

Communication Systems

Prepared by:
K.CHANDRAHAS REDDY
Dept of ECE
Assisitant Professor
Annamacharya University
Rajampet

Analog Communication Systems (ACS)

①

Unit - I

Amplitude Modulation & Demodulation

Elements of Communication Systems :-

Communication :- It is a Process of Conveying or Transferring information from one point (or place) to other point (or place).

Basically Communication system contains "Three" blocks as shown in figure.

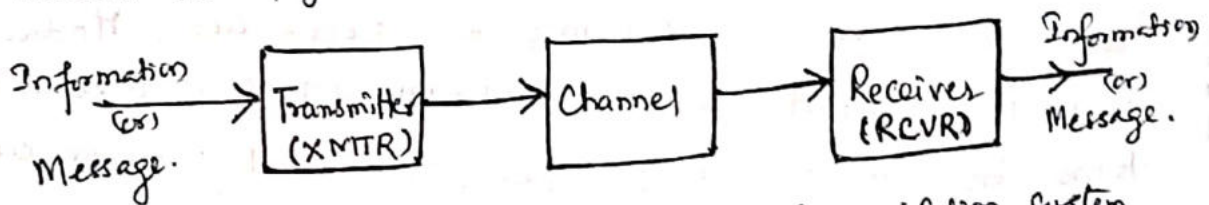


Fig: Basic block diagram of Communication system.

(i) Transmitter :- It Manipulates i/p Information or Message signal such that it is suitable for transmission through Channel.

Functions of Transmitter :-

1. Conversion of information signal from any form into electrical form.

In Communication, information is transferred by means of electrical signal. i.e any form of information signal need to be converted into electrical form.

Example : ① Audio (or) Voice signal Transmission, Microphone is used to convert Audio (or) voice [which are in sound form] into electrical signal.

② Video signal Transmission, Camera is used to convert light signal into electrical signal.

2. Amplification:- Audio (or) Voice (or) Video Signals are natural signals and contain low Power, which cannot travel long distance. These signals are need to be boosted (or) amplified before transmission. ~~Audio~~ Voltage and Power amplifiers are used in the transmitter to boost or amplify the signals.

3. Frequency Translation (Modulation):-

Audio (or) voice (or) Video Signals are contain low frequency signals, which cannot travel long distance. Hence low frequencies are need to be converted into high frequencies. Modulation is a frequency translation technique which is used in the transmitter to convert low frequencies into high frequencies.

4. Transmission or Radiation:-

Either signal need to be coupled to or radiated into the channel. The signal is coupled in the case of channel is physical wire and radiated in the case of channel is free space.

Antenna is a passive device which converts electrical signal into electromagnetic signal. This called "Radiation".

(ii) Channel:- It inter connects transmitter and receiver.

It is two types. i) Physical wire.

ii) Free space.

If physical wire is used in the channel then it is called

"Line Communication" (or) "Wire Communication". Twisted wires, or Coaxial cables (or) Optical fiber cables are used in Wire Communication.

Twisted wires can transmit up to 300 KHz, Coaxial cables can transmit up to 500 MHz and optical fiber cables can transmit up to 6 Hz or 9 Hz. (2)

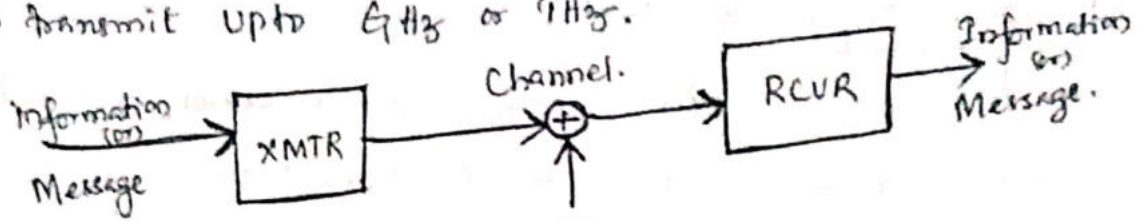


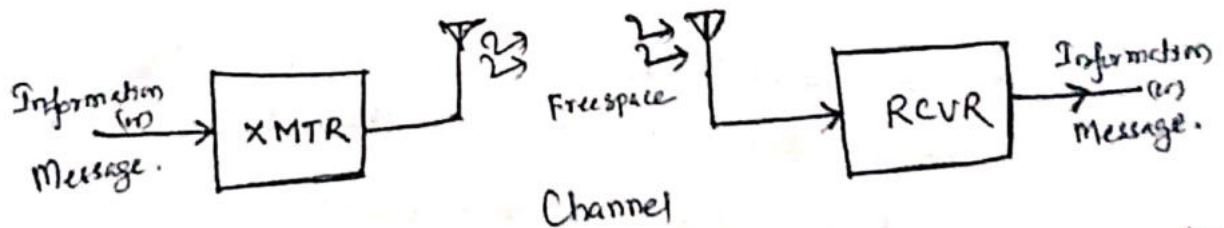
fig: Wire (or) Line Communication.

Limitations associated with Wire or Line Communication

1. It is Cost & Complex
2. Maintenance Problem.
3. Impedance matching Problem.

While transmitting signal from XMTR to RCVR through Channel, some unwanted signal will be added to the information signal and it is called "noise" (or) Noise is unwanted form of signal which is added (or) interfered with information signal.

If free space is used as channel then it is called wireless or lineless (or) Radio Communication. Which is shown in figure.



Transmitter antenna Converts electrical signal into electromagnetic signal and receiver antenna Converts electromagnetic signal into electrical signal.

Here also noise will be added to the information signal while transmitting from transmitter to receiver.

(iii) Receiver :- It manipulates received signal so that information is obtained at the output.

Functions of Receiver :-

1. Reception :- Antenna used for reception, which converts Electromagnetic signal into electrical signal.
2. Selection :- Receiver has to select one ~~than~~ station to which it is tuned among many stations. Tuning circuit is used for selection.
3. Amplification :- Receiver provides voltage and power amplification to the received weak signal.
4. Frequency Translation (Demodulation) :-
Demodulation is a frequency translation technique which converts high frequency signal into low frequency. (or)
It extracts information signal from modulated signal.
5. Conversion of signal :- Receiver converts electrical signal into sound or light signal.

Example : ① Loud speaker converts electrical signal into sound signal (i.e. Audio or voice).

② Picture tube converts electrical signal into light signal.

Signal :— It is a physical quantity which varies with respect to time or space or independent variable or dependent variable. (or)

It is electrical waveform which carries information. It carries information in three forms i.e. Amplitude, Phase and Frequency.

Example :— $m(t) = A \cos(\omega t + \phi)$

Where $A \rightarrow$ Amplitude (or) Peak amplitude. (Volt or Amp)

$\omega \rightarrow$ Frequency ($\frac{\text{rad}}{\text{sec}}$)

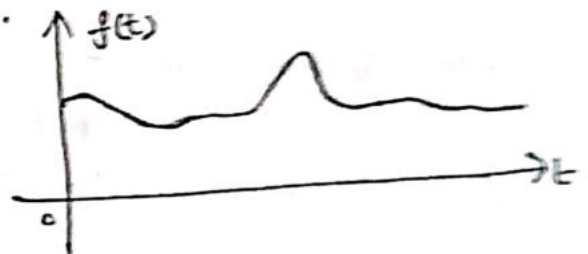
$\phi \rightarrow$ Phase (rad)

Types of Signals :—

1. Analog or Continuous signal
2. Digital signal.

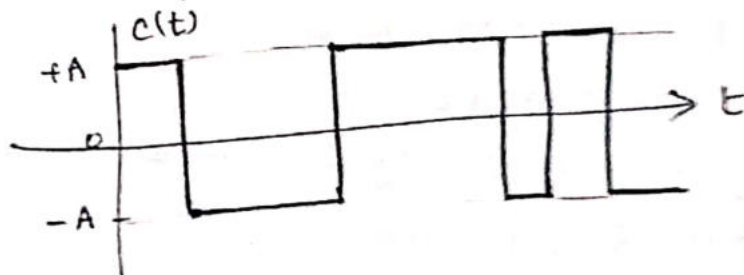
Analog or Continuous signal :— If the signal amplitude of signal continuously varies with respect to time (or) If the signal contains infinite no. of amplitudes then it is called Analog or Continuous signal.

Example : Temperature signal



Digital signal :— If the signal contains only two discrete amplitudes then it is called "Digital" signal.

Example : Computer (or) Internet signal.

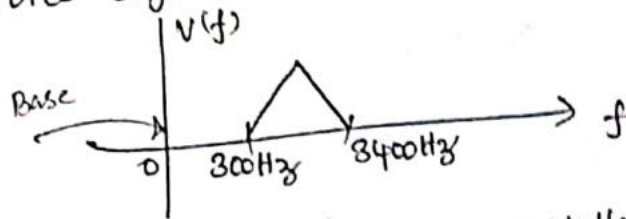


With respect to Communication, Signals are broadly classified into two types. They are,

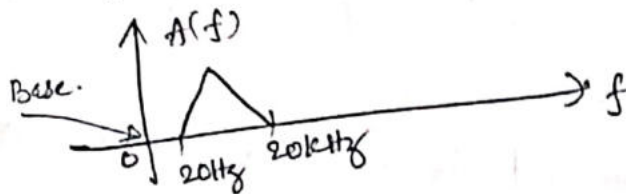
- i) Base Band signal
- ii) Band pass signal.

i) Base Band Signal :— If the signal contains zero frequency (or) near to zero frequency then it is called "Base Band signal". The signals which are generated in real time applications, All are Base band signals.

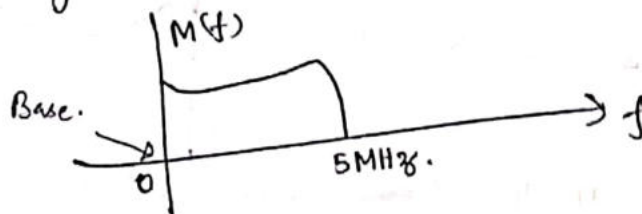
i) Voice signal — (300 Hz - 3400 Hz)



ii) Audio signal — (20 Hz - 20 kHz)



iii) Video signal : — (0-5) MHz



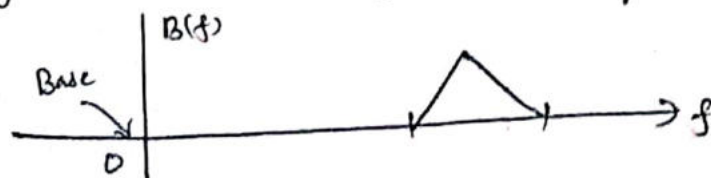
iv) All biomedical signals are base band signals.

Electro Cardogram (ECG)

Electro Encephalography (EEG)

Electro myogram (EMG).

ii) Band Pass Signal :— If the signal contains band of frequencies far away from base (or) zero. Example: AM, FM signals.



Message :- It is sequence of symbols.

Example: i) HAPPY NEW YEAR 2019

ii) THE COST OF MOBILE \$ 500.

Information :- The Content in the message is called "Information".
The information Carried by the symbol is inversely Proportional to Probability of their Occurance.

Information Carried by the symbol 'A' can be represented as

$$I_A \propto \frac{1}{P_A}$$

Where I_A - Information Carried by symbol 'A'.

P_A - Probability of Occurance of symbol 'A'.

Unit of Information :-

Case (i) :- $I_A = \log_2 \left(\frac{1}{P_A} \right) \rightarrow$ Bits. Unit

Case (ii) :- $I_A = \log_{10} \left(\frac{1}{P_A} \right) \rightarrow$ Decits.

Case (iii) :- $I_A = \log_e \left(\frac{1}{P_A} \right) \rightarrow$ Nats.

Fundamental Limitations of Communication Systems :-

1. Technological Problems.

2. Band Width & Noise.

3. Signal to Noise Ratio (SNR).

i) To implement Communication systems, xmitter, Channel & receiver (REVR) are required, which requires Hardware.

ii) Communication system is more cost & Complex.

iii) To establish Communication system, Permission should be taken from federal regulation. Federal regulation allocates frequency bands to establish the Communication system.

2. Band Width & Noise :-

The effect of noise can be reduced by providing more bandwidth in the channel. If more bandwidth is allocated to stations then less no. of stations will be accommodated.

3. Signal to Noise Ratio (SNR) :-

The relation between Channel Capacity, Bandwidth & SNR can be written as

$$C = B \log_2 \left(1 + \frac{S}{N} \right)$$

$\downarrow$ $\downarrow$ $\rightarrow$ SNR.
 Channel Capacity Bandwidth

The Channel Capacity is mainly depends on SNR. Noise should be low to increase Channel Capacity, But it is unavoidable aspect of Communication system.

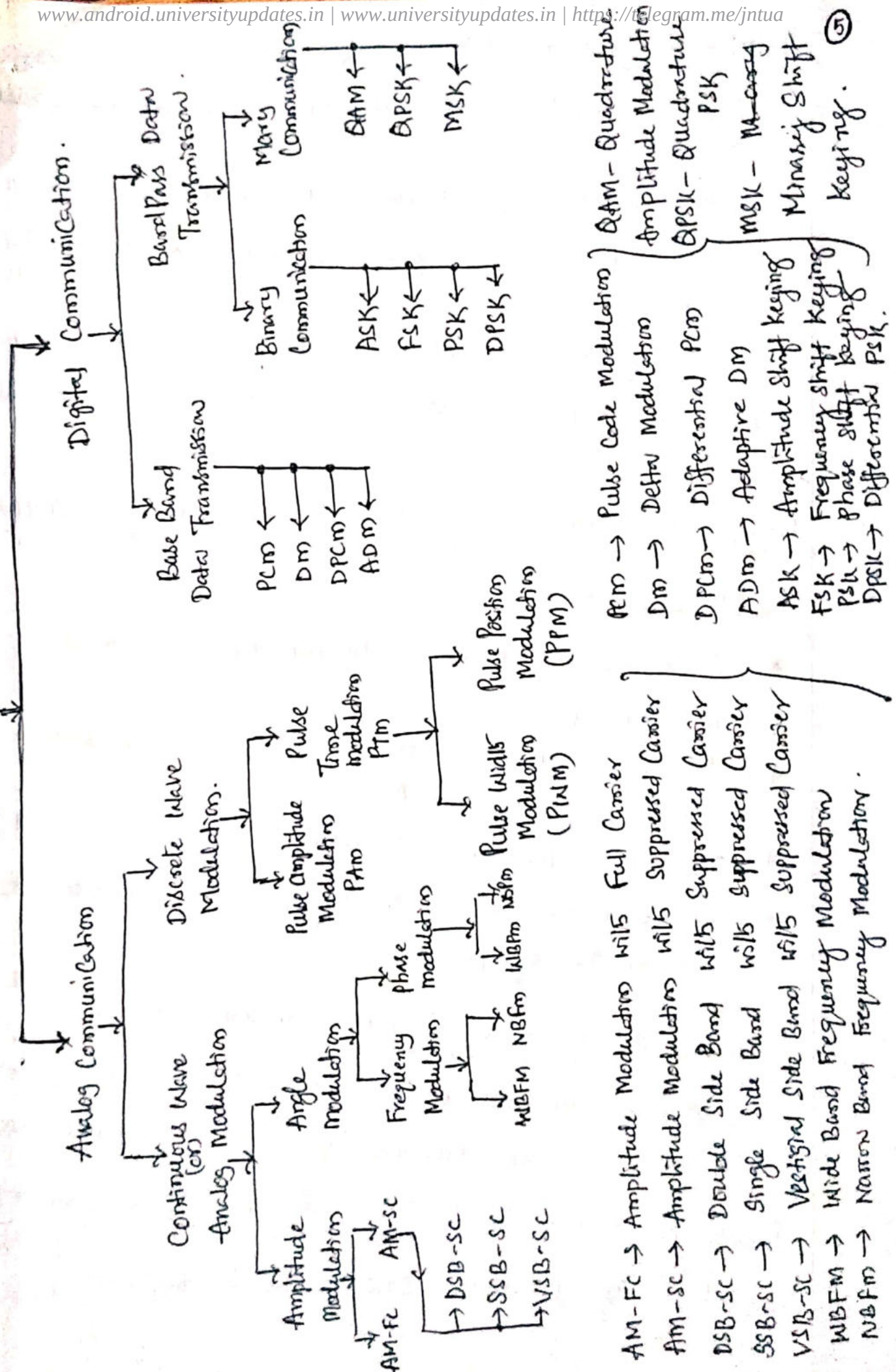
Modulation :- It is a Process of ~~var~~ Varying the Characteristics of high frequency Carrier (Amplitude (or) Frequency (or) phase) in accordance with the instantaneous values of the modulating (or) message (or) Baseband signal.

(or)

It is a frequency translation technique which converts base band signal (or) Low frequency signal into Band pass (or) High frequency signal.

⊛ Modulation is used in the transmitter.

Modulation Methods



Amplitude Modulation :- Amplitude of the Carrier Varied in accordance with the instantaneous values of the modulating signal.

Frequency Modulation :- Frequency of the Carrier Varied in accordance with the instantaneous values of the modulating signal.

Phase Modulation :- phase of the Carrier Varied in accordance with the instantaneous values of the modulating signal.

Benefits of Modulation (or) Need for modulation (or) Application of Modulation :-

- * To reduce the length (or) height of antenna.
- * For Multiplexing
- * For Narrow banding (or) To use a antenna with single (or) same length.
- * To reduce the noise effect.
- * To avoid equipment limitation (or) To reduce the size of the equipment.

To reduce the length (or) height of antenna :-

Antenna radiates electromagnetic signal into freespace.
(i.e Antenna Converts electrical signal into electromagnetic signal)

$$\text{length or height of antenna } l = \lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/sec}}{f(\text{Hz})}$$

Case (i) :- If $f = 3 \text{ kHz} = 3 \times 10^3 \text{ Hz}$ i.e low frequency then

lengths or height of antenna $l = \frac{3 \times 10^8}{3 \times 10^3} = 10^5 \text{ m}$.

$l = 100 \text{ km}$ i.e. lengths of antenna is very high.

Case(ii) :- If $f = 30 \text{ MHz}$ i.e. very high frequency then
 $= 30 \times 10^6 \text{ Hz}$

lengths (or) height of antenna $l = \frac{3 \times 10^8}{3 \times 10^7} = 10 \text{ m}$.

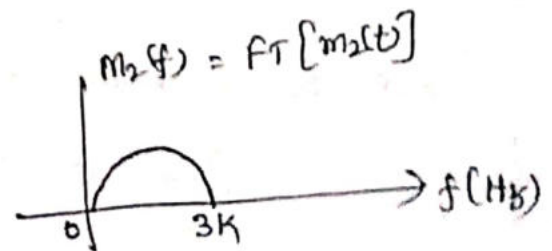
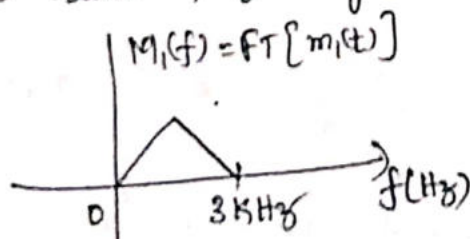
$l = 10 \text{ m}$ i.e. lengths of antenna is very low.

Modulation Converts low frequency into high frequency.
 Hence, Modulation is required to reduced the antenna lengths (or) height.

⊛ For Multiplexing:-

Multiplexing is a Process of transmitting several same message signals through same or single channel.

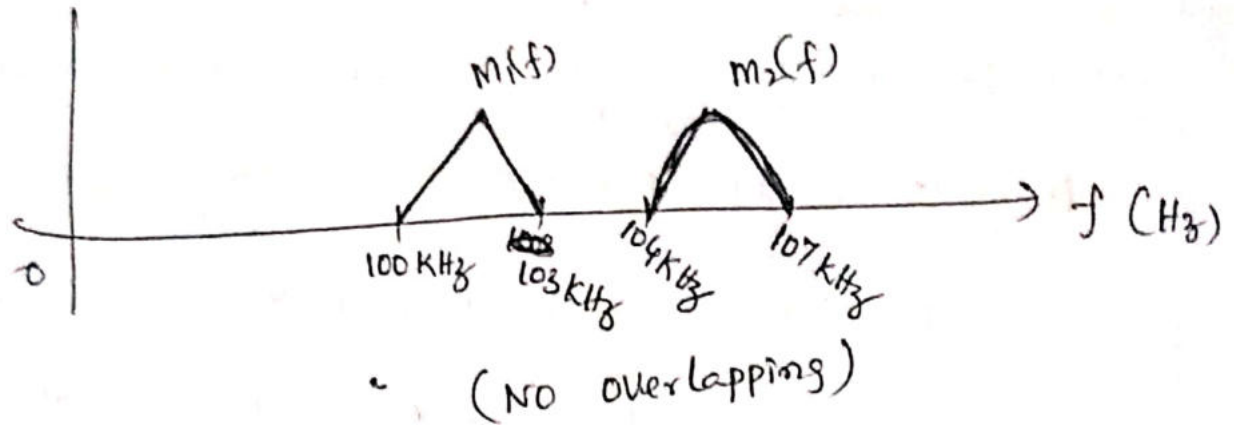
Let us consider two same message signal i.e. with same frequency.



If above two signals are transmitted without Modulation Through Same Channel then the receiver cannot distinguish two signals i.e. overlapped.

If above two signals are transmitted after modulation i.e. by adding different carrier frequencies ~~then~~ Through single (or) same channel then receiver can easily distinguish two signals.

Let $f_{c1} = 100 \text{ kHz}$ is added to signal $m_1(t)$ & $f_{c2} = 104 \text{ kHz}$ is added to signal $m_2(t)$.



Adding ~~the~~ high (or) carrier frequency is called "Modulation".

So, Modulation is required for multiplexing.

⊗ Narrow Banding (or) Use antenna with single length.

Case(i):- Let us consider low frequency signal

i.e. $(3-30) \text{ kHz}$
 $\downarrow \quad \downarrow$
 $f_1 \quad f_2$

length of antenna required to radiate $f_1 = 3 \text{ kHz}$ is

$$l_1 = \frac{c}{f_1} = \frac{3 \times 10^8}{3 \times 10^3} = 10^5 \text{ m} = 100 \text{ km}$$

length of antenna required to radiate $f_2 = 30 \text{ kHz}$

$$l_2 = \frac{c}{f_2} = \frac{3 \times 10^8}{3 \times 10^4} = 10^4 \text{ m} = 10 \text{ km}$$

i.e. Antenna length should vary between 10 km to 100 km which ~~is not possible~~ impractical.

Case (iv):- let us consider carrier frequency for 100 MHz is added to the low frequency signal (i.e. modulation). Now frequency ^{range} of the signal will be $(100.003 - 100.03) \text{ MHz}$.

$\downarrow$ $\downarrow$
 f_1 f_2

length of antenna required to radiate $f_1 = 100.003 \text{ MHz}$ is

$$l_1 = \frac{c}{f_1} = \frac{3 \times 10^8}{100.003 \times 10^6} =$$

length of antenna required to radiate $f_2 = 100.03 \text{ MHz}$ is

$$l_2 = \frac{c}{f_2} = \frac{3 \times 10^8}{100.03 \times 10^6} =$$

$l_1 \approx l_2 =$ i.e. same length antenna is required to radiate the signal after modulation, which is practical.

So, Modulation is required to use antenna with same length to ~~radiate~~ radiate the band of frequencies.

⊗ To reduce the noise effect :-

Noise is an unavoidable aspect of Communication Systems which will be added to the information signal while transmitting from XMITR to RCVR. Noise effect is more on low frequency signal than high frequency signal. Modulation converts low frequency into high frequency. So, Modulation is required to reduce the noise effect.

⑤ To reduce the size of equipments.

The size of the equipments will be reduced if they are operated at high frequencies. Modulation Converts low frequency signal into high frequency signal. So, Modulation is required to reduce the size of the equipments.

(Am or fm-fc Am with full carrier)
Amplitude Modulation — Amplitude of the high frequency Carrier signal in accordance with the instantaneous values of the modulating or message or base band signal.

Amplitude modulation — Single tone modulating signal
 (modulating signal contains single frequency)
 Let us consider modulating signal $m(t) = A_m \cos \omega_m t$
 $= A_m \cos 2\pi f_m t$

and Carrier signal $C(t) = A_c \cos \omega_c t = A_c \cos 2\pi f_c t$

$$A_c \gg A_m \quad \& \quad f_c \gg f_m$$

Amplitude of Carrier is assumed to be constant before modulation and varied after modulation with respect to modulating signal.

$$A_{am}(t) = A_c(t) \cos 2\pi f_c t \quad \text{--- (A)}$$

↳ variable carrier amplitude after modulation.

$$A_c(t) = A_c + m(t) \quad \text{--- (B)}$$

Put eqⁿ. (B) in eqⁿ. (A)

$$A_{am}(t) = [A_c + m(t)] \cos(2\pi f_c t)$$

$$= [A_c + A_m \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

$$= A_c \left[1 + \frac{A_m}{A_c} \cos 2\pi f_m t \right] \cos(2\pi f_c t).$$

$$A_m(t) = A_c \cos(2\pi f_c t) [1 + m_a \cos(2\pi f_m t)]$$

modulation Index $m_a = \frac{A_m}{A_c} = \frac{\text{Peak amplitude of } m(t)}{\text{Peak amplitude of } c(t)}$.

$$A_m(t) = A_c \cos(2\pi f_c t) + \frac{m_a A_c}{2} [2 \cos(2\pi f_c t) \cos(2\pi f_m t)]$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$A_m(t) = A_c \cos(2\pi f_c t) + \frac{m_a A_c}{2} [\cos(2\pi(f_c + f_m)t) + \cos(2\pi(f_c - f_m)t)]$$

↓ Amplitude modulated signal
↓ Un modulated Carrier (or) Carrier
↓ Upper Side Band (USB)
↓ Lower Side Band (LSB)

$$A_m(t) = A_c \cos \omega_c t + \frac{m_a A_c}{2} [\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t]$$

$$= A_c \cos \omega_c t + \frac{A_m}{A_c} \cdot \frac{A_c}{2} [\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t]$$

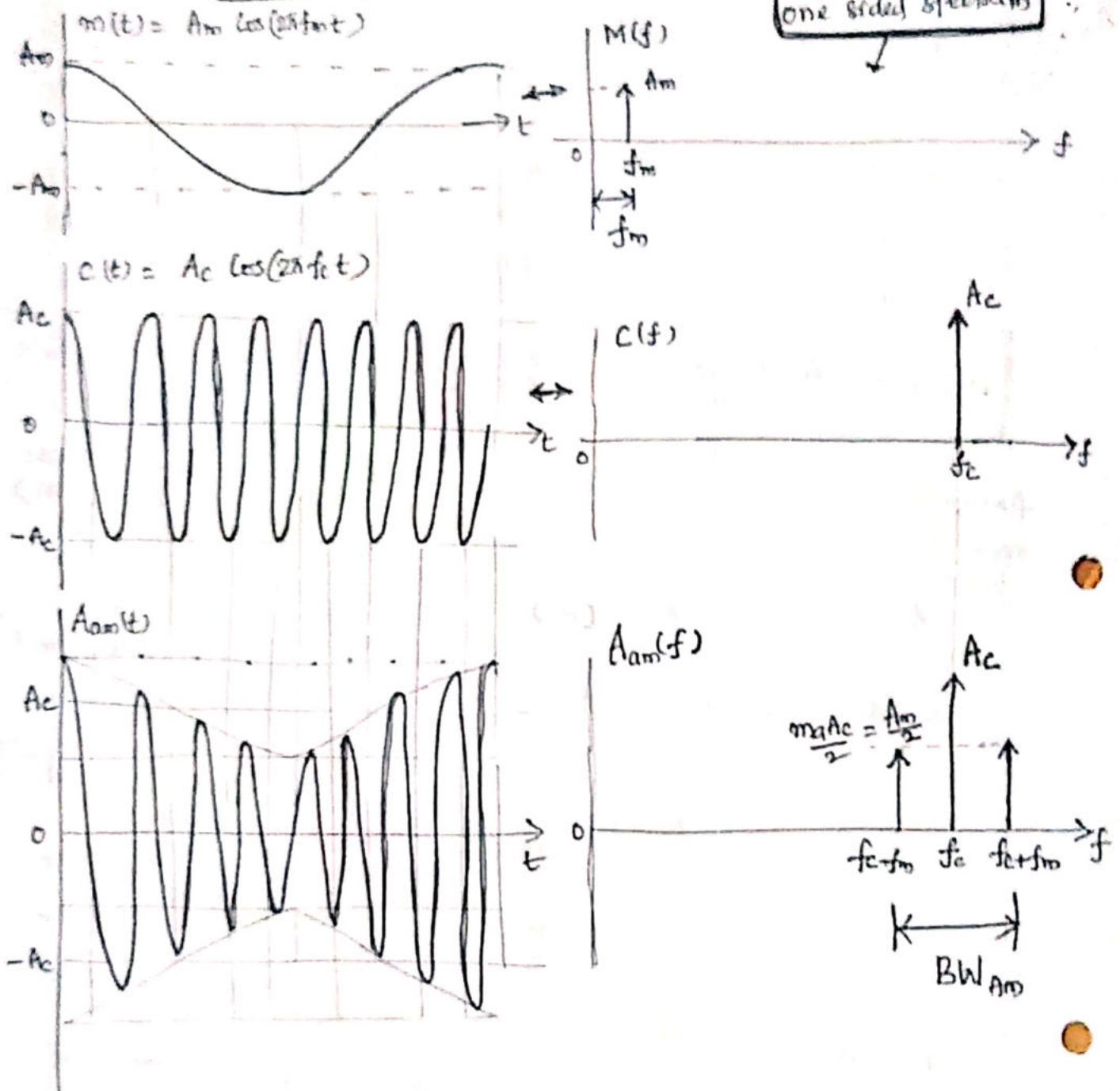
$$= A_c \cos \omega_c t + \frac{A_m}{2} [\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t]$$

Amplitude modulated signal contains both un modulated carrier & modulated carrier (both USB & LSB) because it contains amplitude $\frac{A_m}{2}$ and frequencies $(\omega_c + \omega_m)$ & $(\omega_c - \omega_m)$.

Time Domain

Frequency Domain

one sided spectrum



Band width of modulating signal = f_m Hz (or) N_m and $\frac{sec}{sec}$

Band width of Modulated signal

$$BW_{Am} = f_c + f_m - (f_c - f_m)$$

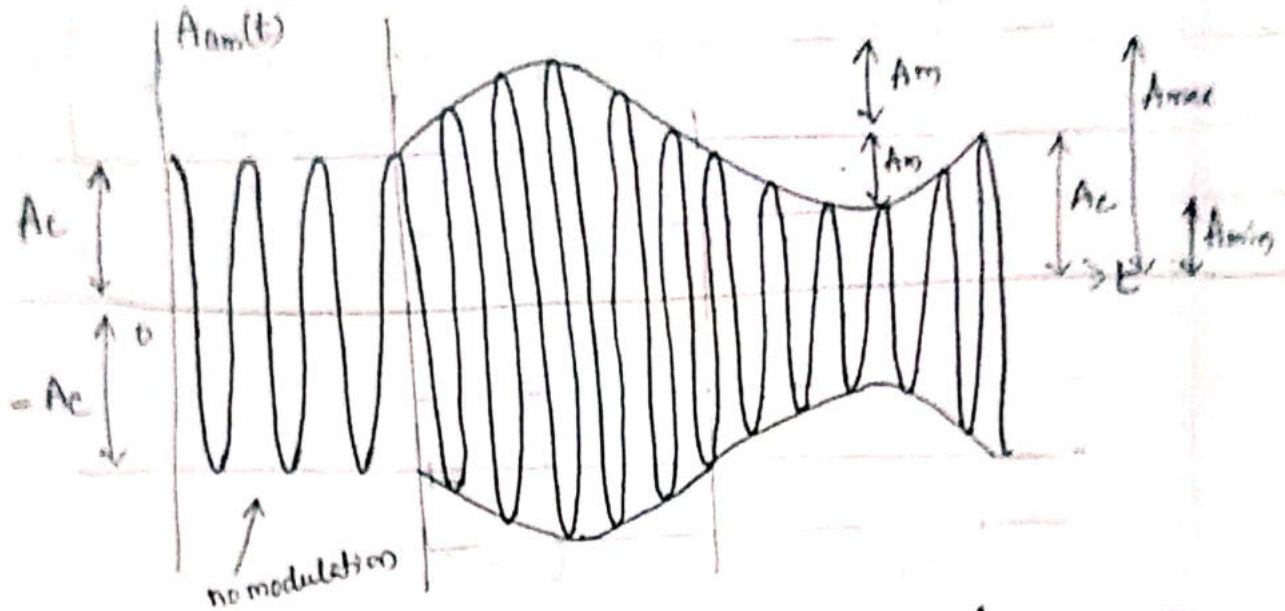
$$= f_c + f_m - f_c + f_m$$

$$BW_{Am} = 2f_m \text{ Hz (or) } 2W_m \text{ and } \frac{sec}{sec}$$

i.e Band width of Amplitude modulated signal is the twice the band width of modulating signal.

Modulation Index (or) Depth of Modulation

①



Modulation Index or Depth of modulation $m_a = \frac{A_m}{A_c}$ — (A)

From the figure

$$A_{max} - A_{min} = 2A_m$$

$$\frac{A_{max} - A_{min}}{2} = A_m \text{ — (B)}$$

$$A_{max} - A_m = A_c$$

$$A_{max} - \left(\frac{A_{max} - A_{min}}{2} \right) = A_c$$

$$A_{max} - \frac{A_{max}}{2} + \frac{A_{min}}{2} = A_c$$

$$\frac{A_{max}}{2} + \frac{A_{min}}{2} = A_c$$

$$\frac{A_{max} + A_{min}}{2} = A_c \text{ — (C)}$$

Put eqⁿ. (B) & (C) in eqⁿ. (A)

$$m_a = \frac{A_{max} - A_{min}}{2} \bigg/ \frac{A_{max} + A_{min}}{2}$$

$$m_a = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{A_m}{A_c}$$

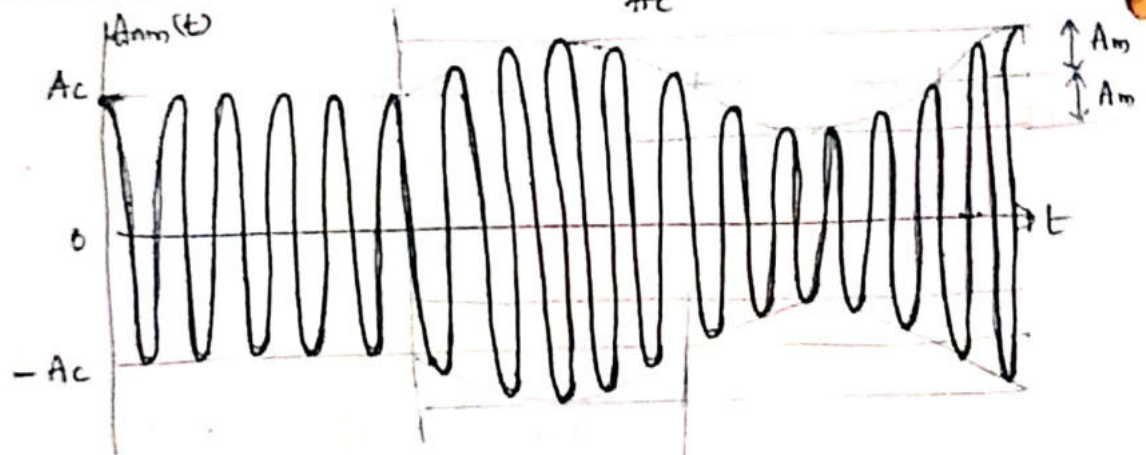
The Percentage of Modulation Index

$$\therefore m_a = \frac{A_m}{A_c} \times 100 = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} \times 100$$

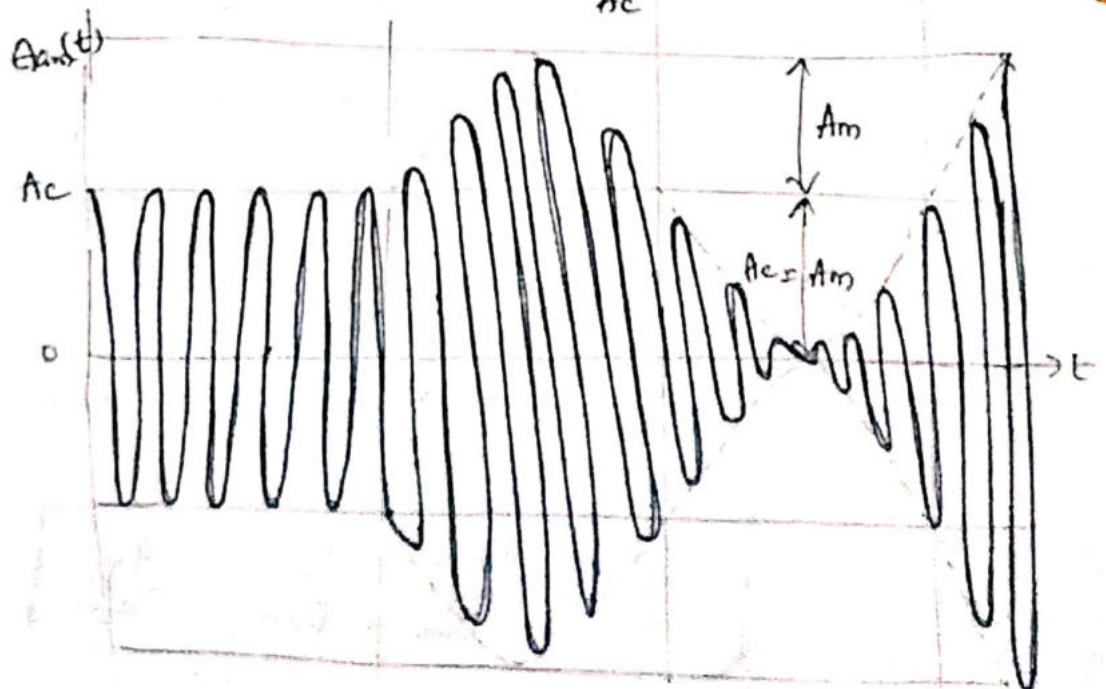
Types of Amplitude Modulation with respect to Modulation Index :-

- ① Under modulation ($m_a < 1$)
- ② Critical Modulation ($m_a = 1$)
- ③ Over modulation ($m_a > 1$)

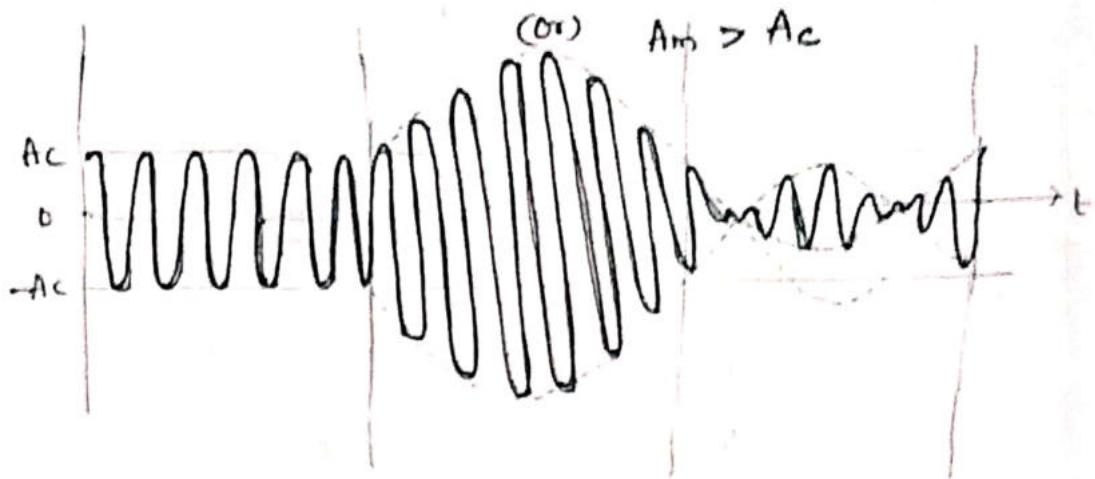
① Under modulation :- $m_a < 1 \Rightarrow \frac{A_m}{A_c} < 1$ (or) $A_m < A_c$



② Critical Modulation :- $m_a = 1 \Rightarrow \frac{A_m}{A_c} = 1$ (or) $A_m = A_c$



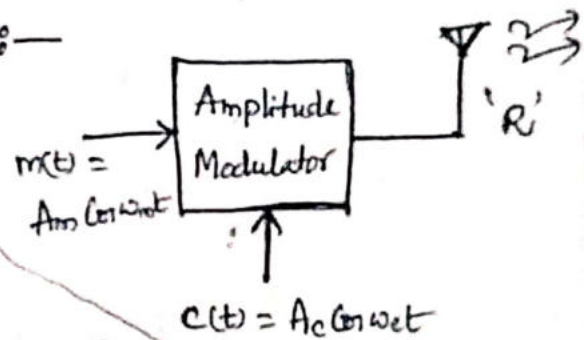
③ Over modulation $m_a > 1 \Rightarrow m_a \geq \frac{A_m}{A_c} > 1$



Power Relation in AM Signal :-

Let us consider AM signal

$$A_{am}(t) = A_c \cos \omega_c t + \frac{m_a A_c}{2} \left[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t \right]$$



Assume that, above equation is voltage waveform is ~~given~~ given to the antenna whose radiating resistance is 'R'.

The Total radiated Power Can be written as

$$P_t = P_{\text{Carrier}} + P_{\text{LSB}} + P_{\text{USB}}$$

$$= \frac{A_c^2}{R} + \frac{m_a^2 A_c^2}{4R} + \frac{m_a^2 A_c^2}{4R} \quad (\text{Peak Power})$$

$$= \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R} + \frac{\left(\frac{m_a A_c}{\sqrt{2}}\right)^2}{4R} + \frac{\left(\frac{m_a A_c}{\sqrt{2}}\right)^2}{4R} \quad (\text{rms Power})$$

$$= \frac{A_c^2}{2R} + \frac{m_a^2 A_c^2}{8R} + \frac{m_a^2 A_c^2}{8R}$$

$$= \frac{A_c^2}{2R} + \frac{m_a^2 A_c^2}{4R} = \frac{A_c^2}{2R} \left[1 + \frac{m_a^2}{2} \right]$$

$$P_t = P_c \left[1 + \frac{m_a^2}{2} \right]$$

where Carrier Power $P_c = \frac{A_c^2}{2R}$

$$\text{Total Power } P_t = P_c + P_{\frac{m_a^2}{2}}$$

$$P_t = P_{\text{carrier}} + P_{\text{sideband}}$$

$$\text{where } P_{\text{sideband}} = \frac{P_c m_a^2}{2}$$

$$\begin{aligned} P_t &= P_{\text{carrier}} + P_{\text{LSB}} + P_{\text{USB}} \\ P_t &= P_c + \frac{P_c m_a^2}{4} + \frac{P_c m_a^2}{4} \end{aligned}$$

Current Relation :— $P_t = P_c \left[1 + \frac{m_a^2}{2} \right]$

$$I_t^2 R = I_c^2 R \left[1 + \frac{m_a^2}{2} \right]$$

$$I_t = \sqrt{I_c^2 \left[1 + \frac{m_a^2}{2} \right]}$$

$$I_t = I_c \sqrt{1 + \frac{m_a^2}{2}}$$

Efficiency :— It is ratio of useful Power and total Power

$$\eta = \frac{P_{\text{useful}}}{P_{\text{total}}} = \frac{P_{\text{sideband}}}{P_t}$$

$$= \frac{P_{\text{LSB}} + P_{\text{USB}}}{P_c + P_{\text{LSB}} + P_{\text{USB}}}$$

$$= \frac{\frac{P_c m_a^2}{4} + \frac{P_c m_a^2}{4}}{P_c + \frac{P_c m_a^2}{4} + \frac{P_c m_a^2}{4}}$$

$$\eta = \frac{\frac{P_c m_a^2}{2}}{P_c + \frac{P_c m_a^2}{2}} = \frac{\cancel{P_c} \frac{m_a^2}{2}}{\cancel{P_c} [1 + m_a^2/2]}$$

$$= \frac{\frac{m_a^2}{2}}{1 + m_a^2/2} = \frac{m_a^2}{2 + m_a^2}$$

The Percentage of efficiency

$$\% \eta = \frac{m_a^2}{2 + m_a^2} \times 100$$

Modulation Index varies b/w 0 & 1 i.e. $0 \leq m_a \leq 1$

maximum efficiency can be obtained at $m_a = 1$

$$\therefore \eta_{\max} = \frac{1^2}{2 + 1^2} \times 100 = \frac{1}{3} \times 100 = 33.33\%$$

$$\therefore \eta_{\max} = 33.33\%$$

Important Points :-

1. Maximum efficiency of AM system is only 33.33%.
2. 33.33% of total Power is existed in both USB & LSB and remaining 66.67% of total Power existed in unmodulated Carrier.
3. Since unmodulated Carrier or Carrier does not ~~carry~~ contain information, It is wasted Power.
4. ~~The~~ Unmodulated Carrier Power can be removed to improve the Power efficiency.
5. If Carrier (or) unmodulated Carrier is removed from AM signal then Power efficiency will be 100%. And % of Power saving is 66.67%.

7. If Carrier is removed from AM signal then it contains:
two (or) Double Side Band with Suppressed Carrier. Hence
it is called "Double Side Band Suppressed Carrier" System
(DSB-SC)

DSB-SC :

- Percentage of Power efficiency is 100%.
- Percentage of Power Saving in DSB-SC with respect to AM is 66.67%.
- Band width of DSB-SC signal is $2W_m$ (rad/sec) or $2f_m$ (Hz)

8. An unmodulated carrier (or) Carrier along with either of side band can be removed to improve efficiency and to reduce band width.

9. If an unmodulated carrier & either of side band is removed from AM signal then it contains single side band. Hence it is called single side band suppressed carrier system (SSB-SC)

SSB-SC :

- The Percentage of Power efficiency is 100%.
- The Percentage of Power Saving in SSB-SC with respect to AM is 83.37%.
- The Percentage of Power Saving in SSB-SC with respect to DSB-SC is 16.67%.
- The Band width of SSB-SC is W_m (rad/sec) or f_m (Hz).

10. If the signal contains one full side band and part of other side band then it is called "Vestigial Side Band Suppressed Carrier" System (VSB-SC)

11. The Performance of VSB-SC is talk b/w DSB-SC & SSB-SC.

Amplitude Modulation - Multitone Modulating Signal (12)

Multitone - If Modulating signal contains more than one frequency component then it is called "Multitone" modulating signal.

Let us consider multitone modulating signal

$$m(t) = A_{m1} \cos 2\pi f_{m1} t + A_{m2} \cos 2\pi f_{m2} t ; A_{m1} > A_{m2} \text{ \& } f_{m1} < f_{m2} ;$$

$$\text{Carrier } c(t) = A_c \cos 2\pi f_c t ; A_c \gg A_{m1} \text{ \& } f_c \gg \omega_{m2} .$$

Carrier signal is assumed to be constant before modulation & varies after modulation with respect to modulating signal.

$$A_{am}(t) = A_c(t) \cos(2\pi f_c t) \text{ --- (A)}$$

$$A_c(t) = A_c + m(t) = A_c + A_{m1} \cos(2\pi f_{m1} t) + A_{m2} \cos(2\pi f_{m2} t) \text{ --- (B)}$$

put eqⁿ. (B) in eqⁿ. (A).

$$A_{am}(t) = [A_c + A_{m1} \cos(2\pi f_{m1} t) + A_{m2} \cos(2\pi f_{m2} t)] \cos(2\pi f_c t)$$

$$= A_c \left[1 + \underbrace{\frac{A_{m1}}{A_c}}_{\rightarrow m_{a1}} \cos(2\pi f_{m1} t) + \underbrace{\frac{A_{m2}}{A_c}}_{\rightarrow m_{a2}} \cos(2\pi f_{m2} t) \right] \cos(2\pi f_c t)$$

$$= A_c \cos(2\pi f_c t) + \frac{m_{a1} A_c^2}{2} \cos(2\pi f_c t) \cos(2\pi f_{m1} t) + \frac{m_{a2} A_c^2}{2} \cos(2\pi f_c t) \cos(2\pi f_{m2} t)$$

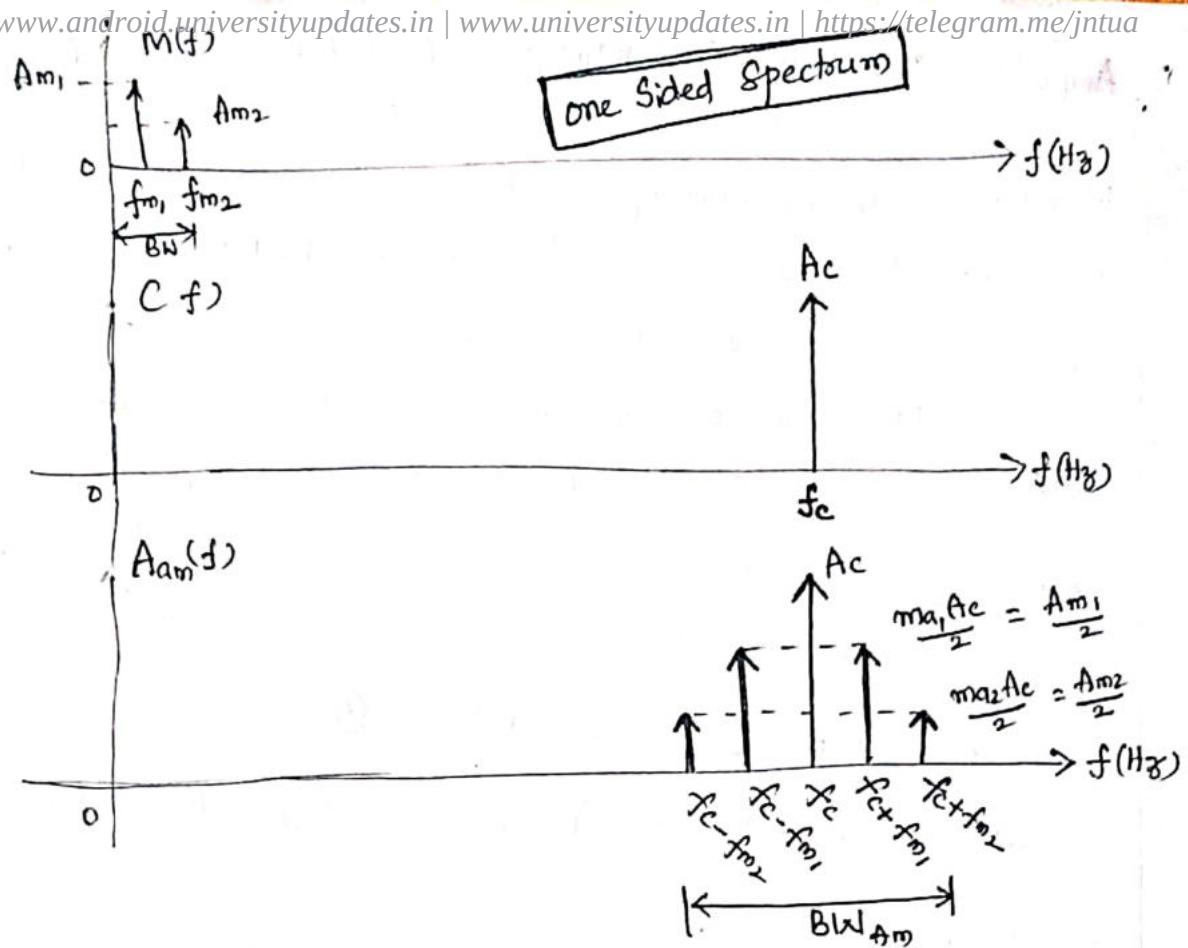
$$A_{am}(t) = A_c \cos(2\pi f_c t) + \frac{m_{a1} A_c}{2} \left[\cos[2\pi(f_c + f_{m1})t] + \cos[2\pi(f_c - f_{m1})t] \right] + \frac{m_{a2} A_c}{2} \left[\cos[2\pi(f_c + f_{m2})t] + \cos[2\pi(f_c - f_{m2})t] \right]$$

(or) USB & LSB corresponding to modulating signal frequency f_{m1}

$$A_{am}(t) = A_c \cos \omega_c t + \frac{m_{a1} A_c}{2} \left[\cos(\omega_c + \omega_{m1})t + \cos(\omega_c - \omega_{m1})t \right] + \frac{m_{a2} A_c}{2} \left[\cos(\omega_c + \omega_{m2})t + \cos(\omega_c - \omega_{m2})t \right]$$

Unmodulated
Carrier
(or)
Carrier

USB & LSB corresponding to
modulating signal frequency f_{m2}



Band width of modulating signal $m(t)$ is $BW = f_{m2}(Hz)$ or $\omega_{m2} \left(\frac{rad}{sec} \right)$

Band width of Amplitude Modulated signal is

$$BW_{AM} = f_c + f_{m2} - (f_c - f_{m2})$$

$$= f_c + f_{m2} - f_c + f_{m2}$$

$$BW_{AM} = 2f_{m2} Hz \text{ (or) } 2\omega_{m2} rad/sec$$

Linear Modulation Technique:-

Band width of amplitude modulated signal increases proportionally with respect to modulating signal bandwidth with factor '2'. Hence amplitude modulation is "Linear modulation Technique".

POWNER RELATION :-

$$\text{Total Power } P_t = P_{\text{carrier}} + P_{\text{LSB}_1} + P_{\text{USB}_1} + P_{\text{LSB}_2} + P_{\text{USB}_2}$$

$$= \frac{A_c^2}{R} + \left(\frac{m_{a1} A_c}{2}\right)^2 \frac{1}{R} + \left(\frac{m_{a1} A_c}{2}\right)^2 \frac{1}{R} + \left(\frac{m_{a2} A_c}{2}\right)^2 \frac{1}{R} + \left(\frac{m_{a2} A_c}{2}\right)^2 \frac{1}{R}$$

$$= \left(\frac{A_c}{\sqrt{2}}\right)^2 \frac{1}{R} + \left(\frac{m_{a1} A_c}{2\sqrt{2}}\right)^2 \frac{1}{R} + \left(\frac{m_{a1} A_c}{2\sqrt{2}}\right)^2 \frac{1}{R} + \left(\frac{m_{a2} A_c}{2\sqrt{2}}\right)^2 \frac{1}{R} + \left(\frac{m_{a2} A_c}{2\sqrt{2}}\right)^2 \frac{1}{R}$$

($\because$ Peak Power)

($\because$ rms Power)

$$= \frac{A_c^2}{2R} + \frac{m_{a1}^2 A_c^2}{8R} + \frac{m_{a1}^2 A_c^2}{8R} + \frac{m_{a2}^2 A_c^2}{8R} + \frac{m_{a2}^2 A_c^2}{8R}$$

$$= \frac{A_c^2}{2R} + \frac{m_{a1}^2 A_c^2}{4R} + \frac{m_{a2}^2 A_c^2}{4R}$$

$$= \frac{A_c^2}{2R} \left[1 + \frac{m_{a1}^2}{2} + \frac{m_{a2}^2}{2} \right]$$

$$= P_c \left[1 + \frac{m_{a1}^2 + m_{a2}^2}{2} \right]$$

$$\boxed{P_{tt} = P_c \left[1 + \frac{m_t^2}{2} \right]}$$

$$\text{Carrier Power } P_c = \frac{A_c^2}{2R} ; m_t^2 = m_{a1}^2 + m_{a2}^2$$

$$\text{In general } m_t^2 = m_{a1}^2 + m_{a2}^2 + m_{a3}^2 + m_{a4}^2 + \dots$$

CURRENT RELATION :-

$$P_{tt} = P_c \left[1 + \frac{m_t^2}{2} \right]$$

$$I_{tt}^2 R = I_c^2 R \left[1 + \frac{m_t^2}{2} \right]$$

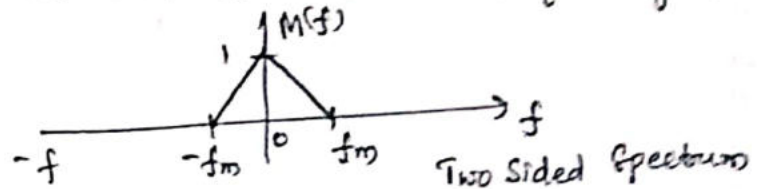
$$I_{tt} = \sqrt{I_c^2 \left[1 + \frac{m_t^2}{2} \right]}$$

$$\boxed{I_{tt} = I_c \sqrt{1 + \frac{m_t^2}{2}}}$$

Power efficiency :- $\therefore \eta = \frac{m^2}{2+m^2} \times 100.$

Amplitude Modulation — Band of frequencies :-

Let us consider modulating signal contains band of frequencies such that it contains maximum frequency f_m Hz as shown below



Carrier signal $c(t) = A_c \cos 2\pi f_c t$; $A_c \gg |m(t)|_{\max}$ & $f_c \gg f_m$.

Amplitude of the carrier is assumed to be constant before modulation & varied with respect to modulating signal $m(t)$.

$$A_m(t) = A_c(t) \cos(2\pi f_c t) \text{ — (A)}$$

$$A_c(t) = A_c + m(t) \text{ — (B)}$$

Put eqⁿ. (B) in eqⁿ. (A)

$$A_m(t) = [A_c + m(t)] \cos(2\pi f_c t)$$

$$= A_c \left[1 + \frac{1}{A_c} m(t) \right] \cos(2\pi f_c t)$$

$$= A_c \left[1 + K_a m(t) \right] \cos(2\pi f_c t)$$

↳ Amplitude sensitivity factor $K_a = \frac{1}{A_c}$ (volt)

$$A_m(t) = A_c \cos(2\pi f_c t) + A_c \cdot K_a m(t) \cos(2\pi f_c t)$$

Take Fourier Transform on both sides.

$$FT[A_m(t)] = FT[A_c \cos(2\pi f_c t)] + FT[m(t) \cos(2\pi f_c t)]$$

$$A_m(f) = FT[A_c \cos(2\pi f_c t)] + FT\left[m(t) \left[\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2} \right]\right]$$

$$= FT[A_c \cos(2\pi f_c t)] + FT\left[\frac{m(t) e^{j2\pi f_c t}}{2}\right] + FT\left[\frac{m(t) e^{-j2\pi f_c t}}{2}\right]$$

$$A_m(t) \Rightarrow FT [A_c \cos 2\pi f_c t] = \frac{A_c}{2} [\delta(f-f_c) + \delta(f+f_c)] \quad (16)$$

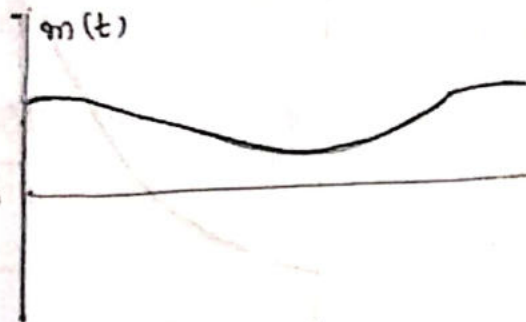
Frequency Shifting Property

$$\text{If } FT [m(t)] = M(f) \text{ then}$$

$$FT [m(t) e^{j2\pi f_c t}] = M(f \pm f_c)$$

$$A_m(f) = \frac{A_c}{2} [\delta(f-f_c) + \delta(f+f_c)] + \frac{1}{2} [M(f-f_c) + M(f+f_c)]$$

Time Domain

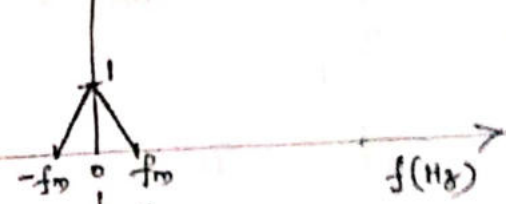


FT
↔

$-f(Hz)$

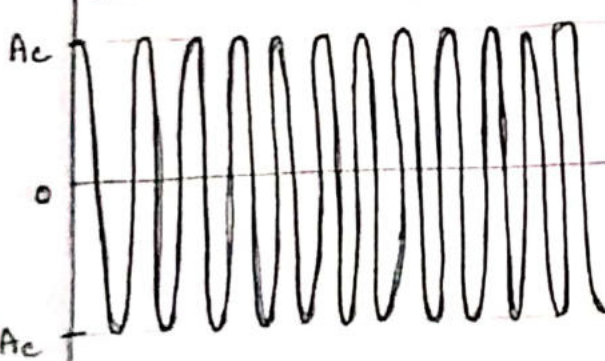
Frequency Domain (Two Sided)

$$M(f) = FT [m(t)]$$



$$C(f) = \frac{A_c}{2} [\delta(f-f_c) + \delta(f+f_c)]$$

$$C(t) = A_c \cos(2\pi f_c t)$$

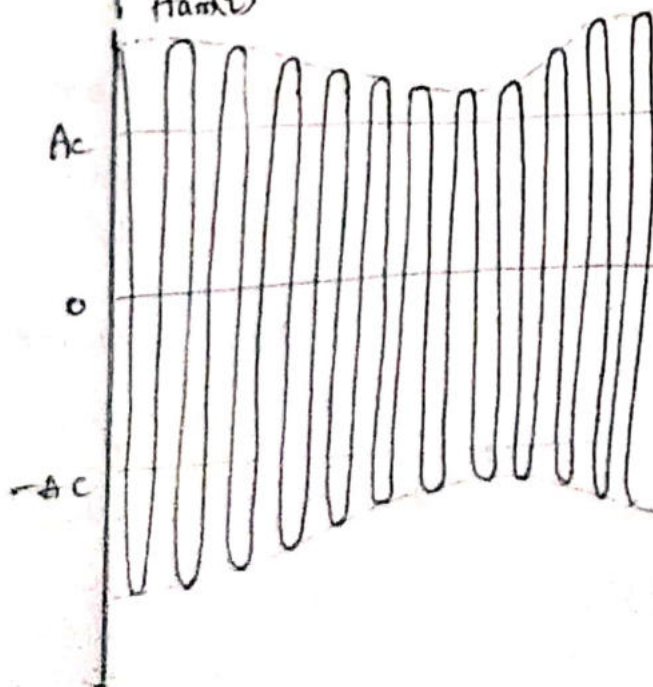


FT
↔

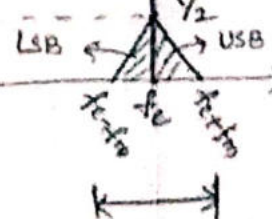
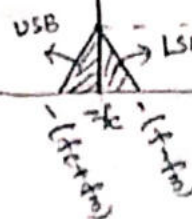
$-f_c$

f_c

$$A_m(t)$$



FT
↔



BW of modulating signal $m(t)$ is

$$BW = f_m(Hz) \text{ or } \omega_m \frac{\text{rad}}{\text{sec}}$$

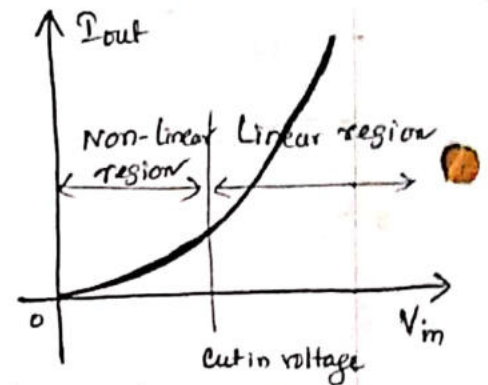
BW of modulated signal $A_m(t)$ is

$$BW_{fm} = f_c + f_m - (f_c - f_m) = 2f_m(Hz) = 2\omega_m \frac{\text{rad}}{\text{sec}}$$

Generation of AM Signal :-

- ① Non Linear or Small Signal Modulator.
 - i) Square Law Modulator
 - ii) Balanced Modulator
- ② Linear or Large Signal Modulator
 - i) Switching Modulator

Here the electronic devices such as diode, transistors, FET, etc., are operated in non linear portion of the V-I characteristics to generate AM signal. V-I characteristics of electronic device is shown in figure.



In non linear region, The relation b/n input voltage & output current is given by

$$I_{out} = a V_{in} + b V_{in}^2 + c V_{in}^3 + \dots$$

Where a, b, c - - are conductance.

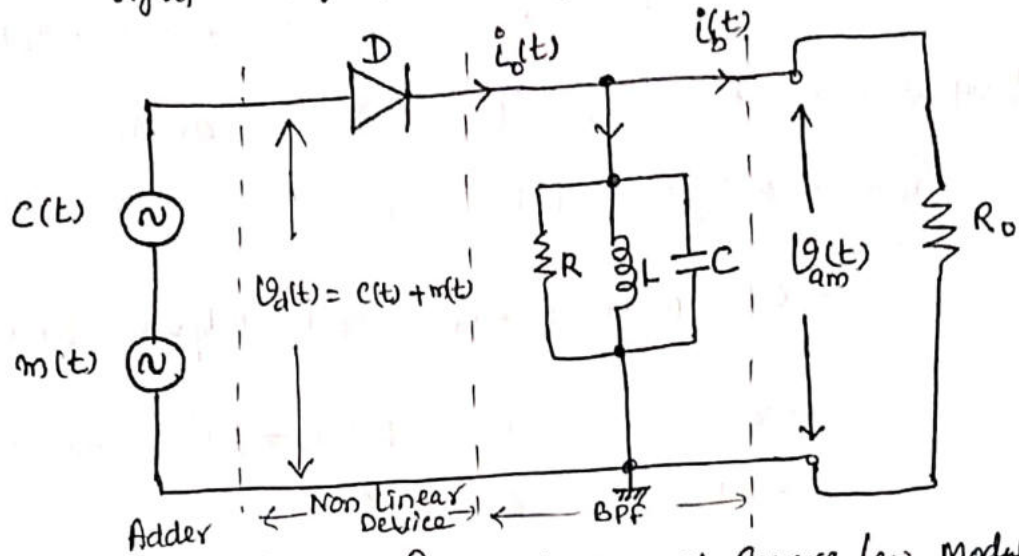
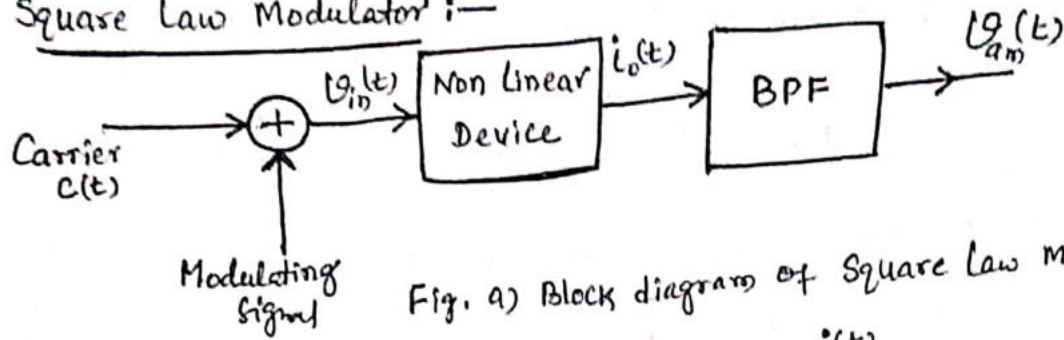
The non-linear region is existed for less voltage. Hence, This type of modulators are called "Small signal modulators".

② Linear or Large Signal Modulator :-

Here the electronic devices such as diode, transistors, FET, etc., are operated in linear portion of the V-I characteristics to generate AM signal.

In linear region, The relation b/n input voltage & output current is given by $I_{out} = a V_{in} = \frac{V_{in}}{R}$, The linear region is existed for large voltages. Hence, This type of modulators are called "Large signal modulators".

(i) Square Law Modulator :-

Operation :-

1. Schematic or block diagram & Circuit diagram using diode are shown in figure a & b respectively.
2. If two voltage sources are connected in series then voltage will be added i.e. it acts as adder.
3. The diode is operated in non-linear region of V-I Characteristics. The relation b/w input voltage & output current is

$$i_o(t) = a V_d(t) + b V_d^2(t) + \dots$$

4. Combination of Resistor, Inductor & Capacitor forms Band pass filter, which passes three frequency components f_c , $f_c + f_m$, & $f_c - f_m$.
5. The voltage across resistor 'R_o' is equal to amplitude modulated signal $V_{am}(t) = i_b(t) R_o$.

Derivation :— Let us consider modulating signal

$$m(t) = V_m \cos(2\pi f_m t) \text{ and}$$

$$\text{Carrier signal } c(t) = V_c \cos \omega_c t$$

Output of adder $V_d(t) = m(t) + c(t)$

$$V_d(t) = V_m \cos(2\pi f_m t) + V_c \cos(2\pi f_c t)$$

Output current of non linear device diode is given by

$$i_b(t) = a V_d(t) + b V_d^2(t) + \dots$$

$$= a [V_c \cos(2\pi f_c t) + V_m \cos(2\pi f_m t)] + b [V_c \cos(2\pi f_c t) + V_m \cos(2\pi f_m t)]^2 + \dots$$

$$= a V_c \cos(2\pi f_c t) + a V_m \cos(2\pi f_m t) + b V_c^2 \cos^2(2\pi f_c t) + b V_m^2 \cos^2(2\pi f_m t) +$$

$$+ 2 b V_c V_m \cos(2\pi f_c t) \cos(2\pi f_m t) + \dots$$

$$= a V_c \cos(2\pi f_c t) + a V_m \cos(2\pi f_m t) + \frac{b V_c^2}{2} + \frac{b V_c^2}{2} \cos(4\pi f_c t) + \frac{b V_m^2}{2} +$$

$$\frac{b V_m^2}{2} \cos(4\pi f_m t) + b V_c V_m [\cos(2\pi(f_c + f_m)t) + \cos(2\pi(f_c - f_m)t)] + \dots$$

BPF eliminates all frequencies, so that O/P current contains only f_c , $f_c + f_m$, and $f_c - f_m$ frequencies.

$$i_b(t) = a V_c \cos(2\pi f_c t) + b V_c V_m [\cos(2\pi(f_c + f_m)t) + \cos(2\pi(f_c - f_m)t)]$$

Voltage across resistor ' R_o ' is given by

$$V_{am}(t) = R_o \cdot i_b(t)$$

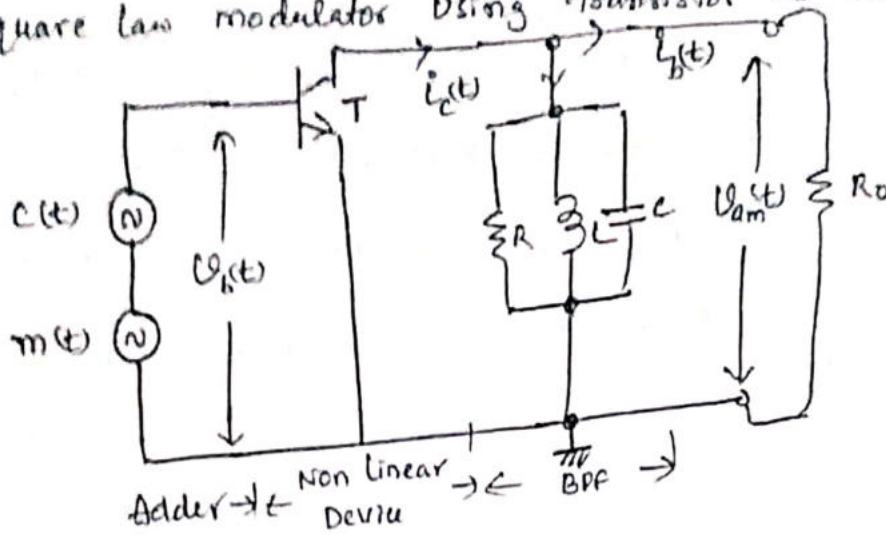
$$V_{am}(t) = R_o a V_c \cos(2\pi f_c t) + R_o b V_c V_m [\cos(2\pi(f_c + f_m)t) + \cos(2\pi(f_c - f_m)t)]$$

$\downarrow$
 Un modulated Carrier
(or)
Carrier

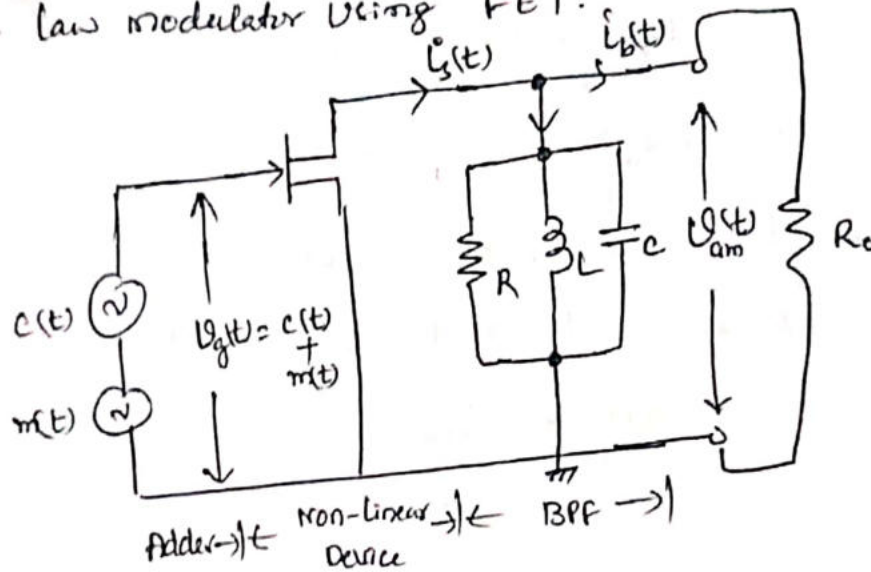
$\downarrow$
 USB

$\downarrow$
 LSB

Square law modulator using transistor as non linear device.



Square law modulator using FET.



(ii) Balanced Modulators :-

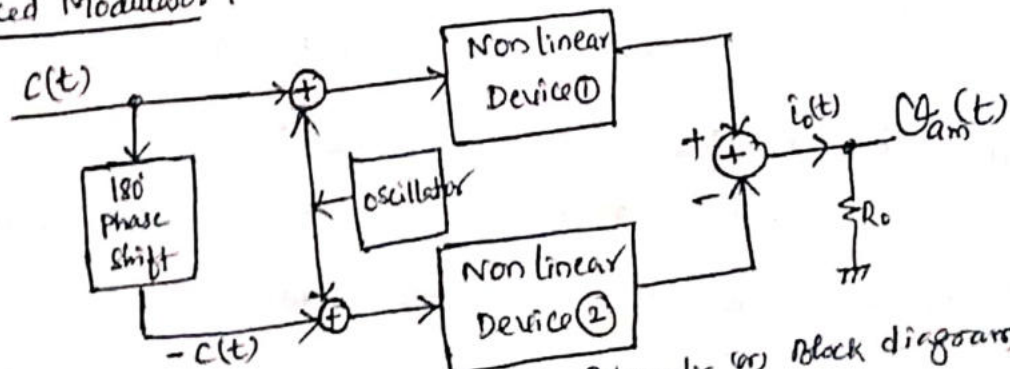


fig. (a) : Schematic (or) block diagram.

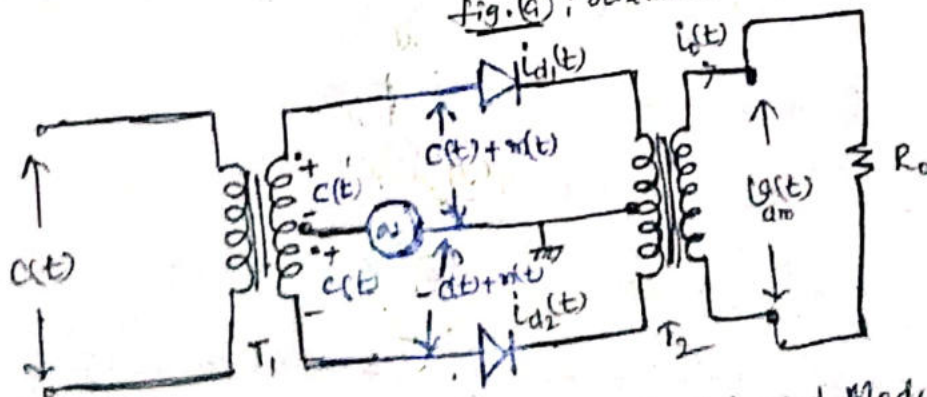


fig. (b). Circuit diagram of Balanced Modulator

Operation :-

1. Schematic or block diagram & Circuit diagram using diodes are shown in figure (A) & (B) respectively.
2. Carrier signal $c(t)$ is applied at the input or Primary of Audio transformer T_1 , which provides $c(t)$ & $-c(t)$ as shown in figure. (Acts as 180° phase shifter)
3. The diodes are operated in non linear region of $V-I$ Characteristics. The relation b/w input & output Current is

$$i_{d_1}(t) = a V_{d_1}(t) + b V_{d_1}^2(t) + \dots$$
4. output ^{Audio} transformer ' T_2 ' acts as BPF which eliminates all frequencies & except f_c , $f_c - f_m$ & $f_c + f_m$ frequencies.
5. Voltage across resistor $V_{am}(t) = I_o(t) \cdot R_o$, which is equal to amplitude modulated signal.

Derivation :- Let Carrier signal $c(t) = V_c \cos(2\pi f_c t)$
 modulating signal $m(t) = V_m \cos(2\pi f_m t)$.

i/p voltage at the anode of diode ' D_1 ' is

$$V_{d_1}(t) = m(t) + c(t) = V_c \cos(2\pi f_c t) + V_m \cos(2\pi f_m t)$$

i/p voltage at the anode of diode ' D_2 ' is

$$V_{d_2}(t) = -c(t) + m(t) = -V_c \cos(2\pi f_c t) + V_m \cos(2\pi f_m t)$$

output Current of Diode ' D_1 ' is

$$\begin{aligned} i_{d_1}(t) &= a V_{d_1}(t) + b V_{d_1}^2(t) + \dots \\ &= a [V_c \cos(2\pi f_c t) + V_m \cos(2\pi f_m t)] + b [V_c \cos(2\pi f_c t) + V_m \cos(2\pi f_m t)]^2 \\ &= a V_c \cos(2\pi f_c t) + a V_m \cos(2\pi f_m t) + a V_c^2 \cos^2(2\pi f_c t) + a V_m^2 \cos^2(2\pi f_m t) \\ &\quad + 2ab V_c V_m \cos(2\pi f_c t) \cos(2\pi f_m t) + \dots \end{aligned}$$

Similarly o/p current of Diode 'D₂' is

$$i_{d2}(t) = -aV_c \cos(2\pi f_c t) + aV_m \cos(2\pi f_m t) + aV_c^2 \cos^2(2\pi f_c t) + aV_m^2 \cos^2(2\pi f_m t) - 2bV_c V_m \cos(2\pi f_c t) \cos(2\pi f_m t)$$

Output currents of non linear devices (Diodes) $i_{d1}(t)$ & $i_{d2}(t)$ are flows in opposite direction. So, the o/p current at the secondary of Audio transformer 'T₂' is

$$i_o(t) = i_{d1}(t) - i_{d2}(t)$$

$$i_o(t) = 2aV_c \cos 2\pi f_c t + 4bV_c V_m \cos(2\pi f_c t) \cos(2\pi f_m t).$$

Voltage across resistor 'R_o' is given by

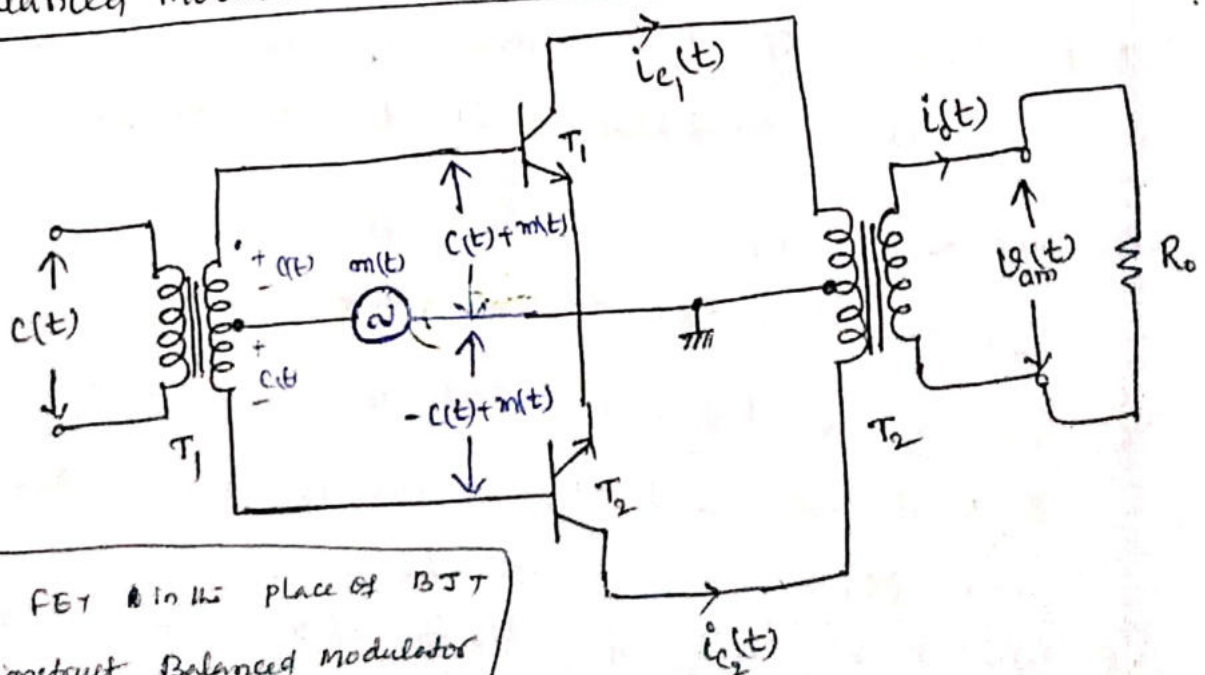
$$V_{am}(t) = i_o(t) \cdot R_o.$$

$$= 2aV_c R_o \cos(2\pi f_c t) + 4bV_c V_m R_o \cos(2\pi f_c t) \cos(2\pi f_m t).$$

$$V_{am}(t) = 2aV_c R_o \cos(2\pi f_c t) + 2bV_c V_m R_o [\cos(2\pi(f_c+f_m)t) + \cos(2\pi(f_c-f_m)t)]$$

Unmodulated Carrier (or) Carrier
Upper Side Band USB
Lower Side Band (LSB)

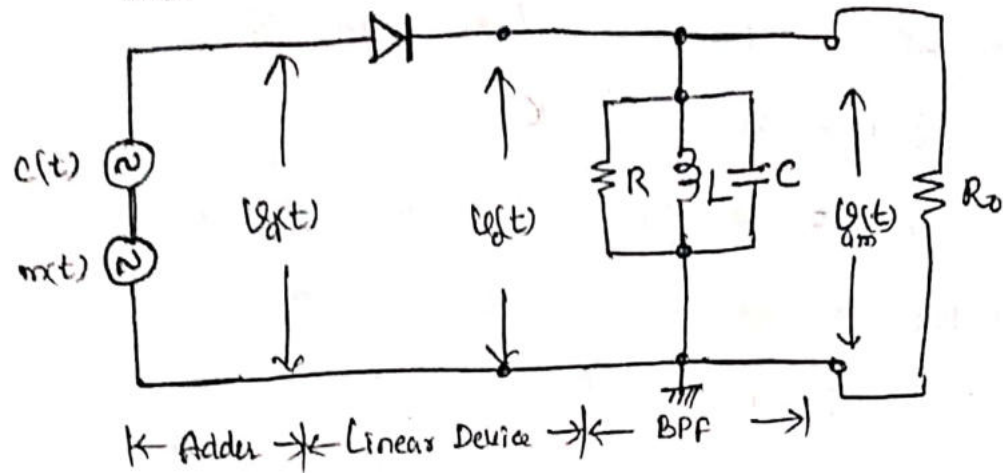
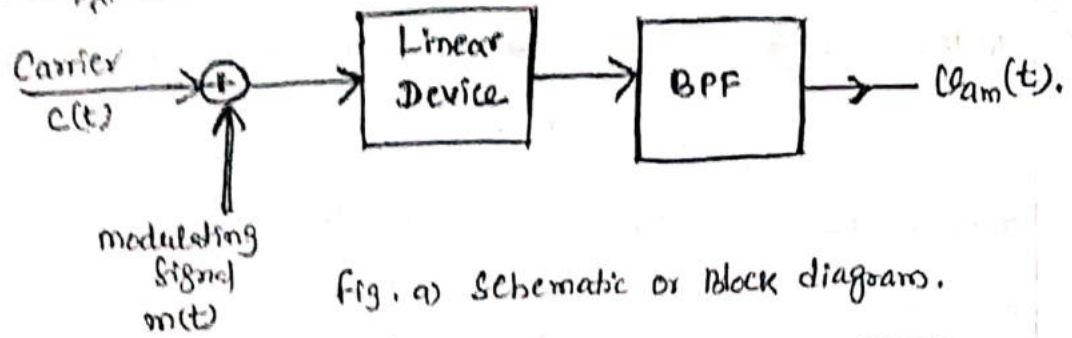
Balanced Modulators Using Transistors :-



* Note

Replace FET in the place of BJT to construct Balanced Modulator Using FET

Switching Modulator :-



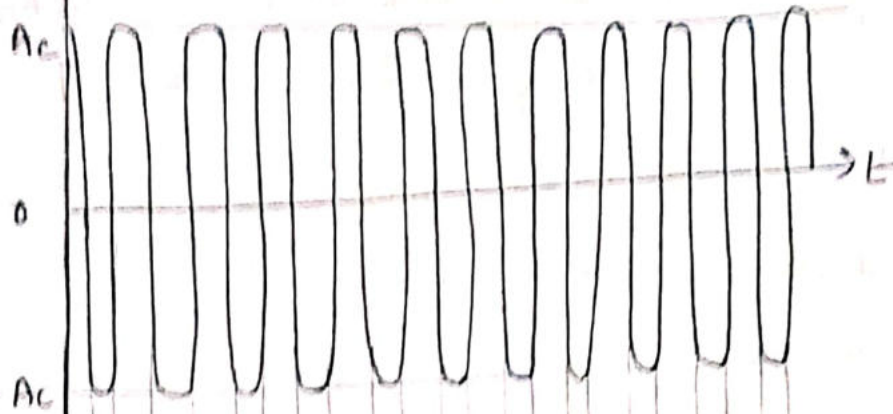
Operation :-

1. Schematic or block diagram & Circuit diagram using diode 'D' are shown in figure.
2. If two voltage sources are connected in series then voltages will be added i.e. it acts as Adder.
3. The diode is operated in linear region of its V-I characteristics i.e. ON (Forward bias) & OFF (Reverse bias).
4. ON or OFF of diode is only depends on Carrier signal amplitude but not on modulating signal.
5. During +ve half cycle, Diode turns 'ON' so that $c(t) + m(t)$ go to the o/p and During -ve half cycle, ~~Diode~~ Diode turns 'OFF' and o/p will be zero.
6. BPF Passes only f_c , $f_c + f_m$ and $f_c - f_m$, so that Amplitude modulated signal is developed across resistor R_o .

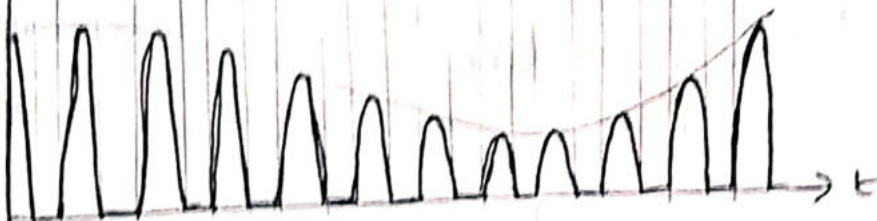
Wave forms :-



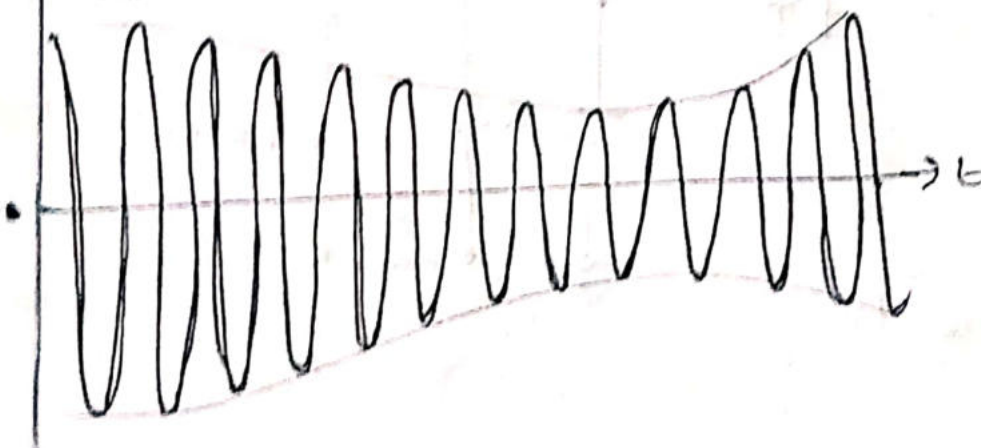
$e(t) = V_m \cos \omega t$



$v_d(t)$ - o/p of Diode



$v_{avg}(t)$



Demodulator or Detector :-

"It is a Process of extracting or detecting modulating signal $m(t)$ from the modulated signal"

(or)

"It is frequency translation technique which converts high or Bandpass signal into low frequency (or) Baseband signal."

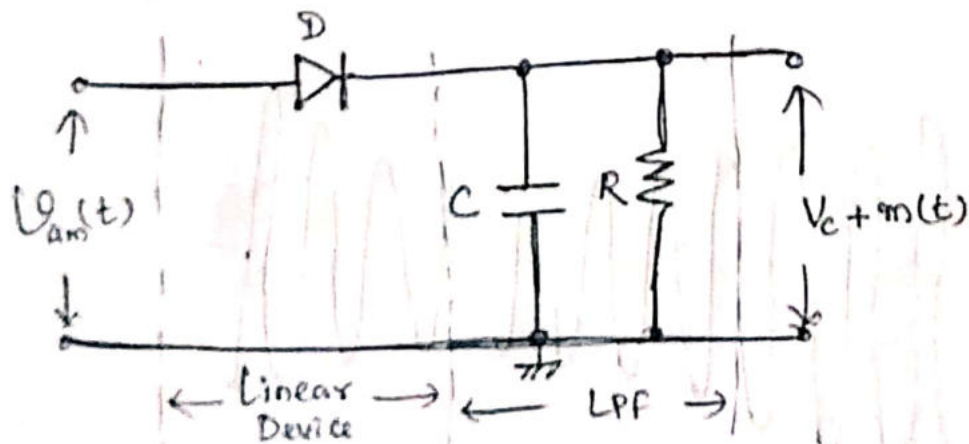
Types of amplitude Demodulators :-

- 1) Envelope Demodulator (or) Detector
- 2) Rectifier Demodulator (or) Detector

① Envelope Demodulator (or) Detector :-



Fig. a) Schematic or Block diagram.



Operation :-

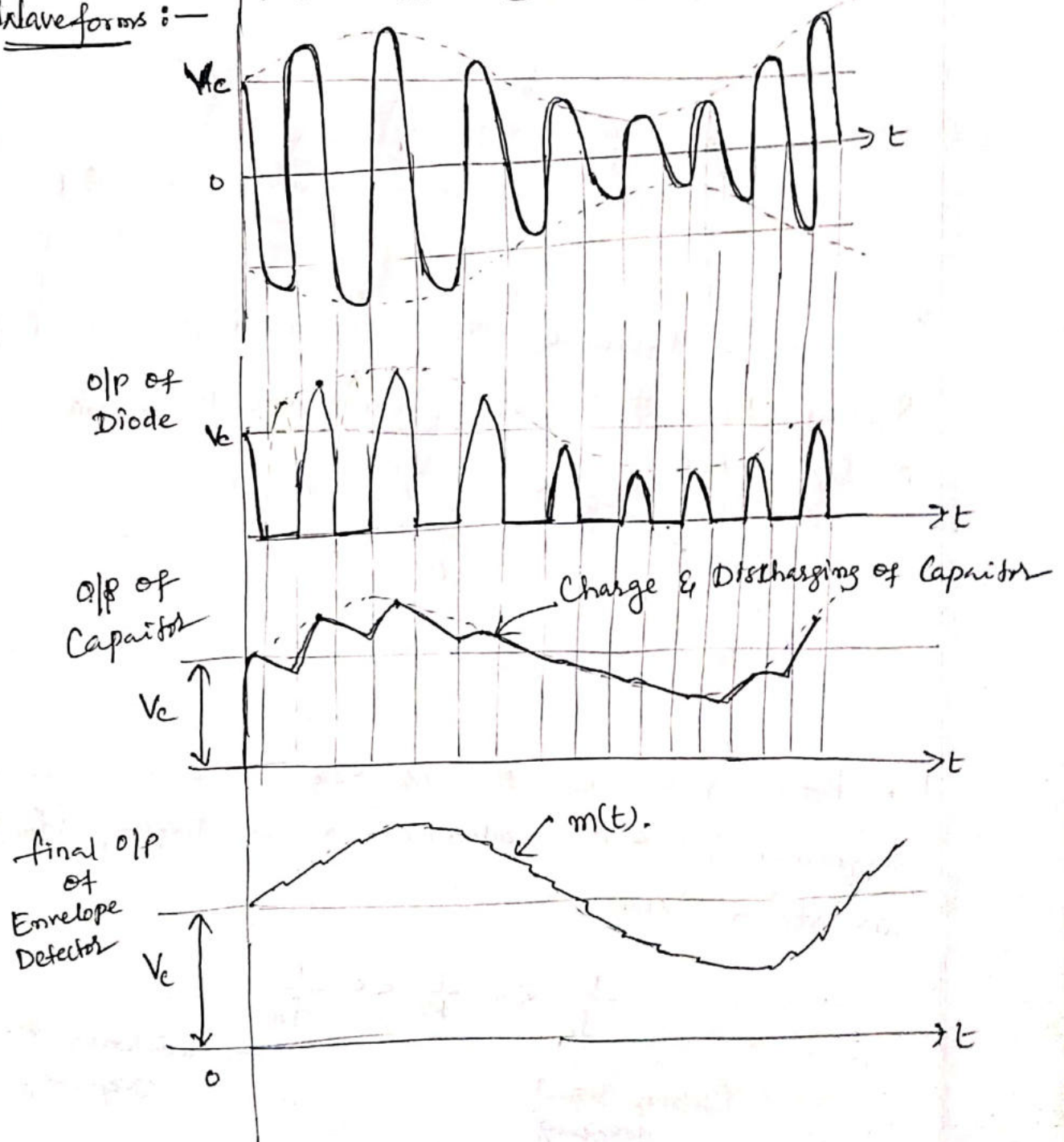
1. The Schematic or Block diagram and Circuit diagram as shown in fig. (a) & (b), to which AM signal is applied as i/p.
2. Diode 'D' is operated in linear region of its $V-I$ Characteristics.

3. Diode turns 'ON' for +ve half cycle and turns 'OFF' for -ve half cycle of i/p AM signal. (19)

4. The Capacitor charges to Peak value of i/p AM Pulse during +ve half cycle ~~and~~ through diode 'D' and Discharges through resistor 'R' during -ve half cycle.

5. Charging & Discharging of Capacitor detects (or) Extracts modulating signal $m(t)$.

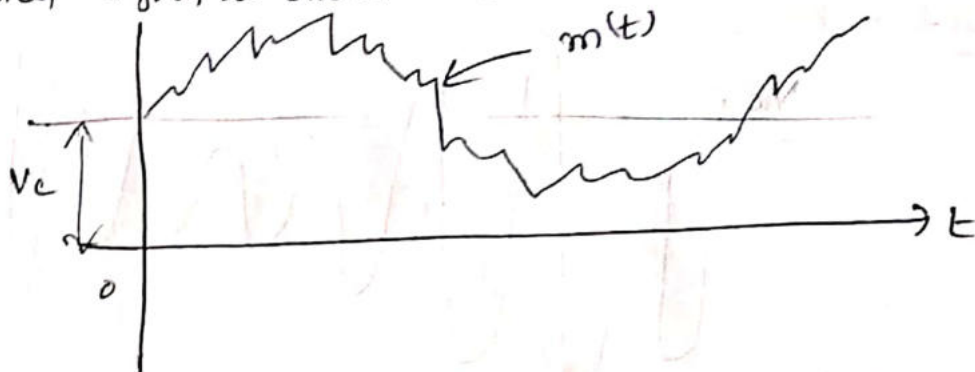
Waveforms :- ~~Amplitude~~ $V_{am}(t) = [V_c + m(t)] \cos 2\pi f_c t$



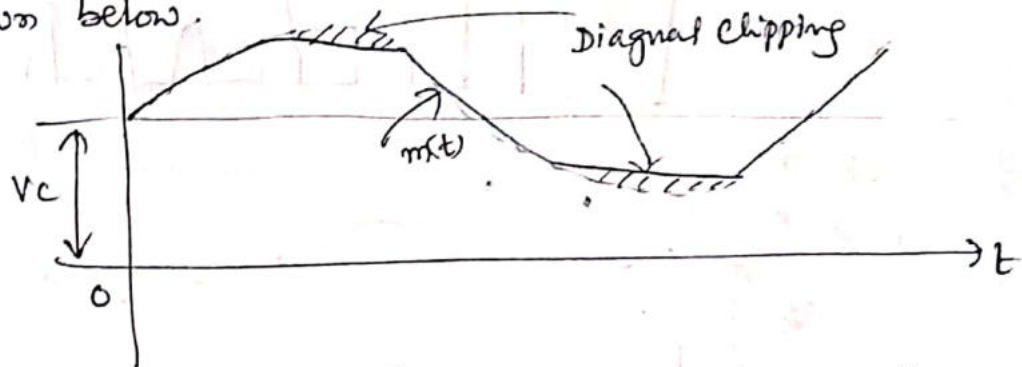
6. By Selecting Proper value of $R \& C$, modulating signal plus DC Component is Obtained at the O/P of Envelope Detector.

7. DC Component is due to unmodulated Carrier & $m(t)$ is due to modulated Carrier (i.e. LSB & USB).

8. If $R \& C$ values are ~~too~~ too low (or) small then Capacitor quickly Charges & discharges, which results 'Spikes' in the detected signal as shown below.



9. If $R \& C$ values are too large (or) high then Capacitor slowly Charges & Discharges, which results 'diagonal clipping' as shown below.

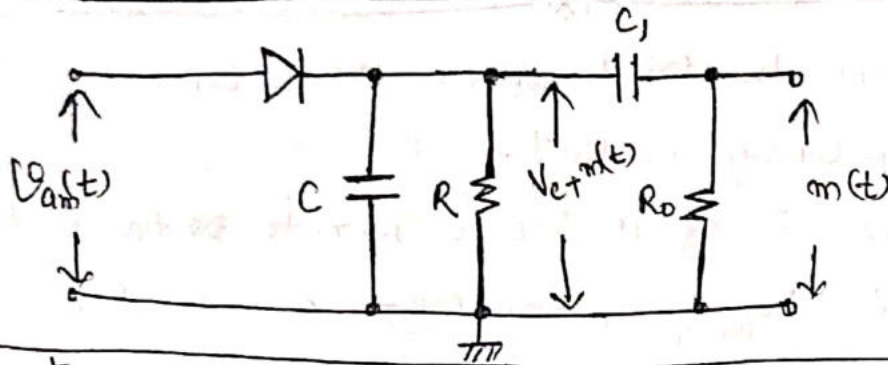


10. Proper value of ' RC ' is Selected based Carrier frequency (f_c) and modulating signal frequency (f_m) as shown below

$$\frac{1}{f_c} \ll \frac{1}{RC} \ll \frac{1}{f_m}$$

$\swarrow$ Carrier signal frequency
 $\searrow$ Modulating signal frequency

② Rectifier Demodulator or Detector :-



Note :

Operation which is discussed in Envelope detector is equally valued in rectifier detector.

1. Output of Envelope ~~det~~ detector contains both DC Component V_c plus modulating signal $m(t)$. And DC Component is not required at the output.
2. Capacitor ' C_1 ' which is shown in the circuit diagram removes the DC Component. So, that output contains only modulating signal $m(t)$.

Application of AM System :-

It is used for Audio transmission in Radio.

Double Side Band Suppressed Carrier (DSB-SC) :-

DSB-SC (or)
DSB (or)
DSB-AM

* It contains two side bands i.e. USB & LSB.

* Power Efficiency is 100% $\eta = \frac{P_{sidebands}}{P_{total}} = \frac{P_{mod}}{P_{mod} + P_{carrier}} \times 100 = 100\%$

* % of Power Saving in DSB-SC w.r. to AM is 66.67%.

* Band width $BW_{DSB-SC} = 2W_m$ (or) $2f_m$ (Hz)

DSB-SC - Single tone modulating signal:-

Let us consider modulating signal

$$m(t) = A_m \cos(2\pi f_m t) \text{ and } f_c \gg f_m \text{ (or)}$$

Carrier signal $c(t) = A_c \cos(2\pi f_c t)$; $A_c \gg A_m$ & $\omega_c \gg \omega_m$

DSB-SC signal can be generated by multiplying $c(t)$ & $m(t)$

i.e. $A_{DSB-SC}(t) = c(t) \times m(t)$

$$= A_c \cos(2\pi f_c t) \times A_m \cos(2\pi f_m t)$$

$$= \frac{A_c A_m}{2} [2 \cos(2\pi f_c t) \cos(2\pi f_m t)]$$

$$A_{DSB-SC}(t) = \frac{A_c A_m}{2} [\cos(2\pi(f_c + f_m)t) + \cos(2\pi(f_c - f_m)t)]$$

↓
USB

↓
LSB

(or) ↑

$$A_{DSB-SC}(t) = \frac{A_c A_m}{2} [\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t]$$

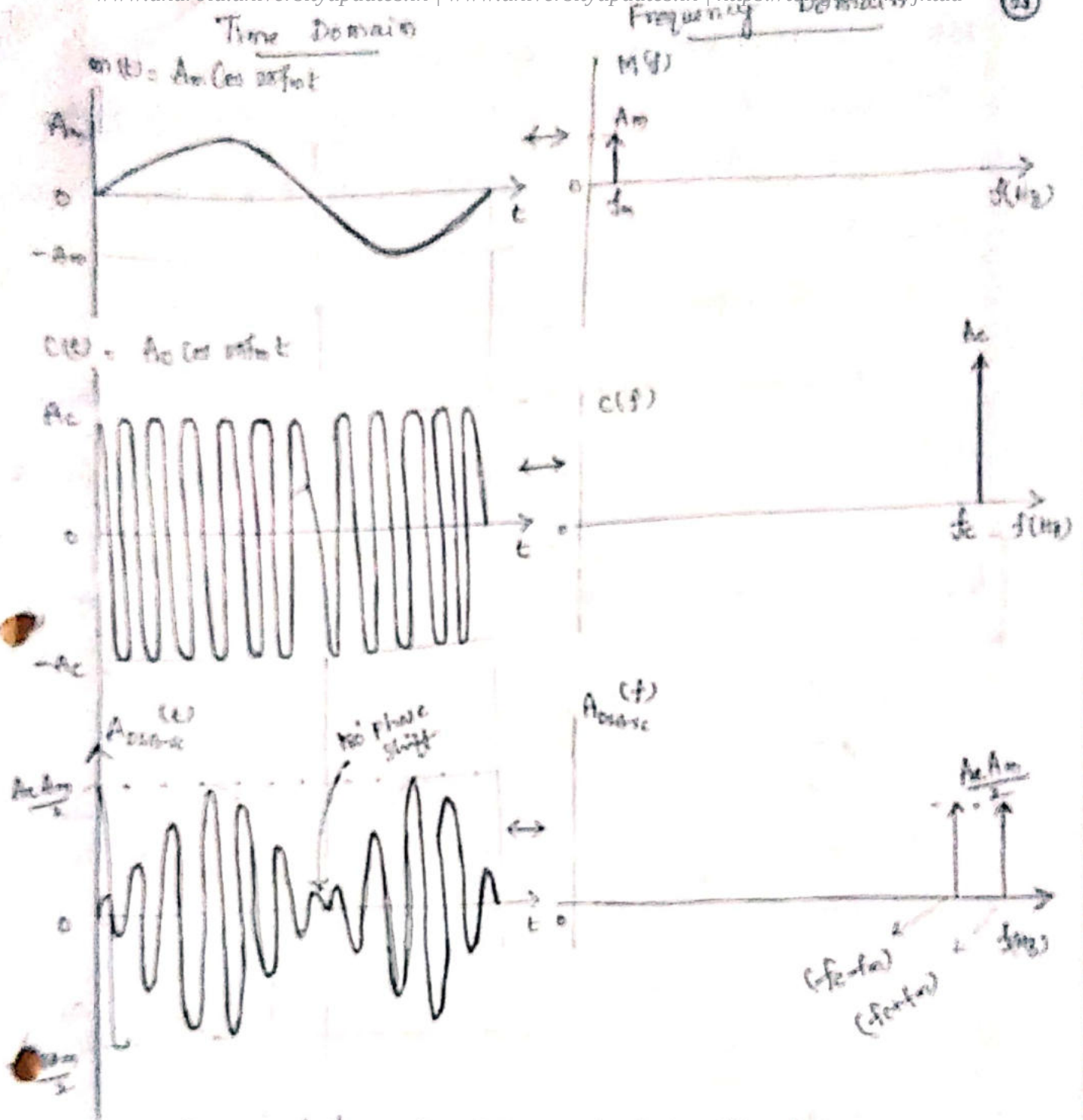
* % of Power Saving in DSB-SC can be written as

$$\% PS = \frac{P_t - P'_t}{P_t} \times 100; \text{ where } P_t = P_c + \frac{P_c m_a^2}{2}$$

$$P'_t = \frac{P_c m_a^2}{2}$$

$$\% PS = \frac{P_c + \frac{P_c m_a^2}{2} - \frac{P_c m_a^2}{2}}{P_c + \frac{P_c m_a^2}{2}} \times 100 = \frac{P_c}{P_c [1 + m_a^2/2]} \times 100$$

$$\% PS = \frac{2}{2 + m_a^2} \times 100 \quad \text{If } m_a = 1 \Rightarrow \% PS = 66.67\%$$



Special feature in DSB-SC is that, The 180° phase shift exist in DSB-SC signal whenever modulating signal crosses time axis.

Band width of DSB-SC is

$$\begin{aligned} BW_{DSB-SC} &= f_c + f_m - (f_c - f_m) \\ &= 2f_m \text{ (Hz)} \text{ or } 2W_m \text{ rad/sec} \end{aligned}$$

DSB-SC - Multitone Modulating Signal :-

$$\text{Let } m(t) = A_{m1} \cos 2\pi f_{m1} t + A_{m2} \cos 2\pi f_{m2} t ; A_{m1} > A_{m2} \text{ \& } f_{m1} < f_{m2}$$

$$c(t) = A_c \cos 2\pi f_c t ; A_c \gg A_{m1} \text{ \& } A_c f_c \gg f_{m2}$$

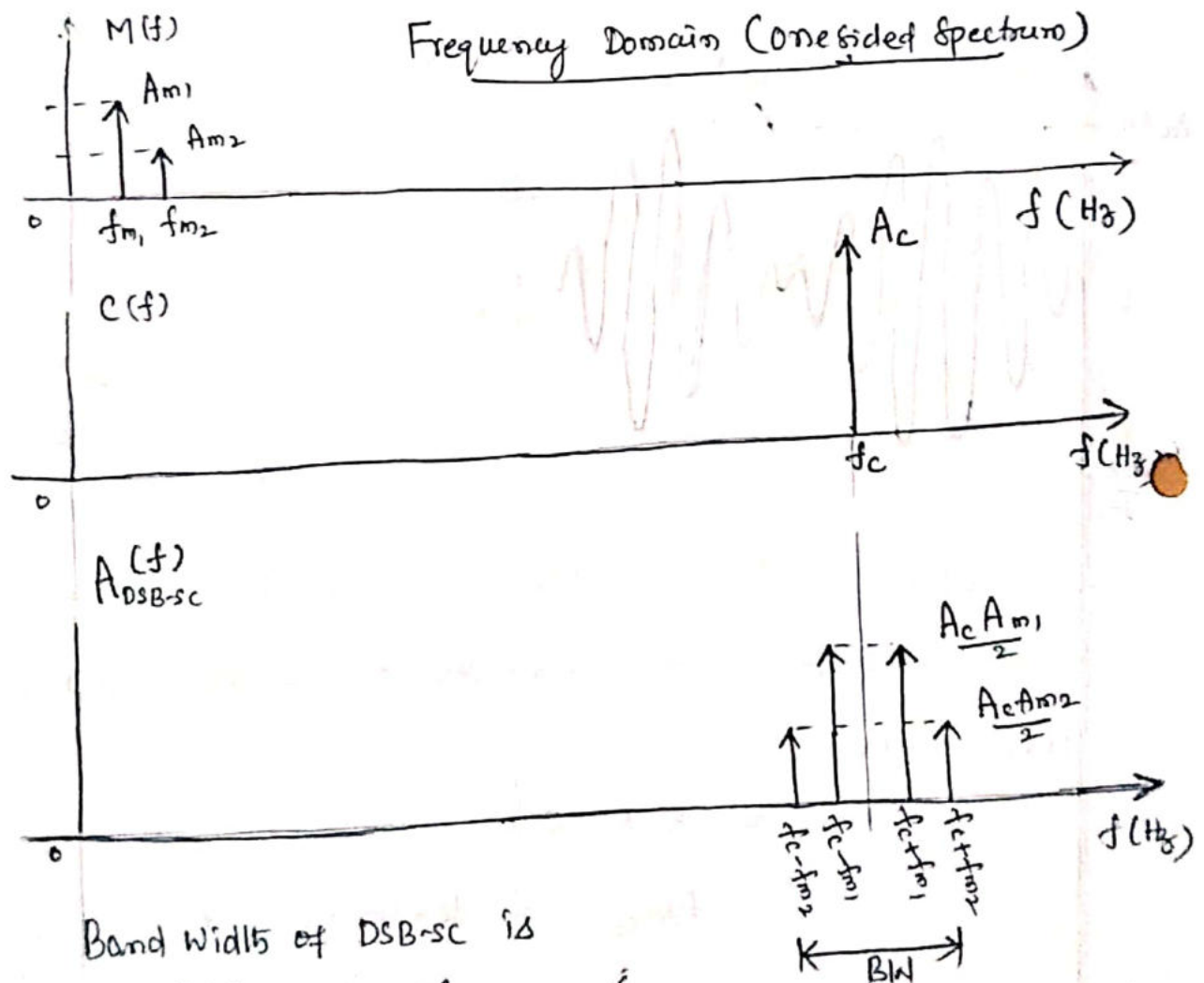
$$A_{\text{DSB-SC}}(t) = c(t) \times m(t)$$

$$= A_c \cos(2\pi f_c t) \times \left[\frac{A_{m1}}{2} \cos(2\pi f_{m1} t) + A_{m2} \cos(2\pi f_{m2} t) \right]$$

$$= \frac{A_c A_{m1}}{2} \cdot 2 \cos(2\pi f_c t) \cos(2\pi f_{m1} t) + \frac{A_c A_{m2}}{2} \cdot 2 \cos(2\pi f_c t) \cos(2\pi f_{m2} t)$$

$$A_{\text{DSB-SC}}(t) = \frac{A_c A_{m1}}{2} \left[\cos(2\pi(f_c + f_{m1})t) + \cos(2\pi(f_c - f_{m1})t) \right] + \frac{A_c A_{m2}}{2} \left[\cos(2\pi(f_c + f_{m2})t) + \cos(2\pi(f_c - f_{m2})t) \right]$$

$$A_{\text{DSB-SC}}(t) = \frac{A_c A_{m1}}{2} \left[\cos(\omega_c + \omega_{m1})t + \cos(\omega_c - \omega_{m1})t \right] + \frac{A_c A_{m2}}{2} \left[\cos(\omega_c + \omega_{m2})t + \cos(\omega_c - \omega_{m2})t \right]$$



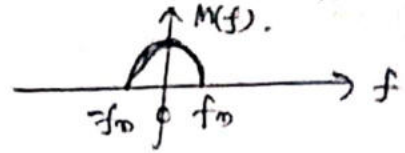
Band Width of DSB-SC is

$$BLN_{\text{DSB-SC}} = f_c + f_{m2} - (f_c - f_{m2}) = 2f_{m2} \text{ Hz or } 2\omega_{m2} \frac{\text{rad}}{\text{sec}}$$

DSB-SC is also linear modulation Technique.

DSB-SC - Band of frequencies :-

Let us consider modulating signal contains band of frequencies such that its maximum frequency f_m Hz



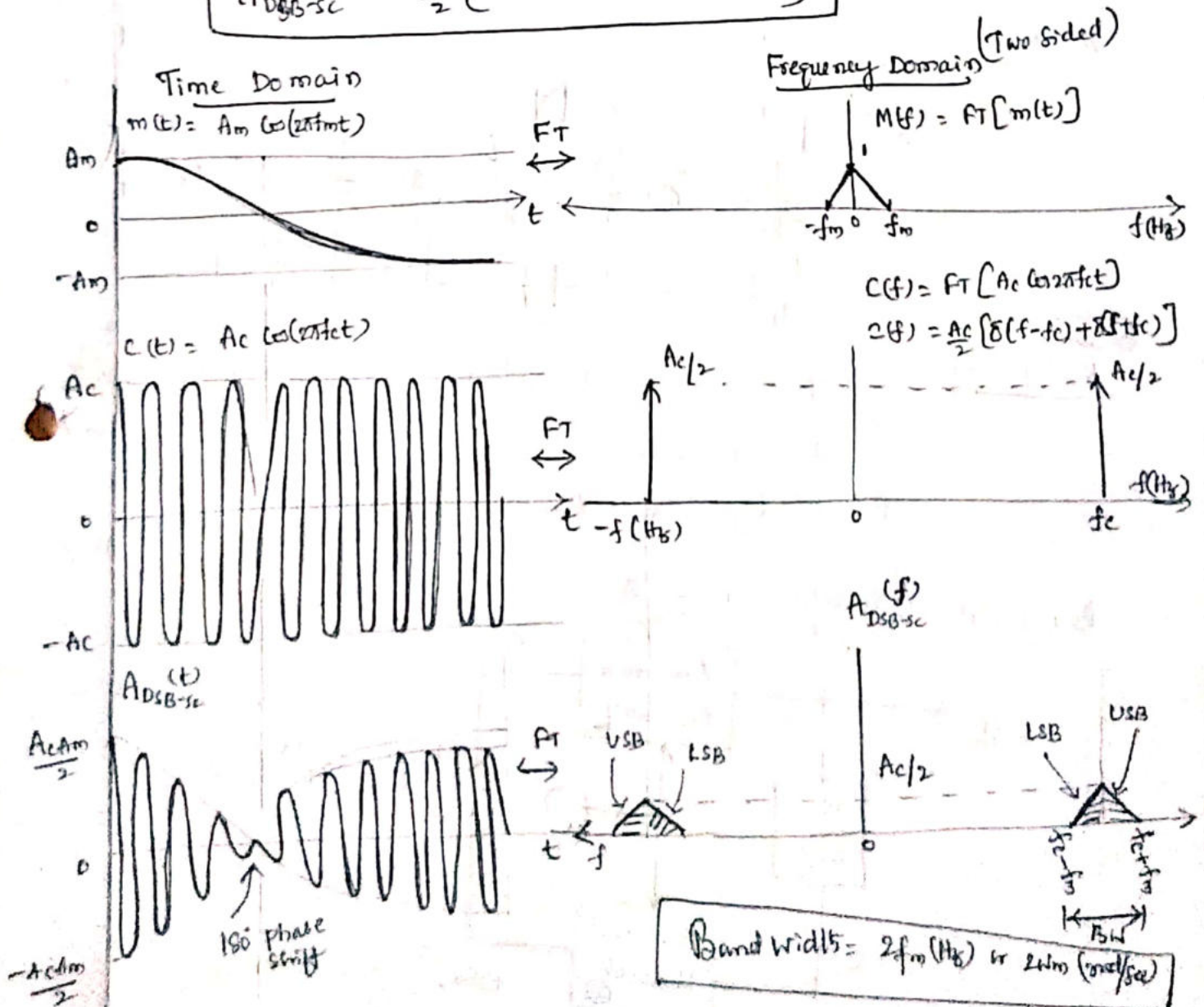
and Carrier signal $C(t) = A_c \cos 2\pi f_c t$; $A_c \gg |m(t)|_{\max}$ & $f_c \gg f_m$.

$$A_{\text{DSB-SC}}(t) = C(t) \times m(t) = A_c \cos(2\pi f_c t) \cdot m(t)$$

Take Fourier Transform on both sides.

$$\begin{aligned} \text{FT}[A_{\text{DSB-SC}}(t)] &= \text{FT}[A_c \cos(2\pi f_c t) m(t)] \\ &= \text{FT}\left[A_c m(t) \left[\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2} \right]\right] \\ &= \frac{A_c}{2} \left\{ \text{FT}[m(t) e^{j2\pi f_c t}] + \text{FT}[m(t) e^{-j2\pi f_c t}] \right\} \end{aligned}$$

$$A_{\text{DSB-SC}}(f) = \frac{A_c}{2} [M(f-f_c) + M(f+f_c)]$$



$$\text{Band width} = 2f_m (\text{Hz}) \text{ or } 2f_m (\text{rad/sec})$$

Generation of DSB-sc Signal:-

Types: ① Small signal or Non Linear modulators.

- i) Square law modulator
- ii) Balanced modulator

② Large signal or Linear modulator

- i) Switching modulator
- ii) Ring modulator
- iii) Frequency mixer

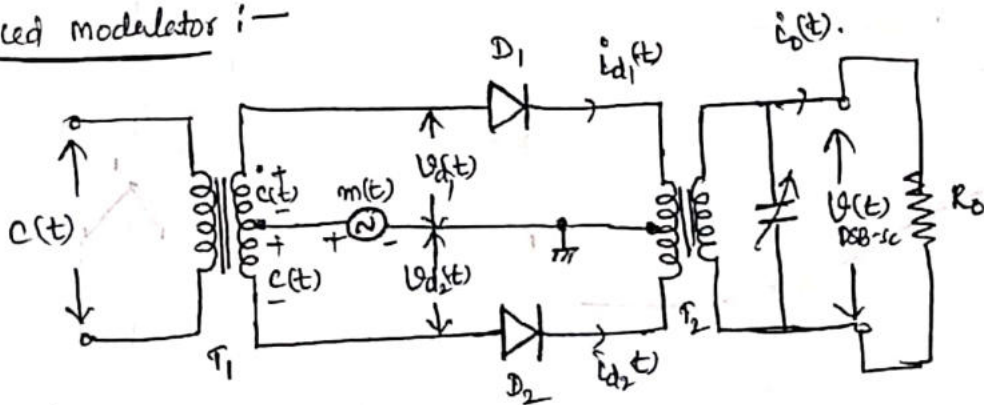
i) Square law modulators:-

⊗ Note:- Square law modulators which we have discussed for generation of AM signal is equally valued for generation of DSB-sc signal.

Modification:- BPF is designed such that it selects only USB & LSB.

(Operation - 4/5 point - AM generation Page no: 15)

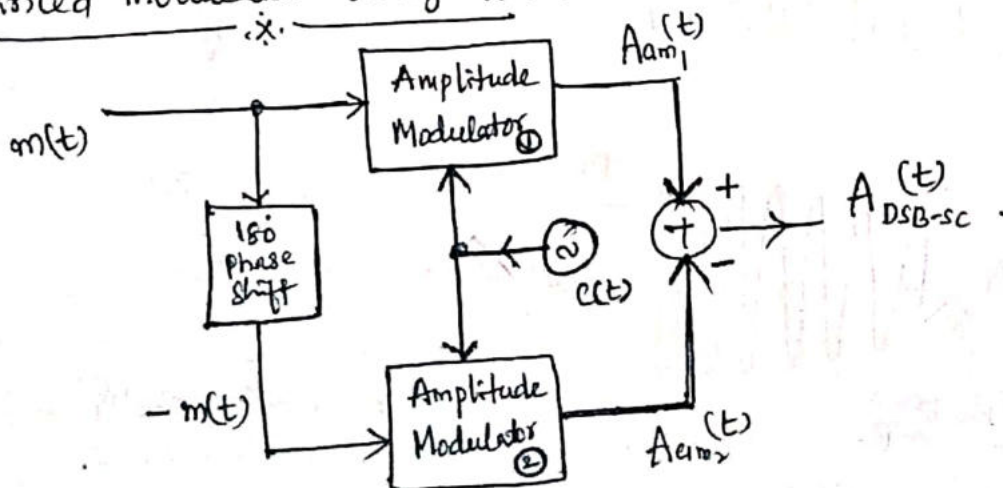
ii) Balanced modulators:-



⊗ Note

Modification:- Capacitor along with Secondary of Audio Transformer T_2 forms 'BPF', which eliminates all other frequencies and passes only USB & LSB. (Operation - 4/5 point in AM generation Page no: 16)

Balanced modulators using AM:-



Operation :-

1. The block diagram of Balanced modulator Using AM is shown in figure, which uses two Amplitude modulators.

2. 90° phase shift network provides (or) generates $-m(t)$.

3. output of Amplitude modulator₁ can be written as

$$A_{am_1}(t) = A_c [1 + K_a m(t)] \cos(2\pi f_c t)$$

where K_a = voltage sensitivity factor $\frac{1}{A_c}$.

4. output of Amplitude modulator₂ can be written as

$$A_{am_2}(t) = A_c [1 - K_a m(t)] \cos 2\pi f_c t$$

5. output of Adder

$$A_{DSB-SC}(t) = A_{am_1}(t) - A_{am_2}(t)$$

$$= A_c [1 + K_a m(t)] \cos(2\pi f_c t) - A_c [1 - K_a m(t)] \cos(2\pi f_c t)$$

$$= A_c \cancel{\cos(2\pi f_c t)} + A_c K_a m(t) \cos(2\pi f_c t) - A_c \cancel{\cos(2\pi f_c t)} + A_c K_a m(t) \cos(2\pi f_c t)$$

$$A_{DSB-SC}(t) = 2 m(t) \cos \omega_c (2\pi f_c t)$$

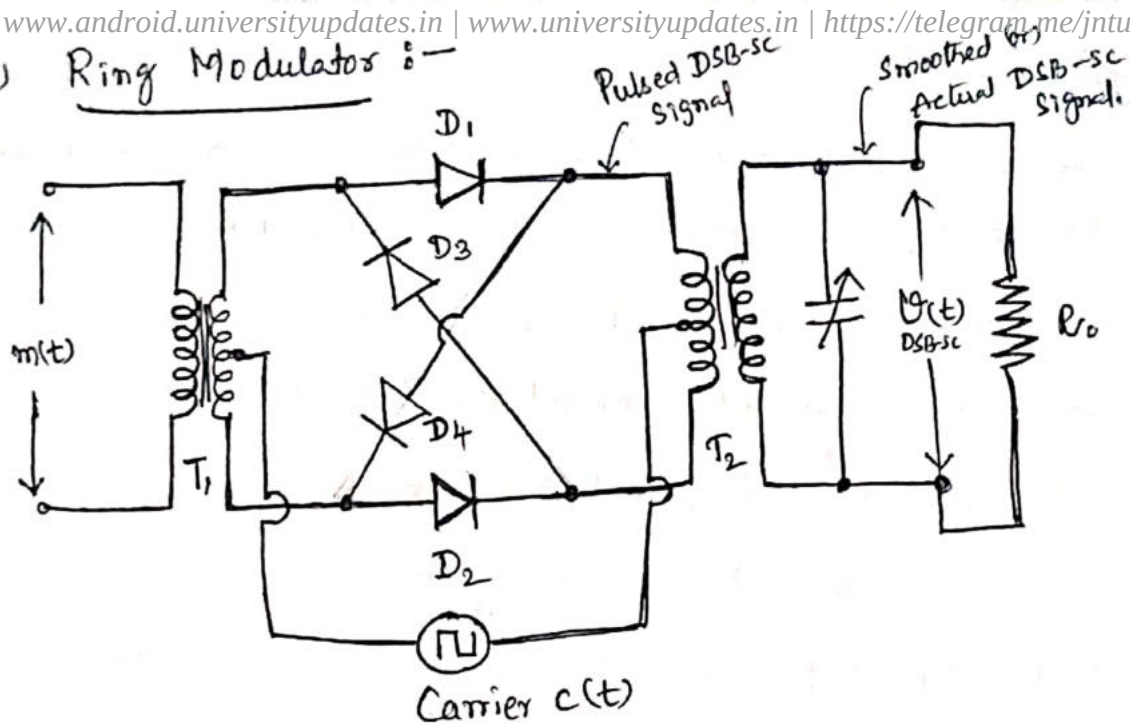
Switching modulator :-

(*) Note :-

modification :- Page no. 17 - AM Generation

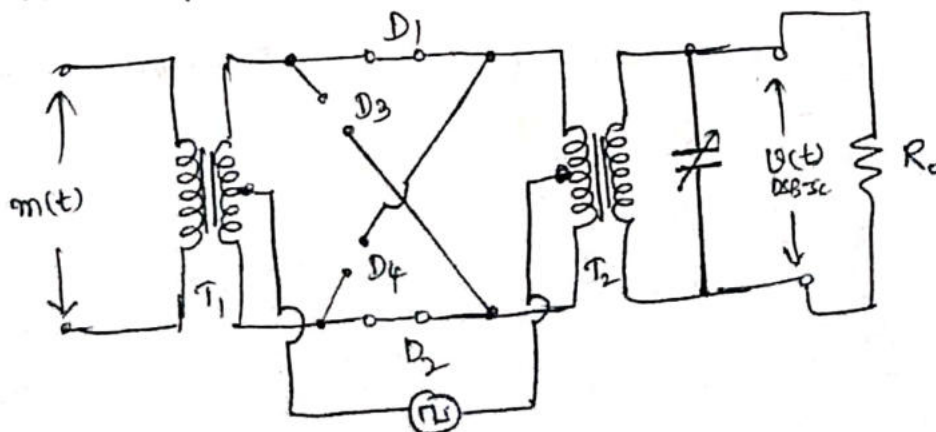
Band Pass filter (BPF) eliminates all frequencies only passes USB $(f_c + f_m)$ & LSB $(f_c - f_m)$.

ii) Ring Modulator :-

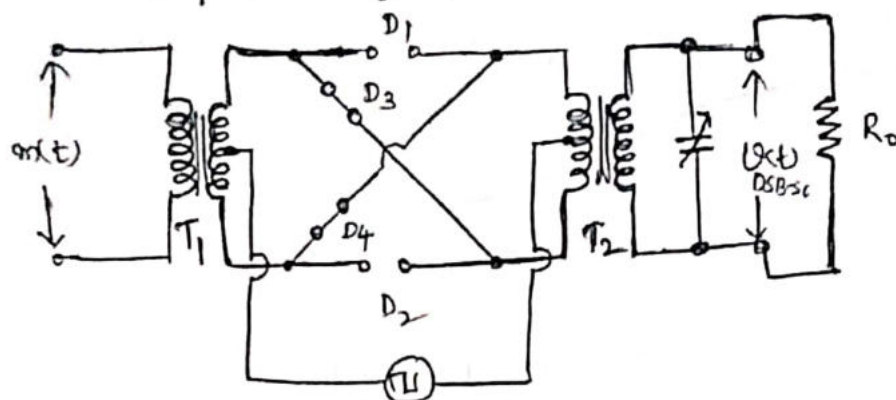


Operation :-

1. The Circuit diagram of Ring modulator is shown in figure, which contain four diodes, Two Audio transformers, Capacitor.
2. Here Square wave is used as Carrier, which is tapped to secondary of Audio T/F T_1 & Primary of Audio T/F T_2 as shown in figure.
3. Carrier signal & modulating signal are selected in such way that ON/OFF of ~~the~~ diodes depends only on Carrier signal amplitude.
4. When the Carrier signal is +ve, diodes D_1 & D_2 are in ON state (i.e forward bias) & D_3 & D_4 are in OFF state (i.e Reverse bias) So that loop (or) ring forms as shown below.



5. When the Carrier is -ve, diodes D_1 & D_2 are in OFF State (i.e. Reverse bias) & D_3 & D_4 are in ON State (i.e. Forward bias) So that loop (or) Ring forms as shown in figure below.



6. Either Carrier +ve or -ve, ring or loop will be formed in the circuit b/w Audio T/F T_1 & T_2 so that Carrier $c(t)$ & modulating signal $m(t)$ are multiplied which generates pulsed DSB-SC signal at the Primary of Audio T/F T_2 .
7. Pulsed DSB-SC signal contains $f_c, 2f_c, 3f_c, \dots, f_c \pm f_m, 2(f_c \pm f_m), \dots$. The Capacitor with Secondary of Audio T/F T_2 forms BPF which selects only $f_c \pm f_m$ i.e. $f_c + f_m$ (USB) & $f_c - f_m$ (LSB), ~~for both~~. This is called Smoothed or Actual DSB-SC signal.

Derivation :- Let us consider modulating signal $m(t)$ & Carrier signal $c(t)$.

Carrier signal $c(t)$ can be represented using Fourier series as

$$c(t) = \frac{4}{\pi} \left[\cos \omega_c t - \frac{1}{3} \cos 3\omega_c t + \frac{1}{5} \cos 5\omega_c t - \frac{1}{7} \cos 7\omega_c t + \dots \right]$$

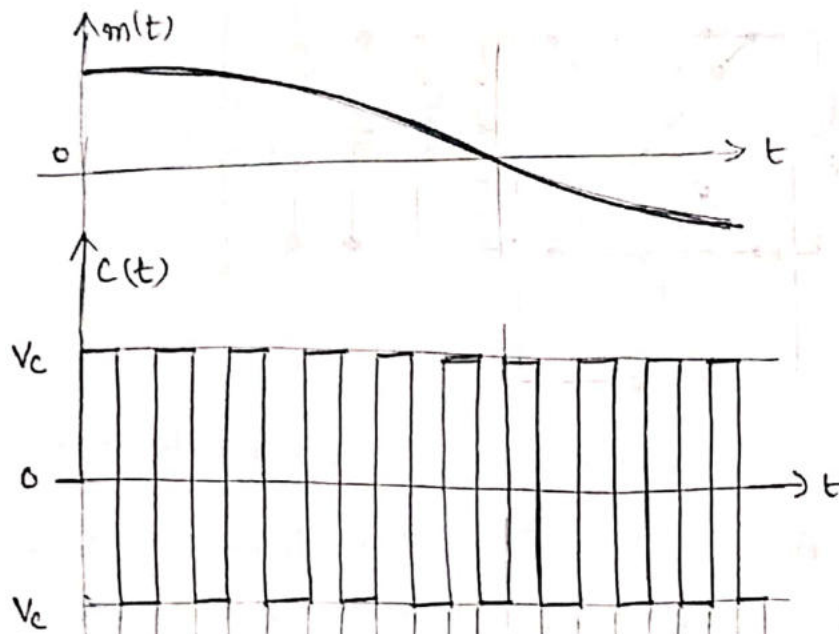
Signal at the Primary of T/F T_2

$$P_{DSB-SC}(t) = c(t) \times m(t)$$

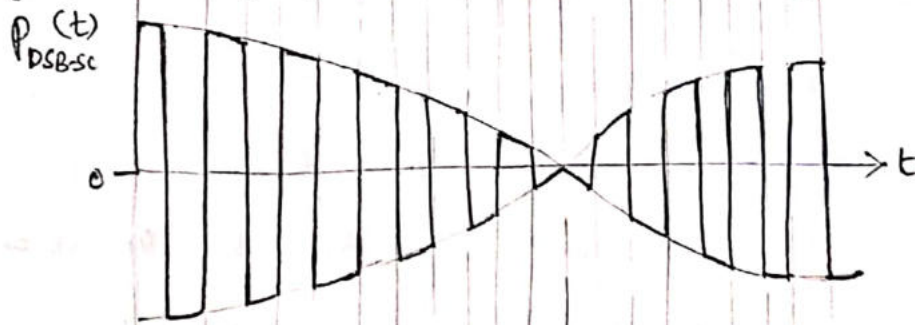
$$= \frac{4}{\pi} \left[m(t) \cos \omega_c t - \frac{m(t)}{3} \cos 3\omega_c t + \frac{m(t)}{5} \cos 5\omega_c t - \frac{m(t)}{7} \cos 7\omega_c t + \dots \right]$$

Signal at the secondary of T/F T_2 will be.

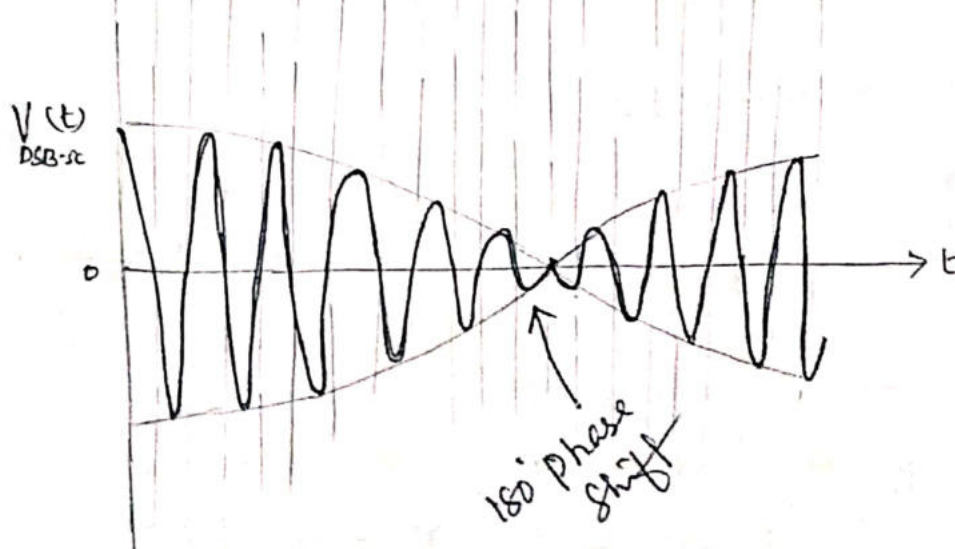
$$A_{DSBsc}(t) = \frac{4}{\pi} \cdot m(t) \cos \omega_c t$$



Signal at Primary of T/F T_2 i.e Pulsed DSB-sc signal.

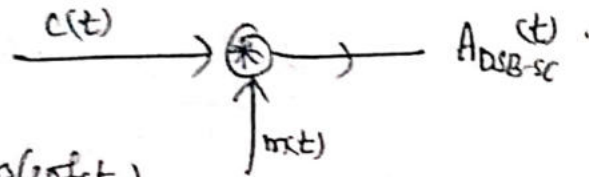


Signal at the secondary of T/F T_2 i.e Smoothed or Actual DSB-sc signal



ii) Frequency Mixer :-

Multiplexer acts as frequency mixer which multiplies two signals and generates sum & difference frequency, which is equal to DSB-SC signal.



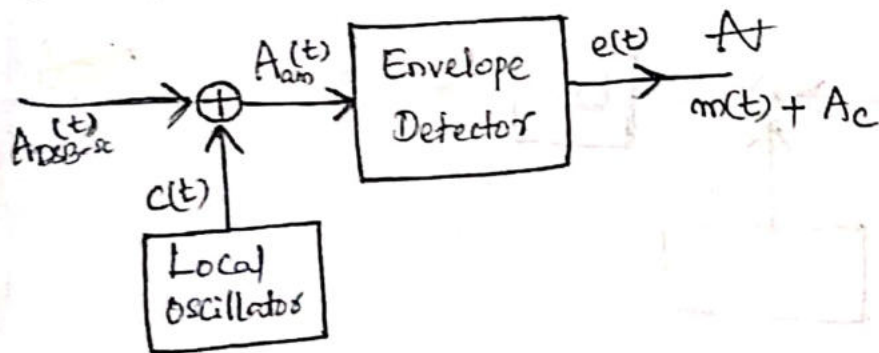
$$\text{Let } c(t) = A_c \cos(2\pi f_c t)$$

$$m(t) = A_m \cos(2\pi f_m t)$$

$$A_{DSB-SC}(t) = c(t) \times m(t) = \frac{A_c A_m}{2} [\cos(2\pi(f_c + f_m)t) + \cos(2\pi(f_c - f_m)t)]$$

Demodulators (or) Detectors for DSB-SC :-

- ① Carrier re-insertion (or) Envelope Detector or Demodulator
- ② Synchronous or Coherent or Homodyne " (or) "
- ③ Costas's Loop Detector or Demodulator
- ④ Square Loop Detector or Demodulator



Operation :-

- ① The block diagram of Carrier re-insertion (or) Envelope Detector is shown in the figure which contains Adder, Local oscillator & an Envelope Detector.
- ② Local oscillator generate carrier signal $c(t) = A_c \cos(2\pi f_c t)$
- ③ Adder generate Amplitude modulated signal by adding locally generated carrier to the incoming DSB-SC signal.

$$A_{DSB-sc}(t) = m(t) \cos(2\pi f_c t)$$

$$c(t) = A_c \cos 2\pi f_c t$$

$$\begin{aligned} \text{O/p of Adder } A_m(t) &= A_c \cos(2\pi f_c t) + m(t) \cos(2\pi f_c t) \\ &= \underbrace{[A_c + m(t)]}_{\text{Envelope}} \cos(2\pi f_c t) \end{aligned}$$

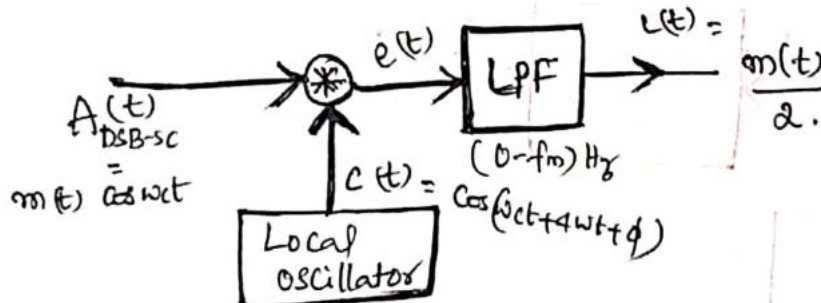
- ④ Envelope Detector extracts envelope of the modulated carrier so that o/p will be equal to

$$e(t) = A_c + m(t)$$

$\downarrow$ modulating signal
 $\downarrow$ DC Component

Question :- ① Discuss the detection of DSB-sc signal using Envelope Detector.
 (or)
 ② Discuss the detection of DSB-sc signal with carrier

- ② Synchronous (or) Coherent (or) Homodyne Detector :-



Operation :-

1. The block diagram of Synchronous or Coherent Detector is shown in the block diagram which contains multiplier, local oscillator and LPF (whose cut-off frequency f_0 should be equal to maximum frequency of the modulating signal $m(t)$)
2. Local oscillator generates carrier frequency $c(t) = A_c \cos(2\pi f_c t)$.
3. Multiplier generates $e(t)$ which is multiplication of $A_{DSB-sc}(t)$ & $c(t)$.

$$e(t) = A_{DSB-SC}(t) \times c(t).$$

$$= m(t) \cos(2\pi f_c t) \times A_c \cos(2\pi f_c t)$$

$$= m(t) \cdot A_c \cdot \cos^2(2\pi f_c t)$$

$$= m(t) A_c \left[\frac{1 + \cos(4\pi f_c t)}{2} \right]$$

$$= \underbrace{A_c \frac{m(t)}{2}}_{\text{Low frequency}} + \underbrace{\frac{A_c}{2} m(t) \cos(4\pi f_c t)}_{\text{high frequency}}.$$

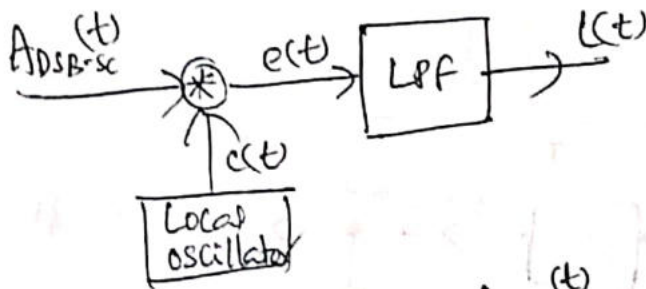
4. Low Pass filter eliminates high frequencies, so that o/p is equal to modulating signal.

$$f(t) = \underbrace{\left(\frac{A_c}{2} \right)}_{\text{magnitude}} m(t)$$

Phase & Frequency Errors in Synchronous or Coherent Detectors :-

Let us consider locally generated carrier has phase & frequency errors (or) difference

$$c(t) = \cos(\omega_c t + 4\omega t + \phi)$$



o/p of multiplier $e(t) = A_{DSB-SC}(t) \times c(t)$

$$= m(t) \cos \omega_c t \times \cos(\omega_c t + 4\omega t + \phi)$$

$$= \frac{m(t)}{2} \left[\cos(2\omega_c t + 4\omega t + \phi) + \cos(4\omega t + \phi) \right]$$

o/p of LPF

$$\boxed{f(t) = \frac{m(t)}{2} \cdot \cos(4\omega t + \phi)}$$

Case (i) :- No phase & frequency error $\Delta\omega = 0$ & $\phi = 0$
o/p of LPF is $L(t) = \frac{m(t)}{2}$.

Case (ii) :- only frequency error $\Delta\omega \neq 0$ & $\phi = 0$
o/p $L(t) = \frac{m(t)}{2} \cdot \cos \Delta\omega t$

The frequency of $m(t)$ is slightly changes due to $\cos(\Delta\omega t)$.
This is called "Beat (or) Warble effect".

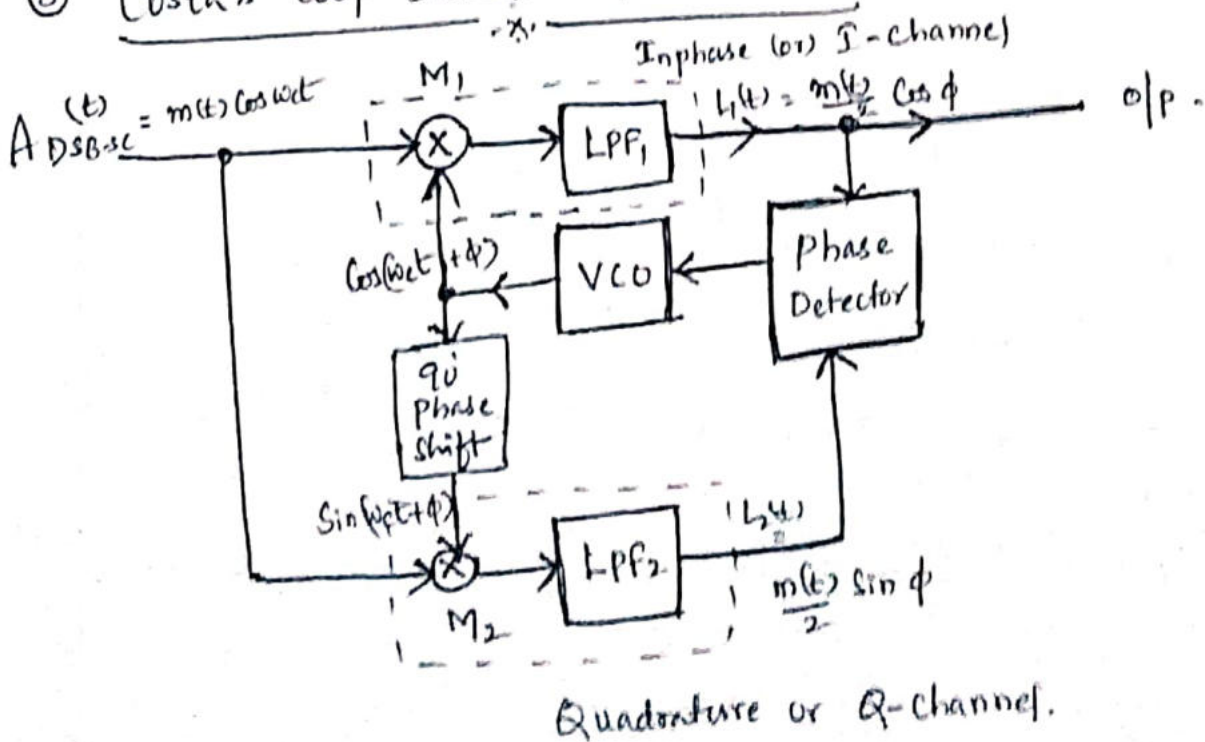
Case (iii) :- only phase error $\Delta\omega = 0$ & $\phi \neq 0$

$$L(t) = \frac{m(t)}{2} \cdot \cos \phi$$

If $\phi = \pi/2$ then $L(t) = 0$

If phase difference b/w incoming signal & locally generated signal is 90° then o/p of Synchronous detector o/p is zero, even if i/p is present. This is called "Quadrature null (or) Donald Duck Voice Effect".

③ Costa's Loop Detector & Demodulator :-



Operation :-

1. The block diagram of Costas's loop detector is shown in fig. which contains I-Channel & Q-Channel Connected Through VCO & Phase Detector in a feed back system.
2. Both I-Channel & Q-Channel contains Synchronous detectors which contains multiplier followed by LPF.
3. Let us consider phase error or difference b/w incoming signal & locally generated signal is ' ϕ ' such that o/p of I-Channel is $\frac{m(t)}{2} \cos \phi$ and o/p of Q-Channel is $\frac{m(t)}{2} \sin \phi$.
4. The phase detector compares (or detects) phase difference b/w $\frac{m(t)}{2} \cos \phi$ & $\frac{m(t)}{2} \sin \phi$ and generates proportional voltage.
 (Voltage Controlled oscillator)
5. The phase or frequency of VCO is varied in accordance with the o/p voltage of phase detector.
6. This process continues until the phase difference goes to zero. The Costas's loop is said to be locked to the incoming signal when $\phi=0$.
7. Once Costas's loop is locked then o/p of I-Channel will be equal to $\frac{m(t)}{2}$ (which is actual o/p) & o/p of Q-Channel will be equal to '0'.

Derivation :- let $A_{DSB-SC}(t) = m(t) \cos \omega_c t$ &

Locally generated Carrier $C(t) = \cos(\omega_c t + \phi)$

Output of multiplier M_1 is

$$M_1(t) = A_{DSB-SC}(t) * \cos(\omega_c t + \phi)$$

$$M_1(t) = m(t) \cos \omega_c t * \cos (\omega_c t + \phi)$$

$$= \frac{m(t)}{2} \left[\cos (2\omega_c t + \cancel{\omega_c t} + \phi) + \cos (\cancel{\omega_c t} + \phi) \right]$$

O/P of LPF is

$$L_1(t) = \frac{m(t)}{2} \cos \phi$$

out put of multiplier M_2 is

$$M_2(t) = A_{DSB-sc}(t) * \sin (\omega_c t + \phi)$$

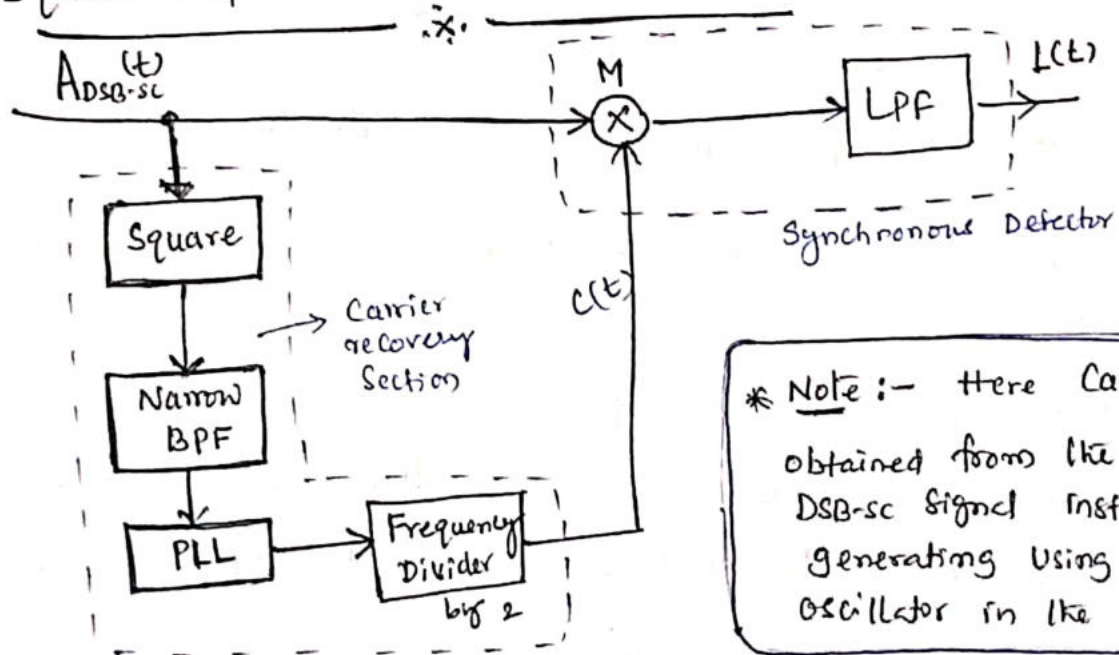
$$= m(t) \cos \omega_c t * \sin (\omega_c t + \phi)$$

$$= \frac{m(t)}{2} \left[\sin (2\omega_c t + \phi) + \sin \phi \right]$$

$$= \frac{m(t)}{2} \sin \phi$$

O/P of LPF₂ is $L_2(t) = \frac{m(t)}{2} \sin \phi$

④ Square loop Detector or Demodulator :-



* Note :- here Carrier is obtained from the received DSB-sc signal instead of generating using local oscillator in the receiver

Operation :-

1. The block diagram of Square loop detector is shown in figure which contains Carrier recovery Section & Synchronous Detector
2. Incoming DSB-sc signal is given to both Synchronous detector and Carrier recovery section.

3. Square Circuit doubles the frequency of the incoming DSB-SC signal & narrow BPF takes narrow band of frequencies at ' $2\omega_c$ '.
4. phase locked loop (PLL) generates (or) selects only ' $2\omega_c$ ' frequency.
5. Frequency divider by 2 generates ' ω_c ' frequency so that ' $\cos \omega_c t$ ' is applied at the I/P of synchronous detector.

Case (i) :- If no phase & frequency errors in the incoming DSB-SC signal, then

$$A_{DSB-SC}(t) = m(t) \cos \omega_c t$$

Locally generated carrier $c(t) = \cos \omega_c t$

$$\text{O/P of multiplier } M(t) = m(t) \cos^2 \omega_c t = \frac{m(t)}{2} + \frac{m(t)}{2} \cos 2\omega_c t$$

$$\text{O/P of Low Pass filter } \boxed{L(t) = \frac{m(t)}{2}}$$

Case (ii) :- If incoming signal contains phase & frequency errors then

$$A_{DSB-SC}(t) = m(t) \cos(\omega_c t + \Delta \omega t + \phi)$$

Locally generated carrier $c(t) = \cos(\omega_c t + \Delta \omega t + \phi)$

$$\text{O/P of multiplier } M(t) = m(t) \cos(\omega_c t + \Delta \omega t + \phi) \times \cos(\omega_c t + \Delta \omega t + \phi)$$

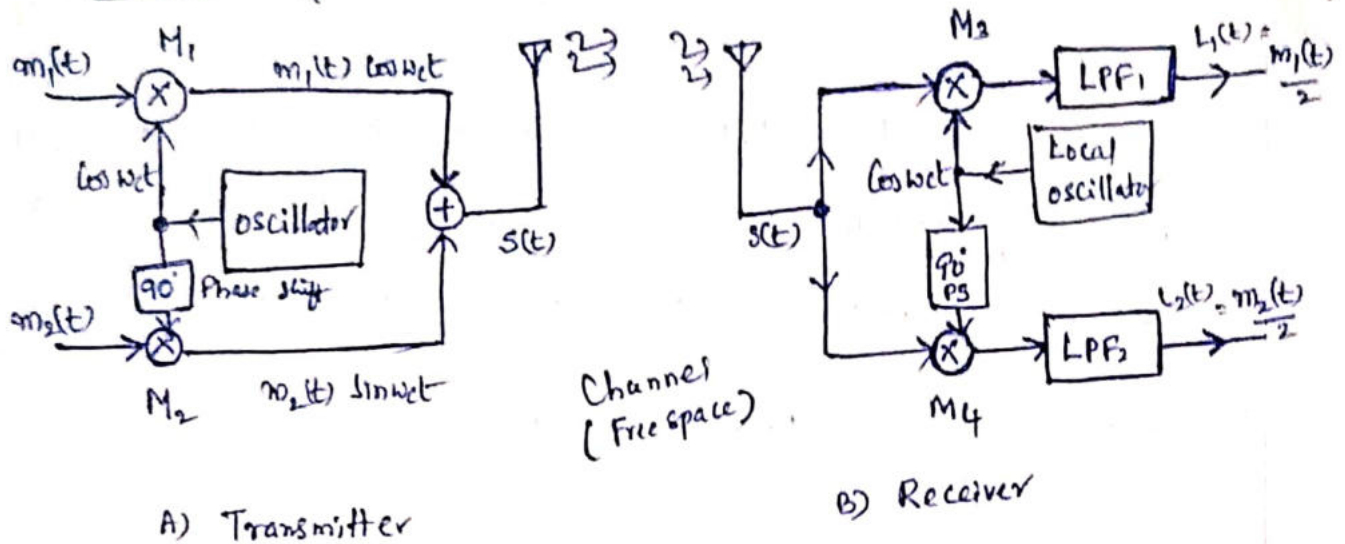
$$= \frac{m(t)}{2} \left[\cos(0) + \cos(2\omega_c t + 2\Delta \omega t + 2\phi) \right]$$

$$\text{O/P of Low Pass filter } \boxed{L(t) = \frac{m(t)}{2}}$$

* Application of DSB-SC :-

Used for Color Transmission in Television using Quadrature Amplitude Modulation (QAM)

Quadrature Amplitude Modulation (QAM) :-



Operation :-

1. The block diagram of QAM with transmitter & Receiver which are connected through free space is shown in figure.
2. Oscillator generates Carrier signal $\cos wt$ & 90° phase shift provides $\sin wt$.
3. Here two Carrier frequencies are used with same frequency ' w_c ' with phase difference 90° . Hence it is called "Quadrature Amplitude modulation" (QAM).
4. Output of multiplier M_1 will be $M_1(t) = m_1(t) \cos wt$ (DSB signal) & output of multiplier M_2 will be $M_2(t) = m_2(t) \sin wt$ (DSB-SC signal)
5. Outputs of two multipliers are added to get QAM signal

$$S(t) = m_1(t) \cos wt + m_2(t) \sin wt$$

6. Two synchronous detectors are used with quadrature phase, so that modulating signals $m_1(t)$ & $m_2(t)$ are obtained at LPF_1 & LPF_2 .

o/p of Multiplier m_3 is

$$M_3(t) = S(t) * \cos wt$$

$$\begin{aligned}
 m_3(t) &= [m_1(t) \cos \omega_c t + m_2(t) \sin \omega_c t] * \cos \omega_c t \\
 &= m_1(t) \cos^2 \omega_c t + m_2(t) \sin \omega_c t \cos \omega_c t \\
 &= \frac{m_1(t)}{2} + \frac{m_1(t)}{2} \cancel{\cos 2\omega_c t}^{2\omega} + \frac{m_2(t)}{2} \cancel{\sin 2\omega_c t}^{2\omega}
 \end{aligned}$$

o/p of LPF₁ is

$$L_1(t) = \frac{m_1(t)}{2}$$

Similarly

o/p of multiplier m_4 is

$$m_4(t) = s(t) * \sin \omega_c t$$

$$\begin{aligned}
 &= [m_1(t) \cos \omega_c t + m_2(t) \sin \omega_c t] \sin \omega_c t \\
 &= m_1(t) \sin \omega_c t \cos \omega_c t + m_2(t) \sin^2 \omega_c t \\
 &= \frac{m_1(t)}{2} \cancel{\sin 2\omega_c t}^{2\omega} + \frac{m_2(t)}{2} - \frac{m_2(t)}{2} \cancel{\cos 2\omega_c t}^{2\omega}
 \end{aligned}$$

o/p of LPF₂ is

$$L_2(t) = \frac{m_2(t)}{2}$$

Single Side Band Suppressed Carrier (SSB-SC) :-

All are same

SSB-SC (or)
SSB (or)
SSB-AM

If the carrier & either of one side band is removed from AM signal then it contains single (or) one side band. Hence it is called "Single Side Band Suppressed Carrier (SSB-SC)" signal.

important points :-

1. It contains only one side band i.e. either USB or LSB.
2. Power efficiency is 100%.

$$\therefore \eta = \frac{P_{\text{useful}}}{P_{\text{total}}} \times 100 = \frac{\frac{P_c m_a^2}{4}}{\frac{P_c m_a^2}{4}} \times 100 = 100\%$$

3. Percentage of Power Saving in SSB-SC with respect to AM is.

83.33 %.

Proof:- % of Power saving can be written as $\% PS = \frac{P_t - P_t''}{P_t} \times 100$

Where $P_t = P_c + P_c \frac{m_a^2}{2}$ & $P_t'' = \frac{P_c m_a^2}{4}$

$$\% PS = \frac{P_c + P_c \frac{m_a^2}{2} - \frac{P_c m_a^2}{4}}{P_c + P_c \frac{m_a^2}{2}} \times 100.$$

$$= \frac{P_c + \frac{P_c m_a^2}{4}}{P_c + P_c \frac{m_a^2}{2}} \times 100$$

$$= \frac{P_c [1 + \frac{m_a^2}{4}]}{P_c [1 + \frac{m_a^2}{2}]} \times 100$$

$$= \frac{1 + \frac{m_a^2}{4}}{1 + \frac{m_a^2}{2}} \times 100$$

$$= \frac{\frac{4 + m_a^2}{4}}{\frac{2 + m_a^2}{2}} \times 100$$

$$\% PS = \frac{(4 + m_a^2)}{2(2 + m_a^2)} \times 100$$

maximum % of Power saving at $m_a = 1$

$$= \frac{(4 + 1)}{2(2 + 1)} \times 100$$

$$\% PS = \frac{5}{6} \times 100 = 83.33\%$$

4. Band width of SSB-SC signal is ' W_m ($\frac{\text{rad}}{\text{sec}}$) or ' f_m ' (Hz).

5. % of Power Saving in SSB-SC with respect to DSB-SC system is 16.67%.

SSB-sc - Single tone modulating signal 8-

(30)

let us consider modulating signal $m(t) = A_m \cos \omega_m t$ & Carrier $c(t) = A_c \cos \omega_c t$

$$A_{DSB-sc}(t) = c(t) \times m(t) = A_c A_m \cos \omega_c t \cos \omega_m t$$

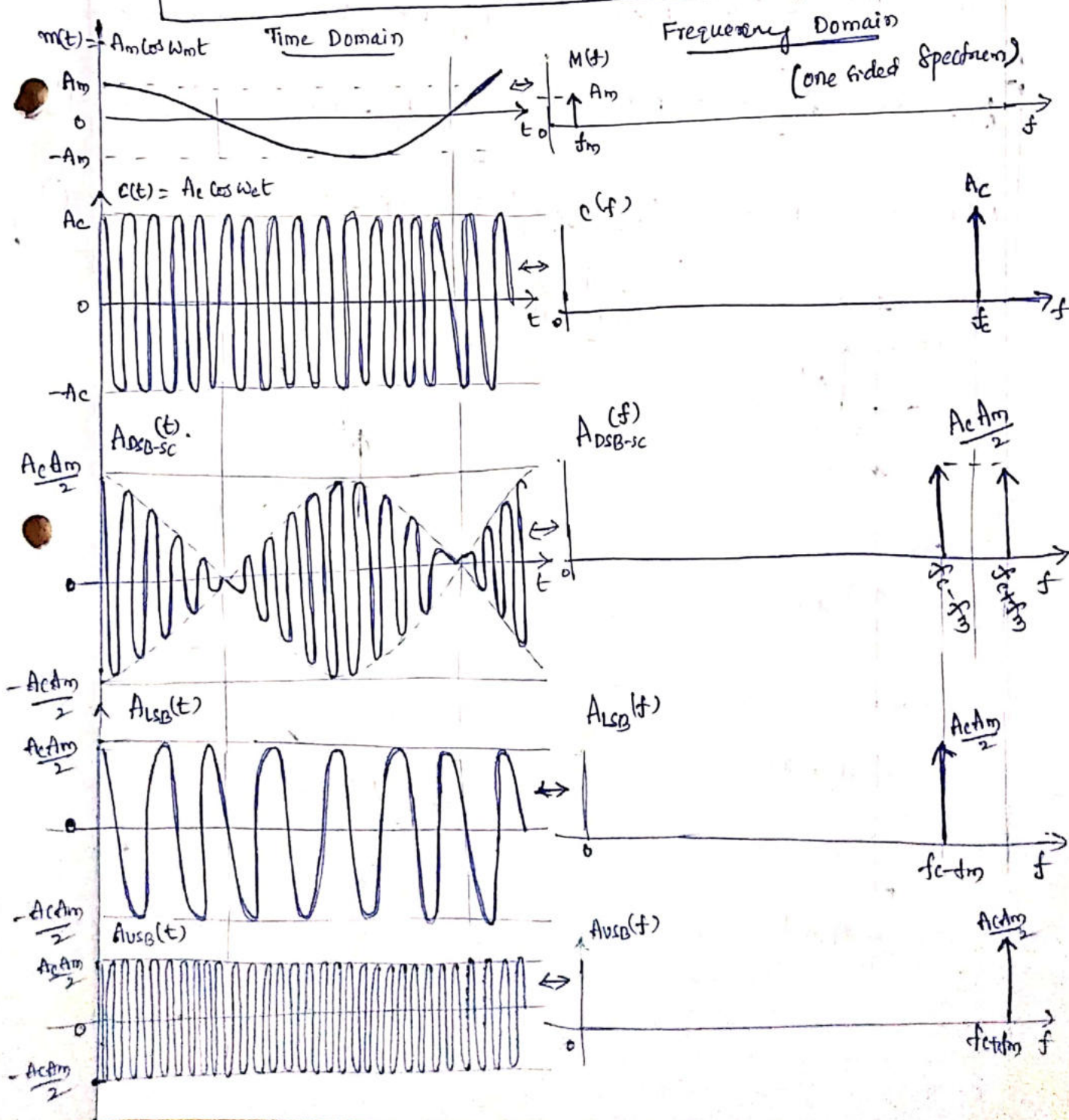
$$= \frac{A_c A_m}{2} \left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t \right]$$

$\downarrow$ USB $\downarrow$ LSB

$$A_{SSB-sc}(t) = A_{USB}(t) = \frac{A_c A_m}{2} \cos(\omega_c + \omega_m)t$$

(or)

$$A_{SSB-sc}(t) = A_{LSB}(t) = \frac{A_c A_m}{2} \cos(\omega_c - \omega_m)t$$



SSB-sc - Multi tone Modulating signal :-

let us consider modulating signal $m(t) = A_{m1} \cos \omega_{m1} t + A_{m2} \cos \omega_{m2} t$;
 $A_{m1} > A_{m2}$ & $\omega_{m1} < \omega_{m2}$.

Carrier signal $c(t) = A_c \cos \omega_c t$; $A_c \gg A_{m1}$ & $\omega_c \gg \omega_{m2}$

$$A_{SSB-sc}(t) = c(t) \times m(t) = A_c \cos \omega_c t * [A_{m1} \cos \omega_{m1} t + A_{m2} \cos \omega_{m2} t]$$

$$= A_c A_{m1} \cos \omega_c t \cos \omega_{m1} t + A_c A_{m2} \cos \omega_c t \cos \omega_{m2} t$$

$$A_{SSB-sc}(t) = \frac{A_c A_{m1}}{2} [\cos(\omega_c + \omega_{m1})t + \cos(\omega_c - \omega_{m1})t] + \frac{A_c A_{m2}}{2} [\cos(\omega_c + \omega_{m2})t + \cos(\omega_c - \omega_{m2})t]$$

(or)

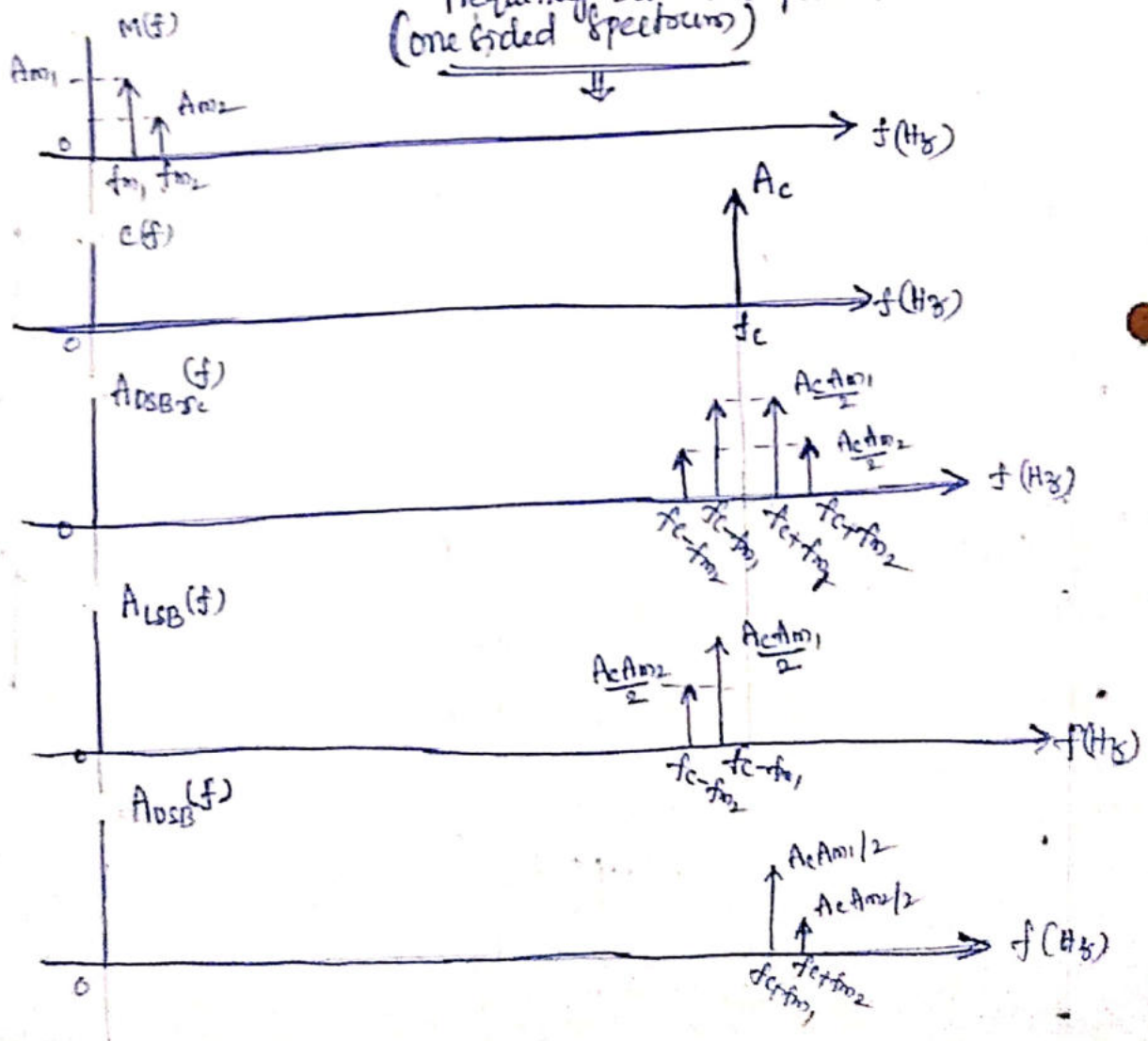
$$A_{SSB-sc}(t) = \frac{A_c A_{m1}}{2} [\cos[2\pi(f_c + f_{m1})t] + \cos[2\pi(f_c - f_{m1})t]] + \frac{A_c A_{m2}}{2} [\cos[2\pi(f_c + f_{m2})t] + \cos[2\pi(f_c - f_{m2})t]]$$

$$A_{SSB-sc}(t) = A_{USB}(t) = \frac{A_c A_{m1}}{2} \cos[2\pi(f_c - f_{m1})t] + \frac{A_c A_{m2}}{2} \cos[2\pi(f_c - f_{m2})t]$$

(or)

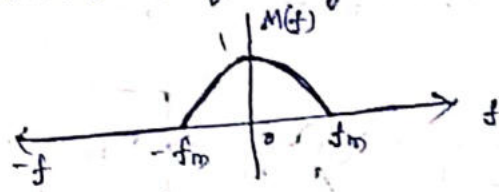
$$A_{SSB-sc}(t) = A_{USB}(t) = \frac{A_c A_{m1}}{2} \cos[2\pi(f_c + f_{m1})t] + \frac{A_c A_{m2}}{2} \cos[2\pi(f_c + f_{m2})t]$$

Frequency Domain representation
(one sided spectrum)



SSB-SC - Band of frequencies :-

Let us consider modulating signal $m(t)$ contains band of frequencies such that ω maximum frequency f_m (Hz) or ω_m (rad/sec) as shown in the figure



Carrier signal $c(t) = \cos \omega_c t$;

The expression for SSB-SC signal can be written as

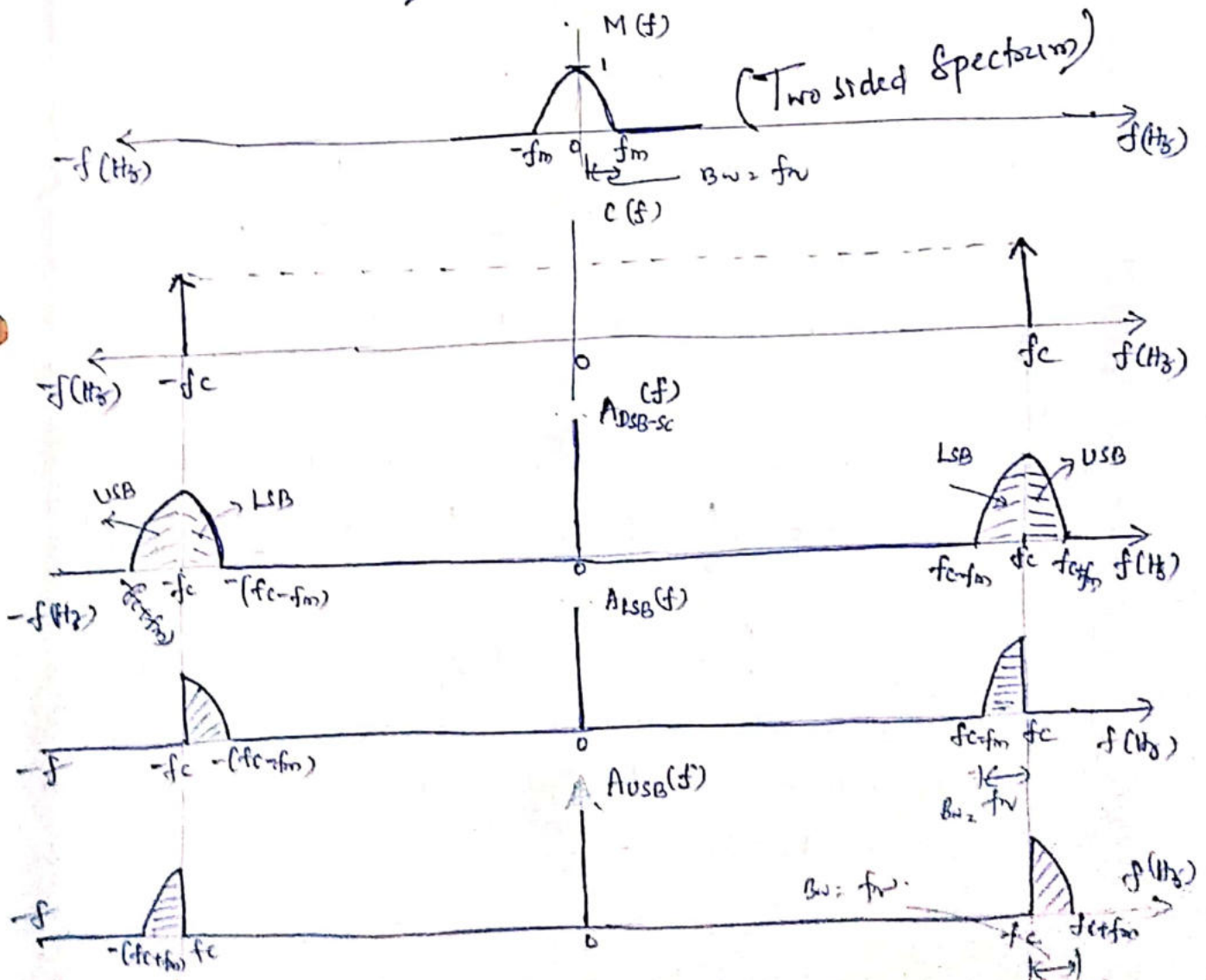
$$A_{SSB-SC}(t) = m(t) \cos \omega_c t \pm m_h(t) \sin \omega_c t$$

$$m_h(t) = HT[m(t)]$$

↑ All frequencies in $m_h(t)$ are 90° phase shifted w.r.t. $m(t)$

'+' → Indicates SSB-SC with LSB.

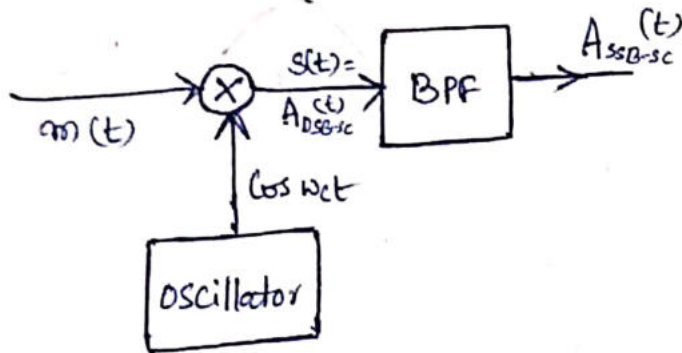
'-' → Indicates SSB-SC with USB.



Demodulation of SSB-sc Signal



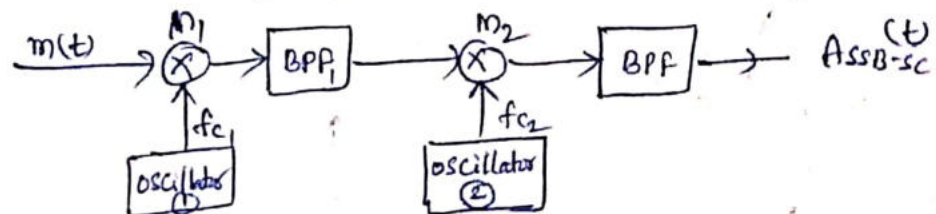
- ① Filter (or) Frequency Discrimination method.
- ② phase shift (or) phase Discrimination method.



Operation :-

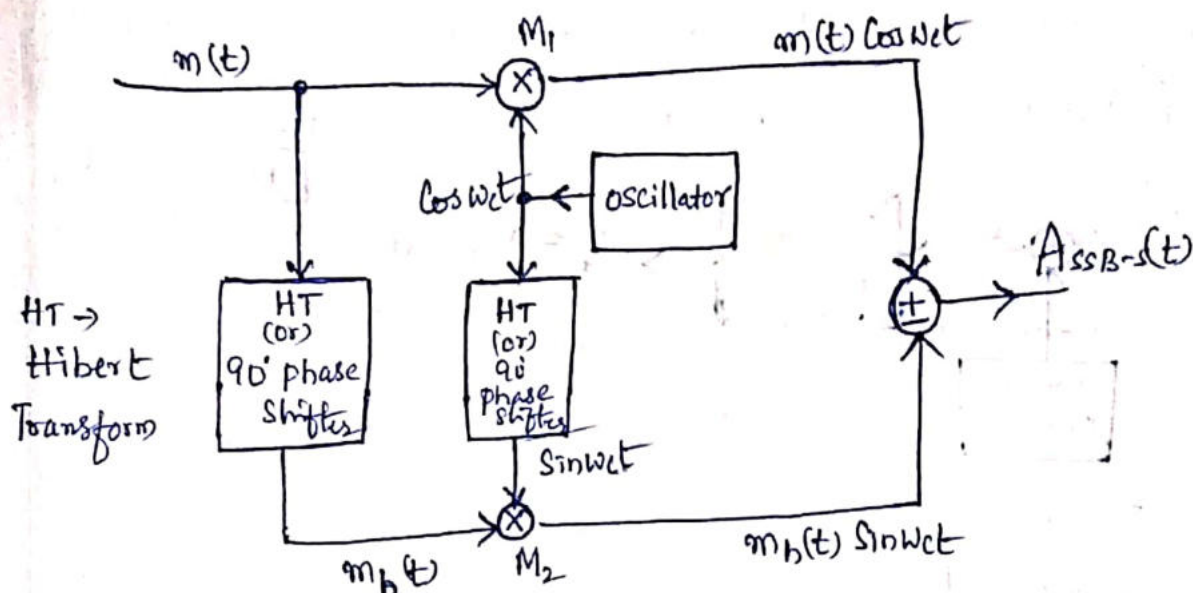
1. Block diagram of Filter (or) Frequency Discrimination method is shown in figure, which contains multiplier, oscillator & BPF.
2. multiplier generates DSB-sc signal by multiplying modulating signal $m(t)$ & Carrier signal $C(t)$.

$$A_{DSB-sc}(t) = m(t) \cos \omega ct.$$
3. DSB-sc signal contains both USB & LSB.
4. BPF is used to eliminate the either of side band.
5. It is very difficult to design BPF to eliminate one side band if frequency gap b/n USB & LSB is less.
6. In such cases two BPFs are used in cascade form as shown below



7. In the case of voice signal, the frequency gap b/n USB & LSB is 600Hz and it is easy to design BPF. Hence, SSB-sc is used to transmit voice signal.

② Phase Shift (or) phase Discrimination method :-



Operation :-

1. The block diagram of phase shift method is shown in figure which contains two multipliers, two HTs (or) 90° phase shifters, one oscillator, and adder.

2. Multipliers are used to generate DSB-SC signals.

$$M_1(t) = A_{DSB-SC}(t) = m(t) \cos wt$$

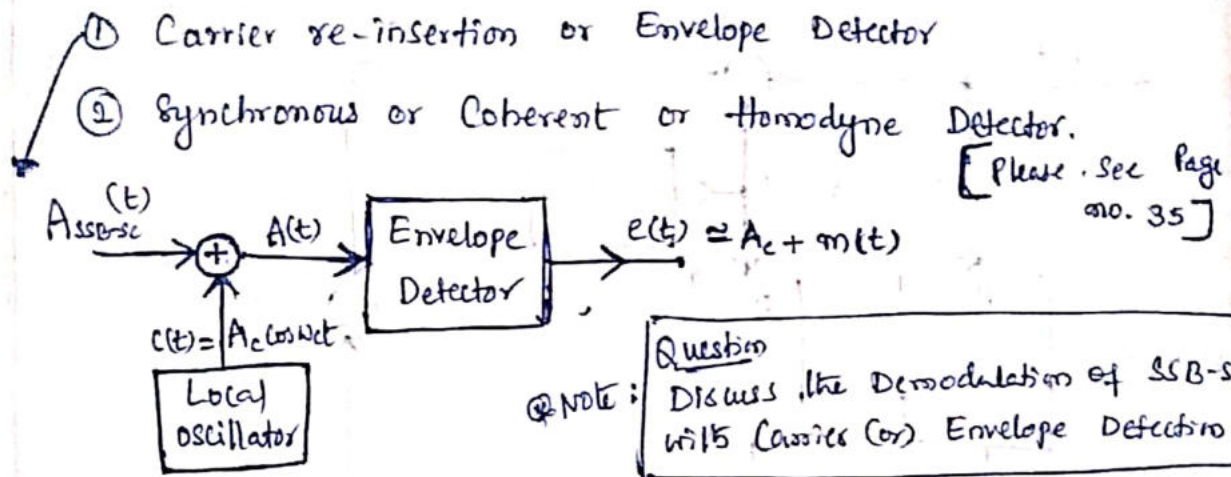
$$M_2(t) = A_{DSB-SC}(t) = m_h(t) \sin wt$$

③ Hilbert Transform Provides 90° phase shift to both modulating signal $m(t)$ & Carrier $c(t)$.

④ Two DSB-SC signals are added by the Adder to generate SSB-SC signal $A_{ssb-sc}(t) = m(t) \cos wt \pm m_h(t) \sin wt$

Drawbacks :-

1. Providing 90° phase shift for all frequencies in $m(t)$ is little bit difficult.



Operation :-

1. The block diagram of Carrier re-insertion or envelope detector is shown in figure which contains Adder, Local Oscillator & Envelope Detector.

2. Let SSB-sc signal $A_{ssb-sc}(t) = m(t) \cos wt \pm m_h(t) \sin wt$

O/p of Adder $A(t) = A_{ssb-sc}(t) + c(t)$

$$= m(t) \cos wt \pm m_h(t) \sin wt + A_c \cos wt$$

$$A(t) = [A_c + m(t)] \cos wt \pm m_h(t) \sin wt$$

Above eqⁿ can be represented in envelope form as

$$A(t) = e(t) \cos[wt + \theta(t)]$$

$\searrow$ Envelope.
 $\searrow$ phase

The Envelope $e(t) = \sqrt{[A_c + m(t)]^2 + m_h^2(t)}$

Phase $\theta(t) = \tan^{-1} \left[\frac{\pm m_h(t)}{A_c + m(t)} \right]$

Envelope detector extracts only envelope. i.e o/p of Envelope detector will be

$$e(t) = \sqrt{[A_c + m(t)]^2 + m_h^2(t)}$$

$$e(t) = \left[A_c^2 + m^2(t) + 2 A_c m(t) + m_h^2(t) \right]^{1/2}$$

$$= \left\{ A_c^2 \left[1 + \frac{m^2(t)}{A_c^2} + \frac{2 A_c m(t)}{A_c^2} + \frac{m_h^2(t)}{A_c^2} \right] \right\}^{1/2}$$

If A_c i.e. Peak amplitude of locally generated carrier is very high then A_c^2 is still very high. so that $\frac{m^2(t)}{A_c^2}$ & $\frac{m_h^2(t)}{A_c^2}$ can be negligible

$$\approx \left\{ A_c^2 \left[1 + \frac{2 m(t)}{A_c} \right] \right\}^{1/2}$$

$$\approx A_c \left[1 + \frac{2 m(t)}{A_c} \right]^{1/2}$$

$$\approx A_c \left[1 + \frac{2 m(t)}{A_c} \right]^{1/2}$$

$$(1+x)^{1/2} \approx 1 + x/2; \text{ if } x \ll 1$$

$$e(t) \approx A_c + m(t)$$

↓ DC Component
 ↓ modulating signal.

Vestigial Side Band Suppressed Carrier System :- VSB-SC (or) VSB or VSB-AM

If Carrier & Part of the either of side band is removed then it is called "VSB-SC" system or signal.

important points :-

1. It contains one full side band & Part of other side band.
2. Power efficiency is 100%.
3. Percentage of Power Saving in VSB-SC with respect to AM is full

b/w DSB-SC & SSB-SC is

$$\% PS_{DSB-SC} < \% PS_{VSB-SC} < \% PS_{SSB-SC}$$

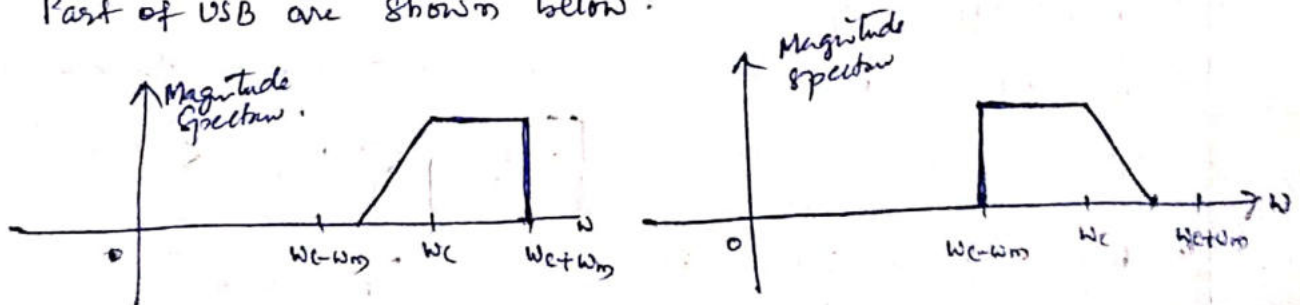
4. Band width of VSB-SC signal

$$W_m < BW_{VSB-SC} < 2W_m$$

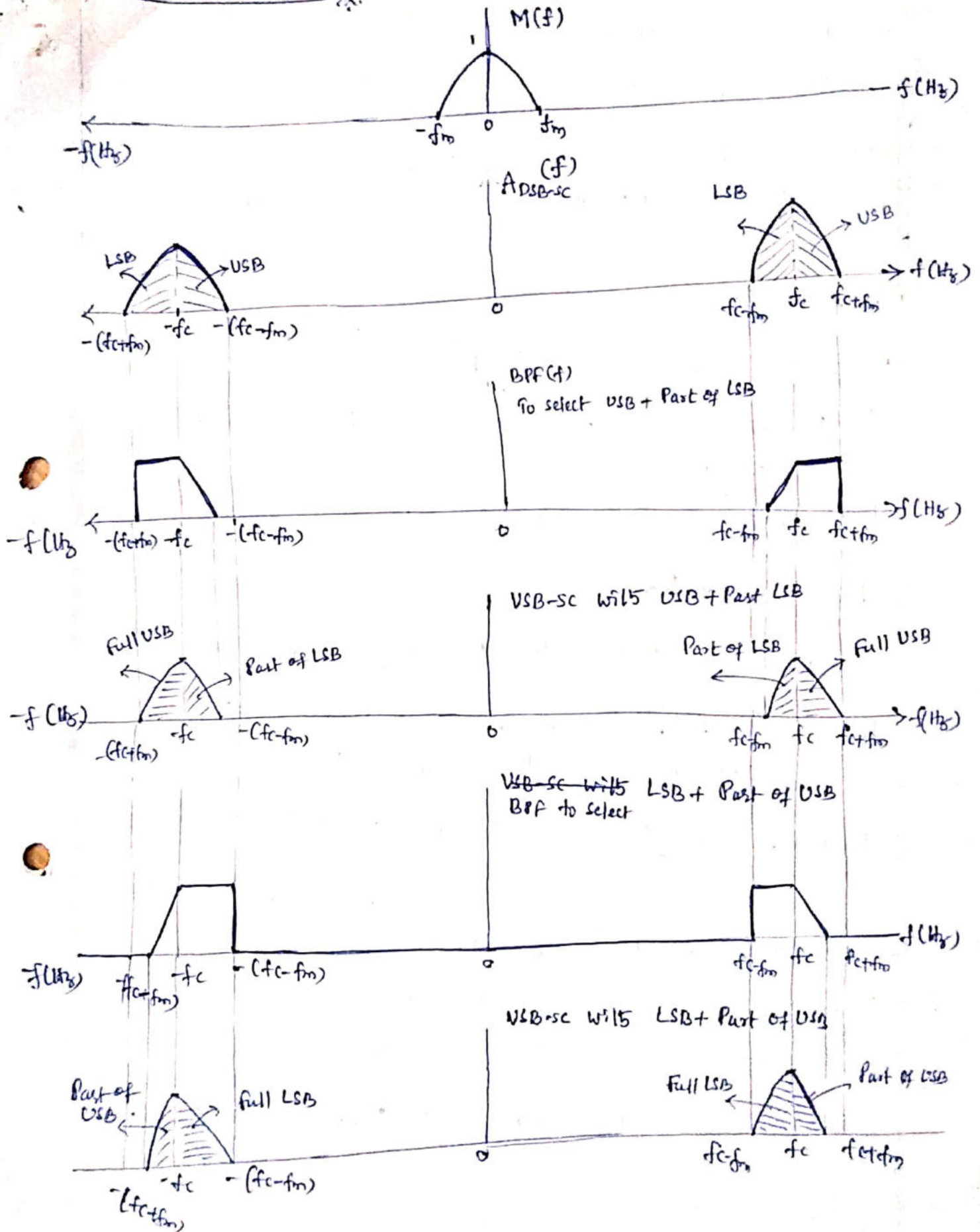
$$i.e. BW_{SS-SC} < BW_{VSB-SC} < BW_{DSB-SC}$$

* Explain about VSB-sc system (or) How VSB-sc system is suitable for Video Transmission?

1. Video signal Very Large bandwidth signal which contains (0-5) MHz. i.e. Very low frequencies & Very high frequencies.
- * 2. Very low frequency signal contains most of useful information.
3. Very low frequencies ~~rise~~ rise the side bands near to the carrier frequency, such that no frequency gap b/w USB & LSB.
4. To reduce the bandwidth of signal, either of one side band to be removed.
5. It is very difficult to remove one complete side band, Because design of BPF is very difficult.
6. Since low frequencies contain more useful information, Any attenuation on the side bands raised by these frequencies, will result phase distortion in the receiver o/p.
7. Therefore a compromise has been made to remove (or) suppress part of the other side band.
8. Now the modulated signal contains one full side band and part of other side band. Hence, This type of modulation is called "Vestigial side band Suppressed Carrier (VSB-sc)".
9. The spectrum of BPF to select USB + part of LSB and LSB + part of USB are shown below.



Frequency Domain representation of VSB-SC signal.



Generation of VSB-SC signal :-

- ① Filter or Frequency Discrimination method.
- ② phase shift or Phase Discrimination method.

Generation methods which are ~~are~~ discussed for the generation of SSB-SC signals are equally valid here with small modification.

① Filter method :-

modification: Page no. 31, 4th point

4. BPF filter is used to select one full side band & part of the other side band.

Points ~~4, 5, 6~~ 5, 6 & 7 are not required.

② Phase shift method :-

modification: Page no. 32, block diagram, phase shift is not equal to 90°

Point 2: $m_1(t) = A_{DSB-SC}(t) = m(t) \cos \omega_c t$ & $m_2(t) = A_{DSB-SC}(t) = m_q(t) \sin \omega_c t$

Point 3, ~~the~~ phase shift provided by the HT is not equal to 90°

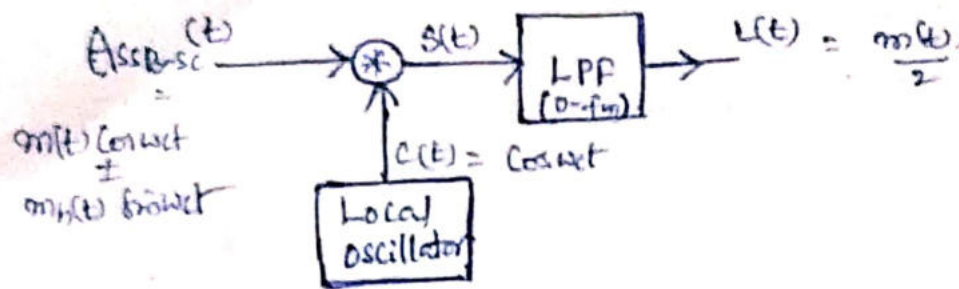
Point 4, $A_{VSB-SC}(t) = m(t) \cos \omega_c t \pm m_q(t) \sin \omega_c t$
 $\downarrow$
 phase shift is not 90°

Demodulation of VSB-SC signal :-

- ① Carrier re-insertion (or) Envelope detector
- ② Synchronous or Coherent or Homodyne detector.

Modifications :- replace $A_{DSB-SC}(t) = A_{VSB-SC}(t)$, $m_h(t) = m_q(t)$.

① Synchronous or Coherent or Homodyne detection of SSB-SC signal. (35)



Operation :-

① The block diagram of synchronous (or) Coherent detector is shown in figure which contains Local oscillator, multiplier, followed by LPF.

2. Incoming SSB-SC signal & locally generated carrier are multiplied by the multiplier.

$$S(t) = A_{ssb-sc}(t) \times c(t).$$

$$= [m(t) \cos \omega_c t \pm m_h(t) \sin \omega_c t] \times \cos \omega_c t$$

$$= m(t) \cos^2 \omega_c t \pm \frac{m_h(t)}{2} \cdot 2 \sin \omega_c t \cos \omega_c t$$

$$= \frac{m(t)}{2} + \frac{m(t)}{2} \cancel{\cos 2\omega_c t} \pm \frac{m_h(t)}{2} \cancel{\sin 2\omega_c t}$$

LPF eliminates high frequencies.

$$\boxed{L(t) = \frac{m(t)}{2}}$$

Phase & Frequency Errors :-

Let us consider locally generated carrier has phase & frequency errors with respect to transmitted carrier.

$$c(t) = \cos(\omega_c t + \Delta \omega t + \phi)$$

$\rightarrow$ phase error
 $\rightarrow$ frequency error

output of the modulator

$$s(t) = A_{\text{SSB-SC}}(t) \times c(t)$$

$$= [m(t) \cos \omega_c t \pm m_h(t) \sin \omega_c t] \cos(\omega_c t + 4\omega t + \phi)$$

$$= \frac{m(t)}{2} \cdot 2 \cos(\omega_c t + 4\omega t + \phi) \pm \frac{m_h(t)}{2} \cdot 2 \sin \omega_c t \cos(\omega_c t + 4\omega t + \phi)$$

$$= \frac{m(t)}{2} \left[\cos(2\omega_c t + 4\omega t + \phi) + \cos(4\omega t + \phi) \right] \pm \frac{m_h(t)}{2} \left[\sin(2\omega_c t + 4\omega t + \phi) + \sin(4\omega t + \phi) \right]$$

High frequencies will be eliminated by the LPF.

$$L(t) = \frac{m(t)}{2} \cos(4\omega t + \phi) \pm \frac{m_h(t)}{2} \sin(4\omega t + \phi)$$

Case (i): - If no phase & frequency error then $4\omega t = \phi = 0$.

o/p of LPF is $L(t) = \frac{m(t)}{2}$

Case (ii): - If only frequency error then $4\omega \neq 0$ & $\phi = 0$.

o/p of LPF is $L(t) = \frac{m(t)}{2} \cos(4\omega t) \pm \frac{m_h(t)}{2} \sin(4\omega t)$

The frequency of $m(t)$ & $m_h(t)$ are slightly changes due to $\cos(4\omega t)$ and $\sin(4\omega t)$. This is called "beat or Warble" effect.

Case (iii): - If only phase error then $4\omega = 0$ & $\phi \neq 0$.

o/p of LPF is $L(t) = \frac{m(t)}{2} \cos \phi \pm \frac{m_h(t)}{2} \sin \phi$.

The phase difference b/w incoming SSB-SC signal & locally generated carrier generates error except $\phi = 0$.

This is called "Quadrature null" (or) "Donald Duck voice" effect.

UNIT-II

(2)

(1)

ANGLE MODULATION.

Angle Modulation:- It is a Process of Varying angle of the Carrier ~~is varied~~ in accordance with the instantaneous values of the modulating signal.

Angle can be varied either by Varying frequency or phase. Hence Angle modulation is two types.

(1) Frequency Modulation.

(2) Phase Modulation.

(1) Frequency Modulation:-

It is a Process of Varying Frequency of the Carrier in accordance with the instantaneous values of the modulating signal.

(2) Phase Modulation:-

It is a Process of Varying Phase of the Carrier in accordance with the instantaneous values of the modulating signal.

Relation b/n angle & Frequency:-

Let us consider Carrier signal $C(t) = A_c \cos(\omega_c t + \phi)$
 $= A_c \cos(2\pi f_c t + \phi)$
 where $\omega_c \rightarrow$ Carrier frequency
 $\phi \rightarrow$ angle or phase.

$$C(t) = A_c \cos[\psi(t)]$$

Where Total phase angle $\psi(t) = \omega_c t + \phi$.

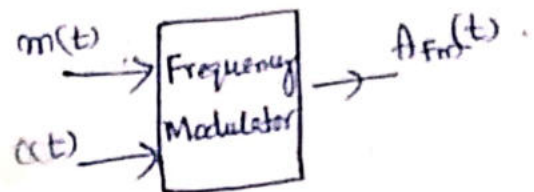
$$\frac{d}{dt} \psi(t) = \omega_c$$

i.e. Frequency can be obtained by derivating angle.

$$\int \omega_c dt = \omega_c t$$

i.e. angle can be obtained by integrating frequency.

Frequency Modulation :-



Frequency Modulator Converts input voltage into frequency i.e. if the amplitude of the modulating signal $m(t)$ changes the frequency at the output. Hence Frequency Modulator is called voltage to frequency converter.

Let us consider Carrier signal $C(t) = A_c \cos \omega_c t$.

The frequency Variation at the output is called instantaneous frequency, which can be expressed as

$$\omega_i = \omega_c + K_f m(t)$$

Where $K_f =$ Frequency Sensitivity factor $\frac{\text{rad/sec (or) Hz}}{\text{volt}}$ (or) $\frac{\text{cycles/sec}}{\text{volt}}$

The angle of the Carrier, After modulation can be written as $\psi_i(t) = \int \omega_i dt = \int (\omega_c + K_f m(t)) dt$

$$\varphi_i(t) = \omega_c t + K_f \int m(t) dt.$$

Frequency modulated signal can be written as

$$A_{FM}(t) = A_c \cos[\varphi_i(t)]$$

$$A_{FM}(t) = A_c \cos[\omega_c t + K_f \int m(t) dt]$$

Frequency deviation in Frequency Modulation :-

The Instantaneous frequency

$$\omega_i = \omega_c + K_f m(t).$$

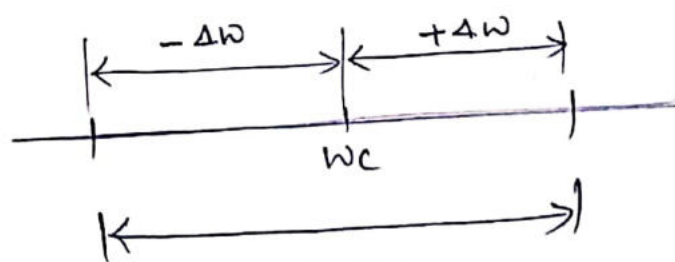
$$= \omega_c + \Delta\omega \quad \rightarrow \text{frequency deviation.}$$

$\Delta\omega = K_f m(t)$, it may be +ve or -ve, that depends on the sign of $m(t)$. Hence, frequency deviation

$$\Delta\omega = \pm K_f m(t).$$

$$\omega_i = \omega_c \pm \Delta\omega$$

And deviation can be represented as shown below



$2. \Delta\omega \rightarrow$ Total frequency Deviation.

Phase Modulation :-

Let us consider modulating signal $m(t)$ & carrier signal $c(t) = A_c \cos \omega_c t$

Phase modulated signal

$$A_{PM}(t) = A_c \cos[\varphi_i(t)]$$

Where $\varphi_i(t) = \omega_c t + K_p m(t)$.

Where $K_p =$ phase sensitivity factor $\left(\frac{\text{rad}}{\text{volt}}\right)$

$$A_{PM}(t) = A_c \cos[\omega_c t + K_p m(t)]$$

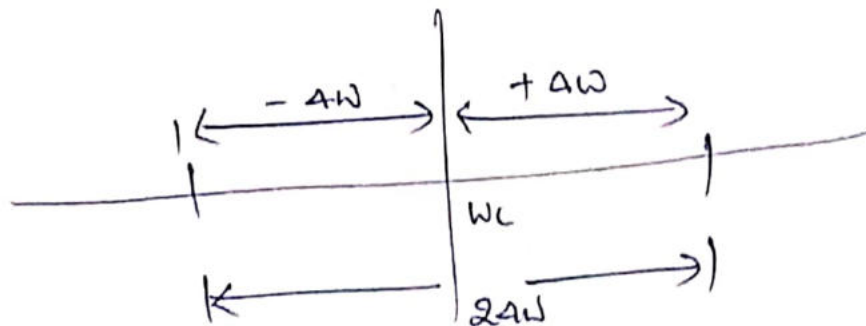
Frequency Deviation in phase modulation:-

$$\omega_i = \frac{d}{dt} \varphi_i(t) = \frac{d}{dt} (\omega_c t + K_p m(t))$$

$$= \omega_c + K_p \frac{d}{dt} m(t)$$

$$= \omega_c + \Delta\omega$$

Frequency Deviation $\Delta\omega = \pm K_p \frac{d}{dt} m(t)$.



→ Total frequency Deviation.

FM - Single tone Modulating Signal:-

Let us consider Carrier $c(t) = A_c \cos \omega_c t$ &

modulating signal $m(t) = A_m \cos \omega_m t$; $A_c \gg A_m$ & $\omega_c \gg \omega_m$.

General form of expression

$$A_{FM}(t) = A_c \cos[\omega_c t + K_f \int m(t) dt]$$

$$\begin{aligned}
 A_{FM}(t) &= A_c \cos \left[\omega_c t + K_f \int A_m \cos \omega_m t \, dt \right] \\
 &= A_c \cos \left[\omega_c t + K_f A_m \frac{\sin \omega_m t}{\omega_m} \right] \\
 &= A_c \cos \left[\omega_c t + \frac{K_f A_m}{\omega_m} \sin \omega_m t \right] \\
 &= A_c \cos \left[\omega_c t + \frac{\Delta \omega}{\omega_m} \sin \omega_m t \right]
 \end{aligned}$$

Where $\Delta \omega = \text{Frequency Deviation} = K_f A_m$

$m_f = \text{Modulation Index} = \frac{\Delta \omega}{\omega_m} = \frac{\text{Frequency deviation}}{\text{max. freq. of } m(t)}$

$$A_{FM}(t) = A_c \cos \left[\omega_c t + m_f \sin \omega_m t \right]$$

Case (i) :- If $m_f \ll 1$ then frequency deviation is less or narrow. Hence, it is called "Narrow Band Frequency Modulation (NBFM)".

Case (ii) :- If $m_f \gg 1$ then frequency deviation is high (or) wider. Hence, it is called "Wide Band Frequency Modulation (WBFM)".

Narrow Band Frequency Modulation (NBFM) :-

General form of FM signal

$$A_{FM}(t) = A_c \cos \left[\omega_c t + K_f \int m(t) \, dt \right]$$

Above signal can be expressed in phasor form as

$$\tilde{A}_{FM}(t) = A_c \cdot e^{j \left[\omega_c t + K_f \int m(t) \, dt \right]}$$

$$A_{FM}(t) = \text{Re} \left[X_{FM}(t) \right]$$

$\uparrow$ Original form $\uparrow$ phasor form.

$$\tilde{A}_{FM}(t) = A_c \cdot e^{j\omega_c t} \cdot e^{j k_f \int m(t) dt} \quad \text{--- (A)}$$

$$e^{j\omega_c t} = \cos \omega_c t + j \sin \omega_c t \quad \text{--- (1)}$$

$$e^{j k_f \int m(t) dt} = \cos \left[k_f \int m(t) dt \right] + j \sin \left[k_f \int m(t) dt \right]$$

if $k_f \ll 1$ then (NBFM)

$$\cos \left[k_f \int m(t) dt \right] \approx 1 ;$$

$$\sin \left[k_f \int m(t) dt \right] \approx k_f \int m(t) dt ;$$

if θ is very small then
 $\cos \theta \approx 1$
 $\sin \theta \approx \theta$

$$e^{j k_f \int m(t) dt} = \left[1 + j k_f \int m(t) dt \right] \quad \text{--- (2)}$$

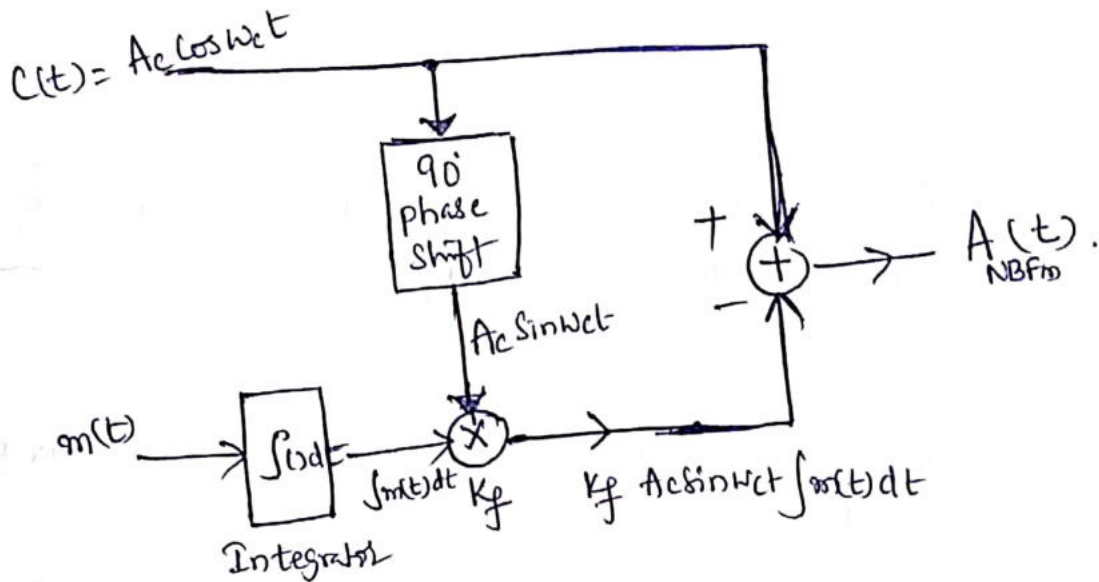
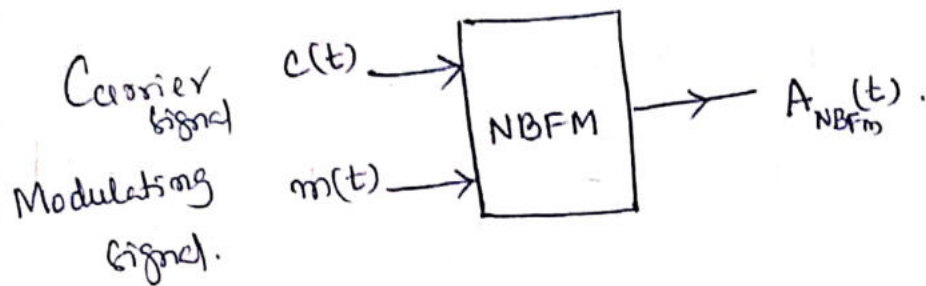
Put eqn. (1) & (2) in eqn. (A).

$$\begin{aligned} \tilde{A}_{FM}(t) &= \tilde{A}_{NBFM}(t) = A_c \cdot [\cos \omega_c t + j \sin \omega_c t] [1 + j k_f \int m(t) dt] \\ &= A_c \cos \omega_c t + j A_c k_f \int m(t) dt + j A_c \sin \omega_c t - A_c \sin \omega_c t \cdot k_f \int m(t) dt \\ &= A_c \cos \omega_c t - k_f A_c \sin \omega_c t \int m(t) dt + j A_c k_f \int m(t) dt + j A_c \sin \omega_c t. \end{aligned}$$

$$A_{NBFM}(t) = \text{Re} \left[\tilde{A}_{NBFM}(t) \right]$$

$$A_{NBFM}(t) = A_c \cos \omega_c t + k_f A_c \sin \omega_c t \int m(t) dt$$

Above eqn. Can be expressed using Block diagrams as.



NBFM - single tone modulating signal :-

Carrier signal $c(t) = A_c \cos w_c t$
modulating signal $m(t) = A_m \cos w_m t$; $A_c \gg A_m$ & $w_c \gg w_m$.

$$\begin{aligned} A_{NBFM}(t) &= A_c \cos w_c t - K_f A_c \sin w_c t \int m(t) dt \\ &= A_c \cos w_c t - K_f A_c \sin w_c t \int A_m \cos w_m t dt \\ &= A_c \cos w_c t - K_f A_c \sin w_c t \cdot A_m \frac{\sin w_m t}{w_m} \\ &= A_c \cos w_c t - \frac{K_f A_m}{w_m} \cdot A_c \sin w_c t \sin w_m t \end{aligned}$$

modulation Index $m_f = \frac{K_f A_m}{w_m} = \frac{\Delta w}{w_m}$

$$A_{NBFM}(t) = A_c \cos w_c t - \frac{m_f A_c}{2} (\sin w_c t \sin w_m t)$$

~~2 sin A cos B~~ $2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

$$A_{\text{NBFM}}(t) = A_c \cos \omega_c t - \frac{m_f A_c}{2} [\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$$

$$A_{\text{NBFM}}(t) = A_c \cos \omega_c t + \frac{m_f A_c}{2} [-\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t]$$

↓ Unmodulated Carrier
↓ LSB
↓ USB

For same Carrier & modulating signal.

$$C(t) = A_c \cos \omega_c t \quad \& \quad m(t) = A_m \cos \omega_m t$$

Am Signal

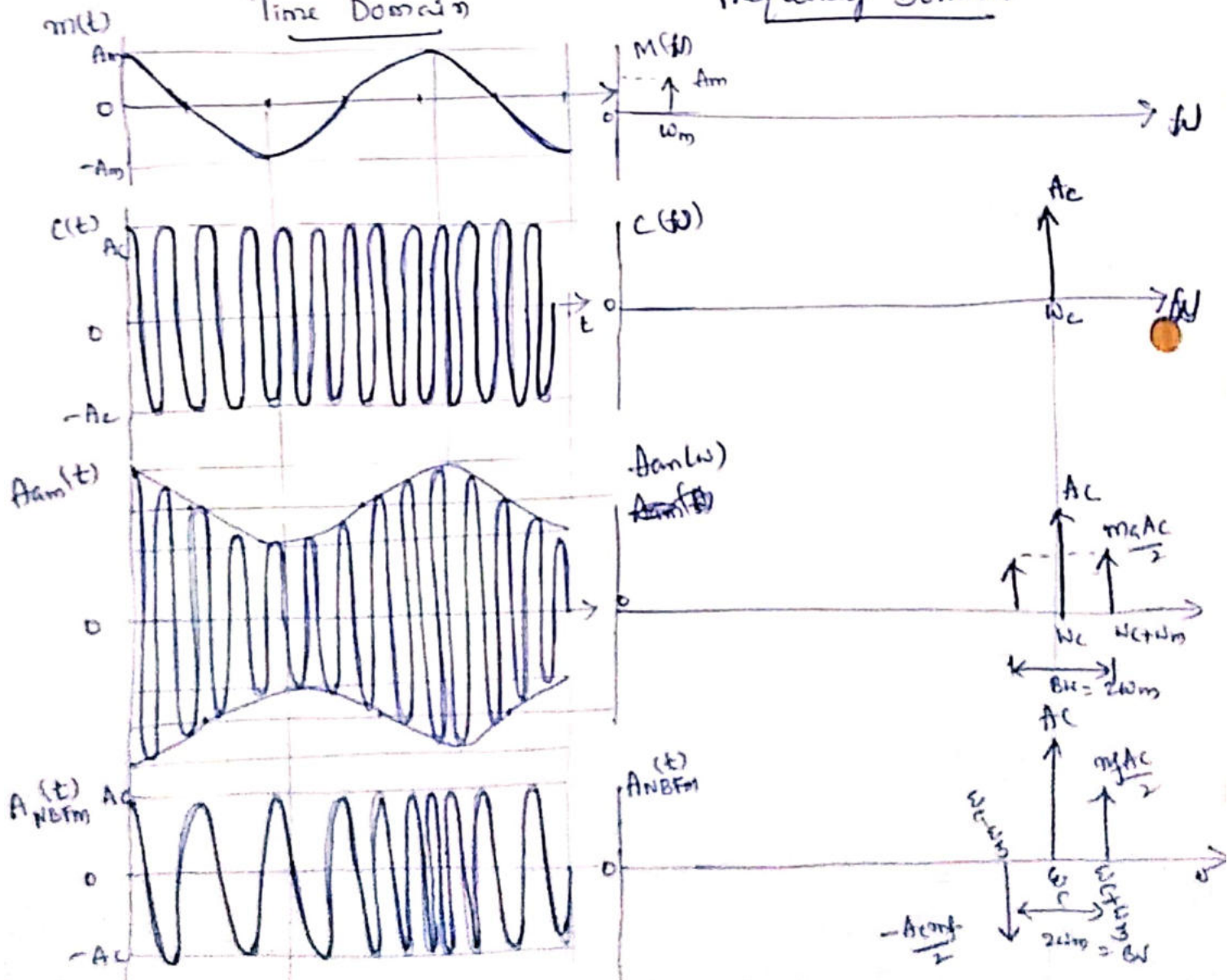
$$A_m(t) = A_c \cos \omega_c t + \frac{m_a A_c}{2} [\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t]$$

↓ Unmodulated Carrier
↓ LSB
↓ USB

For same Carrier & modulating signals, the expression for AM & NBFM are same except LSB is reversed (or) -ve sign with respect to AM signal.

Time Domain

Frequency Domain



Amplitude of the Carrier is Varied in Am & Frequency of the Carrier is Varied in NBfm. Why?

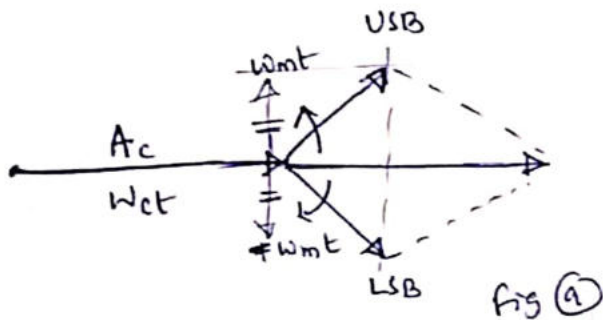
Let us consider Carrier signal $c(t) = A_c \cos \omega_c t$ & modulating signal $m(t) = A_m \cos \omega_m t$; $A_c \gg A_m$ & $\omega_c \gg \omega_m$.

$$A_{NBfm}(t) = A_c \cos \omega_c t + m_f \frac{A_c}{2} \left[-\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t \right]$$

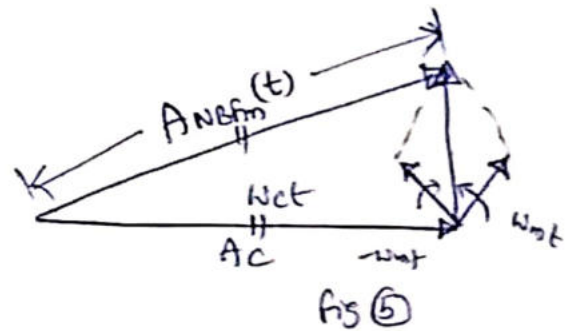
$\downarrow$ Unmodulated Carrier $\downarrow$ LSB $\downarrow$ USB

$$A_{Am}(t) = A_c \cos \omega_c t + m_a \frac{A_c}{2} \left[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t \right]$$

Am - Phasor (or) Vector Diagram



NBfm - Phasor or Vector Diagram



$\leftarrow A_{Am}(t) \rightarrow$

1. Phasor or Vector diagrams of Am & NBfm signal are shown in figure (A) & figure (B) respectively.
2. In Am, Vertical Components of LSB & USB Cancell each other and horizontal Components are added. So that the resultant magnitude is $A_{Am}(t)$ which varies with respect to time. Hence amplitude of the Carrier is Varied.
3. In NBfm, LSB is 180° phase reversed, Horizontal Components are Cancell each other and Vertical Components LSB & USB are added. So that $A_{NBfm}(t) \approx A_c$. Hence amplitude of the Carrier is Constant.

NBfm - Multitone Modulating signal, -

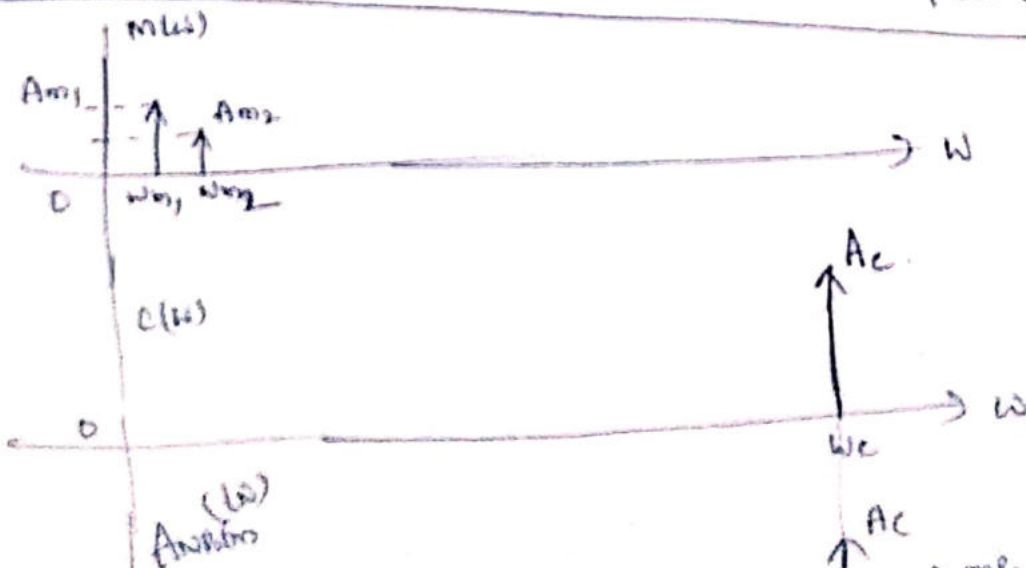
Carrier $c(t) = A_c \cos \omega_c t$
 multitone modulating signal $m(t) = A_{m1} \cos \omega_{m1} t + A_{m2} \cos \omega_{m2} t$

$$A_c \gg A_{m1} \text{ \& \& } \omega_c \gg \omega_{m1}$$

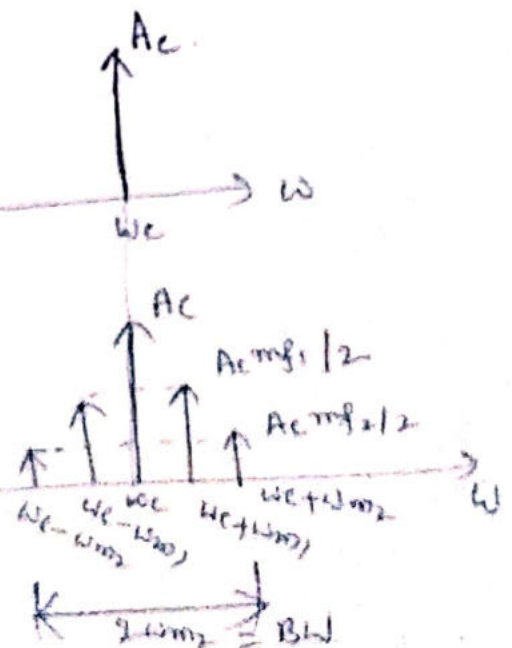
General Expression for NBfm is

$$\begin{aligned} A_{NBfm}(t) &= A_c \cos \omega_c t - k_f A_c \sin \omega_c t \int m(t) dt \\ &= A_c \cos \omega_c t - k_f A_c \sin \omega_c t \int (A_{m1} \cos \omega_{m1} t + A_{m2} \cos \omega_{m2} t) dt \\ &= A_c \cos \omega_c t - k_f A_c \sin \omega_c t \left[\frac{A_{m1} \sin \omega_{m1} t}{\omega_{m1}} + \frac{A_{m2} \sin \omega_{m2} t}{\omega_{m2}} \right] \\ &= A_c \cos \omega_c t - \frac{k_f A_{m1}}{\omega_{m1}} A_c \sin \omega_c t \sin \omega_{m1} t - \frac{k_f A_{m2}}{\omega_{m2}} A_c \sin \omega_c t \sin \omega_{m2} t \\ &= A_c \cos \omega_c t - \frac{m_{f1} A_c}{2} \sin \omega_c t \sin \omega_{m1} t - \frac{m_{f2} A_c}{2} \sin \omega_c t \sin \omega_{m2} t \\ m_{f1} &= \frac{k_f A_{m1}}{\omega_{m1}} = \frac{\Delta \omega_1}{\omega_{m1}} \text{ \& \& } m_{f2} = \frac{k_f A_{m2}}{\omega_{m2}} = \frac{\Delta \omega_2}{\omega_{m2}} \end{aligned}$$

$$A_{NBfm}(t) = A_c \cos \omega_c t + \frac{m_{f1} A_c}{2} [-\cos(\omega_c - \omega_{m1})t + \cos(\omega_c + \omega_{m1})t] + \frac{m_{f2} A_c}{2} [-\cos(\omega_c - \omega_{m2})t + \cos(\omega_c + \omega_{m2})t]$$



 NBfm is Linear Modulation Technique



Wide Band Frequency Modulation (WBFM) - Single tone :-

Let us consider Carrier signal $c(t) = A_c \cos \omega_c t$

modulating signal $m(t) = A_m \cos \omega_m t$

$$A_c \gg A_m \quad \& \quad \omega_c \gg \omega_m$$

General form of Expression.

$$\begin{aligned} A_{FM}(t) &= A_c \cos[\omega_c t + k_f \int m(t) dt] \\ &= A_c \cos[\omega_c t + k_f \int A_m \cos \omega_m t dt] \\ &= A_c \cos[\omega_c t + \frac{k_f A_m}{\omega_m} \sin \omega_m t] \\ &= A_c \cos[\omega_c t + \text{mf} \sin \omega_m t] \end{aligned}$$

phasor (or) vector form

$$\tilde{A}_{FM}(t) = A_c \cdot e^{j[\omega_c t + \text{mf} \sin \omega_m t]}$$

$$\tilde{A}_{FM}(t) = A_c \cdot e^{j\omega_c t} \cdot e^{j\text{mf} \sin \omega_m t} \quad \text{--- (A)}$$

$$e^{j\omega_c t} = \cos \omega_c t + j \sin \omega_c t \quad \text{--- (1)}$$

$e^{j\text{mf} \sin \omega_m t}$ is approximately Periodic signal with line

Period $T_m = \frac{1}{f_m} = \frac{2\pi}{\omega_m}$. And this signal can be

represented using Exponential Fourier Series as

$$e^{j\text{mf} \sin \omega_m t} = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega_m t} \quad \text{--- (2)}$$

$$C_n = \frac{1}{T_m} \int_{t=-\frac{T_m}{2}}^{+\frac{T_m}{2}} e^{j\text{mf} \sin \omega_m t} \cdot e^{-jn\omega_m t} dt$$

Exponential FS

$$f(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega_m t}$$

$$C_n = \frac{1}{T} \int_{t=-\frac{T}{2}}^{+\frac{T}{2}} f(t) e^{-jn\omega_m t} dt$$

$$C_n = \frac{1}{T_m} \int_{t=-\pi/\omega_m}^{+\pi/\omega_m} e^{j m f \sin \omega_m t} e^{-j n \omega_m t} dt$$

let $\omega_m t = x$

$$\omega_m dt = dx \Rightarrow dt = \frac{dx}{\omega_m}$$

if $t = -\pi/\omega_m$ then $x = \omega_m (-\pi/\omega_m) = -\pi$

$t = +\pi/\omega_m$ then $x = \omega_m (+\pi/\omega_m) = +\pi$

$$C_n = \frac{\omega_m}{2\pi} \int_{x=-\pi}^{+\pi} e^{j m f \sin x} e^{-j n x} \frac{dx}{\omega_m}$$

$$C_n = \frac{1}{2\pi} \int_{x=-\pi}^{+\pi} e^{j (m f \sin x - n x)} dx$$

Bessel function.

$$C_n = J_n(mf).$$

$$(2) \Rightarrow e^{j m f \sin \omega_m t} = \sum_{n=-\infty}^{+\infty} J_n(mf) e^{j n \omega_m t} \quad (3)$$

$$(3) \Rightarrow \tilde{A}_{FM}(t) = \tilde{A}_{WBFM}(t) = A_c [\cos \omega_c t + j \sin \omega_c t] \sum_{n=-\infty}^{+\infty} J_n(mf) e^{j n \omega_m t}$$

$$= A_c [\cos \omega_c t + j \sin \omega_c t] \left[\sum_{n=-\infty}^{+\infty} J_n(mf) (\cos n \omega_m t + j \sin n \omega_m t) \right]$$

Real part of Above by w.

$$A_{WBFM}(t) = \text{Re}[\tilde{A}_{WBFM}(t)] = \sum_{n=-\infty}^{+\infty} A_c \cos(\omega_c + n \omega_m) t$$

Properties of $J_n(m_f)$:-

$$1. J_n(m_f) = J_{-n}(m_f) \quad \text{if 'n' is even.}$$

$$= -J_n(m_f) \quad \text{if 'n' is odd.}$$

2. If $m_f \ll 1$ then

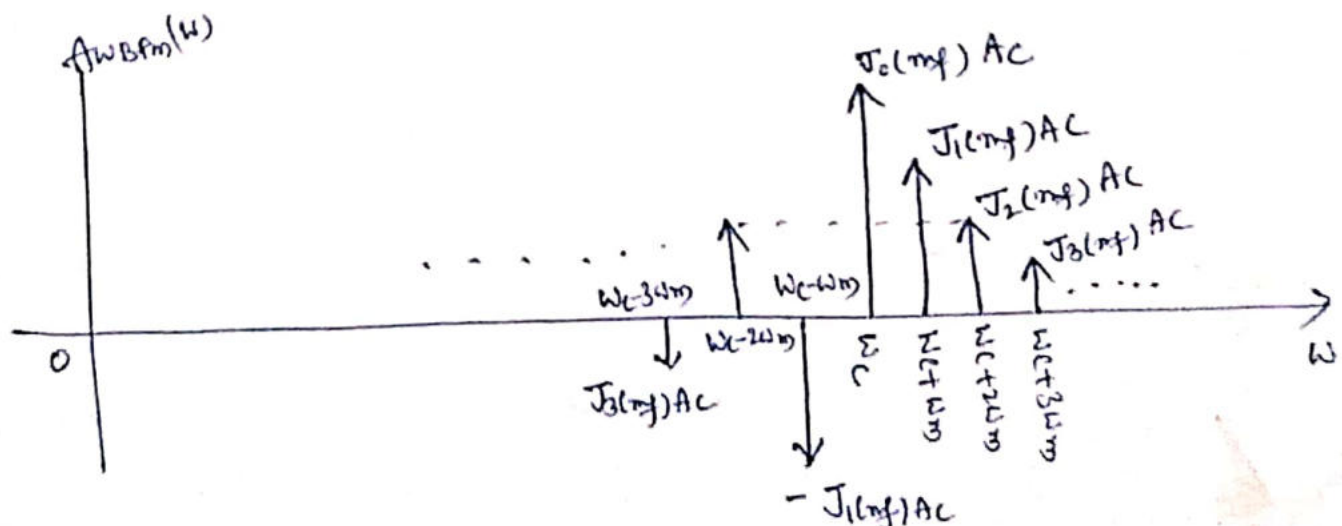
$$J_0(m_f) \approx 1 \quad \& \quad J_1(m_f) \approx m_f/2$$

$$3. \sum_{n=-\infty}^{+\infty} J_n^2(m_f) = 1$$

• WBFm signal can be expanded using Property (1).

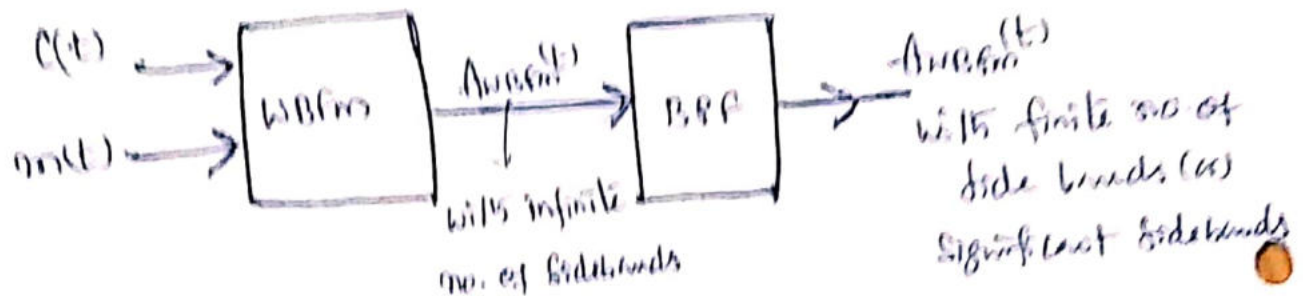
$$A_{WBFm}(t) = A_c \left[J_0(m_f) \cos \omega_c t + J_1(m_f) \cos (\omega_c + \omega_m)t + J_{-1}(m_f) \overset{\cos(\omega_c - \omega_m)t}{\cos (\omega_c - \omega_m)t} \right. \\ \left. + J_2(m_f) \cos (\omega_c + 2\omega_m)t + J_{-2}(m_f) \cos (\omega_c - 2\omega_m)t + \right. \\ \left. J_3(m_f) \cos (\omega_c + 3\omega_m)t + J_{-3}(m_f) \cos (\omega_c - 3\omega_m)t + \dots \right]$$

$$A_{WBFm}(t) = A_c J_0(m_f) \cos \omega_c t + A_c J_1(m_f) \cos (\omega_c + \omega_m)t - J_1(m_f) A_c \cos (\omega_c - \omega_m)t \\ + J_2(m_f) A_c \cos (\omega_c + 2\omega_m)t + J_{-2}(m_f) A_c \cos (\omega_c - 2\omega_m)t + \\ J_3(m_f) A_c \cos (\omega_c + 3\omega_m)t - J_{-3}(m_f) A_c \cos (\omega_c - 3\omega_m)t + \dots$$



Important Points:

1. The magnitude or Amplitude of the carrier is the A_c .
2. WBFM signal contains infinite no. of side bands which are separated by ' f_m ' as shown in figure.
3. The frequency BW of WBFM signal is infinite.
4. The infinite BW can be converted into finite by removing insignificant side bands.
5. If the magnitude or amplitude of side bands are less than 1% of carrier amplitude then it is called insignificant side band.
6. BPF is used to remove the insignificant side band as shown below



7. If $m_f \ll 1$ then $J_0(m_f) \approx 1$ & $J_1(m_f) \approx m_f/2$ and $J_2(m_f) \approx J_3(m_f) \approx J_4(m_f) \approx \dots \approx J_n(m_f) \approx 0$.

$$\begin{aligned}
 A_{WBFM}(t) &= A_c \cos \omega_c t - \frac{m_f A_c}{2} \cos (\omega_c - \omega_m) t + \frac{m_f A_c}{2} \cos (\omega_c + \omega_m) t \\
 &= A_{WBFM}(t)
 \end{aligned}$$

i.e. if $m_f \ll 1$ then WBFM is equal to NBFM.

8. The amplitude of FM remains constant, the Power of the FM signal is same as Unmodulated Carrier Power

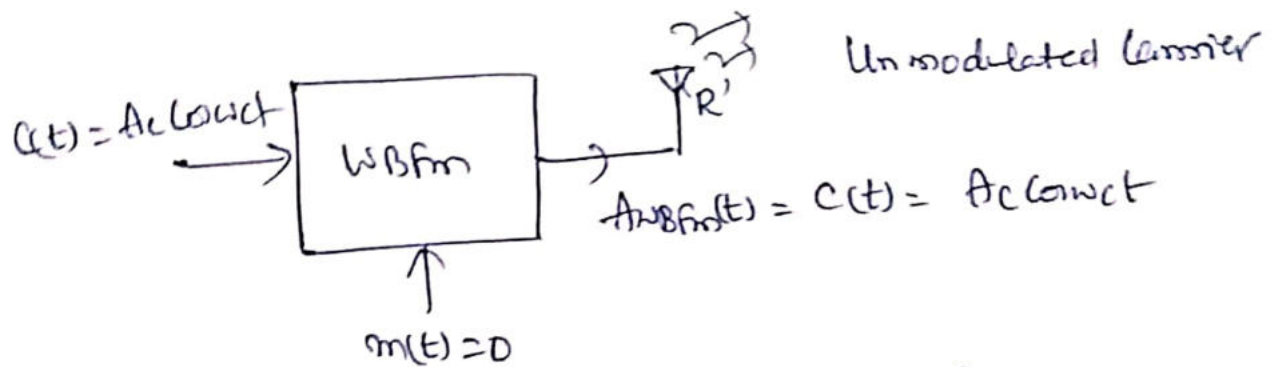
i.e Total Power $P_t = P_c$ (Carrier Power)

Proof:-

$$A_{WBfm}(t) = A_c \cos [\omega_c t + k_f \int m(t) dt]$$

Case (i) :- If modulating signal $m(t) = 0$ then o/p

WBfm modulator $A_{WBfm}(t) = A_c \cos \omega_c t = c(t)$.

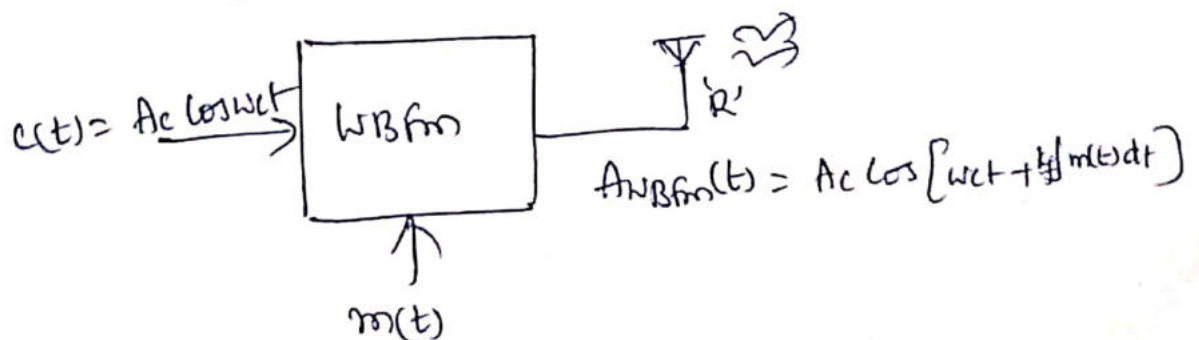


Total Power at the o/p $P_t = \frac{A_c^2}{R}$ (Peak Power)

$$= \frac{(A_c / \sqrt{2})^2}{R} \text{ (rms Power)}$$

$$P_t = \frac{A_c^2}{2R} = P_c$$

Case (ii) :- If modulating signal $m(t) \neq 0$ then



$$A_m(t) = \sum_{n=-\infty}^{+\infty} A_c J_n(mf) \cos((\omega_c + n\omega_m)t)$$

$$\text{Total Power } P_t = \left[\sum_{n=-\infty}^{+\infty} A_c J_n(mf) \right]^2 / R$$

Peak Power

$$P_t = \sum_{n=-\infty}^{+\infty} \left(A_c \frac{J_n(mf)}{\sqrt{2}} \right)^2 / R \quad (\text{rms Power})$$

$$= \sum_{n=-\infty}^{+\infty} \frac{A_c^2 J_n^2(mf)}{2R}$$

$$\sum_{n=-\infty}^{+\infty} J_n^2(mf) = 1 \rightarrow 3^{\text{rd}} \text{ Property}$$

$$P_t = \frac{A_c^2}{2R} = P_c$$

9. The following values are fixed by the CCIR (Consultative Committee for International Radio) or ITU (International Telecommunication Union)

i) Max. frequency deviation $\Delta f = \pm 75 \text{ kHz}$

ii) Max. Allowable BW $= 200 \text{ kHz}$

iii) max. ~~Carrier~~ Carrier frequency drift $= \pm 2 \text{ kHz}$

Comparison b/w Am & Fm :-

Amplitude Modulation

- ① Amplitude of the Carrier is varied with respect to modulating signal $m(t)$.
- ② Modulation Index

$$m_a = \frac{A_m}{A_c}$$
- ③ Modulation Index Varies b/w 0 & 1.
 i.e. $0 \leq m_a \leq 1$
- ④ Total Power is function of modulation Index.

$$P_t = P_c [1 + m_a^2/2]$$
- ⑤ Bandwidth is independent of Modulation Index.
 i.e. $BW_{Am} = 2f_m$ (or) $2W_m$
- ⑥ It contains only two side-bands.
- ⑦ It is Linear modulation Tech.
- ⑧ SNR is low
- ⑨ It covers more (or) large area
- ⑩ It is used in Ionosphere Propagation
- ⑪ Frequency re use is not possible
- ⑫ Less Cost & Circuit Complexity

Frequency Modulation (WBfm)

- ① Frequency of the Carrier is varied with respect to modulating signal $m(t)$.
- ② Modulation Index

$$m_f = \frac{\Delta W}{W_m} = \frac{\text{Frequency Deviation}}{\text{max. freq. of } m(t)}$$
- ③ Modulation Index $\gg 1$
 i.e. $m_f \gg 1$
- ④ Total Power is same as unmodulated Carrier Power

$$P_t = P_c$$
- ⑤ Bandwidth is function of modulation Index.

$$BW_{fm} = 2[m_f + 1]f_m$$
- ⑥ It contains infinite no. of side bands.
- ⑦ It is non linear modulation Tech.
- ⑧ SNR is High
- ⑨ It covers less (or) limited Area
- ⑩ It is Line of Sight Propagation (LoS)
- ⑪ Frequency re use is possible
- ⑫ more Cost & Circuit Complexity

Carson's rule to find BW of NBfm & WBfm signals:-

According to Carson's rule $2(m_f + 1)$ side bands are significant and remaining are insignificant side bands.

$$BW = 2[m_f + 1] f_m \text{ (or) } 2[m_f + 1] W_m.$$

$$m_f = \frac{\Delta f}{f_m} = \frac{\Delta W}{W_m}.$$

Case (i):- If $m_f \ll 1$ then NBfm.

$$BW = 2 \left[\frac{\Delta f}{f_m} + 1 \right] f_m$$

If $m_f \ll 1$ then it is negligible.

$$BW \approx 2f_m \text{ (or) } 2W_m$$

Case (ii):- If $m_f \gg 1$ then WBfm

$$BW = 2[m_f + 1] f_m$$

$$BW \approx 2m_f \cdot f_m.$$

$$\approx 2 \cdot \frac{\Delta f}{f_m} \cdot f_m$$

$$BW \approx 2 \Delta f$$

Carson's rule is used to find the BW of both NBfm & WBfm signals.

Comparison b/w NBFm & WBFm

NBFm

① Modulation index ' m_f ' is Very low
i.e. $m_f \ll 1$

② Frequency deviation is low (or) narrow

③ It contains only two side bands

④ Band width of NBFm is $2f_m$ (or) $2W_m$

⑤ SNR is low

⑥ Less cost & Circuit Complexity

⑦ It is Linear modulation Technique.

⑧ It was used to transmit voice signal in 1G mobile communication

WBFm

① Modulation Index ' m_f ' is Very high.
i.e. $m_f \gg 1$

② Frequency deviation is Very high (or) wider

③ It contains infinite no. of side bands.

④ Band width of WBFm is $2\Delta f$ (or) $2\Delta W$.

⑤ SNR is High.

⑥ more cost & Circuit Complexity.

⑦ It is non Linear modulation Technique.

⑧ It is used to Transmit Audio signal in both radio & TV.

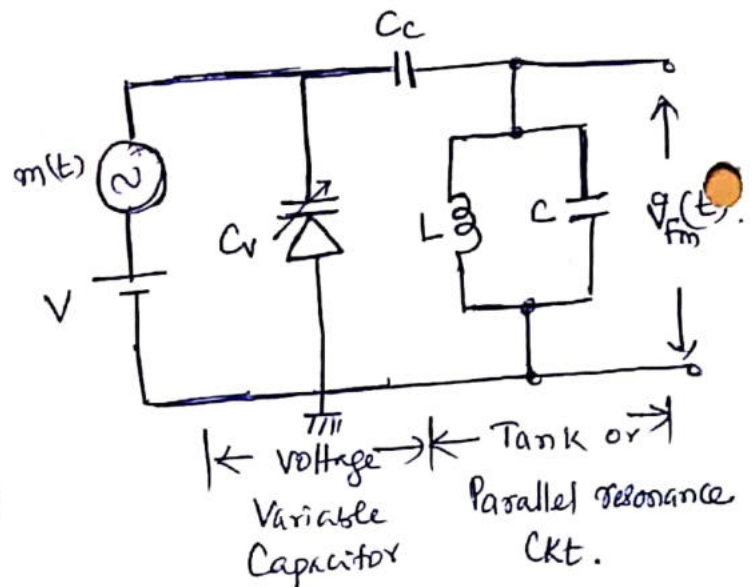
Generation of FM signal :-

- (1) Direct (or) Parameter Variation method
- (2) Indirect (or) Armstrong method.

The required carrier frequency and frequency deviation is directly generated by varying the capacitance of electronic devices. Hence, this is called "Direct or Parameter Variation Method".

Assumptions :-

- (1) The value of Capacitor C_c is very small when compared C_v & C , so that it does not effect the frequency of oscillations.



- (2) The value of voltage 'V' is selected such that Varactor diode is always operated in reverse bias irrespective of $m(t)$.

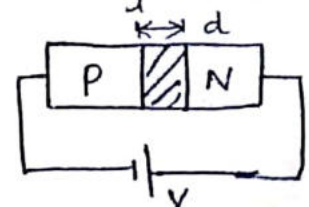
- (3) Tank or Parallel resonance Ckt generates Carrier frequency

$$f_c = \frac{1}{2\pi \sqrt{L(C+C_v)}}$$

Capacitance due to Varactor diode.

- (4) Varactor diode is operated in reverse bias, whose transition Capacitance $C_v = \frac{\epsilon A}{d}$.

where $A \rightarrow$ Area of Depletion layer
 $d \rightarrow$ Width of Depletion layer



Circuit operation :-

1. The Varactor diode is a semiconductor diode whose Junction Capacitance changes with respect to input voltage. Hence it acts as "Voltage Variable Capacitor".

2. Tank or Parallel resonance circuit, generates Carrier frequency.

Case (i) :- modulating signal $m(t) = 0$, ~~not~~ No modulation.

In this case, Constant reverse ~~large~~ bias is applied across Varactor diode, which generates Constant Capacitance ' C_v ' so that modulator generates Unmodulated Carrier frequency.

$$f_c = \frac{1}{2\pi \sqrt{L(C+C_v)}}$$

Case (ii) :- Modulation signal $m(t) \neq 0$ & Amplitude Increasing.

In this case, the Presence of modulating signal increases the Reverse bias on Varactor diode which increases the depletion layer width ' d ', that decreases the Capacitance ~~(Cv)~~ ~~(Cv)~~

($C_v = \frac{\epsilon A}{d}$), so that frequency of the tank circuit

increases. $\uparrow f_c = \frac{1}{2\pi \sqrt{L(C+C_v)}}$ $\downarrow$

$$\boxed{\uparrow m(t) - \uparrow d - \downarrow C_v - \uparrow f_c}$$

i.e. Increasing ~~in~~ amplitude in $m(t)$, increases the Carrier frequency.

Case (iii) :- modulating signal $m(t) \neq 0$ & amplitude decreasing.

In this case, decreasing in the amplitude of modulating signal, decreases the reverse bias on Varactor diode, which increases the depletion layer width ~~case~~ ($C_v = \frac{\epsilon A}{d}$) so that increases the Capacitance C_v value, so that carrier frequency decreases. i.e

$$\boxed{\downarrow m(t) \rightarrow \downarrow d \rightarrow \uparrow C_v \rightarrow \downarrow f_c}$$

Decreasing of modulating signal amplitude, decreases the carrier frequency.

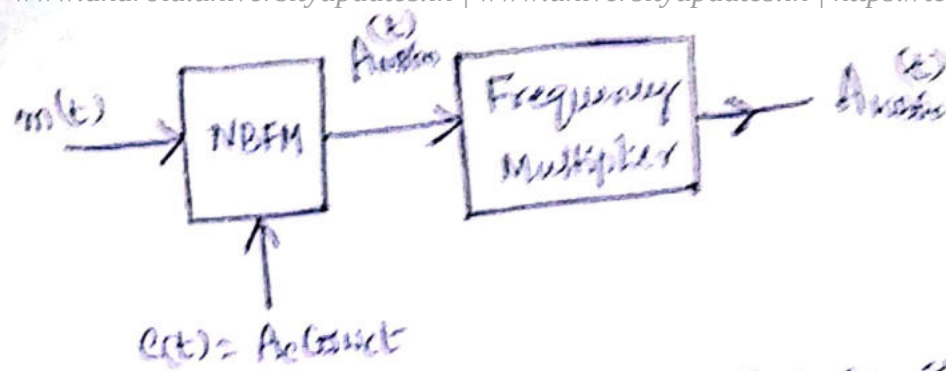
Hence, frequency modulation signal is generated.

Drawbacks :-

- ① Temperature variations may vary the Capacitance of Varactor diode, which is not desirable.
- ② Noise variations in $m(t)$ may vary the Capacitance of Varactor diode.

② Indirect (or) Armstrong Method :-

NBFM signal is generated with less frequency deviation & carrier frequency and then it is converted into WBFM signal. Hence, it is called Indirect (or) Armstrong method.



The block diagram of Indirect method is shown in figure, which contains NBFM modulator & frequency multiplier. Let us consider Carrier signal $c(t) = A_c \cos \omega_c t$ & modulating signal $m(t)$. The general expression of fm signal is

$$A_{nbfm}(t) = A_c \cos \left[\underbrace{\omega_c t}_{\text{Carrier frequency}} + \underbrace{k_f \int m(t) dt}_{\text{frequency deviation}} \right]$$

The output of frequency multiplier

$$A_{wbfm}(t) = \left[A_{nbfm}(t) \right]^n ; \text{ where } n = 2, 3, 4, \dots$$

Let $n=2 \Rightarrow A_{wbfm}(t) = A_{nbfm}^2(t)$

$$= \left[A_c \cos \left[\omega_c t + k_f \int m(t) dt \right] \right]^2$$

$$= A_c^2 \cos^2 \left[\omega_c t + k_f \int m(t) dt \right]$$

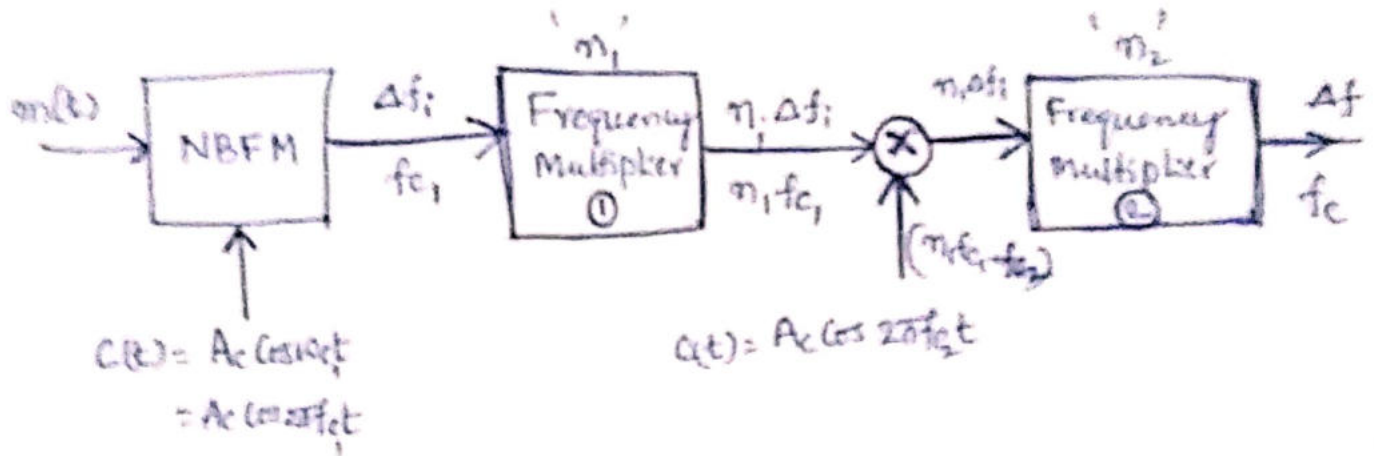
$$= A_c^2 \left[\frac{1 + \cos \left[2\omega_c t + 2k_f \int m(t) dt \right]}{2} \right]$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$A_{wbfm}(t) = \underbrace{\frac{A_c^2}{2}}_{\text{DC Component}} + \underbrace{\frac{A_c^2}{2} \cos \left[2\omega_c t + 2k_f \int m(t) dt \right]}_{\substack{\text{Carrier frequency} \\ \text{is doubled}}} \quad \text{Frequency deviation is doubled.}$$

Thus, NBFM signal is converted into WBFM signal.

If one frequency multiplier is not able to convert (or) generate required carrier frequency and frequency deviation then two frequency multipliers are used in cascade form as shown in figure.



$\Delta f_1 \rightarrow$ Initial frequency deviation.

$f_{c1} \rightarrow$ Initial carrier frequency.

$\Delta f \rightarrow$ Final frequency deviation. $= n_1 n_2 \Delta f_1$

$f_c \rightarrow$ final carrier frequency $= n_2 [n_1 f_{c1} - f_{c2}]$

where $n_1 \rightarrow$ frequency multiplication factor of first stage.

$n_2 \rightarrow$ Frequency multiplication factor of second stage.

Problem:- let $\Delta f_1 = 24.4 \text{ kHz}$, $f_{c1} = 200 \text{ kHz}$, $f_{c2} = 10.925 \text{ MHz}$
 $\Delta f = 75 \text{ kHz}$, $f_c = 90 \text{ MHz}$. find $n_1 = ?$ $n_2 = ?$

$$75 \times 10^3 = n_1 n_2 \times 24.4 \quad \text{--- (1)}$$

$$90 \times 10^6 = [n_1 \times 200 \times 10^3 - 10.925 \times 10^6] n_2 \quad \text{--- (2)}$$

$$n_1 = 48.018 \approx 48$$

$$n_2 = 63.996 \approx 64$$

because n_1 & n_2 are integers.

Frequency Demodulators :- (or) Detectors

It is process of extracting modulating signal $m(t)$ from frequency modulated signal. Two steps are involved in fm detection.

Step 1 :- Conversion of fm signal into Am signal.

Tank or Parallel Resonance Circuit converts fm into Am signal.

Step 2 :- Extraction of modulating signal $m(t)$ from Am signal.

An Envelope detector is used to extract modulating signal $m(t)$ from Am signal.

The detection process of fm signal is also called "Discrimination".

Types of fm Demodulators (or) Detectors (or) Discriminators :-

① Slope Demodulators.

i) Simple (or) Single-tuned Demodulators.

ii) Balanced or Slopper (or) Traxies or Triple Tuned demodulators.

② Phase Difference Demodulators.

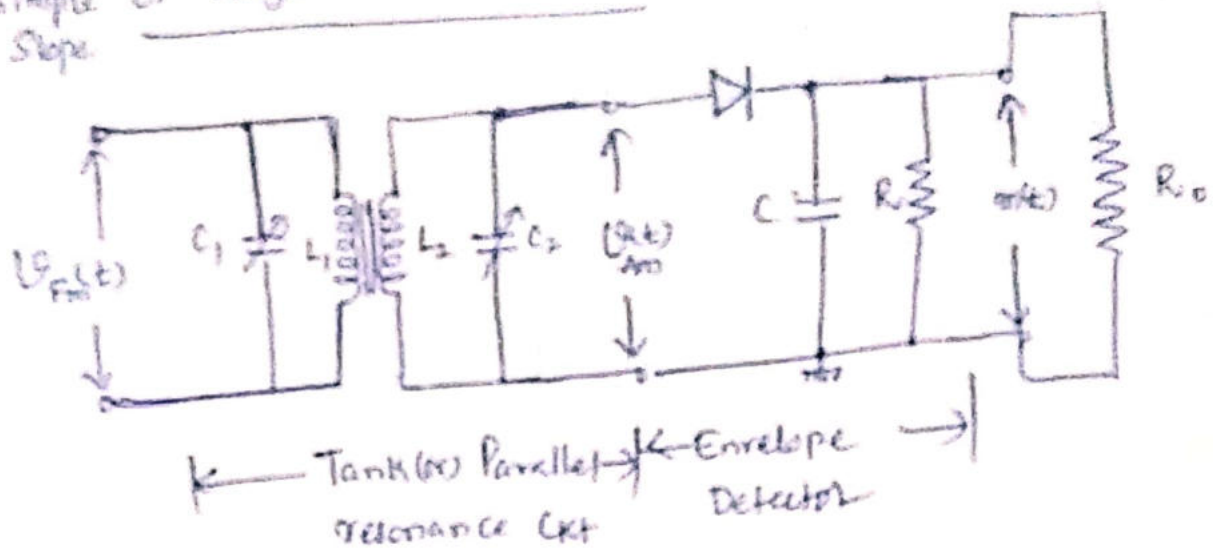
i) Foster Seeley Demodulators.

ii) Ratio demodulators.

iii) Phase locked loop (PLL) Demodulators.

iv) Zero Crossing Demodulators.

① Simple or Single tuned Demodulator :— Slope



Circuit operations:—

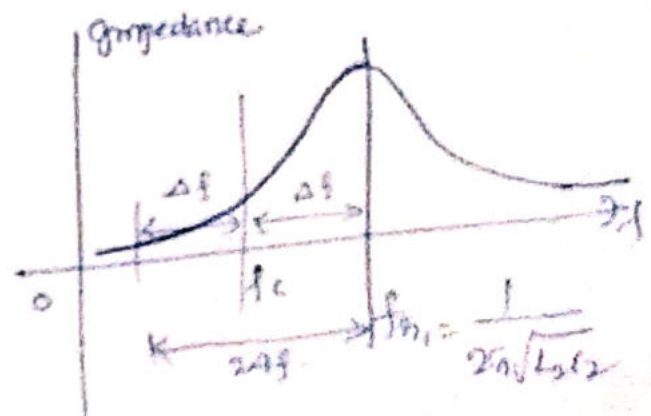
1. The Circuit diagram of Simple Slope or Single tuned fm demodulator is shown in figure which contains Tank (or) Parallel Resonance Ckt & Envelope Detector.
2. The Resonance Circuit formed by L_1 & C_1 are tuned to incoming & carrier frequency.

$$f_{m1} = f_c = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

3. The resonance Circuit formed by L_2 & C_2 are tuned to carrier frequency such that Circuit is operated below the resonance frequency f_{m2} as shown in figure

$$f_{m2} > f_c = \frac{1}{2\pi\sqrt{L_2 C_2}}$$

4. Parallel resonance Circuit Converts fm signal into Am signal.

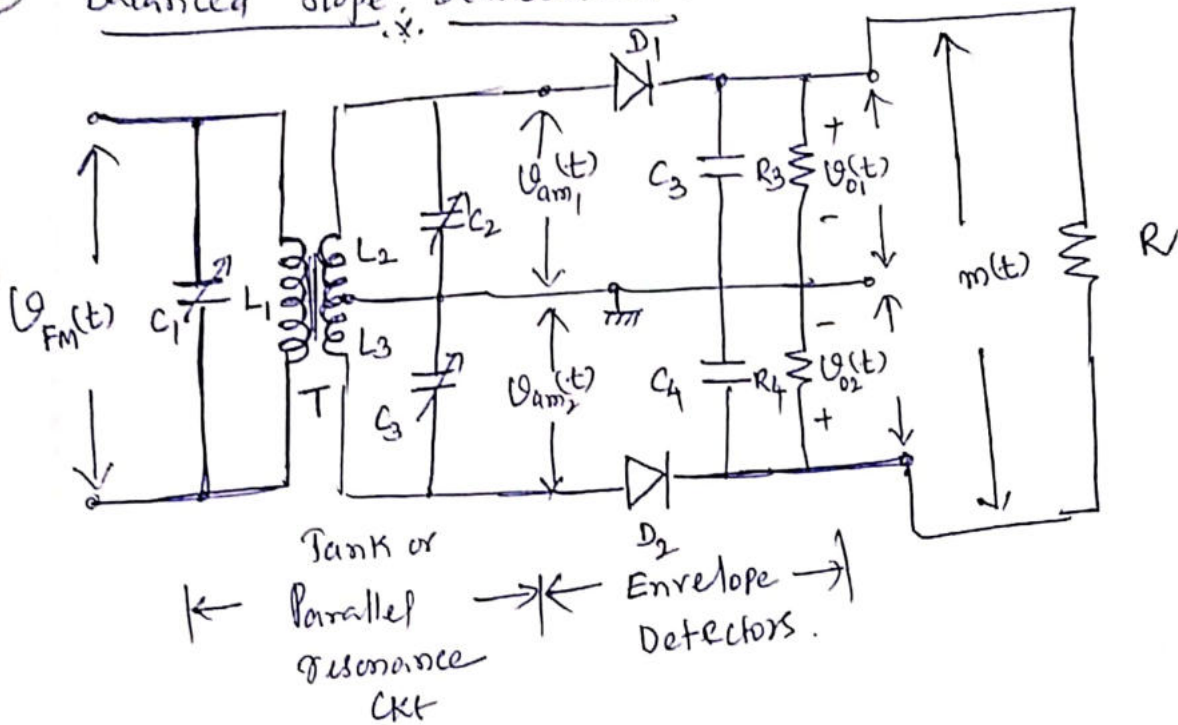


5. An Envelope detector extract modulating signal $m(t)$ from Am signal.

Draw backs :-

- ① Non Linearity introduced due to Inductor & Capacitor.
- ② Amplitude variations in i/p fm signal due to noise may change the o/p of Envelope detector i.e noise effect is more.

② Balanced Slope Demodulator :-



Circuit operation :-

1. The Balanced Slope fm demodulator is shown in figure which contains '3' tuning circuit & '2' Envelope detectors.
2. The resonance circuit formed by \$L_1\$ & \$C_1\$ is tuned to incoming carrier frequency

$$f_c = f_{r1} = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

3. The resonance ckt. formed by \$L_2\$ & \$C_2\$ is tuned to above carrier frequency

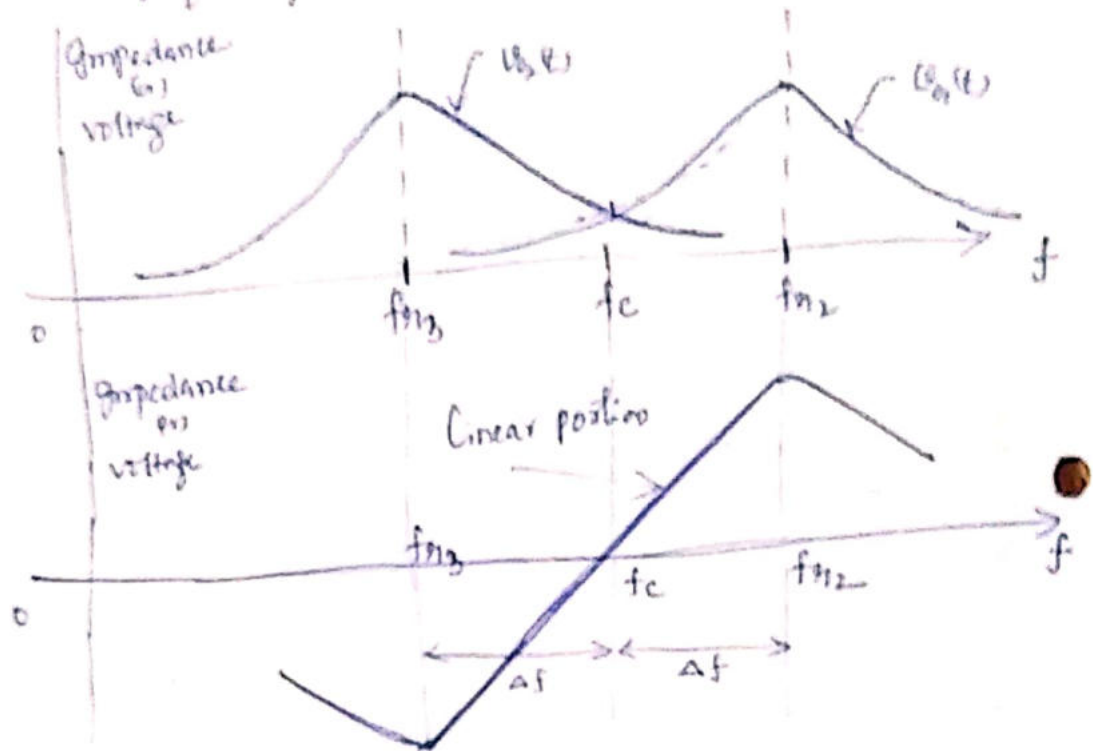
$$f_{r2} = \frac{1}{2\pi\sqrt{L_2 C_2}} > f_c$$

©

4. the resonance circuit formed by L_3 & C_3 are tuned below the carrier frequency

$$f_{m3} = \frac{1}{2\pi\sqrt{L_3C_3}} < f_c$$

5. The overall frequency response resonance circuit is as shown below



6. Parallel resonance circuit converts FM signal into AM signal
7. An envelope detector can extract modulating signals from AM signals.
8. The o/p is equal to $m(t) = V_{o1}(t) - V_{o2}(t)$.

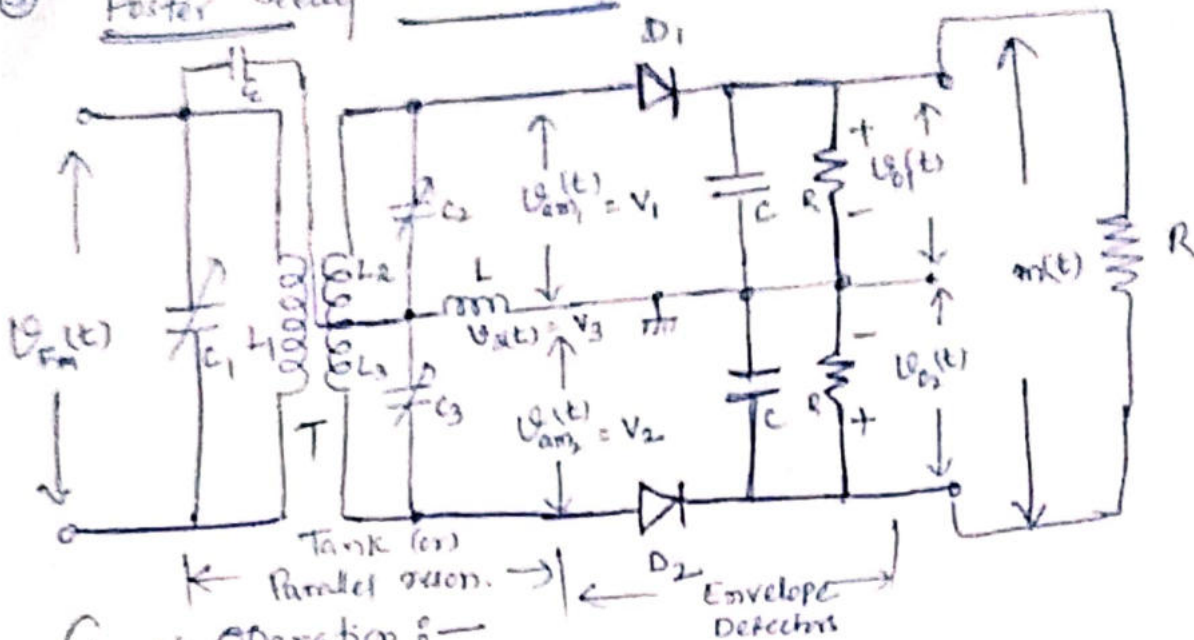
Draw Backs :-

1. Linear Portion is less.

Advantages

1. Linear operation
2. less noise effect

⑤ Foster Seeley Demodulator :-



Circuit Operation :-

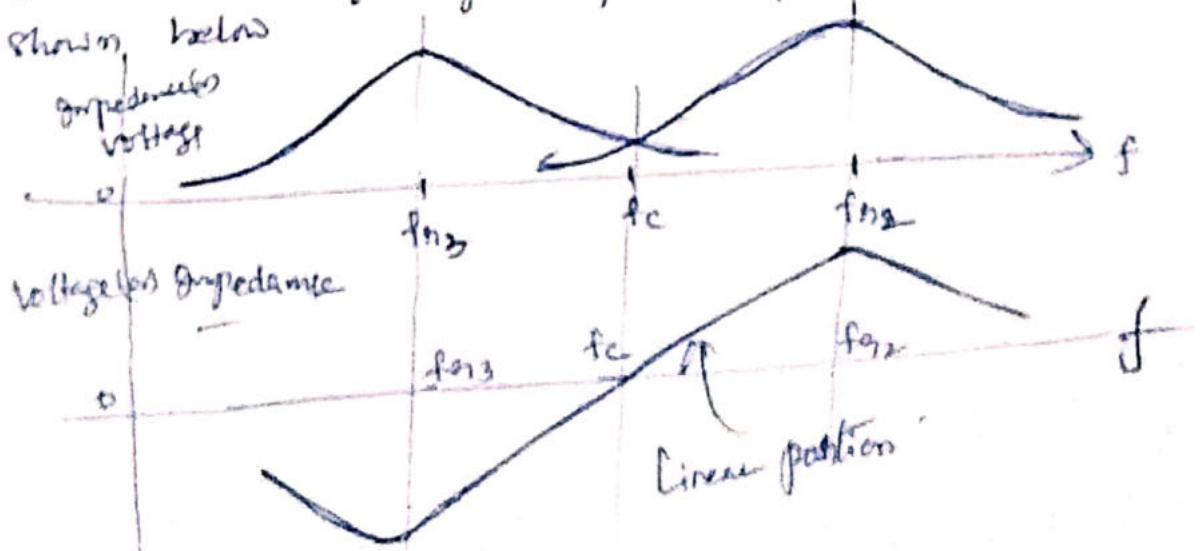
1. The Circuit operation of foster Seeley demodulator is shown figure which contains '3' Parallel resonance Ckts & Two Envelope Detectors.

2. Resonance Circuit is formed by L_1 & C_1 is tuned to incoming Carrier frequency $f_{n1} = f_c = \frac{1}{2\pi\sqrt{L_1C_1}}$

3. Resonance Circuit is formed by L_2 & C_2 is tuned to above Carrier frequency $f_{n2} = \frac{1}{2\pi\sqrt{L_2C_2}} > f_c$

4. Resonance Circuit is formed by L_3 & C_3 is tuned to below Carrier frequency $f_{n3} = \frac{1}{2\pi\sqrt{L_3C_3}} < f_c$

5. The overall frequency response of the circuit is as shown below



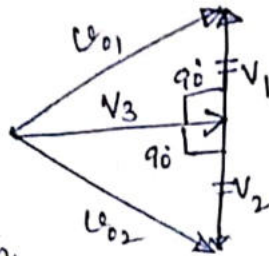
6. The Circuit operation can be explained using phasor or vector diagrams.

Case (i) :- Incoming frequency is equal to Carrier frequency
 $f_i = f_c$

The phasor (or) Vector diagram b/w V_1, V_2 & V_3 will be as shown below.

The angle b/w V_1 & V_3 & V_2 & V_3 is 90° . V_1 & V_2 are Cancels each other.

So that output $m(t) = V_{o1}(t) - V_{o2}(t) = 0$.

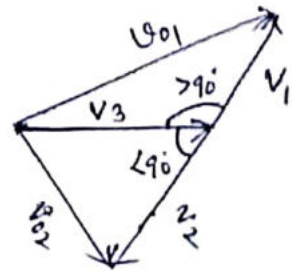


Case (ii) :- Incoming frequency is greater than Carrier frequency
 $f_i > f_c$

The phasor or Vector diagram b/w V_1, V_2 & V_3 will be as shown below

The angle b/w V_1 & V_3 is $> 90^\circ$ & V_2 & V_3 is less than 90° . So, that output

$m(t) = V_{o1}(t) - V_{o2}(t)$ is +ve.

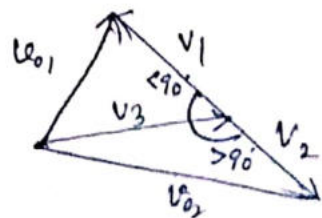


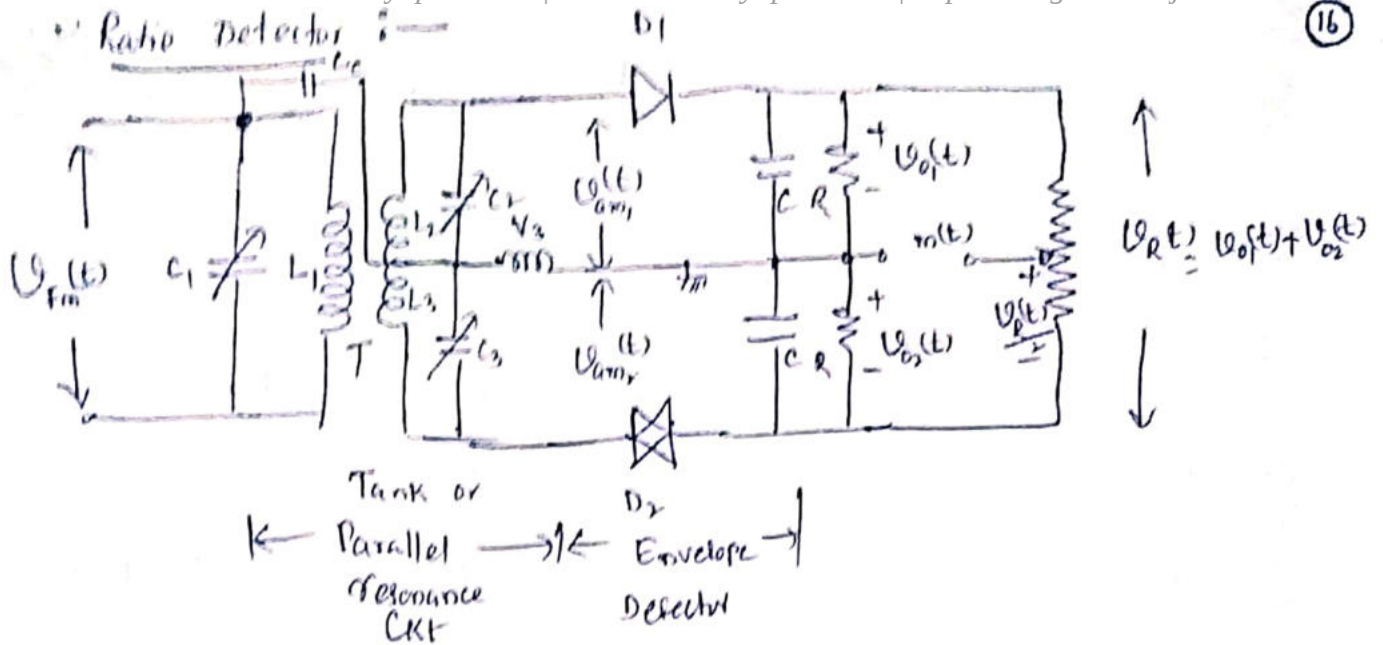
Case (iii) :- Incoming frequency is lesser than Carrier frequency
 $f_i < f_c$

The Phasor or Vector diagrams b/w V_1, V_2, V_3 will be as shown below

The angle b/w V_1 & V_3 is ~~greater~~ ^{less} than 90° & V_2 & V_3 is greater than 90° . So that

Output $m(t) = V_{o1}(t) - V_{o2}(t)$ is negative.





Foster Seeley & Ratio detector are same having Circuit diagram except

- ① Diode D_1 is reversed.
- ② O/p is taken at middle of potentiometer as shown in figure.

$$\begin{aligned}
 \text{O/p } m(t) &= U_{O2}(t) - \frac{U_R(t)}{2} \\
 &= U_{O2}(t) - \left[\frac{U_{O1}(t) + U_{O2}(t)}{2} \right] \\
 &= U_{O2}(t) - \frac{U_{O1}(t)}{2} - \frac{U_{O2}(t)}{2} \\
 &= \frac{U_{O2}(t)}{2} - \frac{U_{O1}(t)}{2}
 \end{aligned}$$

$$m(t) = \frac{U_{O2}(t) - U_{O1}(t)}{2}$$

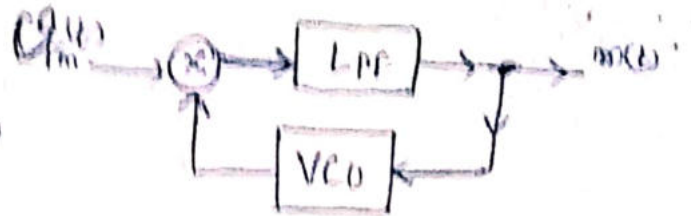
Advantages:-

- ① Noise Spikes will removed at the output, with help of potentiometer.



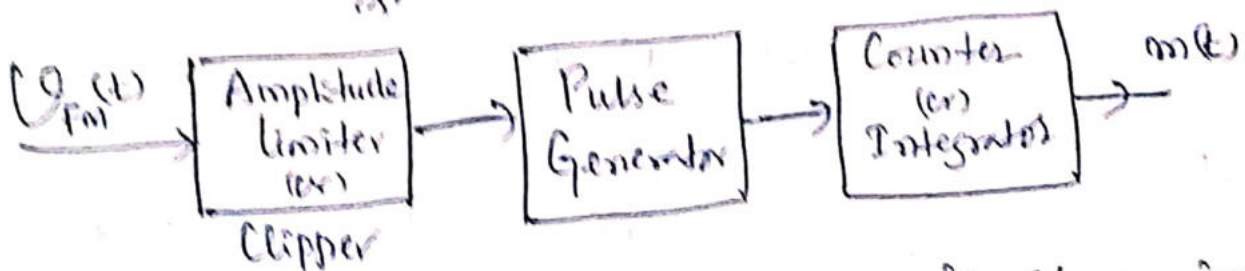
(a)

Phase Locked Loop :- (PLL)



1. Phase Locked loop demodulation is shown in figure which contains multiplier, LPF & VCO (Voltage Controlled Oscillator)
2. PLL is considered as closed loop synchronous detector connected in -ve feedback loop.
3. Multiplier, Multiplies incoming FM signal & locally generated carrier signal and generates sum & difference frequencies.
4. LPF filter removes sum frequency and passes difference frequency to the output.
5. Voltage Controlled oscillator generates variable frequency based on input voltage from LPF.
6. PLL is closed loop synchronous detector, which continuously extracts modulating signal.

(b) Zero Crossing detector :-

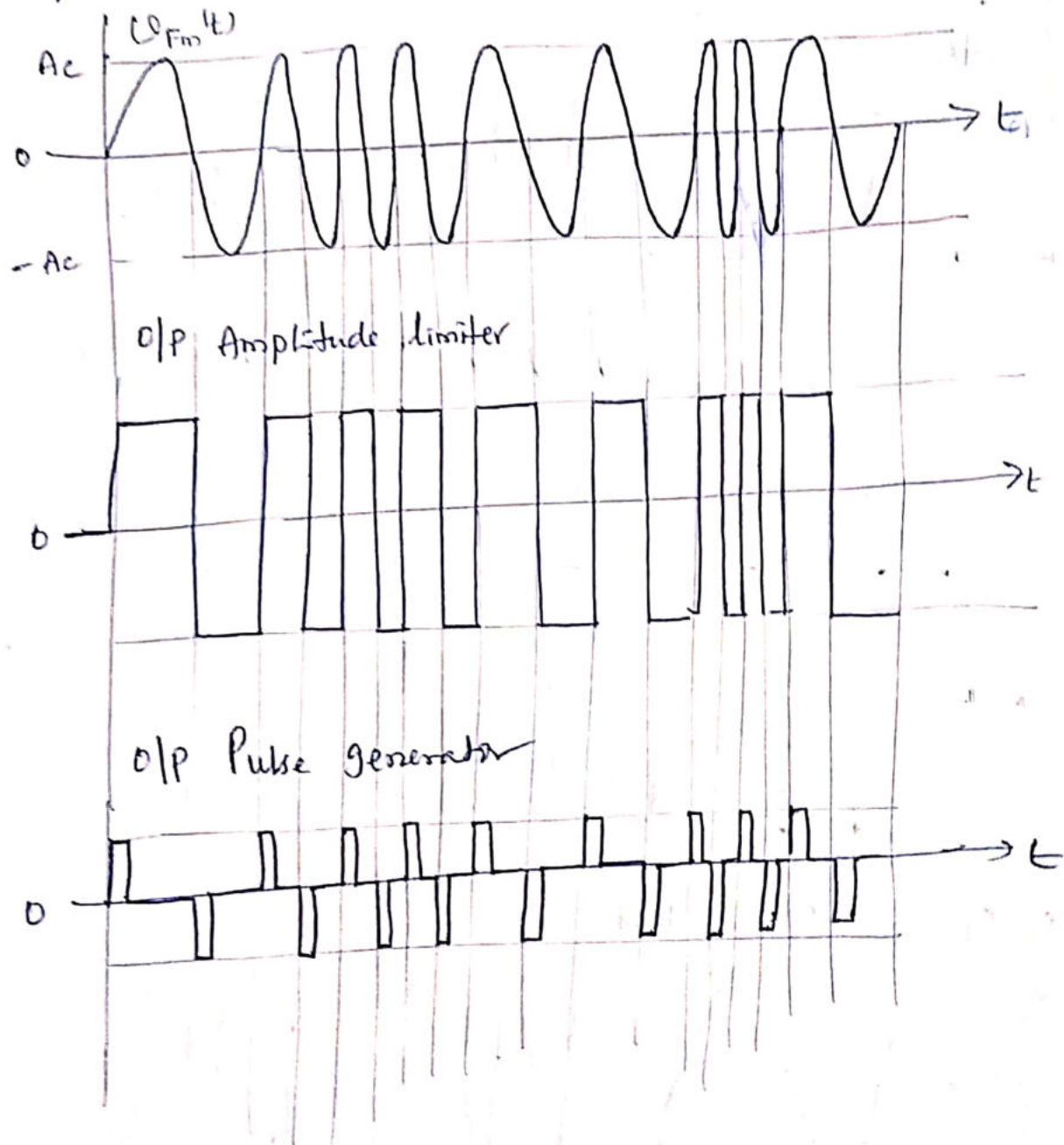


1. Zero Crossing detector is shown in figure which contains Amplitude limiter, Pulse Generator & Counter (or) Integrator.

2. Amplitude Limiter or Clipper Converts Sine(wave) into approximated Square wave by removing +ve & -ve portions.

3. Pulse generator generates -ve & +ve Pulse with Constant width & amplitude at each Zero Crossings as shown in figure.

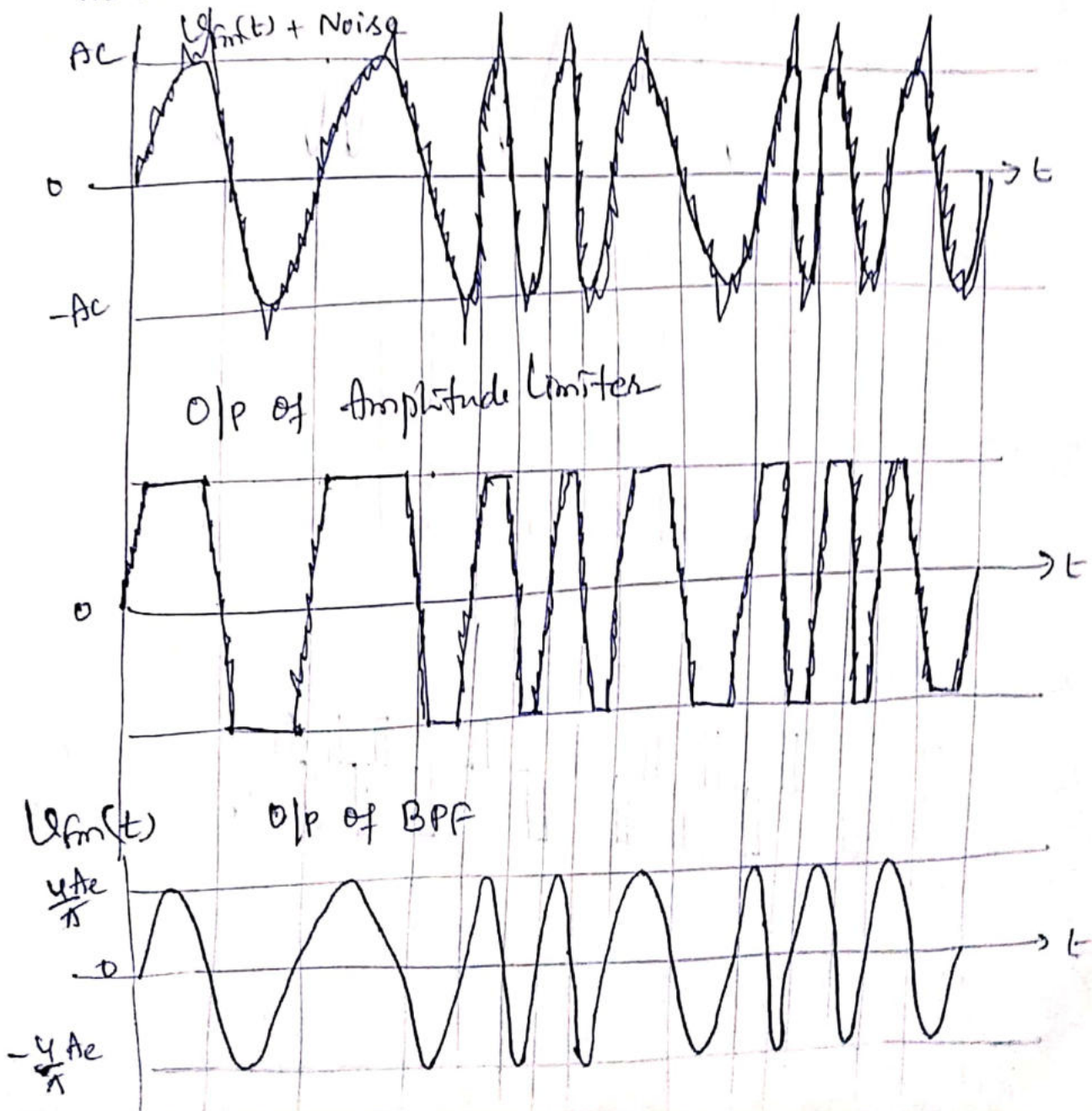
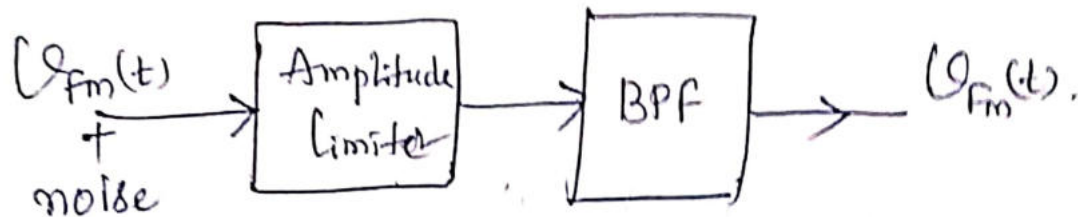
4. Counter (or) Integrator, generates Proportional voltage by Counting no. of Zero Crossing by means of Pulses.



Improvement of Signal to Noise Ratio in FM system

- ① Bandpass Limiter
- ② Pre-emphasis & De-emphasis.

The SNR can be improved using bandpass limiter which consists of Amplitude limiter & BPF filter as shown in figure.



1. Input to the Amplitude limiter is fm signal Plus noise as shown in figure, which can be expressed

as

$$U_{fm}(t) + n(t) = A_c \cos [\omega_c t + k_f \int m(t) dt] + n(t)$$

$$= A_c \cos [\theta(t)] + n(t)$$

Where $\theta(t) = \omega_c t + k_f \int m(t) dt$

2. Amplitude limiter Converts sine or cosine wave in to Approximated Square wave by removing +ve & -ve portions.

3. Amplitude limiter removes noise, Square wave can be represented using Fourier series as.

$$U_{fm}(t) + n(t) = A_c \cdot \frac{4}{\pi} \left[\cos(\theta(t)) - \frac{1}{3} \cos[3\theta(t)] + \frac{1}{5} \cos[5\theta(t)] + \dots \right] + \text{noise}$$

$$= \frac{A_c 4}{\pi} \left[\cos [\omega_c t + k_f \int m(t) dt] - \frac{1}{3} \cos [3\omega_c t + 3k_f \int m(t) dt] - \frac{1}{5} \cos [5\omega_c t + 5k_f \int m(t) dt] + \dots \right] + \text{noise}$$

4. BPF removes 2nd term, 3rd terms and so on and also noise. So that fm signal appears without noise at the O/P of BPF.

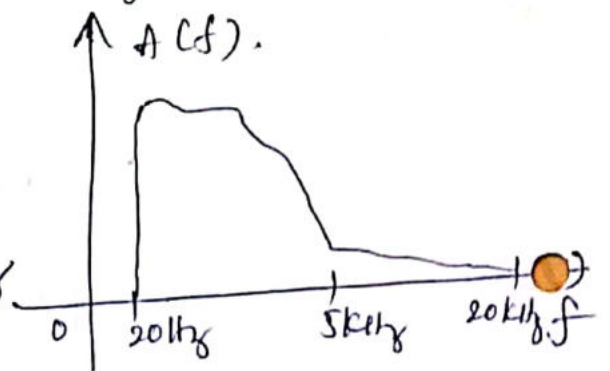
$$U_{fm}(t) = \frac{4A_c}{\pi} \cos [\omega_c t + k_f \int m(t) dt]$$

Thus ~~BPF~~ Bandpass limiter removes noise in fm so that S/N R improved.

② Pre-emphasis & De-emphasis :-

1. Pre-emphasis used in transmitter & De-emphasis used in the receiver.
2. Pre-emphasis is used before modulator & De-emphasis is used after De-modulator.
3. Pre-emphasis is combination of HPF & Amplifier and De-emphasis is Low Pass filter (LPF).
4. Pre-emphasis & De-emphasis are used to improve SNR for system in the case of Audio transmission.

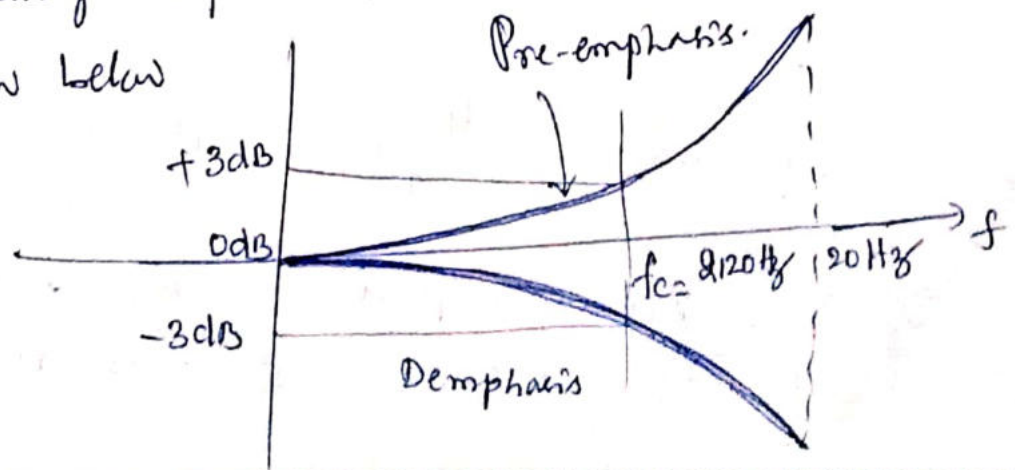
5. Frequency range of Audio signal is 20Hz to 20kHz as shown in figure



6. Side bands raised by high frequency components (5-20)kHz will have lower magnitudes.

7. So, noise effect is more on these frequencies.

8. The frequency response of Pre-emphasis & De-emphasis is as shown below

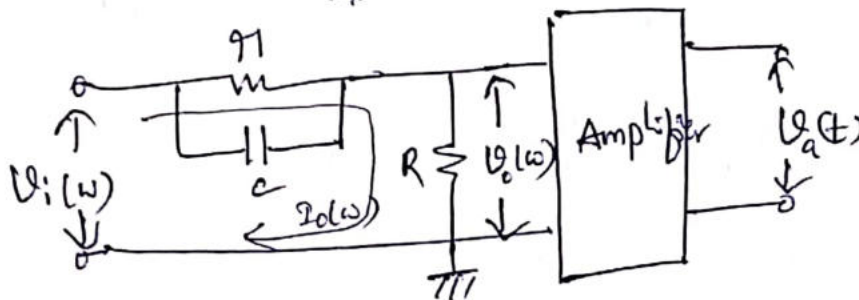


9. Both Pre-emphasis & De-emphasis Circuits will have same Cutoff frequencies

$$f_p = f_c = f_d = 2\pi \times 20 \text{ Hz} = \frac{1}{2\pi RC} = \frac{1}{2\pi C}$$

10. The time constant of the Circuits $C = 75 \mu\text{sec}$.

Derivation for Transfer function :-



Transfer function of Pre-emphasis is

$$H_p(\omega) = \frac{V_o(\omega)}{V_i(\omega)} \quad \text{--- (1)}$$

$$V_o(\omega) = R \cdot I_o(\omega) \quad \text{--- (2)}$$

$$I_o(\omega) = V_i(\omega) \left[\frac{1}{R} + j\omega C \right] \quad \left[\because R \text{ is negligible} \right]$$

$$V_o(\omega) = R \cdot V_i(\omega) \left[\frac{1}{R} + j\omega C \right]$$

$$H_p(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{R}{R} \left[1 + j\omega RC \right] = \frac{R}{R} \left[1 + j \left(\frac{\omega}{\omega_p} \right) \right]$$

$$\boxed{H_p(\omega) = \frac{R}{R} \left[1 + j \left(\frac{\omega}{\omega_p} \right) \right]}$$

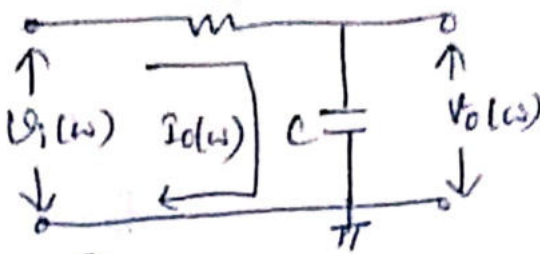
Cut off frequency $\omega_p = \frac{1}{RC} \Rightarrow \pi f_p = \frac{1}{RC}$

$$\boxed{f_p = \frac{1}{2\pi RC}}$$

De-emphasis

T/F of De-emphasis

is $H_d(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$ — (1)



$V_o(\omega) = \frac{I_o(\omega)}{j\omega C}$ — (A)

~~Because~~ $V_i(\omega) = r I_o(\omega) + \frac{I_o(\omega)}{j\omega C}$ — *

$I_o(\omega) \left[r + \frac{1}{j\omega C} \right] = V_i(\omega)$

$I_o(\omega) = \frac{V_i(\omega)}{r + \frac{1}{j\omega C}}$ — (2)

(A) $\Rightarrow V_o(\omega) = \frac{V_i(\omega)}{j\omega C \left[r + \frac{1}{j\omega C} \right]}$

$\frac{V_o(\omega)}{V_i(\omega)} = \frac{1}{j\omega C r + 1} = \frac{1}{1 + j\left(\frac{\omega}{\omega_d}\right)} = H_d(\omega)$

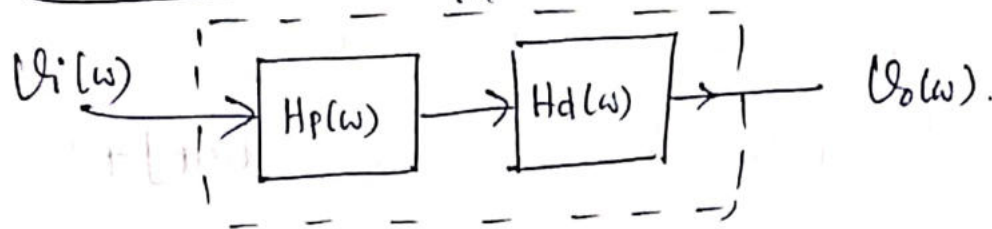
$$H_d(\omega) = \frac{1}{1 + j\left(\frac{\omega}{\omega_d}\right)}$$

Cut-off frequency $\omega_d = \frac{1}{rC}$

$2\pi f_d = \frac{1}{rC}$

$$f_d = \frac{1}{2\pi rC}$$

Cascade Connection of Pre-emphasis & De-emphasis - (20)



Overall T/F $H(\omega) = H_p(\omega) H_d(\omega)$

$\rightarrow$ T/F of De-emphasis
 $\rightarrow$ T/F of Pre-emphasis

$$H(\omega) = \frac{R}{\eta} \left[1 + j \left(\frac{\omega}{\omega_p} \right) \right] \times \frac{1}{\left[1 + j \left(\frac{\omega}{\omega_d} \right) \right]}$$

$$\omega_p = \omega_d = \frac{1}{\eta C}$$

$$H(\omega) = \frac{R}{\eta} \text{ Constant}$$

Above eqⁿ. Says that the frequencies emphasized by the Pre-emphasis and attenuated (or) De-emphasized by the De-emphasis. So, that modulating signal $m(t)$ is produced at the o/p of fm receiver with more SNR.

Thus Pre-emphasis & De-emphasis improve SNR in fm system.

Capture effect :-

Let us consider FM signal with noise can be written as $\phi_{fm}(t) = V_c \cos[\omega_c t + k_f \int m(t) dt] + n(t)$

Where $n(t)$ is Bandpass noise

$$n(t) = n_i(t) \cos \omega_c t \pm n_q(t) \sin \omega_c t$$

$n_i(t)$ & $n_q(t)$ are inphase & Quadrature phase Components of bandpass noise.

Let $m(t) = 0$, ~~because~~ because noise adds to the signal in the form of amplitude.

$$\phi_{fm}(t) = V_c \cos[\omega_c t] + n_i(t) \cos \omega_c t \pm n_q(t) \sin \omega_c t$$

$$\phi_{fm}(t) = [V_c + n_i(t)] \cos \omega_c t \pm n_q(t) \sin \omega_c t$$

Above signal can be represented as using Envelope form as

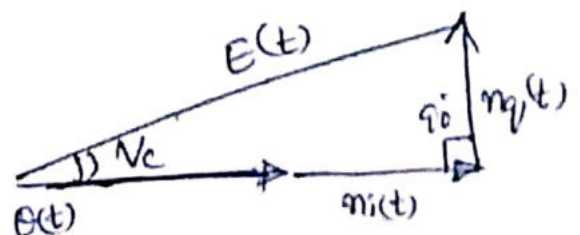
$$\phi_{fm}(t) = E(t) \cos[\theta(t)]$$

$$\text{Envelope } E(t) = \sqrt{[V_c + n_i(t)]^2 + n_q^2(t)}$$

$$\text{Angle } \theta(t) = \tan^{-1} \left[\frac{\pm n_q(t)}{V_c + n_i(t)} \right]$$

Envelope can be ~~to~~ represented using Vector form (or) phasor form as shown below

When FM signals from two XMITRs operated on the nearly same carrier frequency reaches the receiver



Simultaneously, the signal of weak magnitude is suppressed by the strong signal and then FM receiver will

www.android.universityupdates.in | www.universityupdates.in | https://telegram.me/jntua
respond only for the Strong signal. This effect is called "Capture effect".

(21)

Phase modulation :-

Why PM is not used for ~~broad~~ broadcasting?

Let us consider modulating signal $m(t) = A_m \cos \omega_m t$
& Carrier signal $c(t) = A_c \cos \omega_c t$. General form of expression for PM signal is

$$\begin{aligned} A_{PM}(t) &= A_c \cos [\omega_c t + K_p m(t)] \\ &= A_c \cos [\omega_c t + K_p A_m \cos \omega_m t] \\ &= A_c \cos [\varphi_i(t)] \end{aligned}$$

Total angle $\varphi_i(t) = \omega_c t + K_p A_m \cos \omega_m t$

Frequency can be obtained by derivating angle.

$$\omega_i = \frac{d}{dt} \varphi_i(t) = \omega_c + K_p A_m \sin \omega_m t \cdot \omega_m$$

$$= \omega_c \pm K_p A_m \omega_m \sin \omega_m t$$

maximum frequency deviation

$$\omega_i = \omega_c \pm \Delta \omega_{max}$$

$$\Delta \omega_{max} = \left| K_p A_m \omega_m \sin \omega_m t \right|$$

$$\boxed{\Delta \omega_{max} = K_p A_m \omega_m}$$

Frequency Deviation in PM signal is Proportional to both modulating signal amplitude & frequency. Hence, BW of PM signal is Very high. So, it is not used for broadcasting.

Narrow Band phase Modulation (NBPM) :-

Let us consider modulating signal $m(t)$ & Carrier signal $c(t) = A_c \cos \omega_c t$, General form of expression for PM signal

$$A_{PM}(t) = A_c \cos [\omega_c t + K_p m(t)]$$

phasor form representation

$$\tilde{A}_{PM}(t) = A_c e^{j[\omega_c t + K_p m(t)]}$$

$$\tilde{A}_{PM}(t) = A_c \cdot e^{j\omega_c t} \cdot e^{jK_p m(t)}$$

If $K_p \ll 1$ then PM is NBPM.

$$\tilde{A}_{NBPM}(t) = A_c e^{j\omega_c t} \cdot e^{jK_p m(t)} \quad \text{--- (A)}$$

$$e^{j\omega_c t} = \cos \omega_c t + j \sin \omega_c t \quad \text{--- (1)}$$

$$e^{jK_p m(t)} = \cos [K_p m(t)] + j \sin [K_p m(t)]$$

If $K_p \ll 1$ then

$$\cos [K_p m(t)] \approx 1$$

$$\sin [K_p m(t)] \approx K_p m(t)$$

$$e^{jK_p m(t)} = 1 + j K_p m(t) \quad \text{--- (2)}$$

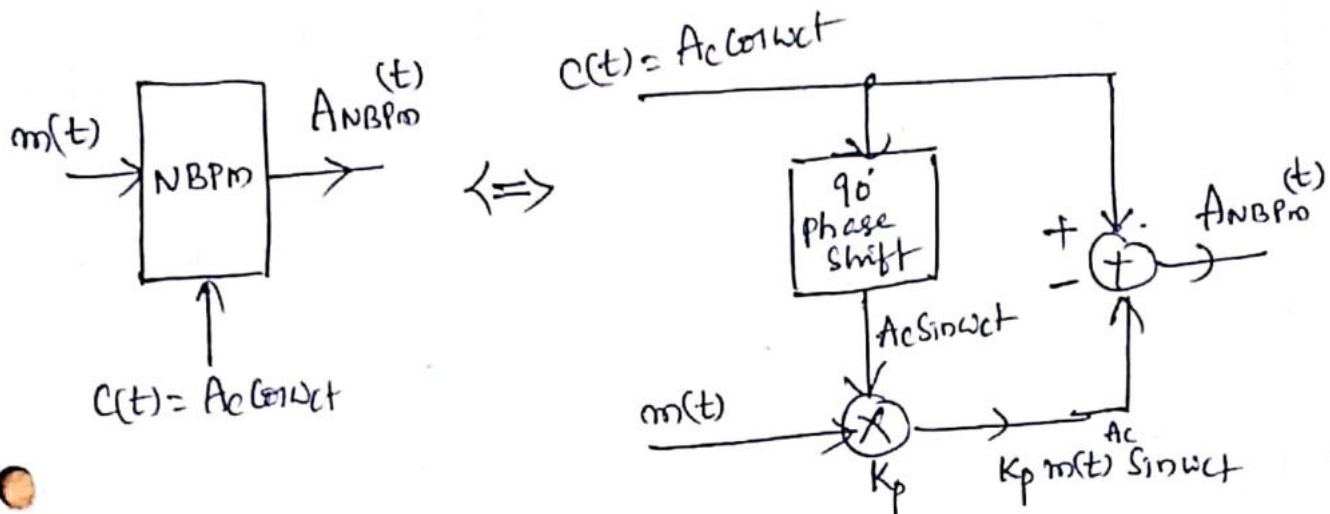
Put eqⁿ (1) & (2) in eqⁿ (A)

$$\begin{aligned} \tilde{A}_{NBPM}(t) &= A_c [\cos \omega_c t + j \sin \omega_c t] [1 + j K_p m(t)] \\ &= A_c [\cos \omega_c t + j K_p m(t) \cos \omega_c t + j \sin \omega_c t - K_p m(t) \sin \omega_c t] \\ &= A_c \cos \omega_c t - K_p m(t) A_c \sin \omega_c t + j K_p m(t) A_c \cos \omega_c t + j A_c \sin \omega_c t \end{aligned}$$

Because if 'θ' is very small then
cos θ ≈ 1
sin θ ≈ θ

Real Part of Above Eqⁿ. is

$$A_{NBPM}(t) = A_c \cos \omega_c t - K_p m(t) A_c \sin \omega_c t$$



NBPM — single tone :—

$$m(t) = A_m \cos \omega_m t \quad \& \quad C(t) = A_c \cos \omega_c t$$

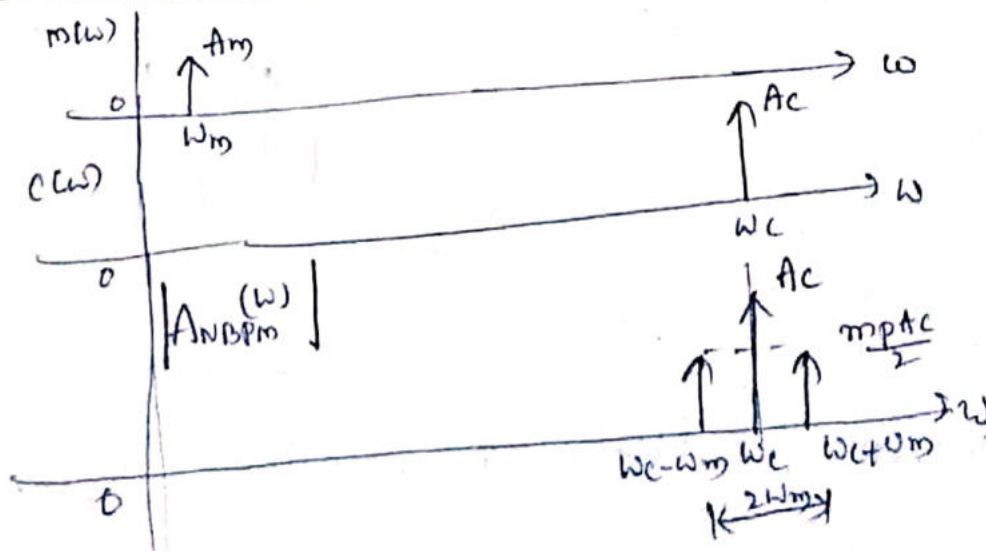
$$A_{NBPM}(t) = A_c \cos \omega_c t - K_p \cdot A_m \cos \omega_m t \cdot A_c \sin \omega_c t$$

$$= A_c \cos \omega_c t - K_p A_m \cdot A_c \sin \omega_c t \cos \omega_m t$$

$$= A_c \cos \omega_c t - \frac{m_p A_c}{2} \cdot 2 \sin \omega_c t \cos \omega_m t$$

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$A_{NBPM}(t) = A_c \cos \omega_c t - \frac{m_p A_c}{2} [\sin(\omega_c + \omega_m)t + \sin(\omega_c - \omega_m)t]$$



NBPM - Multitone :-

$$\text{Let } c(t) = A_c \cos \omega_c t$$

$$m(t) = A_{m1} \cos \omega_{m1} t + A_{m2} \cos \omega_{m2} t$$

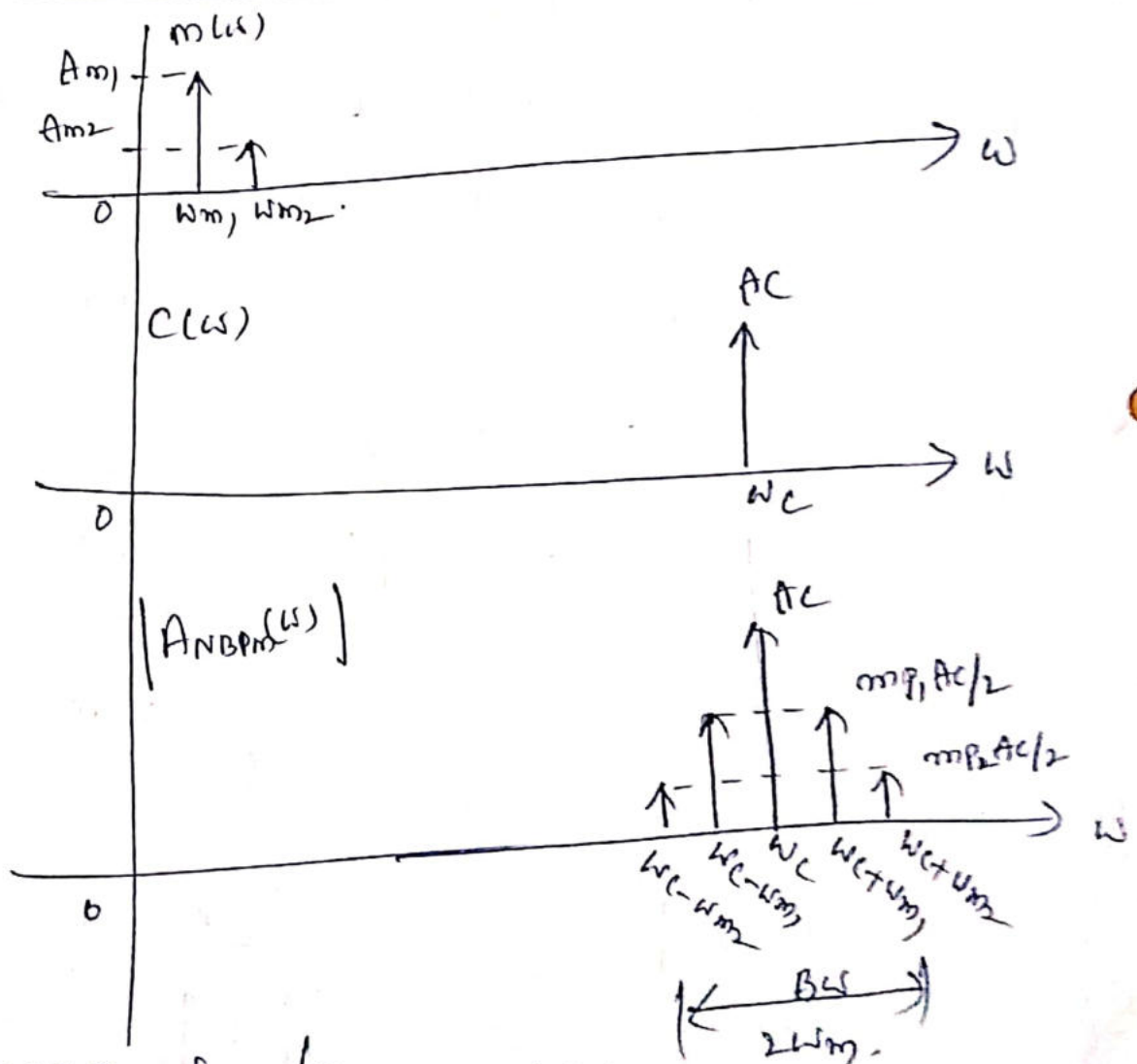
$$A_{NBPM}(t) = A_c \cos \omega_c t - K_p m(t) A_c \sin \omega_c t$$

$$= A_c \cos \omega_c t - K_p A_c \sin \omega_c t [A_{m1} \cos \omega_{m1} t + A_{m2} \cos \omega_{m2} t]$$

$$= A_c \cos \omega_c t - K_p A_{m1} A_c \sin \omega_c t \cos \omega_{m1} t - K_p A_{m2} A_c \sin \omega_c t \cos \omega_{m2} t$$

$$= A_c \cos \omega_c t - m_{p1} A_c \sin \omega_c t \cos \omega_{m1} t - m_{p2} A_c \sin \omega_c t \cos \omega_{m2} t$$

$$A_{NBPM}(t) = A_c \cos \omega_c t - \frac{m_{p1} A_c}{2} [\sin (\omega_c + \omega_{m1}) t + \sin (\omega_c - \omega_{m1}) t] - \frac{m_{p2} A_c}{2} [\sin (\omega_c + \omega_{m2}) t + \sin (\omega_c - \omega_{m2}) t]$$



NBPM is Linear modulation Technique.

Relation b/w Fm & Pm :-

Let us consider modulating signal $m(t)$ and carrier signal $c(t) = A_c \cos \omega_c t$.

$$A_{fm}(t) = A_c \cos \left[\omega_c t + k_f \int m(t) dt \right]$$

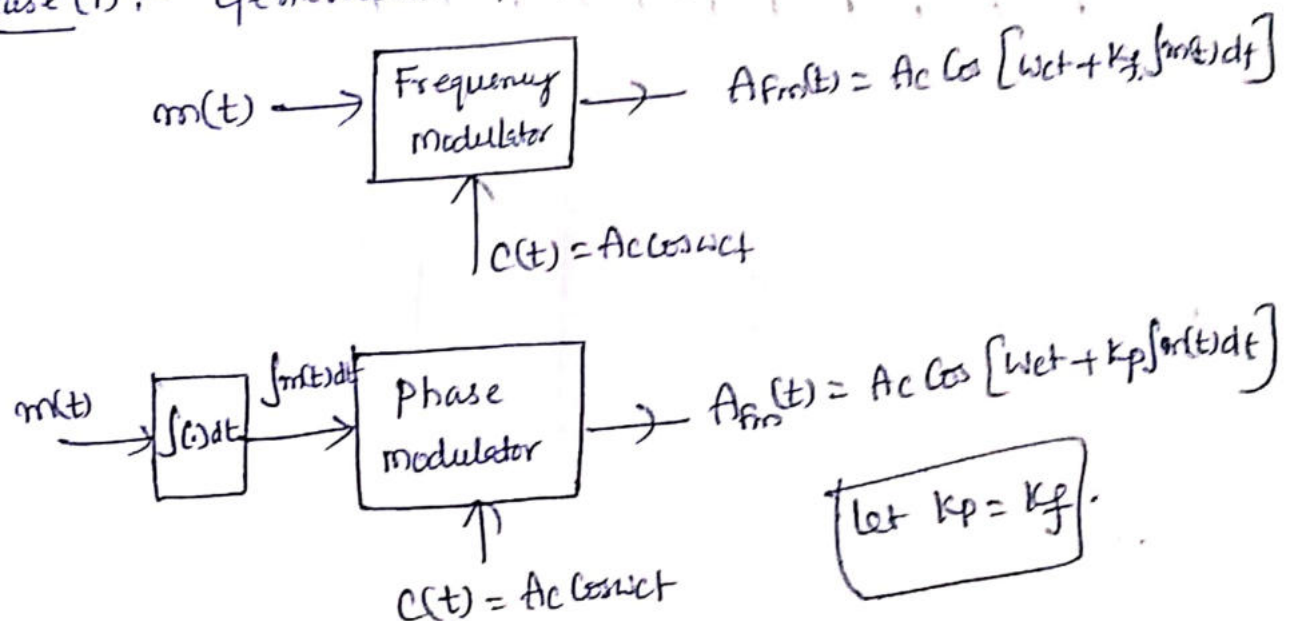
Frequency sensitivity factor
rad/volt

$$A_{pm}(t) = A_c \cos [\omega_c t + k_p m(t)]$$

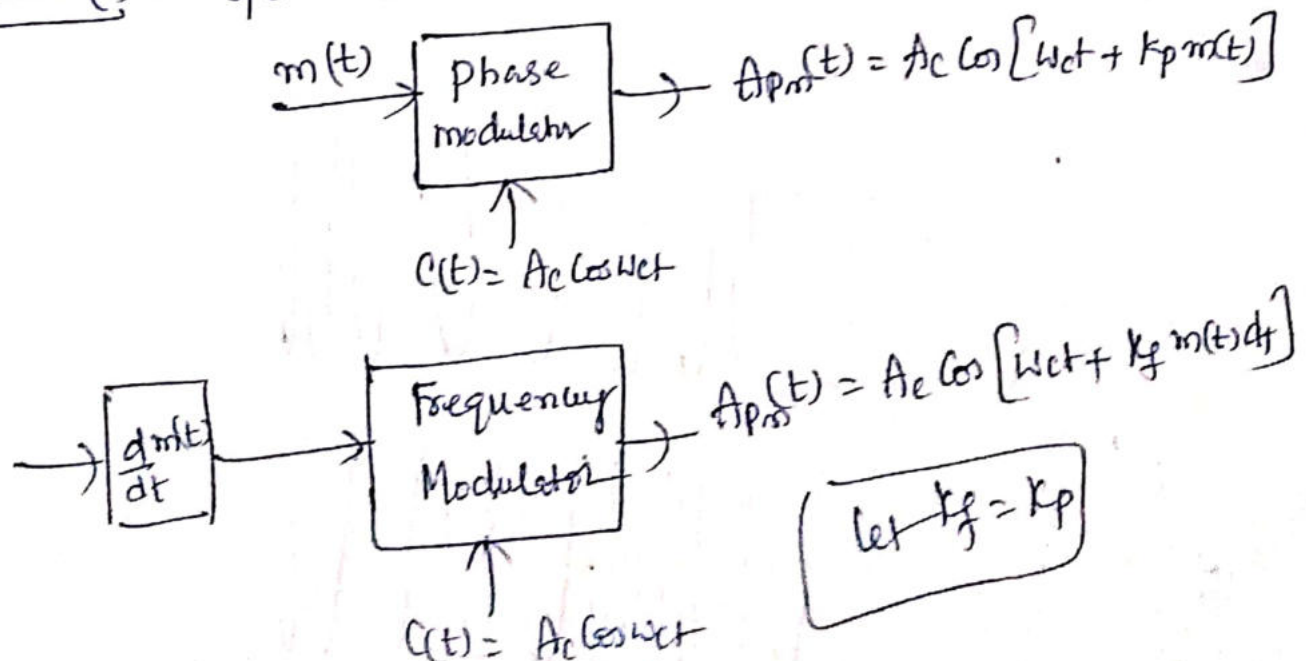
phase sensitivity factor
rad/volt

Conversion b/w fm & pm :-

Case (i) :- Generation of fm signal

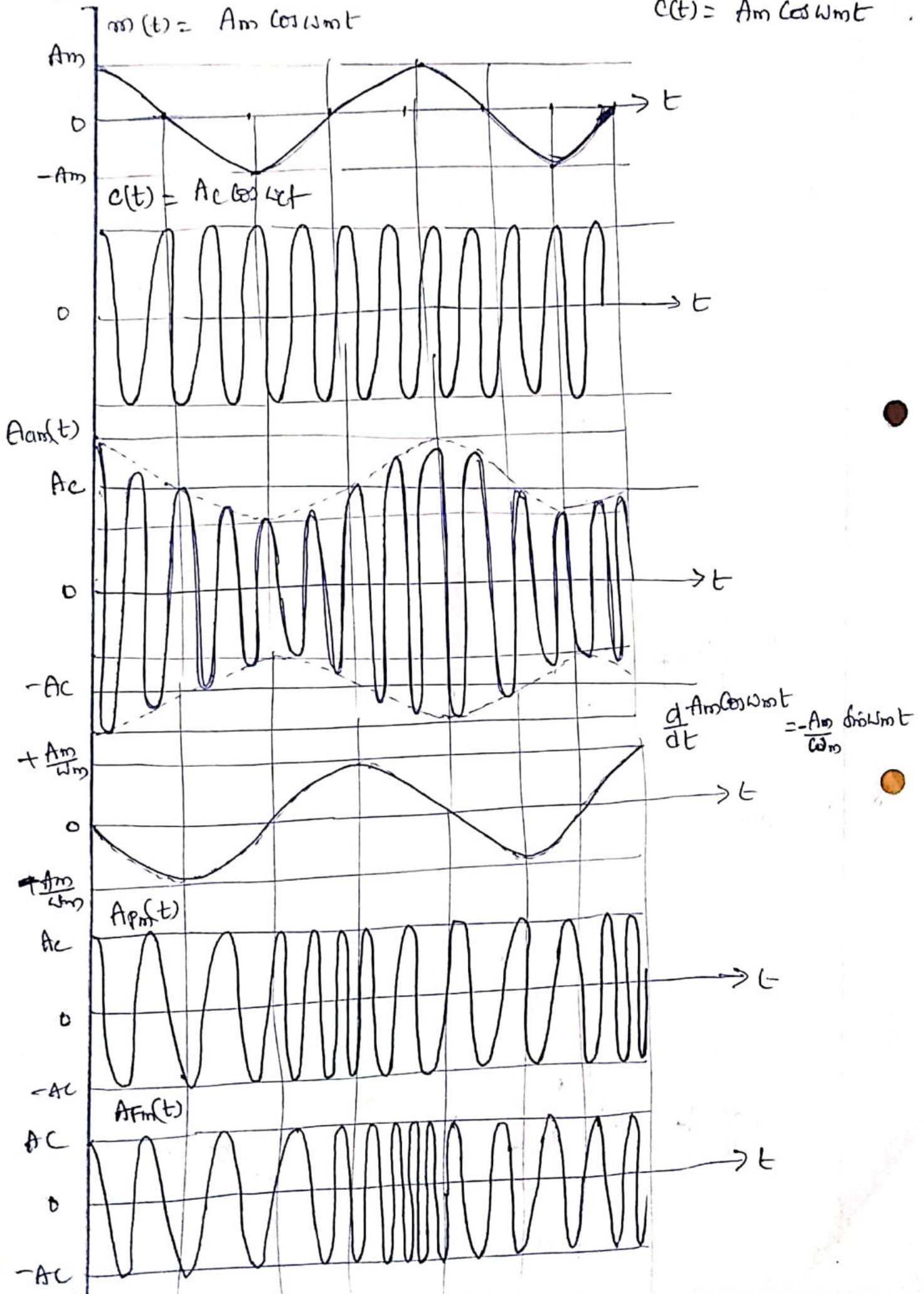


Case (ii) :- Generation of pm signal.

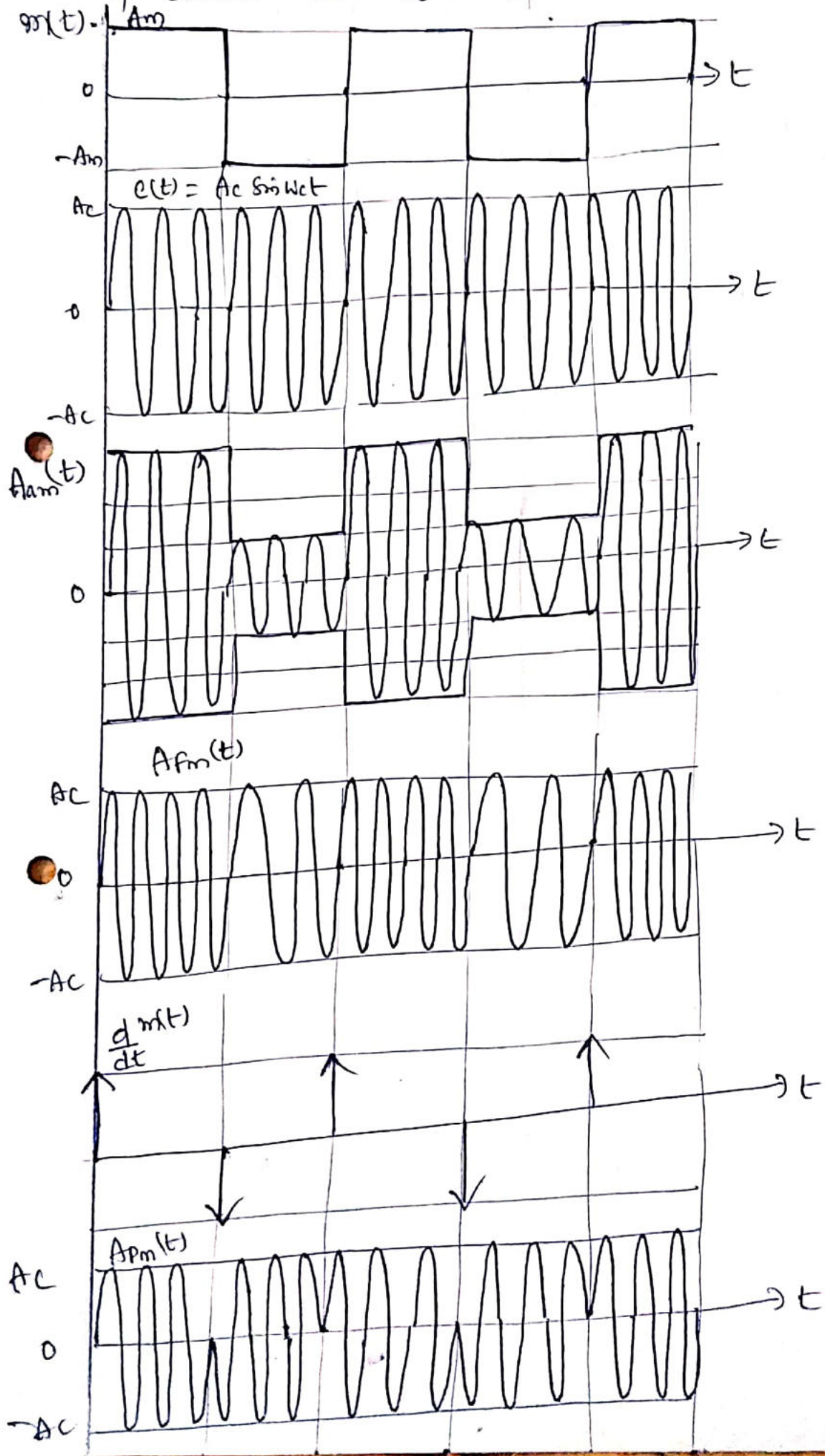


① Draw the AM, FM & PM signal for $m(t) = A_m \cos \omega_m t$

$$c(t) = A_m \cos \omega_m t$$



Q2. Draw the Am, fm & pm signals for $m(t) = \text{Square Wave}$ & Carrier $c(t) = A_c \sin \omega_c t$.



Comparison b/w PM & FM :-

Frequency modulation

- ① Frequency of the Carrier is varied with respect to modulating signal $m(t)$.
- ② Modulation Index

$$m_f = \frac{k_f V_m}{W_m}$$
- ③ $A_{fm}(t) = A_c \cos [w_c t + k_f \int m(t) dt]$
- ④ Integrator is required to generate fm signal using PM modulator.
- ⑤ Band width is low
- ⑥ BW is Proportional to Amplitude of $m(t)$.

$$\Delta W = k_f V_m.$$
- ⑦ It is used to Broad casting Audio signal.
- ⑧ Noise PSD is Parabolic
- ⑨ Less SNR

Phase modulation

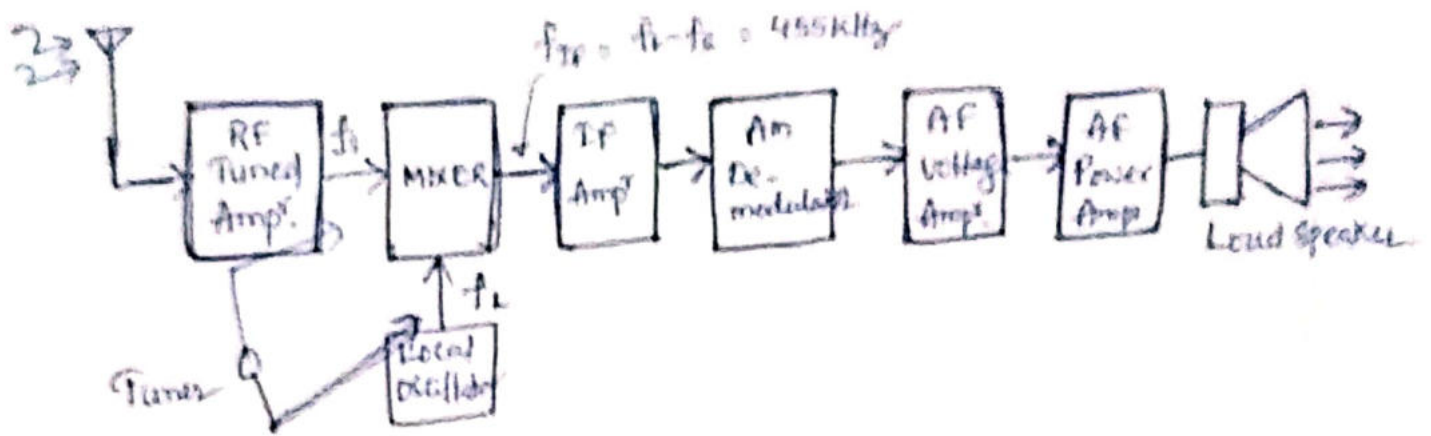
- ① A Phase of the Carrier is varied with respect to modulating signal $m(t)$.
- ② Modulation Index

$$m_p = K_p V_m.$$
- ③ $A_{pm}(t) = A_c \cos [w_c t + k_p m(t)]$,
- ④ Differentiator is required to generate PM signal using FM modulator.
- ⑤ Band width is very high
- ⑥ BW is Proportional to both Amplitude & frequency of $m(t)$.

$$\Delta W = K_p V_m W_m.$$
- ⑦ It is not used for Broad casting.
- ⑧ Noise PSD is Uniform.
- ⑨ more SNR

Super heterodyne AM Receiver :-

25



Heterodyne :- Principle

Mixing two frequencies and generating constant frequency is called "Heterodyne".

Here MIXER, generates sum & difference frequencies b/w f_s & f_L so that $f_{IF} = f_L - f_s$ is appear at the input of IF amplifier.

① Antenna :-

1. Antenna is a Passive device which converts electromagnetic signal into electric signal.
2. Antenna works based on Faraday Law's of Electromagnetic Induction.

② RF Tuned Amplifier :- [Radio frequency amp.]

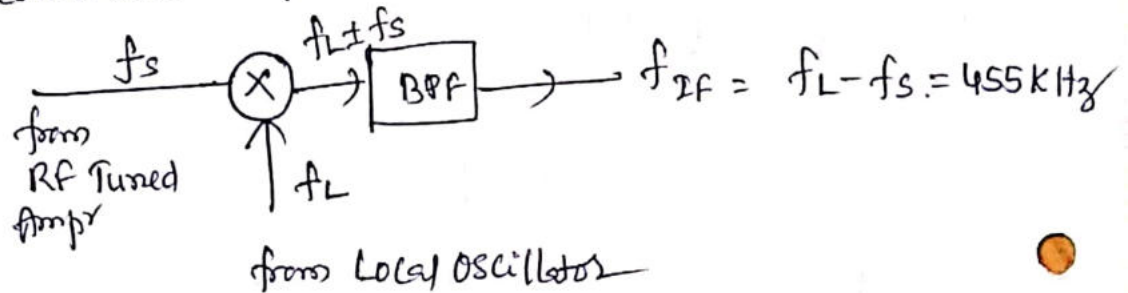
1. It is Broad band amplifier which provide ~~an~~ Pre-amplification for 1100 kHz.
2. It provides very small amplification, Hence it is called "Pre-amplification".

www.android.universityupdates.in | www.universityupdates.in | https://telegram.me/jntua
3. It contains both tuning or Parallel resonance Rkt. and Amplifier.

4. RF Tuned amplifier is responsible for Sensitivity, Selectivity & Image signal rejection.

③ MIXER :-

1. It is combination of multiplier & Bandpass filter as shown below



2. Multiplier generates $f_L \pm f_s$ and BPF selects only difference frequency, which is called "Intermediate" frequency $f_{IF} = f_L - f_s$.

④ Local oscillator :-

1. It is high frequency oscillators such as Hartley or Colpitts or Crystal oscillator.

2. Local oscillator frequency is always greater than Incoming (or) Signal frequency (f_s)

NOTE:- If local oscillator frequency is greater than the incoming signal frequency then less range of Capacitor values are required.

Question
Why local oscillator frequency is greater than ~~incoming~~ incoming signal frequency

④ IF Amplifier :- [IF - Intermediate Amplifier]



1. It is narrow band & high gain amplifier
2. It Provides 90% of total receiver Amplification.
3. The Bandwidth of IF Amplifier is 10 kHz centered at 455 kHz.

(26)

⑤ Demodulator :-

1. It extracts modulating signal from Am signal.
2. An Envelope demodulator is used as demodulator.
3. Fidelity is mainly depends type of demodulator.

⑥ AF voltage Amp :- [AF - Audio frequency Amp.]

1. It is small signal low frequency amplifier.
2. It Provides amplification at [20 Hz - 5 kHz].
3. It is Cascade amplifier i.e. combination of CE amplifier.

⑦ AF Power Amp :-

1. While converting signal from one form to another form, signal should have more power.
2. Power amplifier improves current so that power of signal amplified.
3. Either Class 'A' or Class 'B' (Push Pull) ~~an~~ power amplifiers are employed.

⑧ Loudspeaker :-

It converts electrical signal into ~~an~~ sound signal.

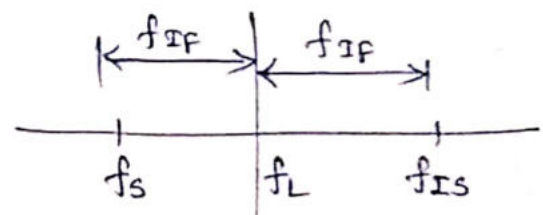
Frequency range :-

- ① Am medium wave Am-mw : (540 - 1640) KHz
- ② Total Band width : (1640 - 540) KHz = 1100 KHz
- ③ Band width of Each station : 10 KHz
- ④ No. of stations available : $\frac{1100 \text{ KHz}}{10 \text{ KHz}} = 110$ Stations
- ⑤ Intermediate Frequency $f_{IF} = 455 \text{ KHz}$.

Image signal & its rejection :-

If the signal frequency is greater than local oscillator frequency by an amount f_{IF} (455 KHz) with respect to 'fs' frequency then signal is known as "Image signal".

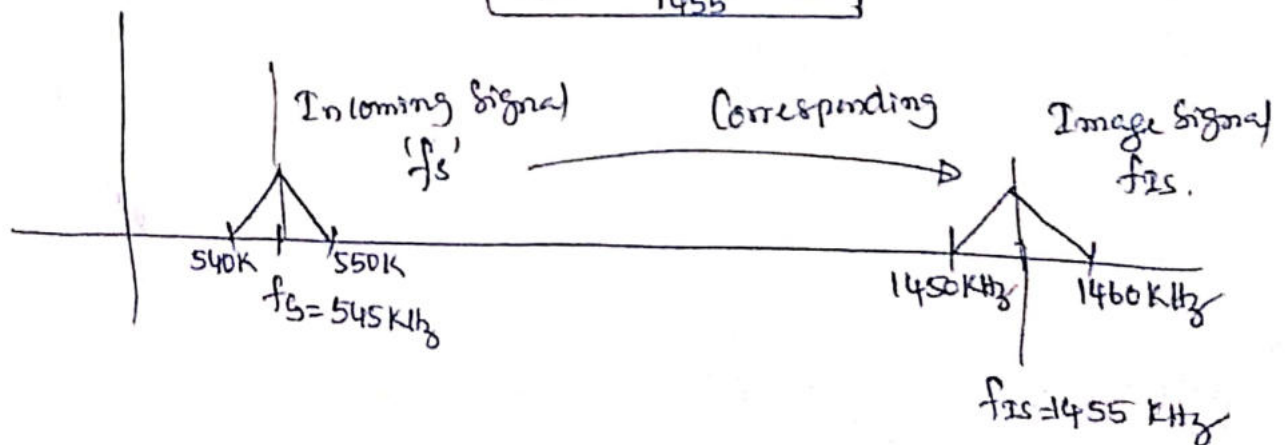
Image signal $f_{IS} = f_s + 2f_{IF}$



Example :- If $f_s = 545 \text{ KHz}$

$$f_{IS} = f_s + 2f_{IF} = 545 \text{ KHz} + 2 \times 455 \text{ KHz}$$

$f_{IS} = 1455 \text{ KHz}$



∴ Image signal can be rejected in the RF tuned Amp, by providing more gain to incoming signal and less gain to Image signal.

The Image rejection ratio (IRR)

$$\alpha = \frac{G_{fs}}{G_{fsI}} = \sqrt{1 + p^2 Q^2}$$

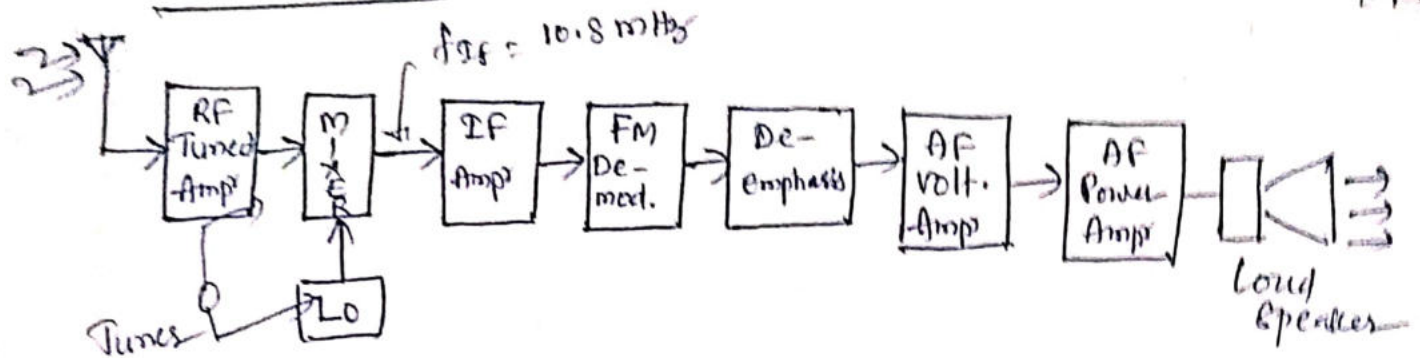
Constant $p = \frac{f_{IS}}{f_s} - \frac{f_s}{f_{IS}}$

Q = Quality factor of resonance Ckt.

Choice of Intermediate frequency (f_{IF}) :-

1. If IF is too high, results in poor selectivity and poor adjacent channel rejection.
2. High value of IF increases tracking difficulties.
3. As the IF lowered, Image signal rejection becomes poor.
4. A very low IF can make the selectivity too sharp, cuts the side bands.
5. If the IF is low, the frequency stability of the local oscillator must be correspondingly higher.
6. The IF must not fall within the tuning range of the receiver, else instability will occur.

Fm - Receiver :-



- i) Frequency range : (88-108) mHz
- ii) Total Band width : 20 mHz
- iii) BW of each station : 200 KHz
- iv) NO. of stations available : $\frac{20 \text{ mHz}}{200 \text{ KHz}} = \frac{20 \times 10^6}{200 \times 10^3} = \frac{2 \times 10^7}{2 \times 10^5} = 100 \text{ Stations.}$
- v) Intermediate frequency $f_{if} = 10.8 \text{ MHz.}$

Frequency Division Multiplexing (FDM) :-

1. Multiplexing : Transmission of several same message signals through same (or) single channel.
2. Two types of multiplexing are employed in communication.
 - ① Time Division Multiplexing (TDM)
 - ② Frequency Division Multiplexing (FDM)
3. In TDM, different time slots are allocated for different stations.
In FDM, different frequency slots are allocated for different stations (i.e. different carrier frequency).

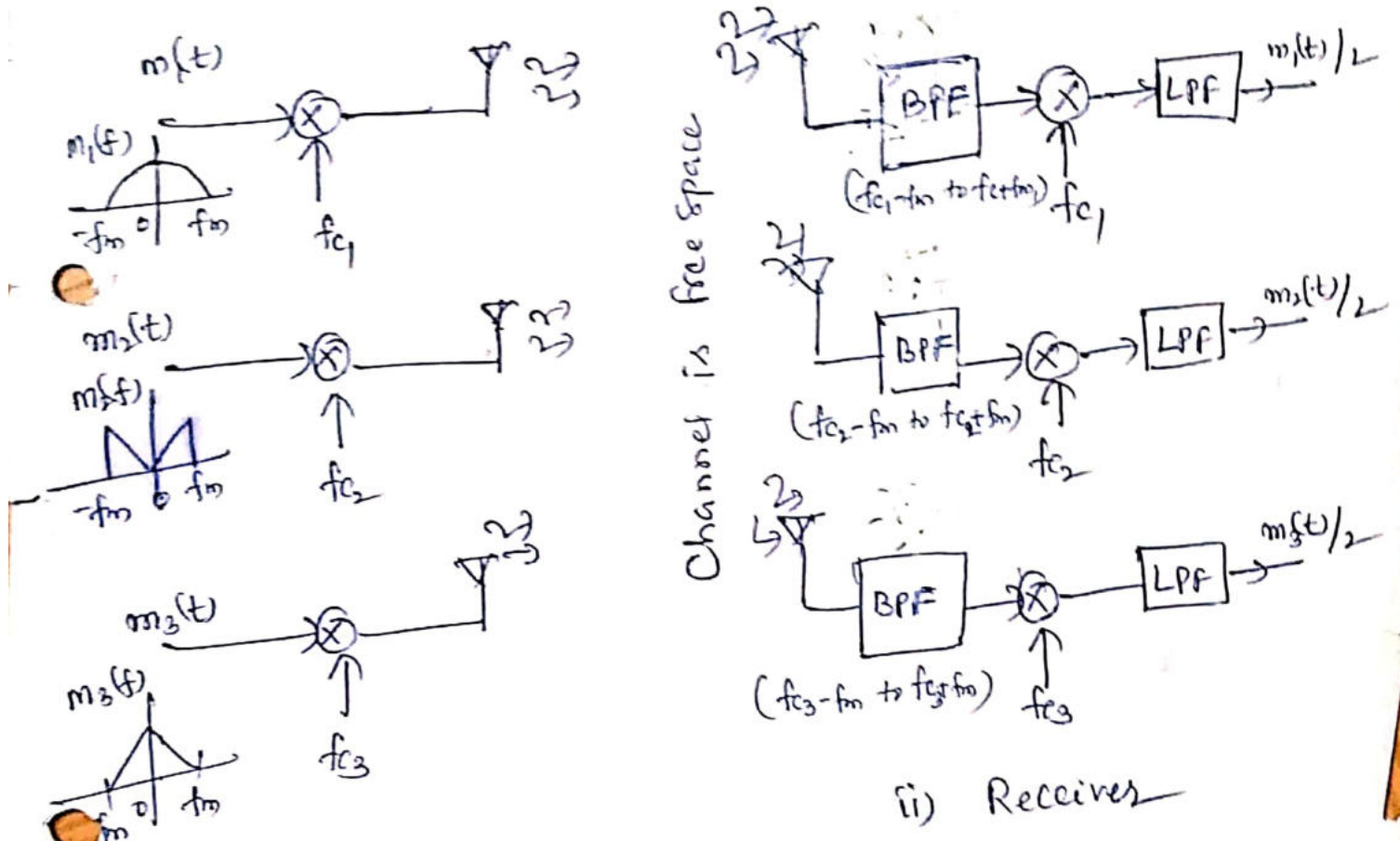
4. TDM is Digital multiplexing Technique

Example: Transmission of Voice signal using Telephone

FDM is Analog multiplexing Technique.

Example: Radio & TV Transmission.

5. Block Diagram of FDM is shown in figure.



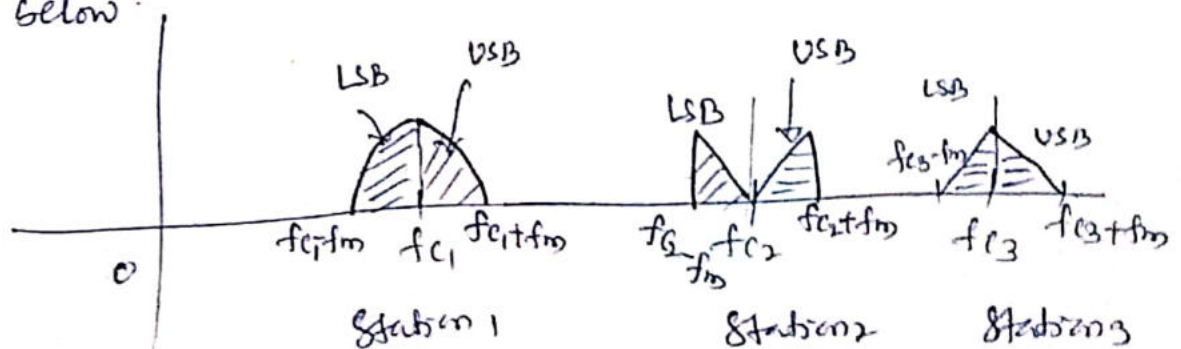
ii) Receiver

i) Transmitter

6) DSB-SC modulator is employed in XMITR & Synchronous

Demodulator is employed in RCVR.

7) Spectrum of different stations will as shown below.



Comparison b/w FDM and TDM

FDM

TDM

- | | |
|---|---|
| ① Different frequencies are allotted for different stations | ① Different time slots are allotted for each station. |
| ② It is Analog multiplexing Type | ② It is Digital multiplexing type |
| ③ It is used in Radio & TV Transmission | ③ It is used in Digital Telephony. |
| ④ more Complexity & cost | ④ Less Complexity & cost |
| ⑤ Less SNR | ⑤ more SNR. |

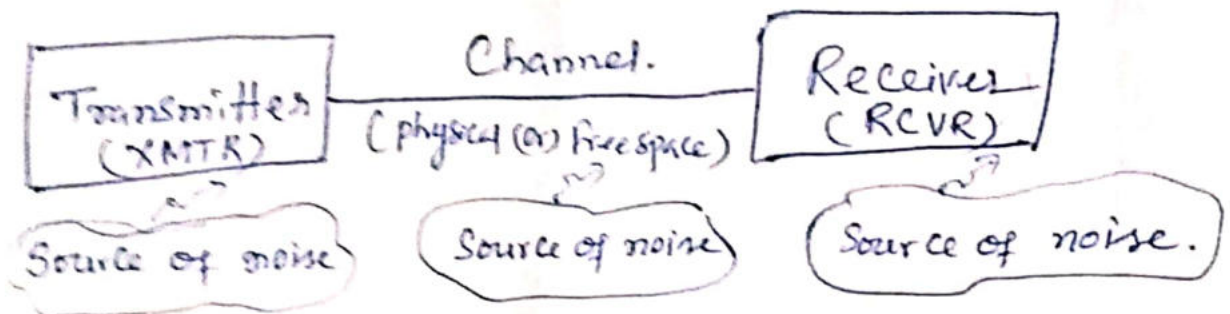
Unit - III

①

Noise Calculation in Analog Modulation Systems

Noise :- It is unwanted form of energy which tends to interfere with modulated signal.

Basic Block Diagram of Communication system :-



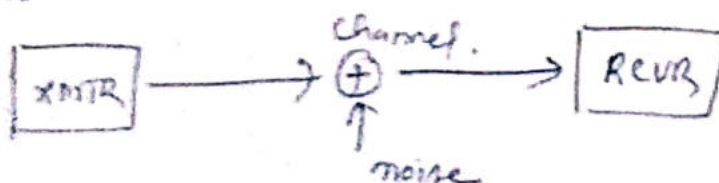
① XMTR Operated at KW or MW of power, hence noise effect is negligible.

② Receiver is operated at μW or nW or pW of power hence noise effect will be more

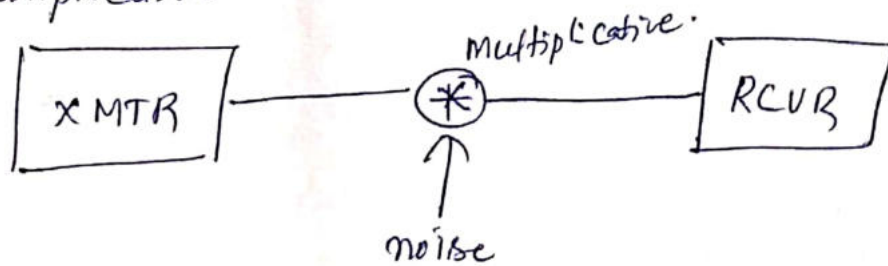
③ Channel ~~now~~ is main or major Source of noise because Channel is Transmission media.

④ Channel may be physical Channel (Two wire or Coaxial cable, optical cable or waveguide etc), or free space.

⑤ If small amount of noise is added by the Channel then it is considered as ~~an~~ Additive Channel (or) noise is additive. i.e



- ④ If large amount of noise added by the Channel. then it is called Multiplicative Channel (or) noise is multiplicative i.e.



- ④ noise generated by the Channel is called "External noise".
- ⑤ noise generated by the Receiver is called "Internal noise".

Types of External noise :-

- ① Cosmic noise - (stars, moon) generates cosmic noise.
- ② Solar noise - Sun generates solar noise.
- ③ Man made noise - noise generated by Automobile ignition, Aircrafts, electric motors, switch gears, welding — is called Man made noise.

Types of Internal noise :-

- ① Shot noise - Random motion of electrons in the S/c Devices generates shot noise.
- ② Thermal (or) Johnson noise :- Random motion of electron in the resistor is called "Thermal noise".

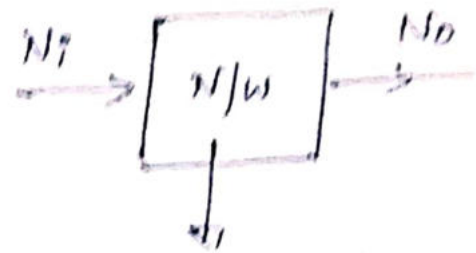
$$V_n = \sqrt{4 K T_0 B R}$$

K - Boltzmann constant
 T_0 - Absolute temp.
 B - Bandwidth
 R - Resistor

(*) Noise Figure (F_n) : It is ratio of Output noise and input noise of an amplifier (or) network,

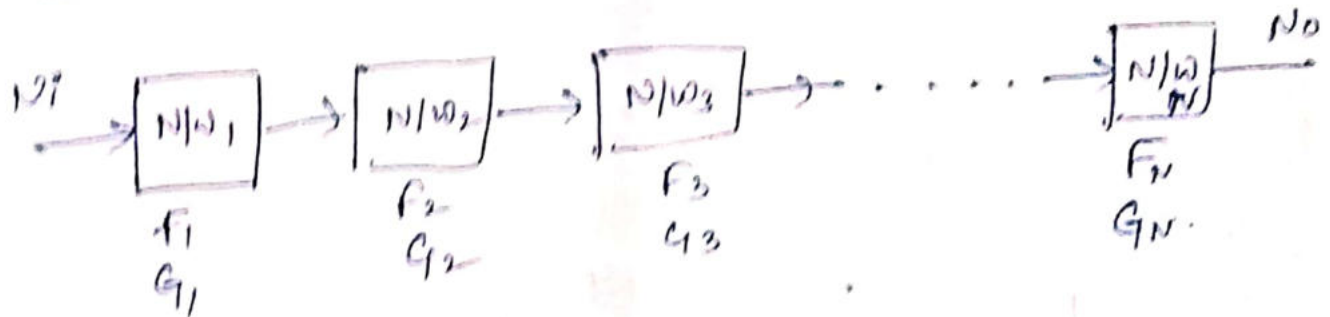
$$F_n = \frac{N_o}{N_i} \quad \begin{array}{l} N_o \rightarrow \text{Output noise} \\ N_i \rightarrow \text{Input noise} \end{array}$$

$$= \frac{N_i + A N}{N_i}$$



$A N$ = noise generated in the network.

Noise Figure of an Cascaded Amplifier (or) network



$$F_n = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \dots + \frac{F_N - 1}{G_1 G_2 \dots G_{N-1}}$$

(*) Noise Temperature (T_e) :-

The noise Temperature is a means for specifying noise in terms of an equivalent Temperature

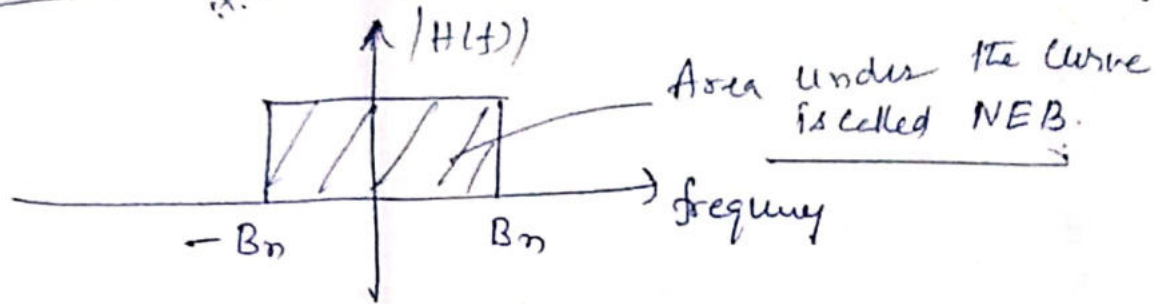
$$T_e = (F_n - 1) T_o$$

F_n — Noise Figure

T_o → Absolute temp.

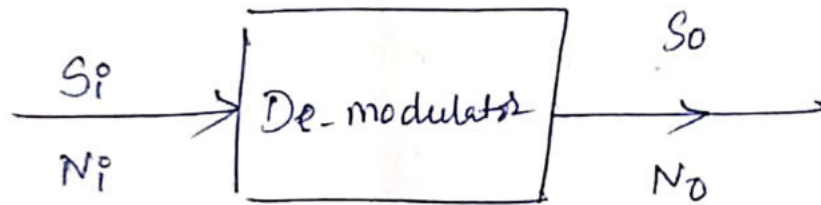
(4)

* Noise Equivalent Bandwidths :-



* Figure of merit (Fom)

It is ratio of output SNR to the input SNR of a Communication system.



$$FOM = \frac{\text{Output SNR}}{\text{Input SNR}} = \frac{\frac{S_o}{N_o}}{\frac{S_i}{N_i}} = \frac{S_o}{N_o} \times \frac{N_i}{S_i}$$

$S_o \rightarrow$ output ^{Signal} ~~noise~~ Power of Demodulator

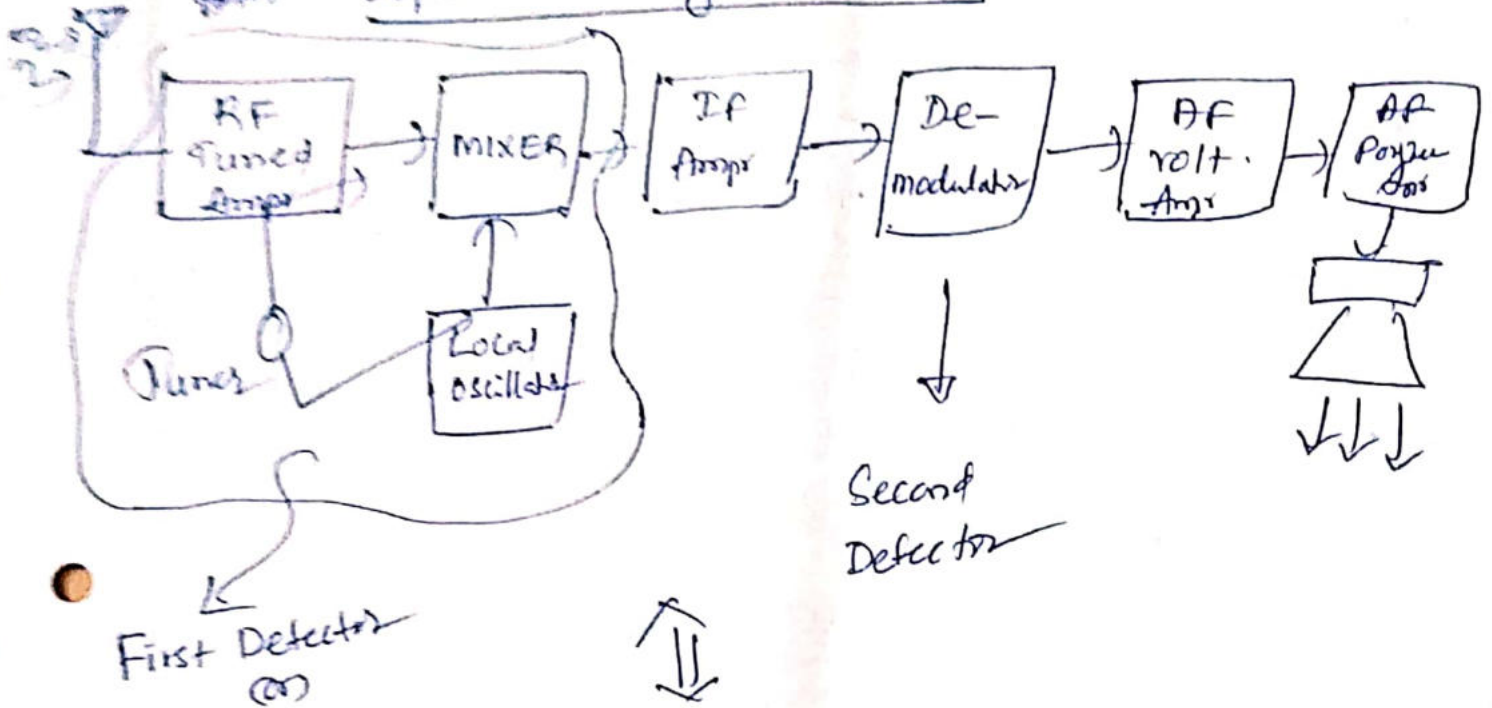
$N_o \rightarrow$ output noise Power of "

$S_i \rightarrow$ Signal Power at the i/p of Demodulator

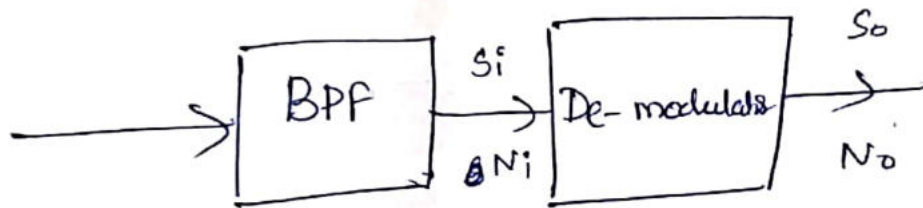
$N_i \rightarrow$ Noise Power at the i/p of Demodulator

Receiver Model for noise Calculation :-

Super Heterodyne AM Receiver

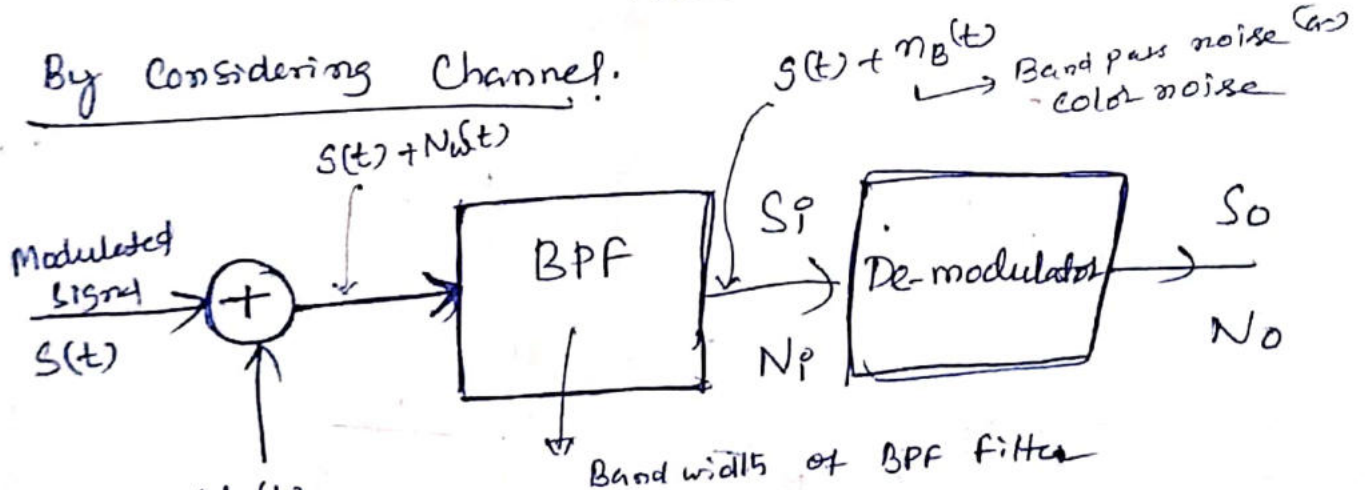


BPF



Receiver

By Considering Channel.



White noise

Case (i) :- AM — $(f_c - f_m)$ to $(f_c + f_m)$
DSB-SC

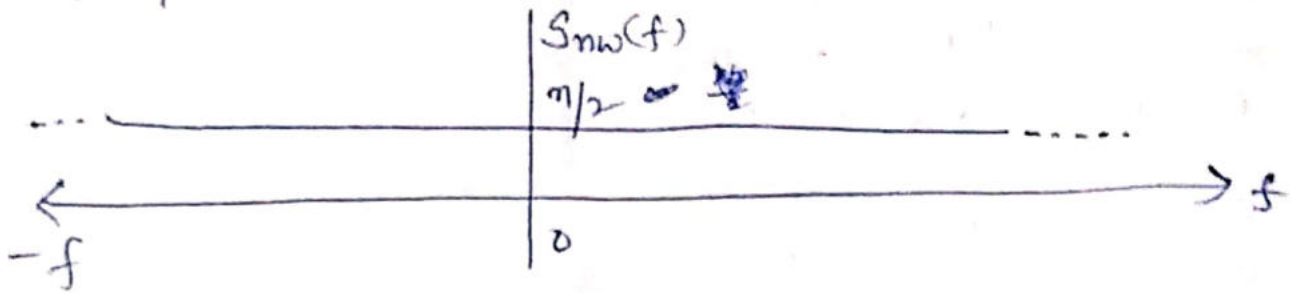
Case (ii) :- SSB-SC — $[f_c - f_m \text{ to } f_c]$ or $[f_c \text{ to } f_c + f_m]$

Case (iii) :- FM — $[f_c - \Delta f \text{ to } f_c + \Delta f]$

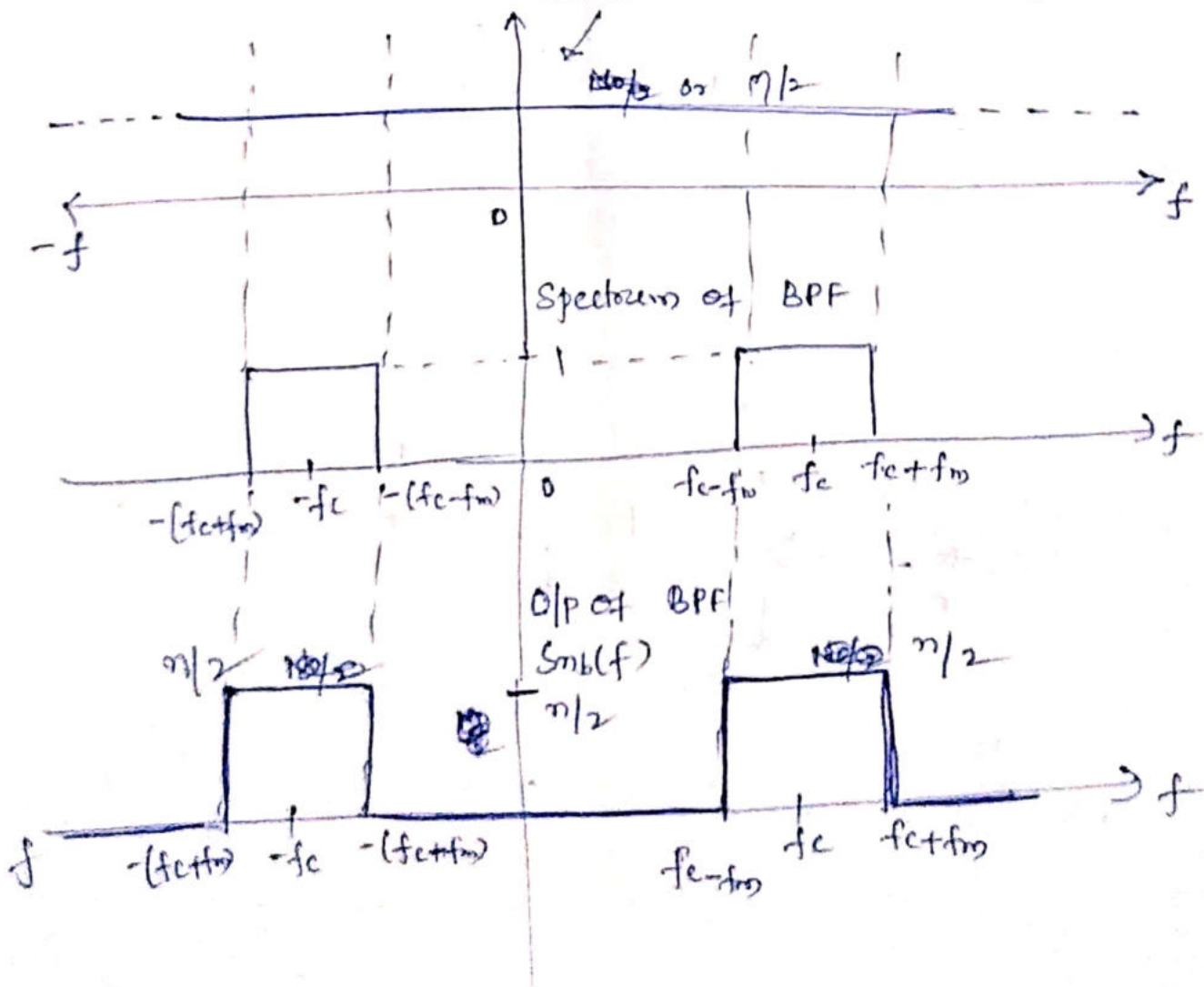
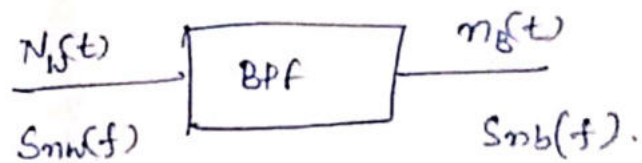
White noise & Band Pass noise

6

White noise :- White noise contains all frequency components whose PSD can be written as.



Band Pass noise :- BPF converts White noise into Band pass noise
(or)
Color noise



Power of Band Pass noise (or) Color noise :-

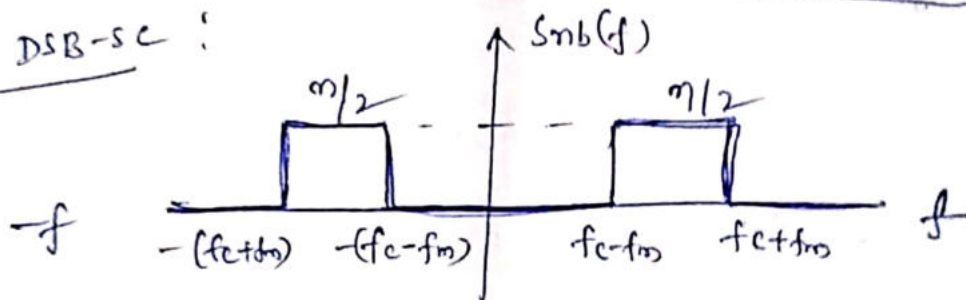
$$\text{Output noise Power of BPF} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \text{PSD } d\omega = \int_{-\infty}^{+\infty} \text{PSD } df$$

PSD is always even function

$$= \frac{1}{2\pi} \times 2 \times \int_{0}^{\infty} \text{PSD } d\omega = 2 \int_{0}^{\infty} \text{PSD } df$$

$$\boxed{\text{Output noise Power of BPF } N_0 = \frac{1}{\pi} \int_{0}^{\infty} \text{PSD } d\omega = 2 \int_{0}^{\infty} \text{PSD } df}$$

For AM or DSB-SC :



$$N_0 = 2 \int_{fc-fm}^{fc+fm} \frac{m}{2} df = 2 \times \frac{m}{2} \times \left[f \right]_{fc-fm}^{fc+fm}$$

$$N_0 = m \left[fc+fm - (fc-fm) \right] = m \cdot (2fm)$$

$$\boxed{N_0 = m B}$$

Bandwidth of AM or DSB-SC

DSB-SC :- $N_0 = m \cdot fm$

fm : $N_0 = m \cdot (2fm)$

Band Pass noise representation :-

① Time Domain Representation

- Quadrature representation
- Envelope representation

② Frequency Domain representation.

Quadrature representation :-

$$n_B(t) = n_i(t) \cos \omega_c t \pm n_q(t) \sin \omega_c t$$

(or)

$$n_B(t) = n_i(t) \cos(2\pi f_c t) \pm n_q(t) \sin(2\pi f_c t)$$

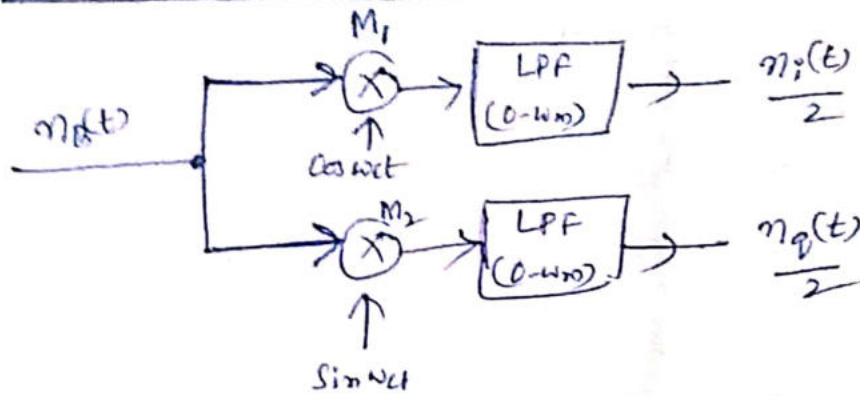
SSB-SC : ~~SSB-SC~~ $SSB-SC(t) = m(t) \cos \omega_c t \pm m_h(t) \sin \omega_c t$

DSB-SC : $DSB-SC(t) = m(t) \cos \omega_c t$

→ $m_i(t)$ = Inphase Component of BP noise

$m_q(t)$ = Quadrature phase Component of BP noise

How to recover $m_i(t)$ & $m_q(t)$ from $m_B(t)$:-



$$m_B(t) = m_i(t) \cos wt \pm m_q(t) \sin wt$$

O/p of multiplier m_1 is $M_1(t) = m_B(t) \times \cos wt$

$$\begin{aligned} M_1(t) &= [m_i(t) \cos wt \pm m_q(t) \sin wt] \cos wt \\ &= m_i(t) \cos^2 wt \pm \frac{m_q(t)}{2} (\sin wt \cos wt) \times 2 \\ &= m_i(t) \left[1 + \frac{\cos 2wt}{2} \right] \pm \frac{m_q(t)}{2} \sin 2wt \\ &= \frac{m_i(t)}{2} + \frac{m_i(t)}{2} \cos 2wt \pm \frac{m_q(t)}{2} \sin 2wt \end{aligned}$$

O/p of LPF will be $\frac{m_i(t)}{2}$

O/p of multiplier ② i.e m_2 is
 $m_2(t) = m_B(t) \times \sin wt$

$$\begin{aligned} &= [m_i(t) \cos wt \pm m_q(t) \sin wt] \sin wt \\ &= 2 m_i(t) \frac{\sin wt \cos wt}{2} \pm m_q(t) \sin^2 wt \\ &= \frac{m_i(t)}{2} \sin 2wt \pm m_q(t) \left[\frac{1 - \cos 2wt}{2} \right] \\ &= \frac{m_i(t)}{2} \sin 2wt \pm \frac{m_q(t)}{2} - \frac{m_q(t)}{2} \cos 2wt \end{aligned}$$

O/p of LPF is $= \frac{m_q(t)}{2}$

Note:- $m_B(t)$, $m_i(t)$ & $m_q(t)$ will have same Power
 i.e $\frac{m_B^2(t)}{2} = \frac{m_i^2(t)}{2} = \frac{m_q^2(t)}{2} = \underline{m_B}$

② Envelope representation of BP noise :-

⑩

$$n_B(t) = n_i(t) \cos \omega_c t \pm n_q(t) \sin \omega_c t$$

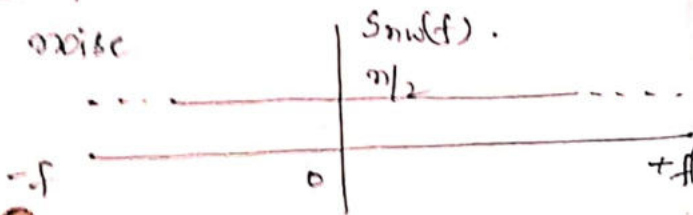
$$= E(t) \cos [\theta(t)]$$

$$\text{Envelope } E(t) = \sqrt{n_i^2(t) + n_q^2(t)}$$

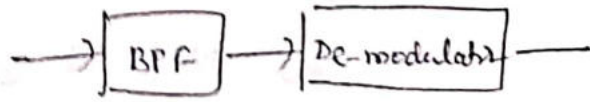
$$\text{Angle } \theta(t) = \tan^{-1} \left(\frac{n_q(t)}{n_i(t)} \right)$$

FOM Cal in DSB-SC - Fundamentals required :-

- ① XMTX is not source of noise.
- ② Channel Adds External noise.
- ③ RXRX adds internal noise.
- ④ Here Channel noise is considered because it is not in our control. & it is considered as white noise



- ⑤ Receiver is modelled as BPF followed by Demodulator



- ⑥ BW of BPF is equal to Signal Bandwidth i.e. AM & DSB-SC $\rightarrow 2f_m$
SSB-SC $\rightarrow f_m$
FM $\rightarrow 2\Delta f$

- ⑦ BPF Converts White noise into Colored noise
 $n_B(t) = n_i(t) \cos \omega_c t \pm n_q(t) \sin \omega_c t$

- ⑧ Power of $n_B(t)$, $n_i(t)$ & $n_q(t)$ are same i.e. $N_0 = \eta \cdot B$

BW	$B = 2f_m$	AM & DSB-SC
	$= f_m$	SSB-SC
	$= 2\Delta f$	FM

- ⑨ All noise Powers S_o, N_o, S_i, N_i are calculated as Total Power (or) Peak Power i.e. Total Power of modulating signal $m(t)$ can be written as $\overline{m^2(t)}$

Total Power (or) Mean Square value of r.v. 'x' can be written as $E[x^2] = \overline{x^2}$

If $m(t) = A_m \cos \omega_m t$ then mean square value or Total Power is $\overline{m^2(t)} = \overline{[A_m \cos \omega_m t]^2}$

$$= A_m^2 \cdot \overline{\cos^2 \omega_m t}$$

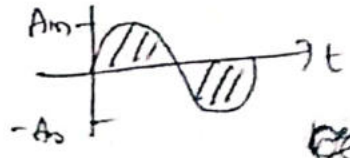
$$= \frac{A_m^2}{2} \cdot \overline{\cos^2 \omega_m t}$$

$$\overline{m^2(t)} = \frac{A_m^2}{2}$$

r.m.s value
 $\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

Mean value of $m(t)$ is

$\overline{m(t)} = 0$ i.e. Avg. value of Sine & Cosine is zero.



Output Signal Power

$$S_o = \overline{\left[\frac{m(t)}{2}\right]^2} = \frac{\overline{m^2(t)}}{4} \quad (4)$$

Output noise Power

$$N_o = \overline{\left[\frac{n_i(t)}{2}\right]^2} = \frac{\overline{n_i^2(t)}}{4}$$

$$= \frac{\eta \cdot B}{4} = \frac{\eta \cdot f_m}{4}$$

$$N_o = \frac{\eta f_m}{2} \quad (5)$$

Output SNR $\frac{S_o}{N_o} = \frac{\overline{m^2(t)}}{4} \times \frac{2}{\eta f_m}$

$$\frac{S_o}{N_o} = \frac{\overline{m^2(t)}}{2\eta f_m} \quad (6)$$

$$\begin{aligned} (A) \Rightarrow F_o m &= \frac{O/P \text{ SNR}}{i/p \text{ SNR}} = \frac{\overline{m^2(t)}}{2\eta f_m} \cdot \frac{4\eta f_m}{\overline{m^2(t)}} \\ &= \frac{\overline{m^2(t)}}{2\eta f_m} \times \frac{4\eta f_m}{\overline{m^2(t)}} \end{aligned}$$

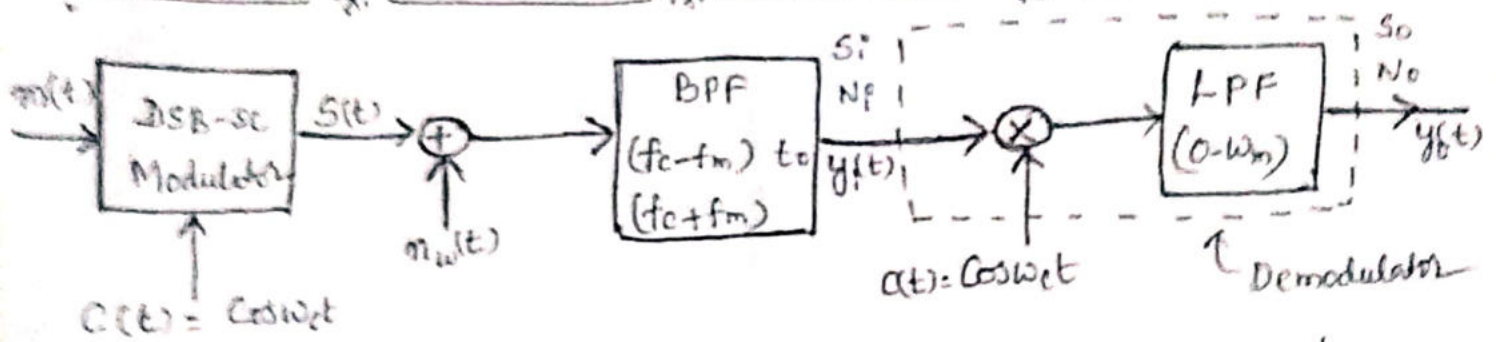
$$F_o m = 2$$

$$F_o m \text{ (dB)} = 10 \log_{10}(2) = 3 \text{ dB}$$

$$F_o m = 3 \text{ dB}$$

Output SNR is twice the i/p SNR

FIGURE OF MERIT (FOM) in DSB-SC System :- (or) SNR in DSB-SC



Transmitter → Channel → Receiver

DSB-SC signal $S(t) = m(t) \cos wt$

Channel: Channel is considered as "Additive Channel" which adds white noise, whose PSD is $\eta/2$.

Receiver: Band width of BPF is $B = 2f_m$

Demodulator — Synchronous Detector which contains multiplier followed by LPF.

$$FOM = \frac{\text{OUT SNR}}{\text{input SNR}} = \frac{S_o / N_o}{S_i / N_i} \quad \text{--- (A)}$$

$$FOM (dB) = 10 \log_{10} (FOM)$$

Output of BPF is

$$y_1(t) = S(t) + n_B(t)$$

$$= \underbrace{m(t) \cos wt}_{S_i} + \underbrace{n_B(t)}_{N_i}$$

$$\text{Input signal Power } S_i = \overline{[m(t) \cos wt]^2}$$

$$= \overline{m^2(t)} \cdot \overline{\cos^2 wt}$$

$$S_i = \frac{\overline{m^2(t)}}{2} \quad \text{--- (1)}$$

$$\text{Input noise Power } N_i = \overline{n_B^2(t)}$$

$$N_i = \eta \cdot B = \eta \cdot 2f_m \quad \text{--- (2)}$$

$$\text{Input SNR } \frac{S_i}{N_i} = \frac{\frac{\overline{m^2(t)}}{2}}{\frac{\eta \cdot 2f_m}{4}} = \frac{\overline{m^2(t)}}{4\eta f_m} \quad \text{--- (3)}$$

Output of Synchronous Demodulator

$$y_0(t) = \text{LPF} [y_1(t) \cdot \cos wt]$$

$$= \text{LPF} [m(t) \cos wt + n_B(t)] \cdot \cos wt$$

$$= \text{LPF} [m(t) \cos^2 wt + n_B(t) \cos wt]$$

$$= \text{LPF} \left[m(t) \left[\frac{1 + \cos 2wt}{2} \right] + [n_B(t) \cos wt + n_Q(t) \sin wt] \cos wt \right]$$

$$= \text{LPF} \left[\frac{m(t)}{2} + \frac{m(t) \cos 2wt}{2} + \frac{n_B(t)}{2} \cos^2 wt \pm n_Q(t) \sin wt \cos wt \right]$$

$$= \text{LPF} \left[\frac{m(t)}{2} + \frac{m(t)}{2} \cos 2wt + \frac{n_B(t)}{2} + \frac{n_B(t)}{2} \cos 2wt \pm \frac{n_Q(t)}{2} \sin 2wt \right]$$

$$y_0(t) = \underbrace{\frac{m(t)}{2}}_{S_o} + \underbrace{\frac{n_B(t)}{2}}_{N_o}$$

Unit - IV

Pulse Modulation & Demodulation

Pulse Modulation (or) Discrete Wave modulation :- It is the process of Varying Characteristics of Pulse (or) Discrete Wave (or) Square wave in accordance with the instantaneous values of the modulating signal.

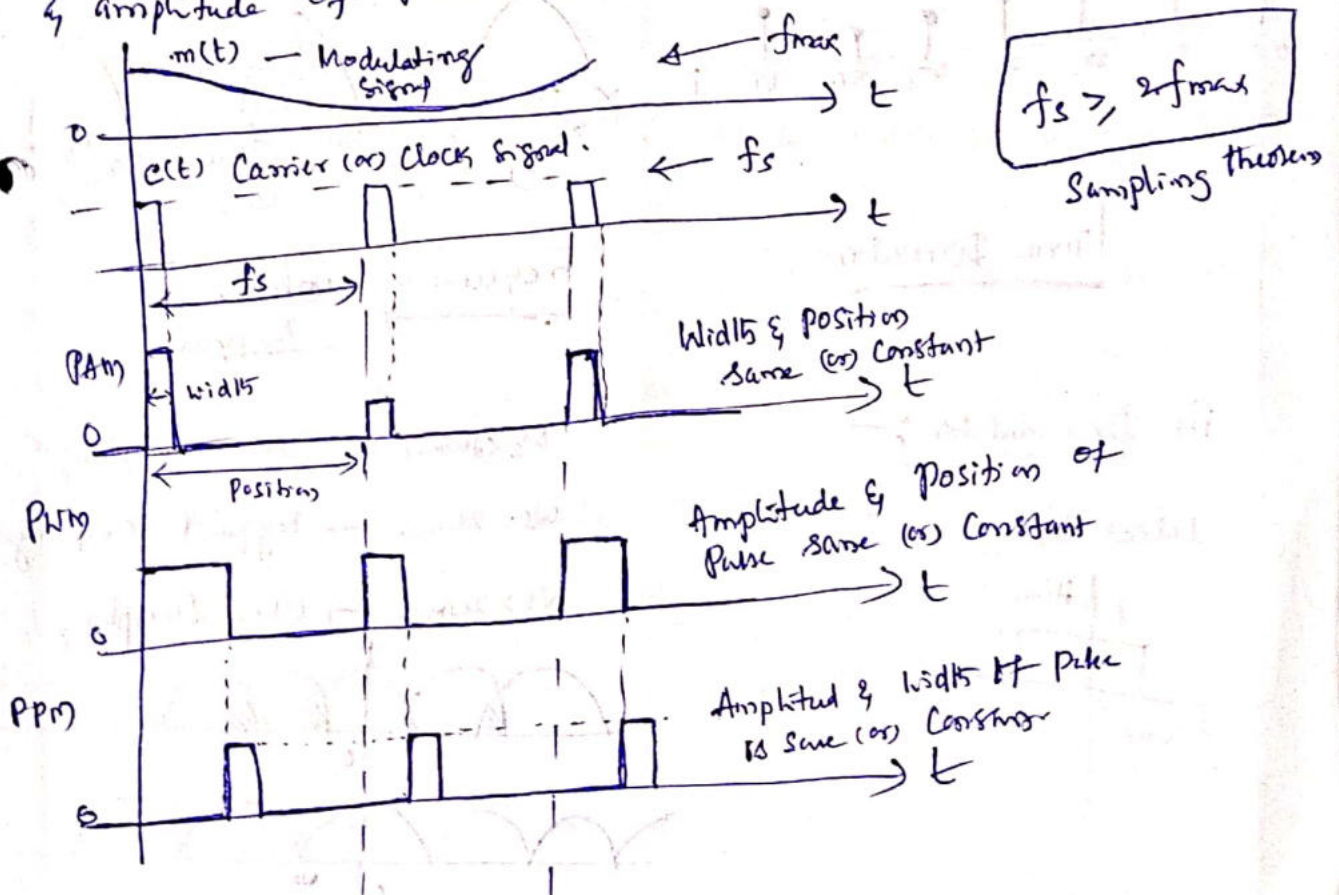
Types :-

1. Pulse Amplitude Modulation (PAM)
2. Pulse Time modulation (PTM)
 - Pulse width modulation (PWM)
 - Pulse Position Modulation

PAM :- It is a process of Varying amplitude of pulse in accordance with the instantaneous values of Modulating signal & width and position of pulse remains same or constant.

PWM :- It is a process of Varying width of pulse in accordance with the instantaneous values of the modulating signal and amplitude & position of the pulse constant or remains same.

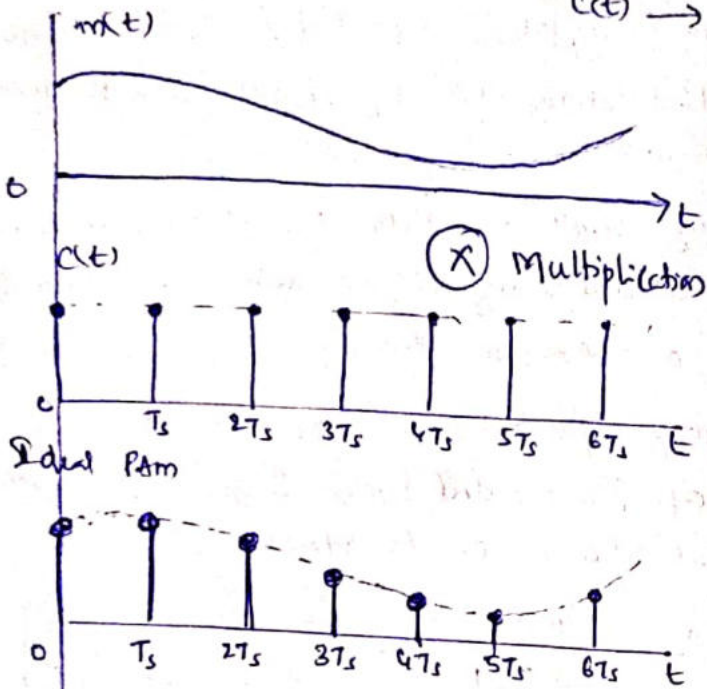
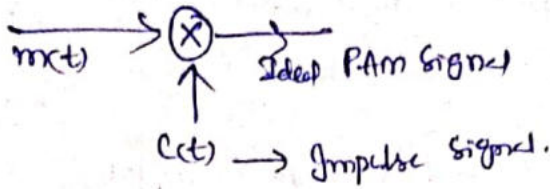
PPM :- It is a process of Varying position of pulse in accordance with the instantaneous values of the modulating signal and width & amplitude of pulse remains same or constant.



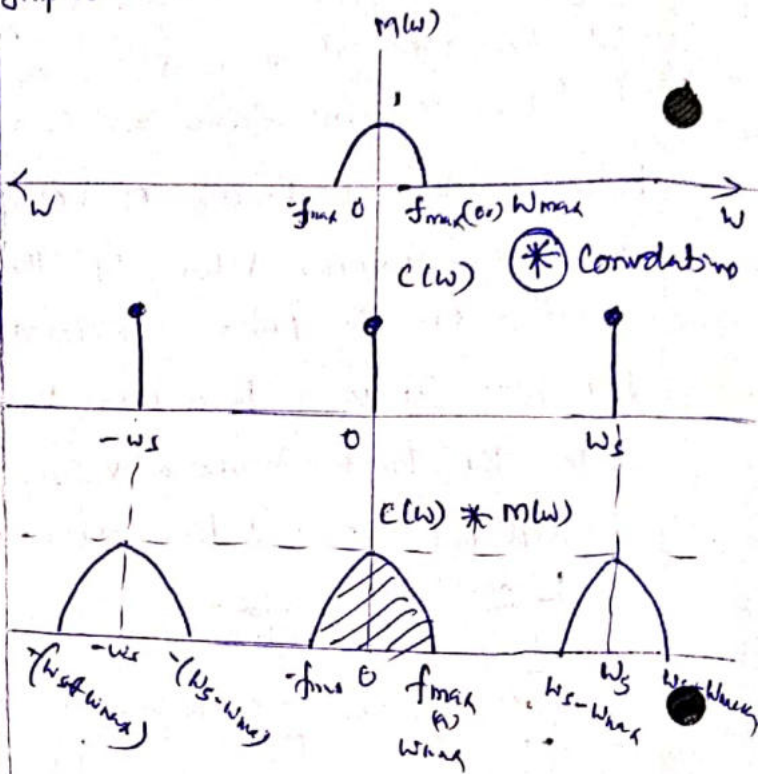
- ① Ideal (or) Instantaneous Sampling (or) PAM
- ② Natural Sampling (or) Natural PAM.
- ③ Flat top Sampling (or) Flat top PAM.

① Ideal (or) Instantaneous Sampling (or) Ideal PAM :-

i) Modulator :-



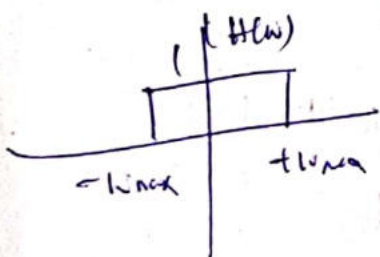
Time Domain



Frequency Domain

ii) Demodulator :-

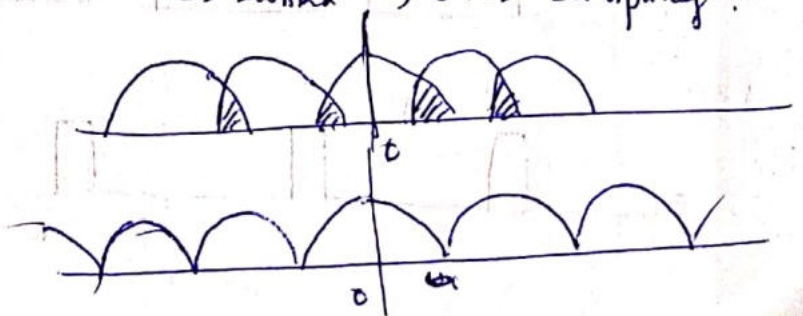
Ideal LPP.



$w_s < 2W_{max} \rightarrow$ Under sampling

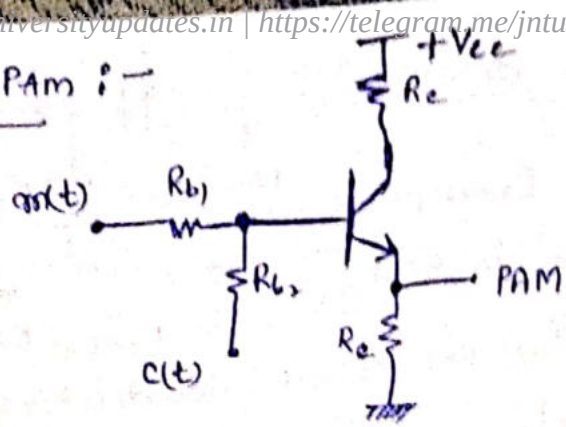
$w_s = 2W_{max} \rightarrow$ Nyquist Sampling

$w_s > 2W_{max} \rightarrow$ Over Sampling.



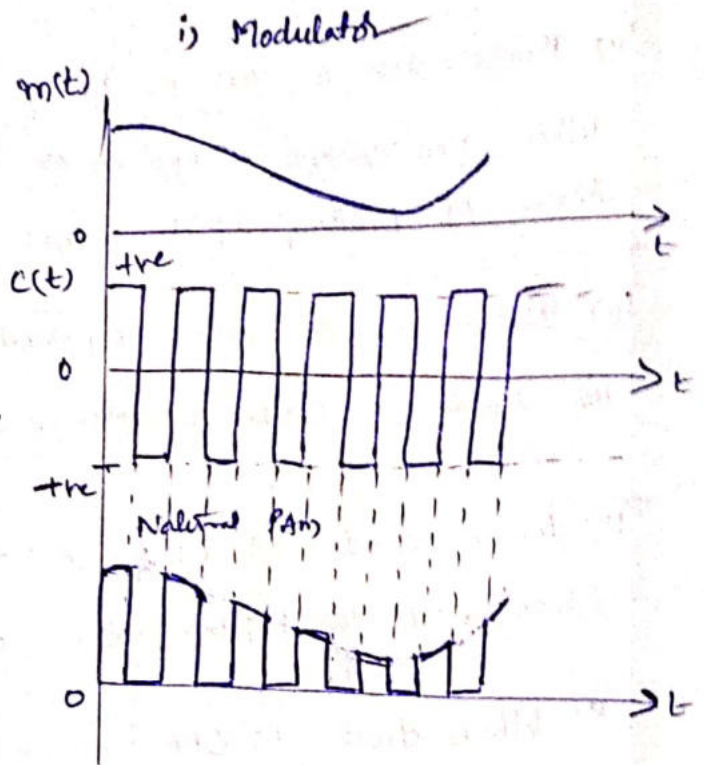
Natural Sampling or Natural PAM :-

1. The Circuit is simple Emitter follower (or) Buffer Switch.
2. Both modulating & clock signal are applied at the base of the Transistor.



3. Transistor is operated b/w Saturation and Cut-off.
4. If Transistor is 'ON' then it goes to Saturation region & if off then it goes to Cut-off region i.e.

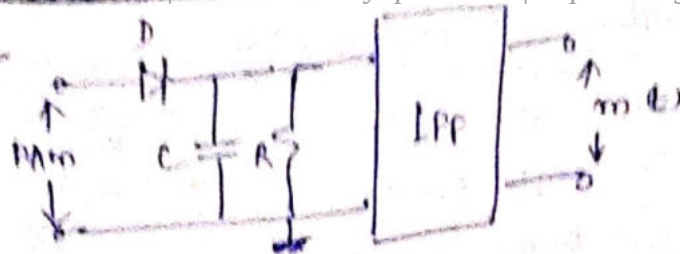
5. ON or off state of Transistor depends only on clock $c(t)$ i.e. $|c(t)|_{\max} \gg |m(t)|_{\max}$



6. When +ve clock is Present, Transistor will be in 'ON' State, In this case both $m(t)$ & $c(t)$ will be multiplied & appears at the O/P.
7. When -ve clock is Present, Transistor will be in 'OFF' State, In this case no output.
8. Above Circuit generates Natural PAM Signal.

Demodulator :-

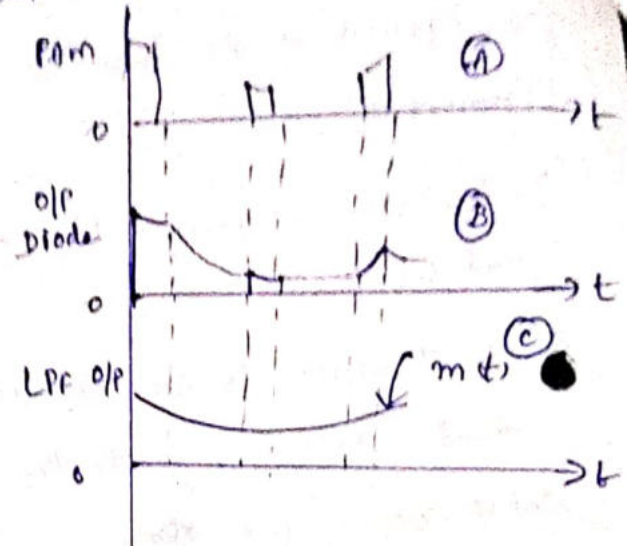
i) An Envelope Detector is used



to detect (or) extract modulating signal $m(t)$ from Natural PAM signal.

ii) Diode acts as ON or OFF, when +ve voltage is applied at the Anode of Diode, it turns ON.

iii) When -ve (or) 0 volt applied at the Anode of Diode, it turns OFF.



iv) When diode is ON i.e. +ve pulse present, Capacitor charges to the Peak value of i/p pulse.

v) When diode is OFF i.e. -ve (or) 0 volt is applied, Capacitor discharges through resistor R .

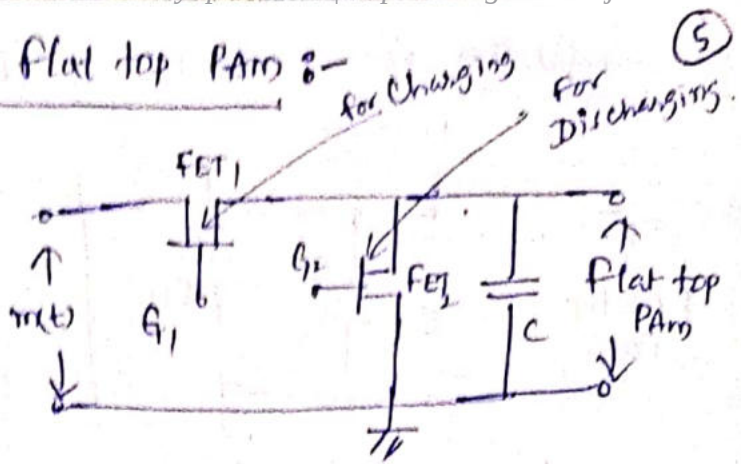
vi) Diode, Resistor & Capacitor ^{combination} detects the envelope of the PAM signal which contains ripples. (As shown in fig (B))

vii) Low Pass filter removes the ripples, ~~and~~ so that Smoothed (or) original $m(t)$ appears at the output.

iii) Flat top Sampling (or) Flat top PAM :-

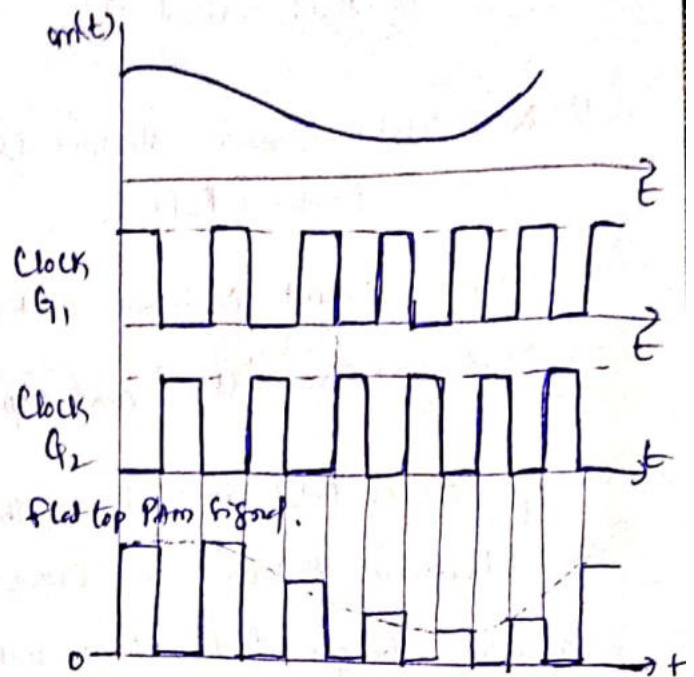
Modulator

1) The Circuit diagram of Flat top PAM modulator is shown in figure.



2) FET₁ is used for Charging & FET₂ is used for Discharging of Capacitor 'C'.

3) Clock 'G₁' is applied to FET₁ and Clock 'G₂' is applied to FET₂ which are inverted, as shown in figure.



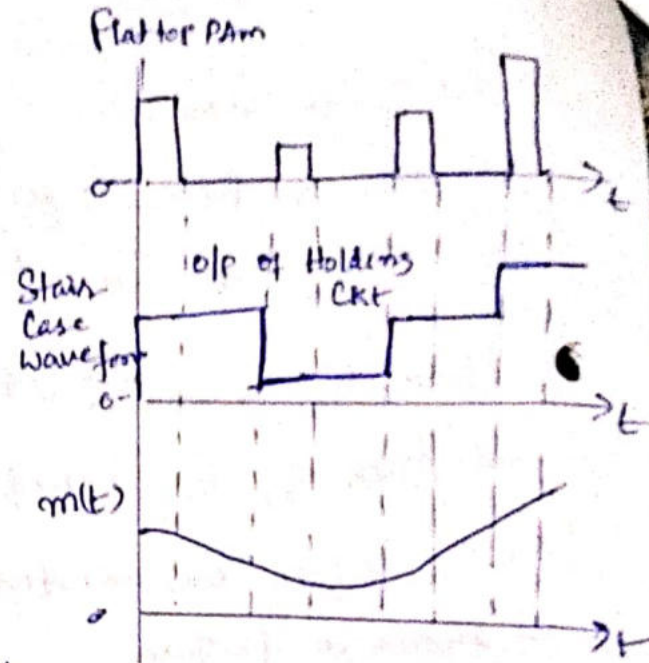
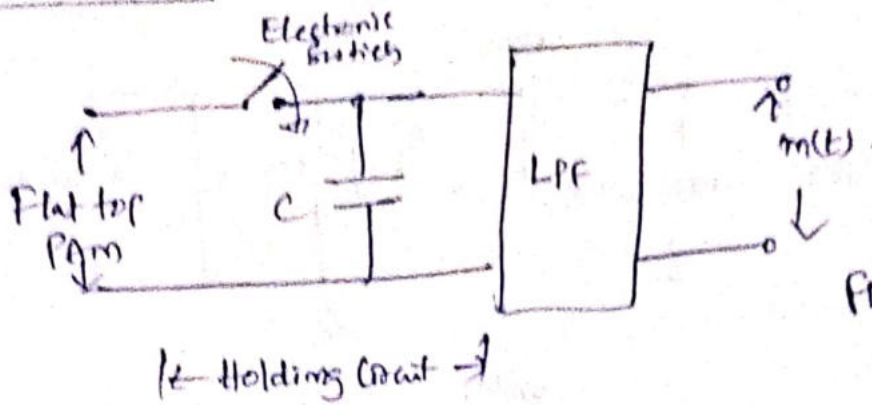
4) FET₁ & FET₂ turns 'on' if +ve voltage is applied at the gate and vice versa.

5) Assume Capacitor 'C' is ideal i.e. Charges & Discharges Very quickly.

6) When 'G₁' is +ve Capacitor charges to Peak value of m(t) because FET₁ is in ON state and Discharges through FET₂ when 'G₂' is +ve.

7) Charging & Discharging of Capacitor generates Flat top PAM signal.

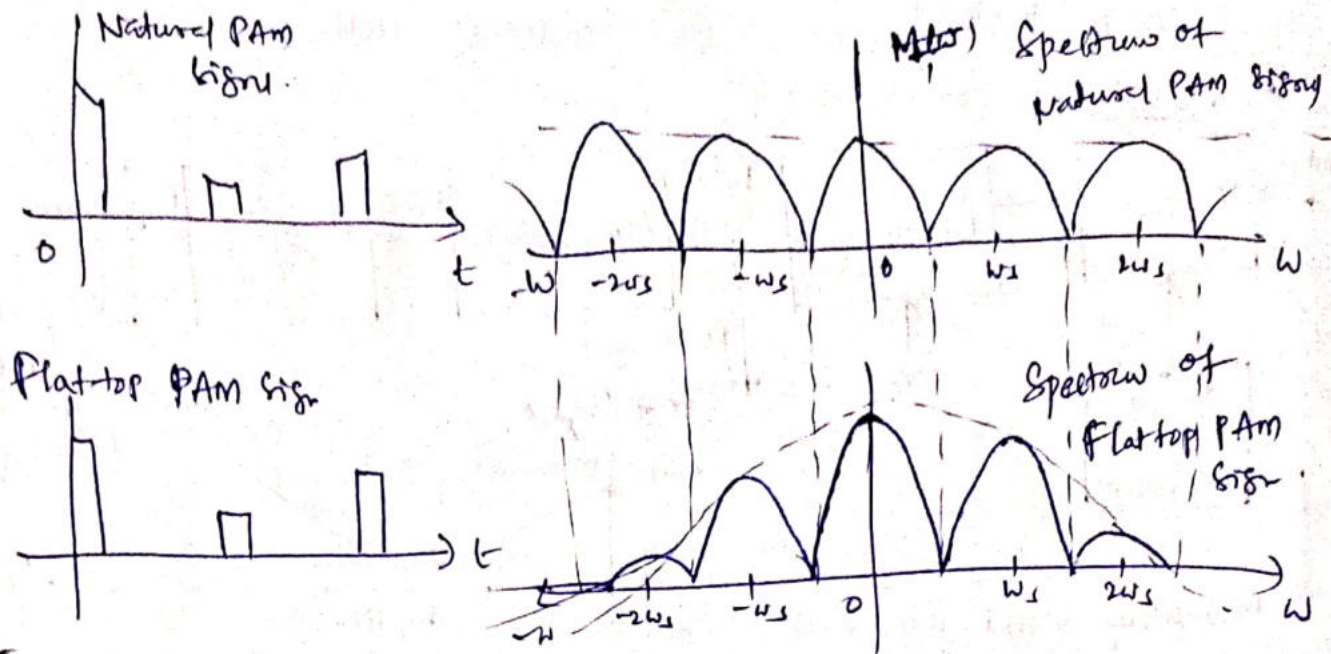
Demodulator :- Holding Circuit



- 1) Demodulator contains Holding Ckt & Low pass Filter (LPF).
- 2) Holding Circuit contains Switch (Diode (or) Transistor) and Capacitor.
- 3) If positive pulse Present at the i/p of Electronic switch then closes so that Capacitor Charges to the Peak value of i/p pulse.
- 4) Capacitor holds the same voltage (or) charged voltage until the Presence of next pulse.
- 5) Holding Circuit generates "Stair Case" Wave form as shown in figure.
- 6) LPF removes the ripples so that Smoothed (or) original $m(t)$ appears at the output.

Spectrum of Natural PAM & Flat top PAM Signals.

(7)



i) The Spectrums of both Natural PAM & Flat top PAM are shown in figure.

ii) Attenuation Present on the some of higher frequency terms in flat top PAM and it is called "Aposture Effect".

iii) This effect is due to lose of some information due to flat top.

iv) But Advantage of flat top is SNR is high.

Applications of PAM :- i) Used in Pulse code modulation & TDM.

ii) used in microcontrollers for generating control signal.

Advantage: i) it requires less BW i.e. $f_s = 2f_{max} = BW$.

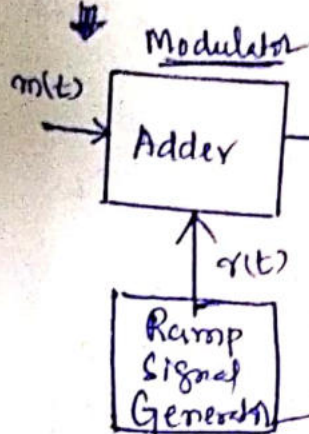
ii) Circuit Complexity is Very Less.

iii) Does not require Synchronization.

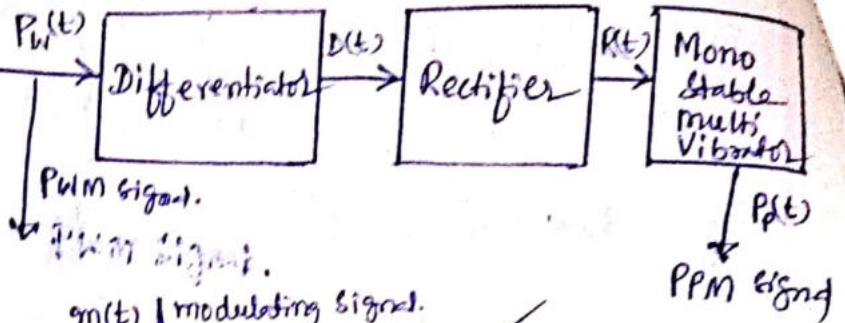
Disadvantage:- i) Less Signal to noise Ratio (SNR)

ii) It requires more Transmission power

① Direct Method



② Indirect method



- ① Modulating signal $m(t)$ and ramp signal $r(t)$ will be added by the Adder as shown in fig ②
- ② ramp pulses will be generated by the ramp generator, whose frequency f_s is adjusted to satisfy the sampling theorem i.e. $f_s > 2f_{max}$

f_{max} - maximum frequency of $m(t)$

- ③ O/P of Adder is compared with DC value ' V_c ' by the Comparator.

- ④ Comparator generates +ve volt when $A(t) > V_c$ and generates -ve volt when $A(t) < V_c$.

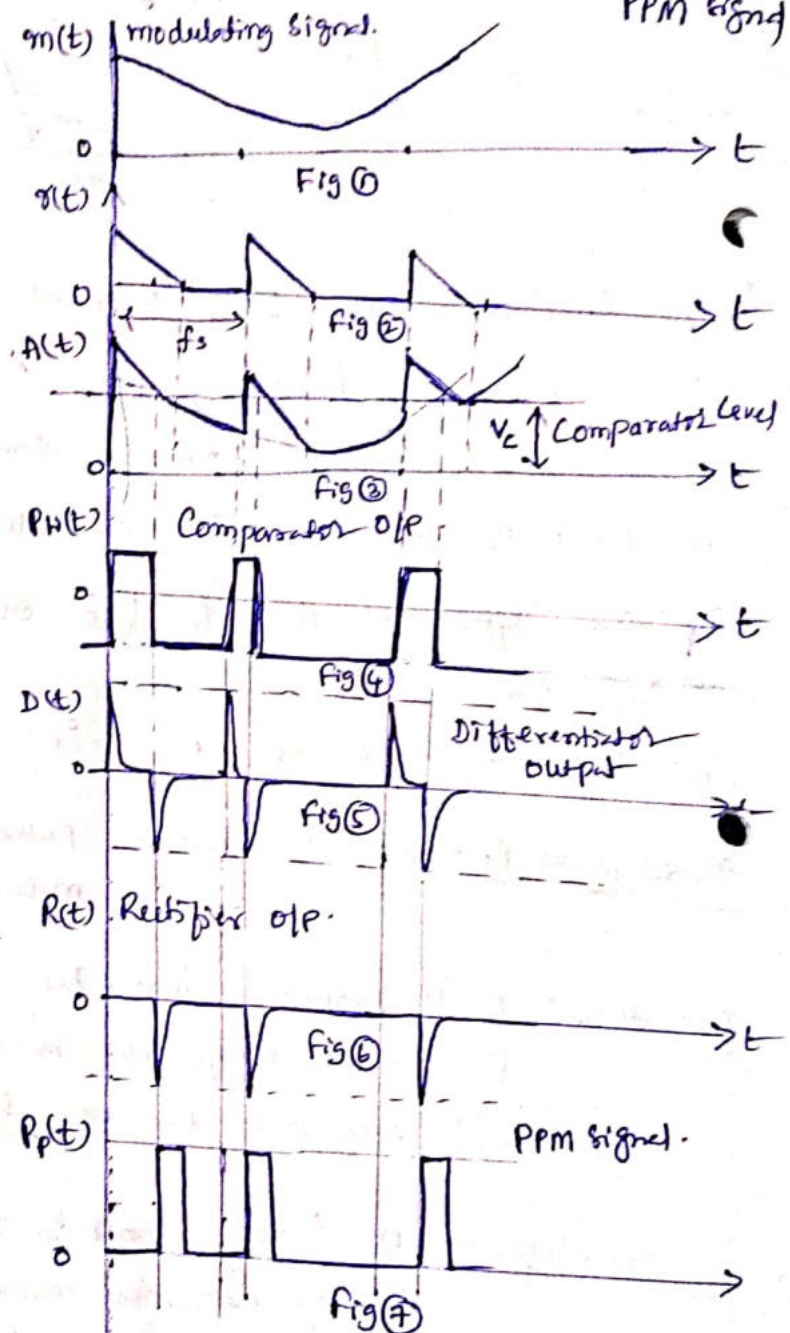
- ⑤ The O/P of Comparator is equal to pulse width modulated signal (PWM).

- ⑥ Differentiator generates +ve & -ve spikes as shown in figure ⑤.

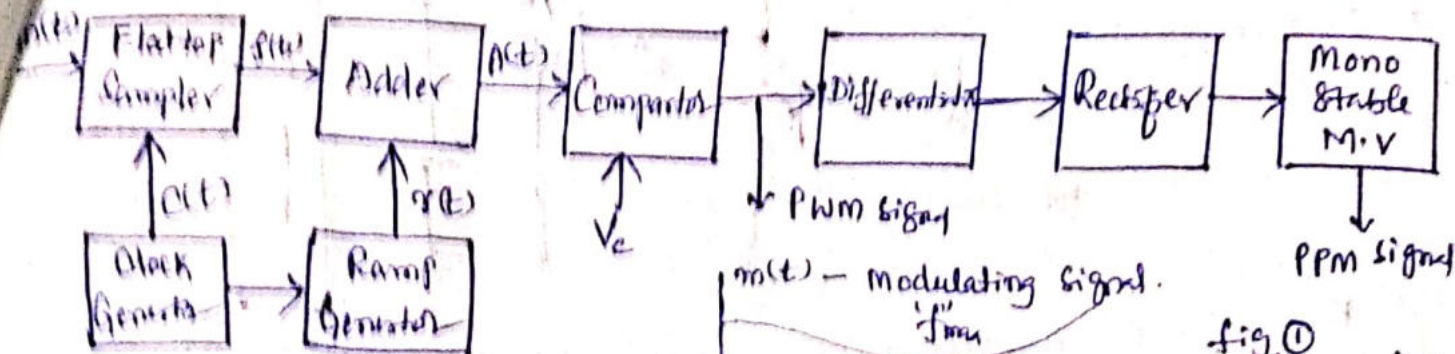
- ⑦ Rectifier remove the spikes as shown in figure ⑥

- ⑧ Monostable multivibrator generates constant amplitude & width pulses for each spikes.

- ⑨ The O/P of Monostable multivibrator is equal to Pulse Position Modulated signal i.e. PPM signal.



Indirect Method - (Modulator)



① Flat top sampler generates flat top sample signal as shown in fig. (1), in which amplitude varies in accordance with modulating signal $m(t)$.

② Clock generator generates clock signal as shown in figure (2), such that $f_s \geq 2f_{max}$.

③ Ramp generator generates one ramp pulse for each clock pulse as shown in figure (3).

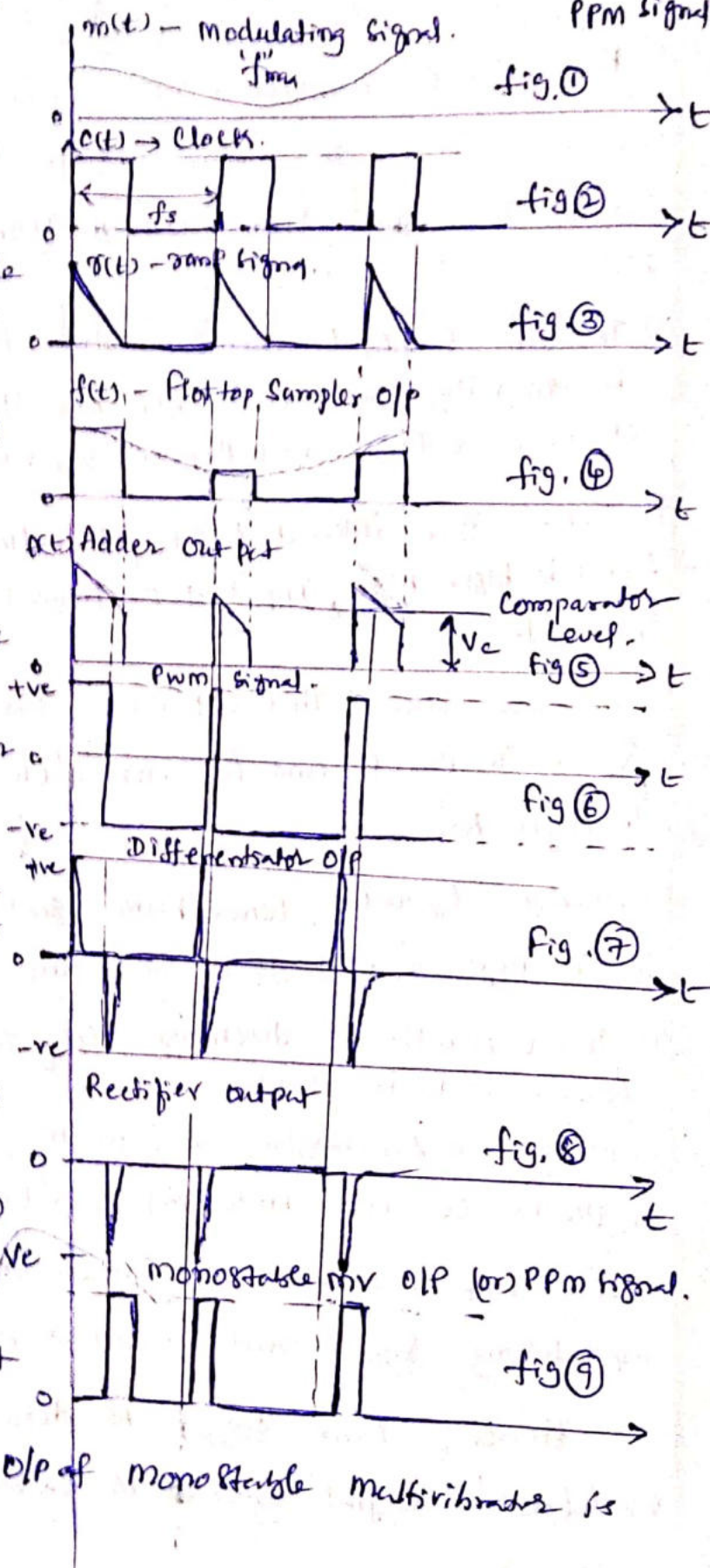
④ Flat top sampler o/p & Ramp generator o/p will adder by the adder as shown in figure (5).

⑤ Comparator compares o/p of Adder with V_c value and generates Pwm signal as shown in fig (6).

⑥ Differentiator generates spikes for Pwm signal as shown in fig (7).

⑦ Rectifier generates allows only -ve pulses as shown in figure (8).

⑧ Monostable multivibrator generates pulses with constant amplitude & width for each -ve spikes. The o/p of monostable multivibrator is equal to PPM signal.

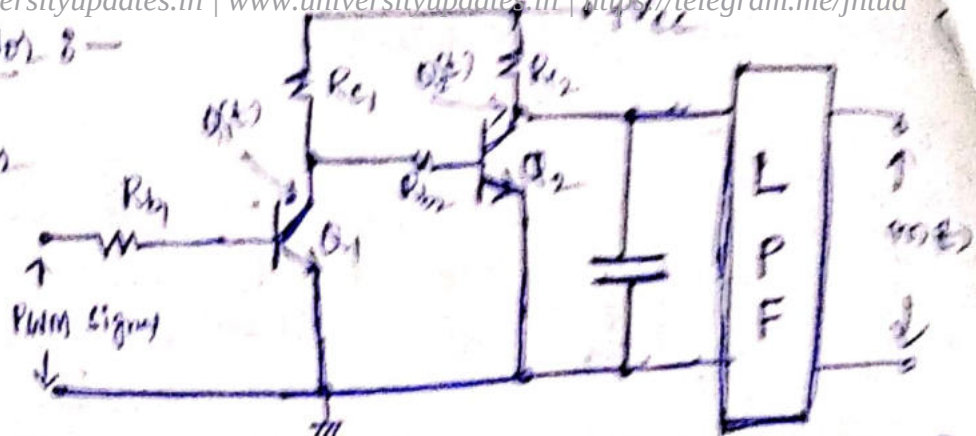


PWM Demodulator :-

1) PWM Demodulator

Circuit diagram is

shown in figure.



2) Here Q_1 & Q_2 Transistors acts as inverter i.e. on or off. If Transistor is in Saturation region then it is considered as 'on' state and If Transistor is in Cut-off region then it is considered as 'off' state.

3) Transistor Q_1 & Q_2 become 'on' when V_{be} is +ve voltage and become 'off' when V_{be} at base either -ve voltage or zero voltage.

4) During Time interval t_1 & t_2 , When PWM signal is high, input to the Transistor Q_2 is low.

5) Therefore during this interval, the Transistor Q_2 is in cutoff and Capacitor 'C' Charges Through R_{c2} .

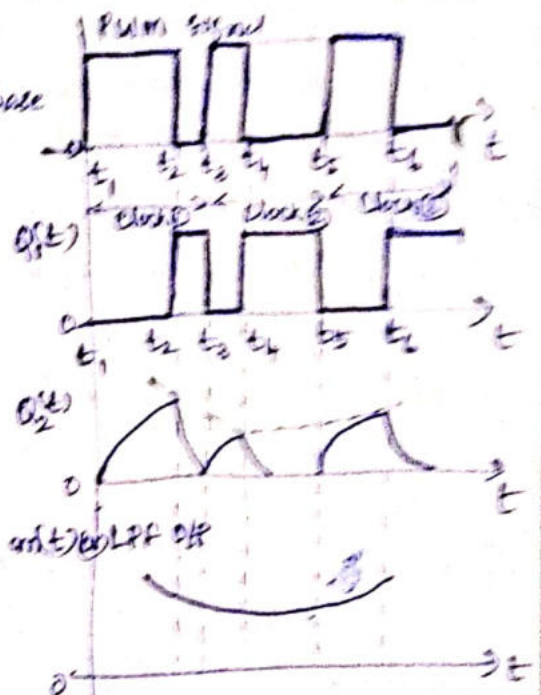
6) During t_2 to t_3 , When PWM signal is low, the input to the Transistor Q_2 is high and it goes to Saturation.

7) The Capacitor 'C' discharges Very rapidly through transistor Q_2 because it is in ON state.

8) Charging & Discharging of Capacitor, Spiky waveform in which width is converted into high & low Envelope.

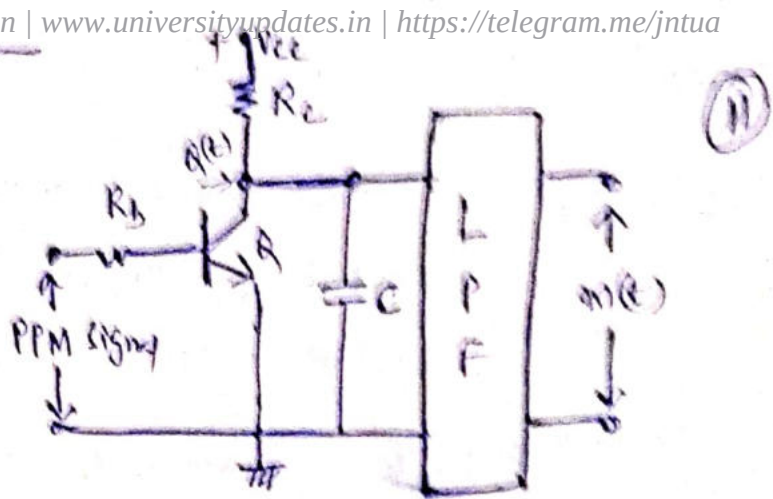
9) The LPF removes the Spikes or ripples, so that modulating signal $m(t)$ appears at the output of the demodulator.

Hence, PWM signal is detected (or) Demodulated. (or) modulating signal $m(t)$ is extracted from PWM signal.

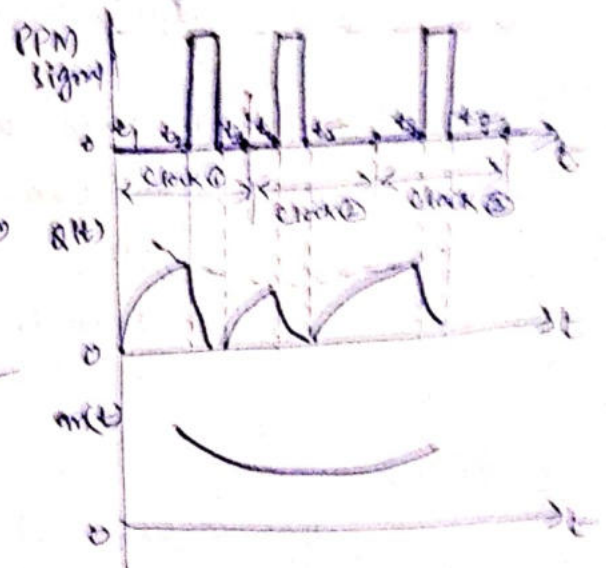


PPM Demodulator

1) A PPM demodulator is shown in figure which contains Transistor Q , collector resistor R_c and Capacitor C etc.



2) This utilizes the fact that the gaps between the pulses of a PPM signal contain the information regarding the modulating signal.



3) During time $t_1 - t_2$, the Transistor is in cutoff and Capacitor C gets charged through R_c .

4) During time interval t_2 to t_3 , the Capacitor discharges through transistor Q and the collector voltage becomes low.

5. The Charging & Discharging of Capacitor, Spiky waveform which approximately is equal to modulating signal $m(t)$.

6. The Spikes (or) ripples will be removed by the LPF so that original modulating signal appears at the o/p of PPM demodulator.

Hence PPM signal is detected or demodulated or modulating signal $m(t)$ is extracted.

PWM

12

- Advantages :-
- i) More Signal to Noise Ratio
 - ii) Does not require Synchronization.
 - iii) requires less Transmission Power

- Disadvantages :-
- i) Transmission BW is high.
 - ii) Circuit Complexity is more

- Applications :-
- 1) It is used in Power electronic applications to control the speed of electric motor or fan etc.
 - 2) used in Embedded systems to control the speed of stepper motor.

PPM :-

- Advantages :-
- i) more Signal to Noise Ratio
 - ii) Requires less Transmission power
 - iii) requires more Transmission BW.

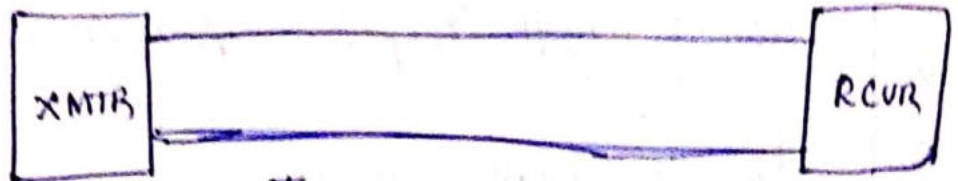
- Disadvantages :-
- i) it requires Synchronization.
 - ii) more Circuit Complexity.

- Applications :-
- 1) It is used to control the position of Satellites.
 - 2) used in RFID applications.

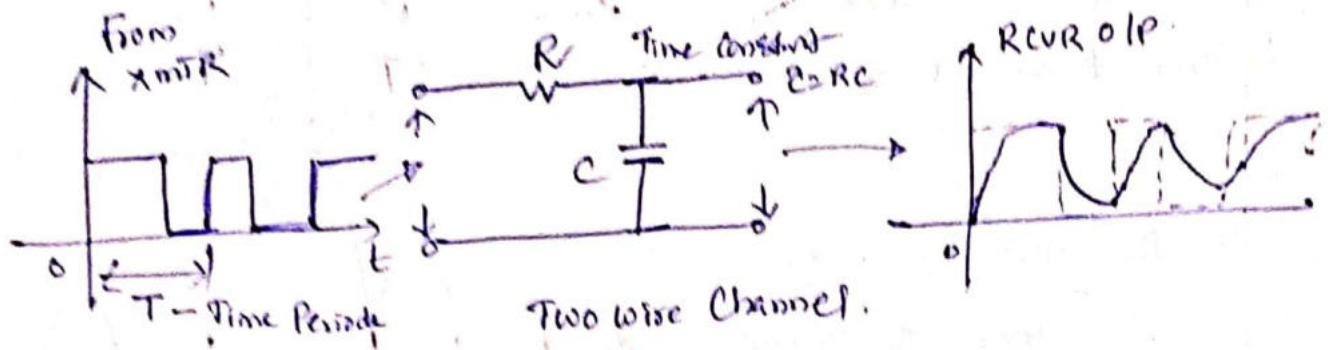
Comparison b/n PAM, PWM & PPM

S.No.	Modulation	Transmitted Power	Transmitted BW	SNR	Synchronization Problems	Hardware Complexity	Applications
1.	PAM	MORE	LESS	LESS	LESS	LESS	Pulse Code Modulation
2.	PWM	LESS	MORE	HIGH	LESS	MORE	Speed control, Electric motors, Fan etc.
3.	PPM	LESS	MORE	HIGH	MORE	MORE	RFID

Cross talk :-



Two wire Channel.



Two wire Channel.

① Usually Pulse Modulation Signals (PAM, PWM, PPM) are transmitted through physical channels. Examples ~~of~~ ^{are} two wire channel, ~~etc.~~ Coaxial Cable etc.,

② Two wire channel acts as Low Pass Filter as shown in fig.

③ Here resistor is function of type of wire, length of wire, area of wire etc., & Capacitor is function of distance between the wire.

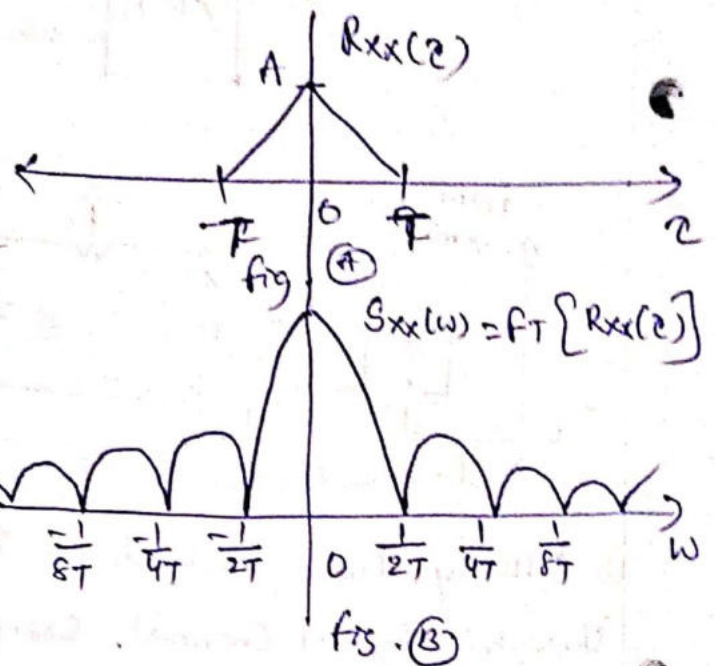
④ If time constant of 'RC' circuit is too large when compared to its time period $\tau \gg T$ i.e. $RC \gg T$ then capacitor slowly charges & discharges.

⑤ Because of this reason amplitude of one pulse enters into other pulse. This is called "Cross-talk".

1) Pulse modulated signals such as PAM, or PWM or PPM are considered as random process because amplitudes of pulses varied randomly in PAM which depends on modulating signal $m(t)$. Similarly pulse width & pulse position are varied randomly in PWM & PPM signals.

2) Power Spectral Density (PSD) of a random process is obtained with help of Auto Correlation function (ACF).

3) Pulse modulation signals PAM, PWM or PPM are some kind of rectangular pulse.



4) The Auto Correlation function (ACF) of rectangular pulse is triangular pulse as shown in figure (B).

5) Power spectral Density can be obtained by taking Fourier Transform of Auto Correlation function (ACF).

6) Fourier Transform of Triangular pulse will be sinc pulse square $S_{xx}(\omega) = FT[R_{xx}(\tau)] = S_A^2(T)$.

7) So, Spectrum of PAM, PWM or PPM signals will be as shown in fig. (B).

Characteristics of the Radio Receiver :-

(15)

The Performance of the receiver can be evaluated based on the following characteristics.

- 1) Sensitivity 2) Selectivity 3) Fidelity.

① Sensitivity :- An ability of the receiver to receive weakest signal is called "Sensitivity" of the receiver.

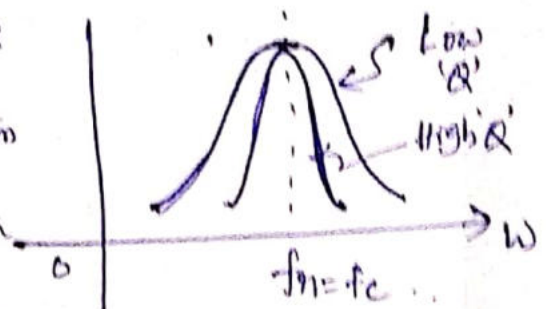
Sensitivity is mainly depends on the amount of amplification provided by the receiver. Sensitivity can be

- Increased by increasing the gain of receiver. The gain of the receiver is depends on RF tuned amplifier, IF amplifier, AF voltage amplifier and AF power amplifier. The unit of sensitivity is milli (or) micro volts.

② Selectivity :- It is defined as the ability of the receiver to select the wanted signal (or) station and reject unwanted signal or station. The selectivity is mainly depends on the tuning circuit present in the RF tuned amplifier.

The main parameter of tuned circuit is Quality factor Q . If Q is high then

Selectivity will be more and vice versa.



As shown in figure. The Quality factor cannot be increased beyond some value, In this case we use double-tuned (or) staggered tuned amplifiers are used to increase the selectivity of the receiver.

③ Fidelity :- It is defined as the ability of the receiver to reproduce the original modulating signal is called "Fidelity". The fidelity of the receiver is mainly depends on the type of demodulator used.

Example :- The performance of the Synchronous ~~receiver~~ Demodulator or ~~detect~~ detector is more when compared to an Envelope detector. Synchronous Detector detects the $m(t)$ with respect to phase and frequency where as Envelope detector detects the $m(t)$ with respect to amplitude. Since the noise adds to signal in the amplitude, the noise effect is more on the envelope detector than the Synchronous detector. But Hardware Complexity of Synchronous detector is more when compared to Envelope Detector.



Unit - V

Information & Channel Capacity

Message : — It is sequence of symbols.

Example : HAPPY NEW YEAR 2020.

INFORMATION : — Content in the message is called "Information".

An Information is statistical Parameter associated with Probability.

An information is inversely proportional to the probability occurrence of symbol.

$$I \propto \frac{1}{P}$$

$$I = \log_x \left(\frac{1}{P} \right) = -\log_x (P)$$

If $x = 2$ then units of information is "bit"

If $x = 10$ then units of information is "Decit"

If $x = e$ then units of information is "nat"

Example : — Let us consider a discrete source generating '4'

Symbols x_1, x_2, x_3 & x_4 with probabilities $P(x_1) = \frac{1}{2}$,

$P(x_2) = \frac{1}{4}$, $P(x_3) = \frac{1}{8}$ & $P(x_4) = \frac{1}{8}$.

Information carried by the symbol x_1 is

$$I_{x_1} = \log_2 \left[\frac{1}{P(x_1)} \right] = \log_2 \left[\frac{1}{\frac{1}{2}} \right] = \log_2 (2) = 1 \text{ Bit}$$

Similarly,

$$I_{x_2} = \log_2 \left[\frac{1}{P(x_2)} \right] = \log_2 \left[\frac{1}{\frac{1}{4}} \right] = \log_2 (4) = 2 \text{ Bit}$$

$$I_{x_3} = \log_2 \left[\frac{1}{P(x_3)} \right] = \log_2 \left[\frac{1}{\frac{1}{8}} \right] = \log_2 (8) = 3 \text{ Bit}$$

$$I_{x_4} = \log_2 \left[\frac{1}{P(x_4)} \right] = \log_2 \left[\frac{1}{\frac{1}{8}} \right] = \log_2 (8) = 3 \text{ Bit}$$

Probability of occurrence of symbols x_3 & x_4 is less than x_1 & x_2 , so, I_{x_3} & I_{x_4} are more than I_{x_1} & I_{x_2} .

www.android.universityupdates.in | www.universityupdates.in | https://telegram.me/jntua

Entropy :- Average information each symbol is called as an "Entropy".

Let us consider a discrete source is generating 'M' symbols $x_1, x_2, x_3, \dots, x_M$ with probabilities $P(x_1), P(x_2), P(x_3), \dots, P(x_M)$. An Entropy is expressed as

$$H = \sum_{k=1}^M P(x_k) \log_2 \left[\frac{1}{P(x_k)} \right] = - \sum_{k=1}^M P(x_k) \log_2 [P(x_k)]$$

Unit of Entropy is Bits/symbol (or) Bits/message

Properties of Entropy :-

1. Entropy is non negative function $H \geq 0$
2. If Discrete source is generating single ^{Symbol} ~~message~~ item entropy $H=0$ i.e. ~~no.~~ no. of symbols $M=1$. $P(x_k)=1$.

$$H = \sum_{k=1}^1 P(x_k) \log_2 \left[\frac{1}{P(x_k)} \right] = 1 \log_2 \left(\frac{1}{1} \right) \\ = \log_2(1) = 0 \Rightarrow \boxed{H \geq 0 \text{ Bits/symbol}}$$

3. Entropy is maximum when probability of occurrence of symbols are equal.

Case(i) :- $M=2$ Discrete source generating two symbols.

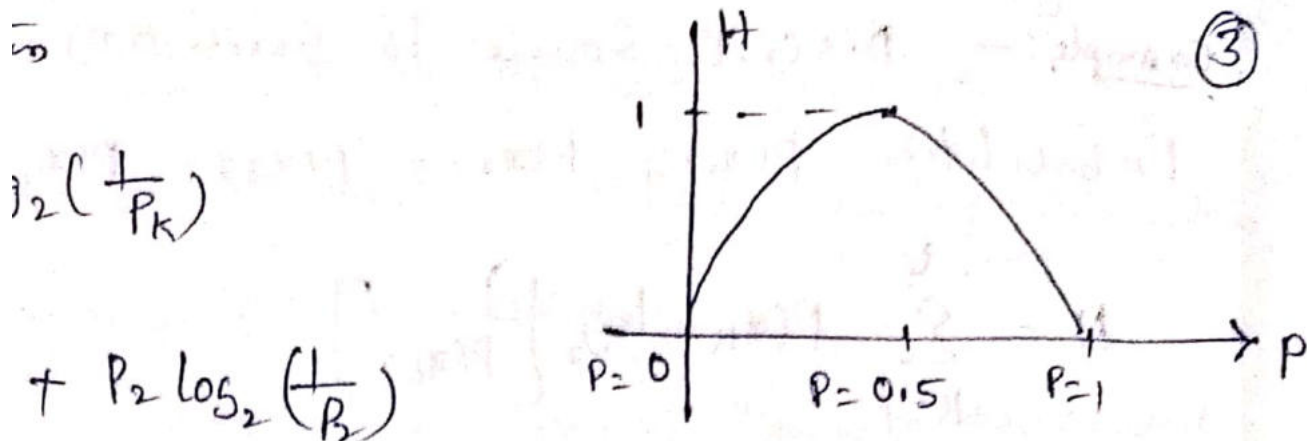
Let us consider discrete source generating two symbols x_1 & x_2 with probabilities $P(x_1)$ & $P(x_2)$. Entropy

$$H = \sum_{k=1}^2 P(x_k) \log_2 \left[\frac{1}{P(x_k)} \right] = P(x_1) \log_2 \left(\frac{1}{P(x_1)} \right) + P(x_2) \log_2 \left(\frac{1}{P(x_2)} \right)$$

Let $P(x_1) = P$ & $P(x_2) = 1-P$.

$$H = P \log_2 \left(\frac{1}{P} \right) + (1-P) \log_2 \left(\frac{1}{1-P} \right)$$

$$\frac{dH}{dP} \geq 0 \Rightarrow -\ln(2) - \log(P) + \ln(2) + \log(1-P) \\ \log P = \log(1-P) \Rightarrow \boxed{P=0.5}$$



$$p_2 \left(\frac{1}{p_k} \right)$$

$$+ p_2 \log_2 \left(\frac{1}{p_2} \right)$$

$$-\left(\frac{1}{2} \right) + \frac{1}{2} \log_2 \left(\frac{1}{2} \right)$$

$$1 + \frac{1}{2} \log_2 \frac{1}{2}$$

$$= 1 \Rightarrow H = 1 \text{ Bit/symbol}$$

The source is generating 'N' symbols

Probability atom $m = N$.

$$= P(x_2) = P(x_3) = \dots = P(x_N) = \frac{1}{N}$$

$$P(x_k) \log_2 \left[\frac{1}{P(x_k)} \right]$$

$$\log_2 \left[\frac{1}{P(x_1)} \right] + P(x_2) \log_2 \left[\frac{1}{P(x_2)} \right] + \dots + P(x_N) \log_2 \left[\frac{1}{P(x_N)} \right]$$

$$\log_2 \left[\frac{1}{1/N} \right] + \frac{1}{N} \log_2 \left(\frac{1}{1/N} \right) + \dots + \frac{1}{N} \log_2 \left(\frac{1}{1/N} \right)$$

Example 1:- Discrete Source is generating '4' symbols

Probabilities $P(x_1) = P(x_2) = P(x_3) = P(x_4) = \frac{1}{4}$.

$$H = \sum_{k=1}^4 P(x_k) \log_2 \left[\frac{1}{P(x_k)} \right]$$

$$= P(x_1) \log_2 \left[\frac{1}{P(x_1)} \right] + P(x_2) \log_2 \left[\frac{1}{P(x_2)} \right] + P(x_3) \log_2 \left[\frac{1}{P(x_3)} \right] + P(x_4) \log_2 \left[\frac{1}{P(x_4)} \right]$$

$$= \frac{1}{4} \log_2 \left(\frac{1}{1/4} \right) + \frac{1}{4} \log_2 \left(\frac{1}{1/4} \right) + \frac{1}{4} \log_2 \left(\frac{1}{1/4} \right) + \frac{1}{4} \log_2 \left(\frac{1}{1/4} \right)$$

$$= \frac{1}{4} \times 4 \log_2 (4) = \log_2 2^2$$

$$H_{\max} = 2 \text{ Bits/Symbol}$$

Symbol	Bits
x_1 —	00
x_2 —	01
x_3 —	10
x_4 —	11

Example 2:- Discrete Source is

generating '4' symbols with probabilities

$P(x_1) = \frac{1}{2}$, $P(x_2) = \frac{1}{4}$, $P(x_3) = \frac{1}{8}$ & $P(x_4) = \frac{1}{8}$.

$$H = \sum_{k=1}^4 P(x_k) \log_2 \left[\frac{1}{P(x_k)} \right]$$

$$= P(x_1) \log_2 \left[\frac{1}{P(x_1)} \right] + P(x_2) \log_2 \left[\frac{1}{P(x_2)} \right] + P(x_3) \log_2 \left[\frac{1}{P(x_3)} \right] + P(x_4) \log_2 \left[\frac{1}{P(x_4)} \right]$$

$$= \frac{1}{2} \log_2 \left(\frac{1}{1/2} \right) + \frac{1}{4} \log_2 \left(\frac{1}{1/4} \right) + \frac{1}{8} \log_2 \left(\frac{1}{1/8} \right) + \frac{1}{8} \log_2 \left(\frac{1}{1/8} \right)$$

$$= \frac{1}{2} \log_2 (2) + \frac{1}{4} \log_2 (4) + \frac{1}{8} \log_2 (8) + \frac{1}{8} \log_2 (8)$$

$$= \frac{1}{2} \log_2 2 + \frac{1}{4} \log_2 2^2 + \frac{1}{8} \log_2 2^3 + \frac{1}{8} \log_2 2^3$$

$$= \frac{1}{2} (1) + \left[\frac{1}{4} \times 2 \right] + \left[\frac{1}{8} \times 3 \right] + \left[\frac{1}{8} \times 3 \right]$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{8} \times 2 = \frac{4+4+6}{8} = \frac{14}{8} = 1.75$$

$$H = 1.75 \text{ Bits/Symbol}$$

Symbol	Bits
x_1 —	1
x_2 —	10
x_3 —	101
x_4 —	111

Rate of Information:- No. of bits generated by discrete source per second is called "rate of information".

rate of information can be written as

$$R = \sigma H$$

Where σ = no. of Symbols generated by the source per second.

$$= \frac{\text{Symbols}}{\text{Sec}} \times \frac{\text{Bits}}{\text{Symbol}}$$

H = Average no. of bits per Symbol (Entropy).

$$R = \frac{\text{Bits}}{\text{Sec}} \Rightarrow \text{Unit of information rate.}$$

Problem ① An event has six possible outcomes with Probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$ & $\frac{1}{32}$. Find the entropy of the system. Also find the rate of information, if there are 16 outcomes per second.

Ans. Given Data

No. of outcomes $M = 6$

Probabilities $P_1 = \frac{1}{2}, P_2 = \frac{1}{4}, P_3 = \frac{1}{8}, P_4 = \frac{1}{16}, P_5 = \frac{1}{32} \text{ \& } P_6 = \frac{1}{32}$

no. of outcomes per second $\sigma = 16 \text{ outcomes/sec.}$

Find Entropy $H = ?$ rate of information $R = ?$

$$\begin{aligned} \text{Entropy } H &= \sum_{i=1}^6 P_i \log_2 \left(\frac{1}{P_i} \right) = P_1 \log_2 \left(\frac{1}{P_1} \right) + P_2 \log_2 \left(\frac{1}{P_2} \right) + P_3 \log_2 \left(\frac{1}{P_3} \right) \\ &\quad + P_4 \log_2 \left(\frac{1}{P_4} \right) + P_5 \log_2 \left(\frac{1}{P_5} \right) + P_6 \log_2 \left(\frac{1}{P_6} \right) \\ &= \frac{1}{2} \log_2 \left(\frac{1}{\frac{1}{2}} \right) + \frac{1}{4} \log_2 \left(\frac{1}{\frac{1}{4}} \right) + \frac{1}{8} \log_2 \left(\frac{1}{\frac{1}{8}} \right) + \frac{1}{16} \log_2 \left(\frac{1}{\frac{1}{16}} \right) \\ &\quad + \frac{1}{32} \log_2 \left(\frac{1}{\frac{1}{32}} \right) + \frac{1}{32} \log_2 \left(\frac{1}{\frac{1}{32}} \right) \end{aligned}$$

$$H = \frac{1}{2} \log_2(2) + \frac{1}{4} \log_2(4) + \frac{1}{8} \log_2(8) + \frac{1}{16} \log_2(16) \\ + \frac{1}{32} \log_2(32) + \frac{1}{32} \log_2(32)$$

$$= \frac{1}{2} \times \log_2(2) + \frac{1}{4} \log_2(2^2) + \frac{1}{8} \log_2(2^3) + \frac{1}{16} \log_2(2^4) \\ + 2 \times \frac{1}{32} \log_2(2^5)$$

$$= \left[\frac{1}{2} \times \log_2(2) \right] + \left[\frac{1}{4} \times 2 \log_2(2) \right] + \left[\frac{1}{8} \times 3 \log_2(2) \right] + \left[\frac{1}{16} \times 4 \log_2(2) \right] \\ + \left[2 \times \frac{1}{32} \times 5 \log_2(2) \right]$$

$$= \left(\frac{1}{2} \times 1 \right) + \left(\frac{2}{4} \right) + \frac{3}{8} + \frac{4}{16} + \frac{10}{32}$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + 1 + \frac{5}{16}$$

$$H = \frac{31}{16} \text{ Bits/Outcome}$$

Rate of information $R = H = \frac{31}{16} \frac{\text{Bits}}{\text{Outcome}} \times \frac{16 \text{ Outcomes}}{\text{Sec}}$

$$R = 31 \text{ Bits/sec}$$

Prob: A Continuous signal is band limited to 5 KHz. The signal is quantized in 8 levels of a PCM system with probabilities 0.24, 0.2, 0.2, 0.1, 0.1, 0.05, 0.05 & 0.05. Calculate the entropy & the rate of information.

Ans: Given Data: Signal's maximum frequency $f_{\max} = 5 \text{ KHz}$
 no. of quantization levels $M = 8$

Probabilities $P_1 = 0.24, P_2 = 0.2, P_3 = 0.2, P_4 = 0.1,$
 $P_5 = 0.1, P_6 = 0.05, P_7 = 0.05 \text{ \& } P_8 = 0.05$
 find $\sigma, R \text{ \& } H$

No. of samples per second $\gamma = f_s = 2 \cdot f_{max}$

(7)

$$= 2 \times 5 \times 10^3 \text{ Samples/sec}$$

$$\gamma = 10 \times 10^3 \text{ Samples/sec}$$

$$\text{Entropy } H = \sum_{i=1}^8 P_i \log_2 \left(\frac{1}{P_i} \right)$$

$$= 0.25 \log_2 \left(\frac{1}{0.25} \right) + 2 \times 0.2 \log_2 \left(\frac{1}{0.2} \right) + 2 \times 0.1 \log_2 \left(\frac{1}{0.1} \right) + 3 \times 0.05 \log_2 \left(\frac{1}{0.05} \right)$$

$$H = 2.74 \text{ Bits/sample}$$

Rate of information $R = \gamma H$

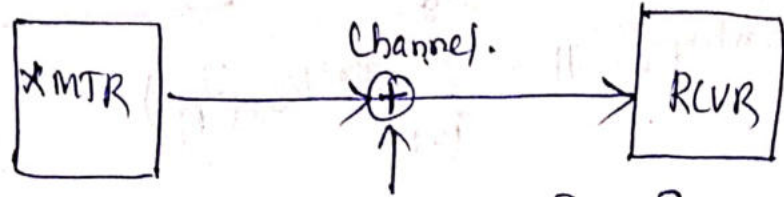
$$= 10 \times 10^3 \frac{\text{Samples}}{\text{Sec}} \times 2.74 \frac{\text{Bits}}{\text{Sample}}$$

$$= 2.74 \times 10 \times 10^3 \text{ Bits/sec.}$$

$$R = 27.4 \text{ KBits/sec}$$

Different types of entropies :-

There are four types of entropies i.e. Marginal, Joint & Conditional entropies associated with X_{MTR}, Channel and R_{CVR}.



$$[X] = [x_1, x_2, \dots, x_M] \quad \text{noise} \quad [Y] = [y_1, y_2, \dots, y_N]$$

Transmitter (X_{MTR}) generates $x_1, x_2, \dots, x_M$ symbols with probabilities $P(x_1), P(x_2), \dots, P(x_M)$ and receiver receives $y_1, y_2, \dots, y_N$ symbols with probabilities $P(y_1), P(y_2), \dots, P(y_N)$ respectively. The probabilities & entropies associated with X_{MTR} & R_{CVR} ~~separately~~ individually is called Marginal Probabilities & marginal entropies. i.e.

$$X_{MTR} \begin{cases} [X] = [x_1, x_2, x_3, \dots, x_M] \\ P[X] = [P(x_1), P(x_2), P(x_3), \dots, P(x_M)] \end{cases}$$

$$R_{CVR} \begin{cases} [Y] = [y_1, y_2, y_3, \dots, y_N] \\ P[Y] = [P(y_1), P(y_2), P(y_3), \dots, P(y_N)] \end{cases}$$

Probabilities & entropies associated with X_{MTR} & R_{CVR} combinedly is called Joint & Conditional probabilities & entropies.

Joint Probability $P[X, Y] =$

	y_1	y_2	y_3	$\dots$	y_N
x_1	$P(x_1, y_1)$	$P(x_1, y_2)$	$P(x_1, y_3)$	$\dots$	$P(x_1, y_N)$
x_2	$P(x_2, y_1)$	$P(x_2, y_2)$	$P(x_2, y_3)$	$\dots$	$P(x_2, y_N)$
x_3	$P(x_3, y_1)$	$P(x_3, y_2)$	$P(x_3, y_3)$	$\dots$	$P(x_3, y_N)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_M	$P(x_M, y_1)$	$P(x_M, y_2)$	$P(x_M, y_3)$	$\dots$	$P(x_M, y_N)$

It is $M \times N$ matrix

Conditional Probabilities.

$P(\frac{x}{y})$ represents

Probability of 'x' given 'y'.

$$P(\frac{x}{y}) = \begin{matrix} & y_1 & y_2 & \dots & y_m \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_m \end{matrix} & \begin{bmatrix} P(\frac{x_1}{y_1}) & P(\frac{x_1}{y_2}) & \dots & P(\frac{x_1}{y_m}) \\ P(\frac{x_2}{y_1}) & P(\frac{x_2}{y_2}) & \dots & P(\frac{x_2}{y_m}) \\ \vdots & \vdots & \ddots & \vdots \\ P(\frac{x_m}{y_1}) & P(\frac{x_m}{y_2}) & \dots & P(\frac{x_m}{y_m}) \end{bmatrix} \end{matrix} \quad m \times n$$

$$P(\frac{x}{y}) = \begin{matrix} & y_1 & y_2 & \dots & y_m \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{matrix} & \begin{bmatrix} \frac{P(x_1, y_1)}{P(y_1)} & \frac{P(x_1, y_2)}{P(y_2)} & \dots & \frac{P(x_1, y_m)}{P(y_m)} \\ \frac{P(x_2, y_1)}{P(y_1)} & \frac{P(x_2, y_2)}{P(y_2)} & \dots & \frac{P(x_2, y_m)}{P(y_m)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{P(x_m, y_1)}{P(y_1)} & \frac{P(x_m, y_2)}{P(y_2)} & \dots & \frac{P(x_m, y_m)}{P(y_m)} \end{bmatrix} \end{matrix} \quad m \times n$$

Similarly

$$P(\frac{y}{x}) = \begin{matrix} & y_1 & y_2 & \dots & y_m \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{matrix} & \begin{bmatrix} P(\frac{y_1}{x_1}) & P(\frac{y_2}{x_1}) & \dots & P(\frac{y_m}{x_1}) \\ P(\frac{y_1}{x_2}) & P(\frac{y_2}{x_2}) & \dots & P(\frac{y_m}{x_2}) \\ \vdots & \vdots & \ddots & \vdots \\ P(\frac{y_1}{x_m}) & P(\frac{y_2}{x_m}) & \dots & P(\frac{y_m}{x_m}) \end{bmatrix} \end{matrix} \quad m \times n$$

$$P(\frac{y}{x}) = \begin{matrix} & y_1 & y_2 & \dots & y_m \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{matrix} & \begin{bmatrix} \frac{P(x_1, y_1)}{P(x_1)} & \frac{P(x_1, y_2)}{P(x_1)} & \dots & \frac{P(x_1, y_m)}{P(x_1)} \\ \frac{P(x_2, y_1)}{P(x_2)} & \frac{P(x_2, y_2)}{P(x_2)} & \dots & \frac{P(x_2, y_m)}{P(x_2)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{P(x_m, y_1)}{P(x_m)} & \frac{P(x_m, y_2)}{P(x_m)} & \dots & \frac{P(x_m, y_m)}{P(x_m)} \end{bmatrix} \end{matrix} \quad m \times n$$

Marginal Entropies

$$H[X] = - \sum_{i=1}^M P(x_i) \log_2 [P(x_i)]$$

$$H[Y] = - \sum_{j=1}^N P(y_j) \log_2 [P(y_j)]$$

Joint Entropy

$$H[XY] = - \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 [P(x_i, y_j)]$$

Conditional Entropies

$$H\left[\frac{X}{Y}\right] = - \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 \left[P\left(\frac{x_i}{y_j}\right) \right]$$

$$H\left[\frac{Y}{X}\right] = - \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 \left(P\left[\frac{y_j}{x_i}\right] \right)$$

Relation between

$H[X]$, $H[Y]$, $H[XY]$, $H\left[\frac{Y}{X}\right]$ & $H\left[\frac{X}{Y}\right]$:-

$$H[XY] = H[X] + H\left[\frac{Y}{X}\right] \quad \& \quad H[XY] = H[Y] + H\left[\frac{X}{Y}\right]$$

Proof of $H[XY] = H[X] + H\left[\frac{Y}{X}\right]$:-

$$H[XY] = - \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 [P(x_i, y_j)]$$

$$= - \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 \left[P(x_i) \cdot P\left(\frac{y_j}{x_i}\right) \right]$$

$$= - \left[\sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 [P(x_i)] + \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 \left[P\left(\frac{y_j}{x_i}\right) \right] \right]$$

$$= - \sum_{i=1}^M P(x_i) \log_2 [P(x_i)] - \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 \left[P\left(\frac{y_j}{x_i}\right) \right]$$

$$\frac{P(AB)}{P(A)} = P\left(\frac{B}{A}\right)$$

$$\log(AB) = \log A + \log B$$

$$H[XY] = H[X] + H\left(\frac{Y}{X}\right)$$

Hence Proved

$$\sum_{i=-\infty}^{+\infty} \sum_{j=-\infty}^{+\infty} P(x_i, y_j) = \sum_{i=-\infty}^{+\infty} P(x_i) \sum_{j=-\infty}^{+\infty} P\left(\frac{y_j}{x_i}\right) = \sum_{i=-\infty}^{+\infty} P(x_i)$$

Proof of $H[XY] = H[Y] + H\left[\frac{X}{Y}\right]$:-

$$H[XY] = - \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 [P(x_i, y_j)]$$

$$= - \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 \left[P(y_j) P\left(\frac{x_i}{y_j}\right) \right]$$

$$= - \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \left[\log_2 [P(y_j)] + \log_2 \left[P\left(\frac{x_i}{y_j}\right) \right] \right]$$

$$= - \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 [P(y_j)] - \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 \left[P\left(\frac{x_i}{y_j}\right) \right]$$

$$= - \sum_{j=1}^N P(y_j) \log_2 [P(y_j)] - \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 \left[P\left(\frac{x_i}{y_j}\right) \right]$$

$$H[XY] = H[Y] + H\left[\frac{X}{Y}\right]$$

If X & Y are statistically independent

$$H\left[\frac{X}{Y}\right] = H[Y] \text{ \& } H\left[\frac{X}{Y}\right] = H[X]$$

$$H[XY] = H[X] + H[Y]$$

Prob: Joint Probability matrix of a noisy channel is given by

$$P(x, y) = \begin{matrix} & y_1 & y_2 & y_3 & y_4 \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{matrix} & \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0.05 \\ 0 & 0 & 0.1 & 0.05 \\ 0 & 0.05 & 0 & 0.1 \\ 0 & 0 & 0 & 0.05 \end{bmatrix} \end{matrix} \quad 5 \times 4$$

find $H[X]$, $H[Y]$,
 $H[XY]$, $H[Y/X]$ &
 $H[Y/X] ?$

Sol.

$$P(x_1) = 0.25 + 0 + 0 + 0 = 0.25 \leftarrow \text{Add first row}$$

$$P(x_2) = 0 + 0.25 + 0 + 0.05 = 0.3 \leftarrow \text{Add second row}$$

$$P(x_3) = 0 + 0 + 0.1 + 0.05 = 0.15 \leftarrow \text{Add third row}$$

$$P(x_4) = 0 + 0.05 + 0 + 0.1 = 0.15 \leftarrow \text{Add fourth row}$$

$$P(x_5) = 0 + 0 + 0 + 0.05 = 0.05 \leftarrow \text{Add fifth row}$$

$$P[X] = [P(x_1) \ P(x_2) \ P(x_3) \ P(x_4) \ P(x_5)] = [0.25, 0.3, 0.15, 0.15, 0.05]$$

125

$$P(y_1) = 0.25 \leftarrow \text{Add first column}$$

$$P(y_2) = 0.3 \leftarrow \text{Add second column}$$

$$P(y_3) = 0.2 \leftarrow \text{Add third column}$$

$$P(y_4) = 0.25 \leftarrow \text{Add fourth column}$$

$$P[y] = \begin{bmatrix} P(y_1) \\ P(y_2) \\ P(y_3) \\ P(y_4) \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.3 \\ 0.2 \\ 0.25 \end{bmatrix}$$

$$\text{Marginal Entropy } H[X] = - \sum_{i=1}^5 P(x_i) \log_2 [P(x_i)]$$

$$= - [P(x_1) \log_2 [P(x_1)] + P(x_2) \log_2 [P(x_2)] + P(x_3) \log_2 [P(x_3)] + P(x_4) \log_2 [P(x_4)] + P(x_5) \log_2 [P(x_5)]]$$

$$= - [0.35 \log_2 (0.35) + 0.3 \log_2 (0.3) + 0.15 \log_2 (0.15) + 0.15 \log_2 (0.15) + 0.05 \log_2 (0.05)]$$

$$\log_2 x = \frac{\log_{10}(x)}{\log_{10}(2)}$$

$$H[X] = 2.0878 \text{ Bits/symbol}$$

$$\text{Marginal Entropy } H[Y] = - \sum_{j=1}^4 P(y_j) \log_2 [P(y_j)]$$

$$= - [P(y_1) \log_2 [P(y_1)] + P(y_2) \log_2 [P(y_2)] + P(y_3) \log_2 [P(y_3)] + P(y_4) \log_2 [P(y_4)]]$$

$$= - [0.25 \log_2 (0.25) + 0.3 \log_2 (0.3) + 0.2 \log_2 (0.2) + 0.25 \log_2 (0.25)]$$

$$= - [2 \times 0.25 \log_2 (0.25) + 0.3 \log_2 (0.3) + 0.2 \log_2 (0.2)]$$

$$H[Y] = 1.97 \text{ Bits/symbol}$$

$$\text{Joint Entropy } H[X,Y] = - \sum_{i=1}^5 \sum_{j=1}^4 P(x_i, y_j) \log_2 [P(x_i, y_j)]$$

$$= - [0.25 \log_2 (0.25) + 0.1 \log_2 (0.1) + 0.25 \log_2 (0.25) + 0.05 \log_2 (0.05) + 0.1 \log_2 (0.1) + 0.05 \log_2 (0.05) + 0.05 \log_2 (0.05) + 0.1 \log_2 (0.1) + 0.05 \log_2 (0.05)]$$

$$= - [2 \times 0.25 \log_2 (0.25) + 3 \times 0.1 \log_2 (0.1) + 4 \times 0.05 \log_2 (0.05)]$$

$$= - [-1 - 0.9966 - 0.8643] = 2.8609$$

$$H[X,Y] = 2.8609 \text{ Bits/symbol}$$

Conditional Probability matrix

$$P\left(\frac{x}{y}\right) = \frac{P(x,y)}{P(y)}$$

$$P\left(\frac{x}{y}\right) = \begin{matrix} & y_1 & y_2 & y_3 & y_4 \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{matrix} & \begin{bmatrix} P\left(\frac{x_1}{y_1}\right) & P\left(\frac{x_1}{y_2}\right) & P\left(\frac{x_1}{y_3}\right) & P\left(\frac{x_1}{y_4}\right) \\ P\left(\frac{x_2}{y_1}\right) & P\left(\frac{x_2}{y_2}\right) & P\left(\frac{x_2}{y_3}\right) & P\left(\frac{x_2}{y_4}\right) \\ P\left(\frac{x_3}{y_1}\right) & P\left(\frac{x_3}{y_2}\right) & P\left(\frac{x_3}{y_3}\right) & P\left(\frac{x_3}{y_4}\right) \\ P\left(\frac{x_4}{y_1}\right) & P\left(\frac{x_4}{y_2}\right) & P\left(\frac{x_4}{y_3}\right) & P\left(\frac{x_4}{y_4}\right) \\ P\left(\frac{x_5}{y_1}\right) & P\left(\frac{x_5}{y_2}\right) & P\left(\frac{x_5}{y_3}\right) & P\left(\frac{x_5}{y_4}\right) \end{bmatrix} \end{matrix}$$

$$P\left(\frac{x}{y}\right) = \begin{matrix} & y_1 & y_2 & y_3 & y_4 \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{matrix} & \begin{bmatrix} \frac{P(x_1, y_1)}{P(y_1)} & \frac{P(x_1, y_2)}{P(y_2)} & \frac{P(x_1, y_3)}{P(y_3)} & \frac{P(x_1, y_4)}{P(y_4)} \\ \frac{P(x_2, y_1)}{P(y_1)} & \frac{P(x_2, y_2)}{P(y_2)} & \frac{P(x_2, y_3)}{P(y_3)} & \frac{P(x_2, y_4)}{P(y_4)} \\ \frac{P(x_3, y_1)}{P(y_1)} & \frac{P(x_3, y_2)}{P(y_2)} & \frac{P(x_3, y_3)}{P(y_3)} & \frac{P(x_3, y_4)}{P(y_4)} \\ \frac{P(x_4, y_1)}{P(y_1)} & \frac{P(x_4, y_2)}{P(y_2)} & \frac{P(x_4, y_3)}{P(y_3)} & \frac{P(x_4, y_4)}{P(y_4)} \\ \frac{P(x_5, y_1)}{P(y_1)} & \frac{P(x_5, y_2)}{P(y_2)} & \frac{P(x_5, y_3)}{P(y_3)} & \frac{P(x_5, y_4)}{P(y_4)} \end{bmatrix} \end{matrix}$$

$$= \begin{bmatrix} \frac{0.25}{0.25} & \frac{0}{0.3} & \frac{0.1}{0.2} & \frac{0}{0.35} \\ \frac{0}{0.25} & \frac{0.25}{0.3} & \frac{0}{0.2} & \frac{0.05}{0.25} \\ \frac{0}{0.25} & \frac{0}{0.3} & \frac{0.1}{0.2} & \frac{0.05}{0.35} \\ \frac{0.1}{0.25} & \frac{0.05}{0.3} & \frac{0}{0.2} & \frac{0.1}{0.25} \\ \frac{0}{0.25} & \frac{0}{0.3} & \frac{0}{0.2} & \frac{0.05}{0.25} \end{bmatrix}$$

$$P\left(\frac{x}{y}\right) = \begin{bmatrix} 1 & 0 & 0.33 & 0 \\ 0 & 0.83 & 0 & 0.16 \\ 0 & 0 & 0.33 & 0.16 \\ 0 & 0.16 & 0 & 0.33 \\ 0 & 0 & 0 & 0.16 \end{bmatrix}$$

Conditional Entropy $H\left[\frac{X}{Y}\right] = - \sum_{i=1}^5 \sum_{j=1}^4 P(x_i, y_j) \log_2 \left[P\left(\frac{x_i}{y_j}\right) \right]$

$$= - \left[0.25 \log_2(1) + 0.1 \log_2\left(\frac{0.5}{0.33}\right) + 0.25 \log_2(0.83) + 0.05 \log_2\left(\frac{0.2}{0.16}\right) \right. \\ \left. + 0.1 \log_2\left(\frac{0.5}{0.33}\right) + 0.05 \log_2\left(\frac{0.2}{0.16}\right) + 0.05 \log_2(0.16) + 0.1 \log_2\left(\frac{0.4}{0.33}\right) \right. \\ \left. + 0.05 \log_2\left(\frac{0.2}{0.16}\right) \right]$$

$$= - \left[0 - 0.1599 - 0.0672 - 0.116 - 0.1599 - 0.116 - 0.1321 - 0.1321 \right]$$

$$H\left[\frac{X}{Y}\right] = 1.0753 \text{ bits/symbol}$$

$$P\left(\frac{Y}{X}\right) = \frac{P(X, Y)}{P(X)} = \begin{bmatrix} \frac{0.25}{0.35} & 0 & \frac{0.1}{0.35} & 0 \\ 0 & \frac{0.25}{0.3} & 0 & \frac{0.05}{0.3} \\ 0 & 0 & \frac{0.1}{0.15} & \frac{0.05}{0.15} \\ 0 & \frac{0.05}{0.15} & 0 & \frac{0.1}{0.15} \\ 0 & 0 & 0 & \frac{0.05}{0.05} \end{bmatrix}$$

$$H\left[\frac{Y}{X}\right] = - \sum_{i=1}^5 \sum_{j=1}^4 P(x_i, y_j) \log_2 \left[P\left(\frac{y_j}{x_i}\right) \right] = \begin{bmatrix} 0.7142 & 0 & 0.2857 & 0 \\ 0 & 0.8333 & 0 & 0.1666 \\ 0 & 0 & 0.666 & 0.333 \\ 0 & 0.333 & 0 & 0.666 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= - \left[0.25 \log_2(0.7142) + 0.1 \log_2(0.2857) + 0.25 \log_2(0.8333) \right. \\ \left. + 0.05 \log_2(0.1666) + 0.1 \log_2(0.666) + 0.05 \log_2(0.333) \right. \\ \left. + 0.05 \log_2(0.333) + 0.1 \log_2(0.666) + 0.05 \log_2(1) \right] \\ = - \left[-0.1213 - 0.1807 - 0.0657 - 0.1292 - 0.0586 - 0.0792 \right. \\ \left. - 0.0792 - 0.0585 \right] = 0.7724 \text{ bits/symbol}$$

find all Entropy!

The Joint Probability Matrix of a noisy channel is given by

$$P(x, y) = \begin{matrix} & y_1 & y_2 & y_3 & y_4 \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{matrix} & \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0.1 & 0.3 & 0 & 0 \\ 0 & 0.05 & 0.1 & 0 \\ 0 & 0 & 0.05 & 0.1 \\ 0 & 0 & 0.05 & 0 \end{bmatrix} \end{matrix}$$

$\rightarrow P(x_1) = 0.25$
 $\rightarrow P(x_2) = 0.4$
 $\rightarrow P(x_3) = 0.15$
 $\rightarrow P(x_4) = 0.15$
 $\rightarrow P(x_5) = 0.05$

$P(y_1) = 0.35$
 $P(y_2) = 0.35$
 $P(y_3) = 0.2$
 $P(y_4) = 0.1$

5x4.

Ans

$$P\left(\frac{x}{y}\right) = \frac{P(x, y)}{P(y)} = \begin{bmatrix} 0.7142 & 0 & 0 & 0 \\ 0.8571 & 0.8571 & 0 & 0 \\ 0 & 0.1428 & 0.5 & 0 \\ 0 & 0 & 0.25 & 0.1 \\ 0 & 0 & 0.25 & 0 \end{bmatrix}_{5 \times 4}$$

$$P\left(\frac{y}{x}\right) = \frac{P(x, y)}{P(x)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 \\ 0 & 0.33 & 0.66 & 0 \\ 0 & 0 & 0.33 & 0.66 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{5 \times 4}$$

$$H[X] = - \sum_{i=1}^5 P(x_i) \log_2 [P(x_i)] = - [0.25 \log_2 (0.25) + 0.4 \log_2 (0.4) + 0.15 \log_2 (0.15) + 0.15 \log_2 (0.15) + 0.05 \log_2 (0.05)]$$

$$H[X] = 2.066 \text{ Bits/Symbol}$$

$$H[Y] = - \sum_{j=1}^4 P(y_j) \log_2 [P(y_j)] = - [0.35 \log_2 (0.35) + 0.35 \log_2 (0.35) + 0.2 \log_2 (0.2) + 0.1 \log_2 (0.1)]$$

$$H[Y] = 1.856 \text{ Bits/Symbol}$$

$$H[X, Y] = - \sum_{i=1}^5 \sum_{j=1}^4 P(x_i, y_j) \log_2 [P(x_i, y_j)] = - [0.25 \log_2 (0.25) + 0.1 \log_2 (0.1) + 0.3 \log_2 (0.3) + 0.05 \log_2 (0.05) + 0.1 \log_2 (0.1) + 0.05 \log_2 (0.05) + 0.1 \log_2 (0.1) + 0.05 \log_2 (0.05)]$$

$$H[X, Y] = 2.665 \text{ Bits/Symbol}$$

$$H[X/Y] = - \sum_{i=1}^5 \sum_{j=1}^4 P(x_i, y_j) \log_2 \left(P\left(\frac{x_i}{y_j}\right) \right)$$

$$= - \left[0.25 \log_2(0.7142) + 0.1 \log_2(0.2857) + 0.3 \log_2(0.8571) \right. \\ \left. + 0.05 \log_2(0.1428) + 0.1 \log_2(0.5) + 0.05 \log_2(0.25) + 0.1 \log_2(1) \right. \\ \left. + 0.05 \log_2(0.25) \right]$$

$$H[X/Y] = 0.809 \text{ bits/symbol}$$

$$H[Y/X] = - \sum_{i=1}^5 \sum_{j=1}^4 P(x_i, y_j) \log_2 \left(P\left(\frac{y_j}{x_i}\right) \right)$$

$$= - \left[0.25 \log_2(1) + 0.1 \log_2(0.25) + 0.3 \log_2(0.75) \right. \\ \left. + 0.05 \log_2(0.33) + 0.1 \log_2(0.66) + 0.05 \log_2(0.33) \right. \\ \left. + 0.1 \log_2(0.66) + 0.05 \log_2(1) \right]$$

$$H[Y/X] = 0.6 \text{ bits/symbol}$$

Check the relations

$$H[X,Y] = H[X] + H[Y/X]$$

$$2.665 = 2.066 + 0.6$$

$$2.665 = 2.666$$

$$H[X,Y] = H[Y] + H[X/Y]$$

$$2.665 = 1.856 + 0.809$$

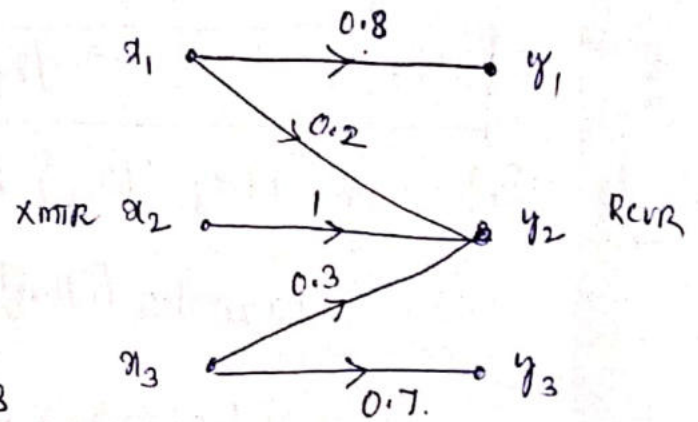
$$2.665 = 2.665$$

A discrete source transmits symbols x_1, x_2 & x_3 with probabilities 0.3, 0.4 & 0.3 respectively. The source is connected to a noisy channel as shown in fig (1). Find all types of entropies.

Ans Given Data

Probabilities of symbols generated at Transmitter side are

$$P(x_1) = 0.3, P(x_2) = 0.4, P(x_3) = 0.3$$



$$P[x] = [P(x_1), P(x_2), P(x_3)]$$

noisy channel

$$P[x] = [0.3, 0.4, 0.3]$$

Conditioned matrix $P\left[\frac{y}{x}\right] =$

	y_1	y_2	y_3
x_1	0.8	0.2	0
x_2	0	1	0
x_3	0	0.3	0.7

3×3

Find $H[x], H[y], H[x, y], H\left[\frac{x}{y}\right]$ & $H\left[\frac{y}{x}\right] = ?$

Joint Probability matrix $P[x, y] = P\left[\frac{y}{x}\right] \times P[x]$

$$= \begin{bmatrix} 0.8 \times 0.3 & 0.2 \times 0.3 & 0 \times 0.3 \\ 0 \times 0.4 & 1 \times 0.4 & 0 \times 0.4 \\ 0 \times 0.3 & 0.3 \times 0.3 & 0.7 \times 0.3 \end{bmatrix}$$

$$P[y] = [P(y_1), P(y_2), P(y_3)]$$

$$= [0.24, 0.55, 0.21]$$

	y_1	y_2	y_3
x_1	0.24	0.06	0
x_2	0	0.4	0
x_3	0	0.09	0.21

3×3

$P(y_1) = 0.24$ $P(y_2) = 0.55$ $P(y_3) = 0.21$

Mutual Information

$$H[X] = - \sum_{i=1}^3 P(x_i) \log_2 [P(x_i)]$$

$$= - [P(x_1) \log_2 [P(x_1)] + P(x_2) \log_2 [P(x_2)] + P(x_3) \log_2 [P(x_3)]]$$

$$= - [0.3 \log_2 (0.3) + 0.4 \log_2 (0.4) + 0.3 \log_2 (0.3)]$$

$$H[X] = 1.571 \text{ Bits / Symbol}$$

$$H[Y] = - \sum_{j=1}^3 P(y_j) \log_2 [P(y_j)]$$

$$= - [P(y_1) \log_2 [P(y_1)] + P(y_2) \log_2 [P(y_2)] + P(y_3) \log_2 [P(y_3)]]$$

$$= - [0.24 \log_2 (0.24) + 0.55 \log_2 (0.55) + 0.21 \log_2 (0.21)]$$

$$H[Y] = 1.441 \text{ Bits / Symbol}$$

$$H[X, Y] = - \sum_{i=1}^3 \sum_{j=1}^3 P(x_i, y_j) \log_2 [P(x_i, y_j)]$$

$$= - [0.24 \log_2 (0.24) + 0.06 \log_2 (0.06) + 0.4 \log_2 (0.4) + 0.09 \log_2 (0.09) + 0.21 \log_2 (0.21)]$$

$$H[X, Y] = 2.053 \text{ Bits / Symbol}$$

$$H\left[\frac{Y}{X}\right] = - \sum_{i=1}^3 \sum_{j=1}^3 P(x_i, y_j) \log_2 \left[P\left(\frac{y_j}{x_i}\right)\right]$$

$$= - [0.24 \log_2 (0.8) + 0.06 \log_2 (0.2) + 0.4 \log_2 (1) + 0.09 \log_2 (0.3) + 0.21 \log_2 (0.7)]$$

$$H\left[\frac{Y}{X}\right] = 0.482 \text{ Bits / Symbol}$$

$$P\left[\frac{X}{Y}\right] = \frac{P[X, Y]}{P[Y]} = \begin{bmatrix} \frac{0.24}{0.24} & \frac{0.06}{0.55} & 0 \\ 0 & \frac{0.4}{0.55} & 0 \\ 0 & \frac{0.09}{0.55} & \frac{0.21}{0.21} \end{bmatrix}$$

$$H\left[\frac{X}{Y}\right] = - \sum_{i=1}^3 \sum_{j=1}^3 P(x_i, y_j) \log_2 \left[P\left(\frac{x_i}{y_j}\right)\right]$$

$$= - [0.24 \log_2 (1) + 0.06 \log_2 (0.109) + 0.4 \log_2 (0.727) + 0.09 \log_2 (0.164) + 0.21 \log_2 (1)]$$

$$H\left[\frac{X}{Y}\right] = 0.612 \text{ Bits / Symbol}$$

$$P\left[\frac{X}{Y}\right] = \begin{bmatrix} 1 & 0.109 & 0 \\ 0 & 0.727 & 0 \\ 0 & 0.164 & 1 \end{bmatrix}$$

Mutual Information :- The information gained about x_i by the reception of y_j is the net reduction in its uncertainty and is known as Mutual Information $I(x_i, y_j)$. And it is expressed as

$$I(x_i, y_j) = \text{Initial Uncertainty} - \text{Final Uncertainty}$$

$$= -\log_2 [P(x_i)] - \left[-\log_2 \left[\frac{P(x_i | y_j)}{P(x_i)} \right] \right]$$

$$= -\log_2 [P(x_i)] + \log_2 \left[\frac{P(x_i | y_j)}{P(x_i)} \right]$$

$$\log_2 \left(\frac{a}{b} \right) = \log_2 a - \log_2 b$$

$$I(x_i, y_j) = \log_2 \left[\frac{P(x_i | y_j)}{P(x_i)} \right]$$

Self information :- It is special case of Mutual Information

$$\text{When } x_i = y_j \Rightarrow I(x_i, x_i) = \log_2 \left[\frac{P(x_i | x_i)}{P(x_i)} \right] = \log_2 \left[\frac{1}{P(x_i)} \right]$$

$$I(x_i, x_i) = \log_2 \left[\frac{1}{P(x_i)} \right]$$

Mutual Entropy :- The Average mutual Information is called "Mutual Entropy".

$$I(X; Y) = I(x_i, y_j) = \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) I(x_i, y_j)$$

$$= \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 \left[\frac{P(x_i | y_j)}{P(x_i)} \right]$$

$$= \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \left[\log_2 [P(x_i | y_j)] - \log_2 [P(x_i)] \right]$$

$$= \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 [P(x_i | y_j)] - \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 [P(x_i)]$$

$$= \sum_{i=1}^M \sum_{j=1}^N P(x_i, y_j) \log_2 [P(x_i | y_j)] - \sum_{i=1}^M P(x_i) \log_2 [P(x_i)]$$

$$I = -H\left[\frac{x}{y}\right] + H[x]$$

$$I(y) = H[x] - H\left[\frac{x}{y}\right]$$

Marginal
Entropy

Condition
Entropy

Similarly

$$I(x; y) = H[y] - H\left[\frac{y}{x}\right]$$

of Mutual Information :- (or) Mutual Entropy :-

Information is symmetric i.e. $I(x_i, y_j) = I(y_j, x_i)$

Information is non-negative i.e. $I(x_i, y_j) \geq 0$

$\Rightarrow$ If $x_i = y_j$ then Mutual information is equal to self information

$$I(x_i, y_j) = I(x_i, x_i) \quad | \quad x_i = y_j$$

Entropy can be obtained by using Self & Conditional
(or) Marginal

i.e. $I(x; y) = H[x] - H\left[\frac{x}{y}\right] = H[y] - H\left[\frac{y}{x}\right]$

Entropy is related to joint Entropy as

$$I(x; y) = H[x] - [H[x, y] - H[y]]$$

$$I(x; y) = H[x] + H[y] - H[x, y]$$

$$= H[y] + H[x] - H[x, y]$$

$$H[x, y] = H[x] + H\left[\frac{y}{x}\right]$$

$$= H[y] + H\left[\frac{x}{y}\right]$$

Q: ① A Transmitter generates 5 symbols x_1, x_2, x_3, x_4 & x_5 with probabilities $P(x_1), P(x_2), P(x_3), P(x_4)$ & $P(x_5)$ and receives y_1, y_2, y_3 & y_4 symbols with probabilities. The joint probability matrix is shown in the Table.

Find Mutual ~~Information~~ Entropy?

Ans:

Mutual Entropy

$$I(x; y) = H[x] - H[x/y]$$

$$I[x; y] = H[x] + H[y] - H[xy] \quad \text{--- (A)}$$

$$P(xy) = \begin{matrix} & y_1 & y_2 & y_3 & y_4 \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{matrix} & \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0.1 & 0.3 & 0 & 0 \\ 0 & 0.05 & 0.1 & 0 \\ 0 & 0 & 0.05 & 0.1 \\ 0 & 0 & 0.05 & 0 \end{bmatrix} \end{matrix}$$

5x4.

$$\begin{aligned} P(x_1) &= 0.25 \\ P(x_2) &= 0.1 + 0.3 = 0.4 \\ P(x_3) &= 0.05 + 0.1 = 0.15 \\ P(x_4) &= 0.05 + 0.1 = 0.15 \\ P(x_5) &= 0.05 \end{aligned}$$

$$\begin{aligned} P(y_1) &= 0.35 = 0.25 + 0.1 \\ P(y_2) &= 0.3 + 0.05 = 0.35 \\ P(y_3) &= 0.1 + 0.05 + 0.05 = 0.2 \\ P(y_4) &= 0.1 \end{aligned}$$

$$\begin{aligned} H[x] &= - \sum_{i=1}^5 P(x_i) \log_2 [P(x_i)] \\ &= - [0.25 \log_2 (0.25) + 0.4 \log_2 (0.4) + 2 \times 0.15 \log_2 (0.15) + 0.05 \log_2 (0.05)] \\ H[x] &= 2.066 \text{ bits/symbol} \end{aligned}$$

$$\begin{aligned} H[y] &= - \sum_{j=1}^4 P(y_j) \log_2 [P(y_j)] \\ &= - [0.35 \log_2 (0.35) * 2 + 0.2 \log_2 (0.2) + 0.1 \log_2 (0.1)] \end{aligned}$$

$$H[y] = 1.857 \text{ bits/symbol}$$

$$\begin{aligned} H[xy] &= - \sum_{i=1}^5 \sum_{j=1}^4 P(x_i, y_j) \log_2 [P(x_i, y_j)] \\ &= - [0.25 \log_2 (0.25) + 3 \times 0.1 \log_2 (0.1) + 0.05 \log_2 (0.05) + 0.3 \log_2 (0.3)] \end{aligned}$$

$$H[xy] = 2.666 \text{ bits/symbol}$$

$$(A) \Rightarrow I(x; y) = 2.066 + 1.857 - 2.666 = 1.257 \text{ bits/symbol}$$

Channel Capacity :-

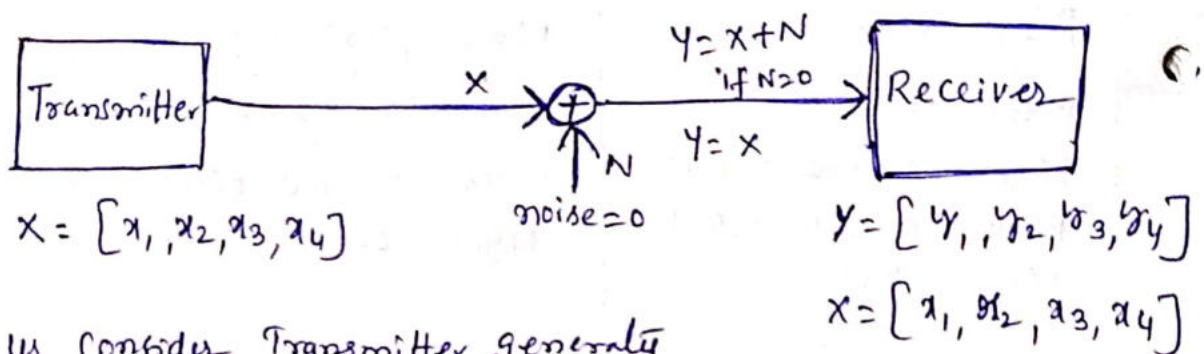
Shannon has introduced a significant concept of Channel Capacity defined as the maximum of mutual Entropy.

$$\text{Channel Capacity } C = \max. [I(X; Y)]$$

$$C = \max [H[X] - H[\frac{X}{Y}]] \text{ Bits/symbol}$$

Some times information rate is referred as Entropy, so, unit of Channel Capacity is Bits/sec.

Channel Capacity of Noise free Channel :-



Let us consider Transmitter generates

four symbols x_1, x_2, x_3 & x_4 with probabilities $P(x_1), P(x_2), P(x_3)$ & $P(x_4)$ respectively. The receiver receives $Y = [y_1, y_2, y_3, y_4]$. Since Noise $N=0$, it receives x_1, x_2, x_3 & x_4 . Hence

$$H[\frac{X}{Y}] = 0.$$

$$\text{Channel Capacity } C = \max. [H[X] - H(X/Y)]$$

$$= \max [H[X]]$$

$$= \log_2 4$$

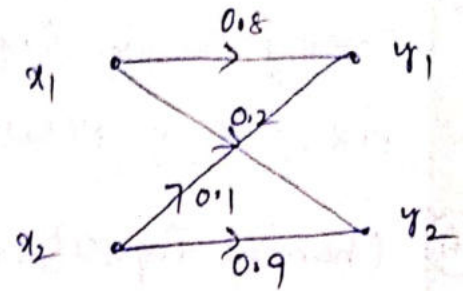
In general if Transmitter generates 'M' symbols then

$$C = \max [H[X]]$$

$$C = \log_2 M \text{ Bits/sec}$$

Find the Channel Capacity of a binary symmetric channel (23)
shown with condition probabilities.

Probabilities of x_1 & x_2 are
0.6 & 0.4 respectively.



(A) Given Data :-

$$P(x_1) = 0.6 \text{ \& } P(x_2) = 0.4$$

$$P\left(\frac{y}{x}\right) = \begin{matrix} & y_1 & y_2 \\ \begin{matrix} x_1 \\ x_2 \end{matrix} & \begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{bmatrix} \end{matrix}$$

$$H[x] = -\sum_{i=1}^2 P(x_i) \log_2 P(x_i)$$

$$= -[P(x_1) \log_2 P(x_1) + P(x_2) \log_2 P(x_2)]$$

$$= -[0.6 \log_2(0.6) + 0.4 \log_2(0.4)]$$

$$= -[-0.4421 - 0.5287]$$

$$H[x] = 0.9708 \text{ Bits/symbol}$$

$$P(xy) = P\left(\frac{y}{x}\right) P(x) = \begin{matrix} & y_1 & y_2 \\ \begin{matrix} x_1 \\ x_2 \end{matrix} & \begin{bmatrix} 0.8 \times 0.6 & 0.2 \times 0.6 \\ 0.1 \times 0.4 & 0.9 \times 0.4 \end{bmatrix} \end{matrix}$$

$$P(xy) = \begin{matrix} & y_1 & y_2 \\ \begin{matrix} x_1 \\ x_2 \end{matrix} & \begin{bmatrix} 0.48 & 0.12 \\ 0.04 & 0.36 \end{bmatrix} \end{matrix}$$

$\rightarrow P(x_1) = 0.6$
 $\rightarrow P(x_2) = 0.4$
 $P(y_1) = 0.52$
 $P(y_2) = 0.48$

$$H(y) = -\sum_{j=1}^2 P(y_j) \log_2 [P(y_j)] = -[P(y_1) \log_2 [P(y_1)] + P(y_2) \log_2 [P(y_2)]]$$

$$= -[0.52 \log_2(0.52) + 0.48 \log_2(0.48)] = -[-0.4905 - 0.5082]$$

$$H(y) = 0.9987 \text{ Bits/symbol}$$

$$H[xy] = -\sum_{i=1}^2 \sum_{j=1}^2 P(x_i, y_j) \log_2 [P(x_i, y_j)]$$

$$= -[0.48 \log_2(0.48) + 0.12 \log_2(0.12) + 0.04 \log_2(0.04) + 0.36 \log_2(0.36)]$$

$$= -[-0.5082 - 0.3670 - 0.1857 - 0.5306] \Rightarrow H[xy] = 1.5915 \text{ Bits/symbol}$$

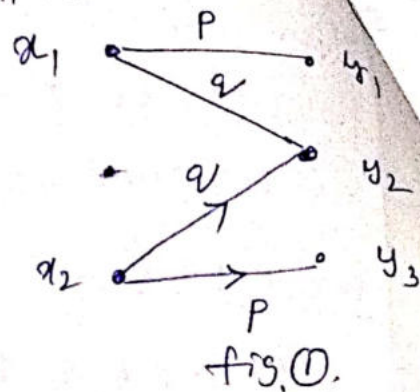
$$H\left[\frac{y}{x}\right] = -\sum_{i=1}^2 \sum_{j=1}^2 P(x_i, y_j) \log_2 \left(\frac{P(y_j)}{P(x_i)}\right) = -[0.48 \log_2(0.8) + 0.12 \log_2(0.2) + 0.04 \log_2(0.1) + 0.36 \log_2(0.9)]$$

$$= -[0.1545 - 0.2786 - 0.1328 - 0.0547] \Rightarrow H\left[\frac{y}{x}\right] = 0.6206 \text{ Bits/symbol}$$

Channel Capacity $C = \max [H[x] - H\left[\frac{y}{x}\right]] = \max [H[x] + H(y) - H(xy)]$

$$C = 0.378 \text{ Bits/sec}$$

- ② Find the Channel Capacity of the following Binary Erasure Channel (BEC) shown in fig. ①.
- $P(x_1) = \alpha$; $P(x_2) = 1 - \alpha$.



- ④ Channel Capacity $C = \max \{ I(X; Y) \}$

$$C = \max [H[X] - H\left[\frac{X}{Y}\right]]$$

$$C = \max [H[X] + H[Y] - H[XY]] \quad \text{--- ①}$$

$$P\left(\frac{X}{Y}\right) = \begin{matrix} & y_1 & y_2 & y_3 \\ x_1 & \begin{bmatrix} p & q & 0 \end{bmatrix} \\ x_2 & \begin{bmatrix} 0 & q & p \end{bmatrix} \end{matrix}$$

$$P(XY) = P\left(\frac{Y}{X}\right)P(X) = \begin{matrix} & y_1 & y_2 & y_3 \\ x_1 & \begin{bmatrix} \alpha p & \alpha q & 0 \end{bmatrix} \\ x_2 & \begin{bmatrix} 0 & (1-\alpha)q & (1-\alpha)p \end{bmatrix} \end{matrix}$$

$$H[X] = - \sum_{i=1}^2 P(x_i) \log_2 [P(x_i)] = - [P(x_1) \log_2 [P(x_1)] + P(x_2) \log_2 [P(x_2)]]$$

$$H[X] = - [\alpha \log_2 \left(\frac{1}{\alpha}\right) + (1-\alpha) \log_2 \left(\frac{1}{1-\alpha}\right)]$$

$$H[Y] = - \sum_{j=1}^3 P(y_j) \log_2 [P(y_j)] = - [\alpha p \log_2 \left(\frac{1}{\alpha p}\right) + q \log_2 \left(\frac{1}{q}\right) + (1-\alpha)p \log_2 \left(\frac{1}{(1-\alpha)p}\right)]$$

$$H[Y] = - [\alpha p \log_2 \left(\frac{1}{\alpha p}\right) + q \log_2 \left(\frac{1}{q}\right) + (1-\alpha)p \log_2 \left(\frac{1}{(1-\alpha)p}\right)]$$

$$H[XY] = - \sum_{i=1}^2 \sum_{j=1}^3 P(x_i, y_j) \log_2 [P(x_i, y_j)]$$

$$H[XY] = - [\alpha p \log_2 \left(\frac{1}{\alpha p}\right) + \alpha q \log_2 \left(\frac{1}{\alpha q}\right) + q(1-\alpha) \log_2 \left(\frac{1}{(1-\alpha)q}\right) + (1-\alpha)p \log_2 \left(\frac{1}{(1-\alpha)p}\right)]$$

$$\begin{aligned} \Rightarrow C &= \max [\alpha \log_2 \left(\frac{1}{\alpha}\right) + (1-\alpha) \log_2 \left(\frac{1}{1-\alpha}\right) + \alpha p \log_2 \left(\frac{1}{\alpha p}\right) + q \log_2 \left(\frac{1}{q}\right) \\ &+ (1-\alpha)p \log_2 \left(\frac{1}{(1-\alpha)p}\right) - \alpha p \log_2 \left(\frac{1}{\alpha p}\right) - \alpha q \log_2 \left(\frac{1}{\alpha q}\right) - (1-\alpha)q \log_2 \left(\frac{1}{(1-\alpha)q}\right) \\ &- (1-\alpha)p \log_2 \left(\frac{1}{(1-\alpha)p}\right)] \\ &= \end{aligned}$$

It states that the maximum Capacity of Channel

$$C = B \log_2 \left[1 + \frac{S}{N} \right] \text{ Bits/sec.}$$

Where C = Channel Capacity (Bits/sec)

B = Band width of Channel (Hz or $\frac{\text{cycles}}{\text{sec}}$ or $\frac{\text{samples}}{\text{sec}}$)

$\frac{S}{N}$ = ~~Signal~~ signal to noise Ratio (SNR)

Proof:- The received signal Power at the i/p of receiver and its mean square value can be written as $\sqrt{S+N}$

Noise Power is N & its mean square value is $\sqrt{N}$.

No. of Levels can be separated without error is

$$M = \frac{\sqrt{S+N}}{\sqrt{N}} = \sqrt{\frac{S+N}{N}} = \sqrt{1 + \frac{S}{N}} = \left[1 + \frac{S}{N} \right]^{\frac{1}{2}}$$

The information can be written as

$$I = \log_2(M) = \log_2 \left[\left(1 + \frac{S}{N} \right)^{\frac{1}{2}} \right]$$

$$I = \frac{1}{2} \log_2 \left[1 + \frac{S}{N} \right]$$

If the Channel transmitting 'K' pulse/sec the Channel

$$\text{Capacity } C = I K = K \cdot \frac{1}{2} \log_2 \left[1 + \frac{S}{N} \right]$$

Nyquist Bandwidth $K = 2B$ (Sampling theorem)

$$C = 2B \cdot \frac{1}{2} \log_2 \left[1 + \frac{S}{N} \right]$$

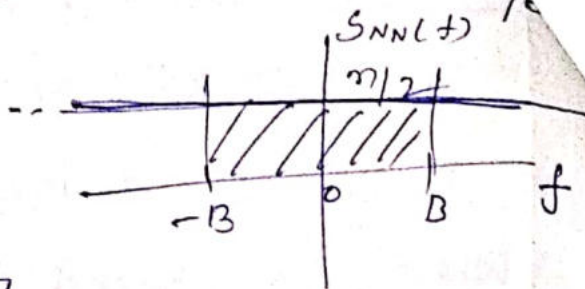
$$C = B \log_2 \left[1 + \frac{S}{N} \right] \text{ Bits/sec}$$

Above equation says that even if increase Band width i.e. $B \rightarrow \infty$, Channel Capacity cannot be ' ∞ ' because Noise also increases.

If the noise PSD is $\eta/2$ then noise power $N = \eta B$

$$N = \int_{f=-B}^{+B} \frac{\eta}{2} \cdot df = \frac{\eta}{2} [f]_{-B}^{+B}$$

$$= \frac{\eta}{2} \cdot 2B \Rightarrow \boxed{N = \eta B}$$



Channel Capacity $C = B \log_2 \left[1 + \frac{S}{\eta B} \right]$

$$= \frac{\eta B}{S} \cdot \frac{S}{\eta} \log_2 \left[1 + \frac{S}{\eta B} \right]$$

$$= \frac{S}{\eta} \lim_{B \rightarrow \infty} \left[\frac{\log_2 \left[1 + \frac{S}{\eta B} \right]}{\frac{S}{\eta B}} \right]$$

$$\lim_{x \rightarrow 0} \frac{\log_2(1+x)}{x} = \log_2 e$$

$$C = \frac{S}{\eta} \cdot (\log_2 e) \rightarrow 1.44$$

$$\boxed{C = B \log_2 [1 + S/N]}$$

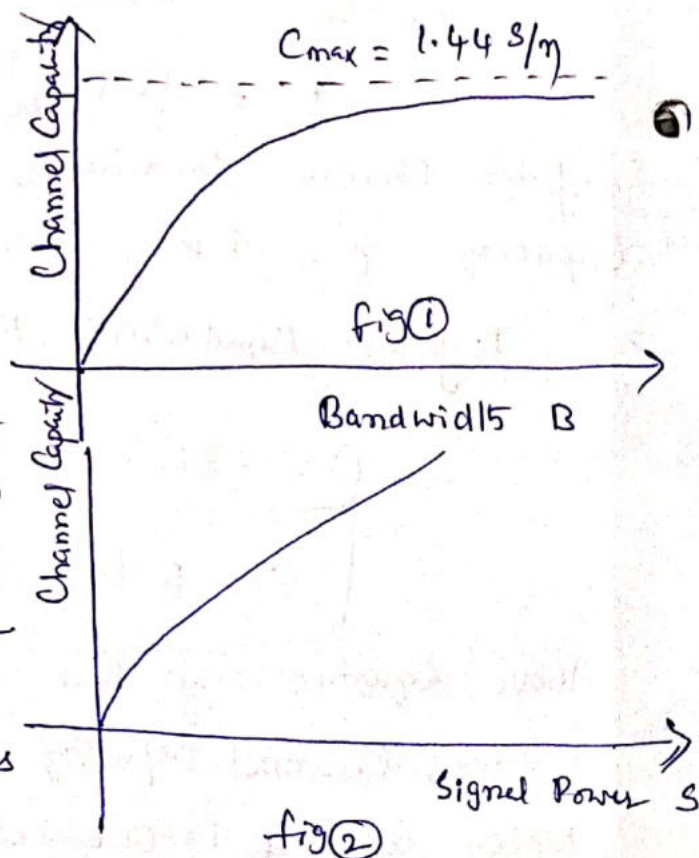
$$N = \eta B$$

$$\boxed{C_{\max} = 1.44 S/\eta}$$

1. Channel Capacity increases with increase of Signal Power as shown fig(2). But Signal Power cannot be increased beyond some value.

2. If Bandwidth is increased $B \rightarrow \infty$, Channel Capacity cannot infinity $C \rightarrow \infty$, because 'N' also increase $N = \eta B$. So, Channel Capacity attains maximum value at $C_{\max} = 1.44 S/\eta$.

3. The fig(1) shows that, There is a Tradeoff b/w Bandwidth & Signal to noise ratio (SNR).



www.android.universityupdates.in | www.universityupdates.in | https://telegram.me/jntua
for a typical telephone line with a signal to noise ratio of 30dB and an audio BW 3 kHz. Find the max data rate! (14)

(A) Given Data

$$\frac{S}{N} = 30\text{dB} \Rightarrow 30 = 10 \log_{10} \left(\frac{S}{N} \right)$$

$$\boxed{\frac{3}{10} = \frac{S}{N}}$$

Bandwidth $B = 3\text{ kHz}$

$$C = B \log_2 \left[1 + \frac{S}{N} \right] = 3 \times 10^3 \log_2 [1 + 1000]$$

$$\boxed{C = 3\text{ MB/sec.}}$$

(2) For Satellite TV Channel with signal to noise ratio of 20dB and a video BW of 10 MHz. find max data rate?

(A) Given Data

$$\frac{S}{N} = 20\text{dB} \Rightarrow 20 = 10 \log_{10} \left(\frac{S}{N} \right)$$

$$\boxed{\frac{2}{10} = \frac{S}{N}}$$

$B = 10\text{ MHz}$

$$C = B \log_2 [1 + S/N] = 66\text{ MB/sec.}$$

(3) A Gaussian Channel has 1 MHz, cal. the Channel Capacity if signal Power to noise Spectral Density is 10^5 Hz .
~~And also find max. information rate!~~

(A) Given Data

Bandwidth $B = 1\text{ MHz}$

$$\frac{S}{\eta} = 10^5\text{ Hz}$$

$$C = B \log_2 \left[1 + \frac{S}{\eta B} \right]$$

$$= 10^6 \log_2 \left[1 + \frac{10^5}{10^6} \right]$$

$$= 10^6 \log_2 \left[1 + \frac{1}{10} \right]$$

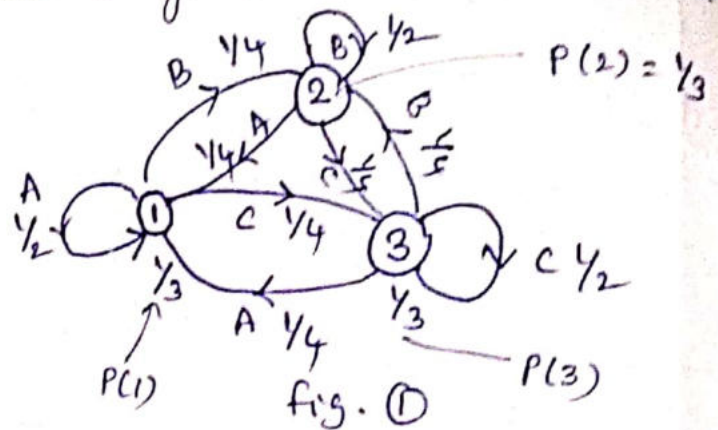
$$= 10^6 \log_2 [1.1] = 10^6 \times 0.13750$$

$$\boxed{C = 137.5\text{ KB/sec}}$$

Markoff Source :- If the information source is represented with state diagram then it is called "Markoff Source".

Example :- Markoff source in fig (1). which contains '3' nodes with

Probabilities $P(1) = \frac{1}{3}$, $P(2) = \frac{1}{3}$ & $P(3) = \frac{1}{3}$. The source generates A, B & C symbols with probabilities are shown on the directed lines.



Markoff source can be represented by using 'TREE DIAGRAM' as shown below.

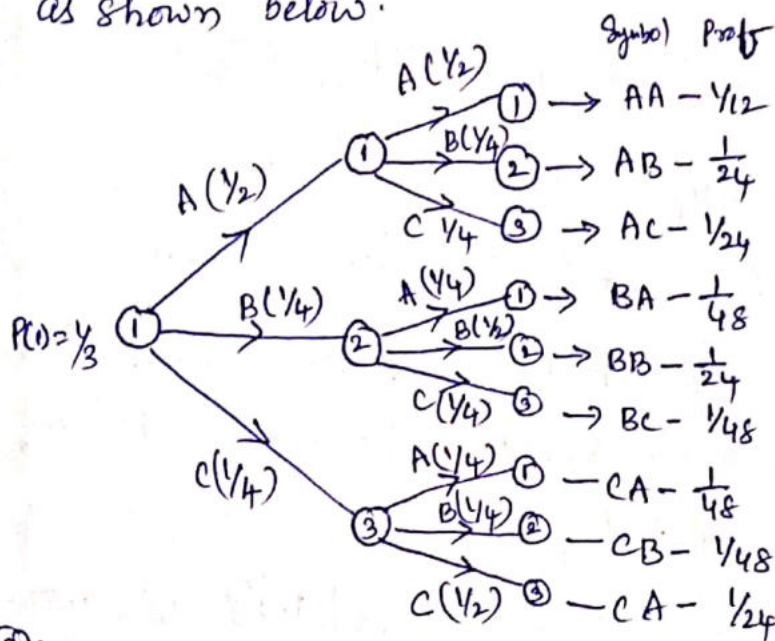


Fig (2): Symbols Emitted Starting at State 1

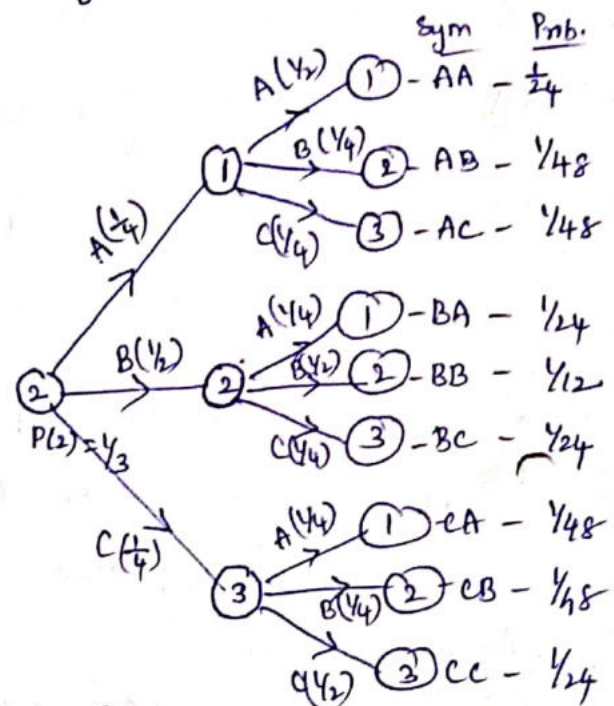


Fig (3): Symbols emitted with state 2

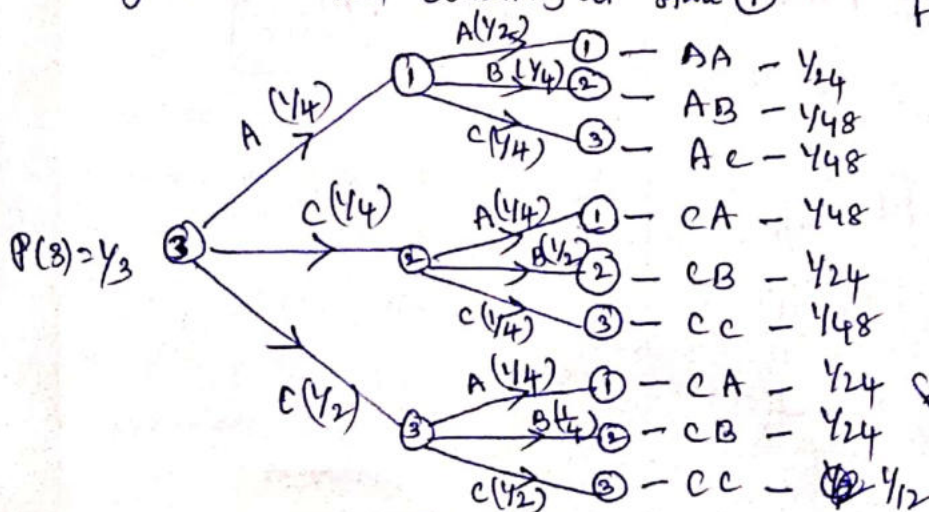


Fig (4): Symbols emitted with state 3

Entropy of Markoff Source :- It is defined as the weighted average of entropy of symbols emitted from each state. (29)

$$H_i = \sum_{j=1}^n P_{ij} \log_2 \left[\frac{1}{P_{ij}} \right] \text{ Bits/symbol.}$$

$i \rightarrow$ State, $n \rightarrow$ no. of nodes, P_{ij} :- The Change in the State of the source from i to j .

Entropy of the source is the Average of entropy of each state i.e. $H = \sum_{i=1}^n P_i H_i = \sum_{i=1}^n P_i \left[\sum_{j=1}^n P_{ij} \log_2 \left(\frac{1}{P_{ij}} \right) \right]$

● Average information content per symbol $\rightarrow$ Prob. that the source is in i 's state

$$G_N = \frac{1}{N} \sum_{i=1}^N P(m_i) \log_2 \left[\frac{1}{P(m_i)} \right]$$

Where P_{mi} = Prob. of sequence m_i of 'N' symbols from source.

Prob:- See the informative source graph. Find the following

i) Source Entropy ii) Avg. info. content per symbol (G_1, G_2, G_3)

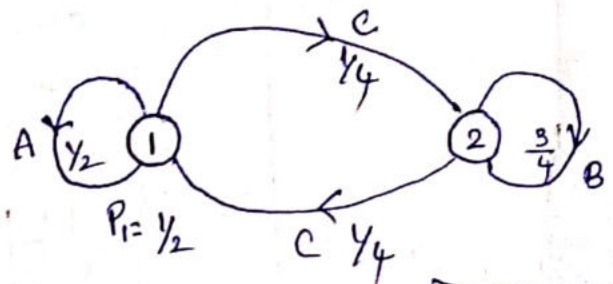
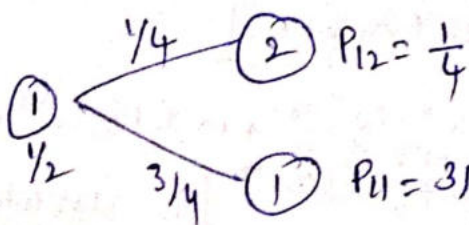
Ans:- Source Entropy $H = \sum_{i=1}^2 P_i H_i$

$$H = P_1 H_1 + P_2 H_2 \quad \text{--- (A)}$$

$$H_i = \sum_{j=1}^2 P_{ij} \log_2 \left[\frac{1}{P_{ij}} \right]$$

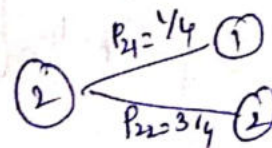
$$i=1 \Rightarrow H_1 = \sum_{j=1}^2 P_{1j} \log_2 \left(\frac{1}{P_{1j}} \right)$$

$$H_1 = P_{11} \log_2 \left(\frac{1}{P_{11}} \right) + P_{12} \log_2 \left(\frac{1}{P_{12}} \right)$$



$$H_1 = \frac{3}{4} \log_2 \left(\frac{4}{3} \right) + \frac{1}{4} \log_2 (4) \Rightarrow H_1 = 0.8112 \frac{\text{Bits}}{\text{Symbol}}$$

$$H_2 = P_{21} \log_2 \left(\frac{1}{P_{21}} \right) + P_{22} \log_2 \left(\frac{1}{P_{22}} \right)$$



$$H_2 = \frac{1}{4} \log_2 (4) + \frac{3}{4} \log_2 \left(\frac{4}{3} \right) \Rightarrow H_2 = 0.8112 \frac{\text{Bits}}{\text{Symbol}}$$

$$H = \frac{1}{2} \times 0.8112 + \frac{1}{2} \times 0.8112 = 0.8112 \frac{\text{Bits}}{\text{Symbol}}$$

Tree Diagram.

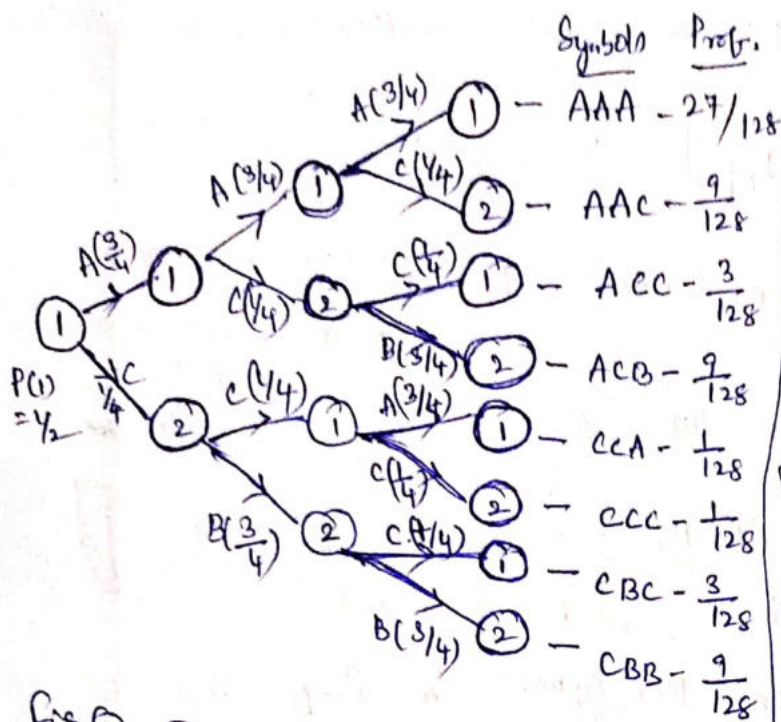


Fig ① Tree diagram starting with State ①

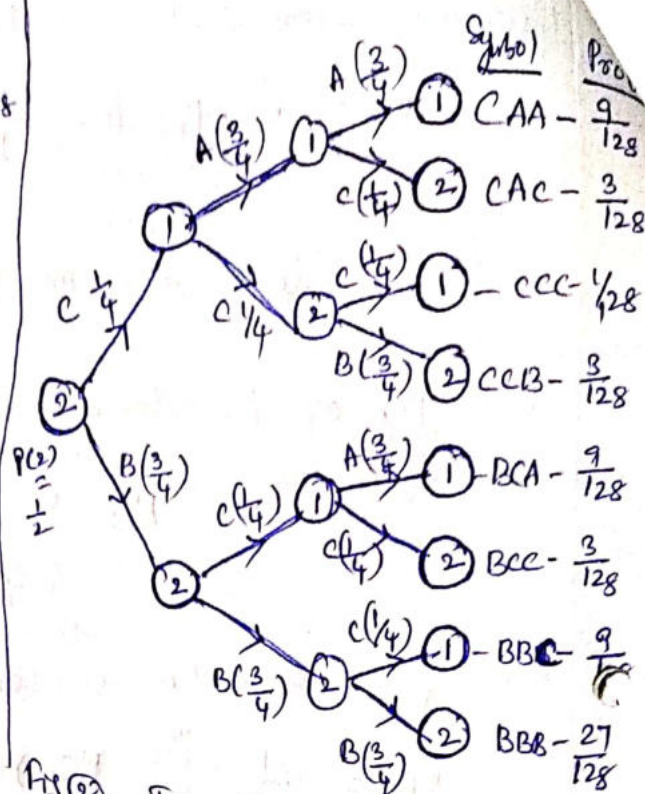


Fig ② Tree diagram starting with State ②.

Message of length m_1	$P(m_1)$	m_2	$P(m_2)$	m_3	$P(m_3)$	m_3	$P(m_3)$
A	$3/8$	AA	$9/32$	AAA	$27/128$	CAA	$9/128$
B	$3/8$	AC	$3/32$	AAC	$9/128$	CAC	$3/128$
C	$1/8$	CC	$1/32$	ACC	$3/128$	CCC	$1/128$
		CB	$3/32$	ACB	$9/128$	CCB	$3/128$
		CA	$3/32$	CCA	$1/128$	CCA	$1/128$
		CC	$1/32$	CCC	$1/128$	BCA	$9/128$
		BC	$3/32$	CBC	$3/128$	BCC	$3/128$
		BB	$9/32$	CBB	$9/128$	BBC	$9/128$
						BBB	$27/128$

N=1 $G_1 = \frac{1}{1} \sum_{i=1}^3 P(m_i) \log_2 \left(\frac{1}{P(m_i)} \right) = \left[\frac{3}{8} \log_2 \left(\frac{8}{3} \right) + \frac{3}{8} \log_2 \left(\frac{8}{3} \right) + \frac{1}{8} \log_2 (8) \right]$

N=2 $G_1 = 1.5612 \text{ Bits/symbol}$ $G_1 = 1.3425 \text{ Bits/symbol}$

$G_2 = \frac{1}{2} \sum_{i=1}^8 P(m_i) \log_2 \left(\frac{1}{P(m_i)} \right) = \frac{1}{2} \left[2 \times \frac{9}{32} \log_2 \left(\frac{32}{9} \right) + 4 \times \frac{3}{32} \log_2 \left(\frac{32}{3} \right) + 2 \times \frac{1}{32} \log_2 (32) \right]$

N=3 $G_2 = 1.28 \text{ Bits/symbol}$

$G_3 = \frac{1}{3} \sum_{i=1}^8 P(m_i) \log_2 \left(\frac{1}{P(m_i)} \right) = \frac{1}{3} \left[2 \times \frac{27}{128} \log_2 \left(\frac{128}{27} \right) + 6 \times \frac{9}{128} \log_2 \left(\frac{128}{9} \right) + 5 \times \frac{3}{128} \log_2 \left(\frac{128}{3} \right) + 3 \times \frac{1}{128} \log_2 (128) \right]$

$G_3 = 1.139 \text{ Bits/symbol}$

Shannon's Coding Theorem (or) Shannon's Channel Encoding Theorem :-

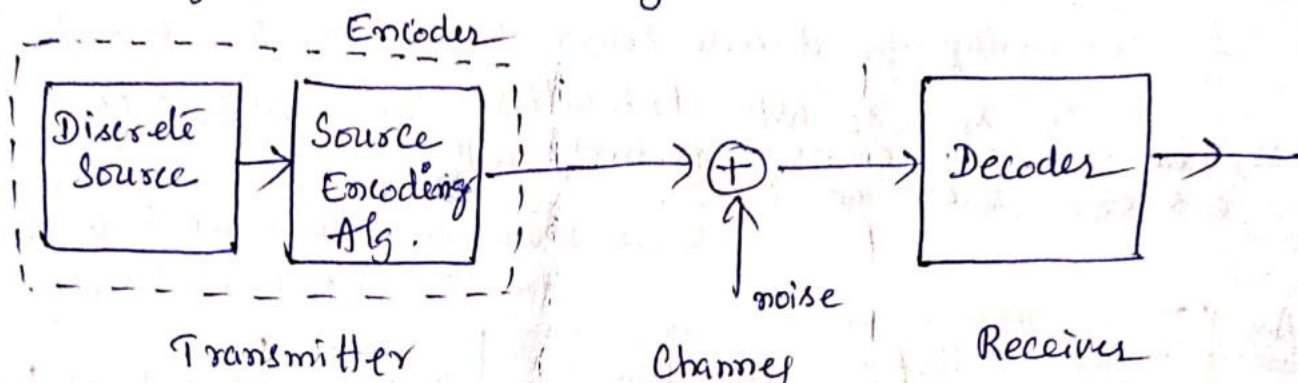
It states that "The maximum rate by which information can be transmitted reliably over the channel is

$$C = \text{Max} [I(X; Y)] = \log_2 M \text{ Bits/sec.}$$

for proof of above theorem — Read Page. no. (22)

Source Encoding Algorithms :-

Source encoding Algorithms are used in the encoder of the transmitter to reduce the redundancy (repeated data or bits) in the digital communication system as shown below.



The redundancy in the coding

$$R = 1 - \eta$$

Q=2 Binary
Q=3 Ternary
⋮

Where efficiency $\eta = \frac{H}{L \log_2 Q}$

$$\text{Entropy } H = - \sum_{k=1}^M P_k \log_2 (P_k) = \sum_{k=1}^M P_k \log_2 \left(\frac{1}{P_k} \right)$$

$$\text{Average Code length } L = \sum_{k=1}^M P_k L_k$$

Types of Source Encoding Alg. :-

① Shannon & Fano Coding

② Huffman Coding

Prob:- Shannon fano Encoding Alg - Procedure:-

- Step 1: Write the Probabilities in descending order
- Step 2: Divide the Probabilities into two subsets of equal Probabilities.
- Step 3: Assign logic '0' for up & logic '1' for Down Subset
- Step 4: Continue Step (2) & (3) until subset contains only one ~~subset~~ Probability.
- Step 5: Cal. efficiency & redundancy.

Prob:- Apply the Shannon fano Coding & find the efficiency & redundancy for discrete source generating six symbols x_1, x_2, x_3, x_4, x_5 & x_6 with Probabilities 0.25, 0.125, 0.0625, 0.0625, 0.25, ~~0.0625~~, 0.0625, 0.0625 & 0.125 respectively with $Q=2$.

→ Divide the sequence (or) Probabilities into two equal parts (or) subsets.

Ans

Symbols	Prob	Algorithm	Code	Code length (L_i)
x_1	0.25 (P_1)	0 0	00	2 (L_1)
x_5	0.25 (P_2)	0 1	01	2 (L_2)
x_2	0.125 (P_3)	1 0 0	100	3 (L_3)
x_8	0.125 (P_4)	1 0 1	101	3 (L_4)
x_3	0.0625 (P_5)	1 1 0 0	1100	4 (L_5)
x_4	0.0625 (P_6)	1 1 0 1	1101	4 (L_6)
x_6	0.0625 (P_7)	1 1 1 0	1110	4 (L_7)
x_7	0.0625 (P_8)	1 1 1 1	1111	4 (L_8)

Average length $L = \sum_{k=1}^8 P_k L_k = \sum_{k=1}^8 P_k L_k$

$$= P_1 L_1 + P_2 L_2 + P_3 L_3 + P_4 L_4 + P_5 L_5 + P_6 L_6 + P_7 L_7 + P_8 L_8$$

$$= (0.25 \times 2) + (0.25 \times 2) + (0.125 \times 3) + (0.125 \times 3) + 0.0625(0.0625 \times 4) + (0.0625 \times 4) + (0.0625 \times 4) + (0.0625 \times 4)$$

$$L = 2.75 \text{ Bits/symbol.}$$

Entropy $H = - \sum_{k=1}^8 P_k \log_2(P_k) = - [0.25 \log_2(0.25) \times 2 + 0.125 \log_2(0.125) \times 2 + 0.0625 \times (\log_2(0.0625) \times 4)]$

$$H = 2.75 \text{ Bits/symbol.}$$

Efficiency $\eta = \frac{H}{L} \times 100 = \frac{2.75}{2.75} = 1 \text{ or } 100\%$

Redundancy $R = 1 - \eta = 0$

Ques: Huffman Coding:—

Procedure:—

Step 1:— Write the Probabilities in the descending order

Step 2:— ^{Assign} Two symbols of lowest probability are assigned logic '0' & logic '1'.

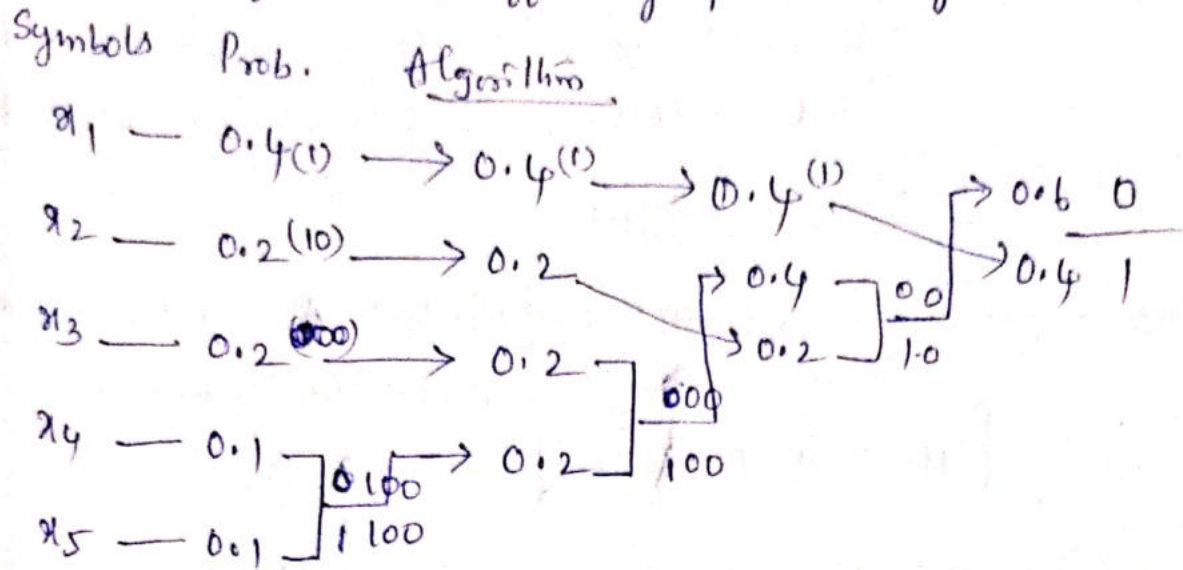
Step 3:— Combine last two probabilities and write Probabilities in descending order

Step 4:— Repeat Step 2 & Step 3 until occurrence of single Prob.

Step 5:— Find efficiency & redundancy.

Prob Apply Huffman coding Alg. for following symbols

x_1, x_2, x_3, x_4, x_5 with Probabilities 0.4, 0.2, 0.2, 0.1, 0.1 respectively. Find efficiency & redundancy.



Symbols	Code	Code length (L_i)	Probabilities (P_i)
x_1	1	1 (L_1)	0.4
x_2	10	2 (L_2)	0.2
x_3	000	3 (L_3)	0.2
x_4	0100	4 (L_4)	0.1
x_5	1100	4 (L_5)	0.1

$$H = - \sum_{i=1}^5 P_i \log_2 P_i = - [0.4 \log_2 (0.4) + 2 * 0.2 \log_2 (0.2) + 2 * 0.1 \log_2 (0.1)]$$

$$H = [0.5287 - 0.9287 - 0.6643] = 2.1217 \text{ bits/symbol}$$

$$L = \sum_{i=1}^5 P_i L_i = (0.4 \times 1) + (0.2 \times 2) + (0.2 \times 3) + (0.1 \times 4) + (0.1 \times 4)$$

$$L = 0.4 + 0.4 + 0.6 + 0.8 = 2.2 \text{ bits/symbol}$$

$$\text{Efficiency } \eta = \frac{H}{L} = \frac{2.1217}{2.2} = 0.9644 \text{ or } 96.44\%$$

$$\text{Redundancy } R = 1 - \eta = 0.0355 \text{ (or) } 3.55\%$$

2. A Discrete Source is generating symbols x_1, x_2, x_3, x_4, x_5 with Probabilities 0.4, 0.1, 0.2, 0.1 & 0.2. Encode using
a) Shannon Fano b) Huffman Cal. efficiency & redundancy with $Q = 2$.

④ Shannon's Fano

Symbols	Prob. (P_i)	Coding	code	no. of bits L_i
x_1	0.4	0	0	1
x_3	0.2	1 0	10	2
x_5	0.2	1 1 0	110	3
x_2	0.1	1 1 1 0	1110	4
x_4	0.1	1 1 1 1	1111	4

$$\text{Entropy } H = - \sum_{i=1}^5 P_i \log_2(P_i) = - [0.4 \log_2(0.4) + 2 \times 0.2 \log_2(0.2) + 2 \times 0.1 \log_2(0.1)]$$

$$H = 2.1217 \text{ bits/symbol}$$

$$\text{Length } L = \sum_{i=1}^5 P_i L_i = (0.4 \times 1) + (0.2 \times 2) + (0.2 \times 3) + (0.1 \times 4) \times 2 = 0.4 + 0.4 + 0.6 + 0.8$$

$$L = 2.2 \text{ bits/symbol}$$

$$\text{Efficiency } \eta = \frac{H}{L \times \log_2 Q} = \frac{2.1217}{2.2 \times \log_2 2} = 0.9644 \text{ (or) } 96.44\%$$

$$\text{Redundancy } R = 1 - \eta = 1 - 0.9644 = 0.0355 = 3.55\%$$

⑤ Huffman code

See page. no. (34)