ANTENNA THEORY AND WAVE PROPAGATION

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Dr. K. Prasad, presently working as a faculty member of Electronics and Communication Dept, in Annamacharya University, one of the reputed technical institutions in Andhra Pradesh. He received his B.Tech in Electronics and Communication engineering from JNTU Hyderabad, and M.Tech from JNTU Anantapur. PhD from Sri satya Sai Univeristy of Technology & Medical sciences, Bhopal.

Dr. Kasigari Prasad is a distinguished academician, researcher, writer, Graphologist, and motivational speaker from India, with a versatile career that bridges science, education, and life skills. With a strong foundation in Electronics and Communication Engineering, he has earned his B.Tech, M.Tech, and Ph.D., and is presently serving as an Associate Professor in the Department of ECE. Over the years, he has guided students through both theoretical and practical aspects of engineering, especially in areas like Antennas, Wave Propagation, Electromagnetic Theory, Analog & Digital Communication, and Electronic Devices & Circuits.

Alongside his engineering expertise, Dr. Prasad has built a reputation as a Life Skills Trainer and Motivational Speaker, inspiring thousands of students and youth across different institutions. In addition to teaching and training, he is deeply committed to research and innovation. He has worked on projects involving antenna design, communication systems, ham radio, IoT, and renewable energy-based applications. He has authored and is authoring books on the Indian Constitution for engineers, communication systems, and ham radio, aiming to make complex subjects simple and accessible for students. He firmly believes in Education Beyond Classrooms, combining technical knowledge with life skills to create well-rounded, capable, and value-driven individuals.

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UNIT I ANTENNA FUNDAMENTALS

CHAPTER OBJECTIVE

In this chapter we will get a thorough knowledge on what is an Antenna, what is mechanism used in antenna. How antenna radiates fields into free space. What are the different antenna parameters that are associated with antenna mechanism? How the shape of an antenna determines the performance of an antenna. What is the effective height, and antenna apertures of an antenna? How the fields oscillate from a dipole antenna. The very basics of antennas are discussed in this chapter.

CHAPTER 1
ANTENNA BASICS

INTRODUCTION

An antenna (also called Aerial, Radiator) is considered as a region of transition between a transmission line and space. Antenna radiates or couple or concentrate or direct electromagnetic energy in the desired or assigned direction. An antenna may be ***isotropic*** (also called omni-directional or non-directional) or *anisotropic* (directional).

How to choose an antenna: There is no specific rule for selecting an antenna for a particular frequency range or application. While choosing an antenna, mechanical and structural aspects are to be taken into account. These aspects include radiation pattern, gain, efficiency, impedance, frequency characteristics, shape, size, weight and look of antennas, and above all their cost.

In some applications (e.g., radars, mobiles), the same antenna may be used for transmission and reception, while in others (e.g., radars, and television) transmission and reception of signals require separate antennas which differ in shape and size and other characteristics. In principles, there is no difference in selection factors relating to transmitting and receiving antennas. The cost, shape and size, etc., make the main difference.

Basic Requirements for transmitting and receiving antenna:

High efficiency and high gain antennas, whereas low side lobes and large signal-to-noise ratio are the key selection aspect for receiving antennas.

Antennas may vary in size from the order of a few millimeters (strip antennas in cellular phones) to 1000's of feet (dish antennas for astronomical observations.)

BASIC ANTENNA PARAMETERS

A *radio antenna* may be defined as the structure associated with the region of transition between a guided wave and a free-space wave, or vice versa. Antennas convert electrons to photons, or vice versa. The basic principle in any type of antenna is that radiation is produced by accelerated (or decelerated) charge. The *basic equation of radiation* may be expressed simply as

Basic Radiation Equation $iL = Qv$

Where

i = time-changing current, A/sec

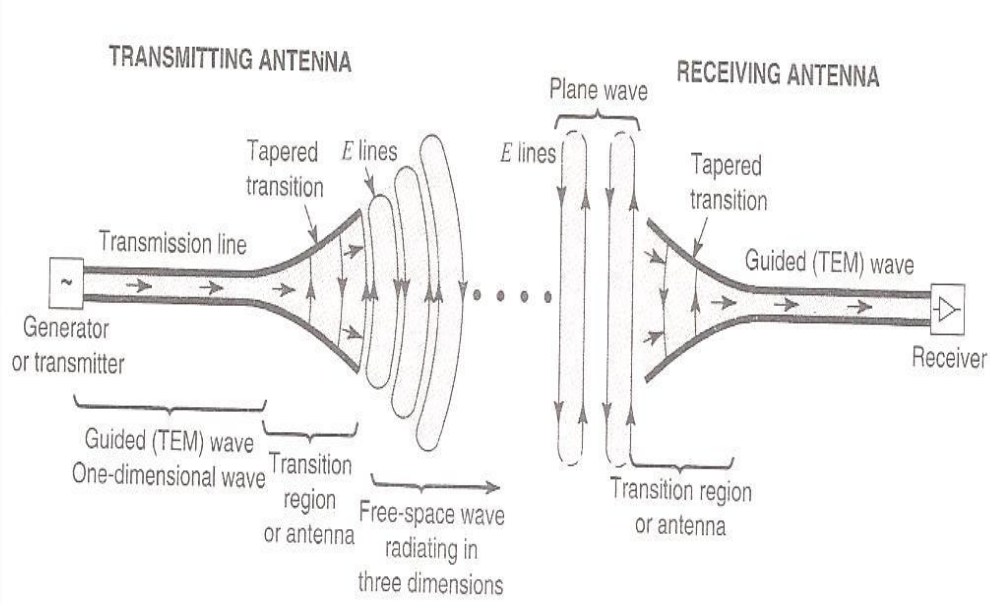
L = length of current element, m

Q = charge, C

V = time change of velocity which equals the acceleration the charge, m s^{-2}

Thus, ***time-changing current and accelerated charge radiates.***

For steady-state harmonic variation, current is used and for transients or pulses, charge is used to analyze the antenna operation. The radiation is perpendicular to the acceleration, and the radiated power is proportional to the square of iL or Qv The two-wire transmission line in Figure is connected to a radio-frequency generator (or transmitter). The transmission line is uniform to some extent and tapers gradually at the ends as shown in the figure



Along the uniform part of the line, energy is guided as a plane Transverse Electro Magnetic Mode (TEM) wave with little loss. The spacing between wires is assumed to be a small fraction of a wavelength. As the separation approaches the order of a wavelength or more, the wave tends to be radiated so that the opened-out line acts like an antenna which launches a free- space wave. The currents on the transmission line flow out on the antenna and end there, but the fields associated with them keep on going.

According to concept of transmission lines, that there would be ***prefect reflection*** of a wave if it is open circuited (OC) or short circuited (SC). Reflections are also present even if there is a slight mismatch at termination or imperfections in the transmission path itself.

An equivalent circuit of a transmission line with loss can be drawn in terms of its resistance (R), inductance (L) and capacitance (C) or only in terms of L and C for a lossless line. The energy of a propagating wave in the transmission line are of two kinds namely Electric field (E) and Magnetic field (H) or voltage (v) and current (i) alike.

The portions of energy shared by Electric field and magnetic field are given with relations $Cv^2/2$ (for electric field and

$Li^2/2$ (for magnetic field).

When Transmission line in open circuited case

Consider a wave propagating in the transmission line and it is open-circuited at the end. As the wave arrives at the OC end, the *current becomes zero* and part of the energy shared by magnetic field becomes (mathematically) zero.

The magnetic field becomes zero so then the electric field is obtained by the expression $Cv^2/2$, the line parameter $c(=\varepsilon A/d)$ does not change unless the area of cross-section 'A', the separation 'd' or the permittivity of the material occupying the space in the transmission line are changed.

The change of voltage is the only possibility by which the additional energy can be carried by the electric field. Thus, the voltage rises at the OC end. The voltage at a point of the OC is now higher. So it tries to move from higher voltage level to lower naturally. So the current starts flowing back.

Transmission line in short circuited case.

Similarly, if the line is short-circuited at the receiving end, the voltage (and electric filed) becomes zero and only magnetic field is present. This time the current rises at the receiving end and there by voltage increases.

In cases of perfect open circuits or perfect short circuits, theoretically there must be perfect reflection. The wave moving in transmission line possesses the property, ***moment-of-inertia***, due to this; *it will take some time to change its direction.* During this time some part of the electromagnetic energy is likely to leak into the space. This process of leakage can be termed as ***radiation***.

In case of an open-circuited parallel wire line, it has more opening at the end of the line. So more time will be taken by the wave to change its direction and thus more energy will leak in to the space during that time. That is there will be more coupling of transmission line to the space.

The maximum radiation will, therefore, occur when the two wires at the end are flared to ***form an*** $\boldsymbol{180^0}$ ***angle***. This process is shown in the figure (1.1).

The transmitting antenna shown in Figure 1.1 is a region of transition from a guided wave on a transmission line to a free-space wave. The receiving antenna in figure is a region of transition from a space wave to a guided wave on a transmission line. "Thus, ***an antenna is a transition device, or transducer, between a guided wave***

and a free-space wave, or vice-versa". The antenna is a device which interfaces a circuit and space.

From **the circuit point of view,** the antennas appear to the transmission lines like a **resistance *R_r*,** *called the **Radiation Resistance**.*

- It is not related to any resistance in the antenna itself but is a resistance coupled from space to the antenna terminals.
- In the transmitting case, the radiated power is absorbed by objects at a distance such as trees, buildings, the ground, the sky, and other antennas.
- In the receiving case, passive radiation from distant objects or active radiation from other **antennas raises the apparent temperature of *R_r*.**

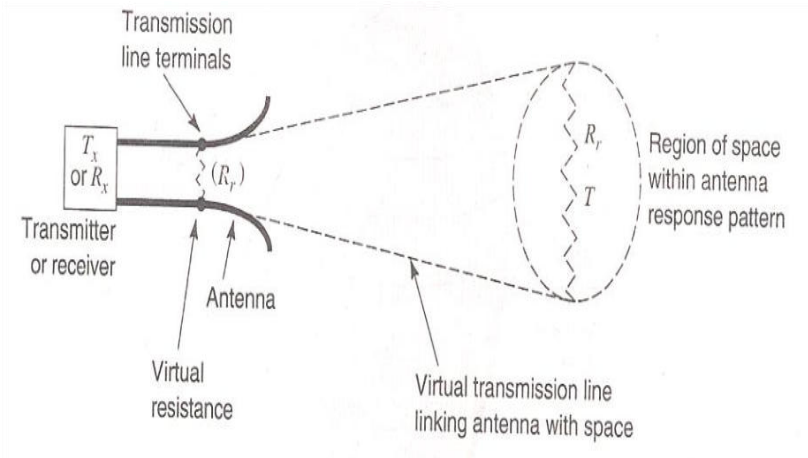


Figure 1.2: Schematic representation of region of space at temperature T

- For lossless antennas, this temperature has nothing to do with the physical temperature of the antenna itself.
- But it is related to the temperature of distant objects that the antenna is "looking at," as suggested in Fig 1.2.
- In this sense, a receiving antenna may be considered as a ***remote-sensing temperature-measuring device.***

In simple words to say, Antenna temperature = temperature of what the antenna is looking at, not the antenna itself.

1.3 ANTENNA PATTERNS

The radiation resistance R_r, and its temperature T_A are simple scalar quantities but radiation patterns are three-dimensional quantities.

Radiation patterns involve the variation of field or power as a function of the spherical coordinates θ and ϕ.

The following figure shows a three-dimensional field pattern with pattern radius ***r*** proportional to the field intensity in direction θ and ϕ The pattern has its *main lobe* (maximum radiation) in the *z* direction ($\theta = 0$) with *minor lobes* (side and back in other directions).

To completely represent the radiation pattern with respect ***to field intensity and polarization*** it requires three patterns:

1. The θ component of the electric field as a function of the angles θ and ϕ or $E_\theta(\theta,\phi)$ (V/ m)
2. The ϕ component of the electric field as a function of the angles θ and ϕ or $E\phi(\theta,\phi)$ (V/ m).

3. The phases of these fields as a function of the angles θ and ϕ or $\delta_\theta(\theta,\phi)$ and $\delta\phi(\theta,\phi)$ (rad or deg).

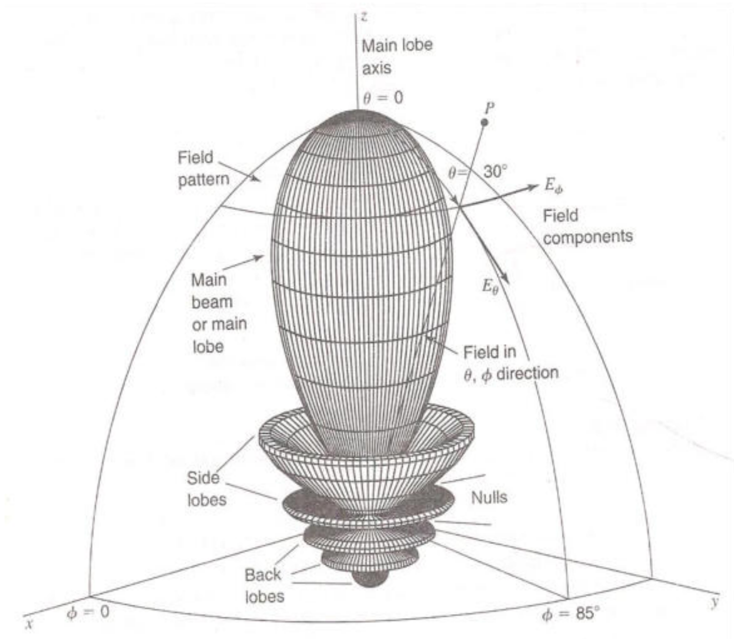


Figure 1.3: 3-D Radiation Pattern of an antenna

Any field pattern can be presented in three-dimensional spherical coordinates, as in above figure or by plane cuts through the main-lobe axis.

Two such cuts at right angles are called the ***principal plane patterns*** may be required but if the pattern is symmetrical around the *z* axis, one cut is sufficient.

The following Figures are principal plane field and power patterns in polar coordinates. The same pattern is presented in rectangular coordinates on a logarithmic, or decibel, scale which gives the minor lobe levels in more detail.

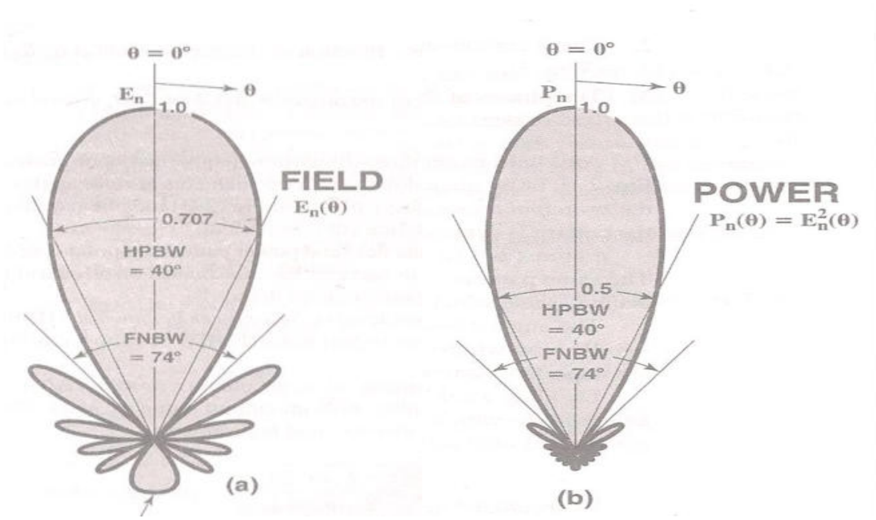


Fig 1.4(a): 2-D field plot **Fig(b) Power plots**

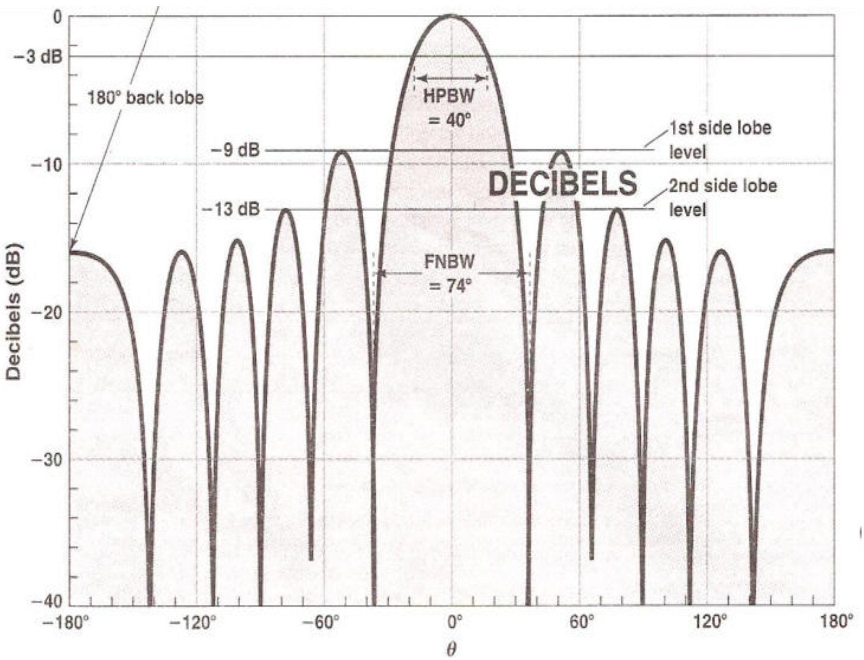


Figure 1.5: Power in decibels plot

The angular beam width at the half-power level or ***half-power beam width*** (HPBW) (or – 3dB beam width) and the ***beam width between first nulls*** (FNBW) as shown in Fig are important pattern parameters.

Dividing a field component by its maximum value, gives ***normalized or relative field pattern*** which is a dimensionless number with maximum value of unity.

Thus, the normalized field pattern for the electric field is given by

$$\textbf{Normalized field pattern} = \mathrm{E}_\theta(\theta,\phi)_n = \frac{\mathrm{E}_\theta(\theta,\phi)}{\mathrm{E}_\theta(\theta,\phi)_{\max}}$$

The half-power level occurs at those angles of θ and ϕ which $\mathrm{E}_\theta(\theta,\phi)_n = 1/\sqrt{2} = 0.707$.

The shape of the field pattern is independent of distance.

Patterns may also be expressed in terms of the ***power per unit area*** [or Poynting vector $S(\theta,\phi)$]. Normalizing this power with respect to its maximum value yields a ***normalized power pattern*** as a function of angle which is a dimensionless number with a maximum value of unity.

Thus, the normalized power pattern is give by

$$\textbf{Normalized field pattern} = p_n(\theta,\phi)_n = \frac{S(\theta,\phi)}{S(\theta,\phi)_{\max}}$$

Where $S(\theta,\phi) =$ Pointing vector $= [E_{\phi}^{2}(\theta,\phi) + E_{\phi}^{2}(\theta,\phi)]/Z_0$, W m^{-2}

$S(\theta,\phi)_{\max}$ = maximum value of $S(\theta,\phi)$, W m^{-2}

Z_0 is intrinsic impedance of space = 376.7 Ω

1.4 BEAM AREA (or) BEAM SOLID ANGLE (Ω_A)

In polar two- dimensional coordinates an incremental area dA on the surface of a sphere is the product of the length $r\,d\theta$ in the θ direction (latitude) and $r \sin\theta\, d\phi$ in the ϕ direction (longitude), as shown in Fig.

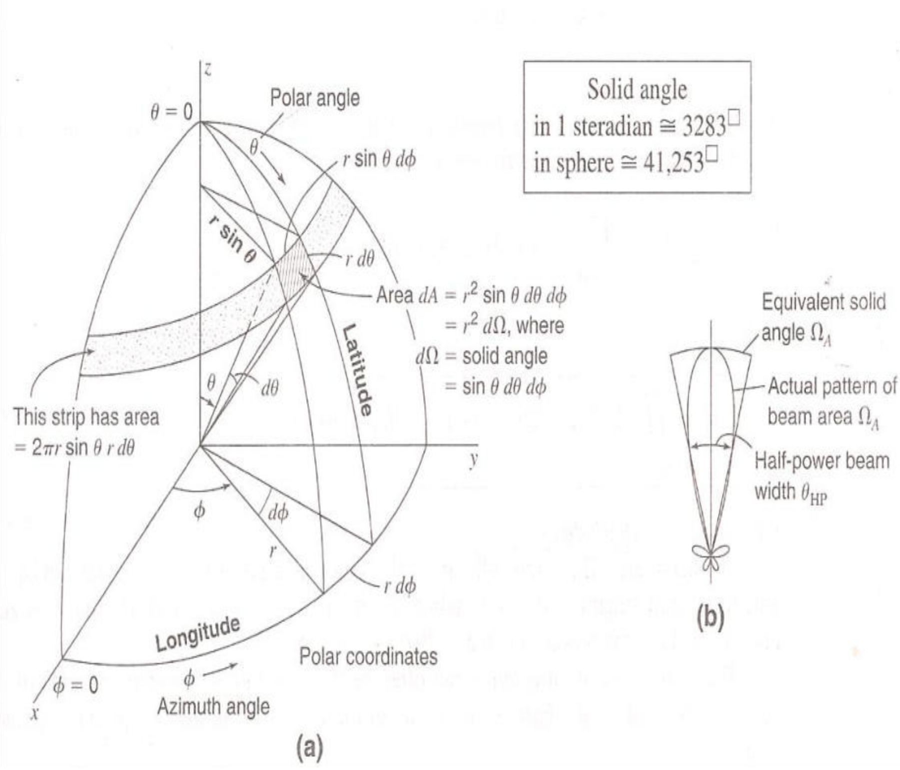


Figure:

Thus,

$$dA = (r d\theta)(r \sin\theta d\phi) = r^2 d\Omega \quad \text{---------------} (1)$$

Where $d\Omega =$ *solid angle* in steradians (sr) or square degrees (□)

$d\Omega =$ solid angle subtended by the area dA

The area of the strip of width $r\, d\theta$ extending around the sphere at a constant angle θ is given by ($2\pi\, r \sin\theta$) ($r\, d\theta$). Integrating this for θ values from 0 to π yields the area of the sphere. Thus,

Area of sphere $= 2\pi r^2 \int_0^{\pi} \sin\theta \; d\theta$

$$= 2\pi\ r^2 [-\cos\theta]_0^{\pi} = 4\pi r^2 \quad \text{--} (2)$$

Where $4\pi =$ solid angle subtended by a sphere, sr

Thus,

1 steradian = 1 sr = (solid angle of sphere) $/(4\pi)$

$= 1 \text{ rad}^2 = \left(\frac{180}{\pi}\right)^2 (\deg^2)$

= 3282.8064 square degrees ------- (3)

Therefore,

4π Steradians $= 3282.8064 \times 4\pi = 41{,}252.96 \cong 41{,}253$ square degrees

. = 41,253□ = solid angle in a sphere ------- (4)

The **beam area or *beam solid angle*** or Ω_A of an antenna is given by the integral of the normalized power pattern over a sphere $(4\pi sr)$

$$\Omega_A = \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi} p_n(\theta,\phi) \sin\theta \; d\theta \; d\phi \quad \text{------- (5)}$$

And

$$\boxed{\Omega_A = \int \int_{4\pi} p_n(\theta,\phi) d\Omega \quad \text{(sr)} \quad \textbf{Beam area}}$$

------- (6)

Where $d\Omega = \sin\theta \; d\theta \; d\phi$, sr.

Beam area (or Beam solid Angle) Ω_A "The beam area Ω_A is the solid angle through which all of the power radiated by the antenna would stream if $p(\theta,\phi)$ maintained its maximum value over Ω_A and was zero elsewhere". Thus the power radiated $= p(\theta,\phi)\Omega_A$ watts.

The *beam area* of an antenna can be described *approximately* in terms of the angles subtended by the *half-power points* of the main lobe in the two principal planes. Thus.

$$\boxed{\textbf{Beam area} = \theta_{HP}\, \phi_{HP} \quad \text{(sr)}} \quad \text{------- (7)}$$

Where θ_{HP} and ϕ_{HP} are the *half-power beam widths* (HPBW) in the two principal planes, minor lobes being neglected.

1.5 RADIATION INTENSITY U (θ, ϕ)

The power radiated from an antenna per unit solid angle is called the ***radiation intensity U*** (watts per steradian or per square degree).

The normalized power pattern can also be expressed in terms of this parameter as the ratio of the radiation intensity $U\ (\theta,\phi)$, as a function of angle, to its maximum value. Thus,

$$p_n(\theta,\phi) = \frac{U(\theta,\phi)}{U(\theta,\phi)_{\max}} = \frac{S(\theta,\phi)}{S(\theta,\phi)_{\max}}$$

Whereas the pointing vector S depends on the distance from the antenna (varying inversely as the square of the distance), the radiation intensity U is independent of the distance.

1.6 BEAM EFFICIENCY

The (total) ***beam area*** Ω_A **(or *beam solid angle*)** consists of the main beam area (or solid angle) Ω_M plus the minor-lobe area (or solid angle) $\Omega_{m.}$ Thus,

$$\boxed{\Omega_A = \Omega_M + \Omega_m}$$

The ratio of the main beam area to the (total) beam area is called the main ***beam efficiency*** ε_M. Thus,

$$\boxed{\textbf{Beam efficiency} = \varepsilon_M = \frac{\Omega_M}{\Omega_A}}$$

The ratio of the minor-lobe area (Ω_m) to the (total) beam area is called the ***stray factor.*** Thus,

$$\boxed{\varepsilon_m = \frac{\Omega_m}{\Omega A} = \textit{Stray factor}}$$

So $$\varepsilon_M + \varepsilon_m = 1$$

1.7 DIRECTIVITY (D) AND GAIN (G)

The ***directivity*** of an antenna is equal to the ratio of the maximum power density $p(\theta,\phi)_{\max}$ (watts/m^2) to its average value over a sphere as observed in the far field of an antenna. Thus

$$D = \frac{P(\theta,\phi)_{\max}}{P(\theta,\phi)av} \quad \textbf{\textit{Directivity from pattern}}$$

------- (1)

<u>The directivity is a dimensionless ratio ≤ 1.</u>

The average power density over a sphere is given by

$$P(\theta,\phi)_{av} = \frac{1}{4\pi}\int_{\phi=0}^{\phi=2\pi}\int_{\theta=0}^{\theta=\pi} P(\theta,\phi)\sin\theta \; d\theta \; d\phi$$

$$= \frac{1}{4\pi}\iint_{4\pi} P(\theta,\phi)d\Omega \qquad (\text{W sr}^{-1})$$

------- (2)

Therefore, the directivity

$$\boxed{D = \frac{P(\theta,\phi)_{\max}}{(1/4\pi)\iint_{4\pi} P(\theta,\phi)d\Omega} = \frac{1}{(1/4\pi)\iint_{4\pi} [P(\theta,\phi)/P(\theta,\phi)_{\max}]d\Omega}}$$

----- (3)

&

$$\boxed{D = \frac{4\pi}{\iint_{4\pi} P_n(\theta,\phi)d\Omega} = \frac{4\pi}{\Omega_A} \quad \textbf{\textit{Directivity from beam area }} \Omega_A}$$

----- (4)

Where $P_n(\theta,\phi)d\Omega = P(\theta,\phi)/P(\theta,\phi)_{\max}$ = normalized power pattern

Thus, the directivity is the ratio of the area of a sphere (4π sr) to the beam area Ω_A of the antenna. ***The smaller the beam area, the larger the directivity D***.

For an antenna that radiates over only half a sphere the beam area $\Omega_A = 2\pi$ sr and the directivity is

$$D = \frac{4\pi}{2\pi} = 2 = 3.01 \text{ dBi} \qquad \text{------- (5)}$$

Where dBi = decibels over isotropic

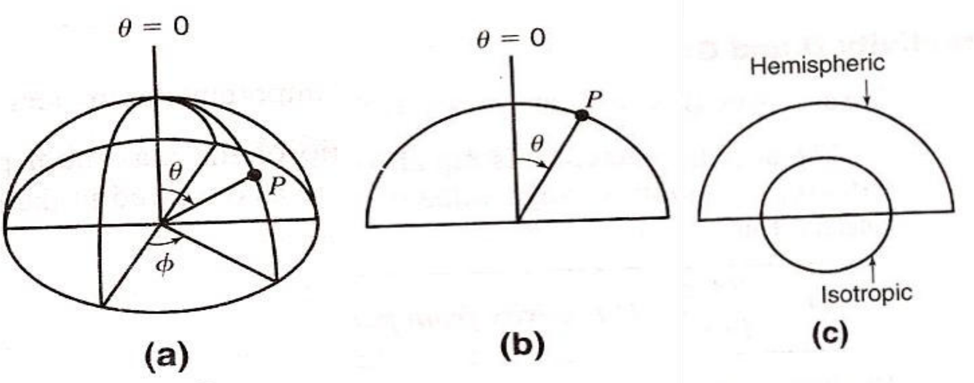


Figure 1.6 (a), (b): Hemispheric Power pattern Fig 1.6 (c): Comparision

Note that,

- The idealized *isotropic antenna* ($\Omega_A = 4\pi$) sr) has the lowest possible directivity $D = 1$.
- All actual antennas have directivities greater than 1 ($D > 1$).
- The simple short dipole has a beam area $\Omega_A = 2.67\pi$ sr and a directivity D =1.5 (= 1.76 dBi).

GAIN

"The ***gain*** G of an antenna is an actual or realized quantity which is less than the directivity D due to ohmic losses in the antenna".

In transmitting, these losses involve power fed to the antenna which is not radiated but heats the antenna structure. A mismatch in feeding the antenna can also reduce the gain.

The ratio of the gain to the directivity is the efficiency ***factor.*** Thus,

$$G = kD \qquad \text{------- (6)}$$

Where k = efficiency factor ($0 \le k \le 1$), dimensionless.

Gain can be measured by comparing the maximum power density of the Antenna under Test (AUT) with a reference antenna of known gain, such as a short dipole. Thus,

$$\textbf{Gain} = G = \frac{P_{\max}(AUT)}{P_{\max}(ref.ant)} \times G\,(\text{Ref.ant.}) \qquad \text{--- (7)}$$

If the half-power beam widths of an antenna are known, its directivity

$$D = \frac{41{,}253}{\theta^o HP \phi^o} \qquad \text{------- (8)}$$

Where

41,253⌉ = number of square degrees in sphere

= 4π $(180/n)^2$ square degrees (⌉)

θ^o HP = half – power beam width in one principal plane

$\phi^o\ HP$ = half – power beam width in other principal plane

By neglects minor lobes in above equation, the directivity can be written as

$$\boxed{D = \frac{40{,}000}{\theta^o HP\phi^o} \quad \textbf{\textit{Approximate directivity}}} \qquad \text{------ (9)}$$

If the antenna has a main half-power beam width (HPBW) = 20° in both principal planes, its directivity $D = \frac{40{,}000}{400}$ = 100 or 20 dBi

This means that the antenna radiates 100 times the power in the direction of the main beam as a non directional, isotropic antenna.

The ***directivity-beam width product*** 40,000 is a rough approximation.

If an antenna has a main lobe with both half-power beam widths (HPBWs) = 2°, its directivity is *approximately*

$$D = \frac{4\pi(sr)}{\Omega_A(sr)} \cong \frac{41{,}253(\deg^2)}{\theta^o HP\phi^o HP} = \frac{41{,}253(\deg^2)}{20^o \times 20^o} \qquad \text{------ (10)}$$

$$\cong 103 \cong 20 \text{ dBi (dB above isotropic)}$$

Which means that the antenna radiates a power in the direction of the main-lobe maximum which is about 100 times as much as would be radiated by a non directional (isotropic) antenna for the same power input.

1.8 DIRECTIVITY AND RESOLUTION

The resolution of an antenna may be defined as equal to half the beam width between first nulls (FNBW)/2 .

- For example, an antenna having pattern with FNBW = 2^o will have a resolution of 1^o
- Half the beam width between first nulls is approximately equal to the half-power beam width (HPBW) or

$$\frac{FNBW}{2} \cong HPBW \qquad \text{------- (1)}$$

The product of the FNBW/2 in the two principal planes of the antenna pattern is a measure of the **antenna beam area**. Thus,

$$\Omega_A = \left(\frac{FNBW}{2}\right)_\theta \left(\frac{FNBW}{2}\right)_\phi \qquad \text{------- (2)}$$

This means that the approximate number *N* of radio transmitters (or radiation sources) spread evenly across the sky, that an antenna can distinguish, is given by

$$N = \frac{4\pi}{\Omega_A} \qquad \text{------- (3)}$$

Where Ω_A = beam area, sr

We know that $$D = \frac{4\pi}{\Omega_A} \qquad \text{------- (4)}$$

So we may conclude that *ideally* the number of point sources an antenna can resolve is numerically equal to the directivity of the antenna or

$$\boxed{D = N} \qquad \text{------- (5)}$$

From the two equations, we can conclude that

- ✓ directivity is the same as the number of beam areas into which an antenna divides the sky.
- ✓ In other words, directivity also represents the number of point sources in the sky that the antenna can clearly separate, assuming the sources are uniformly distributed.

1.9 ANTENNA APERTURES

The concept of aperture can be explained by considering a receiving antenna. Suppose that the receiving antenna is a rectangular electromagnetic horn placed in the field of a uniform plane wave as shown in the following figure 1.7.

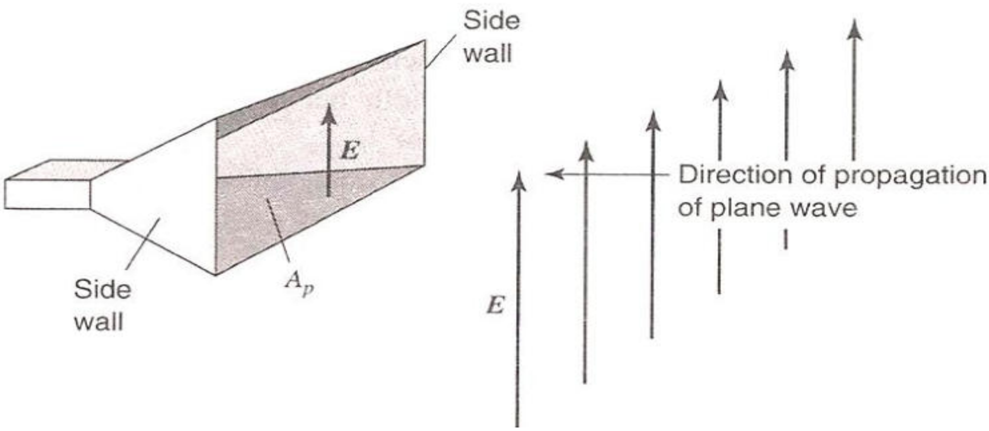


Figure 1.7: Plane wave incident on electromagnetic horn of physical aperture A_p

Let the pointing vector, or power density, of the plane wave be S watts per square meter and the area, or physical aperture of the horn, be A_p square meters.

If the horn extracts or receives all the power from the wave over its entire physical aperture, then

the total power *P* absorbed from the wave is

$$P = \frac{E^2}{Z} A_P = SA_P \qquad \text{(W)} \quad \text{----------- (1)}$$

Hence the electromagnetic horn may be considered as having an aperture, such that

- the total power it collects from a passing wave is proportional to the aperture or area of its mouth.
- But the field response of the horn is NOT uniform across the aperture *A* because *E* at the sidewalls must equal zero.

Thus, the ***effective aperture*** A_e of the horn is less than the ***physical aperture*** A_p as given by

$$\varepsilon_{ap} = \frac{A_e}{A_p} \qquad \textit{Apperture efficiency (dimensionless)}$$

----- (2)

Where ε_{ap} = aperture efficiency.

However, to reduce side lobes, fields are commonly tapered toward the edges, resulting in reduced aperture efficiency.

Conical Pattern: Consider now an antenna with an *effective aperture* A_e, which radiates all of its power in a conical pattern of beam area Ω_A, as shown i

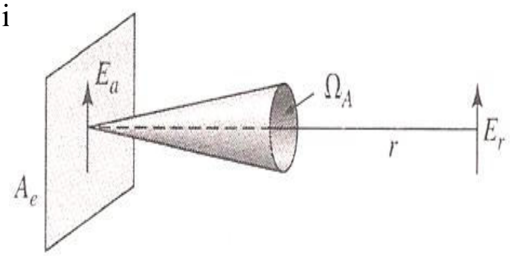


Figure 1.8: Radiation over beam area Ω_A from aperture A_e

Assuming a uniform field E_a over the aperture, the power radiated is

$$P = \frac{E_a^2}{Z_0} A_e \text{ (W)} \qquad \text{-------------- (3)}$$

Where Z_0 = intrinsic impedance of medium (377 Ω for air or vacuum).

Assuming a uniform field E_r in the far field at a distance r, the power radiated is also given by

$$P = \frac{E_r^2}{Z_0} r^2 \Omega_A \text{ (W)} \qquad \text{--------- (4)}$$

Equating (3) and (4) and noting that $E_r = E_a A_e / r\lambda$ yields the aperture –beam-area relation

$$\boxed{\lambda^2 = A_e \Omega_A \quad (\text{m}^2) \quad \textbf{\textit{Aperture-beam-area relation}}}$$

------ (5)

Where $\Omega_A =$ beam area (sr).

Thus, if A_e is known, we can determine Ω_A (or vice versa) at a given wavelength. So the directivity

$$\boxed{D = 4\pi \frac{A_e}{\lambda^2} \quad \textbf{\textit{Directivity from aperture}}}$$

------- (6)

All antennas have an effective aperture which can be calculated or measured. Even the hypothetical, idealized isotropic antenna, for which D=1, has an effective aperture.

$$A_e = \frac{D\lambda^2}{4\pi} = \frac{\lambda^2}{4\pi} = 0.0796\,\lambda^2$$ ------- (7)

❖ <u>The effective aperture of an antenna is the same for receiving and transmitting.</u>

Three expressions have now been given for the *directivity D.* They are

$$\boxed{D = \frac{P(\theta,\phi)_{\max}}{P(\theta,\phi)_{av}} \quad \textit{Directivity from pattern}}$$ (Dimensionless) ------ (8)

$$\boxed{D = \frac{4\pi}{\Omega_A} \quad \textit{Directivity from pattern}}$$

(Dimensionless)------- (9)

$$\boxed{D = 4\pi\frac{A_e}{\lambda^2} \quad \textit{Directivity from aperture}}$$

(Dimensionless) ----- (10)

Maximum power transfer:

When the antenna is receiving electromagnetic waves with a load resistance R_L matched to the antenna radiation resistance R_r *ie.* ($R_r = R_L$), then as <u>much power is reradiated</u> from the antenna as is delivered to the load. This is the condition of ***maximum power transfer*** (antenna assumed lossless).

$$\boldsymbol{P_{load} = S\,A_e} \text{ (W)}$$ -------- (11)

Where S = power density at receiving antenna, W/m^2

A_e = effective aperture of antenna, m^2

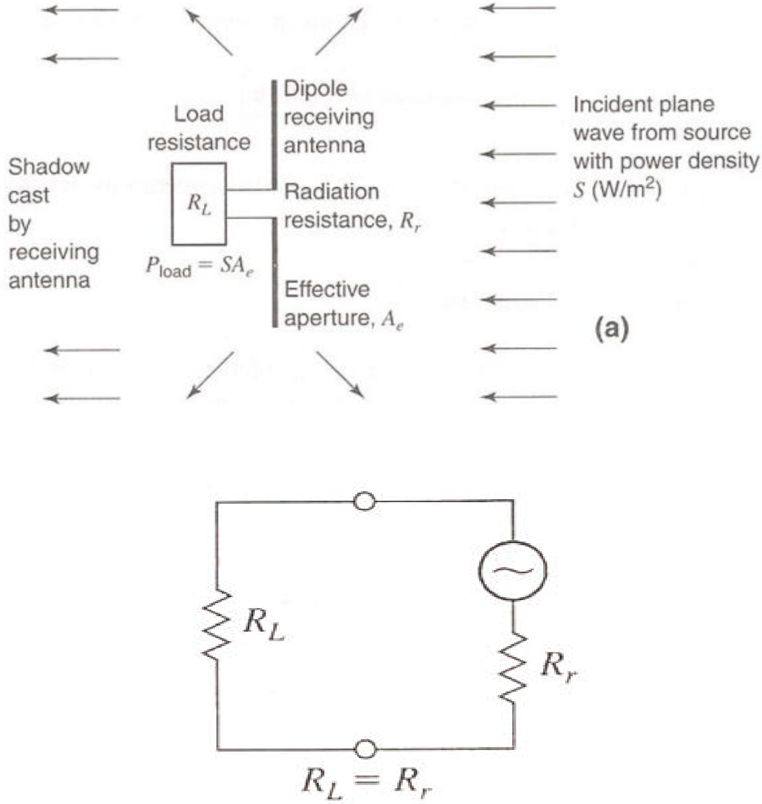


Figure 1.10: Equivalent circuit of receiving antenna matched to load

The ***reradiated power***

$$P_{rerad} = \frac{Powerreadiated}{4\pi} = SA_r \qquad \text{(W)} \quad \text{---------} \ (12)$$

Where A_r = reradiating aperture = A_e, m^2 and

$$\boldsymbol{P_{rerad} = P_{load}} \qquad \text{-----------} \ (13)$$

1.10 EFFECTIVE HEIGHT

To obtain ***effective height h*** of an antenna, Multiply the effective height by the incident field E (volts per meter) of the same polarization that gives the voltage V induced.

$$\text{Thus,} \qquad \boldsymbol{V = h\,E} \qquad \text{------------ (1)}$$

So, the effective height is defined as "the ratio of the induced voltage to the incident field" that is

$$h = \frac{V}{E} \qquad \text{(m)} \qquad \text{------------ (2)}$$

For example from the figure 1.11(a), consider a ***vertical dipole of length*** $l = \lambda/2$ placed in an incident field E, as shown in Fig (a).

If the current distribution of the dipole were uniform, then its effective height would be l.

- But the actual current distribution, ***is nearly sinusoidal*** with an *average value* $2/\pi = 0.64$ (of the maximum)
- so that its effective height $h = 0.64l$ (it is assumed that the antenna is oriented for maximum response). This leads to sinusoidal distribution shown in figure 1.11(a).

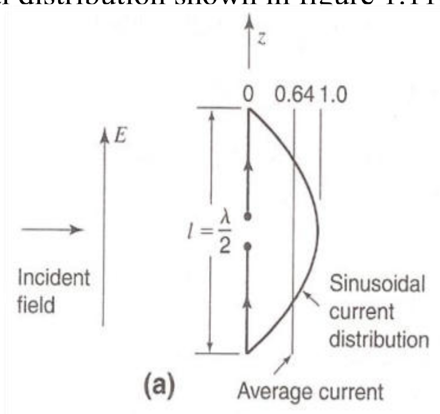


Figure 1.11(a): Dipole of length *l*=λ/2 with sinusoidal current distribution

From the figure 1.11(b), **If the same dipole is used at a longer wavelength** so that its length is only 0.1λ long, then

- the current tapers almost linearly from the central feed point to zero at the ends in a ***triangular distribution,*** as shown in Fig. 1.11(*b*).
- The average current is 1/2 of the maximum so that the effective height is 0.5*l*.

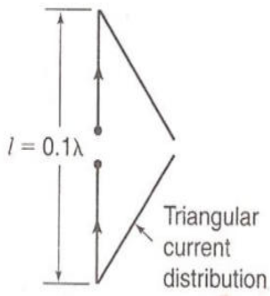


Figure 1.11(b): Dipole of length *l* = 0.1λ with triangular current distribution

Now the effective height can be defined by considering the physical height as follows

$$h_e = \frac{1}{I_0}\int_0^{h_p} I(z)dz = \frac{I_{av}}{I_0}h_p \text{ (m)} \quad \text{------ (3)}$$

Where h_e = effective height, m
h_p = physical height, m
h_{av} = average current, A

For an antenna of radiation resistance R_r matched to its load, the power delivered to the load is equal to

$$P = \frac{1}{4}\frac{V^2}{R_r} = \frac{h^2 E^2}{4R_r} \qquad \text{(W) ------------ (4)}$$

In terms of the effective aperture the same power is given be

$$P = SA_e = \frac{E^2 A_e}{Z_0} \text{ (W)} \quad \text{------------ (5)}$$

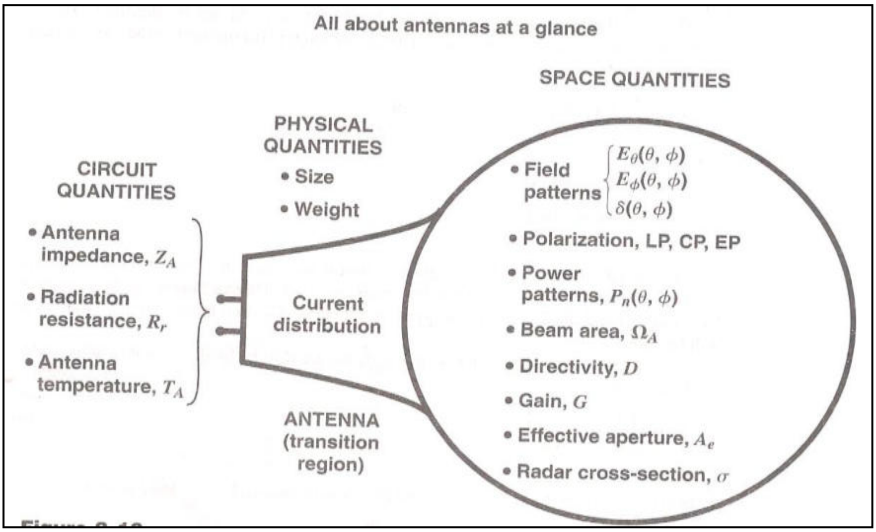
Where Z_0 = intrinsic impedance of space (= 377 Ω) Equating (4) and (5), we obtain

$$h_e = 2\sqrt{\frac{R_r A_e}{Z_0}} \text{ (m)} \quad \text{and} \quad A_e = \frac{h_e^2 Z_0}{4R_r} \text{ (m}^2\text{)} \quad \text{------- (6)}$$

<u>Thus effective height and effective aperture are related via radiation resistance and the intrinsic impedance of space.</u>

Antenna field and power patterns, beam area, directivity, gain, and various apertures are the space parameters of an antenna.

The radiation resistance and antenna temperature are circuit quantity of antenna. An antenna exhibits both of these properties called ***duality of an antenna***. This is shown in the figure **Figure 1.12: Duality of an antenna**



1.11 THE RADIO COMMUNICATION LINK

The concept of the aperture is used to derive the important ***Friis transmission formula***. This Friis transmission formula concept can be explained by the following diagram.

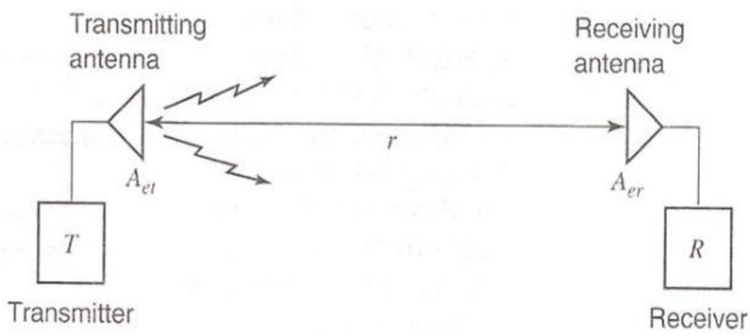


Figure 1.13: A simple radio communication link

From the figure it says that energy is radiating or transmitting from the transmitter to the receiver at a distance ***r***. The apertures of transmitting and receiving antennas are given by A_{et} and A_{er} with the transmitting and receiving powers as P_t and P_r respectively.

Let us consider the transmitting antenna is isotropic, and then the power per unit area available at the receiving antenna is

$$S_r = \frac{P_t}{4\pi r^2} \quad \text{(W)} \quad \text{----------- (1)}$$

If the antenna has gain G_t, the power per unit area available at the receiving antenna will be increased in proportion as given by

$$S_r = \frac{P_t G_t}{4\pi r^2} \quad \text{(W)} \quad \text{----------- (2)}$$

Now the power collected by the lossless, matched receiving antenna of effective aperture A_{er} is

$$P_r = S_r A_{er} = \frac{P_t G_t A_{er}}{4\pi r^2} \quad \text{(W) ----------- (3)}$$

The gain of the transmitting antenna can be expressed as

$$G_r = \frac{4\pi A_{er}}{\lambda^2} \quad \text{(W) ----------- (4)}$$

Substituting this in (3) yields the *Friis transmission formula*

$$\boxed{\frac{P_r}{P_t} = \frac{A_{er} A_{et}}{r^2 \lambda^2} \qquad \textbf{\textit{Friis transmission formula}} \text{ (Dimension less)}} \text{ -- (5)}$$

Where

P_r	= received power, W
P_t	= transmitted power, W
A_{et}	= effective aperture of transmitting antenna, m^2
A_{er}	= effective aperture of receiving antenna, m^2
r	= distance between antennas, m
λ	= wavelength, m

CHAPTER II
INTRODUCTION TO ANTENNA ARRAYS

CONTENTS

Dr. Kasigari Prasad....

INTRODUCTION TO ANTENNA ARRAYS

Antenna Array: An antenna array is a system of similar antennas oriented similarly to get greater directivity in a desired direction.

Antenna arrays are becoming increasingly important in wireless communications.

Advantages of using antenna arrays

- They can provide the ***capability of a steerable beam*** (radiation direction change) as in smart antennas.

- ❖ They can provide ***a high gain*** (array gain) by using simple antenna elements.
- ❖ They provide a ***diversity gain*** in multipath signal reception.
- ❖ They enable array signal processing.

Unlike a single antenna whose radiation pattern is fixed, an antenna ***array's radiation pattern***, called the array pattern, can be changed upon exciting its elements with different currents (both current magnitudes and current phases).

Antenna Arrays (or) Arrays of Antennas

The field radiated by any small linear antenna is un-uniformly distributed in the plane perpendicular to the axis of the antenna.

Fields radiated is

Non uniformly
Distributed

In general any practical antenna radiates ***non- uniformly*** and so they are not all preferable ***for point to point communication.***

To overcome this, the field strength in a desired direction can be increased by properly exciting a group of antennas simultaneously. Such an arrangement is called an ***Antenna Array.*** (or simply Array of Antennas).

An antenna array is a system of similar antennas oriented similarly to get greater directivity in a desired direction. An antenna radiating system consists of several spaced and properly phased antennas.

In many applications it is necessary to design antennas with ***very directive characteristics*** (very high gain) to meet the demands of long distance communication. This can only be accomplished by ***increasing the Electrical size of an antenna***. But in practical the Electrical size improvement ***can be limited***.

So another method is to form an assembly of radiating elements in an electrical and geometrical configuration. This new antenna formed by multi elements is referred to as ***Array.***

In many applications, antennas need to be designed with highly directive characteristics (very high gain) to satisfy the ***requirements of long-distance communication.***

- One way to achieve this is by ***increasing the electrical size*** of a single antenna. However, in practice, increasing electrical size has limitations.

- An effective alternative is to ***arrange multiple radiating elements*** in a proper electrical and geometrical configuration. This new system of multi-element antennas is known as an Antenna **Array**.

Generally, the elements of an antenna array are chosen to be identical, although it is not always compulsory.

Using identical radiators makes the design simpler, convenient, and more practical.

To achieve a highly directive radiation pattern:

- ✓ The fields from array elements must interfere constructively (add up) in the desired direction.
- ✓ At the same time, they must interfere destructively (cancel

each other) in unwanted directions.

Significantly the **overall antenna pattern of arrays** depends on

- Geometrical configuration of the overall array i.e. Linear, Circular, and Rectangular, Spherical etc…
- Relative displacement between the elements
- Excitation amplitude of the individual elements
- Excitation Phase of current element
- The relative pattern of the individual elements.

The ***total field produced*** by an antenna array system at a great distance from it is the ***vector sum of the fields*** produced by the individual antennas of array systems.

The ***relative phases*** of the individual field component depend on the ***relative distance*** of the individual antennas of the antenna array.

An antenna array is said to be ***LINEAR*** if the individual antennas of the array are equally spaced along a straight line.

Individual antennas are termed as ***ELEMENTS***. Therefore linear antenna array is a system of equally spaced elements.

Uniform linear array is one in which the elements are fed with a current of equal magnitude with uniform progressive phase shift along the line.

There are generally three kinds of arrays which are used practically are

- Broad side Array BSA
- End Fire Array EFA
- Increased End Fire Array
- Collinear Array
- Parasitic Array and so on.

POINT SOURCE

An antenna is regarded as a point source or volume less radiator. In other words an antenna is Hypothetical antenna, Isotropic, Omni directional or Non directional antenna which occupies zero volume is considered as POINT SOURCE.

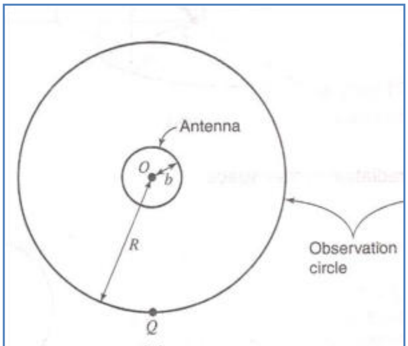
To understand the concept of Antenna Array Theory with point sources very first consider only two point sources separated at a distance *'d'* later extend it to n-point source.

Before knowing about the point source array theory, let's have a glance on what is Point Source and what is its Pattern?

Point Source: An antenna which occupies zero volume is considered as Point Source. A point source is also radiates energy into free space. According to the concept of Antenna Zones, antenna produces two types of fields namely,

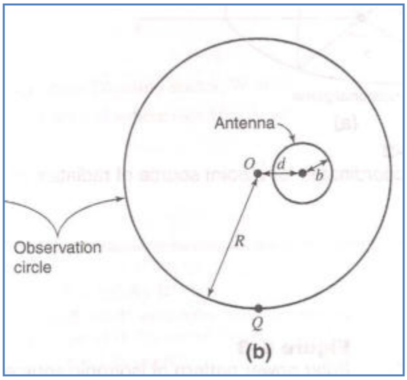
- Near field and
- Far field.

Any point source irrespective of its size it produces these two fields. For continence let us neglect the near field and consider only the far field. Plot an observation circle around the far field at a distance *'R'* to measure the field. This is shown in the following figure.



(a)

From this figure it reveals that the centers of antenna and the observation circle are coincided. In case if ***these centers are shifted by a distance 'd'*** as shown in the following figure then the given consideration are to be taken.



The field pattern of the observation circle has very negligible effect since the distance R is $R >> d, R >> b, R >> \lambda$

But the Phase pattern will differ depending on *d*

If d = 0; then the phase shift is minimum and if d increases then phase shift will increase.

To analyse the Far field pattern of any antenna

We require two patterns of the orthogonal field components namely

$$E_\theta(\theta, \varphi),$$

$$E_\varphi(\theta, \varphi) \text{ and}$$

One pattern of phase difference of these fields $\delta(\theta, \varphi)$

Patterns:

Power Pattern:

Let us consider a transmitting antenna which radiates fields into the free space. Assume the antenna as a point source.

The Rate of energy flow per unit area is defined as poynting vector or power density.

A source that radiate energy uniformly in all directions is called Isotropic source in contrasts to this is Anisotropic Source.

“A graph of S_r at a constant radius as a function of angle is a Poyting Vector or Power density pattern or Simply Power Pattern.

- In 3 dimention the power pattern for an Isotropic source is –Sphere
- In 2 dimention the power pattern for an Isotropic source is –Circle

If S_r is expressed in watts/meter^2 the graph is ***Absolute Power Pattern***

If S_r is expressed in terms of its value in some reference direction the graph is a ***relative power pattern***

Relative Power Pattern = $\frac{sr}{srmax}$

The max value of Relative Power Pattern is 1 unity

"A pattern with maximum of unity is also called ***Normalized Pattern***

Field Pattern:

A pattern showing the variation Electric field intensity at a constant radius 'r' as a function of angle(θ,ϕ)is called Field Pattern.

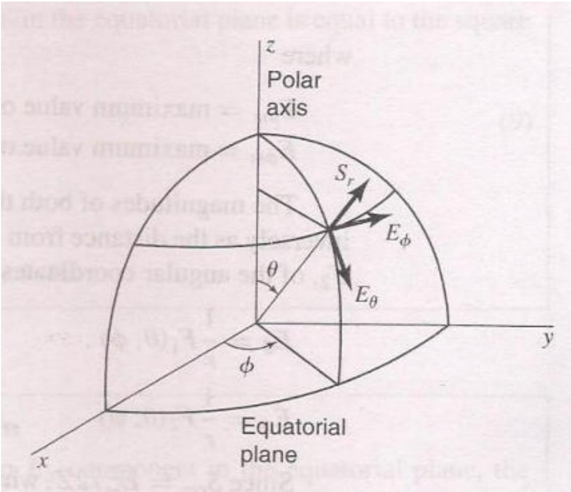
To describe the field of a point source more completely we need to consider the electric field E or magnetic field both as Vectors.

Only far fields are considered for point sources hence E and H are both entirely transverse to the wave direction and are perpendicular to each other are in-phase also E/H=Z=372Ω for free space

Since pointing vector/power density around a point source is everywhere radial that is around direction of axis, it follow Electric field is every where

Transverse.

The ratation of the poynting vector S and the two electric field components are shown in following figure.



The conditions characterizing the far field are then given as follows

Pointing vector radial

Electric field transverse.

The relation between the average pointing vector and the electric field at a point at the far field is

A pattern showing the variation of electric field intensity at a constant radius r as function of angle (   ) is called Field Pattern.

When field intensity is expressed in volts/meter it is called Absolute field pattern.

If the field pattern is expressed in units relative to its value in some reference direction is called Relative field pattern.

The relative field pattern of E component is given by

Phase pattern:

ARRAYS OF TWO ISOTROPIC POINT SOURCES

With the help of two point sources first arrange an antenna array called Array of two isotropic sources. Now to study the nature of these two point source array consider it in five different cases such as

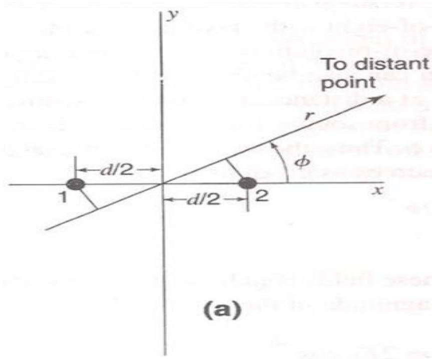
1. Two Isotropic point sources of Same Amplitude and phase
2. Two Isotropic point sources of Same Amplitude but opposite phase
3. Two Isotropic point sources of Same Amplitude and in-phase quadrature
4. Two Isotropic point sources of equal Amplitude and any phase difference

5. Two Isotropic point sources of unequal Amplitude and any phase difference

Case (1): Two Isotropic point sources of Same Amplitude and phase

Consider two isotropic point source having equal amplitudes and are oscillating in same phase.

Let the two point sources. 1 and 2 separated by a distance ***d*** and located symmetrically with respect to the origin of the coordinates as shown in the figure.



(a)

Let the ***origin of the coordinates is taken as the reference for phase***. Then the field from source 1 is retarded by ½ $d_r \cos\theta$ at a distant point in the direction θ, while the field from source 2 is advanced by ½ $d_r \cos\theta$,

where d_r is the distance between the sources expressed in radians;

that is $$d_r = \frac{2\pi d}{\lambda} = \beta d$$

The total field at a large distance ***r*** in the direction $\boldsymbol{\theta}$ is then

$$E = E_{0e}^{--j\psi/2} + E_{0e} +^{j\psi/2} \text{ -------- (1)}$$

Where $\psi = d_r \cos \phi$

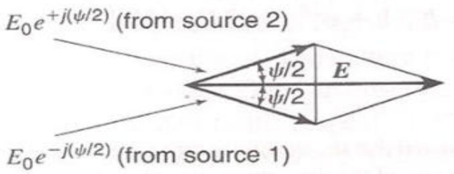
E_0 is the field component due to source 1 , Equation (1) may be rewritten

$$E = 2E_0 \frac{e^{+j\psi/2} + e^{-j\psi/2}}{2} ----(2)$$

Which by **a trigonometric identity is**

$$\mathbf{E} = \mathbf{2E_0} Cos\frac{\psi}{2} 2\, E_0 \cos\left(\frac{d_r}{2}\cos\phi\right) --- (3)$$

This resultant electric field can be obtained from vector sum of the two fields and this shown in the following figure.



Note: The phase of the total field E does not change as a function of ψ.

Normalize equation (3); that means set it to its maximum value then ***2E₀=1.***

Let ***d*** is ***λ/2.*** Then ***$d_r = \pi$.***

Put this conditions in equation (3) it gives

$$E = \cos\left(\frac{\pi}{2}\cos\phi\right) \quad ----- (4)$$

The ***field pattern produced*** by this array is shown in the following figure.

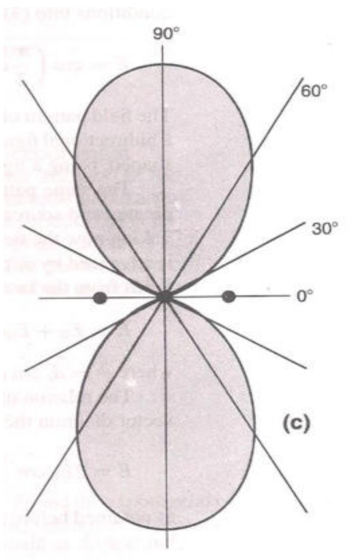
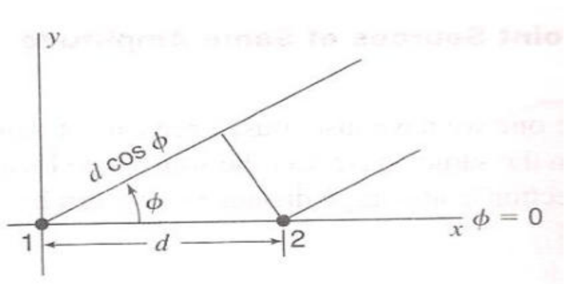


Figure: Radiation pattern

- The pattern is a ***BIDIRECTIONAL FIGURE OF EIGHT*** with maxima along the Y axis.
- ***The space pattern*** is ***DOUGHNUT-SHAPED***, being a figure of revolution of this pattern around the x axis.

Alternate arrangement of case (1)

The same pattern can also be obtained by locating source 1 at the origin of the coordinates and source 2 at a distance d along the positive x axis as shown in the following figure.



Now source 1 becomes reference, field from source 2 in the direction θ is advanced by $d_r \cos\phi$.

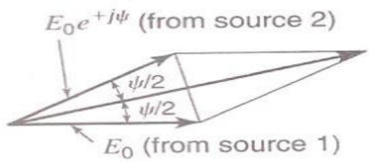
Thus the total field E at a large distance r is the vector sum of the fields from the two sources as given by

Total field due to this two point sources are

$$E = E_0 + E_{oe}^{+j\psi}$$

Where $\psi = d_r \cos\phi$

The relation of this field is indicated by the vector diagram.



From the vector diagram ***the magnitude of the total field*** is

$$E = 2E_0 Cos\frac{\psi}{2} = 2E_0\cos\frac{d_r\cos\phi}{2}$$

as obtained before in(3).

*The phase of the total field E is, however not cons*tan*t in this case but is* $\psi/2$ *, as also shown by rewring* (5) *as*

$$E = E_0(1+e^{j\psi}) = 2E_{0e}{}^{j\psi/2}\left(\frac{e^{j\psi/2}+e^{-j\psi/2}}{2}\right) = 2E_{0e}{}^{j\psi/2}Cos\frac{\psi}{2}$$

Normalizin g by setting $2E_0 = 1$, *then*

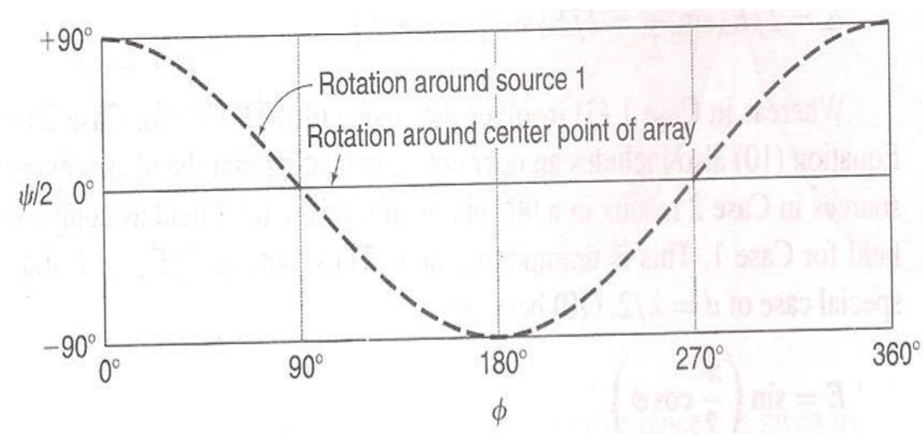
$$E = e^{1\psi/2}\cos\frac{\psi}{2} = \cos\frac{\psi}{2}\angle\psi/2$$

In equation $E = e^{1\psi/2}\cos\frac{\psi}{2} = \cos\frac{\psi}{2}\angle\psi/2$ the term

$\cos\frac{\psi}{2}$ gives amplitude variation and

$e^{j\psi/2}$ gives phase variation with respect to source 1.

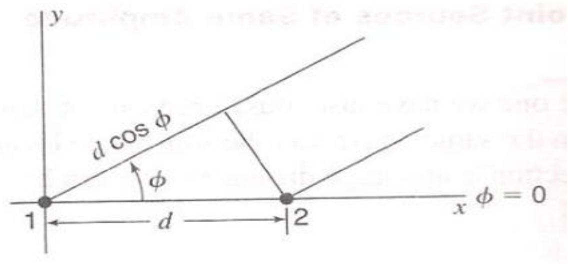
The phase variation for the case of λ/2 spacing ($d_r = \pi$) in shown by the dashed line in the following figure.



Case (2):

Two Isotropic point source of same Amplitude but opposite phase

The two point sources are of equal amplitude but the two sources are in opposite phases.



Let the source be located in Fig 5-15a.

Then the ***total field in the direction φ*** at a large distance r is given by

$$E = E_{0}e^{+j\psi/2} - E_{0}e^{-j\psi/2}$$

from which

$$E = 2jE_0 Sin\frac{\psi}{2} = 2jE_0 \sin\left(\frac{d_r}{2}\cos\phi\right)$$

Here an operator ***j,*** indicating that the ***phase reversal of one of the source*** in case 2 results in ***a 90^0 phase shift of the total field as*** compared with the total field for case 1.

Let put 2j E_0 =1 and considering the special case of d= λ/2, then

$$E = \sin\left(\frac{\pi}{2}\cos\phi\right)$$

In this case the field pattern appers at three cases

- Maximum field at Φ_{max} that is at sin 90, sin270
- Nulls in the field at $\Phi_{null} = \Phi_0$ that is at sin 0, sin180
- Half power in field at Φ at sin 45

So, The direction ϕ_{max} of ***maximum field*** are obtained by equating to $\pm(2k+1)\pi/2$ Thus

$$\frac{\pi}{2}\cos\phi_m = \pm(2k+1)\frac{\pi}{2}$$

$$Where\ k = 0,1,2,3.........$$

$$For\ k = 0, Cos\phi_m = \pm 1\ and\ \phi_m = 0^0\ and\ 180^0$$

$$The\ null\ directions\ \phi_0\ are\ given\ by$$

$$\frac{\pi}{2}\cos\phi_0 = \pm k\pi$$

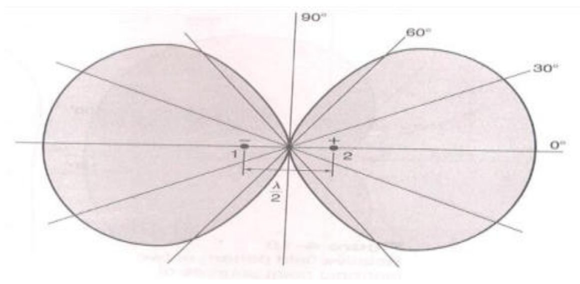
$$For\ k = 0,\ \phi_0 = \pm 90^0$$

$$The\ half\ power\ directions\ are\ given\ by$$

$$\frac{\pi}{2}\cos\phi = \pm(2k+1)\frac{\pi}{4}$$

$$For\ k = 0,\ \phi = \pm 120^0$$

The filed pattern is shown as follows.

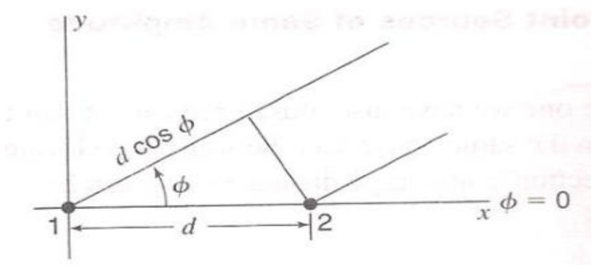


- The ***pattern is*** a relatively ***BROAD FIGURE –OF-EIGHT*** with the maximum field in the same direction as the line joining the source (x axis).

- The ***space pattern*** is a ***figure of revolution of this pattern*** around the x axis.
- This array arrangement is called ***"END-FIRE"*** array.

Case (3): Two Isotropic point sources of the same Amplitude and in phase quadrature :

The two point sources are placed as shown in the figure.



Taking the origin of the coordinates as the reference for phase. Let source I be retarded by 45^0 and source 2 advanced by 45^0.

Then the ***total field in the direction*** ϕ at a large distance ***r*** is given by

$$E = E_0 \exp\left[+j\left(\frac{d_r \cos\phi}{2} + \frac{\pi}{4}\right)\right] + E_0 \exp\left[-j\left(\frac{d_r \cos\phi}{2} + \frac{\pi}{4}\right)\right]$$

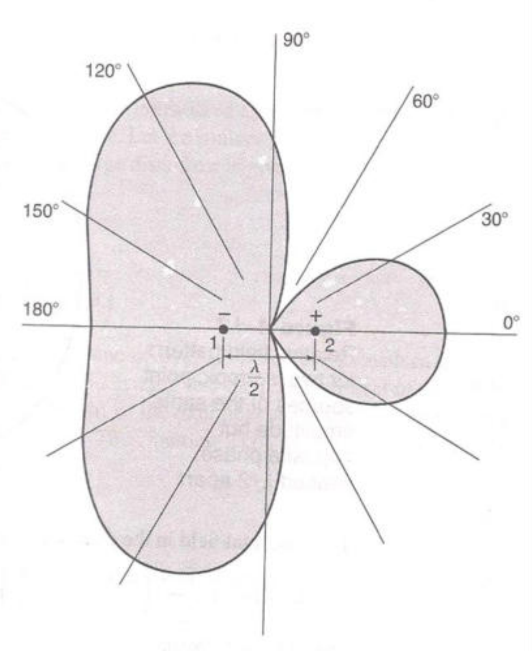
$From(12) we\ obtain$

$$E = 2E_0 Cos\left(\frac{\pi}{4} + \frac{d_r}{2}\cos\phi\right)$$

$Letting\ 2E_0 = 1\ and\ d = \lambda/2, becomes$

$$E = \cos\left(\frac{\pi}{4} + \frac{\pi}{2}\cos\phi\right)$$

The field pattern given is shown as follows..



- The ***space pattern*** is a ***figure-of-revolution of this pattern*** around the x axis.
- Most of the radiation is in the second and third quadrants.

The directions ϕ_m of maximum field are obtained by setting equal to ***kπ***, where k=0,1,2,3…

In this way we obtained

$$\frac{\pi}{4}+\frac{\pi}{2}\cos\phi_m = k\pi$$

$$For\ k=0$$

$$\frac{\pi}{2}\cos\phi m = -\frac{\pi}{4}$$

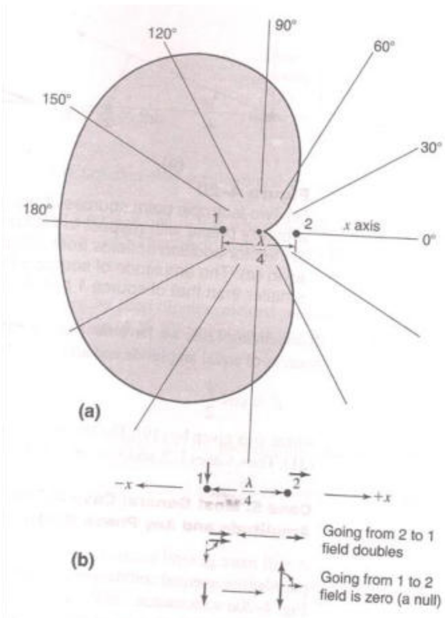
$$and$$

$$\phi_m = 120^0\ and\ 240^0$$

$$If\ the\ spacing\ between\ the\ source\ is\ reduced\ to\ \lambda/4, then$$

$$E=\cos\left(\frac{\pi}{4}+\frac{\pi}{4}\cos\phi\right)$$

The field pattern for this case is shown in the following figure.



- It is a ***CARDIOID-SHAPED, UNIDIRECTIONAL PATTERN*** with maximum field in the negative x direction.
- The ***space pattern*** is a ***figure of revolution of this pattern*** around the x axis.

Case (4):

Two Isotropic point sources of equal Amplitude and any phase difference:

We know that the general phase difference is

$$\psi = d_r \cos \phi + \delta$$

Let $E_1 = E_0 e^{-j\varphi/2}$

$E_2 = E_0 e^{j\varphi/2}$

$E = E_1 + E_2$

$E = E_0 (e^{j\varphi/2} + e^{-j\varphi/2})$

$E = 2 E_0 \cos (\varphi/2)$

$E = E_0 \cos (\varphi/2)$ by normalizing

But we know that $\psi = d_r \cos \phi + \delta$

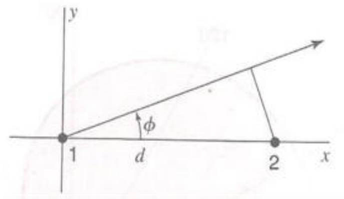
To discuss this, we have to take three special cases when $\delta = 0°$, $\delta = 180°$,

$\delta = 90°$

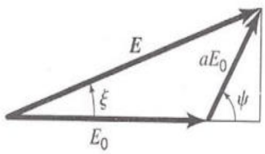
Case (5):

Most general case of two isotropic point source of unequal Amplitude and Any phase Difference

A still more general situation, involving two isotropic point sources, exists when the amplitudes are unequal and the phase difference is arbitrary. Let the source be situated in Fig 5-20a with source 1 at the origin.



Assume that the source 1 has the larger amplitude and that its field at a large distance *r* has amplitude of E_0. Let the field from source 2 be of amplitude a E_0 ($0 \leq a \leq 1$) at the distance r.



Then, referring to Fig, the magnitude and phase angle of the total field E is given by

$$E = E_0 \sqrt{(1 + a\cos\psi)^2 + a^2 \sin^2 \psi\psi} \quad angle \arctan[a \sin \psi /(1 + a \cos \psi)]$$

Where $\Psi = d_r \cos \phi + \delta$ and the phase angle($\angle$) is referred to source 1.

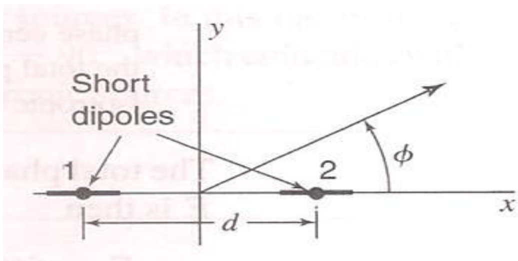
NON ISOTROPIC BUT SIMILAR POINT SOURCES AND THE PRINCIPLE OF PATTERN MULTIPLICATION

Let us arrange an antenna array with non-isotropic point sources.

Any two point sources are said to be **SIMILAR**, if they have the variation with absolute angle ϕ and the ***amplitude and phase of the field is the same***.

Let us consider two point sources which are similar and both sources 1 and 2 have same field patterns given by

$$E_0 = E_0 \, \mathrm{Sin}\phi \quad \text{-----} \; (1)$$



Short diploes oriented parallel to the x axis will produce such pattern.

We know that the total field is

$$E = E_0 \left(e^{j\varphi/2} + e^{-j\varphi/2}\right)$$

Put equation (1) in the above

By normalizing; that is ; set 2E0=1 then it gives

The ***field pattern of the array*** as

$$E = \cos\frac{\psi}{2}$$

So,
$$E = \sin\phi\cos\frac{\psi}{2}$$
$$Where\ \psi = d_r\cos\phi + \delta$$

This result is the same as obtained by multiplying the pattern of the individual source (sin ϕ) by the pattern of two isotropic point sources (cos Ψ).

If the point sources are similar but unequal as in case 5, then their patterns are given as above.

The ***total normalized pattern is***

$$E = \sin\phi\sqrt{(1+a\cos\psi)^2 + a^2\sin^2\psi} \quad \text{---- (3)}$$

Here again, the result is the same as that obtained by multiplying the pattern of the individual source by the pattern of an array of isotropic point sources.

The principle may be applied to arrays of any number of sources provided only that they are similar.

The individual non isotropic source or antenna may be of finite size but can be considered as a point source situated at the point in the antenna to which phase is referred. This point is said to be the ***phase center***.

Pattern of multiplication:

The total field pattern of an array of non isotropic but similar sources is ***the product of*** the individual source pattern and the pattern of an array of isotropic point sources each located at the phase center of the individual source and having same relative amplitude and phase, which the total phase pattern is the sum of the phase patterns of the individual source and the array of isotropic point sources.

The total phase pattern is referred to the phase center of the array. In symbols, the total field E is then

$$\underbrace{E = f(\theta,\phi)\,F(\theta,\phi)}_{Field\ pattern} \quad \underbrace{/\,f_p(\theta,\phi) + F_p(\theta,\phi)}_{Phase Pattern}$$

Where

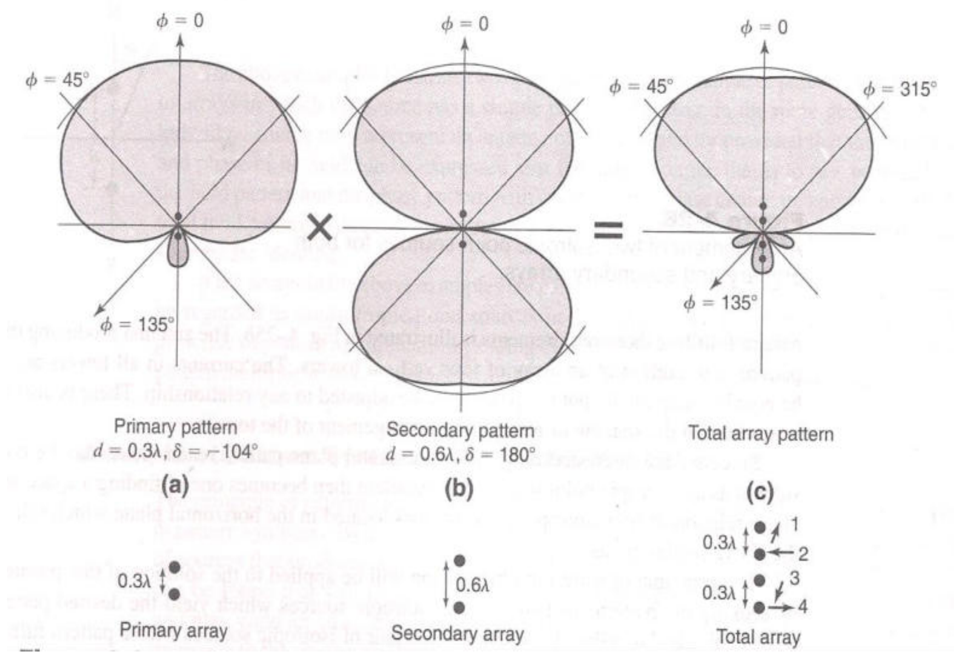
$f(\theta,\phi)$ = field pattern of individual source

$f_p(\theta,\phi)$= phase pattern of individual source

$f(\theta,\phi)$= field pattern of array of isotropic sources

$F_p(\theta,\phi)$= Phase pattern of array of isotropic sources

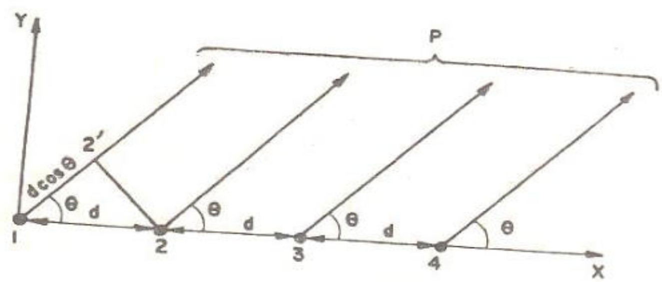
- The principle of pattern multiplication ***applies to space patterns*** as well as to the two-dimensional cases.
- The principle of multiplication of patterns ***provides a Speedy method*** for sketching the pattern of complicated arrays just by inspection. So they are used in design of antenna arrays.
- The ***width of the principle lobe*** and the corresponding width of the ***array pattern are same.***



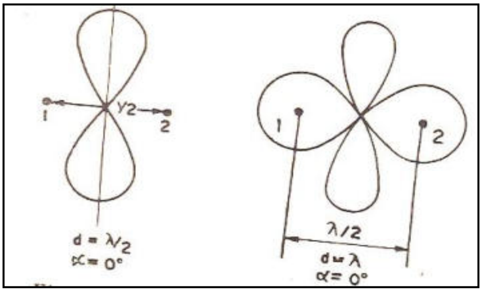
Let us now use the principle to some typical cases.

Radiation Pattern of 4-isotropic elements fed in phase, spaced $\lambda/2$ ***apart:***

Let the 4-elements of isotropic (or non-directive) radiators are in a linear arrays as shown in the figure in which elements are placed at a distance of $\lambda/2$ and are fed in phase. i.e. $\delta = 0$.



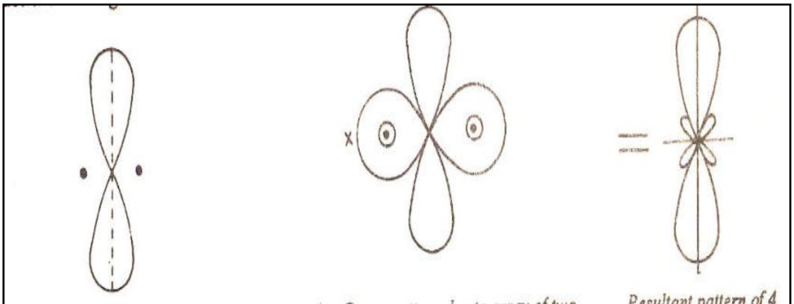
One of the method to get the radiation pattern of the array is to add the fields of individual four elements at a distance point P vectorically but instead an alternative method, using the principle of multiplicity of pattern, will be shown to get the same.



It is already seen that two isotropic point sources, Spaced $\lambda/2$ apart fed in phase provides a ***bidirectional pattern***.

Further, the radiation pattern of two isotropic radiators spaced λ apart, fed in phase is known to be as shown in above figure.

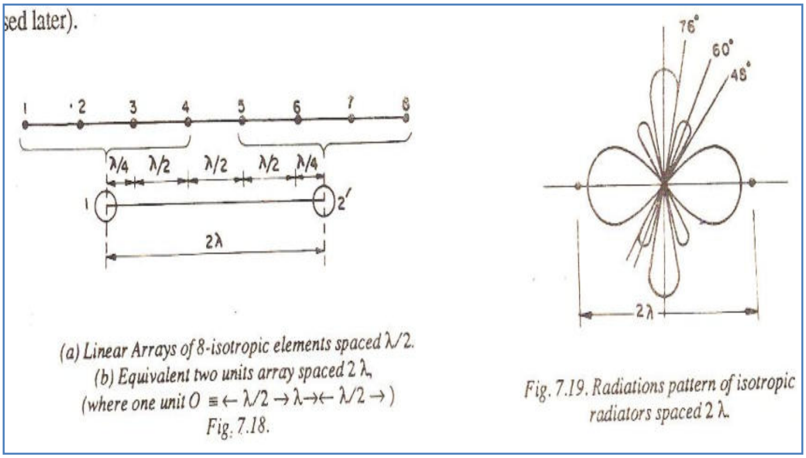
Now elements (1) and (2) are considered as one unit and considered to be placed between the midway of the elements and so also the elements (3) and (4) as another unit assumed to be placed between the two elements as shown in Fig.



Individual (i.e. unit pattern) due to two individual element) | i.e. Group pattern due to array of two (isotropic). | Resultant pattern of 4 isotropic elements.

These two units have the same radiation pattern as two isotropic point sources.

Radiation pattern of 8-isotropic elements fed in phase, spaced $\lambda/2$ apart.



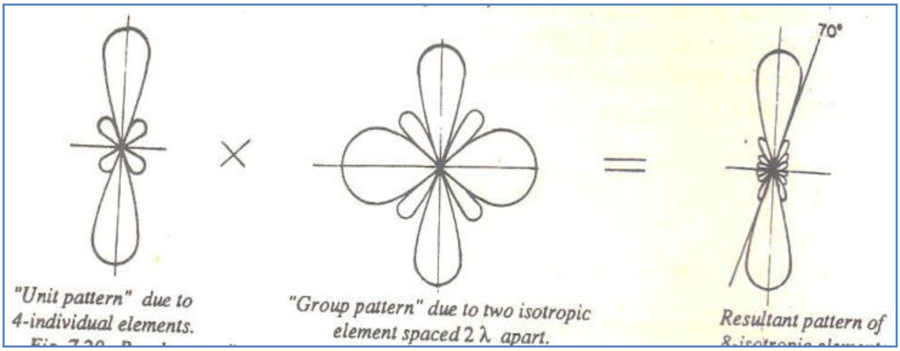
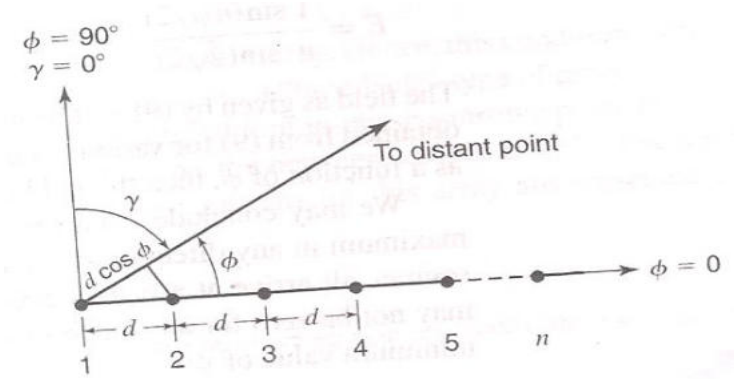


Fig: Radiation pattern of 8 isotropic elements by pattern multiplication

LINEAR ARRAYS OF N ISOTROPIC POINT SOURCES OF EQUAL AMPLITUDE AND SPACING

Let us now proceed to the case of n isotropic point sources of equal amplitude and spacing arranged as a linear array, as shown, where *n* is any positive integer.



The total field E at a large distance in the direction ϕ is given by

$$E=1+e^{j\Psi}+e^{j2\Psi}+e^{j3\Psi}+\ldots+e^{j(n-1)\Psi} \quad \text{------ (1)}$$

The phase angle ξ is with respect to source 1 as phase center.

Where ***Ψ is the total phase difference of the fields*** from adjacent sources as given by

$$\psi = \frac{2\pi d}{\lambda}\cos\phi + \delta = d_r \cos\phi + \delta \quad \text{------ (2)}$$

Where ***δ is the phase difference of adjacent source*** i.e $\phi = 90^0$ source 2 with respect to 1, 3 with respect to 2, etc $\gamma = 0^0$

The amplitudes of the fields from the sources are all ***equal and taken as unity.***

Source 1 is taken as the ***phase reference***. Thus at a distant point in the direction ϕ the field from source 2 is *advanced in phase with respect to source 1 by Ψ* the field from source 3 is advanced in phase with respect to source 1 by 2Ψ etc.

The vector sum of all these fields is given as follows.

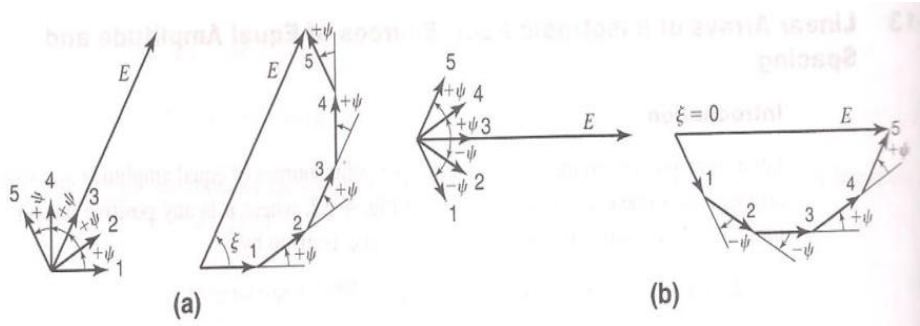


Figure: Vector addition of field at a large distance from the linear

Multiply (1) by $e^{j\Psi}$, giving

$Ee^{j\Psi} = e^{j\Psi} + e^{j2\Psi} + e^{j3\Psi} + \ldots + e^{jn\Psi}$ ----- (3)

Now subtract (3) from (1) and divide by $1 - e^{j\Psi}$, yielding

$$E = \frac{1 - e^{jn\psi}}{1 - e^{j\psi}} \text{--------} (4)$$

Equation (4) *may be rewritten as*

$$E = \frac{e^{jn\psi/2}}{e^{j\psi/2}} \left(\frac{e^{jn\psi/2} - e^{jn\psi/2}}{e^{j\psi/2} - e^{-j\psi/2}} \right)$$

from which

$$E = e^{j\xi} \frac{\sin(n\psi/2)}{\sin(\psi/2)} = \frac{\sin(n\psi/2)}{\sin(\psi/2)} \angle \xi$$

Where ξ *is referred to the field from source* 1.*The value of* ξ *is given by*

$$\xi = \frac{n-1}{2}\psi$$

If the phase is referred to the centerpoint of the array becomes

$$E = \frac{Sin(n\psi/2)}{\sin(\psi/2)}$$

The field as given in above equation will be referred to as the *"Array Factor"*.

In this case the ***phase pattern*** is a ***step function***.

The phase of the field is constant whenever ***E*** has a value but changes sign when ***E*** goes through zero. When $\Psi = 0$; we have

$$\lim_{\psi \to 0} \frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}} = n$$

and by applying limits rule ***E = n.*** This is maximum value that ***E*** that can attain. So the normalized value of the ***total field of*** $E_{max} = n$ is

$$\boxed{E = \frac{1}{n}\frac{\sin(n\Psi/2)}{\sin(\Psi/2)}}$$

This is referred to as ***ARRAY FACTOR***

Array factor concept says that,

We may conclude from the above discussion that

- The field from the array will be a maximum in any direction ϕ for which $\Psi = 0$.
- In other words, the fields from the sources all arrive at a distant point in the same phase when $\Psi=0$.
- In special cases, Ψ may not be zero for any value of ϕ, and in this case the field is usually a maximum at the minimum value of Ψ.

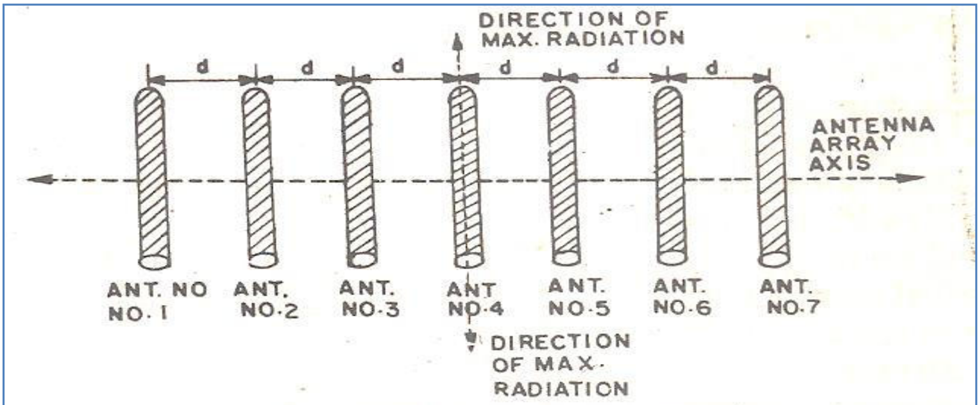
Case (1):

BROADSIDE ARRAY (sources in phase)

Broadside array may be defined as

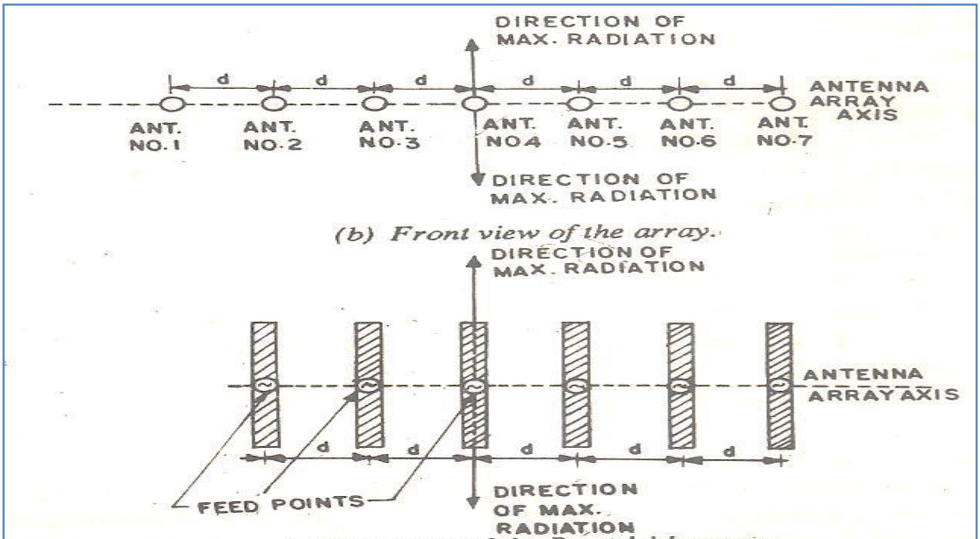
"An arrangement in which the principal direction radiation is perpendicular to the array axis and also to the plane containing the array element".

Broad-side array is one in which a number of identical parallel antennas are set up along a line drawn perpendicular to their respective axes as shown in following figure.



(a) Broadside array arrangement.

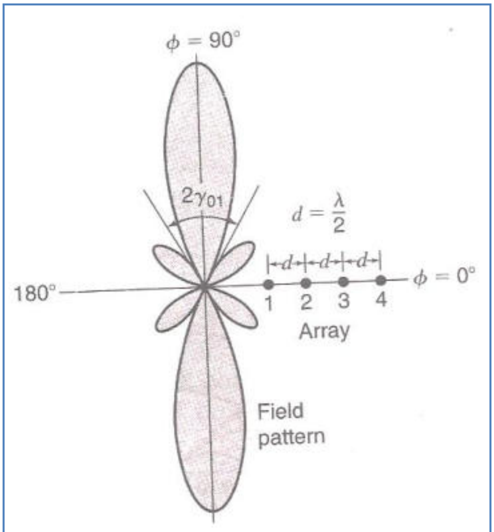
In the broad-side array, individual antennas (or elements) are equally spaced along a line and each element is fed with current of equal magnitude, all in the same.



(b) Front view of the array.

(c) Top view of the Broadside array.

By doing so, this arrangement fires in Broad-side directions (i.e. perpendicular to the line of array axis) where there are maximum radiations and relatively a little radiation in other directions and hence the ***radiation pattern broadside array is bidirectional.***



(a)

The broadside array is bidirectional which radiates equally well in either direction of maximum radiations. Therefore,

The ***bidirectional pattern*** of a broadside array can be ***converted*** into ***unidirectional*** by ***installing an identical array*** behind this array at distance $\lambda/4$ and exciting it by current leading in phase by 90° or $\lambda/2$ radian.

In broad side array ,

δ =0 and so

$\Psi = d_r \cos\phi + \delta$ ----- (1)

To make Ψ =0 it requires that

$\phi = (2k+1)(\pi/2)$, ----- (2)

where k=0,1,2,3…

The field is, therefore, a maximum when

$$\phi = \frac{\pi}{2} \; and \; \frac{3\pi}{2}$$

That is, the ***maximum field is in a direction normal to the array***.

Case (2):

ORDINARY END-FIRE ARRAY

End Fire Array may be defined as "The arrangement in which the principal direction of radiation coincides with the direction of the array axis".

The end-fire array is nothing but broadside array except that individual elements are fed in, out of phase (usually 180^0).

- Thus in the end-fire array, a number of identical antennas are spaced equally along a line and individual elements are fed with currents of equal magnitude but their phases varies progressively along the line in such a way as to make the entire arrangement substantially ***unidirectional.***

An end-fire array is said to form, if two equal radiators are operated in phase quadrature at a distance of $\lambda/4$ apart.

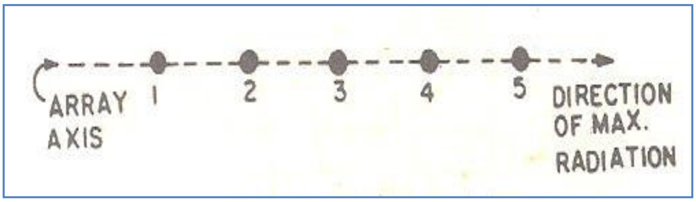


Figure: Front view of an EFA

Let us now find the phase angle between adjacent sources that is required to ***make the field a maximum*** in the direction of the array (ϕ=0).

In EFA, we know the total phase

$\Psi = d_r \cos\phi + \delta$ ----- (1)

Let $\Psi = 0$ then

$0 = d_r \cos\phi + \delta$

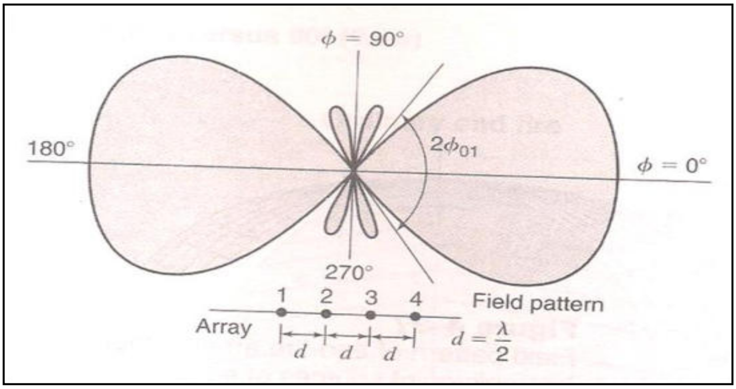
Now at $\phi = 0$; $0 = d_r \cos(0) + \delta$

$$0 = d_r + \delta$$

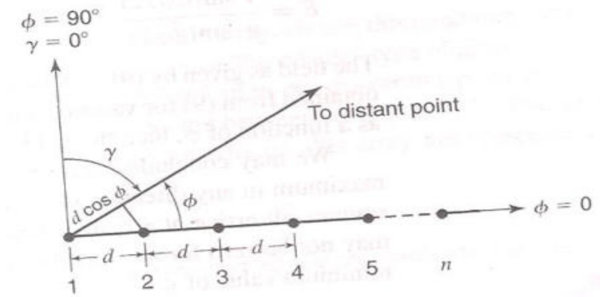
$$\boldsymbol{\delta = - d_r}$$

Hence, for an end-fire array the phase between sources is retarded progressively by the same amount as the spacing between sources in radians.

The Radiation pattern of EFA is shown as follows.



Let us consider four isotropic point sources are arranged on x- axis as shown in figure.



The spacing between sources is $\lambda/2$ and

$$\delta = -d_r$$

$$\text{Where } d_r = \frac{2\pi}{\lambda} d$$

Let d=λ/2 then $\delta = -\pi$. Here ***the resultant field is maximum***

Let d < λ/2 then

The maximum is in the direction of $\phi = 0^0$ when $\delta = -d_r$ and

The maximum is in the direction of $\phi = 180^0$ when $\delta = +d_r$

Case (3):

END-FIRE ARRAY WITH INCREASED DIRECTIVITY (OR) HANSEN WOODYARD EFA

The ordinary EFA for $\delta = -d_r$, produces a maximum field in the direction $\phi = 0$, ***but does not give the maximum directivity***. So in order to attain maximum directivity with the same EFA, just by changing the value of δ it can provide increased Directivity.

Hansen and woodyard EFA gives larger directivity by ***increasing the phase change between sources so that***

$$\delta = -\left(d_r + \frac{\pi}{n}\right)$$

This condition is the condition for "increased directivity" So the total phase difference

$$\Psi = d_r \cos\phi + \delta$$

$$\Psi = d_r \cos\phi - \left(d_r + \frac{\pi}{n}\right)$$

$$\psi = d_r \cos\phi - dr - \frac{\pi}{n}$$

$$\psi = d_r (\cos\phi - 1) - \frac{\pi}{n}$$

The spacing between sources is $d = \lambda/2$

Then $d_r = \frac{2\pi}{\lambda} d$

$d_r = \pi$

We know that

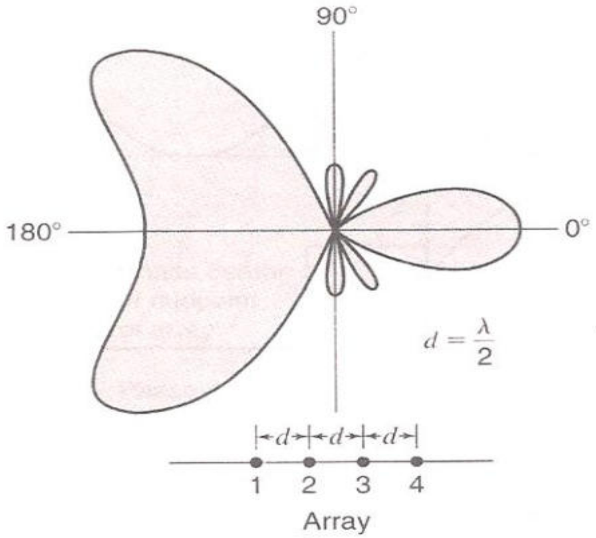
$$\delta = -\left(d_r + \frac{\pi}{n}\right)$$

$$\delta = -\left(\pi + \frac{\pi}{n}\right)$$

When number of sources $n = 4$ then

$$\delta = -\left(\pi + \frac{\pi}{4}\right) = -5\left(\frac{\pi}{4}\right)$$

Now observe the field pattern for δ = - π and δ = -5 $\left(\frac{\pi}{4}\right)$



- ***In Hansen Wood yard EFA*** due to the additional phase difference it yields a considerably ***sharper main lobe*** in the direction $\phi = 0$.
- The ***back lobes in this case are excessively large*** because the large value of spacing results in too great a range inΨ.

❖ In general any increase directivity EFA will have its maximum at $\Psi = -\pi/n$.

❖ In general any increased directivity end-fire array, with maximum at $\Psi = -\pi/2$, has a normalized field pattern given by

$$E = \sin\left(\frac{\pi}{2n}\right)\frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

LINEAR BROADSIDE ARRAYS WITH NON UNIFORM AMPLITUDE DISTRIBUTION

BINOMIAL ARRAYS:

Consider a linear array consisting of five isotropic point sources with λ/2 spacing. If the pattern has equal amplitude and in phase point sources then they may form in broad side array and their radiation pattern has seen in the previous section.

In their radiation pattern it is observed that they consist of some additional minor lobes which are undesirable or unwanted.

So ***To reduce these side lobe levels*** of linear in phase BSA , ***John Stone Stone, has proposed*** that the Sources in the linear array if they have amplitudes proportional to the coefficients of a ***binomial series*** of the form given below.

$$(a+b)^{n-1} = a^{n-1} + (n-1)a^{n-2}b + \frac{(n-1)(n-2)}{2!}a^{(n-3)}b^2 + \cdots$$

Thus for the arrays of three to six sources ***the relative amplitudes*** are given by the following table which is called ***PASCAL TRIANGLE.***

n	Relative amplitudes (Pascal's triangle)
3	1 2 1
4	1 3 3 1
5	1 4 6 4 1
6	1 5 10 10 5 1

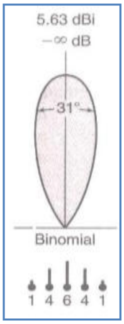
Figure: Pascal triangle

In this triangle any inside number is equal to the sum of the adjacent numbers in row above.

Applying the Binomial distribution to the array of five sources spaced $\lambda/2$ apart, then the sources will have relative amplitudes of

N = 5 1 4 6 4 1

The relative pattern is BINOMIAL



- ❖ This pattern has NO MINOR LOBES but
- ❖ This has increased BEAM WIDTH that is 31°

Even by using EDGE Distribution this increased beam width can be reduced.

In this Edge distribution only the extreme last or edge elements are excited rest of all the in between sources are made to zero amplitudes. Their relative amplitudes are shown below.

N = 1 0 0 0 1

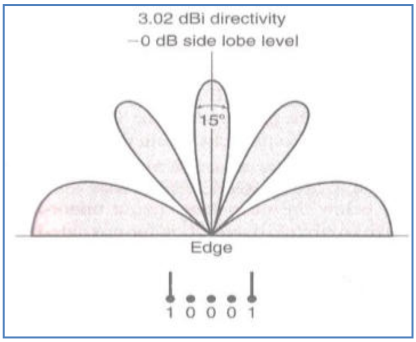


Figure: Edge distribution

For this type of pattern

- the beam width of the major lobe is reduced to 15° but
- it results in Side lobes.

Comparison between the Binomial and Edge distributions are given below.

Distribution	***HPBW***	***Minor lobe amplitude %***
Binomial	31°	0
Edge	15°	100

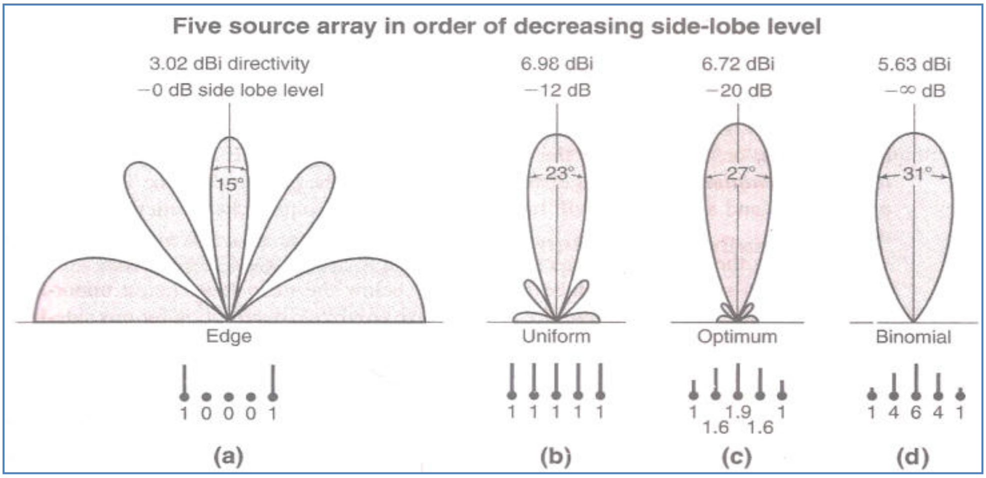
The combination of Binomial and Edge distribution will reduce the beam width of the major lobe but it does not reduce the side lobe level.

An amplitude distribution of this nature for linear in phase BSA proposed by DOLPH which will optimize the relation between beam width and the side lobe level.

- ❖ If the Side Lobe Level is specified then the BWFN can reduced
- ❖ If the BWFN is fixed then Side Lobe Level can be reduced.

Dolphs distribution is based on the properties of Tchebyscheve polynomials. Dolphs distribution includes all the distributions between Binomial and the edge.

- ❖ In fact the Binomial and Edge are special cases of the Tchebyscheve.



An abrupt discontinuity in the distribution results in large minor lobes while a gradually tapered distribution approaching zero at the edge minimizes the discontinuity and minor lobe amplitude.

CHAPTER
3
HF, VHF and UHF
ANTENNAS

Helical Antenna – Helical Geometry, Helix Modes.

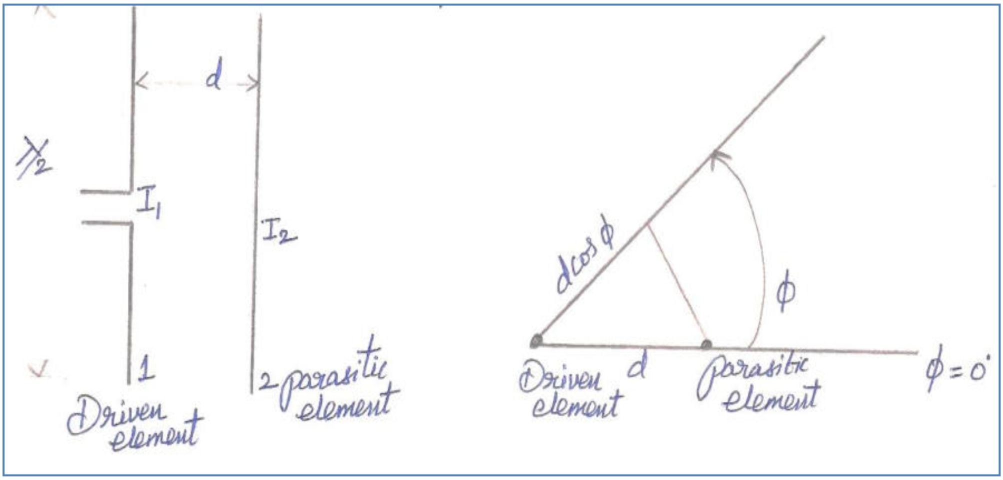
Horn Antennas – Types,
Optimum Horns,
Rectangular Horn Antenna,
Beam width comparison,

ARRAYS WITH PARASITIC ELEMENTS

Antennas can also be constructed with "*parasitic elements*" in which currents are induced by the fields from a driven element. Such elements have no transmission line connection.

Let us consider the case of an array in free space consisting of one driven $\lambda/2$ dipole element (element 1) and one parasitic element (element2), as in Figure.

Consider both elements are vertical so that the azimuth angle ϕ is as shown in figure.



The circuit relations for the elements are

$$V_1 = I_1 Z_{11} + I_2 Z_{12}$$

$$0 = I_2 Z_{22} + I_1 Z_{12}$$

From (2) the current in element 2 is

$$I_2 = I_1 \frac{Z_{12}}{Z_{22}} = I_1 \frac{|Z_{12}|\tau_m}{|Z_{22}|\tau_2} = -I_1 \left| \frac{Z_{12}}{Z_{22}} \tau_m - \tau_2 \right.$$

Or

$$I_2 = I_1 \left| \frac{Z_{12}}{Z_{22}} \xi \right.$$

Where $\xi = \pi + \tau_m - \tau_2$, in which

$$\tau_m = \arctan \frac{X_{12}}{R_{12}}$$

$$\tau_2 = \arctan \frac{X_{12}}{R_{12}}$$

Where

$R_{12} + jX_{i2} = Z_{12} =$ mutual impedance of elements 1 and 2. Ω

$R_{22} + jX_{22} = Z_{22} =$ self-impedance of the parasitic element, Ω

The electric field intensity at a large distance from the array as a function of ϕ is

$$E(\phi) = k(I_1 + I_2 d_r \cos\phi)$$

Where $d_r = \beta d = \frac{2\pi}{\lambda} d$

Substituting (4) for I_2 in (5),

$$E(\phi) = kI_1\left(1 + \left|\frac{Z_{12}}{Z_{22}}\xi + d_r \cos\phi\right.\right)$$

Solving (1) and (2) for the driving-point impedance Z_1 of the driven element, we get

$$Z_1 = Z_{11} - \frac{Z_{12}^2}{Z_{22}} = Z_{11} - \frac{|Z_{12}^2|2\tau_m}{|Z_{22}|\tau_2}$$

The real part of Z_1 is

$$R_1 = R_{11} - \left|\frac{Z_{12}^2}{Z_{22}}\right|\cos(2\tau_m - \tau_2)$$

Adding a term for the effective loss resistance, if any is present, we have

$$R_1 = R_{11} + R_{EL} - \left|\frac{Z_{12}^2}{Z_{22}}\right|\cos(2\tau_m - \tau_2)$$

For a power input *P* to the drive element,

$$h = \sqrt{\frac{P}{R_1}} = \sqrt{\frac{P}{R_{11} + R_{1L} - |Z_{12}^2 / Z_{22}|\cos(2\tau_m - \tau_2)}}$$

And substituting (10) for I_1 in (6) yields the electric field intensity at a large distance from the array as a function of ϕ. Thus,

$$E(\phi) = k\sqrt{\frac{P}{R_{11} + R_{1L} - |Z_{12}^2 / Z_{22}|\cos(2\tau_m - \tau_2)}}\left(1 + \left|\frac{Z_{12}}{Z_{22}}\xi + d_r \cos\phi\right.\right)$$

For a power input P to a single vertical $\lambda/2$ element the electric field intensity at the same distance is

$$E_{HW}(\phi) = kI_0 = k\sqrt{\frac{P}{R_{00} + R_{0L}}}$$

Where

R_{00} = self-resistance of single $\lambda/2$ element, Ω

R_{0L} = loss resistance of single $\lambda/2$ element, Ω

The gain in field intensity (as a function of ϕ) of the array with respect to a single $\lambda/2$ antenna with the same power input is the ratio of (11) to (12). Since $R_{00} = R_{11}$ and letting $R_{0L} = R_{1L}$, we have

$$G_f(\phi)\left[\frac{A}{HW}\right] = \sqrt{\frac{P}{R_{11} + R_{1L} - \left|Z_{12}^2 / Z_{22}\right| \cos(2\tau_m - \tau_2)}}\left(1 + \left|\frac{Z_{12}}{Z_{22}}\xi + d_r \cos\phi\right)\right.$$

When the $\lambda/2$ parasitic as a reflector. When it is capacitive (shorter than its resonant length) it acts as a director.

FOLDED DIPOLE ANTENNA

Folded dipole consists of two half wave dipoles one continuous and other split at the centre, they are folded and joined together in parallel at ends.

The split dipole is fed at the centre by a Balanced transmission line. So the two dipoles have same voltage at the ends.

The Radiation pattern of a Folded dipole and a conventional half wave dipole is same but the input impedence of the Folded dipole is higher.

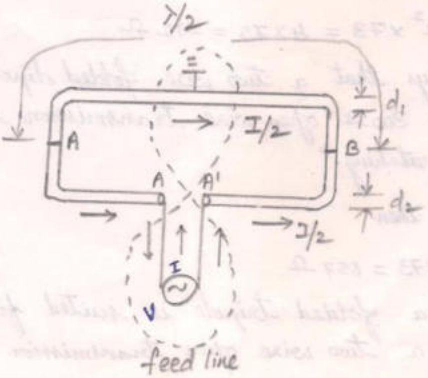
The Folded dipole differs from conventional dipole in two respects or aspects or cases.

(1) Directivity

(2) Broadness in bandwidth.

The directivity of Folded dipole is Bi-directional but because of distribution of currents in folded dipole the Input Impedence becomes higher.

The arrangement of folded dipole is shown as follows.



If the radii of the two conductors are equal, then

- the currents flowing in both conductors are in same direction. ie currents are equal in magnitude & phase.
- The total power in folded dipole is the sum of powers developed in two different dipoles. So the total power of a folded dipole is greater than conventional dipole.
- So input impedence & terminal impedence of folded dipole is greater than conventional dipole.

Consider if the radii of two dipoles are equal, and the the total current fed at terminal AA' is I from fig①. then the currents in each dipoles is $I/2$.

In case if the dipole is straight dipole then the total current will flow through that dipole. Thus with the same power applied, only half the current flows in first dipole and hence input impedence is 4 times the straight dipole.

ie

Input impedence of folded dipole = Square of no. of conductors × Impedence at the conventional dipole.

Eg. If $n=2$ then

$$R_{in} = n^2 \times Z.$$

$$= 2^2 \times 73 = 4 \times 73 = 292\ \Omega$$

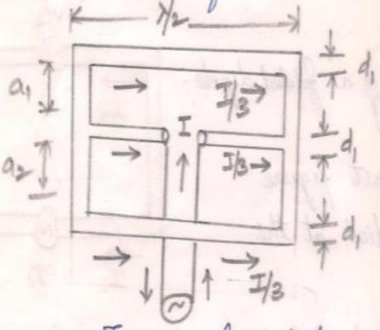
"This says that a two wire folded dipole can be fed with a 300 Ω open wire transmission line without any matching."

Eg. If $n=3$ then

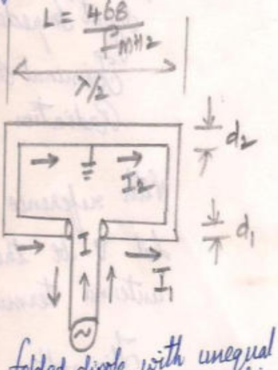
$$R_{in} = 9 \times 73 = 657\ \Omega$$

Thus a folded tripole is suited for matching with a two wire open transmission line of 600 Ω.

ie A Folded dipole antenna has a built in Impedence transforming property. So it is easy to match a transmission line that feeds the antenna.



Three wire folded dipole



folded dipole with unequal radii

The input impedence of a folded dipole can also be changed by " KEEPING THE RADII OF THE TWO DIPOLES UNEQUAL"

- then the currents flows in different directions in folded dipole. This is shown in the following diagram.

* So large current flows in THICKER dipoles and so any required input impedence can be obtained.

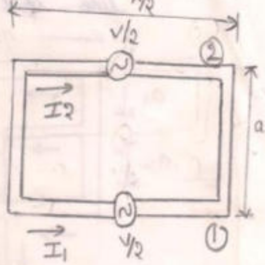
Characteristics of folded dipole:

→* The Folded dipoles does not accept power at any even harmonics of fundamental frequency. Here it act just like a transmission line.

→* The Folded dipole works with low SWR on odd harmonics.

Equation of Input Impedance:

The following figure helps to obtain the equation for
Input Impedance or
Terminal Impedance or
Radiation resistance of a folded dipole.



With reference to the first figure
Let V be the emf applied at the antenna terminals AA'.

This voltage is equally divided in each dipole. So

$$\frac{V}{2} = I_1 Z_{11} + I_2 Z_{12} \quad \text{—①}$$

But $I_1 = I_2$ then

$$\frac{V}{2} = I_1 (Z_{11} + Z_{12}) \quad \text{—②}$$

The two dipoles are very close to each other. then

$$\boxed{Z_{11} \simeq Z_{12}}$$

then

$$\frac{V}{2} = I_1 (2 Z_{11})$$

$$Z = \frac{V}{I_1} = 4 Z_{11} = 4 \times 73$$

$$\boxed{Z = 292\ \Omega}$$

Simillarly for a folded dipole of 3 wires

$$\frac{V}{3} = I_1 (3 Z_{11})$$

$$\frac{V_1}{I_1} = Z = 3^2 Z_{11} = 9 \times 73 = \boxed{657\ \Omega}$$

So generally $\frac{V}{n} = I_1 (n Z_{11})$;

$$\boxed{V = I_1 (n^2 Z_{11})}$$

Impedence Transformation is also possible by making unequal radii of two dipoles, then

$$Z = Z_{11}\left(1+\frac{r_2}{r_1}\right)^2 = 73\left(1+\frac{r_2}{r_1}\right)^2$$

if $r_2 = 2r_1$ then

$$Z = 73\left(1+\frac{2r_1}{r_1}\right)^2 = 73\times 9 = 657\,\Omega$$

The impedence transformation not only depends on the relative radii of conductors but also on

Relative spacing then

$$Z = Z_{11}\left(1+\frac{\log\frac{a}{r_1}}{\log\frac{a}{r_2}}\right)^2 = Z_{11}\cdot Z_{ratio}$$

or

$$Z_r = ITR = \left(1+\frac{\log a/r_1}{\log a/r_2}\right)^2$$

ITR = Impedence Transformation Ratio

" This method is usefull when matching is done with LOW IMPEDENCE ANTENNAS".

Folded dipole : "The Ultra close spaced type of array is called Folded dipole".

Uses of Folded dipoles:

(1) In conjunction with Parasitic elements Folded dipole is used in WIDE BAND OPERATION such as television.

Advantages:

(1) It has high Input Impedence

(2) It has Wide band in frequency

(3) It acts as a built in reactance compensation network.

YAGI-UDA ANTENNA

Yagi Uda antennas are simple and most high gain antennas. Yagi Uda antenna was invented by two scientists Yagi and Uda..

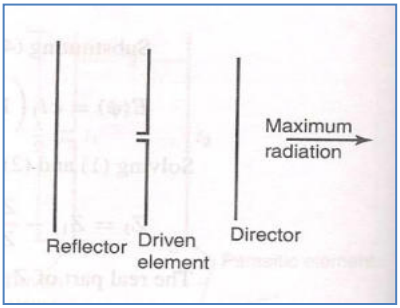


Figure: Yagi Uda antenna elements

Construction or Geometry:

It consists of a driven element, a reflector and one or more Directors. Yagi Uda antenna is an array of a driven element and one or more parasitic elements.

The Driven element is **Resonant Half Wave Dipole** and is generally made up of a metallic rod. The Parasitic elements are also made up of metallic rods and are arranged in parallel to the driven elements at the same line of sight level.

They are arranged **collinearly** and close together as shown in the figure.

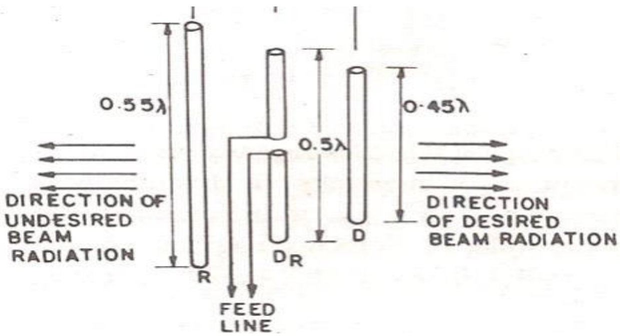
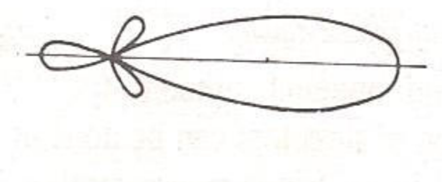


Figure: Yagi Uda antenna Geometry

- Energy is fed to the driven element directly in terms of voltages and it is flown or passed through the parasitic elements.
- Depending on the spacing between the elements the phases and currents are varied .
- The actual spacing between the driven and parasitic elements are order of λ/10, i.e. 0.010 λ to 0.15 λ.



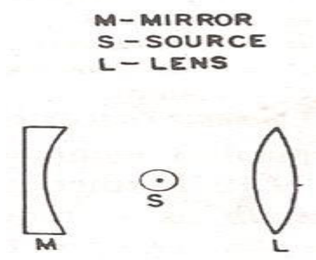
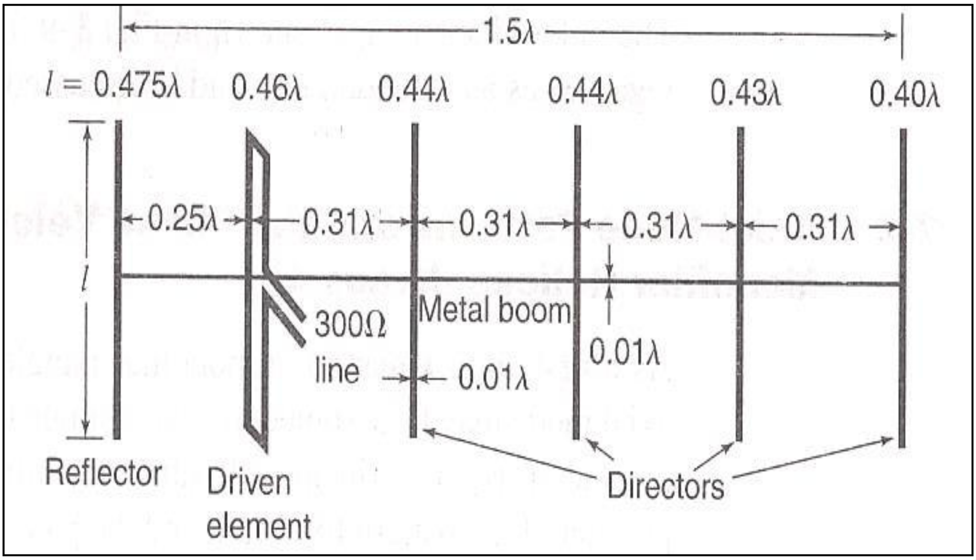


Figure: (a)Radiation pattern of Yagi Uda (b) Arrangement of Yadi Uda



The Parasitic element in front of driven element is known as **Director.** The Parasitic element back to that of driven element is known as **Reflector.**

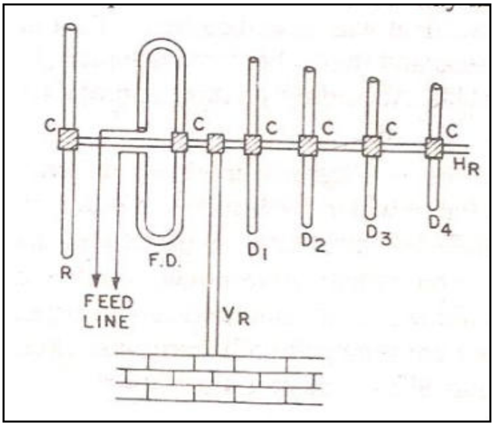
The Reflector is 5% more and the Director is 5% less than the Driven element which is λ/2 at Resonant Frequency**.**

Reflector length =

Driven element length =

Director length =

The Parasitic element can be clamped on a metallic support rod because the middle of each parasitic element the voltage is minimum. That is there has a voltage node.



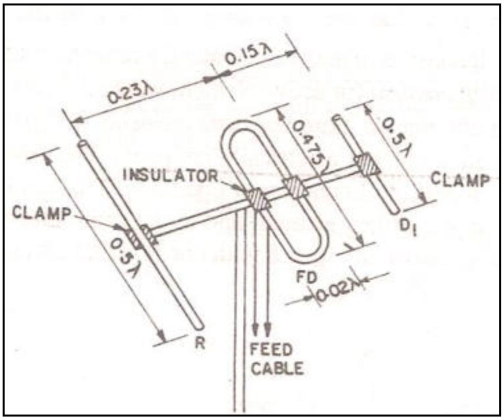


Figure: Practical Yagi Uda antenna Figure: Side View of Yagi Uda

Action of Yagi uda antenna:

- **Shunt feed or the folded dipole** is used to improve the input impedance which is suitable to match the Feed cable.
- To **increase the Directivity** more number of Directors are place in the Yagi uda array**. So additional Gain can be achieved.**
- The spacing between the elements and the length of the parasitic elements determine the **phase of the currents.**

A parasitic element of length greater than or equal to λ/2 will be ***Inductive***. So the phases of the currents will **lag** the induced voltage.

Parasitic element of length less than or equal to λ/2 will be ***Capacitive***. So the phases of the currents will **lead** the induced voltage.

The **spacing** between the two elements must be properly chosen such that good Excitation will be obtained.

❖ If the distance between the Driven element and the Director are greater then greater capacitive reactance is needed to provide correct phasing of parasitic current. Hence the length of the rod is to be **TAPERED** to achieve capacitive reactance.

❖ The <u>Driven element radiated from reflectors to directors.</u> If the spacing between the parasitic elements are not proper then the input impedance will get reduced and in order to overcome this disadvantage Folded dipoles are used which will raise the input impedance.

Characteristics of Yadi uda antennas:

1. If three elements array i.e. one reflector ,one director and one driven element is used then such type of Yagi uda antenna is called ***BEAM ANTENNA.***
2. It is UNI DIRECTIONAL
3. It is of LESS WEIGHT, LOW COST & SIMPLE TO DESIGN.
4. it provides Gain of order 8dB or Front to back ratio of 20 dB.
5. It is known as SUPER DIRECTIVE or SUPER GAIN ANTENNA.
6. It is a FIXED FREQUENCY device.

HELICAL ANTENNA

Helical antenna is the simplest antenna to provide ***circularly polarized waves*** or nearly circularly polarized waves, so that these waves are used in ***Extraterrestrial Communications*** in which satellite relays etc. are involved.

- Helical antenna is ***broad band VHF and UHF*** antenna to provide circular polarization characteristics.

Helical Geometry:

Helical antenna consists of ***a helix of thick copper wire*** or tubing wound in the shape of a ***screw thread*** and used as an antenna in conjunction with a flat metal plate called a ***ground plate.*** It is fed between one end and a ground plane as shown in the below figure.

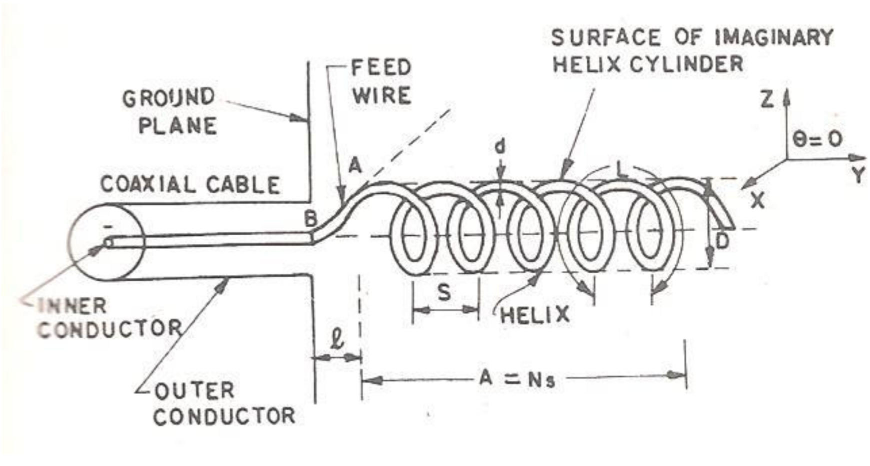


Figure: Helical antenna geometry

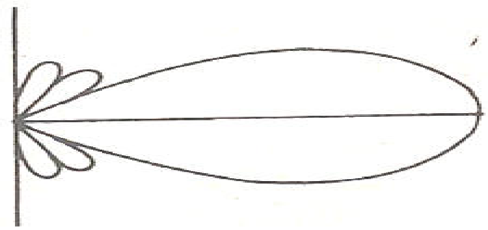


Figure: Radiation Pattern of Helical antenna

The ***ground plane*** is simply made of ***sheet or of screen*** or of radial and concentric conductors. The ***helix is fed by a coaxial cable***. The one end of the helix is connected to the centre conductor of the cable and the outer conductor is connected to the ground plane.

The parameters on which the mode of radiation depends are the diameter of helix D and turn spacing S (center to center).

The **dimensions of the helix** are shown below.

C = Circumference of the helix (π, D)

α = Pitch angle = $\tan^{-1}$ [S/π D]

d = diameter of helix conductor

A = Axial length = NS

N = Number of turns

L = Length of one turn.

l = Spacing of helix from ground plane.

For N turn of helix,

The total length of the antenna is equal to ***NS*** and

Circumference πD.

If one turn of helix is unrolled on a plane surface, the circumference (π, *D*), spacing *S,* turn length *L* and pitch angle α are related by the triangle shown in the above figure.

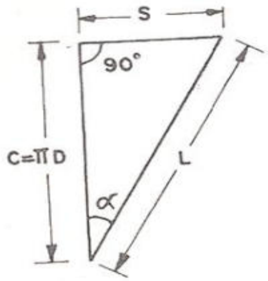


Figure: Interrelation between circumference spacing, turn length and pitch angle

The ***pitch angle*** α is the between a line tangent to the helix wire and the plane normal to the helix axis.

- Pitch angle is an important parameter of the helix and can be calculated from the triangle shown.

The different radiation characteristics are obtained by changing these parameters in relation to wavelength.

Helix action:

Coaxial line is coincident with the helix axis and the feed wire (between point A and B) lies in the plane through helix axis. After the point A the conductor lies in the ***surface of imaginary helix cylinder.*** The helix axial length (A) starts from here .the component of the feed wire length parallel to the axis is "*l*".

This is about a length equal to S/2. The antenna terminal are considered at the point B and all the impedances are referred to this point . the variation of feed wire geometry affects the input impedance of the antenna.

Helix Modes:

A helical antenna may radiate in many modes but prominent modes of radiations are two i.e.

i. Normal or perpendicular mode of radiation and
ii. Axial or end fire or beam mode of radiation.

Normal mode of Radiation

In the normal mode of radiation, the ***radiation field is maximum in broadway*** i.e. in the direction normal to the helix axis and it is circularly polarized or nearly circularly polarized waves.

This mode of radiation is obtained if the dimensions of the helix is small compared with ***wavelength i.e.*** $NL \ll \lambda$. However the ***bandwidth of such a small helix is very narrow*** and the radiation efficiency is low.

The bandwidth and ***radiation efficiency can be increased*** by ***increasing the size of helix*** and to have the current in phase along axis, some type of ***PHASE SHIFTER*** at intervals are required which put practical limitations.

Helical antenna Radiation pattern in normal mode:

The radiation pattern is a combination of the equivalent ***radiation from a short dipole positioned on the same helix axis and a small loop*** which is also coaxial with the helix axis.

It is because ***pitch angle α = 0*** corresponds to ***loop*** and when ***α = 90° the helix becomes a linear antenna*** as shown in the below figure.

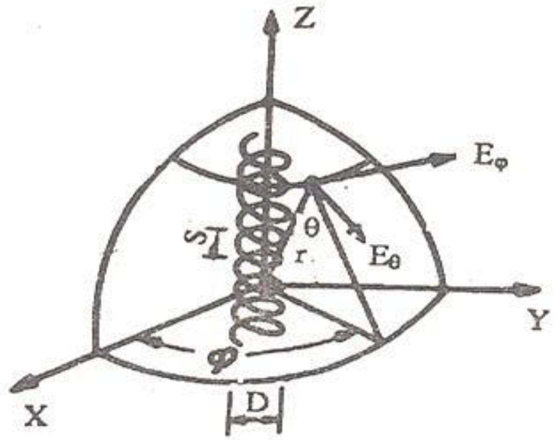
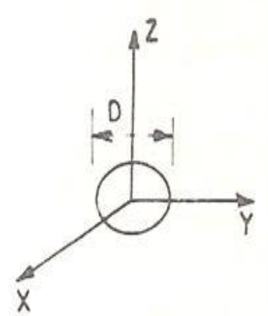


Figure: Helix in 3 dimensional spherical coordinates



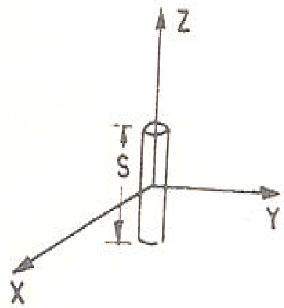


Figure: loop α = 0 Figure: Short dipole S = constant α = 90

Limiting cases of Helix:

The loop and linear antenna are the limiting cases of the helix. Thus in a helix of fixed diameter,

i. S tends to → 0 helix collapses to a loop and
ii. If S is a constant and D tends to → 0, then the helix straightens into a linear conductor (short dipole).

The ***radiation patterns*** of these two loop and dipoles antennas ***are same,*** but their polarizations ***are at right angles to each other*** and their phase angles at any point in the space are at 90˚ apart.

Therefore the ***resultant antenna field*** is either ***circularly polarized or elliptically polarized*** depending upon the field strength ratio or the amplitudes of the two components, which in turn depends on the pitch angle α.

- If α is small, then radiation field is mainly due to loop antenna
- In limiting conditions polarizations of the loop antenna and the dipole antenna are linear.

Finally it is concluded that an helix antenna is consisting of a number of ***small loops and short dipoles connected in series*** in which loop diameter is same as helix diameter and the helix spacing S is same as dipole length. Then

The ***far field of the small loop*** is given by equation

$$E_\varphi = \frac{120\,\pi^2\,[I]\sin\theta}{r}\cdot\frac{A}{\lambda^2}$$

$[I]$ = Retarded current.

r = The distance.

A = Area of loop $= \pi D^2/4$

The ***far field of short dipole*** is given as

$$E_\theta = j\,\frac{60\,\pi\,[I]\sin\theta}{r}\cdot\frac{S}{\lambda}$$

$S = L$ = length of dipole.

The above equations show that there is 90° phase between them due to presence of *j* operator.

Axial Ratio: " The ratio of magnitudes of these equations provides axial ratio (AR) of the Elliptical polarization.

$$AR = \frac{|E_\theta|}{|E_\varphi|} = \frac{\left|\dfrac{j\,60\,\pi\,[I]\sin\theta\cdot S}{\lambda\, r}\right|}{\left|\dfrac{120\,\pi^2\,[I]\sin\theta}{r}\cdot\dfrac{A}{\lambda^2}\right|} = \frac{S\lambda}{2\pi A} = \frac{2S\lambda}{\pi^2 D^2} \qquad \Bigg| \quad \because A = \frac{\pi D^2}{4}$$

$$\boxed{AR = \frac{2S\lambda}{\pi^2 D^2} = \text{Axial Ratio}}$$

When

- AR = 0 ; Elliptical polarization becomes linear horizontal polarization,
- AR = ∞ ; Elliptical polarization becomes linear vertical polarization,
- AR = 1 ; Elliptical polarization becomes circular polarization.

Thus ***for circular polarization***

$$AR = 1 = \frac{|E_\theta|}{|E_\varphi|} \quad \text{or} \quad |E_\theta| = |E_\varphi|$$

$$|2S\lambda| = \left|\pi^2 D^2\right|$$

$$\boxed{\begin{aligned} S &= \frac{\pi^2 D^2}{2\lambda} \\ S &= \frac{C^2}{2\lambda} \end{aligned}}$$

$$\alpha = \tan^{-1}\left(\frac{\pi^2 D^2}{2\lambda \cdot \pi D}\right)$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{\pi D}{2\lambda}\right)}$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{C}{2\lambda}\right)}$$

This is the condition for pitch angle to get circular polarization.

The radiation pattern in normal mode, for circular polarization is shown in the below figure.

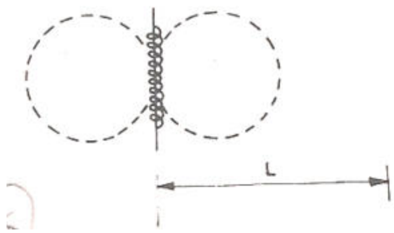


Figure: Radiation for Normal mode of helix

Disadvantage of Normal mode of Helix:

- ✓ This is mode of operation is very narrow in band width
- ✓ Its radiation efficiency is very small.
- ✓ Practically this mode of operation is limited and it is hardly used.

Axial mode or Beam mode of Radiation.

In Axial or beam mode of radiation, the radiation field is ***maximum in the end fire direction***. i.e. along the helix axis and the ***polarization is circular or nearly circular.***

- This mode occurs when the helix circumference (D) and spacing (S) should be in the order of one wavelength.
- This mode is more interesting as it produces a ***broad and fairly directional beam in the axial direction*** with minor lobes at oblique angles.
- The axial mode of radiation is produced very comfortably simply by increasing the helix circumference $\frac{C}{\lambda}$ to the of the order of one wave length and spacing approximately of $\frac{\lambda}{4}$
- The helix is operated in conjunction with a ground plane and is fed by a coaxial cable. The ground plane is at least half wave length in diameter.
- The pitch angle α varies from 12 degrees to 18 degrees and about 14 degrees is optimum pitch angle.

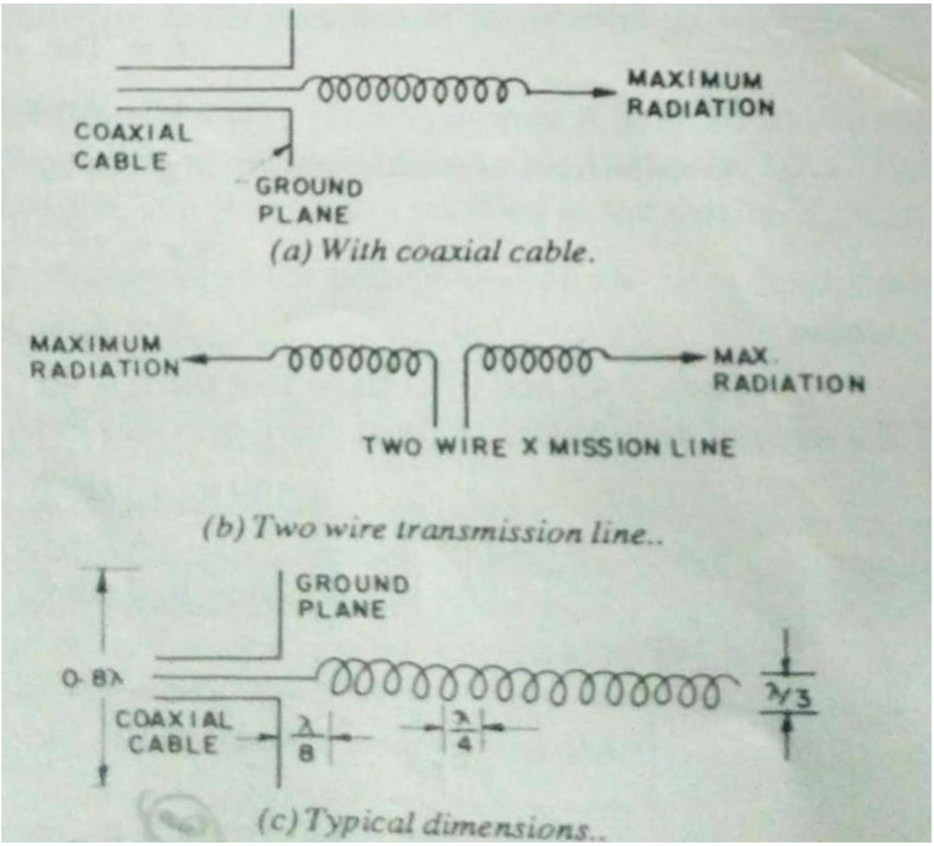


Fig: Arrangement for generating axial mode of helix operation.

The antenna gain and beam width depends upon the helix length NS. The terminal impedance is 100 Ω resistive, at frequency C = λ and at higher and lower frequencies the resistive values changes followed by reactive components.

- ✓ In general, in axial mode the terminal impedance of the helical antenna lies between 100Ω to 200Ω pure resistive.

The terminal impedance of the helix in axial mode with within $\pm 20\%$ approximation, can be given by

$$\boxed{R = \frac{140\,C}{\lambda}\ \text{ohms}}$$

In the axial mode the beam pattern has axial symmetry i.e. it is same in any plane containing the axis. In 3dimensional spherical coordinate with $\theta = 0$ axis coincident with helix axis. The pattern does not depend on angle φ.

Based on number of pattern measurement on helical antenna, the beam width between half power points is given by

$$\boxed{(HPBW)\ \theta_{(-3db)} = \frac{52}{C}\sqrt{\frac{\lambda^3}{NS}}\ \text{degrees}}$$

λ = free space wavelength.
C = circumference.
N = number of turns.
S = spacing.

The beam width between the first nulls is given by

$$\boxed{BWFN = \frac{115}{C}\sqrt{\frac{\lambda^3}{NS}}\ \text{degree}}$$

The maximum directive gain (Directivity) of a axial mode is given by

$$\boxed{D = \frac{15\,NSC^2}{\lambda^3}}$$

And the Axial Ratio is

$$\boxed{AR = 1 + \frac{1}{2N}}$$

And also the Normalized far field pattern is given by

$$E = \sin\left(\frac{\pi}{2N}\right)\cos\theta \cdot \frac{\sin\left(\frac{N\varphi}{2}\right)}{\sin\varphi/2}$$

$$\psi = 2\pi\left[\frac{S}{\lambda}(1-\cos\theta)+\frac{1}{2N}\right]$$

The above formula assumes α = 12 degrees to 15 degrees.

$$N \geq 3$$

$$NS \leq 10 \text{ and}$$

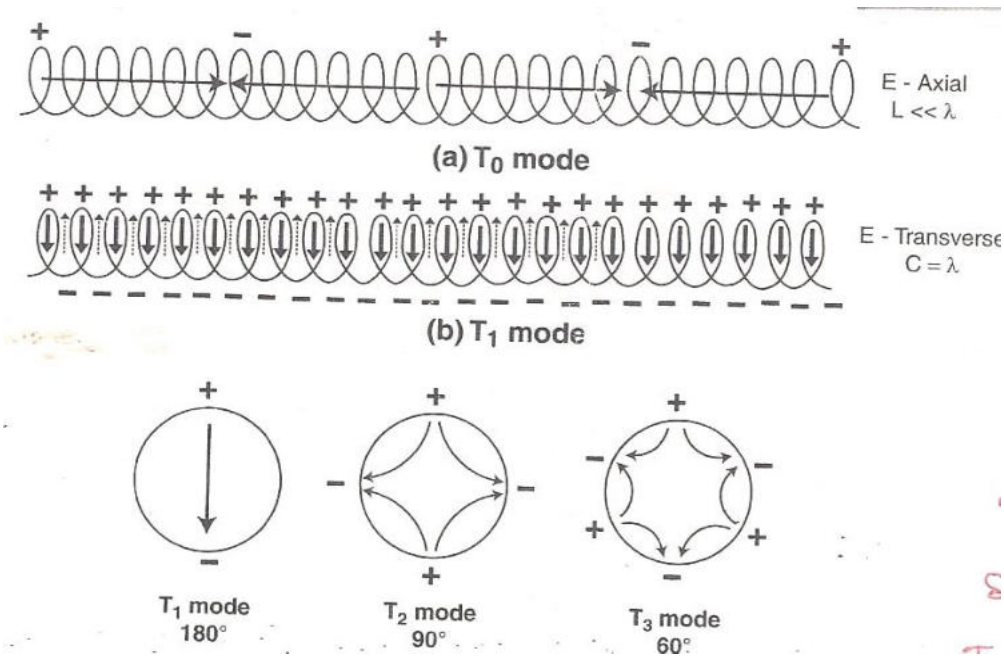
$$C = \frac{3}{4}\lambda \text{ to } \frac{4}{3}\lambda$$

The Helix Modes:

There are two different types of Helix modes namely

1. Transmission (T) Mode
2. Radiation (R) Mode

Transmission (T) Mode: Transmission (T) Mode describes how the electromagnetic wave propagates through an infinite helix.
A variety of different transmission modes are shown in Fig.



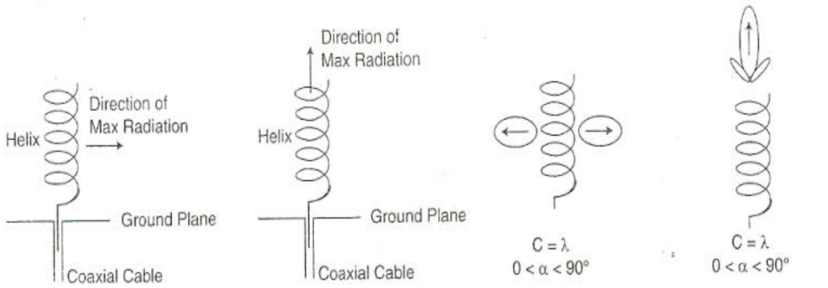
- T_0 is the lowest mode wherein the charges are separated by several turns.
- T_1 is a higher mode than T_0 and the charges are separated by only one turn.
- The modes T_2, T_3, etc., are still higher modes.
- In T_2, charges change their polarity twice in one turn or are separated by 90°, in T_3 by 60° and in T_m by π / m degrees where m is the order of the mode.

Radiation (R) Mode:

It describes the general form of the far field pattern of a finite helical antenna. There are many kinds of R modes modes. The few modes are given as follows.

1. Normal or omni mode of radiation is denoted by R_0 in which the radiation beam is perpendicular to the helix axis.

2. Axial or beam mode of radiation is denoted by R_1 in which the radiation beam is parallel to the helix axis.



(a) R_0 mode (b) R_1 mode (c) T_0 R_1 mode (d) T_1 R_1 mode

Applications of Helical antenna:

1. The dimensions of Helix are critical in normal mode and it limits the band width of antenna and also the Radiation efficiency is limited. But for Axial mode dimensions are not critical and hence Radiation efficiency can be increased. **So Axial mode helical antennas are used to achieve circularly polarized waves.**

2. As Axial mode Helical antennas has wide bandwidth so Directivity and Gain both increases. **So they are used in Space communications because of circular polarization.**

3. Single Helical antenna or array of Helical antenna are useful in **Receiving VHF signals through ionosphere.**

4. Helical antennas are **used in Satellite communications and Space probe communications.**

HORN ANTENNAS

INTRODUCTION:

Horn antenna is considered as a flared-out (or opened-out) waveguide. The function of the horn is to produce a ***uniform phase front*** with a larger aperture than that of the waveguide and hence it gives ***greater directivity***.

The Horn antenna is most widely used simplest form of the microwave antenna. The Horn antenna serves as a feed element for large radio astronomy, communication dishes and satellite tracking throughout the world.

- As it is widely used at microwave frequencies, it may be considered as an ***Aperture antenna.***

"The Horn antenna can be considered as a wave guide with hallow pipe of different cross sections which is flared or tapered into large opening. When one end of the wave guide is excited while other end is kept open, it radiates in open space in all directions. As compared with the radiations in transmission line, the radiation through the wave guide is larger".

In wave guide, the smaller amount of power in the incident wave is radiated, while due to open circuit at other end larger amount of power is reflected back. As one end of wave guide is open circuited, the impedance matching with the free space is not perfect. At the edges of the wave guide, the diffraction takes place which results in poor radiation. Also the radiation pattern is ***NON DIRECTIVE***.

In order to overcome the limitations, the mouth of the wave guide is flared or opened out such that it assumes shape like horn. The advantage of terminating wave guide into an electromagnetic horn is that instead of open circuit at one end of the wave guide, properly shaped gradual transition takes place.

Under the condition of proper impedance matching, the total power incident will be radiated in forward direction. Thus the radiation is increased. As the edges are flared out the diffraction at the edges reduces and thus directivity increases.

TYPES OF HORN ANTENNAS

Several types of horn antennas are shown in the following figures. Antennas that are shown here are rectangular horns. All are energized from rectangular wave guides.

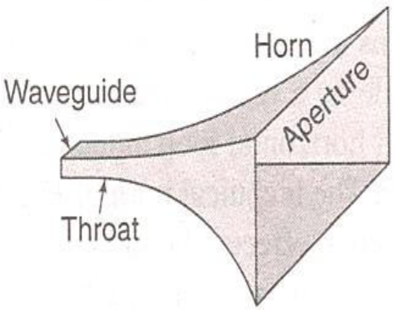
To minimize reflections of the guided wave, the transition region or horn between the waveguide at the throat and free space at the aperture could be given a gradual exponential taper as in Fig (a).

Sectoral horns of rectangular types with a flare in only one dimension are shown in the figures.

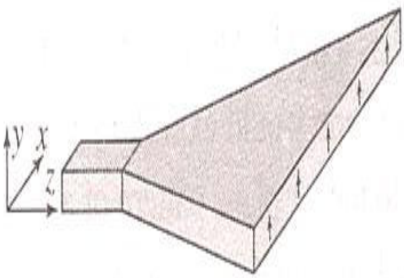
Assume that the rectangular waveguide is energized with a TE_{10} mode wave, the electric field (E in the *y* direction),

Horn in Fig. (*b*) is flared out in a plane perpendicular to E. This is the plane of the magnetic field H. Hence, this type of horn is called a Sectoral horn flared in the *H* plane or simply an ***H-plane Sectoral horn.***

RECTANGULAR HORNS

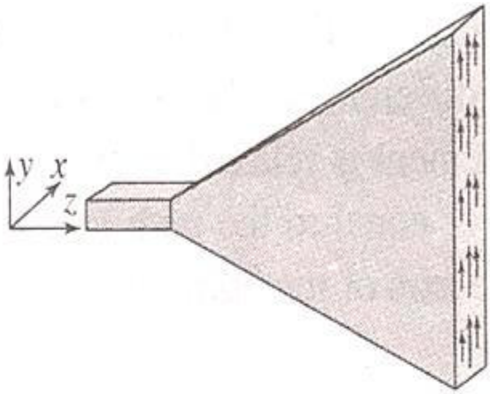


(a) Exponentially tapered pyramidal



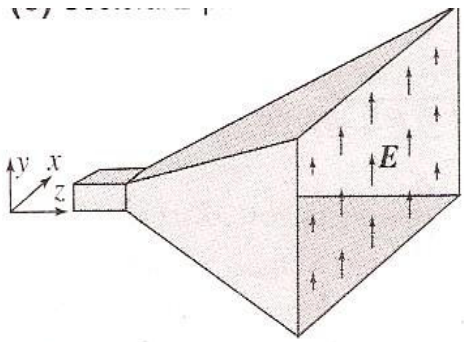
(b) Sectoral *H*-plane

The horn in Fig (c) is flared out in the plane of the electric field E, and, hence, is called an ***E-plane Sectoral horn***.



(c) Sectoral *E*-plane

A rectangular horn with flare in both planes, as in Fig (d), is called a ***pyramidal horn.***



(d) Pyramidal

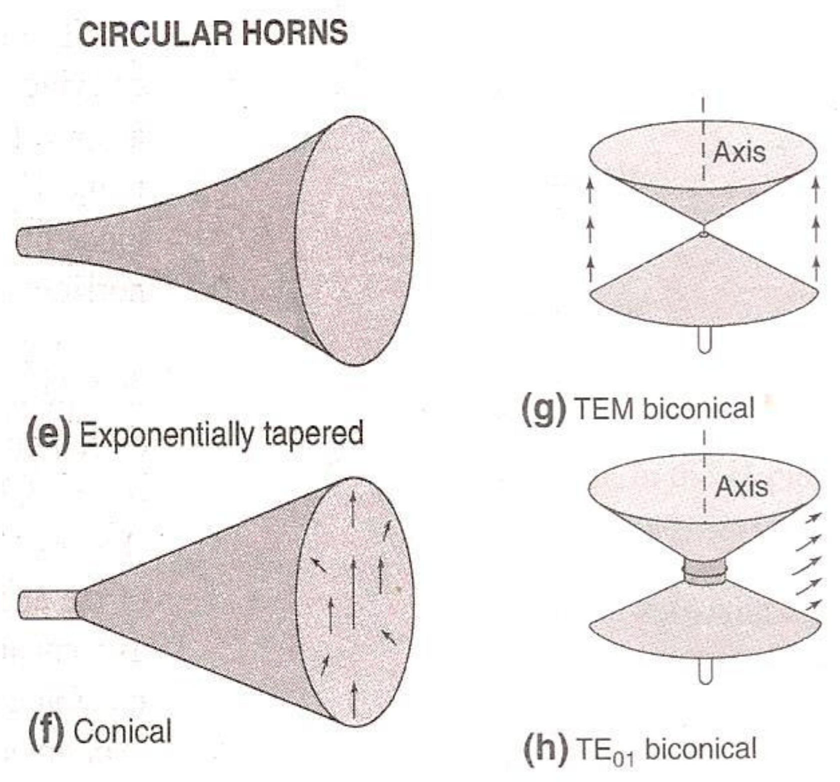
With a TE_{10} wave in the waveguide the magnitude of the electric field is quite uniform in the *y* direction across the apertures of the horns of Fig. *b, c* and *d* but tapers to zero in the *x* direction across the aperture.

This variation of electric and magnetic fields are shown by the arrows at the apertures in Fig. *b, c* and *d*. <u>The arrows indicate the direction of the electric field E.</u>

For small flare angles the field variation across the aperture of the rectangular horns is similar to the sinusoidal distribution of the TE_{10} mode across the waveguide.

The second section of the Horn antennas is Circular horns. These are made from circular wave guides. The dominant mode in circular waveguides is TE_{11} modes. Few Circular Horn antennas are shown in the following figures.

The horn shown in Fig *f* is a ***conical type***. When excited with a circular guide carrying a TE_{11} mode wave, the electric field distribution at the aperture is as shown by the arrows.



The horns in Fig *g* and *h* are ***biconical*** types. This is excited in the TEM mode by a vertical radiator while the one in Fig *h* is excited in the TE_{01} mode by a small horizontal loop antenna.

- *These bi-conical horns are non directional in the horizontal plane.*

FERMAT'S PRINCIPLE:

Principle of equality of path length (Fermat's principles) is applicable to the Horn antenna. Generally it needs a constant phase across the Horn mouth but instead of a constant phase across the Horn mouth, the Horn aperture may be used such that the phase may deviate, but this deviation is less than a specified amount δ, which is equal to the path length difference between a ray traveling along the side and along the axis of the horn.

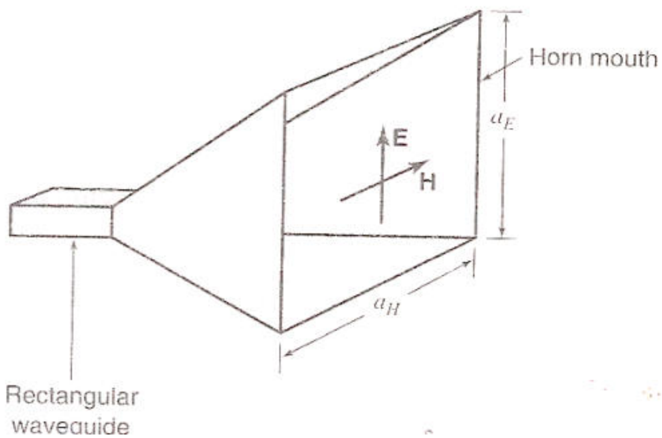


Figure: Pyramidal horn antenna to explain Fermat's principle

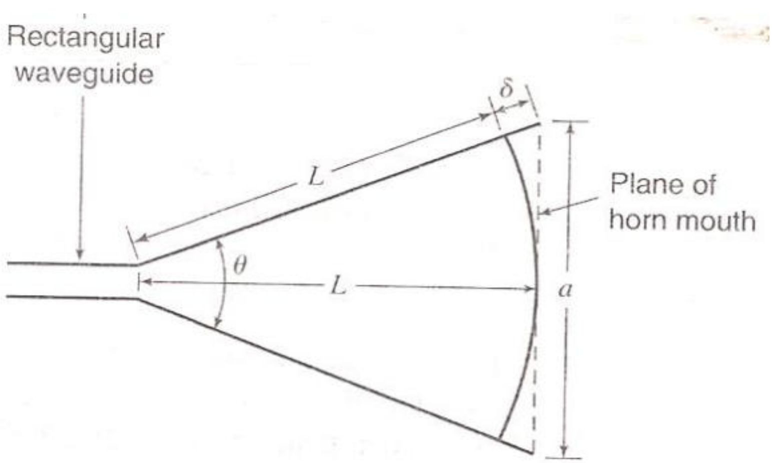


Figure: Cross sectional view to explain Fermat's principle

From the figure we can analyze that

$$\cos\frac{\theta}{2} = \frac{L}{L+\delta} \qquad \ldots\ldots (1)$$

$$\sin\frac{\theta}{2} = \frac{a}{2(L+\delta)} \qquad \ldots\ldots (2)$$

$$\tan\frac{\theta}{2} = \frac{a}{2L} \qquad \ldots\ldots (3)$$

Where

θ = flare angle (θ_E for E Plane, θ_H for H plane), deg
a = aperture (a_E for E plane, a_H for H plane), m
L = horn length, m
δ = path length difference, m

From the geometry we have also that

$$L = \frac{a^2}{8\delta} \quad (\delta << L) \quad \text{and} \qquad \ldots\ldots (4)$$

$$\theta = 2\tan^{-1}\frac{a}{2L} = 2\cos^{-1}\frac{L}{L+\delta} \qquad \ldots\ldots (5)$$

- In the E plane of the horn $\delta \leq 0.25\lambda$.
- In the H plane δ can be larger or about 0.4λ.
- If δ is a sufficiently small fraction of a wavelength, the field has nearly uniform phase over the entire aperture.

OPTIMUM HORNS

To obtain as uniform an aperture distribution as possible, a very long horn with a small flare angle is required. But theoretically the horn should be as short as possible. An *optimum horn* is in between these extremes and has the minimum beam width without excessive side-lobe level (or most gain) for a given length.

- ***For a constant length L,*** the directivity of the horn increases (beam width decreases) as the aperture ***a*** and flare angle θ are increased.

- *If the aperture **a** and flare angle θ become so large that δ **is equal to 180°**,* then the field at the edge of the aperture is in phase opposition to the field on the axis. And this **reduces the directivity by increases side lobes.**

- ***For all very large flare angles the ratio*** $L/(L+\delta)$ **is nearly equal to unity** such that the effect of the additional path length δ on the distribution of the field magnitude can be neglected.

"The maximum directivity occurs at the largest flare angle for which δ does not exceed a certain value (δ_0)". Thus, from (1) the ***optimum horn dimensions*** can be related by

Optimum Horn Dimensions as

$$\delta_0 = \frac{L}{\cos(\theta/2)} - L = \quad \text{Optimum } \delta \quad \text{-------} \ (6)$$

$$L = \frac{\delta_0 \cos(\theta/2)}{1-\cos(\theta/2)} = \quad \text{Optimum Length} \quad \text{-------} \ (7)$$

The value of δ_0 must be usually in the range of 0.1 to 0.4 free-space wavelength.

Let for an optimum horn $\delta_0 = 0.25\lambda$ and that the axial length $L = 10\lambda$ then $\theta = 25^o$. This flare angle then results in the maximum directivity for a 10λ horn.

Limitation of Horn Antenna:

1. Higher order modes of transmission in the horn must be suppressed for uniform aperture illumination.
2. To do so the width of the waveguide at the throat of the horn must be between $\lambda/2$ and 1λ.
3. If the feed given to horn is symmetrical, then even modes are not energized, and the width of the waveguide at the throat of the horn must be between $\lambda/2$ and $3\lambda/2$.

CHAPTER 4

MICROWAVE ANTENNAS

Reflector Antennas
Lens Antennas

REFLECTOR ANTENNAS

INTRODUCTION

Reflectors are widely used to modify the radiation pattern of a radiating element. For example, the backward radiation from an antenna may be eliminated with a plane sheet reflector of large enough dimensions. Several reflector types are shown in the following Figure.

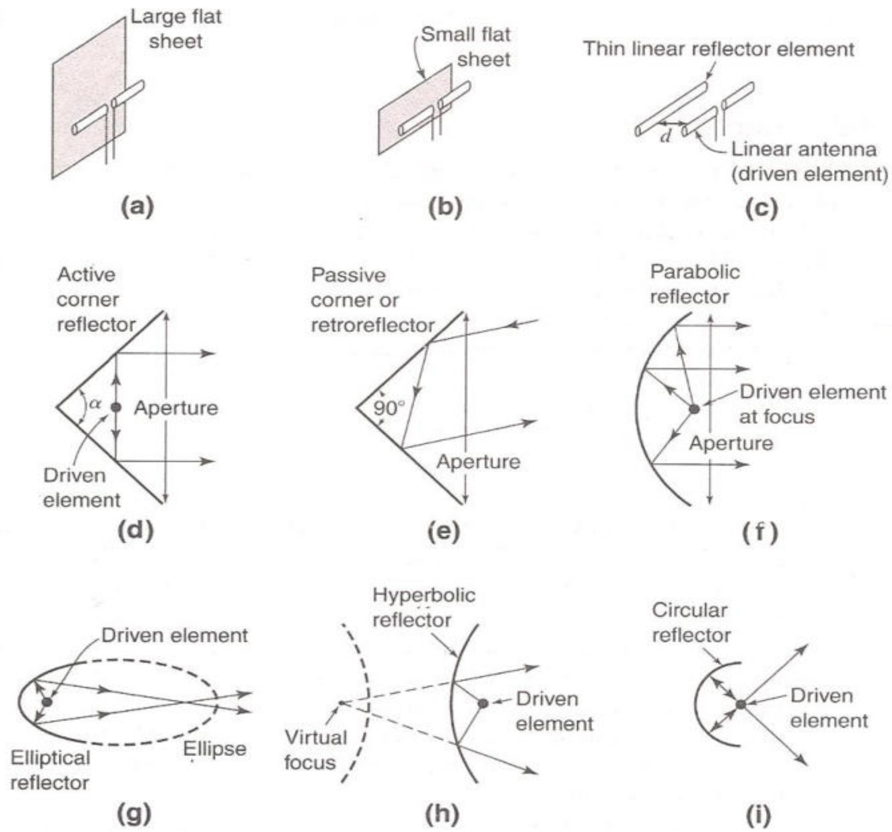
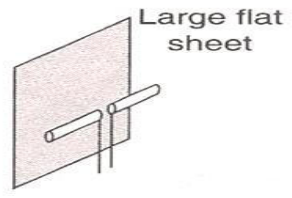


Figure: 5.1 various types of Reflectors

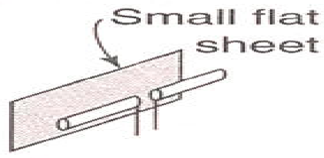
Flat sheet Reflector:

The arrangement in Fig.*(a)* has a large, flat sheet reflector near a linear dipole antenna ***to reduce the backward radiation***



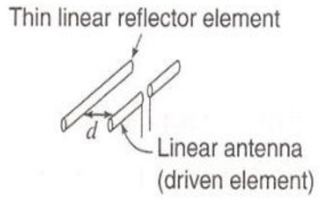
(a)

With small spacing's between the antenna and sheet this arrangement also yields a substantial gain in the forward radiation.



(b)

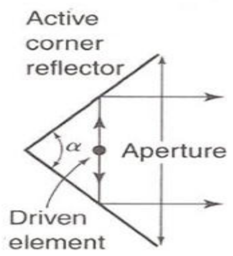
The good and better properties of the sheet reflector can also be obtained with the reflector of reduced in size, which is shown in fig (b)



(c)

The limiting case of flat sheet reflector is shown in the Fig.*(c)*.
Here the sheet has degenerated into a thin reflector element.

- The properties of the large sheet are relatively insensitive to small frequency changes,
- The thin reflector element is highly sensitive to frequency changes.

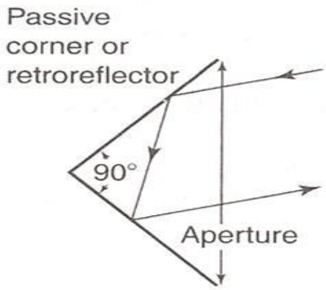


(d)

Corner Reflector:

With two flat sheets intersecting at an angle $\alpha(<180^o)$ as in Figure *(d)*, ***a sharper radiation pattern*** than from a flat sheet reflector ($\alpha = 180^o$) can be obtained.
This arrangement, called an ***Active corner reflector antenna.***

It is most practical where apertures of 1 or 2λ are of convenient size. A corner reflector without an exciting antenna can be used as a ***passive reflector*** or target for radar waves. In this application the aperture may be many wavelengths, and the corner angle is *always* 90^o.

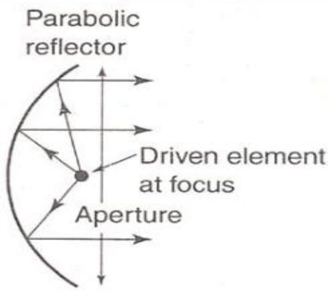


(e)

Reflectors with this angle have the property that an incident wave is reflected back toward its source as in Fig. (*e)*, the corner acting as a ***Retro reflector.***

Parabolic reflectors:

Parabolic reflectors can be used to provide ***highly directional antennas.*** A parabolic reflector antenna is shown in Fig (*f)*.

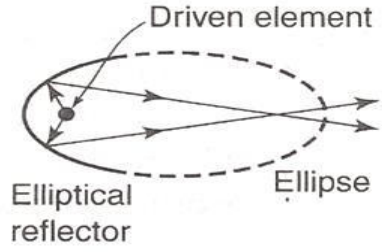


(f)

The parabola reflects the waves originating from a source at the focus ***into a parallel beam***, the parabola transforming the ***curved wave front*** from the feed antenna at the focus into a ***plane wave front.***

Elliptical Reflector:

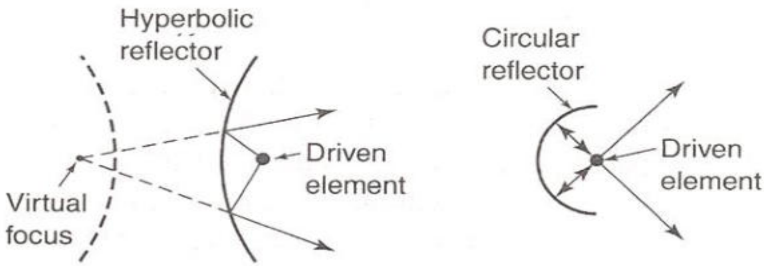
The elliptical reflector shown in Fig *(g)* ***produces a diverging beam with*** all reflected waves passing through the second focus of the ellipse.



(g)

Hyperbolic and Circular Reflectors:

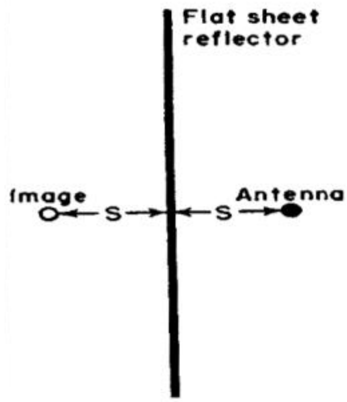
Examples of reflectors of other shapes are the hyperbolic and the circular reflectors are shown in Fig(*h)* and fig (*i)*.



(h) **(i)**

FLAT SHEET REFLECTORS

Method of images technique is used to study and analyze the performance of the reflector antennas. . In this method the reflector is replaced by an image of the antenna at a distance 2S from the antenna, as in Figure.



Assume that there is zero reflector losses, then the gain in field intensity of a $\lambda/2$ dipole antenna at a distance S from an infinite plane reflector is given by

$$G_f(\phi) = 2\sqrt{\frac{R_{11} + R_L}{R_{11} + R_L - R_{12}}}\left|\sin(S_r \cos\phi)\right| \qquad \text{------- (1)}$$

Where $S_r = 2\pi S/\lambda$.

The field patterns of $\lambda/2$ antennas at distances S = λ/4, λ/8, and λ/16 from the flat sheet reflector are shown in Figure. These patterns are calculated from (1) for the case where $R_L = 0$.

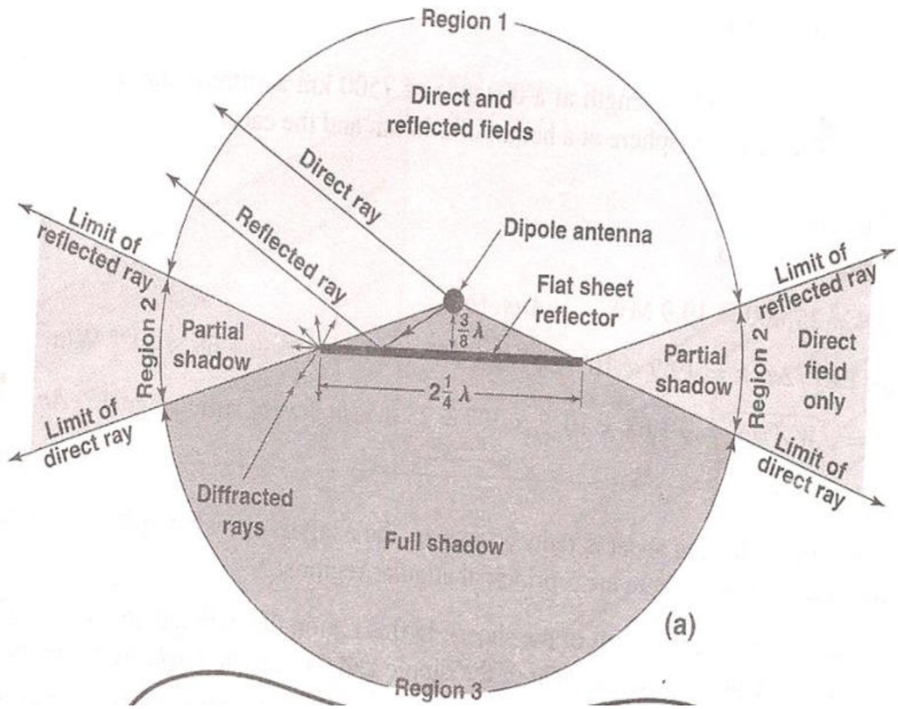
The gain as a function of the spacing S is presented in Fig. for assumed antenna loss resistances $R_L = 0$, 1 and 5 Ω.
These curves are calculated from (1) for $\phi = 0$.

- It is clear that very small spacing can be used effectively, provided that, losses are small. However, the bandwidth is narrow for small spacing's.

- With wide spacing, the gain is less, but the bandwidth is larger. Assuming an antenna loss resistance of 1 Ω, a spacing of 0.125 λ yields the maximum gain.

<u>A large flat sheet reflector can convert a bidirectional antenna array into a unidirectional system.</u>

When the reflection sheet is reduced in size, the analysis is less simple. The situation is shown in Fig.



(a)

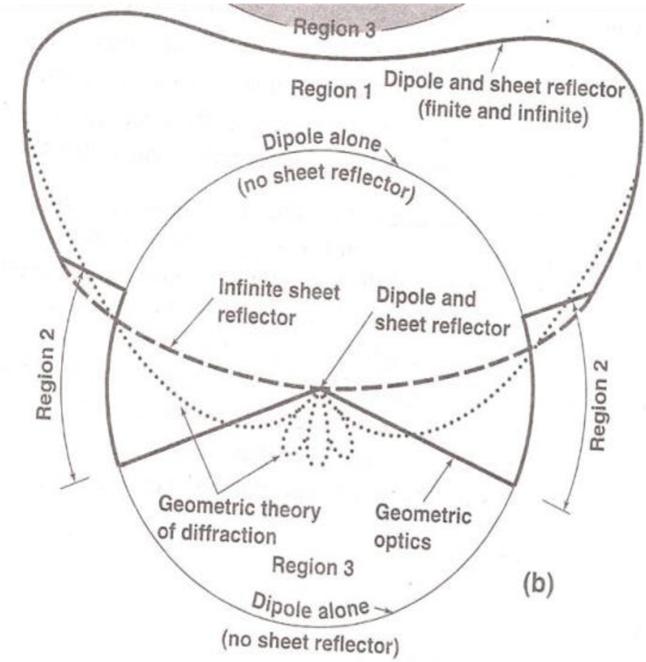
There are 3 principal angular regions:

i. ***Region 1*** (above or in front of the sheet). In this region the radiated field is given by the resultant of the *direct field* of the dipole and the *reflected field* from the sheet.

ii. ***Region 2*** (above or in below at the sides of the sheet). In this region there is only the direct field from the dipole. This region is in the shadow of the reflected field.

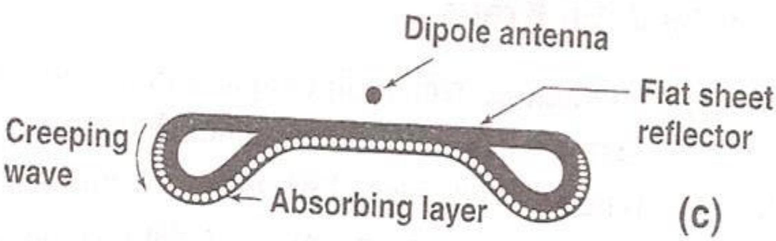
iii. ***Region 3*** (above or behind the sheet). In this region the sheet acts as a shield, producing a full shadow (no direct or reflected fields, only diffracted fields).

- If the sheet is 1 or 2λ in width and the dipole is close to it, image theory accounts completely for the radiation pattern in region 1.
- In region 2, the distant field is dominated by the direct ray from the dipole.
- In the full shadow behind the sheet (region 3) the ***Geometrical Theory of Diffraction (GTD)*** must be used.



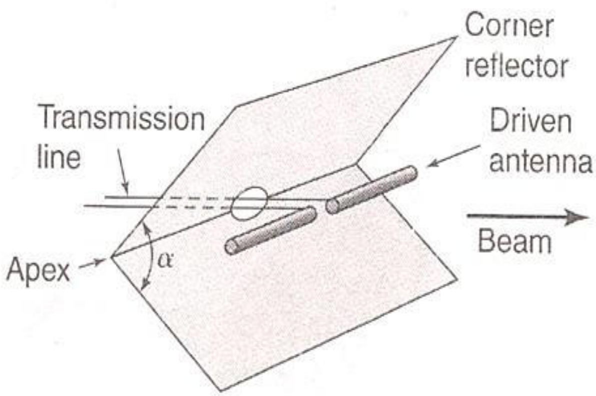
The Geometric Theory of Diffraction (GTD) in a nutshell.

Narrower reflecting sheets result in more radiation into region 3 but this diffracted radiation can be minimized by using a ***rolled edge*** (radius of curvature $< \lambda/4$ and absorbing material, as suggested in Fig.



CORNER REFLECTOR DESIGN

Two flat reflecting sheets intersecting at an angle or corner as shown in figure form an effective directional antenna. When the corner angle $\alpha = 90^o$, the sheets intersect at right angles, forming a ***square-corner reflector.***



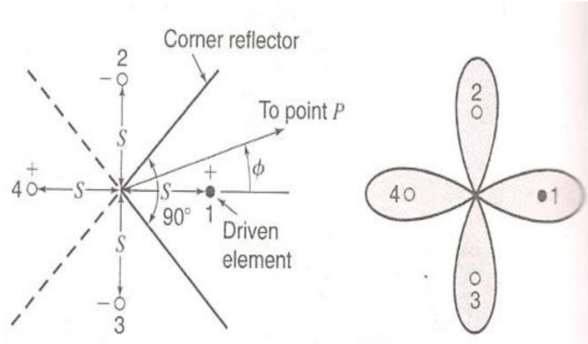
- Corner angles are both greater or less then 90^o.
- A cornet reflector with $\alpha = 180^o$ is equivalent to a flat sheet reflector and may be considered as a limiting case of the corner reflector.

The ***method of images*** can be applied to analyze the corner reflector antenna for angles $\alpha = 180^o / n$, where *n* is any positive integer.

- ✓ Corner angles of 180º (flat sheet), 90º, 60º, etc., can be treated in this method of images way.
- ✓ Corner reflectors of intermediate angle cannot be determined by this method but can be interpolated approximately from the others.

Analysis of 90° Corner Reflector by using Method of Images:

There are 3 image elements, 2,3 and 4, in analyzing 90 degrees corner reflector located as shown in Figure.



The driven antenna 1 and the 3 image have currents of equal magnitude. The phase of the currents in 1 and 4 is the same. The phase of the currents in 2 and 3 is the same but 180° out of phase with respect to the currents in 1 and 4. All elements are assumed to be $\lambda/2$ long.

At the point *P* at a large distance *D* from the antenna, the field intensity is

$$E(\phi) = 2kI_1\left|[\cos(S_r \cos\phi) - \cos(S_r \sin\phi)]\right| \quad \text{------- (1)}$$

Where

I_1 = current in each element

S_r = spacing of each element from the corner, rad

$= 2\pi(S/\lambda)$

k = constant involving the distance *D*, etc.

The emf V_1 at the terminals at the center of the driven element is

$$V_1 = I_1 Z_{11} + I_1 R_{1L} + I_1 Z_{14} - 2 I_1 Z_{12} \quad \text{-------- (2)}$$

Where

Z_{11} = self-impedance of driven element

R_{1L} = equivalent loss resistance of element

Z_{12} = mutual impedance of elements 1 and 2

Z_{14} = mutual impedance of elements 1 and 4

Similar expressions can be written for the emf's at the terminals of each of the images.

Then if *P* is the power delivered to the driven element (Power to each image element is also *P*),

we have from symmetry that

$$I_1 = \sqrt{\frac{P}{R_{11} + R_{1L} + R_{14} - 2R_{12}}} \quad \text{-------- (3)}$$

Substituting (3) in (1) we get

$$E(\phi) = 2k\sqrt{\frac{P}{R_{11} + R_{1L} + R_{14} - 2R_{12}}}\left|[\cos(S_r \cos\phi) - \cos(S_r \sin\phi)]\right|$$

-------- (4)

The field intensity at the point *P* at a distance *D* from the driven $\lambda/2$ element with the reflector removed is

$$E_{HW}(\phi) = k\sqrt{\frac{P}{R_{11} + R_{1L}}} \quad \text{------- (5)}$$

Where k = the same constant as in (1) and (4)

This is the relation for field intensity of a $\lambda/2$ dipole antenna in free space with a power input P and provides a convenient reference for the corner reflector antenna.

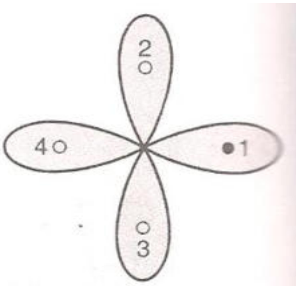
Thus, dividing (4) by (5), we obtain the ***gain in field intensity*** of a square-corner reflector antenna over a single $\lambda/2$ antenna in free space with the same power input, or

$$G_f(\phi) = \frac{E(\phi)}{E_{HW}(\phi)}$$

$$= 2\sqrt{\frac{R_{11}+R_{1L}}{R_{11}+R_{1L}+R_{14}-2R_{12}}}\left|[\cos(S_r\cos\phi)-\cos(S_r\sin\phi)]\right| \quad ..(6)$$

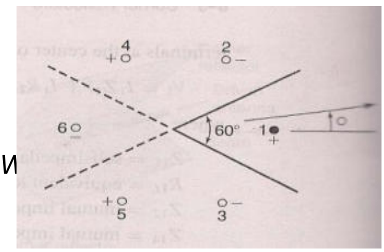
Where the expression in brackets is the ***pattern factor*** and the expression included under the radical sign is the ***coupling factor.***

The pattern shape is a function of both the angle ϕ and the antenna-to-corner spacing *S*. The pattern calculated by equation (6) has 4 lobes as shown in Fig. However, only one of the lobes is real.



Analysis of 60º Corner Reflector by using Method of Images:

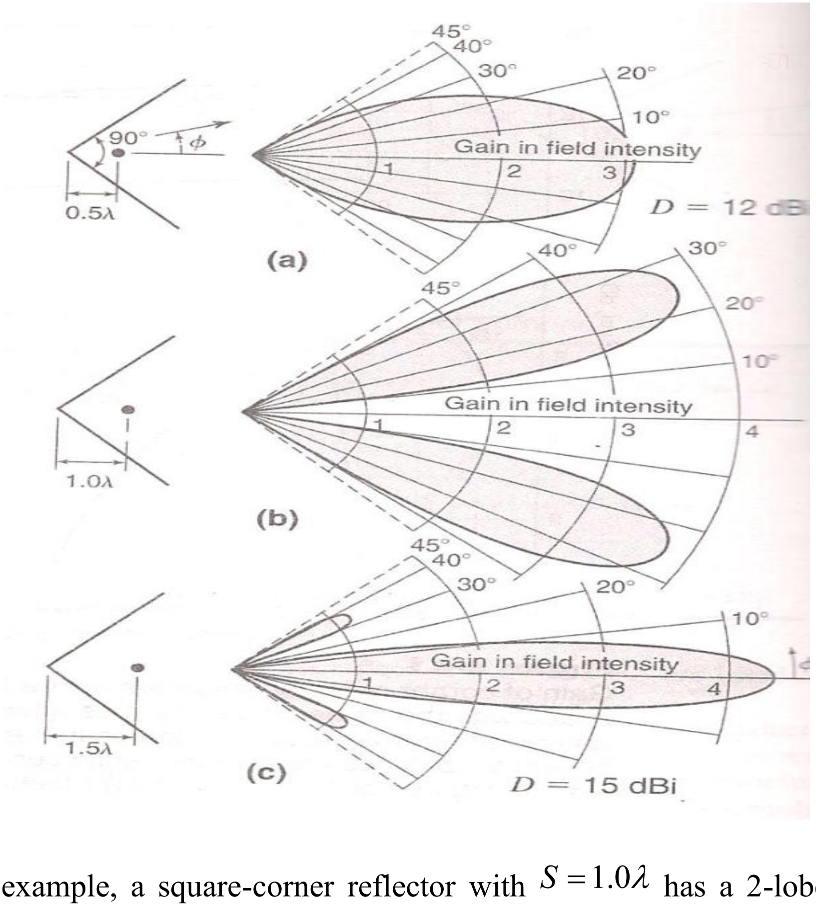
For the 60º corner the analysis requires a total of 6 elements, 1 actual antenna and 5 images as in Fig.



Calculated Patterns of 90° Corner Reflector with reference to spacing S:

The calculated pattern of a 90° corner reflector with antenna-to-corner spacing $S = 0.5\lambda$ is shown in the following Figure.

- ❖ The gain is nearly 10 dB over a reference $\lambda/2$ antenna or 12 dBi. This ***pattern is typical*** if the spacing *S* is not too large.
- ❖ If *S* exceeds a certain value, ***a multiplied pattern*** may be obtained.



For example, a square-corner reflector with $S = 1.0\lambda$ has a 2-lobed pattern as in Fig. (*b*).

If the spacing is increased to 1.5, the pattern shown in Fig. (*c)* is obtained with the major lobe in the $\phi = 0$ direction but with minor lobes present. This pattern may be considered as belonging to a higher-order radiation mode of the antenna. The gain over a single $\lambda / 2$ dipole antenna is 12.9 dB (≈ 15 dBi).

So, the patterns must be restricted to the lower-order radiation mode (no minor lobes), it happens only when *S* lie between the following limits:

α	S
90°	0.25–0.7 λ
180° (flat sheet)	0.1–0.3 λ

ANTENNA WITH PARABOLIC REFLECTORS

BEAM FORMATION BY PARABOLIC REFLECTORS:

A Parabola may be defined as "the locus of a point which moves in such way that its distance from the fixed point (called focus) plus its distance from a straight line (called directrix) is constant".

A parabola with focus F and vertex O is shown in the following figure.

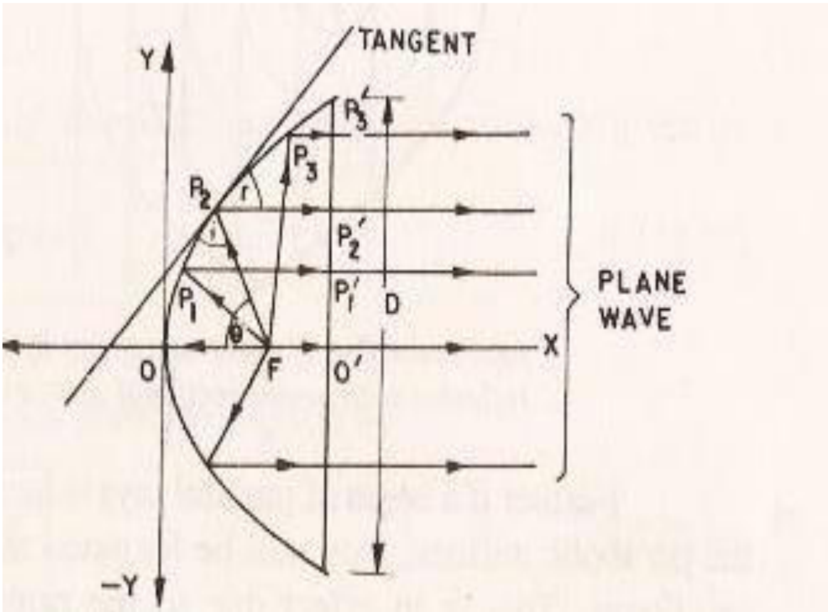


Figure: Geometry if a parabolic reflector

The Parabola is a two dimensional plane curve.

From the figure it is observed that

OF = Focal length $= f$
K = a constant, which depends on the shape of Parabola curve
F = focus
O = vertex

OO' = axis of parabola.

By the definition of parabola we can write

$$FP_1+P_1P_1' = FP_2+P_2P_2' = FP_3+P_3P_3' = \text{Constant (Say K)}$$

The equation of parabola curve in terms of its coordinate is given by

$$Y^2 = 4fx$$

- The open mouth *(D)* of the parabola is known as the ***Aperture.***
- The ratio of focal length to Aperture size (i.e. *f/D*) known as ***" f over D ratio"*** is an important characteristic of parabolic reflector and its value usually varies between **0.25 to 0.5.**

Focusing or beam formation action of parabolic reflector can be understood by considering a source of radiation at the focus.

Let a ray starts from the focus (F) at an angle θ w.r.t parabolic axis (OO') . The curve strikes at point p_2 on the parabola curve.

Let a tangent is drawn at P_2 on the curve. According to law of reflection, the angle of incidence (angle *i*) and angle reflection (angle *r*) will be equal as shown in the above figure.

- This results that the reflected ray is parallel to the parabolic axis, regardless of the particular value of θ that may be considered.

In other words, the entire wave originating from focus will be reflected parallel to the parabolic axis.

- ✓ This implies that the entire wave thus, reaching at the aperture plane is in phase.
- ✓ This shows that a wave front (a surface of constant phase) is created in the aperture plane.

- ✓ Therefore, the rays are parallel to the parabolic axis, because rays are always perpendicular to a wave front.
- ✓ Since all the waves are in phase, so a very strong and concentrated beam of radiation is there along the parabolic axis.

In other words, the entire wave emitting from the source at focus and reflected by parabola are travelling the same distance in same time in reaching the directrix and hence they are in phase.

The principle of equality of path length is maintained between all rays of two wavefronts. Putting in another way where there is path length difference between the two ray cancellation actions will take place.

Hence the geometrical properties of parabola provide excellent microwave reflectors that lead to the production of ***concentrated beam of radiation.***

In fact, parabola converts a spherical wavefront coming from the focus into a plane wavefront at the mouth of the parabola as shown in the figure.

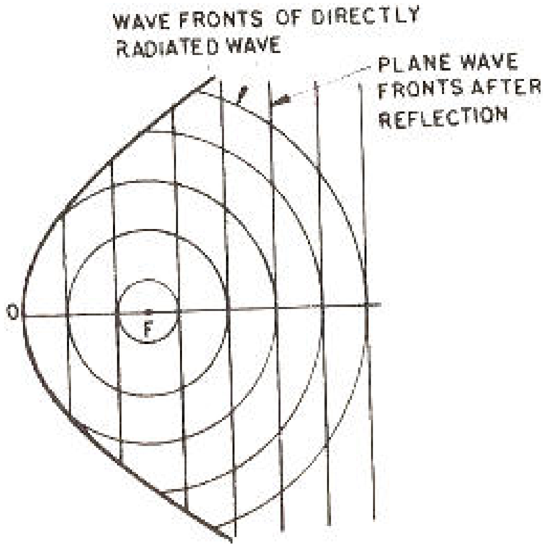


Figure: Plane wave front from spherical wave front.

Some part of the antenna radiation from the focus which is not striking the parabolic curve as spherical wave appears s minor lobes. Obviously this is a waste of power. This is minimized by partially shielding the source as shown in figure.

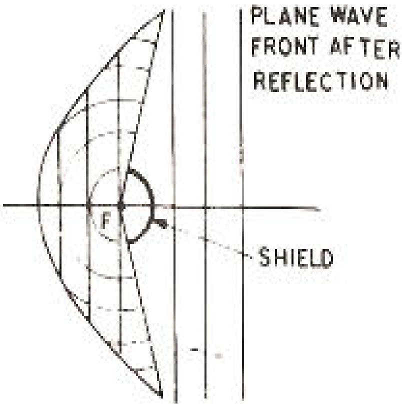


Figure: Parabolic reflector antenna with partially shield.

Future if a beam of parallel rays is incident on the parabolic surface, they will be focused at a point i.e Focus.

The parabolic reflector is directional for reception case also as only rays coming perpendicular to directrix will be focused at the focus and not others due to path length difference.

Parallel rays are known as ***collimated.*** This is shown in the following figure.

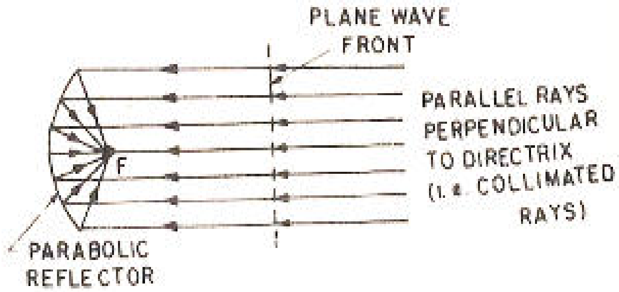


Figure: Focussing by a parabolic reflector.

PARABOLOIDAL REFLECTOR OR MICROWAVE DISH:

A parabola is a two dimensional plane curve. A practical reflector is a three dimensional curved surface. Therefore a practical reflector is formed by rotating a parabola about its axis. The surface so generated is known as paraboloid which is often called as "microwave dish" or "parabolic reflector".

Paraboloid produces a parallel beam of circular cross-section, because the mouth of the paraboloid is circular.

The equation of the paraboloid is given as $\boldsymbol{y^2 + z^2 = 4fx}$

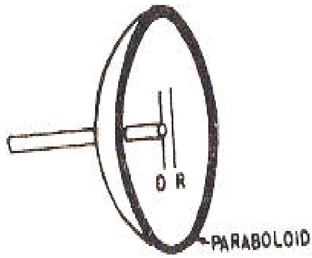


Figure: Paraboloidal reflector

The intersection of any plane perpendicular to x-axis with the paraboloid surface is a circle. In the conventional automobile,(e.g. motor-car headlight, or in search light),this beam forming property is utilized.

The radiation pattern of an antenna employing Paraboloid reflector has a very sharp major lobe accompanied by a number of minor lobes which, of course, are smaller in size.

Figure: Radiation pattern of Paraboloidal

- The narrow major beam is in the direction of paraboloid axis. The actual shape would be like a ***fat cigar.***

If the feed or primary antenna is isotropic, then the paraboloid will produce a beam of radiation.

Assuming the circular aperture is large, the Beamwidth between first null is given by

$$\boxed{BWFN = \frac{140\,\lambda}{D} \text{ degree}} \quad \text{... 9.128 (a)}$$

where λ = Free space wavelength, in m.

D = Diameter of aperture, in m *i.e.* mouth diameter.

The beam-width between first nulls for a large uniformly illuminated rectangular aperture is given by

$$\boxed{BWFN = \frac{115\,\lambda}{L} \text{ (degree)}} \quad \text{... 9.158 (b)}$$

where L = Length of Aperture, in λ

Also width between Half-power points for a large circular aperture is given by

$$\boxed{HPBW \quad \frac{58\,\lambda}{D} \text{ degree}} \quad \text{... 9.128 (c)}$$

Further, the directivity D of a large uniformly illuminated aperture is

$$\boxed{D = \frac{4\pi A}{\lambda^2}} \quad \text{... 9.128 (d)}$$

and for a circular, aperture

$$D = \frac{4\pi}{\lambda^2}\left(\frac{\pi D^2}{4}\right) = \pi^2\left(\frac{D}{\lambda}\right)^2$$

$$\boxed{D = 9.87\left(\frac{D}{\lambda}\right)^2} \quad \text{... 9.128 (e)}$$

where D = Diameter of the aperture, in λ.

In practice, the primary (or feed) antenna is not isotropic and thus does not radiate uniformly which introduces distortion.

- ✓ Besides, the surface of paraboloid is not uniformly illuminated, as there is gradual tapering towards edge.
- ✓ This result in less captures area which is smaller than the actual area i.e. $A_0 = KA$

Where	A_0	is the capture area
	A	is the actual area of the mouth
	K	is a constant

Thus the power gain of the circular aperture paraboloidal with half wave dipoles is given as

$$G_p = \frac{4\pi A_0}{\lambda^2} = \frac{4\pi K A}{\lambda^2} \qquad \because A = \frac{\pi D^2}{4} \text{ for circular Aperture}$$

$$= \frac{4\pi K}{\lambda^2}\left(\frac{\pi D^2}{4}\right) = 0.65 \times \left(\frac{\pi D}{\lambda}\right) = 0.65 \times (3.14)^2 \left(\frac{D}{\lambda}\right)^2 = 6.389 \left(\frac{D}{\lambda}\right)^2$$

$$\boxed{G_p \simeq 6\left(\frac{D}{\lambda}\right)^2}$$

- ✓ Generally the gain of a paraboloidal reflector is a function aperture ratio (D/λ) of the paraboloid.

The **Effective radiated power (ERP)** of an antenna is multiplication of input power fed to antenna and its power gain.

As a numerical example, as is obvious that if actual power fed to paraboloid reflector is 1 watt, then ERP will be 9600 watt. Hence, with the help of paraboloid reflector, extremely large gain and narrow beam widths can be achieved .

For the effective and useful use, a paraboloid reflector must have open circular mouth aperture of minimum 10λ.

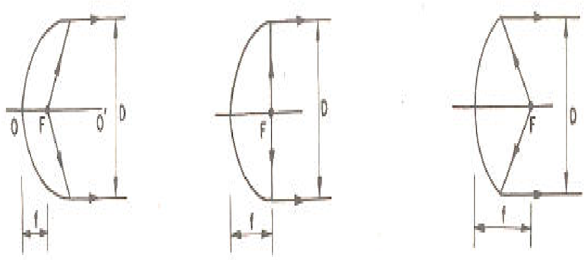
Although construction of paraboloid reflector is possible at lower frequencies e.g. VHF etc. but the size of the mouth would be too large heavy and bulky and hence purposely avoided at television broadcast band etc.

Paraboloid reflector can be designed by keeping the mouth diameter fixed and varying the focal length (f) also.

Three cases possible are shown in figure.

Case (i): the focal length is small such that the focus lies well inside the mouth aperture.

- In this case it is difficult to get a source giving adequately uniform illumination over such a wide angle.



Case(ii): Next when the focal length is large such that focus lies beyond the open mouth (as shown in figure),

- it become difficult to focus all the radiation from the source on the reflector.

Hence, the design which provides maximum gain is compromise between the two cases i.e. the design in which the focus lies in the plane of the open mouth (figure).

Case(iii): By geometry of parabola, It can be shown that when the focus lies in plane of the open mouth the focal length (f) is equal to one fourth of open mouth diameter (i.e. D/4) .

- The radiation beam of an antenna employing paraboloid reflector should be theoretically a pencil-shape.
- This pencil shape beam is almost equal in horizontal and vertical plane.
- However a little amount of control on beam shape may be obtained cutting of the parts of paraboloid.

Spill over and Back Lobes:

The most widely used antenna for microwave is the paraboloidal reflector antenna. Let us consider the following figure.

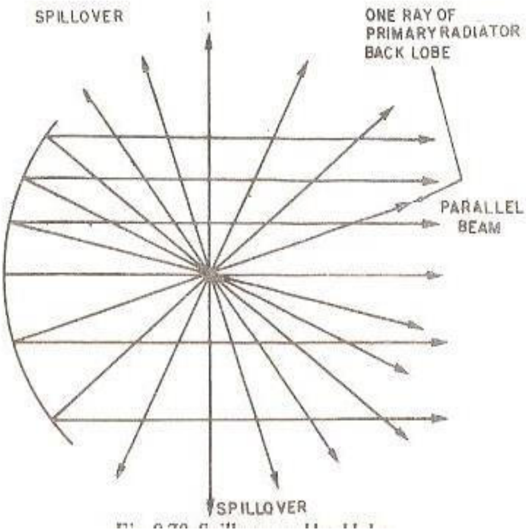


Figure: Spill over and back lobes

In fig an isotropic source is assumed to be situated at the focal point. It is seen that some of the desired rays are not captured by the reflector and these constitute ***"spill-over".***

While receiving spill over increases the noise pick up which is particularly troublesome in satellite ground stations.

Further, some radiations from the primary radiators occur in the forward direction in addition to the desired parallel beam.

- This is known as "***back lobe***" radiation as it is from the back lobe of the primary radiator.
- Back lobe radiations are not desirable as it can interfere destructively with the reflected beam and hence practical radiators are designed to minimize this back lobes.

LENS ANTENNAS

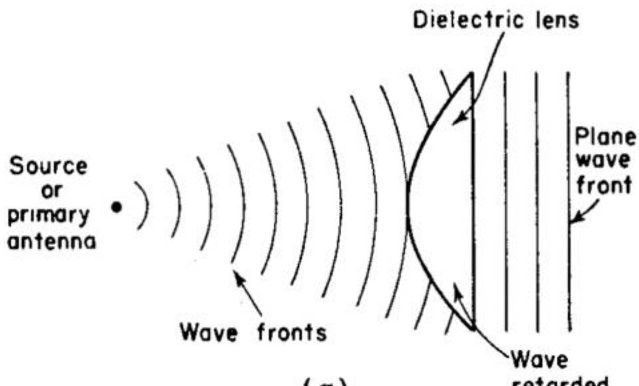
INTRODUCTION

Basically different antennas are used for different applications. LENS antenna is one of the most important antennas used widely.

Lens antennas are preferably used at higher frequencies. They are used for frequency range, starting from 1000MHz to 3000MHz and even at greater frequencies. At Low frequencies Lens antennas become bulky and heavy.

Lens antennas may be divided into two types

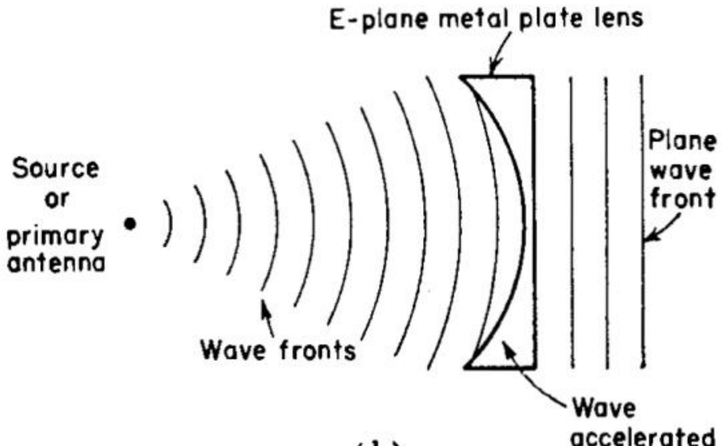
1) ***Delay lenses,*** in which the electrical path length is increased by the lens medium, and
   - In delay lenses, the wave is retarded by the lens medium.
   - Dielectric lenses and H-plane metal-plate lenses are of the delay type.



(a)

The actions of a dielectric lens are shown in the above figure.

2) ***Fast Lenses***: In these type of lens antennas the electrical path length is decreased by the lens medium.
   - E-plane metal-plate lens are of the fast type.

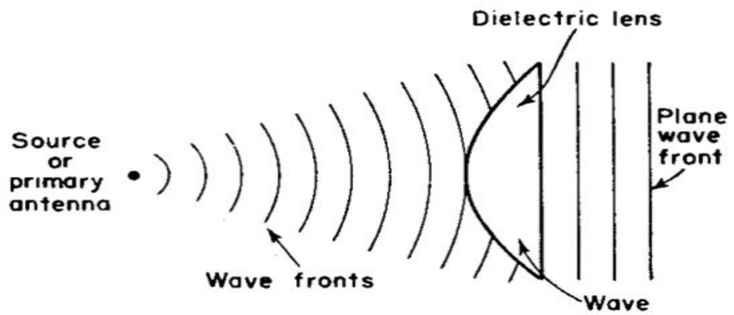


(b)

The actions of E- plane metal-plate lens are shown in the above figure.

FERMAT'S PRINCIPLE (EQUALITY OF PATH LENGTH):

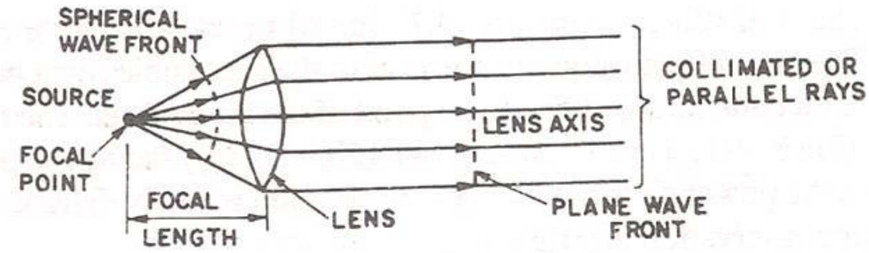
Let us determine the shape ***of the Plano-convex lens*** of figure shown below for transforming the spherical wave front from an isotropic point source or primary antenna into a plane wave front.



(a)

The field over the plane surface can be made everywhere in phase by shaping the lens so that all paths from the source to the plane are of equal electrical length. ***This is the principle of equality of electrical (or optical) path length (Fermat's principle).***

The collimating action of a simple optical lens can be shown from the following figure.

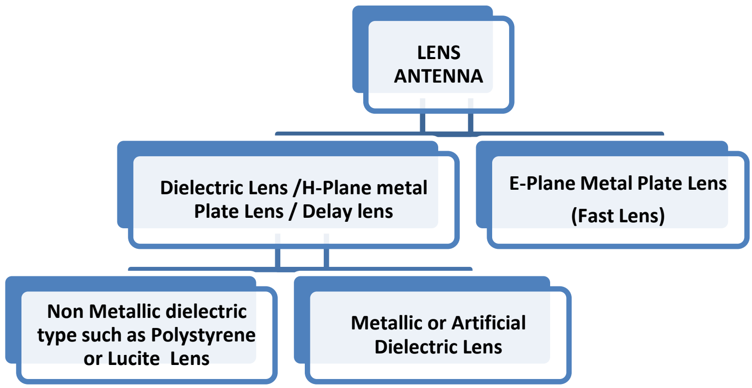


Consider a source is located at the focal point with some focal length from the lens, along the lens axis.

- From the figure it is noticed that collimated or parallel rays is obtained on the right hand side of the lens.
- In other words, it is noticed that a ***divergent beam is collimated*** because refraction takes place as a result of which ray at the center refracted less than at the edges.

TYPES OF LENS ANTENNA

The following figure illustrates the brief classification of Lens antennas.



DIELECTRIC LENS /H-PLANE METAL PLATE LENS / DELAY LENS

The ***dielectric lenses*** may be divided into two groups:

i. Lenses constructed of ***nonmetallic dielectrics***, such as 1ucite or polystyrene
ii. Lenses constructed of ***metallic or artificial dielectrics***

NONMETALLIC DIELECTRIC LENS ANTENNA

Ray analysis method of geometrical optics is used to design the nonmetallic dielectric lenses. This lens is generally used to convert the spherical wave front into the plane wave front.

If this lens is to convert a spherical wave front into plane wave front, then all rays path from the source to the lens should have equal electrical lengths. So a Plano concave lens is used to perform that spherical to plane wave front conversion.

Consider all the ray paths from point 'O' to the plane surface AB of the lens should have equal electrical length according to principle of equality of path lengths. This is shown in the following figure.

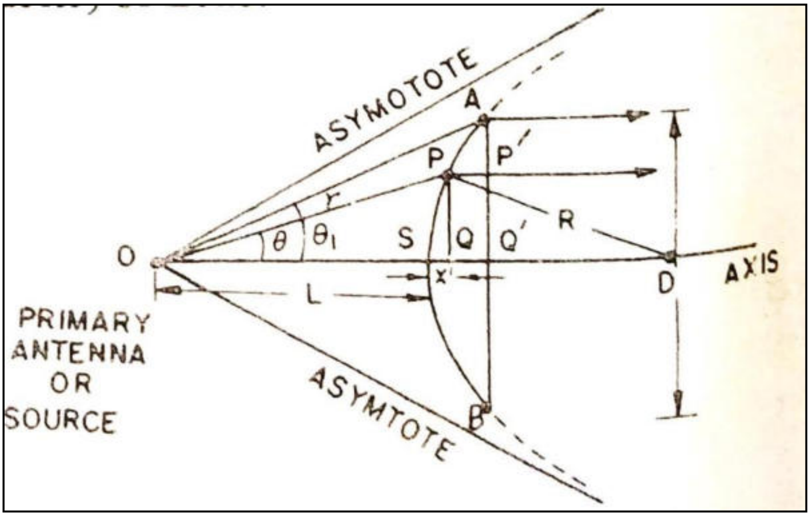


Figure: Raypath in Plano Concave lens

Let the velocity of wave in air and in lens medium be 'c' and 'v' respectively, then,

$$OP + PP' = OS + SQ'$$

$$OP + PP' = OS + SQ + QQ'$$

since $PP' = QQ'$

$$OP + \cancel{QQ'} = OS + SQ + \cancel{QQ'}$$

$$OP = OS + SQ$$

$$\frac{r}{c} = \frac{L}{c} + \frac{x}{v} \qquad \because OS = x$$

$$r = L + \frac{c \cdot x}{v}$$

$$r = L + \left(\frac{c}{v}\right)x$$

$$r = L + \mu x \qquad \because \mu = \frac{c}{v}$$

But $x = OQ - OS$

$x = r\cos\theta - L$

$\therefore$

$$r = L + \mu(r\cos\theta - L)$$

$$r = L + \mu r\cos\theta - \mu L$$

$$r - \mu r\cos\theta = L - \mu L$$

$$r[1 - \mu\cos\theta] = L[1 - \mu]$$

$$\boxed{r = \frac{L(1-\mu)}{1-\mu\cos\theta}}$$

This can also be rewritten as

$$\boxed{r = \frac{L(\mu-1)}{(\mu\cos\theta - 1)}}$$

This equation gives the shapes of the lens. This is the equation of the ***Hyperbola*** whose focal length is *L* and radius of curvature *R*.

$R=L(\mu - 1)$ when θ is small.

If *r* tends to infinity

$$(\mu\cos\theta - 1) = \frac{L(\mu-1)}{r} = \frac{L(\mu-1)}{\alpha} = \frac{\text{Finite}}{\alpha} = 0$$

$$\mu\cos\theta - 1 = 0$$

$$\mu\cos\theta = 1$$

$$\boxed{\cos\theta = \frac{1}{\mu}}$$

One of the focuses of the hyperbola is at 'O'.

$R=L(\mu - 1)$ when θ is small, this equation implies that a hyperbolic lens can be replaced by a Plano concave spherical lens.

LENS ANTENNAS ZONNINGS

It is known that at frequencies less than 1000MHz, the lens antenna is bulky in size and has excessive thickness, which is not suitable. This problem of heavy size can be lens antenna can be solved by using lens antennas with zoned or stepped typed antennas.

The thickness t of such a zoned antennas are given as

$$t = \frac{\lambda}{\mu - 1}$$

The shapes of the zoned or stepped dielectric lenses are shown in the following figure.

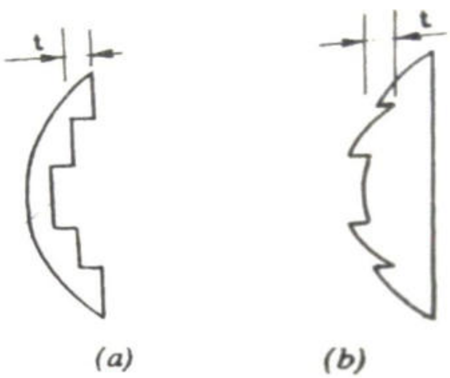


Figure: zoned or stepped dielectric lens

- In a special case, if μ = 1.5, then the thickness would be twice the free space wavelength.
- From the above shown two lenses the second one is preferred as it has good mechanical strength. By using the zoned

dielectric lens the signals are in phase after emergence from the lens.

- The stepped lens antenna has the benefit of reduced weight and less power dissipation.
- At the same time zoned dielectric lens antennas become frequency sensitive.

Chapter 5
WAVE PROPAGATION

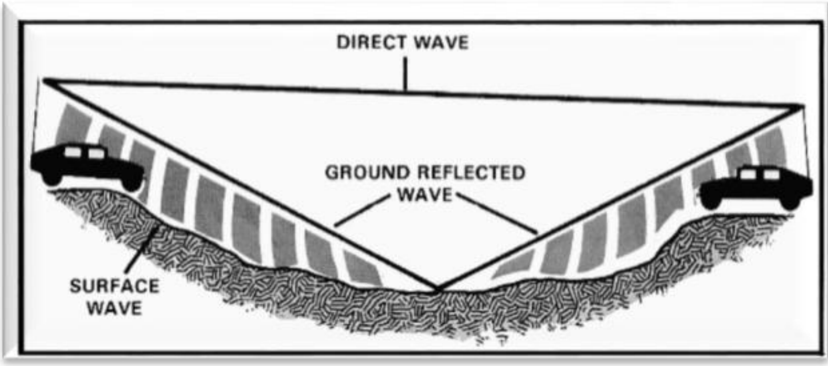
Dr.Kasigari Prasad

Contents

CHAPTER

5

WAVE PROPAGATION



INTRODUCTION

Radio Wave: The term *radio wave* is applied to electromagnetic waves in the frequency range from ***0.001 to 10^{16} Hz***.

In terms of wavelength, the lower limit of radio waves propagated in free space (or vacuum) ***is*** 3×10^{11} ***m and the upper limit is*** 3×10^{-8} ***m.***

The electromagnetic spectrum consists of a frequency range from a ***small fraction of a hertz to 10^{20}*** Hz and even more.

The frequencies which are down to a few thousand of a hertz will experience some natural phenomena. Such frequencies, for example, are generated by fluctuations of the solar electron proton stream as it penetrates the earth's atmosphere. These waves are closely related to magneto-hydro-dynamic waves (mechanical waves produced by the ion plasma of the atmosphere). Lightning discharges also produce these waves.

The lower limit of the frequencies used by transmitters is normally set to be 10^3 Hz. On the higher side, radio wave spectrum presently extends to the limit of 10^{16} Hz.

DEFINITION AND BROAD CATEGORIZATION

Basic Definition:

An electromagnetic wave can be studied in terms of electric and magnetic field vectors ***E*** and ***H***. These vectors obey the mathematical relations

$$\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \quad \text{and}$$
$$\nabla^2 H = \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

respectively.

Wave: "If a physical phenomenon that occurs at one place at a given time and is reproduced at other places at later times, the time delay being proportional to the space separation from the first location, the group of phenomena constitutes a wave.

The phenomena which lead to characteristic modifications of a wave are

- ✓ reflection,
- ✓ refraction,
- ✓ diffraction,
- ✓ absorption and the rotation of plane of polarization.

These are mainly due to variation of *media parameters* (σ, ε and μ) on the way or the shape and characteristics of obstructing objects.

Wave Study may broadly classified as

1. **Guided Waves:** The waves guided by manmade structures such as parallel wire pairs, coaxial cables, waveguides, strip lines, optical fibers, etc. are guided waves.

Applications of guided waves are used in local area networks (LAN), closed circuit TV, interconnections used for providing Internet services, cable networks used by cable TV operators and networking of computers located in the same room, building, locality or campus.

2. **Unguided Waves:** The waves propagating in the terrestrial atmosphere, over and along the earth and in outer space are unguided waves.

The ***applications of the unguided radio waves are***

- in***transmission of information over*** short and long distances through telegraphy and telephony, radio broadcast, television, mobile communication, satellites, antennas, radio location, radio navigation, remote sensing and distance measurements by radio means.

Unguided propagation is also used in ***geophysics,*** in the study of upper atmosphere, ***radio astronomy***, study of activities of sun stars and nebulae inside and outside our galaxy.

CLASSIFICATION OF ELECTROMAGNETIC WAVES

The energy generated by a transmitter is fed to a transmitting antenna which in turn radiates the same into the space. This radiated energy travels all through the space and this mode of travel is termed as electromagnetic wave.

1. GENERAL CLASSIFICATION

Plane wave: In phasor form a plane wave is defined as one for which the equi-phase surface is a plane.

Uniform plane wave: If the equi-phase surface is also an equi-ampliude surface, then the wave is called a uniform plane wave.

- A uniform plane wave progressing in the direction z (say) will have no E_z component. It, has E_x or E_y components only, i.e., in a uniform plane wave E is entirely transverse.
- In a uniform plane wave, E and H are at right angles (orthogonal) to each other

Non-uniform plane wave: In a non-uniform plane wave, the equi phase and equi-amplitude surfaces are neither same nor they are parallel.

- Also, in a non-uniform plane wave, E and H need not necessarily be orthogonal.

Slow wave: When the phase velocity normal to the equi-phase surfaces is less than the velocity of light 'c', the wave is called as a slow wave. In certain microwave devices (e.g., TWT) special structural shapes are employed to slow down the speed of the wave.

Forward wave: A wave traveling in an assigned direction from the point of origin is called forward wave provided there is no obstacle to cause any reflection.

Backward wave: The backward wave is, in general, a reflected wave which results, when a forward wave strikes a reflecting surface.

- The reflection of a wave may be total or partial depending upon the conductivity and roughness of the surface it strikes.
- The reflection also occurs in transmission lines when it is terminated in impedance other than the characteristic impedance of the line.

Traveling wave When a wave is progressing only in one direction and there is no reflected wave present, it is called a traveling wave.

- In such a wave, the maximas or minimas of E and H at different time instants appear at different space locations as shown in the following Figure.

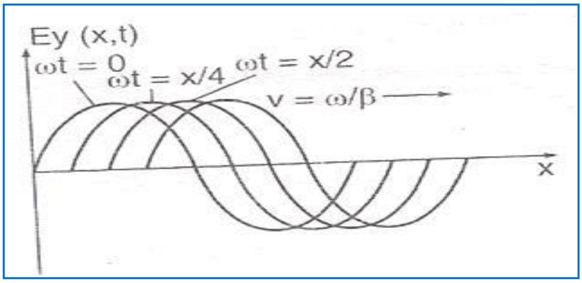


Fig: Travelling Wave

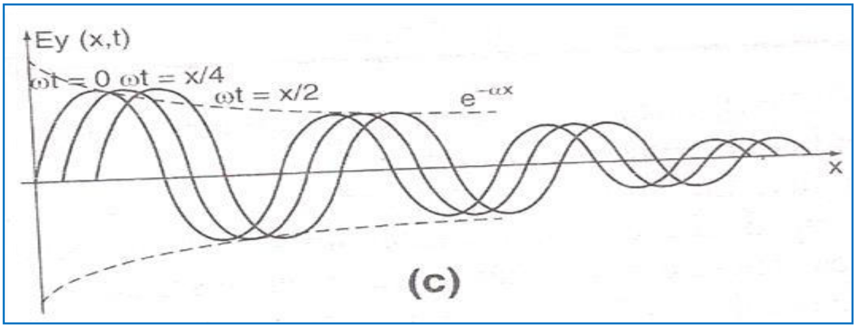


Figure: Travelling wave with attenuation

Standing wave: If both forward and reflected waves are simultaneously present, they combine to result in a wave called standing wave.

- Such a wave does not progress and maximas or minimas (of E and H) for different time instants will appear at the same space location but with varying magnitudes.

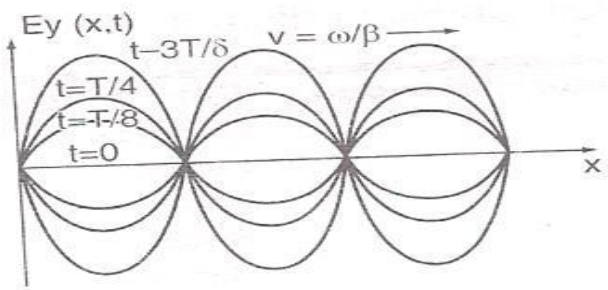


Fig: Standing Wave

Surface wave: If a wave is supported by some kind of surface between two media, it is called a surface wave.

- In other words, a surface wave is one that propagates parallel to the interface and decays vertically to it.
- Ground waves may be composed of surface waves (originating from elevated antennas.

Trapped wave: Sometimes a surface wave is also called a trapped wave because it carries its energy within a small distance from the interface. This wave does not radiate except at discontinuities, such as the termination of the structure.

- A traveling wave carried by a two-wire line with discontinuities placed at regular intervals along the line is a surface wave.
- The ducting phenomenon in space wave mode of propagation also amounts to trapping of the wave within its lower and upper bounds.

Leaky wave: When discontinuities are densely placed along the line, making it a continuous perturbing structure, another type of traveling wave results and it a called a leaky wave. There is some leakage of energy mainly from the top bounds of the duct.

II. CLASSIFICATON BASED ON ORIENTATION OF FIELD VECTOR

A wave may also be classified according to the ***polarization or time-varying behavior of the electric field vector*** E ***at some fixed point in space.*** A wave may be polarized *along x/y* direction or it may be *linearly, circularly* or *elliptically polarized.*

- Besides, if in a wave E is parallel to the boundary surface or perpendicular to the plane of incidence (i.e., a plane containing the incidence ray and the normal to the surface), the wave is termed ***horizontally polarized.***
- In case E is perpendicular to the boundary surface or parallel to the plane of incidence, it is called ***vertically polarized.***
- These waves originate from the horizontal and vertical antennas respectively with reference to the surface of the earth.

However, it is seen that, **E** of a horizontally polarized wave is horizontal, **E** of a vertically polarized wave is not wholly vertical but has some horizontal component.

III. CLASSIFICATION BASED ON THE PRESENCE OF FIELD COMPONENTS

The wave propagating between two parallel planes or waveguides can be of two kinds, namely,

- **Transverse Electric (TE) or H wave &**
- **Transverse Magnetic (TM) or E wave.**

In a transverse electric wave, **E** has no component in the direction that the wave progresses, and in a transverse magnetic wave, **H** has no component in the direction of propagation. Thus, in an H wave, E is entirely transverse, whereas in E wave H is entirely transverse.

- A special case, which results from TM wave, only in case of parallel planes, is that of transverse electromagnetic (*TEM)* also termed as **EH** or **HE** *wave.*

In a TEM wave, **E** and **H** both are transverse and there is no component of **E** or **H** in the direction the wave progresses.

- The use of TEM waves is mainly confined to the parallel plane / parallel wire transmission line.

- Since the cutoff, i.e., the lowest limit of frequency for a TEM wave is zero.

IV. CLASSIFICATION BASED ON MODES OF PROPAGATION

For a long distance unguided waves, the communication link may be established through

- ***Ground waves,***
- ***Space waves* or S*ky waves.***

The selection of a particular mode depends on frequency range and applications. The following table shows different frequency bands and their applications.

Frequency Band	Short Range	Long Range	
		Day	Night
ELF	Ground waves	Ionospheric wave	Ionospheric wave
VLF	Ionospheric wave		
LF			
MF	Ground Waves	Ground Waves	
HF		Ionosphere	
VHF		Ionosphere	Ionosphere
		Tropospheric Wave	Tropospheric
		Direct Wave	Direct Wave
UHF		Tropospheric Wave	Tropospheric Wave
SHF		Direct Wave	Direct Wave
EHF	Direct Wave	Direct Wave	Direct Wave

Table 6.1: Frequencies & their related waves.

DIFFERENT MODES OF WAVE PROPAGATION

The energy radiated from a transmitting antenna may travel all through space with (or without) alteration in its characteristics, after reflection due to variation of media parameters on the way or after reflection from obstructing objects including earth surface to arrive at the receiving antenna. Figure shows many possible propagation paths.

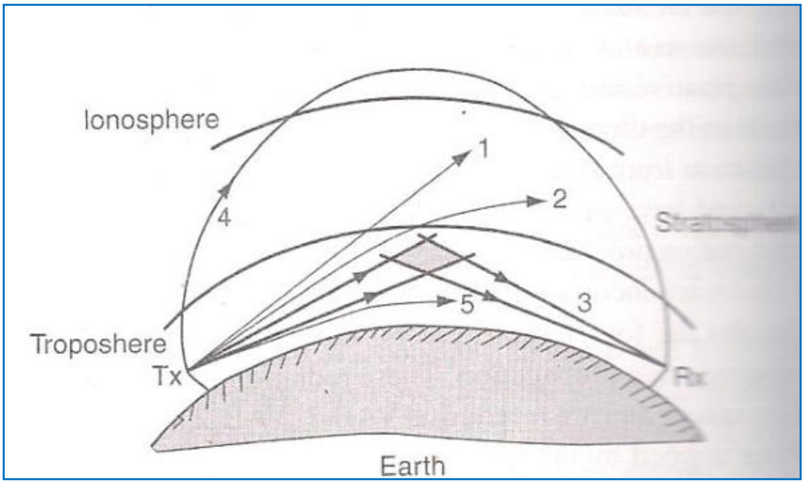


Fig: Different modes of wave propagation

Direct wave: The wave (1) follows a ***straight-line path*** in free space and is sometimes referred as ***direct wave (DW).***

The wave (2) follows a ***curved path*** in view of refraction phenomena in the atmosphere.

Tropospheric communication:

The wave (3) reflected or scattered in the troposphere is termed as ***tropospheric wave***. This mode of propagation is the result of ***irregularities of troposphere***, which extends to nearly 10 to 15 km from the earth surface. The communication utilizing the troposphere waves is called ***tropospheric communication***.

Sky Waves:

The wave (4) reaching the receiver after getting refracted and reflected from the ionosphere, is called *ionospheric or sky wave (SKW)*. This is sometimes also called as ***ionospherically reflected or ionospherically scattered wave.***

- This mode of propagation is used for ***beyond the horizon communication*** or very long distance communication and
- is operated in high frequency (HF) range.
- This mode is also used in over-the-horizon (OTH) radars.

Ground Wave:

The wave (5) propagating over paths near the earth's surface is often referred as *ground wave (GW).*

- Ground (surface) waves are ***vertically polarized*** and exist if antennas are close to earth.
- All ***broadcast signals in daytime*** are ground waves.
- Besides broadcasting, these are also used for ***ground wave radars***.

According to ***Somerfield analysis,*** the ground wave is divided into two parts, namely, the

1) **Surface wave (SUW) and**
2) **Space wave (SPW).**

The space wave predominates at larger distances above the earth, whereas the surface wave is more significant near the earth's surface.

Space wave is made up of the direct wave (comprising the signal reaching the receiver through the straight path from the transmitter) and the ground reflected wave (containing the signal arriving at the receiver after reflection from the surface of the earth). Such waves are used for beyond-the-horizon communication.

- Space waves are the only means of communication **beyond the 30-MHz range.**

A surface wave also includes the energy received as a result of diffraction around the earth’s surface and reflection from the upper atmosphere.

GROUND WAVE PROPAGATION

INTRODUCTION

The waves, which while traveling, glide over the earth's surface, are called *ground waves.*

Ground waves are always ***vertically polarized*** and induce charges in the earth. The number and polarity of these ***charges keep on changing*** with the ***intensity and location of the wave field***. This variation causes the constitution of a current.

In carrying this current, the earth behaves like a ***leaky capacitor***. As the wave travels over the surface, it gets weakened ***due to absorption*** of some of its energy. This absorption, in fact, is the power loss in the earth's resistance due to the flow of current.

This energy loss is partly contributed by the diffraction of energy, downward, from the portion of the wave present somewhat above the immediate surface of the earth. This process is shown in

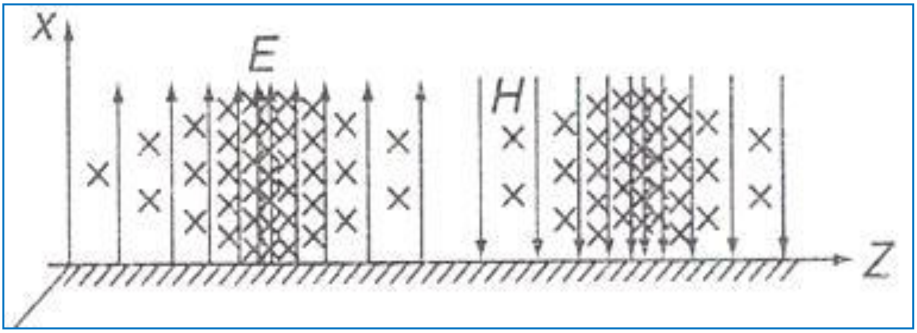
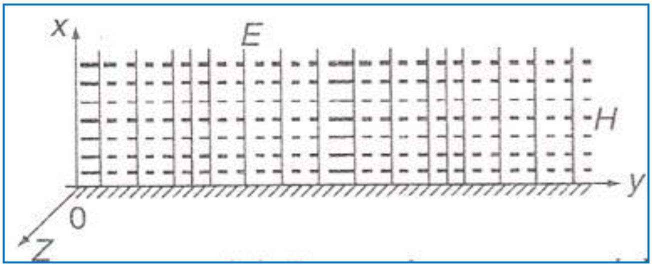


Fig: Side view of the gliding wave



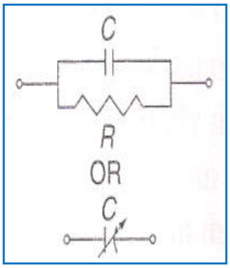


Fig: Front view of the gliding wave **Fig: Earth represented by a leaky capacitor**

Thus, the study of wave propagation can be divided into two parts, i.e., the waves that propagate over

1. Plane earth, and
2. Spherical earth.

PLANE EARTH REFLECTION:

As shown in the figure the elevated transmitting and receiving antennas within the line of sight of each other are placed. And the received ***resultant signal*** is the combination of the signal reaching the receiver through a ***direct path*** and that reaching after ***being reflected by the ground***. These two paths are shown in the Fig.

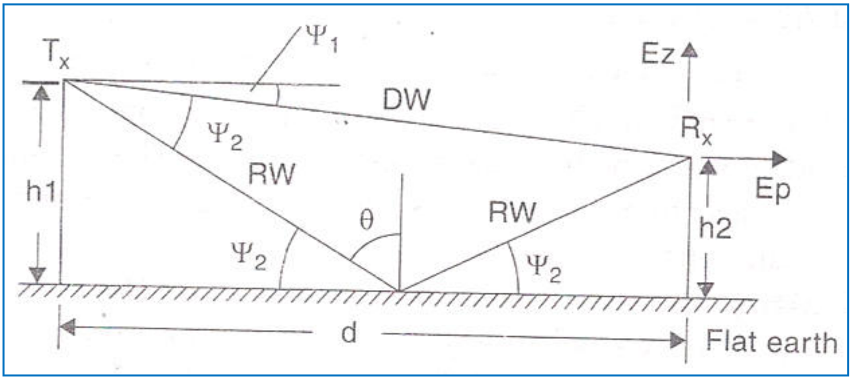


Fig: Direct wave and reflected wave between transmitter and receiver

For a smooth plane and **finitely conducting earth**, the magnitude and phase of the reflected wave differ from that of the incident wave.

When the earth is rough, the reflected wave tends to be scattered and may be much reduced in amplitude compared with smooth earth reflection.

The ***roughness*** is generally estimated by the ***Raleigh criterion*** given by the relation:

$$R = \frac{4\pi\sigma \sin\theta}{\lambda} \quad \text{---------- (1)}$$

Where

σ is the standard deviation of the surface irregularities relative to the mean surface height.

θ is the angle of incidence measured from the normal angle, and

λ is the wavelength.

- If $R < 0.1$, the reflecting surface is considered as being ***smooth.***
- If $R > 10$, the reflecting surface is considered to be ***rough.***

From the rough earth, the reflected wave tends to be scattered and may be much reduced in amplitude compared with that reflected from a smooth surface.

A surface may be considered rough for waves incident at high angles (i.e., large θ). It may approach to be smooth as the angle of incidence approaches the ***grazing angle***(i.e., $\theta \to 0$).

Also, when the incident wave is near grazing over a smooth earth, the reflection coefficient approaches minus one for both polarizations.

For a medium having dielectric constant ε
conductivity σ
Maxwells equations can be written as

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t}$$

this wave equation when propagating experencises to sinusoidal time variation

so; $$\nabla \times H = \varepsilon' \frac{\partial E}{\partial t}$$

where $\varepsilon' = \left[\varepsilon + \frac{\sigma}{j\omega} \right]$ it is a complex quantity.

when is propagating → it has two polarization states.

(i) Horizantal polarization
(ii) Vertical polarization.

The Reflection coefficient for Horizantal polarization is

$$R_H = \frac{\sqrt{\varepsilon_1} \cos\theta - \sqrt{(\varepsilon_2 - \varepsilon_1 \sin^2\theta)}}{\sqrt{\varepsilon_1} \cos\theta + \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2\theta}}$$

The Reflection coefficient for Vertical polarization is

$$R_V = \frac{\frac{\varepsilon_2}{\varepsilon_1} \cos\theta - \sqrt{\left[\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta\right]}}{\frac{\varepsilon_2}{\varepsilon_1} \cos\theta + \sqrt{\left[\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta\right]}}$$

The actual concept behind these equations, is, to say that both R_H and R_V are COMPLEX QUANTITIES given as

$$R_H = |R_H| \angle R_H$$

$$R_V = |R_V| \angle R_V$$

The following figure shows, the variation of magnitude and phase of vertically polarized and the horizontally polarized waves.

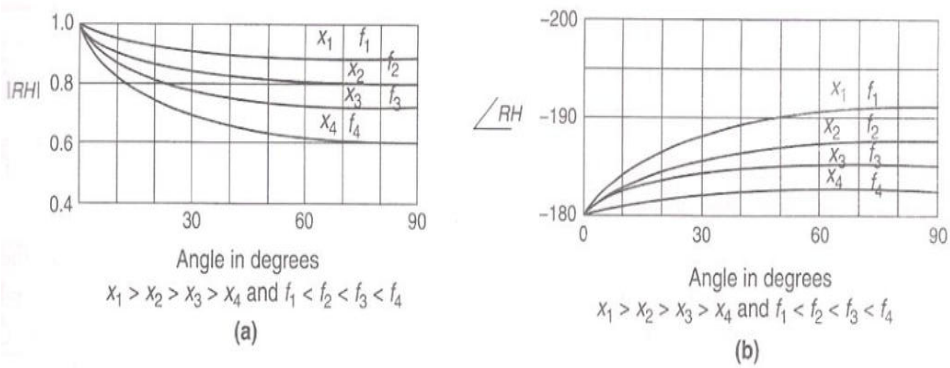


Figure: Variation of horizantally polarized wave (a) magnitude (b) angle

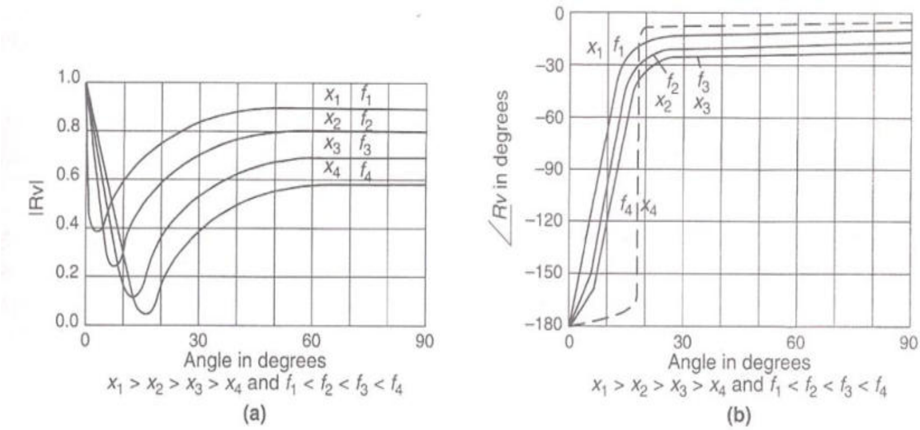


Figure: Variation of vertically polarized wave (a) magnitude (b) angle

When the incident wave is *horizontally polarized*

- The phase of the reflected wave differs from that of the incident wave by nearly 180° for all angles of incidences.
- For angles of incidence near grazing ($(\psi = 0)$, the reflected wave is equal in magnitude but 180° out of phase with the incident wave for all frequencies and for all ground conductivities.

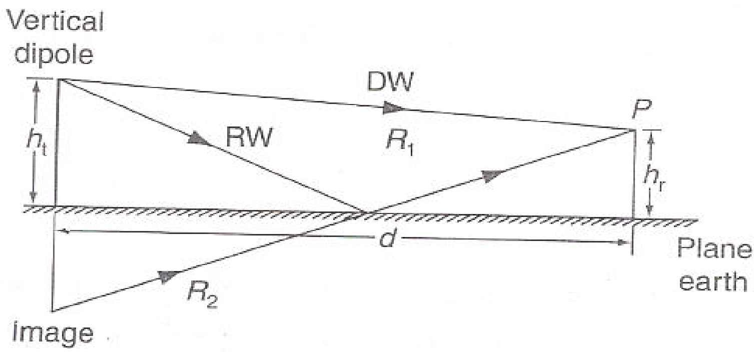
- As the angle of incidence is increased, both the magnitude and phase of the reflection factor change, but not to a large extent. The changes are greater for the higher frequencies and lower ground conductivities.

When the incident wave is *vertically polarized*

- At grazing incidence E, the reflected wave is equal to that of the incident wave and has an 180° phase reversal for all finite conductivities.
- As the angle increases from zero, the magnitude and phase of the reflected wave decrease rapidly. The magnitude reaches a minimum and the phase change goes through -90° at an angle known as *pseudo-Brewster angle* (or just *Brewster angle*) by the analogy of a perfect dielectric case. At angles of incidence above this critical angle, the magnitude increases again and the phase approaches.
- For lower frequencies and higher conductivities, the Brewster angle is less, approaching zero as x becomes much larger than ε_r.
- When the incident wave is normal to the reflecting surface $(\psi = 90^{O})$, it is evident that there is no difference between horizontal and vertical polarization. The reflection coefficients R_V and R_H should have the same value, as E will be parallel to the reflecting surface in both cases. Comparison of these figures illustrates that R_V and R_H have the same magnitude but differ by 180° in phase. This is due to the different positive directions assigned for the reflected waves in two cases.
- For angles of incidence near grazing, a more accurate plot of reflection coefficient is often required. Such curves plotted on logarithmic scales are available.

SPACE WAVE AND SURFACE WAVE

According to ***Sommerfeld,*** the ***ground wave*** can be divided into two parts, a ***space wave***and a***surface wave***.



- The space wave dominates at larger distances above the earth, whereas the surface wave is stronger nearer to the earth's surface.

The expressions given by ***Norton for the electric field*** of an electric dipole above the surface of a finitely conducting plane earth clearly show the separation into space and surface waves.

The field strengths for space and surface waves can be given as

$$E_{total}(space) = \sqrt{\left[E_z^2(space) + E_p^2(space)\right]}$$

$$= j30\beta\, I\,dl\,\cos\psi\left(\left\{\frac{[\exp(-j\beta R_1)]}{R_1}\right\} + R_v\left\{\frac{[\exp(-j\beta R_2)]}{R_2}\right\}\right)$$

$$E_{total}(Surface) = \sqrt{\left[E_z^2(surface) + E_p^2(surface)\right]}$$

$$= j30\beta\, I\,dl\,(1-R_v)\,F\left\{\frac{[\exp(-j\beta R_2)]}{R_2}\right\}$$

In the above relations, u^4 and higher order terms are discarded.

A *surface wave* is also called *Norton surface wave* contains the additional function F representing the attenuation.

The expressions (3) and (4) represent the electric field of a vertical dipole above a finitely conduction plane earth. When the dipole is at the surface of the earth, the expression for the surface-wave part of this field reduces to

$$E_{\text{total}}(\text{surface}) = j30\beta I dl(1 - R_V)F\{[\exp(-j\beta R)]/R\}][a_z(1-u^2) + a_\rho \cos\psi(1 + 0.5\sin^2\psi)\}]u\sqrt{(1 - u^2\cos^2\psi)}\}$$

$$F = \{1 - j\sqrt{(\pi\omega)}e^{-\omega}[e_{rfc}(j\sqrt{\omega})]\}$$

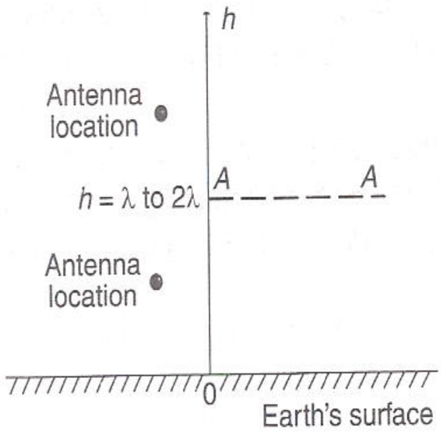
$$\omega = \{-j\beta R u^2(1 - u^2\cos^2\psi)/2\}[1 + \sin\psi/\{u\sqrt{(1 - u^2\cos^2\psi)}\}]^2$$

$$e_{rfc}(j\sqrt{\omega}) = (2/\sqrt{\omega})\int_{i\sqrt{\omega}}^{\infty} e^{-v^2}\,dv$$

When h_t is quite large, the wave is a plane wave and the space wave field is the total ground wave field. When h_t is quite small, the incident wave will not be a plane wave.

Transition between Surface and Space Wave

In case of vertical polarization if the antenna height is less than the barriers *A-A* as shown in figure, the surface wave dominates, so E is not a function of h_t and h_t and the ray action is not present.



Above this barrier, the space wave dominates, ray action (*DR* and *RR*) comes into picture, E is a function of frequency, conductivity and polarization and if σ is finite, h_t is less over the earth surface and large over the sea surface.

In case of a horizontally polarized wave, $h_t = \lambda / 10$ for much smaller σ, even less than for good earth and sea water. Ground wave is almost negligible especially for $f > 30$ MHz.

TILT OF WAVE FRONT DUE TO GROUND LOSSES

Waves moves over the surface of the earth. Initially, electric field E (and hence the displacement current) originating from a vertical antenna can be considered to be entirely perpendicular to the earth. As the wave passes in its travel, it gets weakened due to energy absorption by the earth.

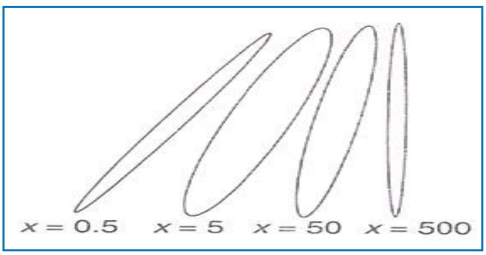


Fig: Elliptic polarization and tilt of E at the earth surface

The farther it travels, the more energy is absorbed and weaker it becomes. The energy absorbed is the result of a current flow beneath (inside) the earth's surface up to a certain depth and the presence of earth resistance.

As shown in above Figure,

- the wave front starts tilting in the forward direction as it progresses.
- The magnitude of tilt will depend upon the conductivity and permittivity of the earth.

The forward tilt of E results in a horizontal component of the current, and hence sufficient power 'P' is dissipated in earth over which the wave is passing.

In general, the components of E parallel and perpendicular to earth will neither be in phase nor will have equal magnitude and thus

> "E above the earth will be will have equal magnitude and thus E above the earth will be ***elliptically polarized.***"

The deeper the current penetrates, smaller is its magnitude.

The surface wave impedance Z_S of earth is given by

$$Z_s = \sqrt{\left[\frac{\omega\mu}{\sqrt{(\sigma^2+\omega^2\epsilon_R)}}\right]} \angle \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\epsilon_R}\right)$$

$$E_h = J_s Z_s$$

$$E_v = \eta_v H$$

$$\frac{E_h}{E_v} = \frac{Z_s}{\eta_v} = \frac{Z_s}{377}$$

The depth of penetration of the current into the ground is the function of the ground constants and the frequency.

At low frequencies,

- ✓ the surface wave is dependent mainly on the conductivity, whereas
- ✓ at higher frequencies a high permittivity is important.
- ✓ Thus, over all frequencies, surface wave is best over sea and worst over dry land.

CURVED EARTH REFLECTION

Generally we considered that the effect of the curvature of the earth is entirely negligible up to a certain distance and all the relations obtained are valid up to this distance given by **[** $d = 50 / (f_{MHz})^{1/3}$**].**

But when this distance gets doubled, the ***errors*** introduced in the estimation of various parameters remain small.

For still greater distances, that is as the wave is propagating for very larger distances, reduction in field strength below the free space value is much more. <u>This high reduction is mainly due to the ***curvature of the earth*** rather than due to losses in the ground.</u>

This is mainly because of ***the bulge of the earth*** which prevents surface waves from reaching the receiver by a straight-line path.

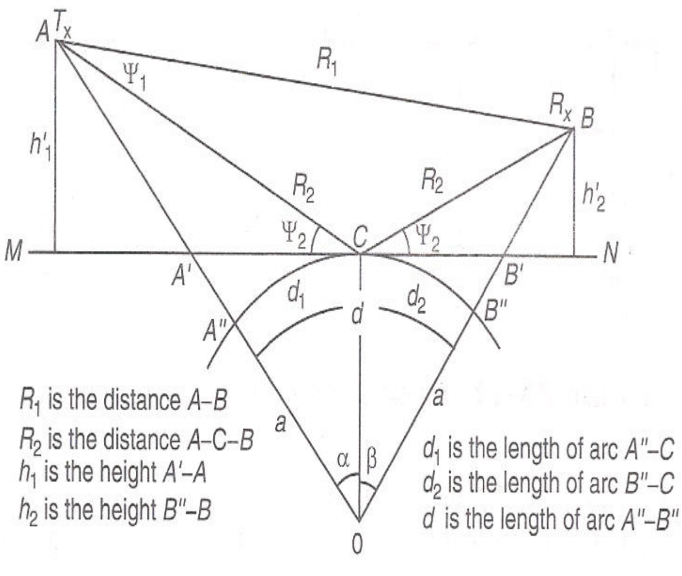
The surface waves arrive at the receiver after

i. diffraction around the earth, or
ii. refraction in the lower atmosphere above the earth.

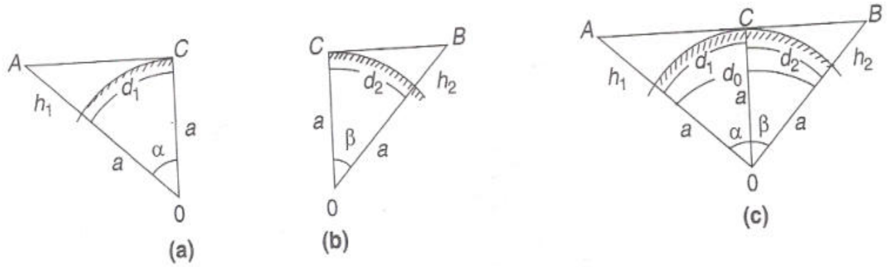
The space-wave propagation too is affected by the earth's curvature. In this case, the wave from the ground is reflected from the curved surface instead of a flat surface. As a result, this wave will have a more **diverged nature** and hence will be weaker while reaching the receiver

As shown in the following Fig, the effective antenna heights $h_1^{'}$ and $h_2^{'}$ are less than the actual antenna heights h_1 and h_2, and thus all equations obtained for flat earth are to be suitably modified.

We considered that this curved earth problem can be solved by applying the Maxwell's equations initially. The problem of spherical earth basically revolves around the question whether transmitting and receiving antennas are within line-of-sight range or not.



To address this problem, consider the following Fig (a). Which shows an elevated antenna A and a point C on the ground.



The problem reduces to finding the distance to visible (optical) horizon. If the radius of the earth is a. antenna height is h_1 and the angle is α then from the right-angled triangle *OAC,*

$$\cos\alpha = \frac{a}{a+h_1} \cong 1 - \frac{h_1}{a} \quad \text{------------ (1)}$$

α in all practical problem is small. Thus for small α ,

We can write from M series equation as

$$\cos\alpha \cong 1 - \frac{\alpha^2}{2} \quad \text{------------ (2)}$$

From (1) and (2),

$$\alpha = d_1 / a = \sqrt{2h_1 / a} \quad \text{------------ (3)}$$

Thus, the horizontal distance is

$$d_1 = \sqrt{2ah_1} \text{ Meters} \quad \text{------------ (4)}$$

Similarly, from Fig. (b),

$$d_2 = \sqrt{2ah2} \quad \text{------------ (5)}$$

If Figures (a) and (b) are joined together by overlapping *OC,* it results in Fig. (c) and the total horizontal distance *d* can be given by

$$d_0 = d_1 + d_2 = \sqrt{2a}\left(\sqrt{h_1} + \sqrt{h_2}\right) \quad \text{------------ (6)}$$

$$d_0 = \sqrt{2 \times 6.37 \times 10^6}\left(\sqrt{h_1(m)} + \sqrt{h_2(m)}\right)$$

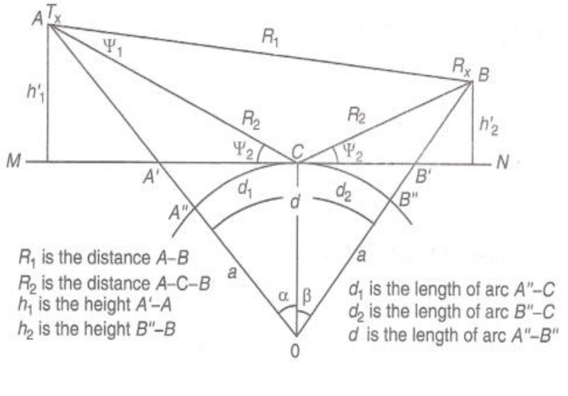
$$= 3.57\left(\sqrt{h_1(m)} + \sqrt{h_2(m)}\right)\text{Km} \quad \text{------------ (7)}$$

The distance d_0 can be termed as line-of-sight (LOS) distance/range.

The curvature of the earth has the following effect on the wave propagation within the LOS range:

1. For fixed antenna heights, the path length difference between *DR* and *RR* will be different from that of flat earth case.
2. The reflection at the convex surface will result in divergence of the *RR* path and hence will reduce the power received via *RR.*

To understand the process, consider Fig.



In the figure

a tangent plane *MN* touching the earth at the point of reflection.

The antenna heights can now be measured from this plane instead of the earth's surface.

The heights $h_1^{'}$ and $h_2^{'}$ so obtained are the reduced heights and can be used for the actual heights h_1 and h_2 wherever they appear in equations.

It is essential that,

The above Figure does not represent parameters in true proportion as heights of antennas are much smaller than the radius of the earth.

Practically, there is little difference between h_1 and $h_1^{'}$ and h_2 and $h_2^{'}$ and the deviations can be written as

$$h_1^{'} = h_1 - \nabla h_1 \quad \text{and} \qquad h_2^{'} = h_2 - \nabla h_2 \qquad \text{------------ (8)}$$

∇h_1 and ∇h_2 are shown as *AA" A'* and *BB" B'*. Since d_1 and d_2 represent the LOS ranges at heights h_1 and h_2 respectively from (4) and (5)

$$\nabla h_1 = d_1^2 / 2a \;\; \text{and} \nabla h_{21} = d_2^2 / 2a \qquad \text{------------ (9)}$$

From (8) and (9)

$$h_1^{'} = h_1 - d_1^2 / 2a \,\text{and}\, h_2^{'} = h_2 - d_2^2 / 2a \qquad \text{------------ (10)}$$

From triangles *OAC and OBC* shown in Fig. 23-12 with angles of incident and reflection being the same,

$(a+h_1)\cos(\alpha+\psi_2)=a\cos\psi_2$ and

$(a+h_2)\cos(\beta+\psi_2)=a\cos\psi_2$

------------ (11)

Equation (11) is justified since h_1 and h_2<<*a,* and $\psi(\psi=\psi_2)$ is the grazing angle *ACM* and *BCN*. From the figure,

$$\tan\psi=\frac{\cos\alpha-\dfrac{a}{a+h_1}}{\sin\alpha}=\frac{\cos\beta-\dfrac{a}{a+h_2}}{\sin\beta} \quad \text{------------ (12)}$$

In the derivation of (12), no assumptions were made.

Therefore, the resulting expression is so rigorous that it cannot be solved analytically and requires graphical or some other approach for getting the solution. It may, however, be simplified since h_1 and h_2 <<*a,* and α and β are also small. Thus, we may set

$$\frac{a}{a+h_1}\cong 1-\frac{h_1}{a} \quad \text{and} \quad \frac{a}{a+h_2}\cong 1-\frac{h_2}{a} \quad \text{------------ (13)}$$

$$\cos\alpha\cong 1-\frac{\alpha^2}{2} \quad \text{and} \quad \cos\beta\cong 1-\frac{\beta^2}{2} \quad \text{------------ (14)}$$

Thus, $$\tan\psi=\frac{h_1/a-\alpha^2/2}{\alpha}=\frac{h_2/a-\beta^2/2}{\beta} \quad \text{---------- (15)}$$

The above equation can also be expressed in terms of distances d_1 and d_2, $(d=d_1+d_2)$ to get

$$\tan\psi=\frac{h_1/a-d_1^2/2}{d_2}=\frac{h_2/a-d_2^2/2}{d_2} \quad \text{---------- (16)}$$

Since in almost all practical cases $h_1 > h_2 d_1$, (16) leads to

$$d_1 = \frac{d_2}{2} + 2\sqrt{\frac{d^2}{12} + \frac{a}{3}\left(h_1 + h_2\right)} \times \cos\left[60^o + \cos^{-1}\frac{ad(h_1 - h_2)}{4\left\{\frac{d^2}{12} + \frac{a}{3}\left(h_1 - h_2\right)\right\}^{3/2}}\right]$$

------------ (17)

When $d < d_0$, the reflection point is located by the equations for flat earth which have the form

$$d_1 = \frac{h_1}{h_1 + h_2} d \qquad \text{and} \qquad d_2 = \frac{h_2}{h_1 + h_2} d$$

------- (18)

The expression for path difference (for flat earth case) given by (5) can be written in the modified from as below.

$$\nabla d = d_2 - d_1 \cong 2h_1 h_2 / d$$

------- (19)

This equation be modified for spherical earth by replacing actual antenna heights h_1 and h_2 by the reduced heights $h_1^{'}$ and $h_2^{'}$. This results in

$$\nabla d \cong 2h_{1'} h_{2'} / d$$

------ (20)

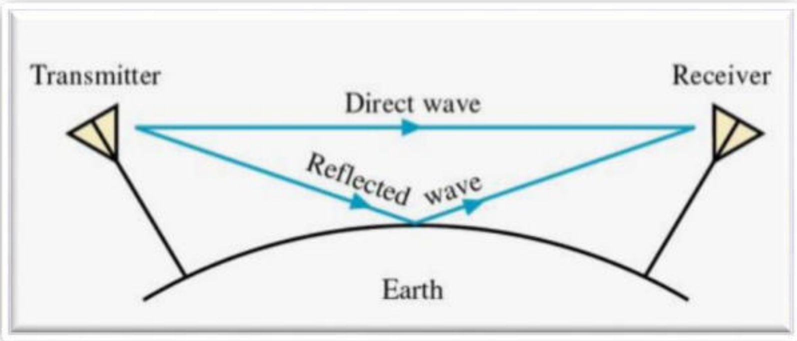
Similarly, in case of flat earth

$$\tan\psi = \frac{h_1 + h_2}{d} \qquad \text{and} \qquad \psi = \frac{h_1 + h_2}{d}$$

------------ (21)

This equation too gets modified in case of spherical earth and can be written as $\psi = \dfrac{h_1^{'} + h_2^{'}}{d}$ ----- (22)

SPACE WAVE PROPAGATION



7.1 INTRODUCTION

The radio waves having high frequencies are basically called as space waves. These waves have the ability to propagate through atmosphere, from transmitter antenna to receiver antenna. These waves can travel directly or can travel after reflecting from earth's surface to the troposphere surface of earth. So, it is also called as Tropospheric Propagation.

Basically the technique of space wave propagation is used in bands having very high frequencies. E.g. V.H.F. band, U.H.F band etc. At such higher frequencies the other wave propagation techniques like sky wave propagation, ground wave propagation can't work. Only space wave propagation is left which can handle frequency waves of higher frequencies. The other name of space wave propagation is line of sight propagation. There are some limitations of space wave propagation.

1. These waves are limited to the curvature of the earth.

2. These waves have line of sight propagation, means their propagation is along the line of sight distance.

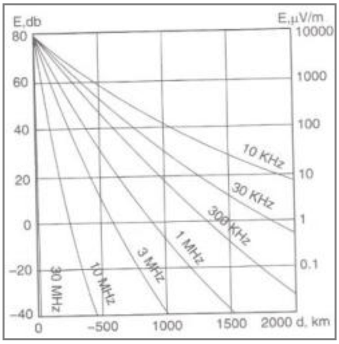
The line of sight distance is that exact distance at which both the sender and receiver antenna are in sight of each other. So, from the above line it is clear that if we want to increase the transmission distance then this can be done by simply extending the heights of both the sender as well as the receiver antenna. This type of propagation is used basically in radar and television communication.

The frequency range for television signals is nearly 80 to 200MHz. These waves are not reflected by the ionosphere of the earth. The property of following the earth's curvature is also missing in these waves. So, for the propagation of television signal, geostationary satellites are used. The satellites complete the task of reflecting television signals towards earth. If we need greater transmission then we have to build extremely tall antennas.

Reflection, diffraction and scattering are the three fundamental phenomena that cause signal propagation in a mobile communication system, apart from Line of Sight communication.

Relation between Signal Strength, distance and frequency:

With the increase of frequency, the attenuation also increases for ground waves. The signal strength at 30 MHz in both the cases will reduce to almost negligible amplitudes after traveling only a very short distance.



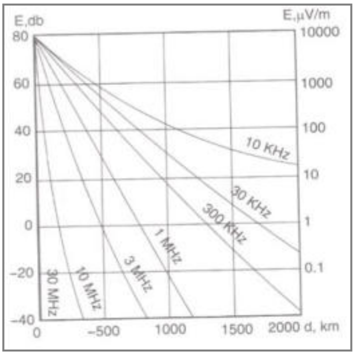


Figure: Relation between signal strength, distance and frequency for poor earth and good earth respectively.

Except in the case of sporadic E layer, the ionosphere too does not reflect energy towards earth at these frequencies. In such a situation, the space wave propagation is the only useful means for any effective and meaningful communication.

7.2 FIELD STRENGTH RELATION

The energy of space-wave travelling from transmitter to receiver is the combination of both partly by *direct wave (DW)* or *direct ray (DR)* and partly by the *reflected wave (RW)*. The net field strength at the receiving antenna will be the vector sum of DW and RW fields. This concept is shown from the following figure.

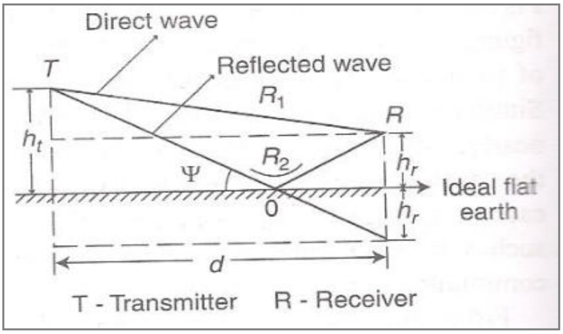


Figure 7.2: Direct wave and the reflected wave.

Up to a certain range of frequencies, the wave travelling through the space shall have negligible attenuation except that ***caused by spreading phenomena***. Also, Direct Ray and Reflected Ray are almost 180° out of phase for both vertically and horizontally polarized waves. Beyond these frequencies (30MHz as shown in figure), waves will be subjected to attenuation by rain, fog, snow, and clouds and due to absorption by gases present in the atmosphere. The field strength of a wave, in general, follows the inverse relation with the distance.

When the distance (d) between transmitting and receiving antennas is sufficiently large in comparison to antenna heights (h_t and h_r), then the incidence angle ψ of the ray on earth is small.

In figure 7.3, the reflection from earth, irrespective of polarization, can be assumed to have no change in magnitude but the phase of the two polarizations is reverse in direction.

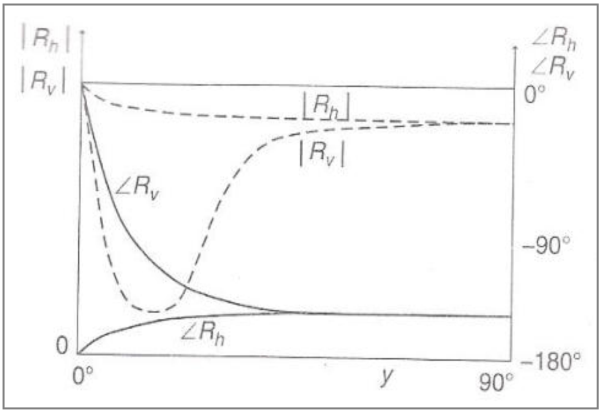


Figure 7.3: Phases of vertical and horizontal polarized wave.

Thus the two waves arriving at the receiver will have equal amplitudes but with different phases.

Let E_0 be the amplitude of DW and RW at a unit distance at a distance d , the amplitudes of both DW and RW reduces to

$E^1 = \frac{Eo}{d}$

Consider the following Fig. 7.4, this shows different parameters of wave propagation. These include the transmitting antenna T located at A , with height h_t , receiving antennas R located at B with height

h_r, R_1 the distance traveled by DW, R_2 the distance traveled by RW both between T and R via 0, i.e., the point from where the wave reflects.

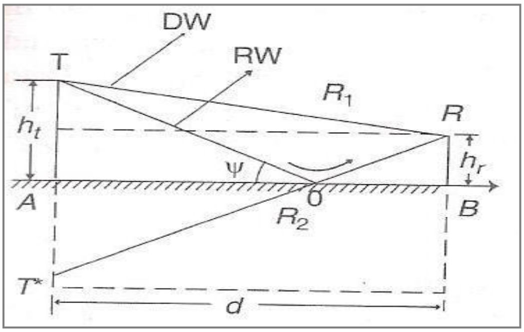


Figure 7.4: Different wave parameters.

Alternatively, it is the distance traveled by reflected ray RR from T^* is the image of the transmitting antenna. Since the earth is assumed to be flat and perfectly conducting, the image will be a perfect replica of the source T and exactly h_t below the ground. The resultant field E can now be obtained from the following Figure 7.5.

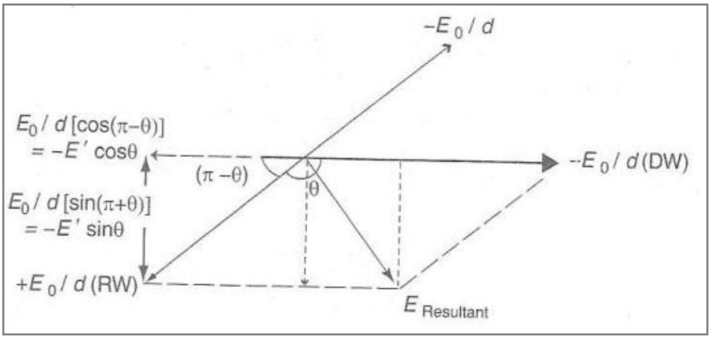


Figure 7.5: Phases of RR and DR.

According to Pythagoras theorem from figure 7.4 we can write

$$R_1^2 = (h_t - h_r)^2 + d^2$$

$R_1 = d[1+(h_t - h_r)^2 / d^2]^{1/2}$ --------- (1)and

$R_2^2 = (h_t - h_r)^2 + d^2$

$R_2 = d[1+(h_t + h_r)^2 / d^2]^{1/2}$ --------- (2)

Assume the distance (d) is much greater than the heights of antennas (h_t and hr), then the wave fronts of the direct and ground-reflected waves are considered to coincide each other at the receiver end. Equations (1) and (2) can then be re-written as

$$R_1 = d[1+(h_t - h_r)^2 / 2d^2]$$

$$= [d+(h_t - h_r)^2 / 2d] \qquad \text{--------- (3) and}$$

$$R_2 = d[1+(h_t + h_r)^2 / 2d^2]$$

$$= [d+(h_t - h_r)^2 / 2d] \qquad \text{--------- (4)}$$

The ***difference in path lengths*** R_2 - R_1 is obtained to be

$$R_2 - R_1 = [(h_t + h_r)^2 - (h_t + h_r)^2] / 2d^2$$

$$R = 2h_t h_r / d \qquad \text{-------- (5)}$$

The ***phase difference*** corresponding to this path difference is

R_2- R_1 $= (2\pi / \lambda)[(2h_t h_r)/d]$ Radians.

R_2-R_1 = $[(4\pi h_t h_r)/\lambda d]$ radians ------ (6)

It is because of this incidence angle ψ, that the direct and indirect reflected waves fail to cancel each other and so the resultant of these two waves is 2 sin $[(2\pi h_t h_r)/\lambda d]$ times the amplitude of one of the waves (i.e., E_0/d).

Thus the field strength E at receiver is

$$E = (2E_0/d)\sin[(2\pi h_t h_r)/\lambda d] \qquad \text{---------} (7)$$

For large distances when $(2\pi h_t h_r)/\lambda d \leq 0.5$, then the sin term in the equation (7) can be replaced by the angle itself and thus equation (7) can be reduces to,

$$E = (2E_0/d)[(2\pi h_t h_r)/\lambda d] = [(4\pi h_t h_r)/\lambda d^2]E_0 \qquad \text{---------} (8)$$

In the above equations, E_0 is the field intensity produced at a unit distance by direct ray emitting from transmitting antenna in the desired direction. E_0 Will depend upon the directivity/gain (G_t) of the antenna and the transmitted power P_t.

For the half-wave elevated transmitting antennas, $E_0 = 137.6\sqrt{P_{kw}}$ mV/m at one mile distance. Assume in all above cases the transmitting and the receiving antennas are Omni directional.

7.3 EFFECTS OF IMPERFECT EARTH

To understand the effect of imperfection of the earth, the following aspects are to be noted.

- The field strength of the DW is $\frac{Eo}{d}$
- The same field strength can be considered for perfectly conducting earth.
- $|R_h|$ and $|R_V|$ both are less than 1 for $\sigma \neq \infty$, Thus, the field strength at a distance d E_d is always be less than E_0/d.

- Besides, $\phi \neq 180^o$, i.e., there is no total phase reversal of RW. Thus RW < DW and the total field is less than that at $\sigma = \infty$.
- The effect is less on Horizontally Polarized Wave than in case of Vertically Polarized Wave, $|R_V| << |R_h|$ at small angles.
- When $\sigma = \infty$, horizontal components of incident electric field E_i and reflected electric field E_r get cancelled at reflected surface and vertical components add together.
- For $\sigma < \infty$, $|R_V| < 1$, neither there is complete cancellation nor complete addition.

The variation of field strength with distance, are shown in the following figure.

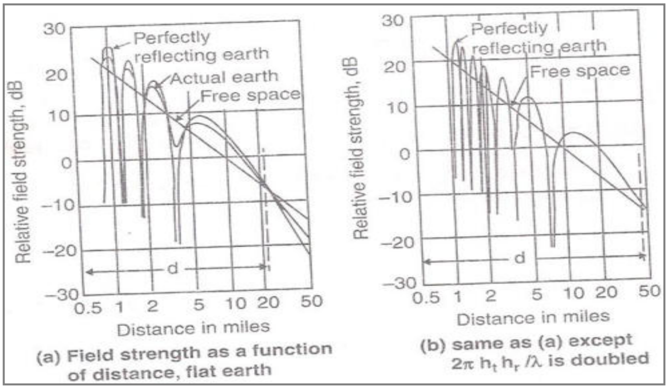


Figure 7.6: Variation of field strength with distance

In Fig (a), d^l is the distance at with free space field and oscillating field for a perfectly conduction earth become equal. It is less than the value that makes the angle $[(2\pi h_t h_r)/\lambda d]$ greater than $\lambda / 6$.

- It can be observed that the field strength oscillates about the value E_0 / d, which corresponds to the strength of the direct ray (often called the free space (FS) wave).

- For a perfectly conducting earth, the maximum amplitude of these oscillations is twice of the free-space value.

These maxima occur at such distances (related to the antenna heights), ***where DW and RW add in phase.*** The minima or nulls have zero amplitude in the case of a perfectly conducting earth, and occur at distances such that the DW and RW cancel each other.

For$d > d'$,Path lengths of DW and RW always differ by less than $\lambda/6$, in such cases electric field E falls rapidly in proportionality with distance square.

For $d > d'$, the angle of incidence is so small that reflection takes place with the reversal of phase and no change in amplitude for both polarizations. The resulting field will be less than the free space value.

In figure (b) By changing h_t, h_r or the frequency the resulting field will be as shown in Fig. (b).

- In this case the field fluctuates more rapidly and the average field strength at any distance is more than in case shown by Fig. (a).

Also, $d'' > d'$ where d'' is the distance at which free space and oscillating fields are equal for perfectly conducting earth.

7.4 EFFECTS OF CURVATURE OF EARTH

Due to Curvature of Earth:

- The effective and actual antenna heights shown in Fig (7.7) get differ. The quantum of difference will depend on the separation between T_x and R_x.
- There is a change in the number and location of maximas and minimas as illustrated in Figure.

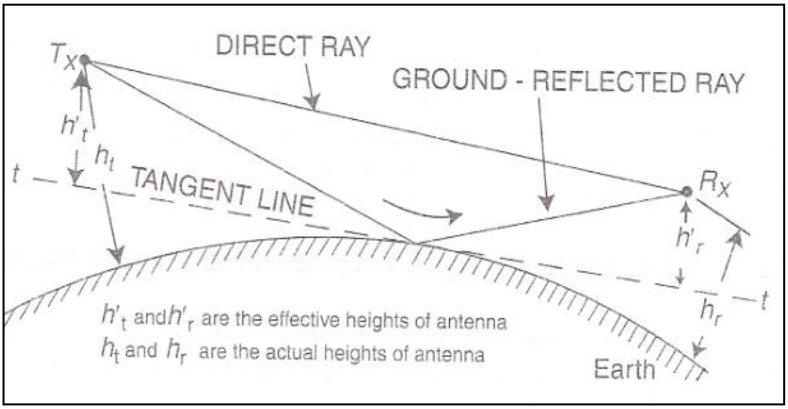
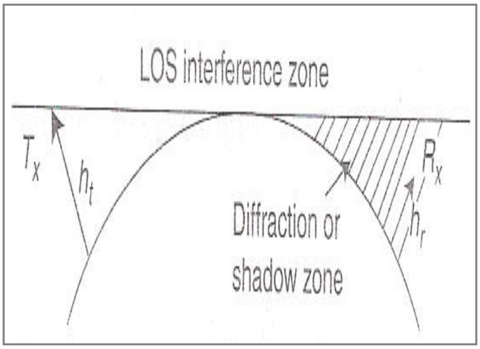


Figure 7.7: DR and RR over curved earth

- There is reduction in d', beyond which the two waves tend to be out of phase.
- The *wave reflected by the ground diverges*. Thus, RW at R_x antenna is *weak*. This effect is less when the incident angle is moderate or large and more when this angle is small. Near grazing angle, the field strength of RW reduces significantly at the receiver by the divergence effect.
- At large distances, for small incidence angles, DW and RW in phase opposition, the resultant E at R_x is significantly greater than that if earth were flat.
- The last two effects of curvature try to neutralize each other.

7.5 EFFECT OF INTERFERENCE ZONE

This effect is shown in Figure wherein if the receiving antenna falls in the ***shadow zone***, logically there should not be any reception. But due the ***diffraction***



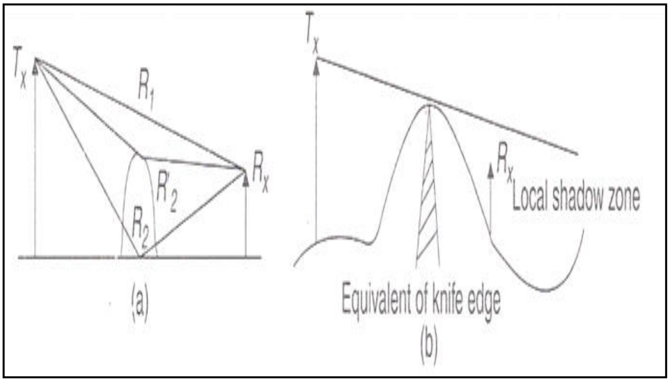
phenomena some signal will reach at the receiver.

7.6 SHADOWING EFFECT OF HILLS AND BUILDINGS

At VHF and above, serious ***disturbances*** in space wave propagation are caused by *trees, buildings, hills and mountains*.

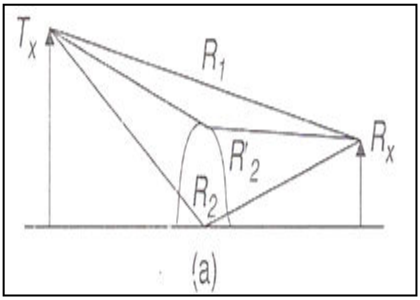
These obstacles cause ***reflection, diffraction and absorption.***

Losses caused by absorption and scattering increase with frequency until f exceeds 3 GHz. Beyond this frequency, building walls and wood become opaque to the waves.



At higher frequencies, the received signal strength is considerably reduced at position on the shadow side of any hill. Above Figure shows the shadowing effect of hills and building.

From Fig. (**a**).



The reduction in R_2 can be seen and thus equations (7) and (8) will yield an altogether different result.

It is not only the reduction in R_2, the obstructive object also scatters the energy.

Therefore, to estimate the real impact the analysis in normally carried out by ***replacing the actual obstruction by an equivalent knife edge*** shown in (b).

7.7 ABSORPTION BY ATMOSPHERIC PHENOMENA

In very high frequency ranges,

- The ***rain attenuates*** the radio wave partly due to ***absorption and partly by scattering.***
- This attenuation is a function of wavelength, permittivity, rain drop diameter and drop concentration, and the losses due to scattering.

Serious attenuation is observed

- ✓ at $\lambda = 3$ cm for heavy rains (not cloud burst) and
- ✓ at $\lambda = 1$ cm for moderate rains,

since attenuation is proportional to the mass of water/unit volume and drop size for cloud and fog smaller than rain drops, serious attenuation occurs below $\lambda = 1$ cm due to clouds and fog.

- ✓ Losses in ice are considerably less than in liquid water.
- ✓ The attenuation by dry hail storm is less than that due to rain except in mm region where it is comparable.
- ✓ As water content in even a heavy snow storm is quite small, so the attenuation caused by snow is always small.
- ✓ Absorption of energy takes place at certain wavelengths due to water vapors and gases generally at λ = 1.33cm and 1.77mm and 1mm.

7.8 VARIATION OFFIELD STRENGTH WITH HEIGHT

The strength of the wave is also get affected with corresponding to the height. The field distribution with respect to the height is shown in the following Figure. Figure (*a*) explains when the earth is considered as flat surface and Fig. (*b*) is for a curved earth.

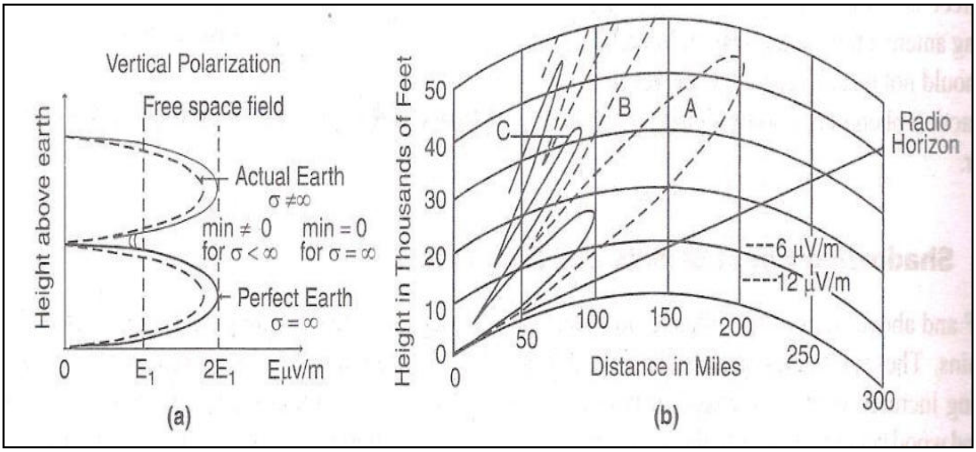


Figure 7.8: variation of field strength with height

The locations of minimas and maximas depend on heights h_t , h_r , frequency and the distance between the transmitter and receiver.

7.9 SUPER REFRACTION

Naturally as height increases from the earths surface the temperature of the atmosphere decreases. In case if the temperature of the atmosphere increases as height increases in reverse fashion and/or the water vapor content decreases rapidly with height, the refractivity gradient will decrease from the specified standard values. This situation is known as **super refraction,** and this makes the radio wave to deflect towards earth surface below its normal path as shown in the figure.

The *refractive index ' n '* (= $\sqrt{\varepsilon}$ for free space is given by the relation:

$$n = 1 + \frac{80}{T} 10^{-6}\left(P_{-} \frac{4800\omega}{T} \right)$$ ------------ (1)

Where, T is the absolute temperature of air, P is the air pressure and ω is the partial pressure of water (humidity) in millibars.

To ***know the concept of refractive index*** the following are to be studied.

- The gradient of the refractive index n is not always uniform.
- It is often divergent from the mean value, particularly in the lower 5 km of the troposphere.
- The variation becomes important if $\lambda < h$; (where h is the height above the ground), since ray paths are dependent on variation of n with height.
- The variation of n leads to the phenomena such as reflection. Refraction, scattering, fading and ducting.
- The duct can be assumed to be a waveguide with leakage.

The actual 'n' is often replaced by a modified index 'N' bearing the relation:

MODIFIED INDEX $N = n + h / a$ ---------- (2)

Relation (2) involves radius of earth 'a' ($a = 6.37 \times 10^6$ m) thus '*N*' accounts for the earth's curvature. *N* is always approximately equal to unity since $h << a$.

A ***new parameter called the refractive modulus 'M'*** is introduced to express this concept.

REFRACTIVE INDEX MODULUS $M = (N - 1) \times 10^6$ -------- (3)

The gradient of N can be written as

$$\frac{dN}{dh}\times 10^6 = \frac{80}{T}\frac{dP}{dh} - \frac{80}{T^2}\left(P + \frac{9600\,w}{T}\right)\frac{dt}{dh} + \frac{80\times 4800}{T^2}\frac{dw}{dh} + \frac{10^6}{a} \quad \text{--- (4)}$$

In this equation, the first term on the right-hand side is always negative and the last term is always positive.

- ✓ Sings of the other two terms depend on atmospheric conditions.

In standard atmosphere, temperature decreases with height @ 6.5°/km and water vapor content w decreases linearly. Thus, the second and third terms both are negative and their values in standard atmosphere are such that $\frac{dN}{dh}$ is positive with value usually taken as $0.118\times 10^{-6}/m$. This value is expressed in terms of $\frac{dM}{dh}$ is given as 0.118M units/m and corresponds to -0.039×10^{-9}.

Under certain atmospheric conditions, $\frac{dT}{dh}$ and $\frac{dw}{dh}$ may greatly differ from standard values, particularly when warm dry air passes over a cool sea surface.

- The air close to water will be cooled and an increase in temperature with the height will result.
- Also, water vapour contents will decrease with height much more rapidly than usual.
- Both of these factors reduce $\frac{dM}{dh}$ which may become negative over a region close to sea surface and result in what is called a ***surface duct.***

Under certain other conditions, $\frac{dM}{dh}$ may assume negative value a little higher in the atmosphere making an ***elevated duct***.

- All conditions which make $\frac{dN}{dh}$ less than the standard values are called *super-standard* and improve radio wave propagation.
- Also, the conditions which make $\frac{dM}{dh}$ greater than the standard values are called *sub-standard making the signals below normal.* The following figure shows different type of refractive index profiles observed.

When $\frac{dM}{dh}$ is negative, the curvature of rays passing through the atmosphere is greater than that of the earth. As a result energy, originated from the antenna and initially directed approximately parallel to the earth surface, tends to be trapped and propagates around the curvature of the earth in a series of hop.

$\frac{dM}{dh}$ =0.036 units/ft for standard atmosphere.

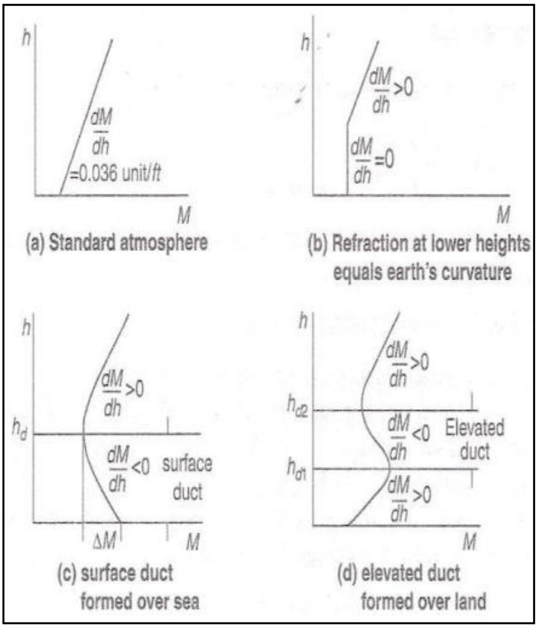


Figure 7.9: different refraction index profiles

Normally, at quite high altitudes n is not a function of height, $\frac{dM}{dh}$=0.048 units/ft.

The ***effect of different rates*** of variation of n on wave propagation is shown in following figure.

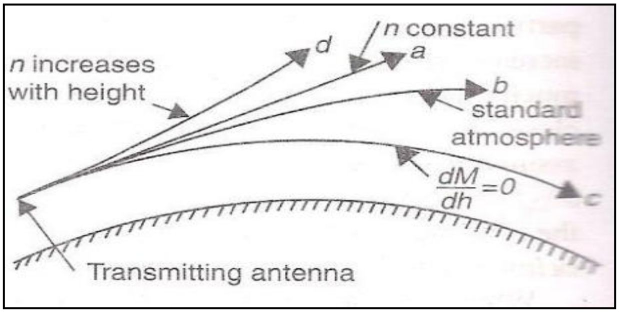


Figure7.10: Effect of variation of n on wave propagation

Thus, the radius of the earth is to be simultaneously adjusted so as to preserve the correct relationship between the ray path and the curvature of the earth. **This adjustment has to be such that the ray path and the curvature are to be seen as parallel and the ray path as a straight line**.

The amount of change in the earth's radius required to achieve the above is to multiply the radius by a factor k, where k is given by

$$k = \frac{Equivalentearthradius}{Actualearthradius} = \frac{0.048}{dM/dh} \quad \text{------------ (5)}$$

In (5), $\frac{dM}{dh}$ represents the change in M with height. For $\frac{dM}{dh} = 0$, k is infinite and for standard atmospheric conditions:

$$\frac{dM}{dh} = 0.036, \quad \text{thus } k = \frac{0.048}{0.036} = \frac{4}{3} = 1.33 \quad \text{------------ (6)}$$

The maximum possible distance at which distance at which direct wave transmission is possible between a transmitting and receiving antennas with heights h_t and h_r is often referred as the ***line of sight***

"(LOS) distance and is equal to the sum of horizontal distances calculated separately for individual antenna heights".

When distance involved is less than LOS value, the path is often referred to as being ***optical***. This is in the sense that the ray can pass directly between transmitting and receiving antennas.

Duct Propagation:

When duct propagation happens, LOS DIFRACTION AND ZONE CONCEPT no longer applies and energy travels long distances with low attenuation.

Ducting is exceptional super-refraction. ***Super-refraction occurs*** when the trajectory of antenna beam bends towards the earth's surface more than normal.

Naturally the antenna beam will tend to increase in height above the earth's surface when moving away from the antenna site because of the earth's curvature.

In a super-refraction situation, the antenna beam could be increasing at a lesser rate with height than normal as the beam moves away from the antenna site or the beam could even be bending back down and getting closer to the earth's surface in spite of the earth's curvature.

It is ducting when the antenna ***beam actually bends closer the earth's surface*** with distance away from the antenna. The bending could be strong enough for the antenna ***beam to bounce*** off the earth's surface.

Features of Ducting:

- ✓ Ducting is caused by strong low level inversions (temperature increases with height).

- ✓ Ducting can also occur when a strong layers of warm and dry air exists in the lower troposphere above very moist air.
- ✓ Ducting causes the antenna to be able to sample much further distances than normal.
- ✓ Ducting increases ground clutter also since the antenna beam remains closer to the earth's surface for a greater distance and can even bend into the earth's surface.
- ✓ Ducting is more common in the morning hours since this time of the day experiences the strongest low-level inversions due to cooling of earth's surface through long wave radiation emission.
- ✓ Ducting can also occur anytime a strong layer exists in the lower troposphere.

The advantages of ducting

- It results increased antenna range.
- It is possible to sample storms further from antenna.
- It is possible to sample lower elevations within storms further from antenna.

The Disadvantages of ducting

The disadvantages of ducting are increased ground clutter and increased anomalous propagation due to antenna beams bouncing energy back from hitting earth's surface or sampling storms beyond the antenna's maximum unambiguous range.

Optical and Radio Horizon:

Consider the following figure. From the point of earths observation. This distance d_0 along LOS is

$$d_0 = \sqrt{2Kah_t} + \sqrt{2Kah_r} \quad \text{------------ (7)}$$

Where h_t and h_r are height transmitting and receiving antennas, a is the earth's radius and K is the factor accounting for refraction due to a uniform gradient of refractivity.

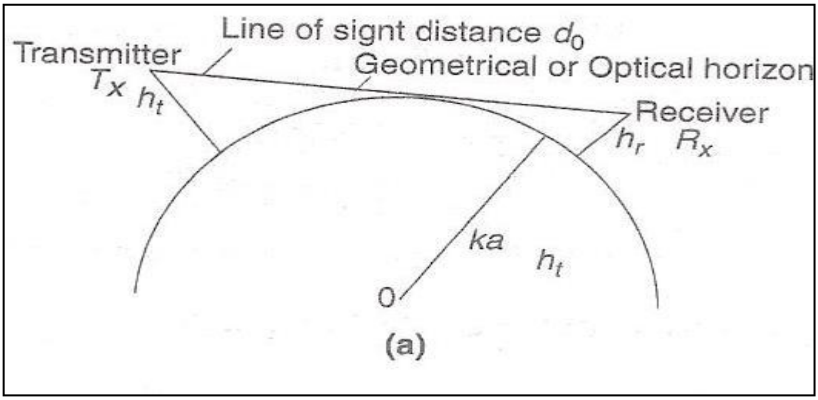
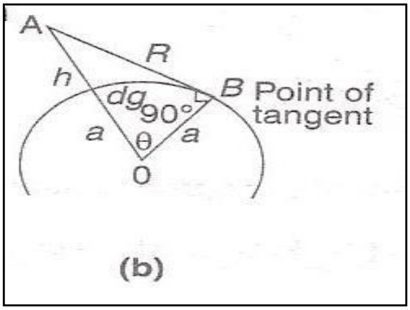


Figure: defines optical horizon

The point of tangency of the LOS with the earth is termed as **geometrical or optical horizon.**

To explain the effect of refraction, consider a transmitting antenna at a height h_t above the earth's surface. The geometrical horizon distance *dg* can be obtained from Fig (b).



From the geometry, with $h_t << a$ such that θ is small for the tangent falling at B,

$R^2 = (a + h_1) + a^2$ -------------- (8)

Geometrical horizon distance *dg*

dg = a θ
dg = a sin θ
sin θ = ?
so, $(h+a)^2 = R^2 + a^2$
$R^2 = (h+a)^2 - a^2$
Sin θ = R/(h + a)
Let h = ht

$$dg = a\theta \approx a\sin\theta = a\frac{\sqrt{(h_t+a)^2-a^2}}{h_t+a} = a = \frac{\sqrt{h_t+2aht}}{h_t+a} \quad \text{-------------- (9)}$$

Since $h_t << a$

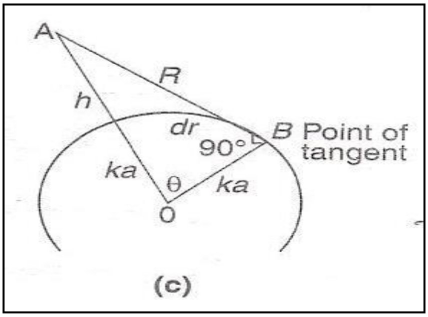
$$dg \approx \sqrt{2ah_t} \quad \text{-------------- (10)}$$

The radio horizon distance dr can be obtained from Fig. (*c)* following the same procedure.

$$dr = ka\theta \approx ka\sin\theta = Ka\frac{\sqrt{(ht+ka)^2-(Ka)^2}}{ht+ka}$$

$$= Ka\frac{\sqrt{h_t^2+2Kah_t}}{h_t+ka} \quad \text{-------------- (11)}$$

$$\approx \sqrt{2Kah_t}$$



(c)

Since $K > 1$, dr has to be greater than dg. Since for standard atmospheric conditions, K = 4/3 the radio horizon distance dr is

$$dr = \sqrt{\frac{4}{3}}\sqrt{2ah_t} \approx 1.155\,dg \quad \text{-------------- (12)}$$

If ΔM is the total decrease in $M$from bottom to the top of the duct the wave length $\lambda = \lambda_{Max}$ at which the duct propagation ceases is given by

$$\lambda_{Max} = 2.5hd\,[\Delta\ M \times 10^{-6}] \quad \text{-------------- (13)}$$

Where, h_d is the height of the duct as shown in figure.

- There is always some leakage of energy from the duct which increases as the ratio of λ / h_d increases.

Ray and Wave Guide concepts of Duct Propagation are shown in the following figure.

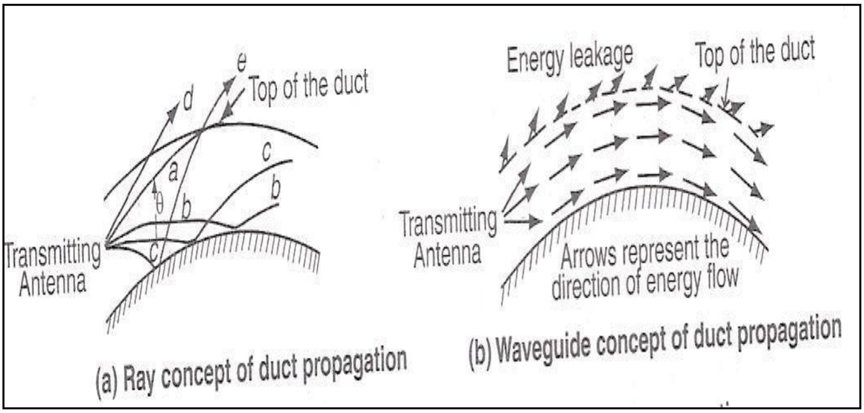


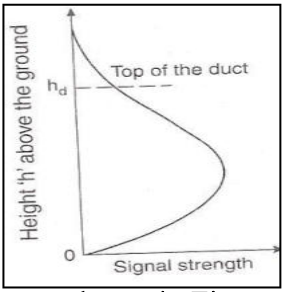
Figure 7.11: Radio and wave concept of the duct propagation

- Duct propagation is limited to UHF and microwave frequencies.

- For duct formation, it is necessary that antenna height remains less than or equal to h_d .
- Ground-based ducts over sea or water stretches occur less frequently and generally temporarily.
- Elevated ducts are always present over oceans or in trade wind belts.

For surface duct of height h_d , λ_{Max} for which trapping occurs is more accurately given by

$$\lambda_{Max} = \frac{8\sqrt{2}}{3}\int_0^{hd} \{N(h) - N(h_d)\}^{1/2} dh$$



The duct formation and its waveguide equivalent are shown in Fig.

SCATTERING PHENOMENA

Reception far beyond the optical horizon in VHF and UHF range is possible due to scatter propagation. Both troposphere and ionosphere continuously changes their refractive index.

This gives rise to local variation in ‘ n ’ of the atmosphere.

Waves passing through such (gradual refractive index changing regions) turbulent regions, the waves get scattered.

- When λ is large the waves scatter in all the directions.
- When λ is small compared to these irregularities then most of the scattering takes place within a narrow cone surrounding the forward direction of propagation of the incident radiation.

To receive scattered signal at a point well beyond the horizon, the transmitting and receiving antennas must be **of high gain**.

The scattering angle should also be as small as possible.

This process is shown in Fig.

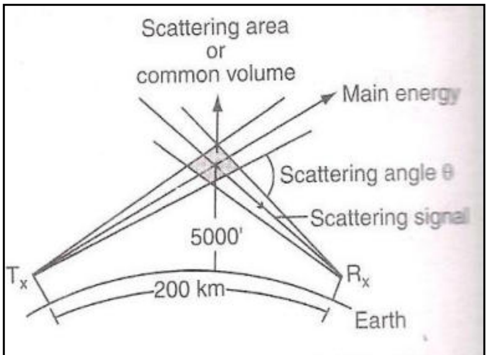


Figure 24–15 Illustration of the scattering process.

Since the scattering process is of random nature, the scattered signals continuously fluctuate in amplitude and phase over a wide range.

The scattering is significant in the following regions:

- **500 MHz onwards with troposphere as the scattering medium.** It is called ***troposphere scattering***. Depending upon the bandwidth of transmitter, its maximum range lies between 300 to 600 km.

- **30 to 50 MHz with ionosphere as medium.** It is called *ionospheric scattering* and mainly occurs in the *E* region with maximum range of about 2000 km. the level of scattered signals in this case is much small, some 10 to 20 dB below the free space signal for the same distance.

TROPOSPHERIC PROPAGATION

Consider an omni-directional antenna which radiates uniformly in all directions. Let the transmitted power be denoted by P_t and the power density (i.e., the power per unit area) in free space at a distance R from the transmitter is denoted by P_{rf} . It will be equal to the transmitted power divided by the surface area $4\pi R^2$ of an imaginary sphere of radius R.

Thus

$$P_{ref} = \frac{P_t}{4\pi R^2} \text{ watts} \qquad \text{------ (1)}$$

If the transmitting antenna is directional with gain G_t the increased power reaching the point of observation as compared to the power that would have been reaching in case of an omni-directional antenna is given by

$$P_{ref} = \frac{P_t G_t}{4\pi R^2} \text{ watts} \qquad \text{------ (2)}$$

At the point of observation, the receiving antenna will capture a portion of this radiated power. If the effective capture area of receiving antenna is A_r,

$$P_{ref} = \frac{P_t G_t A_r}{4\pi R^2} \text{ watts} \qquad \text{------ (3)}$$

The antenna gain G_r and the effective area A_r for receiving antenna bear the following relations:

$$G_r = \frac{4\pi A_r}{\lambda^2} \quad \text{or}$$

$$A_r = \frac{\lambda^2 G_r}{4\pi} \qquad \text{------ (4)}$$

Substitution of A_r in the expression of P_{ref} results in

$$P_{ref} = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2} \text{watts} \qquad \text{------ (5)}$$

If scattering process is involved, P_{ref} will be obtained by multiplying RHS of (5) by an attenuation factor F given by

$$F = \frac{2}{R\sqrt{\pi}} \sqrt{\sigma(\theta)v} \qquad \text{------ (6)}$$

Where $\sigma(\theta)$ is the effective scattering cross-section,
v is the scattering (common) volume and
θ is the scattering angle.

The variation of attenuation factor **F** with distance for a number of frequencies is shown in the figure for easy understanding.

The availability of strong signals can be studied with the help of

- Turbulent scattering theory and
- Layer reflection theory.

1. ***Turbulent Scattering Theory****P:*

According to this theory there is a turbulent variation of refractive index$n$ with height.

- ✓ The twinkling of stars, wavering appearance of objects seen over the earth's surface, heated by the sun, and random erratic appearance of exhaust gases left by the aircraft engines are said to be the result of turbulent variation of n .

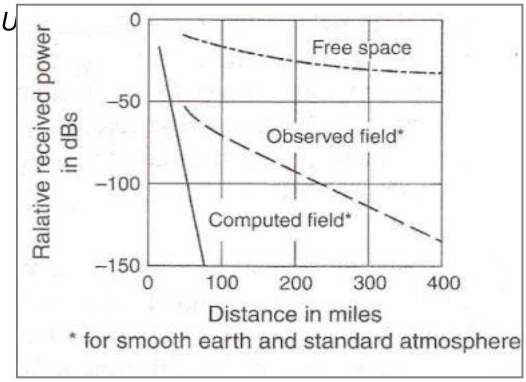


Figure: Relative received power for observed field and computed field

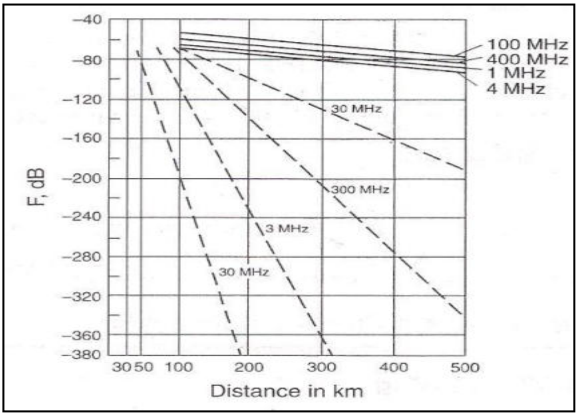
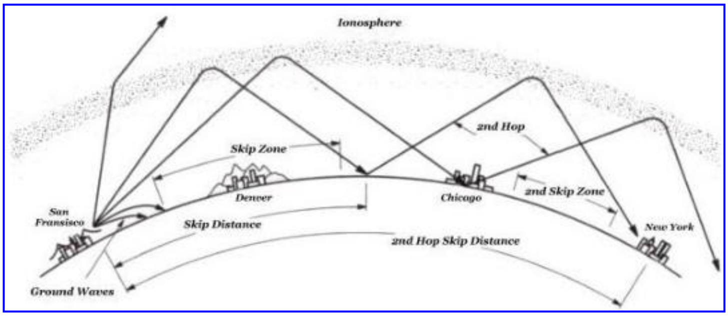


Figure: Variation of attenuation factor F index n with distance.

2. ***Layer Reflection Theory P :***

In this theory, it is presumed that there are a large number of randomly distributed layers with different refractive indices. These layers result in scattering of part of the transmitted energy towards the earth.

SKY WAVE PROPAGATION



INTRODUCTION

The frequencies falling between 3000 hertz (3 kHz) and 300,000,000,000 hertz (300 GHz) are called RADIO FREQUENCIES (abbreviated rf) since they are commonly used in radio communications. This part of the radio frequency spectrum is divided into bands, each band being 10 times higher in frequency than the one immediately below it. This arrangement serves as a convenient way to remember the range of each band.

DESCRIPTION	ABBREVIATION	FREQUENCY
Very low	VLF	3 to 30 KHz
Low	LF	30 to 300 KHz
Medium	MF	300 to 3000 KHz
High	HF	3 to 30 MHz
Very high	VHF	30 to 300 MHz
Ultrahigh	UHF	300 to 3000 MHz
Super high	SHF	3 to 30 GHz
Extremely high	EHF	30 to 300 GHz

Table 8.1: Different frequency bands

Sky Wave propagation mode of wave propagation is related to the ***high-frequency range.*** It finds its application in the ***broadcast services.*** The propagation of sky waves is also called ***ionospheric waves*** that involve the refraction mechanism in the ionosphere.

The electromagnetic waves are launched towards the ionosphere and from there under suitable conditions, they return to the earth due to the ***refraction mechanism***.

STRUCTURAL DETAILS OF THE IONOSPHERE

The ionosphere extends upward from about 31.1 miles (50 km) to a height of about 250 miles (402 km). It contains four cloud-like layers of electrically charged ions, which enable radio waves to be propagated to great distances around the Earth. This is the most important region of the atmosphere for long distance point-to-point communications.

The ionosphere is a region above the earth and is composed of ionized layers. There are, four layers, namely D, E, F_1 and F_2 are assumed to exist at different heights. These layers are shown in Figure.

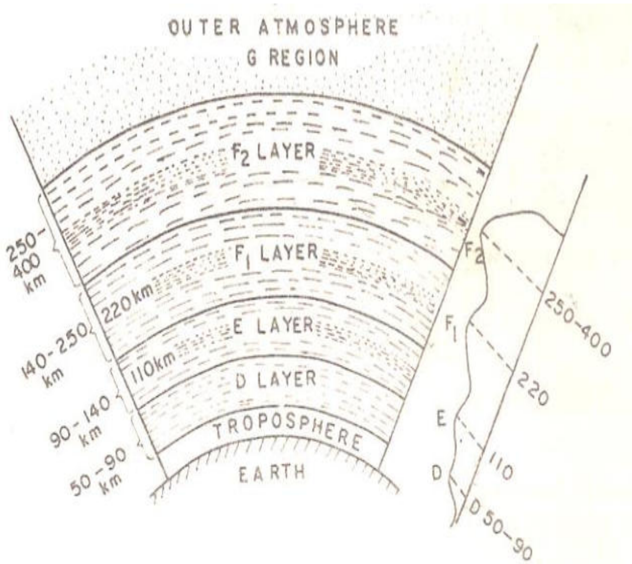


Figure 8.1: structure of Ionosphere.

The ionospheric region extends from about 50km above the earth with maximum ionization density at about 300km.

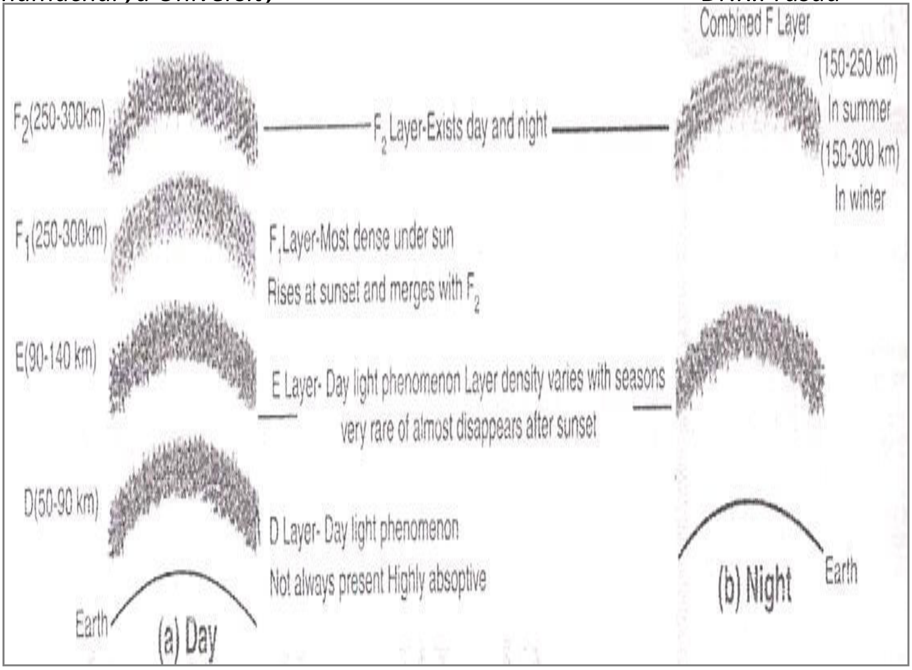


Figure: Ionospheric layers

Ionosphere appears in the form of different layers. The following are basic fundamental basis of ionosphere.

At great heights, the ***ionizing radiations are very intense***, but the atmosphere is rare and there appear few molecules hence ***it consists low ionized densities.***

As height decreases the atmospheric pressure and ionization density increases until a height is reached where the ***ionization density is maximum.***

Below this height the atmospheric pressure continues to increase, but the ionization density decreases because the ***ionizing radiation is absorbed*** almost.

D layer:

- It is lower most region of Ionosphere, It exists between 50 to 90 km above the earth's surface.

- It is a daytime phenomena and is largely absent in the night.
- Ionization in the D layer is low because less ultraviolet light penetrates to this level.
- The ionization density is maximum at noon in this layer.
- Its critical frequency is 100kHz
- At VLF, the space between the D layer and the **ground acts as a huge waveguide**, making communication possible but only with **large antennas and high power transmitters**.
- At LF and MF ranges, this **layer is highly absorptive** and limits daytime communication to about 300 km. it is responsible for much of the daytime attenuation of HF waves.

E layer:

- It exists between 90km to 140 km above the earth's surface, ***with maximum density at about 110 km***.
- E layer is formed by ionization of all gasses by soft X-ray radiation.
- At day time this layer has maximum ionized particles and during night time they have weekly ionized particles.
- It is almost ***constant with little diurnal or seasonal variations.***
- The critical frequency of E layer is 3MHz to 5MHz.
- The size of the E layer is governed by the amount of ultraviolet light from the sun and is uniformly decays with time at night.

- This layer ***permits medium distance communication*** in LF and HF bands.
- At night, the D layer slightly rises and the E layer slightly lowers to form one layer, which is again called **the E layer.**

F Layer:

The region of ionosphere lying between 140 km to 400km is known as F layer. Its average height is around 270km.

- ✓ F layer facilitates long distance sky wave propagation of radio waves during night times.
- ✓ The f layer appears even in night time, because it is the top most layer of ionosphere, it has highly ionized particles.
- ✓ After sun set also some part of the ionized particles present in this layer so it provides propagation of radio waves even in night time.

F_1 layer:

It exists between 150km to 250 km above the earth's surface in summer and 150 to 300 km in winter.

- Its critical frequency is about 5MHz to 7mHz.
- This layer is also ***almost constant*** with little diurnal or seasonal variation.

F_2 layer:

- It exists between 250km to 400 km.
- At night, the F_1 layer slightly rises and the F_2 layer slightly lowers to form one layer, which is ***again called the F_2 layer***.
- It is sometimes also referred as the F layer.

- It is ***more variable in nature***.
- The F_2 layer is responsible for most of the ***HF long-distance communication***.

Sporadic E layer:

- It is the result of an abnormal phenomenon and falls under the category of ***irregular variations***.
- Its occurrence is **quite unpredictable** and is observed both during day and night.
- It occasionally **appears in and around the E layer,** at discrete locations and then disappears.
- Spordic E layer will form during the times of THUNDER STROMS or GEOMAGNETIC disturbances.
- It is also formed due to Meteoric ionization.

It often occurs **in the form of clouds of charged particles of varying size**.

- ✓ In polar regions the sporadic E – layer occurs mainly at night times and shows no pronounced seasonal variations.
- ✓ At Equatorial zone it appears in day time.

It can be so thin that radio waves penetrate it and are returned by the upper layers, or it can extend up to hundreds of km. this layer may appear anywhere between the e and F_1 layers or within the range of the E layer itself.

The negative aspects of the sporadic E layer are that

it sometimes prevents the use of higher, more favorable layers and at some frequencies it may also result in (i) additional absorption,

(ii) Multipath problem, and (iii) additional delay in return of the waves.

Its positive aspects include that **it has greater critical frequency** and thus permits **long-distance communication** at much higher frequencies than the usual ones for well-defined layers.

WAVE PROPAGATION MECHANISM

To understand the refraction mechanism, first assume that the earth's magnetic field is absent. Later the effect of earth's magnetic field on the propagation mechanism can be present.

REFRACTION IN THE ABSENCE OF EARTH'S MAGNETIC FIELD

The waves that reach to ionosphere are reflected back to earth surface, this mechanism can be studied with two interpretations about the bending of ionospheric waves.

Interpretation-I

The wave that is propagating consists of electric and magnetic field vectors ***'E'*** and ***'H'***. These phenomena can be mathematically derived in terms ***Maxwell's equations.***

The ionospheric layer is completely ionized in state and it consists of ionized particles naturally, electrons and protons.

A charge 'Q' that is present in ionosphere exhibits an electric field **'E'** when it is subjected to a force **'F'** with the relation **F** = Q**E**. The force exerted on electrons in ionosphere causes them to vibrate, in ***sinusoidal fashion***, along with the lines of electric flux.

These vibrating electrons vibrate in the ***form of loop*** and results in the loop current I_L . The loop current density J_L is proportional to the velocity of vibration '**U**', and is given with the following relation **J =** ρe U**,** where 'ρe' is the electron charge density.

"**The maximum velocity U**$_{\mathbf{max}}$ **lags behind electric field E**. Thus the current I_L resulting from these vibrating electrons is **inductive in nature".**

We know the Maxwell's equation, $\nabla \times H = \partial D / \partial t = \varepsilon \partial E / \partial t$. This states that in ionosphere, due to presence of these magnetic and electric fields the electrons will obtain some kind of motion and that is called the ***displacement current density*** J_d . Here assume current density J_c is to be zero in free space. Thus, the **displacement current or the capacitive current** (I_d) already exists within the medium due to the presence of electric field vector **E**.

These two different loop and displacement currents (I_L and I_d), are inductive and capacitive in nature. Hence one get subtracted with another, ***resulting in decrease of net current,*** due to the presence of ionized electrons. Since $J_d = \varepsilon \partial E / \partial t$, and there is no change in **E**, the decrease in the net current can be understood by the wave, as it has a change (reduction) of ε .

A decrease in ε will result in ***bending of path of the wave***, from a high electron-density region to a low electron-density region. Since the average velocity of the vibrating electrons is inversely proportional to the frequency 'f', the magnitude of current is greater for lower frequencies and smaller for higher frequencies.

When f is lower enough, the capacitive and inductive currents are equal and the resultant current is zero, amounting to $\varepsilon = 0$.

Interpretation-II

According to the second interpretation, **each vibrating electron acts** as a **small radio antenna extracting energy** from the passing wave. This extracted energy is later re-radiated by these microscopic antennas.

Since I_L due to this re-radiated energy lags ***E*** by 90°, and these electrons, **and act as parasitic antennas,** will offer an **inductive reactance**, and the net effect is to alter the direction in which the resultant energy flows.

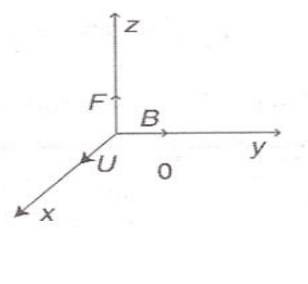
Though ions in the path also behave in the same manner as electrons but since they are heavier, they vibrate at a much slower rate than the electrons, under the influence of the electric field, and hence have negligible effect on the propagation mechanism.

"The above to interpretations ultimately result in SMOOTH BENDING OF THE WAVE towards the earth in the absence of a magnetic field. "

REFRACTION IN THE PRESENCE OF THE EARTH'S MAGNETIC FIELD:

Let us consider there is some effect of earth's magnetic field on the electrons which are in motion in ionosphere. Basically earths magnetic flux density is represented by 'B'.

When force ***F*** is applied on to a charge which is in motion that having a velocity **U,** then the earth's magnetic field **B** effect is given by the relation **F** = Q(**U** $\times$**B**). These vectors are shown in Fig.



At high frequency, a component of magnetic field ***B*** is at right angles to electric field ***E*** of the incident wave, so **vibrating electrons will** follow ***elliptic paths.***

The new electric field produced due to the vibrating charges is given by $\mathbf{E}_{\mathbf{new}} = \mathbf{F}/Q$. This field will have two components, one parallel and the other perpendicular to ***E*** of the incident wave. Thus, the **polarization** of ***E*** will be now **rotated by 90°** in space with respect to the incident wave.

Since some of the portions of such paths have components at right angles to **E** of the wave, **the electrons, from the passing wave, absorb some energy.**

This **energy is re-radiated** with polarization that is rotated by 90° in space with respect to the polarization of the incident wave.

Thus, the **earth's magnetic field** (shown in Fig.) will normally cause a **plane-polarized radio wave** to become ***elliptically polarized*** after it travels some distance in the ionosphere.

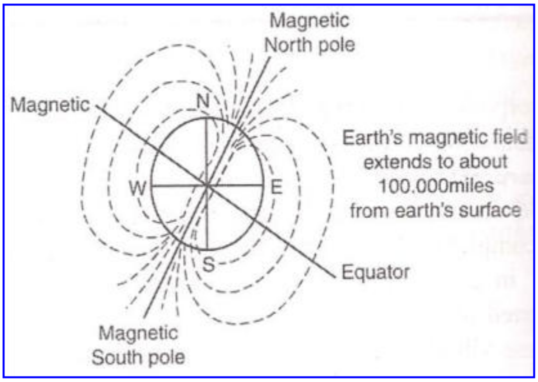


Figure 8.3: The earth's magnetic field and its orientation

The **average velocity** is inversely proportional to the frequency. So at higher frequencies **electrons vibrate** along very ***narrow elliptical paths***, whereas at lower frequencies, the effect on is on greater vibrating electrons.

- As frequency decreases, the amplitude of vibration increases, and the ellipse becomes larger and larger.
- If the frequency is further decreased, a **cyclotron resonance occurs**; the ellipse breaks and electrons start following a **spiral path** of steadily increasing radius.

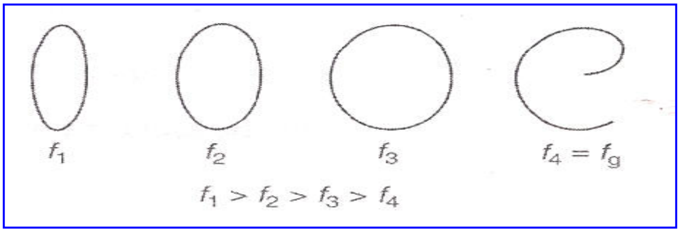


Figure8.4 : Effect of frequency on vibrating electrons.

The frequency at which this resonance occurs is called the ***gyro frequency 'f**$_g$'*.generally it is ($\cong$ 1400 kHz).

At still lower frequencies, electrons follow a ***relatively complicated path*** having components of motion, both in parallel and perpendicular to the plane of polarization.

The earth's ***magnetic field*** causes the ***wave to split*** into two components, namely ***ordinary waves*** and ***extraordinary waves*,** and both of these two waves have elliptic polarization but they rotate in ***opposite directions***.

The two waves will bend with different amounts by the ionized medium in ionosphere and travel in different paths. Their rates of energy absorption and velocities will also differ. This action of splitting is termed as ***magneto-ionic splitting.***

REFRACTION AND REFLECTION OF SKY WAVES BY IONOSPHERE

The radio wave when reaches to Ionosphere from the transmitter on the ground surface, it is experienced to reflection and refraction mechanisms because theionosphereis consisting of different layers with different levels of refractive index. This is shown from the following figure.

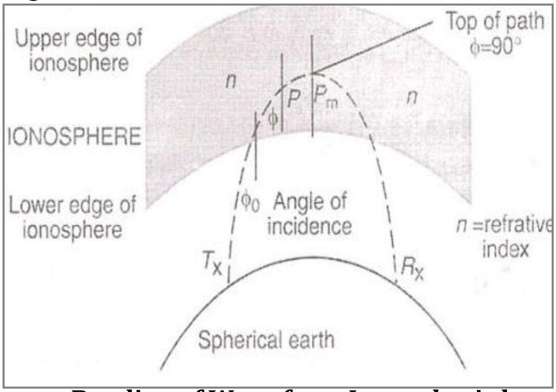


Figure: Bending of Wave from Ionospheric layers

The refractionphenomena in the ionosphere iswith the following relations:

Refractive index $n = \sqrt{\epsilon} = \sqrt{1 - \frac{81N}{f^2}}$ -------- (1)

Snell's law states thar$n_1 \sin\phi = n_2 \sin\phi$

$n_1 \sin\phi = \sin\phi_0$ -------- (2)

Phase velocity $v_p = \frac{c}{n}$ -------- (3)

Where,

n is the refractive index of the ionosphere,

$\varepsilon(=\varepsilon_r)$,is the permittivity relative to free space,

N is the number of electrons per cubic cm,

f is the frequency in kHz,

ϕ is the angle of refraction at P,

ϕ_0 is the angle of incidence at the lower edge of ionosphere

c is the velocity of light.

Let us assume that a plane wave is traveling in the positive Z-direction in ionosphere then it's electric field E has a component only along the x-axis (i.e., E_y and E_z are Zero). So

$$E = E_x \mathbf{a}_x,$$

where $E_x = E \sin \omega t$.

Also, the same wave has only one component of H in y direction, i.e,

$$H = H_y \mathbf{a}_y \ (H_x \text{ and } Hz \text{ are } 0).$$

This wave moves in a region containing free electrons.

The electric field E will exert a force on each electron present in ionosphere given as

$$F_x = eE_x \qquad \text{----------} \ (4)$$

This force results in ***acceleration of electrons*** in the x direction.

From ***Newton's law***,

We also know that $F = ma$

$F = qv$

$F = eV$

So,

$$\frac{md^2x}{dr^2} = -eE_x = -eE\sin\omega\,\mathrm{t} \qquad \text{---------- (5)}$$

Let's find velocity at initial zero condition

$$\text{Velocity}\ \frac{dx}{dt} = -\frac{e}{m}\int E\sin\omega t\ dt = \frac{e}{m\omega}E\cos\omega t \qquad \text{---------- (6)}$$

If there are N electrons per cubic meter in the space, each carrying a charge $-e$ and having mass m, then

The ***current density*** J represented by this motion of electrons is

$$J = -Ne\frac{dx}{dt} = -\frac{Ne^2}{m\omega}E\cos\omega t \qquad \text{---------- (7)}$$

From Maxwell's equations, we know that $J = \sigma E$ ----- (7a)

Equating equation (7) and (7a) we can write $\sigma = \frac{-Ne^2}{m\omega}$ -------- (8)

From Maxwell's equations, we can write

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + \varepsilon\frac{\partial E}{\partial t} \qquad \text{---------- (9a)}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu\frac{\partial H}{\partial t} \qquad \text{---------- (9b)}$$

We assumed that E has only the x component and H has only the y component. Then it results in the following relations.

$$J + \varepsilon_0\frac{\partial E_x}{\partial t} = -\frac{\partial H_y}{\partial z} \qquad \text{---------- (10 a)}$$

$$\mu_0 \frac{\partial E_x}{\partial t} = \frac{\partial E_x}{\partial z} \qquad \text{---------- (10b)}$$

Substitution of Equations (7) and (8) in Eq. (10a) gives

$$-\frac{Ne^2}{m\omega} E\cos\omega t + \varepsilon_0 \frac{\partial}{\partial t}(E\sin\omega t) = -\frac{\partial H_y}{\partial z} \quad \text{---------- (11)}$$

$$\text{Or } \left(\varepsilon_0 - \frac{Ne^2}{m\omega^2}\right)\omega E\cos\omega t = \varepsilon_0\left(1 - \frac{Ne^2}{m\omega^2\varepsilon_0}\right) \quad \text{---------- (12)}$$

$$\omega E\cos\omega t = \varepsilon_0\varepsilon_r\omega t = -\frac{\partial Hy}{\partial z}$$

$$\text{Where } \varepsilon_r = 1 - \frac{Ne^2}{m\omega^2\varepsilon_0} \quad \text{---------- (13)}$$

Since the refractive index $n = \sqrt{\in_r}$

$$n = \sqrt{\left(1 - \frac{Ne^2}{m\omega^2\varepsilon_0}\right)} = \sqrt{\left(1 - \frac{Ne^2}{m\varepsilon_0(2\pi f)^2}\right)} = \sqrt{\left(1 - \frac{Ne^2}{4\pi^2\varepsilon_0 mf^2}\right)} \text{ --(14)}$$

For $e = 1.59 \times 10^{-19}$ c, $m = 9 \times 10^{-3}$ km and f in Hz

$$n = \sqrt{\left[1 - \frac{N(1.59 \times 10^{-19})^2}{4\pi^2 \in_0 (9 \times 10^{-31})f^2}\right]} = \sqrt{\left(1 - \frac{81N}{f^2}\right)} \text{ ---------- (15)}$$

From equation(15), it can be noted that;

- ***Refractive index*** n is a function of frequency so the velocity of the wave is also frequency dependent.

This is the same phenomenon as observed in waveguides, giving rise to two velocities known as *phase velocity* and *group velocity.*

The ***phase velocity***

$$v_p = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{c}{\varepsilon_r} = \frac{c}{\sqrt{1 - \frac{81N}{f^2}}} \text{ (for } \mu_r = 1) \qquad \text{---------- (16a)}$$

The ***group velocity*** $v_g = c\sqrt{1 - \frac{81N}{f^2}}$ ---------- (16 b)

From equation (16), it can be concluded that;

- ❖ The phase velocity is directly proportional to the frequency whereas the group velocity is inversely proportional to the frequency.

Lastly, equations (16a) and (16b) lead to the well-known relation:

$$v_p v_g = c^2$$

The wave when incident towards ionosphere it get reflected back from there to earth surface or may penetrate the ionosphere and left into outer atmosphere. The condition when these mechanisms take place can be analyzed from the above equations.

We know that $n = \sqrt{\epsilon} = \sqrt{1 - \frac{81N}{f\,2}}$

1. For $n > 1$: This condition requires the term $\frac{81N}{f2}$ is to be negative which is not possible. So, ***this condition does not exist.***
2. $n < 1$: This condition requires that $\frac{81N}{f2} < 1$. ***This condition always exists.*** From equation (3) it is clear that v_p ***is always greater than*** c***.***
3. $n = 1$: This condition means that the term $\frac{81N}{f2} = 0$.

In this case, $v_p = c$ and from (3) $\phi = \phi_0$.

4. $n = 0$: This condition requires that the term $\frac{81N}{f2} = 0$ (or) $\frac{81N}{f2} = \frac{N}{f2}$

   At this point, $f = f_c$ (where f_c is termed the *critical frequency*), and the inductive current i_L equals the capacitive current i_c. Also, in this case $v_p = \infty$ and from (3), $\phi_0 = 0$.

5. For $\frac{81N}{f2} > 1$

   $f^2 < 81N, n$ is an imaginary quantity.

   In this case, the ionosphere shall not be able to transmit a wave at such a frequency, instead, the wave will get attenuated.

- When $v_p - c$ is large for $\frac{81N}{f2}$, the wave front advances faster in the region where N is large than that where N is less.

✓ The wave gradually bends and follows the optical law given by equation (2).

✓ If ϕ_0 is smaller, then a higher N is required for the wave to return to the earth.

- For the critical frequency relation $f_c^2 = 81N, f_c$ ***As*** N ***increases, the refractive index*** n ***decreases.***

- For $f \le f_c$, the wave will get reflected back from the layer irrespective of ϕ_0.

- For $f > f_c$, the wave will return only when ϕ_0 is sufficiently small or when ϕ_0 satisfies (2). The wave will penetrate the ionosphere otherwise.

The reflection $n \sin\phi = \sin\phi_0$ gives the path of a ray only when the change of n with height is small in a distance corresponding to λ in the medium, otherwise there is an appreciable reflection as well as refraction and the propagation can no longer be described in terms of a simple ray path.

- The phase velocity v_p is related to c and n by the relation $v_p = \frac{c}{n}$ Since $n < 1$ for an ionized medium, v_p ***is always greater than*** c.

The difference $v_p - c$ is large for large $\frac{N}{f2}$. As a result, when a wave enters the ionosphere, the edge of the wave front in the region of highest electron density will advance faster than the part of the wave front encountering region of lower electron density.

The wave path in the ionosphere is accordingly bent.

This bending of a wave follows optical law (i.e., $n \sin\phi = \sin\phi_0$) shown in Fig8.6.

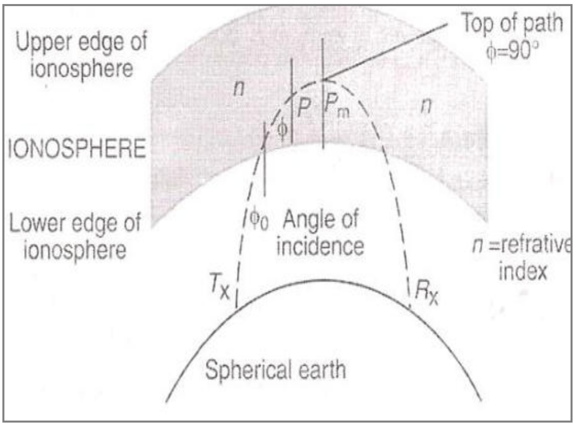
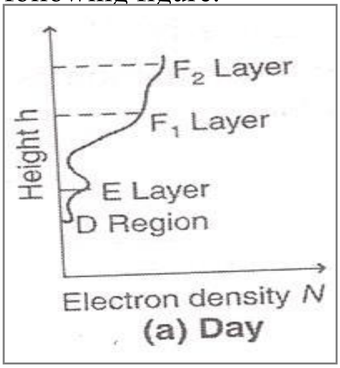


Figure: Bending of the wave in ionosphere

At $\phi_0 = 0$, $n = 0$, the wave penetrates the ionosphere such that $f_c^2 = 81N$.

The variation of electron density ' N ' with height ' h ' is is shown in the following figure.



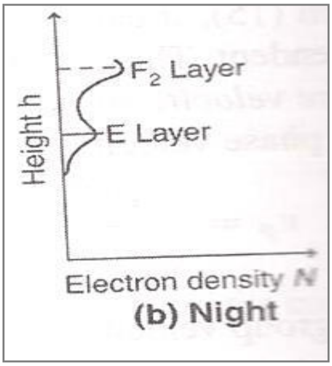


Figure: Electron density during day Figure: during Night

The figure also indicates the relative locations of different Ionospheric layers during day and night.

RAY PATH

The path followed by a wave is termed as ***Ray path***.

The following Figure shows six different paths followed by a wave under different conditions. When $f > f_c$, the effect of the ionosphere depends on the angle of incidence ϕ_0.

From the figure it is observed that

1. When ϕ_0 is relatively large, the wave satisfies the relation $n = \sin\phi_0$. When n drops to less than 1, the wave returns after slight penetration (Ray-1).

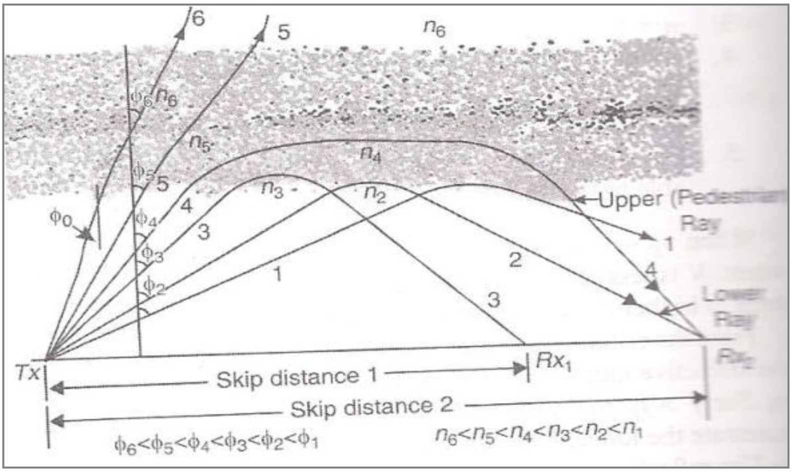


Figure: Different Ray Paths.

2. When ϕ_0 decreases, n decreases, and penetration of the wave increases (Ray-2,3 and 4).
3. When ϕ_0 further decreases, the wave penetrates and crosses the layer (ray-5 and 6).
4. The distance at which the wave returns decreases until $\phi_0 = \phi_c$.

   The angle ϕ_c is called the ***critical angle*** and at this angle, the distance of return is minimum. This distance is called ***skip distance*** (Ray-2,3 and 4).
5. When ϕ_0 further decreases and is less than ϕ_c, the distance of return first increases (Ray-4) and then penetrates the layer.

CRITICAL FREQUENCY

The highest frequency that returns from an ionospheric layer at a vertical incidence is called the ***critical frequency*** for that particular layer.

For a regular layer, it is proportional to the square root of maximum electron density in the layer. The following figure shows the critical frequencies for different ionospheric layers at different instants of time in (a) winter, and (b) summer seasons.

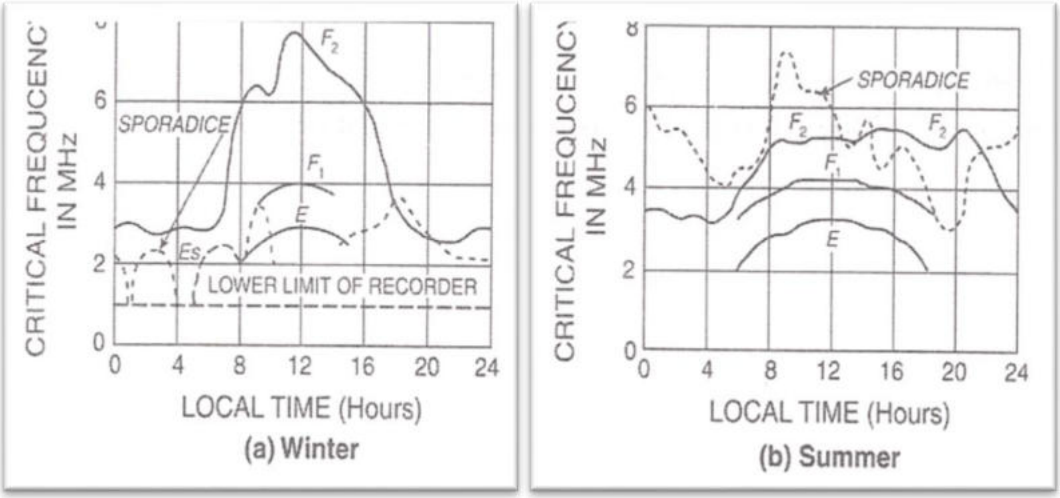


Figure: Critical frequencies for different ionospheric layers.

MAXIMUM USABLE FREQUENCY

The critical frequency f_c is the highest frequency that returns from an ionospheric layer at a vertical incidence. When the frequency exceeds f_c, the wave return will depend upon the angle of incidence at a particular ionospheric layer.

Thus, for a specified angle of incidence, there will be a maximum frequency which will be reflected back.

The maximum possible value of frequency for which reflection takes place for a given distance of propagation is termed as *maximum usable frequency (MUF)* for that distance and for the given ionospheric layer. ***Beyond MUF, the wave will not return.***

The following figure will shows the wave reversal concept to the ground.

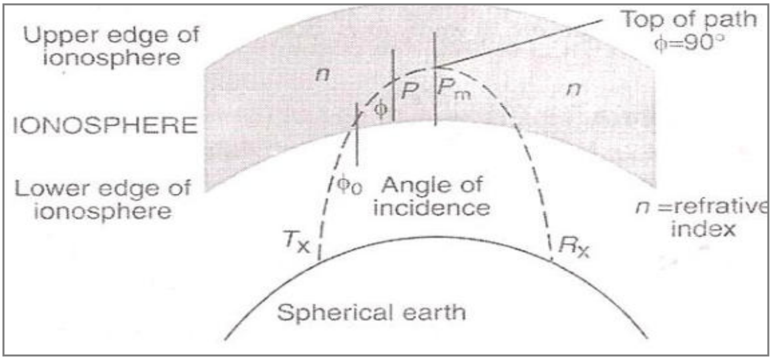


Figure: Showing Wave reversal

For a sky wave to get reflected from the ionosphere, it requires the angle of reflection to be as 90° thus, if ϕ_0 is the incident angle and ϕ_r is the reflection angle, **the refractive index** n can be written as

$$n = \frac{\sin\phi_i}{\sin\phi_r} = \frac{\sin\phi_i}{\sin 90} = \sin\phi_i = \sqrt{1 - \frac{81N_{\max}}{f_{MUF}^2}} \qquad \text{------------- (1)}$$

$$\sin\phi_i = 1 - \frac{81N_{\max}}{f_{MUF}^2} \qquad \text{---------- (2)}$$

We already know that

$$f_c^2 = 81N_{\max} \qquad \text{------------ (3)}$$

Thus, $\sin^2 \phi_i = 1 - \frac{f_c^2}{f_{MUF}^2}$ or

$$\frac{f_c^2}{f_{MUF}^2} = 1 - \sin^2 \phi_i = \cos^2 \phi_i \quad \text{------------- (4)}$$

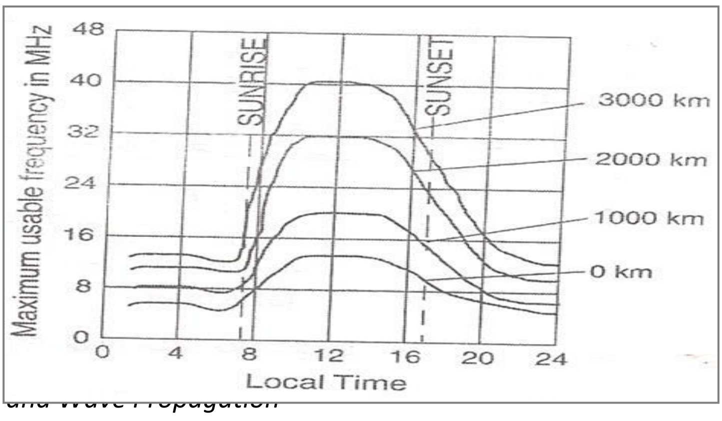
$$f_{MUF}^2 = \frac{f_c^2}{\cos^2 \phi_i} = f_c^2 \sec^2 \phi_i \quad \text{------------- (5)}$$

Finally, we get

$$f_{MUF} = f_c \sec \phi_i \quad \text{------------ (6)}$$

- Equation (6) **is known as *secant law*.**
- It indicates that f_{MUF} **is greater than** f_c **by a factor sec** ϕ_i **.**
- It gives the maximum frequency which can be used for sky wave communication for a given angle of incidence between two locations

The figure shows the maximum usable frequencies at different times destined for coverage of various distances.



. **Figure: MUF at different times of the day.**

LOWEST USABLE FREQUENCY

The frequency below which the entire power gets absorbed is referred to as *lowest usable frequency* (LUF).

OPTIMUM FREQUENCY

The frequency at which there is optimum return of wave energy is called the *optimum frequency* (OF).

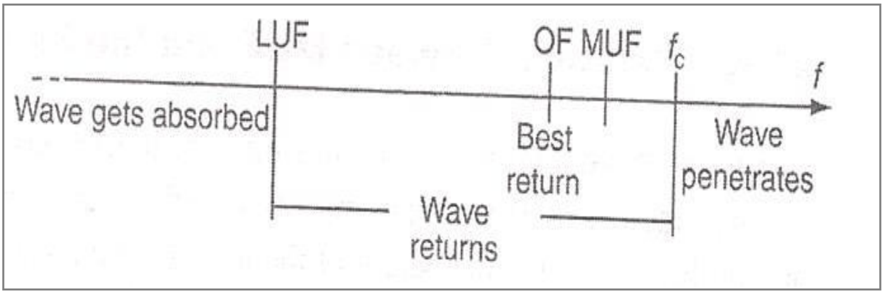


Figure: LUF,OF, MUF

The figure shows the LUF, OF, MUF and the critical frequency f_c on a frequency scale. Limits of all these frequencies are different for different layer.

VIRTUAL HEIGHT:

Virtual height is be defined as

'the height to which a short pulse of energy sent vertically upward and traveling with the speed of light would reach taking the same two-way travel time as does the actual pulse reflected from the Ionospheric layer".

The following figure shows that there is no sharp change of the direction of wave and it starts bending down gradually (from the point *E*) through the process of refraction in the ionosphere.

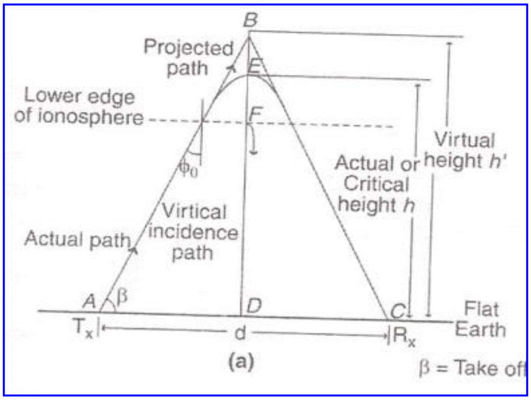


Figure: Actual and Vertical Heights (a) Flat Earth

Just below the ionosphere that is at point *F*, the incident and refracted rays ***follow exactly the same path*** as would have been followed by them if the reflection had taken place from a surface located at a greater height (the point *B*) which is often referred as the ***virtual height.***

If the virtual height of a layer is known, the angle of incidence required for the return of wave to the ground at a selected spot (the point C) can easily be calculated.

The figures give two different cases flat earth, and (b) curved earth.

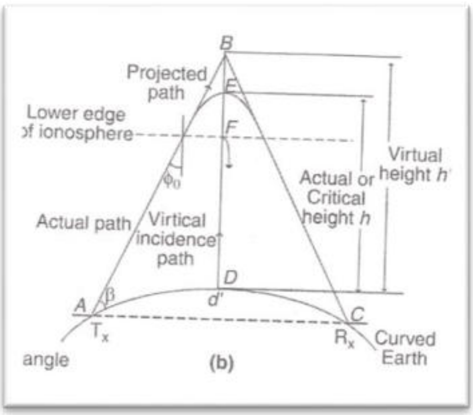


Figure: Actual and Vertical Heights (b) Spherical Earth

On comparison of Fig. (*a*) and (*b*), it can easily be concluded that

- Both the actual height and the virtual height in case of curved earth are less than that for flat earth.
- The virtual height, however, is always greater than the actual height of reflection because the exchange of energy that takes place between the wave and the electrons of the ionosphere causes the velocity of propagation to be reduced.
- The difference between virtual and true heights is influenced by the electron distribution in the regions below the level of reflection. It is generally quite small,

. The following figure gives the virtual heights for different ionospheric layers at different instants of time in (a) winter, and (b) summer seasons.

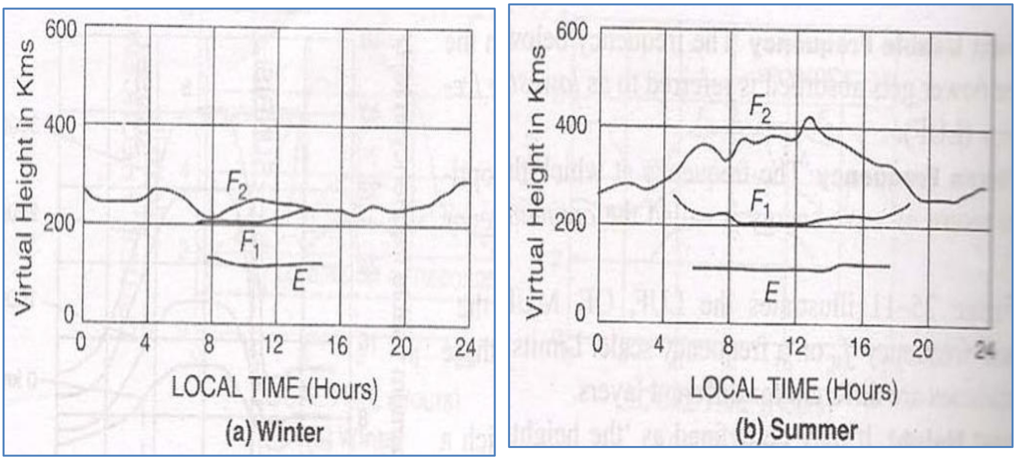


Figure: Virtual heights during winter and summer

SKIP DISTANCE

The minimum distance at which the wave returns to the ground at a critical angle ϕ_c is termed the ***skip distance.***. The skip distance and the maximum usable frequency correspond to each other.

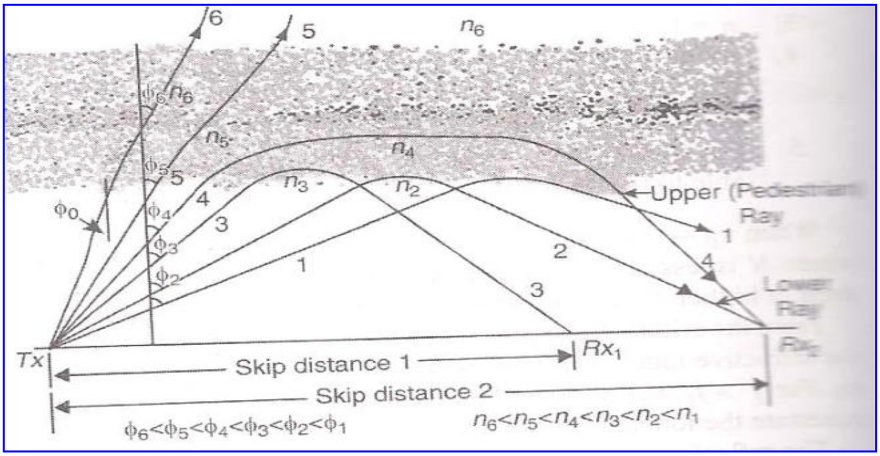


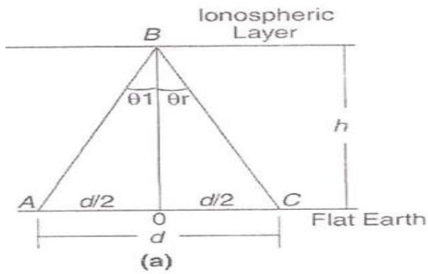
Figure: Different ray paths

RELATION BETWEEN MUF AND THE SKIP DISTANCE

FLAT EARTH CASE:

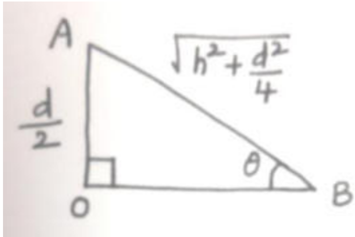
From the following figure it is understood that the ionosphere appear like a mirror surface. Because it is assumed that the ionosphere has sharp ionization density gradient.

Therefore from ionosphere the radio wave experiences mirror ***like reflections.*** For shorter distances,



the earth can be assumed to be flat.

In the figure, h is the height of the ionospheric layer, d is the skip distance, θ_i is the angle of incidence and θ_r is the angle of reflection.



With the help of geometrical considerations we can write

$$\cos\theta_i = \frac{OB}{AB}$$

$$= \frac{h}{\sqrt{h^2 + d^2/4}}$$

$$= \frac{2h}{\sqrt{4h^2 + d^2}} \qquad \text{-------------- (1)}$$

We also know that

$$\frac{f_c^2}{f_{MUF}^2} = \cos^2\theta_i$$

$$\frac{f_c^2}{f_{MUF}^2} = \frac{4h^2}{4h^2 + d^2}$$

$$\frac{f_{MUF}^2}{f_c^2} = \frac{4h^2 + d^2}{4h^2} \qquad \text{----------- (2)}$$

$$\frac{f_{MUF}}{f_c} = \sqrt{\frac{4h^2 + d^2}{4h^2}} = \sqrt{1 + \frac{d^2}{4h^2}} \qquad \text{or}$$

$$f_{MUF} = f_c\sqrt{1 + (\frac{d}{2h})^2} \qquad \text{----------- (3)}$$

Equation (3) gives MUF in terms of skip distance.

Alternatively, from (1),

$$\frac{f_{MUF}^2}{f_c^2} = 1 + \frac{d^2}{4h^2}$$

$$\left(\frac{d}{2h}\right)^2 = \frac{f_{MUF}^2}{f_c^2} - 1$$

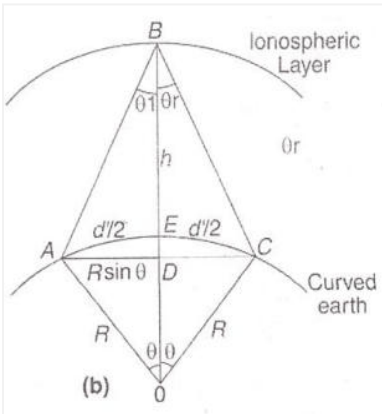
$$d^2 = (2h)^2\left[\frac{f_{MUF}^2}{f_c^2} - 1\right]$$

$$d = (2h)\sqrt{[\frac{f_{MUF}^2}{f_c^2} - 1]}$$ ----------- (4)

Equation (4) gives skip distance with MUF

FOR CURVED EARTH CASE:

The following figureshows the ionized layer and the curved earth.



It is again assumed that the ionospheric layer is ***thin with sharp ionization density*** gradient so as to ***obtain mirror like reflections.***

In this figure, it is observed that 2θ is the angle subtended by the skip distance d ' at the center of the earth.

From the same figure, the following relations are obtained:

Arc $d' = 2R\theta$ - ---- (5)

Angle $2\theta = d'/R$ - ---- (6)

$AD = R\sin\theta$,

$OD = R\cos\theta$,

$BD = OE + EB - OD$

$\mathrm{BD} = R + h - R\cos\theta$ -------- (7)

$$AB = \sqrt{(AD)^2 + (BD)^2} = \sqrt{(R\sin\theta)^2 + (R+h-R\cos\theta)^2} \quad \text{-------- (8)}$$

$$\cos\theta_i = \frac{BD}{AB} = \frac{R+h-R\cos\theta}{\sqrt{(R\sin\theta)^2 + (R+h-R\cos\theta)^2}}$$

$$(\cos\theta_i)^2 = \frac{(R+h-R\cos\theta)^2}{(R\sin\theta)^2 + (R+h-R\cos\theta)^2} \quad \text{---------- (9)}$$

The skip distance d' is maximum when θ is maximum.

- The curvature of the earth limits both MUF and the skip distance.

This limit is obtained when a wave leaves the transmitter ***at a grazing angle OAB = 90^o.***

Under this condition,

$$\cos\theta = \frac{OA}{OB} = \frac{R}{R+h} \quad \text{------------- (10)}$$

Since the actual value of θ is very small, this relation can be expanded as

$$\cos\theta = \frac{R}{R+h} = \frac{R}{R(1+h/R)}$$
$$\cos\theta = (1+h/r)^{-1}$$
$$\cos\theta \approx (1-h/R),$$

since $h/R \ll 1$ ------------- (11)

$$\cos\theta = \sqrt{1-\sin^2\theta} \approx (1-\theta^2)^{1/2} = 1-\theta^2/2$$

for small θ ------------- (12)

From (11) and (12),

$$1-\frac{\theta^2}{2} = 1-\frac{h}{R} \qquad \text{or}$$

$$\theta^2 = \frac{2h}{R}$$ ------------- (13)

From (5) and (13),

$$d'^2 = 4R^2\theta^2 = 4R^2\frac{2h}{R} = 8hR \qquad \text{or}$$

$$d' = \sqrt{8hR}$$ ------------- (14)

From (14), $$h = \frac{d^2}{8R}$$ ------------- (15)

In (14) and (15), d' is the maximum skip distance.

Equation (11) can now be rewritten in view of (15) as

$$\cos\theta = 1 - \frac{h}{R} = 1 - \frac{d'^2}{8R^2} \qquad \text{------------- (16)}$$

From equation (13) and equation (16) we can write

$$\sin\theta \approx \theta = \sqrt{\frac{2h}{R}} = \sqrt{\frac{2d'^2/8R}{R}} = \sqrt{\frac{d'^2}{4R^2}} = \frac{d'}{2R} \text{ ----------- (17)}$$

From (9), (16), and (17)

$$\frac{f_c^2}{f_{MUF}^2} = \cos^2\theta_i = \frac{[R + h - R(1 - d'^2/8R^2)]^2}{(R^2\frac{d'^2}{4R^2}) + \{R + h - R(1 - \frac{d'^2}{8R^2})\}^2}$$

$$= \frac{(h + d'^2/8R)^2}{(d'^2/4) + (h + d'^2/8R)^2} \qquad \text{------------- (18)}$$

$$\frac{f_{MUF}^2}{f_c^2} = \frac{(d'^2/4) + (h + d'^2/8R)^2}{(h + d'^2/8R)^2} = 1 + \frac{d'^2/4}{(h + d'^2/8R)^2} \text{ ----------- (19)}$$

$$f_{MUF} = f_c[1 + \{\frac{d'^2/4}{h + (d'^2/8R)^2}\}]^{1/2} \qquad \text{------------- (20)}$$

Equation (20) gives maximum usable frequency in terms of skip distance.

To get the expression of skip distance in terms of maximum usable frequency, equation (19) can be rewritten as

$$\frac{d'^2}{4} = (h + \frac{d'^2}{8R})^2[(\frac{f_{MUF}}{f_c})^2 - 1] \qquad \text{------------- (21)}$$

$$d' = 2\left(h + \frac{d'^2}{8R}\right)\left[\left(\frac{f_{MUF}}{f_c}\right)^2 - 1\right]^{1/2} \qquad \text{------------- (22)}$$

Similarly, (22) is of quadratic form which will yield the value of the skip distance in terms of maximum usable frequency as given below

$$d' = \frac{2R}{X} \pm 2\sqrt{(R/X)^2 - 2hR} \qquad \text{------------- (23)}$$

Where $X = [(f_{MUF} / f_c)^2 - 1]^{1/2}$ ----------- (24)

MULTI-HOP PROPAGATION

In the following figure the distances between transmitter (T_z) and two receivers (R_{x1} and R_{x2}) were marked as skip distance-1 and skip distance-2.

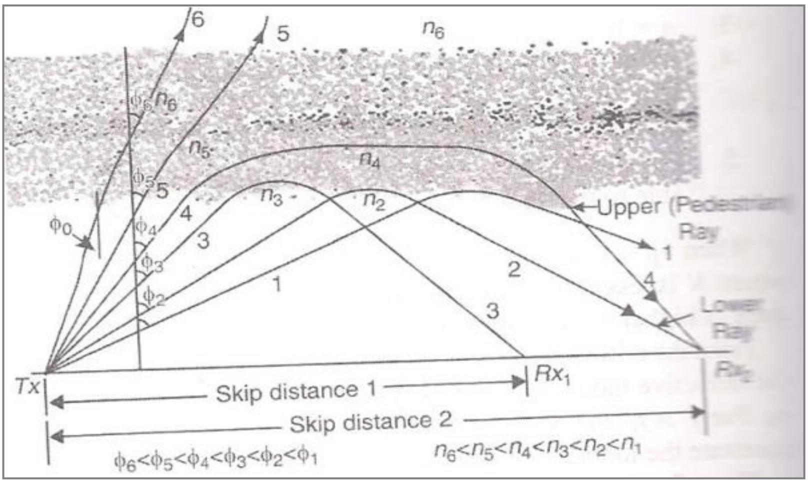


Figure: Showing skip distances

The wave originating from *Tx* arrives at R_{x1} and R_{x2} in its travel, without touching the ground anywhere in between. These distances are termed as ***one hop distances***.

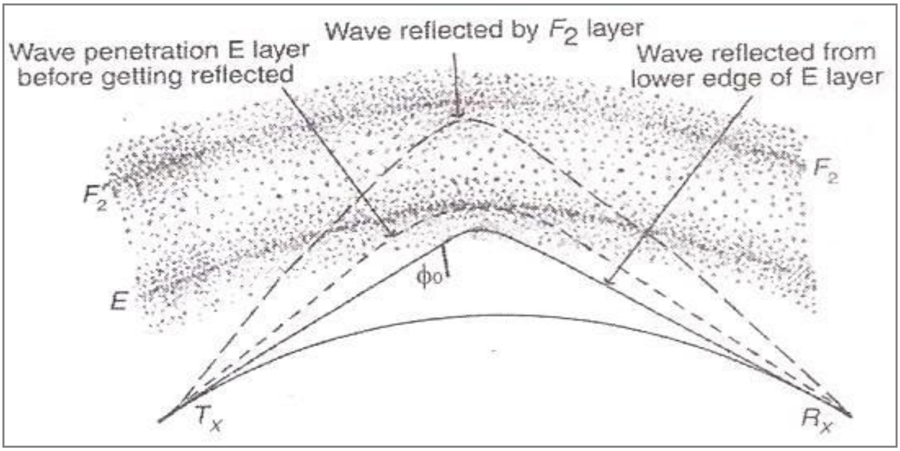


Figure: One hop multi layer propagation

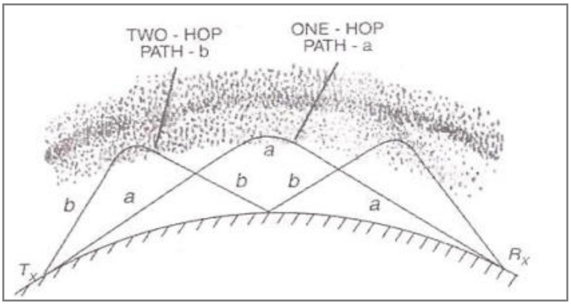


Figure: Skip distance less than half of the distance of the receiver

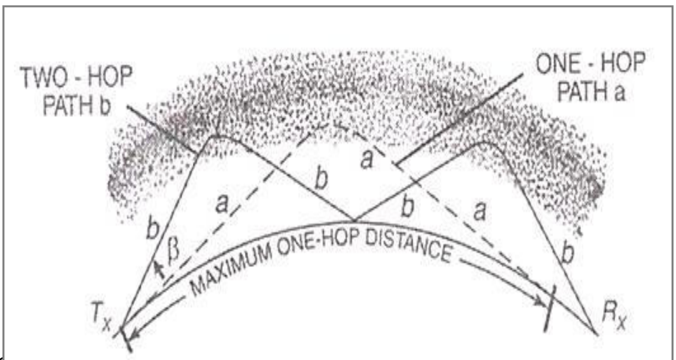


Figure: Distance to the Receiver is greater than maximum possible one hop distance

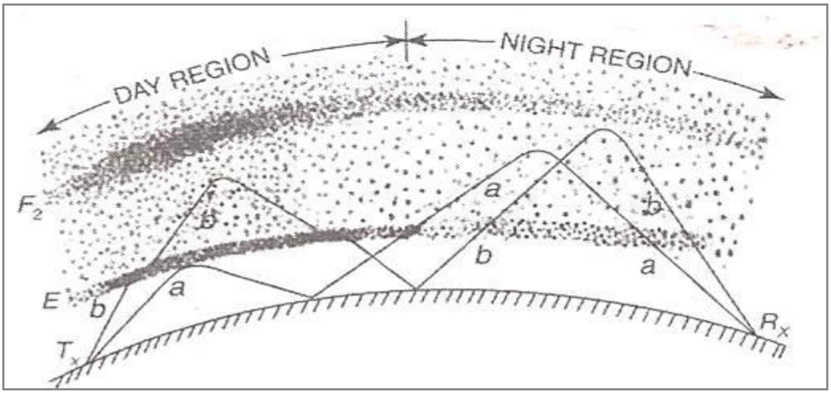


Figure: Multi layer propagation

From the same first figure, it is also shown that the energy may arrive at R_{x2} either through the ray 2 or the ray 4. These rays are termed as ***lower ray (LR) and upper ray (UR).***

- Generally, the ***lower ray*** is preferred for establishing the communication.

The ***upper ray which is also called*** **PEDERSEN RAY** is not very important.

- The upper ray is weaker than the lower ray in terms of its energy contents since over a given solid angle, it spreads more as compared to the lower ray.

The Pedersen ray becomes important only when the lower ray is prevented from reaching the receiver in one hop. This situation arises either when the earth's curvature prevents one hop lower ray or when the distance between the transmitter and receiver is greater than the skip distance.

In such cases, a multi hop system is an alternative for establishing the communication.

Also, if the frequency used falls between critical frequencies of E and F_1 layers and the receiver is beyond the skip distance for E layer, two or even three separate layers may contribute to the propagation of energy.