ANNAMACHARYA UNIVERSITY

EXCELLENCE IN EDUCATION; SERVICE TO SOCIETY

ESTD, UNDER AP PRIVATE UNIVERSITIES (ESTABLISHMENT AND REGULATION) ACT, 2016)

Rajampet, Annamayya District, A.P – 516126, INDIA

CIVIL ENGINEERING

Lecture Notes

on

Structural Analysis

Prepared by

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Asst. Professor

Civil Engineering

Title of the Course:	Structural Analysis
Category:	PCC
Semester:	IV Semester
Couse Code:	24ACIV44T
Branch/es:	CE

Lecture Hours	**Tutorial Hours**	**Practice Hours**	**Credits**
3	0	0	3

Course Objectives:

1. To understand the behavior and analysis of fixed and continuous beams under various loading conditions including effects of support settlement and rotation.
2. To learn the application of energy theorems such as Castigliano's theorems in determining deflections of beams and trusses.
3. To study slope-deflection and moment distribution methods for analyzing continuous beams and single bay portal frames.
4. To gain proficiency in Kani's method for analyzing continuous beams and portal frames with side sway.
5. To understand the concept and construction of influence lines and analyze moving loads on beams for shear force and bending moment.

Course Outcomes:

At the end of the course, the student will be able to

1. Analyze fixed and continuous beams subjected to various loads and interpret the effects of support settlements and rotations.
2. Apply energy theorems including Castigliano's theorems to determine deflections in beams and truss structures.
3. Use slope-deflection and moment distribution methods effectively to analyze continuous beams and portal frames without sway.
4. Employ Kani's method for structural analysis of continuous beams and single-storey portal frames with side sway.
5. Construct influence lines for shear force and bending moment, and analyze beams under moving loads to find critical load positions and maximum effects.

Unit 1 **12**

Fixed Beams & Continuous Beams: Introduction to statically indeterminate beams with uniformly distributed load, central point load, eccentric point load, number of point loads, uniformly varying load, couple and combination of loads – Shear force and Bending moment diagrams – Deflection of fixed beams effect of sinking of support.

Unit 2 **12**

Energy Theorems: Introduction-Strain energy in linear elastic system, expression of strain energy due to axial load, bending moment and shear forces – Castigliano's first theorem Deflections of simple beams.

Analysis of Indeterminate Structures: Indeterminate Structural Analysis – Determination of static and kinematic indeterminacies – Solution of trusses with up to two degrees of internal and external indeterminacies – Castigliano's– II theorem.

Unit 3 **12**

Analysis of Structures by Slope-Deflection Method: Introduction-derivation of slope deflection equations- application to continuous beams with and without settlement of supports - Analysis of single bay portal frames without sway.

Unit 4 **12**

Analysis of Structures by Moment Distribution Method: Analysis of continuous beams – including settlement of supports and single bay, single storey portal frames without sway

Unit 5 Influence Lines and Moving Loads 12

Influence Lines: Definition of influence line for SF, Influence line for BM- load position for maximum SF at a section-Load position for maximum BM at a section single point load, U.D.L longer than the span, U.D.L shorter than the span.

Moving Loads: Introduction maximum SF and BM at a given section and absolute maximum S.F. and B.M due to single Concentrated load U.D.L longer than the span, U.D.L shorter than the span, two point loads with fixed distance between them and several point loads.

Prescribed Textbooks:

1. Analysis of Structures-VolI& Vol IIbyV.N.Vazirani& M.M.Ratwani, Khanna Publications, New Delhi, 16th edition,1994.
2. Theory of Structures by R.S. Khurmi, S. Chand Publishers, 2000.

Reference Textbooks:

1. Mechanics of Structures by S.B.Junnarkar, Charotar Publishing House, 32nd edition, 2016.
2. Theory of Structures by Gupta, Pandit& Gupta; Tat Mc.Graw– Hill Publishing Co. Ltd., New Delhi, 2023.
3. Strength of Materials and Mechanics of Structures- by B.C.Punmia, Khanna Publications, NewDelhi, 2018.
4. Introduction to structural analysis by B.D. Nautiyal, New age international publishers, NewDelhi, 2001.
5. Structural Analysis by V.D.Prasad Galgotia publications,2nd Editions.2011.
6. Analysis of Structures by T.S. Thandavamoorthy, Oxford University Press, New Delhi, 2011
7. Comprehensive StructuralAnalysis-Vol.I&2by Dr. R. Vaidyanathan & Dr.P.Perumal- Laxmi publications pvt.Ltd., New Delhi,4 th edition, 2006.
8. Basic structural Analysis by C.S.Reddy, Tata Mc Grawhill, New Delhi, 3rd edition, 2017.

CO-PO Mapping:

Course Outcomes	Engineering Knowledge	Problem Analysis	Design/Development of solutions	Conduct investigations of complex problems	Engineering tool usage	The Engineer and the World	Ethics	Individual and collaborative teamwork	Communication	Project management and finance	Life-long learning	**PSO1**	**PSO2**
24ACIV44T.1	3	3	3	3	2	-	-	-	-	-	1	3	3
24ACIV44T.2	3	3	3	3	2	-	-	-	-	-	1	3	3
24ACIV44T.3	3	3	3	3	2	-	-	-	-	-	1	3	3
24ACIV44T.4	3	3	3	3	2	-	-	-	-	-	1	3	3
24ACIV44T.5	3	3	3	3	2	-	-	-	-	-	1	3	3

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CIVIL ENGINEERING

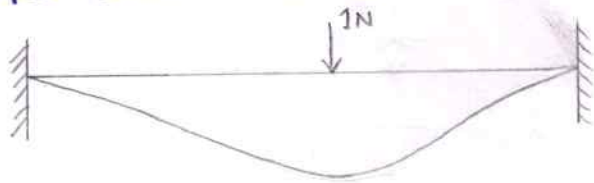
Structural Analysis

UNIT-1

2. Fixed beams & continuous beams.

Fixed beams:-

- A beam whose both ends are fixed is known as a fixed beams. Fixed beam is also called as built-in or encaster beam.
- Incase of fixed beam both its ends are rigidly fixed and the slope and deflection at the fixed ends are zero



Advantages of fixed beams:-

(i) For the same loading the maximum deflection of a fixed beam is less than that of a simply supported beam.

(ii) For the same loading the fixed beam is subjected to lesser maximum bending moment.

(iii) The slope at both ends of a fixed beam is zero

(iv) The beam is more stable and stronger.

Disadvantages of a fixed beam:-

(i) Large stresses are set up by temperature changes.

(ii) special care has to be taken in aligning supports accurately at the same level

(iii) Large stresses are set if a little sinking of one support takes place.

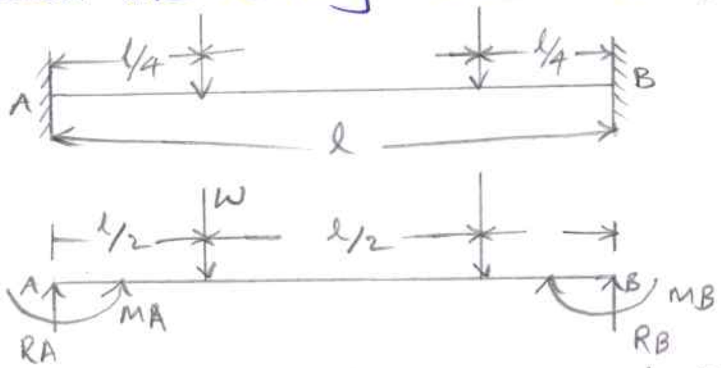
(iv) frequent fluctuations in loading render the degree of fixity at the ends very uncertain.

The beam may be analyzed in the following stages.

(i) Let us first consider the beam as simply supported.

Let V_a and V_b be the vertical reactions at the supports A and B

figure (ib) shows the bending moment diagram for this condition at any section the bending moment M_x is a sagging moment.



(ii) now let us consider the effect of end couples M_A and M_B alone.

Let V be the reaction at each end due to this condition

suppose $M_B > M_A$

Then $V = \frac{M_B - M_A}{L}$

If $M_B > M_A$ the reaction V is upwards at B and downwards at A

Fig (iib) shows the bending moment diagram for this condition.

At any section the bending moment M_x is hogging moment.

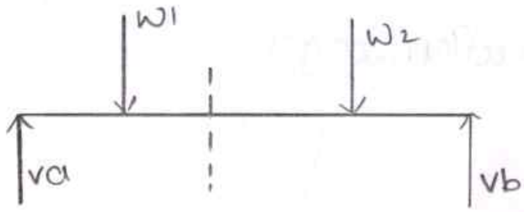
• Now the final bending moment diagram can be drawn by combining the above two BM diagrams as shown in fig (iiib).

Now the final reaction $V_A = V_a - V$

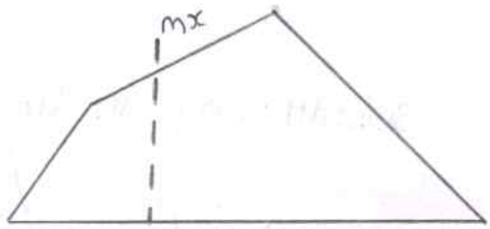
and $V_B = V_b + V$

The actual bending moment at any section x, distance x from the end A is given by

$$EI\frac{d^2y}{dx^2} = M_x - M_x'$$

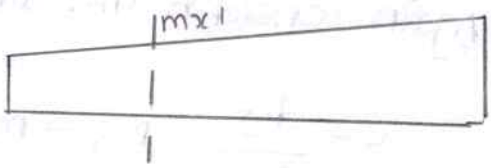


(ia) freely supported condition

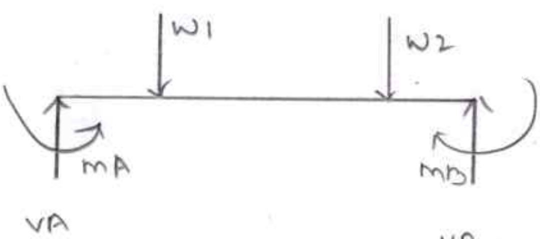


i(b) free B.M.D

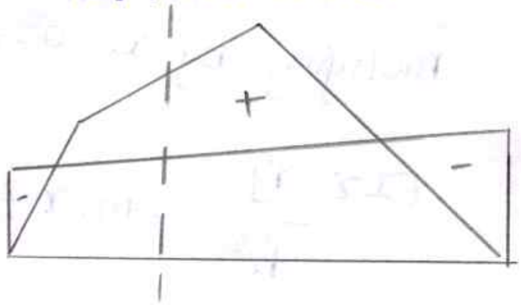
(iia) effect of end couples



i(b) fixed B.M.D.



(iii a) fixed beam



(iiib) Resultant B.MD

$$EI \frac{d^2y}{dx^2} = m_x - m_x'$$

- Integrating, we get
- $$EI\left[\frac{dy}{dx}\right]_0^l = \int_0^l m_x dx - \int_0^l m_x' dx$$
- But at $x = 0, \frac{dy}{dx} = 0$

and at $x = l, \frac{dy}{dx} = 0$

Further $\int_0^l m_x dx$ = area of the free BMD = α

$\int_0^l m_x' dx$ = area of the fixed BMD = α'

substituting in the above equation, we get

$$0 = \alpha - \alpha'$$

$\therefore$ Area of the free BMD = Area of the fixed BMD

Again consider the relation

$$EI \frac{d^2y}{dx^2} = m_x - m_x'$$

multying by x we get

$$EIx \frac{d^2y}{dx^2} = m_x x - m_x' x$$

multying by x we get

$$EIx \frac{d^2y}{dx^2} = m_x x - m_x' x$$

Integrating we get

- $\int_0^l EI x \frac{d^2y}{dx^2} = \int_0^l m_x x \, dx - \int_0^l m_x^1 x \, dx$

$$\therefore EI\left[x\frac{dy}{dx} - y\right]_0^l = a\bar{x} - a^1\bar{x}^1$$

- where $\bar{x}$ = distance of the centroid of the free BMD from A and $\bar{x}^1$ = distance of the centroid of the fixed BMD from A.

- further at $x=0$, $y=0$ and $\frac{dy}{dx}=0$

- and at $x=l$, $y=0$ and $\frac{dy}{dx}=0$

- Substituting in the above relation, we have

$$0 = a\bar{x} - a^1\bar{x}^1$$

$$a\bar{x} = a^1\bar{x}^1$$

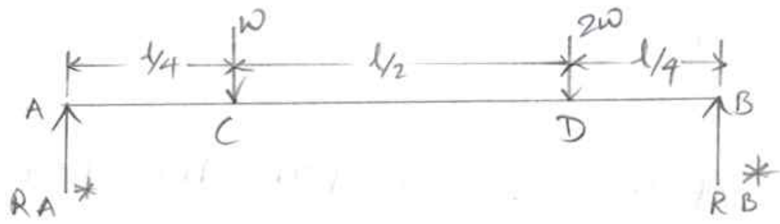
$$(\text{or}) \quad \bar{x} = \bar{x}$$

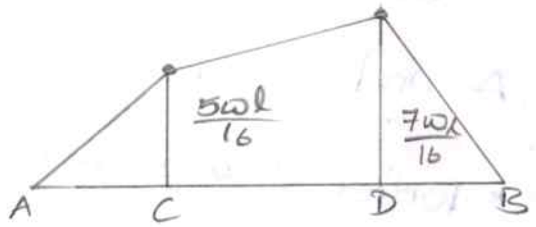
$\therefore$ The distance of the centroid of the free B.M.D from A = The distance of the centroid of the fixed BMD from A

$$\boxed{\begin{aligned} \therefore a &= a^1 \\ \bar{x} &= \bar{x} \end{aligned}}$$

condition 1

① A simply supported beam. subjected to given vertical loads as shown in figure below.





Taking moment about A

$\Sigma m_A = 0$

$\Rightarrow R_B^* \times l + 2w \times 3l/4 + w \times l/4 = 0$

$\Rightarrow R_B^* \times l = \frac{6wl}{4} + wl/4$

$R_B^* = \frac{7w}{4}$

$\Rightarrow R_A^* + R_B^* = w + 2w$

$\Rightarrow R_A^* = 3w \times R_B$

$R_A^* = 3w - \frac{7w}{4}$

$$\boxed{R_A = \frac{5w}{4}}$$

Bending moment:-

$Bm \ @ \ B = 0 \ [R_B \times 0]$

$Bm \ @ \ D = R_B \times l/4 = \frac{7wl}{16}$

$Bm @ C = R_B \times 3l/4 - 2w \times l/2 = \frac{7w}{4} + \frac{3l}{4} - wl = \frac{5wl}{16}$

$$\boxed{M_A = \frac{\omega l}{8} = M_B}$$

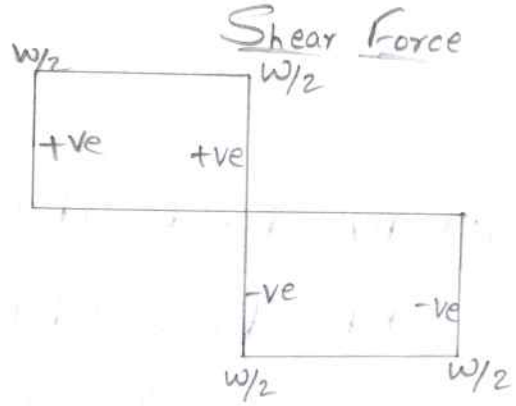
(ii) shear force:-

Taking moments

$\Sigma M_A = 0 \Rightarrow -R_B \times l + \omega \times l/2 + \cancel{M_B} - \cancel{M_A} = 0$

$= R_B \times \cancel{l} = \omega \cancel{l}/2$

$\Rightarrow R_B = \omega/2 = R_A$



$\Rightarrow$ Now shear force

SF @ A = $R_A = \omega/2$

SF @ C = $R_A = \omega/2$

SF @ B = $R_A - \omega = \omega/2$.

(iii) slope and deflection

The bending moment at any section between AC at a distance 'x' from a is given by

$\Rightarrow EI \frac{d^2y}{dx^2} = M_x$

$EI \frac{d^2y}{dx^2} = R_A x - M_A$

$EI \frac{d^2y}{dx^2} = \omega/2 \times (x) - \omega l/8$

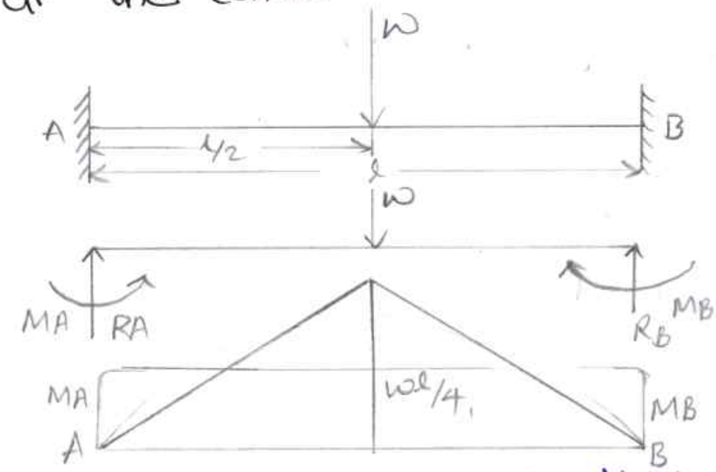
Intigrate the above equation we get:-

$EI \int \frac{d^2y}{dx^2} dx = \int \frac{\omega x}{2} dx - \int \frac{\omega l}{8} dx$

$\therefore$ BM @ $A = R_B \times l - 2w \times \frac{3l}{4} - 2w \times \frac{3l}{4} + w \times l/4 = 0$

Derivation:-

slope and deflection for a fixed beam carry a point load at the centre.



Only horizontal Load should be act on frames not only 2D Structures

Let m_A = fixed end moment at 'A'

m_B = fixed end moment at 'B'

R_A = Reaction at 'A'

R_B = Reaction at 'B'

(i) Bending moment diagram:-

The bending moment incase of simply supported

$\frac{wl}{4}$

$\Rightarrow$ As the member is subjected to symmentric loading the end moments [m_A, m_B] will be same.

$\therefore$ Equating the areas of the two bending moment diagrams

$\Rightarrow$ Area of $\triangle$le ADB = Area of $\square$ (Rectangle) ACFB

$\Rightarrow \frac{1}{2} \times AB \times CD = AB \times AC$

$\Rightarrow \frac{1}{2} \times l \times \frac{wl}{4} = l\, m_A$

$\therefore$ $EI \frac{dy}{dx} = \frac{\omega x^2}{4} - \frac{\omega l}{8}(x) + C_1$

Apply boundary conditions

At $x=0$ $\frac{dy}{dx}=0$

$\Rightarrow 0 = 0 - 0 + C_1 \Rightarrow C_1 = 0$

$\Rightarrow$ subsitute $C_1 = 0$ in slope equation.

$$EI \frac{dy}{dx} - \frac{\omega x^2}{4} - \frac{\omega l x}{8} \rightarrow (1)$$

$\Rightarrow$ Intigrate the above equation we get:-

$$\int \frac{dy}{dx} = y$$

$$\Rightarrow EI(y) = \frac{\omega x^3}{12} - \frac{\omega l x^2}{16} + C_2 \rightarrow (2)$$

$\Rightarrow$ Apply boundary conditions.

At $x=0$, $y=0$

$\Rightarrow 0 = 0 - 0 + C_2 \Rightarrow C_2 = 0$

$$EI(y) = \frac{\omega x^3}{12} - \frac{\omega l x^2}{16} \rightarrow (3)$$

$\Rightarrow$ The above equation represents deflection at any point on the beam.

$\Rightarrow$ the deflection is maximum at the centre of the beam.

$$\boxed{\text{Ax } 'x' = l/2 \text{ . } y = y_{max}}$$

subsitute in equation $\rightarrow$ (2)

$$EI(y_{max}) = \omega/12 \times (l/2)^3 - \frac{\omega l}{16}(l/2)^2$$

$$EIy_{max} = \frac{wl^3}{96} - wl^3/64$$ $\Rightarrow$ $96-64$
$= 32\times6$
$= 192$

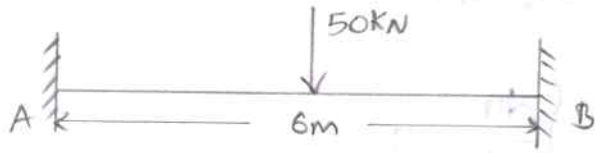
$$\boxed{y_{max} = \frac{wl^3}{192EI}}$$

problem:-

A fixed beam 'AB' of length '6m' is carrying a point load of 50kN at its centre. The moment of interia of beam is $78\times10^6 mm^4$ at $E = 2.1\times10^5 N/mm^2$. determine

(i) fixed end moments at A and B

(ii) Deflection under the load.



sol) (i) $wl/8 = \frac{50\times6}{8} = 37.5$

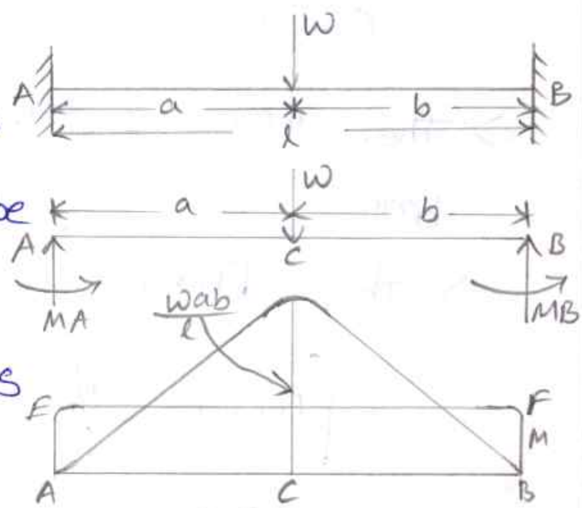
(ii) Deflection.

$$y_{max} = \frac{wl^3}{192EI} = \frac{50\times6^3}{192\times2.1\times10^5\times78\times10^6}$$

$$= 3.43 kN/m.$$

Derivation

slope and deflection for a fixed beam carrying an essentric point load.



The BMD for simply supported beam carrying an essentric point load will be the triangle with maximum, bending BM under the load. and it is equals to $\frac{wab}{l}$.

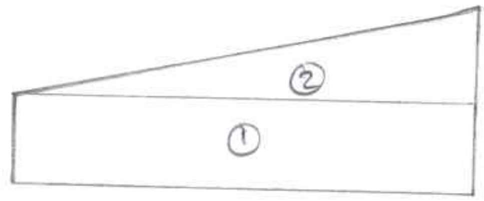
Equating the areas of two bending moment diagrams -

Area of trepezium = Area of triangle

$$\frac{1}{2}(m_A+m_B)\times L = \frac{1}{2}\times L\times\frac{\omega ab}{l}$$

$$\Rightarrow \boxed{m_A+m_B=\frac{\omega ab}{l}} \rightarrow (1)$$

x- end moments



Apply $\boxed{\bar{x}=\bar{x}'}$

$$\bar{x}' = \frac{A_1x_1+A_2x_2}{A_1+A_2}$$

$$\bar{x}' = \frac{(m_A\times L\times l)\times l/2+\left(\frac{1}{2}\times l\,(m_B-m_A)\times 2l/3\right)}{(m_A\times l)+\left(\frac{1}{2}\times l\times(m_B-m_A)\right)}$$

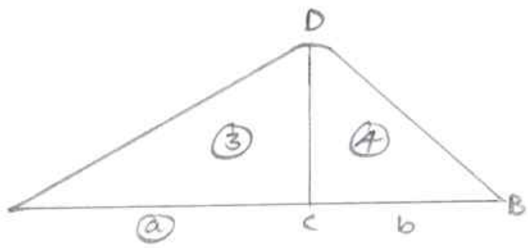
$$= \frac{\frac{m_Al^2}{2}+\frac{(m_B-m_A^2)}{3}l^2}{m_Al+\frac{(m_B-m_A)}{2}}$$

$$= \frac{3m_Al^2+2m_Bl^2-2m_Al^2}{3\cancel{6}\left(\frac{2m_Al+m_Bl-m_Al}{\cancel{2}}\right)}$$

$$= \bar{x}' = \frac{m_Al^2+2m_Bl^2}{3m_Al+3m_Bl}$$

$$\bar{x}' = \frac{(m_A+m_B)l}{3(m_A+m_B)}$$

$$\Rightarrow \frac{A_3x^3+A_4x_4}{A_3+A_4}$$



$$= \frac{(1/2 \times a \times CD)\, 2/3 \times a) + (1/2 \times b \times CD) \times (a + 1/3 \times b)}{1/2 \times a \times CD + 1/2 \times b \times CD}$$

$$= \frac{\cancel{CD/2}\,(a \times 2a/3 + b \times (a + b/3))}{\cancel{CD/2}\,(a+b)}$$

$$= \frac{\frac{2a^2}{3} + ab + b^2/3}{a+b}$$

$$= \frac{2a^2 + 3ab + b^2}{3(a+b)}$$

$$= \frac{(2a^2 + 2ab) + (ab + b^2)}{3(a+b)}$$

$$= \frac{2a(a+b) + b(a+b)}{3(a+b)}$$

$$= \frac{\cancel{(a+b)}(2a+b)}{3\cancel{(a+b)}}$$

$$= \frac{2(a+b)}{3} = \frac{a+a+b}{3}$$

$$\boxed{= \frac{a+l}{3} \quad (\because a+b = l)}$$

$$\bar{x} = \bar{x}'$$

$$\frac{a+l}{3} = \frac{(m_A + m_B)l}{3(m_A+m_B)}$$

$$\Rightarrow (a+l)(m_A+m_B) = (m_A+2m_B)l$$

$$\Rightarrow m_A + 2m_B = \frac{a+l + \frac{\omega ab}{l}}{l}$$

$$\boxed{m_A + 2m_B = \frac{a+l\,(\omega ab)}{l^2}} \rightarrow (2)$$

sub equation (1) & (2)

$$\Rightarrow \cancel{m_A} + m_B - \cancel{m_A} - 2m_B = \frac{\omega ab}{l} - \frac{(a+b)\omega ab}{l^2}$$

$$\Rightarrow -m_B = \frac{\omega ab}{l}\left(1 - \frac{a+l}{l}\right)$$

$$\Rightarrow m_B = \frac{\omega ab}{l}\left(\frac{a+l}{l} - 1\right)$$

$$= \frac{\omega ab}{l}\left(\frac{a+\cancel{l}-\cancel{l}}{l}\right)$$

$$\Rightarrow m_B = \frac{\omega ab}{l}\left(a/l\right) \Rightarrow m_B = \frac{\omega a^2 b}{l^2}$$

subsitute m_B in equation (1) we get

$$m_A + m_B = \frac{\omega ab}{l^2} \rightarrow (1)$$

$$\Rightarrow m_A + \frac{\omega a^2 b}{l^2} = \frac{\omega ab}{l^2}$$

$$m_A = \frac{\omega ab}{l}\left(1 - a/l\right)$$

$$m_A = \frac{\omega ab}{l}\left(\frac{l-a}{l}\right)$$

$$m_A = \frac{\omega ab}{l}\left(b/l\right)$$

$$\boxed{m_A = \frac{\omega ab^2}{l^2}}$$

⇒ slope and deflection.

The bending moment at any section between A and C at a distance of 'x' is given by $EI \frac{d^2y}{dx^2} = m_x$.

$$\Rightarrow \quad EI \frac{d^2y}{dx^2} = m_x$$

$$EI \frac{d^2y}{dx^2} = R_A(x) - m_A \rightarrow 3$$

<u>shear force:-</u>

Taking moments about "A"

$$\Sigma m_A = 0$$

$$\Rightarrow -R_B \times l - m_A + m_B + \omega \times a = 0$$

$$R_B \times l_1 = m_B - m_A + \omega a$$

$$\Rightarrow \boxed{R_B = \frac{m_B - m_A + \omega a}{l}}$$

Then

$$R_A + R_B = 0$$

$$R_A = \frac{\omega - m_B - m_A + \omega a}{l}$$

$$= \frac{\omega l - m_B + m_A - \omega a}{l}$$

$$R_A = \frac{m_A - m_B + \omega(l-a)}{l}$$

$$\boxed{R_A = \frac{m_A - m_B + \omega(b)}{l}}$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = \left[\frac{(m_A - m_B) + \omega b}{l} \times x\right] - m_A$$

$$= \frac{m_A x}{l} - \frac{m_B x}{l} + \frac{\omega b}{l}x - m_A$$

$$= \frac{\omega b}{l}x - [m_A - m_B]\,x/x - m_A$$

$$= \frac{\omega b}{l}x + \left[\frac{\omega ab}{l^2} - \frac{\omega a^2 b}{l^2}\,x/l - \frac{\omega ab^2}{l^2}\right]x/l$$

$$= \frac{\omega b}{l}x - \frac{\omega ab^2}{l^2}(b-a)x - \frac{\omega ab^2}{l^2}$$

$$= \frac{\omega b}{l^3} x \left(l^3 + a(b-a) \right) - \frac{\omega a b^2}{l^2}$$

$$= \frac{\omega b}{l^3} \times \left(l^2 \times ab - a^2 \right) - \frac{\omega a b^2}{l^2}$$

$$= \frac{\omega b}{l^3} \times (a+b)^2 + (ab-a)^2 - \frac{\omega a b^2}{l^2}$$

$$= \frac{\omega b}{l} x \left(\cancel{a^2} + b^2 + 2ab + ab - \cancel{a^2} \right) - \frac{\omega a b^2}{l^2}$$

$$= \frac{\omega b}{l} x \left(3ab + b^2 \right) - \frac{\omega a b^2}{l^2}$$

$$\boxed{EI \frac{d^2y}{dx^2} = \frac{\omega b^2}{l^3} x (3a+b) - \frac{\omega a b^2}{l^2}} \rightarrow (4)$$

Integrate the above equation

$$\Rightarrow EI \frac{dy}{dx} = \frac{\omega b^2}{l^3} (3a+b) \frac{x^2}{2} - \frac{\omega a b^2}{l^2} x + c_1$$

$\Rightarrow$ Apply boundary conditions

$@ \; x = 0, \; \frac{dy}{dx} = 0$

Hence $c_1 = 0$

Sub $c_1 = 0$ in above equation we get.

$$\boxed{EI \frac{dy}{dx^2} = \frac{\omega b^2}{l^3} (3a+b) \frac{x^2}{2} - \frac{\omega a b^2}{l^2} x} \rightarrow 5$$

The above equation represent slope.

Intigrate above equation - ⑤

$$EI(y) = \frac{wb^2}{l^3}(3a+b)\,x^3/6 - \frac{wab^2}{l^2}\,x^2/2 + c_2$$

At $x=0$, $y=0$ Hence $c_2=0$

sub $c_2=0$ in equation.

$$EI(y) = \frac{wb^2}{l^3}(3a+b)\,x^3/6 - \frac{wab^3}{l^2} \times x^2/2$$

$$\Rightarrow EI(y) = \frac{wb^2x^3}{6x^3}(3a+b) - \frac{wab^3x^2}{l^2}$$

⇒ The above equation represent deflection at any point.

⇒ The deflection under the load is optain is substituting

⇒ $\boxed{x=a}$ in the above equation.

$$\Rightarrow EIy_c = \frac{wb^2a^3}{6l^3}(3a+b) - \frac{wab^2a^2}{2l^2}$$

$$= \frac{wb^2a^3}{6l^3}(3a+b) - \frac{wa^3b^2}{2l^2} \times \frac{3l}{3l}$$

$$= \frac{wa^3b^2}{6l^3}(3a+b-3l)$$

$$= \frac{wa^3b^2}{l^3}(\cancel{3a}+b-\cancel{3a}-3b)$$

$$= \frac{wa^3b^2}{6l^3}(b-3b) \quad [3b \text{ is length is more}]$$

$$EIy_c = \frac{wa^3b^2}{6l^3}(3b-b)$$

$$= \frac{wa^3b^2}{6l^3}(2b)$$

$$\boxed{EIy_c = \frac{wa^3b^3}{3l^3}}$$

maximum deflection:-

At $\frac{dy}{dx}=0$ The deflection will be maximum.

$$\therefore EI\frac{dy}{dx} = \frac{wb^2}{2l^3}(b+3a)x^2 - \frac{wab^2}{l^2}x.$$

$$\Rightarrow 0 = \frac{wb^2}{2l^3}(b+3a)x^2 - \frac{wab^2}{l^2}x.$$

$$\Rightarrow \frac{\cancel{w}\cancel{b^2}}{2l^2}(b+3a)\cancel{x^2} = \frac{\cancel{w}a\cancel{b^2}}{\cancel{l^2}}\cancel{x}$$

$$\Rightarrow \frac{(b+3a)x}{2l} = a \Rightarrow (b+3a)x = 2al$$

$$\boxed{x = \frac{2al}{b+3a}} \rightarrow \text{position of maximum deflection.}$$

The above value represents position of maximum deflection.

$\Rightarrow$ substitute "x" in deflection equation in order to get maximum deflection.

$$\boxed{EI = \frac{wb^2}{6l^3}(b+3a)x^3 - \frac{wab^2}{2l^2}x^2}$$

deflection equation

$$\Rightarrow EIy = \left[\frac{wb^2}{6l^3}(b+3a) \times \left(\frac{2al}{b+3a}\right)^3\right] - \left[\frac{wab^2}{2l^2} \times \left[\frac{2al}{b+3a}\right] \times \frac{3l}{3l}\right]$$

$$\Rightarrow EIy = \left[\frac{wb^2}{6l^3}(b+3a) \times \left[\left(\frac{2al}{b+3a}\right)^3\right]\right] - \left[\frac{wab^2}{6l^3} \times \left(\frac{2al}{b+3a}\right)\right] \times 3l.$$

$$\Rightarrow \frac{wb^2}{6l^3}\left(\frac{2al}{b+3a}\right)^2 \left((b+3a)\left[\frac{2al}{b+3a} - 3al\right]\right.$$

$$\Rightarrow \frac{wb^2}{6l^3}\left(\frac{2al}{b+3a}\right)^2 (2al - 3al)$$

$$\Rightarrow \frac{wb^2}{6l^3}\left(\frac{2al}{b+3a}\right)^2 \times (al)$$

$$\Rightarrow \frac{wb^2}{3l^3} \times \frac{\cancel{4}\, a^3l^3}{(b+3a)^2} \Rightarrow \frac{2wa^3b^2}{3(b+3a)^2}$$

$$\Rightarrow EI\,y_{max} = -\tfrac{2}{3} \times \frac{wa^3b^2}{(b+3a)^2}$$

$$\boxed{\Rightarrow y_{max} = \frac{2}{3EI} \times \frac{wa^3b^2}{(b+3a)^2}}$$

problem:-

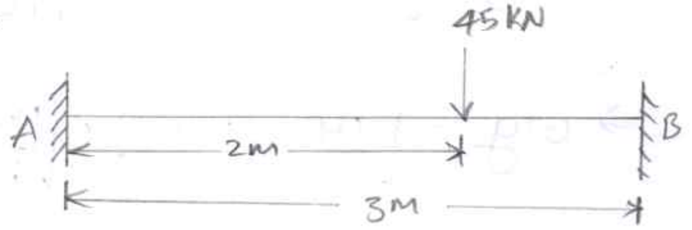
A fixed beam AB of length "3m" carries a point load of 45KN at a distance of 2m from "a" if the flexural rigidity of the beam is 1×10^4 KNm^2 determine

(i) fixed end moments

(ii) Deflection under the load

(iii) maximum deflection.

(iv) position of maximum deflection.



Sol) Given data

Length of the beam = 3m

Load w = 45 KN.

Flexural rigidity $EI = 1 \times 10^4\ KNm^2$

distance of from load "a" = 2m

and "b" = 1m.

Let ma and mb are the fixed end moment "A & B"

$\Rightarrow y_c$ = deflection under the load.

$\Rightarrow y_{max}$ = maximum deflection.

$\Rightarrow$ "x" = position of maximum deflection.

(i) fixed end moments.

$$\boxed{m_A = \frac{wab^2}{l^2}}$$

$$= \frac{45 \times 2 \times 1^2}{3^2} = 10\ KNm$$

$$\boxed{m_B = \frac{wa^2b}{l^2}} = \frac{45 \times 2^2 \times 1}{3^2} = 20\ KNm$$

(ii) Deflection under the load

$$\boxed{y_C = -\frac{\omega a^3 b^3}{3EI l^3}}$$

$$= \frac{45 \times 2^3 \times 1^3}{3 \times 1 \times 10^4 \times 3^3} = 0.44\text{mm}.$$

(iii) maximum deflection.

$$\boxed{y_{max} = \frac{2}{3EI} \times \frac{\omega a^3 b^2}{(b+3a)^2}}$$

$$= \frac{2}{3 \times 1 \times 10^4} \times \frac{45 \times 2^3 \times 1^2}{(1+3 2)^2} = 0.489\text{mm}.$$

(iv) position of maximum deflection.

$$\boxed{x = \frac{2al}{(b+3a)}} = \frac{2(2)(3)}{(1+3\times 2)} = 1.71 \times 10^3$$

$$\boxed{x = 1.71\text{m}.}$$

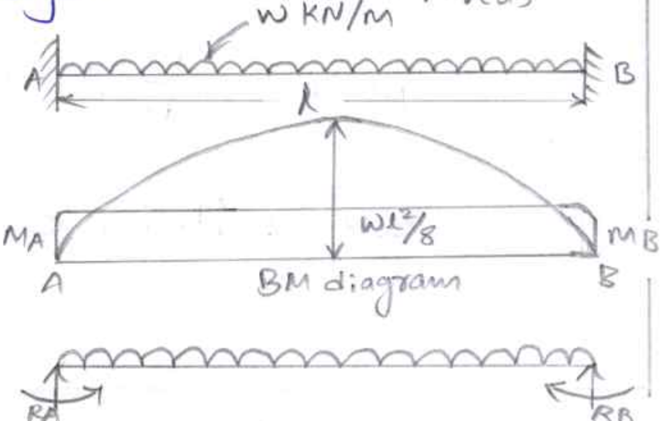
slope and deflection for a fixed beam carrying uniformly distributed load over its entire length.

① → Assume
② Remove extra loads
③ Areas

Area of Parabola = Area of rectangle.

$$2/3 \times AB \times \frac{\omega l^2}{8} = AB \times AC$$

$$2/3 \times l \times \frac{\omega l^2}{8/4} = l \times m_A$$



$$\boxed{M_A = \frac{\omega l^2}{12} = M_B}$$

(ii) Reactions:-

$\Sigma M_A = 0$

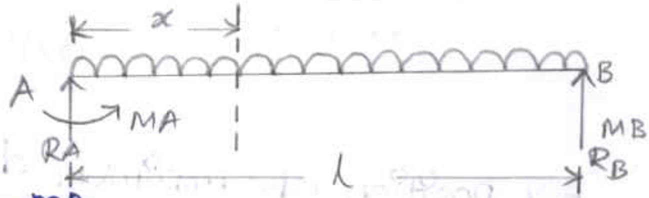
$\Rightarrow -R_B \times l + (\omega \times l) \times l/2 + M_A + M_B = 0$

$\Rightarrow R_B l = \omega l^2/2$

$$\boxed{\Rightarrow R_B = \omega l/2 = R_A}$$

(iii) slope and deflection

consider section $x-x$ @ a distance of x from A



$EI \frac{d^2y}{dx^2} dx$

$\Rightarrow EI \frac{d^2y}{dx^2} = R_A x - (\omega x) \times x/2 - M_A$

$$\Rightarrow \boxed{EI \frac{d^2y}{dx^2} = \frac{\omega l}{2} x - \frac{\omega x^2}{2} - \frac{\omega l^2}{12}} \rightarrow (1)$$

Integrate the equation (1) we get

$EI \frac{dy}{dx} = \frac{\omega l}{2} \times x^2/2 - \frac{\omega x^3}{6} - \frac{\omega l^2}{12} x + C_1 \rightarrow (2)$

At $x=0$, $\frac{dy}{dx} = 0$, $C_1 = 0$

sub C_1 in eq (2)

$$EI\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^2x}{12}$$

$$EI\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^2x}{12} \rightarrow (3)$$

Intigrate the above equation we get

$$EIy = \frac{wlx^3}{12} - \frac{wx^4}{24} - \frac{wl^2x^2}{24} + C_2$$

At $x=0$, $y=0$, $C_2=0$

$$EI(y) = \frac{wlx^3}{12} - \frac{wx^4}{24} - \frac{wl^2x^2}{24} \rightarrow (4)$$

Equation 3 and 4 represents deflection equation and slope equation of the beam respectively If we need identify position maximum deflection.

⇒ Has the member is loaded symmentrically the position of maximum deflection is it centre that is "$x = l/2$"

⇒ To find out the maximum deflection. substitute

$x = l/2$ in equation (4)

$$EIy_{max} = \frac{\omega l}{12}\left(l/2\right)^3 - \frac{\omega}{24}\left(l/2\right)^4 - \frac{\omega l^2}{24}\left(l/2\right)^2$$

$$= \frac{\omega l^4}{96} - \frac{\omega l^4}{384} - \frac{\omega l^4}{96}$$

$$= \frac{\omega l^4}{384}$$

$$y_{max} = \frac{\omega l^4}{384EI}$$

Point of contraflexure:-

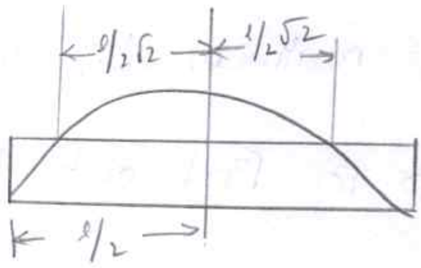
For the bending moment equation should be zero

Hence equating the Bm equation to zero we get

$$0 = \omega l/2\,(x) - \frac{\omega x^2}{2} - \frac{\omega l^2}{12}$$

$$0 = \omega lx - \omega x^2 - \omega x^2/6 \Rightarrow \omega x^2$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$



$$y = \frac{(-l) \pm \sqrt{l^2-4(1)(l^2/6)}}{2 \times 1}$$

$$x = l/2 + l/\sqrt{2} \text{ (or) } x = l/2 - \frac{l/\sqrt{2}}{2}$$

$$x = l/2 + \frac{l}{2\sqrt{3}} \text{ (or) } x = l/2 - l/2\sqrt{3}.$$

Continuous beams

Introduction:-

- Beams are made continuous over the supports to increase structural integrity
- A continuous beams provides an alternate load path in the case of failure at a section.
- In regions with high seismic risk, continuous beams and frames are preferred in buildings and bridges.
- A continuous beam is a statically indeterminate structure.

The advantages of a continuous beam as compared to a simply supported beam are as follows.

(1) for the same span and section, vertical load capacity is more

(2) mid span deflection is less.

(3) The depth at a section can be less than a simply supported beam for the same span. else for the same depth the span can be more than a simply supported beam.

⇒ The continuous beam is economical in material

(4) There is redundancy in load path

⇒ possibility of formation of hinges in case of an extreme event.

(5) Requires less number of anchorages of tendons.

Disadvantages of a continuous beam as compared to a simply supported beam

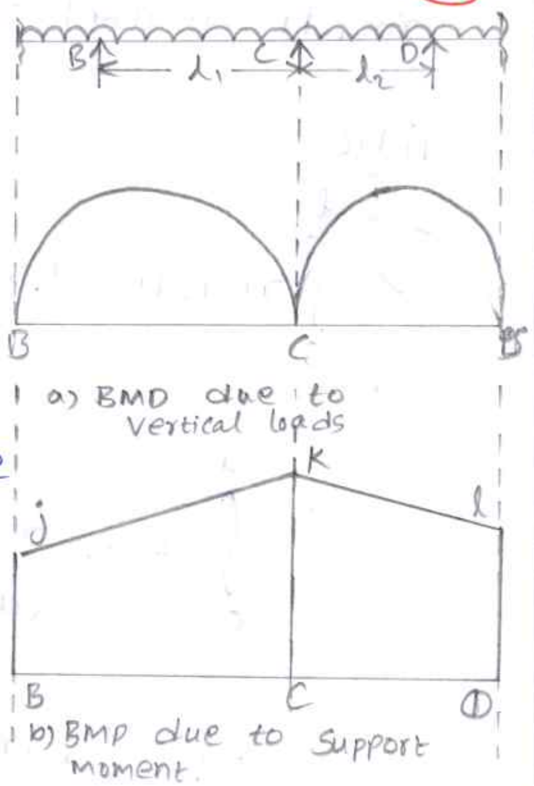
1) Difficult analysis and design procedures
2) Difficulties in construction, especially for precast members.
3) Increased frictional loss due to changes of curvature in the tendon profile.
4) Increased shortening of beam, leading to lateral force on the supporting columns.
5) secondary stresses develop due to time dependent effects like creep and shrinkage, settlement of support and variation of temperature.
6) The concurence of maximum moment and shear near the supports needs proper detailing of reinforcement
7) Reversal of moments due to seismic force require proper analysis and design.

Clapeyron's theorem of three moments

(Focus on intermidiate support)

The figure shows the length of BCD of a continueous beam carrying UDL on it let M_B, M_C and M_D are the support moment at B, C, D respectively.

clapeyron's theorem of three moments



consider span BC

Let m_x equals to bending moment due to vertical load at a distance of 'x' from 'B'.

$\Rightarrow$ m_x' equals to bending moment due to support moments at a distance of 'x' from B (Hogging).

net moment:-

$$\boxed{EI \frac{d^2y}{dx^2} \; m_x - m_x'} \text{(sagging)}$$

$\Rightarrow$ mutiplying with 'x' on both sides.

$$EI(x) \frac{d^2y}{dx^2} = x m_x - x m_x' \longrightarrow ①$$

$\Rightarrow$ Then integrating from zero to l_1

$$\int_0^{l_1} EI(x) \frac{d^2y}{dx^2} = \int_0^{l_1} x m_x - \int_0^{l_1} x m_x'$$

$$EI\left(x\frac{dy}{dx} - y\right)_0^{l_1} = a_1 \bar{x}_1 - a_1' \bar{x}_1$$

$\therefore$ since $m_x dx$ = Area of BMD of length dx & $x m_x dx$ equal to moment of area of BMD of length Bx about

D

$x m_x dx$ = moment of area of BMD of length dx about D

Hence.

$$\Rightarrow \boxed{\int_0^{l_1} x m_x dx = a_1 \bar{x}_1 \rightarrow (2)}$$

$$= \int_0^{l_1} EI(x) \frac{d^2y}{dx} dx = \int_0^{l_1} x m_x dx$$

$$= EI \left(x \frac{dy}{dx} - y \right]_0^{l_1}$$

$$= a_1 \bar{x}_1 - a_1' \bar{x}'$$

$$\Rightarrow EI[(L_1 \theta_c - y_c) - (0 - y_B)] = a_1 \bar{x} - a_1' \bar{x}_1'$$

But deflection @ B and C are zero $y_B - y_c = 0$

$$\boxed{\Rightarrow EI \, L_1 \theta_c = a_1 \bar{x}_1 - a_1' \bar{x}_1' \rightarrow (3)}$$

where $\bar{a}_1$ = area of bending moment due to support moments.

a_1' = Area of trepezium BCKJ

$$= (l_1 x m_B) + [1/2 (l_1 x m_c - m_B)]$$

$$= m_B^2 l_1 + \frac{l_2 m_c}{2} - \frac{m_B l_1}{2}$$

$$= \frac{l_1}{2} m_c + l/2 m_B$$

$\bar{x}_1'$ = distance of CG of area BCKJ from 'B'

$$\bar{x}_1' = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$$

$$= \frac{\left[(m_B \times l_1) \times \frac{l_1}{2}\right] + \left[\frac{1}{2} \times l_1 \times (m_C - m_B) \times \frac{2}{3} \times l_1\right]}{\left[(m_B \times l_1) + \left(\frac{1}{2} l_1 \times (m_C - m_B)\right)\right]}$$

$$= \frac{\frac{m_B l_1^2}{2} + \frac{l_1 (m_C - m_B) \times 2l_1}{6}}{\frac{2m_B \cdot l_1 + l_1 (m_C - m_B)}{2}}$$

$$= \frac{3m_B l_1^2 + 2m_C l_1^2 - 2m_B l_1^2}{\frac{2m_B l_1 + m_C l_1 - m_B l_1}{2}} = \frac{m_B l^2 + 2m_C}{3(m_B l_1 + m_C l_1)}$$

$$= \bar{x}_1' = \frac{l_1^2 (m_B + 2m_C)}{3l_1 (m_B + m_C)} = x_1 = \frac{l_1}{3} \left(\frac{m_B + 2m_C}{m_B + m_C}\right)$$

$\Rightarrow$ substitute the values of a_1 and $\bar{x}_1$ in equation $\rightarrow$ 2

$$EI\theta_C l_1 = a_1 \bar{x}_1 - a_1' \bar{x}_1'$$

$$EI\theta_C l_1 = a_1 \bar{x}_1 - \left(\frac{l_1}{2} (m_B + m_C) \times \frac{l_1}{3} \times \frac{m_B + 2m_C}{m_B + m_C}\right)$$

$$EI\theta_C l_1 = a_1 \bar{x}_1 - \left[\frac{l_1^2}{6} (m_B + 2m_C)\right]$$

$\Rightarrow 6EI\theta_C l_1 = 6a_1\bar{x}_1 - l_1^2(m_B + 2m_C)$

$$\boxed{\Rightarrow 6EI\theta_C \cancel{l_1} = \cancel{l_1}\left(\frac{6a_1\bar{x}_1}{l_1} - l_1(m_B + 2m_C\right)} \rightarrow (3)$$

$\Rightarrow$ consider span CD

similarly considering the span CD taking d as origin and 'x' positive to the left it can be shown that.

$$\Rightarrow \boxed{6EI - \theta_C = \frac{6a_2\bar{x}_2}{l_2} - l_2(m_B + 2m_C)} - (4)$$

Adding 3 & 4 equations

$$0 = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}(l_1)(m_B + 2m_C)$$

$$0 = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2} - l_1m_B - l_1 2m_C - l_2m_D - l_2 2m_C$$

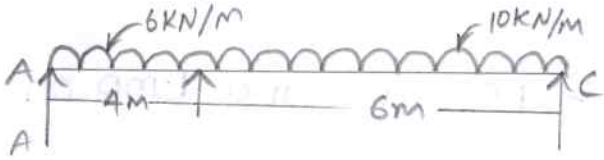
$$\Rightarrow l_1m_B + l_2m_D + 2m_C(l_1 + l_2) = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}$$

$$\Rightarrow \boxed{m_Bl_1 + 2m_C(l_1 + l_2) + m_Dl_2 = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}}$$

problem

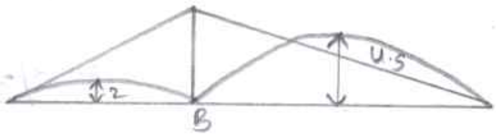
A continuoues beam ABC cover to casigative span AB and BC of length 4m & 6m carries a UDL of 6KN/m and 10 KN/m respectively If the ends A and C are simply supported find the support moment A, B and C also draw the bending moment diagram.

sol) Given data



$l_1 = 4m\ (AB)$

$l_2 = 6m\ (BC)$



$AB\ (UDL_1) = 6\ KN/m$ (Load on span AB)

$BC\ (UDL_2) = 10\ KN/m$ (Load on span BC)

∴ since ends A & C are simply supported the support moments at A and C will be zero.

$m_A = m_C = 0$

∴ To find support moment at B apply clapeyrons equation.

$$m_A l_1 + 2m_B(l_1+l_2) + m_C l_2 = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}$$

$$2m_B(l+l_2) = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}$$

$$\boxed{2m_B(4+6) = \frac{6a_1\bar{x}_1}{4} + \frac{6a_2\bar{x}_2}{6}}$$

$$2m_B = \frac{6a_1\bar{x}_1}{4} + a_2\bar{x}_2 \rightarrow ①$$

$$Bm \text{ on } AB = \frac{\omega_1 l_1^2}{8} = \frac{6\times4^2}{8} = 12 \text{ kN-m.}$$

$$Bm \text{ on } BC = \frac{\omega_2 l_2^2}{8} = \frac{10\times6^2}{8} = 45 \text{ kN-m}$$

$\Rightarrow$ Now a_1 = area Bmd on span AB due to vertical loads.

$$= 2/3 \times AB \times Bm$$

$$= 2/3 \times 4 \times 12 = 32$$

$$\Rightarrow \bar{x}_1 = l_1/2 = 4/2 = 2$$

$$a_2 = 2/3 \times BC \times Bm = 2/3 \times 6 \times 45 = 180$$

$$\Rightarrow \bar{x}_2 = \frac{l_2}{2} = 6/2 = 3$$

Substitute about values in eq ①

$$20m_B = \frac{6\times32\times2}{4} + 180\times3$$

$$20m_B = 636$$

$$m_B = \frac{636}{20} = 31.8 \text{ kN-m.}$$

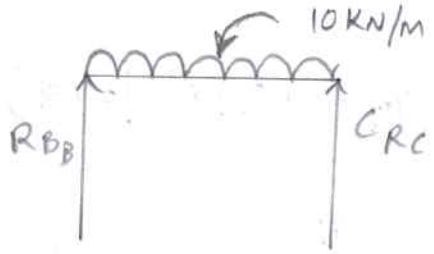
$\Rightarrow$ span AB Taking moments around B.

$$\Rightarrow R_A\times4 - 6\times4\times4/2 + m_B = 0$$

$$R_A\times4 = 6\times4\times4/2\times31.8$$

$4R_A = 16.2$

$R_A = \frac{16.2}{4} = 4.05$



span BC

$R_C \times 6 - 10 \times 6 \times 6/2 + m_B = 0$

$R_C = 24.71 KN$

RB

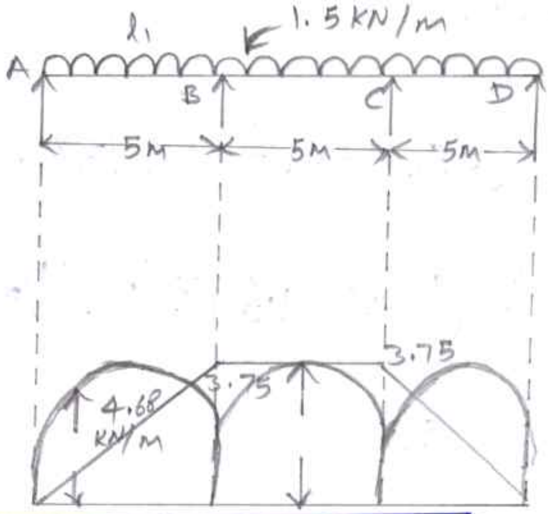
$R_A + R_B + R_C = 6 \times 4 + 10 \times 6$

$R_B = 24 + 60 - 4.05 - 24.7$

$R_B = 55.25 KN$

problem:-2

A continuous beam ABCD simply supported at A, B, C, D is loaded as shown in figure. Find the moments over the beam and draw SFD and BMD.



sol) Given data

$l_1 = 5m$

$l_2 = 5m$

$l_3 = 4m$

For ABC

$$\boxed{m_A l_1 + 2m_B(l_1 + l_2) + m_C l_2 = \frac{6a_1\bar{x}_1}{l_2} + \frac{6a_2\bar{x}_2}{l_2}}$$

$$6m_A + 2m_B(11) + 5m_C = \frac{\cancel{6}a_1\bar{x}_1}{\cancel{6}} + \frac{6a_2\bar{x}_2}{5}$$

$$0 + 22m_C + 5m_C = a_1\bar{x}_1 + \frac{6a_2\bar{x}_2}{5}$$

$$a_1\bar{x}_1 = (1/2 \times 2 \times 12) \times (2/3 \times 2) + (1/2 \times 4 + 2)(2 + (1/3 \times 4))$$

$$= 96.0$$

$$a_2\bar{x}_2 = \left[(1/2 \times 3 \times 9.6) \times (2/3 \times 3)\right] + \left[(1/2 \times 2 \times 9.6) \times (3 + (1/3 \times 2)\right)$$

$$= 64.0$$

$\Rightarrow$ substitute in eq ① we get.

$$22m_C + 5m_C = 96.0 + \frac{6 \times 64}{5}$$

$$22m_C + 5m_C = 172.8 \longrightarrow Ⓐ$$

$\Rightarrow$ span BCD

$l_1 = 5,\ l_2 = 4$

$$a_2\bar{x}_2 = \left((1/2 \times 2 \times 9.6) \times (2/3 \times 2)\right)$$

$$+ (1/2 \times 3 \times 9.6) \times (2 + (1/3 \times 3))$$

$$= 56.$$

$$a_3\bar{x}_3 = 2/3 \times 2 \times 6 \times 4/2 = 32$$

$$m_B(5) + 2m_C(9) + 4m_B = \frac{6 \times 56}{5} \times \frac{6 \times 36}{4}$$

$5M_B + 18M_E = 115.2 \longrightarrow (B)$

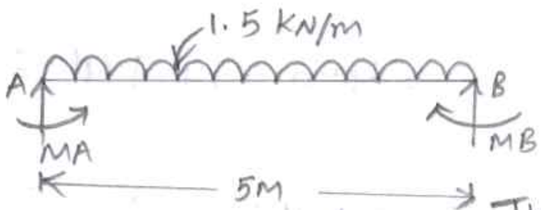
we get

$M_B = 6.831 KN-m$

$M_C = 4.50 KN-m$

Reactions:-

Span AB



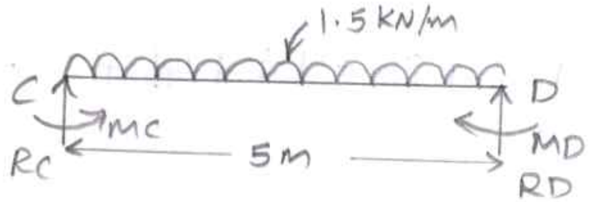
This is Correct

$\Sigma M_B = 0$

$(R_A \times 6) - (9 \times 4) + M_B = 0$

$R_A = 4.86 KN.$

Span CD

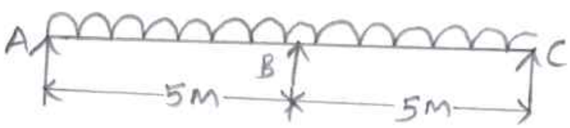


$\Sigma M_C = 0$

$= R_D \times 4 - (3 \times 4 \times 4/2) + M_C = 0$

$R_D = 4.87 KN.$

Span ABC



$= \Sigma M_C = 0$

$= R_A \times 11 + R_B(5) - 9(9) - 8(3) + M_C$

$R_B = 9.41 KN.$

$R_A + R_B + R_C + R_D = 9 + 8 + 3(4)$

$R_C = 9.86 KN.$

Shear force.

$SF @ A = R_A = 4.86\,kN.$

$SF @ B^L = 4.86 - 9 = -4.14\,kN.$

$SF @ B(R) = 4.86 - 9 + 9.41 = 5.27\,kN.$

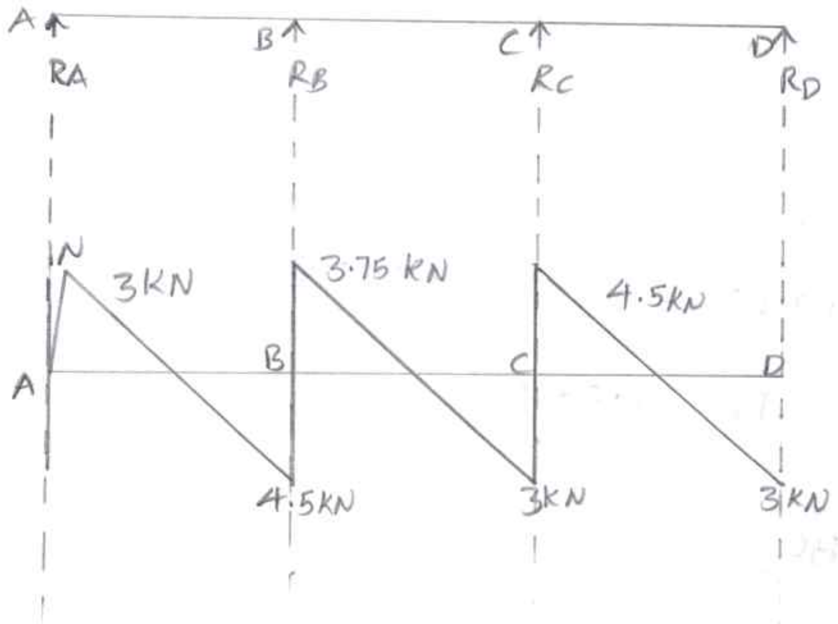
$SF @ C \text{ just left} = 4.86 - 9 + R_B - 8$

$= 4.86 - 9 + 9.41 - 8 = -2.73$

$SF @ C \text{ just right} = 4.86 - 9 + 9.41 + R_C - 8$

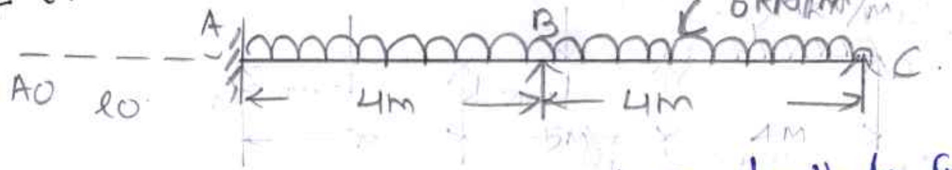
$= 4.86 - 9 + 9.41 + 9.86 - 8 = 7.13\,kN.$

$SF @ D = 4.86 - 9 + 9.41 + 9.86 - 3 \times 4 - 8 = -4.87\,kN.$



problem

A continous beam ABC of uniform section with AB&B as 4m each is fixed at 'A' and simply support at B&C' the beam is carrying a uniformly distribution lead of 6 KN/m run throughout its length find support moments & reactions using three moment theorem. Also draw SFD&BMD.



sol)

considering imaginary point Ao having length to from support 'A'

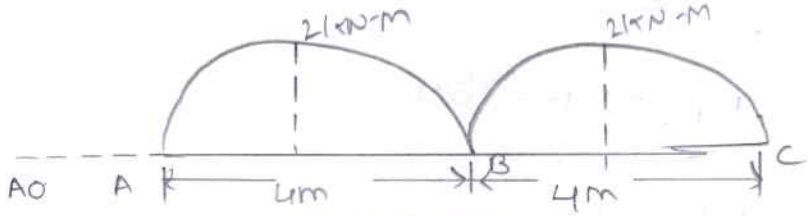
(i) Free bending moment:-

For span AOA = 0

For span AB $= \frac{\omega l^2}{8} = \frac{6\times4^2}{8} = 12$ KN-m

For span BC $= \frac{\omega l^2}{8} = \frac{6\times4^2}{8} = 12$ KN-m

(ii) Draw free bending moment diagram



For span AA = A = 0, $\bar{x} = 0$

For span AB = $A = 2/3 \times 12\times4 = 32$, $x_l = x_R = 4/2 = 2$m.

For span BC = $A = 2/3 \times 12\times4 = 32$, $x_l = x_R = 4/2 = 2$m

(iii) Apply three moment theorem.

For span A_0A & AB:-

$$M_{A0} + 2M_C(l_0+l_1) + M_B(l_1) = -\frac{6A_0\bar{x}_0}{l_0} - \frac{6A_1\bar{x}_1}{l_1}$$

$$0 + 2M_A(0+4) + M_B(4) = 0 - \frac{6\times 32\times 2}{4}$$

$$8M_A + 4M_B = -96$$

$$2M_A + M_B = -24 \rightarrow (1)$$

For span AB & BC:- Support C is simple support $M_C = 0$

$$M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 = \frac{-6A_1\bar{x}_1}{l_1} - \frac{6A_2\bar{x}_2}{l_2}$$

$$4M_A + 2M_B(4+4) + 0 = -96 - 96$$

$$4M_A + 16M_B = -192$$

$$M_A + 4M_B = -48 \rightarrow (2)$$

From eq (1) & eq (2)

$$2(-48 - 4M_B) + M_B = -24$$

$$-96 - 8M_B + M_B = -24$$

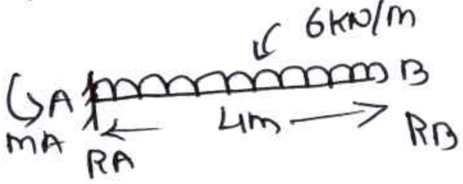
$$-7M_B = 72 \Rightarrow M_B = -10.28\,\text{kN-m}$$

$$M_A = -6.88\,\text{kN-m}$$

Final moments $M_A = -6.88$ kN-m, $M_B = -10.88$ kN-m $M_C = 0$

Reactions:-

For span AB:-



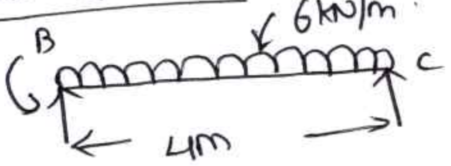
$\Sigma V = 0 \Rightarrow R_A + (R_B)_1 = 6 \times 4 = 24$

$\Sigma M_A = 0 \Rightarrow -(R_B)_1 \times 4 + 6 \times 4 \times 4/2 - M_A = 0$

$(R_B)_1 = 10.28 KN.$

$R_A = 13.72 KN.$

For span BC:-



$\Sigma V = 0 \Rightarrow R_C \times 4 + 6 \times 4 \times 4/2 - M_B = 0$

$R_C = 9.43 KN.$

$(R_B) = 14.57 KN.$

Final reactions:-

$R_A = 13.72 KN.$

$R_B = (R_B)_1 + (R_B)_2 = 10.28 + 14.57 = 24.85 KN.$

$R_C = 9.43 KN.$

ANNAMACHARYA UNIVERSITY

EXCELLENCE IN EDUCATION; SERVICE TO SOCIETY

ESTD, UNDER AP PRIVATE UNIVERSITIES (ESTABLISHMENT AND REGULATION) ACT, 2016)

Rajampet, Annamayya District, A.P – 516126, INDIA

CIVIL ENGINEERING

Structural Analysis

UNIT -2

1. Basic Analysis of indeterminate structures

Introduction:-

When an external load act on a structure, the structure undergoes deformation and hence, the work is done. To resist these external forces, the internal forces develop gradually from zero to final value and work is done. This internal workdone is stored as energy in the structure and it helps the structure to spring basis to the original shape and size, whenever the external loads are removed. provide the material of the structure is still within elastic limit.

when equilibrium is reached, as per the wall known law of conservation of energy, the work done by the external forces must equal the strain energy stored. this concept of energy balance is utilized in structural analysis to develop a member of methods to find deflection of structures. The following methods are finding the deflection of beams & frames.

(i) strain energy/Real work method.

(ii) virtual work/ unit load method.

(iii) castiglianos method.

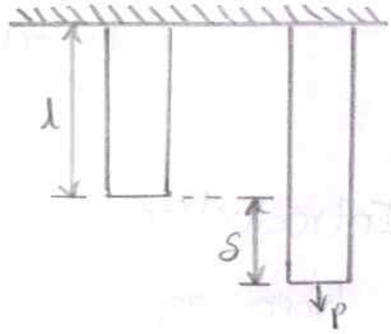
Strain energy:-

when an elastic body is subjected to external forces it will deform. If the elastic limit is not exceeded, the workdone in straining the material is stored in the form of resilience of internal energy this is known as strain energy

(or)

Internal energy stored in a body within elastic limit of elastic body is known as strain energy

strain energy due to axial loading



Let l = Length of member

A = Area of c/s of member.

δ = extension of member

P = External load.

Since the load is applied gradually form zero to 'P' the member is also gradually extended.

∴ External workdone by force (W_e) = Average force x distance

$$= \left(\frac{0+P}{2}\right) \times \delta$$

$$W_e = \tfrac{1}{2} \times P \times \delta \longrightarrow (1)$$

Let internal workdone (or) strain energy $= W_i = U \rightarrow (2)$

From law of conservation of energy.

Internal workdone = External workdone

i.e eq(1) = eq(2)

$$U = \tfrac{1}{2} \times P \times \delta \longrightarrow (3)$$

But we know $\delta = \dfrac{Pl}{AE} \longrightarrow (4)$

$$\left[\begin{array}{l} \therefore \sigma \propto e \\ P/A = E \cdot \dfrac{\delta}{l} \\ \delta = \dfrac{Pl}{AE} \end{array}\right]$$

From eq(3) & eq(4)

$$U = \tfrac{1}{2} \times P \times Pl/AE$$

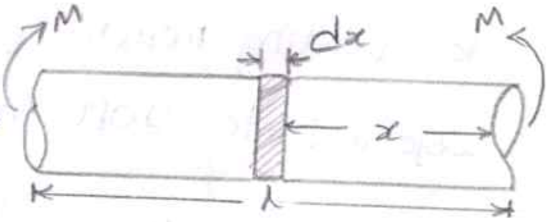
$$\boxed{U = \frac{P^2 l}{2AE}}$$

If however the bar has variable area of c/s consider a small steel bar of length dx and area of c/s A. The strain energy in small element of length dx, is $du = \dfrac{P^2 \cdot dx}{2AE}$

Total strain energy $\boxed{U = \int_0^l \frac{P^2 \cdot dx}{2AE}}$

strain energy due to bending moment:-

consider a member of length (l) subjected to uniform bending moment (m). consider a small element of length 'dx' let 'dθ' is the change in slope.



so strain energy stored in the element

$$du = \frac{1}{2} \times m \times d\theta \rightarrow (1)$$

But we know $\frac{m}{EI} = \frac{d^2y}{dx^2}$

$$\frac{m}{EI} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$\frac{m}{EI} = \frac{d}{dx}(\theta) = \frac{d\theta}{dx} \qquad \left(\because \theta = \frac{dy}{dx}\right)$$

$$\Rightarrow d\theta = \frac{m}{EI} \times dx \rightarrow (2)$$

from eq (1) & eq (2)

$$du = \frac{1}{2} \times m \times \frac{m}{EI} dx$$

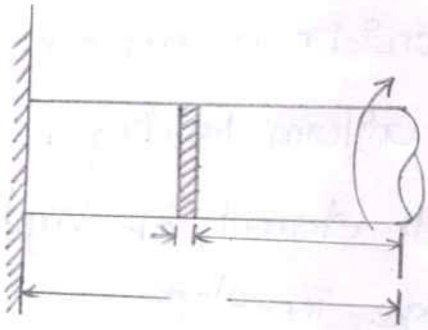
$$\boxed{du = \frac{m^2 dx}{2EI}}$$

The total strain energy

$$\boxed{U = \int_0^l \frac{m^2 dx}{2EI} \quad \text{or} \quad U = \frac{1}{2EI}\int_0^l m^2 dx}$$

Strain energy due to torsion:

consider a shaft of length(l) subjected to twisting moment (T) when torsion is subjected to shaft it will produce twist let 'θ' be the angle of twist.



$\therefore$ work done by external force $(W_e) = \frac{1}{2} \times T \times \theta \rightarrow (1)$

Internal workdone (or) strain energy $(W_i) = U \rightarrow (2)$

From law of conservation of energy

$$eq(1) = eq(2)$$

$$U = \frac{1}{2} \times T \times \theta \rightarrow (3)$$

But we know from torsion equation

$$\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{r}$$

$$\frac{T}{J} = \frac{G\theta}{l}$$

$$\theta = \frac{Tl}{GJ} \rightarrow (4)$$

From eq (3) & eq (4)

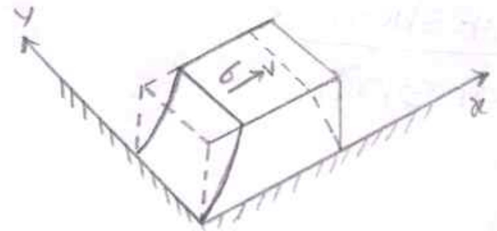
$$U = \frac{1}{2} \times T \times \frac{Tl}{GJ}$$

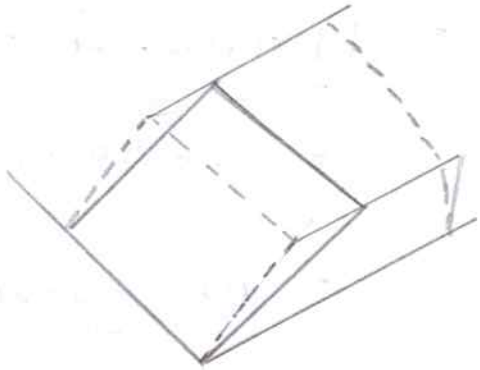
$$\boxed{U = \frac{1}{2} \times \frac{T^2 l}{GJ}}$$

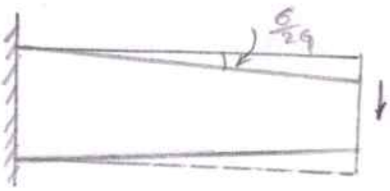
or

$$\boxed{U = \int_0^l \frac{T^2 dx}{2GJ}}$$

strain energy due to transverse shear:







The shear stress on a c/s of beam of rectangular c/s may be found out by the relation

$$\tau = \frac{VQ}{bI_{zz}}$$

where Q = First moment of portion of c/s above the point where shear stress is reqd about NA

V = Transverse shear force

b = Width of section

I_{zz} = MOI of the section about NA.

due to shearstress, the angle between the lines of right angle will change. The shear stress varies across the height in a parabolic manner in the case of rectangular c/s. Also the shear stress distribution is different for different shape of crosssection. However to simplify the computation of shear stress is assumed to be uniform across the cross-section. consider segment of length "dx" subjected to shear stress τ. The shearstress across the c/s may be taken as

$$\tau = k \times V/A \rightarrow (1)$$

where k = factor depend on c/s.

deformation $d\delta = \Delta\gamma \cdot dx \rightarrow (2)$

But we know $G = \frac{\tau}{\Delta\gamma} = \frac{\text{shear stress}}{\text{shear strain}}$

$$\Rightarrow \Delta\gamma = \frac{\tau}{G} \rightarrow (3)$$

from eq (1) & (2)

$$\Delta\gamma = K \times \frac{V}{AG}$$

from eq (1). deformation $d\delta = K \cdot \frac{V}{AG} \times dx$

Total deformation $\delta = \int_0^l K \cdot \frac{V}{AG} dx$.

strain energy $U = \frac{1}{2} \times V \times \delta$

$$U = \frac{1}{2} \times V \times \int_0^l K \cdot \frac{V^2}{AG} dx$$

$$\boxed{U = \int_0^l \frac{KV^2}{2AG} \cdot dx}$$

<u>strain energy</u>:-

(i) Due to axial loading $= \int_0^l \frac{P^2}{2AE} \cdot dx$

(ii) Due to Bending $= \int_0^l \frac{M^2}{2EI} \cdot dx$

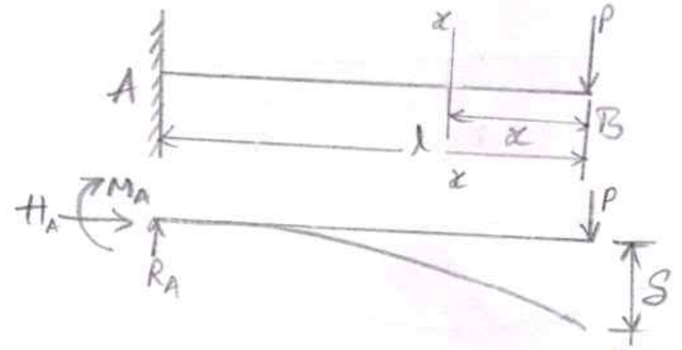
(iii) Due to twisting $= \int_0^l \frac{T^2}{2GJ} \cdot dx$.

(iv) Due to shear $= \int_0^l \frac{V^2}{2AG} dx$.

problem:- 1

Find the deflection of free end of cantilever carrying a point load of free end using strain energy principle

sol)



Now Bm of section x - x from free end $m = Px$

$$\text{strain energy } U = \int_0^l \frac{m^2 dx}{2EI}$$

$$= \int_0^l \frac{(Px)^2 dx}{2EI} = \int_0^l \frac{P^2 x^2 dx}{2EI}$$

$$= \frac{P^2}{2EI} \cdot \int_0^l x^2 \cdot dx$$

$$= \frac{P^2}{2EI}\left(\frac{x^3}{3}\right)_0^l = \frac{P^2}{2EI}\left(\frac{l^3}{3} - 0\right)$$

$$U = \frac{P^2 l^3}{6EI} \rightarrow (2)$$

work done by external load $= \frac{1}{2} \times P \times \delta \rightarrow (2)$

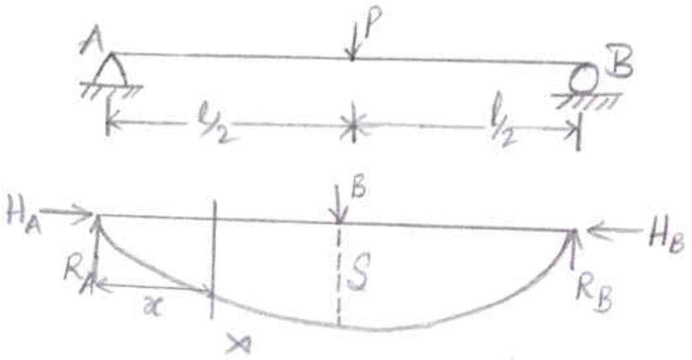
eq(1) = eq(2)

$$\frac{P^{\cancel{2}} l^3}{\cancel{6}EI} = \frac{1}{\cancel{2}} \times \cancel{P} \times \delta$$

$$\boxed{\delta = \frac{Pl^3}{3EI}}$$

Problem②:- A Beam of span 'l' carries a concentrated load 'P' at midspan find central deflection using strain energy principle.

sol)



Reaction at each end $= R_A = R_B = P/2$

Bending moment at section $x-x$, $m = R_A \times x = P/2 \times x$

strain energy stored by half of the beam $= \int_0^{l/2} \frac{m^2}{2EI} dx$

Total strain energy $U = 2\int_0^{l/2} \frac{m^2}{2EI} dx$

$$= 2\int_0^{l/2} \left(P/2\, x\right)^2 \times \frac{1}{2EI} dx$$

$$= \frac{2}{2EI}\int_0^{l/2} \frac{P^2}{4} x^2 \cdot dx$$

$$= \frac{P^2}{4EI}\int_0^{l/2} x^2 \cdot dx = \frac{P^2}{4EI}\left(\frac{x^3}{3}\right)^{l/2}$$

$$= \frac{P^2}{4EI}\left[\frac{\left(\frac{l}{2}\right)^3}{3} - 0\right]$$

$$= \frac{P^2}{4EI}\left[\frac{l^3}{8\times 3}\right]$$

$$U = \frac{P^2 l^3}{96EI} \longrightarrow ①$$

workdone by external force $= \frac{1}{2} \times P \times \delta \rightarrow (2)$

$$eq(1) = eq(2)$$

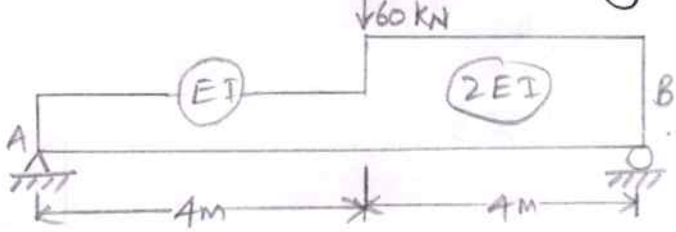
$$\frac{P^2 l^3}{\underset{48}{\cancel{96}} EI} = \frac{1}{\cancel{2}} \times \cancel{P} \times \delta$$

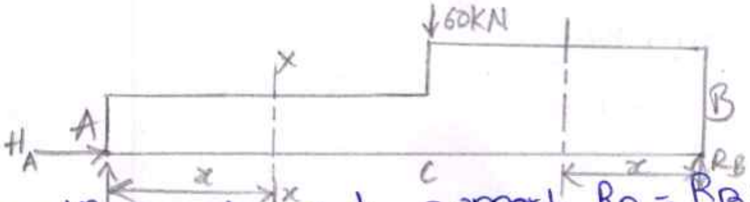
$$\boxed{\delta = \frac{P l^3}{48 EI}}$$

problem:-3

Determine deflection under 60KN load by using strain energy method.

sol)





Reactions at each support $R_A = R_B = 60/2 = 30kN$.

Bm at section x-x from support A $= R_A x = 30x$.

Bm at section x-x from support B $= R_B x = 30x$

strain enery stored in portion AC $= \int_0^4 \frac{m^2}{2EI} dx = \int_0^4 \frac{(30x)^2}{2EI} dx$.

strain enegy stored in position BC $= \int_0^4 \frac{m^2}{2(2EI)} dx = \int_0^4 \frac{(30x)^2}{4EI} dx$.

Total strain energy stored in member.

$$U = \int_0^4 \frac{(30x)^2}{2EI} dx + \int_0^4 \frac{(30x)^2}{4EI} dx$$

$$= \frac{\cancel{960}^{450}}{\cancel{2}EI} \cdot \int_0^4 x^2\,dx + \frac{\cancel{900}}{\cancel{4}EI} \int_0^4 x^2 \cdot dx$$

$$= \frac{450}{EI}\left(\frac{x^3}{3}\right)_0^4 + \frac{225}{EI}\left(x^3/3\right)_0^4$$

$$= \frac{450}{EI}\left(\frac{64}{3}\right) + \frac{225}{EI}\left(\frac{64}{3}\right)$$

$$U = \frac{14400}{EI} \rightarrow (1)$$

But external workdone $W_e = 1/2 \times P \times \delta = 1/\cancel{2} \times \cancel{60}^{30} \times \delta = 30\delta \rightarrow (2)$

$$eq(1) = eq(2)$$

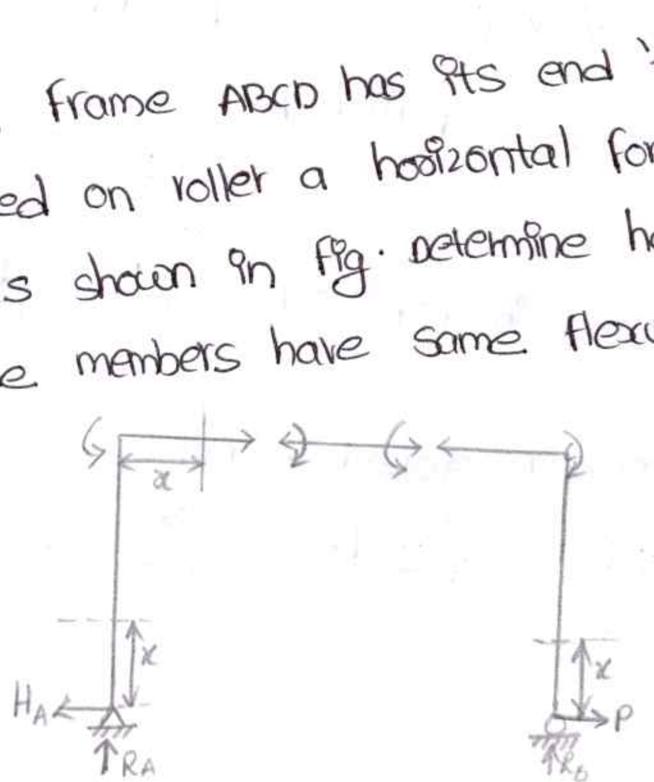
$$\frac{1440}{EI} = 30\delta$$

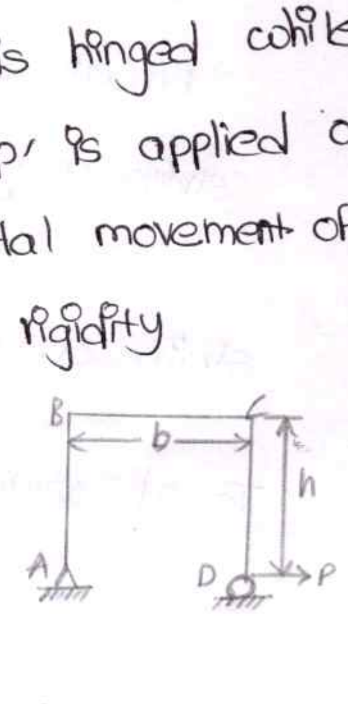
$$\delta = \frac{14400}{30EI}$$

$$\delta = \frac{480}{EI}$$

Problem 4:-

A portal frame ABCD has its end 'A' is hinged while end 'D' is placed on roller a horizontal force 'p' is applied on the end 'D' as shown in fig. Determine horizontal movement of 'D'. Assume the members have same flexural rigidity.





$R_A = R_B = 0,$
$H_A = P$

From above Fig. the Bm expression for various portion are

Portion	AB	BC	CD
origin	A	B	D
limit	o-h	o-b	o-h
mx	$H_A x = Px$	Ph	Px

Total strain energy stored in frame

$$U = \int_0^h \frac{m^2}{2EI}dx + \int_0^b \frac{m^2}{2EI}dx + \int_0^h \frac{m^2}{2EI}dx.$$

$$U = \int_0^h \frac{(Px)^2}{2EI}dx + \int_0^b \frac{(Ph)^2}{2EI}dx + \int_0^h \frac{(Px)^2}{2EI}dx.$$

$$= 2\int_0^h \frac{P^2x^2}{2EI}dx + \int_0^b \frac{P^2h^2}{2EI}dx$$

$$= \frac{\cancel{2}P^2}{\cancel{2}EI}\int_0^h x^2dx + \frac{P^2h^2}{2EI}\int_0^b 1dx$$

$$= \frac{P^2}{EI}\left(\frac{x^3}{3}\right)_0^h + \frac{P^2h^2}{2EI}(x)_0^6$$

$$= \frac{P^2}{3EI}(h^3) + \frac{P^2h^2}{2EI}(b)$$

$$= \frac{P^2h^2}{EI}\left(h/3 + b/2\right) = \frac{P^2h^2}{EI}\left(\frac{2h+3b}{6}\right)$$

$$U = \frac{P^2h^2}{6EI}(2h+3b) \rightarrow (1)$$

External workdone (We) $= \frac{1}{2} \times P \times \delta \rightarrow (2)$

External workdone $(w_e) = \frac{1}{2} \times P \times \delta \rightarrow (2)$

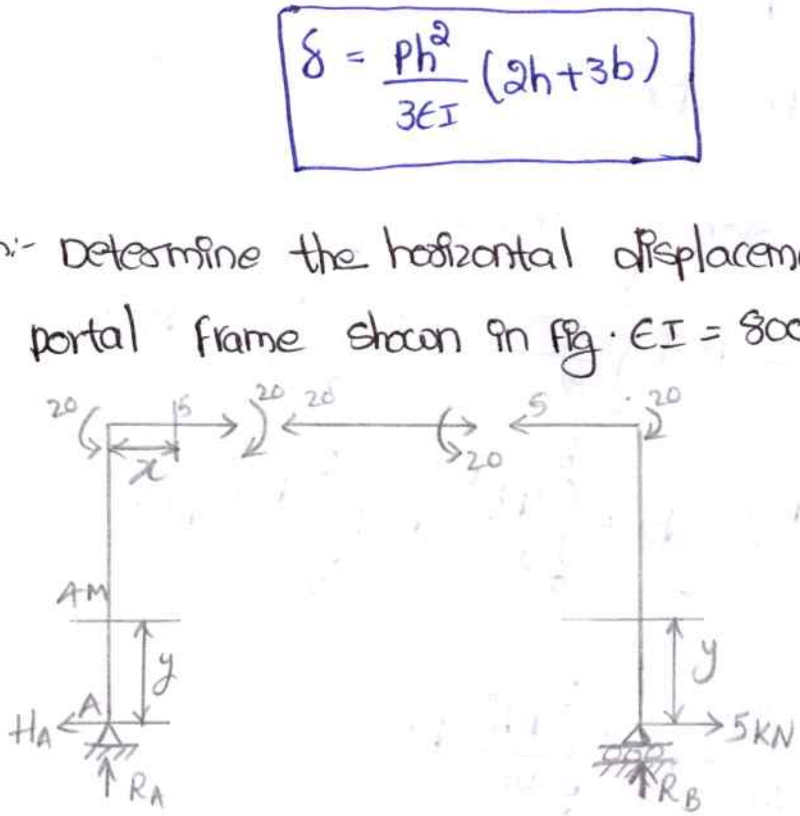
$$eq(1) = eq(2)$$

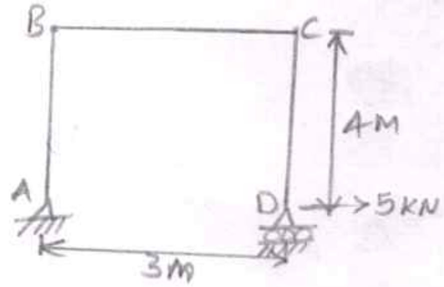
$$\frac{1}{2} \times P \times \delta = \frac{P^2 h^2}{6EI}(2h+3b)$$

$$\boxed{\delta = \frac{Ph^2}{3EI}(2h+3b)}$$

problem:- Determine the horizontal displacement of the roller end 'D' of the portal frame shown in fig. $EI = 8000 kN\text{-}m^2$ throughout

sol)





$R_A = R_B = 0$

$H_A = 5 KN$

From the above the Bm expression for various portions are

Portion	AB	BC	CD
origin	A	B	D
limit	0-4	0-3	0-4
m_x	$5x$	20	$5x$

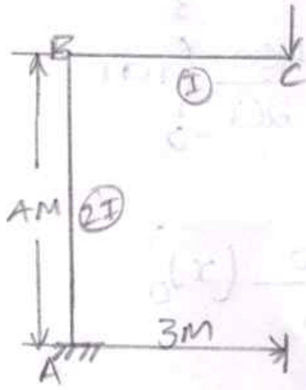
Total strain energy stored in frame

$$U = \int_0^4 \frac{m^2}{2EI}dx + \int_0^3 \frac{m^2}{2EI}dx + \int_0^4 \frac{m^2}{2EI}dx$$

$$U = \int_0^4 \frac{(5x)^2}{2EI}dx + \int_0^3 \frac{(20)^2}{2EI}dx + \int_0^4 \frac{(5x)^2}{2EI}dx$$

problem:- Determine the vertical deflection at point 'c' in the Frame (13)

shown in fig. Take $E = 200 kN/mm^2$, $I = 30 \times 10^6 mm^4$



strain energy method can be convinently used for finding deflection in structures only under the following conditions.

① The structure is subjected to single concentrated load

② Deflection reqd. is at loaded point and only & is in direction of the load.

castigliano's first theorem:-

statement:-

In a linear elastic structure, partial derivative of the strain energy with respect to load is equal to the deflection of the point where the load is acting the deflection being measured in the direction of load.

The load may be force (or) moment mathematically this theorem may be represented by

$$\boxed{\frac{\partial U}{\partial P_i} = \Delta_i \, , \; \frac{\partial U}{\partial m_j} = \theta_j}$$

where U = Total strain energy

P_i, m_j = load, Δ_i, θ_j = deflections.

$$= 2\int_0^4 \frac{(5x)^2 dx}{2EI} + \int_0^3 \frac{400}{2EI} dx$$

$$= \cancel{2} \times \frac{25}{\cancel{2}EI} \int_0^4 x^2 dx + \frac{400}{2EI} \int_0^3 1 dx$$

$$= \frac{25}{EI} \left(\frac{x^3}{3}\right)_0^4 + \frac{200}{EI} (x)_0^3$$

$$U = \frac{25}{EI} \left(\frac{64}{3}\right) + \frac{200}{EI} (3)$$

$$= \frac{25}{EI} \left(\frac{64}{3EI} + 24\right) = \frac{25}{EI} \left(\frac{64+72}{3}\right)$$

$$= \frac{25}{EI} \left(\frac{136}{3}\right)$$

$$U = \frac{1133.33}{EI} \rightarrow (1)$$

External workdone $W_e = \frac{1}{2} \times P \times \delta \rightarrow (2)$

eq (1) = eq (2)

$$\frac{1}{2} \times P \times \delta = \frac{1133.33}{EI}$$

$$\frac{1}{2} \times 2.5 \times \delta = \frac{1133.33}{EI}$$

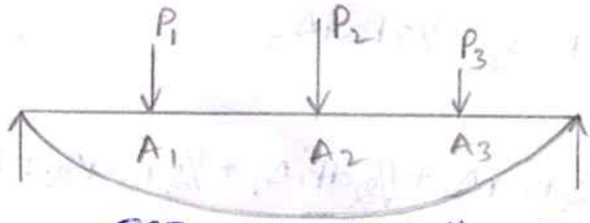
$$\delta = \frac{1133.33}{2.5 EI} = \frac{1133.33}{2.5 \times 8000}$$

$$\delta = 0.0567 m$$

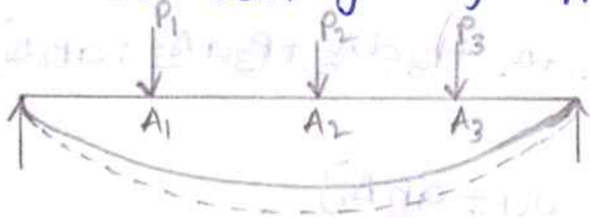
$$\boxed{\delta = 56.67 mm}$$

Proof:-

consider a SSB shown in fig on which loads P_1, P_2 & P_3 are applied gradually. let deflections under the loads P_1, P_2, P_3 be Δ_1, Δ_2 & Δ_3 respectively.



SSB with gradually applied loads.



Beam Subjected to additional load.

Total strain energy $U = \frac{1}{2} P_1 \Delta_1 + \frac{1}{2} P_2 \Delta_2 + \frac{1}{2} P_3 \Delta_3 \rightarrow (1)$

let the additional load dP_1 be added after the loads P_1, P_2 & P_3 let the additional deflection be $d\Delta_1, d\Delta_2$ & $d\Delta_3$

Additional strain energy $du = \frac{1}{2} dP_1 d\Delta_1 + P_1 d\Delta_1 + P_2 d\Delta_2 + P_3 d\Delta_3$

$$dU = P_1 d\Delta_1 + P_2 d\Delta_2 + P_3 d\Delta_3 \rightarrow (2)$$

$$U + du = \frac{1}{2} P_1 \Delta_1 + \frac{1}{2} P_2 \Delta_2 + \frac{1}{2} P_3 \Delta_3 + \frac{1}{2} dP_1 d\Delta_1 + P_1 d\Delta_1 +$$

$$P_2 d\Delta_2 + P_3 d\Delta_3 \rightarrow (3)$$

If $(P_1 + dP_1), P_2$ & P_3 were applied simultaniously strain energy stored

$$= \frac{1}{2} (P_1 + dP_1)(\Delta_1 + d\Delta_1) + \frac{1}{2} P_2 (\Delta_2 + d\Delta_2) + \frac{1}{2} P_3 (\Delta_3 + d\Delta_3) \rightarrow (4)$$

eq(3) = eq(4)

$$U + du = \left[\frac{1}{2} (P_1 + dP_1)(\Delta_1 + d\Delta_1) + \frac{1}{2} \left[P_2 \Delta_2 + d\Delta_2 P_2 \right] + \frac{1}{2} \left[P_3 \Delta_3 + P_3 d\Delta_3 \right) \right.$$

$$= \frac{1}{2}\left[P_1\Delta_1 + P_1 d\Delta_1 + dP_1\Delta_1 + dP_1 d\Delta_1\right] + \frac{1}{2}\left[P_2\Delta_2\right] + \frac{1}{2}\left[P_2 d\Delta_2\right]$$

$$+ \frac{1}{2} P_3\Delta_3 + \frac{1}{2} P_3 d\Delta_3$$

$$= \frac{1}{2} P_1\Delta_1 + \frac{1}{2} P_1 d\Delta_1 + \frac{1}{2} dP_1\Delta_1 + \frac{1}{2} dP_1 d\Delta_1 + \frac{1}{2} P_2\Delta_2 + \frac{1}{2} P_2 d\Delta_2$$

$$+ \frac{1}{2} P_3\Delta_3 + \frac{1}{3} P_3 d\Delta_3$$

$$U + dU = U + \frac{1}{2} P_1 d\Delta_1 + \frac{1}{2} dP_1\Delta_1 + \frac{1}{2} P_2 d\Delta_2 + \frac{1}{2} P_3 d\Delta_3$$

$$\partial U = \frac{1}{2}\left[P_1 d\Delta_1 + P_2 d\Delta_2 + P_3 d\Delta_3 + dP_1\Delta_1\right]$$

$$2dU = \left[\partial U + dP_1\Delta_1\right]$$

$$\partial U = dP_1\Delta_1$$

$$\boxed{\frac{\partial U}{\partial P_1} = \Delta_1}$$ similarlly

$$\frac{\partial U}{\partial P_2} = \Delta_2 - - - -$$

CIVIL ENGINEERING

Structural Analysis

UNIT -3

3. Slope deflection method

Introduction:-

- This method was first proposed by prof. George A. maney in 1915
- It is ideally suited to the analysis of continuous beams and rigid jointed frames.
- Basic unknows like slopes and deflections of joints are found out.
- moments at the ends of a member is first written down in terms of unknown slopes and deflections of end joints.
- considering the joint equilibrium conditions, a set of equations are formed and solutions of these simultaneous equations gives unknown slopes and deflections
- Then end moments of individual members are determined.
- It involves solutions of simultaneous equations, a problem with more than three unknows is considered a difficult problem for hand calculations. Hence this method was sidelined by moment distribution method with the help of computers; solutions for any number of simultaneous equations can be obtained early.
- The development of this method in the matrix from is "stiffness matrix method" (It is commonly used for the analysis of large structures with the help of computes.

Assumptions made in slop-deflection method.

- All joints are rigid.
- The rotations of joints are treated as unknowns.
- Between each pair of the supports the beam section is constant.
- The joint in structure may rotate or deflect as a whole, but the angles between the members meeting at that joint remain the same.
- Distortions due to axial deformations are neglected.
- Shear deformations are neglected.

Sign conventions:-

moments:-

- clockwise moments = $(+)^{ive}$
- Anti - clockwise moments = $(-)^{ive}$

Rotations:-

- clockwise rotations = $(+)^{ive}$
- Anti - clockwise rotations = $(-)^{ive}$

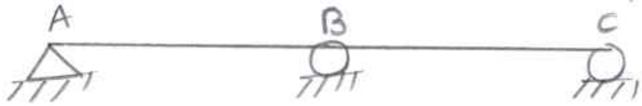
settlements:-

- Right side support is below left side support = $(+)^{ive}$
- Left side support is below right side support = $(-)^{ive}$

Applications of slope deflection equations:-

- Rigid jointed structure can be analyzed.
- continuous beams.
- frames without side sway (Non-sway)
- Frames with side sway (sway)

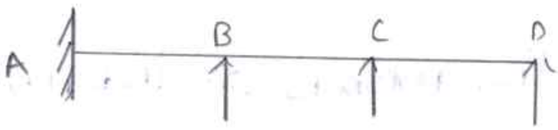
The beam shown in fig is to be analyzed by slope-deflection method. what are the unknowns and to determine them what are the conditions used.



unknowns $\theta_A, \theta_B, \theta_C$.

Equilibrium equations (i) $M_{AB}=0$ (ii) $M_{BA}+M_{BC}=0$ (iii) $M_{CB}=0$

write down the slope deflection equation for a fixed end support.



The slope deflection equation for end A is M_{AB}.

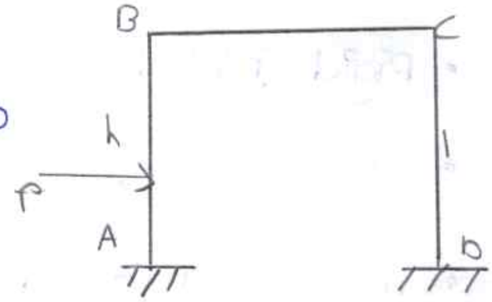
$$M_{AB} = M'_{AB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + \frac{3\Delta}{l}\right]$$

Here $\theta_A = 0$ since there is no support settlement $\Delta = 0$

$$M_{AB} = M'_{AB} + 2EI\left[\theta_B + 3\Delta\right]$$

write down the equilibrium equations for the frame shown in fig.

Unknowns : θ_B, θ_C



equilibrium equations: At B : $m_{BA} + m_{BC} = 0$

At C : $m_{CB} + M_{CD} = 0$

shear equation : $\dfrac{m_{AB} + m_{BA} - Ph}{l} + \dfrac{m_{CD} + m_{DC}}{l} + P = 0$

Limitations of slope deflection method.

- It is not easy to account for varying member sections.
- It becomes very cumbersome when the unknown displacements are large in number.

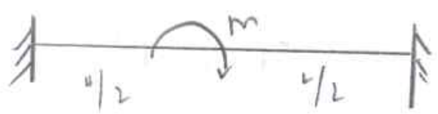
why slope-deflection method is called a displacement method?

- In slope-deflection method, displacements (like slopes & displacements) are treated as unknowns and hence the method is a "displacement method"

Degrees of freedom:-

- In a structure, the numbers of independent joint displacements that the structure can undergoes are known as degrees of freedom.

write the fixed end moments for a beam carrying a central clockwise moment.



Fixed end moments

$m'_{AB} = m'_{BA} = m/4$

Derivation of slope deflection equations.

let 'AB' beam shown in figure be a member of a rigid structure. after loading it undergoes deformations deform slope is shown in figure. will all displacements ($\theta_A, \theta_B, \Delta$) final moments at end "A" and end 'B" are "M_{AB} & M_{BA}" Now we can derive the relation ship between these final end moments and their displacements θ_A, θ_B and Δ"

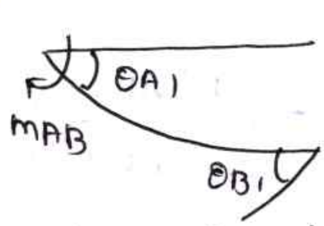
⇒ The development of final moments and deflections involves the following stages.

⇒ Due to given loadings end moments "M_{FAB} & M_{FBA}" develop without any rotations at ends. these moment are similar to end moment in a fixed beam and hence called as fixed end moments.

⇒ settalment "Δ" takes place without any rotations at ends the end moments are developed are

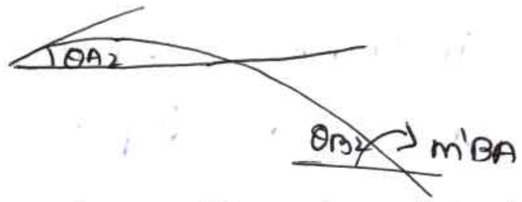
$$\boxed{\frac{6EI\Delta}{l^2}}$$

⇒ moments M'_{BA} comes into account in simple supported beam to cause end rotations "θ_{A1} & θ_{B1}" at A & B respectively



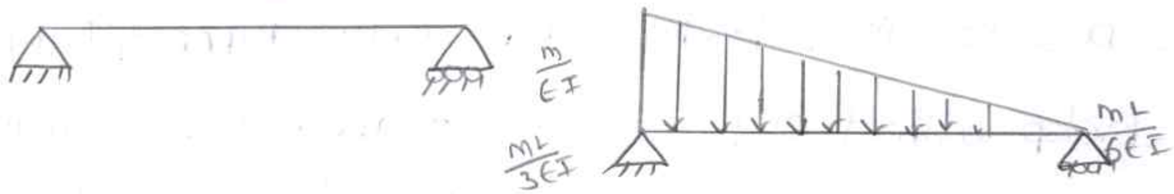
end rotation due to M'_{AB}.

⇒ moments m'_{BA} comes into account in simple supported beam 'AB' to cause end rotations moments "θ_{A2} & θ_{B2}"



End rotation due to m'_{BA}.

⇒ moments m'_{AB} and m'_{BA} gives final rotations θ_A & θ_B to the beam AB. to find the rotations due to applied moment 'm' in a beam without end rotation conjugate beam method may be used.



Now

$$\theta_{A1} = \frac{m'_{AB}L}{3EI}, \quad \theta_{B1} = \frac{m'_{AB}L}{6EI}$$

$$\theta_{A2} = \frac{m'_{BA}L}{6EI}, \quad \theta_{B2} = \frac{m'_{BA}L}{3EI}$$

Hence

$$\theta_A = \theta_{A1} - \theta_{A2} = \left(\frac{m'_{AB}L}{3EI}\right) \cdot \left(\frac{m'_{BA}L}{6EI}\right) \rightarrow (1)$$

$$\theta_B = \theta_{B2} - \theta_{B1} = \left(\frac{m'_{BA}L}{3EI}\right) - \left(\frac{m'_{AB}L}{6EI}\right) \rightarrow (2)$$

(i) $2\theta_A = 2\theta_A = \left(\frac{m'_{AB}L}{3EI}\right) - \left(\frac{m'_{BA}L}{6EI}\right)$

Solve eq ③ & ② we get

$$2\theta_A + \theta_3 = \frac{2m'_{AB}L}{3EI} - \frac{m'_{BA}L}{3EI} + \frac{m'_{BA}L}{3EI} - \frac{m'_{AB}L}{6EI}$$

$$\Rightarrow 2\theta_A + \theta_B = \frac{m'_{AB}L}{EI}\left(2/3 - 1/6\right)$$

$$\Rightarrow 2\theta_A + \theta_B = \frac{m'_{AB}L}{2EI}$$

Similarly

$$m'_{BA} = \frac{2EI}{L}(\theta_A + 2\theta_B)$$

∴ Final moments

$$m'_{BA} = m_{FAB} \Rightarrow \frac{6EI\Delta}{l^2} + m'_{AB}$$

$$= m_{FAB} - \frac{6EI\Delta}{l^2} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= m_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right) \rightarrow Ⓐ$$

$$= m_{BA} = m_{FAB} - \frac{6EI\Delta}{l^2} + m_{BA}$$

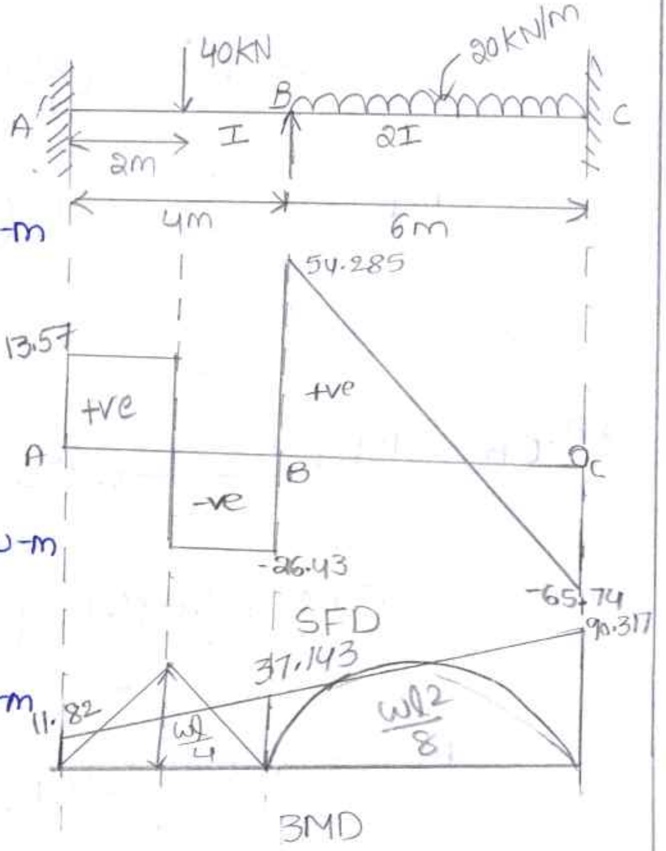
$$= m_{FAB} - \frac{6EI\Delta}{l^2} + \frac{2EI}{l}(\theta_A + 2\theta_B)$$

$$m_{BA} = m_{FBA} + \frac{2EI}{l}\left(\theta_A + 2\theta_B - \frac{3\Delta}{l}\right) \rightarrow B$$

equation "A & B" are known as slope deflection equations.

problem

Analyse the two span continuous beam shown in figure by slope deflection method. and also draw shear force and bending moment diagram.



sol) (i) Fixed end moments:-

$$m_{FAB} = \frac{-wl}{8} = \frac{-40\times4}{8} = -20\,kN\text{-}m$$

$$m_{FBA} = \frac{+wl}{8} = +20\,kN\text{-}m$$

$$m_{FBC} = \frac{-wl^2}{12} = -\frac{20\times6^2}{12} = -60\,kN\text{-}m$$

$$m_{FCB} = \frac{+wl^2}{12} = \frac{20\times6^2}{12} = +60\,kN\text{-}m$$

(ii) slope deflection equation.

$$\Rightarrow M_{AB} = m_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$$

$$= -20 + \frac{2EI}{4}\left(2(0) + \theta_B - 0\right)$$

$$= -20 + \frac{EI}{2}(\theta_B) \rightarrow ①$$

$$\Rightarrow M_{BA} = m_{FBA} + \frac{2EI}{l}\left(\theta_A + 2\theta_B - \frac{3\Delta}{l}\right) \rightarrow ②$$

$$= 20 + \frac{2EI}{\cancel{4}_2}\left(0 + \cancel{2}\theta_B - 0\right)$$

$$= 20 + EI\theta_B \rightarrow ②$$

$$\Rightarrow M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\Delta}{l}\right) \rightarrow ③$$

$$= -60 + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\Delta}{l}\right)$$

$$= -60 + \frac{2EI}{3}(2\theta_B + 0 - 0)$$

$$M_{BC} = -60 + \frac{4EI}{3}(\theta_B) \rightarrow ③$$

$$\Rightarrow M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\Delta}{l}\right)$$

$$= 60 + \frac{2E(2I)}{6}(0 + \theta_B - 0)$$

$$M_{CB} = 60 + \frac{2EI}{3}(\theta_B) \rightarrow ④$$

(iii) Equilibrium equations:-

@ Joint B

$$\Rightarrow 20 + EI(\theta_B) + \left(-60 + \frac{4EI}{3}\right)(\theta_B) = 0$$

$$\Rightarrow \frac{7EI\theta_B}{3} = 40$$

$$\Rightarrow EI\theta_B = 17.143$$

sub $EI\theta_B$ eq in ①, ②, ③ & ④

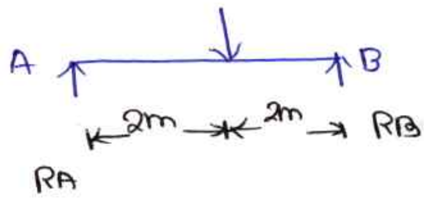
(iv) Final moments:-

$M_{AB} = -11.428$ kN-m $M_{BC} = -37.143$ kN-m

$M_{BA} = 37.143$ kN-m $M_{CB} = 11.428$ kN-m

(iv) Reactions:-

For span AB



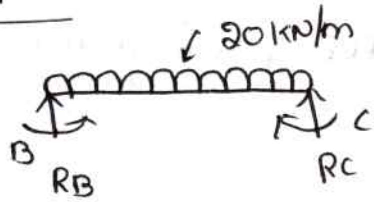
$= (R_A \times 4) - m_A + m_B = 0$

$= (R_A \times 4) - (40 \times 2) - m_A + m_B = 0$

$= (R_A \times 4) - (40 \times 2) - (11.428) + 37.143$

$R_A = 13.571 kN$

For span BC



$\Sigma m_B = 0$

$(R_C \times 6) - (20 \times 6 \times 6/2) - m_C + m_B = 0$

$(R_C \times 6) - (20 \times 6 \times 6/2) - (11.429) + 37.143$

$R_C = 65.71 kN$

$R_A + R_B + R_C = 40 + (20 \times 6)$

$= 40 + 120 - 13.571 - 65.714$

$R_B = 80.71 kN$

Shear force.

SF @ $A = R_A = 13.571\,kN$.

SF @ $B_{\text{Just left}} = R_A - 40 = -26.43\,kN$

SF @ B Just Right $= R_A - 40 + R_B = 54.28\,kN$.

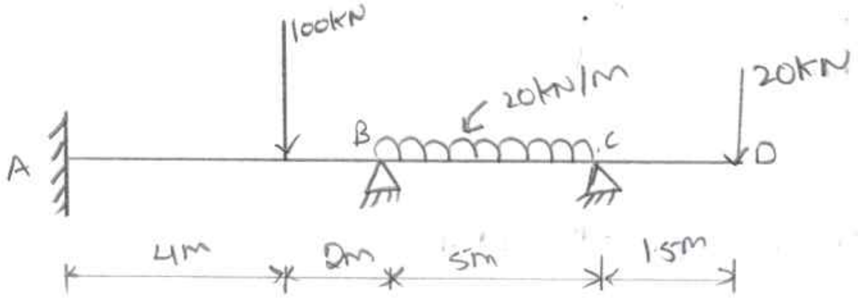
SF @ C Just left $= R_A - 40 + R_B - (20\times6) = 65.71\,kN$.

SF @ C Just Right $= R_A - 40 + R_B - (20\times6) - R_B = 0$.

Problem 2:-

Analyze continuous beam ABCD by slope deflection method and then draw bending moment and SF diagram. Take EI constant.

Sol)



(i) Fixed end moments:-

$$m_{FAB} = \frac{-wab^2}{l^2} = \frac{-100\times4\times2^2}{6^2} = -44.44\,kNm$$

$$m_{FBA} = \frac{+wa^2b}{l^2} = \frac{+100\times4^2\times2}{6^2} = +88.88\,kNm$$

$$m_{FBC} = \frac{+wl^2}{12} = \frac{+20\times5^2}{12} = +41.67\,kNm$$

$m_{FCD} = -20\times1.5 = -30\,kNm$.

(ii) slope deflection equation:-

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B) = -44.44 + 1/3\, EI\theta_B \rightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A) = +88.89 + 2/3\, EI\theta_B \rightarrow (2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C) = -41.67 + 4/5\, EI\theta_B + 2/5\, EI\theta_C \rightarrow (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}(2\theta_C + \theta_B) = +41.67 + 4/5\, EI\theta_C + 2/5\, EI\theta_B \rightarrow (4)$$

$$M_{CD} = -30\,kNm. \rightarrow (5)$$

Now $M_{BA} + M_{BC} = 88.89 + 2/3\, EI\theta_B - 41.67 + 4/5\, EI\theta_B + 2/5\, EI\theta_C$

$$= 47.22 + 22/15\, EI\theta_B + 2/5\, EI\theta_C \rightarrow (6)$$

And $M_{CB} + M_{CD} = +41.67 + 4/5\, EI\theta_C + 2/5\, EI\theta_B - 30$

$$= 11.67 + 2/5\, EI\theta_B + 4/5\, EI\theta_C \rightarrow (7)$$

$EI\theta_B = -32.67$ Rotation @ B anticlockwise.

$EI\theta_C = +1.75$ Rotation @ B clockwise.

(iii) Final moments:-

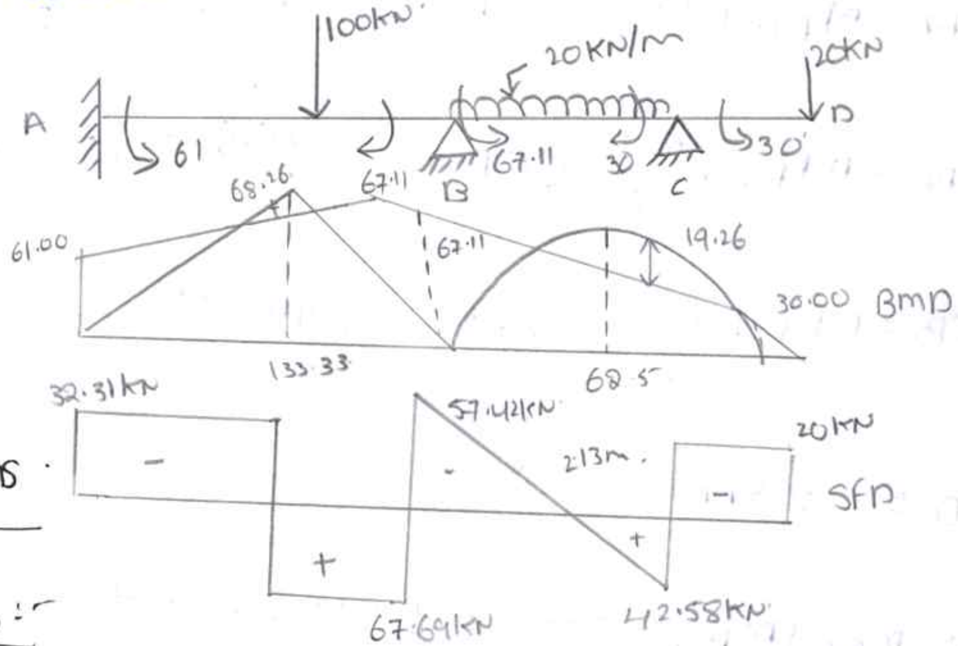
$M_{AB} = -44.44 + 1/2(-32.67) = -61.00\,kNm$

$M_{BA} = +88.89 + 2/3(-32.67) = +67.11\,kNm$

$M_{BC} = -41.67 + 4/5(-32.67) = +2/5(1.75) = -67.11\ kNm$

$M_{CB} = +41.67 + 4/5(1.75) + 2/5(-32.67) = +30.00\ kNm$

$M_{CD} = -30\ kNm$



Reactions:

Span AB:-

$R_B \times 6 = 100 \times 4 + 67.11 - 61$

$R_B = 67.69\ kN$

$R_A = 100 - R_B = 32.31\ kN$

Span BC:-

$R_C \times 5 = 20 \times 5/2 \times 5 + 30 - 67.11$

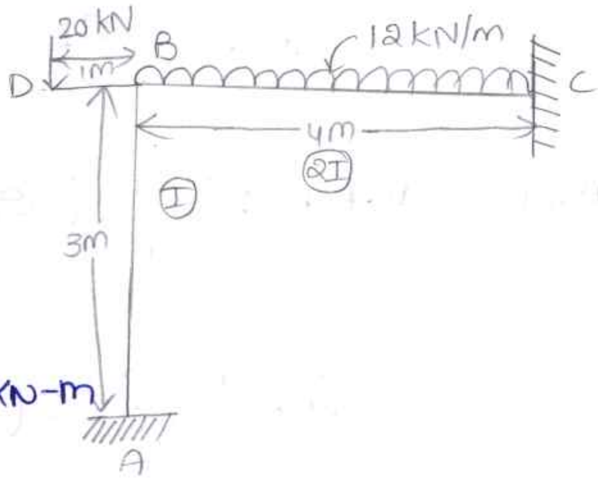
$R_C = 42.58\ kN$

$R_B = 20 \times 5 - R_B = 57.42\ kN$

Frames:-

problem

Analyse the Frame shown in Figure by slope deflection method and draw bending moment diagram



sol) (i) Fixed end moments:-

$M_{FAB} = 0$

$M_{FBA} = 0$

$M_{FBC} = \frac{-\omega l^2}{12} = -\frac{12 \times 4^2}{12} = -16 \text{ kN-m}$

$M_{FCB} = \frac{\omega l^2}{12} = \frac{12 \times 4^2}{12} = 16 \text{ kN-m}$

$M_{FBD} = (20 \times 1) = 20 \text{ kN-m}.$

(ii) slope deflection equations:-

$$\boxed{M_{FAB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)}$$

$$= 0 + \frac{2EI}{3}(2(0) + \theta_B - 0)$$

$$= 0.67\, EI\theta_3 \rightarrow ①$$

$$M_{FBA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\Delta}{l}\right)$$

$$= 0 + \frac{2EI}{3}(2\theta_B + 0 - 0)$$

$$= 1.3EI\theta_B \rightarrow ②$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\Delta}{l}\right)$$

$$= -16 + \frac{\cancel{2}EI}{\cancel{4}}(\cancel{2}\theta_B + 0 - 0)$$

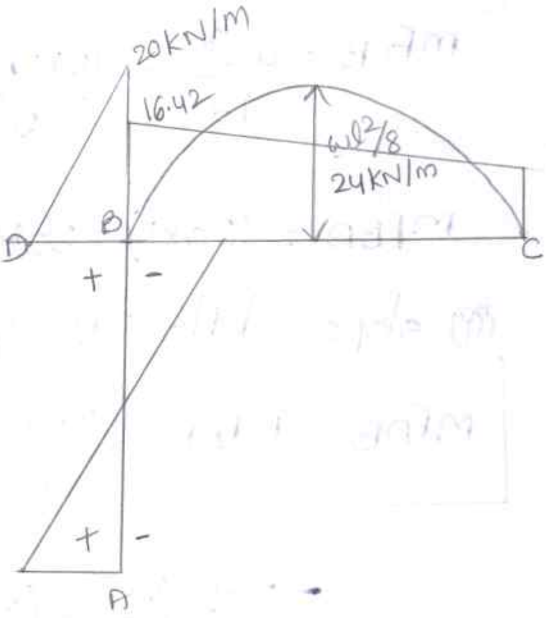
$$= -16 + EI\theta_B \rightarrow (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\Delta}{l}\right)$$

$$= 16 + \frac{2E(2I)}{4}(2(0) + \theta_B - 0)$$

$$= 16 + EI\theta_B \rightarrow (4)$$

$M_{BD} = 20\,kN\text{-}m$



(iii) Equilibrium equations:-

@ Joint B

$$M_{BA} + M_{BC} + M_{BD} = 0$$

$$= 1.3EI\theta_B + 2EI\theta_B - 16 + 20 = 0$$

$$= 1.3EI\theta_B + 2EI\theta_B = 20 + 6 = 0$$

$$= 3.3EI\theta_B = -4$$

$$= EI\theta_B = \frac{-4}{3.3}$$

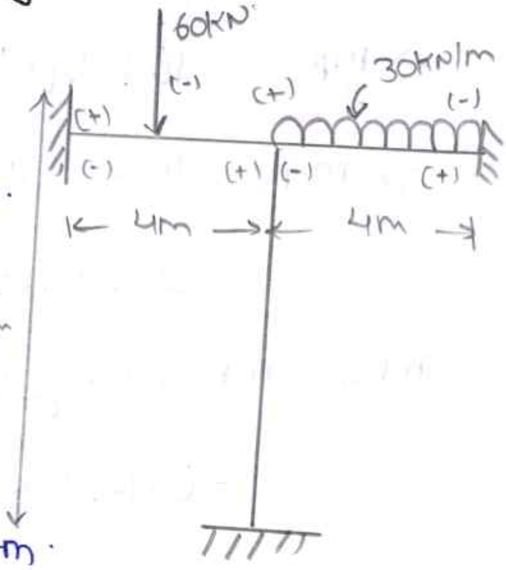
$$= EI\theta_B = -1.212$$

Final moments

$M_{AB} = -0.8\,mm$, $M_{BA} = -1.575$, $M_{BC} = 18.484\,mm$, $M_{CB} = 14.788\,mn$

problem:-2

Analyse the frame shown in figure by slope deflection method and draw bending moment diagram.



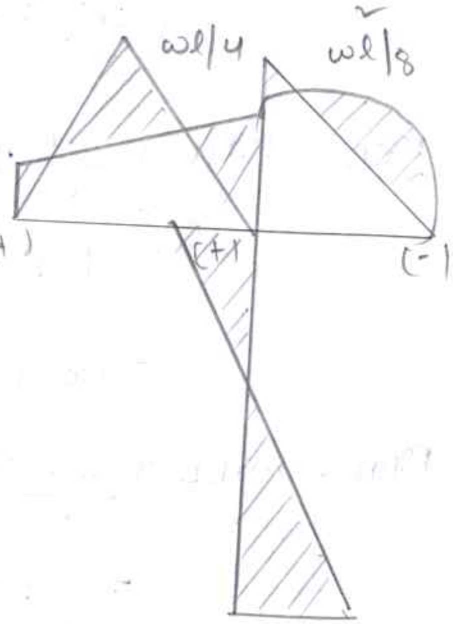
sol) (i) Fixed end moments:-

$$M_{FAB} = \frac{-wl}{8} = \frac{-60\times4}{8} = -30\,KN\text{-}m.$$

$$M_{FBA} = \frac{wl}{8} = \frac{60\times4}{8} = 30\,KN\text{-}m$$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-30\times4^2}{12} = -40\,KN\text{-}m.$$

$$M_{FCB} = \frac{wl^2}{12} = \frac{30\times4^2}{12} = 40\,KN\text{-}m$$



$M_{FBD} = 0$

$M_{FDB} = 0.$

(ii) slope deflection equation:-

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$$

$$= (-30) + \frac{\cancel{2}EI}{\cancel{4}_2}\left(2(0) + \theta_B - 0\right)$$

$$= -30 + 0.5EI\,\theta_B \rightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\Delta}{l}\right)$$

$$= 30 + \frac{2EI}{4}\left(2\theta_B + 0 - 0\right)$$

$\Rightarrow m_{AB} = 2/3\, EI\theta_B = 2/3(-1\cdot 212) = -0\cdot 8 mm$

$\Rightarrow m_{BA} = 4/3\, EI\theta_B = 4/3(-1\cdot 212) = -1\cdot 575 mm$

$\Rightarrow m_{BC} = -16 + 2(-1\cdot 212) = 18\cdot 424 mm$.

$\Rightarrow m_{CB} = 16 - 1\cdot 212 = 14\cdot 788 mm$

$\cdot 50 + EI\theta_B \rightarrow (2)$

$$m_{fBC} = m_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C - 3\Delta/l)$$

$$= (-40) + \frac{2EI}{4}(2\theta_B + \theta_C - 0)$$

$$= (-40) + EI\theta_B + 0\cdot 5EI\theta_C \rightarrow (3)$$

$$m_{CB} = m_{FCB} + \frac{2EI}{l}(2\theta_C + \theta_B - 3\Delta/l)$$

$$= 40 + \frac{2EI}{4}(2\theta_C + \theta_B - 0)$$

$$= 40 + EI\theta_C + 0\cdot 5EI\theta_B \rightarrow (4)$$

$$m_{BD} = m_{fBD} + \frac{2EI}{l}(2\theta_B + \theta_D - 3\Delta/l)$$

$$= 0 + \frac{2EI}{4}(2\theta_B + 0 - 0)$$

$$= EI\theta_B \rightarrow (5)$$

$$m_{DB} = m_{fDB} + \frac{2EI}{l}(2\theta_D + \theta_B - 3\Delta/l)$$

$$= 0 + \frac{\cancel{2}EI}{\cancel{4}_2}(2(0) + \theta_B - 0)$$

$$= 0\cdot 5EI\theta_B \rightarrow (6)$$

(iii) Equilibrium equations:-

$\Rightarrow$ @ Joint B

$m_{BA} + m_{BC} + m_{BD} = 0$

$= 30 + EI\theta_B - 40 + EI\theta_B + 0.5EI\theta_C + EI\theta_B = 0$

$3EI\theta_B + 0.5EI\theta_S = 10 \rightarrow (7)$.

$\therefore$ Assume support C is simply supported

$m_{CB} = 0$

$40 + EI\theta_C + 0.5EI\theta_B = 0$

$\Rightarrow 0.5EI\theta_B + EI\theta_C = -40$.

$EI\theta_B = 10.90$

$EI\theta_C = -45.45$.

(iv) Final moments:-

sub $EI\theta_B$ & $EI\theta_C$ in slope deflection equation.

$m_{AB} = -30 + 0.5EI\theta_B$

$= -30 + 0.5 \times 10.90 = -24.55$

$m_{BA} = 30 + EI\theta_B = 30 + 10.90 = 40.91$

$m_{BC} = -40 + EI\theta_B + 0.5EI\theta_C = -51.825$

$$= 30 + \frac{\cancel{4}EI\theta_B}{\cancel{4}} = 30 + EI\theta_B \rightarrow (2)$$

$$mF_{BC} = mF_{BC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\Delta}{l}\right)$$

$$= (-40) + \frac{2EI}{4}(2\theta_B + \theta_C - 0)$$

$$= -40 + EI\theta_B + 0.5EI\theta_C \rightarrow (3)$$

$$mF_{CB} = mF_{CB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\Delta}{l}\right)$$

$$= 40 + \frac{\cancel{2}EI}{\cancel{4}}(2\theta_C + \theta_B - 0)$$

$$= 40 + \frac{1}{2}EI\theta_B + \frac{4EI\theta_C}{4}$$

$$= 40 + EI\theta_C + 0.5EI\theta_B \rightarrow (4)$$

$$m^F_{BD} = mF_{BD} + \frac{2EI}{l}\left(2\theta_B + \theta_D - \frac{3\Delta}{l}\right)$$

$$= 0 + \frac{2EI}{4}(2\theta_B + 0 - 0)$$

$$= EI\theta_B \rightarrow (5)$$

$$mF_{DB} = mF_{DB} + \frac{2EI}{l}\left(2\theta_D + \theta_B - 3\Delta/l\right)$$

$$= 0 + \frac{\cancel{2}EI}{\cancel{4}2}(2(0) + \theta_B - 0)$$

$$= 0.5EI\theta_B \rightarrow (6)$$

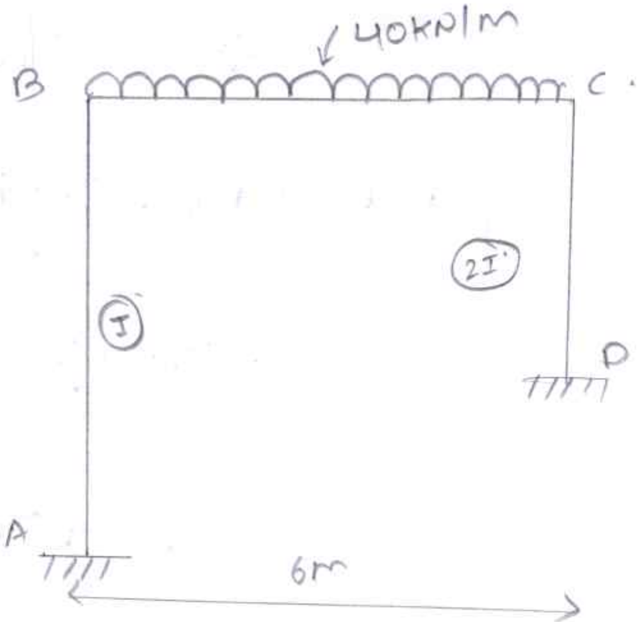
$m_{CB} = 40 + (-45.45) + 0.5(10.90) = 0$

$m_{BD} = EI\,\theta_B = 10.90$

$m_{DB} = 0.5\,EI\theta_B = 0.5 \times 10.90 = 5.45$

problem:-3

Analyse the frame shown in fig slope deflection method and draw BMD.



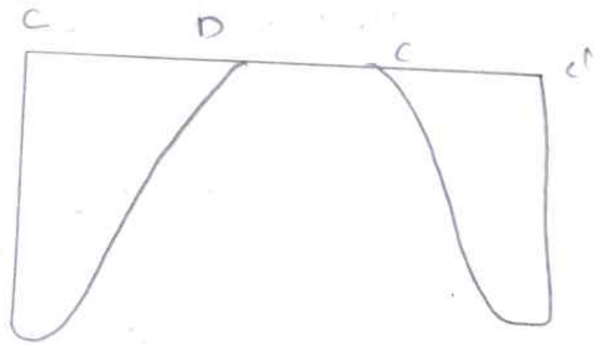
(i) Fixed end moments:-

$m_{FAB} = 0$

$m_{FBA} = 0$

$m_{FBC} = \frac{-\omega l^2}{12} = -120\text{ kN-m}$

$m_{FCB} = \frac{+\omega l^2}{12} = 120\text{ kN-m}$



$m_{FCD} = 0$

$m_{FDC} = 0$

(ii) slope deflection equation:-

$m_{AB} = m_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$

$= 0 + \frac{\cancel{2}EI}{\cancel{4}\,2}\left(2(0) + \theta_B - 3/4\,\delta\right)$

$= 0.5\,EI\theta_B - 0.375\,EI\Delta \rightarrow (1)$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - 3\Delta/l\right)$$

$$= 0 + \frac{2EI}{4}\left(2\theta_B + 0 - 3\Delta/4\right)$$

$$= EI\theta_B - 0.375EI\Delta \rightarrow (2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_C + \theta_B - 3\Delta/l\right)$$

$$= 120 + \frac{2E(2I)}{6}\left(2\theta_C + \theta_B - 3\Delta/l\right)$$

$$= -120 + 1.3EI\theta_B + 0.6EI\theta_C \rightarrow (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI8}{l}\left(2\theta_C + \theta_B - 3\Delta/l\right)$$

$$= 120 + \frac{2E(2I)}{6}\left(2\theta_C + \theta_B - 3\Delta/l\right)$$

$$= 120 + 0.6EI\theta_B + 1.3EI\theta_C \rightarrow (4)$$

$$M_{CD} = M_{FCD} + \frac{2E(2I)}{l}\left(2\theta_C + \theta_D - 3\Delta/l\right)$$

$$= 0 + \frac{2EI(2I)}{6}\left(2\theta_C + \theta_D - 3\Delta/l\right)$$

$$= 1.3EI\theta_C - 0.33EI\Delta \rightarrow (5)$$

$$M_{DC} = M_{FDC} + \frac{2EI(2I)}{l}\left(2\theta_D + \theta_C - 3\Delta/l\right)$$

$$= 0 + \frac{4EI}{6}\left(0 + \theta_C - 3\Delta/l\right)$$

$$= 0.6EI\theta_C - 0.33EI\Delta \rightarrow (6)$$

Take LCm

$$\frac{3(m_A + m_B) + 2(m_C + m_D)}{12} = 0$$

$$3(m_A + m_B) + 2(m_C + m_B) = 0$$

$\Rightarrow 3(0.5EI\theta_B - 0.375EI\Delta + EI\theta_B - 0.375EI\Delta +$

$2(1.3EI\theta_C - 0.33EI\Delta + 0.6EI\theta_C - 0.38EI\Delta) = 0$

$= 4.5EI\theta_B + 3.8EI\theta_C - 3.57EI\Delta = 0 \rightarrow (9)$

Solve 7, 8, & 9 we get

$EI\theta_B = 74.44$

$EI\theta_C = -61.44$

$EI\Delta = 28.44$.

Final end moments:-

$M_{AB} = 26.55$ kN-m

$M_{BA} = 63.77$ kN-m

$M_{BC} = -60.092$ kN-m.

(iii) Equilibrium equations:-

@ Joint B

$m_{BA} + m_{BC} = 0$

$\Rightarrow EI\theta_B - 0.375EI\theta_D - 120 + 1.3EI\theta_B + 0.66EI\theta_C = 0$

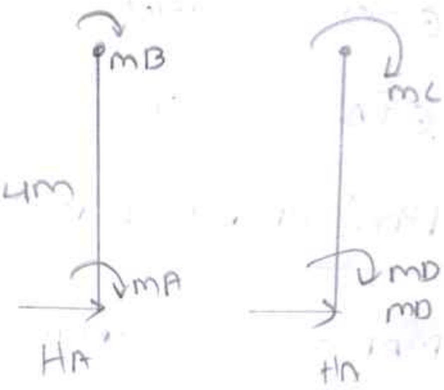
$\Rightarrow 2.3EI\theta_B + 0.66EI\theta_C - 0.375EI\Delta = 120 \rightarrow (7)$

at Joint $m_{CB} + m_{CD} \Rightarrow 0$

$120 + 0.66EI\theta_B + 1.3EI\theta_C + 1.3EI\theta_C - 0.33EI\Delta = 0$

$0.66EI\theta_B + 2.6EI\theta_C - 0.33EI\Delta = -120 \rightarrow (8)$

consider free body diagrams of columns and take moments about top Joints



$\Sigma m_B = 0$

$H_A \times 4 = m_A + m_B$

$\Rightarrow H_A = \dfrac{m_A + m_B}{4}$

$\Sigma m_C = 0$

$H_D \times 6 = m_C + m_D$

$H_D = \dfrac{m_C + m_D}{6}$

Now considering horizontal equilibrium

$H_A + H_D = 0$

$$\frac{m_A + m_B}{4} + \frac{m_C + m_D}{6} = 0$$

CIVIL ENGINEERING

Structural Analysis

UNIT -4

4. Moment distribution method

Introduction:-

- moment distribution is an iterative method of solving an indeterminate structure.
- moment distribution method was first introduced by Hardy cross in 1932.
- moment distribution is suitable for analysis of all types of indeterminate beams and rigid frames
- It is also called a 'relaxation method' and it consists of successive approximations using a series of cycles, each converging towards final result.
- It is comparatively easier than slope deflection method. It involves solving number of simultaneous equations with several unknows but in this method does not involve any simultaneous equations
- It is very easily remembered and extremely useful for checking computer output of highly indeterminate structures
- It is widely used in the analysis of all types of indeterminate beams and rigid frames
- The moment - distribution method was very popular among engineers.
- It is very simple and is being used even today for preliminary analysis of small structures.

- The primary concept used in this methods are
  - → Fixed end moments
  - → Relative or beam stiffness or stiffness factor.
  - → Distribution factor.
  - → carry over moment or carry over factor.

Basic concepts:-

- In moment - distribution method, counterclockwise beam end moments are taken as positive
- The counterclockwise beam end moments produce clockwise moments on the joint.

Note the sign convention:

Anti - clockwise is positive (+)

clockwise is negative (-)

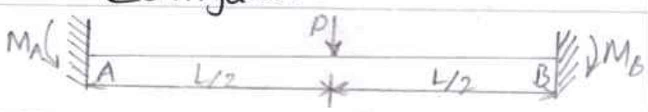
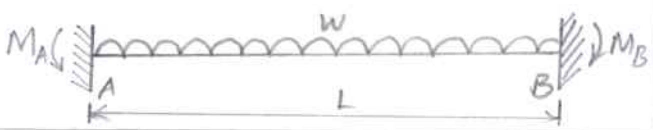
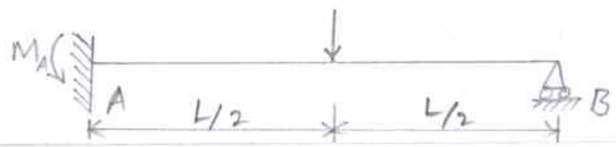
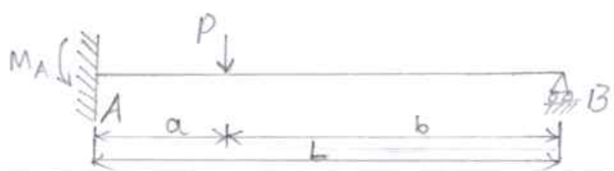
<u>Assumptions in moment distribution method</u>:

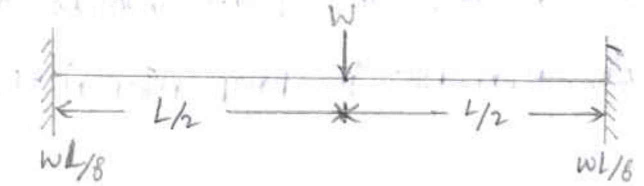
- All the members of the structures are assumed to be fixed and fixed end moments due to external loads are obtained
- All the hinged joints are released by applying an equal and opposite moment.
- The joints are allowed to deflect (rotate) one after the other by releasing them successively.
- The unbalanced moment at the joint is shared by the members connected at the joint when it is released.
- The unbalanced moment at a joint is distributed into the two spans with their distributed factor.

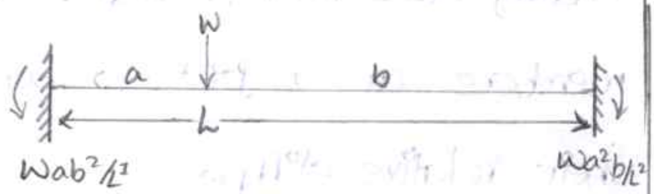
- Hardy Cross method makes use of the ability of various structural members at a joint to sustain moments in proportional to their relative stiffness

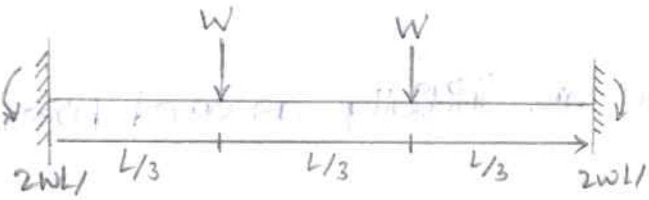
Fixed end moments:-

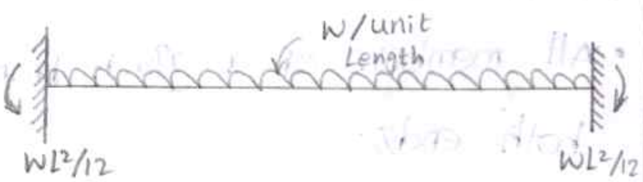
- All members of a given frame are initially assumed fixed at both ends.
- The loads acting on these fixed beams produce fixed end moments at the ends
- FEM are the moments exerted by the supports on the beam ends.
- These (non-existent) moments keep the rotations at the ends of each member zero.

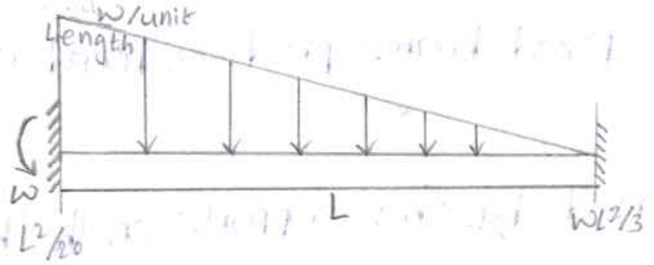
M_A	Configuration	M_B
$+\frac{PL}{8}$		$\frac{PL}{8}$
$+\frac{wL^2}{12}$		$\frac{wL^2}{12}$
$+\frac{Pab^2}{L^2}$		$\frac{Pa^2b}{L^2}$
$+\frac{3PL}{16}$		-
$+\frac{wL^2}{8}$		-
$+\frac{Pab(2L-a)}{2L^2}$		-

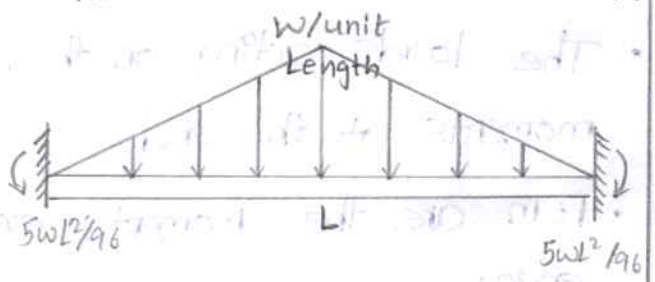

W
L/2
L/2
WL/8
WL/8

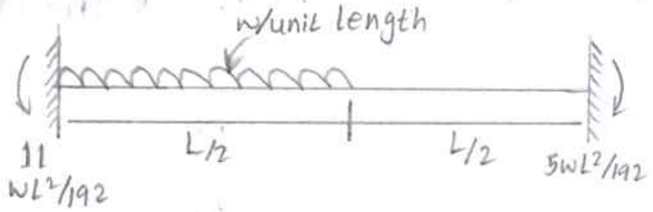

W
a
b
L
Wab²/L²
Wa²b/L²

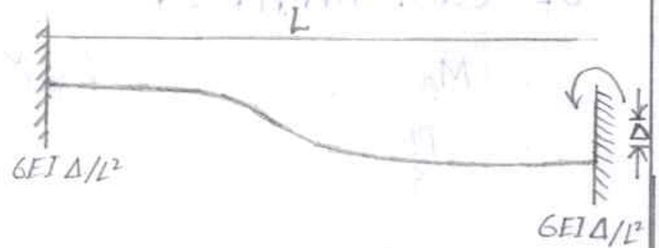

W
W
L/3
L/3
L/3
2WL/
2WL/


w/unit Length
WL²/12
WL²/12


w/unit Length
L
wL²/20
wL²/3


w/unit Length
L
5wL²/96
5wL²/96


w/unit length
L/2
L/2
11 wL²/192
5wL²/192


L
Δ
6EIΔ/L²
6EIΔ/L²


Δ
3EIΔ/L²

Relative or beam stiffness or stiffness factor.

- when a structural member of uniform section is subjected to a moment at one end, then the moment required so as to rotate that end to produce unit slope is called the stiffness of the member.
- stiffness is the member of force required to produce unit deflection
- It is also the moment required to produce unit rotation at a specified joint in a beam or a structure. It can be extended to denote the torque needed to produce unit twist.
- It is the moment required to rotate the end while acting on it through a unit rotation without translation of the far end being.

✓ Beam is hinged or simply supported at both ends.

$$K = 3EI/L$$

✓ Beam is hinged or simply supported at one end and fixed at other end.

$$K = 4EI/L$$

✓ stiffness of members in continuous beams and rigid frames

- stiffness of all intermediate member $K = 4EI/L$
- stiffness of edge members.

✓ If edge support is fixed $K = 4EI/L$

✓ stiffness of edge members $K = 3EI/L$

where

E = Young's modulus of the beam material

I = moment of inertia of the beam.

L = Beam's span length.

Distribution factor:-

- when several members meet at a joint and a moment is applied at the joint to produce rotation without translation of the members, the moment is distributed among all the members meeting at that joint proportionate to their stiffness.
- Distribution factor = Relative stiffness / sum of relative stiffness at the joint.

- If there is 3 members.

Distribution factors:- $k_1/(k_1+k_2+k_3)$, $k_2/(k_1+k_2+k_3)$, $k_3(k_1+k_2+k_3)$

Carry over moment:-

- carry over moment. It is defined as the moment induced at the fixed end of the beam by the action of a moment applied at the other end. which is hinged.

- carry over moment is the same nature of the applied moment.

Carry over factor (co)

- A moment applied at the hinged end B "carries over" to the fixed end "A". a moment equal to half the amount of applied moment and of the same rotational sense C.O = 0.5

problem:-

① find the continuous beam shown in figure by moment distribution method and draw SFD and BMD.

sol) Fixed end moments.

$$Fm_{AB} = \frac{-wab^2}{l^2} = \frac{10\times3\times2^2}{5^2} = -4.8\,kN\text{-}m$$

$$MF_{BA} = \frac{wa^2b}{l^2} = \frac{10\times3^2\times2}{5^2} = 7.2\,kN\text{-}m$$

$$MF_{BC} = -\frac{wl^2}{12} = -\frac{5\times4^2}{12} = -6.667\,kN\text{-}m$$

$$mf_{CB} = wl^2/12 = \frac{5\times4^2}{12} = 6.667\,kN\text{-}m.$$

ii) Distribution table.

Joint	member	stiffness	Total stiffness	$DF = k/\Sigma K$
B	BA	$\frac{4EI}{l} = 0.8EI$	$0.8EI + 1EI = 1.8EI$	$\frac{0.8}{1.8} = 0.44$
	BC	$\frac{4EI}{l} = 1EI$		$\frac{1}{1.8} = 0.56$
				1

(iii) moment distribution table.

supports	A	B	B	C
Distribution Factor		0.44	0.56	
Fixed end moments.	-4.8	7.8	-6.667	6.667
Balancing		-0.234	-0.2398	
carry over factor	$\frac{1}{2}$ -0.117			$\frac{1}{2}$ 0.149
Final moments	-4.917	6.966	-6.965	6.518

(i) Final moments.

$M_{AB} = -4.917$ kN-m

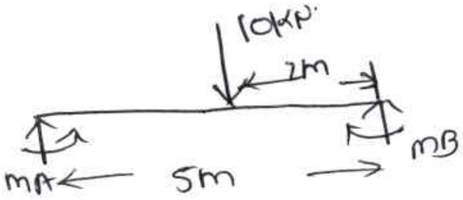
$M_{BA} = 6.96$ kN-m.

$M_{BC} = -6.96$ kN-m

$M_{CB} = 6.518$ kN-m

Reactions at AB span

$\Sigma M_A = 0$



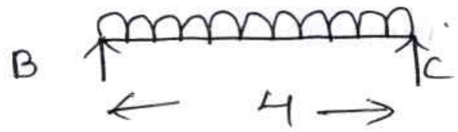
$R_A \times 5 = (10 \times 2) + m_{AB} + m_{BA}$

$R_A \times 5 - (20) - 4.917 + 6.96$

$= -17.957$

$= 3.591$

For span BC



$\Sigma M_B = 0$

$R_C \times 4 - 5 \times 4 \times 4/2 + M_{BC} - M_C$

$= 40 - 6.518 + 6.96$

$4R_C = \frac{39.558}{4}$

$= 9.88$

$R_A + R_B + R_C = 10 + 5 \times 4$

$= 3.602 + R_B + 9.88 = 30$

$= 16.518 \text{ kN}$

Shear force:-

SF at $A = R_A = 3.591$

SF at $B = R_A - 10$

$= 3.591 - 10 = -6.409$

SF at $B(R) = R_A - 10 + R_B$

$= 3.591 - 10 + 18.149 = 11.74$

SF at $C(L) = R_A - 10 + R_B - 5 \times 4 = 3.591 - 10 + 18.149 - 5 \times 4$

$= -8.26$

SF at $C(R) = R_A - 10 + R_B - 5 \times 4 + R_C = 3.591 - 10 + 18.149 - 5 \times 4 + 8.26$

$= 0$

problem:-

(2) Analyze the continues beam by moment distribution method and draw SFD and BMD.

Sol) Fixed end moments:-

$$mF_{AB} = \frac{-wab^2}{l^2} = \frac{-75\times3.5\times1.5^2}{5^2}$$

$$= -23.62 \text{ kN-m}$$

$$mF_{BA} = \frac{wa^2b}{l^2} = \frac{75\times3.5^2\times1.5}{5^2}$$

$$= 55.12 \text{ kN-m}$$

$$mF_{BC} = \frac{-wl^2}{12} = \frac{-15\times4^2}{12} = -20 \text{ kN-m}$$

$$mF_{CB} = wl^2/12 = \frac{15\times4^2}{12} = 20 \text{ kN-m}$$

$$mF_{CD} = -10 \text{ kN-m}.$$

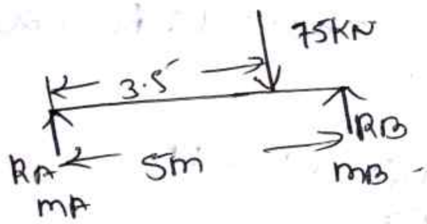
Distribution factor

Joint	member	stiffness	Total stiffness ΣK	$DF = K/\Sigma K$
B	BA	$\frac{4EI}{l} = 3.2EI$	$4.7EI$	0.68
	BC	$\frac{3EI}{l} = \frac{3E(2I)}{4}$ $= 1.5EI$		0.32

moment distribution table					
support	A	B		C	D
Distribution factor		0.67	0.32.		
fixed end moment	-23.62	55.18	-20		-10.00
Balancing factor	-23.625	55.18	-25	+10	-10
		-20.18	-9.63.		
	+10.24				
final moments	-31.265	34.640	-34.63	+10	-10

Reactions:-

at span AB:-



$\Sigma_B = 0$

$R_A \times 5 - (75 \times 1.5) + 33.265 + 34.640$

$= \frac{261.12}{5} = 22.345 \text{KN}.$

for span BC

$\Sigma m_B = 0$

$R_C \times 4 - (15 \times 4 \times 4/2) + m_B - (10 \times 5)$

$R_C = 33.84.$

$\therefore R_A + R_B + R_C = 75 + (1.5 \times 4) + 10$

$22.34 + 33.84 = 91.00$

$R_B = 88.815$

Shear force:-

SF @ $A = R_A = 22.345$ kN.

SF @ $B(L) = R_A - 75 = 22.345 - 75 = -52.64$ kN.

SF @ $B(J) = R_A - 75 \times R_B$.

$= 22.345 - 75 + 88.814 = 36.16$ kN.

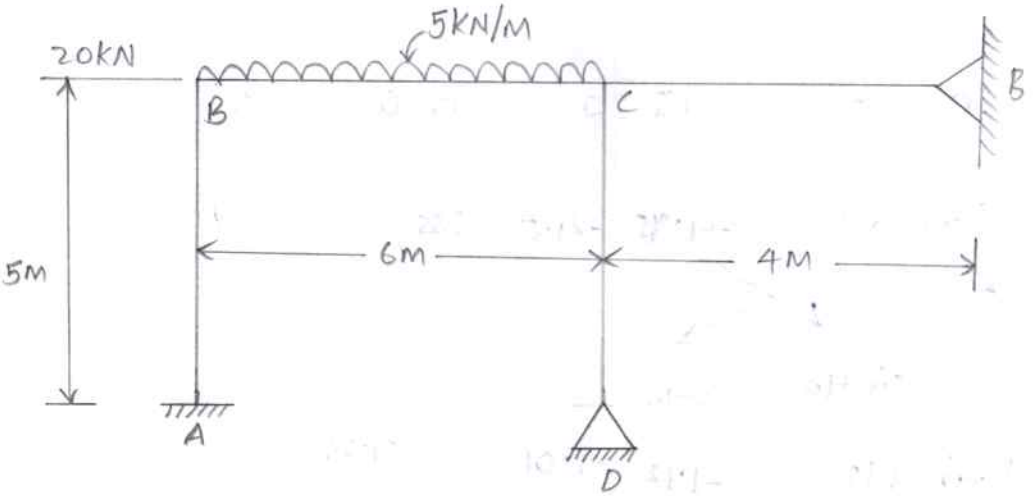
SF @ $C(L) = R_A - 75 + R_B - 15 \times 4$

$= 22.345 - 75 + 88.814 - 15 \times 4 = -23.84$

SF @ $C(R) = -23.84 + R_C = 20$ kN.

SF @ $D = +10 - 10 = 0$ kN.

(3) Analysis the frame shown in figure by moment distribution method and draw BMD Assume EI is constant -



Fixed end moments:-

$mF_{AB} = mF_{BA} = mF_{CD} = mF_{DC} = mF_{CE}\ mF_{EC} = 0$

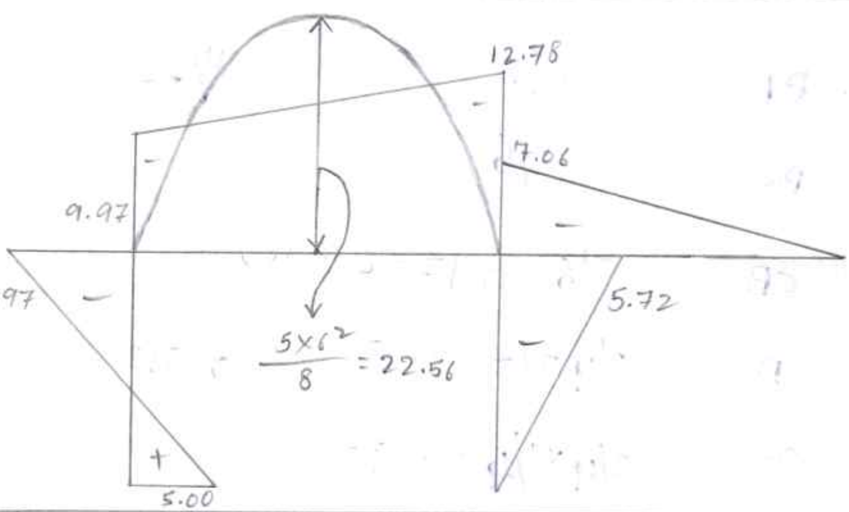
$mF_{BC} = \dfrac{-5\times 6^2}{12} = -15\,kN/m$

$mF_{CB} = +15\,kNm$

Distribution factor

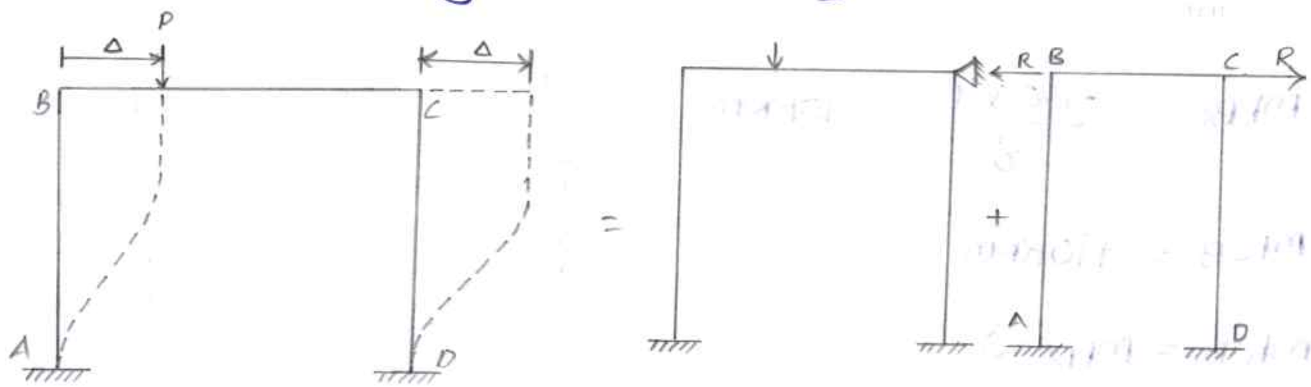
Joint	member	Relative stiffness (K)	ΣK	$DF = \dfrac{K}{\Sigma K}$
B	BA	$I/5$	$11/30\ I$	0.55
	BC	$I/6$		0.45
C	CB	$I/6 = 0.17\,I$		0.33
	CD	$3/4\ I/5 = 0.15\,I$	$0.51\,I$	0.3
	CE	$3/4 \times 1/4 = 0.19\,I$		0.37

JT	A	B		C		D		E
member	AB	BA	BC	CB	CD	DC	DE	EC
DF	0	0.55	0.45	0.33	0.3	1		1
FEM.	0	0	-15	+15	0	0	0	0
Balance		8.25	6.75	-4.95	-4.5	-5.55		
CO	4.13		-2.48	3.48				
Balance		1.36	1.12	-1.12	-1.01		-1.25	
CO	0.68		0.56	0.56				
Balance	0.16		0.25	-0.18	-0.17		-0.21	
CO	0.16		-0.09	0.13				
Balance		0.05	0.04	-0.04	-0.04		-0.05	
CO	0.03							
Final moments	5	9.97	-9.97	12.78	-5.79		0	0



moment distribution method for frames with side sway

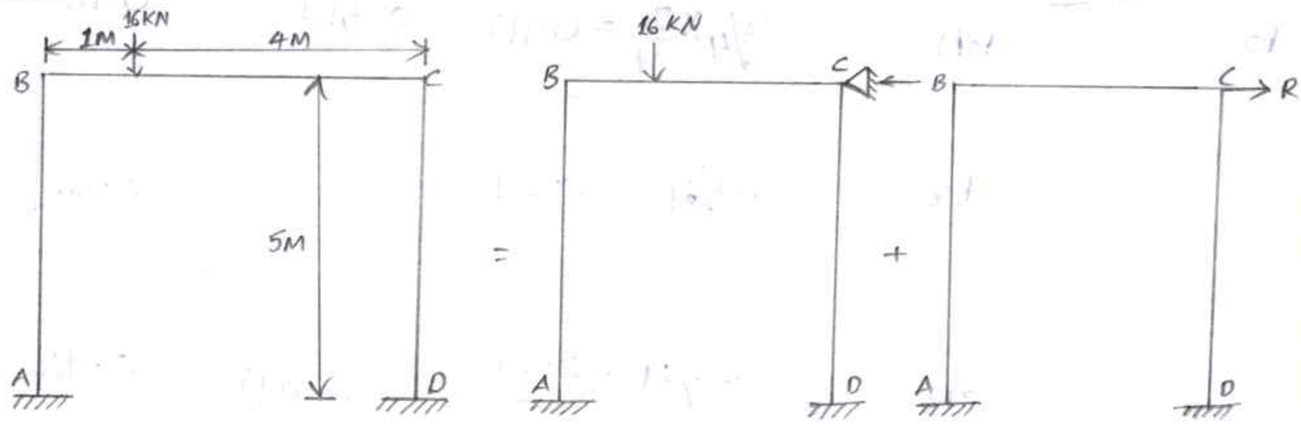
- Frames that are non symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to nonsymmetrical loading have a tendency to side sway



problem

1) Analysis the rigid frame shown in figure by moment distribution method and draw BMD.

145 fig:-



(i) Non sway Analysis:-

First consider the frame held from side sway as shown in figure 2.

FEM calculations:-

$$MF_{AB} = \frac{-10\times 3\times 4^2}{7^2} = -9.8\ kNm$$

$$MF_{BA} = \frac{+10\times 3^2\times 4}{7^2} = 7.3\ kNm$$

$$MF_{BC} = \frac{-20\times 4}{8} = -10\ kNm$$

$$MF_{CB} = +10\ kNm$$

$$MF_{CD} = MF_{DC} = 0$$

Distribution factor:-

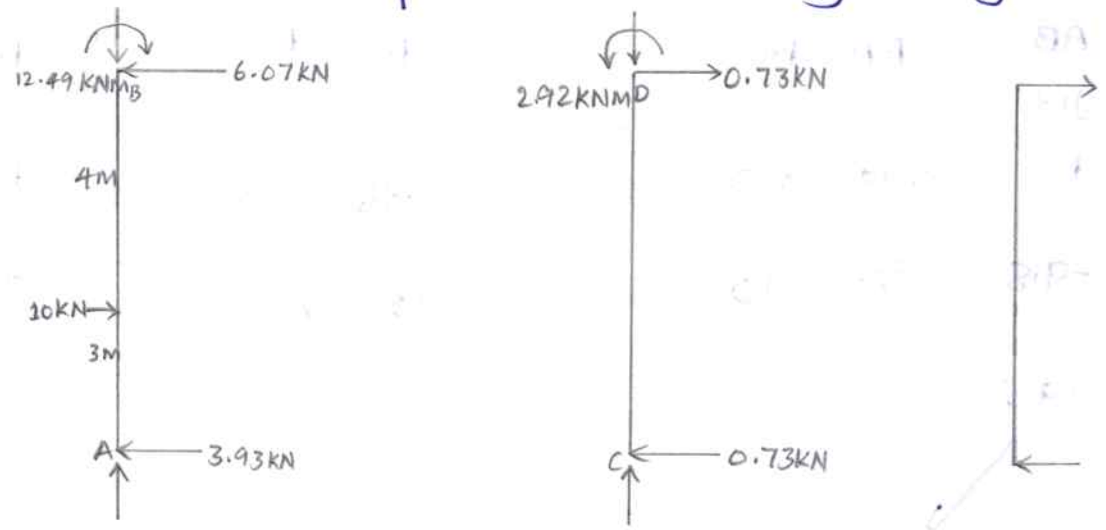
Joint	member	Relative stiffness K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$\frac{3}{4}\times\frac{I}{7} = 0.11I$	$0.61I$	0.18
	BC	$2I/4 = 0.5I$		0.82
C	CB	$2I/4 = 0.5I$	$0.69I$	0.72
	CD	$3/4 \times I/4 = 0.19I$		0.28

moment distribution for non sway analysis:

Joint	A	B		C		D
member	AB	BA	BC	CB	CD	DC
D.F	1	0.18	0.82	0.72	0.28	1
FEM	-9.8	7.3	-10	10	0	0
Release jt	+9.8					
D						
CO		4.9				
Initial moments	0	12.2	-10	10	0	0
Balance		-0.4	-1.8	-7.2	-2.8	
CO			-3.6	-0.9		
Balance		0.65	2.95	0.65	0.25	
CO			0.33	1.48		
Balance		-0.06	-0.27	-1.07	-0.41	
CO			-0.54	-0.14		
Balance		0.1	0.44	0.1	0.04	
Final moments	0	12.49	-12.49	2.92	-2.92	0

FBD of columns:-

• FBD of columns AB & CD for non-sway analysis is shown



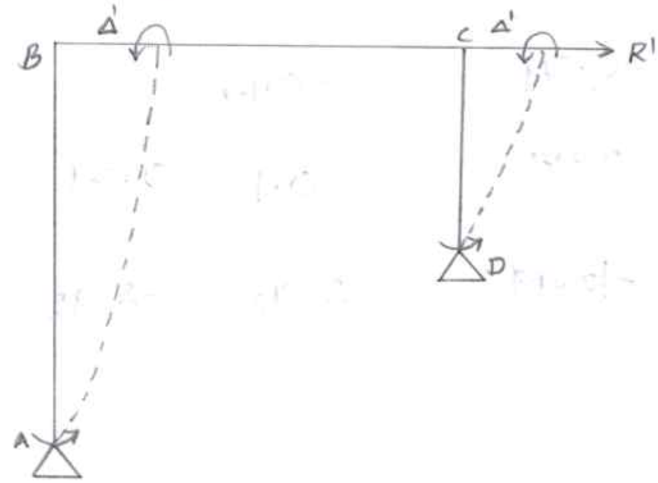
Applying $\Sigma F_x = 0$ for frame as a whole $R = 10 - 3.93 - 0.73$

$= 5.34 (\leftarrow)$ KN.

• Now apply R = 5.34 KN acting opposite as shown in fig for sway analysis

(ii) <u>sway analysis</u>:-

For this we will assume a force 'R' is applied at C causing the frame to deflect Δ as shown in fig.



since ends A & D are hinged and columns AB & CD are of different lengths

$$m'_{BA} = -3EI\Delta/L_1^2$$

$$m'_{CD} = -3EI\Delta/L_2^2$$

$$\therefore \frac{m'_{BA}}{m'_{CD}} = \frac{3EI\Delta'/l_1^2}{3EI\Delta'/l_2^2} = \frac{l_2^2}{l_1^2} = \frac{4^2}{7^2} = 16/49$$

Assume $m'_{BA} = -16$ kNm & $m'_{AB} = 0$

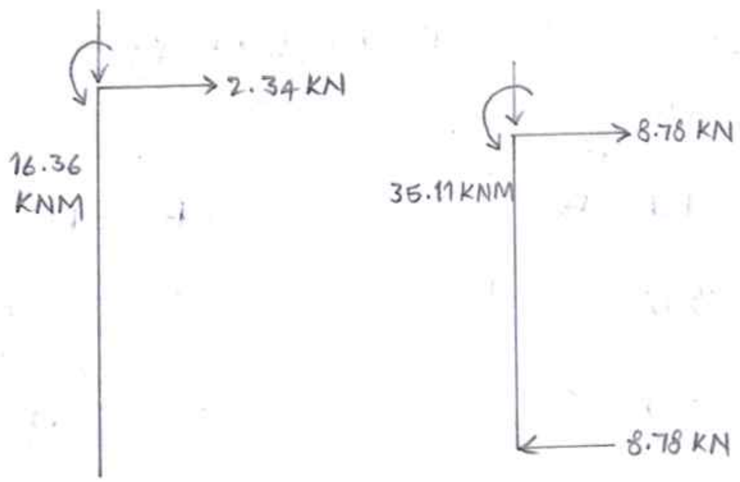
& $m'_{CD} = -49$ kNm & $m_{DC} = 0$

moment distribution table for sway Analysis.

Joint	A	B		C		D
member	AB	BA	BC	CB	CD	DC
D.F	1	0.18	0.82	0.72	0.28	1
FEm	0	-16	0	0	-49	0
Balance		2.88	13.12	35.28	13.72	
CO			17.64	6.56		
Balance		-3.18	-14.46	-4.72	-1.84	
CO			-2.36	-7.23		
Balance		0.42	1.94	5.21	2.02	
CO			2.61	0.97		

Balance		-0.107	-2.14	-0.7	-0.27		
CO			0.35	-1.07			
Balance		0.06	0.29	0.77	0.3		
CO			0.39	0.15			
Balance		-0.07	-0.32	-0.11	-0.04		
Final moments	0	-16.36	16.36	35.11	-35.11	0	

FBD of columns AB & CD for sway analysis moments is shown.



using $\Sigma F_x = 0$ for the entire frame.

$R' = 11.12 kN (\rightarrow)$

Hence $R' = 11.12 kN$ creates the sway moments shown in the above moment distribution table corresponding moments caused by $R = 5.34 kN$ can be determined by proportion.

Thus final moments are calculated by adding non-sway moments and sway moments determined for $R = 5.34 kN$ as shown.

$M_{AB} = 0$

$$M_{BA} = 12.49 + \frac{5.34}{11.12}(-16.36) = 4.63\ \text{KNm}$$

$$M_{BC} = -12.49 + \frac{5.34}{11.12}(16.36) = -4.63\ \text{KNm}$$

$$M_{CB} = 2.92 + \frac{5.34}{11.12}(35.11) = 19.78\ \text{KNm}$$

$$M_{CD} = -2.92 + \frac{5.34}{11.12}(-35.11) = -19.78\ \text{KNm}$$

$M_{DC} = 0$