



ANNAMACHARYA UNIVERSITY

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New Boyanapalli, Rajampet,
Annamayya (Dist), Andhra Pradesh – 516 126



DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

Course: **Electrical and Electronics Measurements**

Course Code: **20A242T**

Branch: **EEE**

Prepared by: **Mr. T.Arun Kumar**

Designation: Assistant Professor

Department: **Electrical and Electronics Engineering**

Unit 5 Smart & Digital Meter

10

Basic Block Diagram of Digital Voltmeter, Advantages and Disadvantages, Resolution and Simple Numerical Problems, Types of DVM's - Successive approximation, dual slope, ramp and integrating type DVM's, Digital frequency meter-Digital multimeter-Digital Tachometer.

Construction and working of smart energy meter-Advantages and Disadvantages

Learning Outcomes: At the end of the unit, the student will be able to:

- Describe the Concept of Working of Different types of DVM's(L2)
- Describe the Construction and Working of Smart Energy Meter(L2)

Prescribed Text Books:

1. E.W. Golding and F.C. Widdis Electrical Measurements and measuring Instruments, 5th Edition, Reem Publications.
2. A.K.Sawhney, Electrical & Electronic Measurement & Instruments, Dhanpat Rai & Co. Publications.

Reference Books:

1. R K Rajput Electrical & Electronic Measurement & Instrumentation., 2nd Edition, S. Chand & Co.
2. H. S. Kalsi Electronic Instrumentation. Tata Graw Hill Mc, 3rd Edition.
3. Reissland, M.U Electrical Measurements: Fundamentals, Concepts, Applications-New Age International (P) Limited, Publishers.

Web Resources:

1. https://onlinecourses.nptel.ac.in/noc19_ee44/preview
2. <https://www.classcentral.com/course/swayam-electrical-measurement-and-electronic-instruments-14032>
3. <https://www.electrical4u.com/>
4. <https://circuitglobe.com/>
5. <https://www.electricaldeck.com/>

Course Outcomes:

Blooms Level of Learning

At the end of the course, the student will be able to

- | | |
|--|----|
| 1. Describe basic requirements and the concepts of electrical measuring instruments. | L3 |
| 2. Measure the energy and power through energy meter and wattmeter | L2 |
| 3. Explain the concept of potentiometers and working of the frequency meters | L3 |
| 4. Measure the resistance, inductance, capacitance and frequency | L2 |
| 5. Explain the principle and operation of Smart and Digital meters | L3 |

CO-PO Mapping:

CO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PS01	PS02
20A242T.1	3	2	-	-	-	-	-	-	-	-	-	-	3	3
20A242T.2	3	2	-	-	-	-	-	-	-	-	-	-	3	3
20A242T.3	3	3	3	-	-	-	-	-	-	-	-	-	3	3
20A242T.4	3	3	2	-	-	-	-	-	-	-	-	-	-	3
20A242T.5	3	3	-	-	3	-	-	-	-	-	-	-	3	3

ELECTRICAL AND ELECTRONICS MEASUREMENTS

T. ARUN KUMAR
ASSISTANT PROFESSOR
ANNAMACHARYA UNIVERSITY



UNIT-1

MEASURING INSTRUMENTS

* Methods of measurements:

The methods of measurement can be classified as,

- a) Direct method
- b) Indirect method
- c) Absolute (or) Fundamental method
- d) Comparative method
- e) Transposition method
- f) Coincidence method
- g) Deflection method
- h) Complementary method
- i) Contact method
- j) ~~Comparative~~ method of null measurement.

a) Direct method

This is a simple method of measurement, in which the value of the quantity to be measured is obtained directly without any calculations.

For example, measurements by using scales, vernier callipers, micrometers etc. This method is widely used in production. This method is not very accurate because it depends on human insensitiveness in making judgement.

b) Indirect method:-

In indirect method the value of quantity to be measured is obtained by measuring other quantities which are functionally related to the required value.

For example, Angle measurement by Sine bar, measurement of screw pitch diameter by three wire method. etc.

c) Absolute (or) Fundamental method:-

It is based on the measurement of the base quantities used to define the quantity.

For example, measuring a quantity directly in accordance with the definition of that quantity (or) measuring a quantity indirectly by direct measurement of the quantities linked with the definition of the quantity to be measured.

d) Comparative method:-

In this method the value of the quantity to be measured is compared with known value of the same quantity (or) other quantity practically related to it.

So, in this method only the deviations from a master gauge are determined. e.g., dial indicators, (or) other comparators.

e) Transposition method:-

It is a method of measurement by direct comparison in which the value of quantity measured is first balanced by an initial known value 'A' of the same quantity, and then the

Value of the quantity measured is put in place of this known value and is balanced again by another known value "B".

If the position of the element indicating Equilibrium is the same in both cases, the value of the quantity to be measured is AB. For example, determination of mass by means of a balance and known weights, using the known double weighing.

f) Coincidence method :-

It is a differential method of measurement in which a very small difference between the value of the quantity to be measured and the reference is determined by the observation of the coincidence of certain lines (or) signals.

For example, measurement by vernier calliper micrometer.

g) Deflection method :-

In this method the value of the quantity to be measured is directly indicated by a deflection of a pointer on a calibrated scale.

h) Complementary method :-

In this method the value of quantity to be measured is combined with a known value of the same quantity. The combination is so adjusted that sum of these two values is equal to predetermined comparison value.

For example, determination of the volume of a solid by liquid displacement.

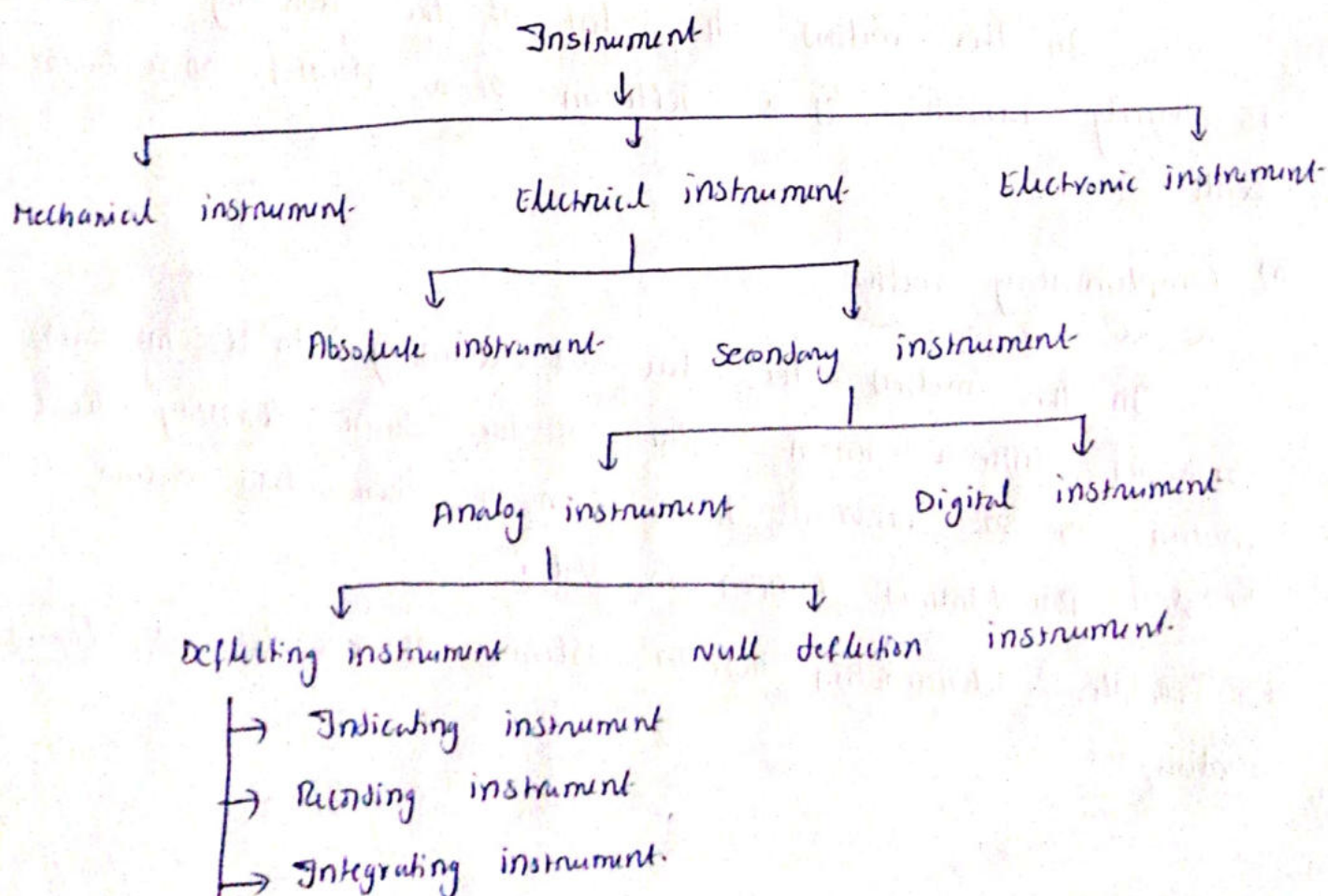
1) Contact method:-

It is a method of direct comparison in which the value of a quantity to be measured is replaced by a known value of the same quantity, so selected that the effects are produced in the indicating device by these two values are same.

2) method of null measurement:-

It is a method of differential measurement. In this method the difference between the value of the quantity to be measured and the known value of the same quantity with which it is compared is brought to zero.

* Classifications of instruments:-



→ The mechanical instrument is used for measuring the physical quantity. This instrument is suitable for measuring the static ~~and~~ and stable conditions because the instrument is unable to give the response to the dynamic condition.

→ The electronic instrument is used for quick response in measuring the value as compared to the electrical and mechanical instrument.

→ The electrical instrument is used for measuring electrical quantities like, current, voltage, power etc viz, ammeter, voltmeter & wattmeter.

→ The electrical instrument again classified into two types.

a) Absolute instrument

b) Secondary instrument.

a) Absolute instrument:-

Absolute instruments are the instruments which give the value of quantity to be measured in terms of instrument constant and deflection.

Ex: Tangent galvanometer, Rayleigh's current balance instrument.

b) Secondary instrument:-

These instruments are so constructed in such a way that the quantity being measured by observing the o/p indicated by instrument.

Ex: Ammeter, voltmeter and wattmeter.

The secondary instruments again classified into two types

* Analog instrument.

* Digital instrument.

* Digital instrument: Digital instrument given the output in digital form.
The instrument is more accurate as compared to the analog instrument because no human error occurs in the readings.

* Analog instrument: Analog instrument is the instrument whose output varies continuously. The analog instrument has the pointer which shows the magnitude of the measurable quantity.

The Analog instrument is further classified into two types

a) Null deflection Type

b) Deflection Type.

a) Null deflection Type:

In this instrument, the null deflection indicates the magnitude of the measured quantity. This instrument has high accuracy and sensitivity. In null deflection instrument, one known and one unknown quantity are used. When the values of the known and the unknown measuring quantities are equal, the pointer shows zero. The null deflection instrument is used in galvanometer.

b) Deflection Type:

The instrument in which the value of measuring quantity is determined through the deflection of the pointer is known as the deflection type instrument.

The deflection type instrument is again classified into three types

- i) Indicating instrument
- ii) Recording instrument
- iii) Integrating instrument

i) Indicating instrument:-

The instrument which indicates the magnitude of electrical quantity at the time of measurement is called as an indicating instrument.

Ex: Ammeter, voltmeter, wattmeter etc.

ii) Recording instrument:-

Recording instruments are those which give the continuous record of variations of electrical quantity over a selected period of time. Ex: ECR.

iii) Integrating instrument:-

Integrating instruments are those which measure the total quantity of electricity delivered in a particular interval of time.

Ex: energy meter.

* characteristics of Instruments :-

→ Instrument characteristics allow users to select the most suitable instrument for specific measuring jobs.

→ There are two basic performance characteristics of measuring instruments.

a) Static characteristics

b) Dynamic characteristics

a) Static characteristics :-
~~~~~

Value of the measured variable change slowly.

b) Dynamic characteristics :-  
~~~~~

Value of the measured variable change very fast.

→ The static characteristics of an instrument is again classified as,

- i) Accuracy
- ii) Precision
- iii) Linearity
- iv) Sensitivity
- v) Dead time
- vi) Dead Zone
- vii) Resolution
- viii) Reproducibility
- ix) Stability
- x) Error

i) Accuracy :-
~ ~ ~

Accuracy of an instrument always indicates the degree of closeness of measured quantity towards the true quantity.

ii) Precision :-
~ ~ ~

The most repeatable value out of set of readings is called as "Precision".

Ex: Among the two Ammeters A and B with their corresponding readings, find a true reading of 2A.

Ammeter-A readings are 1.5A, 1.9A, 1.5A, 1.9A, 1.9A --- Accurate & Precision

Ammeter-B readings are 1.5A, 1.9A, 1.9A, 1.5A, 1.5A --- Precision

Accurate instrument may be precise instrument but precise instrument will not confirm any accuracy. So always the instrument which is both accurate as well as precise is to be preferred.

The Precision can be mathematically expressed as,

$$P = 1 - \left[\frac{x_n - \bar{x}_n}{\bar{x}_n} \right]$$

Where, P = Precision

x_n = value of n^{th} measurement

$\bar{x}_n$ = Average of set of measured values.

Example :- The table shows the set of 5 measurements received in a laboratory. Calculate the precision of the 3rd measurement.

Measurement number	Value of measurement
1	49
2	51
3	52
4	50
5	49

Sol) The average value of set of measurements is,

$$\bar{x}_n = \frac{\text{Sum of readings}}{\text{Number of readings}} = \frac{251}{5} = 50.2$$

The value of 3rd measurement $x_n = 52$ where $n=3$

$$p = 1 - \left[\frac{x_n - \bar{x}_n}{\bar{x}_n} \right] = 1 - \left[\frac{52 - 50.2}{50.2} \right] = 0.964 = 96.4\%$$

This is precision of 3rd measurement.

iii) Linearity :-
~ ~ ~

If the output follows the input with the linear relationship then it is termed as linearity. The linearity is defined as the ability to reproduce the input characteristics symmetrically and linearly. Graphically such relationship between input and output is represented by a straight line.

iv) Sensitivity :-
~ ~ ~

Sensitivity is defined as the ratio of change in output to the unit change in input and it is denoted by "S"

$$\left[\text{Sensitivity} = \frac{\text{change in output}}{\text{unit change in input}} \right]$$

Sensitivity is equal to the slope. For perfect linear scale instrument (MI type), the sensitivity is constant and it is equal to one, whereas the sensitivity for a non-linear instrument (MI Type), the sensitivity varies through out the scale.

The inverse of sensitivity is called "deflection factor"

$$\left[\text{Deflection Factor} = \frac{1}{\text{sensitivity}} \right]$$

Q2: A particular ammeter requires a change of 2A in its coil to produce a change in deflection of the pointer by 5mm. Determine its sensitivity and deflection factor.

Sol The input is current and o/p is deflection

$$\text{Sensitivity} = \frac{5}{2} \Rightarrow 2.5 \text{ mm/A}$$

$$\text{deflection factor} = \frac{1}{2.5} = 0.4 \text{ A/mm}$$

v) Dead time ~ ~ ~

The time taken by the instrument to move the pointer from its rest position is called as "dead time". and the main reason for the dead time in an instrument is inertia.

In all electronic instruments the responsible quantity is voltage. In all electrical instruments the responsible quantity is current.

vi) Dead Zone ~ ~ ~

The minimum input beyond which the response is obtained is termed as "Dead Zone."

vii) Resolution ~ ~ ~ The smallest output that can be detected with clarity is called as "Resolution."

Resolution is given by,

$$R = \frac{\text{Full Scale Value}}{\text{Total number of divisions}}$$

As the total number of divisions increases, the resolution decreases.

Example: A 30 cm scale has 30 uniform divisions. $\frac{1}{20}$ th of a scale division can be estimated with a fair degree of certainty. Determine the resolution of the scale in mm.

Sol: 1 Scale division = $\frac{30 \text{ cm}}{30} = 1 \text{ cm} = 10 \text{ mm}$

$$\text{Resolution} = \frac{1}{20} \times \text{Scale division}$$

$$\Rightarrow \frac{1}{20} \times 10 \Rightarrow 0.5 \text{ mm}$$

viii) Reproducibility:- It is the degree of closeness with which a given value may be repeatedly measured.

ix) Stability:- The ability of an instrument to retain its performance throughout its specified operating life is called as stability.

x) Error:- The deviation of measured quantity from the true quantity is called as error. Error may be positive (or) negative.

$$[\text{Error} = \text{Measured value} - \text{True value}] \quad (1)$$

$$[\text{Error} = \text{True value} - \text{Measured value}]$$

Mathematically it can be expressed as,

$$[e = A_t - A_m]$$

Where $e = \text{Error}$

$A_t = \text{true value of quantity}$

$A_m = \text{measured value of quantity}$

In this expression, the error denoted as " e " is also called as "absolute error." The absolute error does not indicate precisely the accuracy of the measurements.

For example, absolute error of $\pm 1V$ is negligible when the voltage to be measured is of the order of $1000V$ but the same error of $\pm 1V$ becomes significant when the voltage under measurement is $5V$. Hence, generally instead of specifying absolute error, the "relative (or) percentage error" is specified.

Mathematically, relative error can be expressed as,

$$e_r = \frac{\text{absolute error}}{\text{true value}} = \frac{\text{true value} - \text{measured value}}{\text{true value}}$$
$$= \frac{A_t - A_m}{A_t}$$

The percentage relative error is expressed as

$$\% e_r = \frac{A_t - A_m}{A_t} \times 100$$

The accuracy can be expressed as

$$A = 1 - e_r$$

$$\left[A = 1 - \left| \frac{A_t - A_m}{A_t} \right| \right]$$

The percentage accuracy can be expressed as

$$\% a = A \times 100\%$$

Example] The expected value of the voltage to be measured is 150V. However, the measurement gives a value of 149V.
 calculate i) Absolute error ii) percentage error iii) Relative accuracy iv) Percentage accuracy.

Sol] i) Absolute error = $E_t - E_m$
 $= 150 - 149 = \pm 1V$

ii) Percentage error $\Rightarrow$ $\%e_r = \frac{A_t - A_m}{A_t} \times 100 \Rightarrow \frac{1}{150} \times 100$
 $\Rightarrow 0.66\% \Rightarrow 66\%$

iii) Relative accuracy

$$A = 1 - \left| \frac{A_t - A_m}{A_t} \right|$$

$$\Rightarrow 1 - \frac{1}{150} \Rightarrow 0.9933$$

iv) Percentage accuracy

$$A = A \times 100 \Rightarrow 99.33\%$$

* Sources of errors:

In the process of measurement, the errors are bound to occur. If the sources of errors are known, then the efforts can be made to reduce the errors and partly to eliminate them.

The various possible sources of errors are,

$\rightarrow$ Faulty design of the instrument which directly leads to the serious measurement errors.

$\rightarrow$ For the instruments frequent maintenance is necessary. If such maintenance is not done then the errors may occur.

- The unskilled operator of the instrument can cause human errors.
- Improper way of using the instrument can cause error.

* Types of Errors:

There are three types of errors that are classified on the basis of the source they arise from. They are,

- a) Gross errors
- b) Systematic errors
- c) Random errors

a) Gross Errors:

- ✓ The gross errors mainly occur due to carelessness (or) lack of experience of a human being.
- ✓ These cover human mistakes in readings, recordings and calculating results. These errors are also occur due to incorrect adjustments of instruments.
- ✓ These errors are also called personal errors. Some gross errors are easily detected while others are very difficult to detect.

The complete elimination of gross error is not possible but one can minimise them by the following ways.

- i) Taking great care while taking the readings, recordings and calculating the result.
- ii) By increasing the number of experiments, we can reduce the gross error. If each experiment takes different readings at different points, then by taking the average of more readings we can reduce the gross errors.

b) Systematic Errors :-

The Systematic errors are mainly occurs due to the shortcomings of the instrument and characteristics of the material used in the instrument. Such as defective parts, aging effects, Environmental effects etc.

There are three types of Systematic errors.

- i) Instrumental errors
- ii) Environmental errors
- iii) Observational errors.

i) Instrumental errors :-

These errors can be mainly due to following reasons.

* Shortcoming of instruments :-

These are because of mechanical structure of the instruments. For example, friction in the bearings of various moving parts, irregular spring tensions, variation in air gap etc. These errors can be avoided by the following methods.

- Selecting a proper instrument and planning the proper procedure for the measurement.
- Recognizing the effect of such errors and applying the proper correction factors.

* Misuse of instruments :-

A good instrument if used in abnormal way gives misleading results. Poor initial adjustments, improper zero setting etc are the examples of misusing a good instrument. Such things do not cause permanent damage of the instrument but definitely cause serious errors.

* Loading Effect :-

Loading effect due to improper way of using the instrument cause the serious errors. The best example of such loading effect is connecting a well calibrated voltmeter across the two points of high resistance circuit. The same voltmeter connected in a low resistance circuit gives accurate readings. Thus, the error due to the loading effect can be avoided by using an instrument intelligently and correctly.

ii) environmental errors :-

These errors are due to the conditions external to the measuring instrument. The various factors resulting these environmental errors are temperature changes, pressure changes, effect of external fields, ageing of equipment and frequency sensitivity of instrument.

The various methods which can be used to reduce these errors are,

- ✓ using the proper correction factors and using the information supplied by the manufacturer of the instrument.
- ✓ using the arrangements which will keep the surrounding conditions constant. This includes the use of air conditioning, temperature control etc.

iii) observational errors :-

These errors arise due to an individual's bias, lack of proper setting of the apparatus, (or) an individual carelessness in taking observations. The observational errors also include wrong readings due to "Parallax" errors.

c) Random Errors

- The random errors are those errors, which are occur irregularly, and hence are random.
- These can arise due to random and unpredictable fluctuations in experimental conditions.
- Example: unpredictable fluctuations in temperature, voltage supply, mechanical vibrations, errors by the observer taking readings.
- The random errors follow the laws of probability and hence these errors can be analyzed statistically and treated mathematically.
- The only way to reduce these errors is by increasing the number of observations and using the statistical methods to obtain the best approximation of the reading.

* Forces required to operate an Instrument

In case of measuring instruments, the effect of unknown quantity is converted into a mechanical force which is transmitted to the pointer which moves over a calibrated scale. The moving system of such instrument is mounted on a pivoted spindle. For satisfactory operation of any indicating instrument, following systems must be present in instrument.

- i) Deflecting system producing "Deflecting Force" (or) "Deflecting Torque"
- ii) Controlling system producing "Controlling Force" (or) "Controlling Torque"
- iii) Damping system producing "Damping Force" (or) "Damping Torque"

* Deflecting Torque (T_d):

In most of the indicating instruments the mechanical force is proportional to the quantity to be measured is generated. This force or torque causes the moving system to move from its zero position. The system which produces such a deflecting torque is called "deflecting system" and the torque is denoted by ' T_d '. The deflecting torque overcome.

- $[T_d \propto \text{Supply}]$
- 1) The inertia of the moving system
 - 2) The controlling torque provided by controlling system
 - 3) The damping torque provided by damping system.

The deflecting system uses one of the following effects produced by current (or) voltage to produce deflecting torque.

a) Magnetic Effect:-

When a current carrying conductor is placed in a uniform magnetic field, it experiences a force which causes to move it. This effect is used in many instruments like moving iron attraction and repulsion type, permanent moving coil instrument etc.

b) Thermal Effect:-

The current to be measured is passed through a small element which heats it cause rise in temperature which is converted into e.m.f by a thermocouple attached to it.

c) Electrostatic Effect:- When two plates are charged, there is a force between them, which moves one of plates. This effect is used in Electrostatic instruments which are normally voltmeters.

Thus the deflecting system provides the deflecting ^(a) torque (or) operating torque for movement of pointer from its zero position. It acts as the prime mover for the deflection of pointer.

b) controlling Torque:-

There are two purpose of controlling torque.

→ It produces a force equal and opposite to the deflecting force in order to make the deflection of pointer at a definite magnitude. If this system is absent, then the pointer will swing beyond its final steady position and deflection will become indefinite.

→ To bring the pointer to its original position in the absence of it.

At steady state both deflecting and controlling torque are equal in magnitude but acting opposite direction, system produces oscillations which is undesirable. controlling torque is generally provided by

- i) Spring control method
- ii) Gravity control method

i) Spring control method:-

Two Hair Springs are attached to the moving system which exerts controlling torque. To employ spring to the instrument following requirements are essential.

- ✓ The spring should be non-magnetic.
- ✓ The spring should be free from mechanical strain.
- ✓ The spring should have a small resistance, sufficient cross-sectional area.
- ✓ It should have low resistance temperature co-efficient.

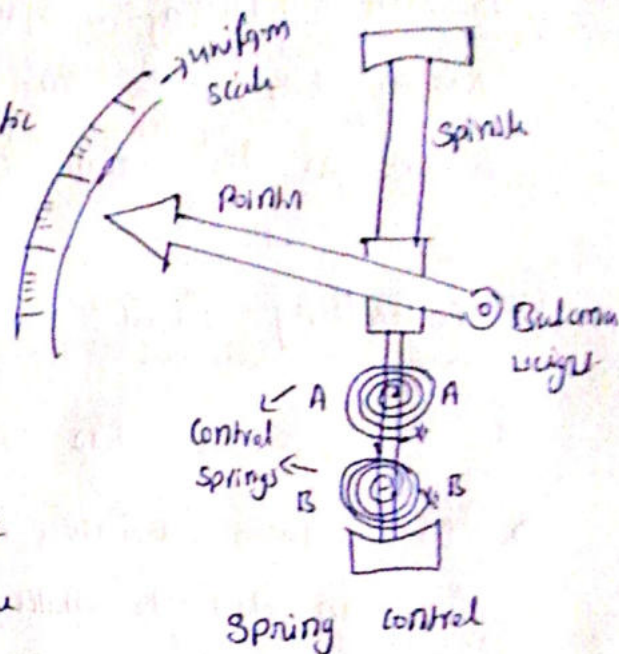
The arrangements of Spring as shown in fig.

→ The springs are made up of non-magnetic materials like silicon bronze, hard rolled Silver (or) copper, platinum silver and German Silver.

→ For most of the instruments, Phosphor bronze Spiral Springs are provided.

→ The controlling torque is provided by the instrument is directly proportional to the angular deflection of pointer.

→ The controlling torque produced by spiral spring is given by.



$$T_c = \frac{E b t^3}{12L} \theta = K_s \theta$$

Where E = Young's modulus of Spring material in N/m^2

t = Thickness in mms.

L = length in mms.

b = depth in mms.

$$K_s = \text{Spring Constant} = \frac{E b t^3}{12L}$$

$$[T_c \propto \theta] \quad [T_d \propto I]$$

At Equilibrium

$$T_d = T_c$$

$$I \propto \theta$$

Thus the deflection is proportional to the current. Hence the scale of the instrument using spring control is uniform.

When the current is removed, due to spring force the pointer comes back to initial position. The spring control is very popular and is used in almost all indicating instruments.

Drawbacks:

→ Ageing effect of Spring with time $K \downarrow$

$$\theta \uparrow \propto \frac{1}{K} \downarrow \quad \text{more deflection}$$

→ Temperature effect

$$T \uparrow \quad L \uparrow$$

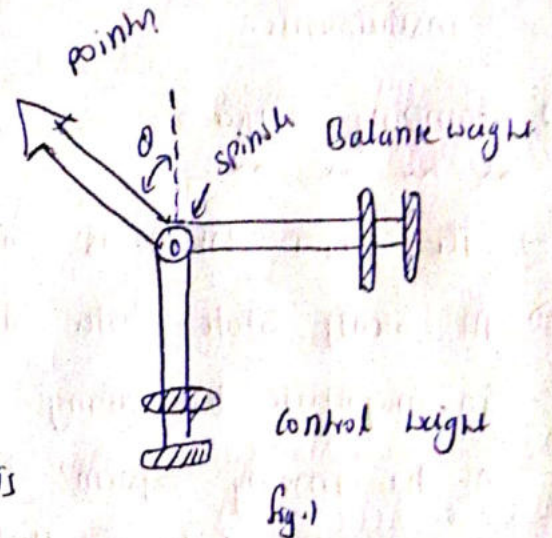
$$\downarrow K \propto \frac{1}{L} \uparrow$$

$\theta \uparrow$ deflection is more

ii) Gravity control method:

→ This type of control consists of a small weight attached to the moving system whose position is adjustable.

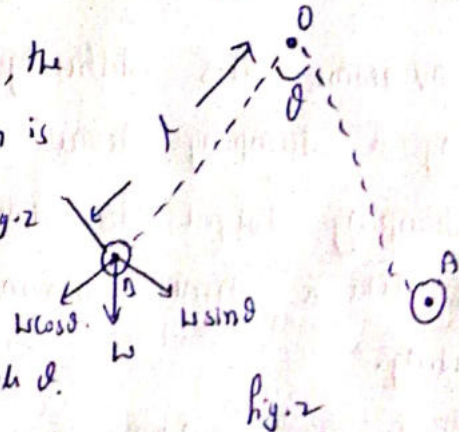
→ This weight produces a controlling torque due to gravity. This weight is called control weight.



→ The Fig. 1 Shows the gravity control system.

→ At the zero position of the pointer, the controlling torque is zero. This position is shown as position-A of the weight in Fig. 2

→ If the system deflects the weight position changes to position-B with angle θ .



→ The component " $W \sin \theta$ " of the weight tries to rotate the pointer back to initial position.

→ The controlling torque is given by

$$T_c = W \sin \theta \times l \quad \Rightarrow \quad T_c = K \sin \theta \quad \text{where } K = Wl$$

$$T_d = K \dot{\theta} \quad I \ddot{\theta} \sin \theta$$

→ As it is a function of $\sin \theta$, the scale of the instrument using gravity control is not uniform.

Advantages

- No aging effect
- Its performance is not dependent on temperature
- It is simple and cheap.

Disadvantages

- non-uniform scale.
- Proper levelling required. In general it is not used for inspecting instruments.

iii) Damping Torque

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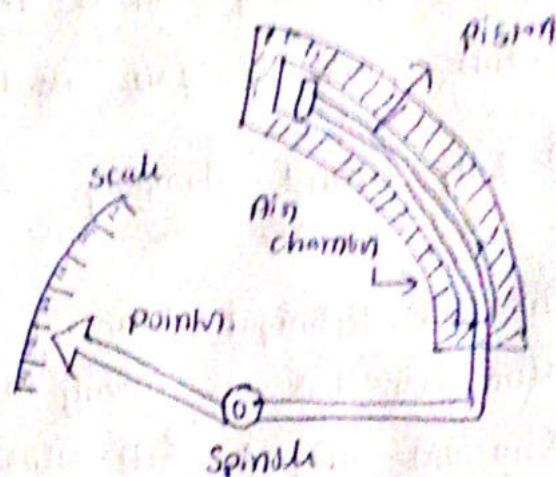
- The torque which is required to reduce the number of oscillations.
- At steady state both deflecting and controlling torques are equal in magnitude but acting opposite direction. However, due to inertia of the moving system the pointer will not come to rest immediately but oscillate about its final deflected position and takes appreciable time to come steady state.
- To overcome this difficulty a damping torque is developed by using a damping device attached to the moving system.
- If damping torque is absent nothing will happen but its like like a time consuming process in order to take each reading.
- The damping torque is produced by the following methods:

- a) Air friction damping
- b) Fluid friction damping
- c) Eddy current damping.



### a) Air Friction Damping Technique :-

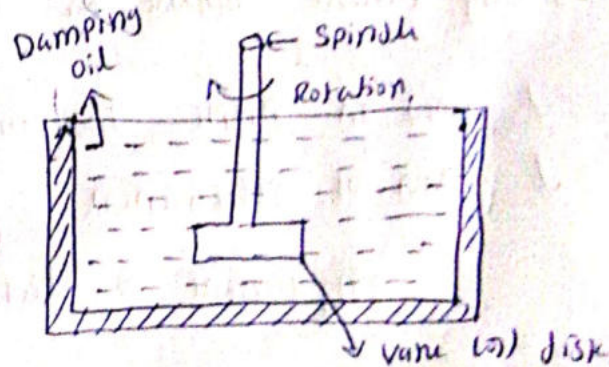
In air friction damping technique an aluminium vane is attached to moving system with the help of link mechanism & placed inside the air container so that friction offered by air will oppose the motion of the pointer.



- used in HS, EMMC, Thermal instrument.
- Pointer where magnetic energy is small.
- It requires less maintenance.
- cost is less. Repeated number of operations we can use.

### b) Fluid Friction Damping Technique :-

Fluid friction damping is more effective form of damping in this technique Aluminium vane is attached to moving system and placed inside the fluid container so that friction offered by the fluid will oppose the motion of the pointer.



- used in ES, wall mounted type instrument.
- Friction offered by liquid is more compared to friction offered by air.

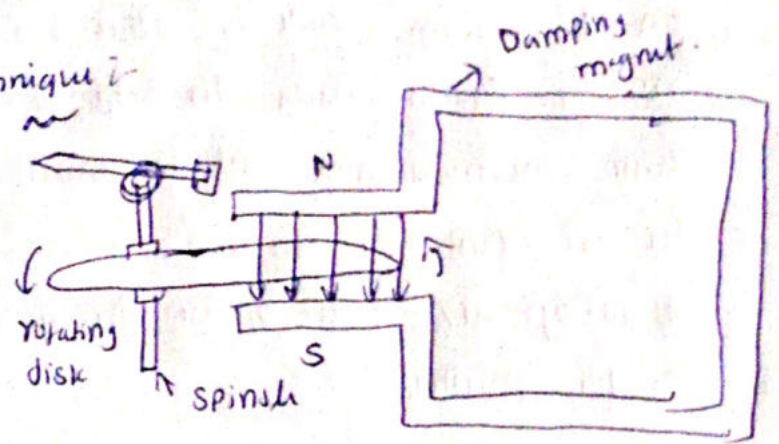
#### Disadvantages :-

- ✓ Cost is more
- ✓ Due to leakage of fluid, it is difficult to keep the instrument clean.
- ✓ Requires more maintenance.
- ✓ Not suitable for repeated number of operation



- ✓ Fluid should not evaporate quickly
- ✓ Fluid viscosity should not change with temperature.
- ✓ Fluid must be good insulation.

### c) Eddy current damping Technique



- In this technique whenever Aluminium disk is moving in permanent magnetic field which cuts the flux produced by the permanent magnet so that an emf is induced in the Aluminium disk.
- Aluminium disk is a metallic one which will offer resistance so that produced emf will send some current known as Eddy current which is circulating through Aluminium disk.
- Eddy current oppose the motion of pointer.

- ✓ Used in PMMC, Induction type instruments (Energy meter)
- ✓ Don't of Effectiveness

Eddy current > Fluid friction > Air friction.

- ✓ Don't of priority.

Eddy current > Air friction > Fluid friction

- ✓ Whenever permanent magnets are available go for Eddy current damping Technique.



## \* Permanent Magnet Moving Coil (or) PMMC Instrument

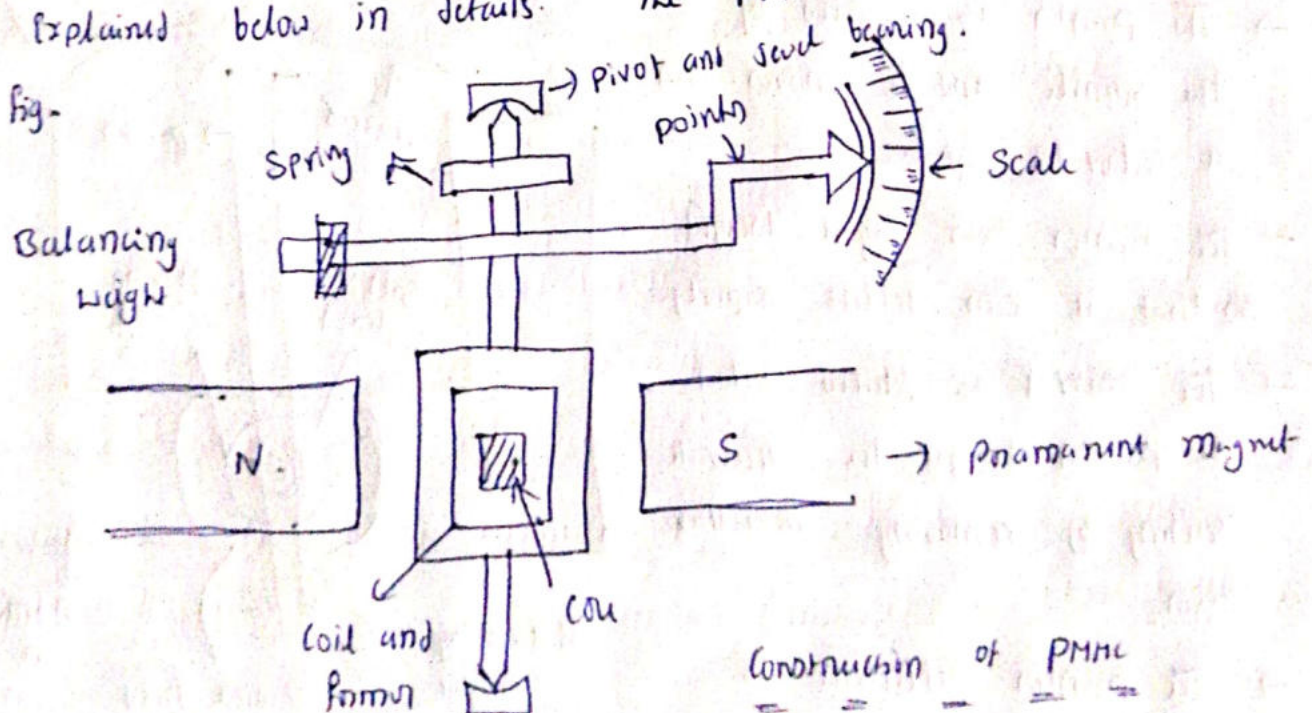
Definition :-

The instruments which use the permanent magnet by creating the stationary magnetic field between which the coils moves is known as the permanent moving coil (or) PMMC instrument.

It operates on the principle that the torque is exerted on the moving coil placed in the field of the permanent magnet. The PMMC instrument gives the accurate result for DC instrument.

### Construction of PMMC instrument

The moving coil and permanent magnet are the main parts of the PMMC instrument. The parts of the PMMC instruments are explained below in details. The PMMC instrument is shown in fig.



### Construction of PMMC

- The moving coil is either rectangular (or) circular in shape.
- It has number of turns of fine wire.



- The coil is suspended so that it is free to turn about its vertical axis.
- The coil is placed in a uniform, horizontal and radial magnetic field of permanent magnet in the shape of a horse-shoe.
- The iron core is spherical if coil is circular and is cylindrical if the coil is rectangular.
- Due to iron core the "deflecting torque" increases and then increasing sensitivity of the instrument.
- The "controlling torque" is provided by two phosphor bronze hair springs.
- The "damping torque" is provided by eddy current damping. It is obtained by movement of aluminium former in the magnetic field of the permanent magnet.

→ The pointer is carried by the spindle and it moves over a calibrated scale.

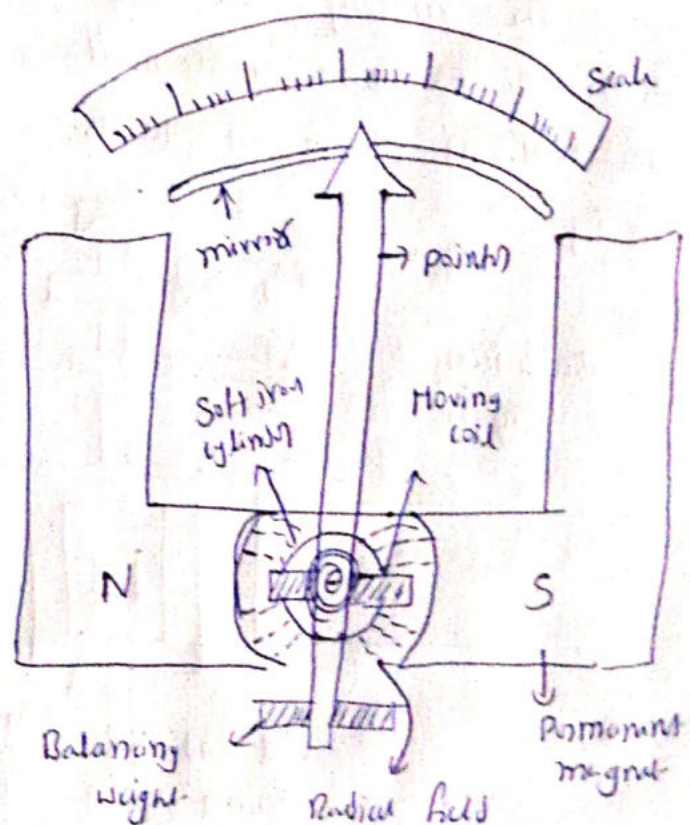
→ The pointer has light weight so that it can deflect rapidly.

→ The mirror is placed below the pointer to get the accurate reading by removing parallax error.

→ The pointer deflection is directly proportional to the current passing through the coil

$$[\theta \propto I]$$

→ The Top-view of PMMC instrument is shown in the fig. 2



PMMC instrument  
Fig. 2



## Torque Equation:-

13

The equation for the developed torque can be obtained from the basic law of "electromagnetic torque."

The deflecting torque is given by

$$T_d = NBA I$$

Where

$T_d$  = Deflecting torque N-m

$N$  = number of turns of the coil

$B$  = Flux density in air gap,  $Wb/m^2$

$A$  = coil area,  $m^2$

$I$  = current in the moving coil, Amp

$$\therefore T_d = k I \quad \text{where } k = NBA = \text{Constant}$$

The controlling torque is provided by the springs and is proportional to angular deflection of the pointer

$$T_c = k \theta$$

Where

$T_c$  = Controlling torque

$k$  = Spring constant,  $Nm/rad$  (or)  $Nm/deg$

$\theta$  = angular deflection,  $rad$  (or)  $deg$

At steady-state

deflection torque = controlling torque

$$T_d = T_c$$

$$k I = k \theta$$

$$\left[ \theta = \left( \frac{k}{k} \right) I \right] \quad (1)$$

$$\left[ I = \left( \frac{k}{k} \right) \theta \right]$$

The pointer deflection can therefore be used to measure current.



- As the direction of the current through to the coil changes, the direction of the pointer also changes.
- Hence such instruments are well suited for the dc measurements.
- In the microammeters and milliammeters upto about 20mA, the entire current to be measured is passed through the coil.
- The springs carry current to the coil.
- For higher currents, the moving coil is shunt by sufficient resistance, while the voltmeters having high ranges use a moving coil together with sufficient series resistance, to limit the current.
- Most dc voltmeters are designed to produce full scale deflection with a current of 20, 10, 5 (or) 1 mA.
- The power requirement of PMMC instrument is very small, typically of the order of 25mW to 200mW.

#### Advantages:-

- ✓ It has uniform scale.
- ✓ The sensitivity is high.
- ✓ It consumes low power.
- ✓ It has high accuracy.
- ✓ Instrument is free from hysteresis error.
- ✓ Extension is possible.

#### Disadvantages:-

- ✓ Suitable for dc measurements only.
- ✓ Aging of permanent magnet and control springs creates errors.
- ✓ Cost is high due to delicate construction.
- ✓ The friction due to jewel-pivot suspension.

Ex: A PMMC instrument has a coil of dimensions  $10\text{mm} \times 8\text{mm}$ . The flux density in the air gap is  $0.5\text{ Wb/m}^2$ . If the coil is wound for 100 turns, carrying a current of  $5\text{mA}$  then calculate the deflecting torque and calculate the deflection if the spring constant is  $0.2 \times 10^{-6}\text{ Nm/deg}$ .

Sol:  $T_d = N B A I$

$$\Rightarrow 100 \times 0.5 \times 10 \times 10^{-3} \times 8 \times 10^{-3} \times 5 \times 10^{-3}$$

$$\Rightarrow 6 \times 10^{-6}\text{ N.m}$$

Now  $T_d = T_c = 100$  at Steady State.

$$6 \times 10^{-6} = 0.2 \times 10^{-6} \times \theta$$

$$\theta = \frac{6 \times 10^{-6}}{0.2 \times 10^{-6}} \Rightarrow 30^\circ$$

$$\{\theta = 30^\circ\}$$

### \* Applications of PMMC Instruments :-

Permanent-magnet moving coil instruments can be used as "ammeters" & with the help of low resistance shunt (or) "voltmeters" with the help of a high series resistance.

The principle of PMMC type instruments has been utilized in the construction of the following.

- 1) A.C. galvanometer
- 2) Fluxmeter
- 3) Ballistic galvanometer.



### a) AC galvanometer

~~AC~~ AC galvanometer which can be used for detecting extremely small AC currents. A galvanometer may be used either as an ammeter (a) voltmeter.

### b) Flux meter

By eliminating the control springs, the instrument can be used for measuring the quantity of electricity passing through the coil. This method is used for fluxmeter.

### c) Ballistic galvanometer

If the control springs of such an instrument are purposely made of a large moment of inertia, then it can be used as a "ballistic galvanometer".

### \* Extension of Range of PMMC instruments

By the extension of range of PMMC instrument can be used as "ammeter" (a) "voltmeter".

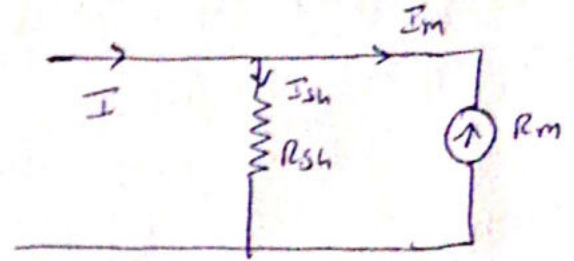
#### i) Ammeter Shunt

→ When the PMMC instrument is used as an ammeter, its range can be extended with the help of low-resistance shunt.

→ The moving coil instrument has a coil wound with very fine wire. It can carry only a few mA safely to give full-scale deflection.

→ For measuring higher current, low resistance is connected in parallel to the instrument to bypass the major part of the current.

→ The figure shows a shunt resistance  $R_{sh}$  connected in parallel with basic meter.



→ The resistance of the shunt can be calculated using conventional circuit analysis.

Here,  $R_{sh}$  = Shunt resistance ( $\Omega$ )

$R_m$  = coil resistance ( $\Omega$ )

$I_m$  = full scale deflection current (A)

$I_{sh}$  = shunt current (A)

→ The voltage drop across the shunt and the meter must be same as they are connected in parallel.

$$I_{sh} \times R_{sh} = I_m \times R_m \rightarrow (1)$$

$$I = I_{sh} + I_m \rightarrow (2)$$

$$I_{sh} = I - I_m \rightarrow (3)$$

→ From Eq. (1)

~~Then~~ 
$$R_{sh} = \frac{I_m}{I_{sh}} R_m$$

$$\left[ R_{sh} = \frac{I_m}{I - I_m} R_m \right] \quad \text{from Eq. (3)}$$

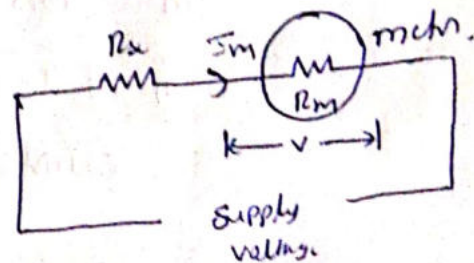
→ The ratio of total current to current in meter is called multiplying power of shunt and it is given by

$$m = \frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}} \Rightarrow \left[ R_{sh} = \frac{R_m}{m-1} \right]$$



## h) Voltage Multipliers

- The range of PMMC instrument, when used as a Voltmeter, can be increased by using a high resistance in series with it.
- For measuring higher voltages, a high resistance is connected in series with the instrument to limit the current in the coil to a safe value.
- This value of current should never exceed the current required to produce full-scale deflection.
- The high resistance connected in series with the instrument is called a "multiplier". In the fig  $R_x$  is the multiplier.
- The value of multiplier required to extend the voltage range is calculated as below.



here  $R_x$  = multiplier resistance ( $\Omega$ )

$R_m$  = meter resistance ( $\Omega$ )

$I_m$  = Full scale deflection current (A)

$V$  = Voltage to be measured (V)

- The voltage across the meter is given by

$$V_m = I_m R_m$$

$$V = I_m (R_m + R_x)$$

Total

$$\therefore R_x = \frac{V - I_m R_m}{I_m} \Rightarrow \frac{V}{I_m} - R_m$$

- Now multiply factor for multiplier

$$m = \frac{V}{I_m R_m}$$

$$m = \frac{V}{I_m R_m} \Rightarrow \frac{I_m (R_m + R_x)}{I_m R_m}$$

$$\left[ m = 1 + \frac{R_x}{R_m} \right]$$

X Errors in PMMC instrument :-

- The basic ~~types~~ sources of errors in PMMC instruments are friction, temperature and aging of various parts.
- To reduce the frictional errors, the ratio of torque to weight is made very high.
- The most serious errors are produced by the heat generated by changes in the temperature. This changes the resistance of the working coil, causing large errors.
- In the case of voltmeters, a large series resistance of a very low-temperature co-efficient resist is used. This reduces the temperature errors.
- The aging of permanent magnet and control springs also cause errors.
- The weakening of magnet and springs cause opposite errors.
- The weakening of magnet cause less deflection with weakness of the control springs cause large deflection, for a particular value of current.
- The proper use of material and packaging during manufacturing can reduce the errors due to weakening of the control springs.



## \* Problems on Extension of Voltmeter and Ammeter :-

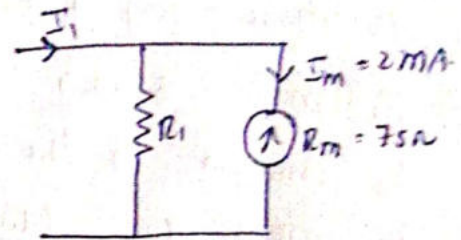
- 1) Design a multirange d.c milliammeter with a basic meter having a resistance  $75\Omega$  and full scale deflection for the current of  $2\text{mA}$ .  
The required ranges are  $0-10\text{mA}$ ,  $0-50\text{mA}$  and  $0-100\text{mA}$ .

Sol, a) The first range  $0-10\text{mA}$

$$I_1 = 10\text{mA}$$

$$I_m = 2\text{mA}$$

$$R_m = 75\Omega$$



$$R_1 = \frac{I_m}{I_1 - I_m} \times R_m \Rightarrow$$

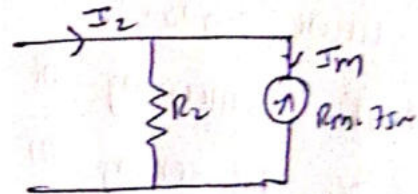
$$\Rightarrow \frac{2}{10-2} \times 75 \Rightarrow 18.75\Omega \Rightarrow [R_1 = 18.75\Omega]$$

b) The second range  $0-50\text{mA}$

$$I_2 = 50\text{mA}$$

$$I_m = 2\text{mA}$$

$$R_2 = \frac{I_m}{I_2 - I_m} \times R_m$$



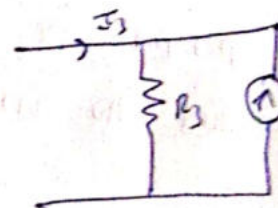
$$\Rightarrow \frac{2}{50-2} \times 75 \Rightarrow [R_2 = 3.125\Omega]$$

c) The third range  $0-100\text{mA}$

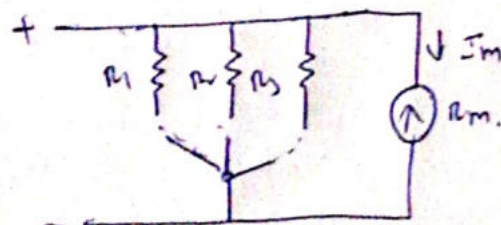
$$I_3 = 100\text{mA}$$

$$I_m = 2\text{mA}$$

$$R_3 = \frac{I_m}{I_3 - I_m} \times R_m \Rightarrow \frac{2}{100-2} \times 75 \Rightarrow 1.53\Omega \Rightarrow [R_3 = 1.53\Omega]$$



The designed multirange ammeter with a selector switch is shown in fig



- Q) A moving coil instrument gives a full scale deflection for a current of  $20\text{mA}$  with a potential difference of  $200\text{mV}$  across it. Calculate: i) shunt required to use it as an ammeter to get the range of  $0-200\text{A}$  ii) multiplier required to use it as a voltmeter of range  $0-500\text{V}$ .

Sol) meter current =  $20\text{mA}$  (for deflection)

Voltage across meter =  $200\text{mV}$

meter resistance  $R_m = ?$

$$V_m = I_m R_m \Rightarrow R_m = \frac{V_m}{I_m} \Rightarrow \frac{200}{20} = 10 \Omega$$

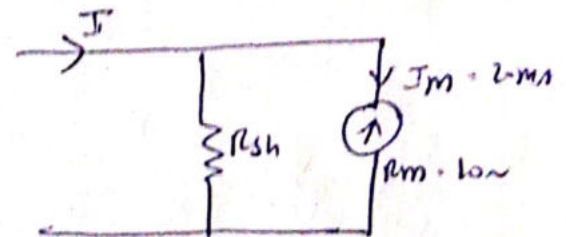
i) For using it as an ammeter

$$I = 200\text{A} \quad I_m = 20\text{mA}$$

$$R_m = 10$$

$$R_{sh} = \frac{I_m}{I - I_m} \times R_m$$

$$\Rightarrow \frac{2 \times 10^{-3}}{200 - 2 \times 10^{-3}} \times 10 \Rightarrow [R_{sh} = 0.001 \Omega]$$



ii) For using it as an voltmeter

$$V = 500$$

$$I_m = 20\text{mA}$$

$$R_m = 10$$

$$R_{sc} = \frac{V}{I_m} - R_m \Rightarrow \frac{500}{2 \times 10^{-3}} - 10$$

$$\Rightarrow 24.99 \text{ k}\Omega$$



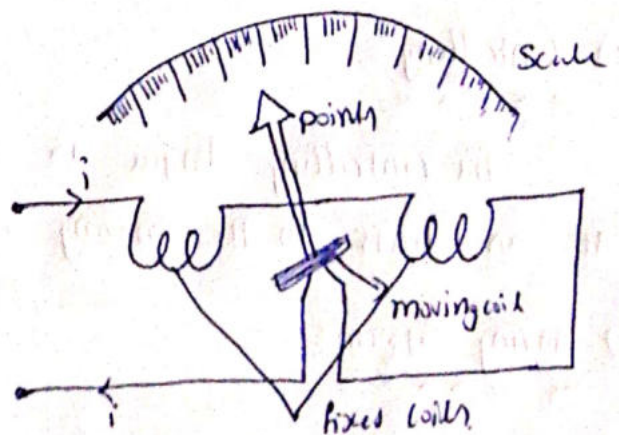
## \* Electrodynamometer type Instrument :-

- The Electrodynamometer is a "transfer type" instrument.
- A transfer type instrument is one that may be calibrated by a D.C. source and then used without modification to measure A.C. measurements.
- Such a transfer instrument has same accuracy for both A.C. and D.C. measurements.
- These instruments are also called "Electrodynamic" (or) "Dynamometer type" instruments.
- An electrodynamometer is a moving-coil instrument in which the operating field is produced not by a permanent magnet but by another fixed coil.
- This instrument can be used either as an ammeter (or) voltmeter but is generally used as a wattmeter.
- The electrodynamometer is a moving coil instrument but the magnetic field is provided by two fixed coils rather than by permanent magnet.
- The schematic diagram of the electrodynamometer instrument is shown below.

Construction :-

Parts of an Electrodynamometer instruments are made.

- Fixed coil
- Moving coil
- Control
- Moving System
- Damping
- Shielding
- Cases and scales.





#### a) Fixed coil :-

- ✓ The field is produced by a fixed coil. This coil is divided into two sections to give a more uniform field near the centre and to allow passage of the instrument shaft.
- ✓ Fixed coils are wound with fine wire for using as voltmeter, while for ammeters and wattmeters it is wound with heavy wire.
- ✓ The coils are usually varnished. They are clamped in place against the coil supports. This makes the construction rigid.

#### b) Moving coil :-

- ✓ The moving coil is wound either as a self-sustaining coil (a) or on a non-metallic former.
- ✓ If metallic former is used, then it would induce eddy currents in it.
- ✓ The construction of moving coil is made light as well as rigid. It is air coil.

#### c) Controlling :-

The controlling torque is provided by springs. These springs act as loads to the moving coil.

#### d) Moving System :-

The moving coil is mounted on an aluminium spindle. It consists of counter weights and pointer. Sometimes a suspension may be used, in case of high accuracy is desired.



### c) Damping :-

The damping torque is provided by air friction and is provided by a pair of aluminium vanes, attached to the spindle at the bottom.

### f) Shielding :-

The field produced by these instruments is very weak. Even earth's magnetic field considerably affects the reading. So Shielding is done to protect it from stray magnetic fields.

### g) Case and Scales :-

Laboratory standard instruments are usually contained in polished wooden (or) metal cases which are rigid. The case is supported by adjustable levelling screws.

→ For measuring using electrodynamic instrument as ammeter, fixed and moving coils are connected in series and carry the same current. A suitable shunt is connected to these coils to limit current through them.

→ For using electrodynamic instrument as voltmeter, fixed and moving coils are connected in series with a high non-inductive resistance. It is most accurate type instrument.

→ For using electrodynamic instrument as wattmeter, the fixed coils act as a current coil and must be connected in series with the load. The moving coil acts as a voltage coil (or) pressure coil and must be connected across the supply terminals.



→ The wattmeter indicates supply power.

→ When current passes through the fixed and moving coils, both the coils produce the magnetic fields.

→ The field produced by the fixed coil is proportional to load current while the field produced by the moving coil is proportional to voltage.

→ As the deflecting torque is produced due to the interaction of these two fields.

\* Why PMMC instruments cannot be used for AC measurements?

The PMMC instrument cannot be used on a.c currents (or) voltage. If a.c supply is given to these instruments, an alternating torque will be developed. Due to moment of inertia of the moving system, the pointer will not follow the rapidly changing alternating torque and will fail to show any reading.

In order for the instrument should be able to read a.c quantities, the magnetic field in the air gap must change along with the change in current. This principle is used in electrodynamic type instrument.



## \* Torque Equation of electrodynamometer type instrument

- Let
- $i_1$  = instantaneous value of current in fixed coil
  - $i_2$  = instantaneous value of current in moving coil
  - $L_1$  = Self inductance of fixed coils
  - $L_2$  = Self inductance of moving coils
  - $M$  = Mutual inductance between fixed and moving coils.

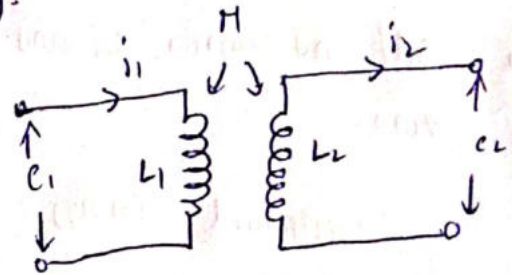
The electrodynamometer instrument can be represented by an equivalent circuit as shown in fig.

The flux linkage of coil-1

$$\phi_1 = L_1 i_1 + M i_2$$

The flux linkage of coil-2

$$\phi_2 = L_2 i_2 + M i_1$$



Now induced EMF

$$e_1 = \frac{d\phi_1}{dt} \quad \text{and} \quad e_2 = \frac{d\phi_2}{dt}$$

$$\text{Electrical input energy} = e_1 i_1 dt + e_2 i_2 dt$$

$$\Rightarrow i_1 d\phi_1 + i_2 d\phi_2$$

$$\Rightarrow i_1 d(L_1 i_1 + M i_2) + i_2 d(L_2 i_2 + M i_1)$$

$$\Rightarrow i_1 L_1 di_1 + i_1^2 dL_1 + i_1 i_2 dM + i_1 M di_2 + i_2 L_2 di_2 + i_2^2 dL_2 + i_1 i_2 dM + i_2 M di_1 \rightarrow \text{①}$$

The energy stored in the magnetic field due to  $L_1$ ,  $L_2$  and  $M$  is given by,

$$\text{Energy stored} = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + i_1 i_2 M$$

$$\text{Change in stored energy} = d \left[ \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + i_1 i_2 M \right]$$

$$\Rightarrow i_1 L_1 di_1 + \frac{1}{2} i_1^2 dL_1 + i_2 L_1 di_2 + \frac{1}{2} i_2^2 dL_2 + i_1 M di_2 + i_2 M di_1 + i_1 i_2 dM \rightarrow (2)$$

From the principle of conservation of energy.

Electrical i/p Energy = Energy stored + Mechanical Energy.

$\therefore$  Mechanical Energy = Electrical i/p Energy - Energy stored.

Subtracting Equation (2) from Eq. (1)

$$\therefore \text{Mechanical Energy} = \frac{1}{2} i_1^2 dL_1 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dM$$

The self inductances  $L_1$  and  $L_2$  are constants and hence  $dL_1$  and  $dL_2$  are zero.

$$\therefore \text{Mechanical Energy} = i_1 i_2 dM$$

If  $T_i$  is the instantaneous deflecting torque and  $d\theta$  is the change in the deflection then,

Mechanical Energy = Mechanical work done

$$i_1 i_2 dM = T_i d\theta$$

$$\left[ T_i = i_1 i_2 \frac{dM}{d\theta} \right]$$

This is the expression for the instantaneous deflecting torque.  
Let us see its operation on 'a.c.' and 'd.c.'

DC operation - For d.c currents of  $I_1$  and  $I_2$

$$T_d = I_1 I_2 \frac{dM}{d\theta}$$

The controlling torque is provided by spring hkn.,

$$T_c = K\theta$$

At steady state

$$T_d = T_c$$



$$I_1 I_2 \frac{dM}{d\theta} = k\theta$$

$$\int \theta = \frac{I_1 I_2}{k} \cdot \frac{dM}{d\theta}$$

Thus the deflection is proportional to the product of the two currents and the rate of change of mutual inductance.

AC operation:-

In ac operation, the total deflecting torque over a cycle must be obtained by integrating  $T_i$  over one period.

Average deflecting Torque over one cycle is,

$$T_d = \frac{1}{T} \int_0^T T_i dt$$

$T$  = time period of one cycle.

$$T_d = \frac{1}{T} \int_0^T I_1 I_2 \frac{dM}{d\theta} dt$$

$$\therefore T_d = \frac{dM}{d\theta} \cdot \frac{1}{T} \int_0^T I_1 I_2 dt$$

Now if the two currents are sinusoidal and displaced by a phase angle  $\phi$  then

$$I_1 = I_{m1} \sin \omega t$$

$$\text{and } I_2 = I_{m2} \sin (\omega t - \phi)$$

$$\therefore T_d = \frac{dM}{d\theta} \cdot \frac{1}{T} \int_0^T I_{m1} \sin \omega t \cdot I_{m2} \sin (\omega t - \phi) d\omega t$$

$$= I_{m1} \cdot I_{m2} \frac{dM}{d\theta} \cdot \frac{1}{T} \int_0^T \sin \omega t \cdot \sin (\omega t - \phi) d\omega t$$

$$= \left( \frac{I_{m1} \cdot I_{m2}}{2} \right) \cos \phi \frac{dM}{d\theta}$$

$$\left[ T_d = I_1 I_2 \cos \phi \frac{dM}{d\theta} \right]$$

Where  $I_1$  and  $I_2$  are the r.m.s values of the two currents,

$$I_1 = \frac{I_{m1}}{\sqrt{2}} \quad \text{and} \quad I_2 = \frac{I_{m2}}{\sqrt{2}}$$

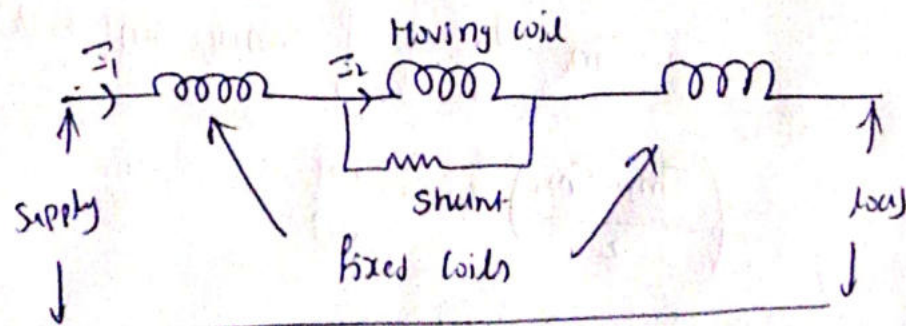
As  $T_c = k\theta$

At steady state  $T_d = T_c$

$$I_1 I_2 \cos \phi \frac{d\theta}{dt} = k\theta$$

$$\left[ \theta = \frac{I_1 I_2}{k} \cos \phi \frac{d\theta}{dt} \right]$$

- Thus the deflection is decided by the product of r.m.s values of two currents, cosine of phase angle and rate of change of mutual inductance.
- If an electrodynamometer instrument is calibrated with a d.c current of 1A and pointer indicates 1A d.c on scale then on a.c the pointer will deflect upto the same mark but 1A in this case will indicate r.m.s value.
- The instrument can be used as an ammeter to measure currents upto 20A. While using as a voltmeter it can have low sensitivity of about 10 to 30V/V.
- The fig a, b, c shows the connections of the Electrodynamometer instrument as ammeter, voltmeter and wattmeter.



Fig(a) Electrodynamometer ammeter.



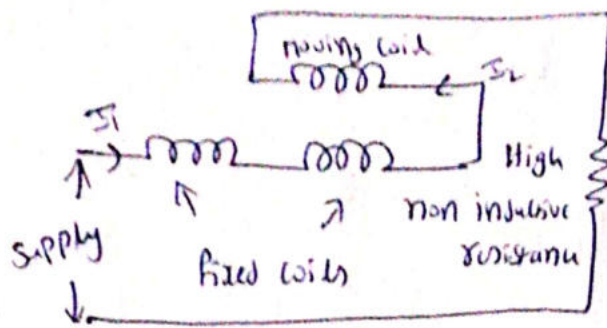


Fig 1b) electrodynamic voltmeter.

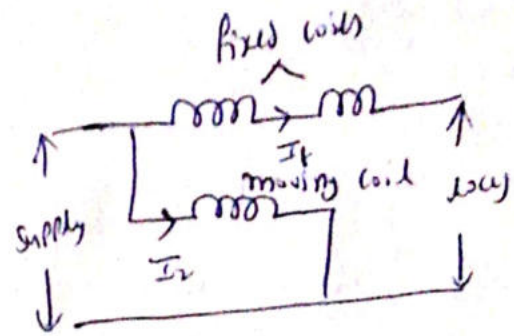


Fig 1c) electrodynamic wattmeter.

### Advantages

~ ~ ~ ~

- These instruments are free from hysteresis and eddy current loss.
- They have a precision grade accuracy.
- low power consumption
- Light in weight.

### Disadvantages

~ ~ ~ ~

- They are more expensive than other instruments.
- They have non-uniform scale
- These instruments are sensitive to overloads and mechanical impacts.

## \* Moving iron instruments

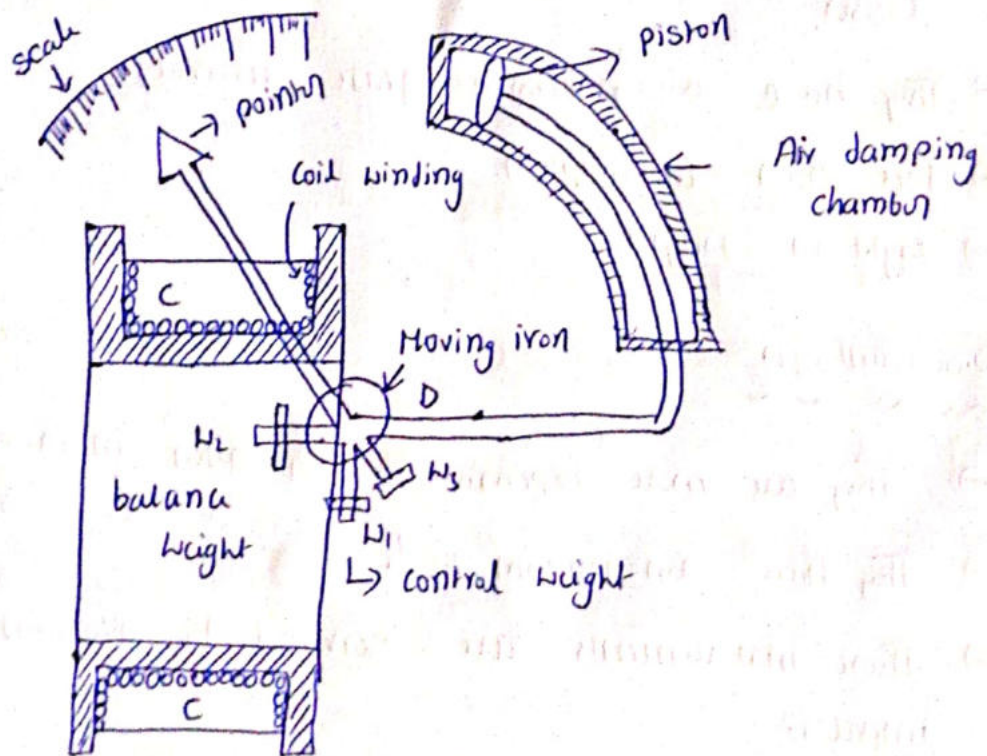
The moving iron instruments are classified as

- i) Moving iron attraction type instruments
- ii) Moving iron repulsion type instruments.

### i) Moving iron attraction type instruments

→ The basic working principle of these instruments is very simple that a soft iron piece if brought near the magnet gets attracted by the magnet.

→ The construction of the attraction type instrument is shown in fig.



### Moving iron attraction type instrument

- It consists of fixed coil 'C' and moving iron piece 'D'.
- The coil is flat and has a narrow slot like opening.
- The moving iron is a flat disc which is eccentrically mounted on the spindle. The spindle is supported between the jewel bearings.



- The spindle carries a pointer which moves over a <sup>23</sup> scale.
- The number of turns of the fixed coil are dependent on the range of instrument.
- For passing large current through the coil only few turns are required.
- The controlling torque is provided by the springs but gravity control may also be used for vertically mounted panel type instrument.
- The damping torque is provided by the air friction.
- A light aluminium piston is attached to the moving system. It moves in a air chamber. The chamber is closed at one end.
- The operating magnetic field in moving iron instruments is very weak. Hence eddy current damping is not used here.

### ii) Moving iron repulsion type instrument:-

These instruments have two vanes inside the coil, the one is fixed and other is movable. When the current flows in the coil both the vanes are magnetized with like polarities induced on the same side. Hence due to the repulsion of like polarities, there is a force of repulsion between the two vanes causing the movement of moving vane.

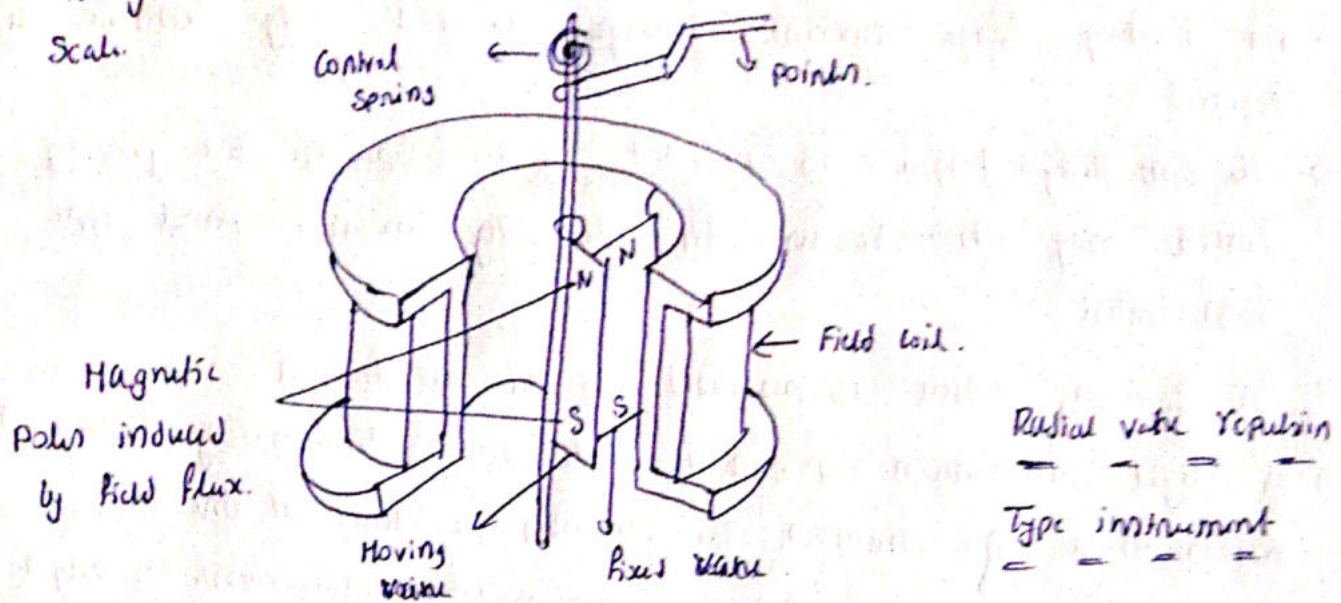
The repulsion type instruments are the mostly used instruments. The two different design of repulsion type instruments are

- a) Radial vane type
- b) Co-axial vane type.



### a) Radial vane Repulsion Type instrument :-

→ Fig shows the radial vane repulsion type instrument. out of the other moving iron mechanisms, this is the most sensitive and has most linear scale.

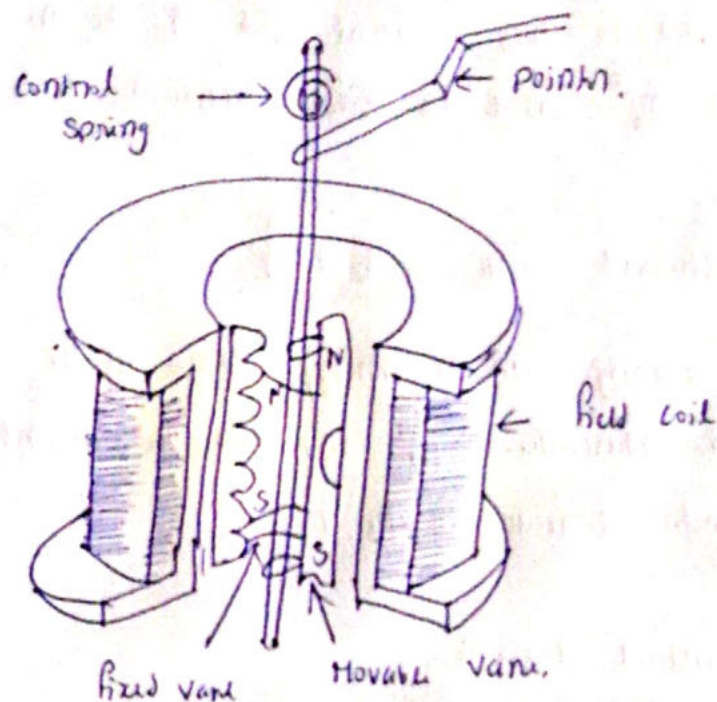


- The two vanes are radial strips of iron.
- The fixed vane is attracted to the coil and the movable vane is attracted to the spindle and suspended in the induction field of the coil.
- pointer is attached to the movable vane.
- Movable vane is curved ~~trapezoidal~~ rectangular in shape.
- When the current flows through the coils the two iron vanes become magnetized with north poles at their upper ends and south poles at their lower ends.
- Even though the current through the coil is alternating, there is always repulsion between the like poles of the fixed and the movable vane. Hence the deflection of the pointer is always in the same direction.
- The deflection is effectively proportional to the actual current and hence the scale is calibrated directly to read amperes (A) volts.



24  
b) Concentric Vane Repulsion Type Instrument

→ Fig Shows the Concentric vane repulsion type instrument.



- The instrument has two concentric vanes. one is attached to the coil frame while the other can rotate co-axially inside the stationary vane.
- Both the vanes are magnetized to the same polarity due to the current in the coil. Thus the movable vane rotates under the repulsive force.
- As the movable vane is attached to the pivoted shaft, the repulsion results in a rotation of the shaft.
- The pointer deflection is proportional to the current in the coil.
- The scale of the instrument is non-uniform in nature.
- Thus whatever may be the direction of the current in the coil, the deflection in the moving iron instruments is in the same direction.
- Hence moving iron instruments can be used for both a.c and d.c measurements.

## Torque Equation of moving iron instrument:-

Consider a small increment in current supplied to the coil of the instrument. Due to this current, let  $\theta$  be the deflection under the deflecting Torque ' $T_d$ '. Due to such deflection, some mechanical work will be done.

$$[\text{Mechanical work} = T_d d\theta]$$

There will be a change in the Energy stored in the magnetic field due to the change in inductance. The inductance is inversely proportional to reluctance of magnetic circuit of the coil.

Let

- $I$ : initial current
- $L$ : instrument inductance
- $\theta$ : deflection
- $dI$ : increase in current
- $dL$ : change in inductance
- $d\theta$ : change in deflection.

Now 
$$e = \frac{d(LI)}{dt}$$

$$\Rightarrow I \frac{dL}{dt} + L \frac{dI}{dt} \quad \dots \text{As both } I \text{ and } L \text{ are changing.}$$

The Electrical Energy supplied is given by

$$e I dt = \left( I \frac{dL}{dt} + L \frac{dI}{dt} \right) I \cdot dt$$

$$\Rightarrow I^2 dL + L I dI \rightarrow \text{--- (1)}$$

The Stored Energy increases from  $\frac{1}{2} L I^2$  to  $\frac{1}{2} (L + dL) (I + dI)^2$

Hence the change in the stored Energy is given by

$$= \frac{1}{2} (L + dL) (I + dI)^2 - \frac{1}{2} L I^2$$



$$\Rightarrow \frac{1}{2} (L + \delta L) (I^2 + 2I\delta I + \delta^2 I^2) - \frac{1}{2} L I^2$$

$$\Rightarrow \frac{1}{2} (L I^2 + 2I L \delta I + \delta^2 I^2 L + I^2 \delta L + 2I(\delta L)^2 + \delta^2 I^2 \delta L) - \frac{1}{2} L I^2$$

neglecting higher order terms

$$I L \delta I + \frac{1}{2} I^2 \delta L$$

The energy is supplied is nothing but increase in stored energy plus the energy required for mechanical work done

$$I^2 \delta L + I L \delta I$$

$$I^2 \delta L + I L \delta I = I \delta I + \frac{1}{2} I^2 \delta L + T_d \delta \theta$$

$$T_d \delta \theta = \frac{1}{2} I^2 \delta L$$

$$\left[ T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} \right]$$

Like the controlling torque is given by

$$T_c = k\theta$$

at steady state condition

$$T_d = T_c$$

$$k\theta = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

$$\left[ \theta = \frac{1}{2} \frac{I^2}{k} \frac{dL}{d\theta} \right]$$

Thus the deflection is proportional to the square of the current through the coil.

### \* Advantages :-

- i) The moving iron instruments can be used for both a.c. & d.c. measurements.
- ii) As the torque to weight ratio is high, errors due to friction are very less.
- iii) There are no current carrying parts in the moving system hence these meters are extremely rugged and reliable.
- iv) These can withstand large loads and are not damaged even under severe overload conditions.
- v) The range of instruments can be extended.

### Disadvantages :-

- i) Scale is not uniform.
- ii) There are serious errors due to hysteresis, frequency changes and stray magnetic fields.
- iii) Power consumption is on higher side.

### \* Errors in Moving iron instruments :-

The various errors in the moving iron instruments, are

#### a) Hysteresis Error :-

Due to hysteresis effect, the flux density for the same current while ascending and descending values is different. While descending the flux density is higher while ascending it is lower. So meter reads higher for descending values of current (or) voltage. So remedy for this is to use smaller iron parts which can demagnetize quickly (or) to work with lower flux densities.



### b) Temperature Error

The temperature error arises due to effect of temperature on the temperature coefficient of the spring. This error is of the order of  $0.02\%$  per  $^{\circ}\text{C}$  change in temperature. Errors can cause due to self heating of the coil and due to which change in resistance of the coil. So coil and series resistance must have low temperature coefficient. Hence manganin is generally used for the series resistances.

### c) Frequency Error

These are related to a.c. operation of the instrument. The change in frequency affects the reactance of the working coil and also affects the magnitude of the eddy currents. This causes errors in the instruments.

### d) Eddy Current Error

When instrument is used for a.c. measurements the eddy currents are produced in the iron parts of the instruments. The eddy current affects the instrument current causing the change in the deflecting torque. This produces the error in the meter readings.

## \* Comparison of controlling Systems

### Gravity control

- i) Adjustable small weight is used which produces the controlling torque.
- ii) Controlling torque can be varied.
- iii) The performance is not temp. dependent.
- iv) The scale is non-uniform.
- v) The controlling torque is proportional to  $\sin \theta$ .
- vi) The readings cannot be taken accurately.
- vii) The system must be used in vertical position only.
- viii) Proper levelling is required as gravity control.
- ix) Simple, cheap but delicate.
- x) Rarely used for indicating.

### Spring control

- i) Two hair springs are used which exert controlling torque.
- ii) Controlling torque is fixed.
- iii) The performance is temperature dependent.
- iv) The scale is uniform.
- v) The controlling torque is proportional to  $\theta$ .
- vi) The readings can be taken very accurately.
- vii) The system need not be necessarily in vertical position.
- viii) The levelling is not required.
- ix) Simple, rigid but costlier compared to gravity control.
- x) Very popularly used for indicating.



## Example problems

- 1) The inductance of a moving iron instrument is given by  $L = (12 + 6\theta - \theta^2) \mu H$ , where  $\theta$  is the deflection in radians from zero position. The spring constant is  $12 \times 10^{-6} \text{ Nm/radians}$ . Calculate the deflection for a current of  $8A$ .

Sol) Given that  $K = 12 \times 10^{-6} \text{ Nm/radian}$   
 $I = 8A$

Deflection  $\theta = \frac{1}{2} \times \frac{I^2}{K} \times \frac{dL}{d\theta}$

$$L = (12 + 6\theta - \theta^2)$$

$$\frac{dL}{d\theta} = (-2\theta + 6) \times 10^{-6}$$

$$\therefore \theta = \frac{1}{2} \times \frac{(8)^2}{12 \times 10^{-6}} \times (-2\theta + 6) \times 10^{-6}$$

$$\theta = 2.667(-2\theta + 6)$$

$$6.334\theta = 16.002 \Rightarrow \theta = 2.526 \text{ radians}$$

$$\Rightarrow \theta = 144.7^\circ$$

- 2) A  $2mA$  mcn with an internal resistance of  $100\Omega$  is to be converted into  $0-150mA$  ammeter. Calculate the value of the shunt resistance required.

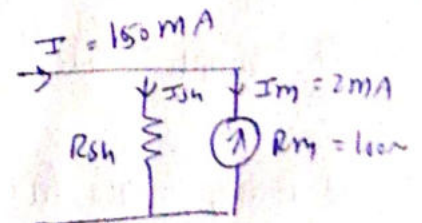
Sol,

$$I_{sh} R_{sh} = I_m R_m$$

$$R_{sh} = \frac{I_m R_m}{I_{sh}}$$

$$R_{sh} = \frac{I_m R_m}{I - I_m}$$

$$\Rightarrow \frac{2 \times 10^{-3} \times 100}{(150 - 2) \times 10^{-3}} \Rightarrow 1.351 \Omega$$



$$I = I_{sh} + I_m$$

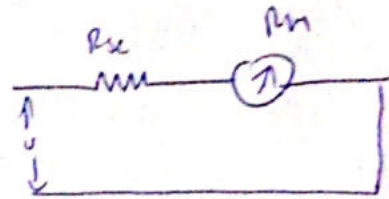
$$I_{sh} = I - I_m$$

- 3) A moving coil instrument gives a full scale deflection with a current of  $40 \mu A$ , while the internal resistance of the meter is  $500 \Omega$ . It is to be used as a voltmeter to measure the voltage range of  $0-100V$ . Calculate the multiplier resistance.

Sol

$$I_m = 40 \mu A, \quad R_m = 500 \Omega$$

$$V = 100V$$



$$I = I_m$$

$$V = I R_x + I R_m$$

$$V = I_m R_x + I_m R_m$$

$$I_m R_x = V - I_m R_m$$

$$R_x = \frac{V - I_m R_m}{I_m} \Rightarrow \frac{V}{I_m} - R_m$$

$$R_x = \frac{100}{40 \times 10^{-6}} - 500 \Rightarrow \underline{\underline{249.95 \text{ k}\Omega}}$$

- 4) Calculate the value of multiplier resistance of  $500V$  range of a dc voltmeter, that uses  $50 \mu A$  meter movement with an internal resistance of  $200 \Omega$ .

Sol

$$R_m = 200 \Omega, \quad I_m = 50 \mu A$$

$$V = 500$$

$$R_x = \frac{V}{I_m} - R_m$$

$$\Rightarrow \frac{500}{50 \times 10^{-6}} - 200$$

$$\Rightarrow 9.99 \text{ k}\Omega$$



## UNIT-2

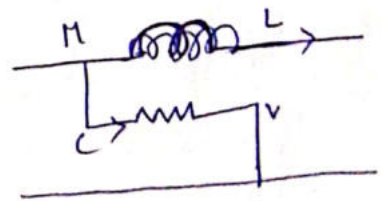
### Measurement of power, Energy & power Factor.

#### \* Power measurement

Electric power is the rate of doing work. It is expressed in Watts. The higher units of power used in practice include kilowatts, megawatts etc.

$P_{\text{watts}} = VI \cos \phi$ , i.e., power of one watt is said to be expended when a source of one volt passes a current of one ampere through a load resistance of one ohm at unity power factor.

The power measurements are made with the help of a wattmeter. Wattmeter is an indicating deflecting type of instrument used in laboratories for measurement of power in various ranges. A wattmeter consists of two coils as shown in the schematic representative fig.



⇒ Current coil

Fig. Wattmeter connection.

The current coil connected in series with circuit and carries the load current. It is designed such that it is wound with 2 to 3 turns of thick wire and hence it has very low resistance.

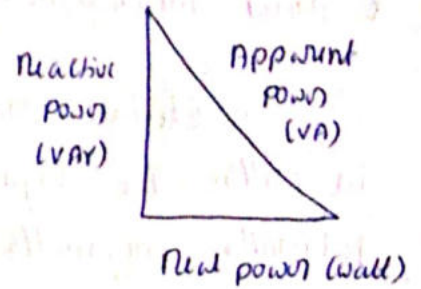
⇒ voltage (or) potential (or) pressure coil

It is connected across the load circuit and hence carries a current proportional to the load current. The total load voltage appears across the potential coil. It is designed such that it is wound with several turns of thin wire and hence it has a very high resistance.

The wattmeter can be a UPF meter or LPF meter depending on the type of the load connected in the measuring circuit. For power measurements in AC circuits, the wattmeter is widely adopted.

The electrical power can be of three forms,

- i) 'Real power' is the power consumed by the resistive loads on the system. It is expressed as true power, absolute power, average power or wattage.
- ii) 'Reactive power' is the power consumed by the reactive loads on the system. It is expressed in reactive volt-ampere (VAR).
- iii) 'Apparent power' is the vector sum of the above two power components. It is expressed in volt amperes (VA).



$$(\text{Apparent power})^2 = (\text{Real power})^2 + (\text{Reactive power})^2$$

$$= (VI \cos \phi)^2 + (VI \sin \phi)^2$$

$$\Rightarrow V^2 I^2 \cos^2 \phi + V^2 I^2 \sin^2 \phi$$

$$\Rightarrow V^2 I^2 (\cos^2 \phi + \sin^2 \phi)$$

$$[\text{Apparent power} = VI]$$

$$\text{Real power (P)} = VI \cos \phi$$

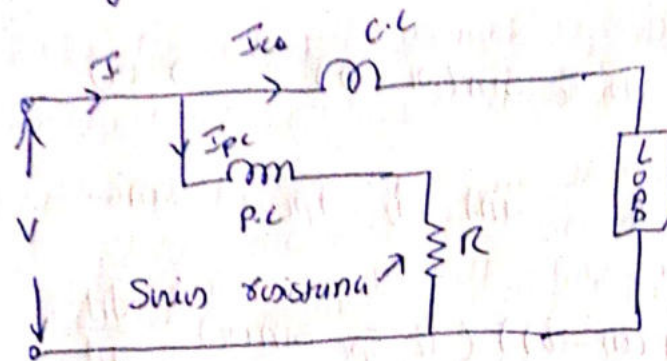
$$\cos \phi = \frac{P}{VI}$$

$$\therefore \left[ \text{Power factor } \cos \phi = \frac{P}{VI} = \frac{\text{Watts}}{VI} \right]$$



## \* Single phase Dynamometer wattmeter:-

- An electrodynamometer type wattmeter is used to measure power.
- It has two coils, one is fixed coil which is current coil and another one is moving coil which is potential coil.
- The current coil carries the current of the circuit while potential carries current which is proportional to the voltage in the circuit.
- The connections of an electrodynamometer wattmeter in the circuit are shown in fig.



Electrodynamometer wattmeter.

- Where
- $I_C$  = Current through current coil
  - $I_P$  = Current through potential coil.
  - $R$  = Series resistance
  - $V$  = R.H.S value of supply voltage
  - $I$  = R.H.S value of current.

### Torque Equation

According to theory of electrodynamic instruments

$$T_s = i_1 i_2 \frac{dM}{d\theta} \rightarrow (1)$$

Let  $V$  = instantaneous voltage =  $V_m \sin \omega t$   
 $= \sqrt{2} V \sin \omega t \rightarrow (2)$

$\left[ V = \frac{V_m}{\sqrt{2}} \right]$  Due to high series resistance, potential coil is treated as purely resistive.

key point: The current  $I_{pc}$  is in phase with  $V$  as potential coil is purely resistive.

$$I_{pc} = \text{instantaneous value} = \frac{V}{R_p}$$

$$i_{pc} = \frac{\sqrt{2}V}{R_p} \sin \omega t \Rightarrow \sqrt{2} I_{pc} \sin \omega t$$

$$i_{pc} = \sqrt{2} I_{pc} \sin \omega t \rightarrow (3)$$

If current coil current lags the voltage by an angle  $\phi$  then its instantaneous value is

$$i_c = \sqrt{2} I_c \sin(\omega t - \phi) \rightarrow (4)$$

Now  $i_1 = i_c$  and  $i_2 = i_{pc}$  hence.

$$T_i = (\sqrt{2} I_c \sin(\omega t - \phi)) (\sqrt{2} I_{pc} \sin \omega t) \frac{dH}{d\theta}$$

$$\Rightarrow 2 I_c I_{pc} \sin(\omega t) \sin(\omega t - \phi) \frac{dH}{d\theta}$$

$$T_i = I_c I_{pc} [\cos \phi - \cos(2\omega t - \phi)] \frac{dH}{d\theta} \rightarrow (5)$$

Averaging torque

$$T_d = \frac{1}{T} \int_0^T T_i d\omega t$$

$$\Rightarrow \frac{1}{T} \int_0^T I_c I_{pc} [\cos \phi - \cos(2\omega t - \phi)] \frac{dH}{d\theta} d\omega t$$

$$\left[ T_d = I_c I_{pc} \cos \phi \frac{dH}{d\theta} \right] \rightarrow (6)$$

For a spring controlled wattmeter

$$T_c = K\theta \rightarrow (7)$$

At steady state condition

$$T_d = T_c$$



$$k_1 \theta = I_c I_{pc} \cos \phi \frac{dH}{d\theta}$$

$$\theta = \frac{1}{k_1} I_c I_{pc} \cos \phi \frac{dH}{d\theta}$$

$$\left[ \theta = k_1 I_c I_{pc} \cos \phi \right] \rightarrow (8)$$

$$\text{When } k_1 = \frac{1}{k} \frac{dH}{d\theta}$$

$$\theta = k_1 I_c \frac{V}{R_p} \cos \phi$$

$$\theta = \frac{k_1}{R_p} V I_c \cos \phi \Rightarrow \left[ \theta = k_2 P \right] \rightarrow (9)$$

$$\text{When } k_2 = \frac{k_1}{R_p} \text{ and } P = V I_c \cos \phi$$

$$\left[ \theta \propto P \right] \rightarrow (10)$$

Thus the wattmeter deflection when calibrated gives the power consumption of the circuit.

\* Low power factor (LPF) wattmeter :-

A low power factor (LPF) wattmeter is an instrument used to measure power in low power factor circuits. This type of wattmeter is employed for power measurement in circuits whose P.F. is less than 0.5.

- Generally, wattmeters are used for measuring power in a circuit.
- But, the power measurement using normal electro-dynamometer wattmeter in a circuit operating at LPF causes inaccuracy in the readings due to the following reasons.

- i) When the ordinary wattmeter is connected to a low power factor circuit. Even though at full excitation of current coil and potential coil, the deflecting torque produced on the moving coil is very small.
  - ii) Wattmeter operating at LPF introduces large errors due to voltage coil inductance.
- Hence some additional features are ~~introduced~~ incorporated into the ordinary electrodynamic wattmeter to overcome these difficulties.
- So that accurate readings can be obtained by the wattmeter when used in LPF circuits.

Construction of LPF is

The construction of a LPF wattmeter is same as that of an electrodynamic type wattmeter with slight modifications for the compensation of errors.

a) Compensation of error due to pressure (or) potential coil current

✓ When the potential coil of the wattmeter is connected across the supply side as shown in fig.

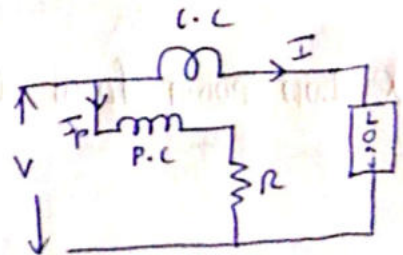


fig.1

✓ In LPF circuits at low power, the current will be very high.

✓ Due to this power loss in the current coil will be very high.

✓ Since the pressure coil is connected across the supply, the total power will be the sum of load power loss and power loss in the current coil.

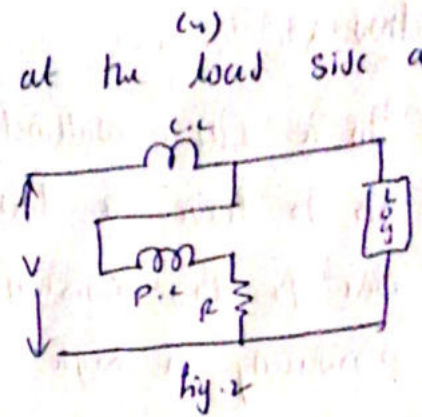
✓ Thus large error will be introduced.



Method-1 :-

Suppose, if the pressure coil is connected at the load side as shown in fig. 2

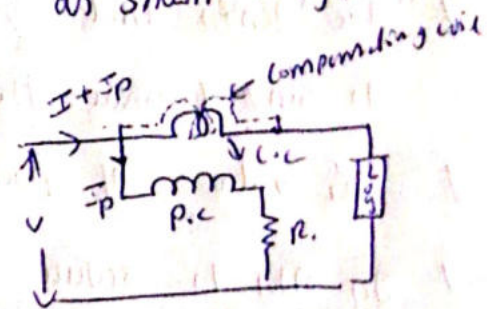
- The total power measured will be not the power loss by the load.



Method-2 :-

- In order to nullify the errors due to pressure coil current, an additional winding known as compensating winding is used and is connected in series with the pressure coil as shown in fig.

- It is identical to the current coil and tightly wound with it to produce a field that acts in opposition to the field of the current coil due to load current.



- The current in the current coil is the sum of load current 'I' and current through the pressure coil ( $I_p$ ).
- The current coil produces a field which is proportional to  $(I + I_p)$  whereas the pressure coil and compensating coil produce a field, proportional to ' $I_p$ '.
- Thus the field produced by the compensating coil neutralizes the field produced by the current coil.
- Hence, the effect of the potential coil current is eliminated and the resultant field is due to current 'I' only.
- Therefore, errors due to pressure coil current are nullified and compensated wattmeter shows no error.

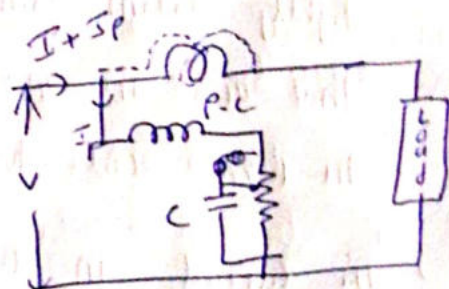


### Method - 3:

- ✓ In a LPF wattmeter, the pressure coil area is increased so as to have a low value of resistance.
- ✓ Due to low resistance, a very high current flows through it producing a high value of operating torque.

### b) Compensation of error due to potential coil inductance:

- ✓ We know that error due to pressure coil inductance is proportional to  $\sin\phi$  (where the magnitude of error is given by  $V I \sin\phi \tan\phi$ )
- ✓ The value of power factor is low and hence the value of  $\phi$  is high.
- ✓ In order to reduce this error, a capacitor is connected across the series resistor of the potential coil as shown in fig.
- ✓ The presence of the capacitor will give a leading phase angle due to which the value of  $\phi$  is small. So the  $\sin\phi$  is small. Hence reducing the error.



- Therefore a modified electrodynamic type wattmeter overcomes the difficulties at LPFs and obtains high accuracy, thus making it a LPF wattmeter.



## \* Difference between LPF and VPF wattmeter

### LPF wattmeter

- i) In the case of LPF wattmeter, the deflecting torque is small.
- ii) The pressure coil current is very high.
- iii) The error caused by potential coil inductance is large.
- iv) Control torque is small.
- v) In LPF wattmeter results are inaccurate.

### VPF wattmeter

- i) In the case of VPF wattmeter, the deflecting torque is high.
- ii) The pressure coil current is small.
- iii) The error caused by potential coil inductance is less.
- iv) Control torque is large.
- v) In VPF wattmeter results are accurate.

## \* Double element dynamometer wattmeter

→ Double element dynamometer wattmeter is used for three-phase, three-wire system and it is called 3-phase electrodynamometer type wattmeter for the measurement of 3-phase power.

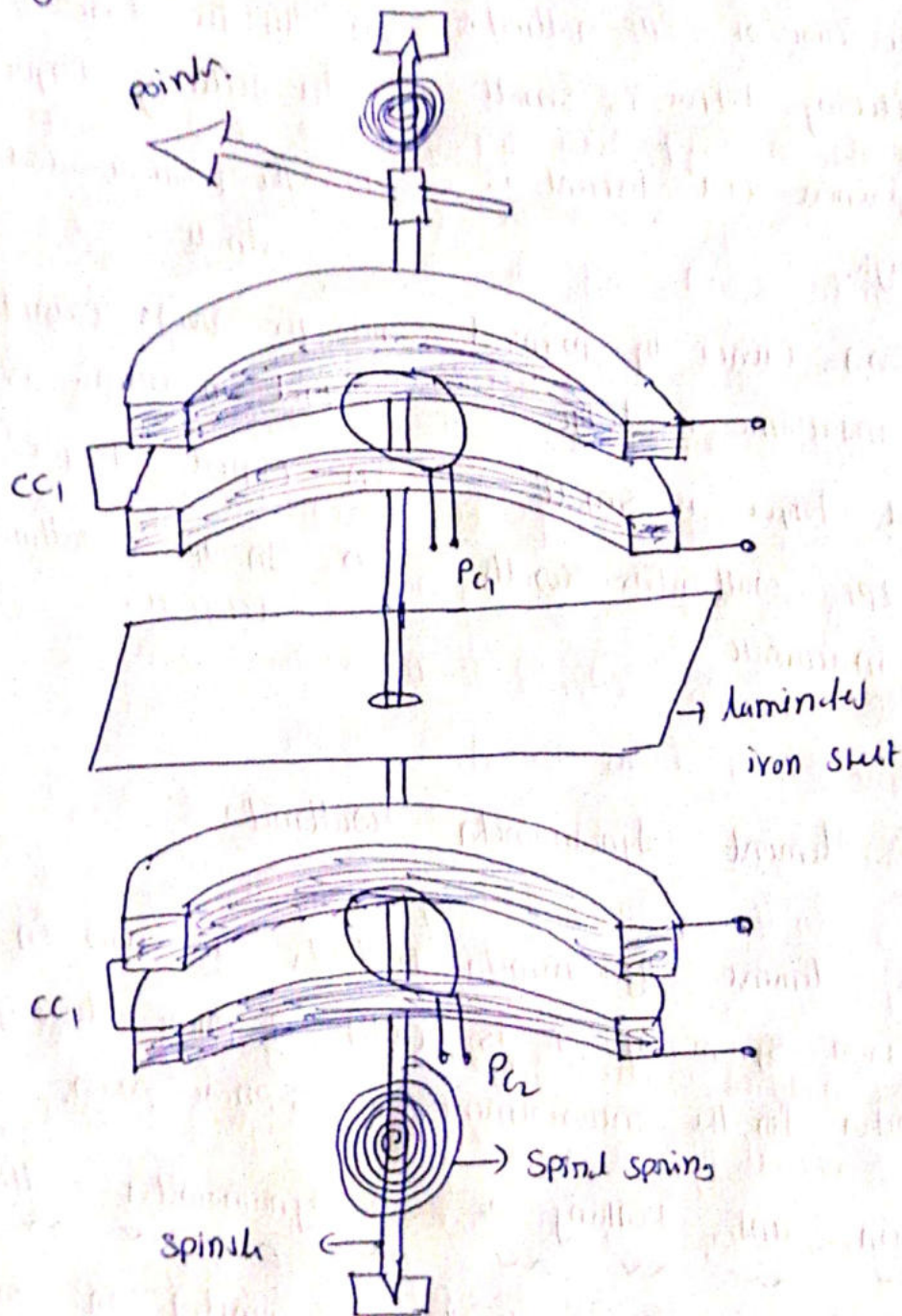
### Construction and Working of 3 $\phi$ dynamometer wattmeter

- ✓ The 3 $\phi$  dynamometer wattmeter is similar in construction to that of a single-phase dynamometer wattmeter. The only difference is it consists of two fixed coils and two moving coils.
- ✓ The fixed coils are connected in series with the lines and moving coils are connected across the lines.



1) Hence the fixed coils are known as current coils ( $CC_1$  &  $CC_2$ ), and moving coils are also known as pressure coils (or) potential coils ( $PC_1$  &  $PC_2$ ).

1) The below fig shows the arrangement of 3 $\phi$  - wattmeter.



1) The combination of a fixed coil and a moving coil is called an element. So, the 3-phase dynamometer wattmeter consists of two elements.

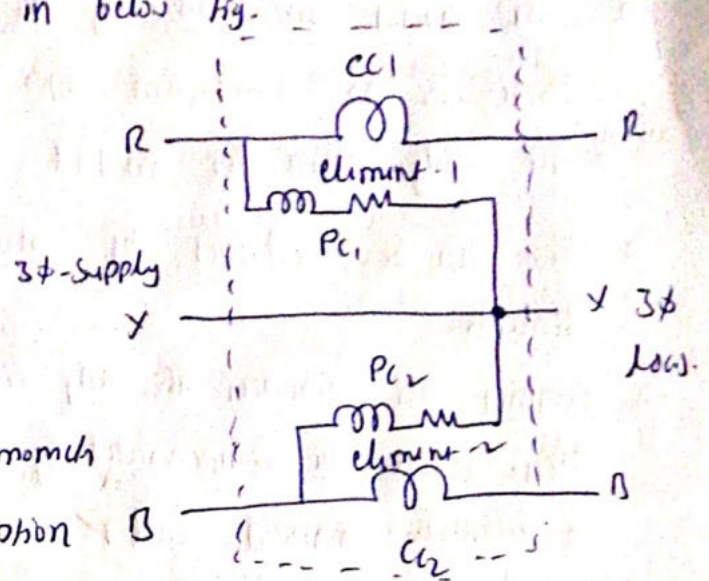
1) Both the potential coils are mounted on a single spindle (so as to integrate the power measured by each element).



✓ The connections of a 3 $\phi$  dynamometer wattmeter are exactly similar to that of the 3 $\phi$  power measurement by using two single phase wattmeters are shown in below fig.

✓ In ~~this~~ <sup>two wattmeter</sup> method, the total power consumed will be the sum of the readings of both wattmeters, which has to be calculated manually.

✓ Whereas, in a 3-phase electrodynamic wattmeter, the total power consumption is indicated directly as the power consumption noticed by both the elements is being integrated internally.



✓ The complete arrangement just behaves as that of two wattmeter method of power measurement.

✓ Each set of fixed coil and moving coil will experience a torque which is proportional to the power measured by that set.

✓ Hence the net torque is the sum of deflecting torque of the two.

$$\text{i.e., } T_{d1} \propto W_1, \quad T_{d2} \propto W_2$$

$$T_d \propto (W_1 + W_2)$$

$$T_d \propto W$$

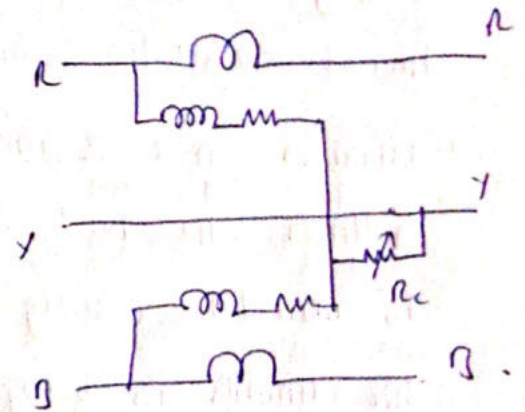
✓ Hence the total deflecting torque of the two sets is proportional to the total power measured by the circuit.



elimination of mutual effect b/w the two elements of wattmeter -

- The power measured by the meter will be correct only when there is no interaction b/w the fields of the both elements i.e., only when the mutual effect b/w the two elements is absent.
- This can be achieved by placing an iron shield in between the elements.

- Further, the presence of any mutual effect will be compensated by connecting a variable resistor called compensating resistor  $R_c$  in series with the potential coil as shown in fig.



- This method of compensating the mutual effect is called "Lorton's method".
- By varying the resistor  $R_c$  the current through the pressure coils are varied which in turn varies the fields.

necessity of pressure coil to be purely resistive -

- The electro-dynamometer type wattmeter consists of two coils i.e., current coil and potential coil.
- The current coil should carry the full line current and hence it will be connected in series with line whereas the pressure coil will be connected across the line.
- The resistance of the current coil should be very less so that, the whole of the current flows through it with fewer losses, whereas the resistance of the potential coil should be very high because of the two reasons.



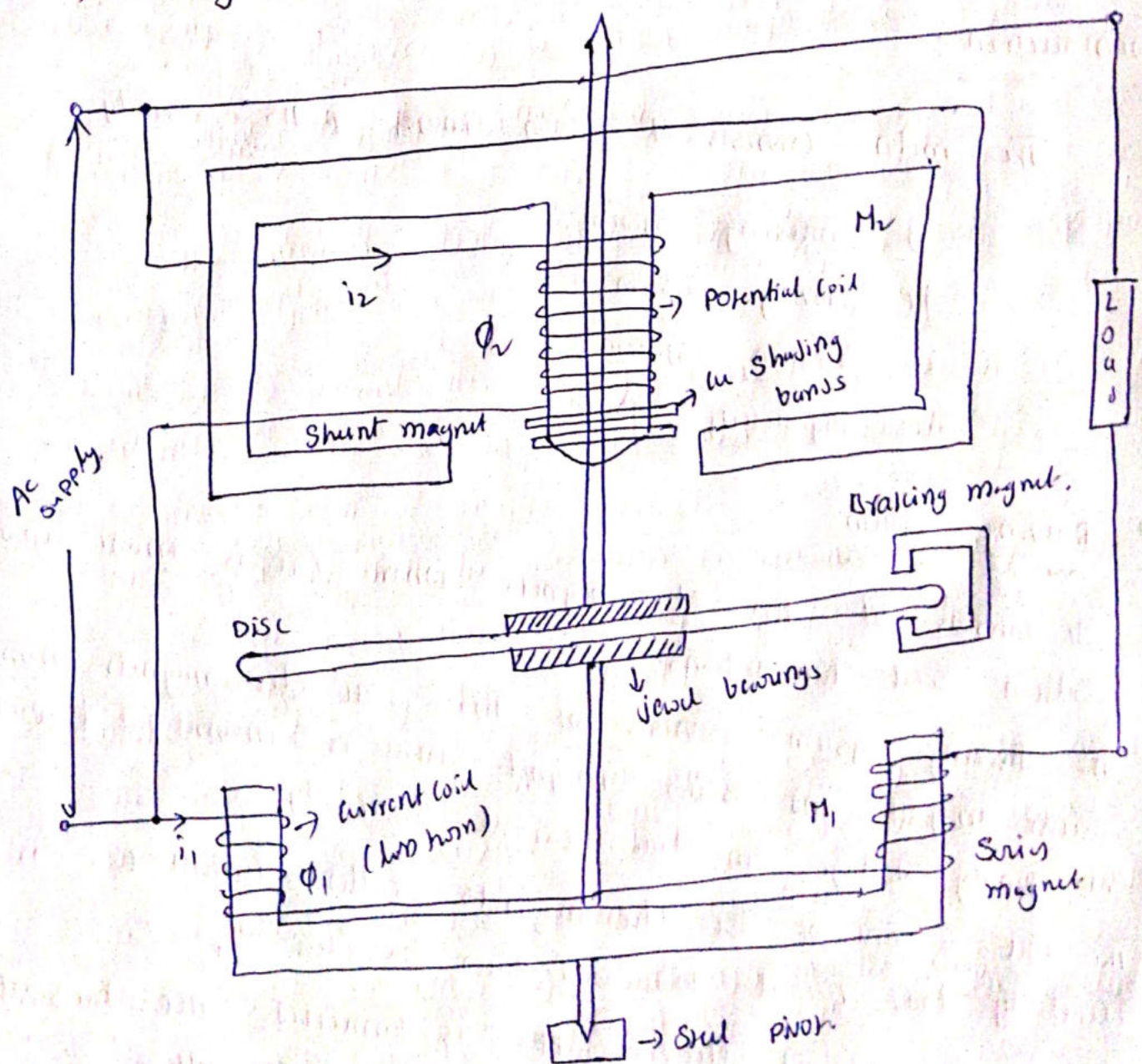
- ✓ Due to very high resistance such that a very small current flows through it and hence  $I^2R$  loss.
- ✓ If the resistance is very high when compared to its reactance, the current will be almost in phase with the voltage on the other hand if the resistance is low, the current will lag behind the voltage with a greater angle and hence this will introduce an error in the readings of the instrument.

Hence from the above two reasons, the primary coil should be purely resistive.



# \* Single phase induction type Energy meter (9)

- Induction type instruments are most commonly used as Energy meters.
- Energy meter is an integrating instrument which measures the quantity of electricity.
- Induction type of Energy meters are universally used for domestic and industrial applications.
- These meters measure the energy in kilo-watt-hours (kwh).
- The fig. 1 shows the induction type single phase Energy meter.



1  $\phi$  induction type Energy meter



- It works on the principle of induction i.e., on the production of eddy currents in the moving system by the alternating flux.
- These eddy currents induced in the moving system interact with each other to produce a driving torque due to which disc rotates to store the energy.
- In the energy meter there is no controlling torque and thus due to driving torque only a continuous rotation of the disc is produced.
- To have constant speed of rotation braking magnet is provided.

Construction :-

The meter consists of four major parts namely,

- i) Driving System
- ii) Moving System
- iii) Braking System
- iv) Registering System.

i) Driving System :-

- ✓ It consists of two electromagnets whose core is made up of Silicon steel laminations.
- ✓ The driving system consists of two AC electromagnets namely, Shunt magnet and Series magnet which is magnetised proportional to supply voltage and load current respectively.
- ✓ The coil of one of the electromagnets called "current coil" is excited by load current which produces the flux  $\phi_1$ .
- ✓ The coil of another electromagnet is connected across the supply and it carries the current proportional to supply voltage. This coil is called "pressure coil" and it produces flux  $\phi_2$ .



✓ The flux produced by shunt magnet is brought in exact quadrature with supply voltage with the help of copper bands whose position is adjustable.

## ii) Moving System:-

- ✓ Light aluminium disc mounted in a light alloy shaft is the main part of the moving system.
- ✓ This disc is positioned in between shunt and series magnets. It is supported b/w jewel bearings.
- ✓ The moving system runs on hardened steel pivot.
- ✓ There is no springs and no controlling torque.

## iii) Braking System:-

- ✓ A permanent magnet is placed near the aluminium disc for braking mechanism. This magnet produces its own field.
- ✓ The disc moves in the field of this magnet and a braking torque is obtained.
- ✓ The position of this magnet is adjustable and hence braking torque is ~~obtained~~ adjusted by shifting this magnet to different radial positions. This magnet is called braking magnet.

## iv) Registering System:-

- ✓ It records continuously a number which is proportional to revolutions made by aluminium disc.
- ✓ By a suitable system, a train of reduction gears, the pinion on the shaft drives a series of pointers.
- ✓ These pointers rotate on round dials which are equally marked with equal divisions.

|        |      |     |    |   |                    |
|--------|------|-----|----|---|--------------------|
| 0      | 2    | 5   | 1  | 2 | 2                  |
| x      | x    | x   | x  | x | x                  |
| 10,000 | 1000 | 100 | 10 | 1 | ( $\frac{1}{10}$ ) |



Winding:-

- Since the pressure coil is carried by shunt magnet  $H_v$ , which is connected across the supply, it carries a current proportional to the voltage.
- The current coil is carried by the series magnet  $H_i$  and it carries load current.
- Both these coils produce alternating fluxes  $\phi_1$  and  $\phi_2$  respectively. These fluxes are proportional to currents in their coils.
- Parts of each of these fluxes link with the disc and induce e.m.f. in it. Due to these emfs eddy currents are induced in the disc.
- The eddy current induced by the magnet  $H_v$  reacts with magnetic field produced by  $H_i$ . Also eddy current induced by the magnet  $H_i$  reacts with magnetic field produced by  $H_v$ .
- Thus each portion of the disc experiences a mechanical force and due to motor action, disc will rotate.
- The speed of the magnet is controlled by C shaped magnet called braking magnet.
- When disc rotates in the air gap, eddy currents are induced in the disc which opposes the cause producing them i.e., relative motion of the disc with respect to magnet. Hence braking Torque  $T_b$  is generated. This is proportional to speed ( $N$ ) of the disc.
- By adjusting the position of this magnet, desired speed of the disc is obtained.
- Spindle is connected to recording mechanism through gears which receive the energy supplied.



Let us study theory behind the working.

<11)

Let  $V$  = Supply voltage

$I_2$  = Current through primary coil proportional to  $V$

$\Phi_2$  = Flux produced by  $I_2$

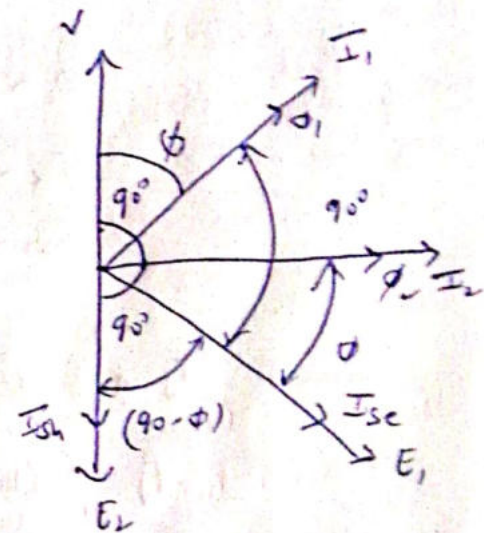
$I_1$  = Current through the current coil

$\Phi_1$  = Flux produced by  $I_1$

→ Now  $I_2$  lags  $V$  by  $90^\circ$  as primary coil is highly inductive and copper shading bands.

→  $I_2$  and  $\Phi_2$  are in phase.

→ While  $I_1$  lags  $V$  by  $\phi$  where  $\phi$  is decided by the load connected.



→  $I_1$  and  $\Phi_1$  are in phase

→ The phasor diagram is shown in fig.

$E_1$  = induced EMF in disc due to  $\Phi_1$

$E_2$  = induced EMF in disc due to  $\Phi_2$

$I_{sh}$  = eddy current due to  $E_1$

$I_{se}$  = eddy current due to  $E_2$

→ The induced EMF lags the respective flux producing it by  $90^\circ$ .

→ The eddy currents are in phase with the induced EMF producing them.

→ Now there is an interaction b/w  $\Phi_1$  and  $I_{sh}$  which produces torque  $T_1$  and interaction between  $\Phi_2$  and  $I_{se}$  which produces torque  $T_2$ .  $T_2$  is opposite in direction to  $T_1$ . Hence net deflecting torque is



$$T_d = T_2 - T_1$$

$$\text{hence } T_2 = \phi_2 I_{sc} \cos \phi$$

$$T_1 = \phi_1 I_{sh} \cos (180 - \phi)$$

$$T_d \propto \phi_2 I_{sc} \cos \phi - \phi_1 I_{sh} \cos (180 - \phi)$$

$$T_d \propto \phi_2 I_{sc} \cos \phi + \phi_1 I_{sh} \cos \phi$$

$$\cos (180 - \phi) = -\cos \phi$$

But

$$\phi_2 \propto I_2 \propto V$$

$$I_{sc} \propto E_1 \propto I_1$$

$$\phi_1 \propto I_1$$

$$I_{sh} \propto E_2 \propto I_2 \propto V$$

$$\therefore T_d \propto = k_1 V I_1 \cos \phi + k_2 I_1 V \cos \phi$$

$$T_d = (k_1 + k_2) V I_1 \cos \phi$$

$$[T_d \propto V I_1 \cos \phi] \quad \text{i.e., power consumed by load.}$$

→ Now braking torque is proportional to Speed  $N$  with which disc rotates.

$$T_b \propto N$$

$$T_b = k_2 N$$

Since

$$T_d \propto V I_1 \cos \phi \Rightarrow T_d = k_1 V I_1 \cos \phi$$

At steady state condition

$$T_b = T_d$$

$$k_2 N = k_1 V I_1 \cos \phi$$

$$N = \frac{k_1}{k_2} V I_1 \cos \phi$$

Total number of revolution is

$$= \int N dt \Rightarrow \int \frac{k_1}{k_2} V I_1 \cos \phi dt$$

$$\Rightarrow \frac{k_1}{k_2} \int (\text{True power}) dt$$



(iv)  
Therefore the total number of revolutions is proportional to the integral of true power i.e., Energy.

#### Advantages:-

- ✓ They can be used over a long period of time with very little maintenance.
- ✓ It is an inexpensive Energy meter and almost used universally for ac measurements.
- ✓ Can be used for the measurement of Energy over a wide range of loads.

#### Disadvantages:-

- ✓ If there is no proper adjustments in the meter, large errors are introduced in readings.
- ✓ The principle of induction can be only in a.c, hence these meters are limited to ac measurements only.

#### \* Errors and Compensation in Energy meter

Energy meter should give correct readings over a period of several years under normal use conditions. Some of the common errors in Energy meter and their remedial measures are discussed below.

##### 1) Phase Error

- It is necessary that the Energy meter should give correct readings on all power factors, which is only possible when the flux setup by shunt lags behind the applied voltage by  $90^\circ$ .
- But the flux due to shunt magnet does not lag behind the applied voltage exactly by  $90^\circ$  because of winding resistance and iron losses.



### Compensation:-

- The flux in the Shunt magnet can be made to lag behind the supply voltage by exactly  $90^\circ$  by adjusting the position of shading band placed at lower part of central limb of the Shunt magnet.
- This adjustment is called as lag adjustment (or) power factor adjustment (power factor compensation)

### 2. Speed Error:-

- Sometimes the speed of the motor is either fast (or) slow resulting in the wrong recording of energy consumption.

### Compensation:-

- An error in the speed of the motor when test on non-inductive load can be eliminated by correctly adjusting the position of the magnet.

### 3) Friction Error:-

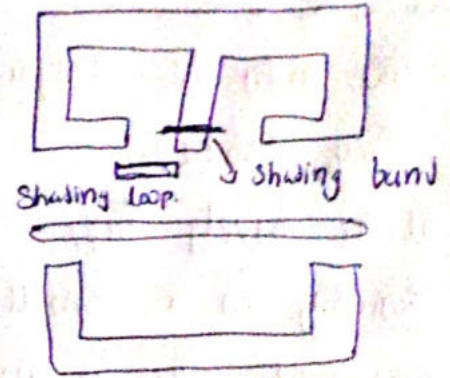
- Frictional forces at the motor bearings and in the registering mechanism cause noticeable error especially at light loads.
- At light loads, the torque due to friction adds considerably to the braking torque on the disc motor and it can cause error in motor recording.

### Compensation:-

- The frictional errors are very high during low loads and this frictional errors overcome by light load adjustment.
- During light loads, the torque produced in the disc is insufficient to overcome the frictional torque, which is high during starting rather than running.



→ For this compensation, a shading loop is placed in the air gap b/w the shunt magnet and the disc, so as to cover a region of the central limb and a pole of the shunt magnet as shown in fig.



→ The shading loop is energized proportional to the supply voltage and the field due to this loop produces more starting torque, which is enough to overcome the frictional torque at the starting.

#### 4) Creeping:-

→ Sometimes the disc of the energy meter rotates slowly but continuously at no load i.e., when the potential coil is excited but with no current flowing in the load. This is called creeping.

#### Compensation:-

→ In order to prevent this creeping at no load, two holes or slots are drilled in the disc on opposite sides of spindle. This causes sufficient distortion of the field.

→ The result is that the disc tends to remain stationary when one of the holes comes under one of the shunt magnets.

#### 5) Temperature:-

→ The errors due to variation in temp<sub>e</sub> are very small, because the various effects produced tend to neutralise one another.

→ The resistance of the disc, potential coil, magnetic circuit and brake magnet are affected by the changes in the temp<sub>e</sub>.

→ Therefore great care is exercised in the design of meter to eliminate the errors due to temp<sub>e</sub> variations.



### 6) Frequency errors:-

- The meter is designed to give minimum error at a particular  $f_{eq}$  (50 Hz).
- If the supply  $f_{eq}$  changes, the reactance of the coils also changes, resulting in a small error.
- Fortunately, this is not of much significance because commercial  $f_{eq}$  are held within close limits.

### 7) Voltage errors:-

- The error due to variation of voltage is very small (0.1 to 0.3%).
- This can be eliminated by the proper design of the magnetic ckt of the shunt magnet.

### \* Power Factor meters:-

- The power in single phase ac circuit is given by,

$$P = VI \cos \phi$$

where  $\cos \phi$  = power factor of the circuit.

- Thus by using precise voltmeter, ammeter and wattmeter in the circuit, the readings of  $V$ ,  $I$  and  $P$  can be obtained. Then power factor can be calculated as,

$$\left[ \cos \phi = \frac{P}{VI} \right]$$

- But this method is not accurate. The errors in all the meters together cause the error in power factor calculations.
- Similarly this method is not suitable for the circuits whose P.f. is varying according to circuit and load conditions.
- Hence it is necessary to have a meter which can directly indicate the P.f. of the circuit.



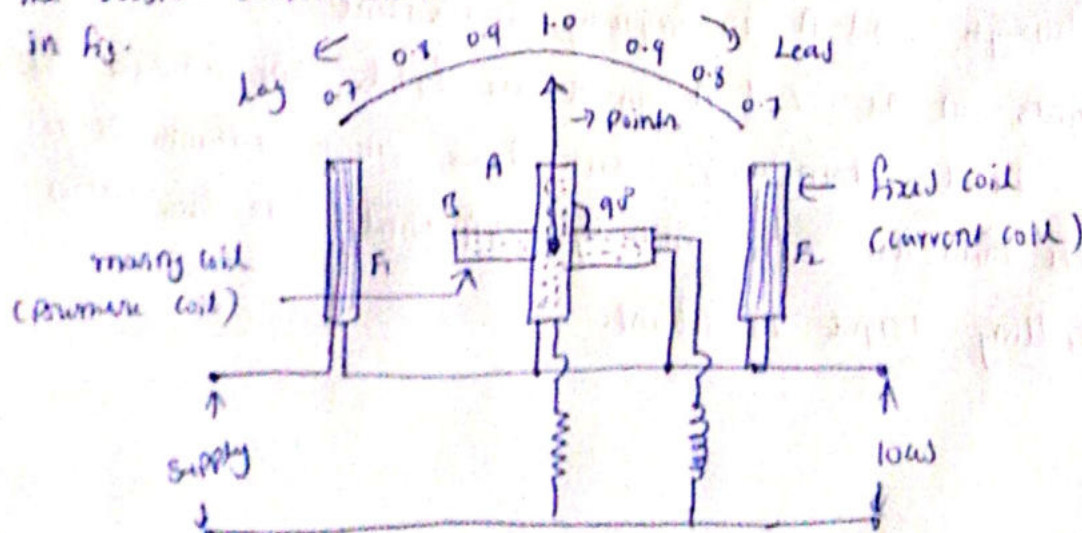
- Such a meter which indicates the instantaneous p.f. of the circuit is called power factor meter.
- Basic construction of p.f. is similar to a wattmeter.
- It consists of two circuits, current circuit and a voltage circuit.
- The current circuit carries current (or) fraction of current in the circuit whose p.f. is to be measured.
- The voltage coil is split into two parallel paths, one inductive and one noninductive.
- The currents in the two paths are proportional to the voltage of the circuit.
- Thus the deflection depends upon the phase difference between the main current through current circuit and the currents in the two branches of the voltage circuit i.e., p.f. of the circuit.

There are two types of power factor meters.

- Electrodynamometer type
- Hotwire type.

#### a) Single phase Electrodynamometer Type P.f. meter

- ✓ The construction of electrodynamometer type p.f. meter is similar to the construction of electrodynamometer type wattmeter.
- ✓ The basic construction of electrodynamometer type p.f. meter is shown in fig.



Single phase dynamometer type P.f. meter.

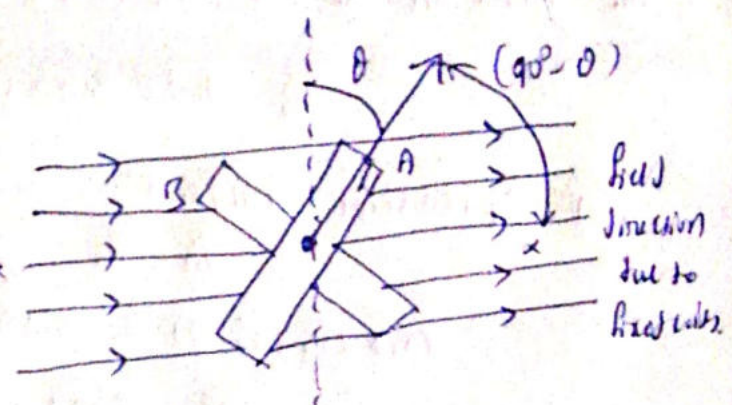


- The  $F_1 - F_2$  are the two fixed coils which are connected in series.
- The  $A - B$  are the two moving coils which are rigidly connected to each other so that their axes are at  $90^\circ$  to each other.
- The moving coils  $A - B$  are move together and carry the pointer which indicates the power factor of the circuit.
- The fixed coils  $F_1 - F_2$  carry the main current in the circuit. If the current is large, the fraction of the current is passed through the coils.
- Thus the magnetic field is produced by the fixed coils is proportional to the main current.
- The moving coils  $A - B$  are identical, which are connected in parallel across the supply voltage and hence called "potential coil(s)" voltage coil.
- The coil  $A$  has non inductive resistance ' $R$ ' in series with it while the coil  $B$  has an inductance ' $L$ ' in series with it.
- The values of  $R$  and  $L$  are so adjusted that the coils  $A$  and  $B$  carry equal currents at normal frequency. so at normal frequency  $R = \omega L$ .
- The current through the coil  $A$  is in phase with supply voltage while the current through coil  $B$  lags the supply voltage by  $90^\circ$  due to highly inductive nature of the circuit.
- Due to ' $L$ ', current through the coil ' $B$ ' is frequency dependent while current through coil  $A$  is frequency independent.
- The currents in the coil  $A$  and  $B$  are equal and produce the magnetic fields of equal strength, which have phase difference of  $90^\circ$  between them. The coils are also mutually perpendicular to each other.
- The controlling torque is absent.



Winding of motor  
~ ~ ~ ~ ~

- Consider the position of the moving system as shown in fig.
- Assume that the current through the coil is lags the voltage exactly by  $90^\circ$



- Also assume that the field produced by the fixed coils is uniform and in the direction of x-x as shown in fig.
- Due to interaction of the fields produced by the currents through various coils, both the coils A and B experience a torque.
- The windings are arranged in such a manner that the torques experienced by coil A and B are opposite to each other.
- Hence the pointer attains an equilibrium position when these two torques are equal.
- The torque on each coil, for a given coil current will be maximum when the coil is parallel to the field produced by  $F_1 - F_2$  i.e., direction x-x

Let  $\phi = \text{power factor angle}$   
 $\theta = \text{Angle of deflection.}$

The  $\theta$  is measured from the vertical axis, in equilibrium position.

Similar to a dynamometer type wattmeter, torque on coil-A is given by,

$$T_A = kVI \cos\phi \cos(90^\circ - \theta) \rightarrow (1)$$

Where k is constant

- The Equation (1) is similar to the torque eqn. of dynamometer type instrument.



→ The torque on B is proportional to  $\cos(90^\circ - \phi)$  i.e.,  $\sin\phi$  and  $\cos\theta$

$$T_B = kVI \sin\phi \cos\theta \rightarrow (2)$$

In equilibrium condition

$$T_A = T_B$$

$$\cos\phi \cos(90^\circ - \theta) = \sin\phi \cos\theta$$

$$\sin\theta = \tan\phi \cos\theta$$

$$\tan\theta = \tan\phi$$

$$[\theta = \phi]$$

→ Thus the angular position of the moving coil is equal to the system power factor angle.

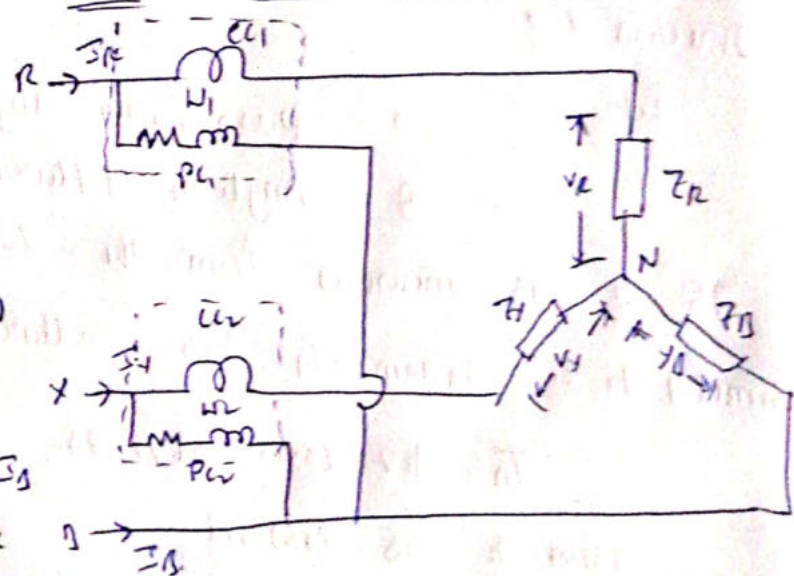
→ The operation of the instrument is dependent on the specific supply frequency.

→ If the freq. is different  $\theta$ , it contains harmonics then inductance of coil will change, due to which there will be serious errors in the instrument's reading.

### \* Two wattmeter method with balanced load condition :-

→ The connection diagram for two wattmeter method of measuring the 3 $\phi$  power of balanced star connected load as shown in fig.

→ Let  $V_R, V_Y, V_B$  and  $I_R, I_Y, I_B$  are the RMS values of phase voltages and phase currents.



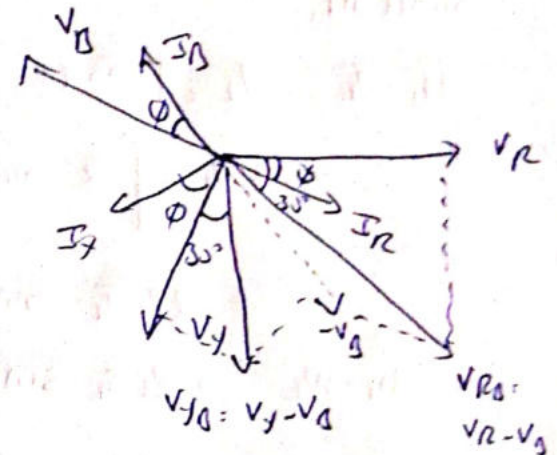
→ If the load is to be considered inductive, the phase currents  $I_R$ ,  $I_Y$  and  $I_B$  lag behind the corresponding phase voltages  $V_R$ ,  $V_Y$  and  $V_B$  by an angle of  $\phi$ .

→ The phasor diagram of such load with a p.f  $\cos\phi$  is shown below.

→ From the phasor diagram, the power measured by the two wattmeters  $W_1$  and  $W_2$  are given by.

$$W_1 = V_{RB} I_R \cos(30^\circ - \phi)$$

$$W_2 = V_{YB} I_Y \cos(30^\circ + \phi)$$



→ Since the load is balanced, for star-connected load

$$\text{line voltages} \Rightarrow V_{RY} = V_{YB} = V_{BR} = V_L$$

$$\text{line currents} = \text{phase currents} \Rightarrow I_R = I_Y = I_B = I_L = I_{ph}$$

→ For balanced load condition

$$W_1 = V_L I_L \cos(30^\circ - \phi)$$

$$W_2 = V_L I_L \cos(30^\circ + \phi)$$

→ Therefore, the total power  $P$  consumed by the 3-phase balanced load will be the sum of two wattmeter readings i.e.,

$$P = W_1 + W_2$$

$$P = V_L I_L [\cos(30^\circ - \phi) + \cos(30^\circ + \phi)]$$

$$[\cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)]$$

$$\Rightarrow 2 V_L I_L \cos 30^\circ \cos \phi$$

$$[P = \sqrt{3} V_L I_L \cos \phi] \rightarrow (1)$$



measurement of P.f using two wattmeter method

From Equation (1)

$$P = W_1 + W_2 = \sqrt{3} V_L I_L \cos \phi \rightarrow (2)$$

Similarly,

$$W_1 - W_2 = V_L I_L [\cos (30^\circ - \phi) - \cos (30^\circ + \phi)]$$

$$\Rightarrow V_L I_L \left[ 2 \sin \left( \frac{60^\circ}{2} \right) \sin \left( \frac{72^\circ}{2} \right) \right]$$

$$\Rightarrow 2 V_L I_L \sin 30^\circ \sin \phi$$

$$W_1 - W_2 \Rightarrow V_L I_L \sin \phi \rightarrow (3)$$

Dividing (3) by (2) we get

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin \phi}{\sqrt{3} V_L I_L \cos \phi}$$

$$\tan \phi = \sqrt{3} \left( \frac{W_1 - W_2}{W_1 + W_2} \right)$$

$$\phi = \tan^{-1} \left[ \sqrt{3} \left( \frac{W_1 - W_2}{W_1 + W_2} \right) \right]$$

$$\therefore \text{P.f} \Rightarrow \cos \phi = \cos \left[ \tan^{-1} \left[ \sqrt{3} \left( \frac{W_1 - W_2}{W_1 + W_2} \right) \right] \right]$$

Ex: 1 Two wattmeters connected to measure the power input to a balanced three phase circuit indicate 2000W and 500W respectively. Find the p.f. of the circuit when

Condition: 1 Both readings are positive

Condition: 2 The 1st reading is obtained after reversing the connections to the current coil of second instrument.

Sol Condition: 1  $W_1 = 2000 \text{ W}$   $W_2 = 500 \text{ W}$

$$\phi = \tan^{-1} \sqrt{3} \left( \frac{W_1 - W_2}{W_1 + W_2} \right) \Rightarrow 46.10^\circ$$

$$\cos \phi = \cos(46.10^\circ) = 0.693$$

Condition: 2  $W_1 = 2000 \text{ W}$   $W_2 = -500 \text{ W}$

$$\phi = \tan^{-1} \sqrt{3} \left( \frac{W_1 - W_2}{W_1 + W_2} \right) = 70.893^\circ$$

$$\cos \phi = \cos(70.893^\circ) = 0.327$$

Ex: 2 A 3 $\phi$  440V motor load has a p.f. of 0.6. Two wattmeters connected to measure the power shows the sum to be 25kW. Find the readings on each instrument?

Sol input power  $P = W_1 + W_2 = 25 \text{ kW}$

Line voltage  $V_L = 440 \text{ V}$

p.f. = 0.6

From the relation  $\tan \phi = \sqrt{3} \left( \frac{W_1 - W_2}{W_1 + W_2} \right)$

$$\tan(\cos^{-1}(0.6)) = \sqrt{3} \left( \frac{W_1 - W_2}{25} \right) \Rightarrow W_1 - W_2 = 19.24 \text{ kW}$$

$$W_1 + W_2 = 25 \text{ kW}$$

on Solving

$$W_1 = 22.12 \text{ kW}$$

$$W_2 = 2.88 \text{ kW}$$



Ex: 3 Two wattmeters are used to measure i/p power to a 1.5 kV, 50 Hz 3 $\phi$  motor running on full load at an efficiency of 85%. Their readings are 250 kW and 80 kW respectively. Calculate the i/p power, p.f, line current and o/p.

Sol

Line voltage  $V_L = 1.5 \text{ kV}$

$$\eta = 85\% = 0.85$$

Wattmeter readings  $W_1 = 250 \text{ kW}$   $W_2 = 80 \text{ kW}$

a) input power (P) =  $W_1 + W_2 \Rightarrow 330 \text{ kW}$

b) P.f  $\Rightarrow \cos \phi = \cos \left[ \tan^{-1} \left( \sqrt{3} \left( \frac{W_1 - W_2}{W_1 + W_2} \right) \right) \right]$

$$\Rightarrow 0.746 \text{ lag.}$$

c) line current  $I_L$

Power  $\Rightarrow P = \sqrt{3} V_L I_L \cos \phi$

$$I_L = \frac{P}{\sqrt{3} V_L \cos \phi}$$

$$\Rightarrow \frac{330 \times 1000}{\sqrt{3} \times 1.5 \times 10^3 \times 0.746}$$

$$\Rightarrow 170.27 \text{ A.}$$

d) output

$$\text{Efficiency} = \frac{\text{O/P}}{\text{i/p}}$$

$$\text{O/P} = 330 \times 0.85$$

$$\text{O/P} \Rightarrow 280.5 \text{ kW}$$

4) Two wattmeters are used to measure the power in a synchronous motor. Each of them indicates 60 kW. If the p.f. be changed to 0.866 leading then determine the reading of the two wattmeters while the total power remains unaltered?

sol) Readings of wattmeters  $W_1 = 60 \text{ kW}$   
 $W_2 = 60 \text{ kW}$

Since both the wattmeters reads same reading & both are +ve, then the motor must be running at VPF

If p.f. is 0.866 lag.

$$W_1 = V_L I_L \cos(30^\circ + \phi)$$

$$W_2 = V_L I_L \cos(30^\circ - \phi)$$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos \phi = 60 + 60 \Rightarrow 120 \text{ kW.}$$

$$W_1 - W_2 = -V_L I_L \sin \phi$$

Given that  $\cos \phi = 0.866 \rightarrow \phi = \cos^{-1}(0.866)$   
 $\phi = 30^\circ$

$$\tan \phi = \frac{1}{\sqrt{3}}$$

From eqn.  $\tan \phi = \frac{\sqrt{3} (W_1 - W_2)}{(W_1 + W_2)}$

$$\frac{1}{\sqrt{3}} = -\frac{\sqrt{3} (W_1 - W_2)}{120}$$

$$W_1 - W_2 = -40$$

$$\text{and } W_1 + W_2 = 120$$

on solving  $W_1 = 40 \text{ kW}$   
 $W_2 = 80 \text{ kW.}$



Problems based on Energy meter:-

- 1) The meter constant of a 220V, 5A, 50Hz, 1 $\phi$  induction type Energy meter is 3275 revolutions per kWh. If the Energy meter is subjected to a full load for one hour then find the speed of the disc in the Energy meter?

Sol, Supply voltage = 220V  
Line current = 5A  
meter constant = 3275 revolutions / kWh  
time period = 1 hr.

$$\text{Power} = VI \cos\phi \Rightarrow 220 \times 5 \times 1 = 1100 \text{ W} \Rightarrow 1.1 \text{ kW.}$$

$$\text{Energy} = \text{Power} \times \text{time} \\ \Rightarrow 1.1 \times 10^3 \times 1 \Rightarrow 1.1 \text{ kWh.}$$

$$\text{Speed (rev)} = \text{meter constant} \times \text{Energy consumed} \\ = 3275 \times 1.1 \\ \Rightarrow 3602.5 \text{ rev} \\ \Rightarrow 3603 \text{ rev.}$$

- 2) An Energy meter is designed to make 100 revolutions of disc for one ~~kWh~~ unit of Energy. Calculate the no. of revolutions made by the disc when connected to load carrying 40A with 230V with 0.4 p.f. for an hour. If it actually makes 360 revolutions then find the percentage error?

Sol, Line current = 40A  
Supply voltage = 230V  
p.f. = 0.4  
time = 1 hr

$$\text{Power consumed} = VI \cos\phi = 230 \times 40 \times 0.4 = 3680 \text{ W.}$$

$$\text{Energy consumed} = \text{Power} \times \text{time} = 3680 \times 1 \Rightarrow 3680 \text{ Wh} \Rightarrow 3.680 \text{ kWh.}$$

## UNIT-3

### Potentiometer & Measurement of Frequency

#### \* De Cromp's potentiometer;

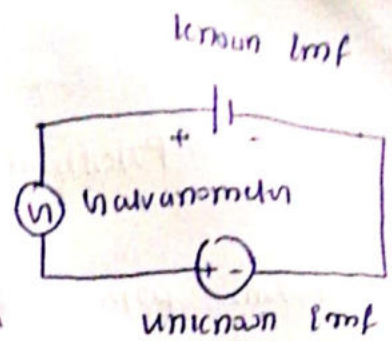
#### \* Introduction

- A "potentiometer" is an instrument used to measure an unknown  $\text{E.m.f}$  which is compared to known  $\text{E.m.f}$ . Thus it is a device used for measurement of unknown  $\text{E.m.f}$  by comparison.
- The unknown  $\text{E.m.f}$  is compared with a known  $\text{E.m.f}$  which is obtained from a standard cell (or) any reference voltage source.
- The main advantage of the comparison method of the  $\text{E.m.f}$  measurement is that the potentiometer is capable of providing high degree of accuracy.
- Basically a potentiometer uses "balance" (or) "null condition" during the measurement of unknown  $\text{E.m.f}$ .
- Actually no current flows in the circuit of the unknown  $\text{E.m.f}$  during the measurement. Thus no power is consumed in such circuit.
- A basic application of the potentiometer is to measure unknown  $\text{E.m.f}$  (or) voltage. It can also be used to determine current.
- The unknown current can be ~~read~~ obtained by using potentiometer by measuring voltage drop across the standard resistor.
- In the field of electrical measurements, a potentiometer is most widely used as standard for the calibration of the voltmeters, ammeters and wattmeters.

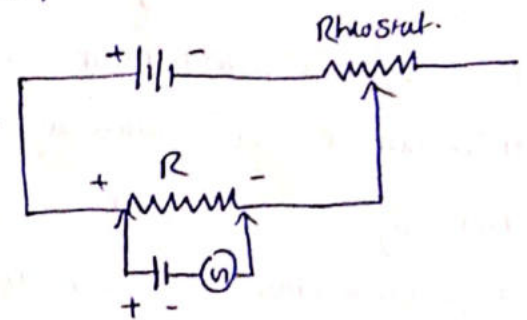


## Principle of potentiometer working:-

- ✓ The potentiometer works on the principle of opposing the unknown  $\mathcal{E}$ -m.f by a known  $\mathcal{E}$ -m.f with the negative terminals of both the  $\mathcal{E}$ -m.f's connected together, while the positive terminals are connected together through a galvanometer, as shown in fig.
- ✓ When the  $\mathcal{E}$ -m.f's are same value, there is no deflection on galvanometer. Thus to measure the unknown  $\mathcal{E}$ -m.f by using above method, with the known  $\mathcal{E}$ -m.f. used must be variable.
- ✓ Another important requirement is that known  $\mathcal{E}$ -m.f should be varied to give a larger number of known values but it is practically very difficult.
- ✓ Hence alternatively, the unknown  $\mathcal{E}$ -m.f is connected in parallel with and in opposition to a voltage drop measured across the resistor is shown in fig.

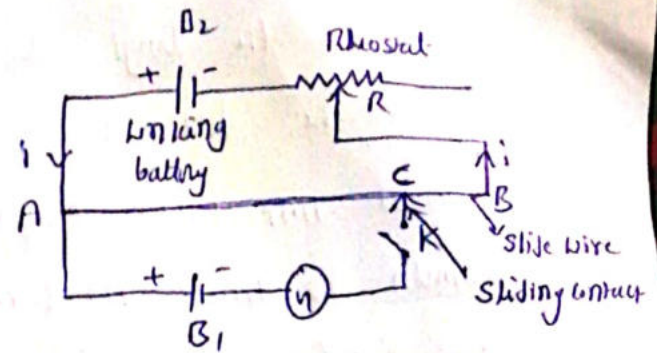


- ✓ The main advantage of this method is that the current in the resistor can be varied easily to obtain any desired voltage with very fine adjustment.
- ✓ The voltage drop across the resistor can be determined by calibrating the resistor with standard cell.
- ✓ The potentiometers are classified as d.c potentiometers and a.c potentiometers. There are various forms of d.c potentiometers which are used widely practically.
- ✓ The basic, simplest type of the d.c potentiometer is the slide wire potentiometer.



## \* \* Slide wire D.C potentiometer

✓ The slide wire d.c potentiometer is the basic and simplest type of the d.c potentiometer as shown in fig.



- ✓ A basic potentiometer circuit consists of a slide wire AB having uniform cross section and unit length and it is made up of manganin.
- ✓ Let  $\rho$  be the resistance per unit length of the slide wire.
- ✓ The battery  $B_2$  supplies a current through the slide wire which is limited with the help of regulating resistance i.e., rheostat.
- ✓ The battery  $B_1$ , whose e.m.f is to be measured is connected in series with a galvanometer 'G' and switch 'K'.
- ✓ When the switch 'K' is opened, the current through slide wire is ' $i$ '.
- ✓ If the sliding contact is at position 'C', let the length AC be  $l$ -units, then the voltage drop across AC is given by ' $i\rho l$ '.
- ✓ Consider that switch 'K' is closed which puts the battery  $B_1$  in the circuit.
- ✓ The battery  $B_1$  whose e.m.f is to be measured is connected such that the voltage drop along the slide wire and e.m.f of  $B_1$  oppose each other.
- ✓ The deflection in the galvanometer 'G' depends on the magnitude of voltage drop across the slide wire portion AC and e.m.f of  $B_1$ .
- ✓ If the voltage drop across length ' $l$ ' of the slide wire is greater than e.m.f of battery  $B_1$ , then the current will flow in the direction A to C through the galvanometer.



- ✓ Similarly if the E.M.F of battery  $B_1$  is greater than the voltage drop across the length  $L$  of the slide wire, then the current will flow in the direction  $C$  to  $A$  through the galvanometer.
- ✓ The most important condition exhibiting the basic principle of the potentiometer is that no current flows through the galvanometer when the two E.M.F's are equal.
- ✓ Generally a scale is provided along with the slide wire which enables to measure the length of portion  $AC$ .
- ✓ When to measure E.M.F of a battery, first adjust the current through slide wire with switch 'K' open. Then insert battery whose E.M.F is to be measured.
- ✓ By closing switch  $K$  adjust sliding contact such that the galvanometer shows zero deflection.
- ✓ Measure the length of the portion of the slide wire with the help of scale provided. Then the unknown E.M.F ' $E$ ' of battery is given by,

$$E = i(rL) \rightarrow (1)$$

Where  $r$  is the resistance per unit length,  $i$  is the current adjusted using rheostat  $R$ .

- ✓ If E.M.F's of the two batteries  $B_1$  and  $B_2$  are to be compared then insert the first battery  $B_1$  in series with the galvanometer and then adjust the sliding contact such that no current flows through the galvanometer.
- ✓ Measure the length of the slide wire portion say  $l_1$ .
- ✓ Repeat the same the procedure with battery  $B_2$  in the circuit.
- ✓ Let the length measured be  $l_2$



✓ Let e.m.f of batteries  $B_1$  and  $B_2$  be  $E_1$  and  $E_2$  respectively, then we can write,

$$E_1 = i(r l_1) \rightarrow (2)$$

$$E_2 = i(r l_2) \rightarrow (3)$$

Thus we can write

$$\frac{E_1}{E_2} = \frac{l_1}{l_2} \rightarrow (4)$$

✓ From the above equation it is clear that the ratio of two lengths gives the ratio of the two e.m.f's.

✓ If one of the batteries used is a standard cell, say battery  $B_2$  of known voltage, the e.m.f of battery  $B_1$  is given by,

$$E_1 = E_2 \left( \frac{l_1}{l_2} \right) \rightarrow (5)$$

⇒ While using basic slide wire potentiometer following precautions must be taken.

✓ The supply battery  $B_2$  should be high capacity so that a constant current flows through the slide wire throughout the measurement.

✓ A small resistance should be used in series with the galvanometer to protect it during the initial adjustments of contact C. It also takes care that no appreciable current is taken from the standard cell.

✓ The accuracy of the measurement depends on how accurately ratio  $(l_1/l_2)$  is determined.

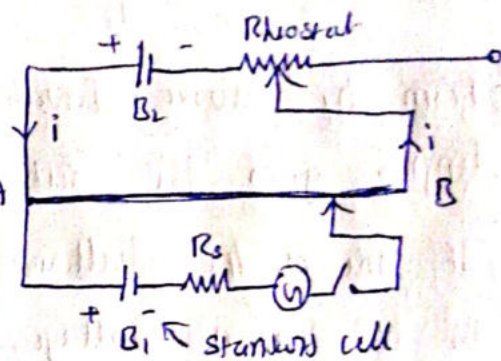


## \* Standardization of potentiometer

→ 'Standardization' of a potentiometer is a process of adjusting the working current supplied by the battery such that the voltage drop across a portion of sliding wire matches with the standard reference source.

→ The practical set up for standardizing a d.c potentiometer is as shown in fig.

→ A battery of sufficient capacity i.e.  $B_2$  is connected in series with rheostat  $R_h$  A which regulates the working current flowing through the slide wire.



→ A standard cell  $B_1$  usually a Weston standard cell of E.M.F 1.0186 volts is connected to galvanometer and a switch  $K$  through a series resistance  $R_s$ .

→ By properly adjusting  $R_s$  full sensitivity of the galvanometer can be obtained.

→ A slide wire total length of 200 cm and resistance of 200  $\Omega$  is connected which is indicated by points A and B.

→ During standardization process, switch  $K$  is closed and the sliding contact is placed at the mark of 101.86 cm along with the slide wire as indicated by point C as shown in fig. Thus we can observe some deflection in the galvanometer.

→ Now by adjusting the value of rheostat  $R_h$  we can get null deflection in the galvanometer.

→ under the condition of null deflection, the voltage drop along 101.86 cm portion of the slide wire equals the E.M.F of standard Weston cell.



- This is nothing but the standardization of a potentiometer and once the potentiometer is standardized, the standard is not disturbed.
- After standardization of potentiometer, it is used as a direct reading potentiometer as the voltage along the slide wire at any point is proportional to the length of the slide wire where the point is obtained by moving sliding contact along the wire to get null deflection in the galvanometer for any battery whose E.M.F. is to be measured.



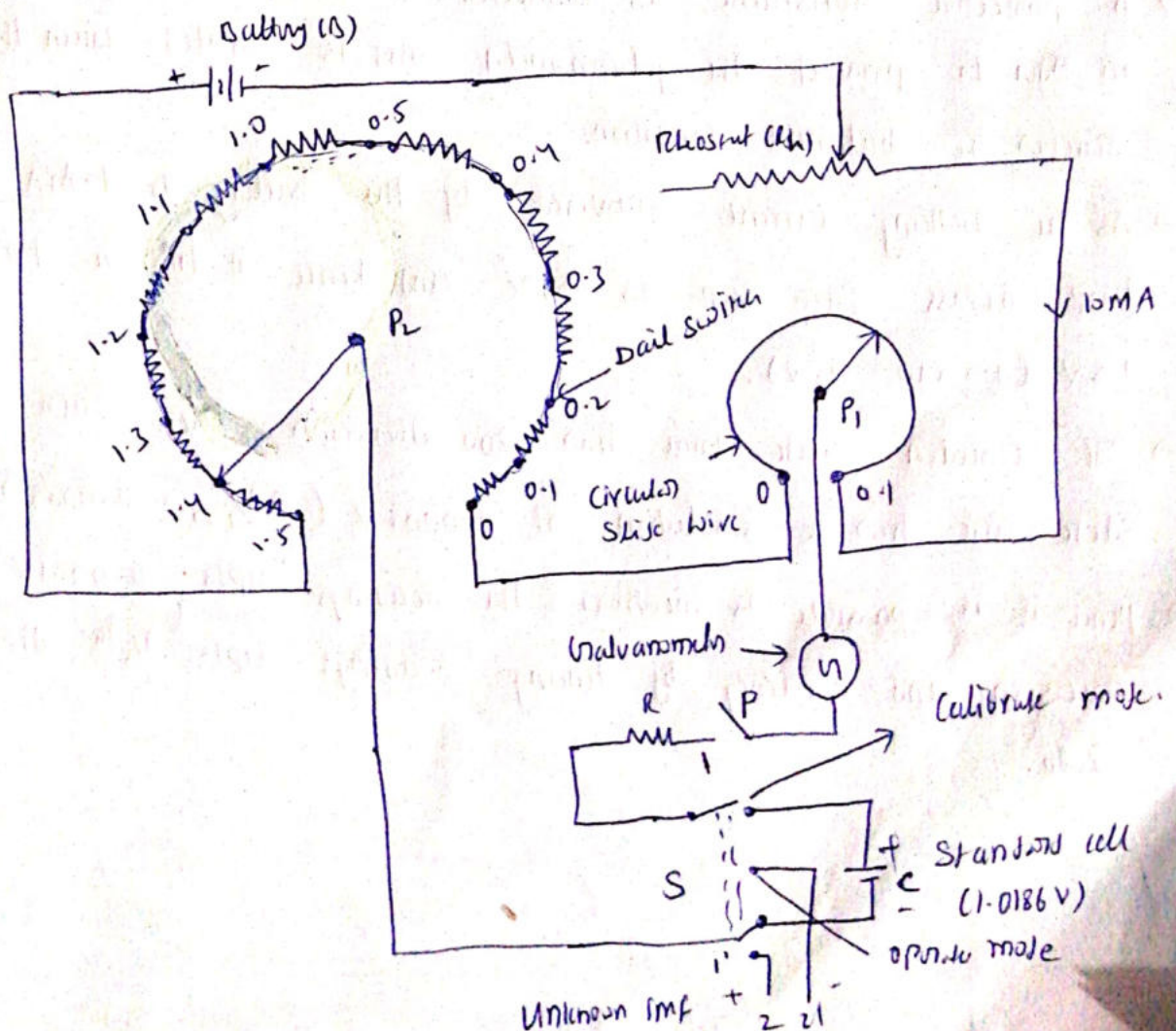
## \* Dc Crompton's potentiometer :-

A potentiometer is a device used for measuring the emf of a cell (or) potential difference b/w two points in a circuit. It works on the principle of comparison i.e., comparing the unknown voltage with the known voltage and makes use of balanced condition.

### Construction:-

→ Dc Crompton's potentiometer is the laboratory-type potentiometer that is used to measure unknown emf effectively with a great degree of precision.

→ Dc Crompton's potentiometer works on the principle of a slide wire potentiometer. In other words, the Dc Crompton's potentiometer is a modified version of a slide-wire potentiometer. It basically consists of a small slide wire which is circular in shape and a dial switch with calibrated resistors, as shown in fig below.





→ In the figure shown,

B = Battery

Rh = Rheostat

G = Galvanometer

R = protective resistance which is of order of  $10k\Omega$ .

S = Double throw switch

C = Standard cell.

→ In De Compton's potentiometer, the dial switch is divided into fifteen steps with each step having a resistance of  $10\Omega$ .

→ Hence, the total resistance of dial switch is equal to  $150\Omega$  ( $15 \times 10\Omega$ ).

→ The slide wire is in the form of circular wire and has a resistance of  $10\Omega$ , with a single turn.

→ A double throw switch is provided for standardization and for measuring the unknown EMF.

→ A protective resistance is connected in series with the galvanometer in order to protect the galvanometer and is shorted when the galvanometer reaches the balanced condition.

→ As the working current provided by the battery is  $10mA$ , the voltage drop across each step is " $0.1V$ " and hence it has a total range of  $1.5V$  ( $1.5 \times 10 = 15V$ ).

→ If circular slide wire has 200 divisions then each division in slide wire has a resolution of  $0.0005V$  ( $\frac{0.1}{200} = 0.0005$ ).

→ Hence it is possible to measure the readings upto  $0.0001V$  with great precision and accuracy by taking readings upto  $\frac{1}{5}$ th division in the scale.



- First, the potentiometer is to be standardized to the standard cell voltage ( $1.0186\text{V}$ ) by keeping the dial switch at IV and slide wire at 0.0186.
- After making these adjustments, switch 'S' is operated in calibrated mode and key 'K' is closed and the rheostat is adjusted in such away that, the galvanometer shows null deflection.
- With this, the potentiometer is standardized to the voltage of standard cell which is connected b/w the terminals 1 and 1'.
- Now the switch is thrown into the operating mode for measuring the unknown emf connected b/w the terminals 2 and 2'.
- The value of unknown emf can be measured directly from the dial switch and circular slide wire, after balancing the galvanometer to show null deflection.
- In this way, an unknown emf can be measured with great precision using DC Crompton's potentiometer.

### Standardization of DC Crompton's potentiometer

The followings are the steps involved in the standardization of DC Crompton's potentiometer.

- i) Settings are made such that, the sum of the voltage across the dial resistors and the slide wire is equal to the standard cell voltage.
- ii) The switch is closed to calibrate the positions and the rheostat is set for null deflection. The galvanometer key is tapped and the resistance is kept in the circuit to protect the galvanometer.



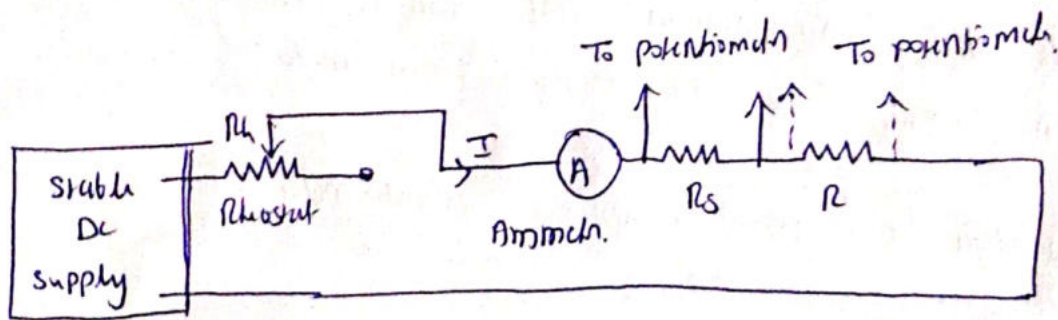
- iii) As soon as zero deflection is obtained, the protective resistance is replaced by short circuit, and then final settings are done for null deflection using a rheostat.

Applications of De Crompton's potentiometer:

- Measurement of resistance
- Measurement of power
- Calibration of wattmeter.
- Calibration of voltmeter.
- Calibration of ammeter.

\* Measurement of unknown resistance using DC potentiometer:-

- The set up for measurement of resistance is as shown in fig.



- A resistor whose resistance is to be measured is connected in series with a standard resistor of resistance  $R_s$ .
- The current through the circuit is supplied by a stable DC supply and it is controlled by a rheostat  $R_h$ .
- The current is adjusted such that the drop across each resistor is of the order of "1V".
- Due to the current 'I', voltages are developed across  $R_s$  and  $R$ .
- Both are then measured by using a DC potentiometer.



Let the voltage across standard resistance be  $V_{RS}$ . Then we can write,

$$V_{RS} = I \cdot R_S \rightarrow (1)$$

Let the voltage across unknown resistance be  $V_R$ . Then, we can write,

$$V_R = I R \rightarrow (2)$$

Dividing Equation (2) by Equation (1)

$$\frac{V_R}{V_{RS}} = \frac{R}{R_S}$$

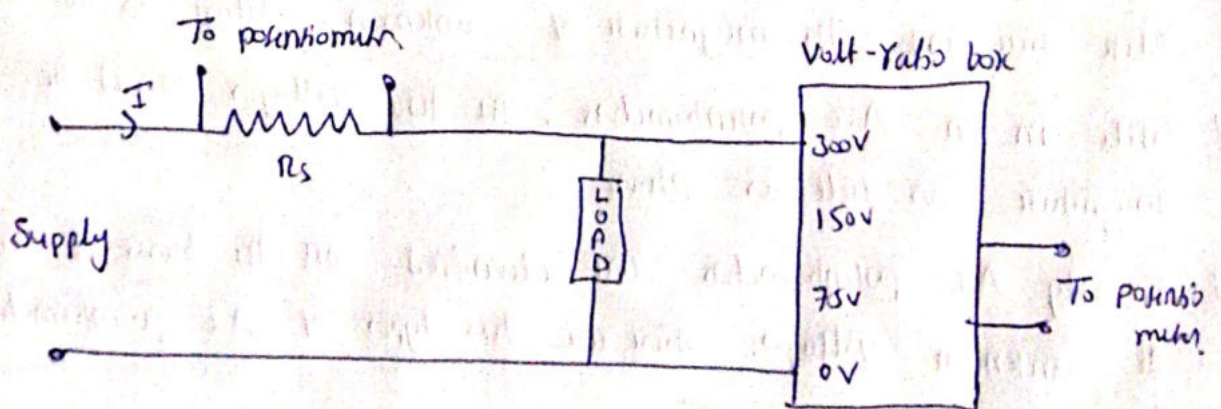
Hence the unknown resistance is given by,

$$R_S = R \left( \frac{V}{V_{RS}} \right) \rightarrow (3)$$

The basic requirement of above measurement method is that the current flowing through the circuit should remain same during measurement of voltage across  $R$  and  $R_S$ . This need can be fulfilled by using a stabilized D.C supply at the i/p.

\* Measurement of current, voltage and power :-

→ The circuit diagram for measurement of power is as shown in fig.



→ The power across the load can be calculated as,

$$P = VI$$

→ In above circuit, voltage across standard resistor  $R_S$  is measured using potentiometer. Let it be  $V_{RS}$ . Then current flowing through it is given by

$$I = \frac{V_{RS}}{R_S} \rightarrow (1)$$



→ In the measurement of voltage 'V' across load, volt-ratio box is connected across load.

→ The o/p of volt-ratio box is then connected to potentiometer but it be 'V'.

\* Then the voltage across the load is given by,

$$[V = k V_R] \rightarrow (2)$$

→ Where k is the multiplying factor of volt-ratio box and  $V_R$  is actual reading of potentiometer when it is connected across volt-ratio box. Thus from Equations (1) and (2) the power is given by,

$$P = VI = k V_R \left( \frac{V_{RS}}{R_S} \right)$$

$$\left[ P = k \left( \frac{V_R V_{RS}}{R_S} \right) \right] \rightarrow (3)$$

\* AC potentiometers:-

= = = = =

✓ The basic principle of operation of d.c potentiometer and an a.c potentiometer is exactly same.

✓ But in a d.c potentiometer, the balance b/w voltage drop across the slide wire and the magnitude of unknown voltage is obtained.

✓ While in an A.c potentiometer, the two voltages should be balanced in magnitude as well as phase.

✓ Broadly A.c potentiometers are classified on the basis of measurement of unknown voltages. There are two types of A.c potentiometers are,

i) Polar type A.c potentiometer

~~~~~  
In which the magnitude and phase angle of unknown voltage are measured on different scales directly. The phase angle is measured with respect to some reference phase. As the voltage measured is represented in polar form as $V \angle \theta^\circ$, the A.c potentiometer is called Polar type A.c potentiometer.

ii) Co-ordinate type a.c. potentiometer

In this the two components of an unknown voltage are measured on two different scales. One of component measured is inphase component while remaining is Quadrature component. Both the components are 90° out of phase with each other. If the inphase component and Quadrature components are represented by V_A and V_B respectively then the magnitude and phase angle of an unknown voltage can be represented as given below,

$$V = \sqrt{V_A^2 + V_B^2} \text{ and}$$

$$\theta = \tan^{-1} \left(\frac{V_B}{V_A} \right)$$

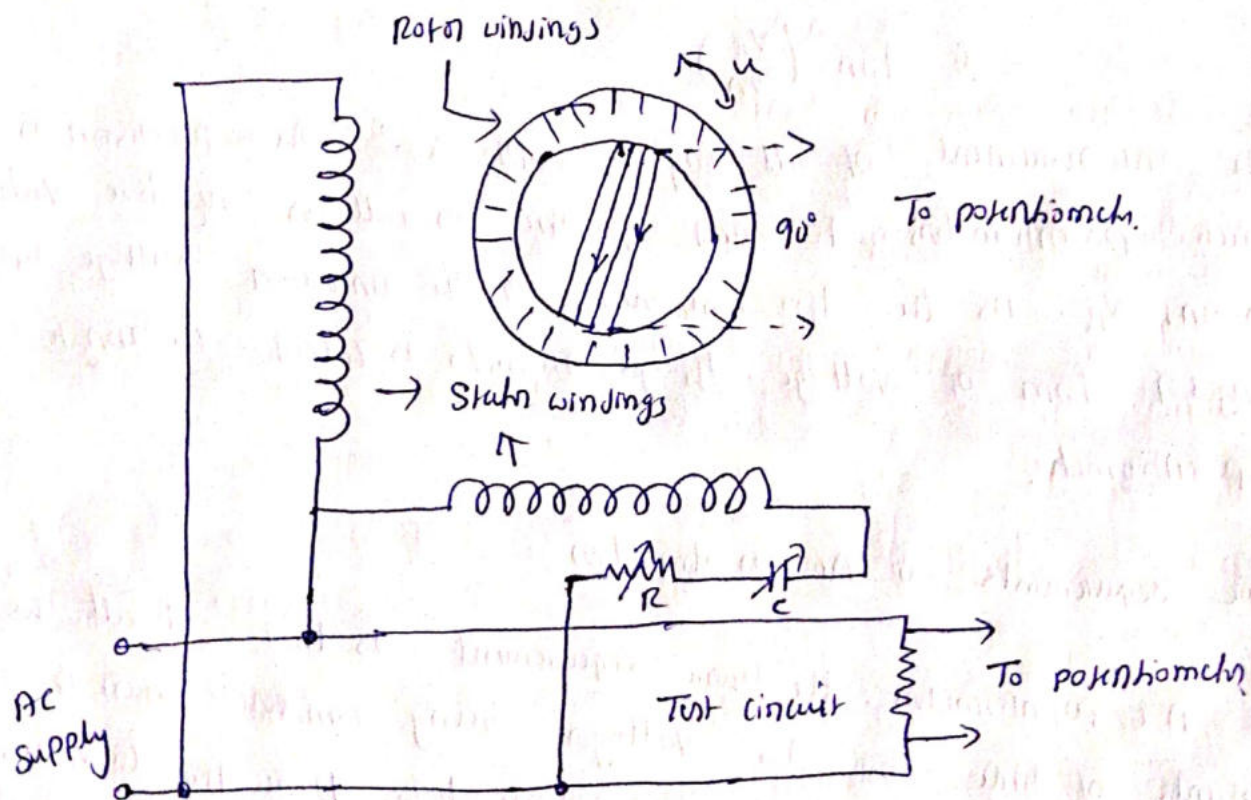
For the measurement of all angles upto 360° , the provision is made in such potentiometers to read positive as well as negative values of V_A and V_B . As the two components of the unknown voltage represent rectangular form of voltage, the potentiometer is called Co-ordinate type A.C. potentiometer.

* Basic requirements of A.C. potentiometer

- ✓ In A.C. potentiometer, the basic requirement is that, at all the instants of time both the voltages being compared must be equal with respect to magnitude and phase both. Hence the current in the potentiometer circuit must have same phase and frequency as compared to the voltage being measured.
- ✓ The A.C. source must supply exactly sinusoidal signal.
- ✓ It is necessary in A.C. potentiometer as the detectors used in it are vibration galvanometers.
- ✓ In A.C. potentiometer, another requirement is the control of stray e.m.f.s in leads and coupling circuits.

* Drysdale - Tinsley polar Type AC potentiometer:-

- Being a polar type, Drysdale - Tinsley a.c. potentiometer measures unknown limit in terms of its magnitude and phase angle.
- It has electrodynamic type ammeter along with DC potentiometer and phase shifting transformer which is operated by single phase AC supply.
- In a phase-shifting transformer, there is a combination of two ring shaped laminated steel stators which are connected perpendicular to each other as shown in fig.



Polar Type potentiometer

- one is directly connected to power supply and other one is connected in series with variable resistor and capacitor.
- The function of the series components is to maintain constant AC supply in the potentiometer by doing small adjustments in it.

- Between the stators, there is a laminated rotor having slots and winding which supplies voltage to the slide-wire circuit of the potentiometer.
- When current starts flowing from stators, the rotating field is developed around the rotor which induces an e.m.f in the rotor winding.
- The phase displacement of the rotor e.m.f is equal to rotor movement angle from its original position and it is related to stator supply voltage.
- The whole arrangement of the winding is done in such a way that the magnitude of the induced e.m.f in the rotor may change but it does not affect the phase angle and it can be read on the scale fixed on the top of the instrument.
- The induced e.m.f in rotor winding by stator winding-1 can be expressed as,

$$E_1 = K I \sin \omega t \cos \phi \rightarrow (1)$$

- The induced e.m.f in rotor winding by stator winding-2 can be expressed as

$$\begin{aligned} E_2 &= K I \sin (\omega t + 90^\circ) \cos (\phi + 90^\circ) \\ &= -K I \cos \omega t \sin \phi \rightarrow (2) \end{aligned}$$

- From equation -1 & 2, we get

$$E = K I (\sin \omega t \cos \phi - \cos \omega t \sin \phi) \rightarrow (3)$$

- Therefore, resulting induced e.m.f in the rotor winding due to two stator windings.

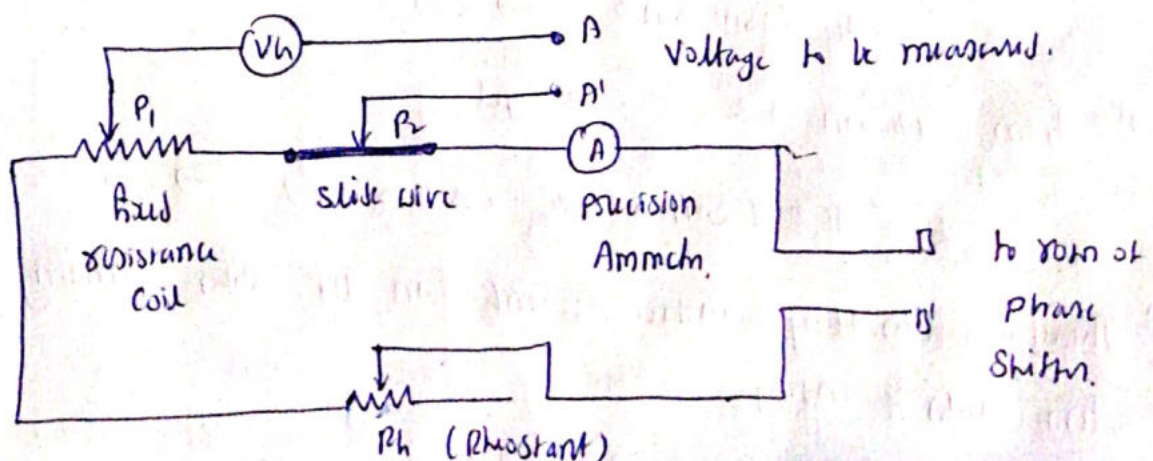
$$\boxed{E = K I \sin (\omega t - \phi)}$$

Standardisation of Drysdale - Tinsley A.C. potentiometer

- ✓ In standardisation of a.c. potentiometer, both d.c. as well as a.c. standardisations are done.
- ✓ The d.c. standardisation is done first by replacing vibration galvanometer by D'Arsonval galvanometer.
- ✓ A standard cell such as " Weston cell " is used for d.c. standardisation. Then by adjusting sliding contacts null deflection in galvanometer is achieved.
- ✓ The reading of precision ammeter included in battery supply is noted.
- ✓ During a.c. standardisation again vibration galvanometer is used.
- ✓ The ammeter still included in the supply circuit but now this circuit is without standard cell.
- ✓ By properly adjusting resistance in the circuit, the rms value of slide wire is made same as that of d.c. current noted in d.c. standardisation.

Measurement of unknown E.M.F.

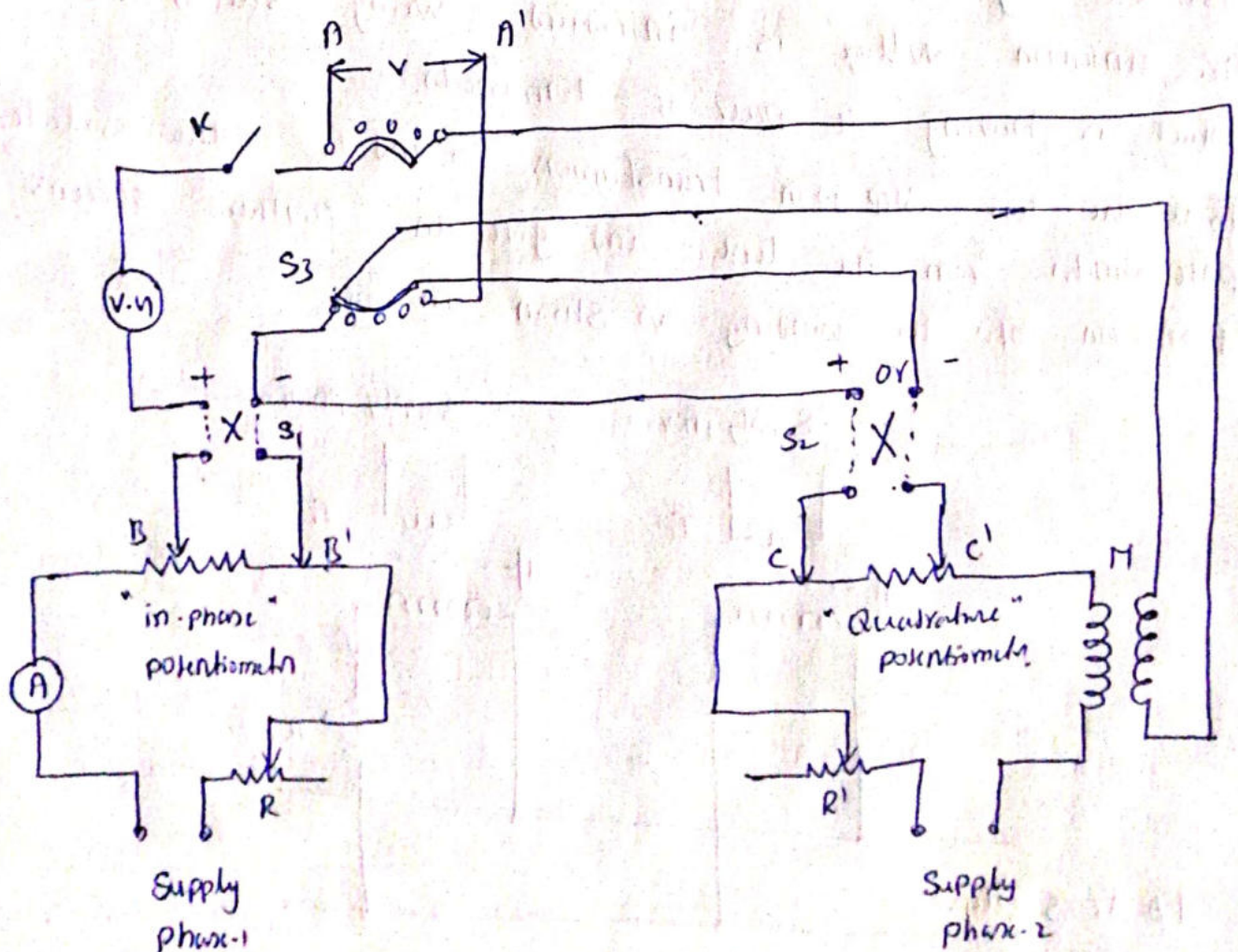
→ The circuit diagram for measurement of unknown E.M.F. using an a.c. potentiometer is as shown in fig.



- An E.m.f to be measured is connected across terminals A-A'
- The sliding contacts P_1 and P_2 and the position of rotor in phase shifter are adjusted simultaneously till the balance is obtained as indicated by the null deflection of vibration galvanometer.
- At balance, the magnitude of the unknown E.m.f is obtained from P_1 and P_2 and the phase angle is obtained from the scale reading which is marked on the top of the instrument.
- Thus the unknown E.m.f can be expressed in polar form as $E \angle \theta^\circ$

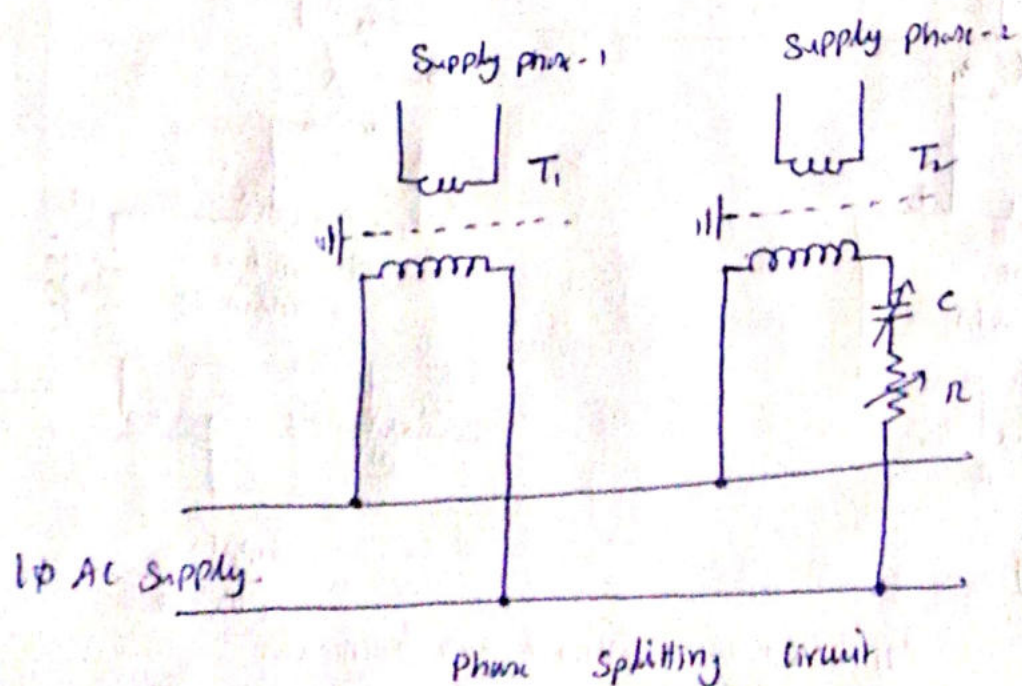
* Grall-Tinsley Co-ordinate type AC potentiometer

→ In co-ordinate type AC potentiometer, two separate potentiometers are caged in one circuit as shown in the figure.



Construction of Co-ordinate type potentiometer

- The first one is named as "in-phase" potentiometer which is used to measure the in-phase factor of an unknown EMF and the other one is named as "Quadrature" potentiometer which measures quadrature part of unknown EMF.
- The sliding contact AA' in the in-phase potentiometer and BB' in quadrature potentiometer are used for obtaining the desired current in the circuit.
- By adjusting rheostat ' R ' and ' R' ' and sliding contacts, the current in the quadrature potentiometer becomes equal to the current in the in-phase potentiometer and the galvanometer shows a null value.
- ' S_1 ' and ' S_2 ' are sign changing switches which are used to change the polarity of the test voltage if it is required for balancing the potentiometer.
- The unknown voltage is introduced using select switch ' S_3 ' which is having 4 pair of terminals.
- There are two step-down transformers T_1 and T_2 with isolate protection b/w the winding as shown in fig.



→ It also supplies '6-8' volts to potentiometers.

→ Now to measure unknown e.m.f its terminals are connected across sliding contacts AA' using switch 'S₂'

→ By doing some adjustments in sliding contacts and rheostat, the whole circuit gets balanced and galvanometer reads zero at the balanced condition.

→ Now the in-phase component V_A of the unknown [mf] is obtained from the in-phase potentiometer and quadrature component V_Q is obtained from quadrature potentiometer.

→ Thus the resultant voltage of the Co-ordinate A.C potentiometer is

$$V = (V_A^2 + V_Q^2)^{1/2}$$

and the phase angle is given by

$$\phi = \tan^{-1} (V_Q/V_A)$$

Standardisation of Co-ordinate type A.C potentiometer

✓ First of all d.c standardisation of potentiometer is done by using standard cell and D'Arsonval type galvanometer.

✓ Then without disturbing this setting, a.c standardisation is done by adjusting slide wire current to give zero deflection.

✓ Then previous galvanometer is replaced by vibration galvanometer and also direct current supply is replaced by a.c supply.

✓ Then the rheostat is adjusted till the current in the quadrature potentiometer wire is same as that in the in-phase potentiometer magnitude wise. Also these two currents must be exactly in quadrature.

* Applications of A.C. potentiometers :-

Similar to the d.c. potentiometers, a.c. potentiometers can be used for the calibration of voltmeters, ammeters. Also they are used for testing of energy meters and wattmeters. Measurement of self resistance of a coil is also possible with the help of a.c. potentiometers.

Following are the few applications of a.c. potentiometers.

i) Calibration of voltmeters

The method of calibration with a.c. potentiometers is very much similar to that with d.c. potentiometers. If the working voltage is less than 1.5V, it can be measured directly. If the voltage is very high then a.c. potentiometer must be used along with volt box.

ii) Calibration of ammeters

The calibration of a.c. ammeters may be carried out by using non-inductive standard resistance and successively noting measurements of various alternating current through it. The process of calibrating ammeters using a.c. and d.c. potentiometers is same.

iii) Testing of energy meters and wattmeters

The practical set up for the testing of wattmeter and energy meter with a.c. potentiometers is similar to that of calibration of wattmeter with d.c. potentiometers.

Ex:1 A slide wire potentiometer has a battery of 4V and negligible internal resistance. The resistance of slide wire is 100Ω and its length is 200cm. A standard cell of 1.0186 V is used for standardisation. The potentiometer and the rheostat is adjusted so that balance is obtained when the sliding contact is at 101.86 cm.

- Find the working current and the rheostat setting.
- If the slide wire has divisions marked in mm and each division can be interpolated to one fifth, calculate the resolution of the instrument.

Sol: a) Since the d.c potentiometer is standardised using standard cell of p.m.f 1.0186V with sliding contact at 101.86 cm; the portion of slide wire of the same length represents a voltage of 1.0186V.

∴ Resistance of 101.86 cm length of wire,

$$\Rightarrow \left(\frac{101.86}{200} \right) \times 100 \Rightarrow 50.93 \Omega$$

$$\therefore \text{Working current } I = \frac{V}{R} \Rightarrow \frac{1.0186}{50.93} \Rightarrow \underline{\underline{20 \text{ mA}}}$$

Total resistance in battery circuit = Resistance of rheostat + Resistance of slide wire.

∴ Resistance of rheostat = Total resistance in battery circuit + Resistance of slide wire

$$\Rightarrow \left[\frac{\text{Battery Voltage}}{\text{Working current}} \right] - 100$$

$$\Rightarrow \left[\frac{4}{20 \times 10^{-3}} \right] - 100 \Rightarrow 100 \Omega$$

b) The measurement range is nothing but total voltage across the slide wire,

$$\begin{aligned}\therefore \text{Range of voltage} &= (\text{working current}) \times (\text{resistance of slide wire}) \\ &= (20 \times 10^{-3}) \times (100) \\ &= 2 \text{ V}\end{aligned}$$

$\therefore$ The length of 200 cm represents 2V so 1mm represents voltage is given by

$$\begin{aligned}x &= 200 \text{ cm} \Rightarrow \frac{200 \times 10^2 \times 10}{10^4} \Rightarrow 200 \times 10^{-3} \\ &\Rightarrow 200 \times 10^{-3} \\ &\Rightarrow 2000 \text{ mm}\end{aligned}$$

$$V = \frac{2}{200} \times \frac{1}{10} \Rightarrow 2 \text{ mV}$$

Since it is possible to read $\frac{1}{5}$ th of 1mm. The resolution of instrument is given by

$$V(1/5) = \frac{1 \times 10^{-3}}{5} \Rightarrow 0.2 \text{ mV}$$

* Frequency Meters :-

The meters which are used in the circuit to indicate the frequency of the supply are called frequency meters.

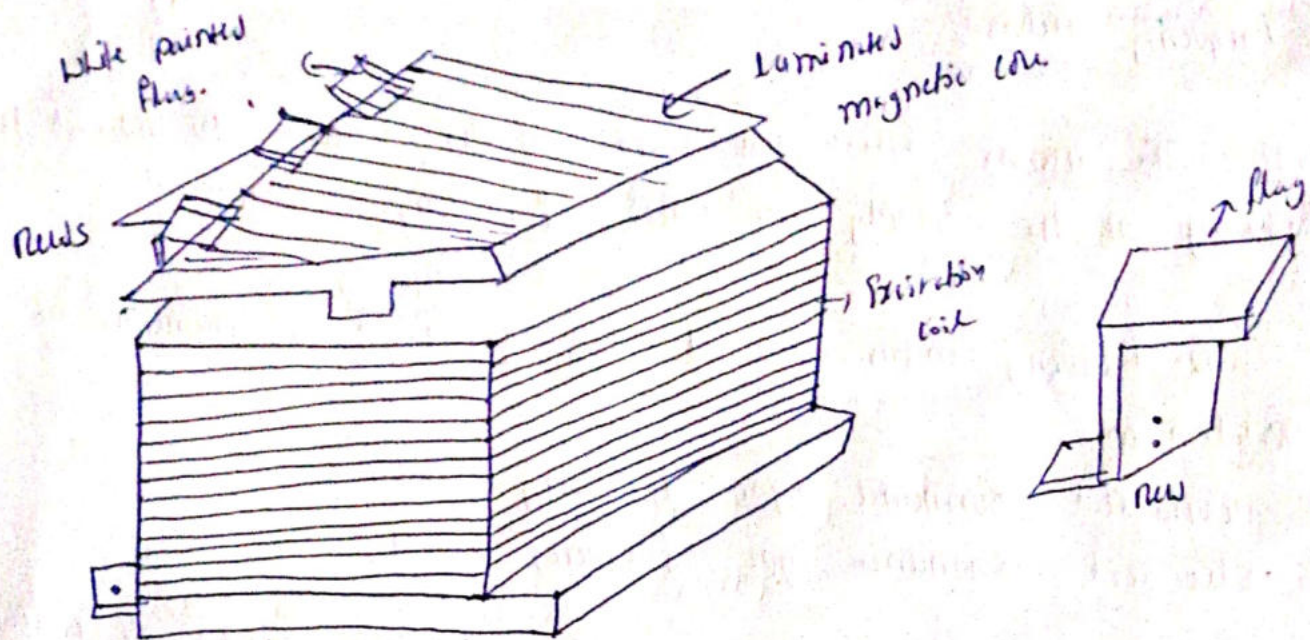
The frequency meters are classified based on the principle of operation as,

- i) Mechanical resonance type frequency meter.
- ii) Electrical resonance type frequency meter.

The mechanical resonance type frequency meter is called vibrating reed type frequency meter. The electrical resonance type frequency meter is called ferro-dynamic frequency meter.

i) Vibrating reed type frequency meter

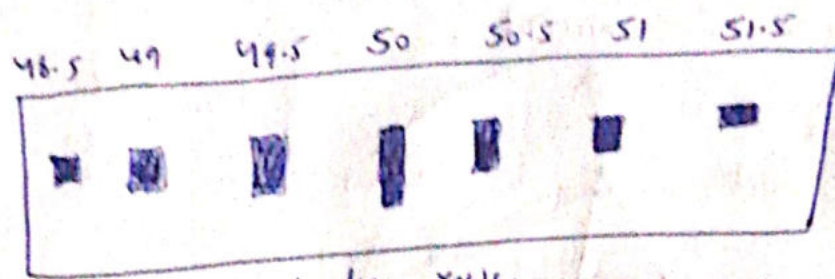
- ✓ This meter works on the principle of mechanical resonance.
- ✓ The meter consists of number of thin steel strips called reeds.
- ✓ The bottom of the reed is rigidly fixed to an electromagnet.
- ✓ The upper part of the reed is free and bent at right angles. This upper part is called as flag.
- ✓ An electromagnet has a laminated iron core, which carries an excitation coil having large number of turns. This coil is connected across the voltage whose frequency is to be measured.
- ✓ The flags are painted light white to have good visibility on the black background.
- ✓ The basic construction of this type of meter and the construction of reed as shown in fig.



Vibrating reed type frequency meter

- ✓ The reeds are manufactured such that their weights and dimensions are different. Hence their natural frequencies of vibration are different.
- ✓ The reeds are arranged in the ascending order of their natural frequencies and the natural frequencies are generally differ by half cycle.
- ✓ So natural frequency of first reed may be 48 Hz, next may be 48.5 Hz, next may be 49 Hz and so on.
- ✓ When meter is connected in the system, the coil carries current i which alternates at the supply frequency. This produces an alternating flux.
- ✓ This flux produces an force of attraction on the reeds which is proportional to square of the current i^2 and hence all the reeds vibrate with a force which varies at twice the supply frequency.
- ✓ But the reed whose natural freq. is twice the freq. of supply voltage will be in resonance and will vibrate most.
- ✓ The tuning in such meters is so precise that for a 1 to 2% change in the frequency away from resonating freq., the amplitude of vibration decreases drastically and become negligible.

✓ Thus when a seed corresponding to '50 Hz' is vibrating with maximum amplitude, other seeds vibrate but with negligible amplitudes which cannot be noticed. This is shown in fig.

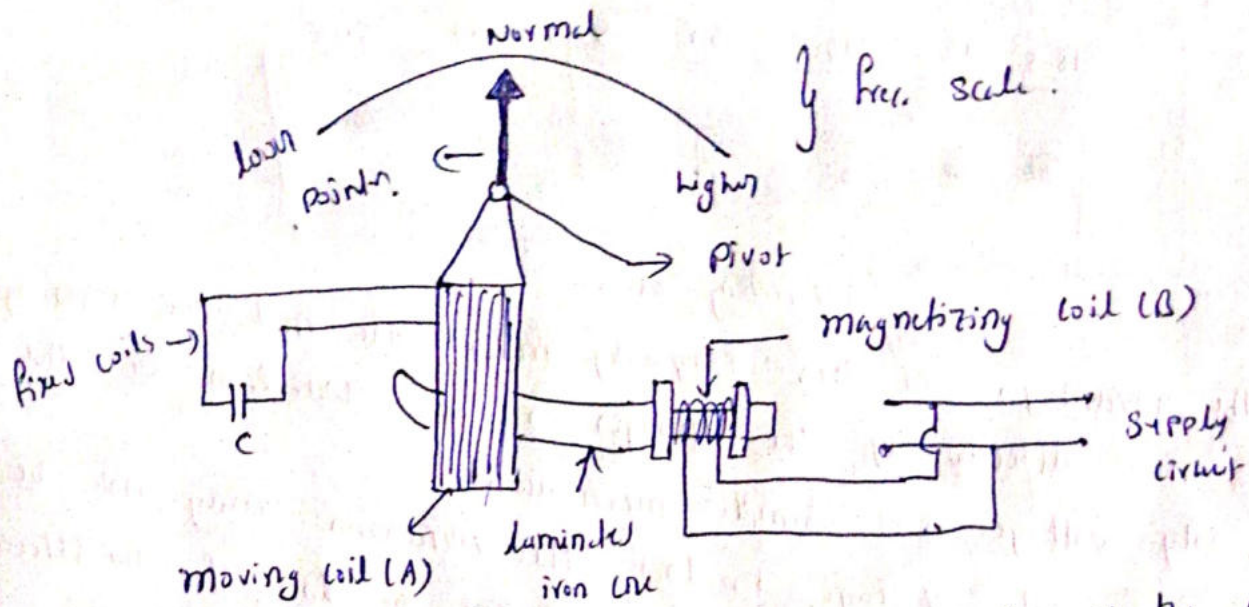


vibrating seeds.

- ✓ The advantage of this frequency meter are that the readings is not affected by the changes in the waveform of the supply voltage and simple mechanism.
- ✓ But if supply voltage is low, the vibrations may not be noticed. So supply voltage should not be low for the effective operation.
- ✓ One more limitation of the meter is that the difference in the frequencies of the adjust seeds is 0.5 only.
- ✓ So reading corresponding to less than half the frequency difference can not be obtained.
- ✓ So precise freq. measurement is not possible.
- ✓ The accuracy of the meter depends on the proper tuning of the seeds.

ii) Electrical Resonance type frequency meter

→ The fig shows the construction of the electrical resonance type



→ It consists of laminated iron core. On one end of the core a fixed coil is wound which is called magnetizing coil.

→ This coil is connected across the supply whose frequency is to be measured. This coil carries current which has same frequency as that of supply.

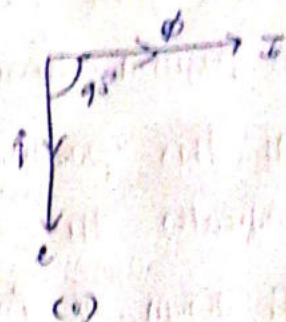
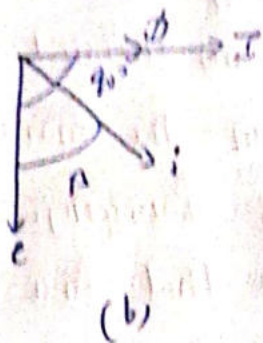
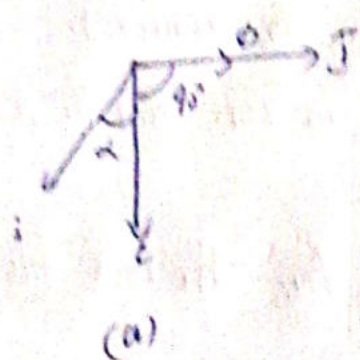
→ On the same core, a moving coil is pivoted which carries a pointer. A capacitor 'C' is connected across the terminals of the fixed coil.

→ Let I = current through the magnetizing coil
 ϕ = flux in the iron core.

→ The flux ϕ is assumed to be in phase with the current I .
 → This flux induces the voltage in the moving coil which always lags flux ϕ by 90° .

→ Let i = current through the moving coil.

- The phase of the current 'i' depends on the inductance of the moving coil and the capacitor 'C'.
- Consider the different cases and the corresponding phasor diagrams to understand the working of the meter as shown in fig a, b, c.



- In fig (a) the circuit of moving coil A is assumed to be inductive, hence current 'i' lags the induced voltage 'e' by an angle ' α '. Hence the torque acting on the moving coil is given by

→ In fig (a)
$$[T_d \propto I_i \cos(90 + \alpha)]$$

- In fig (b) the circuit of moving coil A is assumed to be lagging capacitive, hence current 'i' leads the induced voltage 'e' by an angle ' β '. Hence the torque acting on the moving coil is given by,

$$[T_d \propto I_i \cos(90 - \beta)]$$

- This torque is in opposite direction to the torque produced in case of inductive nature of the moving coil circuit.

- The fig (c) shows the resonance condition where the inductive reactance is equal to the capacitive reactance. So current 'i' is in phase with 'e' and the torque acting on the moving coil is given by

$$[T_d \propto I_i \cos(90^\circ) = 0]$$

- Hence under resonance condition, the torque acting on the moving coil is zero.

- Now, the capacitive reactance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ is constant for a given frequency.
- But the inductive reactance $X_L = \omega L$, not only depends on the frequency but also depends on the position of the moving coil on the scale.
- For a given frequency, moving coil moves in such a way to achieve a position where $X_L = X_C$ and electrical resonance is achieved.
- At this position torque on the moving coil is zero and the pointer indicates the corresponding frequency.
- The design of $f_{n.f.}$ is such that for a normal freq. the coil takes a mean position.
- The capacitor 'C' is chosen such that electrical resonance takes place at this mean position and pointer indicates the normal freq.
- If the frequency is higher than the normal value, the $X_C = \frac{1}{2\pi f C}$ decreases and hence $X_L = 2\pi f L$ must decrease in order to achieve resonance.
- So moving coil moves away from the magnetizing coil on the scale and pointer moves to the right of the mean position, indicating higher frequency.
- If the freq. is lower than the normal value, the $X_C = \frac{1}{2\pi f C}$ increases. So to achieve $X_L = X_C$, the moving coil moves towards the magnetizing coil where inductance increases. Thus pointer moves to the left of the mean position, indicating the lower frequency.
- An important advantage of the instrument is that the greatest sensitivity is achieved as the inductance of the moving coil changes slowly with variation of its position on the scale.

D.C & A.C Bridges* Introduction :-

- The bridges are used for not only the measurement of resistance but also used for the measurement of various component like capacitance, inductance etc.
- A "bridge circuit" in its simplest form consists of a network of four resistance arms forming a closed circuit. A source of current is applied to two opposite junctions. The current detector is connected to other two junctions.
- The bridge circuits use the "comparison measurement" methods and operate on null-indication principle. The bridge circuit compares the value of an unknown component with that of an accurately known standard component.
- In a bridge circuit, when no current flows through the null detector which is generally galvanometer, the bridge is said to be balanced. The relationship between the component values of the four arms of the bridge at the balancing is called "balancing condition" or "balancing equation".
- This "balancing equation" gives us the value of the unknown component.

* Advantages of Bridge Circuit:-

The various advantages of the bridge circuit are,

- i) The balancing equation is independent of the magnitude of the input voltage or its source impedance. These quantities do not appear in the balance equation expression.
- ii) The measurement accuracy is high.
- iii) The accuracy is independent of the characteristics of a null detector and is dependent on the component values.
- iv) The balance equation is independent of the sensitivity of the null detector.
- v) The balance condition remains unchanged if the source and detector are interchanged.

* Types of Bridge:-

The two types of bridge are,

i) D.C. bridge

ii) A.C. bridge.

The D.C. bridge are used to measure the resistance while the A.C. bridge are used to measure the impedances consisting capacitance and inductances.

The D.C. bridge use the D.C. voltage as the excitation voltage while the A.C. bridge use the alternating voltage as the excitation voltage.

The two types of dc bridges are,

- Wheatstone bridge
- Kelvin bridge.

The various types of ac bridges are,

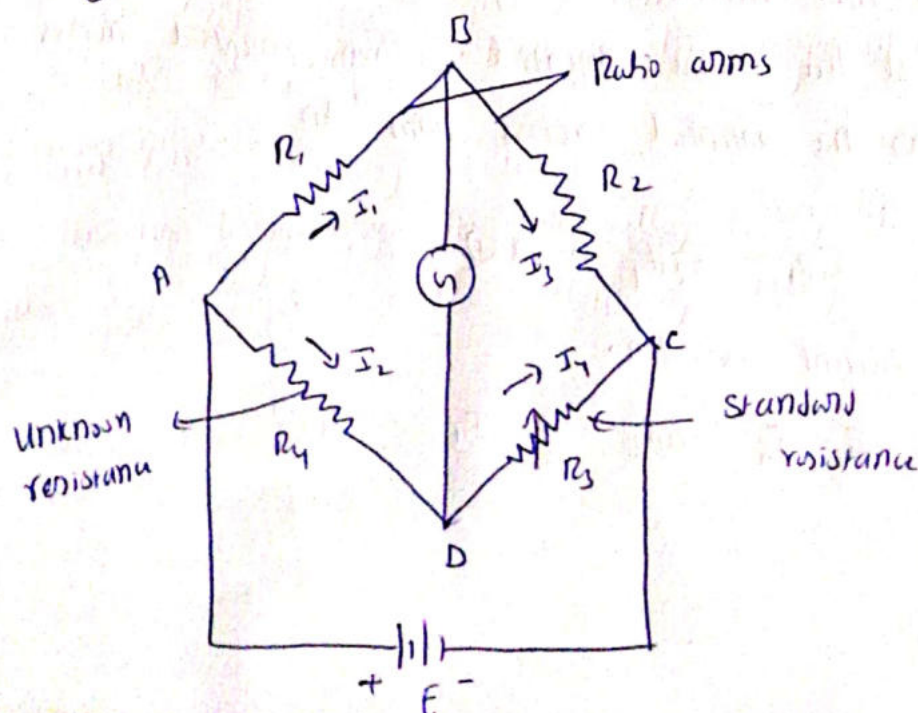
- Capacitance comparison bridge
- Inductance comparison bridge
- Maxwell's bridge
- Hay's bridge
- Anderson bridge
- Schering bridge
- Wien bridge

* Wheatstone Bridge :-

→ For measuring accurately any electrical resistance ^{Wheatstone bridge} is widely used.

→ The bridge consists of four resistive arms together with a source of E.M.F. and null detector. The galvanometer is used as null detector.

→ The fig shows the basic Wheatstone bridge circuit.



- There are two known resistors (R_1, R_2), one variable resistor (R_3) and one unknown resistor connected in bridge form as shown in fig.
- The battery is connected between A and C while galvanometer is connected between B and D.
- By adjusting the variable resistor the current through the galvanometer is made zero.
- When the current through the galvanometer becomes zero, the ratio of two known resistors is exactly equal to the ratio of adjusted value of variable resistance and the value of unknown resistance.
- In this way the value of unknown electrical resistance can easily be measured by using a "Wheat Stone" bridge.

Balance condition :-

When the bridge is balanced, the galvanometer carries zero current and it does not show any deflection. This bridge works on the principle of null deflection (or) null indication.

To have zero current through "galvanometer", the points B and D must be at the same potential. Thus potential across arm "AB" must be same as the potential across arm "AD".

Thus
$$I_1 R_1 = I_2 R_4 \rightarrow (1)$$

As galvanometer current is zero

$$I_1 = I_3 \quad \text{and} \quad I_2 = I_4 \rightarrow (2)$$

Considering the battery path under balance condition, ⁽³⁾

$$I_1 = I_3 = \frac{E}{R_1 + R_2} \rightarrow (3)$$

and $I_2 = I_4 = \frac{E}{R_3 + R_4} \rightarrow (4)$

Substituting equations (3) and (4) in equation (1)

$$I_1 R_1 = I_2 R_4$$

$$\frac{E}{R_1 + R_2} \times R_1 = \frac{E}{R_3 + R_4} \times R_4$$

$$\therefore R_1 (R_3 + R_4) = R_4 (R_1 + R_2)$$

$$R_1 R_3 + R_1 R_4 = R_4 R_1 + R_4 R_2$$

$$R_1 R_3 = R_4 R_2$$

$$\left[\frac{R_1}{R_2} = \frac{R_4}{R_3} \right]$$

This is required balance condition of Wheatstone bridge.

The following points can be observed.

- ✓ It depends on the ratio of R_1 and R_2 hence these arms are called ratio arms.
- ✓ The standard resistance ' R_3 ' can be varied to obtain the required balance.

* Sensitivity of Wheatstone bridge:

- ✓ When the bridge is balanced, the current through galvanometer is zero. But when bridge is not balanced current flows through the galvanometer causing the deflection.
- ✓ The amount of deflection depends on "Sensitivity" of the galvanometer.
- ✓ The sensitivity of can be expressed as amount of deflection per unit current.

$$\left[\text{Sensitivity (s)} = \frac{\text{deflection (d)}}{\text{current (I)}} \right]$$

- ✓ As the current is in microampere and deflection can be measured in mm, radians (or) degrees, the sensitivity is expressed as $\frac{\text{mm}}{\mu\text{A}}$, radians/ μA (or) degrees/ μA .
- ✓ Here is the sensitivity of a galvanometer, more in its deflection for the same amount of current.
- ✓ Another way of representing the galvanometer sensitivity is the amount of deflection per unit voltage across the galvanometer. This is called "voltage sensitivity" of the galvanometer.
- ✓ Mathematically it is denoted as,
$$\left[S_v = \frac{\theta}{e} \right]$$

$e = \text{voltage across galvanometer}$

$\theta = \text{deflection of galvanometer}$
- ✓ It is measured in degree per volts and radians per volts.
- ✓ While the "bridge sensitivity" is defined as the deflection of the galvanometer per unit fractional change in the unknown resistance. It is denoted as S_B .

$$\left[S_B = \frac{\theta}{\Delta R/R} \right]$$

When $\Delta R/R = \text{unit fractional change in unknown resistance.}$

* Measurement Errors :-

24)

The Wheatstone bridge is used to measure the resistance in the range of 1Ω to few megohms. But certain errors occur during the measurement using the Wheatstone bridge.

These errors are follows.

- i) The main error is because of limiting errors of the three known resistances. Hence very precise resistances are required having tolerance of 1% (or) even 0.1% .
- ii) The insufficient sensitivity of the null detector may cause the error.
- iii) Heating Effect :-
When the current passes through the resistances, due to the heating effect (I^2R) the temperature increases. Hence the values of the resistances of the bridge arms change due to heating effect. The excessively high current may cause the permanent change in the resistance values. This may cause serious error in measurement. To avoid this, power dissipation in the arms must be calculated well in advance and currents must be limited to safe value.
- iv) Thermal EMF :- In the galvanometer circuit the dissimilar metals come in contact and generate the thermal EMFs. Such thermal EMFs may cause the errors while measuring low value resistances. To prevent this, more sensitivity galvanometers having copper coils and copper suspension systems are used.
- v) The resistance of leads and contacts exterior to the actual bridge circuit adds the extra resistance and is the major cause of the errors while measuring low resistance values. These errors may be reduced by using another bridge called "Kelvin bridge".

* Applications of Wheatstone bridge

- The Wheatstone bridge is basically a dc bridge and used to measure the resistance in the range of 1Ω to $100\text{ M}\Omega$.
- It is used to measure the dc resistance of various types of wires for the purpose of quality control of wire.
- It is used to measure the resistance of motor winding, relay coils etc.

→

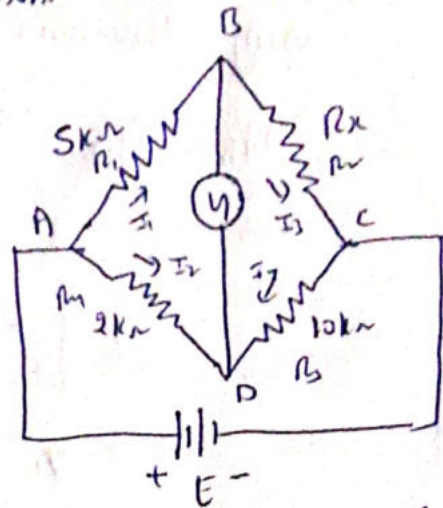
* Advantages of Wheatstone bridge

- The results are not dependent on the calibration and characteristics of galvanometer as it works on null deflection.
- The source e.m.f and inaccuracies due to source fluctuations do not affect the balance of the bridge. Hence the corresponding errors are completely avoided.
- Due to null deflection method used, the accuracy and sensitivity is higher than direct deflection method.

* Limitations of Wheatstone bridge

- ✓ The effect of lead resistance and contact resistance is very much significant while measuring low resistances.
- ✓ The bridge cannot be used for high resistance measurement i.e., measurement in high megaohm range. This is because with such measurement the resistance presented by the bridge becomes so large that the galvanometer becomes insensitive to show any imbalance.
- ✓ Similarly heating effect due to large current also plays a major role. The excessive currents may generate heat which may cause the permanent change in the resistance.
- ✓ The resistance used must be very precise having tolerance up to 1% (or) 0.1%, hence cost is high.

Q1:1) The Wheatstone bridge is shown in the fig. calculate the value of unknown resistance, assuming the bridge to be in balanced condition.



Sol) As per the bridge shown in fig.

$$R_1 = 5k\Omega \quad R_2 = R_x \quad R_3 = 10k\Omega \quad R_4 = 2k\Omega$$

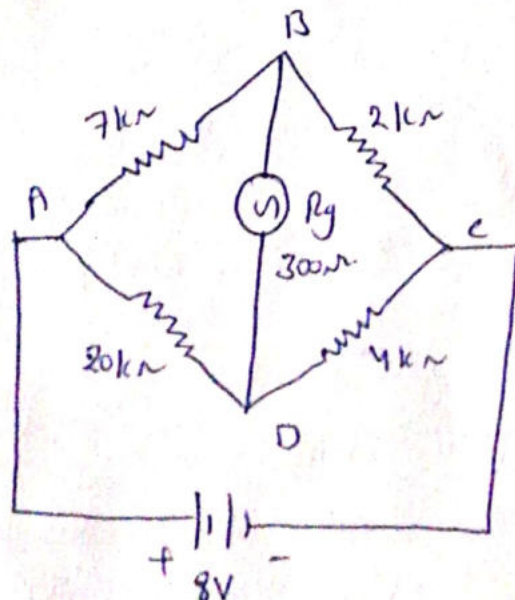
Under balance condition

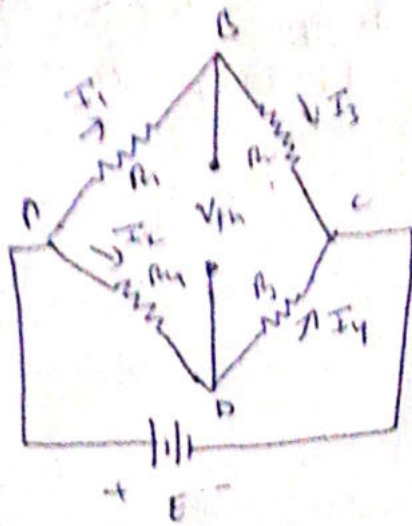
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$R_2 = \frac{R_3}{R_4} \times R_1 \Rightarrow \frac{10k\Omega}{2k\Omega} \times 5k\Omega$$

The unknown resistance is $\Rightarrow 25k\Omega$

Q1:2) calculate the current through the galvanometer for the bridge shown in fig.





$$R_1 = 7k\Omega \quad R_2 = 2k\Omega \quad R_3 = 4k\Omega \quad R_4 = 20k\Omega$$

$$E = 8V$$

using Thevenin's equivalent for I_g .

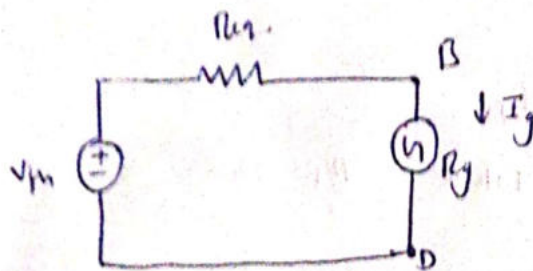
$$V_{th} = V_{BD} = V_{AD} - V_{AB}$$

$$\Rightarrow I_2 R_4 - I_1 R_1$$

$$\Rightarrow \left[\frac{E}{R_3 + R_4} \right] R_4 - \left[\frac{E}{R_1 + R_2} \right] R_1$$

$$\Rightarrow 8 \left[\frac{20}{20+4} - \frac{7}{7+2} \right]$$

$$\Rightarrow 0.444V$$



Thus B is positive w.r.t. D

$$\text{Now } R_{eq} = [R_1 \parallel R_2] + [R_3 \parallel R_4] \dots \text{ with } E \text{ shorted}$$

$$\Rightarrow \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} \Rightarrow 4.88k\Omega$$

$$I_g = \frac{V_{th}}{R_{eq} + R_g} \Rightarrow \frac{0.444}{(4.88 \times 10^3) + (300)} \Rightarrow 85.62 \mu A$$

This is the current through the galvanometer.

* Kelvin Bridge *

→ In the Wheatstone bridge, the bridge contact and lead resistance causes significant error, while measuring low resistance.

→ Thus for measuring the values of resistance below 1Ω , the modified form of Wheatstone bridge is used, known as "Kelvin bridge".

→ The fig shows the basic circuit of the Kelvin bridge.

→ The consideration of the effect of contact and lead resistances is the basic aim of the "Kelvin bridge".

→ The resistance R_y represents the resistance of the connecting leads from R_3 to R_x .

→ The resistance R_x is the unknown resistance to be measured.

→ The galvanometer can be connected to either terminal a, b or terminal c.

→ When it is connected to 'a', the lead resistance R_y gets added to R_x , hence the value measured by the bridge, indicates much higher value of R_x .

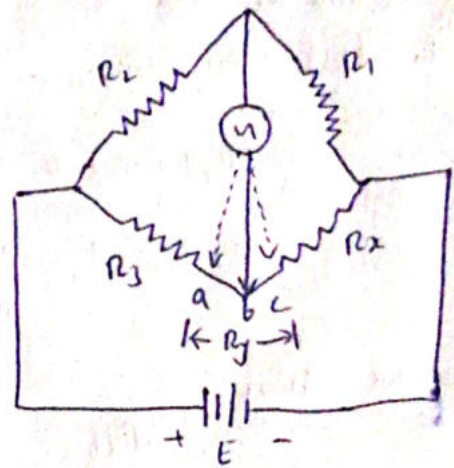
→ If the galvanometer is connected to terminal 'c', then R_y gets added to R_3 . This results in the measurement of R_x much lower than the actual value.

→ The point b is in between the points a and c in such a way that the ratio of the resistance from c to b and that from a to b is equal to the ratio of R_1 and R_2 .

$$\therefore \frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2} \rightarrow (1)$$

Now the bridge balance eqn. in its standard form is,

$$\frac{R_1}{R_2} = \frac{R_3 + R_y}{R_x} \rightarrow (2)$$



→ But R_3 and R_x now are changed to $R_3 + R_{ab}$ and $R_x + R_{cb}$ respectively due to lead resistance.

$$\therefore R_1 (R_3 + R_{ab}) = R_2 (R_x + R_{cb}) \rightarrow (3)$$

$$(R_x + R_{cb}) = \frac{R_1}{R_2} (R_3 + R_{ab}) \rightarrow (4)$$

Now we have

$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$$

$$\frac{R_{cb}}{R_{ab}} + 1 = \frac{R_1}{R_2} + 1 \quad \dots \text{adding 1 on both sides}$$

$$\frac{R_{ab} + R_{cb}}{R_{ab}} = \frac{R_1 + R_2}{R_2} \rightarrow (5)$$

but $R_{ab} + R_{cb} = R_y \quad \dots \text{total lead resistance.}$

$$\frac{R_y}{R_{ab}} = \frac{R_1 + R_2}{R_2} \rightarrow (6)$$

$$\left[R_{ab} = \frac{R_2 R_y}{R_1 + R_2} \right] \rightarrow (7)$$

Now $R_{cb} + R_{ab} = R_y$

$$R_{cb} = R_y - R_{ab} \rightarrow (8)$$

$$\Rightarrow R_y - \frac{R_2 R_y}{R_1 + R_2}$$

$$R_{cb} \Rightarrow R_y \left[1 - \frac{R_2}{R_1 + R_2} \right]$$

$$\left[R_{cb} = \frac{R_1 R_y}{R_1 + R_2} \right] \rightarrow (9)$$

→ now substituting R_{14} and R_{24} in Eqn. (4)

(7)

$$R_1 + \frac{R_1 R_3}{R_1 + R_2} = \frac{R_1}{R_2} \left(R_3 + \frac{R_1 R_3}{R_1 + R_2} \right)$$

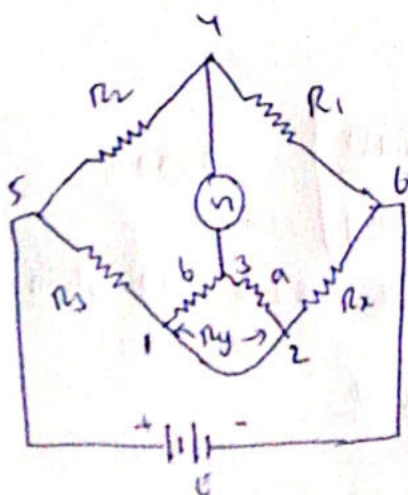
$$R_1 + \frac{R_1 R_3}{R_1 + R_2} = \frac{R_1 R_3}{R_2} + \frac{R_1 R_3}{R_1 + R_2}$$

$$\left[R_x = \frac{R_1 R_3}{R_2} \right] \rightarrow (10)$$

→ Thus Eqn. (10) represents standard bridge balance equation for the Wheatstone bridge. Thus the effect of the connecting lead resistance is completely eliminated by connecting the galvanometer to an intermediate position "b".

→ This principle forms the basis of the construction of "Kelvin's double bridge" which is popularly called "Kelvin bridge".

* Kelvin's Double bridge



→ This bridge consists of another set of ratio arms hence called "double bridge".

→ The fig shows the circuit diagram of Kelvin's double bridge.

→ The second set of ratio arms are the resistances "a" and "b". With the help of these resistances the galvanometer is connected to point-3.

→ The galvanometer gives null indication when the potential of the terminal-3 is same as the potential of the terminal-4.

Then $E_{45} = E_{513} \rightarrow (1)$

Here $E_{45} = \text{potential across } R_2$

$E_{513} = \text{potential across } R_3 \text{ and } b.$

The ratio of the resistances a and b is same as the ratio of R_1 & R_2

$$\frac{a}{b} = \frac{R_1}{R_2} \rightarrow (2)$$

Now $E_{45} = I R_2$

$$\Rightarrow R_2 \times \frac{E}{R_1 + R_2} \rightarrow (3)$$

Consider the path from $S-1-2-b$ back to S through the battery E .

The resistance b/w the terminals $1-2$ is parallel combination of R_y and $(a+b)$

$$E = I [R_3 + (R_y \parallel (a+b)) + R_x]$$

$$E = I \left[R_3 + R_x + \frac{R_y (a+b)}{a+b+R_y} \right] \rightarrow (4)$$

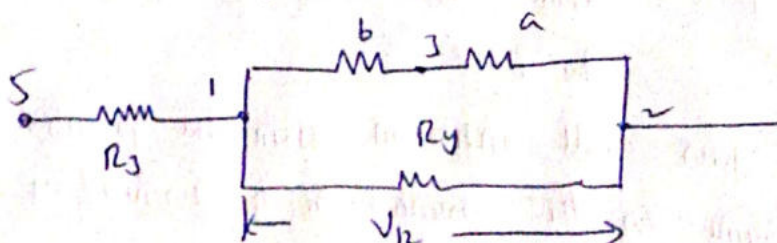
Substituting Eqn. (4) in Eqn. (3)

$$E_{45} = \frac{R_2}{R_1 + R_2} \times I \left[R_3 + R_x + \frac{R_y (a+b)}{a+b+R_y} \right] \rightarrow (5)$$

For E_{513} , consider the path from the terminal S to 2 as shown in

fig.

indicated carries two current



Now from the above fig. we can write.

$$V_{12} = I \times \left[\frac{R_y (a+b)}{a+b+R_y} \right]$$

and $V_{13} = \frac{b}{a+b} \times V_{12}$

$$V_{13} = \frac{b}{a+b} \times I \times \left[\frac{R_y (a+b)}{a+b+R_y} \right] \rightarrow (6)$$

$$\therefore E_{S13} = IR_3 + V_{13}$$

$$E_{S13} = IR_3 + I \times \frac{b}{a+b} \left[\frac{R_y (a+b)}{a+b+R_y} \right]$$

$$E_{S13} = I \left[R_3 + \frac{b}{a+b} \left[\frac{R_y (a+b)}{a+b+R_y} \right] \right] \rightarrow (17)$$

Now $E_{45} = E_{S13}$

... for balancing.

$$\frac{R_1 R_2}{R_1 + R_2} \times \left[R_3 + R_x + \frac{(a+b) R_y}{a+b+R_y} \right] = \left[R_3 + \frac{b}{a+b} \left[\frac{R_y (a+b)}{a+b+R_y} \right] \right]$$

$$R_3 + R_x + \frac{(a+b) R_y}{a+b+R_y} = \frac{R_1 + R_2}{R_2} \left[R_3 + \frac{b}{a+b} \left[\frac{R_y (a+b)}{a+b+R_y} \right] \right]$$

$$R_3 + R_x + \frac{(a+b) R_y}{a+b+R_y} = 1 + \frac{R_1}{R_2} \left[R_3 + \frac{b}{a+b} \left[\frac{R_y (a+b)}{a+b+R_y} \right] \right]$$

$$\cancel{R_3} + R_x + \frac{(a+b) R_y}{a+b+R_y} = \cancel{R_3} + \frac{R_1 R_3}{R_2} + \frac{b R_y}{a+b+R_y} + \frac{b R_1 R_y}{R_2 (a+b+R_y)}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_y}{a+b+R_y} + \frac{b R_1 R_y}{R_2 (a+b+R_y)} - \frac{(a+b) R_y}{a+b+R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{\cancel{b R_y}}{\cancel{a+b+R_y}} + \frac{b R_1 R_y}{R_2 (a+b+R_y)} - \frac{a R_y}{a+b+R_y} - \frac{\cancel{b R_y}}{\cancel{a+b+R_y}}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_1 R_y}{R_2 (a+b+R_y)} - \frac{a R_y}{a+b+R_y}$$

$$\left[R_x = \frac{R_1 R_3}{R_2} + \frac{b R_y}{a+b+R_y} \left(\frac{R_1}{R_2} - \frac{a}{b} \right) \right] \rightarrow (8)$$

but $\frac{a}{b} = \frac{R_1}{R_2}$ then $\frac{R_1}{R_2} - \frac{a}{b} = 0$.

$$\left[R_x = \frac{R_1 R_3}{R_2} \right] \rightarrow (9)$$

This is the standard equation of the bridge balance. The resistance a, b and R_y are not present in this equation. Thus the effect of lead and contact resistance is completely eliminated.

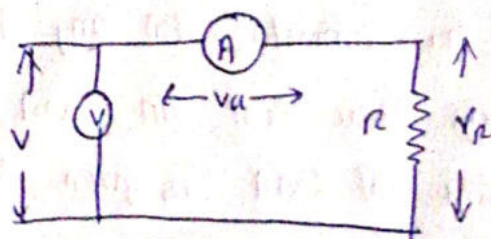
In a typical kelvin's double bridge, the range of a resistance covered is 1Ω to 10Ω with an accuracy of $\pm 0.05\%$ to $\pm 0.2\%$.

* Ammeter-voltmeter method for measurement of resistance ⁽⁹⁾

- Based on the value of resistance, the resistances are classified as low (6Ω), medium (1Ω to $100k\Omega$) and high ($>100k\Omega$) resistance.
- There are various methods used for measuring resistance based on their classification.
- Among these methods, the ammeter-voltmeter method is one of the methods used for measuring low and medium values of resistance.
- This method is the simplest of all, as it requires an ammeter and a voltmeter which will be easily available in the laboratory.
- Also, the connections are very simple and there is no lengthy procedure for the measurement of resistance.
- The ammeter and voltmeter will be connected in series and parallel in parallel.
- The ratio of voltmeter to ammeter reading will give the value of resistance under measurement.
- Based on the relative connections of the ammeter and voltmeter with respect to the unknown resistance there are two methods.

Method 1: Ammeter connected in the side of unknown resistance

- ✓ The below shows the connections for measuring unknown resistance when the ammeter is connected in series with it.



V_A : Voltage across ammeter

V_R : Voltage across unknown resistance.

Let

V : Voltmeter readings

R : unknown resistance

I : Ammeter readings

R_a : internal resistance of ammeter.

R' : Measured value of resistance with the above connections.

✓ In the above circuit, since the ammeter is directly in series with R , the ammeter readings indicate the current through " R ".

✓ The voltmeter is not in parallel with " R ", rather it is connected in parallel with the series combination of unknown resistance and the ammeter.

∴ The voltmeter reading is given by

$$V = V_a + V_R$$

Where $V_a = IR_a$, $V_R = IR$

$$[V = I(R + R_a)]$$

∴ The measured value of resistance.

$$R' = \frac{V}{I} \Rightarrow \frac{I(R + R_a)}{I}$$

$$[R' = R + R_a]$$

∴ True value of resistance

$$[R = R' - R_a]$$

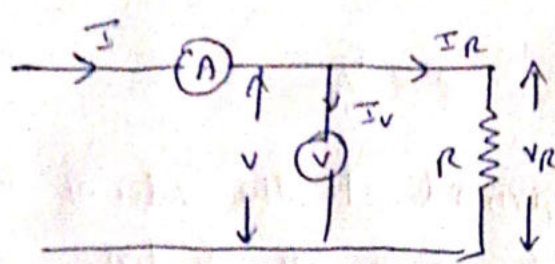
✓ From the above equations, the measured value is not equal to the exact value (or) true value of the unknown resistance. This is because internal resistance of ammeter.

✓ The internal resistance of the ammeter (or) any device cannot be made equal to zero, hence always there exist an error in the measured value of R resistance. The amount of error is given by

$$[e' = \frac{R' - R}{R} \Rightarrow \frac{R_a}{R}]$$

Method-2: Voltmeter connected to the side of unknown resistance (b)

✓ The below shows the connections for measuring the unknown resistance when the voltmeter is connected to the side of unknown resistance.



Let
 V : Voltmeter readings
 I : Ammeter readings
 I_R : Current through unknown resistance.

V_R : Voltage across unknown resistance.

I_v : Current through voltmeter.

R_v : internal resistance of voltmeter.

R'' : measured value of resistance with above connection.

✓ In the above circuit, the voltmeter is connected directly in parallel with unknown resistance, whereas the ammeter is in series with the parallel combination of voltmeter and R .

✓ The voltmeter reads voltage across R , but the ammeter reads the sum of currents through R and voltmeter.

∴ Ammeter reading

$$I = I_v + I_R$$

$$I_R = \frac{V}{R} \quad I_v = \frac{V}{R_v}$$

$$I = V \left(\frac{1}{R_v} + \frac{1}{R} \right)$$

$$\therefore R'' = \frac{V}{I} \Rightarrow \frac{V}{V \left(\frac{1}{R_v} + \frac{1}{R} \right)} \Rightarrow \frac{R R_v}{R + R_v} \Rightarrow \frac{R}{1 + \frac{R}{R_v}}$$

↑ measured value of resistance

$$R = \frac{R'' R_v}{R_v - R''}$$

$$\left[\text{Error} = \frac{R'' - R}{R} \Rightarrow \frac{-R}{R_v} \right]$$

Condition for the selection of limit:-

- ✓ From the Eqn. of Error when the ammeter is connected to the side of unknown resistance,

$$e' = \frac{R_a}{R}, \quad e' \propto \frac{1}{R}$$

- ✓ The error is inversely proportional to the value of unknown resistance, i.e., the higher value of R , the error will be less.

- ✓ Hence this connection is used for measuring of high value of resistance so that error is small and accuracy is more.

- ✓ From the Eqn. of Error when the voltmeter is connected to the side of unknown resistance

$$e'' = -\frac{R}{R_v}, \quad e'' \propto R$$

- ✓ The error is directly proportional to the value of unknown resistance i.e., lower is the value of R , error will be less.

- ✓ Hence connection - 2 is used for measuring low value of resistance in order to have greater accuracy.

* Measurement of High Resistance

(iii)

The measurement of high resistance of the order of hundreds and thousands of megohms is often required in electrical equipments. The examples of such resistances are,

- i) Insulation resistance of components like machines, cables etc.
- ii) Leakage resistance of capacitors.
- iii) Resistance of high circuit elements like vacuum tube circuits.

But there are certain difficulties in measurement of such high resistances. Because of very high resistance, very small currents flow through the measuring circuits, which is very difficult to sense. The various other difficulties are,

⇒ Presence of leakage currents :-

The leakage currents are produced and are of comparable magnitude to the current being measured. Such currents cause errors. These currents depend on humidity and hence are unpredictable. Hence leakage currents must be eliminated from the measurement.

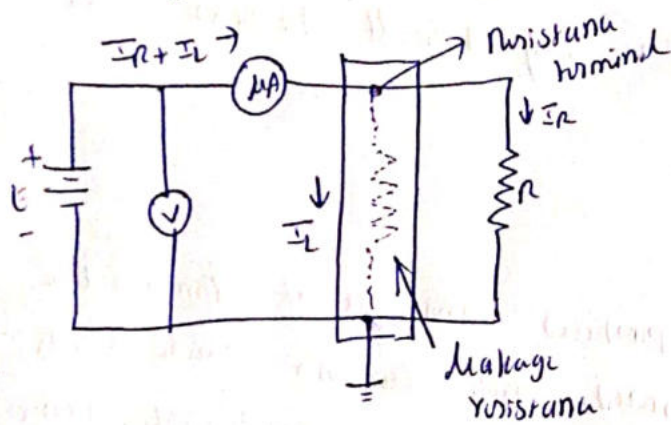
⇒ The stray charges may appear due to electrostatic effect. Such charges and alternating fields can cause serious measurement error.

⇒ One point of the circuit may be connected to earth for accuracy in measurements.

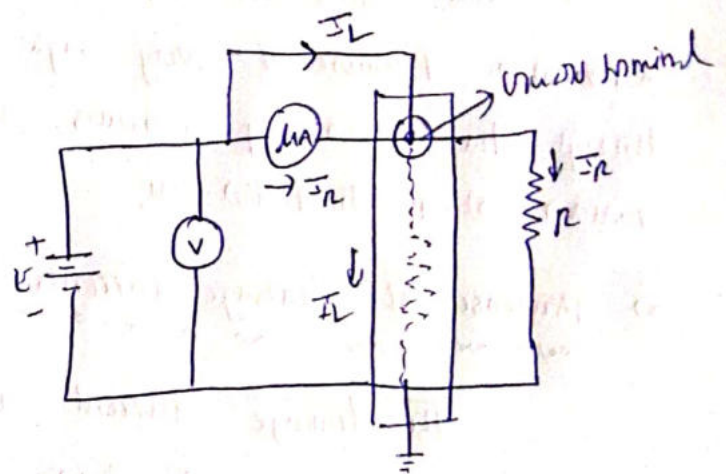
⇒ When the voltage is applied to the insulation resistance, it takes some time for charging and absorbing currents. The measurement should be delayed till these currents vanish completely. In some cases, this may take very long time hence the testing conditions include the time b/w the application of voltage and observation of the reading. This time must be specified for the accuracy.

→ very high voltage is required in order to raise the current magnitudes. The galvanometer should be very sensitive and proper steps must be taken to prevent the damage of galvanometer due to high voltages.

Hence to solve the problem of leakage currents (or) capacitive current we use a guard circuit. The concept of guard circuit is to bypass the leakage current from the ammeter, so as to measure the true resistive current. Figure below shows two connections on voltmeter and micro ammeter to measure " R ", one without guard circuit and one with guard circuit.



a) without guard.



b) with guard.

In the first circuit the μA measures both capacitive and resistive current leading to error in value of R , while in the other circuit the μA reads only the resistive current.

Following few methods used for measurement of high resistance values

- i) Loss of charge method
- ii) Megger
- iii) Megohm bridge method.
- iv) Direct deflection method.

* Loss of charge method

(14)

→ This method is applicable for measurement of resistance greater than $100k\Omega$ to $0.1M\Omega$.

→ There are various examples for high resistance like

✓ Insulation resistance of cables and machines

✓ Vacuum tube resistance.

✓ Leakage resistance of capacitors.

→ The basic idea is based on the charging and discharging of a known value of capacitor.

→ The fig. 1 shows the circuit connection for the loss of charge method.

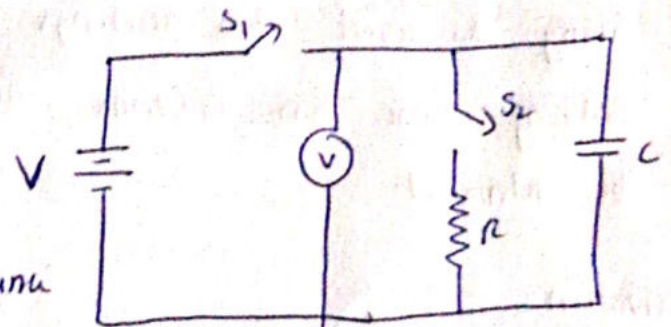


fig. 1

→ where R is the unknown resistance to be measured, C is a known value capacitor placed across R , V is an

electrostatic type voltmeter to measure the voltage across the parallel combination of R and C .

→ The circuit is driven by a battery having a voltage (V).

→ S_1 and S_2 are the switches used for charging and discharging the capacitor.

→ It is assumed that the internal resistance of the voltmeter and leakage resistance of the capacitor is infinite.

→ The sequence of operation can be explained as follows,

When switch S_1 closed and S_2 open 1-

With switch S_1 closed and S_2 open, the capacitor charges to a particular voltage. The voltage across 'C' is measured by the voltmeter 'V' and is noted down.

When switch S_1 open and S_2 closed 2-

With switch S_1 open and S_2 closed, the battery is disconnected and the unknown resistance R gets connected across C. Now the capacitor discharges through R . During this, the capacitor voltage is measured by the voltmeter.

→ Taking the above two readings of voltmeter, we get two equations, solving these two equations, the value of unknown resistance can be determined.

Derivation :-

1) If V_c be the terminal voltage of the capacitor consisting of capacitance 'C' (assumed to be lossless) and Q (Coulomb) be the charge at time 't' sec, then the charging current 'i' is,

$$i = -\frac{dQ}{dt} \Rightarrow -\frac{d(CV_c)}{dt} \Rightarrow -C \frac{dV_c}{dt} \rightarrow (1)$$

$$i = \frac{V_c}{R} \rightarrow (2)$$

Equating Equations 1 and 2 we get

$$\frac{V_c}{R} = -C \frac{dV_c}{dt}$$

$$\frac{dV_c}{dt} = -\frac{V_c}{CR}$$

Applying integration of both sides we get-

$$V_c = -\frac{1}{RC} \int V_c dt$$

→ The voltage across the capacitor (V_c) at time t is given by

$$V_c = V e^{-(t/RC)} \rightarrow \text{discharging}$$

$$\frac{V_c}{V} = e^{-(t/RC)}$$

$$V/V e^{t/RC}$$

$$V_c = V (1 - e^{-t/RC}) \rightarrow \text{charging.}$$

$$\ln \frac{V_c}{V} = -\frac{t}{RC}$$

$$C (\ln(\frac{V_c}{V})) = -\frac{t}{R}$$

$$R = \frac{-t}{C \ln(\frac{V_c}{V})} \Rightarrow \frac{-t}{2.303 C \log(\frac{V}{V_c})}$$

$$\left[R = \frac{0.434t}{C \log_{10}(\frac{V}{V_c})} \right]$$

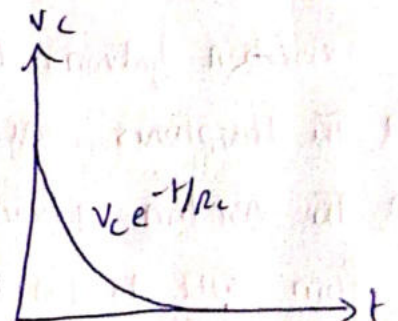
→ The graph shows the variation of V_c with respect to t .

→ If the value of the resistance to be measured is very large, then the time taken for the voltage to fall is quite large. Hence this method is tedious and time consuming.

→ However, the accuracy of the voltage measured ~~depends~~ depends upon methods implemented to reduce errors.

→ In order to obtain high accuracy, a drop in voltage i.e., $V - V_c$ is taken instead of V_c

$$\left[R = \frac{0.434t}{C \log_{10}(\frac{V}{V - V_c})} \right]$$

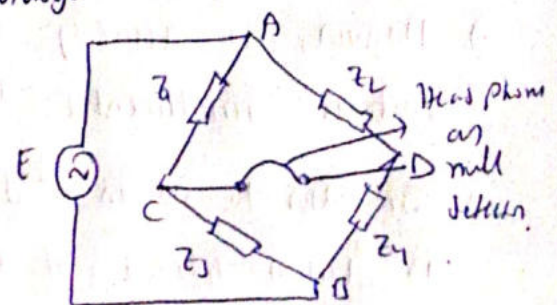


Errors in loss of charge method:

- i) Error due to leakage and Absorption.
- ii) Error due to variation in insulation resistance due to variation in temp.
- iii) Error due to time of application of voltage.

* AC bridges

- An a.c bridge in its basic form consists of four arms, a source of excitation and a balance detector.
- Each arm consists of an impedance.
- The source is an a.c supply which supplies a.c voltage at the required frequency.
- For high frequencies, the electronic oscillators are used as the source.
- The balance detectors commonly used for a.c bridges are headphones, vibration galvanometers.
- The headphones are used as detectors at the frequency of 250 Hz to 4 kHz.
- The vibration galvanometers are useful for low audio frequency range from 5 Hz to 1000 Hz.
- The simple a.c bridge is the outcome of the Wheatstone bridge.
- The impedance at audio and radio frequency range can be easily determined by such simple ac Wheatstone bridge as shown in fig.
- This is similar to d.c Wheatstone bridge.
- The bridge arms are impedances. The bridge is excited by a.c supply and pair of headphones is used as a null detector.
- The null response is obtained by varying one of the bridge arms.



* Maxwell's bridge :-
 = = - =

- The Maxwell's bridge is an AC bridge used to find (or) measure the unknown self-inductance.
- There are different types of ac bridges used for the measurement of self-inductance in which Maxwell's bridge is the most commonly used bridge.
- The bridge uses the principle of null deflection.
- The Equation obtained when the bridge is balanced and it can be used to determine the unknown inductance connected to it.

There are two types of Maxwell's bridge used to find unknown inductance.

- a) Maxwell's inductance bridge.
- b) Maxwell's inductance capacitance bridge.

a) Maxwell's inductance bridge :-
 ~ ~ ~ ~ ~

- ✓ In Maxwell's inductance bridge the unknown self inductance to be measured is compared with the known inductance.
- ✓ Hence the unknown self-inductance and internal resistance of an inductor can be measured with Maxwell's inductance bridge.
- ✓ The circuit diagram is shown in fig.

Let

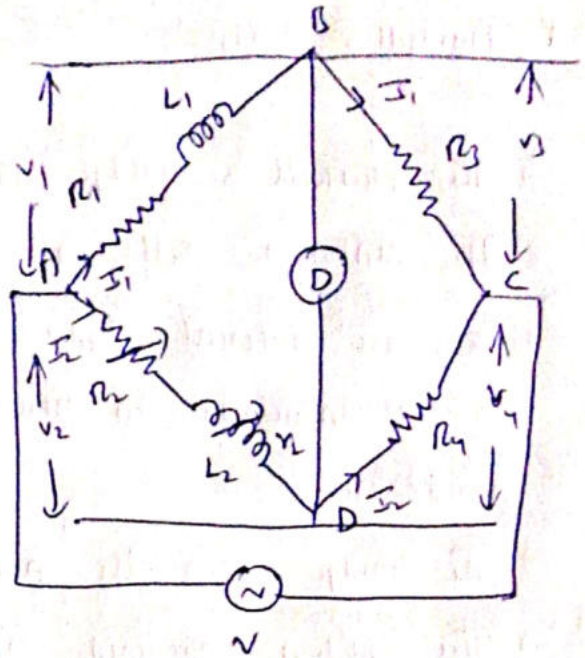
L_1 = unknown inductance to be measured

R_1 = Resistance of unknown inductance.

R_3 & R_4 = Standard non-inductive resistances

R_2 = Standard variable resistor.

L_2 = Standard variable inductance with fixed resistance r_2



✓ From the above fig the impedances of the respective arms are given as

$$Z_1 = R_1 + j\omega L_1, \quad Z_2 = (R_2 + r_2 + j\omega L_2)$$

$$Z_3 = R_3$$

$$Z_4 = R_4$$

✓ Under balanced condition, we have

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + j\omega L_1) R_4 = (R_2 + r_2 + j\omega L_2) R_3$$

$$R_1 R_4 + j\omega L_1 R_4 - R_2 R_3 - r_2 R_3 - j\omega L_2 R_3 = 0$$

$$R_1 R_4 - R_2 R_3 - r_2 R_3 + j\omega (L_1 R_4 - L_2 R_3) = 0$$

✓ on equating the real and imaginary parts on both sides, we get

$$R_1 R_4 - R_2 R_3 - r_2 R_3 = 0$$

$$R_1 R_4 = R_2 R_3 + r_2 R_3$$

$$\left[R_1 = \frac{R_3}{R_4} (R_2 + r_2) \right]$$

also

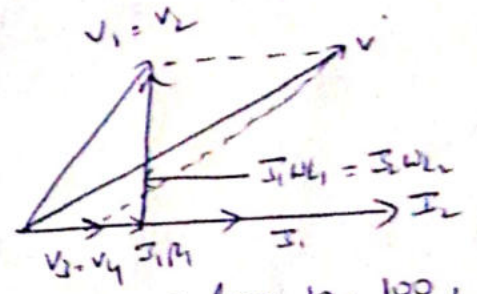
$$L_1 R_4 - L_2 R_3 = 0$$

$$\left[L_1 = L_2 \times \frac{R_3}{R_4} \right]$$

∴ Hence, the unknown self inductance and resistance of the inductor are obtained in terms of known standard values.

Phasor diagram:-

→ In Maxwell's inductance bridge the resistance R_2 is the decade resistance box.



→ The resistors R_3 and R_4 can have their values as from 10, 100, 1000 and 10,000 Ω by the successive adjustment of L_1 and R_2 .

→ The phasor diagram of Maxwell's inductance bridge under balance condition is shown above.

Ex. The arms of an a.c maxwell's bridge are adjusted as, (17)

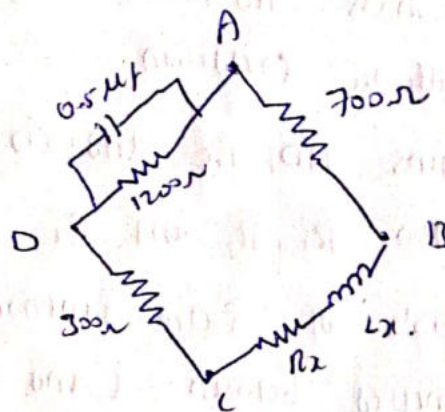
Arm AB: non reactive resistance of 700Ω .

Arm CD: non reactive resistance of 300Ω .

Arm AD: non reactive resistance of 1200Ω in parallel with Capacitor of $0.5\mu F$.

If the bridge is balanced under this condition, find the components of the branch BC.

Sol. The bridge is shown in fig.



$$R_2 (1200) = 700 \times 300$$

$$R_2 = 175\Omega$$

$$L_x = 700 \times 300 \times 0.5 \mu F$$

$$\Rightarrow 105 \text{ mH.}$$

* Anderson's Bridge :-

- The Anderson's bridge is an alternating current bridge, which is used to find the self-inductance of the circuit.
- Actually this bridge is modified form of Maxwell's inductance capacitance bridge for the measurement of self-inductance in terms of the standard capacity of low 'Q' coils ($Q < 1$).
- The bridge uses a fixed capacitor instead of a variable capacitor as seen in Maxwell's bridge and hence gives more accurate results.

* Construction of Anderson's bridge:-

→ The bridge consists of four arms AB, AD, BC, & CD.

→ The arm AB of the bridge consists of unknown inductance L_1 to be measured in series with a non-inductive resistance R_1 and r_1 be the resistance of inductn.

→ The fig shows Anderson's bridge under balance condition.

→ The arms AD, BC and CD consists of standard non-inductive resistances R_2, R_3 and R_4 of known value.

→ It consists of extra junction E to which a variable resistn 'r' is connected between E and D, a fixed capacitn between E and C, and a null detectn between E and B.

operation and theory of Anderson's bridge:-

Let C = Standard capacitn of fixed known value.

R_2, R_3, R_4 = Standard non-inductive resistance of known values.

L_1 = Self inductance to be measured.

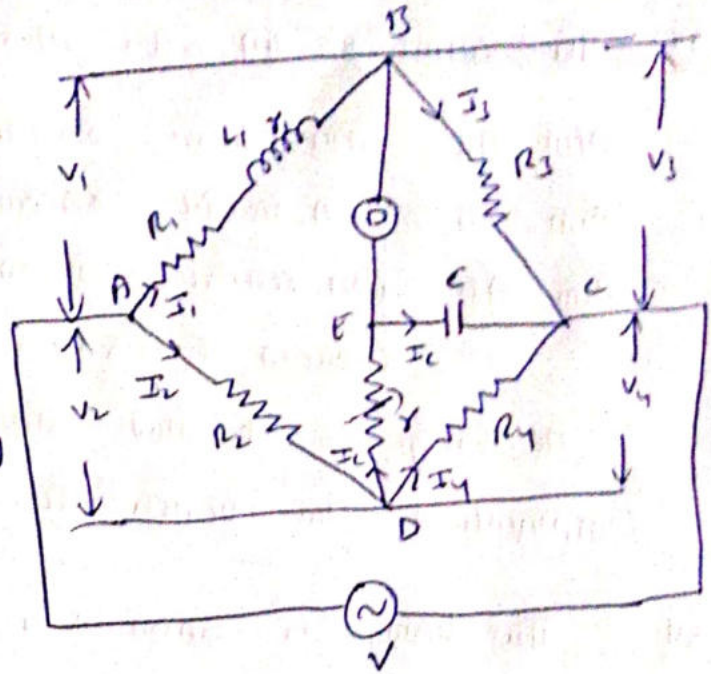
r_1 = Resistance of the unknown inductn.

Under balance condition, we have

$$I_1 = I_3 \rightarrow (1)$$

$$I_2 = I_4 + I_c \rightarrow (2)$$

$$V_3 = V_c \rightarrow (3)$$



$$V_1 = V_2 + V_Y + V_C \text{ and}$$

$$V_4 = V_Y + V_C \rightarrow (5) \quad (17)$$

From eqn. (3) we have

$$V_3 = V_X$$

$$I_3 R_3 = I_1 \times \frac{1}{j\omega C}$$

$$I_C = I_3 (j\omega C R_3)$$

$$= I_1 (j\omega C R_3)$$

($\because$ from eqn. (3))

From eqn. (4) we have,

$$V_1 = V_2 + V_Y$$

$$I_1 (\delta_1 + R_1 + j\omega L_1) = I_2 R_2 + I_C \delta$$

$$I_1 (\delta_1 + R_1 + j\omega L_1) = I_2 R_2 + I_1 (j\omega C R_3) \delta$$

$$I_1 (\delta_1 + R_1 + j\omega L_1 - j\omega C R_3 \delta) = I_2 R_2$$

$$I_2 = \frac{I_1}{R_2} (\delta_1 + R_1 + j\omega L_1 - j\omega C R_3 \delta) \rightarrow (6)$$

From eqn. (5) we have

$$V_4 = V_Y + V_C$$

$$I_4 R_4 = I_C \delta + I_C \times \frac{1}{j\omega C}$$

$$(I_2 - I_C) R_4 = I_C \left(\delta + \frac{1}{j\omega C} \right)$$

($\because I_2 = I_4 + I_C$)

$$I_2 R_4 = I_C \left(\delta + R_4 + \frac{1}{j\omega C} \right)$$

$$I_2 R_4 = I_1 (j\omega C R_3) \left(\delta + R_4 + \frac{1}{j\omega C} \right)$$

$$I_2 = I_1 \left[\frac{j\omega C R_3 \delta}{R_4} + \frac{j\omega C R_3 R_4}{R_4} + \frac{R_3}{R_4} \right]$$

$$I_2 = I_1 \left[\frac{j\omega C R_3 \delta}{R_4} + j\omega C R_3 + \frac{R_3}{R_4} \right] \rightarrow (7)$$

From Equation (6) and (7)

$$\frac{V_1}{R_2} (\delta_1 + R_1 + j\omega L_1 - j\omega C R_3 \delta) = \frac{V_1}{R_4} \left(\frac{j\omega C R_3 \delta}{R_4} + j\omega C R_3 + \frac{R_3}{R_4} \right)$$
$$\delta_1 + R_1 + j\omega (L_1 - C R_3 \delta) = \frac{R_2 R_3}{R_4} + j\omega \left(C R_2 R_3 + \frac{C R_2 R_3 \delta}{R_4} \right)$$

Equating real parts.

$$\delta_1 + R_1 = \frac{R_2 R_3}{R_4}$$

$$\left[R_1 = \frac{R_2 R_3}{R_4} - \delta_1 \right] \quad \text{and}$$

Equating imaginary parts.

$$L_1 - C R_3 \delta = C R_2 R_3 + \frac{C R_2 R_3 \delta}{R_4}$$

$$L_1 = C R_3 \delta + C R_2 R_3 + \frac{C R_2 R_3 \delta}{R_4}$$

$$L_1 = C R_3 \left[\delta + R_2 + \frac{R_2 \delta}{R_4} \right]$$

$$L_1 = \frac{C R_3}{R_4} \left[R_2 R_4 + \delta R_4 + R_2 \delta \right]$$

$$\left[L_1 = \frac{C R_3}{R_4} \left[R_2 R_4 + \delta (R_2 + R_4) \right] \right]$$

Advantages:-

- ✓ Instead of a variable capacitor, a fixed capacitor can be used. This makes the bridge cheaper than Maxwell's bridge.
- ✓ Determination of unknown capacitance in terms of known inductance is also possible.

Disadvantages:-

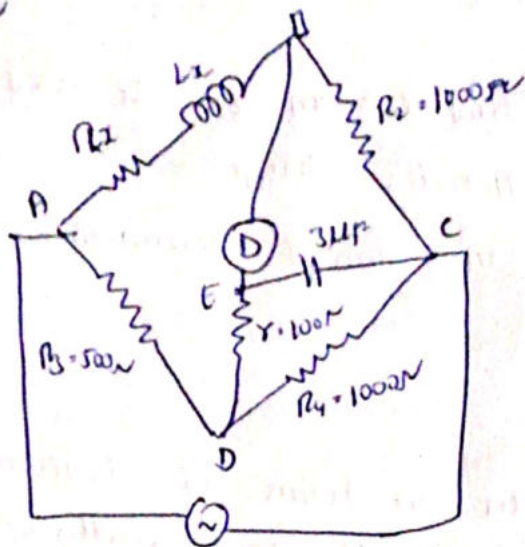
- ✓ The Anderson's bridge is more complex in terms of circuit connections and computations than compared to Maxwell's bridge due to the increase in the number of components used in the circuit.
- ✓ The balance equation calculations are also complicated than Maxwell's bridge calculations.

Ex. An Anderson a.c bridge is as follows,

Arm AB :	Unknown inductance	R_x and L_x
Arm BC :	Non reactive resistance	$R_2 = 1000\Omega$
Arm CD :	Non-reactive resistance	$R_4 = 1000\Omega$
Arm DA :	Non-reactive resistance	$R_3 = 500\Omega$
Arm DE :	Resistance	$r = 100\Omega$
Arm EB :	Detector and a.c Supply	b/w AC
Arm EC :	Capacitor	$C = 3\mu F$

State the expressions for L_x and R_x and find the values of them for given values of elements.

Sol,



$$R_x = \frac{R_2 R_3}{R_4}$$

$$\Rightarrow \frac{1000 \times 500}{1000} = 500 \Omega$$

$$L_x = \frac{C R_3}{R_4} [R_2 R_4 + R_1 (R_2 + R_4)]$$

$$\Rightarrow \frac{3 \times 10^{-6} \times 500}{1000} [1000 \times 1000 + 100 (2000)]$$

$$\Rightarrow 1.8 H$$

* Hay's Bridge :-

→ Hay's bridge is the electrical circuit used for the measurement of self-inductance.

→ It is an alternating current bridge similar to Maxwell's bridge with small modifications.

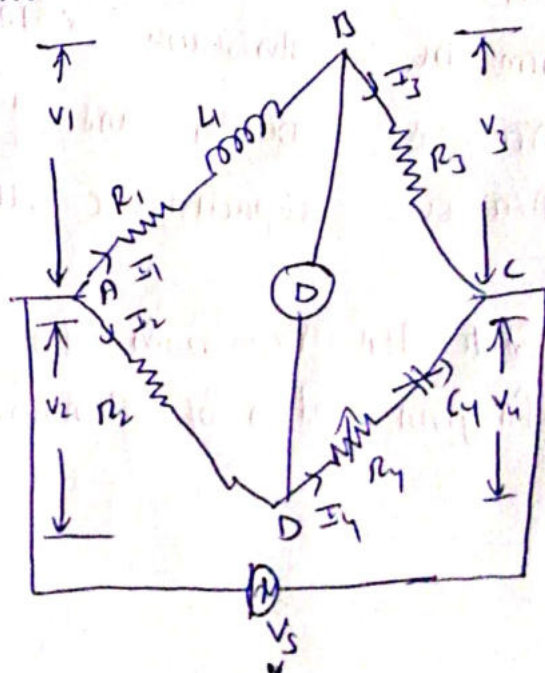
→ The main difference b/w Hay's bridge and Maxwell's bridge is that Hay's bridge employs a resistor in series with the standard capacitor, whereas Maxwell's bridge uses a resistor in parallel with the standard capacitor.

Construction :-

✓ The Hay's bridge is a modified form of Maxwell's inductance-capacitance bridge.

✓ It measures inductance by comparing it with a standard variable capacitance.

✓ The circuit diagram of Hay's bridge is shown in fig.



✓ It consists of an inductor with an inductance L_1 and resistance R_1 in arm AB and non-inductive standard resistances R_2 and R_3 in arms AD and BC respectively, and a known variable standard capacitance C_4 in series with known non-inductive variable standard resistance R_4 in arm CD.

Operation and theory of Hay's bridge

The bridge can be balanced by adjusting the values of R_4 and C_4 . From the bridge circuit.

$$Z_1 = R_1 + j\omega L_1 \quad Z_2 = R_2 \quad Z_3 = R_3 \quad Z_4 = R_4 - j/\omega C_4$$

Under balanced condition,

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + j\omega L_1) \left(R_4 - \frac{j}{\omega C_4} \right) = R_3 R_2$$

$$R_1 R_4 - j \frac{R_4}{\omega C_4} + j\omega L_1 R_4 + \frac{\omega L_1}{\omega C_4} = R_3 R_2$$

$$R_1 R_4 + \frac{L_1}{C_4} + j \left(\omega L_1 R_4 - \frac{R_4}{\omega C_4} \right) = R_3 R_2$$

Equating real parts, and imaginary parts.

$$R_1 R_4 + \frac{L_1}{C_4} = R_3 R_2 \rightarrow (1)$$

$$\omega L_1 R_4 - \frac{R_4}{\omega C_4} = 0$$

$$\omega L_1 R_4 = \frac{R_4}{\omega C_4}$$

$$R_1 = \omega^2 L_1 C_4 R_4 \rightarrow (2)$$

Substituting (2) in (1)

$$(\omega^2 R_4 L_1 C_4) R_4 + \frac{L_1}{C_4} = R_2 R_3$$

$$L_1 \left[\omega^2 R_4^2 C_4 + \frac{1}{C_4} \right] = R_2 R_3$$

$$L_1 \left[\frac{1 + \omega^2 R_4^2 C_4^2}{C_4} \right] = R_2 R_3$$

$$\left[L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 R_4^2 C_4^2} \right] \rightarrow (3)$$

Substituting (3) in (2)

$$R_1 = \omega^2 R_4 \left(\frac{R_2 R_3 R_4}{1 + \omega^2 R_4^2 C_4^2} \right) C_4$$

$$\left[R_1 = \frac{\omega^2 R_2 R_3 R_4 C_4^2}{1 + \omega^2 R_4^2 C_4^2} \right] \rightarrow (4)$$

Now the quality factor of an inductor is given by,

$$Q = \frac{\omega L_1}{R_1}$$

$$Q = \frac{L_1 \left(\frac{R_2 R_3 C_4}{1 + \omega^2 R_4^2 C_4^2} \right)}{\frac{\omega^2 R_2 R_3 R_4 C_4^2}{1 + \omega^2 R_4^2 C_4^2}} \Rightarrow \frac{R_2 R_3 C_4}{\omega R_2 R_3 R_4 C_4^2}$$

$$Q = \frac{1}{\omega R_4 C_4} \Rightarrow \omega R_4 C_4 = \frac{1}{Q}$$

$$\Rightarrow \omega^2 R_4^2 C_4^2 = \frac{1}{Q^2}$$

From Eqn. (3)

$$L_1 = \frac{R_2 R_3 C_4}{1 + \frac{1}{Q^2}}$$

For high Q coils i.e., $Q > 10$, $1/Q^2$ is almost ²¹ negligible.

Hence the above Eqn. reduces to

$$[L_1 = R_2 R_3 C_4]$$

✓ From the above Equations, we can say that for high Q coils the expression for L_1 is free from the frequency term.

✓ For low Q coils $1/Q^2$ cannot be neglected and hence to find L_1 the frequency of source is to be accurately known.

✓ Therefore the bridge suits only for the measurements of inductance of high Q coils.

Advantages:-

→ The expression obtained for the Q -factor of the coil using Hay's bridge is not complicated.

$$Q\text{-factor} = \frac{1}{\omega R_4 C_4}$$

→ From the above Equation, it can be seen that the resistance R_4 is inversely proportional to Q -factor. Thus for high Q -factor the R_4 should be small. Hence this bridge requires low value of R_4 .

Disadvantages:-

→ Not useful for measuring coils having a Q -factor less than 10.

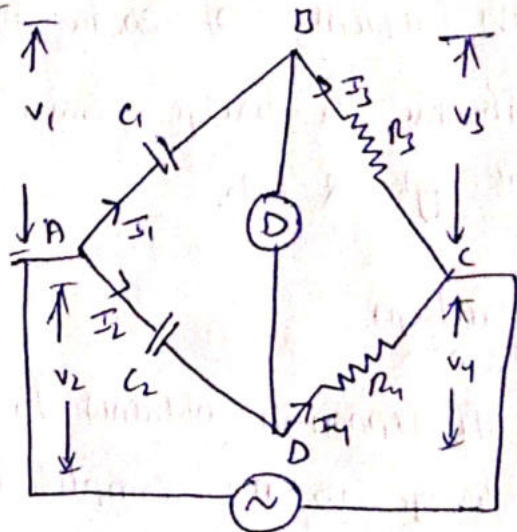
* Measurement of Capacitance *

* DeSauty's bridge *

DeSauty's bridge is the simplest ac bridge circuit used for the measurement of unknown capacitance. The bridge gives a fair degree of accuracy for measuring capacitance over a wide range. It can be also used for comparing two capacitance.

Construction & Theory of DeSauty's bridge

- The bridge consists of four arms.
- Two capacitors C_1 and C_2 are connected between AB and AD respectively.
- The branches BC and CD consist of resistors R_3 and R_4 respectively.
- An AC source is connected between terminals A and C. The circuit diagram of the bridge is shown in fig.
- A null indicator is connected across terminals B and D, which indicates null deflection when the bridge is balanced.



Let

C_1 = unknown capacitance

C_2 = known standard capacitance.

R_3, R_4 = known standard non-inductive resistance.

From the above fig, the impedances in each arm is given as

$$Z_1 = \frac{1}{j\omega C_1}$$

$$Z_2 = \frac{1}{j\omega C_2}$$

$$Z_3 = R_3, \quad Z_4 = R_4$$

When the bridge is balanced, we have

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left(\frac{1}{j\omega C_1}\right) \times R_4 = \left(\frac{1}{j\omega C_2}\right) R_3$$

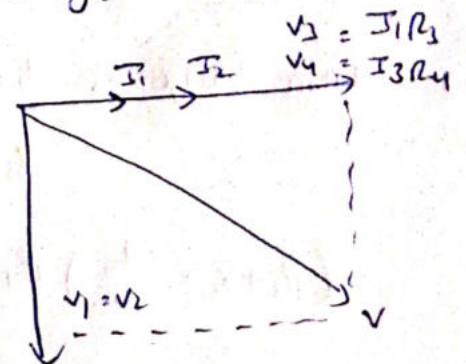
$$\frac{R_4}{C_1} = \frac{R_3}{C_2}$$

$$\left[C_1 = \frac{R_4}{R_3} \times C_2 \right]$$

- It is the easiest method for the measurement of capacitance as it has a simple circuit and only one variable element.
- In order to bring the bridge into a balanced condition, either R_3 or R_4 can be chosen as a variable element.
- The fig shows the phasor diagram for DeSauty's bridge.

→ This method is useful only to measure the capacitance of lossless capacitors.

→ If either of (or both) the capacitors is imperfect, then the balance can never be achieved by varying R_3 or R_4 .

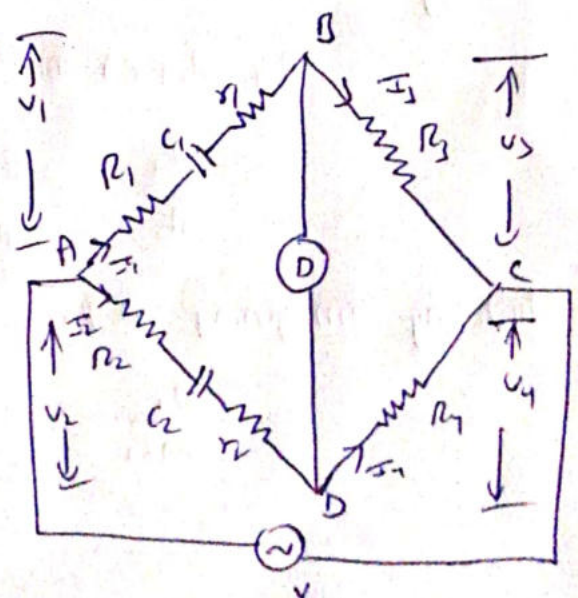


→ This drawback can be overcome by modified DeSauty's bridge.

* Modified DeSauty's bridge:

→ The modified DeSauty's bridge is used for the measurement of unknown imperfect capacitance.

→ It is the modification of DeSauty's bridge. Imperfect capacitance is those which contain dielectric losses.



→ In modified Desauty's bridge, the two capacitors C_1 and C_2 are connected in series with R_1 and R_2 .

→ To measure capacitance with dielectric loss another two resistors r_1 and r_2 are connected to the two capacitors that give the loss component of their respective capacitors.

Let C_1 - unknown imperfect capacitance.

C_2 = known standard capacitance.

R_1, R_2, R_3, R_4 = known non-inductive resistors.

r_1, r_2 = resistance having loss component of two capacitors.

→ From the above fig, the impedances in each arm is given as

$$Z_1 = R_1 + r_1 + \frac{1}{j\omega C_1} \quad Z_2 = R_2 + r_2 + \frac{1}{j\omega C_2} \quad Z_3 = R_3, \quad Z_4 = R_4$$

→ When bridge is balanced, we have

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left(R_1 + r_1 + \frac{1}{j\omega C_1}\right) R_4 = \left(R_2 + r_2 + \frac{1}{j\omega C_2}\right) R_3$$

$$R_4 (R_1 + r_1) + \frac{R_4}{j\omega C_1} = R_3 (R_2 + r_2) + \frac{R_3}{j\omega C_2}$$

Equating real parts,

$$R_4 (R_1 + r_1) = R_3 (R_2 + r_2)$$

$$\frac{R_4}{R_3} = \frac{R_2 + r_2}{R_1 + r_1} \rightarrow \textcircled{1}$$

Equating imaginary parts.

$$\frac{R_4}{j\omega C_1} = \frac{R_3}{j\omega C_2} \rightarrow \frac{R_4}{R_3} = \frac{C_1}{C_2} \rightarrow \textcircled{2}$$

Comparing Eq. (1) & (2) we have

(3)

$$\frac{C_1}{C_2} = \frac{R_2}{R_3} = \frac{R_2 + r_2}{R_1 + r_1} \rightarrow (3)$$

→ The balanced condition in modified Deauty's bridge is determined by changing the resistances R_1, R_2, R_3 and R_4 in the circuit. The dissipation factor 'D' of the two capacitors are,

$$\text{Dissipating factor of } C_1 = D_1 = \tan \delta_1 = \omega C_1 r_1$$

$$\text{Dissipating factor of } C_2 = D_2 = \tan \delta_2 = \omega C_2 r_2$$

$$\text{i.e., } D_1 = \omega C_1 r_1 \text{ and } D_2 = \omega C_2 r_2$$

→ Simplifying Eq. (3) we get

$$\frac{C_1}{C_2} = \frac{R_2 + r_2}{R_1 + r_1}$$

$$C_1 R_1 + C_1 r_1 = C_2 R_2 + C_2 r_2$$

multiplying with 'ω' on both sides

$$\omega [C_1 R_1 + C_1 r_1] = \omega [C_2 R_2 + C_2 r_2]$$

$$\omega C_1 R_1 + D_1 = \omega C_2 R_2 + D_2$$

$$D_2 - D_1 = \omega [C_1 R_1 - C_2 R_2] \rightarrow (4)$$

Also from Eq. (3)

$$C_1 = \frac{R_4 C_2}{R_3}$$

Substituting C_1 in Eq. (4)

$$D_2 - D_1 = \omega \left[\frac{R_4 C_2}{R_3} R_1 - C_2 R_2 \right]$$

$$D_2 - D_1 = \omega C_2 \left[\frac{R_1 R_4}{R_3} - R_2 \right]$$

→ In above eq., if one of the dissipation factor is given, the other dissipation factor can be determined.

→ One of the disadvantage of this method is that the dissipation factor is not precise and accurate because the differences in their resistance quantities are very small.

* Schering Bridge :-

Schering bridge is an ac bridge used for the measurement of capacitance, dielectric loss and power factor of an unknown capacitor. It is the most popular and widely used method for measuring capacitance. The principle of operation is similar to other ac bridges i.e., balancing the bridge and comparing the unknown value with a known value.

a) Low voltage Schering bridge :-

→ The circuit connection of a low voltage Schering bridge is as shown in fig.

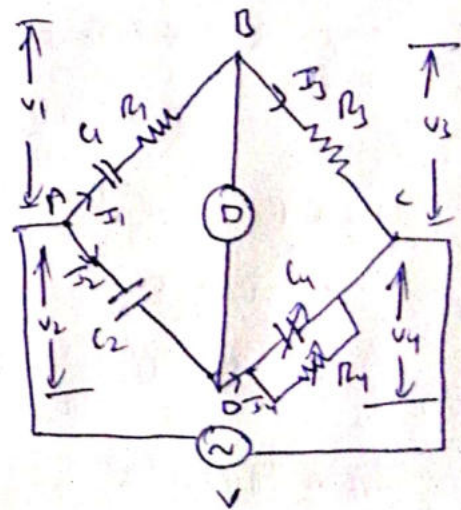
→ The arm AB consists of capacitor C_1 in series with resistor R_1 .

→ The resistor R_1 represents the dielectric loss component of the capacitor C_1 .

→ The capacitor C_2 is the standard capacitor and C_4 is variable one.

→ The resistor R_3 is a non-inductive resistance and R_4 is a variable non inductive resistance connected in parallel with a capacitor C_4 .

→ The supply is connected b/w nodes A and C while the detector is connected b/w B and D.



→ From the above fig, the impedances of the arms of the bridge are,

$$Z_1 = R_1 + \frac{1}{j\omega C_1} \quad Z_2 = \frac{1}{j\omega C_2} \quad Z_3 = R_3 \quad Z_4 = \frac{R_4}{1 + j\omega R_4 C_4}$$

→ under balanced condition of the bridge, we have,

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left(R_1 + \frac{1}{j\omega C_1}\right) \left(\frac{R_4}{1 + j\omega R_4 C_4}\right) = \frac{1}{j\omega C_2} \times R_3$$

$$\frac{(j\omega C_1 R_1 + 1)}{j\omega C_1} \times \frac{R_4}{1 + j\omega R_4 C_4} = \frac{R_3}{j\omega C_2}$$

$$\frac{j\omega C_1 R_1 R_4 + R_4}{(j\omega C_1)(1 + j\omega R_4 C_4)} = \frac{R_3}{j\omega C_2}$$

$$\frac{j\omega C_1 R_1 R_4 + R_4}{(j\omega C_1)(1 + j\omega R_4 C_4)} = \frac{R_3}{j\omega C_2}$$

$$\frac{j\omega C_1 R_1 R_4}{j\omega C_1 + j\omega^2 C_1 R_4 C_4} + \frac{R_4}{j\omega C_1 + j\omega^2 C_1 R_4 C_4} = \frac{R_3}{j\omega C_2}$$

$$\frac{j\omega C_1 R_1 R_4}{j\omega (C_1 + j\omega C_1 R_4 C_4)} + \frac{R_4}{j\omega (C_1 + j\omega C_1 R_4 C_4)} = \frac{R_3}{j\omega C_2}$$

$$j\omega C_1 R_1 R_4 + R_4 = \frac{R_3}{j\omega C_2} \times j\omega (C_1 + j\omega C_1 R_4 C_4)$$

$$j\omega C_1 R_1 R_4 + R_4 = \frac{R_3}{C_2} (C_1 + j\omega C_1 R_4 C_4)$$

$$j\omega C_1 R_1 R_4 + R_4 = \frac{C_1 R_3}{C_2} + \frac{j\omega C_1 R_3 R_4 C_4}{C_2}$$

Equating real and imaginary parts

$$R_4 = \frac{C_1}{C_2} \times R_3$$

$$C_2 R_4 = C_1 R_3$$

$$\left[C_1 = \frac{R_4}{R_3} \times C_2 \right]$$

$$\cancel{C_1 R_4} = \frac{\cancel{C_1} R_4 R_3}{C_2} \quad \Rightarrow \quad \left[R_1 = \frac{C_4}{C_2} R_3 \right]$$

→ now the dissipation factor of the capacitor under measurement is

$$D_1 = \tan \delta$$

$$= \omega C_1 R_1$$

$$\Rightarrow \omega \left(\frac{R_4}{R_3} \times \frac{1}{\omega} \right) \left(\frac{C_4}{C_2} R_3 \right)$$

$$\left[D \Rightarrow \omega C_4 R_1 \right]$$

→ normally C_2 and C_4 are kept constant and balance is obtained by varying R_3 and C_4 , so that the dissipation factor can be directly measured in terms of C_4 .

disadvantages of L.V. Schering bridge:-

- ✓ The detector used is not so sensitive
- ✓ It is quite difficult to obtain a balanced condition.
- ✓ Errors are present in the measurement of small capacitances using a low voltage Schering bridge.

b) High voltage Schering bridge

(15)

→ The H.V. Schering bridge is mainly used for the measurement of small capacitance by using high voltage and high frequency.

→ H.V. Schering bridge is the most suitable method for measurement at high voltage because of following reasons.

- Results are very accurate.
- Detector is very sensitive.
- The errors due to earth capacitance are eliminated using Wagner's earth device.
- The circuit provides greater security for the operator.
- Low power loss

→ The fig shows H.V. Schering bridge.

→ It consists of a step-up transformer that supplies the required high voltage to the bridge from a normal AC supply.

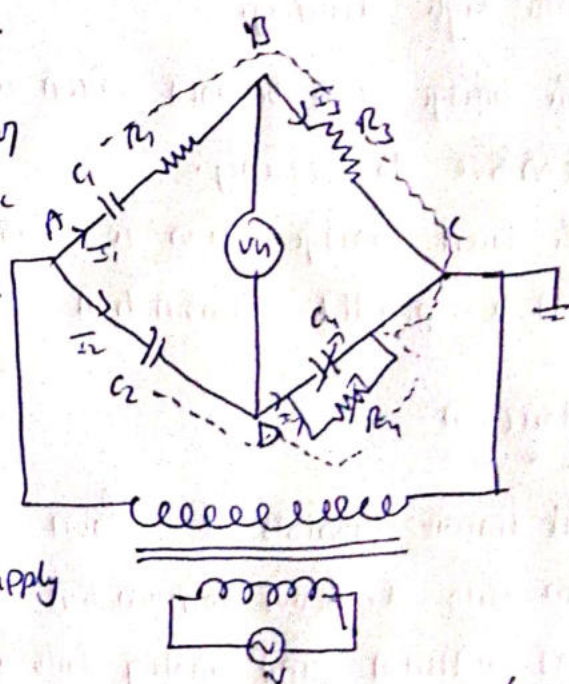
→ The capacitors C_1 and C_2 in the arms AB and AD are specially designed to withstand high voltages and have high impedances at normal supply frequency.

→ The impedances of arms BC and CD are very low when compared with that of arms AB and AD.

→ The currents passing through the bridge will be very small due to high impedance, which results in low power consumption.

→ This makes the bridge to use a sensitive detector.

→ The voltage drops in the arms BC and CD will be very small, as the major part of voltage drops in arms AB and AD due to higher impedances.



→ Point c is earthed, as the controlling elements are present in arms R_x and C_D , it assures the safety of operating person.

Advantages:-

- ✓ used to find the value of capacitance across the bridge.
- ✓ Low cost
- ✓ The balanced S_{eq} does not depend on f_{ac} .

* Wien's bridge:-

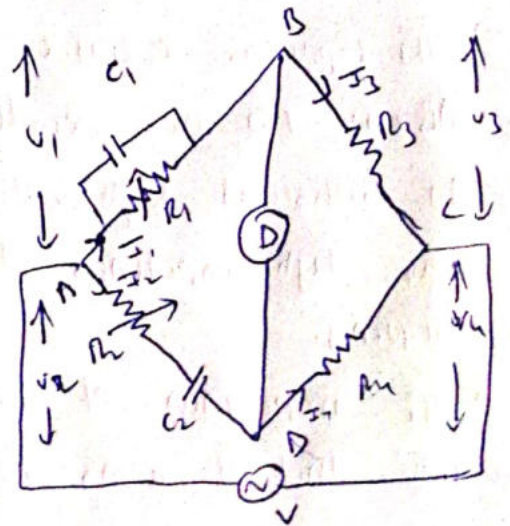
- The Wien's bridge is an AC electrical circuit widely used for measuring of f_{ac} and can also be used for the measurement of capacitance with high accuracy.
- The bridge can be used even at high voltages but the circuit is sensitive to frequency.
- The Wien bridge has a combination of R-C Series in one arm and a parallel combination in another arm.

Construction:-

- ✓ The circuit consists of four arms, one arm with a series combination of resistor and capacitor and another with a parallel combination.

✓ The two other arms comprise a resistance.

✓ The fig shows the circuit diagram of Wien's bridge.



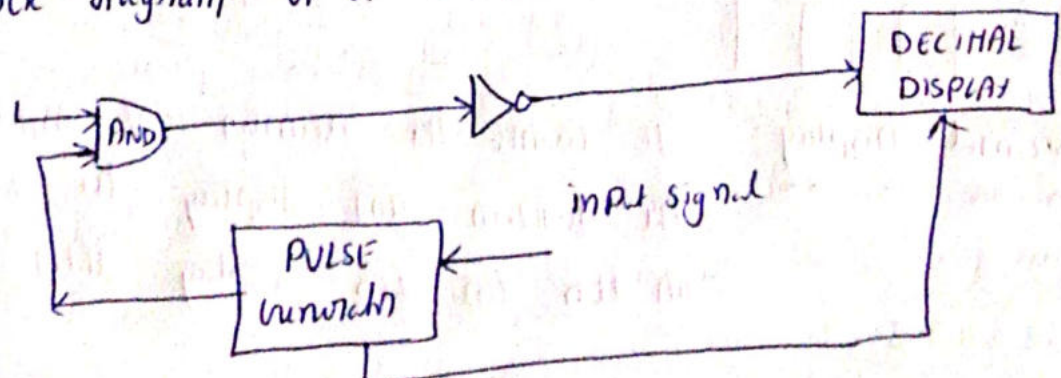
SMART & DIGITAL METER

* Digital voltmeter:

- ✓ Voltmeter is an electrical measuring instrument used to measure the potential difference between two points.
- ✓ The voltage to be measured may be AC (or) DC.
- ✓ Two types of voltmeters are available for the purpose of voltage measurement i.e., Analog and Digital.
- ✓ Analog voltmeters generally contain a dial with a needle moving over it according to the measure and hence displaying the value of the same.
- ✓ With time analog voltmeters are replaced by "digital voltmeters" due to some advantages associated with digital systems.
- ✓ Although digital voltmeters do not fully replace analog voltmeters, still there are many places where analog voltmeters are preferred over digital voltmeters.
- ✓ Digital voltmeters display the value of AC (or) DC voltage being measured directly as discrete numerical instead of a pointer deflection on a continuous scale as in analog instruments.

Working principle of Digital Voltmeter

→ The block diagram of a simple digital voltmeter is shown in below.



A) Input signal
~~~~~

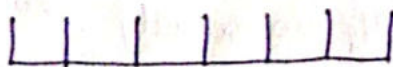
It is basically the signal i.e., voltage to be measured.

b) Pulse generator  
~~~~~

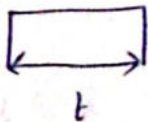
Actually it is a voltage source. It uses digital, analog or both techniques to generate a rectangular pulse. The width and frequency of the rectangular pulse is controlled by the digital circuitry inside the generator while amplitude and rise and fall time is controlled by analog circuitry.

c) AND gate:-
~~~~~

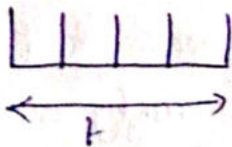
It gives high o/p only when both the i/p's are high. When a train pulse is fed to it along with rectangular pulse, it provides us an o/p having train pulses with duration as same as the rectangular pulse from the pulse generator.



⇒ Train Pulse.



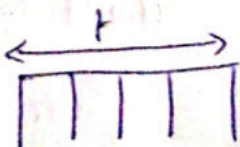
⇒ Rectangular Pulse.



⇒ o/p of AND gate.

d) NOT gate:-  
~~~~~

It inverts the o/p of AND gate.



⇒ o/p of NOT gate.

e) Decimal Display:-
~~~~~

It counts the number of impulses and hence the duration and display the value of voltage on LED (or) LED display after calibrating it.



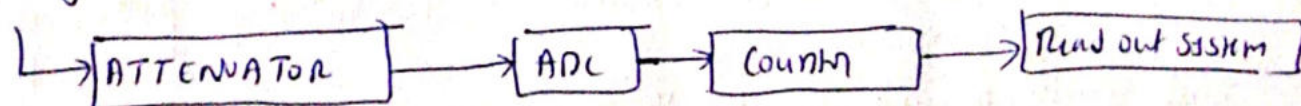
Working of a digital voltmeter :-

2)

- unknown voltage signal is fed to the pulse generator which generates a pulse whose width is proportional to the i/p signal.
- O/p of pulse generator is fed to one leg of the AND gate.
- The i/p signal to the other leg of the AND gate is a train of pulses.
- O/p of AND gate is positive triggered train of duration same as the width of the pulse generated by the pulse generator.
- This positive triggered train is fed to the inverter which converts it into a negative triggered train.
- O/p of the inverter is fed to the counter which counts the number of triggers in the duration which is proportional to the i/p signal i.e., voltage under measurement.
- Thus, counter can be calibrated to indicate voltage in volts directly.

The working of digital voltmeter that it is nothing but an analog to digital converter which converts an analog signal into a train of pulses, the no. of which is proportional to the i/p signal. So a digital voltmeter can be made by using any one of the A/D conversion methods.

i/p signal



on the basis of A/D conversion method used digital voltmeters can be classified as,



- i) Ramp type digital voltmeter.
- ii) Integrating type digital voltmeter.
- iii) potentiometric type digital voltmeter.
- iv) Successive approximation type digital voltmeter.
- v) continuous balance type digital voltmeter.

#### Advantages :-

- Reading of DVMS is easy as it eliminates observational errors in measurement committed by operators.
- Parallel error and approximation error is entirely eliminated.
- Reading can be taken very fast.
- Output can be fed to memory devices for storage and future computations.
- Versatile and accurate.
- Compact and cheap.
- Low power requirements.

#### Disadvantages :-

- It gives some extra features are expensive.
- Speed of operation is limited due to the digitizing circuit.
- It is usually very hard to spot transient voltage spikes etc.

### \* Resolution :-

The resolution of DVM is determined by the number of full scale active digits used in DVM which is given by

$$R = \frac{1}{10^N}$$

Where 'N' represents number of full digits.

Ex: For  $N=3$ ,  $R = \frac{1}{10^3} \Rightarrow 0.001$

### \* Sensitivity :-

Sensitivity is the smallest change in input which a digital meter is able to detect. Hence it is the full scale value of the lowest voltage range multiplied by the resolution of the DVM.

$$S = (FS)_{\min} * R$$

Where  $R$  = resolution

$(FS)_{\min}$  = lowest full scale value of the meter.

NOTE: According to the display of DVM, the various DVM's are,

- i) N digit display DVM
- ii)  $N\frac{1}{2}$  digit display DVM
- iii)  $N\frac{3}{4}$  digit display DVM
- iv)  $N\frac{5}{6}$  digit display DVM.

Where N represents number of full digits in display and  $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}$  digit are used for extending the count range of N digit DVM.



Ex: A  $4\frac{1}{2}$  digit voltmeter is used for voltage measurement. Then

i) Find its resolution

ii) How would 11.87 V be displayed on 10V range.

iii) How would 0.5573 V be displayed on 1V and 10V range.

Sol: a) No. of full digits  $N = 4$

i) Resolution  $R = \frac{1}{10^N} \Rightarrow 0.0001$

ii) Sensitivity of DVM on 1V range is

$$S = (f.s.)_{\max} \times R \Rightarrow S = 1V \times 0.0001 \Rightarrow 0.0001$$

Sensitivity of DVM on 10V range is

$$S = 10V \times 0.0001 \Rightarrow 0.001$$

11.87V can be displayed by  $4\frac{1}{2}$  digit DVM on 10V range is

$$11.870V$$

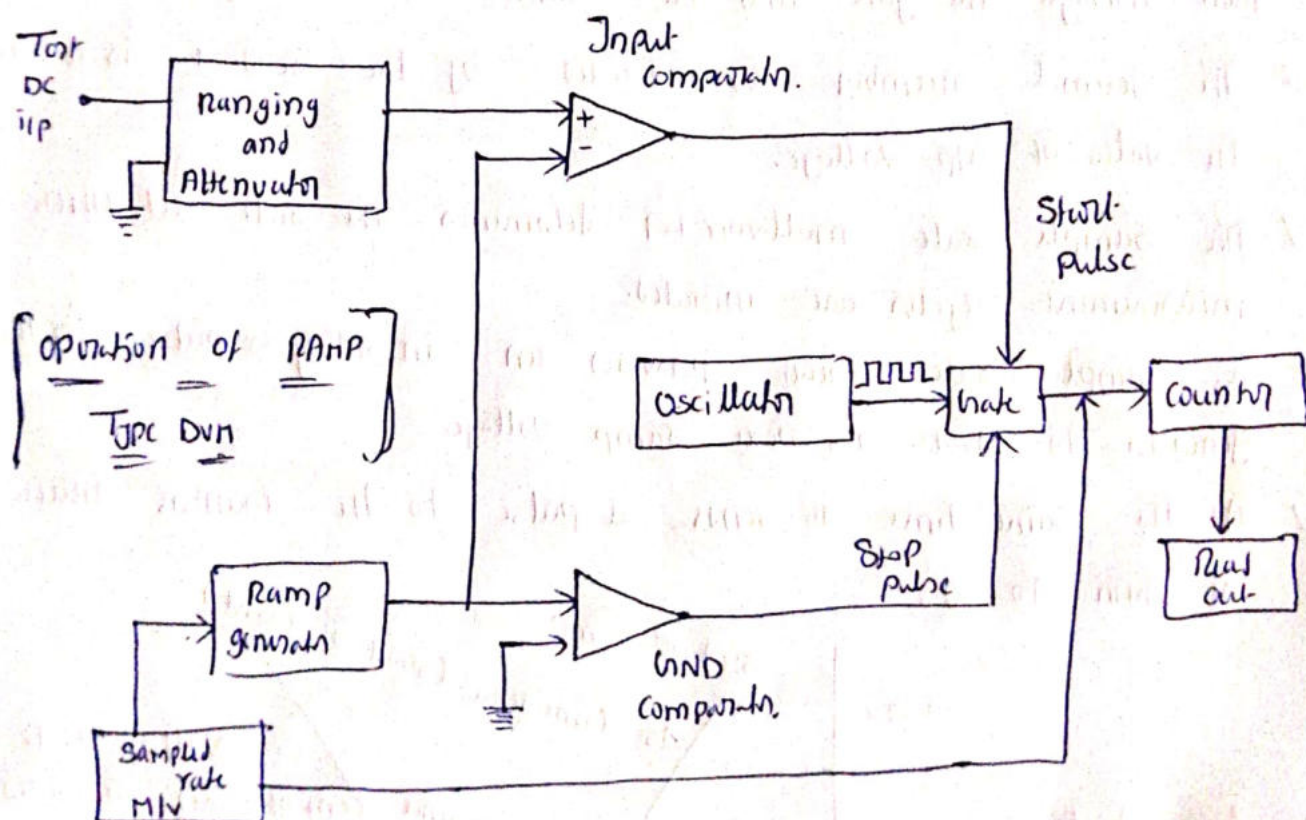
iii) 0.5573V can be displayed by  $4\frac{1}{2}$  digit DVM on 1V range

$$\text{as } 0.5573$$

$$\text{on 10V range as } 0.557V$$

## \* Ramp Type Digital Voltmeter

- The operating principle of a ramp type digital voltmeter is to measure the time that a linear ramp voltage takes to change from level of input voltage to zero voltage (or vice versa).
- This time interval is measured with an electronic time interval counter and the count is displayed as a number of digits on electronic indicating tubes of the o/p readout of the voltmeter.
- The conversion of a voltage value of a time interval is shown in the timing diagram given below.

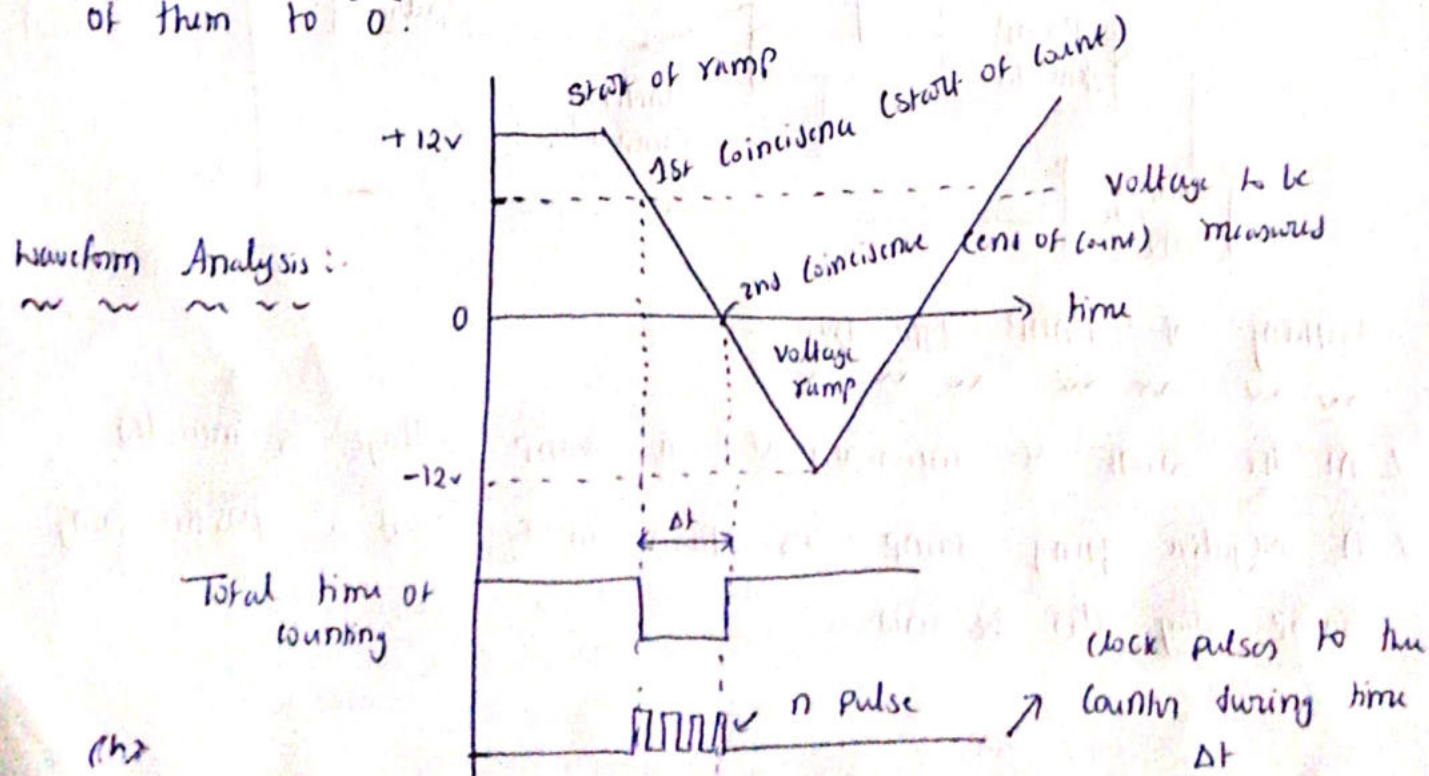


Working of RAMP Type DVM :-

- ✓ At the start of measurement a ramp voltage is initiated.
- ✓ A negative going ramp is shown in Fig.1 but a positive going ramp may also be used.



- ✓ The ramp voltage value is continuously compared with the voltage being measured (unknown voltage).
- ✓ At the instant the value of ramp voltage is equal to that of unknown voltage.
- ✓ The ramp voltage continues to decrease till it reaches ground level.
- ✓ At this instant another comparator called ground comparator generates a pulse (stop pulse) and closes the gate.
- ✓ The time elapsed between opening and closing of the gate is  $T$  as indicated in Fig.
- ✓ During this time interval pulses from a clock pulse generator (oscillator) pass through the gate and are counted and displayed.
- ✓ The decimal number as indicated by the readout is a measure of the value of i/p voltage.
- ✓ The sample rate multivibrator determines the rate at which the measurement cycles are initiated.
- ✓ The sample rate circuit provides an initiating pulse for the ramp generator to start its next ramp voltage.
- ✓ At the same time it sends a pulse to the counters which set all of them to '0'.

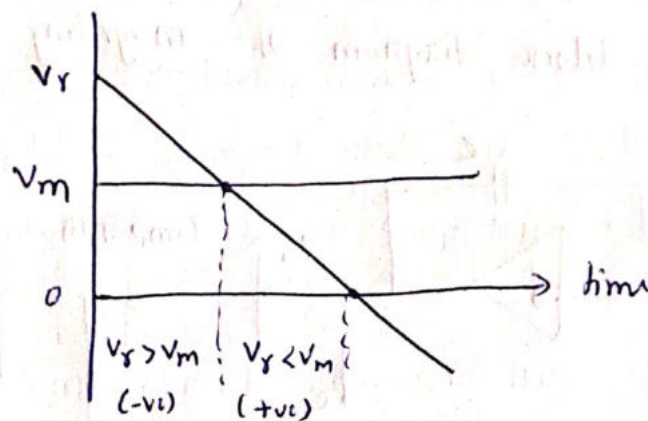




a) At input comparator:-  
~ ~ ~ ~ ~

An unknown voltage  $V_m$  is given to the i/p of non-inverting terminal. When the i/p at inverting terminal of op-amp is less than  $V_m$  at the i/p comparator is positive otherwise it is negative.

When  $V_r$  decreases below  $V_m$  it produces a positive going pulse, this pulse makes the gating circuit to start and the pulse produced by the clock generator passes through the gating circuit. These pulse are counted by decade counter.



b) At ground comparator:-  
~ ~ ~ ~ ~

Ground comparator is also called as zero level detector. So when the  $V_r$  reaches zero and move to negative values, the i/p at inverting terminal is greater than non-inverting terminal.

So o/p of this comparator produces a negative going pulse which makes the gating circuit to stop.

Advantages:-  
~ ~ ~

- The ramp technique circuit is easy to design and its cost is low.
- Also, the o/p pulse can be transmitted over long power lines.

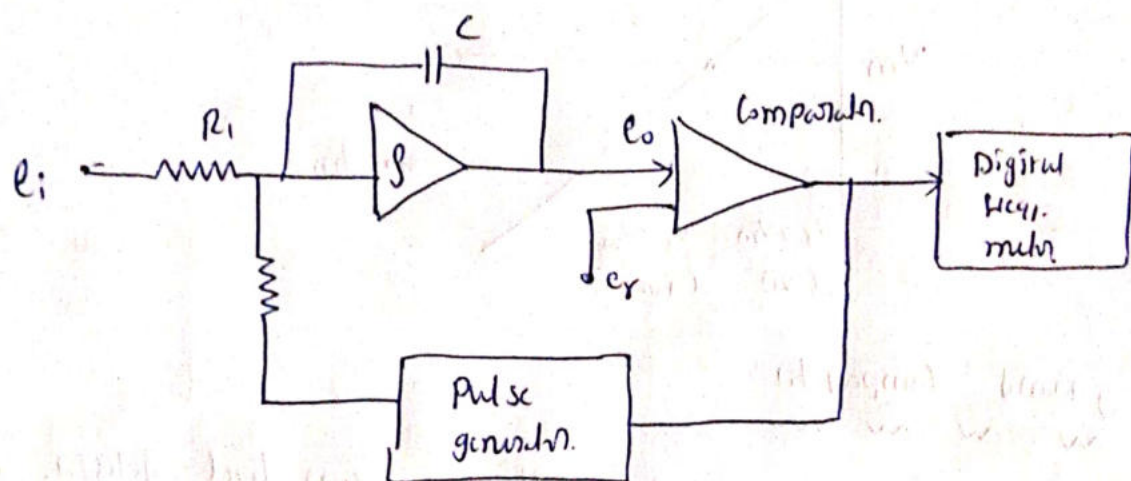
Disadvantages:-  
~ ~ ~

- Large errors are possible when noise is superimposed on the i/p signals.
- O/p filters are required with this type of converter.



## \* Integrating type digital voltmeter

- The integrating type digital voltmeter measures the true average value of the input voltage over a fixed measuring period.
- This voltmeter employs an integration technique which uses a voltage to frequency conversion.
- The voltage to frequency (V/F) converter functions as a feedback control system which governs the rate of pulse generation in proportion to the magnitude of i/p voltage.
- The fig shows block diagram of integrating type digital voltmeter



Working:

- ✓ The i/p voltage produces a charging current  $e_i/R_i$  that charges the capacitor 'C' to the reference voltage  $e_r$ .
- ✓ When  $e_r$  is reached, the comparator changes state, so as to trigger the precision pulse generator.
- ✓ The pulse generator produces a pulse of precision charge content, that rapidly discharges the capacitor.
- ✓ The rate of charging and discharging produces a signal frequency that is directly proportional to  $e_i$ .

✓ During charging

$$e_0 = -\frac{1}{RC} \int_0^{t_1} e_i dt \Rightarrow -e_i \frac{t_1}{RC} \rightarrow (1)$$

During discharging

$$e_0 = -\frac{1}{RC} \int_0^{t_2} e_r dt \Rightarrow -e_r \frac{t_2}{RC} \rightarrow (2)$$

Substituting eqn. (1) from eqn. (2)

$$\Rightarrow -e_r \frac{t_2}{RC} - \left( -e_i \frac{t_1}{RC} \right) = 0$$

$$\Rightarrow \frac{e_i t_1}{RC} - \frac{e_r t_2}{RC} = 0$$

$$\Rightarrow \frac{e_i t_1}{RC} = e_r \frac{t_2}{RC}$$

$$\Rightarrow \left[ e_i = e_r \frac{t_2}{t_1} \right]$$

But in this case,  $e_r$  and  $t_2$  are constants,

$$\text{Let } k_2 = e_r t_2$$

$$\text{Then } e_i = \frac{k_2}{t_1}$$

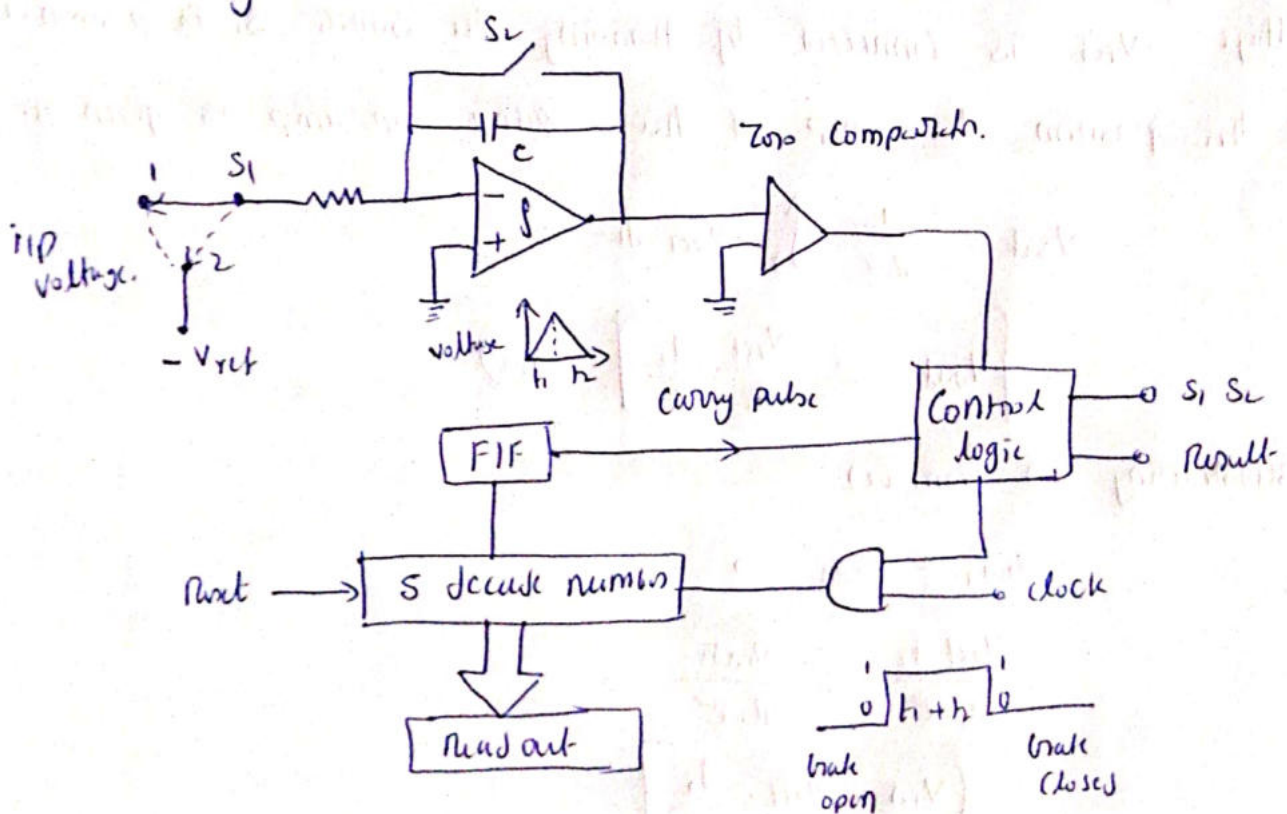
$$\Rightarrow e_i = k_2 \left( \frac{1}{t_1} \right) \Rightarrow \left[ e_i = k_2 (f) \right]$$

✓ The o/p freq. is proportional to the i/p voltage  $e_i$ .



## \* Dual Slope integrating Type digital voltmeter;

- This is the most popular method of analog to digital conversion.
- In the ramp technique, the noise can cause large errors but in dual slope method the noise is averaged out by the positive and negative ramps using the process of integration.
- The basic principle of this method is that the i/p signal is integrated for a fixed interval of time and then the same integrator is used to integrate the reference voltage with reverse slope.
- Hence the name given to the technique is dual slope integration technique.
- The block diagram of dual slope integrating type DVM is shown in the fig.



- It consists of five blocks of an op-amp used as an integrator, a Zero Comparator, clock pulse generator, a set of decimal counters and block of control logic.

→ When the switch  $S_1$  is in position 1, the capacitor  $C$  starts charging from zero level.

→ The rate of charging is proportional to the i/p voltage level. The o/p of the op-amp is given by,

$$V_{out} = -\frac{1}{R_1 C} \int_0^{t_1} V_{in} dt$$

$$\left[ V_{out} = -\frac{V_{in}}{R_1} \cdot \frac{t_1}{C} \right] \rightarrow (1)$$

Where  $t_1$  = time taken for capacitor charging.

$V_{in}$  = i/p voltage

$R_1$  = series resistance

$C$  = capacitance in feedback path

→ After the interval  $t_1$ , the i/p voltage is disconnected and a negative voltage  $-V_{ref}$  is connected by throwing the switch  $S_1$  to position-2

→ In this position, the o/p of the ~~same~~ op-amp is given by.

$$V_{out} = \frac{1}{R_1 C} \int_0^{t_2} -V_{ref} dt$$

$$\left[ V_{out} = -\frac{V_{ref}}{R_1} \cdot \frac{t_2}{C} \right] \rightarrow (2)$$

→ Subtracting (1) from (2)

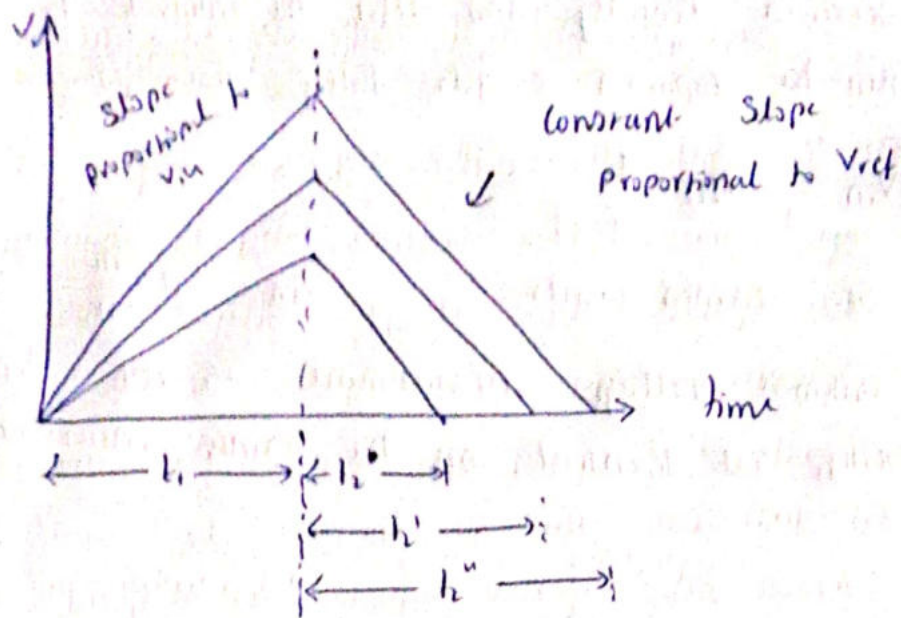
$$V_{out_1} - V_{out_2} = 0$$

$$\frac{V_{ref} t_2}{R_1 C} = \frac{V_{in} t_1}{R_1 C}$$

$$\left[ V_{in} = V_{ref} \cdot \frac{t_2}{t_1} \right]$$

→ Thus, the i/p voltage is dependent on the time periods  $t_1$  and  $t_2$  but not on the values of  $R$  and  $C$ .





- At the start of measurement, the counter is reset to zero, the o/p of the flip-flop is also zero, this is given to the control logic.
- This control sends a signal so as to close an electronic switch to position -1 and integration of the i/p voltage starts, it continues till the time period  $t_1$ .
- As the o/p of the integrator changes from its zero value, the two comparators o/p changes its state.
- This provides a signal to control logic which in turn opens the gate and the counting of the clock pulses starts.
- The counter counts the pulses and when it reaches to 9999, it generates a carry pulse and all digits go to zero.
- The flip flop o/p gets activated to the logic level 1. This activates the control logic.
- This sends a signal which changes the switch position from 1 to 2.
- Thus  $-V_{ref}$  gets connected to op-amp.
- As  $V_{ref}$  polarity is opposite, the capacitor starts discharging.
- The integrator o/p will have constant negative slope as shown in the figure.

→ The o/p decreases linearly and after the interval  $t_2$  attains zero value, when the capacitor  $C$  gets fully discharged.

$$V_{in} = V_{ref} \cdot \frac{t_2}{t_1}$$

in terms of counter counts

$$V_{in} = V_{ref} \cdot \frac{n_2}{n_1}$$

→ Thus the unknown voltage measurement is not dependent on the clock frequency, but dependent on the counts measured by the counter.

Advantages:-

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✓ Excellent noise rejection.

✓ The RC time constant does not affect the i/p voltage measurement.

✓ The capacitor is connected via an electronic switch. This capacitor is an auto zero capacitor and avoids the effect of offset voltage.

✓ High accuracy.

Disadvantages:-

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✓ Speed of operation is slow.