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### **ANNAMACHARYA UNIVERSITY, RAJAMPET**

(ESTD UNDER AP PRIVATE UNIVERSITIES (ESTABLISHMENT AND REGULATION) ACT, 2016
RAJAMPET, Annamayya District, AP, INDIA

Course: Mathematical Foundations of Computer Science

Course Code : 24FMAT11T

Branch : MCA

Prepared by: K.Adisekhar Babu

**Designation:** Assistant Professor

Department: MCA



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RAJAMPET, Annamayya District, AP, INDIA

Title of the Course : Mathematical Foundations of Computer

Science Category : BS

Couse Code : 24FMAT11T

Branch : MCA

Semester : I Semester

Lecture Hours Tutorial Hours Practice Hours Credits
3 0 0 3

#### **COURSE OBJECTIVES:**

The aim is to cover the basics of mathematical logic and to work with sets, relations, and functions. It also involves studying how combinatorial methods are applied in fields like cryptography, coding theory, and algorithm design, as well as understanding generating functions, recurrence relations, and fundamental concepts of graphs and trees and their practical uses

#### **UNIT I Mathematical Logic**

10 Hrs

**Propositional Calculus:** Statements and Notations, Connectives, Well-formed formulas, Truth tables, Tautologies, Equivalence of formulas, Duality Law, Tautological implications, Normal forms, Theory of Inference for Statement calculus.

**Predicate Calculus:** Predicates, Predicative logic, Statement functions, Variables and Quantifiers, Free and Bound variables, Inference theory for Predicate calculus.

UNIT II Set Theory 8 Hrs

**Sets:** Operations on Sets, Principle of Inclusion-Exclusion. Relations-Properties, Operations, Partition and Covering, Transitive closure, Equivalence, Compatibility and Partial ordering, Hasse diagrams. Functions- Bijective, Composition, Inverse functions, Lattice and its Properties.

#### **UNIT III Elementary Combinatorics**

8 Hrs

Basics of Counting, Combinations and Permutations, Enumerating Combinations & Permutations with repetitions, Enumerating Permutations with constrained repetitions, Binomial coefficients, Binomial and Multinomial theorems.

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#### **UNIT IV Recurrence Relations**

8 Hrs

10 Hrs

Generating functions of sequences, calculating coefficients of generating functions, Recurrence relations - Solving recurrence relations by substitution and generating functions, Method of characteristic roots for the Solutions of non-homogeneous recurrence relations.

UNIT V Graph Theory

Definitions, Finite and infinite graphs, Incidence and Degree, Isolated and Pendant vertices, Graph representations- Adjacency and Incidence matrices, Isomorphism, subgraphs, Walk, Path and Circuit, Connected and Disconnected graphs, Components, Multi graphs, Bipartite and Planar Graphs, Euler's formula, Euler graphs, Hamiltonian paths and circuits, Chromatic number, Trees, Spanning trees: DFS, BFS, Minimal Spanning trees: Prim's and Kruskal's algorithms.

#### **TEXT BOOKS:**

- 1. J.P. Tremblay and R. Manohar. Discrete Mathematical Structures with Applications to Computer Science, Tata Mc Graw Hill Education, 1<sup>st</sup> Edition, 2002.
- 2. D. S. Chandra Sekharaiah. Mathematical Foundations of Computer Science, Prism Books, 3<sup>rd</sup> Edition, 2010.

#### **REFERENCE BOOKS:**

- 1. J.L. Mott, A. Kandel and T. P. Baker. Discrete Mathematics for Computer Scientists and Mathematicians, PHI, 2<sup>nd</sup> Edition, 2008.
- 2. C.L. Liu and D.P. Mohapatra, Elements of Discrete Mathematics A computer-oriented approach, Mc Graw Hill India, 3<sup>rd</sup> Edition, 2011.

#### **COURSE OUTCOMES:**

The Student will be able to

- 1. Understand the fundamental concepts of propositional and predicate calculus to construct valid arguments and engage with logical reasoning effectively.
- 2. Utilize operations and principles related to sets, including the Principle of Inclusion-Exclusion, to address real-world problems involving set theory
- 3. Analyze the application of combinatorial mathematics across various fields, such as computer science, cryptography, and optimization, by exploring its principles and techniques for addressing complex challenges
- 4. Solve different types of recurrence relations in computational contexts by applying appropriate methods to model and address various problems effectively.
- 5. Apply graph theory concepts in network analysis.

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#### CO-PO MAPPING:

Course Outcomes	Foundation Knowledge	Problem Analysis	evelopment of Solutions	Modern Tool Usage	Individual and Teamwork	oject Management and Finance	Ethics	Life-long Learning
24FMAT11T.1	2	2	1	-	-	•	-	1
24FMAT11T.2	3	2	1	-	-	-	-	1
24FMAT11T.3	3	3	2	-	-	-	-	1
24FMAT11T.4	3	2	1	-	-	-	-	1
24FMAT11T.5	3	2	1	-	-	-	-	1

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mathematical logical: propositional calcus, statements and notations, connectives, heltormed formulas, truth tables, Tatalogies, equivalent of tormulas, occality law, Tatalogical implications, Normal forms, Theory of Inference, for statement calcus

Predicate calcus: predicates, predicative logic, statement functions, variables, and quantifiers, free and bound variables, Inference theory for predicate calcus.

# : IIT INU

set theory: sets, operations on sets, principle of Inclusion-Exclusion. Relations, properties, operations, partition, and covering, transitive closure, Equivalance, compatability. and partial ordering, Hassie Diagrams. functions: Bijective, composition, imverse functions, dattice and its properties

III TINU

Elementary combinatorics Basics of counting, combinations and permputations, enumerating combinations and permputations with Repititions, Encimerating permputations with constraint Reputitions, Binomial co-efficience; Binomial and multinomial Theorms.

VI TUNU

Recurrance Relations Generating functions of sequences, calculating co-efficients of generating functions, recurrance Relations - solving Recurrance Relations by substitution and generating functions, method of characteristic roots for the solutions of non-homogenous recurrance relations.

V TIMU

Graph Theory: Definitions, finite and infinite graphs, incidence and Degree, Isolated and Pendant vertices, Graph representations. Adjacency and Incidence matrices, whomasha work path, and circuit, connected

Hamiltonion porths and circuit. chromatic number, trees spanning trees: DFS. BFS, minimal spanning trees; prims

and kryskal's Algorithms.

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are paying hospitals to applicable and as in mathematical logic is a foundational aspect of many areas in mfcs. It involves formal systems that help in reasoning about mathematical statements, algorithms and computational Processes.

# Statement (proposition)

- . A statement is a meaningful sentence or declarative sentence, which is either true or lalse, but not both
  - · statements are denoted by capital letters and small letters ine p,Q,R... or p,q,r. - except T,F

- · New Delhi is a capital city of India -> True
  - . Where are you going ?> It is not a statement because. it is a interagartive sentence:

Lygneriche to beautiquette

· close the door > It is not a statement or sentence. because it is a command.

x-2=4. -> It is not a statement because it is depending on the x value.

## Truth value.

9. A statement is said to have truth values (True or False) . The true or false of a statement is called Truth values . The truth value True of a statement is denoted by T and the truth value Faise of a statement It is denoted by F

# Notations

to commonly to represent A various notations are used concepts, structures and operations

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ability party

Bi-conditional

Exi & for 6 born ti

logical And A Logical OR V

. The Negation of Statement (10) of 1001 (10) posociacing the word.

Timplication

aut. Edix Hate Imblication was some of acold taken in the

State 9200 Bi-conditionals 2 2 2000 of dies membrate

connectives

Types of statements

There are two types of statements. They are.

ii. compound statement of the in mel- 29 vovait is

simple statement

A statement which is not cotain any of the connectives is called a simple statement.

SHOT IN STATE OF THE THE THE STATE OF THE ST

DIFFE BEPTE

The integer 5 is a prime number

Anamacharya is a university

compound statement

two or more simple statements can be combined by using logical connectives to form a single statement is called compound statement.

Ex:- Raju is a clever and handsome the opened the book and started to read

Symbolic representation of connectives

Symbol		Nature of compound statement	-form
DOLLE PAR	Negation	NOT (or) Negation	ASTIP CON MAP
2 9111DV	Disjunction	near terms on the sector	so ourse ads
is depot	conjunction	conjunction	by F
$\rightarrow$	Implies (or)	Implication	P->Q
-to 1651626	1700	notations are use	200 inov A
< <b>→</b>	Bi-conditional	it and only if	$P \longleftrightarrow Q$

Negation

· The Negation of statement is formed by introducing the word "Not" at a proper place in the statement or by prefixing the statement with the phrase "It is not that the case that"

. If P denotes a statement, then the negation of P write

P: London is a city

7P: London is not a city

7P: It is not case that london is a city

Truth Table

	Р	7P	3	3	T*	
	T	F	r		3	. 1
distance of the	of Lago	to ball to the	rol stard-	House o		is file (180)

conjunction (n)

A logical connective that represents conjunction. The statement pand Q represents is True only. If both P and Q are True.

EX! P: It is raining Q: It is cold

Pna: It is raining and cold

Truth table

1	ble	0	PNA	nhe av	the for the	ish di	(BEJ4	(-) (1)	3V9)
7	The l	ACTU	TR	177	ar	70	217	23	9
	Т	FI	F	3	1	7	T	1	T
	F	τ_	F	T	7	4	7	7	T
1	F	F	F	7*	7		T	T	7

Disjunction (V)

The disjunction of two statements pand &, if the statement por a which is read as por a (pva) As the truth value False only when both Pand & has the truth value "false" Otherwise it is "True"

Truth Table ((200) (1000)

P	B	PVQ	1
TT	Т	T	1
Ta	F	T	,
F	T	T	d
F	t	F	-

EX P: I will wash my clath.

17(PAQ)V(7PV7Q)-

2. NPANO

3. (NPVB) M(NBVNP)

D	(5)	PNG	7(PNO)	17Ps	78	7PV7Q	avb
T	T	T	F	t	F	F rol-	eroz <b>F</b> oot
T	F	F	T	F	Т		191311To 2
F	T	F	T	Т	F	T	T /97
F	F	F	T	Т	T	т	27174

bather of toward or man a

P	03	NP	NB	NPANO
٦	T	F	F	F
to to	9 FOI	9 kg 45	art trace	two state
KdF!	p <del>f</del> pour	9 1	hat pa	PHP" 20
F	F	T	mal stat	i-corplitio

P	O,	NP	(NPVB)	NB	(NOVNP)	(NPVB) N(NBVNP)
Т	T	F	Т	F	F	paidor disust
T	F	F	F	T	T.	F
F	T	Т	Т	F	T	+
F	F	T	T	T	T	TTT

Implication (or) conditional statement

If pand a cire any two statements then the statement p→0 which is read as "If p then 0" is called conditional statement.

(the statement of when has the truth value faisefand the p truth value tracell at io applied is a slow upplied is

The statement P > 9 has a truth value false. When a has the truth value False and P truth value True, Otherwise it has the truth value T

IJ	P	0,	P -> 0	1107-	act to	indut-	11-11-	ยากรเลยเล
	T	Т	T			(9 4	a) 1	(8-91
	T	(4 Fa)	T (1)		948	0 = 9	0	d
*	F	T	T		T	T	T	T

Ex: P: It is raining

9: The ground is wet

P > 0: If it is raining then the ground is wet

P->0, express as the following ways

i & is necessary for p

in P is sufficient for &

ili Q if p ...

iv ponly it. a

V P->B

Bi-conditional (eqivalance)

If p and Q are any two statement then the statement  $P \iff Q$  which is read as "P. it and only if Q" and abbrevated as "P iff Q" is called Bi-conditional statement

State for 2 - 1

> the statement p > 9 has the truth value T Whenever both p and 9 have identical truth values.

Truth Table.

	D	1	0 ->0	7*	7		3	T
	-	. 9	PZZO	7	T	7.	Ţ	7
	T	T	7	Τ'	T.	1	3	7
	T	F	F				,	
Joseph	F.	T	F	13.617 15.14				
too	Fin	noFbs	terments -	(19/11- 9 FE	ene ana L'an ha	phan Kald	H P	25-0
•			-		410 100	20 21(1	DICHN	5-4

9: A triangle is a polygon with 3 angles.

Person this angles sides if and only if it is an polygon with a angles

construct truth tables for the following statement-formulas

 $(P \rightarrow Q) \land (Q \rightarrow P)$ 

P	0,	P -> 0	B->P	(P>a) n (a>p	)
T	Т	T	T	T. /	

ii mp(PDB) = (TPVTB) (Sytjan - 25) - House Vit [(CTP) n (TPNR))] V [(CONR) V (PNR))] TP -> (Png) ((P > B) NP(NR)) -> (P > R) [(7 (PVCONR))] (>) [CPVO) n(PVR)]  $[(P \rightarrow G)n(P(1R)) \rightarrow (P \rightarrow R)$ AJB (P→9) n(PNR) (P→R) P -> 8 POR TapologiaT T A statement for I mula which is true all possible aministations Statement variables is called Tautology of E Abblepant police attention of the police personal at the principality a ii 7 (PNA) => (7PV7A)

p	9	pno	7(pna)	7P	70	(PV7B)	7(PNB)=(7PV7B)
T	T	Т	F.	F	F	t	7
T	F	F	Т	F	T	T	- IT
F	T	F	T	T	F	T	T
F	F	F	T	T	Т	T	F

P	9	R	TP	TPDR	[((7P) n (7PNR))]	BOR	POR	(conr)v(F	nr))	4) AR
	7	Т	F	F	F	T	T	T		F
	T	F	F	F	t.	F	F	F(Au)	11 4	T
-	F	T	F	F	F	F	Ton	na Tar	0	F
P	F	F	F	F.	F	F	F	-	1"	Т
2	T	T	7	Т	T	T	F	T		F
1	T	F	Т	F	F	F	F	FI	1	T
	F	T	T	T	T	Ł	F	+	12.50	er Flate
	F	F	T	F	F	F	F	F		30

Contradiction is called contempe (ang) - 97 PNA

	[7	(PV(	(GNR))	] ←>[( pve	) n(pvR)]	(D) (a) (a)					
P	9	₽	SINR	(PV(ANR))	(7(PU(anri))	PVA	PVR	((PVB))	(PVR)	A COB	
T	T	T	T	Т	F	Т	T	Т	1 p.y	F . 9	
T	TE	F	F	Ŧ	F	7 -	T,.	, T .	ramid	FACE	
T	F	F	F	T	F (87.1)	T	7	T	77 9	F	
F	T	T	Т	Т	F	7	Т	T	010	F	
t	T	F	F	F	T	T	F.	F.	- (1	Finish	
F	F	F	F	F .	T	F	T C	F		F.	
1 10	1	1	. +		1 2 4 3 1 4	71-9	F 3		2 ( - 4	A 6	

the company of the company of

Tatalogy.

A statement formula which is true all possible combinations of truth values of statement variables is called Tautology or universally valid formula

(on)

If all the entries in the last column of the compound statement case True then the compound statement is called Toutlology EX: PV(7P)

P	70	PV(7P)
2	1- 1	91 / 9F)
T	F	T
F	T	T

contradiction

A compound statement which is false for all possible combination of truth values of a statement variables is called contradiction

EX PAHP)

P	7P	PN(7P)
7	F	F.
F	Т	F

contegency

A compound statement which is neither Tautology nor contradiction is called contengency.

EX(P->B) -> PNB

FP	©,	P-> B	PAS	(P>A) ->(P	na)
Т	7	Т	o Tome	3 (J=9)	94
T	F	F	F	T-	*
F	Т	T	t	t-	
F	F	F	F	F	

prove that the proposition(PAB) -> (PVB) is tautology.

P	8	PNO	PVB	(PNG) -> (PVG)
T	T	T .	T	T
T	F	F	7 10	HONG SALES AND SALES
F	T	F	,T	T
F	F	F	F	T

.: (Pna) -> (pva) is Tautology.

show that ((pva) 170) -> P is tautology or not.

P	6	PVA	78	((PVB) NTB)	((PVB)17B) → P
T	Т	T			TT
T	F	T	T-1-	buse Tot / co	9) ~ T(BT) 1 (8
F	T	T	Ehand	F (3)	$(x) \land (18) \downarrow \rightarrow (P)$
F	F	F	TE	coloipot of (	BEA 917 (01 19)

FOUR 10 EBOPOLITIES : (8=4) = - ((9)) 4 ((219) = 2) } truly made

:  $((PVQ)\Lambda TQ) \rightarrow P$  is Tautology show that  $((P \rightarrow Q)\Lambda(Q \rightarrow R)) \rightarrow (P \rightarrow R)$  is tautology

P	8	R	P->B	$G \rightarrow R$	(P->0) N (0->R)	(P→R)	A->B
T	7	T	Т	T	T	TUVALL	Day Tal DAY
T	T	F	TIVA	F	igr for terms	DAS (ST	19) 21-12 (19)
T	F	T	FT	T	F F	T	TIT
T	F	F	FT	T	9 7 9	TF	TT
F	Т	T	T.7	T	7 T	T	T
F	T	F	TT	F	7 F T	1	T T
	F	T	T	T	T		T
0	F	F	T	T	T	1	

: [(P>B) N(B>R)] >(P>R) is Tautology

prove that (P>O) (>) (TPVO) Tautology or not. P 9 P-> B 7P (P->B) (TPVB) TPVO T F .. (P>B) (>(7PVB) is Tautology show that {(P > (BVR)) 1 (7B)} -> (P > R) is tautology OVR P > (GVR) 70, {(P>(OVR))(70)} (P>R) 0 F . (polotunt fi (avg) - Tang). (ATMENGYA) AT .. ((P > (QUR)) N (7Q) y -> (P -> R) is Tautology show that [(OV(PN7B))]V(7PN7B) is tautology show that  $\{(P \rightarrow (BVR)) \land (B)\} \rightarrow (P \rightarrow R)$  is tautology or not prove that given statement formula : 21 9 - (21/10/9): (PNB) n(7PNB) is contradiction or not Show that ((P--E)) A(G (PV9) -> (PSB) is a contengency. [(BU(PN7B)]V(7PN7B)

T T 7 9 9

: (ce>e) v(e>e) = (e>e) is Toutology

SCP -> (OUR)) n(O)} -> (P->R) P-> (QUR) (P->(QUR)) N(Q) (P->R) A->B Production of a (TEST) (20-1) Statement - Council in the Flower of ATE B (T) IT B F

:: E(P > (BVR)) n(a) ) > (P > R) is not tautology.

(PNB) n (7PNB)

P	0	pna	TPNB	(PNB)N(7PNB)
T	T	T	F	F
F	T	F	7	F
F	F	F	7	F

.: (PNB) n (7PNB) is contradiction

" the solution atto act of the

(PVq) -> (P->q)

P.P	9 2	pvq	p->911	(pvq) ->(p->q) wollow and contractions
T	Т	T	T	D/=> D v((91)119) - (i)
T	F	Т	t	(a= (ava)) = > ((a=a) 1 (a=a)) (it)
F	Т	T	Т	(iii) (PVS) (=> (2V9) (iii)
F	F	F	T	To To (192) (192)

: (pvq) -> (p>q) is a contengency

2 000 V ((91) 19)

Equivalence (<=>)

Two statement formulas A and B is said to be equivalence it and only if both A and B having the same truth values. It is denoted by A <=> B and read as A is equivalent to B

1-	9 <-	=>7848	P	03	TP	TPVB	3	T	7	1
P	(S)	P-> B	Т	T	E	T	*			
T9	(T)2	9111 2910	et drup	agove	9011	o demino	12.0%	1012	nlanos	)
T	F	E.				B+llacar				

9 - 1(a) A ((ava) <- 918 conclusion: Last column of the above truth tables are same : A <=>B> are equivalent show that the statement formula or equivalence using Truth Table. (P→B)<=>(7B→7P) Given that (P -> B) (=>(7B->7P) These statement-formula in the form of A <=>B (or) A = B Here A:P->B -1:00B0:710 →071P2i(2-9) - ((a)11((ava) - 9)3: (POS) P (7POS) 0 P -> B 7.0 F Conclusion: Last column of the above truth tables are same : A < ⇒B are equivalent. p=9) = (pya) show that the following statement formulas or equivalence using (i) (PM(7P))VQ <=> 0 (11) ((P+B) 1 (R+B)) (=> (CPVR)->B) (135 (iii) (PVB) <=>7(7PA7B) 77FF (iv) (P→(B→R)) (PNB) → R) 71,F : (pvq) -> (p-q) is a contengency (PN(7P)) V Q (=> Q (<=>) something Two statement tomails (south) (south) (south) it and could it both A and B having the same trught values of it demorted by A ==> B and read as ==> A is equipalented by FT F SVIT TOP 9

conclusion: Last column of the above truth tables are same

P	03	2	P→B	R->B	((P→B)n(R→B))
T	Т	Т	T	7	Т
7	T	P	T	T	T
T	F	T	F	F	F
T	F	F	F	Т	F
F	T	T	T	Т	Т
t	T	F	Т	T	T
F	F	T	T	F	F
F	F	F	T	T	+

witness	without'-1
4 3	(10)=

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"Is a side in enter of laco

9 - 9/19 (1)

q = q y q (ii)

Ja: communicative loud

900 = 609 (1)

9V8=8V9 (ii)

cupl gvitniso224 ipt.

Je: Distalbutive law

conclusion; Last column of the above touth tables are same

((pvR) → 9)

:((P>B)n(R>B)) (CPVR) >B) is equivalent

3 TV9T = (10 M9) = 17 V76

(PVB) <=>7(7PA7B)

P	0,	PVO
T	T	T
T	F	T
F	T	T
F	F	F

P

T

0

F

F

R

T

F

PVR

B)				ar A	gr = (av9)r (ii)
P	0)	TP	78	TPATE	7(77776)
Т	T	F	F	t.	Jantity law
Т	F	F	T	F	9 I TAG (1)
F	T	Т	F	F	a I ava (ii)
F	F	Т	T	T	9F 9V7

conclusion: Last column of the above truth tables are same. .: (PVB) <=>7(7PN7B) is equivalent.

 $(P \rightarrow (B \rightarrow R)) < \Rightarrow ((P \cap B) \rightarrow R)$ 

L = (dL) Ad (11)

TTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTT	P	6	R	B → R	(P → (B → R)
T T F F T T	T	7	T	T	7
TFTT	T	T	F	F	F
	T	F	T	T	T
TFFT	T	F	F	T	T

(Pna)	$((pnb) \rightarrow R)$	PI
7	T T	,
F	TE FIRE	(i)
F	T = TY9	(1)
-	-	

```
Equivalence of Formulas
It: Double Negation law
     7(7P) = P
I1: Idempotent law
 (i) PAP = P
 (ii) PVP = P
I3: commutative law
   (i) PAS = SAP
   (ii) PVB = Q VP
                            3 - ( gyg )
                                       1 V 4
. Iy: Associative law
  (i) PN(ONR) = (PNO) NR
  (ii) PV (GVE) = (PVB)VR
. Is: Distributive law
 (1) PA(GOVR) = (PAG) V(PAR)
 (ii) PV(anR) = (PVB) n (PVR)
            est, agliciate dissist avoise and to avoise table notations
. Is: De-morgan's law
          Jantoviupo i (a - 1919) (a - 9) A(a - 9).
  (i) 7(PAB) = 7PV7B
                                      (Brage) = > (Brage)
  (ii) 7(PVB) = 7PM7B
In: Identity law
    (i) PAT = P
    (ii) PVF = F
        END = B
 tentereston tast column of the above truth tables stip vatings
                     Cere) => 1(aluand) is committeen
   (i) PN(TP) =F
                     TVF = T
   (ii) PV(TP) = T
                             (2-(009)) <=> ((9-0) < 9)
     (se team) (ann
                            (8-31-1) 9-10 A 8 9
 Iq pominance law
   (i) PAF = F
   (ii) PVT = T
```

```
In: conditional disjunction law
                           8 C- (817) -
    P->B=7PVB
It: Biconditional law: "angry (angriv (canading)) that and
                                               Siver that
    P<->B = (P->B) n(B->P)
    PC-> Q = (PNG) V (TPNT'G) (QNQ) V (QNQ)) V ((QNQ))
i) prove that PV(PNB) = P (Or) PV(PNB) <=> P. is equivalance with-
 out truth table.
    Given that / (ang) = (ava) ag and avitudisting or pribrossA
        According to Absorption law
            P(V(PNB) = P) V(aFA 91) = .
, show that (P > (ank)) = ((p > a) n (p > k)) is equivalance
using without truth tables ) ag wo syllaisozza of pribrossa.
                    = (CIPATIG) N (GVP)) AR
     Given that
         NOW,
                  According to Demorgan's law 7(PVB) = 7PATB
           (P→(GNR))=/((9NB)V(BV9)F)=
     according to conditional disjunctional law. P-> = 3PV.B
           P -> (GNR) & (COPVICE(NR)) =
                       = (9F) V9 CUPS (PV (BNR) = (PVB) N(PVR)
     According to
       Distributive law
                     conditional disjunctional law P>0 = TPVB
        According to
                            \equiv (P \rightarrow \theta) \cap (P \rightarrow R)
    : (P > (ank)) = ((P > a)n(P > k)) - (009) = (10 - 2) - 9)
prove that ((P->0) n(R->0)) <=>((PVR) >0) is equivalance
 using with out truth table.
                                                     GIVED
      Given that
        (CP>B) N(B>B) (8=(BV9)) (9 < B) N(B < 9)
  According to conditional disjunctional law P-> 8 = 7PVB
     Alc Distributive evitud = (3849) (7849) 1 (8495)
                law ropror= (TPATR) NO (ENGAR) = (PVB) A (PVR))
```

```
By using conditional disjunction law P-> 0 = 7PV 0
                  E(PVR) → O.
                                      219-36-9
 Prove that (CTPN (TANR)) V (CANR) V (PNR)) (=> R ( initions)
                            19-19 (0-4) = 0-39
    Given that
      (CTPACTGAR)) V (CGAR) V (PAR))) <=>R
 7PN(TONR) V [(GNR) V(PNR))
  According to Distributive law PN(QUR) = (PNQ)V(PVR)
  AK to Associative = (TPA(TBAR))V((BVP)AR))

= (TPATB) + (BVP)AR

+ (TPATB) + (BVP)AR
   = (TPATB)NR) V((OVP)NR)
 According to Associative law ph(BnR) = (pho)nR
                = (CIPATO) N (QVP)) NR
                                        MOM
 According to Demorgan's law 7(PVB) = 7PN7B
                = (7(PVB) V (BVP)) AR ((912) -9)
 According to commutative law GVP=PVB of priblossa
                = (7(PVA))V(PVA)) AR (900) ~ 9
According to Inverse law PV(7P)=P
                                      at builtions
               = TAR SIE VALLE
                                   Distributive law
  According to Identity law PAT = P.
                                   at philosopa
(P→R) n(a→R) <=> (CPVa)→R)
(P->(B->R)) = (PNB) - (R-1) (1 = -1) = ((9110) - 9):
· 9 Mile drust the trice prize
  Given
                                       Given that
     (P→R) N(B→R) <=> ((PVB)→R) ((0 < 9) ((0 < 9))
      (P->R) ∧ (B->R) ("conditional disjunction law)
         (TPVR) A (96 VR) ( Distributive ) "Holinging of A
  ((949) 11 (OV TOF 19 (9ph Top) V RT ( 9 Demorgants)
```

```
show that P \rightarrow (O \rightarrow R) \equiv (P \cap O) \rightarrow R
                                                         60 11 3
             p > (B > R) c: conditional Disjunction.law
                                                          50001
       TPV (TBVR) (:: Associative)
          (TPN 70) VR 7, pd T paintings and brokens of "A stante
          . 7 (PAB) VR (: Demorgans) Prance 1
              PNB -> R (" conditional Disjunction law)
  show that ((P \rightarrow G) \rightarrow G) \rightarrow (P \rightarrow PVG)
                                ((Arvairna)v/antic (A
             (P->B) -> B.
                               denne Pingevonia a Section of the most
manual mes promoting of (800) vo
  to determine wheather they are touted from The determines
  compare distributions and construct ov (PLV (AL) L)
 increase the si plant (PATA) VB a mitting of the method samprovi
 Desirable and de (PVG) V (TO, VO) amounts and a sale as an ribus mi
         to statement from sis called as AN(AV9) froms.
 doing to sing the mi product in the state of self ast ast
                      mailtannifer of sonia sonia sonia "mus" bas
 INell formed formulas
 A Well formed formula is a string of symbols form a formal language that is symbolically correct according to
                  Flementony producters pine, nend, party
 rules
  Atomic statement
    If p'is a statement variable then p is a hiell formed
                   the com of variables and their megantic
  formula
  => p is a statement 7p is a Well formed formula of 19119
  paranthesis
    paranthesis are used to ensure the correct grouping and
  order of operations in a formula.
   EX: (PA(Q->R)) is a well formed formula
  => If A&B care well formed (formulas) statements then A&B
   (ANB)(AVB), (A->B) and (A <>B) are well formed formulas
                   The product of statement variables of
  but not both at the same time is (BVA). IT (PVA) if (iTT) X35
        (P) (P) CPVQ;)) " L + 3P & W. 291d Diroy 1 900 9190 +T
 [(1 FOR two variables P.A. the min-terms are in the min-terms
      Two formulas A and A are said to be duals of each other
```

```
obtain 7(PVO) => (PNO)
                                                           PESE (PNG) V(7PN76)
 <=> [7(PVB) n(PNB)] V[77 (PVB) n(7(PNB)]
(=> [(TPNTG)) N (PNG)) V (PVG) N (TPVTG)]
<=> (TPN7BAPAB) V [PN(TPV7B) V (BA(TPV7B)]
(7PA7BAPAB) V (CPA7P) V (PA7B)) Y (BA7P) V (BA7B))
 (TPATEMPNE) V (PATP) V (PATE) V (GATP) V (GATE)
                    sum of elementary products.
  obtain 7(P -> (anR))
                                                                              217 PO T TO 1807 14 34 34 19F
                                                        Figure 2 mag | In more a registronically a
                7(P -> (ONR))
                                                        ( " " " ) senior to men will not proper
        >> T (TP V (OnR))
                                        The section is a community of the section of the se
       C=> PA (76, UR)
        <=> (PMB) V (PMR)
                                                   bis joint five Monmal Lamis (or Someth ele
          sum of elementary products.
                                                                  products is eatled as phili
  Conjuctive Normal forms (CNF)
         A formula which consists of a product of elementary sums is
called a conjuctive Normal forms
                                                                                                                    2000/3789
            CNF = product of elementary sum
          EX: (PVB)A(TPVB)A(PVTB)A(TPVTB) (2 491)A4 (=)
 obtain CNF of proposition product (BA9) v (91779)
                                              THE METER OF THE PROPERTY OF THE PROPERTY OF
          <=> PA(7PVB) P->B = (7PVB)-A(8-93)-9
      ( ( ) (PVA)) ( (AVA)) ( (AVA)) ( (AVA)) ( )
                             product of elementary sum (avars) var
 obtain CNF of (CP-) B) NTB) = 9P (D) V(9A D) A9F) V9F (D)
                (CP>a) Ma) >7P [(qnone) V(qnoner)] vgr (s
      <=> 7((P>A) ATA)) VTP [(900)) V(900097)] V9T
       2007 (CTPVG) NTG)) VTP (900) (900) (900) TO
       come of eternioning beautiful (Bulland) (Chua)
       (CPVG) 1 (76VG)) V 7P
                                                      obtain a sum of simple of the parties
      (PVGV7P) A(TAVAV7P)
                                   product of elementary sum. 119-27
                                                                       (AMALIA (ALABITA CE
```

```
(CP > B) MTP) -> 7B
                             annual and a second of
   <=> 7((P>O) 17P))V79
   (=> 7((7PVG)17P)) V7B
                                 8119
   (CPAB) VP) UTB
   ((PVG) A (TPVP)) VTG
   (PVGV76)A(7PVPV7G)
              product of elementary sum.
 obtain CNF of 7(PVB) => (PNB)

⇒ T(PVB) → (PNB) ∧((PNB) → T(PVB))

                                  PC>B= (P>B) n(B->P)
(=> 7 (7(PVB)) V (PNB)) N (7(PNB)V7(PVB))
((PVB) V(PNB)) A ((TPV7B) V(TPATB))
((PVOVP) A (PVOVO)) A (6PV TOVTP) A (7PV TOVTO))
(PVG) 1 (PVG) 1 (7PV7G) 1 (7PV7G)
(pvo) 1 (7PV76)
              product of elementary sum.
Principle Disjunction Normal Form (PDNF) (or) sum of products
                              cononical form.
   A formula which consists of sum of min-terms only
is known as PDNF
 method to tinding PDNF
  By Truth Table.
For every truth value T of the given formula, select the min-term which also have the value T
using the fruth obtain the PDNF of the following statement
 formulas
    ① P→B ① P<→B ① PVB ① 7(PAB)
                      Obtain the PLATE OF (TP > P) A (E, -> P) US
                   minterms
               e laubang PAG ina na mikah huamanan ai meng luark
                                         papposition someth
             (FIR W. E. E-10) S of (ENVAX) V (ETY(VAXES)
                                              plator Ware
                      TPNO
                      7PN78.
      (>) (PNB) V(PNB) V(7PA7B)
```

Obtain CNF of ((P->B)17P) ->7B.

```
The second secon
         P(-> B
                                                                                                                                min-terms
                                                                 P <-> 0
           P
                                                                                                                                          PNO
                                                                                                                                                                                                                                                                     The state of
                                                                                                                                                                                                                                      al virana iliyanna da
                                                                                                                                                                                                                          CONTRACTOR ASSESSED
                                                                                                                                                                           a Francisco de la desperación y
                                                                                                                               TPATES
                                                                                                                                                                                                        In the little of the many that the
     (PNG) V (TPATE)
                                                        sum of min-terms.
                                                                                                                                              PVO
                                                                                                                                                                                         · Surface to the second of the
                                                                                                                        min-terms
                                                                       PVB
                                        0
                                                                                                                                               Dializa Managera and Alexandra
                                                                                                                              PAG
                                                                                                                                                                                                   PATE
                                          F
                                                                                                                                                                                                                                                                  TOVALLY VICTORY
                                                                                                                       7PAS
            (=> (PNO) V (PNTO) V (TPNO)
                                                                                          sum of minterms
                                                                                                                                                                                           and the second of the first state of the second
          7(PAG)
                                                                                                                                                                                                                                                                                Trong or William Cont.
                                                                                                                                                                                  Min-Terms
                                                                       PNA
                                                                                                             7(PAB)
         T T F PN-8 -
                                                                                                               T PATS
          FIT IF TO THE TOTAL TO THE TOTAL DOLL OF THE TOT
                                                                                                                                                       TPATO
            <>> (PATA)V(TPAA)V. (TPATA) - sum of minterms
obtain the PDNF of (7P->R) 1 (O<->P) using Touth table.
                                                                                                                                                                            20mg 45mg 16mg 16mg 16mg 16mg
            P-> [(PNB) 17(7BV7P)]
   show that the cononical form of sum of products (PDNF) of the
  proposition formula
                                 ((7XAY)V7Z) V(XAYAZ) is \(\int(0,2,3,4,6,7)\)
            Truth Table
```

( ) ASITY ( BASTY (SALE) 24

```
72 (TXNY)VTZ XNY XNVNZ-AVB: Min-Terms
                                                XVXVA.
                                                XMYMIZ
                                               XATYATZ
                                              TXNYNZ
                                              TXNYNTZ
                                               TXATYATZ
C=> (XNYNZ)V(XNYNTZ)V(TXNYNTZ)V(TXNYNZ)V(TXNYNZ)
     V(7XATYX7Z)
 <=> (TTT) V(TTF) V (TFF) V (FTT) V (FTF) V(FFF)
 (111) V(110) V(100) V(011) V (010) V(0000)
 ( 7 V 6 V 4 V 3 V 2 V 0
 C=> E(0,2,3,4,6,7)
 Show that the PDNF of [PV(7P > (OV(70 -> R))] is &
(A) (1/2,3,4,5,6,7)
         TP 79 70->R (QV(70->R)) TP->(QV(70->R) AVB Min-Terms
                                                PAGAR
                                                PAGATR
                                               PATONR
                                               PATGATR
                                               TPAGAR
                                               TPAGATE
                             (a TAETI V (BATT) L (angTPATGAR
                        aming - giar to mus
(PAGAR) V(PAGATR) V(PATGAR) V (PATGATR) V (TPAGAR) V (TPAGATR) V
    (TPATGAR)
 <=> (TTT) V(TTF) V(TFT) V(TFF) V(FTF) V(FFF) V(FFT)
 - CENTRIE (100) V (110) V (100) V (100) V (001) V (001) V (111) C
                    -> mappelermentory products which are conti
<=> 7 V 6 V 5 V 4 V 3 V 2 V 1
                                      4 - 4 TAT X 3
EFAS WITH (SCAPING (SCAPE)) (III & pol
```

scan of the above steps until all elementary products to

```
POR TP (TP-)R) (O-)P) AND MIN-TEIMS

TTT. P T T PAGAR

TTFT F T F

TFT F T F

FTT T F

FTT T F

FFT T F

FFT T F

FFF T T

FFF T
```

```
P \rightarrow (CPNG) \Lambda 7 (7GV7P)]

P G P \Lambda G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 7 G 8 G 7 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G 9 G
```

v(armenor) vigo - sum of min-terms

Rules without using Truth Table.

> obtain DNF (100) / (110) / (110) / (100) / (101) / (101) / (111) (111)

-> propelementary products which are contradiction EX PATP = F

→ If P; and TP; are missing in an elementary product & replace by & with (&API) V (&ATPI) (:. &ATET]

( PABLAGE)

-> Repeat the above steps until all elementary products to sum of min-Terms

```
PDNIF of p <> 0, without using Truth Table.
 obtain
  P \leftarrow > G \leftarrow > (P \rightarrow G) \land (G \rightarrow P)
                                             PATPEF
        (TPVB) A (TBVP)
                                             GATOSEF
        ((TPVB) NTB) V (TPVB) NP)
        (TPATO) V(GATO) V(TPATP) V (GAP)
        €> (7PATO) VFVFV(GAP)
        <>> (TPNTA) VCAMP)
                       sum of min-terms.
                       ravali Marria raseni i
 obtain PDNF OF 7PVO
   TPVB (=> (TPAT) V (GAT)
                                          PVTP = T
       ( CTPN (GV TO)) V (GN (PVTP))
                                          av7a=T
       (TPAG)V(TPATG)V(GAP)V(GATP)
       (=> (TPAG) V (TPATG) V (GAP)
                    sum of minterms.
 Find the PDNF ofp (P->0)117(70 V7P)) without using Truth table
        P-> (CP-> B) N7 (7BV7P))
    ( P > (CTPVA) A (AAP))
    <>> TPV ((TPVB) NCBAP))
                                       91B2B
    (=> TPV (TPN(GAP) VG(GAP))
                                        FAP=F
     TPV ((TPNGAP) V (GAGAP))
                                        FVP=P
                                       TVP=P
     STPV((FNB)V(GAP)
     STATE OF TOUR PYCENPY)
                                     19r 91.
     G> TPV (BAP)
    (TPAT) V (GAP)
     E> (7PA (GV7B)) V (GAP)
    (TPNB) V (TPNTB) V (BAP)
 Principle conjuctive Normal Formicon) product of sums cononical form.
    A formula which consists of product of max terms only is
   called as PCNF of given tormula
   known
  Method to obtain PCNF, also (alve) A (aver) to amore all aintedo
 For every truth value F of the given formula, select the
By Freth Table
```

max term which also has the value F. of to 34 the works

problems

```
Find the PCNF tox the following statement tormulas
a) P <>> a (b) P V a <>> (Pn B) (c) TP 17 a
                 B P CO
                                                                       Maxterms
                                                               PVTO
                                                                                                 1243111111111 -- --
                        .. P <> B <> (PV7B) A (TPVB)
                                                     product of max-terms
      PVO -> (PNO)
  P
                                                     PNB -> (PnB) Max-terms
                                PVO
                                                               - 2 1H49 1 - T
  The transfer of the transfer o
                                                                                        17 49 17 17 19 19 19 19 19
  F
                     .. PVO -> (PNO) (PVO))
                                                              product of max-terms
                                      1 944
                                                                               CONTRACTOR GARAGES VIEWS
                                    9 - 97 -
     TPATE
                                    9-1-1-
                                                                                             (910) V (201) VIT ->
                                                                                                       Max-terms
                                    7P
                                                                            7PA 70
                                                       70
                     0
                                                                                                             PVO TARY OF
                                                                                                            PV78 "TAGE"
                                                                                       TPVS POST
                                                                            Frankly (arnar) v (anar) <=>
   profile conjuctive clounced sourced product of course signals
       .. TPATES (=> (PVE) N (PVTE) NCIPVED) IN DILLINGTON
                                                                        product of max-terms.
                                                                                                                                                      COOMS
 obtain the PCNF O+ (7PVO) 1 (PV70) using truth table of bootsm
                                                                                                                                             sy-hutth lab le
 Obtain the PCNF of (TPV7B) 1 (TPVB) 1(PV7B)
a. show that thepc NF of the formula p > (OnR))n((7P > (70, n7R)) is
```

2 meldera

```
ONR P->(ONR) 75 TR 7517R 7P 7P->(TONTR)
         FRYENT IT
  P>(BAR)A(TP>(7BATR)
                          Max-Terms
                          PVAVTR
                        PVTOVR
                                      191 (36) 96-5311 (36)
                         PV78V7R
        (BIVATVALIA
                       2007PV GVR + 1111
                         TPVBVTR
                          TPVTGVR
(PVGVTR) A (PVTGVR) A (PVTGVTR) A (TPVGVR) A (TPVGVR) A
                                   (TPV7BVR)
 (=> (TTF) A (TFT) A (TFF) A (FTT) A (FTF) A (FFT)
 <>> (110) \( (101) \( (100) \( (011) \)
 (001) V (010) V (011) V (000) V (101) V (110)
 6 INZMBAUMSAG
(1,2,3,4,5,6)
 obtain penf of (TP >R) n (Ox->P) using truth table
 obtain the cononical product of sum of the proposition-formulae.
 PV(TP -> (GV(7G->R))
  i) 7X N(7YVZ)
  ii) (PAB) V(7BAR)
```

```
TXN (7YVZ)
                                     Max-Terms
                       TX TXN(TYVZ)
             74
                                     XVYVZ
             F
                       F XVYV72
                    ar Falla Falla XV7YVZ
                            X V TYV TZ
                                    TXVYVZ
                        PRODUCTION OF THE BOTTON
   (=> (XVYVZ) A (XVYVTZ) A (XVTYVZ) A (XVTYVTZ) A (XVTYVZ)
: (7 × 1 (7 4 × 2)
               product of max -Terms
                             Tacanharagia (ave) del
  (PNB) V(TOAR)
                              MEDIA FRANCIA N (AVE) CO
                    PNO (PNO)V(TONR)
              TANK
          70
                                   chiain pentr of travely
                                     L ATATE CO
                                 (-1,79-1 x (-1470) ->
                         ((9 A) var) A ((arna var) ->
                STVENT NAVITEDIA ((ariginal (avery)) <=
                           (First) A farrella (aver) -s
                         annust were to haulouss
                          (9=2) A (9-97) FOR THOSE DID ALGERTH)
             HEBLAD
                           ((0 < 9) 11(9 = 3)) 11 (9 v9) (0)
             91 - 109 1
              T-7 P
                           ((BV91) A ((764P) A (949))
                      (PVR)VF) A (CIGVP)VF) A (TPVG)VF)
     ((SLAND) V(CEVIE)) V(CLEAD) N(CLEAD)) V(CLEAD))
  OS (PVRVB) A (PVRV7B) A (TBVPVR) A (TBVPVTR) A ((TPVBVP) A
                                      (ALVENAL)
      (PVRVB) A(PVRVPB) A(TGVPVPR) A(TPVGVR) A (TPVGVR)
                     product of max-terms.
                              obtain pour of (PNB) v(TPVB)
```

```
obtain PCNF of (PNB) V (7PN7B) without Truth Table.
     (PAB) V (TPATO)
 (cons) VTP) A (cons) VTB)
( PV7P) N (BV7P) N (PV7B) N (BV7B)
 < >> F M(BV7P) M (PV7B) MF.
 <=> (AV7P) A (PV7A)
            product of max-terms.
Obtain PCNF Of (7P->B) A (B=>P)
   (7P→ a) 1 (A=>P)
 (PVG) 1 (B > P)1(P->G)
                                               (240m) v (eng)
(PVB) A (TOVP) A (TPVB)
            product of max-terms.
obtain PCNF of 7(PVB)
     ALVAL C
                                               GNIGEF
    <>> (TPWF) A (TQVF)
   <=> (TPV (GNTG)) N (TGV (PNTP))
   (CTPVB) N (TPVTB)) N (CTQVP) N (TQVTP))
   < >> (TPVB) N (TPVTB) N (TBVP)
            product of max-terms
"obtain PCNF for (7P->R) N(B=>P)
                                       GATBEF
   (PVR) n ((B-P)n (P->B))
                                       TPVF= TP
                                       TVF = T
   (PVR) N ((TBVP) N (TPVA))
  ((PVR)VF) A ((TOVP)VF) A((TPVB)VF)
  (IPVR) K(GATG)) N(CTOVP) W(RATR)) N(CTPVB) W(RATR))
 CON (PURVE) A (PURVIE) A (TOUPUR) A (TOUPUTR) A ((TPVOUR) A
          (TPV GVTR)
(-> (PVRVA) A(PVRV7A) A(TAVPV7R)A(TPVAVR) A (TPVAVTR)
                 product of max-terms.
 obtain PCNF of (PNB) V(7PVB)
```

G> (PN-GV7P) A (PN-GV7B) C> (PV7P) A (B, V7P) A (PV7B) A (QV7B) FA(GV7P)A(PV7G)AF costs of a control of the control of the second billion product of max terms obtain PCNIF of (PNB) VC7PNR) Districted by the little of the following the track the (PNG) V(TPAR) borthers are but at LBT of ((PNB)VF)V((IPNR)VF) (=>(PNB)V(RATE))V((TPAR)V(QATB)) (=>(pvavR) n (pvavzp) n (zpwRva) n (zpvRva) <=> (PVOVR) N (PVOVTR) N (TPVRVO) N (TPVRVTO) product of max terms Determine intentions of a constantion of delication to delication had premises (301) - 9:5 3-9:41 (1 (angla-4 and 8-1 a から一日

Theory of Interence. for statement calculus:

consider H1, H2, H3 ---- the of the given statements and C. C is also an another statement.

Premise:-

The set of all statements #1, #2, ---- #th except C statement is called premise.

conclusion:

conclusion is a statement which is obtained from a set

It we start from premises and proceed to conclusion such process is called Interence.

The theory associated with the rules of inference is called as Inference theory

Note: premises means set of assumptions, axioms and hypothesis Argument:

consider a set of premises HI, Hz, --- thound a conclusion a then the argument is denoted by (HIAHZAH3A ---- AHA) -> C valid argument (or) valid conclusion.

It a conclusion is derived from a set of premises by using the rules of reasoning then it is called valid conclusion

To determine meather a conclusion logically follows from given premises. We use the following methods.

1. Truth Table Method.

2. Without using Touth Table.

Truth Table Method

let HI, Hz ---- the are the premises and c be a conclusion we say that c is a valid conclusion. It and only if thround HINHIAM H3M---- Attn) -> Cols a tautology

Determine weather the conclusion c follows logically from the premises 1) H1: P → B C: P → (PNB)

Pagnoise

Conclusion:

is called premies.

Hic is a Tactology

: C follows logically from HI

is so the conclusion is valid.

Determine weather the conclusion c-follows logically from the given premise

The section of the continue of the single of

conclusion is of missioners and all the statement

H. AH; C (s a Tautology

```
1 Hi: P-> B , H2: 7P C: B NOV
2 H1: 7P, H2: PVB C:B V
3 H1: P->0, H2:10 C:P N.V
  HI: TPVB, H2:700ATR), H3 TR, C; TP V
               7P HINHL HINH, >C
        PJO
  PB
  FF
       HINHZ ic is not a tautology
          is so the conclusion is not valid
                                   state with pales to dis-
2. H1:7P, H2: PVG C:0
                                                  Ci allia
   P & TP PV & HINH2 HINH2 -> Channel A
                                                   21139
  Toutening it is proposed in a first proposed in The deginary
                   Rather of Japineations us keine of sende
                                            4 - and HI.
       HINHZ ic is a Tauto logy
                                            Sc and I'l
          .so the conclusion is valid
                                             13: P=>PVB
                               control miles
                                             Ja: 6 => PVG
  H1: P-> B , H2: 7B
                         HINH2 HINH2 -> C
                 +12
                                           B < 91 19 10 10 10
  PB P > B
                 70
                                           BET CB SI
                                          9 = (B=9) [ II
                                       BI = (0 = 9)T : 8I
                             Total Panisher and = 8,9 :pt
       HINH2: c is not a Tautology
            "so the conclusion is not valid" ac a grant
                        Its Page 2 Page Tourstine how
                          TIN: PARIPORE, B-SK DKIICHANIA
                                ILE: 6-8, 6-8 (6008) -8
```

Determine meather the conclusion cfollows logically from the

following premises.

	ŧ	11:7	PVB	H2 h(	BATR)	H3:7 R	oct ?	7P	70 10	18.	to strong to seem!	e pateur	sofad slight
	P	0,	R	79	TPVB	<del>11</del> 3	Q AT D	Y/2 7(GATR					
۱	·	T	7	E	4	F	F	T			e 1 1 , 14		
	T	T	E	F	Т	7	T	F			, n		
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ľ	F	F	F	T	T	T	F.	T	T		7		3
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ı				,, 50	Wie e	Min		Freday	Trans.	11)			
ľ	With	our	t usi	ing Tr	uth Tak								
ı	Rule	P	Given	psemise						1 4	9 /41 : [1	1 9-1	14 - 6
	A	pre	mise	may	be into	roduc	ed at	ciny pi					
	Justine Dayled in			piemise			**	3	3				4
ı	70	utc	Uogi	cal in	nplicati	ans a	mû be	intro	turned s	n th	o dexi	vation	
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					ions (	אלא	ales o	Title	ence.	1	7	. 7	7
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	.Is:				Additi								
	'Jų '					1 A . A.		*	9.4	ો (શે	Tam.	8=9:	:13
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	I(2: :I(3:	P-	70,	6->R	Modu 7P N =>P ->R	R ni	instive	αω					
	. TIP.	PA	0) / 1	,	,								
1	TIS:	P-	>K, (	S)-7K.	=> CPVE	17 61	`						

```
Equivalence.
· E1: 7(7P) = P Double negation law
                                          TADAM TONIO
.Ez: PNB => ONP } commutative laws
                                       3-3,4-1,871
· Ey: PN(BNR) <=> (PNB)NR & Associative
                                  A PLINT
ES: PV(OVR) <=> (PVO) VR y laws
.EG: PN(GVR) <=>(PNG) V(PNR) } Distributive
.ET: PV(GNR) <=> (PVG) N(PVR) J law
E8: 7(PNB) <=> 7PV7B / Demorgan's laws
.E9: 7(PVB) <=> 7PN7B / Demorgan's laws
                                                 (1) 4517
· E10: PVP <=> P
                                    9 1110 2 20 9 100 100.
· EII : PAPED P
 EIZ: RV(PNTP) <=> RVF<=>R
E13: RA (PV7P) <=> RAT <=> R
                                (+) raigo siva (3) (a,E)
EIY: RV (PV7P) (> RVT <>T
                            A. EVR Ingically followed the
EIS: RN (PATP) <=>RNT<=>F
                        where that R.VS -fellower legically from the
. F16: P-> 0 <=> 7PVB
                               THE COUNTY POR PROPERTY SAN
E17: 7(P-> 6) <=> PATA
                                       ava
                               2/1/24
· E18: P> B<=> 78->7P
                               Palus transcor (ca) fulle P
·EIq: P > (B > R) <=> (PNB) -> R
E20: 7(P=0) 2=> P=0" (C)(1)) | 01119 11 (E) (E)
· E21: PZQ (>>B) N(B->P) 9 OUT STARLIF (D) (D)
· E 22: P=> Q <=> (PD B) V (TPATB)
                          Less tables dead (2) kp. 13
Note: PYTP = T
                          $ 6 4 CE) (ALLE) > 612 8010 6.
     PATP = F
show that R is a valid inference from the premises p->0,0,7R, and p
  position steps Given vipadi a Reasonuallot pilasipol 249 ...
                              Rulep Min 4 9 3 3 79
 $14
          (1)
                          grule P
 £2 9
          (2)
                 PS
                        Rule + (1)(2) -> P,P-> 8 => 8
 &1,24 Tre- or 3)
          64 Dagg 21 Bom (8),(1) Rule P
  244
                   R Trulet (3)(4) > P,P > 0 =>0
          (2)
                                           (Modus ponens)
  $42,44
```

```
show that sorr is tautalogically implied by (PVB) n(P->R) n(B->S)
                           - Est Titl) = P touble regertion att
  Given premises
                          Est Pos -> Our porton differ inter-
     PVG, P->R, B->S
£13 (1)
           PVB
                   RuleP
Siy (2)
                   Rulet -> 7P->0 = 77(P) VO TP-> 0 = PVO
           7P->B
£34 (3)
          B->S
                       usri (979)/(879) /->
                 Rule7 (2)(3) P->B,B->R -> P->R (Transitive law)
81,34 (4)
          7P->S
                 Rule T (u) 78->7P=P->8
           P->75
€64 (6) P→R
                 Rule P
€1,3,63 (1) 75→R Rule T(5)(6) P->6,0->R => P->R(Transitive Rule)
                                EISTERN (PVTP) CE RETCEPR
£1,3,63 (8) SVR RULET (7)
      is sur logically tollowed from the given premises 19 pp
show that RVS follows logically from the premises CVD, CVD-7H,
7H -> (ANTB) and (ANTB) -> RVS
                                        FIG : P-> B <=> TPVB
                                     EIT: 7(P>A) <=> PATS
 (1)
             CVD
                      Rule P
                                       1 EIR: P-3 B C=5 76 -779
 {2} (2)
                      Rule P
                               9 = (809) (9 = 8) (9 18) = R
             CVD->7H
                    Rule T ((1) (2) modus Ponens PIP-38=8.
£1,2} (3)
             THIS PER POSTA (BETTA BITTAKHT
       (4)
 843
                             (3), (4) modus ponens
 8,2,43 (S)
              ANTB
                       Rulet
                                                Mate: balls
            (ANTB) -> RVS Rule P.
 $ 6 4 (6)
                                                PATAG
show that Ris a valled interest enough a property of and billing a start work
     .. RVS logically followed from the given premises
 show that RMLPVA) is a valid conclusion from the given premises
 PVB, A > R, P -> M,7M.
                       901119
                                                        114
                                                (1)
  Sil
        (1)
                        Rule P
                TM
                                      BK-9
                                               (5)
                                                        62 3
  823
       (2) PSMOON PULLED
  (2 (19 (10) 2 11 booms
              TM->7P RuleT (contratve = p->0 <=>70->7P)
                       9 SINA
                         Rule 7 (1), (3) modus ponens
  S1,24 (4)
                 7P
                                                      P133
              Propose RulesP
  (Modus poness
                                                    2014 CA
```

```
689 (8) B->R RuleP
 & 42,5,83 (9) R Rule T (7)(8)
 &1,2,5,83 (10) RA(PVG) RULET (9),(5) -> P/G => PAG
.. RA(PVB) logically followed from the given premises
show that 79, p-> 9 implies conclusion 7P
 213 (1) 70 Rule P
 623 (3) 79->7P RuleT (2) P->9 (>70->7P (contrapostive)
 €1,23 (4) TP RuleT (1),(3) modus ponens
                           P,P >B =>B.
 :. IP logically followed from the given premises
show that P-> 0, 0 -> 7R, R, PV (JAS) => JAS
 213
               P->B Rule P
        (1)
 &29 ·(2) 0,→7R RuleP
              P->7R Rule T (1712) Transitive Rule
 £1,23 (3)
                                    P->0, 0, ->R => P->R
 &424 (4) R->7P RuleT (3) contrapositive
                                    P->8=>79-77P
                        Rule P
 8 53
         (5)
                 R
 &1,2,53 (6) TP Rule P (4)(5) modusponens
      y (7) PY(JNS) Rule P. Housen inos signis to transition
 ad at blos (8) TP > JAS II RULE JIL(7) P>B<=> TPVB + 1091+0 AL
 $1,2,5,74 (9) - JASIA Rule T. (6), (8) modus ponensp. p->0 =>01
    .. Ins logically tollowed from the given premises:
 Show that 7P follows logically from the premises 7 (PN701),
show that is a valid argument from the premises it cove hound
 7(TPAS) P->0, (TOVR) ATR, 7(TPAS)
       (1)
```

the marrises (pvs) nump) are consistent

consistency of premises.

9-7-7-4-1 - ---

(3) (3) The premises HI, H2 ---- Hn of an argument said to be In atleast one possible situation.

and the four than the district of

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The premises #1, H2 - - +10 of an argument are said to be inconsistent". It their conjunction HIAHIA --- the is false in all possible situations ine HIAH2A---- AHD Then -> F 35.18 let the premises are PVB, and TP CTP is consistent with the help of touth table.

P. B. PVB 7P (PVB), 1 (7P) F (2Agras, 11'A (9ve)) and an result wester

The premises (pva) n(TP) are consistent let the premises Pampans is in-consistent

```
9 7P 7PAG (P)NEPAG)
  T To Fard of management and substitution (2) test to other 8 112 Antitude
                             d april 31 - 14 (8) 683
  Fat's Ft 11 Transa author (80/(a)) along at 10 48,04413
   : (P) n(TPN on) is in consistent in sing dend (a)
show that P→B,P→R,B→TR and P are inconsistent without
Touth Table.
               admentalization of the property of the following soft is
                   RULEP
 (1) P>B
 629 (2). B→7R RuleP
\{l_{1}2\} (3) P \Rightarrow 7R Rule T (1),(2) Transitive Rule P \Rightarrow \theta, P \Rightarrow R \Rightarrow R \Rightarrow R
 244 (4) Proposeule P
 &1,2,43 (5) 7R Rule T (3),(4) modus Ponysens
                                         2892 for Mant Ant
  &63 (6) P->R Rule P
  (6) (7) 7R >> 7P RULET (6) P>B = 7R>>7B
  &1,2,4,6} (8) TP RuleT (5),(7) modus ponens
  €1,2,4,64 (9) PATP RUIET (4),(8) P/S = PAS.
  Hence we get contradiction
 ithe given premises are inconsistent.
show that the tollowing premises are inconsistent.
 If the contract is valid, then john is liable for penalty. If john is
liable for penalty, the will go bank rupt. If the bank will loan him
 money, the Will not go bank rupt. As a matter of act the contract
 is valid and the bank mill loan him money.
 cet-the given statements are.
    V: The contract is valid.
    L: John is liable for penalty
    M: Bank will loan him money
let the given premises are.
    V->L, L->B, M->7B VAM
   &13 (1) V-> L RUICP
  €2} (2) L>13 Rulep
   81,24 (3) V-B Rule7 (1),(2), transitive Rule
                                     p->0,0->R=>P->R
```

```
Rule T (4) simplification law BnP= B
     é 49 (6)
                                                                    m
                                                                                                      Rule T(3),(5) modus ponens p, p->6 => 8
    £1,2,49 (7)
                                                                   B
                                                                                                          Rule P
  289
                                         (8) M >7B
                                                                                                        Rule T(6),(8) modus ponens P,P->0=>0
                                                                           78
 £+,2,4,84 (Q)
                                                                                                               RuleT(7)(9) PNS=PNS .
& tr=14,89 (10) BA7B
                       and the Property of an illustration of a second to be designed
           .; The given set of premises is inconsistent.
  Show that the tollowing premises are inconsistent
  If Jack misses many classes through illness, then he tails high
 school. It Jack fails high school then he is uneducated. It Jack
  reads a lot of books, then he is not uneducated. Jack misses
many classes through illness and reads a lot of books
                           m: Jack misses
                           H
                                                                                               The transfer of the section of the s
                          B
                                                                                     $ 12 July 69 (5) (5) 10 (5) (5) (5) (5) (5) (5)
                          E
                     Hence was got combined with
                                                                                                                    attraction of the section of reality of the
                                                                               I WITH WIND TO STANDED TO BE STANDING TO BE STANDING TO BE STANDING TO STANDIN
       physical plant of the market and address of the parent of the first of the tell of soft the
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```
Rules c p (conditional proof) deeduction Theorom)
  Rule CP is generally used if the conclusion is in the torm of
R->S.
 In such case R is taken as additional premise and s is derived
show that R->s can be derived from the given premises
 P-> (B->s), TRVP and B
                                gotha Dries Col Est
          R Rule P (Additional Premise)
  814
     CI)
          TRVP Rule Profession 1919
  $23 (2)
           R->P Rule 7 (2) P->8=7PVB
 £21 (3)
 {1,23 (4) P RULET (1),13) modeus ponens P,P→B =>B
 &sy (s) p->(0,->s) Rule p
&1,2,5 y (6) 0 -> s Rule T (4)(5) modus ponens p,P-> B => B
 & 73 (7) & bodis Rule Pinni hallon di analogia da las las
é 112,5,74 (8) S Rule 7 (6),(7) modus ponens P, P=>9=>9
                                    in mile facility gratical a
 &1,2,5,73 (9) R->S Rulecp (1)(8).
show that p >s can be derived from the premises 7PVB,
           easternass and most ultrafically term the given pasernises
 TOVR, R->S
             P Rule P (Additional premise)
7PVB Rule P
                                                 19 101/
  813
 $ 24 (2) 7PVB
 824 1 (3) P-> B RUIET (2) TP- B= TPVB
 &1,24 (4) Brig Rulet (1)(3) modus ponens
        (5) TOVR Rule P
  653
           92 image Rule T (5) TOVR = Residence nonens
  253
         (6)
                     RuleT (4)(6) modus ponens
                                                 115
  $1,2,53 (7)
 { 8 y (8) R→S Rule P (7)(ε) modius ponens
                                                 113
                                                 113
  8112, 5,84 (9)
  &1,2,5,83 (10) P->S Rule CP (1),(9)
                                                 103
 It there was a ball game, then travelling was difficult. It
they arrived on time., then travelling was not difficult. They
 arrived on time. Therefore there was no ball game show that
the statements constitute a valid argument.
```

Raises e presenting in the second of a salary R: They arrived on time. range of it generally and it has conclusive it has placed as 90 sing Given premises are basished to be to be the country and the food marry is a same daylead P->B, R->7B, R Show that a set out the dealer instance in the person feet Eig (1) P->9 Rule P 2 har 1/2 - 3 -9 623 (2) R-> 78 RuleP (3) R Rule Position of Signa 1 (1) 113 2,33 (4) 70, Ryle T (2), (3) modus ponens \$1,2,34 (5) TP Rule T (1)(4) modus Tollens are as to 1 am ingression a tribits retire of the forth In direct method of proof The method which uses rule up and concept of inconsistent at set of premises is called indirect method of proof Procedure. > Assume that the conclusion cistalse and consider as 70 as an additional premise -> From the given premises together with additional premise derive a contradiction -> Thus c follows logically from the given premises Note: (assembled todoughphat a aima d (10 For a direct proof taking the given premises and derive the conclusion for an indirect proof taking the negation of conclusion and using the given premises we derive the contradiction By indirect proof prove that 70, P->0, PVR->R Given conclusion R include TR as a new premise (1) PVR Rule P (1) 214 (2) TP->R RULE T (1) P-> 0 = TPVB 214 (13 (3) TR >P Rule J. (2) contrapositive TP >70 = 0 ->R (u) TR Rule, P. (New premise) 844 (1,4) (5) P Rule T (3),(4) modus ponens 264 10(6) 11P->00 Relle Portionest route parties parties parties (1,4,63 (7) 00 Rule T (5), (6) 310 910 1910 1910 Rule Por May a surfusano las de las sel-18) 78

```
there the given premise along with new premise leads to
   contradication
                                 Blown trace and agreement grant and
   1. 70,P→B, PVR =>R
using indirect method of proof show that P-> a, B->R, 7(PAR),
                            emorpolisticare in a mary a real in
 PVR => R.
show that 7(PNB) follows 7PN7Busing Indirect Proof.
                                               accompand to the
                JEGUES.
                                             Stpirited 12
                                         National Internation
                                           9-17-2.25
                                9 201174 20 4 (1) 1/15
                                9 31112 9 4 4 10 153
           stud antification (2)(1) Totals and rat of the
         200 - 1 - 1 200 A
     I there is a volte construction from the give premise
              hammangers but of a st good mail our southeasteld team
               entire than all mode grows began in this present All of
                             and waster by the waste Hilling of
                             premium in the track of the in-
  Page , 16, 19 en chits inits a centuary thengets accar
          S. suchin does not get a freezent
                               9 9/129
                               9 9 108
TE BERGE TOTAL TOTAL TOTAL
```

Verity validity of the tollowing

- 1. Every square is a rectangle
- 2- Every Rectangle is a parallelogram
- .. Every square is a parallelogram

Let p: square

9: Rectangle

R: Paralle logram

P->8, 8->R, P->R

214 (1) P→B Rulep

223 (2) B->R RUIEP

 $\{1,2\}$  (3)  $P \rightarrow R$  Rule T(1)(2) Transitive Rule  $P \rightarrow 8$ ,  $S \rightarrow R = P \rightarrow R$ 

:. P-> R is a valid conclusion-from the given premise

IN CASTO A LA MONTO DATE STORY

Contradication

10 149 POR DE 9 DE

Test Wineather the tollowing is a valid argument

- 1. If sachin hits a centuary then he gets a car
- 2. sachin does not get atree car
- . . sachin does not hit a centuary

P->0, 70, 7P 9: sachin hits a centuary thengets acar

£13 (1) P→9 Rule P

62) (2) 78 Rule P

{1,23 (3) 7P RuleT (1)(2) modus Tollens 70,P→0=7P

Predicate calculus.

statement: A statement is a meaningful sentence. statements recontain two parts sement with put be solder plans is their it

I made to subject it is (LE) A made us y bone E & to tagmentate 2. predicate Guerriffess.

ex: sekhar is a good boy book of stranger attached another actiones por predicate an about "and a dort", "amaz", "to de les comments subject! a serie some " presence of actra sup out or recorne odi

The past of a statement which contains noun or pronoun is known as subject alight son tosays sugar x4

predicate:

The remaining part of the statement except subject is known as predicate.

Notation

h cimiyeascal Gardantiffer-Gics), where Gis a predicate and six a subject.

All Birds have Wings

Williams & Louper D.

Predicate logic.

the logic based upon the analysis of predicates in any istatement is called predicate logici in (1) for all a seprecents each of the following phages with baye the M- place predicate

A predicate requiring on (m>0) names or objects is called M- Place predicate. V dons 197 x prove 103 10 107" "andt diex" Amulya "is renstudent y ding prova roa" " rodt douz "

there the predicate is a one-place predicate because of it is related to one object in Amulya. Sca) EXE For every seat intumbers was se

Maveen is talled than royu

there the predicate T is taller than in a two-place predicate because it is related to two objects naveen & Note: A universal quantificate generally tollowed the (Tin) Telling

A simple statement function.

A simple statement function of one variable is defined an expression consisting of a predicate symbol and an individual variable, such as a statement? Hunction becomes a statement when the variable is replaced by the name of any object. We can toom a compound statement function.

Predicate calculus.

ting int on material at

Carrier of C

Predicate (egic.

EX: G(X,4): x is greater than 4

If both x andy replaced by the names of objects, hie get a statement. It x = 3 and y=12 then G(3,2):3 is greater than 2 Quantifiers:

certain statements involve words That indicate quantity such as "all, "some", "none" or one". These words help to determine the answer to the question. "Howmany" since these words indicate quantity, they are called quantitiers.

Ex: some mens are tails

All Birds have Wings

No dirbalbon is perfectly round

Quantitiers are divided into two types

2. Existential Quantifier.

Universal Quantifier

the quantifiers all in the universal quantifier we shall denoted by \tag{x} (or) (x) and we get it as "For all x". The symbol for all x represents each of the following phrases which have the same meaning A pardicate arquiring an in-obtainer or cities

1. For all x 2. For every x "3. For each x " For every x "4. For every x is s. such that " 6. For every each x is such that "6. All x are such that" 7. for any ox "observed and and and of others out asset

Ex: For every real number x, x2 >0

symbolic form

tions the predicate T is tilles than it, at the place Predicate because it is related to two of x 183 x out of

Note: A universal quantifieris generally tollowed the connective of actions transcribed Market

May cen it talles if meavour

ban Extrafil apples are red muit tramature alquie . an individual variable, such as a confermation silver to the for any object. The can town a compand statement tanetion.

symbolictorm

For all x if x is a apple then x is red the quantifier "some" is the existential quantifier. Existential Quantifier: It is denoted by fx. (There exist) The symbol, there exist represent each of the following phrases which have the same meaning EXIDE A (XIII) X F 1. FOX some X - with Arrive a forest of 2. Some x such that 4. There is an x such that called bound marinbles. 5. There is atleast one x such that Exi Fox some integers x, x+3=0

symbolic Form 1 d babanoul and road and darden statistor A Fire Various teis called free variable 于x(x), x+3=0 Note: The existential quantifiers generally followed by the connectives and wish plately formula plumot testome en Ex: some parallelograms are square. P(x): x is a parallelogram Box1: xis a square 10: 918 piros brenos Fire vosidales; 7.5 Excepe of the variable [SEX2] xt (60)  $f_{x} p(x) \wedge s(x)$ negation of quantitiers! 2014 betweentalant & over slav 1. Universal epecification on (x) = f(x) = ((x)) T 2. Universal generalization dexiance 4 (1x) + xE)T 3- Existensial specification of Rule Es Write each of the following in symbolic form a. All men are good Generalization b. No men are good c. some men are good. 20 oling d some men are not good. be true the the this early of the following obtain be decreased obtain Sur Deservense in the contract For all x, if x is a man then exist good of such 21 (3)9

+x (mcx) -> G(x)

can be expressed at the pex

Forsome x x is a man and x is good:

Jx (mcx) n G(2)]

For some x xis a man and xis not good 

7x [m(x) n 7G(x)]

Bound variables

A variable which has been bounded by the quantifiers is called bound variables. Se Thore to attent! and Y S of stadt - a

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Free variables

Since M. Streenshift Names and the A variable which has not been bounded by the quantifiers is called tree variable 0 1 . Table Park 1 11

scope of the quantifiers The smallest formula immediately following (For all x or there exist x) +x con)+x is scalled scope of quantitiers some principal strong arms and

Ex: - \x (P(x,y))

Bound variable :x Free variables; 2,4 scope of the variable P(x,y)

Inference theory of predicate calcule we have 4 fundamental rules they are

- 1. Universal specification or Rule US
- 2. Universal generalization or Rule UG ( )
- 3. Existensial specification or Rule Es
- 4 Existensial specification or Rule EG Generalization

Rule Us .

The state of the s It a statement is in the town of txp(x) is assumed to be true. Then the universal quantitier can be dropped to obtain P(c) is true for arbitary element c in the universe. These rule can be expressed as +xp(x) The [mex) - Calk)

If a statement p(c) is true for every element of c in the universe then the universal quantifier will be prefixed to obtain  $\forall x p(x)$  is True: These rule can be expressed as p(c), for all c

they (x) (x) - replied (x) (x) x X (x) (x)

Rule ES

If a statement is in the torm of then (there exist x) fx P(x) is assumed to be True. There exists an element c in the universe such that P(c) is true. These rule can be expressed as fx P(x)

P(c) for some c

FADIT NO 23 JC - CKIT

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Jx D(x)

257 1 1114 (55)

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€ 8 }

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- E Housellei-

(11)

RULE EG

If a statement pcc) is true for some c in the universe then fxp(x) is true These rule can be expressed as p(c) for some c fxp(x)

Note: DElimination of quantifiers can be done by using rules of specification is called as Rule US & Rule 65

2) To prefix the correct quantifier me need the rules of generalization is called as rule UG & Rule EG.

prove that the validity of the following arguments.

2. All kings are men 23 silos

.. All kings are telliable.

let

mix): x is a men

FIX): or is taliable

k(x): xis king

Symbolic form of the given arguments

+x(m(x) →F(x)) demonstrations on appoint inter

Ax(K(x) -> m(x)] = mand

VX [K(X) ->F(X)]

```
RU10 US -
\{1\} (1) +2 (m(x) \rightarrow F(x)) Rule p
Ely, (2) (mix) > FCX)
234 (3) +x (k(x) -> m(x)) Rule P
                                          1 (1) rot (1) q
      (4) K(c) \rightarrow m(c)
 634
                             RuleUs
(1,3) (5) K(C) -> F(C) Rule + (2)(4) Transitive Rule
                                   P->B,B->RE P->R
 €1,34 (6) +x(K(x) → F(x)) Rule UG
  of Palas agradi in the to much nith of at transmissing Dott
verify the validity of the following arguments
    1. Tigers cire dangerous animals
     2. There are Tigers "19 4 - 26 bassanges od ans
       i. There are dangerous animals
     let
         TCX): x is a Tiger
                                                A J STOR
 e zravious ocx); x is a dangerous animal
  symbolic form of given arguments and i (x) 9 x f and
           \forall x [T(x) \rightarrow D(x)] (x) q x \neq 0 or q x \neq 0
  to setur potal (xxx)) Elimitection of quantifiers and cone out station of notions of
           specification is called as Rule US & CX) of KE
  2) To prefix the consect quentifies me need the sules of
          +x (T(x) > D(x)) = Rule P = noitestiorenep
613
     (1)
           T(E) -> D(E) Rule US (1)
     (2)
613
                             Rule Para in the man and the state of
     (3)
           Jx T(x)
{3}
                              Rule ES Mont you applied HAVE
      (4)
£3 y
              T(C)
                             Rule T (2), (4) modus ponens
6434 (5)
                                           P, P>8=8
              D(C)
                             Rule FGD 21 X CXIM
             Jx D(x)
£1,34 (6)
                              FIX); of is tairable
Given an argument which will establish the validity of the
                         Symbolic form of the given arguments
following.
    1. All integers are rational numbers
    2. some integers are powers of three
```

.: some rational numbers are power of three.

NA [K[X] DE(X)

```
let
                   is freezent upone perfectable to a first of the first of the
       I(x): x is a integer
       R(x): Il is a Rational number
       p(x): x is a power3
  symbolic form of given arguments
                               the after one of security t
           ₩X [I(X) -> R(X)]
                               StableHold with the Shell Shell
           Jx [I(x) A P(x)]
                            Mineral March 1975 To the Control of
           Jx [R(x) A P(x)]
                            tech and the standard make a Virginia
               +x(I(x) -> R(x)) Pule P
   813
         (1)
                I(C) -> R(C) Rule US(1)
         (2)
  213
                JX (ICX) N PCX)] Rule P.
  633
         (3)
                ICC) NP(C) Rule ES (3)
  633
         (4)
                               Rule P
                JX (R(X) 1 P(X)]
         (2)
  654
                                 Rule ES (5)
                 RUC) A PCC)
        (6)
  253
                               Rule 7(4) simplification law
                 I(c)
         (2)
  633
                                            PNB=P
                 P(C)
                                Rule T(u) simplification law
         (6)
   633
                                            ONP = O
   21,34
         (7)
                 RICO
                               Rule 7 (2)(5) modus poneous
   $1,33 (8) R(C) n P(C)
                              Rule 7 (6)(7) P/B=> PNB.
   &1,33 (a) fx (R(x)n P(x)) Rule EG
using cp or otherwise obtain the following implication for all
  \forall x(P(x) \rightarrow B(x)), (x)(R(x) \rightarrow 7B(x) \Rightarrow (x)(R(x) \Rightarrow 7P(x))
                                  Rulep
   \mathcal{L}(y) (1) (x)(P(x) \rightarrow g(x))
                                  Rule US
              P(X) -> O(C)
   613 (2)
   { 33 (3) (x)(p(x) -> 70(x))
                                   Rulep
    234 (4) p(1) -> 70, (c)
                                   Rule Us
                                  RuleT (U) P->B=7B-77P
    633 (5) B(C) → TR(C)
                                           contrapositive
    € 1/34 (6) P(C) -> 7R(C)
                                  RUILET (2)(5) AIC
                                          Transitive rule
     &113 y (7) R(() →7P(()
                                  Rule T6) contra positive
     &1,34 (8) +x (R(x) ->7P(x)] Rule UE,
```

## UNIT IT SET THEORY

set: set is a collection of Mell-defined objects and it is represented by { } denoted by letters A,B --- so, on

Ex: collection of students in the class

Representation: There are two ways to represent the sets.

1. Roster Form

2. set builder Form

Roster Form: Roster Form notation is a complete list of all the elements of the set.

Ex: find the list of even numbers between 2 to 40 S= 22,4,6,8---- 403

Set builder Form! It is used to write in the torm of variables  $Ex: B = \{x/2 \le x \le 46 \text{ and } x \text{ even } y$ 

The vertical bar is read as such that

Subset: It every element is a set A is also element of a set B then A is called subset of B. It is denoted by ACB

Note: The null set is a subset of every set and every set is a subset of itself

EX: If 
$$A = \{1,3,4,5,8,9\}$$

$$B = \{1,2,3,5,7\}$$

$$C = \{1,5\} \text{ then } C \subseteq A, C \subseteq B$$

Types of sets

1. Null set: The set with no elements is called an empty set or null set. A null set is denoted by  $\phi$ . The null set is a subset of every set.

2. Universal set: All the sets are considered to be subsets of one particular set. This set is called the universal set. The universal set.

Disjoint sets: Two sets are said to be disjoint it they have, common element in both sets 9110 (EX; It A : d1,2,33, B=d4,5,63) to an eit partians=do bodreson spilinel- all profilesons daidu phoniliodomis 2 to 2 officity out to notice set in 2 tropies cartesian product: (\*) 2 10 10 93 23 49 49 let AnB are two sets the set of all ordered pair carb) where a belongs to A&b belongs to B is called contesian product It is denoted by AXB 180 A1 - 1811-181 = 180 A1 Ex: let A. d1,2,33, B= d4,53 Then aradid AXB = { (1,4), (1,5), (2,4), (2,5), (3,4) (3,5)} BXA = {(4,1), (4,2), (4,3), (5,1) (5,2) (5,3) } \*\* Operations officets in stagement to our = (804) In set theory operations involve various ways of combinding, comparing and manipulating sets 1. UNION (U): The union of two sets A and B is a set containing all elements that are in either A, B or both (without duplicates) giver in the relicions of the carte. Notation: AUB EX: It A = d 1, 2, 39, B = d 3, 4, 5 } then AUB - 61, 2, 3, 4, 5 } 2. Intersection (n): The intersection of two sets A and B is a set containing only the elements that are in both A and B 2 appropriate analytic upurs 2 (vi Notation: AnB EXT I+ A = (1,2,34, B= 63,4,54 then ANB (34 3. Difference (-): The difference of sets A and B is the set of elements that are in a but not in B 1) = (3) n DC = (HAT)a Notation: A-B (00) A1B EX: It A = {1,2,34, B= {3,4,5} then A-B {1,2} = (=) AT) 4. complement: A, A', A' The complement, of a set A includes all elements in the universal set U that are not in A By inclusion & exclusion painciple Notation: A aria (Har) EX: It U; & 4,2,3,4,5 } and A = & 4,2,3 } then A = & 4,5 } 5. Symmetric Difference: (A) The symmetric difference of A and B

is the set of elements that are in either A or B but not in

EX: It A= (1,2,3) and B (3,4,5) and then ABB: (1,2,4,5) principles of Inclusion and Exclusion: The inclusion exclusion principles is counting technique which generalizes the familiar method of obtaining the no of elements in the union of two tinite sets, symbollically expressedas CONTRACTOR PRODUCT n(AUB) = n(A) +n(B)-n(ANB) the resident (0) in the property of a better (0) 1AUB1 = 1A1+1B1 - 1ADB1 TO ATT OF THE BEST AND TO I A Where n(A) = No-of elements in A n(B) = NO of elements in B n(AUB) = NO of elements in both A and B n(ANB) = No of common elements in A and B for any finite sets A,B,C we have number = n(Anc) - n(Bnc) - n(Bnc) - n(Bnc) the post of the A star must be noticed the CARBAC ) word find the no-of mathematics students at a college taking atleast one of the following languages Telugu, Hindi and English given in the following data. i) 65 study telugu, 20 study telugus Hindi ii) 45 study Hindi, 25 study tempus english iii) 42 study english, 150 study Hindi & English principal iv) 8 study all three languages ANA Incidnation EY: It A - (1,2,3), B-(3,4,5) Then AND (3) Given D(T)=65 8. Difference con sets of sets of sets of child set of elements that are in a but not in B n(E)=42 n(thH)= 20 BIG (60) B-B (60) FILE n(TAE)= 25, (1) 8-A and (2,0,8) = 8, (1,2,1) - A +T 14 4. complement : if a la a the complement, of a se & standard and elements in the universell set. U that are not in By inclusion & Exclusion principle band to some of the state of the party the par 5. Symmetric Difference (A) The symmetric difference of Acond B

is the set of elements that are in either A or B but not in

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and Address of the transferred the same of the same of
                Description
                                                        Notation.
                                                                          The Control of the American Property of the Control of the Control
                    NOT A
               one of A and B
                                                                                                                                                              AUB
            Both A and Billian at someones And and a 191
             only A but not B A4 H 259 THORONDON AN BE SAT BE
            only B but not A
                                                                                                                                                           ANB
                                                                                                                                                                                                Doute Hoyla
                                                                                                                                                            ANB
           Neither A nor B
                                                                                                                                                      ANB (OT)(ANB)
          Exactly one of A and B
                                                                                                                                                       AUBUC
           Atleast one A, B, &C
                                                                                                                                                       ANBACCONT
              All OF ABC
                                                                                                                                            ANBAC
             None of A,B,C
                                                                                                                          BUAJANBAZ
         only a but not B and C
                                                                            Formulae
                                                                                                              - 100 - Q1+35-23
               A = 101 - 1A1
          IANBI = [AUB]
                                                                                                                                                                      13 0
                                = 101-1AUB1
      [ANB] = [A] - [ANB]
   A sample of 80 people revealed that 25 like cinema and 60 like TV programs. Find the no-of people who like both cinema
            [A nB] = |B| - |AnB|
                                                                                                                                                   AUB = 141+ 181-1810
    and TV programs.
                         A be the set of people who like cinema
                         B be the set of people who like TV programs
             Given dataf(10) Endrod pd aldizivib 2 sapatni to da 1804
                                   n(AUB) = 80
                                         n(A) = 25
                                          n(B) = 60
                                                                                                                                                     (800) - 181+101 - 1800)
                             n(AAB)
                                                                                                                                                                           U- 41 +88 =
                n(AUB) = n(A)+n(B)-n(ANB)
                                                                                                                                                                                                43
                                80 = 25 + 60 - n(ANB)
tet A and B be two sets then a supsets st= (8 and) a to
         a relation (or) Binary relation from R: A - & symbollically
        The no of people watching cinema and Tv programs are 5 = 1
```

Formulas.

A certain computer center employee 100 computer programmers of this 47 can program in FORTAN languages 35 in PASCAL and 23 can program in both languages. How many can program in neither of these two languages. She of Annell Let A be the set of programmers in FORTAN DA MOS B be the set of programmers in pascall a plan only B but ret A 309 Given that AAA state a configura n(A)= 47 Exactly one of a mod B n(B)= 35 Affect one A.E. . n(ANB)= 23 All of MAC n (AnB) = n(AUB) None of AiBic =101-n(AUB) only A but not Dand C > |U| - n(A)+n(B)-n(ANB) onlucato) = 100 - 47 + 35 - 23 [A] - [O] = A laual : laual = 4 | aua | - | u1 = | Determine the no of positive integers < 100 which are divisible by 3017. 1800 - 181 - 180 AT let A be the set of numbers divisible by 3 B be the set of numbers divisible by 7 We know that 1AUB) = [AI + 1BI-1A NB] tal= no of integers divisible by 3 = 100 = 33

1B1= no of integers divisible by 7 = 100 = 14

IANBI. no. of integers divisible by both(3100)7 = 100

IAUBI = [AI + IBI - IANB]

= 33+14-4

= 43

(BAAB)

n(AUB) = n(A)+n(B)-n(AUB) 80 = 25 + 60 -D(ADB)

00 = (8)0

Relation

let A and B be two sets then a subset of AXB, is called a Relation (or) Binary Relation from R: A -> B symbollically RSAXBO and carbine R Where as A & be B.

Domain of the Relation

It is denoted by Dom R= & Black and ca, b) ER.

Range of Relation.

The set of all the second elements of the ordered pairs in a relation R is called the Range of Relation. It is denoted by Rank = folbeB and (a,b) ER

\*\* operations on relations

UNION of Relations: The union of two relations R and s sets A and B is the relation containing all pairs that are in either R(OX)S

traditioning to the transiquest

(combine two relations without duplicates)

RUS

Ext R= ((1,2) (2,3) & and S= ((2,3)(3,4)) RUS = { (1,2), (2,3) (3,4) }

Intersection of Relations:

The intersection of two relations R and s on sets A and B is is the relation containing all pairs that are in both Rands (common elements in both relations)

Ex: If R= ((1,2)(2,3) and S= ((2,3)(3,4)) then Rns = ( (2, 3) }

Difference of Relations.

The difference between two relations R and s on sets A and B is the relation containing all pairs that are in R but not in S

Motation R-S

Ex: I+ R= {(1,2), (2,3)} and s.f(2,3), (3,4)}

then R-S. & 1,29

composition of Relations

The composition of two relations R on AXB and (AXX)s on BXC is a relation on AXC

It contains pairs (a,c) such that there exists an element beb where (a,b) er and (b,c) es

Renoge of Relation Exi It R = {(1,2) (2,3) } and S = {(2,4) (3,5)} SOR: {(1,4)(2,5)}

The partied by the partial participation of the

## compliment of a relation:

let R" be a relation from A to B then the (relation) compliment of R is denoted by R cor R' and is defined as R = (AXB)-R

Ex: let A = { (a,b) & B: {1,2,3} and the relation R from A to B is given by R = { a,1)(a,3)(b,2)(b,3)} then find R

R: (AXB)-R , of (a,1) (a,2) (a,3) (b,1) (b,2) (b,3) - R Intersection of telephone: 2 q ca,2) (b,1) }

the first term of the first te

the first of the second of the

## converse Relation:

let "R" be a relation from A to B then the converse of R is denoted by R is a relation from B to A and is defined CLS

R = { (b,a) a EA, b EB and (a,b) ERY. pifference of kelantions.

- Notation r

composition of kelations or Water the same of a second to the second with

5 K O F C 10 10 10 12 1 10 13

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FLIS LIFE OF

2 Sersial in the relation of the glad internals

```
consider the sets A = & ABCY, B= {123} and the relations R = {(a,1),
    (b,1), (c,2), (c,3) y and s= {(a,1)(a,2)(b,1)(b,2) } From A to B
     Determine R, S, RUS, RNS and converse of RIS
                                                                              Reflexive Ji(a.ca) Lak . V avea
       Let AXB
                 = e((A,1) (A,2) (A,3) (B,1) (B,2) (B,3) (C,1) (C,2) (C,3) }
      S NOW PER SER (CIT) ( 2) S NOW E AXB ER (CIT) ( 2) S PONDO ON STORY OF THE MOON R
      9 d or blo ( CA12) (A13) (B12) (B13) (C11) 4 anixology of xollogical
                   3 = AXB-S (B13) (C1) (C12) (C13) 38 (11) -A 11 (X)
RUS = \{(C_{12})(b_{11})(b_{11})(b_{12})(b_{12})\}
\{(a_{11})(b_{11})(c_{12})(c_{13})(a_{12})(b_{12})\}
                                                                                               H(a,b)exthen(ba) ex
         ens = {(a,1) (b,1) }
   converse of R & (1,a) (1,b) (2,c) (3,c) 3

converse of S & (1,a) (2,a) (1,b) (2,b) 3
consider the sets A=(1,2,34 and the relations R=(1,1) (1,2)
 (2/3) (3,1) and S = { (3,1) (3,2) (3,3) (2,1) } on A. then
  compute R & RUS RNS and converse of S. Distanting
                                                                                                                                                    Cub C A:
      Let AXA
               - d (1,1)(1,2)(1,3)(2,1)(2,2)(2,3)(3,1)(3,2)(3,2)
         NIPW (ε, ε) (ε,
        3 = AXA -S
         2i 9 (19dt (23/1) (4.2)(11) 3=9 ban (5141) 6 A 4I 73

converse of R (11/1) (2,11), (3,2) (1,3) (1,2) 4

converse of S (1,3) (2,3) (3,3) (1,2) 4

converse of S (1,3) (2,3) (3,3) (1,2) 4
                                                                                                         Representation of Relatings
                                                                                    mainten of materix Repleser distri
                                                                                                   a bit- people action of the contestion.
                                                                              1. Relation of matrix Representation.
  set foden didinal a one for = 120,00 ? - A HE
```

Properties of Relations

Reflexive Relation: A relation R on a set A is said to be Reflexive. It (a, a) GR, + a EA

Ex: If A= (1,2,3 3 and R= ((1,1)(2,2)(3,3)(1,2)3

Then R is Reflexive because (1,1)(2,2)(3,3) are all in R

2. Irreflexive Relation: A refation R on a set A is said to be irreflexive. If (a,a) ∉ R For any a ∈ A.

is irreflexive

3. Symmethic: A relation R on a set A is said to be symmetric it (a, b) ER then (b,a) ER

Ex: It A =  $\{1,2,3\}$  and R =  $\{(1,2)(2,1)(2,3)(3,2)\}$  then

R is symmetric because whenever (aib)  $\in \mathbb{R}$ , (bia)  $\in \mathbb{R}$  is

Ex:  $(1,2) \in \mathbb{R}$  Then  $(2,1) \in \mathbb{R}$ 

4. Anti-symmetric: A relation R on a set A is said to be antisymmetric: If a≠b and (arb) ∈ R then (b,a) ∉ R. For all a,b ∈ A.

If (a,b) ∈ R then (b,a) ∈ R, a=b

5x: If  $A = \{1,2,3\}$  and  $R = \{(1,2),(1,3),(2,3)\}$  them R is antisymmetric because there are no pairs (a,b) and (b,a) in R where  $a \neq b$ .

If R= ((1,1) (2,2) (3,3) & when a=b

5. Transitive: A relation R on a set A is said to be transitive if  $(a_1b)\in R$  and  $(b_1c)\in R$  then  $(a_1c)\in R$   $\forall a_1b_1c\in R$   $(a_1b)\in R$  and  $(b_1c)\in R$  then  $(a_1c)\in R$   $\forall a_1b_1c\in R$  is transitive because  $(1,2)\in R$   $(2,3)\in R$  then  $(1,3)\in R$   $\forall (1,2,3)\in R$ .

Representation of Relations

1. Relation of matrix Representation 2. Di-graph representation

It A = { a, a, a, a, a, a, a, and B = { b1, b2, b3 --- bny are

matrix. This is denoted by Mr. Cmij] mxn

if (ai, bi) ER

if (ai, bi) ER

EX: IH A = { a1, a2, a4 }

B= &b1,b2,b3,b43 then the relation R from A to, B is given by R = {(a1,b1), (a1,b4), (a2,b2), (a2,b3) (a3,b)}
Now relation matrix.

$$m_{R} = a_{2} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 1001 \\ 0110 \\ 1010 \end{bmatrix}_{3X4}$$

Di-graph

A relation can be represented partially pictorial by drawing its Di-graph follows

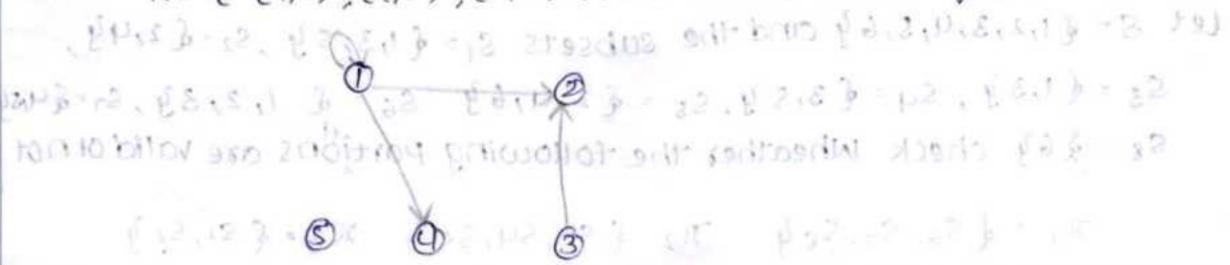
1. Draw a small circle cos) A bullet too each element of A. and table the circle with the corresponding element of A. These circles are called vertices or Nodes

\$ A \$1 P \$1 P \$1.2 \$ ... \$ ... \$2.2 P ... \$5 P .

- 2. Draw an arrow from the vertex a to bit and only it will ber these arrow is called an edge
- 3. It pair ca, a) ex then draw a directed edge from a to a such edge is called a loop
  - 4. This pictorial representation of R is called Di-graph 60)
    Directed graph R

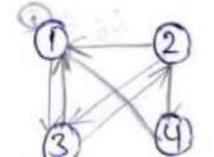
Exize A= & 1,2,3, (1,5 & then a relation R on A is given by

R, & (1,1), (1,2), (1,4) (3,2) . Draw the di-graph of R



EXT IFA= (1,2,3,44 and R= (11,1) (1,3) (2,1) (2,3) (2A) (31) (3,2) (4,1) is a relation R on A the draw the digraph R.

best vitton nothings in



let R = {(1,1) (1,2) (2,3) (3,3) (3,4) } be a reloction on A = {1,2,3,4}. Draw the di-graph of R. obtain R2 and draw the di-graph of R2



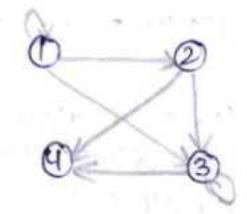
R2 is the composite of R withit self and it consists of ordered pairs (aic) such that (aib) ER, and (bic) ER

$$R^2 = ROR$$
=  $\{(1,1)(1,2)(2,3)(3,3)(3,4)\}$ 0  $\{(1,1)(1,2)(2,3)(3,3)\}$ 

the first transfer of the first transfer of

the state of the s

= {(1,1)(1,2)(1,3)(2,3)(2,4)(3,3)(3,4)}



## Partition and covering

Let S be a non-empty set, S1, S2, S3 and --- -- Sn cire the subsets of s the collection of subsets si is a portition of s If and only if the first of the garden In St.

i) Si = p for each;

ii) Sins; = 0 for i +;

(ii)  $S_i \cap S_j = \emptyset$  for  $i \neq j$ (iii)  $U^m \cdot S_i = S$  Where  $U^m$  i = 1

Let S= {1,2,3,4,5,6} and the subsets S1= {1,3,5}, S2= {2,44, S3 = (1,3 ), S4 = (3,5 ), S5 = (2,4,6 ) S6 = (1,2,3 ), S7 - (4,5) S8 = 263 check wheather the following partions are valid or not

71, = { S6, S7, S84 712= { S3, S4, S53 753= { S1, S2 } 2) ( ) " SI + O ( 12 1 142) ( 2, C ) ( 1, C ) ( 2, C ) ( 1, C ) ( 2, C ) ( 1, C ) , A SI ( 1, C ) , A SI ( 1, C ) ( 1, C ) ( 1, C ) ( 1, C ) , A SI ( 1, C ) , A SI ( 1, C ) ( S6= { 1,2,34 37= { 4,54 in se se 6640 3 noitolesso

S1 ≠ Ø ... S8 ≠ Ø S6 = 0 Ist condition scristicd.

```
that is no bullery or
        (S6, S8) = &1,2,34n &64 = 0
    (57,58) = {4,5} n {6} = $
                    Ind condition is also satisfied
                                                                                        Section 30 to the little of th
    111 0 m si = S
              (S6, 57, 58) = S6.US7US8; = {1,2,340{4,540£63
                         1 1/2,3,4,5,6,3 = S 1 1/2,3,4,5,6,3 = S
and the secondation is satisfied
                                                                                                                   en et mellos d'initial camille.
             .. The sesses is a valid partition
           7 = (S3, S4, S53
                                                                                                                  S5 = & 2,4,64
                                                      11) Sy = & 3,5 3
       i) Si + 0
                                                                                                                               S5 = 0
                S3 = & 1,3 4
                                                                         SH+ O
                     S3 + 0
                              Ist condition satisfied
      ii) sinsj = ¢
             [53 NS4] = {1,33 n {3,54 = {34+0
            (S3 N S5) = & 1,33 N & 2,4,6 3 = 0
              [Synss] = & 3,5 yn & 2,4,6 y = 0
                               Ind condition is not satisfied.
       (ii) U SI=S
                                 [ 53, 54, 55] = 53USY USS = { 1, 34, 83, 54, 82, 4, 64
                                                                        - 61,3,5,2,4,64
                                                                 = §1,2,3,4,5,63 = S
                 in ad condition is satisfied
                       1. N2 = S3 Sc1 S5 is not a valid partition
             Tl3 - & S1, S2 }
   iti)
                                                                                                 sinsi = \phi
           i) si + 0
                                                                                                        (SINS2) - { 1,3,5 } n { 2,4} = $
                SI= &1,3,59 S2 = &2,43
                                                                s2 + 0
                                                                                                           Ind condition is satisfied
                       SIFO
```

1st condition is satisfied

```
covering of a set.
                           Let s be a given set and A = { a, a2, a3, -- any where each A;
    U" Ai = S. Ai is a subset of s then set A is called a
 covering of sets and the subsets A1, A2, A3 ---- Am one said
to be coverset of S
  Ex: Let s = {ABC} and consider the following collections of
subsets A = & ca,b) cb,c) & B = & calla,c) & c = & (a), (b,c) &
 D= {(a,b,c)} E= {(a),(b), cc)} f= {(a), (a,b), (a,c)} among
them which subsets are covering of s and which are partitions.
 consider A = {(a,b) (b,c) & contains two subsets
         Al= faiby
         A2 = 4 b16 4
        Um 81 = S
      AIUAz = {aiby U fbicy
            = {a,b, c} = s
                           B= { (a) (a,c) }
 B1 = { a4
                      B2 = { a1 c }
                      $35° me has been been a little
 BIUB = Eagle CI, Cy # 5
 C= { (a), (b, ())
                                       2 12 7 (11
  C1 : { a }
c2. { a, c }
                          F= ((a), (a,b), (a,c) }
 civez, fay v (a, c)
                          Fi sal
      . & a, c y + s
                           Fz jarby
   D = & (a, b, c }
                           F3 = { a, c }
                 AC TO WAY !
                          FIUF2UF3 = &a,b, C3 = 5
  fa, b, c 3 = S
                                        E = {(a),(b),(c)}
   Ei= fab
   E2 = 4 by
                              Lawrence of modifilation "!
   E3 2 6 ( 4
```

-x\*\*\* Equivalence Relations

A Relation R on a set A is said to be equivalence relation on A-It it is satisfies reflexive, symmetric and transitive.

let x = { 1,2,3,4 4 and R = { (1,1) (1,4) (4,1) (4,4) (2,2) (2,3) (3,2) (3,3) 4 check wheather equivalence is relation or not.

Given

R= { (1,1) (1,4) (4,1) (4,4) (2,2) (2,3) (3,2) (3,3) }

Reflexive relation

If AEX then (a,a)ER

ine (1,1) ER, (2,2) ER, (3,3) ER, (4,41) ER

.. R is reflexive

symmetric relation

It (a,b) ER then (b,a) ER

ine (1,4) ER -> (4,1) ER (2,3)ER -> (3,2) ER

.. R is symmetric

Transitive relation

It (a,b) ER, (b,c) ER then (a,c) ER

i.e (1,1) ER (1,4) ER -> (1,4) ER (1,4) ER (4,1) ER -> (1,1) ER (2,2) ER (2,3) ER -> (2,3) ER (2,3) ER (8,2) ER -> (2,2) ER (3,3) ER (3,2) ER > (3,2) ER . Ris Fransitive.

r. Given Relation R is equivalence Relation

2. Verify the Relation R = ((1)) (2,2) (3,3) (4,4) (3,1) (2,1) you the set a = £1,2,3,43 is equivalence or not

Given

R= { (1,1) (2,2) (3,3) (4,4) (3,1) (2,1) }

Reflexive relation

Fi (x-1) E pd staletyth ie If AEX then (a,a) ER

(11) ED (2.2) ER (3,3) ER, (4,4) ER (3,1)

If (a,b) er then (b,a) er

i. R is not symmetric

Transitive Relation)

ine If (a,b) ER, (b,c) ER then (a,c) ER

 $(3,3) \in \mathbb{R} (3,1) \in \mathbb{R} \longrightarrow (3,1) \in \mathbb{R}$ 

(GET (BES) ) | Committee (Committee) | Committee (Comm

Carried a possible of the

the second of the second of the second of

(3,1) ER (1,1) ER -> (3,1) ER

(2,2) ER (2,1) ER -> (2,1) ER

(2,1) ER (1,1) ER -> (2,1) ER

(2,2) ER

.. R is a transitive

i. Given relation R 1s not equivalence Relation.

3. Let x = { 1,2,3, -- 7 } and R = { (x,y) / x-y is divisible by 3 } show that R is equivalence relation.

The state of the s

let R= {(x,y)/x-y is divisible by 3} in other words (a,y) ER it there exists an integer k such that x-y= 3k

Reflexive relation

tx ex, (x-x)=0 is divisible by-3 -> (x,x) ER +acex

since o is divisible by 3 me have cxix) ERHXEX i. Ris Reflexive

no filles (that i pure) (see ) that the fill a nothing of party

A relation is symmetric (2,4) ER the (4,2) ER suppose (ouy)er this means x-4 = 3k for some integer K

(y-x)=-(x-y)=-3K y-x is also divisible by 3 (x,y) er then x-y is divisible by 3 -(y-x) is divisible by 3 (y-x)also divisible

by 3 (y, x) € R 11 11

Transitive relation. let (x/y) ER and (y/z) ER then (x/z) ER Me know that a-y is divisible by 3 y-z is divisible by 3 x-y+y-z is divisible by 3 R. BCK x-z is divisible by 3 (x-z) ∈ R ". Ris transitive. Transitive closure. Let x be a finite set containing n elements and R be a relation on x Then the transitive closure relation of ax is denoted by Rt such that ean be defined as R' = RUR UR OULLE RO 10 3112010 SVINSON SOIT It R = {(1,2) (2,3) (3,3) & be a relation on A = {1,2,3}. Find the transitive closure of R. Given data A= {1,2,34 has 3 elements. so the transitive closure of Ris Rt = RUR2 UR3 THE HE SENDED IN LINES AND THE R2 ROR · ((1,2) (2,3) (3,3) 4 0 2 (1,2) (2,3) (3,34 Given dutin 21 90022012 of (1,31) (-2,3) (-3,3) ( (9,6) (6,0) (0,d) (d,0) ) - A R3, ROR2 = {(1,2)(2,3)(3,3)40((1,3)(2,3)(3,3)} (9, b) (b, 2) (1,3) (2,3) (3,3) (3,3) (4 ) (b, 2) (2,d) (d, 2) 3. of (0,0) (bd) (0,0) }-R+= RUR2UR3 - ( ( ( ( ( ) ( ) ( ) ( ) ) ) ( ( ( ) ¿ (0,0) (b,0) }. Rt = {(1,2) (1,3) (2,3)(3,3)} Let x = \(\ell\_{1/2}, \frac{3}{4}\) and R=\(\ell\_{1/2}\) (\frac{2}{3}) (\frac{3}{3}) (\frac{3}{4}) \(\frac{4}{3}\) (\frac{3}{3}) \(\frac{3}{4}\) (\frac{3}{3}) \(\frac{3}{3}\) (\frac{3}{4}) \(\frac{4}{3}\) (\frac{3}{3}) (\frac{3}{3}) (\frac{3}{3}) \(\frac{3}{3}\) (\frac{3}{3}) (\frac{3}) (\frac{3}{3}) (\frac{3}{3}) (\frac{3}{3}) (\frac{3 Given data 909 tg g (ane) g 0(9, b), (b)) R=16((12)(2,3)(3,4)4

```
TO THE PROPERTY OF THE PARTY OF
              R2 ROR
                        = {(112)(2,3)(3,4)}0 {(1,2)(2,3)(3,4)}
            R2 26 (1,3) (2,4)4
                                                                                         the state of the state of
        R3 = ROR2
                                                                                             - pd old state at a
                      · ((1,2)(2,3)(3,440 Q(1,3)(2,4))
                      = { (1,3) (2,4) (1,4) 4
           RY ROR Did themself a projection to select p and x tell
            3 6 (1,2) (2,3) (3,43 0 6 (14) C
                                                                                                                                       an harrish ad and took ding
                       = $ 64
           The transitive closure of Rt= RUR2UR3UR4:
                 · ((1,2)(2,3)(3,4) y ((1,3)(2,4) y U ((1,4) y U (0)
     EC112) (1,3) (1,4) (2,3) (2,4) (3,4) 4
                                                                                                                                                                                                                   Given data
   If R= {(a,b) (b,c) (c,d) (d,e) & and A = {a,b,c,d,e} then find
   the transitive closure of R.
                                                                                                                                                                                                                P RCR
                                                                          ( E12) (213) (213) $ (213) (213) (213) (213)
        Given data
                                      R = & ca, b) (b, c) (c,d) (d,e) }.
                                      A= (a,b,c,d,e) has 5 elements. so the transitive closure is
             R+ RUR2UR3UR4UR5 (811) 31 (818) (818) (811) = 809 89
                    R'= ROR
                             = { (a,b) (b,c) (c,d) (d,e) } of(a,b) (b,c) (c,d) (d,e) }
                              - of (a,c) (b,d) (c,e) }
                                                                                                                                                                        Pt. RUR'UR
(E16) (E15) 2012 E16) (E16) (E
                               · { ca,d) cb,e)}
                                                                                                 pt = ((1,2) (1,3) (2,3)(3,3)}
 & carb) (b,c) (c,d) (d,e) } O { (a,d) (b,e) }
                                                                                                                                                                            The transitive closure of
                                                                                                                                                                              Rt - RUR 2 UR 3 UR 5
                                   & care)y
                                                                                                                        10 & care) & (carp) (cb,0) (cd), (d,e)
                                                                                                                                                                                       ar ) ( b, d ) ( c, e) & ( ) & ( a, d) ( b, e)
                     PSTROPY
```

compatability and partial order. and all parties are the second of the second compata bility relations.

A relation R on a set A is said to be compatability relation if it satisfies reflexive property and symmetric property

A compatability relation is represented by &

Let A = \$1,2,3 & and let the relation R = \$ (1,1) (2,2) (3,3) (1,2)(2,3)(2,1)(3,2)(1,3)(3,1)4 define the A. Check wheather this relation is compatability relation or not?

let

distriction it is a city A= 91,2,33 R = & (1,1)(2,2)(3,3)(1,2)(2,3)(2,1)(3,2)(1,3)(3,1)}

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.3VITIZADIT

For any two Integers out

i) Reflexive property:

It AEA then caraler {(1,1)(2,2)(3,3)}€R

3 vix 817-36 21 < : .. R is Reflexive R satisfies reflexive property

ii) symmetric property (do)

It carb) ER then (bra) ER (bid) , 93 (dio) II (i (112) ER then (271) ER (1010) 19 (din) II (1) (2,3) ER then (3,2) ER DEd, dED ti 311 (1,3) ER then (3,1) ER

if a sb, bec then ase

: R scitisties symmetric property

In this given R satisfies reflexive and symmetric property because R is compettability relations (3,0) 93 (d,0) ti

· Partial ordering Relation.

A relation R on a set P is said to be partial ordering relation. It and only if If R is reflexive, anti symmetric and transitive notions paire by site of the state of the stat

Partial ordered set (01) POSET \*\*\*

It partial ordering relation is a partial ordering on a setp. then the ordered pair (P, < ) is called a pair tratordered

totally ordered relation. In a late of the for every two elements let (pr < ) is partially ordered set. It for every two elements a, b ep. we have either a < b (00) b < a (comparable). Then partial simple exdesing on p and (P. S.) is

```
EX!
  It P= (1,2,3,43 then (p,s) is totally ordered set.
                                                                                                         is triffed being directly
PY'
  show that the relation "greater than or equal to is a partially
 ordering relation on a set of Integer.
             Z= {----3,-2,-1,0,1,2,3---3
   A relation greater than or equal to is a partially ordering relation
  it it satisfies 3 properties.
              (i) > is Reflexive
             (ii) > is Anti-symmetric
           (ili) > is Transitive
                                                                               71/2017 - 11/2019 9
     Reflexive.
            for every integer a , a= 2 then ca, a) eR
                ie 121,222,323, ER
                                                                       1 Toff Tof 1/ Com and 1/2 1
                  :. > is reflexive.
     Antisymmetric.
        For any two integers arb it (arb) & z 1999
        i) If (a,b) ER, (b,d) ER-then a=b
        ii) It carb) er, (b,a) er then atb
             ire if azb, b ≥ a then a = b month significant
                                > is anti-symmetric
                                           preemon sid amongs 2911-11102 g a
    Transitive.
      For any three integers a, b, c if carbic) ez
          if (a,b) & R (b,c) & R) then (a,c) & R, 1100 21 9 0211100000
         enterior at the place of the pl
         in the given relation z is partially ordering relationsvitizant
       If P= { 1,2,3,4,6,123 R= {(x,y) ER' | x divides yy prove that
     given relation is partial ordering Relation or not
       Given data 109 p bello, 12 3 = 4) 109 berebro ett 1911 1915
                            R= {(1,1) (1,2) (1,3) (1,4) (1,6) (1,12) (2,2) (2,4) (2,6)
    2 trasmele out p(2,12) (3,3) (3,6) (3,12) (4,4) (4,12)(6,6) (6,12)
 and ep. we have either a = b (a) b = a feathable on postial
```

.. R satisfies Reflexive property. tylenthest averaged of Anti-symmetric If (a,b) ER and (b,a) ER then a +b iel (1,3)  $\in \mathbb{R}$   $\Rightarrow$  (3,1)  $\notin \mathbb{R}_{2}$  in 1900) 200000 from 2000 and which is (1/2) ER => (2,1) ER (1,4) ER => (4,1) €R The state of the second section of the second sections. (1,6)ER => (6,1) &R (1,12) ER => (12,1) & R 1997/199 .. R satisfies anti-symmetric property Referred (n. her and a said Transitive , assured maintains profession principles of a right If carb) er (bic) er then carc) er (Des Chi) ER Ch2) ER then Ch2) ER Remove Rediently (1,2) ER (2,2) ER then (1,2) ER (1,3) ER (3,3) ER then (1,3) ER y 112.121 (1,4) ER (4,4) ER then (1,4) ER REPUENCE ECTIONIS (116) ER (616) ER then (116) ER (1/12) ER (12,12) ER then (1,12) ER) (3,83 (1,13) 340003 (2,2) ER (2,4) ER then (2,4) ER SALLISTE SAUCHER (2,4) ER (4,12) ER then (2,12) ER. (2,6) ER (6,12) ER then (2,12) ER (3,8) er (3,6) er then (3,6) tr .. R satisfies Transitive property .. R satisfies reflexive, Antisymmetric and Transitive Margaria Diagram pro perty i. Ris S A partial ordering relation on a set p can be represented Hassie Diagram. by means of adiagram is known as Hassie Diagram. 1. In such a diagram each element of p is represented by a(.) 2. Itx y then is drawn below y and it y covers oc. 01,0 then a andy are connected by a straight line (-) 3. It x < y and y does not cover x. then x and y eire connected directly by a single line. e:In hassie diagrams me don't draw loops Note: 2. It there is an edge from a vertex to vertex b. and b to-c

```
411 v 2048 " 1 1 1 1 2 2 2 2 1 2 1 1 2 2 4 4
Rules.
1. Remove Reflexive
                                             Apti-symmethic
2. Remove Transitive
3. Arrange elements vertically
4. praw lines omit arrows label the elements.
                                             STATE R
Let P= 1,1,3,43 and = be the relation "less than or equal to "or" ="
then draw the hassie diagram.
                                  Given
       P= { 1,2,3,4 }
       R= {(a,b)/a,bep and a sby
      Here R is partially ordering relation because R is retlexive
  antisymmetric and Transitive. For hassie diagram
  Remove Reflexive.
        R= ((1,1) (1,2) (1,3) (1,4) (2,2) (2,3) (2,4) (3,3) (3,4)
                          (113) FR (373) ER - 11100 (1-100
            (4,4) 4
                          CIMPER INVINEE Then (SA)EF
  Remove Reflexive
                         (1,6) ET (6,6) ER -1601 (1.6) GR
 Remove ((1,1) (2,2)(3,3)(4,4))
                        (2,2)ep (2,0) ee then (1,0) ee
 Remove Transitive.
      (1,2) ER (2,3) ER => (1,3) ER (1,0) 9 (1,0)
      (1,3) ER (3,4) ER => (1,4) ER 9) (3) 9) (3,5)
      (243) ER (341) ER > (241) ER. 1 3 (3,8) 40 (8,8)
                      in R sortisties Transitive property
    9 pi (201,21), (228); (3,4) }
   Hassie Diagram
                                           Proporty
                                          S 27 A .. ..
           A postial ordering relation on a set is can be
         by means of adjagram is known as Hassi piagram.
( ) pd b 2) 1323 13 1 2 1 36 3 and the relation & and x, y = x, X < y
and if x divides y draw the hassie diagram.
      2 x 2 2, 3, 6, 12, 24, 36 3 pd betsected our phone in court
       R = \{(x,y) \mid x,y \in X \text{ and } x \mid y \}

R = \{(2,2) (2,8)(2,12) (2,24) (2,36) (3,3) (3,6) (3,12)
5-ot d. bono. d. (3, 24) (3,36) (6,6) (6,12) (6,24) (6,36) (12,12)
```

```
Remove Reflexive
    Remove, ((2,2)(3,3)(6,6)(12,12)(24,24)(36,36)4
                       present the series of the series of the
 Remove Transitive
      (2,6) \in \mathbb{R} (6,12) \in \mathbb{R} \to (2,12) \in \mathbb{R}
      (2,12) ER (12,24) ER > (2,24) ER
      (2,36) ER (6,36) ER > (2,36) ER
      (3,6) ER (6,24) ER -> (3,24) ER
      (3,6) ER (6,12) ER -> (3,12) ER
      (3,6) ER (6,36) ER -> (3,36) ER
      (6,12) er (12,24) er -> (6,24) er
      (6,3€) ER (12,36) ER → (6,36) ER
                                 THE STATE OF STATE OF
    X = 2(2,6) (3,6)(6,12) (12,24) (12,36) 
                        39(11,E) 12,12) ER (12,12) ER -14 EN (3,11) ER
Hassie diagram
                       Chinks (suppr) se then tapent e
                       97((36) and 95(0,03 95(0,0)
                       3 / way 200 17 13 (way 20) 23 (way 3).
                         progond avinamon asitaitos 9:
 program svitizaner bab sirtemapathan, svirgitos 29183thaz 9 ..
                                          3 >219;
Draw the hassie diagram p = { 1,2,3,5,6,10,15,30}, 1.
cet A = $ 1,2,3,6,12,24% on A Define the partial ordering relation R
by a Rb itt and only itay b. draw hassie diagram.
      A: {1,2,3,6,12,24} A= (3,13 modi- 9=(3,2) A=(2,033)
   Given
     R= ((1)(1)(1)2)(1/3)(1/6)(1/12)(1/24)(2/2)(2/6)(2/12)(2/24)(3/6)
   (3,12)(3,24)(6,12)(6,24)(12,12)(12,24)(24,24)(3,12)

(6,6)

Hexive property

if ACA then (a,a) ER)

i.e (1,1) ER (2,2) ER (3,3) ER (6,6) ER (12,12) ER, (24,24) ER
 Reflexive property
          .. R soutisties retiexive property; (8) (8) (8) (8)
     if cab) ER and (b, a) & R then a + b.
 Anti-symmetric property
```

10 (1,2) ER =) (2,1) ER

```
Remove tellesits
       (1,12) ER => (12,1) FR
       (1,24) ER => (24,1) ER
    .: R satisfies anti-symmetric property
                                         Demoye Transfilling
                     C2,63-F (6 11)(E - 10)
 Transitive property.
   If carb) er (bic) er then carc) er
      CI,1) ER C1,2) ER then C1,2) ER
      (1,3) ER (3,3) ER then (1,3) ER
(1,6) ER (6,6) ER then (1,6) ER
     (1/12) ER (12/12) ER then (1/12) ER
     (1,24) ER (24,24) ER then (1,24) ER
     (2,2) ER (2,6) ER then (2,6) ER (1)
     (2,2) ER (2,12) ER then (2,12) ER (8,12) (8,13)
     (2/12) ER (12,24) ER then (2,24) ER
     (3,3) ER (3,6) ER then (3,6) ER (10) (3,6) (3,8)
     (3,12) ER (12,12) ER then (3,12) ER
                                            Hassie diagram
     (3,24) ER (24,24) ER then (3,24) ER
     (6,6) ER (6,12) ER then (6,12) ER
     (6,24) ER (24,24) ER then(6,24) ER
    .. R satisfies Transitive property
  . R satisfies Reflexive, Antisymmetric and Transitive property
         : Riss
 Remove Reflexive
Remove & (1/1)(2/2)(3,3)(6,6)(12/12)(24/24)3
                    199 S. All School A 195 & 13, 11, 3, 7, 11 3 = A 935
 Remove Transitive
   (1,2) ER (2,6) ER then (1,6) ER
                                                  (19VIE)
 (3,8) (1,3) CRII (3,12) ER then (1,12) ER (105,51,3.8.4) 3-19
     (1,6) ER 100 (6,24) ER then (6,24) ER 1) (8,1) (8,1)
     (2,6) ER (6,12) ER then (2,12) ER 3) (148)
     (2,12) ER (12,24) ER then (2,24) ER (3)
     (3,6) ER (6,12) ER then (3,12) ER
                                        perlexive property
93 (12 (3,12) ER (12,24) er then (3,24) ER
     (6,12) ER (12, 24) eR, other (6,24) ER (12)
                                   ine (1,1) ∈ R (2,2) ∈ R
A= ( C42) C1,3) (2,6) (3,6) (6) (6) (2) (12) (12,24) }
                                   E SCHISTIES I
                     H couple R and co, a) & R then a #
                                        Anti-symmetric bapp
```

```
Draw the hassie diagram . p & 1,2,3,5,6,10,15,303. /.
          Si com a branch of
       P= (112,3,5,6,10,15,30)
      R. & CI,1) (1,2) (1,3) (1,5) (1,6) (1,10) (1,15) (1,30) (2,2) (2,3) (2,5)
         (2,6) (2,10) (2,15) (2,30) (3,3) (3,6) (3,15) (3,30) (5,5)
         (5,10) (5,15) (5,30),(6,30) (10,10) (10,30) (15,$5) (15,30) (30,30)
                           (6,6)
  Remove Retlexive:
    Remove: { (1,1) (2,2) (3,3) (5,5)(6,6) (10,10) (35,15) (30,30)}
  Remove Transitive
         (1,2) ER (2,6) ER then (1,6) ER
                                                          WHAT STA
         (1,3) ER (3,15) ER then (1,15) ER
         (1,5) ER (5,30) ER then (1,30) ER
                    (6130) ER then (2130) ER
         (2,6) ER
                    (6,30) ER them (3,30) ER
         (3,6) ER
                    (10130) ER then (5,30) ER
   P. & (112) (113) (115) (110) (216) (2,10) (3,6) (3,15) (5,10) (6,15) (6,30) &
          (5,10) ER
                                        1, 2, 3, 4, 6, 9, 12, 18, 136
      (10130) CIS, 30)
.: For every pair of the elements in the Ogiven Poset both was and GUB
           exist hence ween say that given poly is a lattice.
                                 I do braitsh statzagh
                                                 (= 11, 1, 30)
                          attrisibility relation defined
    R. ((1,1) (1,2) (1,1) (1,1) (1,30) (2,2) (2,30) (3,3) (3,30)(5,5)
    A lattice is a partially ordered set (L, <), in which every
lattice
pair of elements such as (a,b) = L as a greatest lower bound (G/LB)
and a least upper bound chilists) (2,2) (8,8)(5,2) (1,1) } . gyomes
 LUB
   The CUB (supremum) of a subsettaiby CL is denoted by
  avb (01) aublina &b is called the sum of a and b con joint.
 GLB.
  The GLB (infimum) of a subset (a,b) cl is denoted by
```

and (or) and (or) areb is called the product of a and b or

Check wheather the given poset of a hassie diagram is lattice or not

	LOB .	(3)	2	3	6
ida	net ros	-,1,	2 ,	3	4
	2	2	2	6	6
	3	3	6	3	TG (0
	6	6	6	6	6

1 112 - 61 7 10 1 61 7 16 - 511 1
13/01/3/3/13/13/13/13/13/13/13/13/
(5/2) (5/12) (5/12) (5/12) (0/5) (6/12)
(3)
Remove Reflexive:

PERMOVE: ( (111) (212) (213) (212) (213) (212) (213)

Permove Tagnostifive

GUB Table	9	
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GLB	1	2	3	6	
t	ţ	ţ		1,	
2	,	2	ι	2 =	
3 (	End.	(12)	-3(	3	
6	•	2	3	6	

(2,6) (3,6) (4,6) (1,10) (2,6) (2,6) (3,6) (3,6) (4,6)

.: For every pair of the elements in the given poset both LUB and GLB

exist hence We can say that given poset is a lattice.

Let  $L = \{1, 2, 3, 5, 30\}$  and R be the relation "is divisible defined on L show that L is Laticee or not?

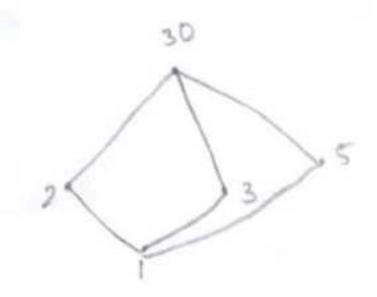
Given L= {1,2,3,5,304

R= is the divisibility relation defined on L oreo

R. ((1,1) (1,2) (1,3) (1,5) (1,30) (2,2) (2,30) (3,3) (3,30)(5,5)
(5,30) (30,30)4

(a Remove Reflexive) respect of (a,b) = L as ordinated to the series of elements such as (a,b) = L as ordinated to the series of elements such as (a,b) = L as ordinated to the series of (1,1)(2,2)(3,3)(5,5)(30,30) }) sound a least upper bound of (30,30)(2,2)(1,1) }, syomes

Remove Francitive 21 12 pd10 ) tozdoz o to (mumo qua) aus out
toici, () to (2730) & Romo (1/30) & R



GLB P	ioud lo	. 201	3	25	30
1	•	1	1	1	1
2	1	2	1	1	20
3	t	t	3	1	3
. 5	1	t	t.	5	5/-
30	(	2	3,	5	30

et (L.S.) to a Inttice tox (c	
- programme committee a	
ar are and ara se	
Little of the first transfer	
valadyn han and dan	ú
group on asidai, 2226.	(3

	503	0.5	1. 7.8	441	
LUB	1	2	3	5	30
1	1	2	3	50	30/-
2	2	2	30	30	30
3	3	30	3	30	30
5	5	30	30	30	30
30	30	30	30	30	30

ovidvo) bas (sad) ab = sa(dam) previous realization of an (avb) = a and av(axb) = a

for every pair of the elements in the given poset both LUB and GIUB exist. hence we can say that given poset is a lattice.

show that (D6,1) is a lattice or not (or)

show that D6, 1 that means factors of 6 is a lattice or not.

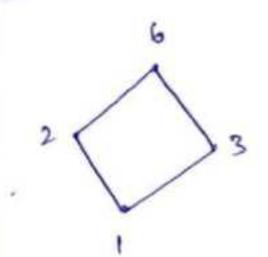
Factors of 6:21,2,3,63 R, {(1,2)(1,3)(1,6)(2,2)(2,6)(3,3)(3,6)(6,6)}

Remove Reflexive

Remove = e((,1)(2,2)(3,3)(6,6)}

remove mansitive:

(1,2) ER (2,6) ER (1,6) ER



properties of lattice.

let (L, <) be	α	lattice	for	(a,b,c)el, Lsatisfies	-tallacation 1 10
conditions:		the little	100	(apopular, L'satisties	Hollowing 31

1) Idempotant property.

ana = a and ava = a +ael

2) commutative property

anb , bna and avb , bva , tarbel

3) Associative property

(anb) nc = an (bnc) and (avb)vc = av(bvc) + cabic) & L

4) Absorption property

an (avb) = a and av(anb)=a

	2	£	2	1	901
<b>₩ Q</b> ,	b€	LE		ĭ	.1
	30	30	2	2	2
30	3.0	3	30	3	٤
30			30	S	2.
30	3.0	30	30	30	30

30

exists and recovery point of the song that it are property and to sing prove too

Crown afford the first and the country waste.

remine will be the first and the first threshold by it is all the books.

Factors of 6:21,2,3,63

R. (((,2)(1,3)(1,6)(2,2)(2,6)(3,3)(3,6)(6,6))

Remove Reflexive

Remove . ((1,1)(2,2)(3,3)(6,6))

tomove monsitive:

(1,2) ER (2,6) ER (1,6) ER

UNIT-TO

## Elementary combinatorics

Basics of counting.

testing 15 - Marie

The basics of counting are foundational principles in combinatorics that help determine the no of mays events (or) outcomes can occur. These principles are used to solve problems involving arrangements, selections and combinations

There are two tundamental rules cos) counting

1. sum rule ( Disjunctive rule )

2. product rule (sequential rule)

sum rule.

If there are two tasks and the first task can be done in m'ways and the second task can be done in n' mays, and the tasks cannot be happen at the same. time. then the total no of mays one of the two tasks ti cositz can be performed in mth ways

EXI: If there are 5 boys and 4 girls in a class then there are 5+4 = 9 mays of selecting one student as a class representative ceither a boy or agirl)

Ex2: suppose a hostel library has 12 books on maths, to books on physics, 16 books on computer science and Il books on electronics suppose a student wishes to choose one of these books for study: The no of inlays 2 in philich he can choose a book is 12+10+16+11=49 mays

product rule:

suppose that two task to and to be performed one after the other. If to can be performed in hi different mays and for each of these mays. to can be performed in no difficult mays or different mays. Then both of tasks trand to can be performed simultaneously in hin, hays

Ex. suppose a person has three shirts and five ties in how many manys a person can choose a shirt and a tle.

sol: The no of mays a person can choose a shirt and a tie is 3x5= 15 ways Explanation

let sississ be a shirts

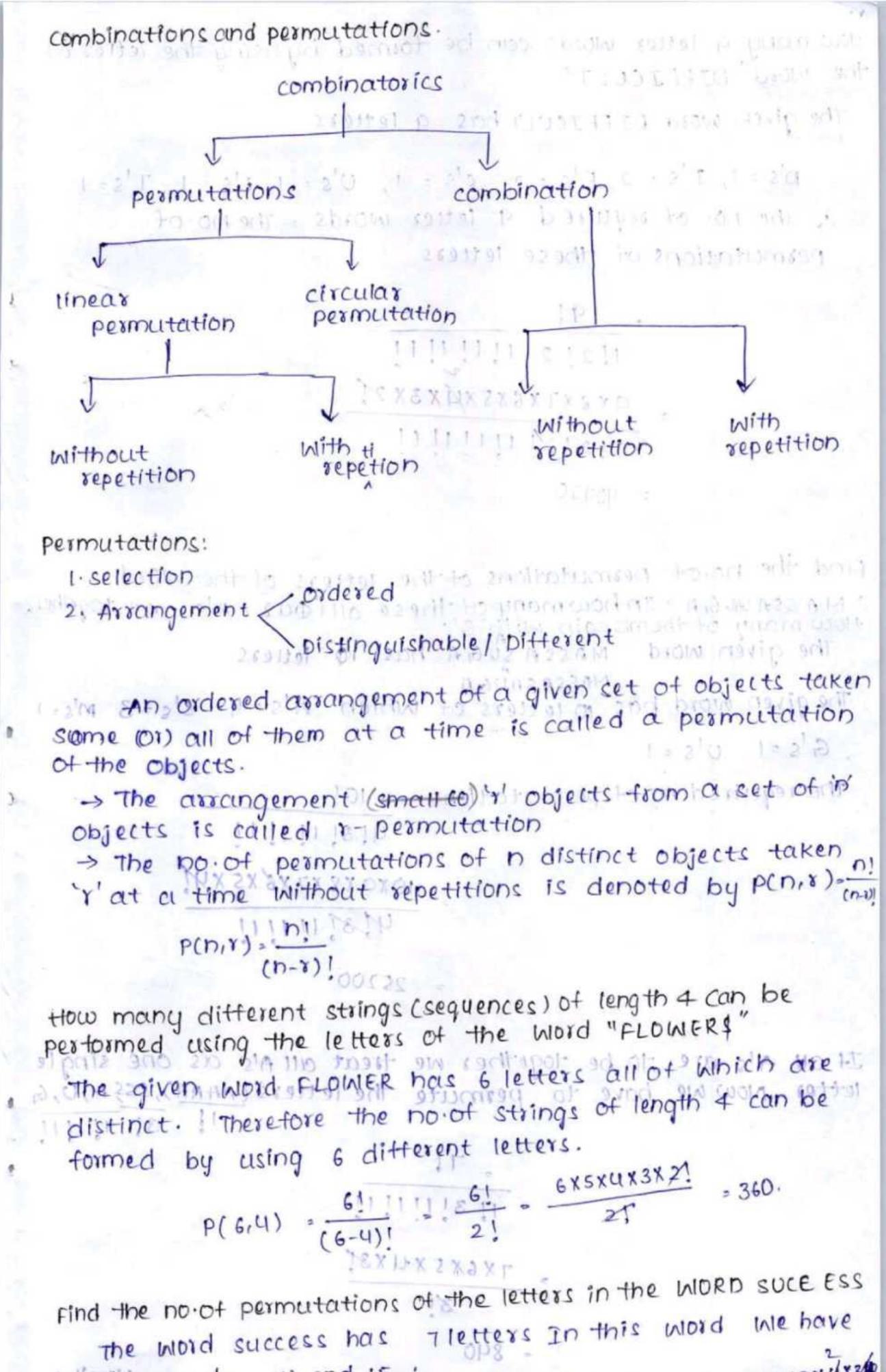
ToTz, T3, T4, T5 be 2 ties

S2 71 S3 71 S1 71 S3 T2 S2 72 SI TZ

S2 T3 SI T3

S3 73

In how many ways we construct sequence of 4 symbols (letters) in which the tirst two are english letters and next two are single digit numbers it 1. Modigit cor) Letter can be repeated 2. Repetition of letters and digits is allowed. We have 26 letters in english and 10 digits (i.e o to a) whe need to construct sequence of 4 letters in which first two are english letters and next 2 are single digit numbers. i) friend traitment pass to stude to building of 26 25 simmus. ad one with the act box 22010 and ass and to in 19 26 X 25 X 10 X 9 wint booss and bout appoint of anob It no digit or letter can be depeated then the no otion different ways = 26x25x10x9 = 58500 ways th (costs can be performed in mater ii) A. A. 9 9 strip to brue 2113d & orio erecit HE 1 X3 1026 26 10 900 postroplas to symbol o ++ 8 9810 989011-If repetition is allowed then the no. of different mays is eq: 26 x 26 x 10 x 10 = 67,600 mays. In how many ways can 3 different coins be placed in two Cyou can put first coin in any of the 2 purses it gives you two opportunities you can put tirst second coin any of the 2 purses it gives all byour two opportunities want seed to done not bon supply you can put third (coin any of the) apply the substitution of the Note: If you have in coins and 2 purses then the no of ways nooms be placed in 2 different purses 20 - 915sol: The no. of inlays a person con choose a shirt and a tie is 3x5 = 15 Mays asthmaday. 2+ 11d2 D ad 22 12 13 191 7, Tz, T3, Tu, Tc be 3 ties ST 12 ST 12



3 Sign 211cis 210 and 18 in areatt 2 attin aipped noited X6X5XU136

AA. How many 9 letter words can be formed by using the letter of the word" DIFFICULT" 201 softenucions The given word DIFFICULT has a letters D'S=1, I'S=2 F'S=2, C'S=1, U'S=1, L'S=1 T'S=1 .. The no. of required 9 letter mords = The no. of permutations of these letters 9! gaitatumeng 1; 2! 2! 1! 1! 1! 1! = QX8X7X6X5XXXXXXX Bir. 11/21/21 11 11 11 11 11 distant 00711) 400 Millbout Cidititoger = 90720 find the no. of permutations of the letters of the infording MASSASAUGIA" In how many of these allfour Als are together? The given word MASSA SUGA has 10 letters The given mord has no letters of which A's = 4 08's = 3 m's=1 signification of the most of the second . G'S=1 U'S=1 of the objects. the required no of permutations = 10,190000000 off moit of the tip strate of stratego in day of permitted to to additions to depoted by PCDIS (1977) A181 114111 How mound different staings (sequences) of length 4 can be It all A's are to be together me treat all A's as one single to permette the letters CAAAA), SSS M, U, G letter now we have formed by using 6 different letters. P(64) (6-4)! 21 - 25 TXGX5XUX3T thing the never permutations of the letters somble bitters some to an extreme avad sini broin sint of examinate 2001 2200000 broin of For permittation begin with s there are a places to till with the

```
ithe no of permutations begin withs = 91
                                                                                                       4! 2! 1! 1! 1!
        FRED FX LD FX Drope (x) A ad of benifest 41,2 letter in protosories
                                                                                                          = 7560
Find the value of n so that P(n12) = 90?
                                                                                                                                                                              27214
      The sequence and proton care be denoted blopen and
                                                                ( .... no .... no , no , no , no .... on ) is
          - n! = 90
               (n-2)!
                                                                                                                                Umportunit Francisco
          -> n(n-1)(n-2)! =90
                                                                       "x 3 - + "x + x + 1 - " / x - 1 ) - - - 1
                      (n-z)!
                                                              TATA STATE OF A DEATH TATE OF A STATE OF A S
            => n^2 - n = 90
              -> n2-n-90 =0
              -> n2-10n+9n-90=0, 3 --- + 2 x x + x -1 - (x +1) = - .2
                => n(n-10) + 9(n-10) = 0
                 5. 6-10° b= -33 - 1-55+38 - 1-55+38 -1-6 (XH) = -1-6 (XH)
                    - n=10
                                                                                                     6 (1-2) - E c(n+2-1, 2) x3
                                                                                           (1+x)"- 3 (-1) c (11+x-1, 1) x
                                                                                                                    T (HEXTY & COMB) of
                                                                                                           (1-x) = = (-1) c(h,3)x3
                                                                        (+ 2+3 + --- +h - n(n+1)

1 + 2 + 3 + --- +h 2 2 2 2 2 2 2 2 3 4 1)
                                                                                [(1+0) - 60+ --- + 2 + 2 + 5 + 5 + 5
                                                            9. atartart....tar". all-10" it 1 < 1
```

2

## 4 Recurrence Relations

Generating functions of sequence

Let  $A = \{a_n\}_{n=0}^{\infty} = (a_0a_1, \dots, a_n, \dots)$  be the sequence of then its generating function is defined to be  $A(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n + \dots = \sum_{n=0}^{\infty} a_nx^n$ 

## Note:

The sequence an: n=otox can also be denoted by fany or (an) or <an> or {a1} or {a1} or {a1} or {a21, ......an, ......}

## Important Formulas

1. 
$$\frac{1}{1-x}$$
 -  $(1-x)^{-1}$  =  $1+x+x^2+\cdots$   $\sum_{i=0}^{\infty} x^i$ 

2. 
$$\frac{1}{1-\alpha x} - (1-\alpha x)^{-1} - 1 + \alpha x + \alpha^2 x^2 + \dots \leq \alpha^3 x^3$$

3. 
$$\frac{1}{1+x}$$
 =  $(1+x)^{-1}$  -  $1-x+x^2+x^3+ - ... \times (-1)^{2}(x)^{2}$  |  $(-1)^{2}(x)^{2}$  |  $(-1)^{2}(x)$ 

$$8 \quad 1+2+3+----+h \cdot \frac{n(n+1)}{2}$$

$$1^{2}+2^{2}+3^{2}+----+h^{2} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$1^{3}+2^{3}+3^{3}+----+h^{3} \cdot \left[\frac{n(n+1)}{2}\right]^{2}$$

9. 
$$a + ar^2 + \dots + ar^{n-1}$$
,  $a(1-r)^n$  if  $r < 1$ 

$$\begin{cases} a(1-r)^n & \text{if } r < 1 \\ a(1-r) & \text{if } r > 1 \end{cases}$$

n 1-2

```
11. e^{x} > 1 + \frac{x}{1!} + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \cdots
                                                                                                                                                 REDIED
 1. Find the generating functions for the sequence (1,1,1,1,1,1--)
                                                                              all explanations of the property of
      Given that
             ao=1, a1=1, a1=1, a3=1, ---- 0+ x 10 + x 10
... The generating function is
                          A(x) 2 a0 + a1x + a2 x2 + a3 x3 + ----
                                     = 1+1x+1x2+1x3+----
                                      = 1+ x + x2 + x3 + . ----
                                       · (1-x)-1
  2. Find the generating function for the tollowing sequences
                                                                                            P = 20 - 10 9-1
          (i) 1,2,3,4 - ---
                   ap=1, a,=2, a,=3, a3=4 -----
             Given
            .. The generating function is
                             A(x) = a0 + a1 x + a2 x2 + a3 x3 + ----
                                         = 1+2x+3x2+4x3+---
                                          = (1-x1)-2
2
          (ii) 1,-2,3,-4 -....
                     Given
                             ap=1, a,=-2, a2=3, a3=-4---
                                                                                                                                                   GIVED
                  .: The generating tunction is
                               A(x): ao + alx + a2x2+ a3x3+ patters asp setti
                                           = 1= 2 x 1+13 x2 - 4x3 - x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = + x = +
                                           = (1+x) -- - + x+ x0+x+1 =
         (iii) 0,1,2,3
                                                                                               - (1-x) - x2
                   Given
                             a_0 = 0, a_1 = 1, a_2 = 2, a_3 = 3
                        : The generating function is
                                     A(x)= a0 + a1x + a2x2 + a3x3 + - ...
                                                  - 0+ 1x + 2x2+ 3x3 -----
                                                          21+2x2+3x3 ----
```

```
IV 0,1,-2,3,-4----
  Given
   a_0=0, a_1=1, a_2=-2, a_3=3, a_4=-4
   i. The generating function is
    A(x), ao+a,x+a,x2+a3x3+ayx4-----
        = x (1-2x+3x2-4x3)
         > x(1+x)-2
  fany = 644
  i.e a = a = a = a = 4
                                        CHIVELD
  i. The generating function is
      A(x)= a0+a1x+a1x2+a3x3+a4x4----
                              Builtine dend aut. "
         = 4+4x+4x2+4x3------
         · 4(1+x+x2+x3----)
         = 4(1-2)-1
· Find the generating tunction for the following sequences.
   1,1,0,1,1,1 ---.
                                      GIVED
                  Op 1, T, = 2, d, = 3, Ol - 1
  Given
     ao=1, a1=1, a2=0, a3=1, a4=1, a5=1=== 9dT :
   :. The generating function is to the form
      A(x) = a0+a1x+ a2x2+a3x3+a4x4 ----
          = 1+x+0x2+x3+--- (x+1) =
          , 1+x+0+x3+ ---.
          - (1-x)-1-x2
Kr.
        generating function for the sequences
   an = 9,5" nzo . - - 21 moitant politoranap ant ...
       A(2) - & an 2 hert 1 10 + 20 - (10)
                 -- 13x + 3x + x1 +-
                a shan
```

$$q = (5x)^{\frac{1}{2}}$$

$$= q(1+51+(5x)^{2}+(5x)^{3}+\cdots)$$

$$= q(1+5x)^{-1}$$

$$a_{1}=q_{1} + x_{2}$$

$$= x_{1} + x_{2}$$

$$= x_{2} + x_{3}$$

$$= x_{1} + x_{2} + x_{3}$$

$$= x_{2} + x_{3}$$

$$= x_{1} + x_{2} + x_{3}$$

$$= x_{2} + x_{3} + x_{4}$$

$$= x_{1} + x_{2} + x_{3}$$

$$= x_{2} + x_{3} + x_{4}$$

$$= x_{1} + x_{2} + x_{3}$$

$$= x_{2} + x_{3} + x_{4}$$

$$= x_{1} + x_{2} + x_{3} + x_{4}$$

$$= x_{2} + x_{3} + x_{4}$$

$$= x_{1} + x_{2} + x_{3} + x_{4}$$

$$= x_{1} + x_{2} + x_{3} + x_{4}$$

$$= x_{1} + x_{2} + x_{3} + x_{4}$$

$$= x_{2} + x_{3} + x_{4} + x_{4} + x_{4} + x_{4} + x_{4} + x_{4} + x_{4$$

```
Find the sequences generated by the following functions
(1) (2+x)3
 (x+4)"- nco yoxn+ nc, y'xn-1+nc2 y'xn-2 + --- +ncn yhxo
 A(x)=(2+x)3
     = 3c_0 2^0 x^3 + 3c_1 2^1 x^{3-1} + 3c_2 2^2 x^{3-2} + 3c_3 2^3 x^{3-3}
     1+x^3+32.x^2+3.4x+1.8.1
      = x^3 + 6x^2 + 12x + 8
      = 8+12x+6x2+x3
      a0=8, a1=12 a2=6, a3=1
 The sequence of the function (2+x)3 = 8,12,6,1 ----
il)(2+x)4
exty) = nco yox + nc, y' x n-1+ nc, y2 x 2 x n-2+ - + ncn yn x
  A(x)= (2+2)4
  =402 x4+40,21 x4-1 + 40,22 x+40,3 2 x +
                      4cy 2 x ...
    = 1.x4 + 4.2.x3 + 6.4.x + 4.8. x + 1.16.1
    = x^4 + 8x^3 + 24x^2 + 32x^4 + 16
    = 16+32x+24x2+8x8+x4
     ao=16, a1:32, a2=24, a3=8, a4=1
 The sequence of the function (2+x)4 = 16,32,24,8,1.
ifi) 2x2 (1-x)1
      2x^{2}(1+x+x^{2}+x^{3}+---)
       222+2x3+2x4+2x5+----
    a0=0, a1=0, a2=2, a3=2 ----
 The sequence of the function is 0, 0, 2, 2, 2, 2 ----
```

```
iv) 3x3+ e2x
                             THE REST AND THE REST OF THE REST OF THE REST
       3 \times \frac{3}{1!} + (1 + \frac{2 \times 1}{1!} + \frac{(2 \times 1)^{2}}{2!} + \frac{(2 \times 1)^{3}}{3!} + ----)
       1 + 8x + 2x^2 + \frac{(2x)^3}{3!} + 3x^3 + - \frac{1}{3!}
       1+2x+2x^2+\frac{13}{26}x^3+\cdots
       ao=1, a,=2, a, =2 as= 13 ----
   Find the co-efficient of x27 in (x4+x5+x6+.---)
        Given
            (x4+x5+x6 ----)5
         (x4)5 (1+x+x2+---)5 01 x(01,01+1)3
     101x11 x20 (1-x) 15 = (01,111) = 21 1 x to 10101 111100 out
      x 20 5 (C(n+x-1, x)x
           x20 & C(5+8-1, 8) x 3
               & c(u+r, r) x x x 20
               8=0
                               20 tr
               £ c( u+x, x) 2
                                                        Given
               8=0
            put r=7 i.e z 27
- C(4+7,7) x
          .. The coefficient of 2 is occording
                                                        0-8
                     C(11,1)
                  => (11-7))7!
                                     x (x (n-x)! x
                 => _11 x 10 x 9 x 8 x 75
                         417
                   The coefficient of 2 to coefficient of 2 is
```

```
Find the co-efficient of x10 in (1+x+x2+----)2
      (1+x+x^2+---)
 Given
    (1-x)-2
     \leq c(n+r-1,r) x^r
     E C(2+r-1, r) x

    C(1+8,8) x<sup>8</sup>
    C(1+8,8) x<sup>8</sup>

    put Y = 10
        C(1+10,10)x10 (---+*x+x+1) 2,4x
   The coefficient of x 10 is c(11,10) = 10! 11x10!
                     K(6 (11-10)] 10] or 1! 10;
                     F ( F 1 - F = 17 3 17 50 W
find the co-efficient of x10 in
                     E C( U+3, x) 3
  Given
    -3
     E C(n+r-1, r)x 21 12 x 10 troisitteos odt:
       c(3+x-1, x) xx
    x c(2+7, x) x 1 [[[-11]
                     JEXBXDXOIXII <=
    820
   put Y= 10
       C(2+10,10) x 10
                                        12 X 11 X 101
  The coefficient of 210 is c(12,10): -
                                         2/101
```

```
Find the coefficient of x32 in (1+25+219)10
 Note:
 1. The coefficient of x"y 12 in (x+y)" is _n!
               Where 7,472 = D
 2. The coefficient of x_1^{11} x_2^{12} x_3^{13} - x_1^{14} in (x_1 + x_2 + \dots + x_4)
    15 x | 82 + 83! - 8+! 81+82+83+84 --- 8 = D.
      The co-efficient of (1^{x_1}(x^5)^{x_2}(x^9)^{x_3} in (1+x^5+x^9)^{10} is
          \frac{10!}{\eta_1! + \eta_2! + \eta_3!} Where \eta_1 + \eta_2 + \eta_3 = 10
  ant a get co-efficient of x32 hie take
    2)30ivara 21x, 200-(12+73) sao or sampas so to mest "a
               11=10-(2+3) --- 10, 0,00 20 20 dous 2000
               11=5
                                          (10)
  The coefficient of x 32 in at 10+ dottolar amorniage A
  of the previous teams! Eligible sequence handled an air as
                                 3/12/13/200127 200Ward aut to
                                  10 x 9 x 8 x 7 x 6 x 8 1 1 50 40
     Note: Reconstance. Refetississ also called difference equation
   Exis If an denotes no tengen of a general progression
 Find the coefficient of 225 in (1+23+25) nommon ditin
argust and enallest subscript appears in the training on one
                           Where 11+12+13 7100-1
             10!
           11 1 + 12 1 + 13 !
      To get coefficient of x25 we take 1960
                 11 = n - (12 +83)
```

the co-efficient of x25 in.

(1+x3+x8)10 = 10! (18 - 5 | 3! 2! mai 17-900 BOT .

10 x q x 8 x 7 x 6 x 5 1.

51 31 2!

5040 + minimum 00 90T

2 2520.

Recurrence Relation painting is to thingities of the or. A recurrance relation is a formula that relates the n'n term of a sequence to one or more of its previous terms such as ao, a, a2 ---- an-1 (01)

A recurrance relation for the sequence ao, a, a, a, and is an equation that express an in terms of oneor more of the previous terms in the sequence namely do, a, d2an-1 for all integers n >1

Note: Recurrance Relation is also called difference equation Ex1: It an denotes not term of a general enterior progression with common ratio r. then an = r (an-1)

**XXXX** Order of Recurrance Relation

The order of recurrence relation is the difference between. largest and smallest subscript appear in the relations

Ext an - an-1 = 0 n-(n-1) = 0+ (++) = 9 19 (1) 128 + 108 + 118 M-N+1=0

TO get coefficient of x st vie talle rabro

cinear recurrance relation: (88+ 87) - 11 - 17 suppose n and k are non-negative integers. A relation for n≥ K

where ci(n), (2(n), .... ck(n) are functions of n and fin) also constants is said to be linear recurrance relation.

The state of the s

- 1. If cocn) and ckcn) are not equal to zero then the relation is said to be linear recurrance relation.
- #2. If fn=0 in eq 1 then the relation is said to be homogeneous linear recurrance relation

EX! an-an-1+2an-1=0

3. It t(n) = 0 in eq 0 then the relation is said to be non-linear homogeneous recurrance relation (or) non-homogeneous recurance relation

ex: an -an-1 + 2an-1 = 8 00 3ld Dups on 120 +139

the recurrence relation for = fn-1 + Fn-2, n>2 with Fibonacci Recurance Relation.

initial conditions to =fi=1 is known as Fibonacci

Poccurrance Polation. Reccurrance Relation.

\*methods to solve Recurrance Relation.

There are 3 methods to solve the recurrance relations they are

\* 1. substitution method + sp sp an app ai gent the

2. Generating function method.

3. characteristic roots method.

substitution method.

In this method the recurrance relation for an is used repeatedly to solve for a general expression for an in terms of n.

O solve the recurrance relation an = an-1+f(n) for n >1 by using substitution method; (Ha)

Given

 $an = an - 1 + f(n) \rightarrow 0$ 

put n= 1 in eqn O we get,

a1 = a0+ f(1)

putn=2 in eqn 1 we get = E+ 1-00 = 0

a2 = a, + + (2) + 3p sw (1) rips ni 1=1 +ug

GIVED

Put n=3 in eqn () we get 1 50 651 han a3 = a2 + +(3) and (13) 2 ao+ f(1) +f(2) +f(3) acitoles. Similarly an = a0+fu)+f(2)+f(3)+-----2. solve the recurrance relation an = an-1+n2 Where 90=7 by using substitution method. Given maltolar samprin are automapomal smanil-non an = an-1 + n2 - or Ossans sas sussas pomod - don put n=1 in ean O we get - 1-ans + 1-an-an :x3 Fibring in the second of the s dring ar = ao + 112 put n=2 in eqn D we get 2001 miera to solve the start error e error error az = 7+12+22 SUD DAGIL put n=3 in ean @ we get bodiem noitutitaduz.1" 2. Grenerating Function method. 3. characteristic roots method. 2+ 10 = 80 beau 21 ap 101-7+12+22+32

This method the recentioned relation 15 used d to amost 27+1<sup>2</sup>+2<sup>2</sup>+3<sup>2</sup>+--··  $> 7 + \frac{n(n+1)(2n+1)}{n(n+1)(2n+1)}$ the recurrance relation of an -1+3 where an = 1 by using substitution method, top ow o apo at 1 - a tug Given (1) + + 00 = 10 90=1 an = an-1+3" -> OP 9W Onpeni senting put n=1 in eqn (s) ++, D=, D

put 
$$n=2$$
 in eq  $0$  wile get

 $a_2 = a_1 + \frac{1}{3^2}$ 
 $= [+3^1 + 3^2]$ 

put  $n=3$  in eq  $0$  wile get

 $a_3 = a_2 + 3^3$ 
 $= [+3^1 + 3^2 + 3^3]$ 

similarly

 $a_1 = a_0 + \frac{1}{1-3} + \frac{1}{1-3} + \frac{1}{1-2} + \frac{1}{1-2} + \frac{1}{2-3} + \frac{1}{3-2} + \frac{1$ 

```
solve the recurrance relation an = c (an-1)+fin) for n > 1
                                                                                                                                                                   an-1+2,0.3
                   GIVED
                                                                                                                                                                     E 1.18 +1 .
                               an=c(an-1)+fin) ->0
                                                                                                                                         PUT DES in eg ( Mie get
                    put n=1
                               a1 = ((a0)+fu)
                                                                                                                                                                       03 = 01 + 3
                      Put n=2 (a1)++(2)
                               a2 = c(((a0)++(1))++(2)
                                                                                                                                              1 3 + 3 + 3 + 3 + 3 *
                     PILL 0 = c'aota(1)++(2)
                                                                                                                                                                                                               similarly
                               a3, c(a2)++(3)
          cimility.cccdao+cf(1)+f(2))+f(3)+ E+E+OD=nD
                    a_{n}, c^{2}a_{0}+c^{2}f(1)+cf(2)+1+(3) = +\epsilon+1

a_{n}, c^{n}a_{0}+c^{n-1}+(1)+c^{n-2}f(0)+--+1cf(n-1)+f(n)
      ·an: chao+ & en-8+(8)
solve the recurrence relation an = an-1 + n.3", ao=1
             Given
                               a_n = a_{n-1} + n \cdot 3^n \rightarrow 0 (1) - 1 + 1-00 = 0
          Put n=1
                                                                                                                      put bel in eqn ( nie qet
                            a, = a0 + (1) 3
          Put n = 2 1 + (D. 31 5.1)
                                                                                                                    put n=2 in eqn (
                           a, = 1+(1).31+(2).32
           put p = 2^3 \frac{1}{5 \cdot 1} + 00 = 8 \cdot 0 \frac{1}{3} + 1 \frac{1}{(1+c)} + 10 = 0
                                                                                                                                                      Put n=3 in eqn (1)
            similarly
                                   an= 1+1.3'+2.32+3.33+--++ 1.3" = 0
    solve the recurrance relation a_{n} = a_{n-1} + \frac{1}{2} + \frac{1}{2
              1+Given + + + + + + + + + + + + + + + + + =
                                                an = an-1 + 2
                                                                                                                                             THO +6 -
                             put n=
```

$$a_{1} = a_{0} + \frac{1(2)}{2}$$

$$put n_{2} = a_{1} + \frac{2(2+1)}{2}$$

$$= a_{0} + \frac{1(2)}{2} + \frac{2(3)}{2}$$

$$= a_{0} + \frac{1(2)}{2} + \frac{2(3)}{2}$$

$$= a_{0} + \frac{1(2)}{2} + \frac{2(3)}{2} + \frac{3(4)}{2}$$

$$= a_{0} + \frac{1(2)}{2} + \frac{1(2)}{2} + \frac{1(2)}{2} + \frac{3(4)}{2}$$

$$= a_{0} + \frac{1(2)}{2} + \frac{1(2)}{2} + \frac{3(4)}{2} + \frac{1(2)}{2} + \frac{1(2)}{2} + \frac{1(2)}$$

Given

$$a_1 = 2a_{1-1} + 1 \rightarrow 0$$
 $a_{i+1} = 2a_{1+1} + 1 \rightarrow 0$ 
 $a_{i+1} = 2a_{i+1} + 1 \rightarrow 0$ 
 $a$ 

Characteristic Rootmethod.

Let antcian-i +czan-z + - - - + czan-z =0 -> 0, n=K ck +0 be a linear recourrance relation of degree k

- 1. Then the characteristic equation is  $t^{k+1}+c_1t^{k-1}+c_2t^{k-2}+c_2t^{k-2}+c_2t^{$
- 2. since eqn @ is of degree k find 'k' roots

  Let d1, d2, d3 --- dk are the roots of eq2) in characteristic

  roots are

  - 2) If did --- dk are equal roots then an=(ci+cin+
    cin² + --- + ck·n k-1) dn is the solution of eq. (D)

The sense of then an = x (cicosno

solve the reccurrance relation an - 3an-1 +2an-2 =0 for n > 2 using characteristic root method. q = (a +) : - (a - +) + Given an - 3an-1 + 2an-2 = 0 ->0 equation of is the second order homogeneous reccurrance ADDRESS ALLOCASODERON 1-5 GORALIOS TORREDOS SCIT. relation The characteristic equation is  $t^2 - 3t + 2 = 0$ t2 2t-t+2=0 Put he o in como we get t (t-2) -1 (t-2) =0 000 01760 3 - 11 17 to = 10 t=1, t=2 The roots are real and distinct :. The general solution of homogeneous reccurrance relation is  $\left|a_n = c_1 l^n + c_2 l^n\right|$ solve an-6an-1 + 9an-2 =0 - (1) aps bas (3) aps paivios eqn (3) X 2 - 2014 20/2 = 10 Given  $a_{n}-6a_{n-1}+9a_{n-2}=0 \rightarrow 0^{-1}$ eq 0 is the second order homogeneous reccurrance the characteristic equation is relation t2-3t-3t+19=60 aps ai 2911/101/ 12 9/10/1/20102 t(t-3)-3(t-3)=0 t=3, t=3 the roots are real and equal .. The general solution of homogeneous reccurrance relation is [an=(c1+c27)3 Qn: 7(5) + 3(2) solve the reccurrance relation an- Tan-1 + 10 an-2 = 0, n>2 with a0=10, a=41 by using characteristic root method. GIVED ean D is the second order homogenous receive ance relation

\*
$$\{t^2 - 5t - 2t + 10 = 0$$
  
 $+(t-5) - 2(t-5) = 0$   
 $t=5$ ,  $t=2$   
The two roots are real and distinct  
The general solution of homogenous reccurance relation is
$$a_n = c_1 + c_2 + c_3 + c_4 + c_5 +$$

Put n=0 in egn @ we get a0 = c1 + c2 = 0 ao = c1 + c2 = 10 → 3

putn=1 in eqn @ we get a,= 5c,+ 2c2 => 5c,+2c2=41->4)

1=1=1/+(5-1)+

Bolving ean 3 and ean (1)

egn 3 x2 = 2C1+26/2 = 20 Given (1) . 5C1 + 2/2 = 41 وم ال اله و ووروسط وعطوع المصوروك ووروس عود وع عود وع relation the characteristic equation is

C1=7

substitute ci values in eqn @ weget.

C1+C2 = 10 7+02 = 10

E=+.8-+ 93000 10012 010 100 100 100 2100 2100 9ctt the general solution of homogenerus

Cz = 3/1

an = 7(5)"+3(2)"

selation is (an = (c, + c, ))

Given

t(t-3)-3(t-3)=0

solve the reccurrance relation an = Tain-1 + 16 an-2 -12an-3 =0 for n 23 with the initial conditions a0=1, a,=4 & a2-8

Given

 $a_0=1$ ,  $a_1=4$ ,  $a_2=8$   $a_0=1$ ,  $a_0=1$ , a

equation o is the third order homogeneous rectumance

t3-7t2+16t-12=0 (a) Land (a) the particles t=2 23-7(2)2+16(2)-12  $2^{3}-7(2)^{2}+16(2)^{2}-12$  8-28+32-12=-26+26=0t=2 now t=2 is a root we can divide the cubic equation by t=2 using synthetic division 2013

t²-5t+6=0 + mps mi 29mlov 29 \$15 9tutilez t(t-3)-2(t-3)=0 t2-3t-2++6=0

The two roots t = 3,2 Is equal and another root is 3 is distinct root

put n=0 in eqno weget

$$a_0 = c_1 + c_3$$

$$c_1 + c_3 = 1 \rightarrow 3$$

put n=1 in eqn @ weget

$$a_1 = (c_1 + c_2)_2' + c_3 3'$$
 $a_1 = (c_1 + c_2)_2' + c_3 3'$ 
 $a_1 = 2c_1 + 2c_2 + 3c_3 \Rightarrow 2c_1 + 2c_2 + 3c_3 \Rightarrow 0$ 

CHANGE

6 = 3

put n=2 in eqn & we get

2 in eqn & we get

$$a_2$$
,  $(c_1 + 2c_2) 2^2 + c_3 3^2 - (c_1 + 2c_2 + 2c_3 = 8^{-1})$  (S)

 $a_2$  =  $uc_1 + 2c_2 + 2c_3 \Rightarrow uc_1 + 2c_2 + 2c_3 = 8^{-1}$  (S)

equation 1 is the second order books and in a moissupe ean (1) x 4 = 28c(1) +862+ 12c3 = 16 401 +8/2 +903 = 8 = 5++5-4

```
1 - 7 + 1 (6 t " - 2)
   solving ean 3 and 6
                                    to2 2 - T(2) + 16(2) -12
   ean 3 x3 = 3c1+3/3 =3
             C) (=) (-)
Nion t= 1 is a root but can itself 2+=121/pic equationably i=2
                               using synthetic division
                 C1=5
     substitute a value in eqn 3
             C1+ C3 = 1
                             0 2 -10 12
             5+ C3 = 1
               C3 = -4
      substitute c18 c3 values in eqn (4)
                                 12-3t-16=0
           201 + 202 + 303 = 4
                               1 (4-3)=0
           2(5)+2(2+3(-4)=4
  E 21 took 10 + 2 C2 4 = 12 0= 4 0 = 1 00 = + 2100 0 out out
                                       t=3,2=
                 2C_2 - 2 = 4
                                        took toottalb 21
                   2C2 = 6 - "E E) + "c(11) + (2)= 90
                    C2 = 3
             (5+3)(2)+(+4)(3) (5 = 1=8)+1)
                           bat w= 1 in edu @ Medet
solve a_n = (2a_{n-1} + -2a_{n-2}) for n \ge 2 Woth a_0 = 1, a_1 = 2
  Given
                           put has in ean @ wie get
        ao=1, a1=2
   (3) _ an= (2an-1-2an-2) -> (2) + c (24+10) ...
                  az= uci +8cz +9cz => uci+8cz
      a_{n-2}a_{n-1}+2a_{n-2}=0
 equation 1 is the second order homogenous reccurrance
    the characteristic equation for eq 00 is = 1 x 10 mps
  relation
```

Here a=1, b=-2, c=2 2 ± 1(-2)2-4(1)(2) 1=V-1 = 2± √4-8 = 2 + \-4  $\frac{2 \pm i \cdot 2}{2} = 1 \pm i$ : Gis is an = 8" (cicosno + cz sinno) 0 - tan-1 (4/2) = tan-1 (1/1), 71/4 where & = | Ltil  $=\sqrt{1^2+1^2}$ -> an=(V2)" Ccicos mx + c2sin mx) > 0 put n=0 in eqn @ we get ao = cicoso + c2 sin a c1+0 = a0 C1=1 put  $n=1 \Rightarrow a_1 = (\sqrt{2}) \left( c_1 \cos \frac{\pi}{4} + c_2 \sin \frac{\pi}{4} \right)$ =  $2 = \sqrt{2} \left( C_1 + C_2 + C_2 + C_2 \right)$ 2 = 1/2 (C1 /2 + C2 /2) 2 · 1/2 (C1+(2) = 2 K. T -> c1 + c2 = 2 The General solution is an  $=(\sqrt{2})^n(\cos n\frac{\pi}{4} + \sin n\frac{\pi}{4})$ C2 = 2-C1 => 2-1=1

tte

Non-homogeneous recumance relation.

Where +(n) +0

The general solution of nonhomogenous linear reccurrance relation is

$$a_n = a_n + a_n$$

an is the solution of non-homogeneous recturance relation which can be obtained by taking f(n)=0 in eq. the particular solution can be obtained on the nature of f(n) to determine the particular solutions whe use the following cases

case 1: If f(n) is in the form of f(n) = g(n). an where g(n) = botbint - --- + bm n<sup>m</sup> and if 'a' is not a root of a characteristic equation then the pasticular solution is in the form of  $a_n^{(p)} = A_0 + A_1 n + A_2 n^2 - --- + A_m n^m )a^m$ Note.

1.  $f(n)=2 \Rightarrow 2.1^n$  Here  $f(n)=g(n)=a^n$ Here a=1 is not a root and g(n) is a constant so the particular solution is  $a_n=A_0-1^n$  (ex)  $a_n=A_0$ 

2.  $f(n) = n \cdot 2^n$ . Here f(n) is in the torm of  $f(n) = g(n) \cdot a^n$ . Where g(n) = n and the a = 2, Here a = 2 is not a characteristic root and g(n) has upton then the particular solution is in the torm of  $a_n = (A_0 + A^n) \cdot 2^n$ 

case2:

1. solve the reccurrance relation an- $9a_{n-1}+20a_{n-2}=1$  by using characteristic roots method.

Given that  $a_{n-}-9a_{n-1}+20a_{n-2}\to 0$ 

eqno is a second order non-homogenous reccurrance relation.

.: The general solution of non-homogenous reccurrance

relation is the an ist an pomort and to mortille through out it

zi noitplar

H'+ "2,0 + "0,0 = 00

To find an ch)

The homogeneous reccurrance relation of ean O is

 $a_{n}-qa_{n-1}+20a_{n-1}=0$ 

the characteristic equation is

 $t^{2}-qt+20=0 \longrightarrow 2$   $t^{2}-5t-4t+20=0$  t(t-5)-4(t-5)=0  $t^{2}-6t+20=0$   $t^{2}-6t+20=0$   $t^{2}-6t+20=0$   $t^{2}-6t+20=0$   $t^{2}-6t+20=0$   $t^{2}-6t+20=0$   $t^{2}-6t+20=0$   $t^{2}-6t+20=0$ 

.: The roots are real and distinct.

The general solution of homogeneous reccurance relation is

 $a_n^{(n)} = c_1 4^n + c_2 5^n$ 

The homogeneous recourance relation of ean (9) and or of Given

 $a_n - qa_{n-1} + 20a_{n-2} = 1$   $a_n - qa_{n-1} + 20a_{n-2} = 1$ The character(stic equation is

Here f(n) = 1  $f(n) = 1 \cdot 1^n$   $f(n) = 1 \cdot 1^n$ 

This is in the form of f(n) = g(n), a(n) + f(n) + f(n) = g(n).

Where g(n) = 1 a = 1

Here since from eqn@ a=1 is not a root of the characteristic equation and gen) is a constant? so that other particular

TO THE PERSON OF substitute an= Ao in ean O A0-9A0+20A0=1 Frank as visa 12 AO -1 OF CADOS - ANT -40 esmontos 12 umbagomado as a cobre la redes o el mage relation .. substitute to value in pasticular solution STREET, The demand of the contraction of the contra an = /12 of dollar .. The general solution of non-homogeneous linear recurrance relation is an = an + an de las Officentes an = c, 4 + c25 + 1/2 the characteristic equation is solve the reccurrance relation ant 4an-1+4an-2 given 0=1, a, =2 (E) - 0 - 00 F + 10 - 1 D = 0 C + 311 - 32 - 5 Given  $a_{n} + 4a_{n-1} + 4a_{n-2} = 8 \rightarrow 0$ ean of is a second order nonhomogeneous recourrance .. The general solution is in to rolling in solution of an=an th) +an (P) 2) noitoise 15 5 + an 5 = (4) to find an (h): the homogeneous reccurance relation of ean Ois an+4an-1+4an-2=0 characteristic equation is something of the characteristic equation is 1 = (m) + every t2+4++4=0 ->0 +2+2++2++2++4/5.9 app = (11)++0 moof out ai 2i zidt + (++2)+2 (++2)=0 t = (copp gradual t = -2, -2 Here since from equal a=1 is not a robbook of oute entracteristic the generalion and and is a constant sison pointaine

To find 
$$a_n^{(p)}$$

Given

 $a_n + 4a_{n-1} + 4a_{n-2} = 8$ 

Here  $f(n) = 8$ .

 $f(n) = 8 \cdot 1^n$ 

Here since from eqn  $\emptyset$   $\alpha = 1$  is not a root of a characteristic equation and gcn) is a constant. So that the pasticular solution is

 $a_n^{(p)} = A_0 \cdot 1^n$ 
 $a_n^{(p)} = A_0$ .

 $a_n^{(p)} = A_0$ .

 $a_n^{(p)} = A_0$ .

And  $a_n^{(p)} = A_0$ .

2 = (-2) = -202 + 8/9

2-2+2-125

as brille or  $2C_2$ ,  $-\frac{2}{9} + \frac{8}{9} - 2$ - Given Sa contillation of the contillation -2+8-18 9 1 2 = 7 dot-Since there are to the control of th 2C2: -12 11-0A-1 (0) 2 C2 = -12 9 - 12 0A - (9) S = -42 3xx1 = -2 Brips of of an autitedus. substitute c, and c, in eqn 3 3 - QAP  $a_n = \left(\frac{1}{9} + \left(\frac{-2}{3}\right)n\right)(-2)^n + 8/9$ p/8 04 solve the reccurrance relation an-san-1 +6an-2 = 1 solve the reccurrance relation an-an-1-6an-2 = -30 with as i. The general collition is Given that egn Dis a second order non-homogenous reccurrance relation. an-5an-1+6an-2=1 -0 (9) on = (c1 + c2) (-2) + 8/9 = 21 xix. din to 2.p ant .: an = an tan (P) put noo in egn 3 Do=(CC1+C2)) (-2) + 8 | Totind an (h) D13+10=0D The HI. V. Y of egn ( is an = 5an-1+6an-2 =0 1 = p13 + 10 The characteristic equation is  $\frac{1}{p} = \frac{9}{8} - 1 = 10$ t2-5t+6 =0 t2-3t-2t-16=0 Put nel in egn 3 t (t-3)-2(t-3)=0 at = ((1) ((2)) + 8/9 t = 2/3 The roots are real and distinct plat soc - 125 - = c The gis of the R. R. 15 2 - (-2) -2 -2 (2 + 8/9 an(h) D+C3

```
to find an (P)
     GIVED
        an-5an-1+6an-2=1
     ttere fin) = 1
         f(n) > 1.10
 This is the form of fcn)= qcn).an
      Where gin)=1
             a=1
 Here since from eqn@ a=1 is not a root of the characteristic
equation angent is a constant so that the pasticulas solution is
            an LP) - Ao 1 "
            an = Ap
   substitute an= Ao in egn ()
          AO-SAO+GAO = 1
              7-AU = 1
                A0=1/2
   substitute to value in particular solution
             an = 1/2
  ". The q's of N.H.R.R
           = an = an +an (P)
          an= c12 + C23 +/
   Given
ean 1 is a second order non homogeneous reccurrance relation
     .. The general solution is
          an =an tan
 Totind an in):
     The H. 8.8 of eqn 1 is
          an- an-1-60 n-2 =0
```

the two roots are real and distinct.

The general solution is

$$a_n^{(h)} = c_1 3^h + c_2 (-2)^n$$
  
To find  $a_n^{(P)}$ 

Given

Herefin) = -30

+(n),-30.1 n

Here since trom eqn 10 00=20 is not a voot of a characteristic equation and gcn) is a constant, so that the particular solution , zi

, 16

anth), Ao

substitute an-Aoin can O

substitute to value in particular solution

.. the general solution is

put n=0 in egn 3

put n=1 in eqn 3 Edentification of the section of the section and the section and the section of t alminito 55361726255 anouneported for to nothing income out in (45 to to the do -5 = 3(15) - 262 - 5 -5 : 45 - 29 - 5 to find on 45-262-5 = 5 202 =15+400 s ascraverusses aussanopomaci a 21. Ompamora 20, 45 O= Late of too and Tine cheanttear the equation is C2 . US substitute c, and c, in eq 3 O- OIT IT -T an = 15 + 42 (-2) - 5 0 = 01 + +E - ra - + 0=(=1)=-(1+1)= 2,6-4 the works are real and distinct The general solution of the Honogeneous secturation 2 i noith algo an contantos (9) an brill-or CHEVED an-100n-1 + 100n-2 = 40 Here fin) 4 => +cn)= 1.4" no casp elast to most off mi 21 21dt Here a=q qunz=1 Since a=4 is not a root of characteristic equation qualis constant. so that the pasticular solution is ap A A CO Substitute ans 404" in eq ( AG-7A0 4 + 10 A0 4 = 47

solve the reccurance relation an- 7an-1+10an-2 = 4h

Given

eand is the second order non-homogeneous reccurrance relation in the general solution of non homogeneous reccurrance relationis

an = an +an (P)

To find an (h)

Fromegno is a homogeneous reccurrance relation

The characteristic equation is

$$t^{2}-7t+10=0$$
 $t^{2}-5t-2t+10=0$ 
 $t(t-2)-5(t-2)=0$ 
 $t=2,5$ 

the roots are real and distinct.

The general solution of the Homogeneous reccurrance relation is

Totind an (P)

Given

Here f(n): 4" => +(n): 1.4"

This is in the form of ton1 = genz. an

Here a = 4 gin1=1

since a=4 is not a root of characteristic equation gin) is constant. so that the particular solution is

substitute an= A04" in eq 0

Divide through 1, n-2

$$\frac{a^{m}}{a^{n}} = a^{m-n}$$

$$A0 4^{n-(n-2)} - 7A0 4^{n-1-(n-2)} + 10A0 4^{n-2-(n-2)}$$

$$A0 4^{n-n+2} + 10A0 4^{n-2-(n-2)}$$

$$A0 4^{n-n+2} + 10A0 + 10A$$

.. The general solution of nonhomogeneous reccurrance relation is

solve the seccurrance relation SK-3SK-1-4SK-2 = 4K Given

can o is the second order non-homogeneous reccurrance relation

.. The general solution of non homogeneous reccurrance relation is

To find an ch) from eqn® is a homogeneous reccurance relation

$$an - 3a_{n-1} - 4a_{n-2} = 0$$
characteristic
the equivition is

(1 m n +2 4 + + + + + = 0 + (+-4) +1 (+-4) = 0 11-2 - (n-1) +1 1.Ac 4 ((t-74)/(t-1)) -0 (1,47-1)) + + + 4,1 The roots are real and distinct The general solution of Homogeneous reccurrance relation is in OA (" 1an (h) = C14n + C210 = A04 - 7404 + 1CA040 4 to find an (P) 16A0 - 28A0 + 10A0 GIVED an -3an-1 -4an-2 = 40 -276 = 16 Here +(n) , 4" -> +(n) = 1.4" substitute An -- 8 in Presticetian scolution g(n) = 1 since 0=4 is not a root of characteristic equation gen) is constant 150 that the particular solution is off a not a = 4 an . Anun .. The general solution of nothermongling out reservations A04" - 3A04"- 4A04"-2 40 relation is through 4n-2 Divide  $\frac{du}{du} = \frac{1}{2u^{1/2}} = \frac{1}{2u^{$ 404/-14-5) - 3404 - 1-16/-1) . K-X-16/-2) - 446/-2 - 320-32  $\Rightarrow \alpha_n \cdot 3\alpha_{n-1} - \alpha_{n-2} = \alpha^n \rightarrow 0$ out the second order non-pointed exigents, have signed ation : the general solution of non homogeneous of and the general solution of non homogeneous of a od relation relation is substitute to =16 in particular solution. (P) (N) · 00 (4) - 40.49 · LI.Un  $a_{n} - 3a_{n-1} - 4a_{n-2} = 0$  (9)  $a_{n} + a_{n} + a_{n} = 0$ C.47+C. 17+16.47

solve the reccurrance relation an-6an-1 +8an-2 = n. un where Given a6=5, a=22 atil -at (-cr-dia an-6an-1 +8an-2 = n.4" -> 0 ean o is a second order non homogeneous reccurrance relation. the general solution for non homogeneous reccurrance relation an = anch) +ancp) To Find an Ch); Find the form From egn O is a homogeneous reccurrance relation  $a_{n-1} + 8a_{n-2} = 0$ The characteristic equation is  $t^2 - 4t - 2t + 8 = 0$   $\longrightarrow 0$ + 0A 400A) 3 + P [A+ 0]A ( - 0) = 0 + = 2, 4 The roots are real and distinct of + MAT-INTERIAL The general solution of the Homogeneous reccurrance selation tenast + "diate - sauct dones - diasit dones" = an = c127+ c24" all = alase - 18 = 148+ = 148+ = 148+ Totind an (P) 131 - 11A31+1A3+0A8 iven  $a_{n-1} + 8a_{n-2} = n \cdot 4^n \rightarrow 0$ Given This is in the form of fun; gin; an ttere fing: n.un Heregini= n a=4 oA = 0=1+ 0A since from ean @ a=4 is a root of characteristic equation with multiplicity q=1 substitute An = -1 and A (= 1 in pasticulas solution gen) has upton an - (App + App 2) 40 The particular solution is an = n (40 + A, n) y n (41+ 1-) - (9) " ncn-1)4"

```
substitute an= (Aon+Ain2)4" in eqn(1)
 (Aon + Ain2)4"-6 (Ao(n-1) + Ai(n-1)2) 4"+8 (A6 (n-2)+
A1(n-2)2) 4 n-2 = n.4 n
                   Openium and and and
 Divide through 4n-2
(Aon+Ain2)4" 6 (Ao(n-1)+Ai(n-1)2)4n-1
   4n-2
  4n-1+8 (A0 (n-2) +A1 (n-2)2) +4n-2 n.4n
         4 h-2 min 12336 3 100 10000 41h-2 31 10 100 11014
= (AON +AIN2) 4 (h2)
- 6(AO(+AI) (n2-2N+1)) 4 1/1-(1/2)
  +8[A0-+A0 +A1(n2+4-40)] 4 N-1 - N+2 = n.4 N-12
=(AON+AIN2)4 -6(AON-AO +AIN2-2AIN +AI]4 +8(AON-1 AO+
 AIN +4AI - 4AIN ] 4° = n. 45 air in ban loss 350 21506 941
= [16A6n+16A1n2 - 24A6n+24A0-24A1n2+48A1n+-24A1+88n-
16A6 + 8h12 +32A1 -32A1 = 16D
 = 8AO+8A,+16AIN=16N
                                        ap batter
= 8(A0+A1)+16A1D=16D
                                         GIVER
                        an-6an-1 +8an-2 = n.cl
   equating the coefficients
                          Here fing: D. a
              8(A0+A1)=0
    16 A, = 16
                Ao+A1=0 arot at ai 21 21dT
     A1 = 1
                 Heregan) = n a1-20A <= 0=1+0A
 since from ean @ a = 4 is a root of characteristic
    A0=-1, A1=1
                      equation with multiplicity q=1
 substitute Ao = - 1 and A 1 = 1 in posticulas solution
                              den) per mato u
     an = (Aon + Ain2) 4h
                            The pasticulas solution is
     an (P) = (-n+n2)4"
                      a h ( u + 4+ 0+) a - as
         · n(n-1)4n
```

```
an=c12n+c24n+n(n-1)4n -> 3
          sub n=0 in eqn 3
                                                                                        OF FOUND OF
            ao = C120+C240+0(0.1)40
                                                                                                               an built of
             a0 = C1+C2
                                                                                                                         713 VI A
              C1+C2 = 8 -> 4
                                                             tetons - fab = contate and an
        sub n=1 in eqn (1)
            a= c(2)'+c2(4)'+1(1.1)4
             a,= 2c,+4c2 "0.(a)p.(a)= (a)= 10 man mile (i) 21 21aT
             2C1+4C2 = 22 ->5
                                                                                     The Forticulos courtion (s
      solving 3 and 5
      eqn 4 x2 => 2/1 +2C2 = 16 - A-CAL
               5 x1 -> 2/c1+4c2=22
                                                e) e) on pa cii an aturitzana
      +(2-11)1A-0A-13+[1-120] =1-16(1-11)1A+0A) =- -1, A+-(1A+0A
                                                                                  A = (0-2) ] + 30 - 30 + 1
                                               c2 = -6 3
     subc, = 3 in eqn & C2 = 3 Teat 14-01 At 04 de de la late
                   C1+C2=8 015 - 08= [(0+-++0)
                   C1+3=8
substitue C_{i=5}^{-5} C_{i=5}^{-5}, C_{2=3}^{-3} C_{2}^{-3} 
                  a_{n-5}a_{n-1}+6a_{n-2}=3n^{2}-2n+1\rightarrow 0
        Given
   eqD is a secondorder non homogenous reccurrance relation
    The general solution of non homogeneous reccurance
      relation is + and the coefficients of the an + an + an theant of an an = an theant of the coefficients of the an + an theant of the an + an theant of the anti-
                                                                                            2AJO BOY
                           (6)
    To find an
   From ean O is homogenous reccurrance relation
                                                                                                 12 - 3/2
                        an-5an-1 +6an-2 =0
            The characteristic equation is not prize quation is
                                                                                  2A1-14A2 = -2
                           t2-5++6 =0
                                                                         2A1-(W)(3)0-2
                       t2-3t-2t+6 =0
                                                                                   2A1-21 -- 2
                          (t-3)-2(t-3)=0
```

```
the general solution of homogeneous reccurrance is
       an = C12"+C, 3"
                            2 + 3 + 0 ) 0 = 2 + 0 + 0 + 0 = 00
 To find an (P)
     Given
       a_{n-5}a_{n-1}+6a_{n-2}=3n^{2}-2n+1
     Here +(n) = 3n^2 - 2n + 1 = > (+(n) = 3n^2 - 2n + 1 \cdot 1^n)
 This is in the torm of f(n)= g(n).an
    Here g(n) = 3n2-2n+1, an=1 => a=1
 The particular solution is
                                    solving (3) and
         an (P) = A0 + A1D + A2D2 = 1 3 C+ 10 C C CX + 1109
                             SE DRITHER STIXE
  substitute an in ean o
   AO+AIN+A2 N2 -5 (AO+AI (N-1)+A2 (N-1)2]+6[A6AI (N-2)+
   A2 (n-2)2]=3n2-2n+1
 = AO+AIN+A2N2-5[AO+AIN-AI+A2(N2+2N)]+6[AO+AIN-2AI+A2
                      (n^2+4-4n)] =3n^2-2n+1
-AO+AIN+A2N2-5A6-5AIN+5A1-5A2N2-5A2+10A2N+6A0+6AIN-
12A1+6A2D2+24A2-24A2D = 3D2-2D+1
= 2A0+2A1D+2.A2D2-7A1+19A2-14A2D=3D2-2D+1 (19VI)
  (2AB-7A1+19A2)+2A1D+2A2D2_1UA2D=3D2_2D+10
                                     eg O is a second of
 2A0-7A1+19A2 +2A1N+2A2N2-14A2 = 3N22N+1
 2A0-7A1+19A2+(2A1-14A2) n+2A2 n2=3n2-2n+1
    comparing the coefficients n2 them + 10 = 10
          2A, 1 = 312
                                             no balt or
            Prom egn () is homogenous securitance RECIAS
   compasing the coefficients of n then 1-and-an
             A2 = 3/,
          2A1-14A2 = -2
                                 12-5++6 = 0
          2 A1 - (W) (3) = -2
           2A1-21 = -2
                             0-(8-1)-2(4-3)-0
```

```
Sed And 19 by our pd Dags to mest does Dags plaisting
       3 - 3-4 - 40 3 0 C+ 4 1-40 3 P-4,40 5
               E on x " + Ou + "x on + du a" x on 3
           x, D-0D-(- - +8x, D+ x, D+ x, D+ co)
                                 - ACE)- A0-CEX
                   E an-1 x - a, x + a, x + a, x + a, x - in E
                  (--+ xxxx+ xxx+xxx) x =
                   SOD-(-+ x x D+ x D+ x,D)x .
                                 -X(A(X))-OOX
                   - x (Acria,x+a,x+ - ....
                                      -x'(E((X))
 Generating function method + [100 - (1) H & anzh (0) - (1) A

Let the generating function be A(x) = & anzh (0)
                  I)A. XOCHIODP + antaix + dix + dix + ton = LXDA
 multiply each term of the sequence of reccurrance relation
  by x and taking the &
 > write each term in the form of ACX)
-> Find Acx) decompose Acx) into postial-fractions. it
    necessary (xp-1) op
 -> Express Acx) as a sum of series from this we can get
   An by equating the co-efficience of xnon
solve the reccurrance relation an-aan-1 + 20an-2 = 0
 an=3 a1=-10.
  Given
       an-9an-1 +20an-2 = 0 (x=0) & + (xxx-1) A - (x) A
```

multiply eqno each term of eqno by 20 and take the &  $\frac{d}{dx} a_n x^n - 9 \frac{d}{dx} a_{n-1} x^n + 20 \frac{d}{dx} a_{n-2} x^n = 0 \implies 3$  $\tilde{\xi}$  an  $z^n$  =  $a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$ = (ao+a,x+a,x2+a,x3+---)-ao-a,x = A(x)-a0-a1x  $\leq a_{n-1}x^n$ ,  $a_1x^2 + a_2x^3 + a_3x^4 + \dots$ = x (a1x+a2x2+a2x3+---) = 2 (a, x + a, x2 + a, x3 + - )-a6) = x (A(x))-aox  $\leq \alpha_{n-2} x^n$   $\alpha_0 x^2 + \alpha_1 x^3 + \alpha_2 x^4 + \dots$ · x2 (Ao + a, x+ a, x2+ ----. · x2 (A(x))  $A(x) = a_0 - a_1 x - 9 (x A(x) - a_0 x) + 20 (x^2 A(x)) = 0$   $A(x) = a_0 - a_1 x - 9 (x A(x) + 9a_0 x) + 20 (x^2 A(x)) = 0$ A(x)[1-9x+20x2]-a62a1x+9a6x=0100+ done plaitium by a cond taking the  $A(x)(1-qx+20x^2) = a_0+a_1x-qa_0x=0$   $A(x)(x) + a_0x + a$  $\frac{1}{A(x)} = \frac{a_0 + a_0 + a$ + 100 9W 21dt (1- 9x)+aix 10 mu2 1-9x+(20x2 22319)(3 <-(1-9x) (1-5x) A(x)= A(1-4x) + B(7-5x)0 = 1001+ 1-10P-10

 $\frac{x}{\epsilon}$  an  $x^n - n = 4^n x^n + B = 5^n x^n$   $\frac{x}{n = 0}$   $\frac{x}{n = 0}$   $\frac{x}{n = 0}$ Equating the coefficients of xn an=A4n+Bsn -> put n=0 in ean 4 THE THE PART PART OF THE a0=A(4)+B(5) L --- + x , o + x, o r ad ) x ap=A+B A+B=3->(5) (STA) X . Q, = A(4) + B(5) put n=1 in egn 4 4A1+5B = -10 ->6 (F)A X)3-11 (K)A Aca) an exact) = 0 solving 5 & 6 We get ean 5 x y = 4/A + 413 = 12 00- (x)-1)(x)A egn6 x1 = 4A +5B = -10 ACAD = Cto -B = 22 B=-22 (A) (A) (A) sub b=-22 in eqn 5 ACRIE A CHELL A+B=3 A-22 = 3 ACA) = A E TO 6 A = 3+22 A = 25 COUNT - WE XUE A = 25, B = -22 sub A and B values in (9) an= 25(4) -22(5) x anx = 25 & 4 x = 22 & 5 x x n solve the reccurrance relation an-6an-1=0 for nzorand ao=1 by using generating function method. Given Let the generating function be  $A(x) = \frac{2}{8} a_n x^n \rightarrow 0$ an-6an-1=0 .-> 0 multiply eqn & each term by xn and take the &  $\xi a_n x^n - 6\xi a_{n-1} x^n = 0 \rightarrow 3$ 

GIVED

NOW

$$= (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots) - a_0$$

$$+ A(x) - a_0$$

$$= (a_0 + a_1 x^4 + a_1 x^2 + a_2 x^3 + \cdots) - a_0$$

$$+ x(a_0 + a_1 x + a_2 x^2 + \cdots)$$

$$+ x(A(a_0) + a_1 x + a_2 x^2 + \cdots)$$

$$+ x(A(a_0) + a_1 x + a_2 x^2 + \cdots)$$

$$+ x(A(a_0) + a_1 x + a_2 x^2 + \cdots)$$

$$+ x(A(a_0) + a_1 x + a_2 x^2 + \cdots)$$

$$+ x(A(a_0) + a_1 x + a_2 x^2 + \cdots)$$

$$+ A(x) = a_0 - 6x A(x) = 0$$

$$+ A(x) = a_0 - 6x A(x) = 0$$

$$+ A(x) = a_0 - 6x A(x) = 0$$

$$+ A(x) = \frac{a_0}{1 - 6x}$$

$$+ A(x) = \frac{A}{1 - 6x}$$

$$+ A(x) = \frac{A}{1 - 6x}$$

$$+ A(x) = A = \frac{A}$$

MOIN

Given

Let the generating function be A(x),  $\stackrel{>}{\geq}$  an  $\stackrel{*}{\sim}$ multiply eand each term by in and take the & n= a tox  $z = a_1 x^1 = a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots$ · (ao+a,2+a222+a323+ --- )-ao-a,x FOR IN THE COLD OF ME GET = A(2)-a0-a12 · x (a1x+a2x2+a3x3----) book (3) 91/102 , x (a, x + a, x2+ a, x3+---)-ao IN = SUF Are x (A(x))- a0 x - x (A(2)) - x ao £ an-2 xn, ao x2 + a, x3 + a2 x4+ --- +2 , x2 (A0+a,x+a2x2+--...) n=2 subtitute these values in ean 3 A(x) = 00 - 0, x -7 (xA(x) - 00x) + 10(x2 A(x))=0 A(2) - a0-a1 x - 7x A(x)+7 xa0 + 10x2 A(x)]=0 A(x)-(1-7x+10x2)=00-7x06+01x B-10-3 = a6-7xa0+a,x A(X) 1-7x+10x2)+ (s)(s) = (D)  $= \frac{a_{0}-7xa_{0}+a_{1}x}{(1-2x)(1-5x)}$ + B 1-2 X A (1-2x) + B (1-5x) A(X) = A & 2 n x n + B & 5 n x n

n= 0

```
Equating the xh co-efficients
   an=A2n+B5n -> @ de pod meste masse pres plaitium
         put n=0 in eqn @ we get
                     an = A+B
                                                         THE PARTY OF THE RED TO STATE OF THE PARTY O
                      ab = A+B
                      A+B=10 -> (S)
        Put no1 in egn @ we get
                                                                                       Fire-grant Eld ...
                          Q1 = 2A +5B
                        2A+5B= 41→®
          solve (5) and (6)
 6 -> 2A + 5/B = 41
                                              3'A = 9+12 , n + 1 5 , D + 1 00 10 10 10 3
                                                 A- 43 + - 10,0 + x,0+ 0A3 x.
                                                 A = 3 Eaps at mount of statitute
             substitue A = 3 in egh 3-(x) A - x - 00 - (x)
                            3+ B = 10 L( E) A XOI + ODE F + (E) A KT - X 10 OD - (E)A
                                                        x10+00/01-00=('x01+x1-1)-(x)A
                       a_{h=}(3)(2)^{n} + (7)(5)^{n}
solve the general solution tox an using generating function
      an - 7an-1 + 12an-2 =0 With a0 = 10-a, = 41
         an-6an-1+12an-2-5an-3=0
         Given
                     an-7an-1+12an-2 = 01-30
     let the generating function be A(x) = & anx 1/20
    Multiply egn O each term by & and take n= 2tad ( & A
```

$$\sum_{n=2}^{\infty} a_{n}x^{n}, (a, x^{2} + a_{3}x^{3} + a_{4}x^{4} + \cdots) - a_{0} - a_{1}x$$

$$= (a_{0} + a_{1}x + a_{3}x^{2} + a_{3}x^{2} + \cdots) - a_{0} - a_{1}x$$

$$\Rightarrow A(x) = a_{0} - a_{1}x$$

$$\sum_{n=2}^{\infty} a_{n-1}x^{n} \cdot (a_{1}x^{2} + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{0} + a_{2}x^{2} + a_{2}x^{3} + \cdots) - a_{0}$$

$$= (a_{0} + a_{2}x^{2} + a_{2}x^{3} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{2}x^{2} + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{1}x + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{1}x + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{1}x + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{1}x + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{1}x + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{1}x + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{1}x + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{1}x + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{0}$$

$$= (a_{1}x + a_{1}x + a_{2}x^{3} + a_{3}x^{4} + \cdots) - a_{$$

Solve eqn s and 
$$\textcircled{S}$$

solve eqn s and  $\textcircled{S}$ 

eqn s xy .  $\forall A + \forall B - \forall D$ 

6 .  $3A + \forall B - \forall D$ 

6 .  $3A + \forall B - \forall D$ 

6 .  $3A + \forall B - \forall D$ 

6 .  $3A + \forall B - \forall D$ 

1A = -1

A =  $\frac{1}{1}$ 

. Substitute A = 1 eqn  $\textcircled{S}$ 

-1+B = 10

B = 10+1

Substitute B=11

substitute B=11

substitute B=10

G= (-1)(3), +(11)(4),

```
= x2 (a0+a,2+a2x2+a3x3+---)-a0+110+10+10
                        = x2 (A(X)) - XQO
              Ean-3 x = aox + a, x + a, x + a2x + a3x + x = x = 1 = 1 = 1
                                               = x3(a0+a, x+a, x2+a3x3+x0)
                                                 , x3 (A(X))
                 substitute these values in eq 3
                A(x)-00-0,x-0,x2-6(xA(x)-00x-0,x)+12(x1(A(x)-x00-5)
                          5 (x3 A(x)) = 0
                                                                                                       C---+K+X+11E
                 AX- a0 -a1x-a2 2 - TXAGX)-700X-701X + 12x2A(X) - 12x00 - 5x3A(X)
                     A(x) (1-1x+12x2-5x3) = 00-0,x-0,x+700x+70,x+12x00 = 0
                       A(x) (1-72+12x2-5x3) = 00=81x202x +1000x out attribute
                                                                                  A(X), QUE 81X5-Q1(X2+1900X-00-(X)A
                                                                                                             (1-7x+12x2-5x32NE-0D-(X)A
                                                                                                    (1-3x) (1-4x) (1-7x)
                                                                     A(1-3x)+B(1-4x)-(1-7x); (XE-1)(x)A
                                    AX. A = 30x0+B = 40x0 - C = 40x0 (IE-1) (X)A
Solve the recourrance relation on = 3an-1 +24 with a=1 by
      using generating tunctions method.
          8- 4- AE an 13an-17-25-> 1+1
                   multiply eq @ each term by xp and take n= 1 to & &
                multiply eq ( ) each term of \mathbb{Z}^{1} and \mathbb{Z}^
```

$$= (a_0 + a_1 x + a_1 x^2 + a_3 x^3 + - - -) - a_0$$

$$= A(x) - a_0$$

$$= A(x) - a_0$$

$$= (a_0 + a_1 x + a_2 x^2 + - - -)$$

$$= x(a_0 + a_1 x + a_2 x^2 + - - -)$$

$$= x(A(x))$$

$$= x + x^2 + x^5 + - - - -$$

$$= x(1 + x + x^2 + - - -)$$

$$= x(1 + x + x^2 + - - -)$$

$$= x(1 - x)^{-1}$$

$$= x(1 - x)^{-1}$$

$$= x(1 - x)^{-1}$$

$$= x(1 - 3x) - 2x + a_0$$

$$= A(x) - a_0 - 3x(A(x)) - 2x + a_0$$

$$= A(x)(1 - 3x) = \frac{2x}{1 - x} + 1$$

$$= A(x)(1 - 3x) = \frac{2x}{1 - x} + 1$$

$$= A(x)(1 - 3x) = \frac{2x}{1 - x} + 1$$

$$= A(x)(1 - 3x) = \frac{x + 1}{1 - x}$$

$$= \frac{x + 1}{1 - x}$$

$$= A(x) = \frac{x + 1}{1 - x}$$

$$= A($$

= 
$$\underbrace{\xi}_{n=0}^{2} x^{n} + 2 \underbrace{\xi}_{n=0}^{2} (3x)^{n}$$
 $\underbrace{\xi}_{n=0}^{2} x^{n} + 2 \underbrace{\xi}_{n=0}^{2} 3^{n} x^{n}$ 
 $\underbrace{\xi}_{n=1}^{2} x^{n} + 2 \underbrace{\xi}_{n=0}^{2} 3^{n} x^{n}$ 

compare the coefficients of  $x^{n}$ 
 $\underbrace{\xi}_{n=0}^{2} x^{n} + 2 \underbrace{\xi}_{n=0}^{2} 3^{n} x^{n}$ 
 $\underbrace{\xi}_{n=0}^{2} x^{n} + 2 \underbrace{\xi}_{n=0}^{2} x^{n}$ 
 $\underbrace{\xi}_{n=0$ 

Given

Let the generating function be 
$$A(x)$$
,  $\leq an x^n \rightarrow 0$   
multiply eq0 each term by  $x^n$  and take  $n=1$  to  $d \leq an x^n - 3 \leq a_{n-1} x^n = 4 \leq x^n \rightarrow 3$   
 $A(x) = 1$ 

$$\stackrel{d}{\in}$$
  $a_1 x^1 = a_1 x^1 + a_2 x^2 + a_3 x^3 + --- = a_0$   
 $\stackrel{d}{\in}$   $a_1 x^1 = a_2 x^2 + a_3 x^3 + --- = a_0$   
 $\stackrel{d}{\in}$   $a_1 x^1 = a_2 x^2 + a_3 x^3 + --- = a_0$ 

$$\sum_{n=1}^{\infty} a_{n-1} x^{n} = a_{0} x + a_{1} x^{1} + a_{2} x^{3} + \cdots$$

$$x(a_{0} + a_{1} x + a_{2} x^{2} + \cdots)$$

$$x(A(x))$$

$$\frac{1}{2} x^{n}$$
,  $x + x^{2} + x^{3}$   
 $x^{n}$   $x + x^{2} + x^{3}$   
 $x(1+x+x^{2}+---)$   
 $x(1-x)^{-1}$ 

substitute the values in egn @

$$A(x), \frac{3x+1}{(1-x)(1-3x)}$$

$$A(x), \frac{A}{1-x} + \frac{B}{1-3x}$$

$$A(x), \frac{A}{1-x} + \frac{B}{1-x}$$

$$A(x), \frac{A}{1-x} + \frac{A}{1-x}$$

$$A(x), \frac{A}{1-x} + \frac{$$

$$A(\chi) = \frac{-2}{1-\chi} + \frac{3}{1-3\chi}$$

3

$$A(x) = 2(1-x)^{-1} + 3(1-3x)^{-1}$$

$$2 = 2(1-x)^{-1} + 3(1-3x)^{-1}$$

$$2 = 2(1-x)^{-1} + 3 = 2(3x)^{-1}$$

compase the co-efficients of xn (-217 3.3")

Given that

$$a_{n-1}q_{a_{n-1}}+20_{a_{n-2}}=-20 \rightarrow 0$$

egn D is a second order non-homogeneous reccurrance relation:

1. The general solution of non-homogeneous reccurrance relation is

an = anth) + antp)

to find anth)

the homogeneous reccurrance relation of ean 10 is

The characteristic equation is

$$t^{2}$$
- 9t+20=0  $\rightarrow \mathcal{D}$   
 $t^{1}$ -5t-4t+20=0  
 $t(t-5)$ -4(+-5)=0

t=4,5

the roots are real and distinct the general solution of homogeneous recourrance relation is

to find an (P) Given an-9an-1 +20an-1 = -20 Here +(n) = -20 f(n): - 80.1 n there since from eqn® a=1 is not a root of a characteristic equation and gun) is a constant so that the particular solution is an LP) . Ao. 1 n anlp), Ao. substitute an. Ao in egn O A0 - 9A0 + 20A0 = -20 A0 = 20 10 - 75 3 substitute Ao value in particular solution an(P) -10/6 =. The general solution of non-homogeneous linear reccurrance relation is an = an thitan (P) an = c14" + c2 57(-5/3) . . . . 12 90 - C140+C250 00= C17 CT CI+CI > 40 SX5 5C1 +5/2, 205 n=1 a1= c141+ c25' 4c1+5/2=10 C1= 195

 $Q_1 = 4C1 + 5C_2$   $Q_1 = 4C1 + 5C_2 = 10$   $Q_1 = 195 + C_2$   $Q_1 = 195 + C_2$   $Q_1 = 195 + C_2$ 

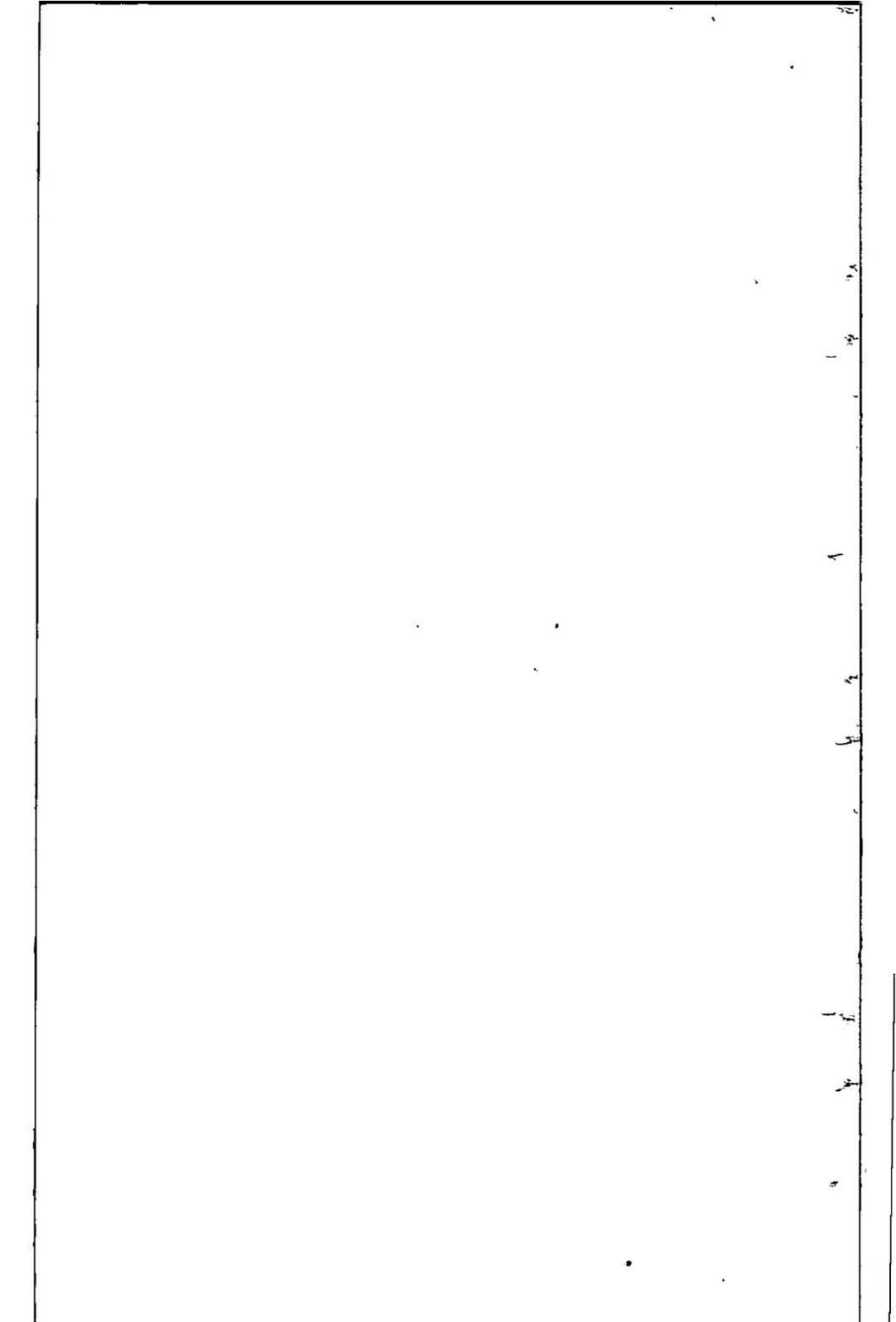
Given 
$$a_{n}-qa_{n-1}+a_{0}a_{n-2}=0$$
 $a_{0}>41$ ,  $a_{1}=10$ 

Let the generating function be  $a(x)=\sum_{n=0}^{\infty}a_{n}x^{n}\rightarrow 0$  ( ) intribly eqn(0) each term by  $x^{n}$  and take the  $\sum_{n=0}^{\infty}a_{n}x^{n}$  -  $a_{1}x^{2}+a_{2}x^{3}+a_{4}x^{4}+\cdots$  (  $a_{0}+a_{1}x+a_{2}x^{2}+a_{3}x^{3}+a_{4}x^{4}+\cdots$  )  $a_{0}-a_{1}x$ 
 $\sum_{n=2}^{\infty}a_{n-1}x^{n}$  ,  $a_{1}x^{2}+a_{2}x^{3}+a_{3}x^{4}+a_{4}x^{4}+\cdots$  )  $a_{0}-a_{1}x$ 
 $\sum_{n=2}^{\infty}a_{n-1}x^{n}$  ,  $a_{1}x^{2}+a_{2}x^{3}+a_{3}x^{4}+a_{4}x^{4}+\cdots$  )  $a_{0}-a_{1}x$ 
 $\sum_{n=2}^{\infty}a_{n-1}x^{n}$  ,  $a_{1}x^{2}+a_{2}x^{3}+a_{3}x^{4}+a_{4}x^{4}+\cdots$  )  $a_{0}-a_{1}x$ 
 $\sum_{n=2}^{\infty}a_{n-1}x^{n}$  ,  $a_{1}x^{2}+a_{2}x^{3}+a_{3}x^{3}+a_{4}x^{4}+\cdots$  )  $a_{0}-a_{1}x$ 
 $\sum_{n=2}^{\infty}a_{n-1}x^{n}$  ,  $a_{1}x^{2}+a_{2}x^{3}+a_{2}x^{4}+a_{3}x^{3}+\cdots$  )  $a_{1}x^{2}+a_{2}x^{4}+a_{3}x^{3}+\cdots$  )  $a_{1}x^{2}+a_{2}x^{4}+a_{3}x^{3}+\cdots$  )  $a_{1}x^{2}+a_{1}x^{2}+a_{2}x^{4}+a_{3}x^{3}+\cdots$  )  $a_{1}x^{2}+a_{1}x^{2}+a_{2}x^{2}+a_{3}x^{3}+\cdots$  )  $a_{1}x^{2}+a_{1}x^{2}+a_{2}x^{2}+a_{2}x^{2}+a_{3}x^{3}+\cdots$  )  $a_{1}x^{2}+a_{1}x^{2}+a_{2}x^{2}+a_{3}x^{3}+\cdots$  )  $a_{1}x^{2}+a_{1}x^{2}+a_{2}x^{2}+a_{3}x^{2}+\cdots$  )  $a_{1}x^{2}+a_{1}x^{2}+a_{2}x^{2}+a_{2}x^{2}+a_{3}x^{2}+\cdots$  )  $a_{1}x^{2}+a_{1}x^{2}+a_{2}x^{2}+a_{3}x^{2}+\cdots$  )  $a_{1}x^{2}+a_{1}x^{2}+a_{2}x^{2}+a_{3}x^{2}+\cdots$  )  $a_{1}x^{2}+a_{1}x^{2}+a_{2}x^{2}+a_{1}x^{2}+a_{3}x^{2}+\cdots$  )  $a_{1}x^{2}+a_{1}x^{2}+a_{2}x^{2}+a_{3}x^{2}+\cdots$  )  $a_{1}x^{2}+a_{1}x^{2}+a_{2}x^{2}+a_{3}x^{2}+\cdots$  )  $a_{1}x^{2}+a_{1}x^{2}+a_{2}x^{2}+a_{3}x^{2}+\cdots$  )  $a_{1}x^{2}+a_{1}x^{2}+a_{2}x^{2}+a_{3}x^{2}+\cdots$  )  $a_{1}x^{2}+a_{1}x^{2}+a_{2}x^{2}+a_{2}x^{2}+a_{3}x^{2}+\cdots$  )  $a_{1}x^{2}+a_{1}x^{2}+a_{2}x^{2}+a_{3}x^{2}+a_{3}x^{2}+$ 

(1-4x)(1-5x)

```
A(Z) > A (I-UX) -11 B (1-5X)-1
  ACX) > A & 4 x D + B & 5 2 D
 E anxh -A & 4 h2 h + B & 5 hxh
equating the coefficients of zn=an. A4n+B5n >0
  put n=oin egn @
       Q0= A(4)0+ B(5)0
         00 . A+ B
           A+B=41 -> 5
 put nei "in ean S
       at = A(4) + B(5)
         4A++5B = 10 →0
 solving s. E6 we get
      5A+15B -205
                                        t2- 9t+2020
5 X5
      (-) 4A+5B= 10
                                       t2-5t-ut+10=0
ᡌ
          1A= 195
A . 195
                                       +(t-5)-4(t-5)=0
                                         t>4,5.
   substitue A = 19 t in eqn (5)
      A+B=41
    195+8=41
      B=41-195
B=-154
```

1



Graphi-A graph G is defined as a pair of sets (VIE). It is denoted by G=(V,E) & G(V,E) & G

where V & V(G) = set of all vertices in G

E & E(G) = set of all edges connecting the vertices in G |V| = number of vertices |E| = number of edges.

Noti: - Vertices are also called modes. 81 points.

Here V(G)={a,b,c,d} E(G)={e,e2,e3,e4} of {a,b},{b,c},{e,d},{a,d}}

Adjacent vertices: Two vertices u and v are said to be adjacent if there is an edge between u and v

Here u is adjacent to v and v is adjacent from u

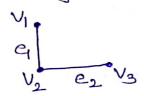
Incident edge: The edge'e that joins the vertices u and v

is said to be incident on each of its end points u and v.

Eg: u e

Here'e' is called incident edge.

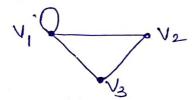
Adjacent edges: - Two non-parallel edges are said to be adjacent if they are incident on a common vertex.



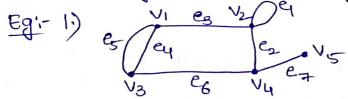
Here  $V_2$  is a common vertex of  $e_1, e_2$ .

So, e, e, e2 are adjacent edges

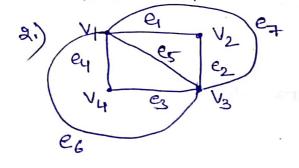
Self loop of loop: Loop is an edge drawn from a vertex to itself. (81) An edge associated with a vertex pair (Vi,Vi) is called a loop or self loop.



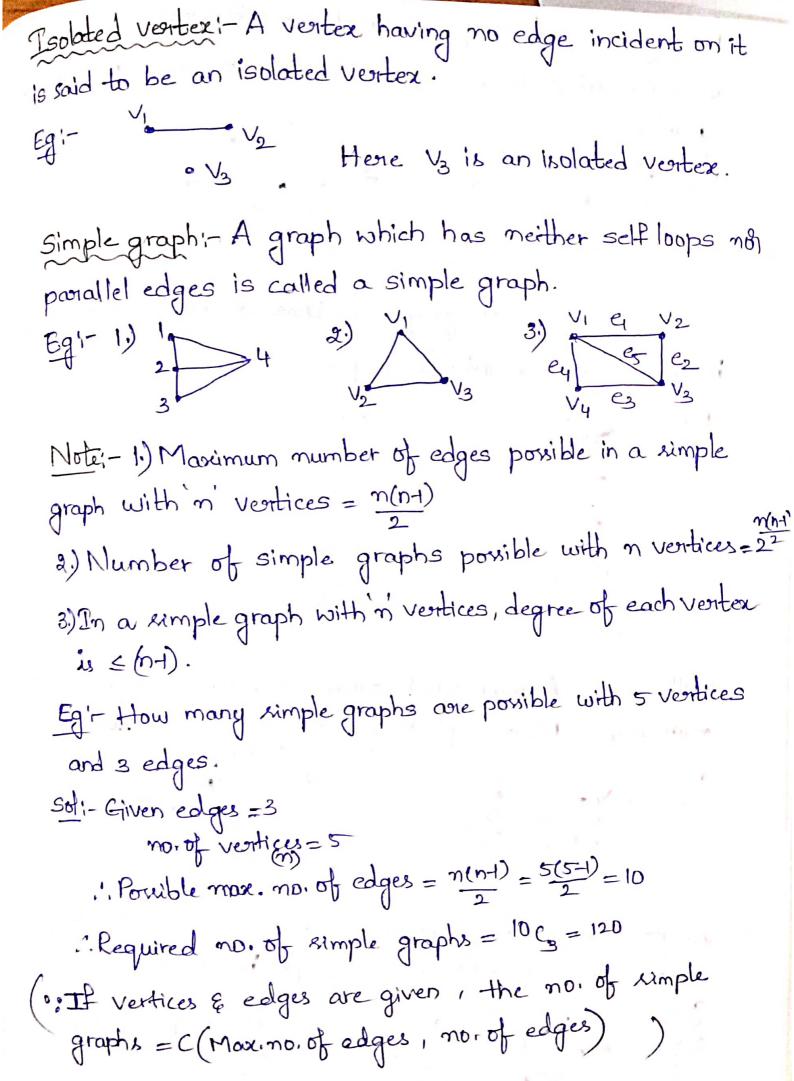
Same end points then the edges are called parallel edges (3) If there are more than one edge associated with a given pair of vertices, then these edges are called parallel edges of multiple edges.

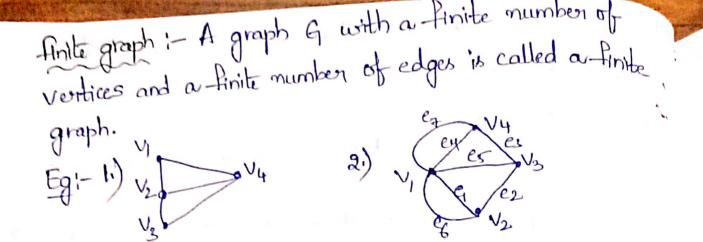


Here eyes are parallel edges

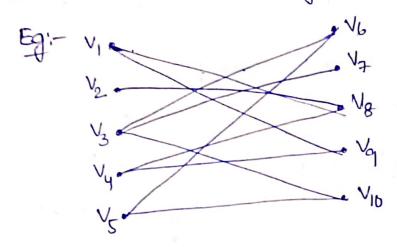


Here es, e6, e7 ane multiple or parallel edges.

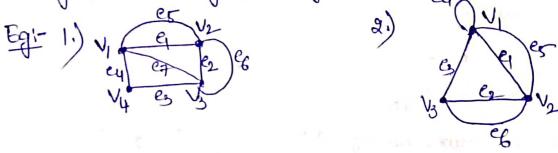




Infinite graph: A graph of that is not a finite graph.



Multigraph: A graph which contains some parallel edges is called multigraph. Multigraph may or may not contain self loops.



Weighted graph: A graph in which weights of costs one assigned to every edge of the graph is called weighted graph.

To this graph 213,415,619

Egi- A 3 7 E8 P 819 are weights of edges.

Directed graph of digraph: - A graph in which every edge is directed is called directed graph & digraph. Adirected graph contains an ordered pair of vertices denoted py (wv). The first element 'u' is called initial mode or stanting vertex. The second element 'v' is called terminal mode or ending vertex. Vertex set, Y= {a,b,c,d} Edge set, E = { (a,b), (a,c), (a,d), (r,b), (d,c)} Undirected graph & Non-directed graphi- A graph in which every edge is undirected is called undirected graph i.e., there is no specific direction to represent the edges. Undirected graph contains unoidered pair of vertices denoted as flung Vestex Set, V={aibicid} Edge set, E= {fa,b}, {b,c}, {c,d}, {d,a}} Mired graph' - A graph in which some edges are directed and some edges are undirected is called a mixed graph.

Degree of a venter for undirected graph: The number of edges incident on a venter V and with loops counted truice is called degree of Nortex V and is denoted by deg (u) & deg (v) & d(v)

Eg: - V, - d(v) = 4

 $E_{q'} - V_{1} = V_{2} - d(V_{1}) = V_{3} - d(V_{3}) = 3$   $d(V_{4}) = 3$   $d(V_{4}) = 2$ 

Pendent vertex. A vertex of degree one is called a pendent vertex and an edge incident on a pendent vertex is called a pendent edge.

Egi-V, e2 V3 e4 V5 V5 V7

Here the vertices  $V_6$  &  $V_7$  are isolated vertices  $V_1$ ,  $V_5$  are pendent vertices  $e_{11}e_{5}$  are pendent edges.

Mole: A vertex of degree zero is called an isolated vertex.

Indegree and out degree of vontex to a directed graphi-Indegree of a vertex: The Indegree of a vertex V in a directed graph is the number of edges which one coming of a vertex V is denoted by deg(v) of d(v) of Indeg(v). Outdegree of a vertex: The outdegree of a vertex V in a directed graph is the number of edges which are going out from the vertex V i.e., number of outgoing edges. Dutdegree of a vertex V is denoted by degt(v) or d'(v) or out deg (V). Note: For aloop, we can take indegree = 1 & outdegree = 1 591- V<sub>1</sub> V<sub>3</sub> V<sub>4</sub> Verten Indegree Outdegree Totaldegree

Matrix representation of a graph: A graph can be represented by using a matrix. There are two ways to represent the graph by a matrix.

1) Adjacency matrix 2) Incidence matrix

$$A_{G_1} = \begin{bmatrix} a & b & c \\ b & 0 & 0 & b \\ c & 0 & 1 & 0 \end{bmatrix}$$

2) Incidence matrix!

i) for non-directed graphi- Let G be graph with moverlikes and m edges. Then the incidence matrix is denoted by In is defined by matrix.

I(G) = [ajj]nxm, where aj = {1, when y is incident with ej

2) 
$$e_1$$
  $e_2$   $v_2$   $v_3$   $v_4$   $v_5$   $v_6$   $v_6$   $v_6$   $v_6$   $v_7$   $v_8$   $v$ 

ii) for digraph or directed graph: Let  $G_1 = (V, E)$  be a graph with n vertices and m edges without loops, then the incidence matrix  $I_{G_1}$  is defined by  $I(G_1) = (b_{ij})_{n,m}$  defined by  $b_{ij} = (1, iP, V_i)$  is the initial vertex of the edge  $e_i$  of iP iP iP is the final vertex of  $e_i$ 

Egl- write the Incidence matrix for established

I(G) = b + 0 + 0

c 0 + 0 + 0

d 0 0 0 -1

1)-find the incidence matrix 
$$\frac{1}{2}$$
  $\frac{1}{2}$   $\frac{1}{$ 

Bipartite graph I- A graph G=(V,E) is said to be bipartite graph if the vertex set V can be partitioned into two disjoint subsets VI and V2 such that every edge in E come a vertex in V1 and a vertex in V2.

Complete bipartite graph:—The complete bipartite graph on and n vertices denoted by Kmin is the graph whose vertex vertices in which there is an edge between each pair of the complete bipartite graphs of VI & V2, where V1 is in V1 & V2 is in V2.

The complete bipartite graphs k213, k3,5 are as follows:

fundamental theorem of graph theory of Handshaking-theorem:The G=(V,E) be an undirected graph with e' edges then & deg(Vi) = 2e The sum of the degrees of all vertices in an undirected graph is even. Proof: - Verify the above theorem by the following graph eg V1 By V2
eg eg eg eg V= (V11 V21 V31 V4 } E= { e1 (21 (31 e41 e5, e6) deg(V1) = 4 deg (V2) = 3 deg(V3)=3 deg(V4)=2 ... Endeg (vi) = 4+3+3+2 = 12 2|E| = 2x number of edges = 2x6=12

1) How many vertices are needed to construct a graph with 7 edges in which each vertex is of degree 2.

Soll Let the nequired no. of vertices be n'

Given edges, e = 7

Each vertex is of degree = 2

By Handshaking theorem,  $\underset{\longrightarrow}{\text{Alg}}(V_i) = 2e$   $\Rightarrow d(V_i) + d(V_2) + - + d(V_n) = 2x7$   $\Rightarrow 2 + 2 + - - - + 2(by n + 1) = 14$  $\Rightarrow 2n = 14 \Rightarrow n = 7$  2) How many edges are there in a graph with 10 vertices of each of degree 6.

Soll- Let e be the number of edges.

The given graph contain 10 vertices of each of degree 6.

By Handshaking theorem, & d(vi) = 2e

Regular graph: A graph in which all vertices are of same degree is called a regular graph & simply regular.

Eg.- Vi

Here degree of each vertex is 3.

Null graph: - A graph G=(V,E) is said to be a null graph if E=\$ 81

A graph & contains no edges is called null graph.

Eg:- . VI

v20 . v3 is a null graph since there is no edges bettheevs. the vertices.

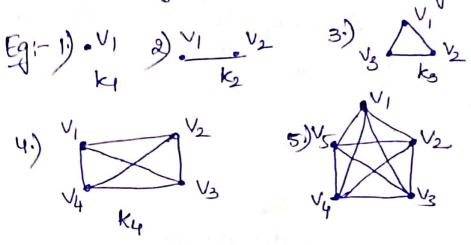
graph is (n-1)

2.) The maximum number of edges in a simple graph with in vertices is n(n+)

X

Complete graph: - A simple graph & is said to be complete graph, if every vertex of G is connected with every other vertex of Gile, if Gicontains only one edge between each pain of distinct vertices.

The complete graph with 'n' vertices is denoted by Kn. A complete graph Kn has exactly non-1) edges.



Isomorphism: - Two graphs q and 9' are said to be isomorphic to each other if there is a one-one correspondence between their vertices and between their edges such that the incidence relationship must be preserved.

Verification of isomorphism between the graphs.

1) The same number of vertices

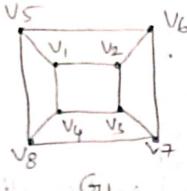
2) The same number of edges

3) An equal number of vertices with a given degree. If any of the above criteria fails, the graphs are not isomorphic.

1) show that the following graphs are isomorphic (G12) B12= 6. P. 9. 18, 5. E. } Sol: - G1= {a,b,c,d,e's N21 = 5 = no. of verticuin 6 14,1=5 = no. of vertices in Gy | E2 = 6 = 11 edges in G2 IEI = 6 = no. of edges in Gig i) | \ | = | \ \ \_ | 11) |E1= |E2| 111) In G11 Vertex degree Vertex degree We should have a >s, d >g, and e >x we take b->p and c->t Adjacency matrix of GII, AGII = afo Adjacency matrix of G12,

-> AGII = AGI2 => f 10 one-one, onto and also preserves adjacency. .. G, and Go are isomorphic.

2) show that Gi, Go are isomorphic



AGII =

i) Number of vertices 1V1 = 8 , 1V2 = 8 . . . | V1 | = | V2 | 2 mg and a

11) No. of edges

|E1 = 12 , |E2 = 12 ,. |E| = |E2|

In 
$$G_1$$
,  $d(v_1) = 3$   
 $d(v_2) = 3$   
 $d(v_4) = 3$   
 $d(v_6) = 3$   
 $d(v_6) = 3$   
 $d(v_6) = 3$   
 $d(v_8) = 3$ 

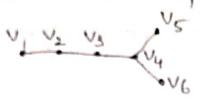
In Ga, d(p) = 3  $d(q_i) = 3$ d(8)=3 d(s)=3 d(+)=3 d(4)=3 d(v)=3

 $d(\omega)=3$ 

We should have VI->P, V2->Q, V8->W, V4->S, V5->U, VL >V, V2 >X, VB >t

46-	, ,	+	,	0					
Thus,	V	Vi	42	V3	Vy	V5	V <sub>6</sub>	VZ	Vg
1.12	P(v)	P	9	W	S	u	V	8	t,
1		'						The second second	

3) check the isomorphism for the following graphs



The second secon

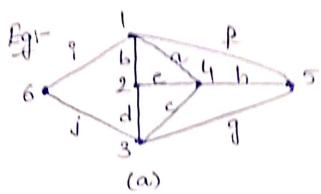
$$3d!$$
  $d(v_1)=1$   
 $d(v_2)=2$   
 $d(v_4)=3$   
 $d(v_6)=1$   
 $d(v_6)=1$ 

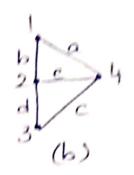
$$d(p)=1$$
 $d(q)=2$ 
 $d(s)=3$ 
 $d(s)=2$ 
 $d(4)=1$ 
 $d(u)=1$ 

We conclude that there are 2 pendent vertices adjacent to V4 and there is only one pendent vertex adjacent to 7. So, a contradiction exists.

. The two graphs are not isomorphic.

subgraph: A graph H is sold-to be a subgraph of a graph G1, if all the vertices and all the edges of H are in G1 and each edge of H has the same end vertices in H as in G1. It is denoted by HCG1





i. Graph b is a subgraph to graph a.

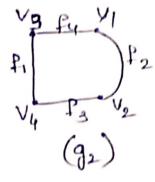
Note: 1) Every graph is a subgraph of itself.

2) A subgraph of a subgraph of G Is a subgraph of G.

3) A single vertex in a graph G is a subgraph of G.

4) A single edge in G, together with its end vertices is a subgraph of G.

5) Two subgraphs g, and g, of a graph 'G' age soid to be edge-disjoint if g, and g, do not have any edges in



Walk: A finite alternating sequence of vertices and edges (no suspetition of edge allowed) beginning and ending with vertices suchthat each edge is incident with the vertices preceding and following it, is called a walk of edge train of chain. i.e., no edge appears molethon once in a walk.

c de P (GV)

Here Viavabraduyerafus is a walk

Note: 1.) The vertices with which a walk begins and ends are called terminal vertices of terminal points.

2.) A walk is said to be closed walk if the terminal points are same.

VICV3 bV2 aV1 is a closed walk

3) A walk which is not closed is called an open walk. In the above Eg VICU3bV2 is a open walk.

Path:- An open walk in which no vertex appears morethan once is called a path of simple path of dementary path. Note: - 1) In the graph (Gi) the path Maybyd 4 is of length 3.
2) An edge that is not a self-loop is a path of length 1. length 3.

3) A self loop is a walk but not a porth.

Circuit: A path of length greater than I with no repeated edges and whose end vertices are equal is called a circuit. A circuit may have repeated vertices other than the end vertices.

Cycle: - A cycle is a circuit with mo other repeated vertices except its end vertices.

Eg:-1) V, Q V2 V, bV2 aV, 10 a circuit

2) dia V2 V1 av2bv3 cv 4 d V1 is a cisacit

Every vertex is a circuit is of degree 2.

Every self loop is a circuit but every circuit is not a self loop.

Connected graph! - A graph Gi is said to be connected if there is atleast one edge between every pair of vertices in Gi.

Eg1-V1 e4 e5 V4 e1 e3 V2 e2 V3

10 ai Viervaeavsea V4 es 45 lis (a) pathing and n.E. (1 - dal)

To the steer is a view of

ducke of their ago and with

Disconnected graph: A graph G is said to be disconnected graph:

formation of components !- If Gis a connected graph, then G contains only one connected component and it is equal to G. If G is a disconnected graph. Consider a vertex V in G which is not joined by any path. The vertex V and all the vertices of G that are joined by some paths to V together with all the edges incident on them form a component G1. similarly consider the ventex u which is not in G1 and incident with the edges on them to Asm Giz. Continue this procedure to find the components. Note: - 1.) If G is connected, then G is the connected

component. 2) A disconnected graph G1 consists of two of more connected

and to a some or fair.

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ample to the project of girls for the prost of

components.

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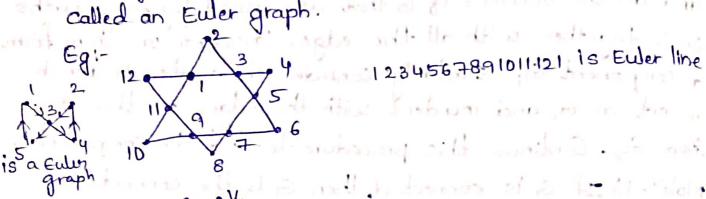
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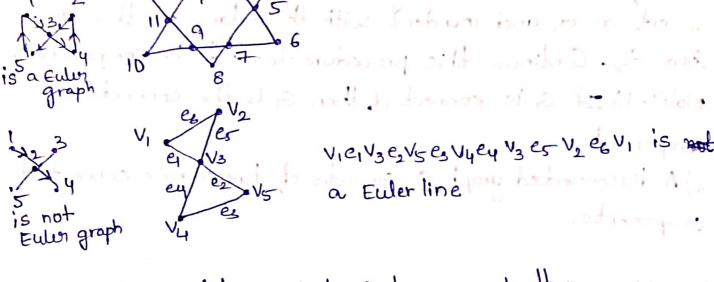
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Euler line: - Let Gr be a graph. A closed walk nunning through every edge of the graph Grexactly once is called an Ewler line of Ewley circuit

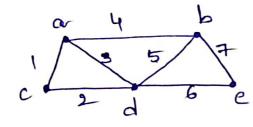
Euler graph: - A graph Gi that contains Euler line is. to emilion who the kin called an Euler graph.





Vieiv3e2v5e3v4e4v3e5v2e6vi is not a Euler line

Unicursal line! - Let G be a graph then an open walk orunning through all edges of the graph G exeactly once is called an unicursal line



alceds a 4 b 5 d 6 e 7 b

Unicurral graph :- A connected graph that contains unicuosal line is called a unicursal graph.

Euler path: - A path of graph G is called Euler path if it includes each edge of G exactly once & it contains every edge of G Egi- A B Euler path bln B and D c is B-D-C-B-A-D.

Euler theorem:

A given connected graph G is an Euler graph iff if all vertices of G are of even degree.

Proof: - 1) Suppose GIs an Euler graph

In tracing this walk everytime we meet a vertex vit goes through 'two' new edges.

By using one edge which we entered, another edge which we are going to leave.

from the first vertex, we "existed" and we can "enter" Into that vertex at the last.

- . . Every vertex in a graph G is of even degree
- ii) Suppose all vertices of Grane of even degree. Construct a Euler line starting-from vertex V.
- "Every vertex is of even degree,
  we can enter into every vertex and exit from the same
  vertex.

Tracing continues until reaching the vertex V.

... Gr contains an Euler line.

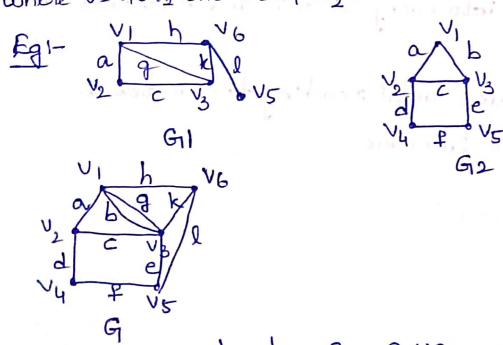
Note: - 1) If G is a graph with n' vertices then G is obtained from kn by simply deleting the edges of G.

2) If G has 'n' vertices and 'p' edges then G has n' vertices

and n(n-1)-p edges

31) Let G1 and G2 be simple graphs, then G11 and G12 agre isomorphic iff G11 and G12 agre isomorphic.

Union of graphs: Let  $G_1 = (V_1 E_1)$  and  $G_{12} = (V_2 E_2)$  be two graphs. Then the union of  $G_1$  and  $G_{12}$  is defined as  $G_1 U G_{12} = (V_1 E)$  where  $V = V_1 U V_2$  and  $E = E_1 U E_2$ .



clearly G= GIUG2

Intersection of graphs: Let  $G_{1} = (V_{1}, E_{1})$  and  $G_{1} = (V_{2}, E_{2})$  be two graphs. Then the intersection of  $G_{1}$  and  $G_{2}$  is defined as  $G_{1} \cap G_{2} = (V, E)$  where  $V = V_{1} \cap V_{2}$   $\xi$   $E = E_{1} \cap E_{2}$ .

Eg:-for the above example, GING1 = 201

Ring sum of graphs: Let  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  be two graphs. Then the oring sum of two graphs  $G_1$  and  $G_2$  is defined as  $G_1 \bigoplus G_2 = (V_1 E)$ , where  $V = V_1 \cup V_2$  and  $E = (E_1 \cup E_2) - (E_1 \cap E_2)$ 

= edges that either in G1 & G12 but not both.

$$\frac{\text{Eq'}-2}{2}\frac{\text{G}}{\text{Eq'}}-2\frac{\text{G}}{\text{Eq'}}$$

Sum of two graphs: - V(G1+G2) = V(G1)+V(G2)

E(G1+G12) = Edges which are in G1 and G12 are obtained by joining each vertex of G1 to each vertex of G12

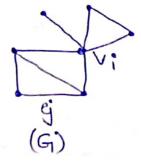
Decomposition: A graph G is said to have decomposed into two subgraphs G1 and G2 if G1UG2 = G1 and G1NG2 = \$\frac{1}{2}\$ i.e., some of the vertices may occur both in G1 and G2.

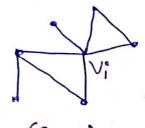
In decomposition, the Isolated vertices are disregarded.

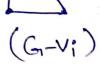
Deletion: - If Vi is a vertex in graph G. Then G-Vi denotes a subgraph of G obtained by deleting Vi from G. Deletion of a vertex always implies the deletion of all edges incident on that vertex.

If ej is an edge in G, then G-ej is a subgraph of G obtained by deleting ej from G. Deletion of an edge does not imply deletion of its end vertices

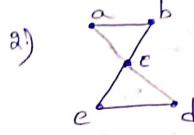
· · G - g = G + g

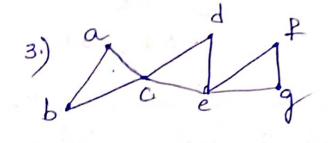






Ambitrarily traceable curve: - Let Gi be a graph and v be a venter. The graph is said to be arbitrarily traceable graph-from the ventex V if an Ewler line is always obtained when one follows any walk from the vertex vaccolding to the single rule that whenever one arrives at a vertex one shall select any edge initial vertex vican be extended the faith of a graph must pass through the initial vertex vican be extended vertex. is ambitrarily traceable graph from VI & V2 a) is arbitrarily traceable graph from c.

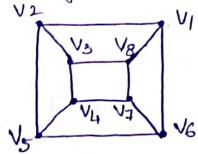




3.) a from any vertex every since from any vertex every cycle does not have a consumon cycle does not have a consumon

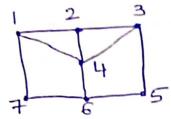
Note: - An Euler graph Gr is arbitrarily toraceable from vertex V in Gr iff every circuit in Gr Contains V.

Hamiltonian cycle: In a connected graph a closed walk that visits every vertex of the graph G exactly once except the starting and ending vertices. It is called Hamiltonian cycle or circuit.

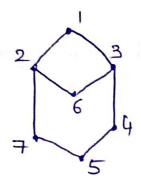


V1V2V3V4V5V6V7V8Y1

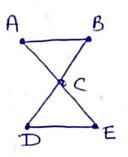
Hamiltonian graph: - A graph containing a Hamiltonian graph.



Hamiltonian Path: - In a connected graph an open walk the visits every vertex of the graph & exactly once. But if contains an open walk.



1-2-6-3-4-5-7



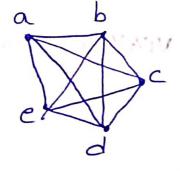
A-B-C-D-E

(8) If we remove any one edge from a Hamilton with then we left with a path. This path is called a Hamiltonian path.

# Problem

1) Identify five different Hamiltonian cycles of the following undirected graph





five different Hamiltonian cytes age on-b-c-d-e-ar

- Note

  1.) Clearly by definition, a Hamiltonian path in a graph Gr
  traverses through every vertex of Gr.
- 2) Il a graph G1 that contains a Hamiltonian clucuit then G1 contains a Hamiltonian path.
- 3) there exists graph with Hamiltonian paths that have no Hamiltonian circuits
- 4) The length of a Hamiltonian path in connected graph of n vertices is 'n+1'.
- 5) In a Hamiltonian circuit of path every vertex appears exactly once. Hence Hamiltonian circuit cannot include a self-loop or a set of parallel edges.

At he do not proper a path a series,

recording to the conference of the sale of the classical and a second section of

Traveling salesman problem: This problem is related to Hamiltonian cincuit.

Problem: A salesman required to visit a number of cities during his trip. Given the distances between the cities, In what 8 der should the salesman travels so as to visit every city precisely once and return to his home city, with the minimum mileage traveled?

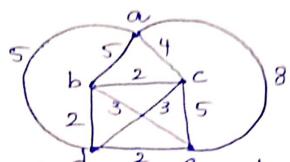
Solution: Represent the cities by vertices and the Gloods between them by edges. Then we get a graph i) In this graph to every edge e' there corresponds a real number w(e). Such graph is called a weighted graph. Here w(e) is called as the weight of the edge e'.

ii) If each of the cities has a good to every other city, we have a complete weighted graph. The graph has numerous Hamiltonian circuits and we are to select the Hamiltonian circuit that has the smallest sum of distances.

iii) The no. of different Hamiltonian circuits in a complete graph of 'n' ventices is (n-1)!

iv.) First we list all the (n+)! Hamiltonian circuits that are possible in the given graph. Next calculate the distance traveled on each of these Hamiltonian circuits. Then select the Hamiltonian circuit with the least distance. This provide the solution for the travelling salesman problem.

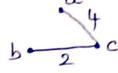
exactly once and returns to its storting point with minimum distance storting at vertex a and d'



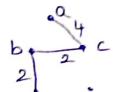
Consider Hamiltonian Einquit achdea sol:- stepl select minimum distance edge path faic?



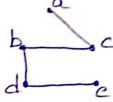
step@ select minimum distance from c'path = {a,c,b}



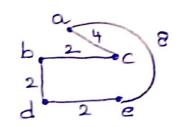
step3) Path = {a,c,b}



step@ Path = {a,c,b,d,e}



step Path = {aicibidieia}

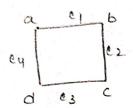


.'.Total <del>minimum</del> distance = 18

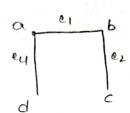
#### spanning tree

A spanning tree is a tree that connects all the vertices of a graph with the minimum possible no of edges . Thus a spanning tree is always defined for a graph and it is always a subset of that graph.

EX



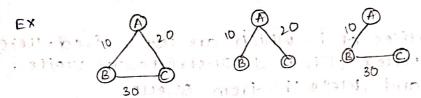
connected graph Gi Tree

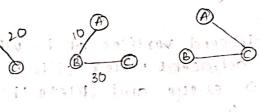


spanning services in the si

Minimum spanning tree

A minimum spanning tree is a subset of edges of a connected undirected graph that connects all the vertices together with the minimum possible total edge weight





BFS (Breadth First search)

BFS is used for searching spanning tree of the graph we use queue data structure with maximum size of total number of Vertices in the graph to implement BFS · see and real forming the best best the open to the see of

BFS Algorithm

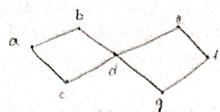
step 1: Define a queue with maximum size of total number of vertices in the graph

step2: select any vertex as starting vertex for traversal. Visit that vertex and insert it into the queue ..

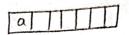
steps: visit all the unvisited adjacent vertices of the vertex which is at front of the queue and insert them into the Queue

step 4: when there is no new vertex to visit from the vertex at front of the queue then delete that vertex from gueue

steps: repeat step3 and step4 until queue becomes empty step 6: when queue becomes empty. Inal we get spanning tree by removing unused edges from the graph

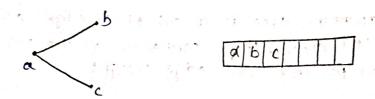


By using a Brs algorithm to tind a spanning tree of the following graph G take a queue with size + no of vertices = 7 step: select the vertex at as the starting vertex and insert at into succes

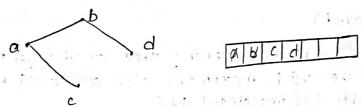


a

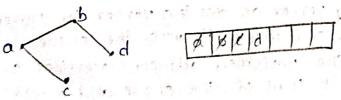
steps: visit all adjacent vertices of 'a' which are not visited the adjacent vertices of 'a' are band c. Insert newly visited vertices bandc into queue and delete 'a' trom queue



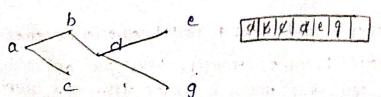
steps: visit all adjacent vertices of b which are not visited, there the unvisited adjacent vertex of b is d. Insert newly visited vertex d into queue and delete b' from queue



step 4: visit all unvisited adjacent vertices of cottere unvisited vertex of cois do it we visit do from cotten cycle will be formed, so we cannot visit do from coand also delete cofrom queue



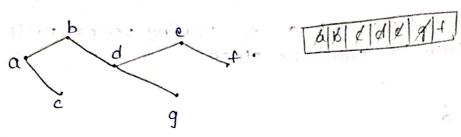
steps: visit all unvisited vertices of differe unvisited vertices of difference unvi



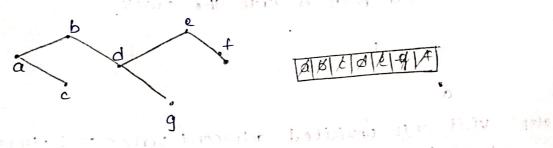
steps: Visit all unvisited adjacent vertices of e-tiere unvisited. vertex of e is f. ansert bewly visited vertex 4 into queue and delete e from quelle



step7: visit all unvisited adjacent vertices of gittere unvisited adjacent vertex of gisf. If we visit f from g then cycle Will be formed so we cannot visit & from g and also delete q trom quelle



steps: visit all unvisited adjacent vertices of fintere no unvisited vertex of f, so me delete f from Quelle water parties of decidal bear of



DFS (Depth First search)

DFS is used for finding spanning of the graph. We use stack data structure with maximum size of total number of vertices in the graph to implement Drs

## DFS algorithm

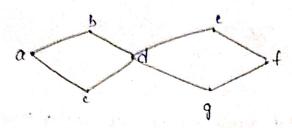
step1: Define a stack with maximum size of total number of

step2: select any vertex as starting point for traversal visit that vertex and push it on to stack

step3: Visit any one of the unvisited adjacent vertex of the veitex which is at top of the stack and push it on boll the for four sources to the stack

step4: repeat step3, until there is no new vertex to visit from the vertex which is at top of the stack

steps: when there is no new vertex to vicit then use back tracking and poplatiete) one vertex from the stack steps steps 3,4, and 5 until stack becomes empty



By using DFs algorithm to find a spanning tree of the

Take stack with size = no-of vertices = 7

and push 'o' onto the stack E

å

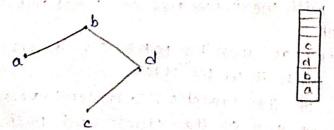
vertices of larare bandc. We have to visit either borc visit b and push b onto the stack



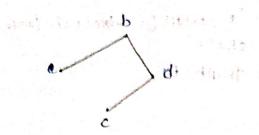
steps: visit any unvisited adjacent vertex of bittere the unvisited adjacent vertex of b is d visit d and push d onto the stack



step 4: visit any unvisited adjacent vertex of differe unvisited odjacent vertices of dare cierq will visit either q or core visit cand push conto the stack

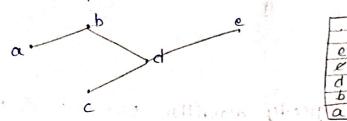


steps: visit any unvisited adjacent vertex of c. there unvisited adjacent vertex of c. is lat. If the visit lat from c -then cycle will be formed. So we cannot visit a from c now c will be deleted from stack and we use back track to d.



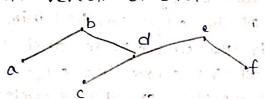


step 6; Atter back-track to d, visit any unvisited adjacent vertex of d. Here unvisited adjacent vertex of d are gande . We q ore wisit e and pushe onto the stack either

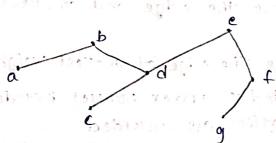




etep7: visit any unvisited adjacent vertex of e Here unvisited adjacent vertex of elsf. visit fand push fonto the stack



steps: visit any unvisited adjacent vertex of for Here unvisited and push g onto the stack vertex of f is 9



stepa; visit any unvisited adjacent vertex of gottere unvisited adjucent verter of g is d- If we visit d from g. then cycle will be formed, so wie cannot visit d from g. Now g will be deleted from stac K

Now back track tof . There is no any unvisited adjacent vertex of f, so + will be deleted from stack

Now back track to e. There is no any unvisited adjacent vertex of e, so e will be deleted from stack

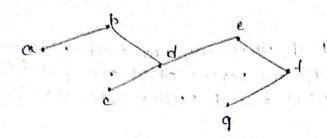
Now backtrack to do there unvisited vertex of dis go If we visit 9 from d'then cycle will be formed so me cannot visity now d will be deleted from stack

Now backtrack to b. There is no unvisited adjacent vertex of b, so b will be deleted from the stack

Now backtrack to 'a' there unvisited adjacent vertex of 'a' is c lif we visit a tromial then cycle will be formed.

All the vertices are detected in the stack.

The spanning tree of given graph is



Prim's algorithm

prim's algorithm is a greedy algorithm that is used to find the minimum spanning tree from a graph. Prim's algorithm finds the subset of edges that includes every vertex of the graph, such that the sum of the weights of the edges can be minimized

prim's algorithm starts with the single node and explores all the adjacent nodes with all the connecting edges at every step. The edges with minimum weights crossing no cycles in the graph get selected:

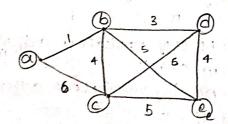
Algorithm

step1: select any vertex & choose the edge and smallest inleight from G

Step2: At each stage, choose the edge of smallest weight joining

a vertex already included to vertex not yet included step 3: continue until all vertices are included

to tind the minimal spanning tree for the following inleighted using prims algorithm



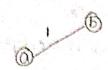
the beautiful the state of the

step1: select any vertex from the given graph. Now, consider the vertex a

(a)

step 2: From the vertex a concider the adjacent vertex with the minimum weighted edge such that they do not form any loops or cycle

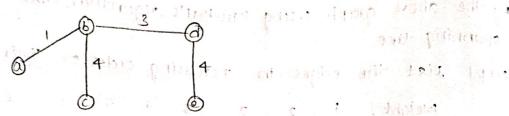
Now draw an edge from a to b



steps: Now, next concider the minimum weighted edge from the vertex a and b and the next minimum weighted edge is blog

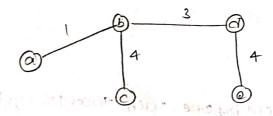


step 4. Next, consider the minimum meighted edge from the vertex a bound of the next minimum weighted edge is btoc and ctod.



steps. Next, consider the minimum weighted edge from the verted ine c-e if we add this edge it forms a loop so we cannot add this edge sphe hampion municipal sale thates

By using the prim's algorithm the minimum spanning tree is



weight of the spanning tree is

### kruskalls algorithm

kruskalls algorithm is the concept that is introduced in the Graph theory of discrete mathematics It is used to discover shortest path between two points in a connected weighted graph

star to and a top water tool and

#### Algorithm

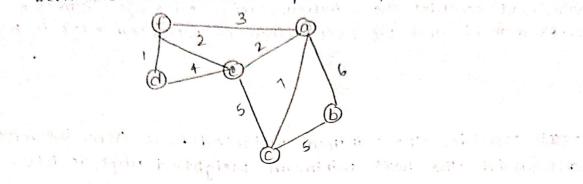
step 1: Remove all loops and parallel edges

step2: Arrange the edges of G1 in increasing order of their Weights

step3: choose an edge of minimum weight If there is more than one edge of minimum aleight then choose arbitrarily one of these edges

step 4: At each stage choose the not selected edge of minimum meight whose inclusion will not create a cycle

steps: It G has n vertices then stop after not edges have been choosen otherwise repeat step 4



By the above graph using kruskalls algorithm tind the minimum spanning tree

steps list the edges in increasing order of their weight

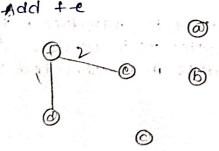
W	ineight	1	2	2	3	4	5	5	6 7	
114 910	Edge	f-d	-t-e	e-a "	-f-a	d-e	e-c	c-b	a-b a-C	-

minimum weighted edge fid the step 2

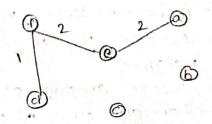


next add the above edges one by one such that no cycle The Mark in the major with the trip forme d

step 3: Add the next minimum weighted edge there two edges having the weight 2, so we can add them arbitarily such that they do not form a cycle



now add an edge

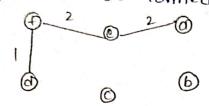


edge then cycle will be formed so we cannot add an edge



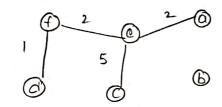
steps: Next minimum meighted edge is de But it me add this edge then cycle mill be tormed so me cannot add an edge de

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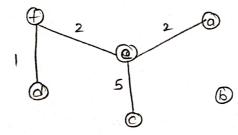


these edges one after another such that they do not form a cycle

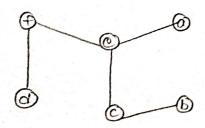
Now add an edge e-c



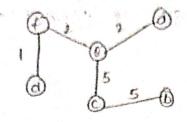
Now add an edge c-b



stept; Add next minimum weighted edge is a-b we cannot add this edge because cycle will be formed



steps: Add next minimum weighted edge a-c we cannot add this edge because cycle will be tormed



this is minimum spanning tree weight of minimum spanning tice . 1+1+2+5+5 -15

ရေးသည်။ နေနေက <del>အနေ့ချေးတွင်</del> တွင် မြို့သည် မြို့သည် မြို့သည် လေသများ သည် ကို တွေးသွင်း

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