 2. Mechanical Gauges.

1. Manometers : manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as

(a) Simple Manometers (b) Differential Manometers.

2. Mechanical Gauges : These are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight.

commonly used mechanical pressure gauges :

(a) Diaphragm P.G. (b) Bourdon P.G. (c) Dead-weight P.G. and (d) Bellows P.G.

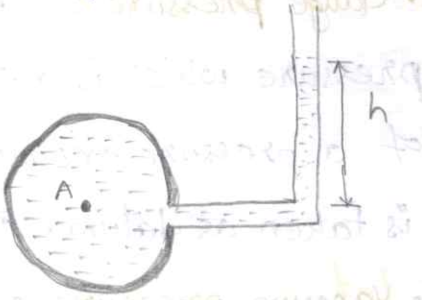
Simple Manometers :

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmospheric. Common types of simple man

① Piezometer ② U-tube Manometer ③ Single column manometer.

1. Piezometer :

It is the simplest form of manometer use for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in fig.

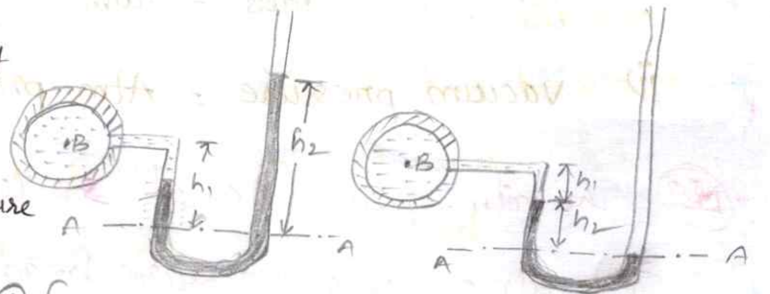


The rise of liquid gives the pressure head at that point. If at a point A, the height of liquid say water is h in piezometer tube, then pressure at A

$$P = \rho \times g \times h \ N/m^2$$

2. U-tube Manometer :

It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atm. as shown.



ⓐ For gauge pressure ⓑ For vaccum pressure

The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of liquid whose pressure is to be measured.

ⓐ **For gauge pressure :** let B is the point at which pressure is to be measured, whose value is P, the datum line is A-A.

let h_1 – height of light liquid above datum line.
h_2 – " heavy " " "
S_1 – sp. gr. of light liquid, S_2 – sp. gr. of heavy liquid.

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

$$P + \rho_1 g h_1 = \rho_2 g h_2$$

$$P = (\rho_2 g h_2 - \rho_1 g h_1)$$

ⓑ **For vaccum pressure :** left column $= \rho_2 g h_2 + \rho_1 g h_1 + P$.
right colum $= 0$.

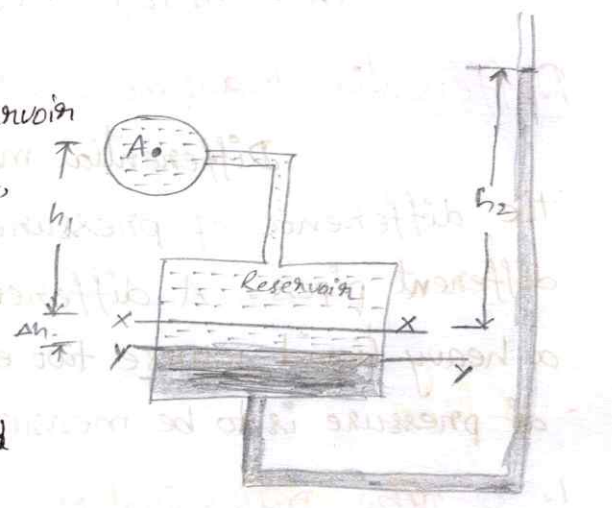
$$P = -(\rho_2 g h_2 + \rho_1 g h_1)$$

3. Single Column Manometer:

Single column manometer is a modified form of a U-tube manometer in which a reservoir, having a large c/s area as compared to the area of the tube is connected to one of the limbs of the manometer as shown in fig. Due to large c/s area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined.

a) Vertical single column manometer:

Let X-X be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.



Let Δh = Fall of heavy liquid in reservoir

h_2 = Rise of heavy liquid in right limb.

h_1 = height of centre of pipe above X-X.

P_A = pressure at A, which is to be measured.

A = c/s area of reservoir. S_1 = sp. gr. of liquid in pipe

a = c/s area of right limb. S_2 = sp. gr. heavy liquid.

Fall of heavy liquid in reservoir will cause rise of heavy liquid in right limb

$$A \times \Delta h = a \times h_2$$

$$\Delta h = \frac{a \times h_2}{A}$$

Now consider the datum line Y-Y. Then pressure in right limb above Y-Y

$$= \rho_2 g (\Delta h + h_2)$$

pressure in left limb above Y-Y $= \rho_1 g (\Delta h + h_1) + P_A$

Equating the pressures, we have

$$\rho_2 g (\Delta h + h_2) = \rho_1 g (\Delta h + h_1) + P_A$$

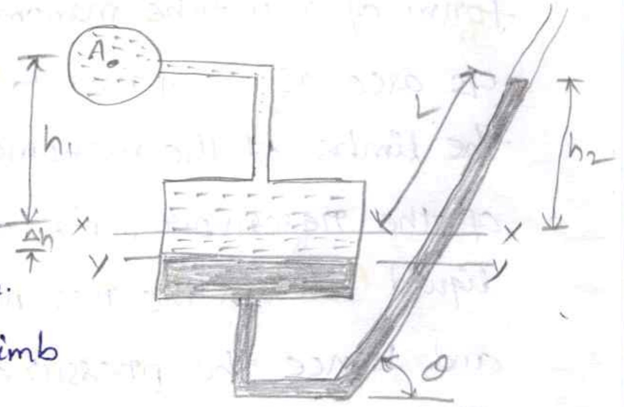
$$P_A = \frac{a \times h_2}{A} [\rho_2 g - \rho_1 g] + \rho_2 g h_2 - \rho_1 g h_1 \quad \left[\Delta h = \frac{a h_2}{A}\right]$$

As area A is very large as compared to a, hence $\frac{a}{A}$ becomes very small.

Then $$P_A = \rho_2 g h_2 - \rho_1 g h_1$$

ⓑ Inclined single column Manometer :

This manometer is more sensitive. due to inclination the distance moved by the heavy liquid in the right limb will be more.



Let L - length of heavy liquid moved from XX

θ = Inclination of right limb with horizon.

h_2 = vertical rise of heavy liquid in right limb from X-X = $L \times \sin\theta$.

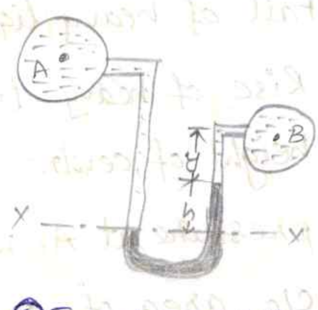
$$P_A = h_2 \rho_2 g - h_1 \rho_1 g. = \rho_2 g (L \sin\theta) - h_1 \rho_1 g.$$

Differential Manometers :

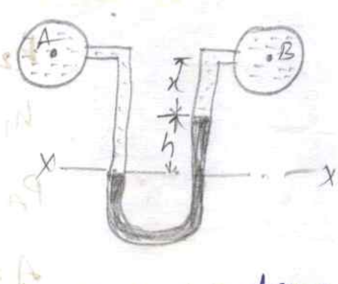
Differential manometers are the devices used for measuring the difference of pressure between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Mostly commonly types of diff. MM. are two.

1. U-tube Differential Manometer :

In Fig(a), the two points A & B are at different level and also contains liquids of different sp. gr. These points are connected to the U-tube differential manometer.



ⓐ Two pipes at different levels



ⓑ A and B at same level.

Let pressure at A & B are P_A and P_B.

Let h = Difference of mercury level in the U-tube.

y = Distance of the centre of B, from the mercury level in right limb.

x = " " A, " " "

ρ_1 = density of liquid at A, ρ_2 = density of liquid at B.

ρ_g = Density of heavy liquid or mercury.

Taking datum line at X-X :

pressure above X-X in the left limb = $\rho_1 g (h + x) + P_A$

" " right limb = $\rho_g g h + \rho_2 g \times y + P_B$

Equating the two pressure, we have

$$\rho_1 g (h+x) + P_A = \rho_g g h + \rho_2 g y + P_B$$

$$P_A - P_B = \rho_g g h + \rho_2 g y - \rho_1 g (h+x)$$

Difference of pressure A and B = $hg(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$.

In Fig(b), the two points A and B are at the same level and contains the same liquid of density ρ_1, then.

pressure above X-X in right limb $= \rho_g h g + \rho_1 g x + P_B$

" " Left limb $= \rho_1 g(h+x) + P_A$

Equating the two pressure.

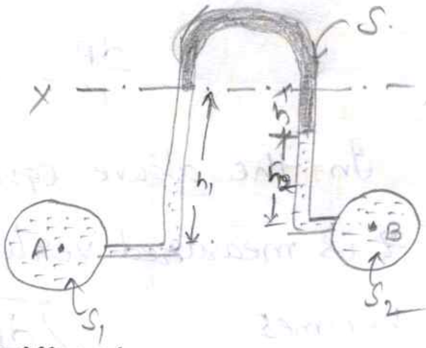
$$\rho_g g h + \rho_1 g x + P_B = \rho_1 g (h+x) + P_A$$

$$P_A - P_B = \rho_g . g h + \rho_1 g x - \rho_1 g (h+x)$$

$$P_A - P_B = g h (\rho_g - \rho_1).$$

2. Inverted U-tube Differential Manometer:

It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured.



It is used for measuring difference of low pressures.

Let $h_1 =$ Height of liquid in left limb below the datum line X-X.

$h_2 =$ height of liquid in right limb.

$h =$ difference of light liquid.

$\rho_1 =$ Density of liquid at A $\quad \rho_2 =$ density of liquid at B.

$\rho_s =$ density of light liquid.

$P_A =$ pressure at A $\quad P_B =$ pressure at B.

Taking X-X as datum line. Then pressure in the left limb below X-X

$$= P_A - \rho_1 g h_1$$

pressure in the right limb below X-X $= P_B - \rho_2 g h_2 - \rho_s g h$

Equating the two pressure.

$$P_A - \rho_1 g h_1 = P_B - \rho_2 g h_2 - \rho_s g h.$$

$$P_A - P_B = \rho_1 g h_1 - \rho_2 g h_2 - \rho_s g h.$$

Pressure at a point in compressible fluid :

For compressible fluids, density (ρ) changes with the change of pressure and temperature. Such problems are encountered in aeronautics, Oceanography and meteorology where we are concerned with atmospheric air where density, pressure and temperature changes with elevation. Thus for fluids with variable density, the equation $\frac{\partial p}{\partial z} = \rho g$ cannot be integrated, unless the relationship between P and ρ is known. For gases the equation of state is

$$\frac{P}{\rho} = RT \Rightarrow \rho = \frac{P}{RT}$$

$$\frac{dp}{dz} = w = \rho g = \frac{P}{RT} \times g.$$

$$\frac{dp}{P} = \frac{g}{RT} dz$$

In the above equation, z is measured vertically downward. But if z is measured vertically up, then $\frac{dp}{dz} = -\rho g$ and hence above eq. becomes

$$\frac{dp}{P} = -\frac{g}{RT} dz.$$

1. Isothermal Process :

If temperature T is constant which is true for isothermal process. equation $\frac{dp}{P} = -\frac{g}{RT} dz$ can be integrated as

$$\int_{P_0}^{P} \frac{dp}{P} = -\int_{z_0}^{z} \frac{g}{RT} dz = -\frac{g}{RT} \int_{z_0}^{z} dz$$

$$\log \frac{P}{P_0} = -\frac{g}{RT} [z - z_0]$$

Where P_0 is the pressure where height is z_0. If the datum line is taken at z_0, then $z_0 = 0$ and P_0 becomes the pressure at datum line.

$$\log \frac{P}{P_0} = -g/RT \; z$$

$$\frac{P}{P_0} = e^{-gz/RT}$$

The pressure at a height z is given by $P = P_0 e^{-gz/RT}$.

Adiabatic Process : If temperature: T is not constant but the process follows adiabatic law then the relation between pressure and density is given by $\frac{P}{\rho^k} = \text{Constant} = C.$

where k is ratio of specific constant.

$$\therefore \rho^k = \frac{P}{C} \Rightarrow \rho = \left(\frac{P}{C}\right)^{1/k}.$$

from eq. $\frac{dp}{dz} = -\rho g = -\left(\frac{P}{C}\right)^{1/k} g.$

$$\frac{dp}{\left(P/C\right)^{1/k}} = -g\,dz.$$

Integrating, we get $C^{1/k}\left[\frac{P^{-\frac{1}{k}+1}}{-\frac{1}{k}+1}\right]_{P_0}^{P} = -g(z)_{z_0}^{z}$

$$C^{1/k} = \left(\frac{P}{\rho^k}\right)^{1/k} = \frac{P^{1/k}}{\rho}$$

$$\left[\frac{P^{1/k}}{\rho}\;\frac{P^{-\frac{1}{k}+1}}{-\frac{1}{k}+1}\right]_{P_0}^{P} = -g(z-z_0)$$

$$\frac{k}{k-1}\left[\frac{P}{\rho}\right]_{P_0}^{P} = -g(z-z_0)$$

$$\frac{k}{k-1}\left[\frac{P}{\rho} - \frac{P_0}{\rho_0}\right] = -g(z-z_0)$$

If datum line is taken at z_0, where pressure, temperature and density are P_0, T_0 & ρ_0. then $z_0 = 0$.

$$\frac{k}{k-1}\left[\frac{P}{\rho} - \frac{P_0}{\rho_0}\right] = -gz$$

$$\frac{P}{\rho} = \frac{P_0}{\rho_0} - gz\left(\frac{k-1}{k}\right) = \frac{P_0}{\rho_0}\left[1 - \frac{\rho_0}{P_0} gz \left(\frac{k-1}{k}\right)\right]$$

$$\frac{P}{\rho} \cdot \frac{\rho_0}{P_0} = \left[1 - \frac{k-1}{k} gz \frac{\rho_0}{P_0}\right]$$

$$\frac{P}{\rho^k} = \frac{P_0}{\rho_0^k} \text{ or } \left(\frac{\rho_0}{\rho}\right)^k = \frac{P_0}{P} \text{ or } \frac{\rho_0}{\rho} = \left(\frac{P_0}{P}\right)^{1/k}$$

$$\frac{P}{P_0}\left(\frac{P_0}{P}\right)^{1/k} = \left[1 - \frac{k-1}{k} gz \frac{\rho_0}{P_0}\right]$$

$$\left(\frac{P}{P_0}\right)^{1-\frac{1}{k}} = \left[1 - \frac{k-1}{k} gz \frac{\rho_0}{P_0}\right]$$

$$P = P_0\left[1 - \frac{k-1}{k} gz \frac{\rho_0}{P_0}\right]^{\frac{k}{k-1}}$$

$$P = P_0\left[1 - \frac{k-1}{k}\frac{gz}{RT_0}\right]^{\frac{k}{k-1}} \qquad \because \frac{P_0}{\rho_0} = RT_0 \text{ or } \frac{\rho_0}{P_0} = \frac{1}{RT_0}$$

Manometric liquids :

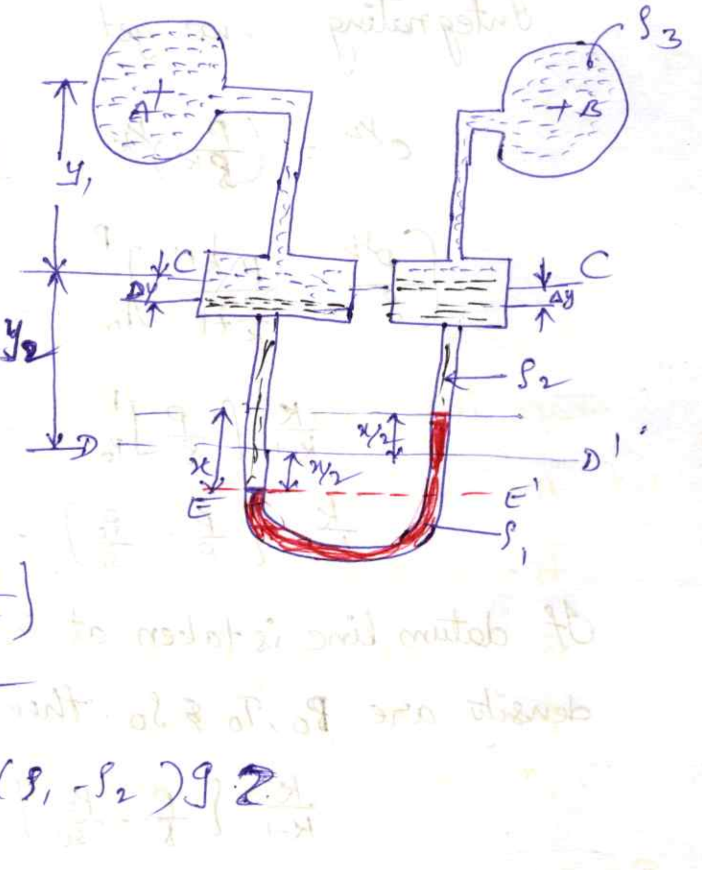
Desirable characteristics of manometric liquid :

→ Low viscosity, i.e. capability of quick adjustment with pressure change.
→ Low coefficient of thermal expansion; min density changes with temp.
→ Low vapour pressure, i.e. little or no evaporation at ambient conditions.
→ negligible surface tension and capillary effects.

Micromanometer : for the measurement of very small pressure differences, (or) for the measurement of pressure difference with very high precision. Special-forms of manometers called micromanometers are used.

$P_A > P_B$

$$A(\Delta y) = a\left(\frac{x}{2}\right)$$



$$P_A + \rho_3 g(y_1 + \Delta y) + \left(y_2 - \Delta y + \frac{x}{2}\right)\rho_2 = P_B + x\rho_1 g + \left(y_2 - \frac{x}{2} + \Delta y\right)\rho_2 g + (y_1 - \Delta y)\rho_3 g$$

$$P_A - P_B = \left[\rho_1 g - \rho_2 g\left(1 - \frac{a}{A}\right) - \rho_3 g \frac{a}{A}\right]$$

After neglecting $\frac{a}{A}$,

$$P_A - P_B = x(\rho_1 - \rho_2)g$$

Hydrostatic forces on surfaces:

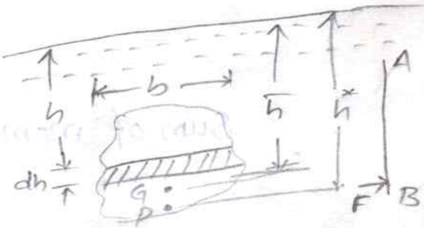
Total pressure: It is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces. This force always acts normal to the surface.

Centre of pressure: It is defined as the point of application of the total pressure on the surface. There are 4 cases of submerged surfaces on which the total pressure force and centre of pressure is to be det. The submerged surfaces may be:

1. vertical plane surface,
2. Horizontal plane surface,
3. Inclined plane surface,
4. Curved surface.

1. Vertical plane surface submerged in liquid:

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in fig.



Let A = Total area of the surface

$\bar{h}$ = Distance of C.G. of the area from free surface of liquid.

G = Centre of gravity of plane surface

P = centre of pressure.

h^* = Distance of centre of pressure from free surface of liquid.

ⓐ Total pressure (F): The total pressure on the surface may be deter. by dividing the entire surface into no. of small parallel strips. The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small strip.

Consider a strip of thickness dh and width b at a depth of h from free surface of liquid as shown in fig.

pressure intensity on the strip, $p = \rho g h$

Area of the strip, $dA = b \times dh$

Total pressure force on strip, $dF = p \times \text{Area} = \rho g h \times b \times dh$.

$\therefore$ Total pressure force on whole surface $= F = \int dF = \int \rho g h \times b \times dh$;

$$\int b \times h \times dh = \int h \times dA$$

= Moment of surface area about the free surface of liquid

$= A \times \bar{h}$

$$F = \rho g A \bar{h}$$

(b) Centre of pressure (h^*): centre of pressure is calculated by using the "principle of Moments", which states that the moment of the resultant force about an axis is equal to sum of moments of the components about same axis. The resultant force F is acting at P, at a distance h^* from free surface of liquid as shown.

Hence moment of force F about free surface of liquid $= F \times h^*$.

Moment of ~~strip~~ of force dF, acting on strip about free surface of liquid

$= dF \times h$

$= \rho g h \times b \times dh \times h$

Sum of moments of all such forces about free surface of liquid.

$$F = \int \rho g h \, b \, dh \, h$$

$$I_0 = \int h^2 dA = \int b h^2 dh$$ = moment of Inertia of the surface about free surface of liquid.

Sum of moments about free surface $= \rho g I_0$

$$F \times h^* = \rho g I_0$$

$$F = \rho g A \bar{h}$$

$$\rho g A \bar{h} \times h^* = \rho g I_0$$

$$h^* = \frac{\rho g I_0}{\rho g A \bar{h}} = \frac{I_0}{A\bar{h}}$$

By the theorem of parallel axis, we have $I_0 = I_G + A\bar{h}^2$

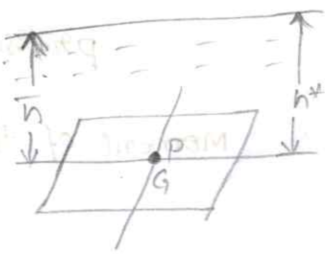
$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}$$

i) Centre of pressure (h^*) lies below the centre of gravity of vertical surface.

ii) The distance of centre of pressure from free surface of liquid is independent of the density of the liquid.

② Horizontal plane surface submerged in Liquid :

consider a plane horizontal surface immersed in a static fluid. As every point of the surface is at the same depth from the free surface of the liquid, The pressure intensity will be equal on the entire surface and equal to, $P = \rho g h$; h is depth of surf.

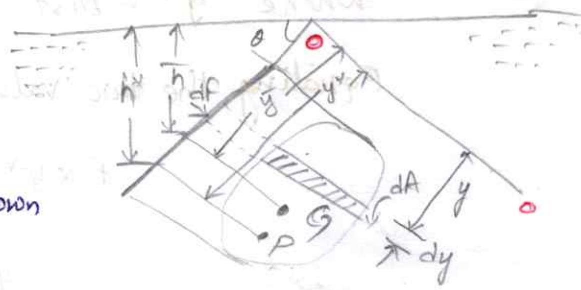


Let A = Total area of surface

Then total force, F on the surface $= P \times Area = \rho g h A = \rho g A \bar{h}$

③ Inclined plane surface submerged in Liquid :

Consider a plane surface of arbitrary shape immersed in a liquid in such a way that the plane of the surface makes an angle θ with the free surface of liquid as shown



Let plane of surface, if produced meet the free liquid surface at O. Then O-O is the axis perpendicular to the plance of surface.

Let $\bar{y}$ = dist. of the C.G. of the inclined surface from O-O.

y^* = dist. of the centre of pressure from O-O.

Consider a small strip of area dA at a depth 'h' from free surface and at a distance y from the axis O-O as shown in fig.

pressure intensity on the strip, $P = \rho g h$.

pressure force, dF on the strip, $dF = P \times$ Area of strip $= \rho g h \times dA$.

Total pressure on the whole area, $F = \int dF = \int \rho g h \, dA$

But from fig. $\frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*} = \sin\theta$.

$h = y\sin\theta$

$F = \int \rho g y \sin\theta \times dA = \rho g \sin\theta \int y \, dA$ =

But $\int y \, dA = A\bar{y}$ where $\bar{y}$ = Dist. of C.G. from axis O-O. ②

$F = \rho g \sin\theta \, \bar{y} \times A = \rho g A \bar{h}$ ②.

Centre of pressure (h^*) :

pressure force on the Strip, $df = \rho g h \, dA = \rho g y \sin\theta \, dA$.

Moment of the force dF, about axis O-O.

$$= dF \times y = \rho g y \sin\theta \, dA \times y = \rho g \sin\theta \, y^2 dA.$$

Sum of moments of all such forces about O-O $= \int \rho g \sin\theta \, y^2 dA$

$\int y^2 dA$ = MOI of the surface about O-O $= I_0$

Sum of moments of all forces about O-O $= \rho g \sin\theta \, I_0$

Moment of total force, F, about O-O is also given by $= F \times y^*$.

where y^* = Dist. of centre of pressure from O-O.

Equating the two values given by equations.

$$F \times y^* = \rho g \sin\theta \, I_0$$

$$y^* = \frac{\rho g \sin\theta \, I_0}{F} = \frac{h^*}{\sin\theta}, \quad (F = \rho g A \bar{h})$$

and I_0 by the theorem of parallel axis $= I_G + A\bar{y}^2$.

$$\frac{h^*}{\sin\theta} = \frac{\rho g \sin\theta}{\rho g A \bar{h}} \left[I_G + A\bar{y}^2 \right]$$

$$h^* = \frac{\sin^2\theta}{A\bar{h}} \left[I_G + A\bar{y}^2 \right]$$

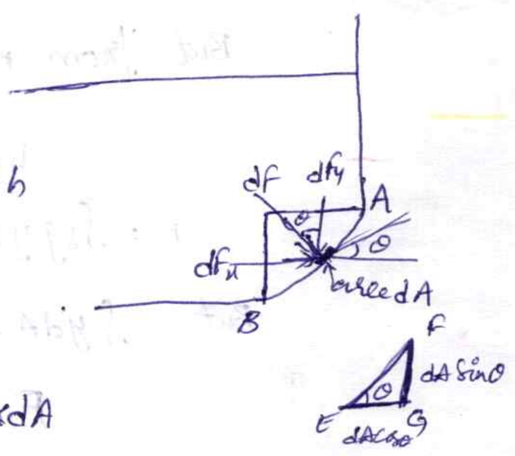
$$\frac{\bar{h}}{\bar{y}} = \sin\theta, \quad (\text{or}) \quad \bar{y} = \frac{\bar{h}}{\sin\theta}$$

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

01/07/2017

Curved surface Sub-merged in liquid :

Consider a curved surface AB, sub-merged in a static fluid as shown in Fig.



Let dA is the area of a small strip at depth of h from water surface.

pressure intensity on area dA is $= \rho g h$

pressure force $dF = p \times \text{Area} = \rho g h \times dA$

This force dF acts normal to the surface

Hence total pressure force on the curved Surface should be

$$F = \int \rho g h \, dA$$

But here as the direction of the forces on the small are not in the same direction, but varies from point to point. Hence integration of eq. for curved surface is impossible. The problem can, however, be solved by resolving the force dF intowo components dF_x & dF_y in the x and y direct

$$F = \sqrt{F_x^2 + F_y^2}, \quad \tan\phi = \frac{F_y}{F_x}$$

$$dF_x = dF \sin\theta = \rho g h \, dA \sin\theta$$

$$dF_y = dF \cos\theta = \rho g h \, dA \cos\theta$$

$$F_x = \int dF_x = \int \rho g h \, dA \sin\theta$$

$$F_y = \int dF_y = \int \rho g h \, dA \cos\theta$$

In these expressions $dA \sin\theta$ and $dA \cos\theta$ represent repre the vertical and horizontal projections of the elementay area dA consequently $\rho g \int h \, dA \sin\theta$ represents total pressure force on projected area of curved surface on the verticall plane. The point of application of horizontal component F_x is at the centre of pressure of the projected area.

Further $\rho g \int h \, dA \cos\theta$ represents the total pressure on projected area of curved surface on the horizontal plane and it equals the weight of liquid lying that portion. weight of liquid extending from the curved surface of the free surface of the liquid. The point of application of component F_y acting vertically downward is at centroid of the liquid volume above the curved surface.

Buoyancy : When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the force of buoyancy or simply buoyancy.

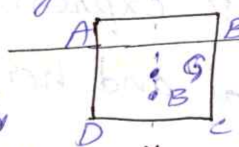
Centre of Buoyancy: It is defined as the point, through which the force of buoyancy is supposed to act. As the force of buoyancy is a vertical force and is equal to weight of the fluid displaced by the body, the centre of buoyany will be centre of gravity of fluid displaced.

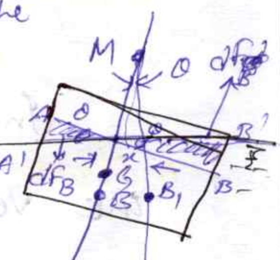
Metacentre : It is defined as the point about which a body starts oscillating when the body is tilted by a small angle.

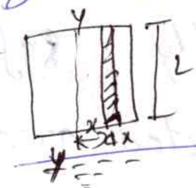
Meta-centric height : The distance MG i.e. the distance between the meta-centre of a floating body and centre of gravity of the body is called meta-centric height.

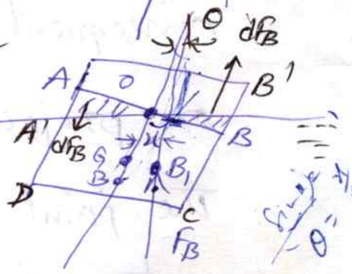
Analytical method for meta-centre height :

Fig. shows the position of a floating body in equilibrium. The location of centre of gravity and centre of buoyancy in this position is at G and B. The floating body is given small angular displacement in the clockwise direction. The new centre of buoyancy is at B_1. The vertical line through B_1 cuts the normal axis at M. Hence M is metacentre and GM is meta centric height.









Consider a small strip of thickness dx at a distance x from O as shown in fig.

Area of strip $= x \times \theta \times dx$

volume of strip $= x \times \theta\, dx \times L$

weight of strip $= \rho g \cdot x\, x\, \theta L dx$

Moment of weight about Y-Y $= (\rho g x \theta L dx) x$

Total moment $= M = \int \rho g x^2 \theta\, dx L$

Total buoyant force $F_B = \rho g$ volume

moment due to movement of centre of buoyany from B to B' $= F_B \times BB'$

$= F_B \times BM \times \tan\theta$

$= \rho g \text{ volum} \times BM\theta$

equating

$$\rho g V BM \theta = \int \rho g x^2 \theta dx L$$

$$BM = \frac{\int x^2 dA}{V}$$

$\int x^2 dA = I_{yy}$ moment of inertia of the plane of floatation about centroidal axis perpendicular plane of rotation

$$GM = BM - BG$$

$$\boxed{GM = \frac{I_{yy}}{V} - BG}$$

M is above G → Stable equilibrium

M is below G → Unstable equilibrium

M = G → Neutral equilibrium

B is above G → Stable equilibrium

B is below G → Unstable equilibrium

B coinciding G → Neutral equilibrium

ANNAMACHARYA UNIVERSITY

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Rajampet, Annamayya District, A.P – 516126, INDIA

CIVIL ENGINEERING

Fluid Mechanics, Hydraulics & Hydraulic Machinery

UNIT-2

UNIT-III

Laws of fluid friction.

friction: The opposing force, which acts in the opposite direction of the movement of the upper body, is called "friction force".

① static friction ② dynamic friction.

① static friction: when the body is in rest condition then the friction force exerted by the body is called "static friction".

② Dynamic friction: when body is in moving condition then the friction force exerted by the body is called Dynamic friction.

i) sliding friction (ii) Rolling friction (iii) pivot friction.

A pipe is a closed conduit through which the fluid flows under pressure. Pipe runs full and the fluid has no free surface. flow is then at a pressure above or below the atmosphere, and this pressure generally varies along the pipe line. when the fluid flows through the piping system, some of the potential energy [head] is lost to over come **hydraulic** resistance.

i) viscous friction effects associated with fluid flow.

ii) The local resistances which result from flow disturbances caused by

→ Sudden expansion and contraction of pipe c/s.

→ obstructions in the form of valves, elbows and other pipe fittings.

→ curves and bends in the pipelines.

→ Entrance and exit losses.

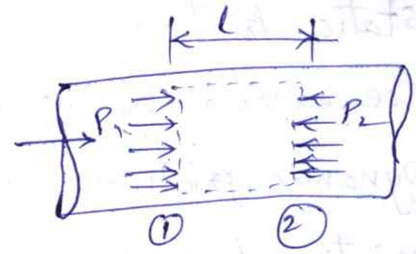
The local resistances are essentially due to change in velocity either in magnitude or direction or both.

The frictional pressure drop associated with fluid flow is called the major pipe losses, while the contributions of pipe fitting are collectively referred to as minor pipe loss.

Darcy weisbach Equation for head loss due to friction:

The friction loss for the turbulent flow through a pipe can be evaluated by the force balance on the control volume.

Let P_1 and P_2 be the intensities of pressure at sections ① and ② respectively. Then the propelling force on the following fluid between two sections is $= (P_1 - P_2) A$



The frictional resistance force can be written as $= f'(PLV^2)$

where $V \rightarrow$ avg flow velocity, l is distance b/w two chosen sections, $P \rightarrow$ wetted perimeter, f' is a non-dimensional factor whose value depends upon the material and nature of the pipe surface.

equilibrium conditions

$$(P_1 - P_2) A = f' P L V^2$$

$$\left(\frac{P_1 - P_2}{\gamma}\right) = (2g) \frac{f'}{\gamma} \left(\frac{P}{A}\right) \frac{l V^2}{2g}$$

$$h_f = \left(\frac{2gf'}{\gamma}\right) \frac{l V^2}{\left(\frac{A}{P}\right) 2g}$$

where $\frac{A}{P}$ called hydraulic mean depth y_m.

The term $\left(\frac{l}{y_m} \frac{V^2}{2g}\right)$ has units as those of h_f. evidently then the term $\left(\frac{2gf'}{\gamma}\right)$ is dimensionless quantity and can be replaced by another constant f.

$$h_f = f \cdot \frac{l}{y_m} \frac{V^2}{2g}$$

for circular pipe running full, $A = \frac{\pi}{4} d^2$; $P = \pi d$

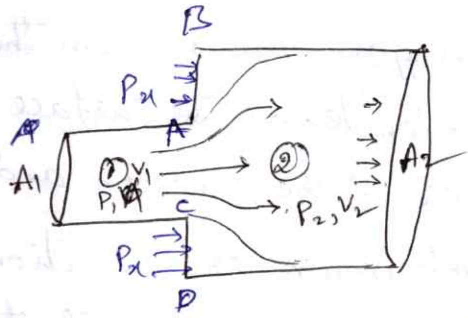
$$y_m = \frac{d}{4}$$

$$h_f = \frac{4 f l V^2}{2gd}$$

This ~~result~~ is Darcy weisbach equation and holds good for all types of flows provided a proper value of f is chosen. The f is called Darcy coefficient of friction.

Minor head losses.

① Sudden enlargement :



Consider two c/s of stream. section 1-1 through the plane of expansion, and section 2-2 which makes the end of the region of extensive turbulence caused by fluid seperation. from momentum considerations.

$$P_1 A_1 + P_x (A_2 - A_1) - P_2 A_2 = \rho Q (V_2 - V_1)$$

Neglecting any radial acceleration of fluid in the plane of change in section, the velocity at the annular wall AB and CD is very small and so the pressure there can be assumed to be equal to the pressure of incoming fluid, i.e $P_x = P_1$ and then the momentum equation transforms to

$$(P_1 - P_2) A_2 = \rho Q (V_2 - V_1)$$

$$\frac{P_1 - P_2}{\gamma} = \frac{Q}{A_2 g}(V_2 - V_1)$$

Bernoulli's equation [h_{exp} = head loss due to expansion.]

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_{exp}.$$

$$h_{exp} = \left(\frac{P_1 - P_2}{\gamma}\right) + \frac{V_1^2 - V_2^2}{2g}$$

$$h_{exp} = \frac{V_2(V_2 - V_1)}{g} + \frac{V_1^2 - V_2^2}{2g} = \frac{(V_1 - V_2)^2}{2g}.$$

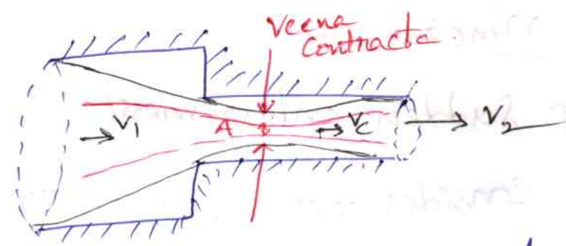
$$h_{exp} = \frac{(V_1 - V_2)^2}{2g}.$$

from continuity eq. $V_2 = \frac{A_1 V_1}{A_2}$

$$h_{exp} = \left[1 - \left(\frac{A_1}{A_2}\right)^2\right] \frac{V_1^2}{2g} = K_{exp} \frac{V_1^2}{2g}.$$

i.e. the head loss is a function of incoming velocity head. when area A_2 is very large as compared with A_1, velocity V_2 can be assumed to be zero.

Sudden contraction :



① a converging flow wherein the stream of flow leaves the surface at the corner of the junction and attains a minimum cross-sectional area at the veena-contracta. At vena-contracta, the effective flow area becomes considerably less than the cross-sectional area of the small-dia pipe. The accelerating flow is stable, boundary layer has no chance to seperate and consequently there is little loss of head between the entrance and the vena-contracta.

② a diverging flow downstream from vena-contracta where in the stream expands and ultimately assumes uniform flow over the entire cross-section of the narrow pipe. During the sudden and unguided expansion, vortices are formed between the mainstream and wall of the pipe. Eddy formation and the consequent energy dissipation are essentially responsible for most of the head loss.

The loss of head in a pipe contraction is thus caused mainly by the turbulence created by abrupt expansion of the flow just after it has passed through vena-contracta, it can be approximated by the abrupt expansion equation

$$h_{com} = \frac{(V_c - V_2)^2}{2g}$$

where V_c is the velocity at vena-contracta.

$A_c = C_c A_2$, where C_c is coefficient of contraction.

$A_2 V_2 = A_c V_c = C_c A_2 V_c$ (or) $V_c = \frac{V_2}{C_c}$

$$h_{com} = \frac{\left[\left(\frac{V_2}{C_c}\right) - V_2\right]^2}{2g} = \left[\frac{1}{C_c} - 1\right]^2 \frac{V_2^2}{2g}$$

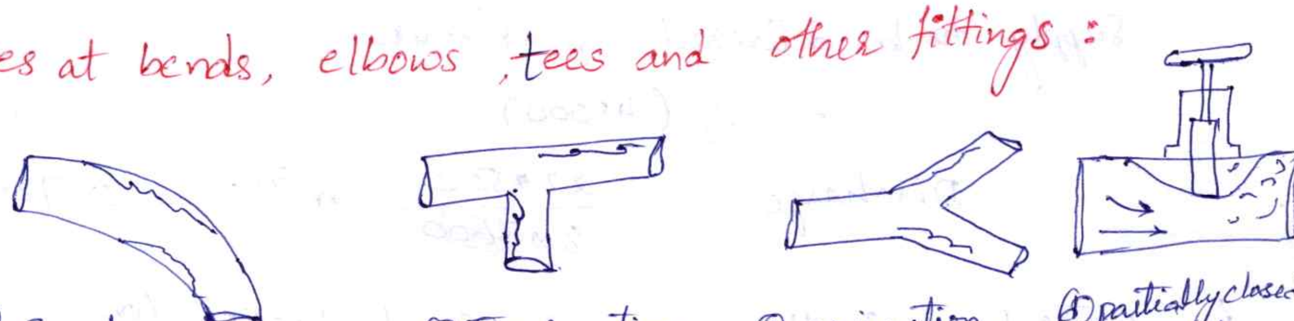
$$h_{com} = K_{con} \frac{V_2^2}{2g}$$

The contraction loss coefficient K_{con} depends essentially on the rate of contraction, i.e on area ratio $\frac{A_2}{A_1}$ and to some extent on the flow Reynolds number.

when upstream area A_1 is very large compared with downstream area A_2, resistance coefficient $k \simeq 0.5$. The flow situation then corresponds entrance to a pipeline from a reservoir of sufficient size, head loss is then called the pipe entrance loss, The corresponding resistance coefficient, K_{ent} for the various mouthpieces are

* Sharp edged mouth pieces, $K_{ent} = 0.5$
* Re-entrance of Borda mouth pieces $K_{ent} = 1.0$.
* Rounded or Bell-mouthed entrance. $K_{ent} = 0.05$.

losses at bends, elbows, tees and other fittings :

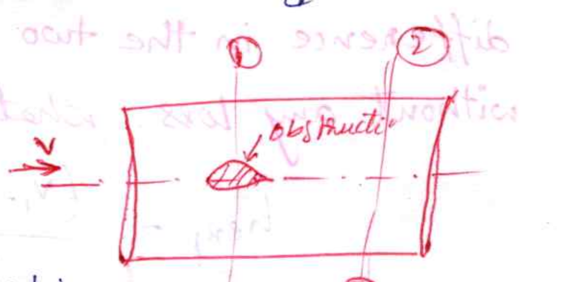


(a) Bend (b) Tee junction. (c) V-junction. (d) partially closed valve.

The figure depicts The flow pattern regarding separation and eddying in The region of separations in bends, tees and valves. The resulting head loss due to energy dissipation can be prescribed by the relation, $h = K\frac{V^2}{2g}$ where V is the avg velocity the resistance coefficient K depends on parameters defining the geometry of the section and the flow.

Loss coefficient of a bend is primarly a function of the total angle of bend and the ratio $\frac{r}{d}$ where r is the radius of the bend and d is the pipe dia. Experiments shows that for 90° smooth bend, minimum loss is experienced when $\frac{r}{d} = 2.5-5$.

losses due to obstruction



V → velocity of liquid in the pipe

A → area of pipe

a → max area of obstruction.

(A-a) = flow area of liquid at section 1-1

$$h = \frac{(V_c - V)^2}{2g},$$

V_c → is velocity at vena contracta.

$A_cV_c = AV \Rightarrow V_c = \frac{AV}{C_c(A-a)}$

$$h = \frac{V^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]^2$$

② Water is supplied to a town of 3,00,000 inhabitants from a reservoir 5 km away from the town. The per capita consumption of water per day is 150 litres and half of the daily supply is pumped in 8 hours. The highest and lowest water levels in the reservoir are 150m and 100m respectively and delivery end of the main supply pipe is at 25m above the reference level. If the head required at the delivery end is 15, make calculations for the dia of supply main. Take friction coefficient $4f = 0.04$ in darcy eq.

Total water supply $= 300000 \times (150 \times 10^{-3})$

$= 45000 m^3$

Supply to be delivered in 8 hours

$= \frac{1}{2}(45000)$

$$\text{Discharge} = \frac{22500}{8 \times 3600} m^3/s = 0.7812 m^3/s.$$

i) water level in the reservoir is at highest level.

Head available $= (150 - 25 - 15) = 110m$.

$$110 = \frac{4flv^2}{2gd} \Rightarrow \boxed{d = 0.62m}$$

(10^3)

13824

117.

ii) water level in reservoir at lowest level.

Head $= 100 - 25 - 15 = 60m$.

$d = 0.7m$.

→ Horizontal pipe, 10cm dia, is joined by sudden enlargement to a 15cm dia pipe. water is flowing through it at the rate of $2 m^3/min$. find the loss of head due to abrupt expansion and pressure difference in the two pipes. If the change of section is gradual without any loss, what would be the change in pressure.

$$h_{exp} = \frac{(V_1 - V_2)^2}{2g}$$

$V_1 = \frac{Q}{A_1} = 4.25 m/s$, $V_2 = 1.88 m/s$

$$h_{exp} = \frac{(4.25-1.88)^2}{2\times 9.81} = 0.286\,m.$$

from Bernoulli's eq.

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + y_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + y_2 + h_{exp},$$

$$\frac{P_2 - P_1}{\gamma} = \frac{v_1^2 - v_2^2}{2g} - h_{exp} = 0.455\,m \text{ of water.}$$

(*) A 15cm dia pipe is attached to a 10cm dia pipe by means of a flange in such a manner that axes of the two are in a straight line. The water flows through the arrangement at a rate of $2\,m^3/min$. The pressure loss at the transition as indicated by differential gauge lengths on water-mercury manometer connected b/w two pipes equal 8cm. Calculate head loss and coefficient of contraction.

$$\frac{P_1 - P_2}{\gamma} = h(G-1)$$

$$\frac{P_1 - P_2}{\gamma} = 0.08(13.6-1) = 1.008\,m. \text{ of water.}$$

$$V_1 = \frac{Q_1}{A_1} = \frac{(2/60)}{\frac{\pi}{4}(0.15)^2} = 1.88\,m/s.$$

$$V_2 = \frac{Q_2}{A_2} = 4.25\,m/s.$$

Applying Bernoulli's eq.

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + y_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + y_2 + h_{com}.$$

$$h_{com} = \frac{P_1 - P_2}{\gamma} + \frac{V_1^2 - V_2^2}{2g} = 1.008 + \frac{(1.88)^2 - (4.25)^2}{2\times 9.81}$$

$$h_{com} = 0.268\,m.$$

$$h_{com} = \left(\frac{1}{C_c} - 1\right)^2 \frac{V_2^2}{2g} \Rightarrow C_c = 0.649.$$

(Q) Water flows through 15 cm dia horizontal pipe at a velocity 2.5m/s. If a circular solid plate of 10cm dia is placed in the pipe to obstruct the flow. Determine head loss. $C_c = 0.6$.

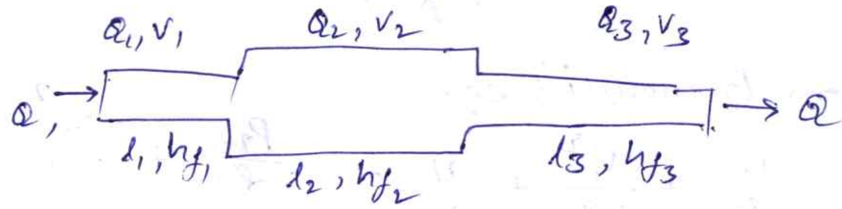
$$A = \frac{\pi}{4}(0.15)^2 = 0.0176\,m^2, \quad a = 0.00785\,m^2$$

$$C_c = 0.6, \quad A - a = 0.00981\,m^2$$

$$h_f = \frac{v^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]^2 = 1.274\,m.$$

pipes in Series and in parallel.

In series



In series arrangement, 2 or more pipes of diff dia are connected with one another to form single pipeline.

for such arrangement Q is same

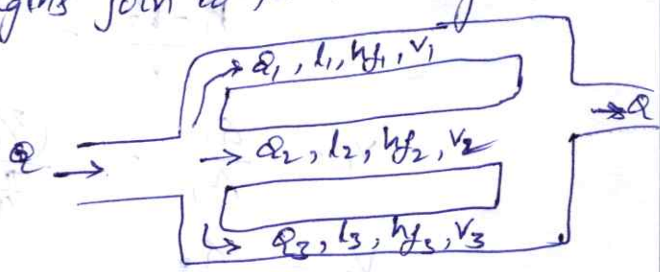
$$Q = Q_1 = Q_2 = Q_3$$

total loss of head through entire system is sum of the losses in all individual pipes and fittings.

$$h_f = h_{f_1} + h_{f_2} + h_{f_3} + \dots$$

[neglecting minor transition losses]

→ In parallel arrangement, 2 or more pipes branch out from a single pipe and after equal or unequal lengths join to form a single pipe.



* $Q = Q_1 + Q_2 + Q_3$.

* $h_{f_1} = h_{f_2} = h_{f_3} = h_{f_1}$.

Concept of equivalent pipe: Designers of conduit systems analyse the pipe flow by replacing the series combination by a single pipe of uniform dia which would have the same head loss and the discharge rate. The pipe is called equivalent pipe and the uniform dia of the equivalent pipe is known as equivalent dia of the series or compound pipe.

Let l_1, l_2, l_3 etc. represent the lengths and d_1, d_2, d_3 etc. denote the respective dia of the different pipes constituting the series arrangement. Then neglecting minor losses, total head loss $h_f = h_{f_1} + h_{f_2} + h_{f_3}$.

$$h_f = \frac{f_1 l_1 Q_1^2}{3 d_1^5} + \frac{f_2 l_2 Q_2^2}{3 d_2^5} + \frac{f_3 l_3 Q_3^2}{3 d_3^5}$$

By continuity of flow $Q_1 = Q_2 = Q_3 = Q$, $f_1 = f_2 = f_3 = f$.

$$h_f = \frac{fQ^2}{3}\left[\frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5}\right]$$

Let l_e denote the length & d_e → dia of equivalent pipe which carries same discharge.

$$h_f = \frac{f l_e Q^2}{3 d_e^5} = \frac{fQ^2}{3 d_e^5}\left[\frac{l_e}{d_e^5}\right]$$

$$\frac{l_e}{d_e^5} = \left(\frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5}\right)$$

(*) Two pipes each 250 m long are available for connecting to a reservoir from which a flow of $0.08\ m^3/s$ is required. The pipe dia are 10 cm and 20 cm respectively. Compare the head loss through the system if the pipes constitute a series and parallel arrangement. Assume $f = 0.01$.

When pipes are connected series.

$$(h_f)_s = h_{f_1} + h_{f_2} = \frac{f_1 l_1 Q^2}{3 d_1^5} + \frac{f_2 l_2 Q^2}{3 d_2^5}$$

$$= \frac{0.01 \times 250 \times 0.08^2}{3\,(0.1)^5} + \frac{0.01 \times 250 \times 0.08^2}{3\,(0.2)^2}$$

$$(h_f)_s = 549.86\ m.$$

ii) for parallel arrangement

$$Q = Q_1 + Q_2 = 0.08, \qquad h_{f_1} = h_{f_2} \Rightarrow \frac{f_1 l_1 Q_1^2}{3 d_1^5} = \frac{f_2 l_2 Q_2^2}{3 d_2^5}$$

$$Q_2 = \left(\frac{d_2}{d_1}\right)^{5/2} Q_1 = 5.66\, Q_1$$

$$Q_1 = 0.012\ m^3/s, \quad Q_2 = 0.068\ m^3/s.$$

$$(h_f)_p = \frac{f_1 l_1 Q_1^2}{3 d_1^5} = 12.04\ m.$$

$$\frac{(h_f)_s}{(h_f)_p} = 45.67$$

→ A piping system consists of 3 pipes arranged in series.

	L	Dia
AB	2000 m	40 cm
BC	1500 m	30 cm
CD	1000 m	20 cm

Transform the system to (i) an equivalent length of 30 cm dia pipe

(ii) an equivalent dia fol 4500 m long pipe.

(i)

$$\frac{l_e}{d_e^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5}$$

$$l_e = (0.3)^5 \left[\frac{2000}{(0.4)^5} + \frac{1500}{(0.3)^5} + \frac{1000}{(0.2)^5} \right]$$

$l_e =$ [illegible] m.

(ii) $l_e = 4500$

$$\frac{4500}{d_e^5} = \left[\frac{2000}{(0.4)^5} + \frac{1500}{(0.3)^5} + \frac{1000}{(0.2)^5} \right]$$

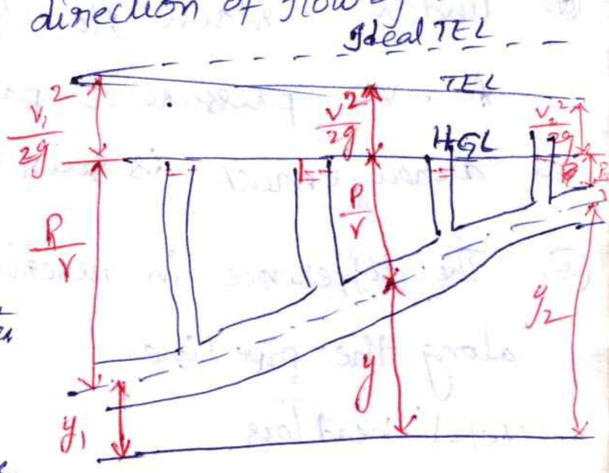
$d_e = 0.4089\,m$.

Siphon: A siphon is a long bent pipe which is used to carry water from a reservoir at a higher elevation to another reservoir at a lower elevation when the two reservoirs are separated by a hill (o) high level ground in between.

Hydraulic gradient line & Total Energy lines:

Total energy gradient line and hydraulic gradient line (HGL) are the graphical representations of the longitudinal variation in total head and piezometric head at salient points of a pipeline.

Total head (or energy per unit weight) with respect to any arbitrary datum is prescribed by the summation of pressure head, velocity head, and datum head y. Because of friction effects associated with fluid flow and local resistance arising from the pipe transitions and fittings, a part of the energy is dissipated. Evidently there is loss of head, and energy drops in the direction of flow by an amount equal to the head loss.



Summation of pressure head ($\frac{P}{\gamma}$) and datum head at any point represents the piezometric head. If piezometer tubes are installed at several points along the pipeline, the rise of liquid in the tube would correspond to the piezometric head. HGL is then prescribed by a line connecting the values of piezometeric head at successive points along the piping system.

following points need consideration when constructing these lines:

i) Since velocity is essentially zero in a reservoir, HGL & EGL are coincident with the liquid surface.

ii) At entrance to the pipe from the reservoir, entrance losses are experienced and EGL drops. There is also fall in HGL due to the liquid now flowing with a certain velocity. likewise, the losses at exit from the pipe cause an abrupt step down in these lines.

iii) Because of pipe friction and all minor losses, EGL slopes downward in the direction of flow

iv) HGL may rise or fall depending upon velocity & pressure changes which are dictated by a variation in c/s of the pipes comprising the pipe system. for example velocity would decrease with pipe expansion.

(v) An abrupt rise in EGL & HGL would occur if mechanical energy (pressure) is supplied to the liquid by means of a pump.

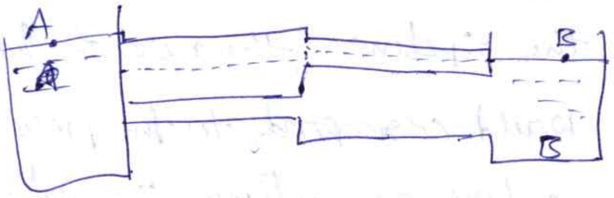
vi) for a pipe having uniform physical characteristics (dia, roughness) the head loss per unit of length will be constant. Slope of Energy line is called energy gradient & $\frac{d}{dl}\left(\frac{P}{\gamma}+\frac{v^2}{2g}+z\right)$

vii) The HG & TGL are straight sloping lines, irrespective of the pipelines being straight or curved.

(viii) under certain flow situations the pipeline may rise above HGL. A -ve pressure or partial vacuum then exists within the pipe. an arrangement is said to constitute a Siphon. 7.6 m of wat

(ix) The difference in reservoir levels equals the sum of all head losses along the pipe line.

Total head loss



$$\frac{0.5V_1^2}{2g} + \frac{4f_1l_1V_1^2}{2gd_1} + \frac{(V_1-V_2)^2}{2g}$$

Entry loss, friction in Pipe (1), enlargment

$$+ \frac{4f_2l_2V_2^2}{2gd_2} + \frac{V_2^2}{2g}$$

friction in Pipe (2), Exit loss

(*) Two reservoirs are connected by a pipeline which is 15 cm in dia for the first 5 m and 25 cm in dia for the remaining 15 m. Entry to and exit from the pipe is sharp, and the water surface in the upper reservoir is 7.5 m above that in the lower reservoir. Represent the layout and tabulate the head losses by assuming $f = 0.01$ for both pipes. Calculate the flow rate through the arrangement and draw HGL & TEL.

$$V_1 = \frac{A_2}{A_1}V_2 = \left(\frac{d_2}{d_1}\right)^2 V_2 = 2.78\,V_2$$

i) loss at entry $= 0.5\frac{V_1^2}{2g} = \frac{3.87\,V_2^2}{2g}$

ii) friction in 15 cm pipe $\Rightarrow h_{f_1} = \frac{4fl_1V_1^2}{2gd_1} = 10.3\frac{V_2^2}{2g}$

iii) loss at enlargement $= \frac{(V_1 - V_2)^2}{2g} = 3.17 \frac{V_2^2}{2g}$.

iv) friction in 25cm pipe. $= \frac{4fl_2 V_2^2}{2gd_2} = 2.4 \frac{V_2^2}{2g}$.

v) loss at exit $= \frac{V_2^2}{2g} = 1.00 \frac{V_2^2}{2g}$.

Total loss of head $= (3.87 + 10.3 + 3.17 + 2.4 + 1.0)\frac{V_2^2}{2g} = 20.74 \frac{V_2^2}{2g}$

Applying Bernoulli's eq. A and B.

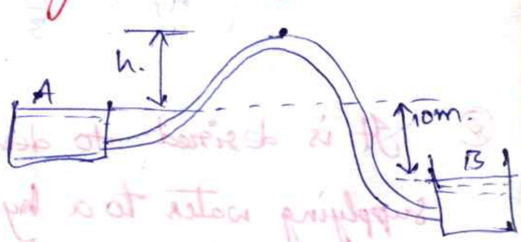
$$\frac{P_a}{\gamma} + y_a + \frac{V_a^2}{2g} = \frac{P_b}{\gamma} + y_b + \frac{V_b^2}{2g} + \text{losses}.$$

$$y_a - y_b = h_{loss}$$

$$7.5 = 20.74 \frac{V_2^2}{2g} \Rightarrow V_2 = 2.67 \text{ m/s}$$

$$Q = A_2 V_2 = 0.131 \text{ m}^3/\text{s}.$$

⊛ water from a main canal is siphoned to a branch canal over an embankment by means of wrought iron pipes of 9cm dia. The length of pipe upto the summit is 25m and total length is 65m. water surface elevation in the branch canal is 10m below that of the main canal.
i) How many pipes are needed if total water conveyed is 60 lit/sec
ii) what is max permissible height of summit above the water level in the main canal so that the water pressure of summit may not fall below 0.2 bar absolute. barometeric reading is 10m of water.
$f = 0.0075$



$$10 = \frac{0.5V^2}{2g} + \frac{4flV^2}{2gd} + \frac{V^2}{2g}$$

$V = 2.91$ m/s.

$Q = \frac{\pi}{4}(0.09)^2 \times 2.91 = 18.5$ Lit/s.

No. of pipes $= \frac{60}{18.5} = 3.24$

Thus 4 no. of pipes of 9cm dia are needed.

Bernoullis eq.

$$\frac{P_a}{\gamma} + \frac{V_a^2}{2g} + y_a = \frac{P_b}{\gamma} + y_b + \frac{V_b^2}{2g} + \text{entry loss} + \text{friction loss}$$

$$10 + y_a + 0 = \frac{0.2\times10^5}{9810} + y_b + \frac{2.91^2}{2\times9.81} + \frac{0.5\times2.91^2}{2\times9.81} + \frac{4\times0.0075\times25 ...}{2\times9.81\times0.0 ...}$$

$y_b - y_a \approx 3.72\,m$

$\boxed{h = 3.72\,m}$

Hydraulic transmission of power:

let H is the total head at the source & h_f be the head loss in transmit...

Then head available at the outlet $h = H - h_f$.

$$h_f = \frac{4flV^2}{2gd}$$

$$h = H - \frac{4flV^2}{2gd}$$

$$P = \gamma Q h = \gamma Q\left(H - \frac{4flV^2}{2gd}\right)$$

$$P = \gamma\left(\frac{\pi}{4}d^2\right)\left[HV - \frac{4flV^3}{2gd}\right]$$

for maximisation of power transmitted $\frac{dP}{dV} = 0$

$$\boxed{H = 3h_f}$$

Transmission efficiency $(\eta_t) = \frac{\text{Power delivered at outlet of pipe}}{\text{power supplied at inlet to pipe}}$

$$\eta_t = \frac{\gamma Q(H - h_f)}{\gamma Q H} = \frac{H - h_f}{H}$$

$$\left(h_f = \frac{H}{3}\right) \Rightarrow \eta_t = \frac{2}{3} = 66.67\%$$

(*) It is desired to develop 1000 kW of power at 85% efficiency by supplying water to a hydraulic turbine through a horizontal pipe 500 m long. Determine the necessary flow rate and the min dia of pipe to carry that discharge. Water is available at a head of 150 m. Take $f = 0.006$.

$$h_f = \frac{H}{3} = \frac{150}{3} = 50\,m$$

$$h = H - h_f = 150 - 50 = 100\,m$$

$$P = \frac{1000}{0.85} = 1176.47\times10^3\,W$$

$$P = \gamma Q H \Rightarrow Q = 1.2\,m^3/s$$

$$h_f = \frac{4flV^2}{2gd} = \frac{flQ^2}{3.5} \Rightarrow d = 0.492\,m$$

pipe network : A pipe network is an interconnected system of pipes forming several loops or circuits. The examples of such networks of pipes are the municipal water distribution systems cities and laboratory supply system. In such system, it is required to determine the distribution of flow through the various pipes of the network. The following are necessary conditions for any network of pipes.

(i) The flow into each junction must be equal to the flow out of the junction

(ii) The algebraic sum of head losses round each loop must be zero. i.e. In each lopp, the loss of head due to flow in clockwise must be equal to the loss of head due to flow in anticlockwise direction.

iii) The head loss in each pipe is expressed as $h_f = rQ^2$!

$$h_f = \frac{4flQ^2}{3d^5} = rQ^2 \qquad \left(r = \frac{4fl}{3d^5}\right)$$

Hardy cross Method

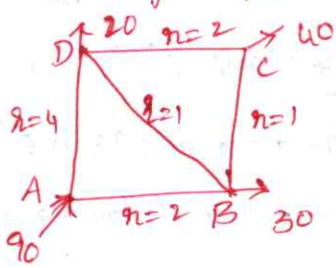
1. In this method a trial distribution of discharges is made but in such a way that continuity eq. is satisfied at each junction (or node).
2. with assumed values of Q, the head loss in each pipe is calculated.
3. Now consider any loop, The algebraic sum of head losses round each loop much be zero.
4. Now calculate the net head loss around each loop considering the head loss to be positive a clockwise flow & -ve in anticlockwise flow.

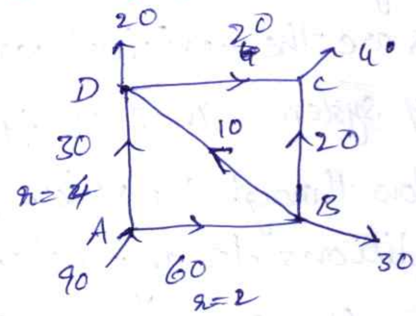
If net head loss due to assumed values of Q round the loop is zero, then assumed values of Q is correct. But if the net head loss due to assumed values of Q is not zero, then the assumed values of Q are corrected by introducing a correction ΔQ for the flows,

$$\Delta Q = -\frac{\Sigma r Q_0^n}{\Sigma r n Q_0^{n-1}} \quad , \quad \text{for } n=2 \quad \Delta Q = \frac{-\Sigma r Q_0^2}{\Sigma 2 r Q_0} \text{ \&}$$

5. If the values of ΔQ comes out to be +ve, then it should be added to the flows in clockwise direction & substracted for anticlockwise dir.
6. Some pipes may be common to two circuits then two corrections are applied.

Q Calculate the discharge in each pipe of the network as shown in fig. The network consists of 5 pipes. The head loss is given by $h_f = rQ^2$.





First trial

Loop ADB

	r	Q_0	$h_f = rQ_0^2$	$2rQ_0$
AD	4	30	3600	240
DB	1	10	~~-1×10~~ -100	~~100~~ 20
AB	2	60	-7200	240
			$\Sigma = -3700$	$\Sigma = 500$

$$\Delta Q = -\frac{(-3700)}{500} = 7.4$$

AD = 30 + 7.4 = 37.4

AB = 60 − 7.4 = 52.6

BD = 10 − 7.4

Loop DCB

	r	Q_0	rQ_0^2	$2rQ_0$
DC	2	20	800	80
CB	1	20	−400	40
BD	1	10	100	20
			$\Sigma = 500$	$\Sigma = 140$

$$\Delta Q = \frac{-500}{140} = -3.6$$

DC = 20 − 3.6 = 16.4

BC = 20 + 3.6 = 23.6

BD = 2.6 − 3.6 = −1

Second trial

	r	Q_0	rQ_0^2	$2rQ_0$
AD	4	37.4	5595	299.2
DB	1	1	1	2
AB	2	52.6	−5533.5	210.4

$$\Delta Q = \frac{-62.54}{511.6} = -0.1$$

	r	Q_0	rQ_0^2	$2rQ_0$
DC	2	16.4	537.9	65.6
BC	1	23.6	−556.9	47.2
BD	1	1	−1	2

$$\Delta Q = -\frac{(20)}{114.8} \simeq 0.2$$

AD = 37.3 from A to D

AB = 52.7 from A to B

DB = 0.7 from D to B

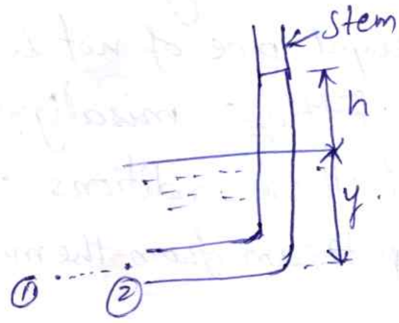
DC = 16.6 from D to C

BC = 23.4 from B to C.

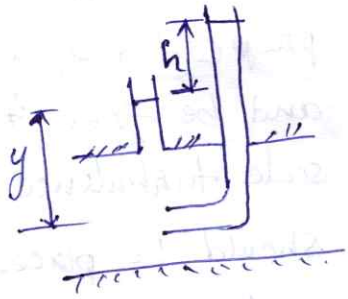
pitot tubes

It is elementary form, a pitot tube consists of a L-shaped tube, a tube bent through 90° and with ends unsealed. One limb called the body is inserted into the flow stream and aligned with the direction of flow, the other limb called stem, is vertical and open to atmosphere.

Applying bernoulli's eq. to pt ① & ②



ⓐ



ⓑ

$$y_1 + \frac{v_1^2}{2g} + \frac{p_1}{\gamma} = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + y_2$$

The flowing fluid is brought to state of zero velocity at the nose of tip of the tube and therefore $v_2 = 0$, $y_1 = y_2$

$$\frac{P_2 - P_1}{\gamma} = \frac{v_1^2}{2g}$$

The pressure P_1 in the undisturbed free stream flow is called the static pressure P_s and pressure P_2 at the stagnation point 2 where velocity is zero is referred as the total pressure.

$$\frac{v_1^2}{2g} = \left(\frac{P_t - P_s}{\gamma}\right)$$

for fig ⓑ $P_s = \gamma y$, $P_t = \gamma(y+h)$, $\Rightarrow \frac{v_1^2}{2g} = \frac{\gamma(y+h) - \gamma y}{\gamma} = h$

$$v = \sqrt{2gh}$$

Venturimeter: The venturimeter was invented by clemens Henshel in 1887 and has been named in the honour of an Italian engineer Venturi. The venturimeter consists of

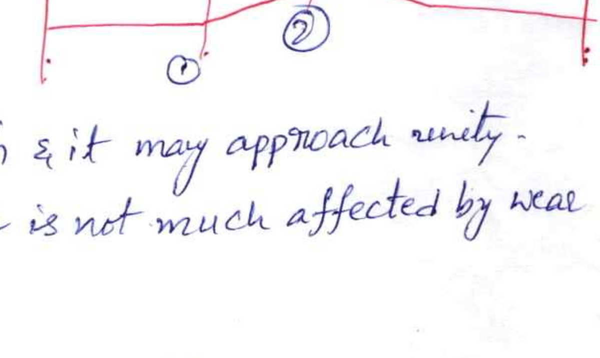
1. cylindrical entrance section: This section has the size of pipe to which it is attached. for satisfactory operation, the venturimeter should be preceded by a straight pipe of not less than 5 to 10 pipe diameters and be free from fittings, misalignment and other sources of large scale turbulence. If these conditions cannot be met, straightening vanes should be placed upstream from the meter for reduction of rotational motion in flow.

ii) Converging conical section: The converging take place at angle of $21° \pm 2°$. The velocity of fluid increases as it passes through the converging section and correspondingly the static pressure falls.

iii) Throat: This is a cylindrical section of min area. At this section the velocity is maximum and pressure is min. The throat dia is usually b/w $\frac{1}{2}$ to $\frac{1}{4}$ of inlet dia. length of throat equal to its dia.

iv) Diverging section in which there is a change of stream area back to the entrance area. The recovery of KE by its conversion to pressure is nearly complete and so the overall pressure loss is small. To accomplish a max recovery of KE, the diffuser section is made with an included angle of $5°$ to $7°$. This angle has to be kept less so that flowing fluid has least tendency to seperate out from boundary of section. The angle of the diverging cone may be kept as high as $14°$.

Advantages & limitations.



→ High pressure recovery is attainable, i.e. loss of head due to installation in the pipelines is small. due to low value of losses, the coefficient of discharge is high & it may approach unity.

→ Because of smooth surface, the meter is not much affected by wear and tear.

→ Ideally suited for large flow of water.

→ Long laying length, space requirements are more.

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + y_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + y_2$$

$$\frac{(P_1 - P_2)}{\gamma} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$A_1V_1 = A_2V_2$

$$\left(\frac{P_1 - P_2}{\gamma} = h\right)$$

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{A_2V_2}{A_1}\right)^2}{2g} = \frac{V_2^2}{2g}\left(1 - \left(\frac{A_2}{A_1}\right)^2\right)$$

$$Q_{th} = A_2 V_2 = \frac{A_2\sqrt{2gh}}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} = \frac{A_1A_2\sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$

$$Q_{actual} = C_d Q_{th} = \frac{C_d A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$

Orifice meter: The orifice meter consists of a thin, circular metal plate with a hole in it. The plate is held in the pipeline between two flanges called orifice flanges. The flow characteristics of the orifice differ from those of a nozzle in that min section of the stream-tube occurs not within the orifice but downstream from orifice edge. section of min area called vena contracta. and min pressure exists at this section.

Advantages & limitations.

- → Low intial cost, ease of installation & replacement.
- → less space require compared to venturi
- → can be used in wide range of pipes. [1.25cm to 150cm]
- → pressure recovery is poor; overall pressure loss varies from 40 to 90% of differential pressure.
- → coefficient of dischaye has low value.
- → necessity of providing straightening vanes upstream.

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + y_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + y_2$$

$$\frac{V_2^2 - V_1^2}{2g} = \frac{P_1 - P_2}{\gamma}$$

$$V_2 = \frac{1}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \times \sqrt{2gh}$$

$$C_V = \frac{V_{actual}}{V_{th}} \Rightarrow V_2 = C_V \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}}$$

$$Q = \frac{C_V A_2 \sqrt{2gh}}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}}$$

Area of jet at vena contra = $C_c \times$ area of orifice.

$A_2 = C_c A_2$

$$Q = \frac{C_c C_V A_2 A_1}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}.$$

$$Q = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}.$$

(P) A venturimeter 20 cm dia at inlet & 10 cm throat dia is laid with axis horizontal. used for measuring flow of oil of Sp gr 0.8. The difference of levels in U-tube man is 180 mm of Hg. & 11.52×10^3 kg of oil collected in 4 min. Calcul C_d.

$A_1 = \frac{\pi}{4}(0.2)^2 = 0.0314$, $A_2 = 0.00785$

$h = x\,(S_m - 1) = 0.18\left(\frac{13.6}{0.8} - 1\right) = 2.88$ m of oil.

Discharge $= \frac{11.52 \times 10^3}{4 \times 60} \cdot \frac{1}{800} = 0.06\ m^3/s$. ($m = \rho Q$)

$$Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh} = 0.0609\ m^3/s$$

$C_d = 0.985$

④ Petrol of specific gravity 0.8 is flowing through a pipe inclined at 30° to the horizontal in upward direction. A venturimeter is fitted in this 25cm dia pipe the ratio of areas of main and throat is 4 and throat is at a dist. of 1.2m from inlet along its length. The U-tube differential manometer connected to the inlet and throat section registers a steady reading of 5cm of Hg. tubes above the Hg being full of water. find discharge & pressure difference in KPa. $C_d = 0.95$

$$\frac{A_1}{A_2} = 4, \quad A_1 = 0.04906\ m^2,$$

$$h = x(S_m - 1) = 0.05\left(\frac{13.6}{0.8} - 1\right) = 0.8\ m\ of\ oil.$$

$$Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh} = 0.0477\ m^3/s.$$

$$h = \frac{P_1 - P_2}{\gamma} + (y_1 - y_2)$$

$$y_2 - y_1 = 1.2 \sin 30° = 0.6$$

$$0.8 = \frac{P_1 - P_2}{\gamma} - 0.6 \Rightarrow \boxed{P_1 - P_2 = 13.3\ KPa}$$

⑤ An orifice plate of orifice dia 10cm has been fitted into a 25cm dia pipe that conveys oil of sp. gravity 0.8. The pressure differential on the two sides of orifice plate is measured by mercury-oil differential manometer. If the gauge shows a deflection of 80cm of Hg. Cal. oil discharge in LPS. $C_d = 0.65$.

$$A_1 = \frac{\pi}{4}(0.25)^2 = 0.0491\ m^2, \quad A_2 = 0.00785\ m^2$$

$$h = x(S_m - 1) = 0.8\left(\frac{13.6}{0.8} - 1\right) = 12.8\ m\ of\ oil.$$

$$Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh} = 81.4\ ltr/sec.$$

⑥ An orifice meter with orifice dia 15cm is inserted in a pipe of 30cm dia. The pressure diff. is 50 cm of Hg. find Q of oil of sp. gr. 0.9, $C_d = 0.64$

$$h = x\left(\frac{S_n}{S_L} - 1\right) = 0.5\left(\frac{13.6}{0.9} - 1\right) = 7.055\ m\ of\ oil.$$

$$Q = C_d \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} = 137.4\ ltrs/sec.$$

Ⓠ find velocity of the flow of an oil through a pipe, when difference of mercury level in U-tube is 100mm. $C_v = 0.98$ & sp-gr oil = 0.8.

$$h = x\left(\frac{S_h}{S_L} - 1\right) = 0.1\left(\frac{13.6}{0.8} - 1\right) = 1.6\text{m of oil}.$$

$$V = C_v\sqrt{2gh} = 0.98\sqrt{2\times1.6\times9.81} = 5.49\text{ m/s}.$$

Ⓠ A pitot-tube is inserted in a pipe of 30cm dia. The static pressure in pipe is 10cm of Hg. The stagnation pressure at the centre of the pipe, recorded by the pitot tube is 0.981 N/cm². Calculate rate of flow of water through pipe, if the mean velocity of flow is 0.85 times the central velocity. Take $C_v = 0.98$.

$d = 0.3\text{m}$, $a = 0.07068\text{m}^2$

Static pressure head = 100 mm of Hg (vacuum)

$$= -\frac{100}{1000}\times13.6 = -1.36\text{ m of water}.$$

Stagnation pressure = 0.981 = 0.981×10^4 N/m²

$$\text{Stagnation pressure head} = \frac{0.981\times10^4}{9810} = 1\text{m}.$$

h = stagnation − static pressure

$= 1 - (-1.36) = 2.36$ m of water.

$$V \text{ at centre} = C_v\sqrt{2gh} = 0.98\sqrt{2\times9.81\times2.36} = 6.68\text{m/s}$$

$$\bar{V} = 0.85\times6.68 = 5.67\text{ m/s}.$$

Rate of flow = $\bar{V}$ × area of pipe = $5.67\times0.07068 = 0.4\text{ m}^3/\text{s}$.

Classifications of orifices:

→ orifices are classified as small & large orifice depending upon size of orifice and head of liquid from the centre of orifice. If head is more than 5 times depth of orifice, is called small orifice.

→ Based on C/s are i) circular ② triangular ③ Rectangular ④ Square orifice

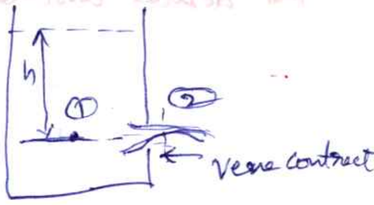
→ Based on shape of upstream edge of orifices i) Sharp edged orifice (ii) Bell-mouthed

→ Based on nature of discharge (i) free discharging ② Drowned (or) Sub-merged orif.

→

Orifices and mouthpieces.

Flow through An orifice.



Consider a tank fitted with a circular orifice in one of its sides as shown in fig.

Let H be the head of the liquid above the centre of the orifice. The liquid flowing through the orifice form a jet of liquid whose area of c/s is less than that of orifice. The are of jet of fluid goes on decreasing and at a section c-c, the area is min. This section is appr. at a distance of half of dia of the orifice. The section is called vena contracta.

applying Bernoulli's eq ① & ②

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + Z_2$$

$$\left(\frac{P_1}{\gamma} = H\right) \quad , \quad \frac{P_2}{\gamma} = 0 \text{ atmosp}$$

v_1 is very small in comparison to v_2 as area of tank is very large.

$$H = \frac{V_2^2}{2g} \Rightarrow V_2 = \sqrt{2gh} \quad \text{[the oritical velocity]}$$

Hydraulic coefficients

① **Coefficient of velocity (C_v)**: It is defined as the ratio b/w the actual velocity of a jet of liquid at vena-contracta and theoritical velocity of jet.

$$C_v = \frac{V}{\sqrt{2gH}} , \qquad C_v \text{ varies} \to 0.95 \text{ to } 0.99.$$

$C_v = 0.98$ for sharp edge orifices.

② **Coefficient of contraction:** It is defined as ratio of the area of jet at vena-contracta to area of the orifice.

$$C_c = \frac{a_c}{a} = \frac{\text{area at vena-contracta}}{\text{area of orifice}}$$

C_c - varies - 0.61 to 0.69.

③ C_d : =

$$C_d = \frac{\text{Actual velocity}}{\text{theoritical vel.}} \times \frac{\text{Actual area}}{\text{theoriti area}}$$

$$C_d = C_v \times C_c.$$

$C_d \to 0.61$ to 0.65 for orifice

for general $C_d = 0.62$

(5) The head of water over the centre of an orifice of dia 20mm is 1m. The actual discharge through the orifice is 0.85 litr/s. Find C_d.

$d = 0.02 m$, $a = 0.000314 m^2$,

$H = 1m$, $Q = 0.85$ ltrs/s.

$V_{th} = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1} = 4.429 m/s$.

$Q_{th} = V_{th} \times A_{th} = 4.429 \times 0.000314 = 0.00139 m^3/s$.

$C_d = \frac{0.00085}{0.00139} = 0.61$

(6) The head of water over an orifice of dia 100mm is 10m. The water coming out from orifice is collected in a circular tank of dia 1.5m. The rise of water level in this tank is 1.0m in 25 sec. Also the coordinates of a point on the jet, measured from vena-contracta are 4.3m horizontal & 0.5m vertical. Find C_d, C_v & C_c.

$H = 10 m$, $d = 0.1 m$. $a = 0.007853 m^2$

$D = 1.5 m$.

$A = \frac{\pi}{4}(1.5)^2 = 1.767 m^2$

Rise of water $h = 1m$, $t = 25$ sec

$x = 4.3$, $y = 0.5 m$

$V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 10} = 14 m/s$.

$Q_{th} = V_{th} \times$ Area of orifice $= 14 \times 0.007854 = 0.1099 m^3/s$.

$Q_{actual} = \frac{A \times h}{t} = \frac{1.767 \times 1}{25} = 0.07068$.

$C_d = \frac{Q_{actual}}{Q_{th}} = \frac{0.07068}{0.1099} = 0.64$

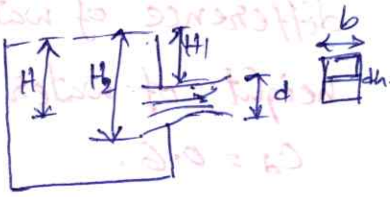
$C_v = \frac{x}{\sqrt{4yH}} = \frac{4.3}{\sqrt{4 \times 0.5 \times 10}} = 0.96$

$C_d = C_c C_v \Rightarrow C_c = 0.669$

Discharge through Large rectangular orifice:

Area of strip = $b \times dh$

Theoritical velocity through Strip = $\sqrt{2gh}$.



$$dQ = C_d (b \times dh)\sqrt{2gh}$$

$$Q = C_d b \sqrt{2g} \int_{H_1}^{H_2} \sqrt{h}\, dh = \frac{2}{3} C_d b \sqrt{2g} \left(H_2^{3/2} - H_1^{3/2}\right)$$

(P) A rectangular orifice 0.9m wide and 1.2m deep is discharging water from a vessel. The top edge of the orifice is 0.6m below the water surface in the vessel. Cal. the discharge through orifice if $C_d = 0.6$ and % error if orifice is treated as a small orifice.

$b = 0.9m$, $d = 1.2m$, $H_1 = 0.6m$. $H_2 = H_1 + d = 1.8m$.

$C_d = 0.6$.

$$Q = \frac{2}{3} C_d\, b \sqrt{2g} \left[H_2^{3/2} - H_1^{3/2}\right] = 3.109\, m^3/s.$$

for small orifice $\qquad h = H_1 + \frac{d}{2} = 1.2m.$

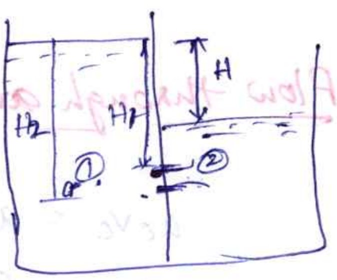
$$Q_1 = C_d a \sqrt{2gh} = 3.144\, m^3/s.$$

$$\% \text{ error} = \frac{3.144 - 3.1097}{3.1097} = 1.1\%$$

Discharge through fully submerged orifice:

Height of water above centre of orifice on upst/sd.

$$= H_1 + \frac{H_2 - H_1}{2} = \frac{H_1 + H_2}{2}$$



H. D/S $\Rightarrow \frac{H_1 + H_2}{2} - H$

Applying Bernoullis eq. ① & ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

$$\frac{P_1}{\gamma} = \frac{H_1 + H_2}{2}, \quad \frac{P_2}{\gamma} = \frac{H_1 + H_2}{2} - H$$

$$\frac{V^2}{2g} = H \Rightarrow V_2 = \sqrt{2gH}.$$

Area of orifice = $b \times (H_2 - H_1)$

$$Q = C_d\, b (H_2 - H_1) \sqrt{2gH}.$$

(P) find discharge through a fully submerged orifice of width 2m if the difference of water levels on both sides of the orifice be 50cm. The height of water from top and bottom of the orifice are 2.5m & 2.75m. $C_d = 0.6$.

given $b = 2m$, $H = 50cm$, $H_1 = 2.5m$, $H_2 = 2.5m$, $C_d = 0.6$

$$Q = C_d\, b\, (H_2 - H_1)\sqrt{2gH} = 0.9396\ m^3/s.$$

* Time of emptying a tank through an orifice at its bottom.

$$T = \frac{2A\sqrt{H_1}}{C_d\, a\sqrt{2g}}$$, $H_1 \to$ Intial height, $a \to$ area of orifice.

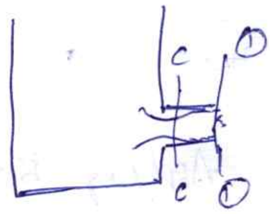
$$T = \frac{2A\left[\sqrt{H_1} - \sqrt{H_2}\right]}{C_d\, a\sqrt{2g}}$$

(P) A circular tank of dia 4m contains water upto 5m. The tank is provided with an orifice at its bottom of dia 0.5m. find time taken by water from 5m to 2m. ii) for completely empty. $C_d = 0.6$.

$$T_1 = \frac{2A(\sqrt{H_1} - \sqrt{H_2})}{C_d\, a\sqrt{2g}} = 39.5\,sec.$$

$T_{empty} = 107.7$ Sec.

Flow through an External cylindrical mouthpiece :



$a_c V_c = a_1 V_1$

$V_c = \frac{a_1 V_1}{a_c}$

$C_c = \frac{a_c}{a_1}$ = coefficient of contraction

$C_c = 0.62$ we got $\frac{a_c}{a_1} = 0.62$

$V_c = \frac{V_1}{0.62}$

jet of liquid from section C-C suddenly enlarges at ①-① Due to sudden enlargement $h_L = \frac{(V_c - V_1)^2}{2g}$, $h_L = \frac{0.375\, V_1^2}{2g}$.

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z + h_L$$

$$H = \frac{V_1^2}{2g} + 0.375\frac{V_1^2}{2g}$$

$$v_1 = \sqrt{\frac{2gH}{1.375}} = 0.855\sqrt{2gH}.$$

$$C_v = \frac{0.855\sqrt{2gH}}{\sqrt{2gH}} = 0.855.$$

C_c for mouthpiece $= 1$

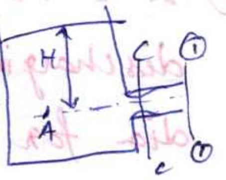
$$C_d = C_c \times C_v = 1 \times 0.855 = 0.855.$$

(Q) An external cylindrical mouthpiece of dia 150mm is discharging water under a constant head of 6m. Dete. the discharge and absolute pressure head of water at vena-contracta. Take $C_d = 0.855$, $C_c = 0.62$. $P_{atm} = 10.3$ m of water.

$d = 0.15m$, $a = 0.01767 m^2$ $H = 6.0m$, $C_d = 0.855$.

$H_a = 10.3m$.

$$Q = C_d a\sqrt{2gH} = 0.855 \times 0.01767\sqrt{2\times9.81\times6} = 0.164 m^3/s.$$



Bernoulli's eq.n (A) & (C)

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_c}{\gamma} + \frac{V_c^2}{2g} + Z_c$$

$$\frac{P_A}{\gamma} = H_a + H, \quad V_A = 0,$$

$$H_a + H = H_c + \frac{V_c^2}{2g}$$

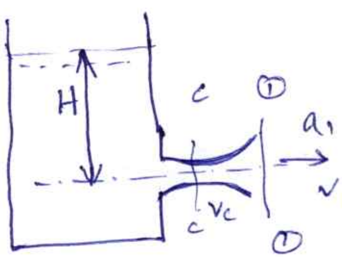
$$H_c = H_a + H - \frac{V_c^2}{2g} \Rightarrow \left(V_c = \frac{V_1}{0.62}\right)$$

$$H = 1.375\frac{V_1^2}{2g} \qquad \frac{V_1^2}{2g} = 0.7272H.$$

$$H_c = H_a + H - 1.89H = 10.3 - 0.89\times6 = 4.96\,m.$$

(Q) flow through a convergent – Divergent mouth piece:

If mouthpiece converges upto vena-contracta and then diverges. Then the type of mouth piece is called C-D-MP.



$$\frac{P}{\rho g} + \frac{V^2}{2g} + Z = \frac{P_c}{\rho g} + \frac{V_c^2}{2g} + Z_c$$

$$\frac{P}{\rho g} = H_a, \quad v = 0, \quad Z = H, \quad \frac{P_c}{\rho g} = H_c, \quad Z_c = 0.$$

$$H_a + 0 + H = H_c + \frac{V_c^2}{2g} + 0.$$

$$V_c = \sqrt{2g(H_a + H - H_c)}$$

$$\frac{P_c}{\rho g} + \frac{V_c^2}{2g} + Z_c = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 \qquad Z_c = Z_1$$

$$H_c + \frac{V_c^2}{2g} = H_a + \frac{V_1^2}{2g}$$

$$H_c + \frac{V_c^2}{2g} = H + H_a$$

$$H_a + \frac{V_1^2}{2g} = H + H_a$$

$$V_1 = \sqrt{2gH}$$

$$a_c V_c = V_1 a_1$$

$$\frac{a_1}{a_c} = \frac{V_c}{V_1} = \frac{\sqrt{2g(H_a + H - H_c)}}{\sqrt{2gH}} = \sqrt{1 + \left(\frac{H_a - H_c}{H}\right)}$$

$$Q = a_c\sqrt{2gH}$$ $a_c \rightarrow$ area at vena-Contracta.

(P) A convergent divergent mouthpiece having throat dia of 4.0 cm is discharging water under a constant head of 2.0m dete the max outer dia for max discharge. Find max discharge also. Take $H_a = 10.3$ m.w. H_{sep} = 2.5m of water (absolute).

$d_c = 4$ cm, $\quad a_c = \frac{\pi}{4}(4)^2 = 12.566\ cm^2$

$H = 2$ m.

$$Q_{max} = a_c\sqrt{2gH} = 7871.5 \times 10^{-6}\ m^3/s.$$

$$\frac{a_1}{a_c} = \sqrt{1 + \frac{H_a - H_c}{H}} = \sqrt{1 + \frac{10.3 - 2.5}{2}}$$

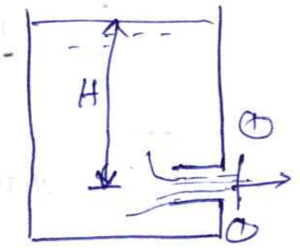
$$= 2.2135$$

$$\frac{d_1}{d_c} = \sqrt{2.21} = 1.48$$

$$d_1 = 5.95\ cm.$$

Flow through Internal (or) Re-entrant on Borda's mouthpiece.

i) Borda's mouthpiece Running free.



Dischage $= Q = C_d a \sqrt{2gH}$

$= 0.5 a \sqrt{2gH}$

$C_c = 0.5, \quad C_v = 1, \quad C_d = 0.5$

ii) Borda's mouthpiece running full.

$C_v = \frac{\sqrt{gH}}{\sqrt{2gH}} = \frac{1}{\sqrt{2}} = 0.707$

$C_d = C_c C_v = 1 \times 0.707 = 0.707$

$Q = C_d a \sqrt{2gH} = 0.707 a \sqrt{2gH}$

(P) An internal mouthpiece of 80mm dia is discharging water under a constant head of 8m. find discharge through mouthpiece, whe i) running free ii) running full.

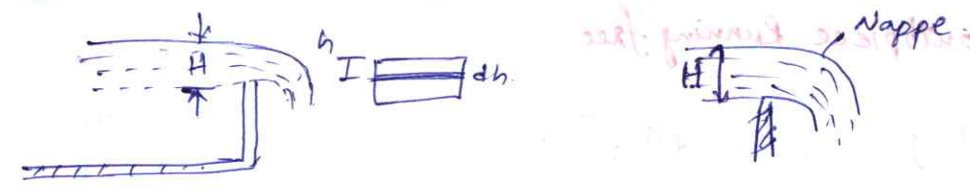
Dia $= d = 80mm, \quad a = \frac{\pi}{4}(0.08)^2 = 0.005026 \, m^2$

$H = 4m,$

i) $Q = 0.5 \, a \sqrt{2gH} = 22.26$ Ltrs/Sec.

ii) $Q = 0.707 \, a \sqrt{2gH} = 31.47$ Ltrs/Sec

Discharge over a rectangular Notch (or) weir.



Notch

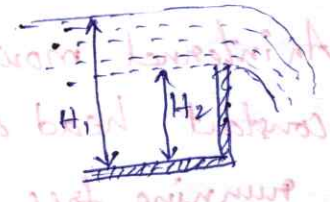
$$dq = C_d \times (L \times dh)\sqrt{2gh}.$$

$$Q = \int_0^H C_d L\sqrt{2g}\,(\sqrt{h})\,dh = \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2}$$

Q. Determine the height of a rectangular weir of length 6m to be built across a rectangular channel. The max. depth of water on upstream side of the weir is 1.8m and discharge is 2000 litres/s. Take $C_d = 0.6$

$L = 6m,\ H_1 = 1.8m;\ Q = 2m^3/s$

$C_d = 0.6.$



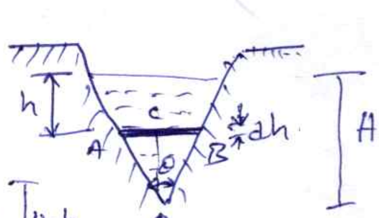
$$Q = \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2}$$

$H = 0.328m.$

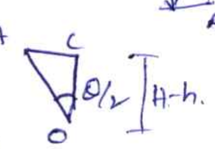
Height of weir $= H_2 = H_1 - H = 1.8 - 0.328 = 1.472m.$

Discharge over a triangular Notch or weir.

$\tan\theta/2 = \dfrac{AC}{OC} \Rightarrow$



$AC = (H-h)\tan\theta/2$



width of strip $AB = 2AC = 2(H-h)\tan\theta/2$

Area of strip $= 2(H-h)\tan\theta/2\ dh.$

$$dQ = C_d \times 2(H-h)\tan\frac{\theta}{2}\sqrt{2gh}\ dh$$

Total discharge $$Q = \int_0^H 2C_d(H-h)\tan\theta/2\,\sqrt{2gh}\ dh.$$

$$= 2C_d\tan\frac{\theta}{2}\sqrt{2g}\left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$$

$$= \frac{8}{15} C_d \tan\frac{\theta}{2} \times \sqrt{2g}\ H^{5/2}$$

for right-angled v-notch, $C_d = 0.6$, $\theta = 90°$.

$$Q = 1.417\,H^{5/2}$$

(3) water flows over a rectangular weir 1m wide at a depth of 150 mm and afterwards passes through a triangular right-angled weir. Take C_d for rect – 0.62, for triangular 0.59. find depth over the triangular weir.

for rectangular weir, length $L = 1m$, $H = 150mm = 0.15m$.

$C_d = 0.62$, $\theta = 90°$,

$$Q_{rect} = \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2} = \frac{2}{3}(0.62)\, 1 \times \sqrt{2 \times 9.81}\,(0.15)^{3/2}$$

$$Q_{rect} = 0.10635\ m^3/s.$$

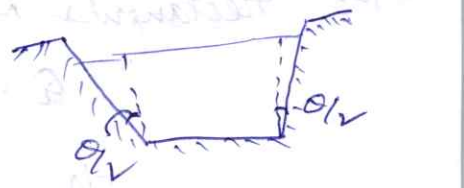
$$Q_{triangle} = \frac{8}{15} C_d \sqrt{2g}\, \tan\frac{\theta}{2}\, H^{5/2}$$

$$H = 0.3572\ m.$$

Advantages of triangular Notch or weir.

1. The expression for discharge for a right-angled v-notch or weir is very simple
2. for measuring low discharge, a triangular notch gives more accurate results
3. In case of triangular notch only one reading i.e H.
4. ventilation of a triangular notch is not necessary.

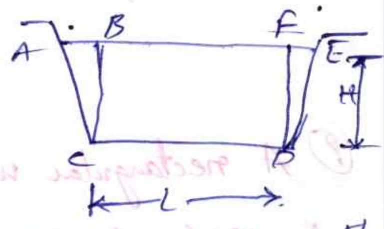
Discharge over a trapezoidal Notch or weir.



$$Q = Q_{rect} + Q_{triangle}$$

$$Q = \frac{2}{3} C_{d_1} L \sqrt{2g}\, H^{3/2} + \frac{8}{15} C_{d_2} \tan\frac{\theta}{2} \sqrt{2g}\, H^{5/2}$$

(4) find the discharge through a trapezoidal notch which is 1m wide at top and 0.4m at the bottom and is 30cm height. The head of water on the notch is 20cm. Assume C_d for rect. = 0.62, C_d tria = 0.6.



$$\tan\frac{\theta}{2} = \frac{AB}{BC} = \frac{\left(\frac{AE - CD}{2}\right)}{BC} = \frac{0.3}{0.3} = 1$$

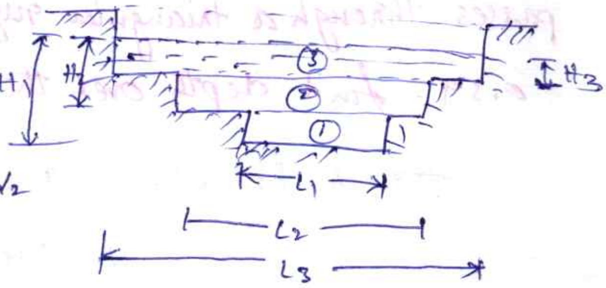
$$Q = \frac{2}{3} C_{d_1} L \sqrt{2g}\, H^{3/2} + \frac{8}{15} C_{d_2} \tan\theta/2 \sqrt{2g}\, H^{5/2}$$

$$= \frac{2}{3} \times 0.62 \times 0.4 \times \sqrt{2 \times 9.81}\,(0.2)^{3/2} + \frac{8}{15}\, 0.6 \times 1 \times \sqrt{2 \times 9.81}\,(0.2)^{5/2}$$

$$Q = 90\ liter/s.$$

Discharge over a Stepped Notch

$$Q = \frac{2}{3} C_d L_1 \sqrt{2g}\,[H_1^{3/2} - H_2^{3/2}] + \frac{2}{3} C_d L_2 \sqrt{2g}\,(H_2^{3/2} - H_3^{3/2}) + \frac{2}{3} C_d L_3 \sqrt{2g}\,[H_3]^{3/2}$$



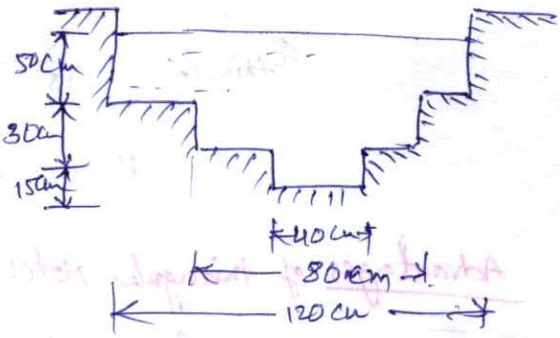
(P) The fig shows a stepped notch. find discharge through the notch C_d for all $= 0.6$.

$L_1 = 40cm$, $L_2 = 80cm$, $L_3 = 120cm$

$H_1 = 50 + 30 + 15 = 95cm$.

$H_2 = 50 + 30 = 80cm$.

$H_3 = 50cm$



$Q = Q_1 + Q_2 + Q_3$

$Q_1 = 154\ l/s$ $Q_2 = 530\ l/s$, $Q_3 = 776.7\ litre/s$.

$Q = 1460\ ltr/s$.

Effect on discharge over a notch or weir due to error in measuring head.

for rectangular notch or weir.

$$Q = \frac{2}{3} C_d L \times \sqrt{2g}\, H^{3/2} = KH^{3/2}$$

$$\frac{dQ}{Q} = \frac{K\frac{3}{2}\sqrt{H}\,dH}{KH^{3/2}} = \frac{3}{2}\frac{dH}{H}$$

error of 1% in measuring H will produce 1.5% error in disch

for triangular weir or Notch.

$$\frac{dQ}{Q} = \frac{5}{2}\frac{dH}{H}$$

(P) A rectangular notch 40cm long is used for measuring a discharge of 30 ltr/s. An error of 1.5mm was made, while measuring the head over the notch. Calculate percentage error in discharge. Take $C_d = 0.6$.

$L = 40cm$, $Q = 30 lit/s$, $dH = 1.5mm$, $C_d = 0.6$

$$Q = \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2}$$

$H = 12.16cm$.

$$\frac{dQ}{Q} = \frac{3}{2}\frac{dH}{H} =$$

1.85%

(Q) Time required to empty a reservoir (or) A Tank with a rectangular notch

$$T = \frac{3A}{C_d \times L \times \sqrt{2g}} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right]$$

Time for triangular weir (or) Notch $= \frac{5A}{4 C_d \tan\frac{\theta}{2} \sqrt{2g}} \left[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}} \right]$

(P) find the time req. to lower ~~the lower~~ the water level from 3m to 2m in a reservoir of dimension 80x80m. by rectangular notch of length 1.5m. $C_d = 0.62$

$H_1 = 3$, $H_2 = 2m$, $A = 80 \times 80 = 6400 m^2$

$L = 1.5m$, $C_d = 0.62$

$$T = \frac{3A}{C_d L \sqrt{2g}} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right] = 605 \text{ sec.}$$

If triangular notch

$$T_{triangular} = \frac{5A}{4 C_d \sqrt{2g} \tan\frac{\theta}{2}} \left[\frac{1}{(H_2)^{3/2}} - \frac{1}{(H_1)^{3/2}} \right] = 469 \text{ sec?}$$

Velocity of approach :

velocity of approach is defined as the velocity with which the water approaches or reaches the weir or notch before it flows over it. Thus if V_a is the velocity of approach, then additional head $h_a = \frac{V_a^2}{2g}$ due to velocity of approach. is acting on the water flowing over the notch. Then initial height of water over the notch becomes $(H + h_a)$ and final height becomes equal to h_a.

$$V_a = \frac{Q}{\text{Area of Channel}}$$

$$Q = \frac{2}{3} C_d L \sqrt{2g} \left[(H_1 + h_a)^{3/2} - h_a^{3/2} \right]$$

(P) water is flowing in a rectangular channel of 1m wide & 0.75m deep. find the discharge over a rectangular weir of crest length 60cm, if the head of water over the crest of weir is 20cm and water from channel flows over the weir. $C_d = 0.62$. Take velocity of approach into consideration.

$$Q = \frac{2}{3} C_d L \sqrt{2g} H_1^{3/2} = \frac{2}{3}(0.62) 0.6 \times \sqrt{2 \times 9.81} \times (0.2)^{3/2} = 0.0982$$

$$V_a = \frac{Q}{A} = \frac{0.0982}{0.75} = 0.1309 \text{ m/s}.$$

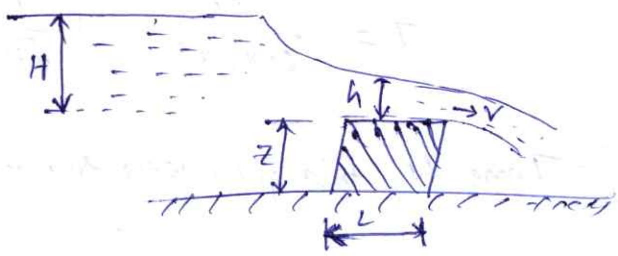
$$h_a = \frac{V_a^2}{2g} = 0.000873 \text{ m}.$$

$$Q = \frac{2}{3} \times 0.62 \times 0.6 \times \sqrt{2 \times 9.81} \left[(0.2 + 0.00087)^{3/2} - (0.00087)^{3/2} \right]$$

$= 0.09881 \text{ m}^3/\text{s}$.

Discharge over a Broad-Crested weir :

A weir having wide crest is known as broad-crested weir.



Let H = height of water above the crest

L - Length of crest

If 2L > H, the weir is called broad-crested weir

If 2L < H, the weir is called narrow-crested weir

Let h = head of water at the middle of weir which is constant

V = velocity of flow over the weir.

$$0+0+H = 0+\frac{V^2}{2g}+h.$$

$$V = \sqrt{2g(H-h)}$$

$$Q = C_d \times L\times h \times \sqrt{2g(H-h)} = C_d L\sqrt{2g(Hh^2-h^3)}$$

discharge will be max, if (Hh^2-h^3) is max.

$$\frac{d}{dh}(Hh^2-h^3) = 0 \quad (d) \qquad 2hH - 3h^2 = 0 \Rightarrow \boxed{h=\frac{2}{3}H}$$

$$Q_{max} = C_d L\sqrt{2g\left\{H\left(\frac{2}{3}H\right)^2-\left(\frac{2}{3}H\right)^3\right\}} = C_d L\sqrt{2g}\sqrt{\frac{4}{27}H^3}$$

$$Q_{max} = 1.705\, C_d L H^{3/2}$$

(P) a) A broad-crested weir of 50 m length, has 50cm height of water above its crest. find max discharge. Take $C_d = 0.6$. Neglect velocity of app.

(b) If velocity of approach is to be taken into consideration. find max disch when channel c/s area 50 m² on u/s side.

i) Neglecting velocity of approach $Q_{max} = 1.705\, C_d L H^{3/2} = 18.084\, m^3/s.$

ii) $V_a = \frac{Q}{A} = \frac{18.084}{50} = 0.36\ m/s.$

$$h_a = \frac{V_a^2}{2g} = 0.0066\,m$$

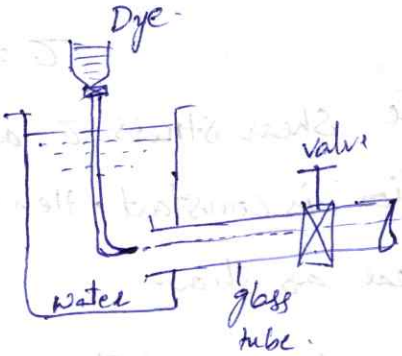
$$Q_{max} = 1.705 \times C_d \times L\left[(H+h_a)^{3/2} - h_a^{3/2}\right]$$

$$Q_{max} = 18.412\ m^3/s.$$

UNIT-IV

Laminar & Turbulent Flow.

Reynolds Experiment :



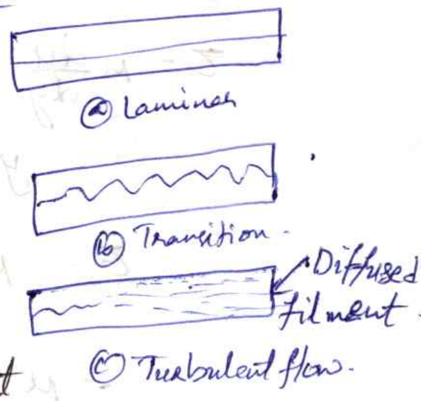
The apparatus consists of

i) water tank at constant head

ii) Small tank containing some dye

iii) A glass tube having bell-mouthed entrance at one end & regulating valve at other ends.

The water from the tank was allowed to flow through the glass tube. The velocity of flow was varied by the regulating valve. A liquid dye having same specific weight as water was introduced into the glass tube as shown in fig.



observations

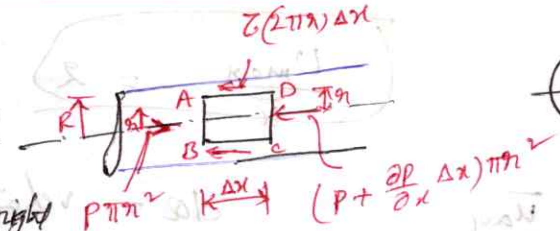
i) when velocity of flow was low, the dye filament in the glasstube was in the form of a straight line. This straight line of dye filament was parallel to the glass tube which was case of laminar flow.

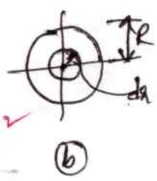
ii) with increase of velocity of flow, the dye filament was no longer a straight line but it became a wavy one as show in fig (b).

iii) with further increase of velocity of flow, the wavy dye-filament broke up and finally diffused in water as shown fig (c). Turbulent flow.

flow of viscous fluid through circular pipe :

$$Re = \frac{\rho V D}{\mu}$$





Consider a horizontal pipe of radius R. The viscous fluid is flowing from left to right in the pipe as shown.

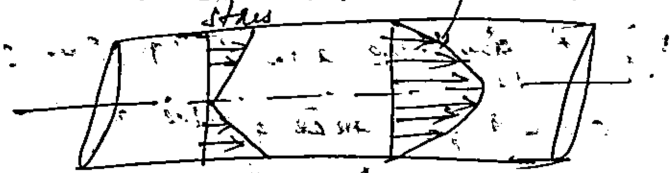
(a) consider a fluid element of radius r, sliding in cylindrical fluid element of radius (r+dr). let length of fluid element be Δx. If 'p' is the intensity of pressure on face AB, intensity of pressure on face CD will $(P + \frac{\partial P}{\partial x}\Delta x)$. Then forces acting on the fluid element

1. pressure force, $(P \times \pi r^2)$ on AB
2. pres $(P+\frac{\partial P}{\partial x}\Delta x)\pi r^2$ on CD
3. Shear force $(\tau \times 2\pi r \Delta x)$ on surface of fluid element.

$$p\pi r^2 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2 - \tau(2\pi r)\Delta x = 0.$$

$$\therefore \tau = -\frac{\partial p}{\partial x}\left(\frac{r}{2}\right)$$

The shear stress τ across a section varies with r as $\frac{\partial p}{\partial x}$ across a section is constant. Hence shear stress distribution across a section is linear as shown.



i) velocity distribution.

$\tau = \mu \frac{du}{dy}$, y is measure from the pipe wall.

$y = R - r$, $dy = -dr$

$$\therefore \tau = \mu \frac{du}{-dr} = -\mu \frac{du}{dr}$$

$$-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x}\frac{r}{2} \Rightarrow$$

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 + c$$

$r = R$, $u = 0$, $c = -\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2$

$$u = -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right]$$

ii) Ratio of max. velocity to Avg. velocity.

$(r=0)$

$$U_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2$$

$$\frac{U_{max}}{\bar{u}} = 2$$

$\bar{u}$: dQ = vel at r × area of ring element

$$= u \times 2\pi r\,dr$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right] 2\pi r\,dr$$

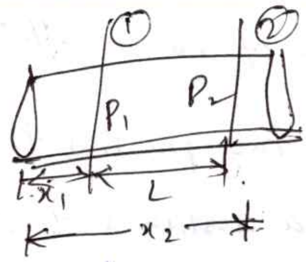
$$Q = \int dQ = \int_0^R \left(-\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2 - r^2)\right) 2\pi r\,dr$$

$$\bar{u} = \frac{Q}{A} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2$$

iii) Drop of pressure for a given length L of pipe;

$$\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2 \quad \left(-\frac{\partial p}{\partial x} = \frac{8\mu\bar{u}}{R^2}\right) \qquad (I)$$

$$-\int_2^1 dp = \int_2^1 \frac{8\mu\bar{u}}{R^2} dx$$



$$-[P_1 - P_2] = \frac{8\mu\bar{u}}{R^2}[x_1 - x_2] \text{ or}$$

$$(P_1 - P_2) = \frac{8\mu\bar{u}L}{(D/2)^2}$$

$$(P_1 - P_2) = \frac{32\mu\bar{u}L}{D^2}$$

$$\frac{(P_1 - P_2)}{\rho g} = \frac{32\mu\bar{u}L}{\rho g D^2} = h_f \quad \text{— (1)}$$

This equation is called Hagen Poiseuille formula.

(P) A crude oil of viscosity 0.97 poise and relative density 0.9 is flowing through a horizontal circular pipe of dia 10cm and of length 10m calculate difference of pressure at the two ends of the pipe, if 100kg of oil is collected in tank in 30 sec.

$\mu = 0.97$ Poise $= 0.097$ NS/m²

$G = 0.9$, $L = 10m$, $D = 0.1m$.

mass of oil/sec $= \frac{100}{30}$ kg/s $= \rho_0 Q = 900Q$

$Q = 0.0037$ m³/s.

$$\bar{u} = \frac{Q}{Area} = \frac{0.0037}{\frac{\pi}{4}(0.1)^2} = 0.471 m/s.$$

$$Re = \frac{\rho VD}{\mu} = \frac{900 \times 0.471 \times 0.1}{0.0971} = 436 \quad < 2000$$

flow is laminar

$$P_1 - P_2 = \frac{32\mu\bar{V}L}{D^2} = \frac{32 \times 0.097 \times 0.471 \times 10}{(0.1)^2} = 1462 \text{ N/m}^2$$

(T) An oil having viscosity of 1.43 Poise and sp gravity of 0.9 flows through a pipe 2.5cm dia and 300cm long, at one-tenth of critical velocity for which Re is 2500. find (a) velocity of flow through the pipe (b) head loss in meter of oil across the pipe length. required to maintain the flow (c) power req to overcome viscous resistance of flow of oil.

(a) $\mu = 1.43 = 0.143$ Pa.Sec.

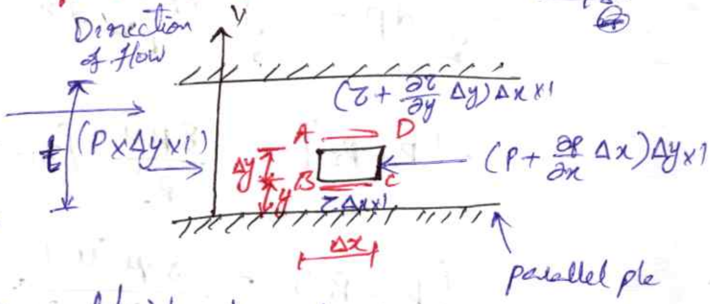
$$Re_c = \frac{V_c d \rho}{\mu} \Rightarrow V_{critical} = \frac{Re_c \mu}{d\rho} = \frac{2500 \times 0.143}{0.025 \times 900} = 15.8 m/s$$

$V_{avg} = \frac{1}{10}(15.8) = 1.58$ m/s.

(b) $P_1 - P_2 = \frac{32\mu V_{avg} L}{d^2} \Rightarrow h_f = 3.59$ m of oil.

(c) $P = \gamma Q h_f = (900 \times 9.81)(\frac{\pi}{4}(0.025)^2 \times 1.583) \times 3.59 = 24.?$ W

Flow of viscous fluid between Two parallel plates :



Consider two parallel fixed plates kept at a distance 't' apart as shown. A viscous fluid is flowing b/w these two plates from left to right. Consider a fluid element of lengths Δx and thickness Δy at a distance y from the lower fixed plat. If P is the intensity of pressure on face AB of the fluid element then pressure on face CD will be $(P + \frac{\partial P}{\partial x}\Delta x)$. Let τ is the shear stress acting on the face BC then the shear stress on face AD will be $(\tau + \frac{\partial \tau}{\partial y}\Delta y)$. If width of the element in the direction ⊥ to paper is unity. Then forces acting on the fluid elemet are.

1. The pressure force $(P \times \Delta y \times 1)$ on face AB.
2. pressure force $(P + \frac{\partial P}{\partial x}\Delta x)(\Delta y \times 1)$ on face CD
3. The shear force $(\tau \times \Delta x \times 1)$ on face BC.
4. The shear force $(\tau + \frac{\partial \tau}{\partial y}\Delta y)\,\Delta x \times 1$ on face AD

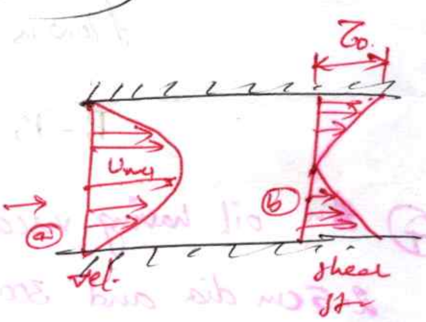
for steady and uniform flow there is no acceleration and hence resultant force in the direction of flow is zero.

$$P(\Delta y \times 1) - (P + \frac{\partial P}{\partial x}\Delta x)\Delta y \times 1 - \tau(\Delta x \times 1) + (\tau + \frac{\partial \tau}{\partial y}\Delta y)\Delta x \times 1 = 0$$

$$-\frac{\partial P}{\partial x}\Delta x \Delta y + \frac{\partial \tau}{\partial x}\Delta y \Delta x = 0$$

$$-\frac{\partial P}{\partial x} + \frac{\partial \tau}{\partial y} = 0 \quad (\text{or}) \quad \boxed{\frac{\partial P}{\partial x} = \frac{\partial \tau}{\partial y}}$$

i) Velocity Distribution :



$$\tau = \mu \frac{du}{dy}$$

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial y}\left(\mu \frac{du}{dy}\right) = \mu \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial P}{\partial x}$$

Integration

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial P}{\partial x} y + C_1$$

$$u = \frac{1}{\mu}\frac{\partial P}{\partial x}\frac{y^2}{2} + C_1 y + C_2$$

(i) $y = 0,\ u = 0$; (ii) at $y = t,\ u = 0$ $\qquad C_2 = 0.$

$$C_1 = -\frac{1}{2\mu}\frac{\partial P}{\partial x} t$$

$$u = -\frac{1}{2\mu}\frac{\partial P}{\partial x}\left[ty - y^2\right]$$

ii) Ratio of Max velocity to Avg velocity: The velocity is max at $y = t/2$

$$U_{max} = \frac{-1}{2\mu}\frac{\partial p}{\partial x}\left[t \times \frac{t}{2} - \left(\frac{t}{2}\right)^2\right]$$

$$U_{max} = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{t^2}{2} - \frac{t^2}{4}\right] = \frac{-1}{8\mu}\frac{\partial p}{\partial x} t^2$$

dQ = v. at y × Area of Strip

$$= \frac{-1}{2\mu}\frac{\partial p}{\partial x}(ty - y^2)(dy \times 1)$$

$$Q = \int_0^t dQ = \int_0^t \left(-\frac{1}{2\mu}\frac{\partial p}{\partial x}(ty - y^2)\right)dy$$

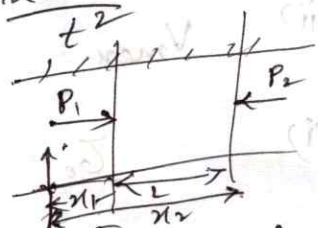
$$Q = \frac{-1}{12\mu}\frac{\partial p}{\partial x} t^3$$

$$\bar{u} = \frac{Q}{Area} = \frac{\frac{1}{12\mu}\frac{\partial p}{\partial x} t^3}{(t \times 1)} = \frac{-1}{12\mu}\frac{\partial p}{\partial x} t^2$$

$$\boxed{\frac{U_{max}}{\bar{u}} = \frac{3}{2}}$$

iii) Drop of pressure head for a given length:

$$\bar{u} = \frac{-1}{12\mu}\frac{\partial p}{\partial x} t^2 \quad (or) \quad \frac{\partial p}{\partial x} = \frac{12\mu\bar{u}}{t^2}$$



$$\int_2^1 dp = \int_2^1 \frac{-12\mu\bar{u}}{t^2} dx$$

$$(P_1 - P_2) = -\frac{12\mu\bar{u}}{t^2}[x_1 - x_2] = \frac{12\mu\bar{u}}{t^2}(x_2 - x_1)$$

$$(P_1 - P_2) = \frac{12\mu\bar{u}L}{t^2}$$

$$h_f = \frac{P_1 - P_2}{\rho g} = \frac{12\mu\bar{u}L}{\rho g t^2}$$

iv) Shear stress distribution:

$$\tau = \mu\frac{\partial u}{\partial y} = \mu\frac{\partial}{\partial y}\left[\frac{-1}{2\mu}\frac{\partial}{\partial x}(ty - y^2)\right]$$

$$\tau = \mu\left[-\frac{1}{2\mu}\frac{\partial p}{\partial x}(t - 2y)\right]$$

$$\tau = -\frac{1}{2}\frac{\partial p}{\partial x}[t - 2y]$$

when $y = 0$, (or) t

$$\tau_0 = -\frac{1}{2}\frac{\partial p}{\partial x} t$$

(P) A container full of oil has a horizontal parallel crack in its end wall which is 500 mm wide and 50 mm thick in the direction of flow. The pressure difference between two faces of the crack is 10 kPa and the crack forms a gap of 0.4 mm b/w parallel surfaces.

Calculate (i) volume of oil leakage per hour through the crack.
ii) max. leakage velocity (iii) shear stress and velocity gradient at the boundary. Take S.G. = 0.85, μ = 1.8 Poise.

$\mu = 1.8\,\text{Poise} = 0.18\ Ns/m^2$, $t = 0.4\,mm = 0.0004\,m$.

$l = 50\,mm = 0.05\,m$.

$P_1 - P_2 = 10\times10^3\ Pa$.

$$(P_1 - P_2) = \frac{12\mu V_{av} l}{t^2}$$

$$V_{avg} = \frac{(P_1-P_2)t^2}{12\mu L} = \frac{10^4\times(0.0004)^2}{12\times0.18\times0.05} = 0.0148\ m/s$$

i) oil leakage = clear area × avg. velocity
$= (0.05\times0.0004)\times0.0148 = 2.96\times10^{-6}\ m^3/s$.

ii) $V_{max} = \frac{3}{2}V_{avg} = 0.022\ m/s$.

iii) $$\tau_0 = -\frac{1}{2}\left(\frac{\partial p}{\partial x}\right)t = \left(\frac{10\times10^3}{0.05}\right)\times\frac{0.004}{2} = 40\ N/m^2$$

$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0} \Rightarrow \frac{du}{dy} = \frac{\tau_0}{\mu} = \frac{40}{0.18} = 222/\text{sec}$$

(2) water flows b/w two large parallel plates at a distance of 1.6 mm apart. Determine i) max velocity (ii) pressure drop per unit length iii) shear stress at walls of plates. if avg. velocity is 0.2 m/s

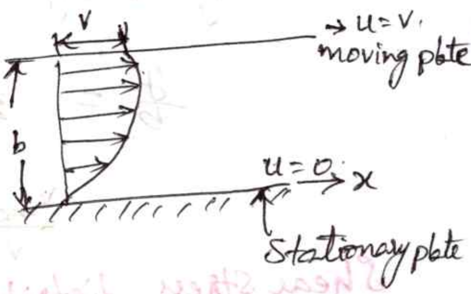
Sol i) max. velocity $= \frac{3}{2}u_{avg} = \frac{3}{2}\times 0.2 = 0.3\ m/s$.

ii) $$(P_1 - P_2) = \frac{12\mu u_{avg} L}{t^2} = \frac{12\times0.001\times0.2}{(0.0016)^2} = 937\ Pa/m$$

iii) $$\tau_0 = \frac{1}{2}\frac{\partial p}{\partial x}\times t = 0.749\ Pa$$

Laminar uni-directional flow between parallel plates having Relative motion :

Consider the flow field between two parallel fat plates seperated by a small gap b such that the lower plate is fixed and upper plate moves uniformly in its own platne with a velocity V.



Boundary conditions : u = 0 at y = 0, and u = V at y = b.

$$\frac{dp}{dx} = \mu \frac{d^2u}{dy^2}$$

$\frac{dp}{dx}$ is independent of y, by integration.

$$u = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{y^2}{2} + C_1 y + C_2$$

u = 0, at y = 0, $C_2 = 0$,

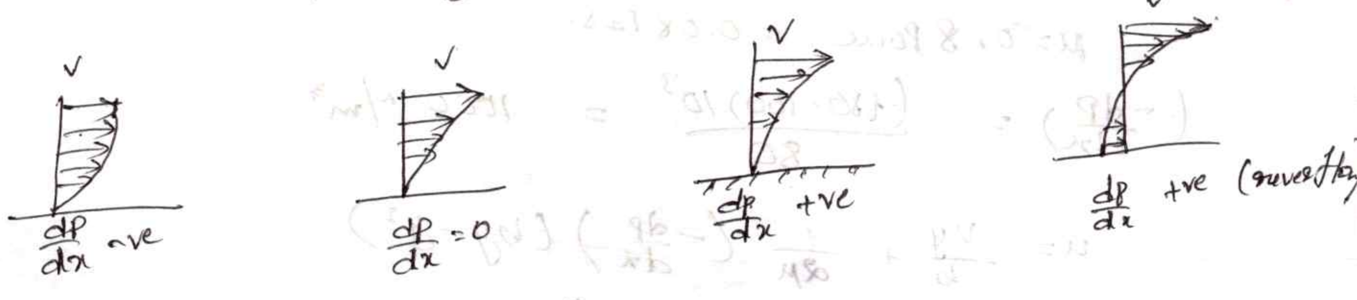
u = V, at y = b, $C_1 = \frac{V}{b} - \frac{1}{2\mu}\left(\frac{dp}{dx}\right) b$.

$$u = \frac{Vy}{b} - \frac{1}{2\mu}\left(\frac{dp}{dx}\right)(by - y^2)$$

velocity distribution depends on both V and $\frac{dp}{dx}$.

If $\frac{dp}{dx} = 0 \Rightarrow u = \frac{Vy}{b}$ this particular case known as Simple shear flow.

$$\frac{u}{V} = \frac{y}{b} + \phi \frac{y}{b}\left(1 - \frac{y}{b}\right) \qquad \phi = \frac{b^2}{2\mu V}\left(-\frac{dp}{dx}\right)$$



$$Q = \int_0^b u\,dy = V\int_0^b \left[\frac{y}{b} + \phi\frac{y}{b}\left(1 - \frac{y}{b}\right)\right]dy.$$

$$= \left(\frac{1}{2} + \frac{\phi}{6}\right)Vb.$$

$$\text{Avg. velocity} = \frac{Q}{\text{Area per unit width}} = \frac{Q}{(b \times 1)} = \left(\frac{1}{2} + \frac{\phi}{6}\right)V.$$

for Simple shear flow, $\frac{dp}{dx} = 0$, $\phi = 0$,

$$V_{avg} = \frac{V}{2}$$

$$\frac{du}{dy} = \frac{d}{dy}\left(v\left(\frac{y}{b} + \phi\frac{y}{b}\left(1-\frac{y}{b}\right)\right)\right) = \frac{v}{b} + \frac{v\phi}{b}\left(1-\frac{2y}{b}\right) = 0$$

$$\frac{y}{b} = \frac{1}{2} + \frac{1}{2\phi}$$

$$\frac{V_{max}}{V} = \frac{(1+\phi)^2}{4\phi}$$

Shear stress distribution:

$$\tau = \mu\frac{du}{dy} = \mu\frac{V}{b} + \left(-\frac{dp}{dx}\right)\left(\frac{b}{2} - y\right)$$

$$\frac{b\tau}{\mu V} = 1 + \phi\left(1 - \frac{2y}{b}\right)$$

(i) for simple shear flow $\frac{dp}{dx} = 0$, $\phi = 0$, $\boxed{\tau = \frac{\mu V}{b}}$

ii) At $y = \frac{b}{2}$ i.e. at the centre of The flow passage, the shear stress is independent of the pressure gradient ϕ.

Q. Two parallel plates kept 0.01m apart have laminar flow of oil b/w them. Taking dynamic viscosity of oil to be 0.8 poise, determine the velocity distribution, discharge and shear stress on the upper plate that moves horizontally at relative velocity 1m/s with respect to the lower plate which is stationary. further the pressure drops in flow direction from 180 kPa to 100 kPa over a distance of 80m.

$\mu = 0.8$ Poise $= 0.08$ Pa.S.

$$\left(-\frac{dp}{dx}\right) = \frac{(180-100)10^3}{80} = 1000 \text{ N/m}^3$$

$$u = \frac{Vy}{b} + \frac{1}{2\mu}\left(-\frac{dp}{dx}\right)(by - y^2)$$

$$= \frac{1}{0.01}y + \frac{1}{2\times 0.08}(1000)(0.01y - y^2)$$

$$= 162.5y - 6250y^2$$

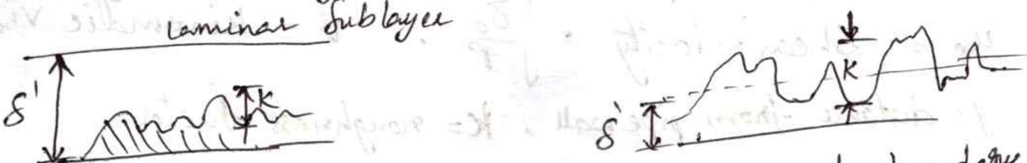
ii) $Q = \int_0^b u\,dy = \int_0^b (162.5y - 6250y^2)\,dy = 0.0060$

iii) $\tau = \mu\frac{du}{dy} = 0.08(162.5 - 12500y)$

at $y = 0.01$ m, $\tau = 3$ N/m²

Hydrodynamically smooth and rough boundaries:

Let k is the avg height of the irregularities projecting from the surface of a boundary as shown fig. If the value of k is large for a boundary then boundary is called rough boundary and if the value of k is less, then boundary is known as smooth boundary, in general.



ⓐ Smooth boundary ⓑ Rough boundary.

for turbulent flow analysis along a boundary, the flow is divided in two postions. The first postion consists of a thin layer of fluid in the immediate neighbourhood of the boundary, where viscous shear stress predominates while the shear stress due to turbulence is negligible. This postion is known as laminar sub layer. The height upto which the effect of viscosity predominates in this zone is denoted by δ'. The second postion of flow, where shear stress due to turbulence are large as compared to viscous stress is known as turbulent zone.

If the avg height k of the irregularities, projecting from the surface of a boundary is much less than δ', the thickness of laminar sublayer as shown fig ⓐ, the boundary is called smooth boundary.

Now, if the Reynolds number of the flow is increased then the thickness of laminar sub-layer will decrease. If the thickness of laminar sub-layer becomes much smaller than the avg height k of irregularities of the surface as shown fig ⓑ. The boundary will act as rough boundary.

① If $\frac{k}{\delta'} < 0.25$ → boundary is called smooth.

② If $\frac{k}{\delta'} > 6$, the boundary is rough.

③ If $0.25 < \frac{k}{\delta'} < 6$ boundary is in transition.

In terms of roughness Reynolds number $\frac{u_* k}{\nu}$

$u_* = \sqrt{\frac{\tau_0}{\rho}}$

Shear velocity =

① $\frac{u_* k}{\nu} < 4$, boundary smooth

② $4 < \frac{u_* k}{\nu} < 100$ — transition.

③ $\frac{u_* k}{\nu} > 100$, boundary is rough.

velocity distribution for turbulent flow is

$$\frac{u}{u_*} = 5.75 \log_{10} \frac{u_* y}{\nu} + 5.5 \quad \text{for smooth pipes.}$$

$$= 5.75 \log_{10} (y/k) + 8.5 \quad \text{for rough pipes.}$$

where u = velocity at any point in turbulent flow

u_* = Shear velocity = $\sqrt{\frac{\tau_0}{\rho}}$, ν = kinematic viscosity of fluid

y = distance from pipe wall, k = roughness factor.

velocity distribution in terms of avg velocity is:

$$\frac{\bar{U}}{u_*} = 5.75 \log_{10} \frac{u_* R}{\nu} + 1.75 \quad \text{for smooth pipes.}$$

$$= 5.75 \log_{10} \frac{R}{k} + 4.75 \quad \text{for rough pipes.}$$

Coefficient of friction is given by

$$f = \frac{16}{Re} \quad \dots \text{for laminar flow,}$$

$$= \frac{0.0791}{(Re)^{1/4}} \quad \text{for turbulent flow in smooth for } Re \geq 4000 < 10^5$$

$$= 0.008 + \frac{0.05525}{(Re)^{0.257}} \quad \text{for } Re < 10^5, \ngtr 4\times10^7.$$

Thickness of laminar Sub-layer $\delta' = \frac{11.6 \times \nu}{u_*}$

value of coefficient of friction 'f' for rough pipe is given by

$$\frac{1}{\sqrt{4f}} = 2 \log_{10}\left(\frac{R}{k}\right) + 1.74$$

(1) The two reservoirs with surface level difference of 20 m are to be connected by 1 m dia pipe 6 km long. What will be the discharge when a cast iron pipe of roughness $k = 0.3$ mm is used? What will be the % increase in discharge if cast iron pipe were to be repleted by steel pipe of roughness $k = 0.1$ mm? Neglect local losses.

Case (i)

$$\frac{1}{\sqrt{4f}} = 2\log_{10}\frac{0.5}{0.0003} + 1.74 = 8.1837$$

$$f = \frac{1}{4}\left(\frac{1}{8.1837}\right)^2 = 0.00373$$

$$h_f = \frac{flQ^2}{3d^5} \Rightarrow 20 = \frac{0.00373 \times 6000 \times Q^2}{3 \times (1)^5}$$

$$Q = 1.637 \text{ m}^3/\text{s}$$

ii)

$$\frac{1}{\sqrt{4f}} = 2\log_{10}\frac{0.5}{0.0001} + 1.74 = 9.138$$

$$f = \frac{1}{4}\left(\frac{1}{9.138}\right)^2 = 0.003$$

$$h_f = \frac{flQ^2}{3d^5} \Rightarrow 20 = \frac{0.003 \times 6000 \times Q^2}{3(1)^5}$$

$$Q = 1.826$$

$$\% \text{ in discharge} = \frac{1.826 - 1.637}{1.637} \times 100 = 11.54\%$$

(P) A pipeline 12 cm in dia and 100 m long conveys water at a rate of 0.075 m³/s. The avg height of the surface is 0.012 cm and coefficient of friction is 0.005. Calculate the loss of head, wall shearing stress, centre line velocity and nominal thickness of laminar sublayer.

$$V = \frac{Q}{A} = \frac{0.075}{\frac{\pi}{4}(0.12)^2} = 6.64 \text{ m/s}$$

$$h_f = \frac{4flv^2}{2gd} = \frac{4 \times 0.005 \times 1000 \times (6.64)^2}{2 \times 9.81 \times 0.12} = 373.4 \text{ m}$$

$$u_* = v\sqrt{\frac{4f}{8}} = 6.64 \times \sqrt{\frac{4 \times 0.005}{8}} = 0.332 \text{ m/s}$$

$$u_* = \sqrt{\frac{\tau_0}{\rho}} \Rightarrow \tau_0 = \rho u_*^2 = (0.332)^2 1000 = 110.224$$

rough

$$\frac{u}{u_*} = 8.5 + 5.75\log_{10} y/k \Rightarrow$$

$y = R = 6$ cm, $u = V_{max}$.

$$V_{max} = 0.33\left[8.5 + 5.75\log_{10}\frac{0.06}{0.012}\right] = 7.968 \text{ m/s}$$

(2) A smooth pipe of dia 80mm and 800m long carries water at the rate of 0.480 m³/min. Calculate the loss of head, wall shearing stress, centre line velocity, velocity and shear stress at 30mm from pipe wall. Also calculate thickness of laminar sub-layer. Take $\nu = 0.015$ stokes.

$d = 0.08\text{m}$, $L = 800\text{m}$.

$Q = 0.008\ \text{m}^3/\text{s}$.

$\nu = 0.015 \times 10^{-4}\ \text{m}^2/\text{s}$

$$V = \frac{Q}{Area} = \frac{0.008}{\frac{\pi}{4}(0.08)^2} = 1.59\ \text{m/s}$$

$$Re = \frac{\rho V d}{\mu} = \frac{1.591 \times 0.08}{0.015 \times 10^{-4}} = 8.485 \times 10^4$$

$Re > 4000$, flow is turbulent

$$f = \frac{0.0791}{(Re)^{1/4}} = \frac{0.079}{(8.48 \times 10^4)^{1/4}} = 0.004636$$

$u_* = V\sqrt{\frac{f}{2}}$

$u_* = \sqrt{\frac{\tau_0}{\rho}}$

$V\sqrt{\frac{f}{2}} = \sqrt{\frac{\tau_0}{\rho}}$

$V^2\frac{f}{2} = \frac{\tau_0}{\rho}$

$\tau_0 = \frac{\rho f V^2}{2}$

(1) $$h_f = \frac{4f \cdot L V^2}{2gd} = 23.42\ \text{m}$$

(2) $$\tau_0 = \frac{f \rho V^2}{2} = 0.004636 \times \frac{1000}{2} \times (1.59) = 5.86\ \text{N/m}^2$$

(3) $$\frac{u}{u_*} = 5.55 + 5.75 \log \frac{u_* y}{\nu}$$

$$u_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{5.866}{1000}} = 0.0765\ \text{m/s}$$

(4) $$y = d/2 = \frac{0.08}{2} = 0.04\text{m}.$$

at $y = 0.04$ $u = u_{max}$

$$\frac{u_{max}}{0.0765} = 5.55 + 5.75 \log\left(\frac{0.07655 \times 0.04}{0.015 \times 10^{-4}}\right)$$

$u_{max} = 1.8\ \text{m/s}$

(4) $$\tau = -\left(\frac{\partial p}{\partial x}\right)\frac{r}{2} \Rightarrow \tau_0 = -\frac{\partial p}{\partial x}\left(\frac{R}{2}\right)$$

$$\frac{\tau}{\tau_0} = \frac{r}{R} \Rightarrow \tau = \frac{\tau_0 r}{R} =$$

$$\tau_{r=0.01} = \frac{5.866 \times 0.01}{0.04} = 1.46\ \text{N/m}^2$$

$y = 3\text{cm}$ $$\frac{u}{u_*} = 5.55 + 5.75 \log \frac{u_* y}{\nu}$$

$u = 1.25\ \text{m/s}$

$$\delta' = \frac{11.6 \times \nu}{u_*} = \frac{11.6 \times 0.015 \times 10^{-4}}{0.0765} = 0.2\ \text{mm}$$

CIVIL ENGINEERING

Fluid Mechanics, Hydraulics & Hydraulic Machinery

UNIT-3

UNIT-II

Fluid Kinematics

Kinematics is defined as that branch of science which deals with motion of particles without considering the forces causing the motion. The velocity at any point in a flow field at any time is studied in this branch of fluid mechanics. once the velocity is known, then the pressure distribution and hence forces acting on the fluid can be determined.

The fluid motion is described by two methods. They are i) Lagrangian Method and ii) Eulerian Method.

In the langrangian method, a single fluid particle is followed during its motion and its velocity, acceleration, density etc., are described.

In case of Eulerian method, the velocity, acceleration, pressure, density etc, are described at a point in flow field. This is commonly used.

Types of Fluid flow:

1. Steady and Unsteady Flows: Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density etc. at a point do not change with time. Thus for steady flow, mathematically we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0, \quad \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0, \quad \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

where (x_0, y_0, z_0) is a fixed point in fluid field.

Unsteady flow is that type of flow, in which the velocity, pressure or density at a point changes with respect to time. for unsteady flow

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \quad \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0 \text{ etc.}$$

② Uniform and Non-uniform Flows: Uniform flow is defined as that type of flow in which the velocity at any given time doesnot change wrt space.

$$\left(\frac{\partial V}{\partial s}\right)_{t=\text{Const}} = 0 \quad ; \quad \begin{aligned} \partial V &= \text{change of velocity} \\ \partial s &= \text{length of flow in the direction } s. \end{aligned}$$

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus mathematically, for non-uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t=\text{Constant}} \neq 0.$$

3. Laminar and Turbulent flows :- Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles move in zig-zag way. Due to the movement of fluid particles in a zig-zag way, the eddies formation takes place which are responsible for high energy loss. for a pipe flow, the type of flow is determined by a non-dimensional number $\frac{VD}{\nu}$ called the Reynold number.

where D = Dia of pipe, V = mean velocity of flow in pipe

ν = kinematic viscosity of fluid.

If the Reynold number is less than 2000, the flow is called laminar. If the Reynold number is more than 4000, it is called turbulent flow. If R lies b/w 2000 & 4000, the flow may be laminar or turbulent.

4. Compressible and Incompressible flows : Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ) is not constant for the fluid.

for compressible flow $\rho \neq$ constant.

Incompressible flow is that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible. Mathematically,

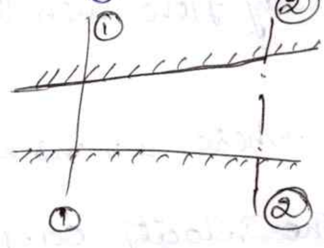
ρ = constant.

5. Rotational and Irrotational Flows : Rotational flow is that type of flow in which the fluid particles while flowing along stream-lines, also rotate about their own axis. And if the fluid particles while flowing along streamlines, donot rotate about their own axis then that type of flow is called irrotational flow.

$$w_z = \frac{1}{2}\left(\frac{dv}{dx} - \frac{du}{dy}\right), \quad \text{If } \frac{dv}{dx} = \frac{du}{dy}$$

Continuity Equation.

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant. Consider two cross-sections of a pipe as shown in fig.



Let V_1 = Avg velocity at c/s 1-1

ρ_1 = Density at section 1-1

A_1 = Area of pipe at section 1-1

& $\rho_2, A_2 V_2$ are corresponding values at section 2-2

Then rate of flow at section 1-1 $= \rho_1 A_1 V_1$

2-2 $= \rho_2 A_2 V_2$

According to law of conservation of mass

Rate of flow at section 1-1 = Rate of flow at section 2-2.

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Above Eq. is applicable to the compressible as well as impressible fluids and is called continuity Equation. If the fluid is incompressible, then $\rho_1 = \rho_2$ continuity equation reduces to $\{A_1 V_1 = A_2 V_2\}$.

Stream line: Stream line is an imaginary line drawn in the flowing fluid such that the tangent drawn at any point indicates the direction of velocity. Stream line gives avg velocity of no. of fluid particles.

$$\frac{dx}{u} = \frac{dy}{v}$$ for 2-D eq. of stream line.

Path line: It is path traced by individual fluid particle at different instants of time. Path lines may cross but stream lines donot cross.

Streak line: It is the instantaneous picture of the positions of all the fluid particles that have passed through a fixed point in the flow field.

path line: It is the locus of a fluid particle as it moves along

Velocity potential function And Stream Function.

Velocity potential function: It is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction. It is defined by ϕ (phi).

Mathematically, the velocity, potential is defined as $\phi = f(x, y, z)$ for steady flow such that

$$\left\{ u = -\frac{\partial \phi}{\partial x} \;;\; v = -\frac{\partial \phi}{\partial y} \;;\; w = -\frac{\partial \phi}{\partial z} \right\}$$

where u, v and w are the components of velocity in x, y and z direction resp.

The velocity components in cylindrical polar coordinates interms of velocity potential function are given by

$$u_r = \frac{\partial \phi}{\partial r} \;;\quad u_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$$

u_r = velocity component in radial direction (i.e in r direction)

u_θ = velocity " " tangential direction (i.e in θ direction)

The continuity eq. for an incompressible steady flow is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$.

$$\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial z}\right) = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

Equation is a Laplace equation.

For two-dimension case, equation reduces to $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$

properties of the potential function:

rotational components are $w_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$

$$w_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$$

$$w_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$$

$$w_z = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial x}\right)\right] = \frac{1}{2}\left[-\frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial x \partial y}\right]$$

If ϕ is a continuous function, then $\frac{\partial^2 \phi}{\partial x \partial y} = \frac{\partial^2 \phi}{\partial y \partial x}$;

$$w_x = w_y = w_z = 0$$

when rotational components are zero, the flow is called irrotational

→ If velocity potential satisfies the Laplace eq. it represents steady incompressible irrotational flow.

Stream Function : It is defined as the scalar function of space and time, such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction. It is denoted by ψ (psi) and defined only for two dimensional flow. For steady flow it is defined as $\psi = f(x, y)$ such that

$$\frac{\partial \psi}{\partial x} = v \quad ; \quad \frac{\partial \psi}{\partial y} = -u.$$

velocity components in cylindrical polar co-ordinates interms of stream function are given as

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \phi} \quad \text{and} \quad u_\theta = -\frac{\partial \psi}{\partial r}$$

The continuity eq. for 2-D flow is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$.

Substituting u & v values

$$\frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right) = 0 \quad (\text{or}) \quad -\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial x \partial y} = 0$$

The flow may be rotational or irrotational.

The rotational component w_z is given by $w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

$$w_z = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial y}\right)\right] = \frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right]$$

For irrotational flow $w_z = 0$; the above eq. becomes $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$.
which is Laplace equation for ψ.

- If stream function (ψ) exists, it is possible case of fluid flow which may be rotational or irrotational.
- If stream function (ψ) satisfies the Laplace equation, it is a possible case of an irrotational flow.

Stream Tube : If streamlines are drawn through a closed curve, they form a boundary surface across which fluid cannot penetrate. Such a surface bounded by streamlines is a sort of tube, and is known as a streamtube.

Equipotential Line : A line along which the velocity potential ϕ is constant, is called equipotential line.

For equipotential line $\phi =$ constant

$$d\psi = 0$$

But $\phi = f(x, y)$ for steady flow

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = -u dx - v dy$$

For equipotential line $d\phi = 0$

$$-(u dx + v dy) = 0 \Rightarrow \frac{dy}{dx} = \frac{-u}{v}$$

$\frac{dy}{dx}$ = Slope of equipotential line.

Line of constant stream function : (Stream line)

$$\psi = \text{constant}, \quad d\psi = 0$$

But $d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = + v dx - u dy$ $\left[\because \frac{\partial \psi}{\partial x} = v ; \frac{\partial \psi}{\partial y} = -u\right]$

for a line of constant stream function.

$$d\psi = 0 \text{ or } v dx - u dy = 0$$

$$\frac{dy}{dx} = \frac{v}{u}$$ [Slope of stream line]

Flow Net : A grid obtained by drawing a series of equipoten lines and stream lines is called a flow net. The flow net is an important tool in analysing two-dimensional irrotational flow problems.

Relation between stream function and potential function.

$$\left\{ \begin{aligned} \frac{\partial \phi}{\partial x} &= \frac{\partial \psi}{\partial y} \\ \frac{\partial \phi}{\partial y} &= -\frac{\partial \psi}{\partial x} \end{aligned} \right\}$$

Uses : Rate of seepage loss, Seepage pressure, Uplift pressure, exit gradi

Fluid Dynamics

Dynamics of fluid flow is the study of fluid motion with the forces causing flow. The dynamic behaviour of the fluid flow is analysed by the Newton's second law of motion, which relates the acceleration with forces. The fluid is assumed to be incompressible and non-viscous.

Equations of motion :

According to Newton's seconds law of motion the net force F_x acting on a fluid element in the direction of x is equal to mass m of the fluid element multiplied by the acceleration a_x in the x-dir

$$F_x = m \cdot a_x$$

In the fluid flow, the following forces are present.

i) F_g, Gravity force

ii) F_P, the pressure force

iii) F_v, force due to viscosity

iv) F_t, force due to turbulence

v) F_c, force due to compressibility.

Thus the net force $F_x = (F_g)_x + (F_P)_x + (F_v)_x + (F_t)_x + (F_c)_x$.

ⓐ If the force due to compressibility, F_c is negligible, the resulting force $F_x = (F_g)_x + (F_P)_x + (F_v)_x + (F_t)_x$ and equation of motions are called "Reynold's equations of motion."

ⓒ If flow is assumed to be ideal, viscous force (F_v) is zero and equation of motions are known as Euler's equation of motion.

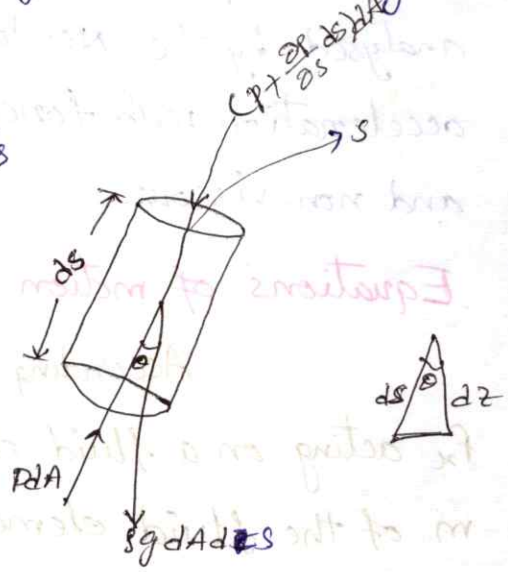
$$F_x = (F_g)_x + (F_P)_x$$

ⓑ F_t is negligible, the resulting equations of motion are known as Navier-Stokes equation.

$$F_x = (F_g)_x + (F_P)_x + (F_v)_x$$

Euler's equation of motion:

This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line as:



Consider a stream line in which flow is taking place in s-direction as shown. Consider a cylindrical element of c/s dA & length ds. The forces acting on the cylindrical element are

1. Pressure force PdA in direction of flow.
2. pressure force $(p+\frac{\partial p}{\partial s}ds)dA$ in opposite direction of flow.
3. weight of the element $\rho g dA ds$

Let θ is the angle between the direction of flow and the line of action of the weight of element. The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element x acceleration in the direction s.

$$PdA - (P+\frac{\partial p}{\partial s}ds)dA - \rho g\, dA ds \cos\theta$$

$$= \rho\, dA ds \times a_s \quad \text{--- (1)}$$

where a_s is the acceleration in the direction of s.

$$a_s = \frac{dv}{dt} \quad \text{where } v \text{ is a function of } s \text{ and } t.$$

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t}$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$, $a_s = \frac{v\partial v}{\partial s}$

substituting the value of a_s in eq-(1) and simplifying the eq.

$$-\frac{\partial p}{\partial s}ds dA - \rho g\, dA ds \cos\theta = \rho\, dA ds \frac{v\partial v}{\partial s}$$

÷ by $\rho ds dA$;

$$\frac{\partial p}{\rho \partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0$$

$\cos\theta = \frac{dz}{ds}$

$$\frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + v\frac{dv}{ds} = 0 \quad (\text{or})$$

$$\frac{dp}{\rho} + g dz + v dv = 0 \rightarrow \text{Euler's equation of motion.}$$

Bernoulli's equation from Euler's Equation.

Bernoulli's equation is obtained by integrating the Euler's eq. of motion as

$$\int \frac{dp}{\rho} + \int g\,dz + \int v\,dv = \text{constant}.$$

If flow is incompressible, ρ is constant and

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}.$$

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}. \rightarrow \text{Bernoulli's equation.}$$

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$\frac{v^2}{2g}$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.

01/7/2017

Assumptions

- → flow is along streamline
- → Ideal fluid flow
- → flow is steady
- → Incompressible fluid.

→ In a flow field point where v becomes '0' is called stagnation point.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

for a real fluid flow the bernoulli's equation.

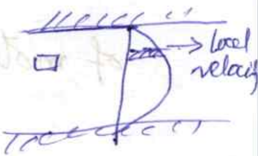
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

h_L → loss of head.

Kinetic energy factor: It is correction factor apply to KE based on avg velocity to get actual kinetic head. It is denoted by α.

$$\text{avg velocity} = \frac{\int_A v\,dA}{A}$$

$$TOKE = \frac{\rho}{2}\int_A u^3 dA$$



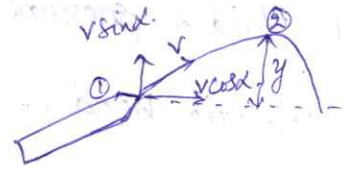
$$\alpha = \frac{1}{V_{avg}^3 A}\int_A u^3 dA$$

$$KE_{avg} = \alpha (\dot m)\frac{V^2}{2} = \frac{\rho}{2}\alpha A V^3$$

i) $\alpha = 2$ for laminar flow with parabolic velocity distribution.

ii) $\alpha = 1.02$ to 1.15 for general turbulent flow.

Applications of Bernoulli's equation:

1) 

pressure at every point is atmosphere

Since there is no acceleration horizontal velocity doesnot change.

Applying Bernoulli's equation b/w ① & ②

$$0 + \frac{v^2}{2g} + 0 = 0 + \frac{v^2\cos^2\alpha}{2g} + y.$$

$$y = \frac{v^2\sin^2\alpha}{2g}$$

$$x = (v\cos\alpha)t$$

$$y = (v\sin\alpha)t - \frac{1}{2}gt^2$$

$$y = (v\sin\alpha)\frac{x}{(v\cos\alpha)} - \frac{1}{2}g\left(\frac{x}{v\cos\alpha}\right)^2$$

$$y = x\tan\alpha - \frac{x^2 g\sec^2\alpha}{2v^2}$$

$$\text{Range} = \frac{v^2}{g}\sin 2\alpha.$$

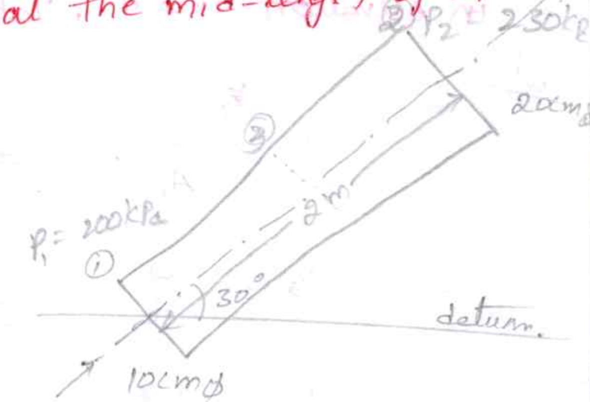
Body force: Distributed over entire mass (or) volume of the element. It is usually expressed per unit mass of the element (or) medium upon which the forces act. Eg: gravitational force, Electromagnetic force fields etc.

Surface force: force exerted on the fluid element by its surroundings through direct contact at the surface.

Surface force has two components: Normal force, Shear force.

Eg: friction, Stress

(P) A ~~2m~~ 2m long pipeline tapers uniformly from 10cm dia to 20cm at its upper end. The pipe centre line slopes upwards at an angle of 30° to the horizontal and flow direction is from smaller to bigger cross-section. If the pressure gauges installed at the lower and upper ends of the pipeline read 200 kPa and 230 kPa respectively, determine the flow rate and the fluid pressure at the mid-length of the pipeline. Assume no energy losses.



Applying Bernoulli's equation b/w ① & ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$Z_1 = 0$

$Z_2 = 2 \sin 30^\circ = 2(\tfrac{1}{2}) = 1\,m.$

$$\frac{200 \times 10^3}{9810} + \frac{V_1^2}{2g} + 0 = \frac{230 \times 10^3}{9810} + \frac{V_2^2}{2g} + 1$$

$$\frac{V_1^2 - V_2^2}{2g} = \frac{(230-200) \times 10^3}{9810} + 1 = 4.058$$

from continuity equation $A_1 V_1 = A_2 V_2$

$$V_2 = \left(\frac{A_1}{A_2}\right) V_1 \Rightarrow V_2 = \frac{V_1}{4}$$

$V_1 = 9.215\ m/s.$

Discharge, $Q = A_1 V_1 = \frac{\pi}{4}(0.1)^2 \times 9.125 = 0.0723\ m^3/s.$

At the mid length of pipeline, $D_3 = 15cm = 0.15\,m$

$a_1V_1 = a_3V_3$

$V_3 = \left(\frac{a_1}{a_3}\right)V_1 \quad \frac{\frac{\pi}{4} D_1^2}{\frac{\pi}{4} D_3^2}$

$$V_3 = \left(\frac{D_1}{D_3}\right)^2 V_1 = 4.095\ m/s.$$

Applying Bernoulli's equation b/w ① & ③

$$\frac{200 \times 10^3}{9810} + \frac{(9.125)^2}{2 \times 9.81} + 0 = \frac{P}{\gamma} + \frac{(4.035)^2}{19.6} + 1\sin 30$$

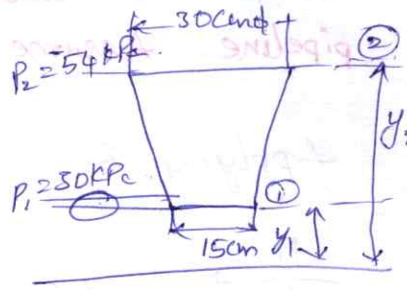
$P = 2.29 \times 10^5\ N/m^2 = 2.29$ bar

1

(1) Water is flowing upwards through a pipeline having diameters of 15 cm and 30 cm at the bottom and upper ends respectively. When a discharge of 50 l/s is passed through the pipe line, the pressure gauges at the bottom and upper section read 30 kPa & -54 kPa respectively. If the friction loss in the pipe is 2 m, determine the difference in elevation head. Take specific weight of water is 9.81 kN/m³.

Sol

$$V_1 = \frac{Q}{A_1} = \frac{0.05}{\frac{\pi}{4}(0.15)^2} = 2.83\ m/s$$



$$V_2 = \frac{Q}{A_2} = \frac{0.05}{\frac{\pi}{4}(0.3)^2} = 0.707\ m/s$$

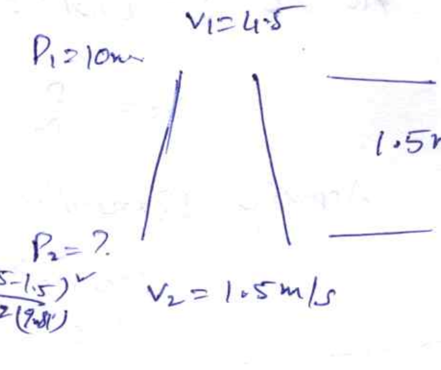
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + y_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + y_2 + h_f$$

~~$y_2 - y_1 = 6.78\ m$~~

$$\frac{30\times10^3}{9810} + \frac{(2.83)^2}{19.6} + y_1 = \frac{-54\times10^3}{9810} + \frac{(0.707)^2}{19.6} + y_2 + 2$$

$$y_2 - y_1 = 6.78\ m$$

(2) Conical tube 1.5 m long is fixed vertically with its smaller end upwards, and it forms a part of pipe line. Water flows down the tube and measurements indicate that velocity is 4.5 m/s at smaller end and 1.5 m/s at larger end and pressure head is 10 m of water at the upper end. Loss of head in tube is expressed as $\frac{0.3(V_1 - V_2)^2}{2g}$. Calculate pressure head at lower end.



$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + y_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + y_2 + \text{losses}$$

$$\frac{P_2}{\gamma} = 10 + \frac{4.5^2 - 1.5^2}{2\times9.81} + (1.5 - 0) - 0.3\frac{(4.5-1.5)^2}{2(9.81)}$$

$= 10 + 0.917 + 1.5 - 0.1376$

$= 12.27$ m of water

$= 120.46\ kN/m^2$

Momentum of fluid in motion [Impulse - momentum Relationship]

Momentum principle states that : "The time rate of change of momentum is proportional to the impressed force and takes place in the direction in which force acts."

Momentum is the product of mass and velocity of the body and represents energy of motion stored in a moving body.

Mathematically $\quad f = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$

for a constant fluid mass $dm = 0$.

$$f = m\frac{dv}{dt} \quad \text{or} \quad \boxed{f\,dt = m\,dv}$$

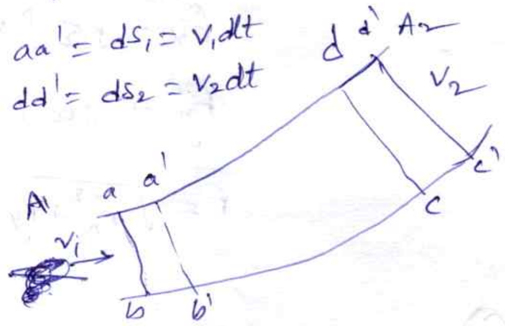
$f\,dt$ represents the impulse of applied force

$m\,dv$ represents change in momentum.

Impulse - momentum theorem states that : "the impulse due to force acting on a fluid mass in a small interval of time is equal to change in the momentum of the fluid mass."

$aa' = ds_1 = V_1 dt$

$dd' = ds_2 = V_2 dt$



Consider steady flow of fluid through a diverging stream tube. flow can be assumed to be uniform and normal to the inlet and outlet areas. The fluid mass has avg. velocity V_1 and density ρ_1 at entrance. the corresponding values at exit are V_2 and ρ_2.

Under effect of external forces on the stream, the mass of fluid in the region abcd shift to new position a'b'c'd' after a short time interval dt. Because of gradual increase in flow area in the direction of flow, velocity of fluid mass and momentum gradually decreased.

Fluid mass within the region abb'a' = fluid mass within region cc'd'd

$$\rho_1 A_1 ds_1 = \rho_2 A_2 ds_2$$

momentum of fluid mass contained in the region abb'a'

$$= (\rho A_1 ds_1) V_1 = (\rho_1 A_1 V_1 dt) V_1$$

Momentum cc'd'd $= (\rho_2 A_2 ds_2) V_2 = (\rho_2 A_2 V_2 dt) V_2$

change in momentum

$$= (\rho_2 A_2 V_2 dt) V_2 - (\rho_1 A_1 V_1 dt) V_1$$

$A_1 V_1 = A_2 V_2 = Q,$

for steady incompressible flow $\rho_1 = \rho_2 = \rho$.

$\therefore$ change of momentum $= \rho Q (V_2 - V_1)\, dt$

By impulse-momentum principle

$$F dt = \rho Q (V_2 - V_1)\, dt$$

$$F = \rho Q (V_2 - V_1)$$

This is the basic momentum-flux equation. ρQ is called mass flux.

force F and velocity V_1 and V_2 are all vector quantities and can be resolved into components in the directions of x and y. If θ_1 and θ_2 inclination with horizontal of the centre line of the pipe at ab and cd

then components of V_1 and V_2 along x-axis – $V_1 \cos\theta_1$ and $V_2 \cos\theta_2$

along y-direction $V_1 \sin\theta_1$ and $V_2 \sin\theta_2$

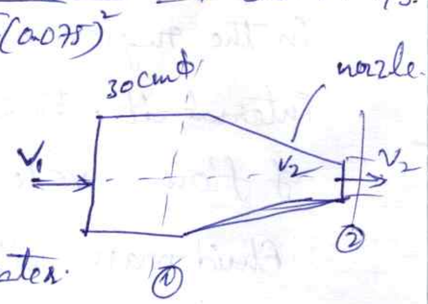
$$F_x = \rho Q [V_2 \cos\theta_2 - V_1 \cos\theta_1]$$

$$F_y = \rho Q [V_2 \sin\theta_2 - V_1 \sin\theta_1]$$

$$F = \sqrt{F_x^2 + F_y^2}$$

(1) A 30cm dia horizontal pipe terminates in a nozzle with the exit dia of 7.5cm. If the water flows through the pipe at a rate of $0.15 m^3/s$ what force will be exerted by the fluid on the nozzle.?

$V_1 = \dfrac{0.15}{\frac{\pi}{4}(0.3)^2} = 2.12$ m/s, $V_2 = \dfrac{0.15}{\frac{\pi}{4}(0.075)^2} = 33.97$ m/s.



$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + y_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + y_2$$

$$\frac{P_1}{\gamma} = \frac{V_2^2 - V_1^2}{2g} = 58.58 \text{ m of water.}$$

$$P_1 = 574.67 \times 10^3 \ N/m^2$$

Impulse momentum eq. then gives force along x-axis.

$$F_x = \rho Q (V_1 - V_2) + P_1 A_1 - P_2 A_2$$

$$= -4.77 + 40600. - 0$$

$F_x = 35822$ N $= 35.8$ kN.

② A fireman holds a water hose ending into a nozzle that issues a 20 mm diameter jet of water. If the pressure of water in the 60mm dia hose is 700 kPa. find the force experienced by the fireman.

Impulse momentum equation gives

$$F_x = \underbrace{\rho Q\,[V_1 - V_2]}_{\text{dynamic force}} + \underbrace{P_1A_1 - P_2A_2}_{\text{static force}}$$

from continuity equation $\frac{\pi}{4}(0.06)^2 V_1 = \frac{\pi}{4}(0.02)^2 V_2$

$$\boxed{V_2 = 9V_1}$$

Applying bernoulli's eq:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + y_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + y_2$$

$$\frac{P_1}{\gamma} = \frac{V_2^2 - V_1^2}{2g} = \frac{80V_1^2}{19.6}$$

$V_1 = 4.183$ m/s.

$V_2 = 37.65$ m/s, $Q = A_1V_1 = 0.0118\ m^3/s$.

$$F = 11.8\,[4.183 - 37.65] + 700\times10^3 (0.06)^2\frac{\pi}{4} - 0$$

$= 1.58$ kN.

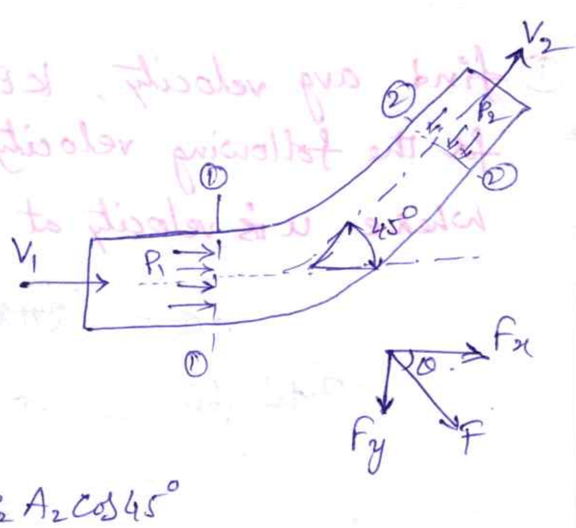
③ A 30 cm dia pipe carries water under a head of 20 meters with a velocity of 3.5 m/s. If the axis of the pipe turns through 45° find the magnitude and direction of the resultant force on the bend.

$V_1 = V_2 = 3.5$ m/s.

$Q = A_1V_1 = \frac{\pi}{4}(0.3)^2(3.5)$

$= 0.247\ m^3/s$

$P_1 = P_2 = \gamma h = 9810\times 20 = 196200\ N/m^2$



force along x-direction.

$$F_x = \rho Q\,[V_1 - V_2\cos 45°] + P_1A_1 - P_2A_2\cos 45°$$

$$= ~~9810~~\ 247\,[3.5 - 3.5\times 0.707] + 196200\times\frac{\pi}{4}(0.3)^2 - 196200\,\frac{\pi}{4}(0.3)^2\,0.707$$

F_x = ~~4318 kN~~ 4378 N

force along y-axis.

$$F_y = \rho Q(0 - V_2 \sin 45^\circ) - P_2 A_2 \sin 45^\circ$$

$$= 247(0 - 3.5 \times 0.707) - 196200 \times \frac{\pi}{4}(0.3)^2 \times 0.707$$

$$= -10419.4\ N\ (\downarrow).$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(4318.1)^2 + (10419.4)^2} = 11.27\ kN.$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{10419}{4318}\right) = \tan^{-1}(2.4) = 67^\circ 29'.$$

Momentum correction factor (β): The effect of non-uniform velocity distribution on the momentum flux is taken care of by using factor β, called momentum correction factor (β).

$$M.\ \text{flux per unit time} = m \times v = (\rho\, dA\, u)\, u = \rho\, dA\, u^2$$

$$\text{total MF per unit time} = \int \rho u^2 dA$$

momentum flux calculated based on avg velocity V &

$$= \beta(\rho A V)V = \beta \rho A V^2$$

$\rho = \frac{m}{V}$, $m = \rho V$

$$\beta \rho A V^2 = \int \rho u^2 dA$$

$$\beta = \frac{1}{AV^2}\int u^2 dA$$

i) $\beta = 1.33$ for laminar flow with parabolic velocity distribution

ii) $\beta = 1.01$ to 1.07 for turbulent flow.

① find avg velocity, KE correction factor and momentum correction fact for the following velocity profile in a circular pipe. $u = V_m\left[1 - \left(\frac{r}{R}\right)^2\right]$ where u is velocity at r, V_m is velocity at the pipe axis.

$$\delta Q = (2\pi r\, dr)\, v$$

Total flow
$$Q = \int_0^R 2\pi r V_m \left(1 - \frac{r^2}{R^2}\right) dr.$$

$$= 2\pi V_m \left(\frac{R^2}{4}\right) = \frac{\pi V_m R^2}{2}$$

$$Q = AV$$

Avg. velocity $= \frac{V_m}{2}$

ii) KE factor

$$\alpha = \frac{1}{AV^3}\int u^3\, dA$$

$$= \frac{1}{AV^3}\int V_m^3\left(1-\left(\frac{r}{R}\right)^2\right)^3 2\pi r\, dr$$

$$= \frac{2\pi V_m^3}{AV^3}\left[\int \left(r - \frac{r^7}{R^7} + \frac{3r^5}{R^4} - \frac{r^7}{R^6}\right]dr\right.$$

$$= \frac{2\pi V_m^3}{AV^3}\left(\frac{R^2}{8}\right)$$

$\alpha = 2.$ $\quad$ $V = \frac{V_m}{2}$, $A = \pi R^2$

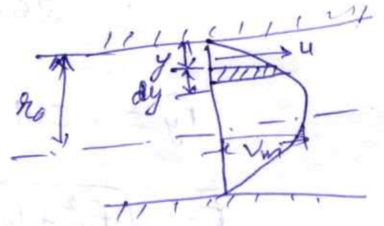
iii)

$$\beta = \frac{1}{AV^2}\int u^2\, dA = \frac{1}{AV^2}\int V_m^2\left(1-\left(\frac{r}{R}\right)^2\right)^2 2\pi r\, dr$$

$\beta = 1.33$

② velocity distribution for a flow in a pipe is given $u = V_m\left(\frac{y}{r_0}\right)^{1/7}$ where u is local velocity at a distance y from pipe wall, V_m is maximum flow velocity. find avg velocity, KE correction factor, MCF.

– consider elementary ring of thickness dy at a distance y from the pipe wall, at a distance $(r_0 - y)$ from the pipe axis.



flow rate through the elemental ring = elemental area x local velocity

$$= 2\pi(r_0 - y)\, dy\, u.$$

Total flow $Q = \int_0^{r_0} 2\pi u(r_0 - y)\, dy = \int_0^{r_0} 2\pi V_m \left(\frac{y}{r_0}\right)^{1/7}(r_0 - y)\, dy$

$$Q = \frac{2\pi V_m}{r_0^{1/7}}\left[\frac{7}{8} r_0^{15/7} - \frac{7}{15} r_0^{15/7}\right]$$

$$= 2\pi V_m\left(\frac{49}{120} r_0^2\right)$$

$Q = AV = \pi r_0^2 V$ ⇒

$$V = \frac{49}{60} V_m$$

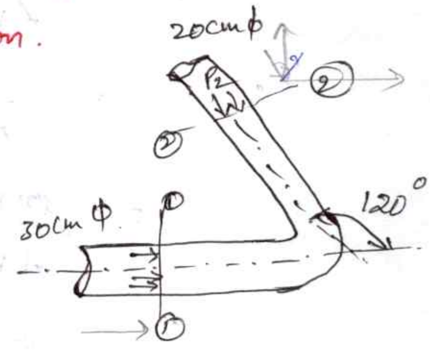
ii) KE.CF :

$$\alpha = \frac{1}{AV^3}\int_0^{r_0} u^3\, dA = \frac{1}{AV^3}\int_0^{r_0} V_m^3\left(\frac{y}{r_0}\right)^{3/7} 2\pi(r_0 - y)\, dy$$

$\alpha = 1.06.$

$$\beta = \frac{1}{AV^2}\int u^2\, dA = \frac{1}{AV^2}\int_0^{r_0} V_m^2\left(\frac{y}{r_0}\right)^{2/7} 2\pi(r_0 - y)\, dy = 1.02$$

① Estimate the force exerted on the bend as shown in fig. Discharge is $0.25 \, m^3/s$, volume of bend $= 0.1 m^3$. pressure at entrance $= 60 KPa$. The exit is 2m above the entrance section.



$$V_1 = \frac{0.25}{\frac{\pi}{4}(0.3)^2} = 3.54 \, m/s.$$

$$V_2 = \frac{0.25}{\frac{\pi}{4}(0.2)^2} = 7.96 \, m/s.$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + y_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + y_2$$

$$P_2 = 14970 \, N/m^2$$

force along x-axis.

$$f_x = \rho Q \left[V_1 - V_2 \cos 120^\circ \right] + P_1 A_1 - P_2 A_2 \cos 120^\circ$$

$$= 6354 \, N$$

$$F_y = \rho Q \left[0 - V_2 \sin 120^\circ \right] - P_2 A_2 \sin 120^\circ - \text{weight of water in bend}$$

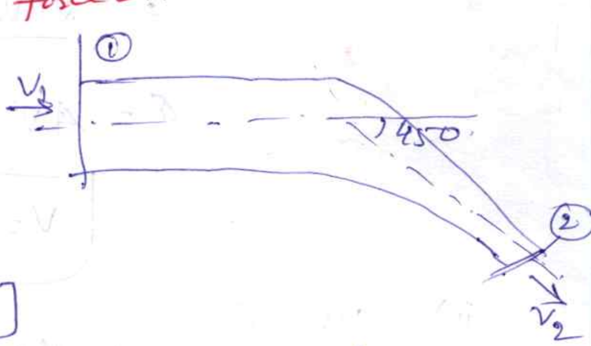
$$= 250 \left[0 - 7.96 \times 0.866 \right] - 14970 \times \frac{\pi}{4}(0.2)^2 \times 0.866 - 0.1 \times 9810$$

$$F_y = -3111.4 \, N.$$

$$F = \sqrt{F_x^2 + F_y^2} = 7075 \, N$$

$$\tan\theta = \frac{F_y}{F_x} \Rightarrow \theta = 26.09^\circ$$

② A pipe bend placed in a horizontal plane tapers from 50 cm dia at inlet to 25cm dia at outlet. An oil of density $850 \, kg/m^3$ enters the reducing bend horizontally and gets turned through 45° clockwise direction. discharge is $0.45 \, m^3/s$ pressure at inlet $40 \, kN/m^2$, at outlet $23 \, kN/m^2$. Calculate resultant force on the bend.



$V_1 = 2.29 \, m/s$ $(\theta = -45^\circ)$

$V_2 = 9.17 \, m/s.$

$$F_x = \rho Q \left[V_1 - V_2 \cos\theta \right] + P_1 A_1 - P_2 A_2 \cos\theta$$

$$= 850 \times 0.45 \left[2.29 - 9.17 \times 0.707 \right] + 40 \times 10^3 \times \frac{\pi}{4}(0.5)^2 - 23 \times 10^3 \frac{\pi}{4}(0.25)^2 \cos(-45^\circ)$$

$$= -1603.6 + 7850 - 797$$

$$= 5448 \, N.$$

$$F_y = \rho Q[0 - V_2 \sin\theta] - P_2 A_2 \sin\theta$$

$$= 850 \times 0.45 [0 - 9.17 \sin(-45^\circ)] - 23 \times 10^3 \times \frac{\pi}{4}(0.25)^2 \sin(-45^\circ)$$

$$= 3279.6 (\uparrow) N$$

$$F = \sqrt{F_x^2 + F_y^2} = 6358.8 N$$

$$\tan\theta = \frac{F_y}{F_x} \Rightarrow \theta = 31.04^\circ$$

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CIVIL ENGINEERING

Fluid Mechanics, Hydraulics & Hydraulic Machinery

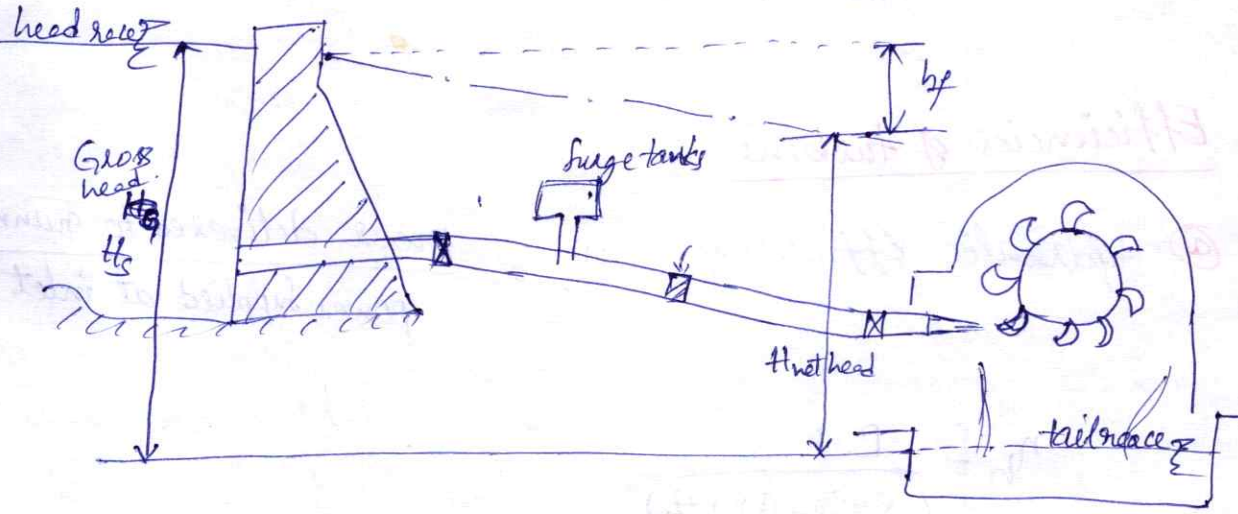
UNIT-4

UNIT-IV

Hydraulic Turbines

Hydraulic turbines are the machines which use the energy of water (hydro-power) and convert into mechanical energy. As such these may be considered as hydraulic motors & primemovers. The mechanical energy developed by a turbine is used in running an electric generator which is directly coupled to the shaft of turbine. The electric generator thus develops electric power which is knownas hydro-electric power.

Layout of a hydro-electric power plant :



The water surface in the storage reservoir is known as head race level or simply head race. water from the storage reservoir is carried through penstock or canals to the powerhouse. In some installations smaller reservoirs known as fore bays are also provided. A forebay is essentially a storage reservoir at the head of the penstocks. The purpose of a forebay is to temporarily store water when it is not required by the turbine and supply the same when required.

The water passing through the turbine is discharged to the tail race. The tail race is the channel which carries water away from the power house after it has passed through the turbine.

Head and Efficiencies of Hydraulic Turbines.

(i) Gross head: It is defined as the difference b/w the head race level and tail race level when no water is flowing. As such the gross head is often termed as static head & total head and it may be represented by H_s.

ii) Net of effective head: It is the head available at the entrance to the turbine. It is obtained by substracting from the gross head all the losses of head that may occurs as water flow from the head race to the entrance of turbine. The losses of head are mainly due to friction occuring in penstocks, canal etc.

$$H = H_s - h_f$$

Efficiencies of turbine

(a) Hydraulic efficiency: $\eta_h = \dfrac{\text{power delivered to runner}}{\text{power supplied at inlet}} = \dfrac{R.P}{W.P}$

$$\eta_h = \frac{R.P}{(\rho g(Q+\Delta Q)H)}$$

(b) Mechanical efficiency: (η_m):

$$\eta_m = \frac{\text{power available at the turbine shaft}}{\text{power developed by the runner}} = \frac{SP}{RP}$$

(c) volumetric efficiency (η_v): $\eta_v = \dfrac{Q}{(Q+\Delta Q)}$

volumetric efficiency is the ratio of quantity of water actually striking the runner and quantity of water supplied to the turbine.

(d) Overall efficiency (η_o):

$$\eta_o = \frac{\text{power available at turbine shaft}}{\text{Net power supplied at turbine entran}}$$

$$\eta_o = \frac{P}{\rho g(Q+\Delta Q)H}, \quad \eta_o = \eta_h \times \eta_m.$$

Classification of turbines :

① According to action of water flowing through the turbine runners.

i) **Impulse turbine** : In an impulse turbine, all the available energy of water is converted into KE or velocity head by passing it through a contracting nozzle provided at the end of penstock. Some examples are Pelton wheel, Turgo impulse wheel, Jonval turbine, Banki turbine, Girard turbine. etc.

ii) **Reaction Turbine** : In a reaction turbine, at the entrance to the runner, only a part of the available energy of water is converted into kinetic energy and substantial part remains in the form of pressure energy. As water flows through the runner change from pressure to KE takes place gradually. As such the pressure at inlet to the turbine is much higher than the pressure at outlet and it varies throughout the passage of water through turbine. For this gradual change of pressure to be possible the runner in this case must be completely enclosed in an air-tight casing and passage is entirely full of water throughout the operation of turbine. The difference of pressure b/w inlet & outlet of runner is called reaction pressure. hence the turbine called reaction turbines.

Examples : Fourneyron, Thomson, Francis, Kaplan, propeller etc.

② According to main direction of flow of water in the runner :

i) **Tangential flow** : In a tangential flow turbine the water flows along the tangent to the path of rotation of the runner. (Pelton wheel)

ii) **Radial flow Turbine** : In this the water flows along the radial direction and remains wholly and mainly in the plane normal to the axis of rotation, as it passes through the runner.

a) Inward radial flow : The water enters at the outer circumference and flows radially inwards towards the centre of the runner. (Eg: old Francis, Thomson)

b) Outward radial flow : water enters at the centre and flows radially outwards towards the outer periphery of the runner.

Eg: Fourneyron turbine

iii) **Axial flow turbine**: In an axial flow turbine the flow of water through the runner is wholly and mainly along the direction parallel to the axis of rotation of the runner.

Eg: Jonval, Propeller, kaplan turbine etc.

iv) **mixed flow turbine**: In this, water enters the runner at the outer periphery in the radial direction and leaves it at the centre in the direction parallel to the axis of rotation of the runner. Eg: modern francis turbine.

Based on head & quantity of water required:

i) **High head turbines**: high head turbines are those which are capable of working under very high heads ranging from several hundreds meters to few thousand meters. In practical Pelton wheel has so far been used under highest head of 1770m (5800ft)

ii) **Medium head Turbines**: which are capable of working under medium heads ranging from about 60 to 250m. These are required relatively large amount of water.

Eg: modern francis turbine

iii) **Low head turbines**: which are capable of working under the heads less than 60m. These require large quantity of water

Eg: Kaplan & other propeller turbines.

According to Specific speed: (N_s)

The specific speed of a turbine is the speed of a geometrically similar turbine that would develop one kilowatt power when working a head of one metre.

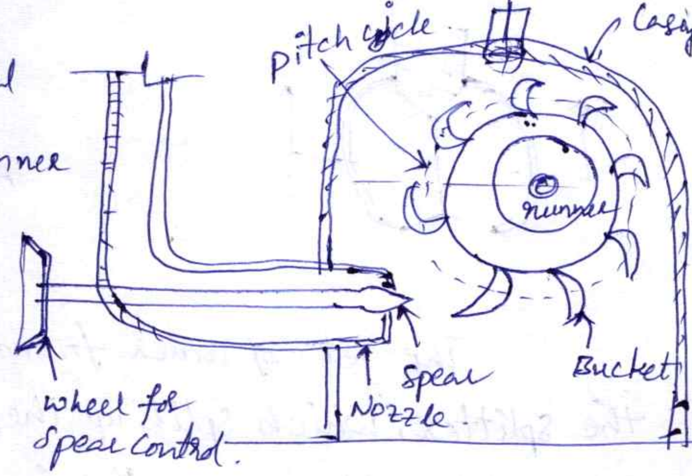
i) N_s varying from 8.5 to 30 - pelton wheel with single jet & upto 43 for pelton wheel with double jet.

ii) N_s varying from 50 to 340 - francis turbine

iii) N_s - 255 to 860 - kaplan & other propeller turbines.

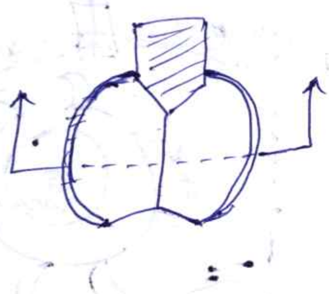
Pelton wheel: It is named after lester A. Pelton (1829-1908) American engineer who contributed much to its development.

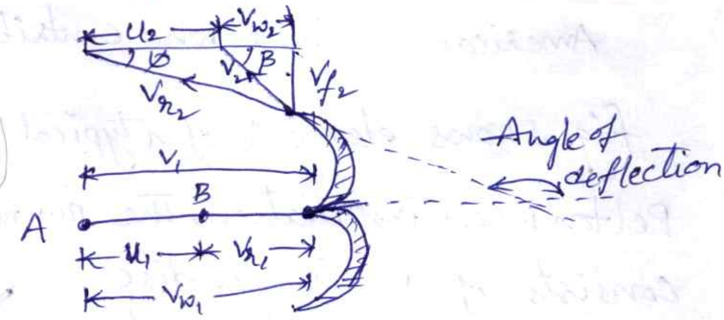
fig shows elements of a typical Pelton wheel installation. The runner consists of a circular disc with a no. of buckets evenly spaced round its periphery. The buckets have a shape of double semi-ellipsoidal cups.



Each bucket is divided into two symmetrical parts by a sharp edged rigde known as splitter. One or more nozzle are mounted so that each directs a jet along a tangent to the circle through the centres of the buckets called the pitch circle. The jet of water impinges on the splitter, which divides the jet into two equal portions, each of which after flowing round the smooth inner surface of bucket leaves it at its outer edge. The buckets are so shaped that the angle at the outlet tip varies from 10° to 20° (usually 15°) so that the jet of water gets deflected through 160° to 170°. The advantage of having a double cup-shaped buckets is that the axial thrusts neutralise each other, being equal and opposite, and hence the bearings supporting the wheel shaft are not subjeted to any axial or end thrust.

velocity Triangles and workdone for pelton wheel:





The jet of water from the nozzle strikes the bucket at the splitter, which splits up the jet into two parts. These parts of the jet, glides over the inner surface and comes out at outer edge.

V_1 = velocity of jet at inlet $= \sqrt{2gH}$

$$u = u_1 = u_2 = \frac{\pi D N}{60}$$

velocity triangle at inlet will be a straight line where

$$V_{r_1} = V_1 - u_1 = V_1 - u, \quad V_{w_1} = V_1$$

$$\alpha = 0^\circ \text{ and } \theta = 0^\circ$$

from velocity triangle at the outlet

$$V_{r_2} = V_{r_1} \text{ and } V_{w_2} = V_{r_2}\cos\phi - u_2$$

force exerted by the jet of water in the direction of motion is given by the eq.

$$F_x = \rho A V_1 [V_{w_1} + V_{w_2}]$$

As the angle β is an acute angle. Also this is the case of series of vanes, the mass of water striking is $\rho A V_1$ and not $\rho A V_{r_1}$.

area of jet $= A = \frac{\pi}{4}(d)^2$

workdone by the jet on the runner per second $= F_x \times u$

$$= \rho A V_1 [V_{w_1} + V_{w_2}] \times u \text{ Nm/s}$$

$$\text{power} = \frac{\rho A V_1 [V_{w_1} + V_{w_2}] u}{1000} \text{ kW}$$

workdone/s per unit weight of water $= \dfrac{\rho A V_1 [V_{w_1} + V_{w_2}] u}{\rho A V_1 \times g} = \dfrac{1}{g}(V_{w_1} + V_{w_2})u$

KE of jet per second $= \frac{1}{2}(\rho A V_1) V_1^2$

Hydraulic efficiency $\eta_h = \frac{\text{workdone per second}}{\text{KE of jet per second}}$

$$\eta_h = \frac{\rho A V_1 [V_{w_1} + V_{w_2}] u}{\frac{1}{2} (\rho A V_1) V_1^2} = \frac{2 [V_{w_1} + V_{w_2}] u}{V_1^2}$$

Now, $V_{w_1} = V_1$, $V_{r_1} = V_1 - u_1 = V_1 - u$, $V_{r_2} = V_1 - u$

$V_{w_2} = V_{r_2} \cos\phi - u_2 = V_{r_2} \cos\phi - u = (V_1 - u)\cos\phi - u$

$$\eta_h = \frac{2[V_1 + (V_1 - u)\cos\phi - u] u}{V_1^2}$$

The efficiency will be max for a given value of V_1 when

$$\frac{d}{du}(\eta_h) = 0 \quad (or) \quad \frac{d}{du}\left(\frac{2u(V_1 - u)(1+\cos\phi)}{V_1^2}\right) = 0$$

$$2V_1 - 4u = 0 \quad (or) \quad u = \frac{V_1}{2}$$

$$(\eta_h)_{max} = \left(\frac{1+\cos\phi}{2}\right)$$

Working proportions of pelton wheel.

i) The velocity of the jet at inlet is given by $V_1 = C_v \sqrt{2gH}$

$C_v \rightarrow$ Coefficient of velocity $= 0.98$ (or) 0.99

$H \rightarrow$ net head on turbine.

ii) The velocity of wheel (u) is given by $u = \phi\sqrt{2gH}$

ϕ = Speed ratio. The value of speed ratio varies from 0.43 to 0.48.

iii) The angle of deflection of the jet through buckets is taken at 165° if no angle of deflection is given.

iv) The mean diameter or the pitch diameter D of the pelton wheel is given by $u = \frac{\pi D N}{60}$ (or) $D = \frac{60 u}{\pi N}$

v) <u>Jet Ratio</u>: It is defined as the ratio of the pitch diameter (D) of the pelton wheel to the diameter of jet (d). It is denoted by '<u>m</u>' and is given by

$$m = \frac{D}{d} = [12 \text{ for most cases}]$$

vi) Number of buckets on runner is given by

$$Z = 15 + \frac{D}{2d} = 15 + 0.5\,m$$

$m \rightarrow$ jet ratio

(Q) A pelton wheel has a mean bucket speed of 12 m/s and is supplied with water at a rate of 750 ltr per second under a head of 35m. If the bucket deflects the jet through an angle of 160°. find power developed by the turbine and its hydraulic efficiency. Take $C_v = 0.98$. Neglect friction in the bucket. Also find overall efficiency of turbine if its mechanical eff. is 80%.

$Q = 750 \, lt/s = 0.75 \, m^3/s$, $H = 35 m$

$V = C_v \sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 35} = 25.68 \, m/s$.

$u = 12 \, m/s$. $\phi = 180 - 160 = 20°$

$P = \rho Q [(V-u)(1+\cos\phi)] \times u$

$= 238816 \, W$.

$\eta_h = \frac{2u(V-u)(1+\cos\phi)}{V^2} = 96.6\%$

$\eta_o = \eta_h \eta_m = 77.3\%$.

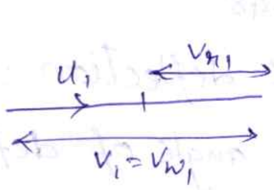
(Q) A pelton wheel operates under a head of 400m with a speed ratio of 0.5 and flow ratio of 0.98. The buckets deflect the jet through an angle of 160°. Determine the power developed per unit weight of water flow.

$H = 400 m$

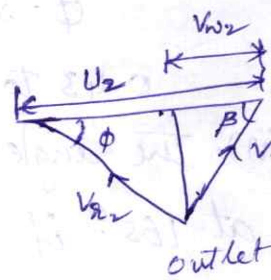
$\phi = 180 - 160 = 20°$

Speed ratio $= 0.5$

flow ratio $= 0.98$



Inlet



outlet

velocity of jet at inlet $V_1 = C_v \sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 400} = 86.82$

Tangential velocity of wheel is $U = k_u \sqrt{2gH} = 0.5\sqrt{2 \times 9.81 \times 400}$

$U = 44.29 \, m/s$.

$V_{w_1} = V_1 = 86.82 \, m/s$

$V_{r_1} = V_1 - u_1 = 42.53 \, m/s$.

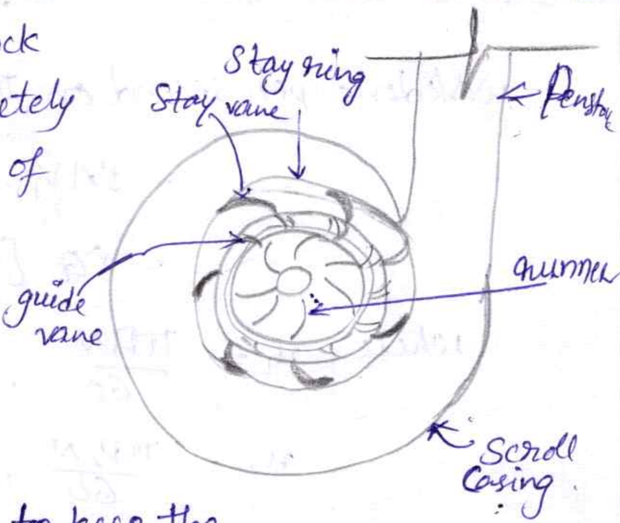
from outlet triangle, $V_{w_2} = U_2 - V_{r_2}\cos\phi = 4.33 \, m/s$

$P = (V_{w_1} - V_{w_2}) U = (86.82 - 4.33) \, 44.29$

$= 3653.48 \, N\text{-}m/kg$.

Francis Turbine :

The water from the penstock enters a scroll casing which completely surrounds the runner. The purpose of the casing is to provide an even distribution of water around the circumference of the turbine. runner, maintaining an approximately constant velocity for water. In order to keep the velocity of water constant throughout its path around the runner, the cross-sectional area of casing is gradually decreased.



Stay ring (or) Speed ring: from the scroll casing the water passes through a speed ring (or) stay ring. this consists of an upper and lower ring held together by series of fixed vanes called stay vanes. The no. of stay vanes is usually taken as half the no. of guide vanes.

→ It directs water from scroll casing to the guide vanes.

→ further it resists the load imposed upon it by the internal pressure of water and weight of the turbine and electrical generator and transmits the same to the foundation.

Guide vanes (or) wicket gates: from speed ring the water passes through a series of guide vanes provided all around the periphery of the runner. The function of guide vanes is to regulate the quantity of water supplied to the runner and to direct water on to the runner at an angle appropriate to the design. These are airfoil shaped. The guide vanes are operated either by means of a wheel.

The main purpose of the various components so far described is to lead the water to the runner with a min loss of energy. The runner of a francis turbine consists of a series of a curved vanes (about 16-24 number) evenly arranged around the circumference in the annual space between 2 plates. The vanes are so shaped that water enters the runner radially at the outer periphery and leaves it axially at the inner periphery.

Inward Radial flow Turbine:

workdone per second on the runner by water is given by

$$= \rho A V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]$$

$$= \rho Q [V_{w_1} u_1 \pm V_{w_2} u_2]$$

where $u_1 = \frac{\pi D_1 N}{60}$, $D_1 \rightarrow$ outer dia of runner

$u_2 = \frac{\pi D_2 N}{60}$, $D_2 \rightarrow$ Inner dia of runner

$N \rightarrow$ rpm.

workdone per unit weight of water per second.

$$= \frac{\text{workdone per second}}{\text{weight of water striking per second}}$$

$$= \frac{\rho Q [V_{w_1} u_1 \pm V_{w_2} u_2]}{\rho Q g} = \frac{1}{g} [V_{w_1} u_1 \pm V_{w_2} u_2]$$

In the above eq +ve is taken if β is acute angle.
-ve is taken if β is obtuse angle.

If $\beta = 90°$. then $V_{w_2} = 0$. and then workdone per second per unit weight of water striking is $= \frac{1}{g} [V_{w_1} u_1]$

$$\text{Hydraulic efficiency} (\eta_h) = \frac{RP}{WP} = \frac{(V_{w_1} u_1 \pm V_{w_2} u_2)}{gH}$$

$$\eta_h = \frac{V_{w_1} u_1}{gH}$$

Degree of Reaction: Degree of reaction is defined as the ratio of pressure energy change inside a runner to the total energy change inside the runner. It is represented by "R".

$$R = \frac{\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right)}{\left(\frac{V_{w_1} u_1 - V_{w_2} u_2}{g}\right)}, \quad V_{w_2} = 0$$

$$R = \frac{\left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right)}{\left(\frac{V_{w_1} u_1}{g}\right)}$$

working proportions :

① Speed ratio : It is defined as $= \frac{u_1}{\sqrt{2gH}}$, $u_1 \rightarrow$ tangential velocity at inlet

$K_u \rightarrow$ 0.6 to 0.9.

② Flow ratio : The ratio of the velocity of flow at inlet (V_{f_1}) to the velocity given $\sqrt{2gH}$ is known as flow ratio & it $\frac{V_{f_1}}{\sqrt{2gH}}$

ψ_f = 0.15 to 0.3

③ Discharge of turbine : The discharge through a reaction radial flow turbine is given by $Q = \pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$

$D_1 \rightarrow$ dia at inlet, $B_1 \rightarrow$ width of runner at inlet

$V_{f_1} \rightarrow$ velocity of flow at inlet

If thickness of vanes are taken into consideration, then the area through which flow takes place is given by $(\pi D_1 - n \times t)$

$n \rightarrow$ no. of vanes on runner, $t =$ thickness of each vane.

iv) Head on the turbine is given by $H = \frac{P_1}{\rho g} + \frac{V_1^2}{2g}$.

v) Radial discharge : This means angle made by absolute velocity with tangent on the wheel is 90° and component of whirl velocity is zero. Radial discharge at outlet means $\beta = 90^\circ$ & $V_{w_2} = 0$, while radial discharge at outlet means $\alpha = 90^\circ$, $V_{w_1} = 0$

vi) If there is no loss of energy when water flows through the vanes then we have

$$H - \frac{V_2^2}{2g} = \frac{1}{g}\left(V_{w_1} u_1 \pm V_{w_2} u_2\right]$$

i) Ratio of width B of the runner to dia D of the runner $n = \frac{B}{D}$.

n ranges from 0.1 to 0.45.

(8) An inward flow turbine has exit dia 1m and its breadth at inlet is 250mm. If velocity of flow at inlet is 2 m/s, find mass of water passing through the turbine per second. Assume 10% of area of flow is blocked by blade thickness. If speed of runner is 210 rpm, guide blades make an angle of 10° to the wheel tangent - find (i) runner vane angle at inlet (ii) velocity of wheel at inlet. iii) Absolute velocity of water leaving guide vanes iv) relative velocity

$D_1 = 1m$, $B_1 = 250mm = 0.25m$ $V_{f_1} = 2 m/s$ $\alpha = 10°$

$u_1 = \frac{\pi D_1 N}{60} = 10.99$

Area of flow $= \pi D_1 B_1 - 0.1 \pi D_1 B_1 = 0.7068 m^2$

mass of water passing per second $= \rho A V_{f_1} = 1000 \times 0.7068 \times 2$
$= 1413.6$ kg/s

$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{2}{V_{w_1}} \Rightarrow V_{w_1} = \frac{2}{\tan\alpha} = \frac{2}{\sin 10} = 11.34 m/s$

$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} \Rightarrow \theta = 80°$

$u_1 = 10.99$, $\sin\alpha = \frac{V_{f_1}}{V_1} \Rightarrow V_1 = 11.517 m/s$

$V_{r_1} = \frac{V_{f_1}}{\sin\theta} = 2.03 m/s$

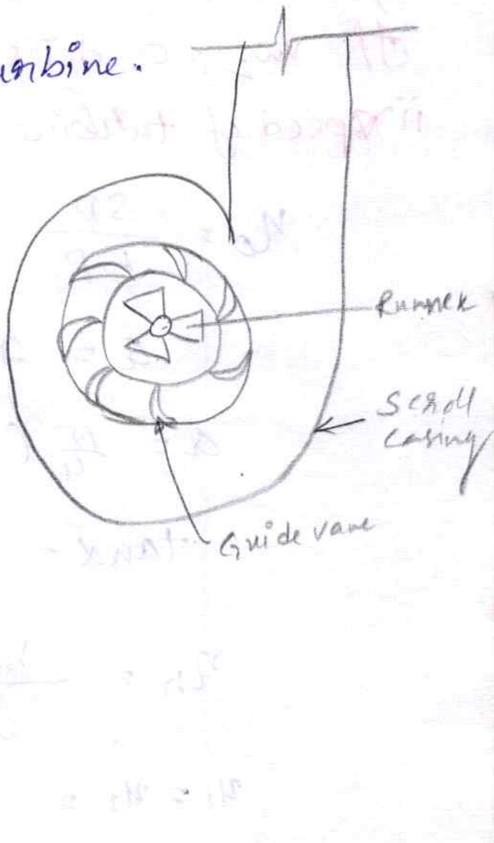
Axial flow Reaction Turbine [Kaplan turbine]

For the axial flow reaction turbine, the shaft of the turbine is vertical. The lower end of the shaft is made larger which is known as hub or boss. The vane are fixed on the hub and hence hub acts as a runner for axial flow reaction turbine.

When the vanes are fixed to the hub and they are not adjustable, the turbine is knowns as propeller turbine. But if vanes on the hub are adjustable, the turbine is known as a Kaplan turbine, after the name of V. Kaplan. This turbine is suitable where large quantity of water at low head is availab.

The main parts of Kaplan Turbine

1. Scroll casing
2. Guide vanes mechanism
3. Hub with vanes or runner of the turbine.
4. Draft tube.



$$Q = \frac{\pi}{4}(D_o^2 - D_b^2)V_{f_1} = \frac{\pi}{4}(D_o^2 - D_b^2)\psi\sqrt{2gH}, \quad \psi = 0.7$$

$D_o \rightarrow$ outer dia of runner, D_b - Dia of hub.

$V_{f_1} \rightarrow$ velocity of flow at inlet.

① The peripheral velocity at inlet and outlet are equal

$$u_1 = u_2 = \frac{\pi D_o N}{60}, \quad D_o \rightarrow \text{outer dia}$$

② $V_{f_1} = V_{f_2}$

③ Area of flow at inlet = Area of flow at outlet $= \frac{\pi}{4}(D_o^2 - D_b^2)$

④ $n = \frac{D_b}{D_o}$, n varies 0.35 to 0.60

Ⓐ Kaplan turbine working under a head of 20m develops 11772 kW shaft power. The outer dia of runner = 3.5m. hub dia - 1.75m. The guide blade angle at the extreme edge of the runner is 35°. the η_h and η_o are 88% & 84% resp. If $V_{w_2} = 0$ determine: Runner vane angles at inlet & outlet ii) Speed of turbine.

$$\eta_o = \frac{SP}{WP} \Rightarrow 0.84 = \frac{11772 \times 10^3}{\gamma Q H}$$

$$Q = 71.428 \, m^3/s.$$

$$Q = \frac{\pi}{4}(D_o^2 - D_b^2)V_{f_1} \Rightarrow V_{f_1} = 9.9 \, m/s.$$

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} \Rightarrow V_{w_1} = 14.14 \, m/s.$$

$$\eta_h = \frac{V_{w_1} u_1}{gH} \Rightarrow u_1 = 12.21 \, m/s$$

$$u_1 = u_2 = \frac{\pi D_o N}{60} \Rightarrow N = 66.6 \, rpm.$$

$$\tan\phi = \frac{V_{f_2}}{u_2} \Rightarrow \phi = 39°$$

$$\theta = 78° \Rightarrow \tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1}$$

Design of Pelton wheel: It means the following data is to be determined
① Dia of jet (d) ② dia of wheel (D). ③ width of bucket = 5d.
④ Depth of bucket = 1.2d & ⑤ no. of buckets on the wheel.

⊛ A pelton wheel is required to develop 6MW when working under a head of 300m. It rotates with a speed of 550 rpm. Assuming jet ratio as 10 and η_o as 85%. Calculate (i) dia of wheel (ii) quantity of water req. (iii) no. of jets. Assume suitable values for velocity coefficient & speed ratio.

$V = C_v\sqrt{2gH} = 0.98\sqrt{2\times9.81\times300} = 75.18\ m/s$.

$u = \phi\sqrt{2gH} = 0.46\sqrt{2\times9.81\times300} = 35.29\ m/s$.

$u = \frac{\pi DN}{60} \Rightarrow D = 1.225\ m, \Rightarrow \frac{D}{d} = 10 \quad d = 0.1225 m.$

$\eta_o = \frac{SP}{WP} \Rightarrow 0.85 = \frac{6\times10^6}{\gamma Q H}$

$Q = 2.398\ m^3/s.$

$Q = n\frac{\pi}{4}d^2V \Rightarrow n = 2.7$

hence no. of jets = 3.

Revised jet dia $\quad 2.398 = 3\frac{\pi}{4}d^2\, 75.18 \Rightarrow d = 0.1164.$

⊛ A radially inward flow turbine working under a head of 10m and running at 250 rpm develops 185 kW at turbine shaft. At inlet tip of the runner vane, the peripheral velocity of wheel is $0.9\sqrt{2gH}$ and radial velocity of flow is $0.3\sqrt{2gH}$. H being effective head on the turbine. If η_o = 78% & hydraulic losses amount to 12% of total head. determine (i) dia of runner & its width at inlet, (ii) guide blade angle at which water is directed to runner & inlet angle of the runner vane.

(i) $u_1 = 0.9\sqrt{2gH} = 12.61\ m/s. \quad u_1 = \frac{\pi D_1 N}{60} \Rightarrow D_1 = 0.96m$

flow velocity $V_{f_1} = 0.3\sqrt{2gH} = 4.2\ m/s.$

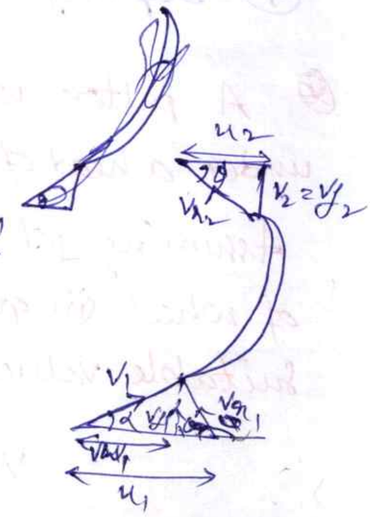
$$\eta_0 = \frac{SP}{WP} = \frac{185 \times 10^3}{\gamma Q H} = 0.78$$

$Q = 2.4\ m^3/s$.

$Q = \pi D_1 B_1 V_{f_1} \Rightarrow B_1 = 0.19\ m$.

ii) As discharge is radial $\beta = 90^\circ$

$V_2 = V_{f_2}$, $V_{w_2} = 0$.



workdone per unit weight of water $= \frac{V_{w_1} u_1}{g}$

from energy balance

Head supplied $= \frac{V_{w_1} u_1}{g} + 0.12H$

$V_{w_1} = \frac{0.88 gH}{u_1} = 6.84\ m/s$.

$\tan\alpha = \frac{V_{f_1}}{u_1} = \frac{4.2}{6.84} = 0.614 \Rightarrow \alpha = 30^\circ 30'$

$\tan(180-\theta) = \frac{V_{f_1}}{u_1 - V_{w_1}} = 0.7229 \Rightarrow$

$180 - \theta = 36^\circ \Rightarrow \theta = 143^\circ 57'$

② A propeller type reaction turbine develops 95 MW at 195 rpm under a 28 m head and with $\eta_0 = 85\%$. Speed ratio 2, flow ratio = 0.63, $D_b = 35\%$ of D_0. Calculate D_0, D_b, discharge, power available at shaft, No. of turbines needed.

$u = \phi\sqrt{2gH} = \frac{\pi D_0 N}{60}$

$2\sqrt{2 \times 9.81 \times 28} = \frac{\pi D_0\, 195}{60}$

$D_0 = 4.59\ m$

flow velocity $V_f = 0.63\sqrt{2 \times 9.81 \times 28} = 14.77\ m/s$.

$Q = \frac{\pi}{4} D_0^2 (1 - n^2) V_f = 214.35\ m^3/s$ $[n = 0.35]$

$P = (\gamma Q H)\eta_0 = (9810 \times 214.35 \times 28)(0.85)$

$= 50\ MW$.

No. of turbine needed $= \frac{95 \times 10^6}{50 \times 10^6} = 1.9 \approx 2$

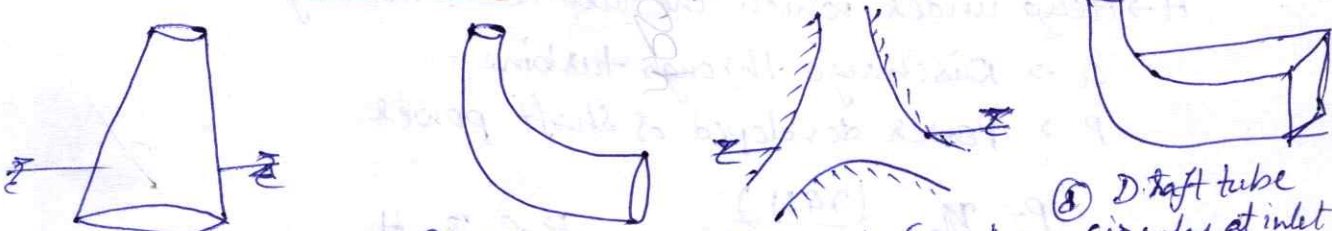
$N_s = \frac{N\sqrt{P}}{H^{5/4}} = 64.41$.

Draft tube : It is a pipe of gradually increasing area which connects the outlet of the runner to the tail race. It is used for discharging water from the exit of the turbine to the tail race. This pipe is called draft-tube.

→ It permits a negative head to be established at the outlet of the runner and there by increase the net head on turbine

→ It converts a large amount of KE rejected at the out let of the turbine into useful pressure energy.

Types of Draft-tubes :



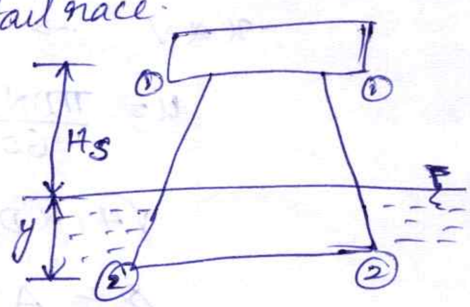
(a) Conical Draft-tube. (b) Simple Elbow (c) Moody Spreading tube (d) Draft tube circular at inlet rectangular at outlet

Derivation

Consider a ~~capital~~ draft-tube as shown in fig

H_s = Vertical height of draft-tube above tail race.

y → distance of bottom of draftube from tail race.



Applying Bernoulli's eq section 1-1 and section 2-2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + (H_s + y) = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + 0 + h_f$$

$$\frac{P_2}{\gamma} = \frac{P_a}{\gamma} + y$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + H_s = \frac{P_a}{\gamma} + \frac{V_2^2}{2g} + h_f$$

$$\frac{P_1}{\gamma} = \frac{P_a}{\gamma} - H_s - \left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_f\right]$$

Efficiency of draft-tube (η_d) = $\dfrac{\text{Actual Conversion of KE head into pressure}}{\text{Kinetic head at inlet of draft tube}}$

$$(\eta_d) = \frac{\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - h_f}{\left(\frac{V_1^2}{2g}\right)}$$

Specific Speed : It is defined as the speed of a turbine which is identical in shape, geometrical dimensions, blade angles, gate opening etc. with actual turbine but of such size that it will develop unit power when working under unit head. It is denoted by symbol N_s.

Derivation of the Specific speed :

$$\eta_0 = \frac{\text{Shaft power}}{\text{water power}} = \frac{P}{\gamma Q H}$$

H → Head under which the turbine is working

Q → Discharge through turbine

P → Power developed or shaft power

$$P = \eta_0 (\gamma Q H) \qquad P \propto Q H$$

D → dia of actual turbine, N → speed of actual turbine.

u → tangential velocity of turbine. N_s → Specific speed

V → Absolute velocity of water.

$$u \propto V \quad \rightarrow \quad V \propto \sqrt{H} \qquad u \propto \sqrt{H}$$

$$u = \frac{\pi D N}{60} \qquad u \propto DN$$

$$\sqrt{H} \propto DN \quad (\text{or}) \quad D \propto \frac{\sqrt{H}}{N}$$

$$Q = AV \quad , \quad A \propto BD \Rightarrow A \propto D^2$$

$$V \propto \sqrt{H} \quad \Rightarrow \quad Q \propto D^2 \sqrt{H}$$

$$Q \propto \left(\frac{\sqrt{H}}{N}\right)^2 \times \sqrt{H}$$

$$\propto \frac{H}{N^2} \times \sqrt{H} \propto \frac{H^{3/2}}{N^2}$$

$$P \propto \frac{H^{3/2}}{N^2} \times H \qquad P = k\frac{H^{5/2}}{N^2}$$

If $P = 1, H = 1, N = N_s \Rightarrow N_s^2 = k$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

If P is taken in metric horsepower then N_s is in M.K.S.

But P is taken in Kilowatts., N_s is in SI units.

Significance of Specific Speed : specific speed plays an important role for selecting the type of the turbine. Also the performance of a turbine can be predicted by knowing the specific speed of turbine.

S NO.	Specific speed MKS	Specific speed SI	Types of turbine
1.	10 to 35	8.5 to 30	pelton wheel with single jet
2.	35 to 60	30 to 51	pelton wheel with two or more jets.
3.	60 to 300	51 to 225	francis turbine.
4.	300 to 1000	255 to 860	Kaplan (or) propeller turbine

(Q) A turbine is to operate under a head of 25 m at 200 rpm. The discharge is 9 m^3/s. If $\eta_o = 90\%$ determine (i) N_s, (ii) P and (iii) type of turbine.

$$\eta_o = \frac{P}{\gamma Q H} \Rightarrow P = 0.90\,(9810)(9)(25) = 1986.5\,kW$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = 159.4\ rpm. \quad (\text{francis turbine})$$

Runway Speed : for a turbine working under max head and full gate opening, if the external load suddenly drops to almost zero value and at the same time the governing mechanism of the turbine also fails, then the turbine runner will tend to race up and it will attain the maximum possible speed. This maximum or limiting speed of the turbine runner is known as runway speed. for safe design the various rotating components are designed for the runway speed.

for pelton wheel → 1.8 to 1.9 times its normal speed.

francis turbine → 2 to 2.2

kaplan → 2.5 to 3 to its normal speed

(Prob. 4.15) The inner and outer diameter of an inward flow reaction turbine are 0.5 and 1 m respectively. The vanes are radial at inlet and discharge is also radial. The head on the turbine is 32 m and the inlet guide vanes angle is 10°. Assuming the velocity of flow as constant and equal to 3 m/s, find speed of the runner, the vane angle at the outlet, and the hydraulic efficiency.

$D_1 = 1m, \quad D_2 = 0.5m, \quad H = 30m$

$V_{w_1} = u_1, \quad V_{r_1} = V_{f_1}, \quad V_{f_1} = V_{f_2}$

Discharge is radial $V_{w_2} = 0, \quad V_2 = V_{f_2} = 3 m/s.$

$$\tan\alpha = \frac{V_{f_1}}{u_1}$$

$$\tan 10 = \frac{3}{u_1} \Rightarrow u_1 = 17.01 m/s.$$

$$u_1 = \frac{\pi D_1 N}{60}, \quad N = \frac{60 \times 17.01}{\pi \times 1} = 324.86 rpm.$$

$$V_2 = \frac{\pi D_2 N}{60} = 8.5 m/s.$$

$$\tan\phi = \frac{V_{f_2}}{u_2} \Rightarrow \phi = 19.43°$$

$$\eta_h = \frac{V_{w_1} u_1}{gH} = 92.17\%$$

Unit quantities of Hydraulic turbines

Unit speed: It is defined as the speed of a geometrically similar turbine working under a unit head.

$$u \propto V$$

$$V \propto \sqrt{H} \Rightarrow u \propto \sqrt{H}$$

$$u = \frac{\pi D N}{60} \Rightarrow u \propto N$$

$$N \propto \sqrt{H} \Rightarrow N = K_1 \sqrt{H}$$

when $H = 1m$, $N = N_u$ (unit speed)

$$N_u = K_1 \sqrt{1} = K_1$$

$$N = N_u \sqrt{H} \Rightarrow \boxed{N_u = \frac{N}{\sqrt{H}}}$$

Unit Discharge: The unit discharge is the flow rate the turbine would have when working under a unit head.

$$Q \propto \sqrt{H}$$

$$Q = K_2 \sqrt{H}$$

when $H = 1$, $Q = Q_u$

$$Q_u = K_2 \sqrt{1} \Rightarrow K_2 = Q_u$$

$$Q = Q_u \sqrt{H}$$

$$Q_u = \frac{Q}{\sqrt{H}}$$

Unit power: The unit power is defined as the power generated by a geometrically similar turbine working under a unit head.

$$P = \eta_o \gamma Q H$$

$$P \propto V H$$

$$P \propto \sqrt{H}\, H$$

$$P = K_3 H^{3/2}$$

when $H = 1$, $P = P_u \Rightarrow P_u = K_3$

$$P = P_u H^{3/2}$$

$$\boxed{P_u = P / H^{3/2}}$$

(8) A turbine with an overall efficiency of 85% is to be installed in a hydroelectric plant. The head and discharge available at the plant are 25m and 45 m^3/s respectively. If the specific speed of the turbine is 230, determine the unit speed, unit discharge and unit power.

$$\eta_o = \frac{P}{\gamma Q H}$$

$$P = 9380.8 \text{ KW}$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$N = 132.75 \text{ rpm}.$$

$$N_u = \frac{N}{\sqrt{H}} = 26.55$$

$$Q_u = \frac{Q}{\sqrt{H}} = 9$$

$$P_u = \frac{P}{H^{3/2}} = 75.046.$$

Model Testing of Turbines : [Geometrical Similarity]

In order to have an idea about the performance of the actual turbine in advance, a small scale model of the turbine which is geometrically simillar to the actual turbine is first prepared. The various linear dimensions of the model turbine bear the same proportion to their corresponding dimensions of the actual turbine. The model turbine is then tested under a known head, speed and flow rate and its output as well as efficiency are determined. from these test results it will be possible to predict the performance of actual turbine.

The parameter $\frac{Q}{ND^3}$ is known as discharge number (or) flow number

Head number $= \frac{gH}{N^2D^2}$

power number $= \frac{P}{\gamma HND^3}$

By combining these parameters, their alternative expression may be obtained.

discharge number $= \frac{Q}{D^2\sqrt{gH}}$

power $= \frac{P}{\rho g^{3/2} H^{3/2} D^2}$

(a) A water turbine develops 130 kW at 230 rpm. under a head of 16m. Determine the scale ratio and the speed of a similar machine which will generate 660 kW when working under a head of 25m.

$$\frac{P_1}{D_1^2 H_1^{3/2}} = \frac{P_2}{D_2^2 H_2^{3/2}}$$

$$\frac{D_2}{D_1} = 1.612$$

$$\frac{N_1 D_1}{\sqrt{H_1}} = \frac{N_2 D_2}{\sqrt{H_2}}$$

$$N_2 = 178.35 \text{ rpm}$$

(b) The following data were obtained from the main characteristics of a Kaplan turbine of runner dia 1m, $P_u = 30.695$, $Q_u = 108.6$; $N_u = 63.6$. Estimate the runner dia, the discharge and the speed of a similar runner working under a head of 30m and developing 2000 kW. Determine the specific speed of the runner.

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3} \;; \quad \frac{N_1 D_1}{\sqrt{H_1}} = \frac{N_2 D_2}{\sqrt{H_2}},$$

$$\frac{N_1 \sqrt{P_1}}{H_1^{5/4}} = \frac{N_2 \sqrt{P_2}}{H_2^{5/4}}$$

$$\frac{P_1}{D_1^2 H_1^{3/2}} = \frac{P_2}{D_2^2 H_2^{3/2}}$$

$$D_2 = 0.6297\ m.$$

$$\frac{N_1 D_1}{\sqrt{H_1}} = \frac{N_2 D_2}{\sqrt{H_2}}$$

$$N_2 = 553.2\ rpm.$$

$$\frac{Q_1}{D_1^2\sqrt{H_1}} = \frac{Q_2}{D_2^2\sqrt{H_2}}$$

$$Q_2 = 235.86\ m^3/s.$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = N_u\sqrt{P_u}.$$

$$N_s = 63.6\sqrt{30.695} = 352.36$$

$$N_s = 352.36.$$

(4) A francis turbine develops 20MW under a head of 30m and at a speed of 150 rpm and gives an efficiency of 85%. If a model $\frac{1}{5}$th the size of prototype is tested under a head of 5m, what must be its speed, power and dischage to run under similar condition?

$$\frac{D_m}{D_p} = \frac{1}{5}$$

$$P_p = 20\ MW = 20\times10^6\ W$$

$$N_m = 150\ rpm,\quad H_p = 30m,\quad \eta_o = 0.85$$

$$H_m = 5m.$$

$$\eta_o = \frac{P_p}{\rho Q_p g H_p} \Rightarrow Q_p = 79.95\ m^3/s.$$

$$\left(\frac{H}{N^2D^2}\right)_m = \left(\frac{H}{N^2D^2}\right)_p$$

$$\frac{\sqrt{H_m}}{N_m D_m} = \frac{\sqrt{H_p}}{N_p D_p}$$

$$N_m = N_p \times \frac{D_p}{D_m} \sqrt{\frac{H_m}{H_p}} = 612.37 \text{ rpm}$$

$$\left(\frac{P}{N^3 D^5}\right)_m = \left(\frac{P}{N^3 D^5}\right)_p$$

$$\frac{P_m}{N_m^3 D_m^5} = \frac{P_p}{N_p^3 D_p^5}$$

$$P_m = 13.6 \text{ KW.}$$

$$\left(\frac{Q}{ND^3}\right)_m = \left(\frac{Q}{ND^3}\right)_p$$

$$Q_m = 0.326 \text{ m}^3/s$$

UNIT - V

Centrifugal pumps.

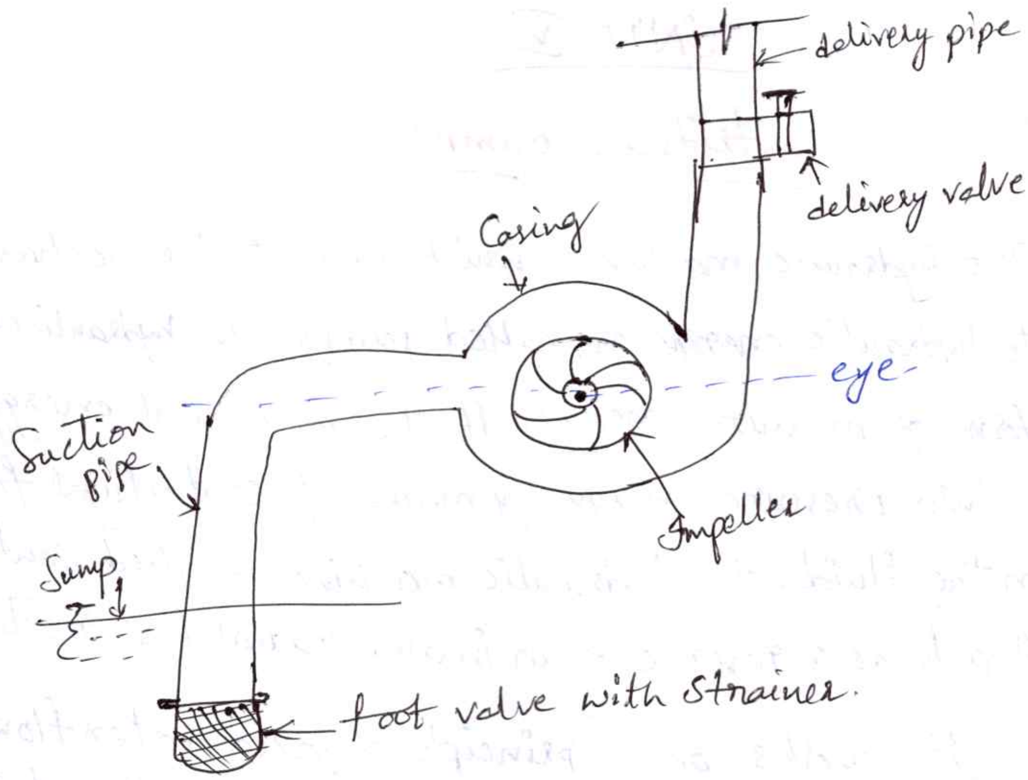
The hydraulic machines which convert the mechanical energy into hydraulic energy are called pumps. The hydraulic energy is in the form of pressure energy. If the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, the hydraulic machine is called centrifugal pump. It acts as a reverse of an inward radial flow reaction turbine.

principle: It works on the principle of forced vortex flow which means that when a certain mass of liquid is rotated by an external torque, the rise in pressure head of the rotating liquid takes place. The rise in pressure head at any point of the rotating liquid is proportional to the square of tangential velocity of the liquid at that point. Thus at the outlet of the impeller, where radius is more, the rise in pressure head will be more and the liquid will be discharged at the outlet with a high pressure head. Due to this pressure head, the liquid can be lifted to a high level.

Main parts of a centrifugal pump:

1. Impeller
2. Casing
3. Suction pipe with a foot valve and strainer
4. Delivery pipe.

① Impeller: It is a wheel or rotor which is provided with a series of backward curved blades or vanes. It is is mounted on a shaft which is coupled to an external source of energy which imparts the required energy to the impeller thereby making it to rotate.



② Casing : It is an airtight chamber which surrounds the impeller. It is similar to the casing of a reaction turbine.

③ Suction Pipe : It is a pipe which is connected at its upper end to the inlet of the pump or to the centre of impeller which is known as eye. The lower end of the suction pipe dips into liquid in a sump from which the liquid is to be pumped.

The lower end of the suction pipe is fitted with a foot valve and strainer. The liquid first enters the strainer which is provided in order to keep the debris away from the pump. It then passes through the foot valve to enter the suction pipe. A foot valve is a non-return or one way type of valve which opens only in the upward direction. As such the liquid will pass through the foot valve only upwards and it will not allow the liquid to move downwards back to the pump

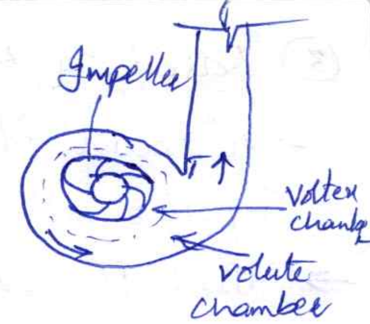
④ Deliver Pipe : It is a pipe which is connected at its lower end to the outlet of the pump and it delivers the liquid to the required height. Just near the outlet of the pump on the delivery pipe a delivery valve is invariably provided..

Types of centrifugal pumps:

According to the type of casing provided.

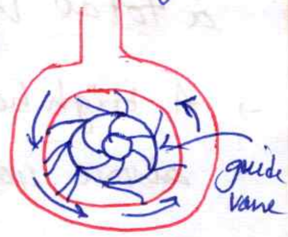
① Volute pump.

② Diffuser & turbine pump.



① Volute pump: In a volute pump the impeller is surrounded by a spiral shaped casing which is known as volute chamber. The shape of the casing is such a way that the crosssectional area of flow around the periphery of the impeller gradually increases from the tongue T towards the delivery pipe. This increase in the crosssectional area results in developing a uniform velocity throughout the casing, because as the flow progresses from the tongue T towards the delivery pipe, more and more liquid is added to the stream from the periphery of the impeller.

② Diffuser chamber: In this, the impeller is surrounded by a series of guide vanes mounted on a ring called diffuser ring. The diffuser ring and the guide vanes are fixed in position. The adjacent guide vanes provide gradually enlarged passages for the flow of liquid.



The liquid after leaving the impeller passes through these passages of increasing area, wherein velocity of flow decreases, pressure increases.

These pumps which are provided with diffuser ring and guide vanes very much resemble a reversed turbine and they are also known as turbine pumps.

③ Based on no. of Impellers per shaft

According to no of impellers provided the pumps may be classified as single-stage and multi-stage pumps. A single stage pump has only one impeller mounted on shaft. A multistage centrifugal pump has two or more impellers connected in series which are mounted on the same shaft and are enclosed in the same casing.

③ Based on Relative direction of flow of liquid through impeller

→ A "radial flow pump" is that in which the liquid flows through the impeller in the radial direction only.

→ In Mixed flow pumps the liquid flows through impeller axially as well as radially, that is there is a combination of radial and axial flows.

→ In axial flow pumps the flow of liquid through the impeller is in the axial direction only.

④ Disposition of Shaft : The centrifugal pumps may be designed with either horizontal or vertical disposition of shaft. Generally pumps are provided with horizontal shafts. However, for deep wells and mines the pumps with vertical shaft are more suitable because the pumps with vertically disposed shafts occupy less space.

⑤ According to Head [working head]:

→ A "Low Head" pump is one which is capable of working against a total head upto 15m.

→ A Medium Head pump is that which is capable of working against a total head more than 15m but upto 40m.

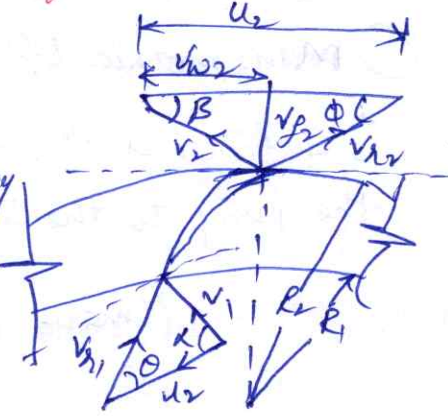
→ A High head pump is capable of working against a total head above 40m. Generally high head pumps are multi-stage pumps.

⑥ According to no. of entrances to the impeller

→ In "Single suction pump" liquid is admitted from a suction pipe on one side of the impeller.

→ In "Double suction pump" liquid enters from both sides of the impeller. A double suction pump has an advantage that by this arrangement the axial thrust on the impeller is neutralised. Further it is suitable for pumping large quantities of liquid since it provides large inlet area.

Work done by the centrifugal pump (by impeller) on water:



In this work is done by the impeller on the water. The ~~exp~~ water enters the impeller radially at inlet for best efficiency of the pump, which means the absolute velocity of water at inlet makes an angle of 90° with the direction of motion of impeller at inlet. Hence $\alpha = 90^\circ$, $V_{w_1} = 0$. For drawing velocity triangles, the same notation are used as that for turbines.

Let N = speed of impeller in rpm

D_1 = dia of impeller at inlet

u_1 = Tangential velocity of impeller at inlet $= \frac{\pi D_1 N}{60}$

$D_2 \rightarrow$ dia at outlet

$$u_2 = \frac{\pi D_2 N}{60}$$

As the water enters the impeller radially which means the absolute velocity of water at inlet is in the radial direction and hence $\alpha = 90^\circ$, $V_{w_1} = 0$.

Workdone by water on the runner per second per unit weight of

$$\text{water} = \frac{1}{g}\left[V_{w_1} u_1 - V_{w_2} u_2\right]$$

$=$ − [work done in case of turbine]

$$= -\frac{1}{g} V_{w_2} u_2$$

Workdone by impeller on water per second $= \frac{\rho Q g V_{w_2} u_2}{g}$

$$Q = \pi D_1 B_1 \times V_{f_1} = \pi D_2 B_2 V_{f_2}$$

Efficiencies and losses:

(i) **Manometric Efficiency**: The manometric efficiency η_{mano} is defined as the ratio of the manometric head developed by the pump to the head imparted by the impeller to the liquid.

$$\eta_{mano} = \frac{g H_m}{V_{w_2} u_2}$$

$$\eta_{mano} = \frac{H_m}{H_m + \text{losses in pump}}$$

If Q is the volume of the liquid actually delivered per second by the pump and γ is the specific weight of the liquid then

$$\eta_{mano} = \frac{\gamma Q H_m}{\gamma Q \left(\frac{V_{w_2} u_2}{g}\right)} = \frac{\text{output of the pump}}{\text{power imparted by impeller}}$$

(2) **Mechanical Efficiency**: It is defined as the ratio of the power actually delivered by the impeller to the power supplied to the shaft by the prime mover or motor.

$$\eta_{mech} = \frac{\gamma (Q + \Delta Q) \left(\frac{V_{w_2} u_2}{g}\right)}{\text{power given to the shaft}} = \frac{\left(\frac{V_{w_2} u_2}{g}\right)}{\text{energy head given to shaft}}$$

(3) **Volumetric Efficieny**: It is defined as the ratio of the quantity of liquid discharged per second from the pump to the quantity passing per second through the impeller.

$$\eta_v = \frac{Q}{(Q + \Delta Q)}$$

(4) **Overall Efficiency (η_o)**: It is defined as the ratio of the power output from the pump to the power input from the prime mover driving the pump.

$$\eta_o = \frac{\gamma Q H_m}{\text{power given to the shaft}}$$

$$\eta_o = \eta_{mano} \times \eta_v \times \eta_{mech}$$

Losses :

① **Hydraulic losses :** The hydraulic losses that may occur within the pump consist of the following

(i) Shock or eddy losses at the entrance to and the exit from impeller

ii) friction losses in the impeller.

iii) friction and eddy losses in the guide vanes and casing.

other hydraulic losses consists of friction and other minor losses in the suction pipe and delivery pipe.

② **Mechanical losses :** The mechanical losses occur in the centrifugal pump on account of the following.

→ Disc friction between the impeller and liquid

→ Mechanical friction of the main bearings and glands.

③ **Leakage losses :** In centrifugal pumps as ordinarily built, it is not possible to provide a completely water tight seal between the delivery and suction spaces. As such there is always a certain amount of liquid which slips or leaks from high pressure to the low pressure points in the pump and it never passes through the delivery pipe. The liquid which escapes or leaks from a high pressure zone to low pressure zone carries with it energy which is subsequently wasted in eddies. This loss of energy due to leakage of liquid represents the Leakage loss.

Specific Speed of the Centrifugal Pump (N_s) :

The specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump which would deliver one cubic metre of liquid per second against a head of one metre. It is denoted by N_s.

Expression :

$Q = A \times V = \pi D \times B V_f$ (or) $Q \propto D \times B \times V_f$

$B \propto D$

$Q \propto D^2 \times V_f$

$$u = \frac{\pi D N}{60} \Rightarrow u \propto DN$$

$$u \propto V_f \propto \sqrt{H_m}$$

$$\sqrt{H_m} \propto DN \text{ (or) } D \propto \frac{\sqrt{H_m}}{N}$$

$$Q \propto \frac{H_m}{N^2} V_f$$

$$Q \propto \frac{H_m}{N^2} \sqrt{H_m}$$

$$Q = k \frac{(H_m)^{3/2}}{N^2}$$

where k is constant of proportionality

If $H_m = 1m$, $Q = 1m^3/s$, $N = N_s$.

$$1 = k \frac{1}{N_s^2} \Rightarrow k = N_s^2$$

$$Q = N_s^2 \frac{H_m^{3/2}}{N^2}$$

$$N_s = \frac{N\sqrt{Q}}{(H_m)^{3/4}}$$

Minimum Starting Speed of a Centrifugal pump:

If the pressure rise in the impeller is more than or equal to the manometric head (H_m), the centrifugal pump will start delivering the water. otherwise, the pump will not discharge any water though the impeller is rotating.

Head due to pressure rise in impeller $= \frac{u_2^2}{2g} - \frac{u_1^2}{2g}$

The flow of water will occur only if

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} \geq H_m.$$

But $\eta_{man} = \frac{gH_m}{V_{w_2} u_2} \Rightarrow H_m = \eta_{man} \frac{V_{w_2} u_2}{g}$

for minimum speed, we must have

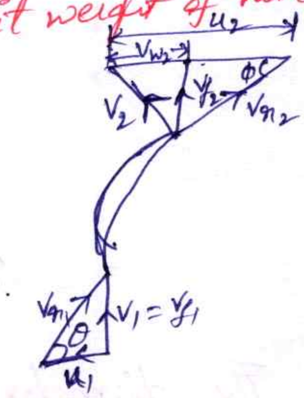
$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = \eta_{man} \frac{V_{w_2} u_2}{g}$$

$$u_1 = \frac{\pi D_1 N}{60}, \quad u_2 = \frac{\pi D_2 N}{60}$$

$$\frac{1}{2g}\left(\frac{\pi D_2 N}{60}\right)^2 - \frac{1}{2g}\left(\frac{\pi D_1 N}{60}\right)^2 = \eta_{man}\frac{V_{w_2}\pi D_2 N}{g\,60}$$

$$N = \frac{120\,\eta_{man}\,V_{w_2}\,D_2}{\pi\,(D_2^2 - D_1^2)}$$

(Q) The internal & external dia of the impeller of a centrifugal pump are 200mm & 400mm resp. The pump is running at 1200rpm. The vane angles of the impeller at inlet and outlet are 20° & 30° respec. The water enters the impeller radially and velocity of flow is const. Determine the workdone by the impeller per unit weight of water.



$D_1 = 0.2m,\ D_2 = 0.4,\ N = 1200\ rpm$

$\phi = 30^\circ,\ \theta = 20^\circ,\ \alpha = 90^\circ,\ V_{w_1} = 0$

$V_{f_1} = V_{f_2}$

$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi(0.2)(1200)}{60} = 12.56\ m/s.$

$u_2 = \frac{\pi D_2 N}{60} = 25.13\ m/s$

$\tan\theta = \frac{V_{f_1}}{u_1} \Rightarrow V_{f_1} = u_1\tan\theta = 12.56\tan 20^\circ = 4.57\ m/s.$

$V_{f_1} = V_{f_2}$

$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} \Rightarrow \tan 30^\circ = \frac{4.57}{25.13 - V_{w_2}}$

$V_{w_2} = 17.215\ m/s.$

workdone by impeller per kg of water per second is given by

$= \frac{1}{g}V_{w_2}u_2 = \frac{17.215 \times 25.13}{9.81} = 44.1\ \frac{Nm}{N}$

(Q) A centrifugal pump delivers water against a net head of 14.5m & a design speed of 1000rpm. The vanes are curved back to an angle of 30° with the periphery. The impeller dia is 300mm. & outlet width is 50mm. Determine the discharge of the pump if manometric efficiency is 95%.

$H_m = 14.5m,\ N = 1000\ rpm,\ \phi = 30^\circ,\ D_2 = 0.3m,\ B_2 = 0.05m$

$\eta_{man} = 0.95 = \frac{gH_m}{V_{w_2}u_2}$; $u_2 = \frac{\pi D_2 N}{60} = 15.7\ m/s.$

$0.95 = \frac{9.81 \times 14.5}{V_{w_2} \times 15.7} \Rightarrow V_{w_2} = 9.54\ m/s.$

$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} \Rightarrow V_{f_2} = 3.556$

$Q = \pi D_2 B_2 V_{f_2} = \pi(0.3)(0.05)(3.55) = 0.1675\ m^3/s.$

(Q) The diameters of an impeller of a centrifugal pump at inlet and outlet are 30 cm & 60 cm respectively. The velocity of flow at outlet is 2.0 m/s and the vanes are set back at an angle of 45° at outlet. Determine the min starting speed of the pump if the $\eta_{man} = 70\%$.

$D_1 = 0.3m,\ D_2 = 0.6m,\ V_{f_2} = 2m/s,\ \phi = 45^0,\ \eta_{man} = 0.7$

$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} \Rightarrow V_{w_2} = u_2 - 2 = \frac{\pi D_2 N}{60} - 2 = 0.03141N - 2$

minimum starting speed $N = \frac{120\,\eta_{man} \times V_{w2}\, D_2}{\pi(D_2^2 - D_1^2)}$

$N = \frac{120 \times 0.7 \times (0.0314N - 2)\,0.6}{\pi\,(0.6^2 - 0.3^2)}$

$N = 137.2$ rpm.

(Q) The diameter of a centrifugal pump, which is discharging 0.03 m³/s of water against a total head of 20m is 0.4 m. The pump is running at 1500 rpm. Find the head, discharge and ratio of powers of a geometrically similar pump of dia 0.25m when it is running at 3000 rpm.

$Q_1 = 0.03\,m^3/s \quad H_{m_1} = 20m, \quad D_1 = 0.4m$

$N_1 = 1500$ rpm, $D_2 = 0.25m$, $N_2 = 3000$ rpm,

let Head on similar group $= H_{m_2}$, Discharge $= Q_2$

$$\left(\frac{Q}{D^3N}\right)_1 = \left(\frac{Q}{D^3N}\right)_2$$

$$\frac{0.03}{(0.4)^3 1500} = \frac{Q_2}{(0.25)^3 3000} \Rightarrow Q_2 = 0.01465\ m^3/s.$$

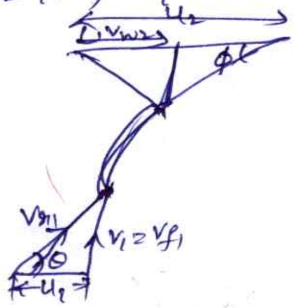
$$\left(\frac{\sqrt{H_m}}{DN}\right)_1 = \left(\frac{\sqrt{H_m}}{DN}\right)_2$$

$$\frac{\sqrt{20}}{0.4 \times 1500} = \frac{\sqrt{H_{m2}}}{0.25 \times 300} \Rightarrow H_{m_2} = 31.25m$$

$$\left(\frac{P}{D^5N^3}\right)_1 = \left(\frac{P}{D^5N^3}\right)_2 \Rightarrow \frac{P_1}{P_2} = \frac{D_1^5 N_1^3}{D_2^5 N_2^3} = 1.31$$

(27) A centrifugal pump has external and internal impeller dia as 600mm & 300mm respectively. The vane angle at inlet and outlet are 30° and 45° respectively. If the water enters the impeller at 2.5 m/s find speed of the impeller in rpm & work done per kN of water

$D_2 = 0.6m$, $D_1 = 0.3m$, $\theta = 30°$, $\phi = 45°$, $V_1 = 2.5 m/s$.



$V_1 = V_{f1}$

$V_{f1} = V_{f2}$

$$\tan\theta = \frac{V_1}{u_1}$$

$$u_1 = \frac{V_1}{\tan\theta} = \frac{2.5}{\tan 30°}$$

$$u_1 = 4.33 m/s$$

$$\frac{\pi D_1 N}{60} = 4.33 \Rightarrow N = 275.6 rpm, \quad u_2 = \frac{\pi D_2 N}{60} = 8.66$$

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}} \Rightarrow \tan 45 = \frac{2.5}{8.66 - V_{w2}}$$

$$V_{w2} = 8.66 - 2.5 = 6.16$$

$$u_2 = 8.66$$

$$\text{work done per N of water} = \frac{1}{g} V_{w2} u_2 = \frac{8.66 \times 6.16}{9.81}$$

$$= 5.44 \frac{Nm}{N}$$

(28) A centrifugal pump delivers water against a net head of 10m at a design speed of 1000 rpm. The vanes are curved backwards and make an angle of 30° with the tangent at the outer periphery. The impeller diameter is 30cm and has a width of 5cm at the outlet. Determine the discharge of the pump if the manometric efficiency is 95%.

$N = 1000 rpm$, $H_m = 10m$, $\eta_{man} = 95\%$, $\phi = 30°$

$D_2 = 0.3m$ $B_2 = 5 cm = 0.05$

$$\eta_{man} = \frac{g H_m}{V_{w2} u_2}, \quad u_2 = \frac{\pi D_2 N}{60} = \frac{\pi (0.3)(1000)}{60} = 15.7 m/s.$$

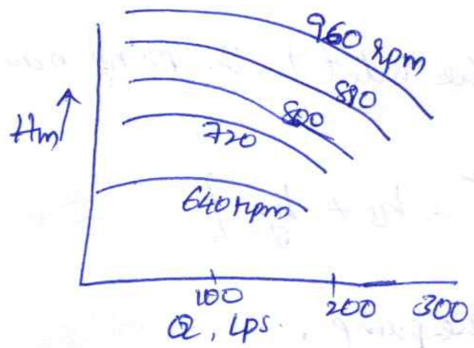
$$0.95 = \frac{9.8 \times 10}{V_{w2} \times 15.7} \Rightarrow V_{w2} = 6.57 m/s.$$

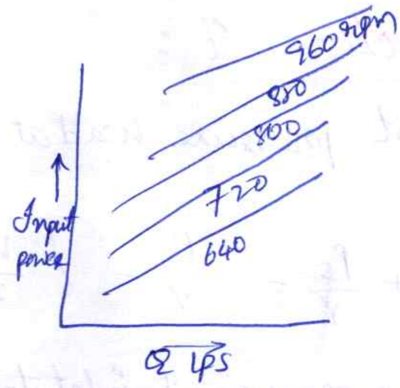
$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}} \Rightarrow V_{f2} = \tan 30 (15.7 - 6.57) = 5.27$$

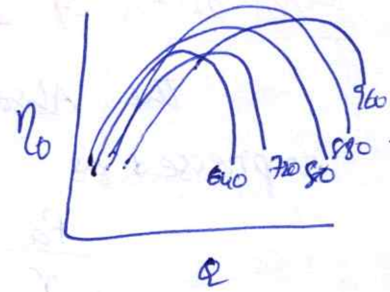
$$Q = \pi B_2 D_2 V_{f2} = \pi (0.05)(0.3)(5.27) = 0.248 m^3/s.$$

Performance of pumps – Characteristic curves.

ⓐ **Main and operating characteristics:** In order to obtain the main characteristic curves of a pump it is operated at different speeds. for each speed the rate of flow Q is varied by means of a delivery valve and for the different values of Q the corresponding values of manometric head H_m, Shaft power P, and η_o are measured or calculated.

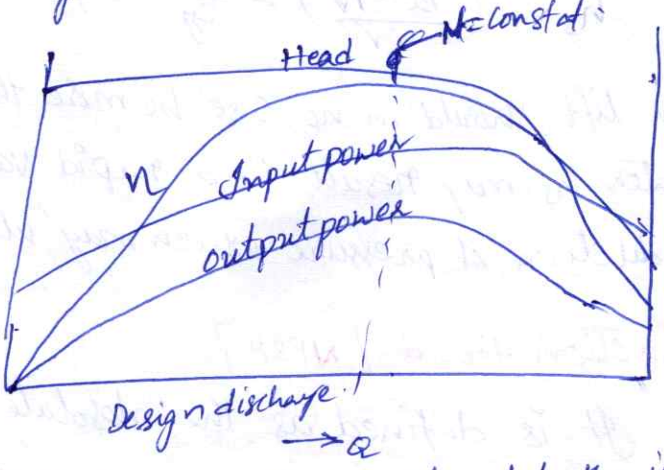






H_m vs Q, P vs Q, η_o vs Q Curves for different speeds are main characteristic curves.

During the operation a pump is normally required to run at a Constant speed, which is its designed speed. As such that particular set of main characteristics. which corresponds to the designed speed is mostly used in the operations of a pump and is therefore, known as operating characteristics.



ⓑ **Constant Efficiency curves:** In order to plot the iso-efficiency curves horizontal lines representing constant efficiencies are drawn on the η_o v/s Q curves.

ⓒ **Constant head and Constant discharge Curves:** These curves are useful in determining the performance of a variable speed pump for which the speed constantly varies.

In such cases if the head H_m is maintained constant then as the speed N varies the rate of flow Q will vary. As such a plot of Q v/s N can be prepared which can be used to determine the speeds required to discharge varying amounts of liquid at a constant pressure head.

Limitation of Suction lift :

The Absolute pressure head at the inlet to the pump may be expressed as

$$\frac{P_a}{\gamma} + \frac{P_s}{\gamma} = \frac{P_a}{\gamma} - \left[\frac{V_s^2}{2g} + h_s + h_{fs} \right]$$

where $P_s \rightarrow$ gauge pressure at inlet to the pump,

h_{fs} is head lost at foot valve, strainer and suction pipe.

It is not possible to create at the pump inlet, an absolute pressure lower than the vapour pressure of liquid. Thus if P_v is vapour pressure of liquid in absolute units, then in the limiting case $(P_a + P_s) = P_v$, hence from above eq the limiting value of suction lift h_s is obtained as

$$h_s = \left(\frac{P_a - P_v}{\gamma} \right) - \frac{V_s^2}{2g} - h_{fs}.$$

The suction lift should in no case be more than that given by above eq because a greater h_s may result in a rapid vaporization of liquid due to the reduction of pressure which may ultimately lead to Cavitation

Net positive Suction Head [NPSH]

It is defined as the absolute pressure head at the inlet to the pump, minus the vapour pressure head corresponding to the temperature of liquid pumped, plus velocity head at this point

$$NPSH = \left(\frac{P_a}{\gamma} + \frac{P_s}{\gamma} \right) - \frac{P_v}{\gamma} + \frac{V_s^2}{2g}$$

$$NPSH = \frac{P_a}{\gamma} - \frac{P_v}{\gamma} - h_s - h_{fs}$$

$$H_{sv} = \frac{P_a}{\gamma} - \frac{P_v}{\gamma} - h_s - h_f = NPSH.$$

In otherwords, NPSH also be defined as the head required to make the liquid to flow through suction pipe to impeller.

Cavitation in centrifugal pumps.

If the pressure at the suction side of the pump drops below the vapour pressure of the liquid then the cavitation may occur. The cavitation in a pump can be noted by a sudden drop in efficiency and head.

Thoma's Cavitation Factor

$$\sigma = \frac{\left(\frac{P_a - P_v}{\gamma}\right) - (h_s + h_{fs})}{H_m}$$

$$\sigma = \frac{NPSH}{H_m} = \frac{H_{sv}}{H_m}$$

$$\sigma_c = 0.103 \left(\frac{N_s}{1000}\right)^{4/3}$$

(Q) At what height from water surface a centrifugal pump may be installed in the following case to avoid cavitation. atmospheric pressure 101 kPa. vapour pressure 2.34 kPa, inlet and other losses in suction pipe 1.55m. effective head of pump 52.5m. and cavitation parameter $\sigma = 0.118$.

$$\sigma = \frac{\left(\frac{P_a - P_v}{\gamma}\right) - (h_s + h_{fs})}{H_m}$$

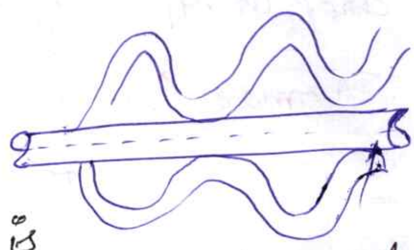
$$0.118 = \frac{\left(\frac{101\times10^3 - 2.34\times10^3}{9810}\right) - (h_s + 1.55)}{52.5}$$

$$h_s = 2.312\,m$$

Pumps in Series - Multistage pumps 2.

A multistage pump consists of two or more identical impellers mounted on the same shaft, and enclosed in the same casing. All the impellers are connected in series, so that liquid discharge with increased pressure from one impeller to passes through the connecting passages to the inlet of the next impeller and so on, till the discharge from the last impeller passes into the delivery pipe.

Total head $H = n(H_m)$.

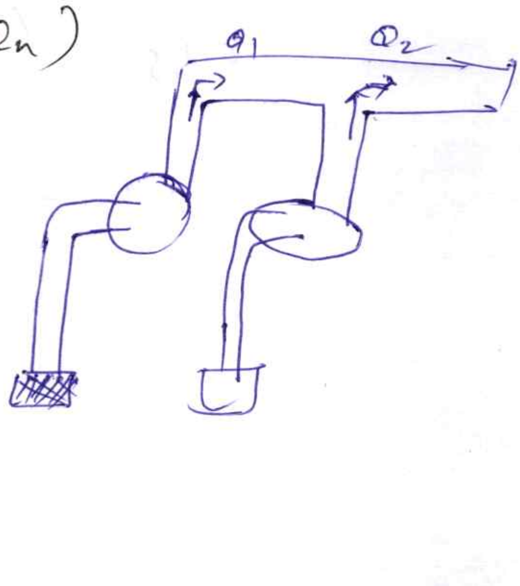


pumps in parallel:

when a large quantity of liquid is required to be pumped against a relatively small head, then it may not be possible for a single pump to deliver the required discharge. In such case two or more pumps are used which are so arranged that each of these pumps working separately lifts the liquid from a common sump and delivers it to a common collecting pipe through which it is carried to the required height. Since in this case each of the pumps delivers the liquid against the same head, the arrangement is known as pumps in parallel. If $Q_1, Q_2, Q_3 \cdots Q_n$ are discharge capacities of n pumps arranged in parallel then total discharge

$$Q = (Q_1 + Q_2 + Q_3 + \cdots Q_n)$$

$$Q = Q_1 + Q_2$$



(*) A three-stage centrifugal pump has impellers 375 mm dia and 18 mm wide at outlet. The outlet vane angle is 45° and the vanes occupy 8% of the outlet area. The manometric efficiency is 84% and the overall efficiency is 75%. What head the pump will generate when running at 900 rpm, discharge 60 ltr/s? What is the input power?

$$V_{f_1} = \frac{Q}{\pi D_1 B_1 \cdot k_1} = \frac{60 \times 10^{-3}}{(\pi (0.375)(0.018) \times 0.92)} = 3.08 \text{ m/s}$$

$$u_1 = \frac{\pi D_1 N}{60} = 17.67 \text{ m/s}$$

$$\tan 45^\circ = \frac{V_{f_1}}{u_1 - V_{w_1}} \Rightarrow V_{w_1} = 14.59 \text{ m/s}$$

$$\eta_{mano} = \frac{g H_m}{V_{w_1} u_1} \quad (\text{or}) \quad 0.84 = \frac{9.81 H_m}{14.59 \times 17.66}$$

$$H_m = 22.06 \text{ m}$$

Total head $H = 3H_m = 3(22.06) = 66.18 \text{ m}$.

$$\eta_o = \frac{\gamma Q H}{P} \Rightarrow P = 51.938 \text{ kW}$$

Types of hydropower plants :

According to the Storage being provided or not hydropower plants maybe classified as (i) ~~Run~~

① Run-of-river plants,

② Reservoir or storage plants

③ pumped Storage plants

④ Tidal plants

① Run-of-river plants : run of river plants are those which utilize the flow as it comes, without any Storage being provided. These are low head plants.

load factor : It is defined as the ratio of the average load during a certain period to the max or peak load during that period.

~~for~~ The load factor of a power plant would vary greatly with the character of the load. In highly industrialised areas the load factors will be high but in residential areas the load factors may be as low as 25 to 30%.

Utilization factor : [plant-use factor] It is defined as the ratio of the peak load developed during a certain period to installed capacity of the plant. for hydro electric plant, utilization factor varies from 0.4 to 0.9.

Capacity factor : [plant factor] It is defined as ratio of the energy that the plant actually produces during any period to the energy that it might have produced if operated at full capacity throughout this period.

Capacity factor will be identical with load factor when the maximum or peak load just equals the plant capacity.

C.P varies 0.25 to 0.7.

① 2 turbo-generators each of capacity 25000 kW have been installed at a hydel power station. During a certain period the load on the hydel plant varies from 15000 kW to 40,000 kW. Calculate (i) total installed capacity ② load factor ③ plant factor ④ utilization factor.

i) Total installed capacity $= 2 \times 25000 = 50,000$ kW.

② Load factor $= \dfrac{\left(\frac{15000+40,000}{2}\right)}{40,000} = 68.75\%$.

③ Plant factor $= \dfrac{\text{Energy actually produced}}{\text{Max. energy which can be produced}}$

$$PF = \frac{\left(\frac{15000+40,000}{2}\right)}{50,000} = 0.55 \quad 55\%$$

④ $U.F = \dfrac{40,000}{50000} = 80\%$.

CIVIL ENGINEERING

Fluid Mechanics, Hydraulics & Hydraulic Machinery

UNIT-5

UNIT - II

Open channel flow

Flow in open channel is defined as the flow of a liquid with a free surface. A free surface is a surface having constant pressure such as atmospheric pressure.

Classification of flow in channels.

(1) **Steady flow and Unsteady flow:** If the flow characteristics such as depth of flow, velocity of flow, rate of flow at any point in open channel flow do not change w.r.t time, the flow is said to be steady flow. Mathematically

$$\frac{\partial V}{\partial t} = 0, \quad \frac{\partial Q}{\partial t} = 0, \quad \frac{\partial y}{\partial t} = 0.$$

If at any point in open channel flow, the velocity, depth & rate of flow changes w.r.t time, the flow is said to be unsteady flow. Mathematically

$$\frac{\partial V}{\partial t} \neq 0, \quad \frac{\partial y}{\partial t} \neq 0, \quad \frac{\partial Q}{\partial t} \neq 0.$$

(2) **Uniform and Non-uniform flow:** If for a given length of the channel, the velocity of flow, depth of flow, slope of the channel and cross-section remain constant, the flow is said to be uniform. On the other hand, if for a given length of channel, the velocity of flow, depth of flow etc, do not remain constant, the flow is said to be non-uniform flow, mathematically.

$$\frac{\partial y}{\partial s} = 0, \quad \frac{\partial V}{\partial s} = 0 \rightarrow \text{uniform flow.}$$

$$\frac{\partial y}{\partial s} \neq 0, \quad \frac{\partial V}{\partial s} \neq 0 \rightarrow \text{Non-uniform flow.}$$

(3) **Laminar and Turbulent flow:** The flow in open channel is said to laminar if Re is less than 500 & 600.

$$Re = \frac{\rho V R}{\mu}, \quad \begin{array}{l} V \rightarrow \text{mean velocity of flow} \\ R \rightarrow \text{hydraulic mean depth.} \end{array}$$

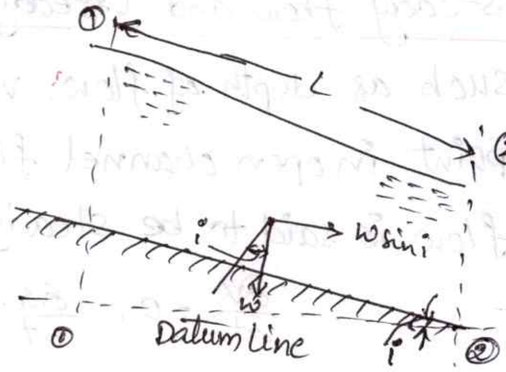
If Re is more than 2000, the flow is said to be turbulent in open channel flow. If Re lies 500 to 2000, the flow is considered to be in transition state.

④ Sub-critical, critical and super critical flow:

The flow in open channel is said to be sub-critical if the froude number (F_r) is less than 1.0. $F = \frac{v}{\sqrt{gD}}$.

If $F=1$ then flow is critical flow. If $F>1$ the flow is called super critical.

Discharge through open channel by chezy's formula.



Consider uniform flow of water in a channel as shown in fig. As the flow is uniform, it means velocity, depth of flow and area of flow will be constant for a given length of channel. Consider sections 1-1 and 2-2. i = Slope of bed

The weight of water between sections 1-1 and 2-2.

w = Specific weight of water × volume of water.

$= \gamma AL$.

Component of w along direction of flow $= w \times \sin i = \gamma AL \sin i$

Frictional resistance against motion of water $= f \times$ Surface area $\times (v)^2$

frictional resistance against motion $= f(PL)v^2)$

The forces acting on the water b/w 1-1 and 2-2.

1. Component of weight of water along direction of flow.
2. friction resistance against flow of water.
3. Pressure force at section 1-1 } same and in opposite direction
4. Pressure force at section 2-2.

$\gamma AL \sin i - f PLV^2 = 0$

$\gamma AL \sin i = f PLV^2$

$V^2 = \frac{\gamma AL \sin i}{f PL} = \frac{\gamma}{f} \times \frac{A}{P} \sin i$

$V = \sqrt{\frac{\gamma}{f}} \times \sqrt{\frac{A}{P} \sin i}$, $\frac{A}{P} = m$, $\sqrt{\frac{\gamma}{f}} = C$

$V = C\sqrt{m \sin i}$, C = chezy's constant.

for small values of i $\tan i = \sin i = i$

$$V = C\sqrt{mi}$$

Discharge $Q = A \times V = A \times C\sqrt{mi}$.

Bazin's formula : $C = \dfrac{157.6}{1.81 + \dfrac{K}{\sqrt{m}}}$ (chezy's constants)

where K = Bazin's constant & depends on roughness of surface

m = hydraulic mean depth.

page - 703 modi & seth.

manning's formule : $C = \frac{1}{N} \cdot m^{1/6}$

$N \to$ manning's constant.

$m \to$ Hydraulic mean depth.

Ganguillet - kutter Formula

$$C = \dfrac{23 + \dfrac{0.00155}{i} + \dfrac{1}{N}}{1 + \left(23 + \dfrac{0.00155}{i}\right)\dfrac{N}{\sqrt{m}}}$$

N = Roughness coefficient known as kutter's constant.

i = slope of the bed

m = hydraulic mean depth.

(P) find the slope of the bed of a rectangular channel of width 5m when depth of water is 2m and rate of flow is given as 20 m³/s. Take Chezy's constant, C = 50.

given $b = 5m$, $d = 2m$, $Q = 20 m^3/s$, $C = 50$.

$Q = AC\sqrt{mi}$

$A = b \times d = 2 \times 5 = 10 m^2$

$m = \frac{A}{P} = \frac{10}{b+2d} = \frac{10}{5+(2\times 2)} = \frac{10}{5+4} = \frac{10}{9} m.$

$20 = 10 \times 50 \sqrt{\frac{10}{9} i} \Rightarrow i = \frac{1}{694.44}$ Ꝗ.

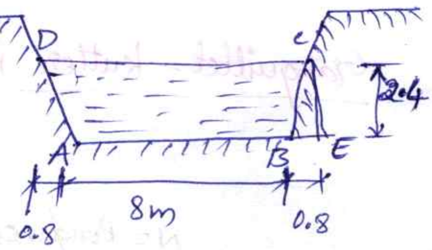
→ find the discharge through a trapezoidal channel of width 8m and side slope of 1 horizontal to 3 vertical. The depth of flow water is 2.4m and value of chezy's constant, C = 50. The slope of the bed of the channel is given 1 in 4000.

$b = 8m$,

Side slope = 1 hd to 3 V.

$d = 2.4m$, $C = 50$

$i = \frac{1}{4000}$;



$BE = 2.4 \times \frac{1}{3} = 0.8$

Top width

$CD = AB + 2BE = 8 + 2 \times 0.8 = 9.6 m.$

Area of trapezoidal ABCD $= \frac{(AB + CD)\, CE}{2} = 21.12 m^2$

wetted perimeter, $P = AB + BC + AD = AB + 2BC$

$BC = \sqrt{BE^2 + CE^2} = \sqrt{(0.8)^2 + (2.4)^2} = 2.529 m.$

$P = 8 + 2 \times 2.529 = 13.058 m.$

$m = \frac{A}{P} = \frac{21.12}{13.058} = 1.617 m.$

$Q = AC\sqrt{mi} = 21.12 \times 50 \sqrt{1.617 \times \frac{1}{4000}} = 21.23$ Ꝗ

(P) find the discharge through a rectangular channel of width 2m, having a bed slope of 4 in 8000. The depth of flow is 1.5m take N - 0.012.

$b = 2m$, $d = 1.5m$, $A = bd = 3m^2$

$P = b + d + b = 2 + 1.5 + 1.5 = 5m.$ $m = \frac{A}{P} = \frac{3}{5} = 0.6$ Ꝗ

$i = \frac{4}{8000} = \frac{1}{2000}$ $N = 0.012$

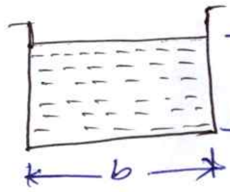
$C = \frac{1}{N} m^{1/6} = \frac{1}{0.012} \times (0.6)^{1/6} = 76.54$. $Q = AC\sqrt{mi}$

$Q = 3 \times 76.54 \sqrt{0.6 \times \frac{1}{2000}} = 3.977 m^3/s.$

Most Economical section of Channels :

A section of a channel is said to be most economical when the cost of the channel is minimum. But the cost of construction of a channel depends upon the excavation and the lining. To keep the cost down or minimum, the wetted perimeter, for a given discharge, should be minimum. This condition is utilized for determining the dimensions of a economical sections of channels.

⇒ most Economical rectangular channel :



The condition for most economical section, is that for a given area, the perimeter should be minimum. Consider a rectangular channel as shown.

width of channel = b, depth = d.

Area of flow, $A = b \times d$. — ①

wetted perimeter, $P = b + d + d = b + 2d$. — ②

from eq ① $b = \frac{A}{d}$,

② ⇒ $P = \frac{A}{d} + 2d$

for most economical section, P should be min.

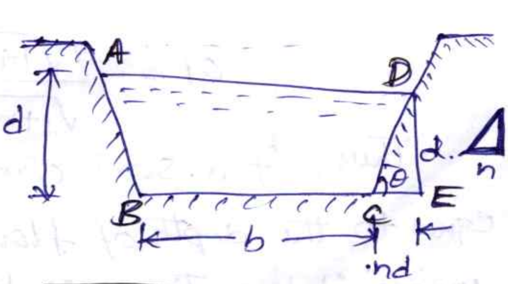
$\frac{dP}{d(d)} = 0 \Rightarrow \frac{d}{d(d)}\left[\frac{A}{d} + 2d\right] = 0$

$-\frac{A}{d^2} + 2 = 0 \Rightarrow A = 2d^2$

$A = bd = 2d^2 \Rightarrow \boxed{b = 2d}$

hydraulic mean depth, $m = \frac{A}{P} = \frac{b \times d}{b + 2d} = \frac{2d^2}{4d} = d/2$

→ Most Economical Trapezoidal channel.



The trapezoidal section of a channel will be most economical, when it is wetted perimeter is min.

side slope is given as 1V to n H.

$Area = \frac{b + (b + 2nd)}{2} \times d$ ⇒ $\boxed{\frac{A}{d} - nd = b}$

wetted perimeter, $P = b + 2\sqrt{n^2d^2 + d^2} = b + 2d\sqrt{1+n^2}$

$$P = \frac{A}{d} - nd + 2d\sqrt{1+n^2}$$

$$\frac{dP}{d(d)} = 0 , \quad \frac{d}{d(d)}\left[\frac{A}{d} - nd + 2d\sqrt{1+n^2}\right] = 0$$

$$-\frac{A}{d^2} - n + 2\sqrt{1+n^2} = 0$$

$$\frac{A}{d^2} + n = 2\sqrt{1+n^2}$$

$$\frac{b+2nd}{2} = d\sqrt{1+n^2}$$

$\frac{b+2nd}{2}$ = Half of topwidth.

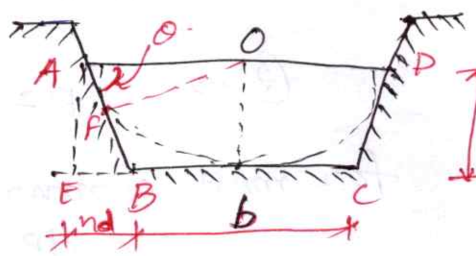
$d\sqrt{1+n^2}$ = one of the sloping side.

ii) Hydraulic mean depth $(m) = \frac{A}{P} = \frac{(b+nd)\,d}{2(b+nd)} = \frac{d}{2}$.

iii) If three sides of the trapezoidal section of most economical section are tangential to the semi-circle described on water line.

θ = angle made by sloping side with horizontal

O = centre of top width AD.



Draw OF ⊥ to sloping side AB.

ΔOAF is right angle triangle and angle OAF = θ

$$\sin\theta = \frac{OF}{OA}$$

In ΔAEB, $\sin\theta = \frac{AE}{AB} = \frac{d}{\sqrt{d^2+n^2d^2}} = \frac{1}{\sqrt{1+n^2}}$

$$\sin\theta = \frac{1}{\sqrt{1+n^2}}$$

$OF = OA \times \frac{1}{\sqrt{1+n^2}}$, AO = half of top width = $\frac{b+2nd}{2}$

$OF = \frac{d\sqrt{1+n^2}}{\sqrt{1+n^2}} = d$ depth of flow.

Thus, if a semi-circle is drawn with O as centre and radius equal to the depth of flow d, the 3 sides of most economical trapezoidal section will be tangential to the semicircle.

Hence the conditions for the most economical trapezoidal section are

1. $\frac{b+2nd}{2} = d\sqrt{1+n^2}$; ② $m = d/2$

3. A semi circle drawn from O with radius equal to depth of flow will touch the 3 sides of the channel.

Best side slope for most economical Trapezoidal Section:

$A = (b+nd)d$; $b = \frac{A}{d} - nd$ 2.

$P = b + 2d\sqrt{1+n^2} = \frac{A}{d} - nd + 2d\sqrt{1+n^2}$

for the most economical trapezoidal section, the depth of flow, d and area A are constant. Then n is the only variable. Best side slope will be when section is most economical & in other words, P is min. For P to be min, we must have $\frac{dP}{dn} = 0$ 2.

$$\frac{d}{dn}\left[\frac{A}{d} - nd + 2d\sqrt{1+n^2}\right] = 0$$

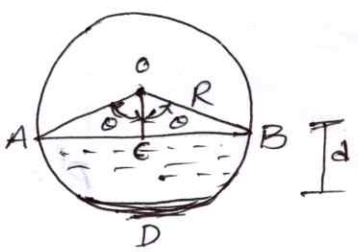
$$-d + \frac{2d}{\sqrt{1+n^2}} \times \frac{1}{2}\,2n = 0.$$

$$2n = \sqrt{1+n^2}$$

$n = \frac{1}{\sqrt{3}}$ (or) $\tan\theta = \frac{1}{n} = \sqrt{3} \Rightarrow \tan 60^\circ$ 2

$\boxed{\theta = 60^\circ}$

Flow through circular Channel: 2.



The flow of a liquid through a circular pipe, when the level of liquid in the pipe is below the top of the pipe is classified as an open channel flow. The rate of flow through circular channel is determined from the depth of flow and angle substended by the liquid surface at the centre of circular channel.

d - depth of water, 2θ = angle substended by water surface AB at centre in radians.

$R \rightarrow$ radius of channel.

$A = \pi r^2$

$P = \frac{2\pi R}{2\pi} \times 2\theta = 2R\theta$

A = Area ADBA = Area of sector OADBO − Area of △ ABO.

$$= \frac{\pi R^2}{2\pi} \times 2\theta - \frac{AB \times OC}{2} = R^2\theta - \frac{2BC \times CO}{2}$$

$$= R^2\theta - \frac{2R\sin\theta \times R\cos\theta}{2}$$

$$= R^2\left[\theta - \frac{\sin 2\theta}{2}\right]$$

hydraulic mean depth, $m = \frac{A}{P} = \frac{R^2\left(\theta - \frac{\sin 2\theta}{2}\right)}{2R\theta} = \frac{R}{2\theta}\left(\theta - \frac{\sin 2\theta}{2}\right)$

$Q = AC\sqrt{mi}$ 2.

(8) A rectangular channel 4m wide has depth of water 1.5m. The slope of the bed of the channel is 1 in 1000 and value of Chezy's Constant C=55. It is desired to increase the discharge to a max by changing the dimensions of the section for constant area of cross-section, slope of the bed and roughness of the channel. Find the new dimensions of the channel and increase in discharge.

$C=55$, $b=4m$, $d=1.5m$. $A=4\times1.5=6m^2$, $i=\frac{1}{1000}$ 2.

$P=d+2b=7m$. $m=\frac{A}{P}=\frac{4}{7}=0.857$ 2.

$Q=AC\sqrt{mi}=6\times55\sqrt{0.857\times\frac{1}{1000}}=9.66\,m^3/s$.

for max discharge for a given area, slope of bed and roughness

b' = new width of channel, d' = new depth of flow.

$A=b'd'=6m^2$, $b'd'=6$.

Also for max discharge $b'=2d'$,

$2d'd'=6 \Rightarrow d'^2=\frac{6}{2}=3.$ 2.

$d'=\sqrt{3}=1.732\,m$.

$b'=2\times1.732=3.464\,m$

$P'=d'+2b'=6.928$

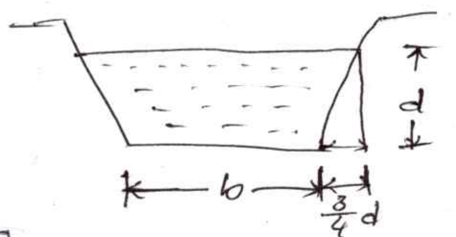
$m'=\frac{A}{P'}=\frac{6}{6.928}=0.866\,m$.

Max. discharge $Q'=AC\sqrt{m'i}=9.71\,m^3/s$.

Increase in discharge $=Q'-Q=9.71-9.66=0.05\,m^3/s$

(9) A trapezoidal channel has side slopes of 3 horizontal to 4 vertical and slope of its bed is 1 in 2000. Determine the optimum dimensions of the channel, if it is to carry water at 0.5 m³/s. Take C=80 22

$n=\frac{\text{Horizontal}}{\text{vertical}}=\frac{3}{4}$, $i=\frac{1}{2000}$



$Q=0.5\,m^3/s$, $C=80$.

$\frac{b+2nd}{2}=d\sqrt{1+n^2}$

$\frac{b+2(\frac{3}{4}d)}{2}=d\sqrt{1+(\frac{3}{4})^2} \Rightarrow$ $b=d$

$Q=AC\sqrt{mi}$, $m=d/2$ for most economical section.

$0.5=A(80)\sqrt{\frac{d}{2}\times\frac{1}{2000}} \Rightarrow A=1.75d^2 \Rightarrow d=0.55m$

$b=d=0.55m$.

(a) A trapezoidal channel with side slopes of 3 horizontal to 2 vertical has to be designed to convey 10 m^3/s at a velocity of 1.5 m/s, so that the amount of concrete lining for the bed and sides is minimum. find (i) wetted perimeter and (ii) slope of the bed if mannings N = 0.014.

$n = \frac{\text{Horizontal}}{\text{vertical}} = \frac{3}{2} = 1.5$, $Q = 10\ m^3/s$, $V = 1.5\ m/s$, $N = 0.014$

$\frac{b + 2nd}{2} = d\sqrt{1+n^2}$

for $n = 1.5$, $\frac{b + 2(1.5)d}{2} = d\sqrt{1+(1.5)^2}$ => $b = 0.6d$.

Area of trapezoidal section, $A = (b+nd)d = 2.1d^2$

$A = \frac{\text{Discharge}}{\text{velocity}} = \frac{Q}{V} = \frac{10}{1.5} = 6.67\ m^2$

$2.1d^2 = 6.67$ => $d = 1.78m$.

$b = 0.6d = 1.07m$.

wetted perimeter, $P = b + 2d\sqrt{1+n^2} = 1.07 + 2(1.78)\sqrt{1+1.5^2}$

$P = 7.48m$.

~~$C = \frac{1}{N} m^{1/6} =$,~~

for most economical trapezoidal section, hydraulic mean depth m,

$m = \frac{d}{2} = \frac{1.78}{2} = 0.89m$

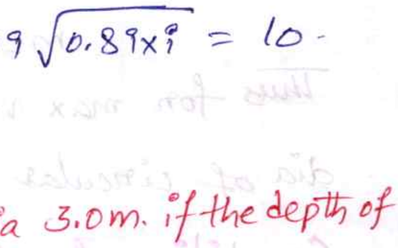
$C = \frac{1}{0.014} \times (0.89)^{1/6} = 66.09$

$Q = AC\sqrt{mi} = 6.67 \times 66.09\sqrt{0.89 \times i} = 10$.

$i = \frac{1}{1729.4}$ 2.

(a) Find discharge through a circular pipe of dia 3.0m. if the depth of water in the pipe is 1.0m. and the pipe is laid at a slope of 1 in 1000. Take value of C = 70.

$D = 3.m$, $R = \frac{3}{2} = 1.5m$, $i = \frac{1}{1000}$, $C = 70$



from fig -> $OC = OD - CD = 1.5 - 1 = 0.5m$

$AO = R = 1.5m$.

$\cos\theta = \frac{OC}{AO} = \frac{1}{3}$ => $\theta = 70.53^\circ$ => $70.53 \times \frac{\pi}{180} = 1.23$ radians.

$P = 2R\theta = 2 \times 1.5 \times 1.23 = 3.69m$.

$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) = 1.5^2\left[1.23 - \frac{\sin(70.53) \times 2}{2}\right] = 2.06m^2$

$m = \frac{A}{P} = 0.55$, $Q = AC\sqrt{mi} = 3.407\ m^3/s$.

Condition for max velocity for circular section:

$$V = C\sqrt{mi} = C\sqrt{\frac{A}{P} i}$$

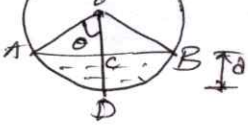
The velocity of flow through a circular channel will be maximum when hydraulic mean depth m is maximum for a given value of C & i.

$$\frac{d\left(\frac{A}{P}\right)}{d\theta} = 0 \qquad A = R^2\left[\theta - \frac{\sin 2\theta}{2}\right]$$

$$P = 2R\theta$$

$$\frac{d}{d\theta}\left\{\frac{R^2\left[\theta - \frac{\sin 2\theta}{2}\right]}{2R\theta}\right\} = \frac{d}{d\theta}\left\{\frac{R^2}{2}\left[1 - \frac{\sin 2\theta}{2}\right]\right\} = 0$$

$$\frac{\left(P\frac{dA}{d\theta} - A\frac{dP}{d\theta}\right)}{P^2} = 0 \Rightarrow P\frac{dA}{d\theta} - A\frac{dP}{d\theta} = 0.$$



$$\tan 2\theta = 2\theta.$$

$$2\theta = 257°30' \qquad \theta = 128°45'$$

$$d = OD - OC = R - R\cos\theta = R[1 - \cos\theta] = 0.81D$$

$$m = \frac{A}{P} = \frac{R^2\left(\theta - \frac{\sin 2\theta}{2}\right)}{2R\theta} = \frac{R}{2\theta}\left[\theta - \frac{\sin 2\theta}{2}\right]$$

for $\theta = 128°45' = 128.75 \times \frac{\pi}{180} = 2.247$ radians.

$$m = \frac{R}{2(2.247)}\left[2.247 - \frac{\sin 257°30'}{2}\right] = 0.611R$$

$$m = 0.3D$$

Thus for max velocity, the hydraulic mean depth is equal to 0.3 times dia of circular channel.

Condition for Max. Discharge for circular section:

$$Q = AC\sqrt{mi} = AC\sqrt{\frac{A}{P} i} = C\sqrt{\frac{A^3}{P} i}$$

Q will be max for constant values of C and i, when $\frac{A^3}{P}$ is max

$\frac{A^3}{P}$ will be max when $\frac{d}{d\theta}\left(\frac{A^3}{P}\right) = 0$

$$\frac{P\,3A^2\frac{dA}{d\theta} - A^3\frac{dP}{d\theta}}{P^2} = 0 \;; \qquad \frac{dP}{d\theta} = 2R, \quad \frac{dA}{d\theta} = R^2[1 - \cos 2\theta]$$

$$2\theta = 308°, \quad \theta = 154°$$

$$d = OD - OC = R - R\cos\theta \simeq 0.95D.$$

(P) A Concrete lined circular channel of dia 3m has a bed slope of 1 in 500. workout the velocity and flow rate for conditions of i) Max velocity, (ii) Max discharge, Take C = 50.

$D = 3m, \quad i = \frac{1}{500}, \quad C = 50,$

(i) for Max velocity, $\theta = 128^\circ 45' \simeq 128.75 \times \frac{\pi}{180} \simeq 2.247$ radians.

$P = 2R\theta = 2 \times 1.5 \times 2.247 = 6.741 m.$

$A = R^2 \left(\theta - \frac{\sin 2\theta}{2}\right) = 1.5^2 \left[2.247 - \frac{\sin(2 \times 128.75)}{2}\right) = 6.15 m^2$

$m = \frac{A}{P} = \frac{6.15}{6.7} = 0.912$

$v = C\sqrt{mi} = 50 \times \sqrt{0.912 \times \frac{1}{500}} = 2.135 \; m/s.$

$Q = AV = 6.15 \times 2.135 = 13.138$

ii) for max discharge. $\theta = 154^\circ = \frac{154 \times \pi}{180} = 2.68$ radians.

$A = R^2 \left[\theta - \frac{\sin 2\theta}{2}\right] = 1.5^2 \left[2.68 - \frac{\sin(2 \times 154)}{2}\right] = 6.9 m^2$

$P = 2R\theta = 2 \times 1.5 \times 2.68 = 8.06 m.$

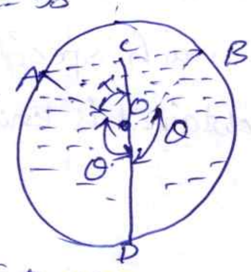
$m = \frac{A}{P} = \frac{6.9}{8.06} = 0.8599$

$V = C\sqrt{mi} = 50 \times \sqrt{0.8599 \times \frac{1}{500}} = 2.07 \; m/s.$

$Q = AV = 6.93 \times 2.0735 = 14.37 \; m^3/s.$

(Q) Calculate the quantity of water that will be discharged at a uniform depth of 0.9m in a 1.2m dia pipe which is laid at a slope 1 in 1000. Assume C = 58.

$D = 1.2m, \quad R = 0.6m, \quad i = \frac{1}{1000}, \quad C = 58$



$OC = CD - OD = 0.9 - R = 0.3m.$

$OA = R = 0.6, \quad \cos\alpha = \frac{0.3}{0.6} = \frac{1}{2}$

$\alpha = 60^\circ, \quad \theta = 180 - \alpha = 120^\circ = 120\frac{\pi}{180} = 0.667\pi$

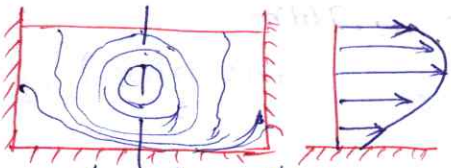
$P = 2R\theta = 2.526 m,$

$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) = 0.913 m^2$

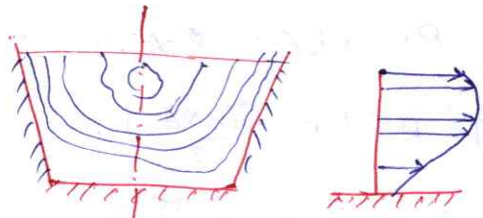
$Q = AV =$

Velocity distribution in a channel section :

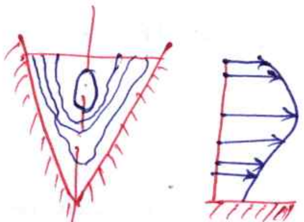
The Velocity of flow at any channel section is not uniformly distributed. The non-uniform distribution of velocity in an open channel is due to the presence of a free surface and the frictional resistance along the channel boundary. The general patterns for velocity distribution as represented by lines of equal velocity.



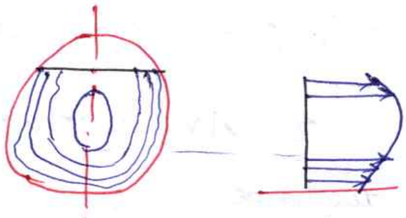
rectangular section.



trapezoidal Channel



Triangular chamel



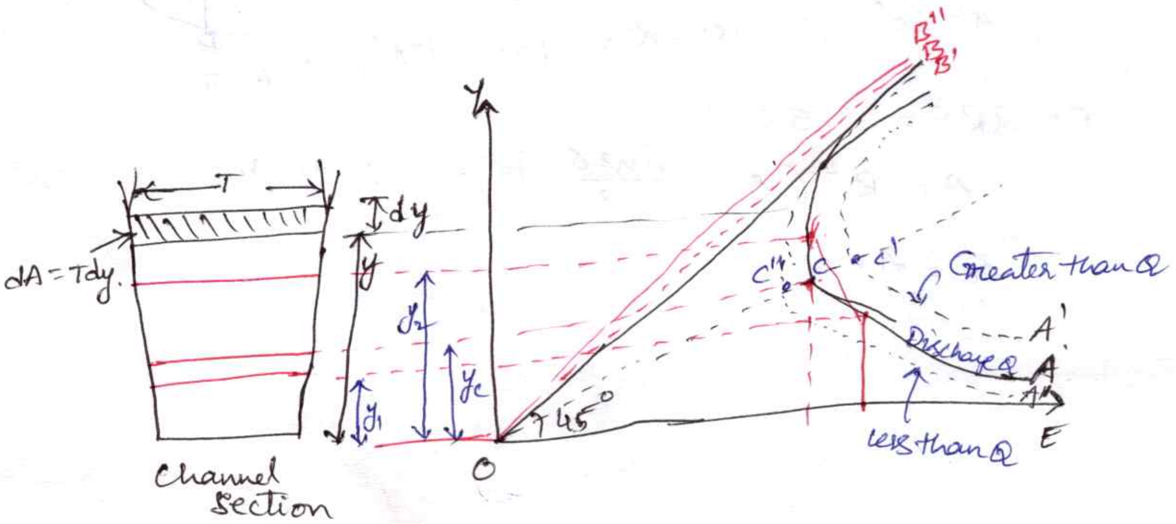
Circular channel.

Specific Energy and Critical Depth

The specific energy of flow at any channel section is defined as the energy per unit weight of water measured with respect to the channel bottom as the datum. Thus the specific energy E at any section is the sum of the depth of flow at that section and velocity head.

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2}$$

Specific energy is a function of the depth of flow only. Thus for a given channel section and discharge above equation may be represented graphically in which specific energy is plotted against the depth of flow. The curve so obtained is known as specific energy curve.



It can be seen from the specific energy curve that, there is one point c on the curve which has a minimum specific energy, there by indicating that below this value of the specific energy the given discharge cannot occur. The depth of flow at which the specific energy is minimum is called critical depth y_c. Similarly the velocity of flow at the critical depth is known as critical velocity V_c. for any other value of specific energy there are two possible depths, one greater than the critical depth and the other smaller than critical depth, at which a given discharge can occur with the same specific energy. These two depths for given specific energy are called the alternate depths:- y_1 smaller than y_c and y_2 greater than y_c. For any depth of flow greater than the critical depth, the specific energy increases with increase in depth. Moreover when the depth of is greater than the critical depth, the velocity of flow is less than the critical velocity for the given discharge. Hence the flow at the depths greater than critical depth is known as "subcritical flow" or tranquil flow. On the other hand when the depth of flow is less than the critical depth, the specific energy increases as the depth of flow decreases. When the depth of flow is less than critical depth, the velocity of flow is greater than the critical velocity. The flow is ~~calle~~ known as supercritical flow or rapid flow.

For a given discharge the condition for min specific energy can be obtained by differentiating eq $E = y + \frac{Q^2}{2gA^2}$ w.r.t y.

$$\frac{dE}{dy} = 0 \Rightarrow \frac{d}{dy}\left[y + \frac{Q^2}{2gA^2}\right] = 0$$

$$1 - \frac{2Q^2}{2gA^3}\left(\frac{dA}{dy}\right) = 0$$

Since Q is a constant and A is function of y. As shown in fig, in a channel section if T is the top width of flow then differential water area dA near the free surface is equal to Tdy. i.e $dA = Tdy$. (or) $\frac{dA}{dy} = T$

$$1 - \frac{Q^2T}{gA^3} = 0 \Rightarrow \boxed{\frac{Q^2}{g} = \frac{A^3}{T}}$$

$V = \frac{Q}{A}$, $D = \frac{A}{T}$; $\frac{V^2}{g} = D \Rightarrow \frac{V}{\sqrt{gD}} = 1$

Critical flow and its computation:

When the depth of flow of water over a certain reach of a given channel is equal to the critical depth y_c, the flow is described as critical flow or in critical state.

The critical depth for a given discharge Q is the depth y_c corresponding to which the crossectional area A and top width T of the channel section are such that the value of A^3/T is given by

$$\left(\frac{A^3}{T}\right)_c = \frac{Q^2}{g} \Rightarrow A\sqrt{\frac{A}{T}} = Z_c = \frac{Q}{\sqrt{g}}$$

Various characteristics of the critical state of flow through a channel.

- → The specific energy is a minimum for a given discharge.
- → The discharge is maximum for a given specific energy.
- → The specific force is minimum for a given discharge.
- → The discharge is maximum for a given specific force.
- → The velocity head is equal to half the hydraulic depth in a channel of small slope.
- → Froude number is equal to unity.

$Z \rightarrow$ section factor Z.

$$Q_c = \sqrt{\frac{A^3}{T}} \times \sqrt{g} = Z\sqrt{g}.$$

(a) Critical flow in Rectangular Channels:

It is assumed that q represents the discharge per unit width of the channel section then the total discharge Q passing through a channel of rectangular section of bottom width B may be expressed as $Q = B \times q$. or $q = \frac{Q}{B}$ corresponding to a critical depth of flow y_c the area of rectangular channel section $A = B \times y_c$,

$$\frac{qB}{\sqrt{g}} = By_c\sqrt{\frac{By_c}{B}}$$

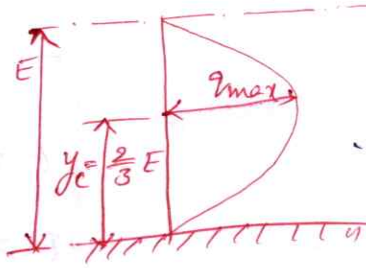
$$\frac{q^2}{g} = y_c^3 \Rightarrow y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

specific energy $E = y + \frac{(qB)^2}{2g(By)^2} = y + \frac{q^2}{2gy^2}$

$$q = \sqrt{2g(E-y)y^2}$$

for a given specific energy E, the condition for dischage per unit width q to be max is obtained by putting $\frac{dq}{dy} = 0$.

$$\frac{dq}{dy} = \frac{\sqrt{2g}\,(2yE - 3y^2)}{2\sqrt{(E-y)y^2}} = 0$$



$$\boxed{E = \frac{3}{2}y}$$

$$\boxed{q = \sqrt{gy^3}}$$

$$\boxed{y_c = \frac{2}{3}E}$$

(4) Trapezoidal channel section: for a channel of trapezoidal section no explicit expressions for y can be obtained, but the following expressions in terms of dimensionless parameters may be developed which can be used for the computation of the critical depth y_c. Thus for a Channel section of bottom width B and side slope n horizontal to 1 vertical.

$$\frac{Q^2 n^3}{g B^5} = \frac{\left(\frac{B}{n y_c} + 1\right)^3}{\left(\frac{B}{n y_c}\right)^5 \left(\frac{B}{n y_c} + 2\right)}$$

$$\frac{En}{B} = \frac{\left(3\frac{B}{n y_c} + 5\right)}{2\frac{B}{n y_c}\left(\frac{B}{n y_c} + 2\right)}$$

(5) for triangular section:

$$y_c = \left(\frac{2Q^2}{g n^2}\right)^{1/5}$$

$$\boxed{y_c = \frac{4}{5}E}$$

* for parabolic channel section: $y_c = \left(\frac{27 Q^2}{8 g k^2}\right)^{1/4}$

$$\boxed{y_c = \frac{3}{4}E}$$

2.

(*) A trapezoidal channel has a bottom width of 6m and side slopes of 2 horizontal to 1V. If the depth of flow is 1.2 m at a discharge of 10 m³/s. Compute specific energy and critical depth.

$$A = (B+ny)y = (6+2(1.2))1.2 = 10.08 m^2$$

$$E = y + \frac{1}{2g}\left(\frac{Q}{A}\right)^2 = 1.2 + \frac{1}{2\times9.81}\left(\frac{10}{10.08}\right)^2$$

$E = 1.25 m$.

for critical flow $\frac{Q^2}{g} = \frac{A^3}{T}$

$$\frac{(10)^2}{9.81} = \frac{((6+2y_c)y_c)^3}{6+4y_c}$$

$y_c = 0.611 m$.

(*) Calculate the possible depths of flow at which a discharge of 26.67 cumec may be carried in a rectangular channel 3.5m wide with a specific energy equal to 2.74m.

$$E = y + \frac{Q^2}{2gA^2} = y + \frac{Q^2}{2g(By)^2}$$

$y^3 - 2.74y^2 + 2.96 = 0 \Rightarrow y = 2m, 1.64, -0.9m$.

(*) water flows at a velocity of 1m/s and a depth of 2m in an open channel of rectangular cross section 3m wide. At a certain section the width is reduced to 1.8m and the bed is raised by 0.65m. will the upstream depth be affected? If so, to what extent.

$A = 3\times2 = 6m^2$, $V = 1 m/s$, $Q = 6\times1 = 6 m^3/s$.

$$E_1 = y_1 + \frac{V_1^2}{2g} = 2.0 + \frac{(1)^2}{2\times9.81} = 2.051 m.$$

→ The upstream depth will not be affected if
$E_1 \geq E_c + 0.65$

$E_c = \frac{3}{2}y_c \rightarrow$ $y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{\left(\frac{6}{1.8}\right)^2}{9.81}\right)^{1/3} = 1.042m$

$E_c = \frac{3}{2}(1.042) = 1.563 m$.

$E_c + 0.65 = 1.563 + 0.65 = 2.213 m$.

Since $E_1 < E_c + 0.65$, the u/s depth will be affected and there will be heading up of water at the upstream section to such an extent that the flow at the contracted section will be in critical state.

let y be new depth of flow at upstream.

$$v = \frac{6}{3 \times y} = \frac{2}{y}$$

$$E_1 = y + \frac{(2/y)^2}{2 \times 9.81} = y + \frac{0.204}{y^2}$$

$$E_2 = E_c + 0.65 = 2.213m$$

for no loss of energy b/w two sections

$$E_1 = E_2$$

$$y + \frac{0.204}{y^2} = 2.213 \Rightarrow y = 2.17m$$

(a) velocity measuring instruments:

(a) Pitote Tube

b) Current meter

c) floats.

Non-uniform flow in channels

Dynamic Equation of gradually varied flow: The dynamic equation for gradually varied flow can be derived from the basic energy eq with the following assumptions.

ⓐ The uniform flow formulae may be used to evaluate the energy slope of a gradually varied flow and the corresponding coefficients of roughness developed primarly for uniform flow are applicable to gradually varied flow also.

Thus, (S_f), GVF $= \left(\frac{Vn}{R^{2/3}}\right)^2$ – Mannings.

(S_f), GVF $= \left(\frac{V}{C\sqrt{R}}\right)^2$ – Chezy's n.

ⓑ The bottom slope of the channel is very small.

ⓒ The channel is prismatic.

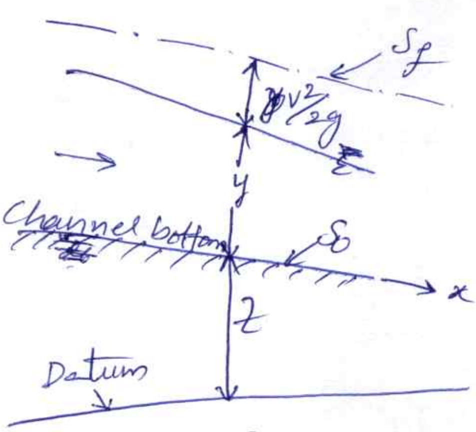
ⓓ The energy correction factor α is unity.

ⓔ The pressure distribution in any vertical is hydrostatic.

(f) The roughness coefficient is independent of the depth of flow and it is constant throughout the channel reach considered.

Considering a short reach of channel having gradually varied flow as shown in fig. The energy equation at any section may be written as

$$H = \frac{V^2}{2g} + y + z$$



GVF

$$H = \frac{Q^2}{2gA^2} + y + z$$

$$\frac{dH}{dx} = \frac{d}{dx}\left(\frac{Q^2}{2gA^2}\right) + \frac{dy}{dx} + \frac{dz}{dx}$$

$$\frac{dH}{dx} = \frac{-Q^2}{gA^3}\frac{dA}{dx} + \frac{dy}{dx} + \frac{dz}{dx}$$

$\frac{dH}{dx}$ is the slope of the energy gradient and hence $\frac{dH}{dx} = -S_f$.

$\frac{dz}{dx}$ is the slope of channel bed & $\frac{dz}{dx} = -S_0$ [−ve for S_f and S_0 indicates that as x increases H and z decreases,

$\frac{dy}{dx}$ is slope of water surface wrt the channel bottom.

$$\frac{dA}{dx} = \frac{dA}{dy}\frac{dy}{dx} = T\frac{dy}{dx},$$

$$-S_f = -\frac{Q^2 T}{gA^3}\frac{dy}{dx} + \frac{dy}{dx} - S_0$$

Solving, $\frac{dy}{dx}$, the following D.E for water surface slope can be obtained,

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2 T}{gA^3}} = \frac{S_0 - S_f}{1 - F_r^2}$$

It is the basic differential equation for the gradually varied flow. It may be observed that when $\frac{dy}{dx} = 0$; then $S_0 = S_f$ and water surface is parallel to channel bottom thus representing a uniform flow.

→ when $\frac{dy}{dx}$ is +ve water surface is rising.

→ when $\frac{dy}{dx}$ is −ve water surface is falling.

Dynamic equation for GVF in wide Rectangular Channel:

for a wide rectangular channel of width B the hydraulic radius $R = \frac{By}{B+2y} = \frac{By}{B} \simeq y$.

according to Mannings formula $Q = \frac{1}{N}(By)\, y^{2/3} S_f^{1/2}$

$$Q = \frac{1}{n}(By_n)(y_n)^{2/3} S_0^{1/2}$$

It may be noted that the hydraulic radius has been replaced by depth of flow and n is assumed to be same for uniform and non-uniform flow. $\frac{S_f}{S_0} = \left(\frac{y_n}{y}\right)^{10/3}$

Chezy's formula is used insted of Mannings. the value of

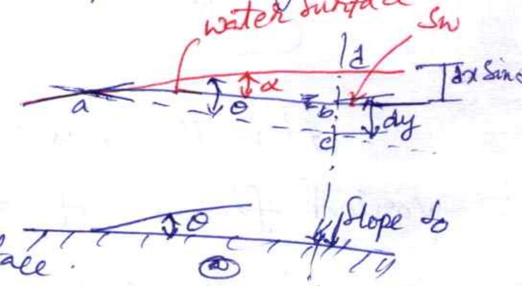
$$\frac{S_f}{S_0} = \left(\frac{y_n}{y}\right)^3$$

$$\frac{Q^2 T}{gA^3} = \frac{Q^2 B}{g(BT)^3} = \frac{q^2}{gy^3} = \left(\frac{y_c}{y}\right)^3$$

$$\frac{dy}{dx} = S_0 \frac{1 - \left(\frac{y_n}{y}\right)^{10/3}}{1 - \left(\frac{y_c}{y}\right)^3} = S_0 \frac{1 - \left(\frac{y_n}{y}\right)^3}{1 - \left(\frac{y_c}{y}\right)^3}.$$

Relation b/w water Surface Slopes and Channel Bottom Slope:

$\frac{dy}{dx}$ represents the slope of water surface with respect to the channel bottom. But often water surface slope S_w w.r.to horizontal may be required to be determined. As such a relation between water surface slope S_w, the channel bottom slope S_o and slope $\frac{dy}{dx}$ may be developed which facilitates determination of S_w when S_o and $\frac{dy}{dx}$ are known. for rising water surface as shown in fig. ⓐ

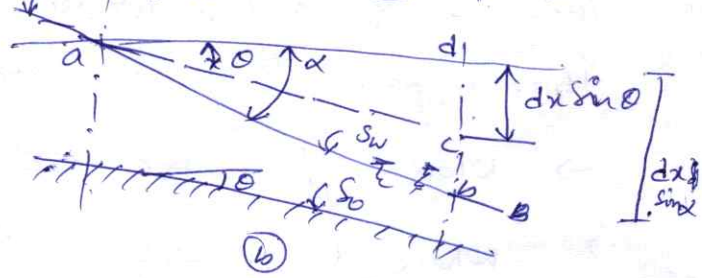


for Δabd ⇒ $S_w = \sin\alpha$

$$S_w = \frac{bd}{ba} = \frac{cd - cb}{ba}$$

$$\frac{cd}{ba} = \sin\theta = S_o, \quad \frac{cb}{ba} = \frac{dy}{dx}$$

$$S_w = S_o - \frac{dy}{dx}$$



However in this case if the water surface is such that point b lies above point d, then $\frac{dy}{dx} > S_o$ and

$$S_w = \frac{dy}{dx} - S_o.$$

for falling water surface as shown in fig ⓑ

abd, $S_w = \sin\alpha = \frac{bd}{ba} = \frac{cd + ba}{ba}$, $\frac{cd}{ba} = \sin\theta = S_o$,

$$\frac{cb}{ba} = \frac{dy}{dx} \Rightarrow \boxed{S_w = S_o + \frac{dy}{dx}}$$

Classification of channel Bottom slopes:

① **Critical slope**: The channel bottom slope is designated as critical when the bottom slope S_o is equal to the critical slope S_c i.e $S_o = S_c$. Thus in this case the normal depth of flow will be equal to the critical depth. i.e $y_n = y_c$

② **Mild slope**: The channel bottom slope is designated as mild when the bottom slope S_o is less than critical slope S_c. i.e $S_o < S_c$. The application of mannings or Chezy's formula will then indicate that when bottom slope is mild, $y_n > y_c$.

iii) Steep Slope : The channel bottom slope is designated as steep when the bottom slope S_0 is greater than critical slope i.e $S_0 > S_c$. Again the application of manning's or chezy's formula will indicate that when the bottom slope is steep, the normal depth of flow is less than critical depth $y_n < y_c$.

iv) Horizontal Slope : when the channel bottom slope is equal to zero, $S_0 = 0$ the bottom slope is designated as horizontal. $y_n = \infty$.

v) Adverse Slope : when the channel bottom slope instead of falling rises in the direction of flow it is designated as an adverse slope. Thus in a channel with adverse bottom slope S_0, is less than zero ($S_0 < 0$) or it is -ve. Obviously for an adverse-slopped channel ~~slopped~~ the normal depth of flow y_n is imaginary or it is non-existent.

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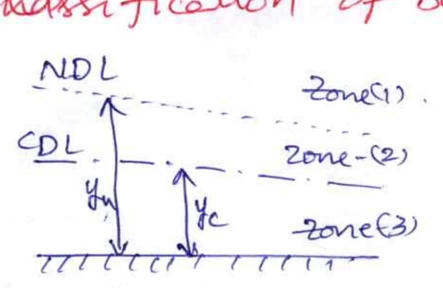
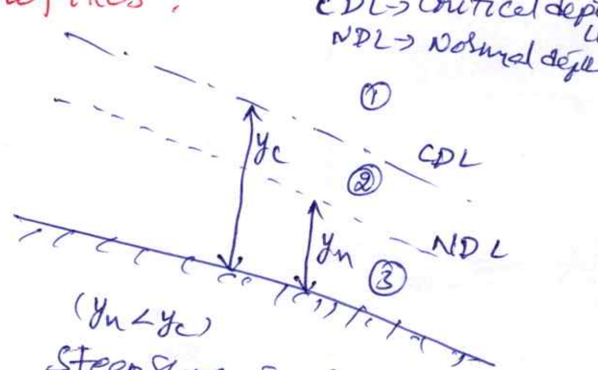
Modi & Seth

Classification of Surface profiles :

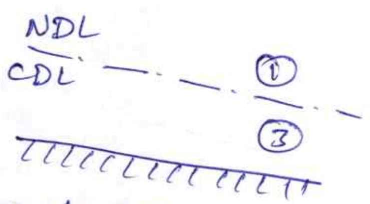
CDL -> Critical depth line
NDL -> Normal depth line



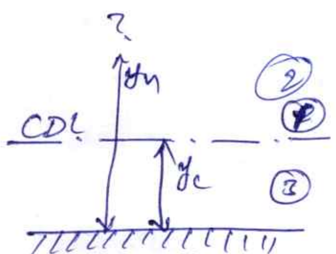
Mild slope, ($S_0 < S_c$)
($y_n > y_c$)



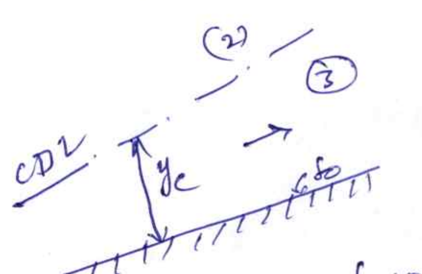
($y_n < y_c$)
Steep Slope $S_0 > S_c$



Critical Slope $S_0 = S_c$
$y_n = y_c$



Horizontal
$S_0 = 0$



Adverse Slope $S_0 < 0$

The various water surface profiles occuring in the channels are designated wr to the bottom slopes of the channel. Thus surface profiles which occur in mild-sloped channels are known as M-curves, those occure in steep-slopped channel are known as S-curves, those occure in critical slope channels are known as C-curves.

Those which occur in adverse-slopped channels are known as A-curves, those which occur in horizontal channel are known as H-curves.

Integration of the varied flow equation.

In practice it is often required to determine the distance upto which the surface profile of gradually varied flow extends. In order to solve the problems of this type it is necessary to integrate the dynamic eq. of gradually varied flow.

(a) The step method

(b) The graphical integration method (c) The direct integration method.

(c) Bresse's method: It is applicable in case of very wide rectangle channels.

$$\frac{dy}{dx} = S_0 \frac{1-\left(\frac{y_n}{y}\right)^3}{1-\left(\frac{y_c}{y}\right)^3}$$

Now if $\frac{y}{y_n} = u$; then $dy = y_n du$ and

$$dx = \frac{y_n}{S_0}\left[1-\left(1-\left(\frac{y_c}{y_n}\right)^3\right)\frac{1}{1-u^3}\right] du$$

$$x = \frac{y_n}{S_0}\left[u-\left(1-\left(\frac{y_c}{y}\right)^3\right)\int\frac{du}{1-u^3} + \text{Const}\right]$$

$$\int\frac{du}{1-u^3} = \left[\frac{1}{6}\log_e\frac{u^2+u+1}{(u-1)^2} - \frac{1}{\sqrt{3}}\cot^{-1}\left(\frac{2u+1}{\sqrt{3}}\right)\right]$$

$$\left(\frac{y_c}{y_n}\right)^3 = \frac{C^2 S_0}{g}, \quad C \rightarrow \text{Chezy's constant}.$$

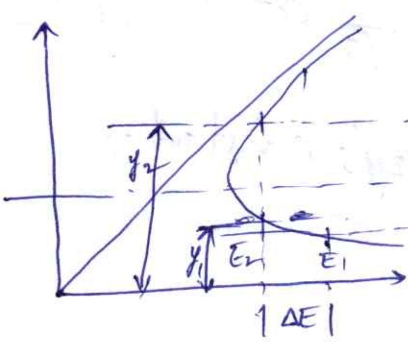
Hydraulic Jump : It is defined as sudden and turbulent passage of water from super critical state to subcritical state. It has been classified as rapidly varied flow, since the change in depth of flow from rapid to tranquil state is in an abrupt manner over a relatively short distance. The flow in a hydraulic jump is accompanied by the formation of extremely turbulent rollers and there is a considerable dissipation of energy.

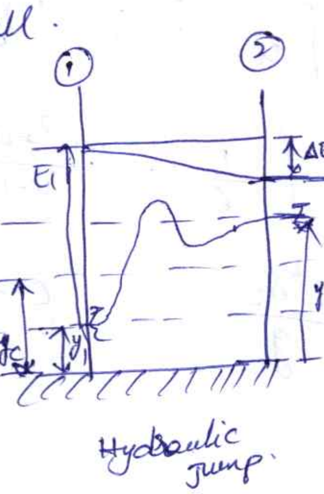
Assumptions

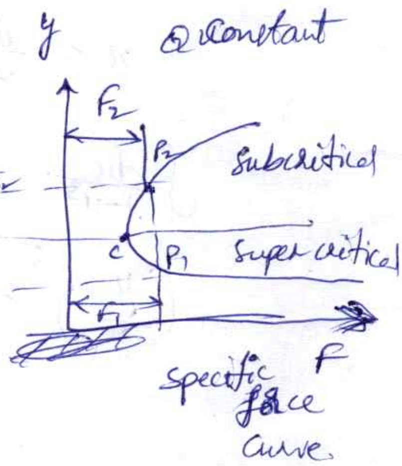
-> before and after jump flow is uniform and pressure distribution is hydrostatic

-> The length of the jump is small so that the losses due to friction on channel floor are small and neglected

-> The channel floor is horizontal or slope is so gentle that the weight component of water mass comprimising the jump is negligibly small.

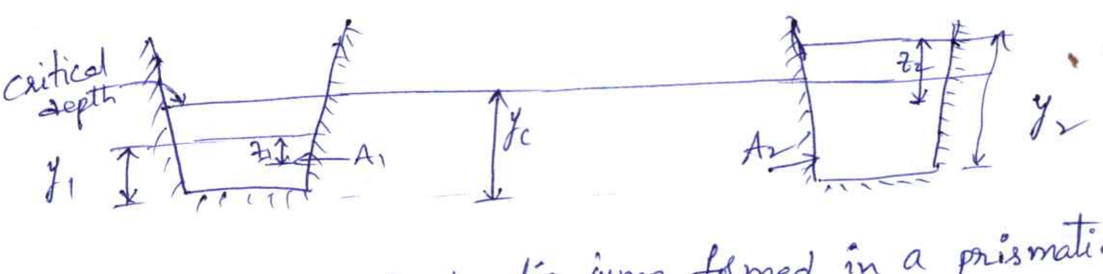






$$E = y + \frac{Q^2}{2gA^2}$$

$$F = \frac{Q^2}{gA} + A\bar{z}$$



Consider hydraulic jump formed in a prismatic channel with horizontal floor carrying a discharge Q as shown in fig. Let the depth of flow before the jump at section 1 be y_1 and depth of flow after the jump at section 2 be y_2. The depth y_1 is intial depth and y_2 is known as sequent depth.

The only external forces acting on the mass of water between the sections 1 and 2 are the hydrostatic pressures P_1 and P_2 at sections 1 & 2; according to momentum equation.

$$(P_2 - P_1) = \rho Q (V_1 - V_2)$$

$$\gamma A_2 \bar{z}_2 - \gamma A_1 \bar{z}_1 = \frac{\gamma}{g} Q \left[\frac{Q}{A_1} - \frac{Q}{A_2} \right]$$

$$\left(\frac{Q^2}{gA_1} + A_1 \bar{z}_1 \right) = \left(\frac{Q^2}{gA_2} + A_2 \bar{z}_2 \right)$$

$\left(\frac{Q^2}{gA} + A\bar{z} \right)$ is called the specific force, designated by F. Thus if F_1 and F_2 represent specific force at sections 1 and 2 respectively

$$F_1 = F_2$$

y_1 and y_2 are known as conjugate depths which indicates same specific force.

Hydraulic jump in Rectangular channels.

for rectangular channel the above equation can be further simplified and, $A_1 = By_1$, $A_2 = By_2$, $\bar{z}_1 = \frac{y_1}{2}$, $\bar{z}_2 = \frac{y_2}{2}$, $q = \frac{Q}{B}$

$$\frac{Q^2}{gBy_1} + (By_1)\left(\frac{y_1}{2}\right) = \frac{Q^2}{gBy_2} + (By_2)\left(\frac{y_2}{2}\right)$$

$$\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$$

$$y_2^2 + y_2 y_1 - \frac{2q^2}{gy_1} = 0$$

$$y_2 = \frac{-y_1}{2} + \sqrt{\left(\frac{y_1}{2}\right)^2 + \frac{2q^2}{gy_1}}$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8q^2}{gy_1^3}}\right]$$

$$y_2 = \frac{y_1}{2}\left[-1 + \sqrt{1 + 8F_1^2}\right]$$

when the conjugate depths are known the energy loss ΔE in a hydraulic jump may computed as

$$\Delta E = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

$$\Delta E = \frac{q^2}{2g}\left[\frac{1}{y_1^2} - \frac{1}{y_2^2}\right] - [y_2 - y_1]$$

$\frac{q}{2g}(y_1 y_2)(y_1 + y_2)\frac{(y_2^2 - y_1^2)}{y_1^2 y_1^2}$

$$\Delta E = \frac{1}{4} \frac{y_1 y_2 (y_1 + y_2)(y_2^2 - y_1^2)}{(y_1 y_2)^2} - (y_2 - y_1)$$

$$\Delta E = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$$

The height of the jump is difference between the depths after and before the jump $= h_j = (y_2 - y_1)$

The length of the jump may be defined as the distance measured from the front face of the jump to a point on the surface immediately downstream from the roller. However, the length of the jump cannot be determined analytically. Generally for rectangular channel the length of the jump varies between 5 and 7 times height of jump.

$$L_j = (5 \text{ to } 7)\, h_j$$

① Types of hydraulic jump:

1. for $F_{r_1} = 1$ to 1.7, the water shows undulations and jump is called an undular jump.
2. For $F_{r_1} = 1.7$ to 2.5, the jump formed is called weak jump.
3. for $F_{r_1} = 2.5$ to 4.5, jump formed is known as oscillating jump.
4. $F_{r_1} = 4.5$ to 9, jump formed is well stabilized and is called steady jump. for this jump energy dissipation ranges from 45 to 70%
5. $F_{r_1} = 9.0$ and larger the jump formed is called a strong jump.

② Applications of Hydraulic jump:

1. It is a useful means of dissipating excess energy of water
2. It raises the water level in the channels for irrigation.
3. It increases the weight on an apron of hydraulic structure.
4. It increases the discharge through a sluice by holding back the tailwater
5. It may be used for mixing chemicals in water.

Q. A trapezoidal channel having bottom width 8 m side slope 1:1, carries a discharge of 80 m³/s. Find the depth conjugate to intial depth of 0.75 m before the jump. Also determine loss of energy.

$$A_1 = (B + ny_1)y_1 = [8 + 0.75 \times 1]0.75 = 6.56\, m^2$$

$$\bar{Z}_1 = \frac{y_1}{6}\frac{(3B + 2ny_1)}{(B + ny_1)} = 0.364\, m.$$

$$A_2 = (8 + y_2)y_2, \quad \bar{Z}_2 = \frac{y_2}{6}\frac{(24 + 2y_2)}{(8 + y_2)}$$

$$\frac{(80)^2}{9.81 \times 6.56} + (6.56 \times 0.364) = \frac{(80)^2}{9.81(8 + y_2)y_2} + \frac{y_2^2}{6}(24 + 2y_2)$$

$$y_2 = 4.066\, m.$$

$$E_1 = y_1 + \frac{Q^2}{2gA_1^2} = 0.75 + \frac{(80)^2}{2 \times 9.81(6.56)^2} = 8.33\, m.$$

$$E_2 = 4.2\, m, \quad \Delta E = E_1 - E_2 = 4.13\, m.$$

Q. hydraulic jump occurs in a 90° triangular channel. Derive equation relating the two depths and the flow rate. If the depths before and after the jump in the above channel are 0.5 m and 1.0 m. determine the flowrate and obtain the froude numbers before and after the jump.

$$\frac{Q^2}{gA_1} + A_1\bar{Z}_1 = \frac{Q^2}{gA_2} + A_2\bar{Z}_2$$

If y_1 and y_2 are the depths before and after the jump.

$$A_1 = y_1^2, \quad \bar{Z}_1 = \frac{y_1}{3}, \quad A_2 = y_2^2, \quad \bar{Z}_2 = \frac{y_2}{3}$$

$$\frac{Q^2}{g}\left[\frac{1}{y_1^2} - \frac{1}{y_2^2}\right] = \frac{1}{3}\left[y_2^3 - y_1^3\right]$$

$$\frac{Q^2}{gy_1^5}\left[\frac{\left(\frac{y_2}{y_1}\right)^2 - 1}{(y_2/y_1)^2}\right] = \frac{1}{3}\left[\left(\frac{y_2}{y_1}\right)^3 - 1\right]$$

$$\frac{Q^2}{gy_1^5} = \frac{v_1^2}{gy_1} = F_{r_1}^2 \quad \& \quad \frac{y_2}{y_1} = r$$

$$3F_{r_1}^2 = \frac{r^2(r^3 - 1)}{(r^2 - 1)},$$

$$y_1 = 0.5, \quad y_2 = 1, \quad r = \frac{1}{0.5} = 2$$

$$Q = 0.977\, m^3/s, \quad F_{r_1} = 1.764$$

(*) In a flow through a rectangular open channel for a certain discharge the Froude numbers corresponding to the two alternate depths are F_{r_1} and F_{r_2}. Show that $\left(\frac{F_{r_2}}{F_{r_1}}\right)^{2/3} = \frac{2+F_{r_2}^2}{2+F_{r_1}^2}$

let y_1 and y_2 be the alternate depths

$$Q = Vby = V_1 b y_1 = V_2 b y_2$$

$$E = y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$y_1\left(1+\frac{F_{r_1}^2}{2}\right) = y_2\left(1+\frac{F_{r_2}^2}{2}\right)$$

$$\frac{y_1}{y_2} = \frac{1+\frac{F_{r_2}^2}{2}}{1+\frac{F_{r_1}^2}{2}} = \frac{2+F_{r_2}^2}{2+F_{r_1}^2}$$

$$F_r = \frac{V}{\sqrt{gy}} = \frac{Q}{by\sqrt{gy}}$$

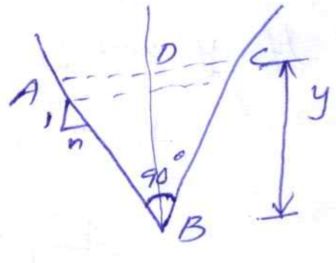
$$F_r^2 = \frac{Q^2}{b^2 g y^3} \Rightarrow F_{r_1}^2 = \frac{Q^2}{b^2 g y_1^3}$$

$$F_{r_2}^2 = \frac{Q^2}{b^2 g y_2^3}$$

$$\frac{F_{r_2}^2}{F_{r_1}^2} = \frac{y_1^3}{y_2^3} \Rightarrow \left(\frac{F_{r_2}}{F_{r_1}}\right)^{2/3} = \frac{y_1}{y_2}$$

obtain an expression for hydraulic radius for hydraulically efficient triangular channel in terms of depth of flow.

$$\text{Area} = \frac{1}{2} y \times ny = \left(\frac{1}{2}ny^2\right)2 = ny^2$$



$$R = \frac{A}{P} = \qquad A = ny^2 \quad \left(n = \frac{A}{y^2}\right)$$

$$P = 2y\sqrt{1+n^2}$$

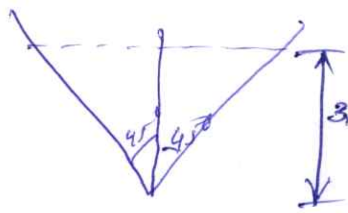
$$P = 2y\sqrt{1+\left(\frac{A}{y^2}\right)^2} = 2y\sqrt{\frac{A^2}{y^4}+1}$$

$$\frac{dP}{dy} = 0 \Rightarrow \frac{d}{dy}\left(2\sqrt{\frac{A^2}{y^2}+y^2}\right) = 0$$

$$-\frac{2A^2}{y^3} + 2y = 0 \qquad n = \frac{A}{y^2} = 1 \qquad \Rightarrow \theta = 90^\circ$$

$$y^2 = A$$

(a) Water flows through a triangular channel of vertex angle 90° as shown in fig. If the slope of the bed of the channel is $\frac{1}{3000}$, find the rate of flow. Assume Chezy's constant as 50.



$y = 3m$

$i = \frac{1}{3000}$, $C = 50$

$A = my^2 = 1(3)^2 = 9\ m^2$

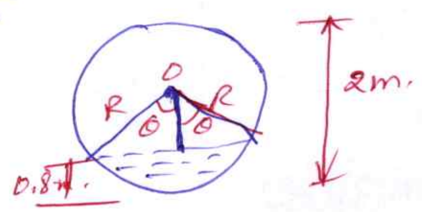
$P = 2y\sqrt{1+n^2} = 8.48m.$

$R = \frac{A}{P} = \frac{9}{8.48} = 1.06m,$

$Q = AC\sqrt{Ri} = 9 \times 50\sqrt{1.06 \times \frac{1}{3000}} = 8.459\ m^3/s.$

(*) A circular channel of 2m diameter carries a water at a depth of 0.8m. If the bed slope of the channel is 1/1500. Find discharge through the chanel. Take C = 50.

$D = 2m,\ R = 1m,\ y = 0.8m.,\ i = \frac{1}{1500}$



$\cos\theta = \frac{R-y}{R} = \frac{1-0.8}{1} = \frac{0.2}{1}$

$\theta = 1.369$ rad.

$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) = 1^2\left(1.369 - \frac{\sin(2 \times 1.369)}{2}\right)$

$A = 1.173\ m^2$

$P = 2R\theta = 2.738$

$R_w = \frac{A}{P} = 0.428m$

$Q = AC\sqrt{Ri}$