ANNAMACHARYA UNIVERSITY, RAJAMPET

(ESTD UNDER AP PRIVATE UNIVERSITIES (ESTABLISHMENT AND REGULATION) ACT, 2016

RAJAMPET, Annamayya District, AP, INDIA

Course: Mathematical Foundations of Computer Science

Course Code : 24FMAT11T

Branch : **MCA**

Prepared by: **K.Adisekhar Babu**

Designation : **Assistant Professor**

Department : **MCA**

Title of the Course	:	**Mathematical Foundations of Computer Science**
Category	:	**BS**
Couse Code	:	**24FMAT11T**
Branch	:	**MCA**
Semester	:	**I Semester**

Lecture Hours	Tutorial Hours	Practice Hours	Credits
3	0	0	3

COURSE OBJECTIVES:

The aim is to cover the basics of mathematical logic and to work with sets, relations, and functions. It also involves studying how combinatorial methods are applied in fields like cryptography, coding theory, and algorithm design, as well as understanding generating functions, recurrence relations, and fundamental concepts of graphs and trees and their practical uses

UNIT I Mathematical Logic 10 Hrs

Propositional Calculus: Statements and Notations, Connectives, Well-formed formulas, Truth tables, Tautologies, Equivalence of formulas, Duality Law, Tautological implications, Normal forms, Theory of Inference for Statement calculus.

Predicate Calculus: Predicates, Predicative logic, Statement functions, Variables and Quantifiers, Free and Bound variables, Inference theory for Predicate calculus.

UNIT II Set Theory 8 Hrs

Sets: Operations on Sets, Principle of Inclusion-Exclusion. Relations**-**Properties, Operations, Partition and Covering, Transitive closure, Equivalence, Compatibility and Partial ordering, Hasse diagrams. Functions**-** Bijective, Composition, Inverse functions, Lattice and its Properties.

UNIT III Elementary Combinatorics 8 Hrs

Basics of Counting, Combinations and Permutations, Enumerating Combinations & Permutations with repetitions, Enumerating Permutations with constrained repetitions, Binomial coefficients, Binomial and Multinomial theorems.

UNIT IV Recurrence Relations 8 Hrs

Generating functions of sequences, calculating coefficients of generating functions, Recurrence relations - Solving recurrence relations by substitution and generating functions, Method of characteristic roots for the Solutions of non-homogeneous recurrence relations.

UNIT V Graph Theory 10 Hrs

Definitions, Finite and infinite graphs, Incidence and Degree, Isolated and Pendant vertices, Graph representations- Adjacency and Incidence matrices, Isomorphism, subgraphs, Walk, Path and Circuit, Connected and Disconnected graphs, Components, Multi graphs, Bipartite and Planar Graphs, Euler's formula, Euler graphs, Hamiltonian paths and circuits, Chromatic number, Trees, Spanning trees: DFS, BFS, Minimal Spanning trees: Prim's and Kruskal's algorithms.

TEXT BOOKS:

1. J.P. Tremblay and R. Manohar. Discrete Mathematical Structures with Applications to Computer Science, Tata Mc Graw Hill Education, 1st Edition, 2002.
2. D. S. Chandra Sekharaiah. Mathematical Foundations of Computer Science, Prism Books, 3rd Edition, 2010.

REFERENCE BOOKS:

1. J.L. Mott, A. Kandel and T. P. Baker. Discrete Mathematics for Computer Scientists and Mathematicians, PHI, 2nd Edition, 2008.
2. C.L. Liu and D.P. Mohapatra, Elements of Discrete Mathematics – A computer-oriented approach, Mc Graw Hill India, 3rd Edition, 2011.

COURSE OUTCOMES:

The Student will be able to

1. Understand the fundamental concepts of propositional and predicate calculus to construct valid arguments and engage with logical reasoning effectively.
2. Utilize operations and principles related to sets, including the Principle of Inclusion-Exclusion, to address real-world problems involving set theory
3. Analyze the application of combinatorial mathematics across various fields, such as computer science, cryptography, and optimization, by exploring its principles and techniques for addressing complex challenges
4. Solve different types of recurrence relations in computational contexts by applying appropriate methods to model and address various problems effectively.
5. Apply graph theory concepts in network analysis.

CO-PO MAPPING:

Course Outcomes	Foundation Knowledge	Problem Analysis	evelopment of Solutions	Modern Tool Usage	Individual and Teamwork	oject Management and Finance	Ethics	Life-long Learning
24FMAT11T.1	2	2	1	-	-	-	-	1
24FMAT11T.2	3	2	1	-	-	-	-	1
24FMAT11T.3	3	3	2	-	-	-	-	1
24FMAT11T.4	3	2	1	-	-	-	-	1
24FMAT11T.5	3	2	1	-	-	-	-	1

UNIT-I:

Mathematical logical: propositionalcalcus, statements and notations, connectives, Welformed formulas, Truth Tables, Tatalogies, equivalent of formulas, Duality law, Tatalogical implications, Normal forms, Theory of Inference, For statement calcus

Predicate calcus: predicates, predicative logic, statement functions, variables, and quantifies, free and bound variables, Inference theory for predicate calcus.

UNIT II:

set theory: sets, operations on sets, principle of Inclusion-Exclusion. Relations, properties, operations, partition, and covering, Transitive closure, Equivalance, compatability. and partial ordering, Hassie Diagrams.

Functions: Bijective, composition, inverse functions, lattice and its properties

UNIT III

Elementary combinatorics Basics of counting, combinations and permutations, Enumerating combinations and permutations with Repititions, Enumerating permutations with constraint Reputitions, Binomial co-efficience; Binomial and multinomial Theorms.

UNIT IV

Recurrance Relations Generating functions of sequences, calculating co-efficients of generating functions, Recurrance Relations - solving Recurrance Relations by substitution and generating functions, method of characteristic roots for the solutions of non-homogenous recurrance Relations.

UNIT V

Graph Theory: Definitions, Finite and infinite graphs, incidence and Degree, Isolated and Pendant vertices, Graph representations. Adjacency and Incidence matrices, Graph representations. Adjacency and Incidence matrices,

Graph representations. Adjacency and Incidence matrices. and circuit, connected

Hamiltonion paths and circuit, chromatic number, Trees, spanning Trees: DFS, BFS, minimal spanning Trees; prims and Kruskal's Algorithms.

Mathematical logic is a foundational aspect of many areas in MFC's. It involves formal systems that help in reasoning about mathematical statements, algorithms and computational processes.

Statement (proposition)

- A statement is a meaningful sentence or declarative sentence, which is either true or false, but not both
- statements are denoted by capital letters and small letters i.e P, Q, R... or p, q, r... except T, F

Ex: $2+4=6 \rightarrow$ True

$5+2=8 \rightarrow$ False

- New Delhi is a capital city of India $\rightarrow$ True
- Where are you going ? $\rightarrow$ It is not a statement because. it is a interagative sentence.
- close the door $\rightarrow$ It is not a statement or sentence. because it is a command.

$x-2=4$. $\Rightarrow$ It is not a statement because it is depending on the x value.

Truth value.

- A statement is said to have truth values (True or False)
- The true or false of a statement is called Truth values
- The truth value True of a statement is denoted by T and the truth value False of a statement It is denoted by F

Notations

A various notations are used to commonly to represent concepts, structures and operations

Ex: $\{\ \}$ or ϕ

logical And $\wedge$
Logical OR $\vee$
Logical NOT $\sim$ (or) $\neg$
Implication $\Rightarrow$
Bi-conditional $\longleftrightarrow$

connectives

statements can be connected or combined by using

Types of statements

There are two types of statements. They are.

i. simple statement

ii. compound statement

Simple statement

A statement which is not cotain any of the connectives is called a simple statement.

EX:- Raju is a clever boy

The integer 5 is a prime number

Annamacharya is a university

compound statement

Two or more simple statements can be combined by using logical connectives to form a single statement is called compound statement.

EX:- Raju is a clever and handsome

He opened the book and started to read

Symbolic representation of connectives

Symbol	Connective word	Nature of compound statement	symbolic form
~ or ¬	Negation	NOT (or) Negation	¬P (or) ~P
∨	Disjunction	Disjunction	P∨Q
∧	conjunction	conjunction	P∧Q
→	Implies (or) If then	Implication	P → Q
⟷	Bi-conditional	if and only if	P ⟷ Q

Negation

· The Negation of statement is formed by introducing the word "NOT" at a proper place in the statement or by prefixing the statement with the phrase "It is not that the case that"

· If P denotes a statement, then the negation of P write

EX: P: London is a city

¬P: London is not a city

¬P: It is not case that, London is a city

Truth Table

P	¬P
T	F
F	T

Conjunction (∧)

A logical connective that represents conjunction. The statement P and Q, represents is True only. If both P and Q are True.

EX: P: It is raining

Q: It is cold

P∧Q: It is raining and cold

Truth table

P	Q	P∧Q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (∨)

The disjunction of two statements P and Q, if the statement P or Q which is read as P or Q (P∨Q) As the truth value False only when both P and Q has the truth value "False". Otherwise it is "True"

Truth Table

P	Q	P∨Q
T	T	T
T	F	T
F	T	T
F	F	F

EX P: I will wash my cloth.

1 $\neg(P\wedge Q)\vee(\neg P\vee \neg Q)$

2. $\sim P \wedge \sim Q$

3. $(\sim P \vee Q)\wedge(\sim Q \vee \sim P)$

P	Q	$P\wedge Q$	$\neg(P\wedge Q)$ (a)	$\neg P$	$\neg Q$	$\neg P\vee\neg Q$ (b)	$a\vee b$
T	T	T	F	F	F	F	F
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

P	Q	$\sim P$	$\sim Q$	$\sim P\wedge\sim Q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

P	Q	$\sim P$	$(\sim P\vee Q)$	$\sim Q$	$(\sim Q\vee\sim P)$	$(\sim P\vee Q)\wedge(\sim Q\vee\sim P)$
T	T	F	T	F	F	F
T	F	F	F	T	T	F
F	T	T	T	F	T	T
F	F	T	T	T	T	T

Implication (or) conditional statement

If P and Q are any two statements then the statement $P\rightarrow Q$ which is read as "If P then Q" is called conditional statement.

(The statement Q when has the truth value false(F) and the P truth value True(T) at)

The statement $P\rightarrow Q$ has a truth value False when Q has the truth value False and P truth value True, otherwise it has the truth value T

P	Q	$P\rightarrow Q$
T	T	T
T	F	F
F	T	T

EX: P: It is raining

Q: The ground is wet

$P \to Q$: If it is raining then the ground is wet

$P \to Q$ express as the following ways

i Q is necessary for P

ii P is sufficient for Q

iii Q if P

iv P only if Q

v $P \to Q$

Bi-conditional (eqivalance)

If P and Q are any two statement then the statement $P \rightleftarrows Q$ $P \longleftrightarrow Q$ which is read as "P if and only if Q" and abbrevated as "P iff Q" is called Bi-conditional statement

→ the statement $P \leftrightarrow Q$ has the truth value T whenever both P and Q have identical truth values.

Truth Table.

P	Q	$P \rightleftarrows Q$
T	T	T
T	F	F
F	T	F
F	F	T

EX: P: A triangle has 3 sides

Q: A triangle is a polygon with 3 angles.

$P \rightleftarrows Q$: A triangle has 3 sides if and only if it is a polygon with 3 angles

* construct truth tables for the following statement formulas

$(P \to Q) \wedge (Q \to P)$

P	Q	$P \to Q$	$Q \to P$	$(P \to Q) \wedge (Q \to P)$
T	T	T	T	T

ii $\neg(P \wedge Q) \rightleftarrows (\neg P \vee \neg Q)$

$[((\neg P) \wedge (\neg P \wedge R))] \vee [((Q \wedge R) \vee (P \wedge R))]$

$\neg P \rightarrow (P \wedge Q)$

$((P \rightarrow Q) \wedge P(\wedge R)) \rightarrow (P \rightarrow R)$

$[(\neg (P \vee (Q \wedge R))] \longleftrightarrow [(P \vee Q) \wedge (P \vee R)]$

$[(P \rightarrow Q) \wedge (P \wedge R)) \rightarrow (P \rightarrow R)$

P	Q	R	$P \rightarrow Q$	$P \wedge R$	A $(P \rightarrow Q) \wedge (P \wedge R)$	B $(P \rightarrow R)$	$A \rightarrow B$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	F	F	F	T
F	T	T	T	F	F	T	T
F	T	F	T	F	F	T	T
F	F	T	T	F	F	T	T
F	F	F	T	F	F	T	T

ii $\neg(P \wedge Q) \rightleftarrows (\neg P \vee \neg Q)$

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P$	$\neg Q$	$(\neg P \vee \neg Q)$	$\neg(P \wedge Q) \rightleftarrows (\neg P \vee \neg Q)$
T	T	T	F	F	F	F	F
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	F
F	F	F	T	T	T	T	F

iii $[((\neg P) \wedge (\neg P \wedge R))] \vee [((Q \wedge R) \vee (P \wedge R))]$

P	Q	R	$\neg P$	$\neg P \wedge R$	A $[((\neg P) \wedge (\neg P \wedge R))]$	$Q \wedge R$	$P \wedge R$	B $[((Q \wedge R) \vee (P \wedge R))]$	$A \vee B$
T	T	T	F	F	F	T	T	T	T
T	T	F	F	F	F	F	F	F	F
T	F	T	F	F	F	F	T	T	T
T	F	F	F	F	F	F	F	F	F
F	T	T	T	T	T	T	F	T	T
F	T	F	T	F	F	F	F	F	F
F	F	T	T	T	T	F	F	F	T
F	F	F	T	F	F	F	F	F	F

$\neg P \rightarrow (P \wedge Q)$

P	Q	$\neg P$	$P \wedge Q$	$\neg P \rightarrow (P \wedge Q)$
T	T	F	T	T

$[\neg(P\vee(Q\wedge R))] \longleftrightarrow [(P\vee Q)\wedge(P\vee R)]$

P	Q	R	$Q\wedge R$	$(P\vee(Q\wedge R))$	$\neg(P\vee(Q\wedge R))$ (A)	$P\vee Q$	$P\vee R$	$(P\vee Q)\wedge(P\vee R)$ (B)	$A\longleftrightarrow B$
T	T	T	T	T	F	T	T	T	F
T	T	F	F	T	F	T	T	T	F
T	F	T	F	T	F	T	T	T	F
T	F	F	F	T	F	T	T	T	F
F	T	T	T	T	F	T	T	T	F
F	T	F	F	F	T	T	F	F	F
F	F	T	F	F	T	F	T	F	F
F	F	F	F	F	T	F	F	F	F

** Tautology.

A statement formula which is true all possible combinations of truth values of statement variables is called Tautology or universally valid formula

(or)

If all the entries in the last column of the compound statement are True then the compound statement is called Tautology

EX:- $P\vee(\neg P)$

P	$\neg P$	$P\vee(\neg P)$
T	F	T
F	T	T

contradiction

A compound statement which is false for all possible combination of truth values of a statement variables is called contradiction

EX $P\wedge(\neg P)$

P	$\neg P$	$P\wedge(\neg P)$
T	F	F
F	T	F

* contengency

A compound statement which is neither Tautology nor contradiction is called contengency

EX $(P\to Q)\to P\wedge Q$

P	Q	$P \to Q$	$P \cap Q$	$(P \to Q) \to (P \cap Q)$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	F	F

prove that the proposition $(P \wedge Q) \to (P \vee Q)$ is tautology.

P	Q	$P \cap Q$	$P \vee Q$	$(P \cap Q) \to (P \vee Q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

$\therefore (P \cap Q) \to (P \vee Q)$ is Tautology.

show that $((P \vee Q) \wedge \neg Q) \to P$ is tautology or not.

P	Q	$P \vee Q$	$\neg Q$	$((P \vee Q) \wedge \neg Q)$	$((P \vee Q) \cap \neg Q) \to P$
T	T	T	F	F	T
T	F	T	T	T	T
F	T	T	F	F	T
F	F	F	T	F	T

$\therefore ((P \vee Q) \wedge \neg Q) \to P$ is Tautology

show that $[(P \to Q) \wedge (Q \to R)] \to (P \to R)$ is tautology

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \wedge (Q \to R)$ (A)	$(P \to R)$ (B)	$A \to B$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$\therefore [(P \to Q) \wedge (Q \to R)] \to (P \to R)$ is Tautology prove that $(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$ Tautology or not.

P	Q	$P \rightarrow Q$	$\neg P$	$\neg P \vee Q$	$(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

$\therefore (P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$ is Tautology

Show that $\{(P \rightarrow (Q \vee R)) \wedge (\neg Q)\} \rightarrow (P \rightarrow R)$ is tautology

P	Q	R	$Q \vee R$	$P \rightarrow (Q \vee R)$	$\neg Q$	(A) $\{(P \rightarrow (Q \vee R)) \wedge (\neg Q)\}$	(B) $(P \rightarrow R)$	$A \rightarrow B$
T	T	T	T	T	F	F	T	T
T	T	F	T	T	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	F	F	T	T
F	T	F	T	T	F	F	T	T
F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T

$\therefore \{(P \rightarrow (Q \vee R)) \wedge (\neg Q)\} \rightarrow (P \rightarrow R)$ is Tautology

show that $[(Q \vee (P \wedge \neg Q)] \vee (\neg P \wedge \neg Q)$ is tautology

show that $\{(P \rightarrow (Q \vee R)) \wedge (Q)\} \rightarrow (P \rightarrow R)$ is tautology or not

prove that given statement Formula.

$(P \wedge Q) \wedge (\neg P \wedge Q)$ is contradiction or not

$(p \vee q) \rightarrow (P \Leftrightarrow Q)$ is a contengency.

$[(Q \vee (P \wedge \neg Q)] \vee (\neg P \wedge \neg Q)$

P	Q	$\neg Q$	$(P \wedge \neg Q)$	(A) $(Q \vee (P \wedge \neg Q))$	$\neg P$	(B) $\neg P \wedge \neg Q$	$A \vee B$
T	T	F	F	T	F	F	T
T	F	T	T	T	F	F	T
F	T	F	F	T	T	F	T
F	F	T	F	F	T	F	F

$\{(P \to (Q \vee R)) \wedge (Q)\} \to (P \to R)$

P	Q	R	$Q \vee R$	$P \to (Q \vee R)$	A: $(P \to (Q \vee R)) \wedge (Q)$	B: $(P \to R)$	$A \to B$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	F
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	T
F	F	F	F	T	F	T	T

$\therefore \{(P \to (Q \vee R)) \wedge (Q)\} \to (P \to R)$ is not tautology.

$(P \wedge Q) \wedge (\neg P \wedge Q)$

P	Q	$P \wedge Q$	$\neg P \wedge Q$	$(P \wedge Q) \wedge (\neg P \wedge Q)$
T	T	T	F	F
T	F	F	T	F
F	T	F	T	F
F	F	F	T	F

$\therefore (P \wedge Q) \wedge (\neg P \wedge Q)$ is contradiction

$(p \vee q) \to (p \to q)$

p	q	$p \vee q$	$p \to q$	$(p \vee q) \to (p \to q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	T

$\therefore (p \vee q) \to (p \to q)$ is a contengency

Equivalence ($\Leftrightarrow$)

Two statement formulas A and B is said to be equivalence if and only if both A and B having the same truth values. It is denoted by $A \Leftrightarrow B$ and read as A is equivalent to B

Ex $P \to Q \Leftrightarrow \neg P \vee Q$

P	Q	$P \to Q$
T	T	T
T	F	F

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	T
T	F	F	F

conclusion: Last column of the above truth tables are same

$\therefore A \Leftrightarrow B$ are equivalent

show that the statement formula orequivalence using Truth Table.

$(P \to Q) \Leftrightarrow (\neg Q \to \neg P)$

Given that

$(P \to Q) \Leftrightarrow (\neg Q \to \neg P)$

These statement formula in the form of $A \Leftrightarrow B$ (or) $A \equiv B$

Here $A : P \to Q$

$B : \neg Q \to \neg P$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\neg Q$	$\neg P$	$\neg Q \to \neg P$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

conclusion: Last column of the above truth tables are same

$\therefore A \Leftrightarrow B$ are equivalent.

show that the following statement formulas or equivalence using Truth tables

(i) $(P \wedge (\neg P)) \vee Q \Leftrightarrow Q$ TF7F

(ii) $((P \to Q) \wedge (R \to Q)) \Leftrightarrow ((P \vee R) \to Q)$ $^{\epsilon 7,3F}$

(iii) $(P \vee Q) \Leftrightarrow \neg(\neg P \wedge \neg Q)$ 77FF

(iv) $(P \to (Q \to R)) \Leftrightarrow ((P \wedge Q) \to R)$ 77,F

$(P \wedge (\neg P)) \vee Q \Leftrightarrow Q$

P	Q	$\neg P$	$(P \wedge (\neg P))$	$(P \wedge (\neg P)) \vee Q$
T	T	F	F	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	F

conclusion: Last column of the above truth tables are same

$\therefore (P \wedge (\neg P)) \vee Q \Leftrightarrow Q$ is equivalence.

ii $((P \to Q) \wedge (R \to Q)) \Leftrightarrow ((P \vee R) \to Q)$

P	Q	R	$P \to Q$	$R \to Q$	$((P \to Q) \wedge (R \to Q))$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	F
F	F	F	T	T	T

P	Q	R	$P \vee R$	$((P \vee R) \to Q)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	F
T	F	F	T	F
F	T	T	T	T
F	T	F	F	T
F	F	T	T	F
F	F	F	F	T

conclusion : last column of the above truth tables are same

$\therefore ((P \to Q) \wedge (R \to Q)) \Leftrightarrow ((P \vee R) \to Q)$ is equivalent

$(P \vee Q) \Leftrightarrow \neg(\neg P \wedge \neg Q)$

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$\neg(\neg P \wedge \neg Q)$
T	T	F	F	F	T
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	T	F

conclusion: last column of the above truth table's are same

$\therefore (P \vee Q) \Leftrightarrow \neg(\neg P \wedge \neg Q)$ is equivalent.

$(P \to (Q \to R)) \Leftrightarrow ((P \wedge Q) \to R)$

P	Q	R	$Q \to R$	$(P \to (Q \to R))$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T

$(P \wedge Q)$	$((P \wedge Q) \to R)$
T	T
F	F
F	T
F	T
F	T

Equivalence of Formulas

I_1: Double Negation law

$\neg(\neg P) \equiv P$

I_2: Idempotent law

(i) $P \wedge P \equiv P$

(ii) $P \vee P \equiv P$

I_3: commutative law

(i) $P \wedge Q \equiv Q \wedge P$

(ii) $P \vee Q \equiv Q \vee P$

I_4: Associative law

(i) $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$

(ii) $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$

I_5: Distributive law

(i) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

(ii) $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

I_6: De-morgan's law

(i) $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$

(ii) $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$

I_7: Identity law

(i) $P \wedge T \equiv P$

(ii) $P \vee F \equiv F$

$F \vee P \equiv P$

I_8: Inverse law

(i) $P \wedge (\neg P) \equiv F$

$\neg P \vee F \equiv \neg P$

$T \vee F \equiv T$

(ii) $P \vee (\neg P) \equiv T$

I_9 Dominance law

(i) $P \wedge F \equiv F$

(ii) $P \vee T \equiv T$

** I_{11}: conditional disjunction law

$$P \rightarrow Q \equiv \neg P \vee Q$$

I_{12} : Biconditional law :

$$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$$
$$P \leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$$

i) prove that $P \vee (P \wedge Q) \equiv P$ (or) $P \vee (P \wedge Q) \Leftrightarrow P$ is equivalance without truth-table.

Given that

According to Absorption law

$$\boxed{P \vee (P \wedge Q) \equiv P}$$

Show that $(P \rightarrow (Q \wedge R)) \equiv ((P \rightarrow Q) \wedge (P \rightarrow R))$ is equivalance using without truth table.

Given that.

Now,

$(P \rightarrow (Q \wedge R)) \equiv$

According to conditional disjunctional law. $P \rightarrow Q \equiv \neg P \vee Q$

$$P \rightarrow (Q \wedge R) \equiv \neg P \vee (Q \wedge R)$$

According to Distributive law $\equiv (\neg P \vee Q) \wedge (\neg P \vee R)$ $[P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)]$

According to conditional disjunctional law $P \rightarrow Q \equiv \neg P \vee Q$

$$\equiv (P \rightarrow Q) \wedge (P \rightarrow R)$$

$$\therefore (P \rightarrow (Q \wedge R)) \equiv ((P \rightarrow Q) \wedge (P \rightarrow R)).$$

prove that $((P \rightarrow Q) \wedge (R \rightarrow Q)) \Leftrightarrow ((P \vee R) \Rightarrow Q)$ is equivalance using with out truth table.

Given that

$((P \rightarrow Q) \wedge (R \rightarrow Q))$

According to conditional disjunctional law $P \rightarrow Q \equiv \neg P \vee Q$

A/C Distributive law $\equiv (\neg P \vee Q) \wedge (\neg R \vee Q)$

$\equiv (\neg P \wedge \neg R) \vee Q$ $(P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R))$

By using conditional disjunction law $P \to Q \equiv \neg P \vee Q$

$$\equiv (P \vee R) \to Q.$$

Prove that $((\neg P \wedge (\neg Q \wedge R)) \vee ((Q \wedge R) \vee (P \wedge R))) \Leftrightarrow R$

Given that

$$((\neg P \wedge (\neg Q \wedge R)) \vee ((Q \wedge R) \vee (P \wedge R))) \Leftrightarrow R$$

$$\neg P \wedge (\neg Q \wedge R) \vee [(Q \wedge R) \vee (P \wedge R))$$

According to Distributive law $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \vee R)$

$$\equiv (\neg P \wedge (\neg Q \wedge R)) \vee ((Q \vee P) \wedge R))$$

~~Ak to~~ ~~Associative~~ law $\cancel{P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R}$

$$\cancel{\equiv ((\neg P \wedge \neg Q) \vee (Q \vee P) \wedge R}$$

$$\equiv ((\neg P \wedge \neg Q) \wedge R) \vee ((Q \vee P) \wedge R)$$

According to Associative law $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$

$$\equiv ((\neg P \wedge \neg Q) \vee (Q \vee P)) \wedge R$$

According to Demorgan's law $\neg(P \vee Q) = \neg P \wedge \neg Q$

$$\equiv (\neg(P \vee Q) \vee (Q \vee P)) \wedge R$$

According to commutative law $Q \vee P = P \vee Q$

$$\equiv (\neg(P \vee Q) \vee (P \vee Q)) \wedge R$$

According to Inverse law $P \vee (\neg P) = P$

$$\equiv T \wedge R \to I_8$$

According to Identity law $P \wedge T = P$.

$$\equiv R \to I_7$$

$(P \to R) \wedge (Q \to R) \Leftrightarrow ((P \vee Q) \to R)$

$(P \to (Q \to R)) \equiv (P \wedge Q) \to R$

$(P \to Q) \to Q \Leftrightarrow P \vee Q.$

Given

$$(P \to R) \wedge (Q \to R) \Leftrightarrow ((P \vee Q) \to R)$$

$(P \to R) \wedge (Q \to R)$ ($\because$ conditional Disjunction law)

$(\neg P \vee R) \wedge (\neg Q \vee R)$ ($\because$ Distributive)

$(\neg P \wedge \neg Q) \vee R$ ($\because$ Demorgan's) show that $P \to (Q \to R) \equiv (P \wedge Q) \to R$

$P \to (Q \to R)$ (∵ conditional Disjunction law

$\neg P \vee (\neg Q \vee R)$ (∵ Associative)

$(\neg P \vee \neg Q) \vee R$

$\neg (P \wedge Q) \vee R$ (∵ Demorgans)

$P \wedge Q \to R$ (∵ conditional Disjunction law).

show that $((P \to Q) \to Q) \Leftrightarrow P \vee Q$

$(P \to Q) \to Q$.

$\neg (P \to Q) \vee Q$

$\neg (\neg P \vee Q) \vee Q$

$(\neg (\neg P) \wedge \neg Q) \vee Q$

$(P \wedge \neg Q) \vee Q$

$(P \vee Q) \wedge (\neg Q \vee Q)$

$(P \vee Q) \wedge F$

$(P \vee Q)$

Well formed formulas

A well formed formula is a string of symbols form a formal language that is synthatically correct according to rules

Atomic statement

If 'P' is a statement variable then P is a well formed formula

⇒ P is a statement $\neg P$ is a well formed formula

paranthesis

paranthesis are used to ensure the correct grouping and order of operations in a formula.

Ex: $(P \wedge (Q \to R))$ is a well formed formula

⇒ If A & B are well formed (formulas) statements then A & B $(A \wedge B)$, $(A \vee B)$, $(A \to B)$ and $(A \leftrightarrow B)$ are well formed formulas

Ex: i) $\neg (P \wedge Q)$, $\neg (P \vee Q)$

ii) $(P \to (P \vee Q))$

iii) $[(P \to Q) \wedge (Q \to R)]$

Duality law:

Two formulas A and A^* are said to be duals of each other obtain $\neg(P \vee Q) \rightleftarrows (P \wedge Q)$

$P \leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$

$\Leftrightarrow [\neg(P \vee Q) \wedge (P \wedge Q)] \vee [\neg\neg(P \vee Q) \wedge (\neg(P \wedge Q))]$

$\Leftrightarrow [(\neg P \wedge \neg Q) \wedge (P \wedge Q)] \vee [(P \vee Q) \wedge (\neg P \vee \neg Q)]$

$\Leftrightarrow (\neg P \wedge \neg Q \wedge P \wedge Q) \vee [P \wedge (\neg P \vee \neg Q) \vee (Q \wedge (\neg P \vee \neg Q))]$

$\Leftrightarrow (\neg P \wedge \neg Q \wedge P \wedge Q) \vee ((P \wedge \neg P) \vee (P \wedge \neg Q)) \vee ((Q \wedge \neg P) \vee (Q \wedge \neg Q))$

$\Leftrightarrow (\neg P \wedge \neg Q \wedge P \wedge Q) \vee (P \wedge \neg P) \vee (P \wedge \neg Q) \vee (Q \wedge \neg P) \vee (Q \wedge \neg Q)$

sum of elementary products.

obtain $\neg(P \rightarrow (Q \wedge R))$

$\neg(P \rightarrow (Q \wedge R))$

$\Leftrightarrow \neg(\neg P \vee (Q \wedge R))$

$\Leftrightarrow P \wedge (\neg Q \vee R)$

$\Leftrightarrow (P \wedge \neg Q) \vee (P \wedge R)$

sum of elementary products.

Conjuctive Normal forms (CNF)

A formula which consists of a product of elementary sums is called a conjuctive Normal forms

CNF = product of elementary sum

EX:- $(P \vee Q) \wedge (\neg P \vee Q) \wedge (P \vee \neg Q) \wedge (\neg P \vee \neg Q)$

obtain CNF of $P \wedge (P \rightarrow Q)$

$\Leftrightarrow P \wedge (\neg P \vee Q)$ $\quad P \rightarrow Q \equiv \neg P \vee Q$

$(\Leftrightarrow (P \vee \neg P) \wedge (P \vee Q))$

product of elementary sum.

obtain CNF of $((P \rightarrow Q) \wedge \neg Q) \rightarrow \neg P$.

$((P \rightarrow Q) \wedge \neg Q) \rightarrow \neg P$

$\Leftrightarrow \neg((P \rightarrow Q) \wedge \neg Q)) \vee \neg P$

$\Leftrightarrow \neg((\neg P \vee Q) \wedge \neg Q)) \vee \neg P$

$\Leftrightarrow ((P \wedge Q) \vee Q) \vee \neg P$

$\Leftrightarrow ((P \vee Q) \wedge (\neg Q \vee Q)) \vee \neg P$

$\Leftrightarrow (P \vee Q \vee \neg P) \wedge (\neg Q \vee Q \vee \neg P)$

product of elementary sum.

Obtain CNF of $((P \to Q) \wedge \neg P) \to \neg Q$.

$$((P \to Q) \wedge \neg P) \to \neg Q$$
$$\Leftrightarrow \neg((P \to Q) \wedge \neg P)) \vee \neg Q$$
$$\Leftrightarrow \neg((\neg P \vee Q) \wedge \neg P)) \vee \neg Q$$
$$\Leftrightarrow ((P \wedge Q) \vee P) \vee \neg Q$$
$$\Leftrightarrow ((P \vee Q) \wedge (\neg P \vee P)) \vee \neg Q$$
$$\Leftrightarrow (P \vee Q \vee \neg Q) \wedge (\neg P \vee P \vee \neg Q)$$

product of elementary sum.

obtain CNF of $\neg(P \vee Q) \rightleftarrows (P \wedge Q)$

$$\Leftrightarrow \neg(P \vee Q) \to (P \wedge Q) \wedge ((P \wedge Q) \to \neg(P \vee Q))$$

$P \leftrightarrow Q = (P \to Q) \wedge (Q \to P)$

$$\Leftrightarrow \neg(\neg(P \vee Q)) \vee (P \wedge Q)) \wedge (\neg(P \wedge Q) \vee \neg(P \vee Q))$$
$$\Leftrightarrow ((P \vee Q) \vee (P \wedge Q)) \wedge ((\neg P \vee \neg Q) \vee (\neg P \wedge \neg Q))$$
$$\Leftrightarrow ((P \vee Q \vee P) \wedge (P \vee Q \vee Q)) \wedge ((\neg P \vee \neg Q \vee \neg P) \wedge (\neg P \vee \neg Q \vee \neg Q))$$
$$\Leftrightarrow (P \vee Q) \wedge (P \vee Q) \wedge (\neg P \vee \neg Q) \wedge (\neg P \vee \neg Q)$$
$$\Leftrightarrow (P \vee Q) \wedge (\neg P \vee \neg Q)$$

product of elementary sum.

Principle Disjunction Normal Form (PDNF) (or) sum of products cononical form.

A formula which consists of sum of min-terms only is known as PDNF

method to finding PDNF

By Truth Table.

For every Truth value T of the given formula, select the min-term which also have the value T

using the truth obtain the PDNF of the following statement formulas

(i) $P \to Q$ (ii) $P \longleftrightarrow Q$ (iii) $P \vee Q$ (iv) $\neg(P \wedge Q)$

P	Q	$P \to Q$	minterms
T	T	T	$P \wedge Q$
T	F	F	-
F	T	T	$\neg P \wedge Q$
F	F	T	$\neg P \wedge \neg Q$.

$$\Leftrightarrow (P \wedge Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$$

$P \leftrightarrow Q$

P	Q	$P \leftrightarrow Q$	min-terms
T	T	T	$P \wedge Q$
T	F	F	–
F	T	F	–
F	F	T	$\neg P \wedge \neg Q$

$\Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$

sum of min-terms.

$P \vee Q$

P	Q	$P \vee Q$	min-terms
T	T	T	$P \wedge Q$
T	F	T	$P \wedge \neg Q$
F	T	T	$\neg P \wedge Q$
F	F	F	–

$\Leftrightarrow (P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q)$

sum of min terms

$\neg(P \wedge Q)$

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	Min-Terms
T	T	T	F	$\cancel{P \wedge Q}$ -
T	F	F	T	$P \wedge \neg Q$
F	T	F	T	$\neg P \wedge Q$
F	F	F	T	$\neg P \wedge \neg Q$

$\Leftrightarrow (P \wedge \neg Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$ sum of minterms

Obtain the PDNF of $(\neg P \to R) \wedge (Q \leftrightarrow P)$ using Truth table.

$P \to [(P \wedge Q) \wedge \neg(\neg Q \vee \neg P)]$

* Show that the canonical form of sum of products (PDNF) of the proposition formula

$((\neg X \wedge Y) \vee \neg Z) \vee (X \wedge Y \wedge Z)$ is $\sum(0, 2, 3, 4, 6, 7)$

Truth Table

x	y	z	$\neg x$	$\neg x \wedge y$	$\neg z$	$(\neg x \wedge y) \vee \neg z$ (A)	$x \wedge y$	$x \wedge y \wedge z$ (B)	$A \vee B$	Min-Terms
T	T	T	F	F	F	F	T	T	T	$x \wedge y \wedge z$
T	T	F	F	F	T	T	T	F	T	$x \wedge y \wedge \neg z$
T	F	T	F	F	F	F	F	F	F	-
T	F	F	F	F	T	T	F	F	T	$x \wedge \neg y \wedge \neg z$
F	T	T	T	T	F	T	F	F	T	$\neg x \wedge y \wedge z$
F	T	F	T	T	T	T	F	F	T	$\neg x \wedge y \wedge \neg z$
F	F	T	T	F	F	F	F	F	F	-
F	F	F	T	F	T	T	F	F	T	$\neg x \wedge \neg y \wedge \neg z$

$\Leftrightarrow (x \wedge y \wedge z) \vee (x \wedge y \wedge \neg z) \vee (\neg x \wedge y \wedge \neg z) \vee (\neg x \wedge y \wedge z) \vee (\neg x \wedge y \wedge \neg z)$
$\vee (\neg x \wedge \neg y \wedge \neg z)$

$T = 1$
$F = 0$

$\Leftrightarrow (TTT) \vee (TTF) \vee (TFF) \vee (FTT) \vee (FTF) \vee (FFF)$

$\Leftrightarrow (111) \vee (110) \vee (100) \vee (011) \vee (010) \vee (000)$

$\Leftrightarrow 7 \vee 6 \vee 4 \vee 3 \vee 2 \vee 0$

$\Leftrightarrow \Sigma(0, 2, 3, 4, 6, 7)$

** Show that the PDNF of $[P \vee (\neg P \rightarrow (Q \vee (\neg Q \rightarrow R)))]$ is Σ (A) $(1, 2, 3, 4, 5, 6, 7)$

P	Q	R	$\neg P$	$\neg Q$	$\neg Q \rightarrow R$	$(Q \vee (\neg Q \rightarrow R))$	$\neg P \rightarrow (Q \vee (\neg Q \rightarrow R))$ (B)	$A \vee B$	Min-Terms
T	T	T	F	F	T	T	T	T	$P \wedge Q \wedge R$
T	T	F	F	F	T	T	T	T	$P \wedge Q \wedge \neg R$
T	F	T	F	T	T	T	T	T	$P \wedge \neg Q \wedge R$
T	F	F	F	T	F	F	T	T	$P \wedge \neg Q \wedge \neg R$
F	T	T	T	F	T	T	T	T	$\neg P \wedge Q \wedge R$
F	T	F	T	F	T	T	T	T	$\neg P \wedge Q \wedge \neg R$
F	F	T	T	T	T	T	T	T	$\neg P \wedge \neg Q \wedge R$
F	F	F	T	T	F	F	F	F	-

$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge \neg R) \vee$
$(\neg P \wedge \neg Q \wedge R)$

$\Leftrightarrow (TTT) \vee (TTF) \vee (TFT) \vee (TFF) \vee (FTT) \vee (FTF) \vee (FFT)$

$\Leftrightarrow (111) \vee (110) \vee (101) \vee (100) \vee (011) \vee (010) \vee (001)$

$\Leftrightarrow 7 \vee 6 \vee 5 \vee 4 \vee 3 \vee 2 \vee 1$

$\Leftrightarrow \Sigma(1, 2, 3, 4, 5, 6, 7)$

$(\neg P \to R) \wedge (Q \leftrightarrow P)$

P	Q	R	$\neg P$	$(\neg P \to R)$ (A)	$(Q \leftrightarrow P)$ (B)	$A \wedge B$	min-Terms
T	T	T	F	T	T	T	$P \wedge Q \wedge R$
T	T	F	F	T	T	T	$P \wedge Q \wedge \neg R$
T	F	T	F	T	F		
T	F	F	F	T	F		
F	T	T	T	T	F		
F	T	F	T	F	F		
F	F	T	T	T	T		
F	F	F	T	F	T		

$P \to [(P \wedge Q) \wedge \neg(\neg Q \vee \neg P)]$

(A) (B)

P	Q	$P \wedge Q$	$\neg Q$	$\neg P$	$\neg Q \vee \neg P$	$\neg(\neg Q \vee \neg P)$	$[(P \wedge Q) \wedge \neg(\neg Q \vee \neg P)]$	$A \to B$
T	T	T	F	F	F	T	T	T
T	F	F	T	F	T	F	F	F
F	T	F	F	T	T	F	F	T
F	F	F	T	T	T	F	F	T

min-Terms

$P \wedge Q$.

–

$\neg P \wedge Q$

$\neg P \wedge \neg Q$

$\iff (P \wedge Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$

sum of min-Terms

Rules without using Truth Table.

→ obtain DNF

→ Drop elementary products which are contradiction

EX $P \wedge \neg P = F$

→ If P_i and $\neg P_i$ are missing in an elementary product α, replace by α with $(\alpha \wedge P_i) \vee (\alpha \wedge \neg P_i)$ $[\because \alpha \wedge T \equiv T]$

→ Repeat the above steps until all elementary products to sum of min-Terms problems

Obtain PDNF of $P \leftrightarrow Q$ without using Truth Table.

$P \leftrightarrow Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$ $\quad P \wedge \neg P \equiv F$

$\Leftrightarrow (\neg P \vee Q) \wedge (\neg Q \vee P)$ $\quad Q \wedge \neg Q \equiv F$

$\Leftrightarrow ((\neg P \vee Q) \wedge \neg Q) \vee (\neg P \vee Q) \wedge P)$

$\Leftrightarrow (\neg P \wedge \neg Q) \vee (Q \wedge \neg Q) \vee (\neg P \wedge P) \vee (Q \wedge P)$

$\Leftrightarrow (\neg P \wedge \neg Q) \vee F \vee F \vee (Q \wedge P)$

$\Leftrightarrow (\neg P \wedge \neg Q) \vee (Q \wedge P)$

sum of min-terms.

Obtain PDNF of $\neg P \vee Q$

$\neg P \vee Q \Leftrightarrow (\neg P \wedge T) \vee (Q \wedge T)$ $\quad P \vee \neg P \equiv T$

$\Leftrightarrow (\neg P \wedge (Q \vee \neg Q)) \vee (Q \wedge (P \vee \neg P))$ $\quad Q \vee \neg Q \equiv T$

$\Leftrightarrow \underline{(\neg P \wedge Q)} \vee (\neg P \wedge \neg Q) \vee (Q \wedge P) \vee \underline{(Q \wedge \neg P)}$

$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (Q \wedge P)$

sum of minterms.

Find the PDNF of $P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$ without using Truth table

$P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$

$\Leftrightarrow P \rightarrow ((\neg P \vee Q) \wedge (Q \wedge P))$

$\Leftrightarrow \neg P \vee ((\neg P \vee Q) \wedge (Q \wedge P))$ $\quad P \cap P \equiv P$

$\Leftrightarrow \neg P \vee (\neg P \wedge (Q \wedge P) \vee Q (Q \wedge P))$ $\quad Q \cap Q \equiv Q$

$\Leftrightarrow \neg P \vee ((\neg P \wedge Q \wedge P) \vee (Q \wedge Q \wedge P))$ $\quad F \wedge P \equiv F$

$\Leftrightarrow \neg P \vee ((F \wedge Q) \vee (Q \wedge P))$ $\quad F \vee P \equiv P$

$\Leftrightarrow \neg P \vee (F \vee (Q \wedge P))$ $\quad T \vee P \equiv P$

$\Leftrightarrow \neg P \vee (Q \wedge P)$

$\Leftrightarrow (\neg P \wedge T) \vee (Q \wedge P)$

$\Leftrightarrow (\neg P \wedge (Q \vee \neg Q)) \vee (Q \wedge P)$

$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (Q \wedge P)$

Principle conjuctive Normal Form(or) product of sums cononical form.

A formula which consists of product of max terms only is called as PCNF of given formula known

Method to obtain PCNF

By Truth Table

For every truth value F of the given formula, select the max term which also has the value F.

Find the PCNF for the following statement formulas

(a) $P \leftrightarrow Q$ (b) $P \vee Q \leftrightarrow (P \cap Q)$ (c) $\neg P \wedge \neg Q$

P	Q	$P \leftrightarrow Q$	Maxterms
T	T	T	–
T	F	F	$P \vee \neg Q$
F	T	F	$\neg P \vee Q$
F	F	T	–

$\therefore P \leftrightarrow Q \Leftrightarrow (P \vee \neg Q) \wedge (\neg P \vee Q)$

product of max-terms

$P \vee Q \rightarrow (P \cap Q)$

P	Q	$P \vee Q$	$P \cap Q$	$P \vee Q \rightarrow (P \cap Q)$	Max-terms
T	T	T	T	T	–
T	F	T	F	F	$P \vee \neg Q$
F	T	T	F	F	$\neg P \vee Q$
F	F	F	F	T	–

$\therefore P \vee Q \rightarrow (P \cap Q) \Leftrightarrow (P \vee \neg Q) \wedge (\neg P \vee Q)$

product of max-terms

$\neg P \wedge \neg Q$

P	Q	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	Max-terms
T	T	F	F	F	$P \vee Q$
T	F	F	T	F	$P \vee \neg Q$
F	T	T	F	F	$\neg P \vee Q$
F	F	T	T	F	–

$\therefore \neg P \wedge \neg Q \Leftrightarrow (P \vee Q) \wedge (P \vee \neg Q) \wedge (\neg P \vee Q)$

product of max-terms.

Obtain the PCNF of $(\neg P \vee Q) \wedge (P \vee \neg Q)$ using Truth Table

Obtain the PCNF of $(\neg P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (P \vee \neg Q)$

2. Show that the PCNF of the formula $P \rightarrow (Q \cap R))\cap((\neg P \rightarrow (\neg Q \cap \neg R))$ is

P	Q	R	$Q\wedge R$	$P\to(Q\wedge R)$	$\neg Q$	$\neg R$	$\neg Q\wedge\neg R$	$\neg P$	$\neg P\to(\neg Q\wedge\neg R)$
T	T	T	T	T	F	F	F	F	T
T	T	F	F	F	F	T	F	F	T
T	F	T	F	F	T	F	F	F	T
T	F	F	F	F	T	T	T	F	T
F	T	T	T	T	F	F	F	T	F
F	T	F	F	T	F	T	F	T	F
F	F	T	F	T	T	F	F	T	F
F	F	F	F	T	T	T	T	T	T

$P\to(Q\wedge R)\wedge(\neg P\to(\neg Q\wedge\neg R))$	Max-Terms
T	-
F	$P\vee Q\vee\neg R$
F	$P\vee\neg Q\vee R$
F	$P\vee\neg Q\vee\neg R$
F	$\neg P\vee Q\vee R$
F	$\neg P\vee Q\vee\neg R$
F	$\neg P\vee\neg Q\vee R$
T	-

$\Leftrightarrow (P\vee Q\vee\neg R)\wedge(P\vee\neg Q\vee R)\wedge(P\vee\neg Q\vee\neg R)\wedge(\neg P\vee Q\vee R)\wedge(\neg P\vee Q\vee\neg R)\wedge(\neg P\vee\neg Q\vee R)$

$\Leftrightarrow (TTF)\wedge(TFT)\wedge(TFF)\wedge(FTT)\wedge(FTF)\wedge(FFT)$

$F=1$
$T=0$

~~$\Leftrightarrow (110)\wedge(101)\wedge(100)\wedge(011)$~~

$\Leftrightarrow (001)\wedge(010)\wedge(011)\wedge(100)\wedge(101)\wedge(110)$

$\Leftrightarrow 1\wedge 2\wedge 3\wedge 4\wedge 5\wedge 6$

$\Leftrightarrow \Pi(1,2,3,4,5,6)$

obtain PCNF of $(\neg P\to R)\wedge(Q\leftrightarrow P)$ using Truth Table

$P\vee(\neg P\to(Q\vee(\neg Q\to R)))$

obtain the cononical product of sum of the proposition formulae.

i) $\neg X\wedge(\neg Y\vee Z)$

ii) $(P\wedge Q)\vee(\neg Q\wedge R)$

$\neg x \wedge (\neg y \vee z)$

x	y	z	$\neg y$	$\neg y \vee z$	$\neg x$	$\neg x \wedge (\neg y \vee z)$	Max-Terms
T	T	T	F	T	F	F	$x \vee y \vee z$
T	T	F	F	F	F	F	$x \vee y \vee \neg z$
T	F	T	T	T	F	F	$x \vee \neg y \vee z$
T	F	F	T	T	F	F	$x \vee \neg y \vee \neg z$
F	T	T	F	T	T	T	–
F	T	F	F	F	T	F	$\neg x \vee y \vee z$
F	F	T	T	T	T	T	–
F	F	F	T	T	T	T	–

$\therefore (\neg x \wedge (\neg y \vee z))$

$$\Leftrightarrow (x \vee y \vee z) \wedge (x \vee y \vee \neg z) \wedge (x \vee \neg y \vee z) \wedge (x \vee \neg y \vee \neg z) \wedge (\neg x \vee \neg y \vee z)$$

product of max-Terms

$(P \wedge Q) \vee (\neg Q \wedge R)$

P	Q	R	$\neg Q$	$\neg Q \wedge R$	$P \wedge Q$	$(P \wedge Q) \vee (\neg Q \wedge R)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	T	T	F	T
T	F	F	T	F	F	F
F	T	T	F	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	F	T
F	F	F	T	F	F	F

obtain PCNF of $(P\wedge Q)\vee(\neg P\wedge\neg Q)$ without Truth Table.

$(P\wedge Q)\vee(\neg P\wedge\neg Q)$

$\Leftrightarrow ((P\wedge Q)\vee\neg P)\wedge((P\wedge Q)\vee\neg Q)$

$\Leftrightarrow (P\vee\neg P)\wedge(Q\vee\neg P)\wedge(P\vee\neg Q)\wedge(Q\vee\neg Q)$

$\Leftrightarrow F\wedge(Q\vee\neg P)\wedge(P\vee\neg Q)\wedge F$

$\Leftrightarrow (Q\vee\neg P)\wedge(P\vee\neg Q)$

product of max-terms.

obtain PCNF of $(\neg P\rightarrow Q)\wedge(Q\rightleftarrows P)$

$(\neg P\rightarrow Q)\wedge(Q\rightleftarrows P)$

$\Leftrightarrow (P\vee Q)\wedge(Q\rightarrow P)\wedge(P\rightarrow Q)$

$\Leftrightarrow (P\vee Q)\wedge(\neg Q\vee P)\wedge(\neg P\vee Q)$

product of max-terms.

obtain PCNF of $\neg(P\vee Q)$

$\Leftrightarrow \neg P\wedge\neg Q$

$\Leftrightarrow (\neg P\vee F)\wedge(\neg Q\vee F)$ $\quad Q\wedge\neg Q\equiv F$

$\Leftrightarrow (\neg P\vee(Q\wedge\neg Q))\wedge(\neg Q\vee(P\wedge\neg P))$

$\Leftrightarrow ((\neg P\vee Q)\wedge(\neg P\vee\neg Q))\wedge((\neg Q\vee P)\wedge(\neg Q\vee\neg P))$

$\Leftrightarrow (\neg P\vee Q)\wedge(\neg P\vee\neg Q)\wedge(\neg Q\vee P)$

product of max-terms

** obtain PCNF for $(\neg P\rightarrow R)\wedge(Q\rightleftarrows P)$

$Q\wedge\neg Q\equiv F$

$\neg P\vee F\equiv\neg P$

$T\vee F\equiv T$

$\Leftrightarrow (P\vee R)\wedge((Q\rightarrow P)\wedge(P\rightarrow Q))$

$\Leftrightarrow (P\vee R)\wedge((\neg Q\vee P)\wedge(\neg P\vee Q))$

$\Leftrightarrow ((P\vee R)\vee F)\wedge((\neg Q\vee P)\vee F)\wedge((\neg P\vee Q)\vee F)$

$\Leftrightarrow ((P\vee R)\vee(Q\wedge\neg Q))\wedge((\neg Q\vee P)\vee(R\wedge\neg R))\wedge((\neg P\vee Q)\vee(R\wedge\neg R))$

$\Leftrightarrow (P\vee R\vee Q)\wedge(P\vee R\vee\neg Q)\wedge(\neg Q\vee P\vee R)\wedge(\neg Q\vee P\vee\neg R)\wedge((\neg P\vee Q\vee R)\wedge$
$(\neg P\vee Q\vee\neg R)$

$\Leftrightarrow (P\vee R\vee Q)\wedge(P\vee R\vee\neg Q)\wedge(\neg Q\vee P\vee\neg R)\wedge(\neg P\vee Q\vee R)\wedge(\neg P\vee Q\vee\neg R)$

product of max-terms.

obtain PCNF of $(P\wedge Q)\vee(\neg P\vee Q)$

$\Leftrightarrow (P\wedge Q \vee \neg P) \wedge (P\wedge Q \vee \neg Q)$

$\Leftrightarrow (P \vee \neg P) \wedge (Q \vee \neg P) \wedge (P \vee \neg Q) \wedge (Q \vee \neg Q)$

$\Leftrightarrow F \wedge (Q \vee \neg P) \wedge (P \vee \neg Q) \wedge F$

$\Leftrightarrow (Q \vee \neg P) \wedge (P \vee \neg Q)$

product of max terms

obtain PCNF of $(P\wedge Q) \vee (\neg P \wedge R)$

$(P\wedge Q) \vee (\neg P \wedge R)$

$\Leftrightarrow ((P\wedge Q) \vee F) \vee ((\neg P \wedge R) \vee F)$

$\Leftrightarrow ((P\wedge Q) \vee (R \wedge \neg R)) \vee ((\neg P \wedge R) \vee (Q \wedge \neg Q))$

$\Leftrightarrow (P \vee Q \vee R) \wedge (P \vee Q \vee \neg R) \wedge (\neg P \vee R \vee Q) \wedge (\neg P \vee R \vee \neg Q)$

$\Leftrightarrow (P \vee Q \vee R) \wedge (P \vee Q \vee \neg R) \wedge (\neg P \vee R \vee Q) \wedge (\neg P \vee R \vee \neg Q)$

product of max terms

Theory of Inference for statement calculus:

consider H_1, H_2, H_3 ------ H_n of the given statements and C. C is also an another statement.

Premise:-

The set of all statements H_1, H_2, ---- H_n except C statement is called premise.

Conclusion:

conclusion is a statement which is obtained from a set of premises

Inference.

If we start from premises and proceed to conclusion such process is called Inference.

The theory associated with the rules of inference is called as Inference theory

Note:- premises means set of assumptions, axioms and hypothesis

Argument:

consider a set of premises $H_1, H_2, \ldots H_n$ and a conclusion C Then the argument is denoted by $(H_1 \wedge H_2 \wedge H_3 \wedge \ldots \wedge H_n) \rightarrow C$

Valid argument (or) valid conclusion.

If a conclusion is derived from a set of premises by using the rules of reasoning then it is called valid conclusion

To determine weather a conclusion logically follows from given premises. We use the following methods.

1. Truth Table Method.
2. Without using Truth Table.

Truth Table Method

let $H_1, H_2 \ldots H_n$ are the premises and C be a conclusion We say that C is a valid conclusion If and only if H_1 and $(H_1 \wedge H_2 \wedge H_3 \wedge \ldots \wedge H_n) \rightarrow C$ is a tautology

Determine weather the conclusion C follows logically from the premises given

1) $H_1: P \rightarrow Q \quad C: P \rightarrow (P \wedge Q)$

P	Q	$P \rightarrow Q$	$P \wedge Q$	$P \rightarrow (P \wedge Q)$	$H_1 \rightarrow C$
T	T	T	T	T	T
T	F	F	F	F	T
F	T	T	F	T	T
F	F	T	F	T	T

$H_1 : C$ is a Tautology

$\therefore$ C follows logically from H_1

$\therefore$ so the conclusion is valid.

Determine weather the conclusion C follows logically from the given premise

1. $H_1: P \rightarrow Q \quad H_2: P \quad C: Q \quad H_1 \wedge H_2 \rightarrow C$

P (H_2)	Q	$P \rightarrow Q$ (H_1)	$H_1 \wedge H_2$	$H_1 \wedge H_2 \rightarrow C$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

$H_1 \wedge H_2 : C$ is a Tautology

Determine wheather the conclusion C follows logically from the following premises.

1 $H_1: P \to Q$, $H_2: \neg P$ $C: Q$ N.V.

2 $H_1: \neg P$, $H_2: P \vee Q$ $C: Q$ V

3 $H_1: P \to Q$, $H_2: \neg Q$ $C: P$ N.V.

** 4 $H_1: \neg P \vee Q$, $H_2: \neg(Q \wedge \neg R)$, H_3 $\neg R$, $C: \neg P$ V

P	Q	$P \to Q$ (H_1)	$\neg P$ (H_2)	$H_1 \wedge H_2$	$H_1 \wedge H_2 \to C$
T	T	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	F

$H_1 \wedge H_2 : C$ is not a Tautology

$\therefore$ so the conclusion is not valid

2. $H_1: \neg P$, $H_2: P \vee Q$ $C: Q$

P	Q	$\neg P$ (H_1)	$P \vee Q$ (H_2)	$H_1 \wedge H_2$	$H_1 \wedge H_2 \to C$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

$H_1 \wedge H_2 : C$ is a Tautology

$\therefore$ so the conclusion is valid

$H_1: P \to Q$, $H_2: \neg Q$ $C: P$

P	Q	$P \to Q$ (H_1)	$\neg Q$ (H_2)	$H_1 \wedge H_2$	$H_1 \wedge H_2 \to C$
T	T	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	T	F

$H_1 \wedge H_2 : C$ is not a Tautology

$\therefore$ so the conclusion is not valid

$H_1: \neg P \vee Q \quad H_2: \neg(Q \wedge \neg R) \quad H_3: \neg R \quad C: \neg P$

P	Q	R	$\neg P$	$\neg P \vee Q$ (H_1)	$\neg R$ (H_3)	$Q \wedge \neg R$	$\neg(Q \wedge \neg R)$ (H_2)	$H_1 \wedge H_2 \wedge H_3$	$H_1 \wedge H_2 \wedge H_3 \rightarrow C$
T	T	T	F	T	F	F	T	F	T
T	T	F	F	T	T	T	F	F	T
T	F	T	F	F	F	F	T	F	T
T	F	F	F	F	T	F	T	F	T
F	T	T	T	T	F	F	T	F	T
F	T	F	T	T	T	T	F	F	T
F	F	T	T	T	F	F	T	F	T
F	F	F	T	T	T	F	T	T	T

$\therefore H_1 \wedge H_2 \wedge H_3$ is a Tautology

$\therefore$ so the conclusion is valid.

Without using Truth Table

Rule P $\Rightarrow$ Given premise

A premise may be introduced at any point in the derivation

Rule T $\rightarrow$ Derived premise

Tautalogical implications may be introduced in the derivation

Rules of Implications (or) Rules of Inference.

- $I_1: P \wedge Q \Rightarrow P$ } conjuctive simplification
- $I_2: P \wedge Q \Rightarrow Q$
- $I_3: P \Rightarrow P \vee Q$ } Addition laws
- $I_4: Q \Rightarrow P \vee Q$
- $I_5: \neg P \Rightarrow P \rightarrow Q$
- $I_6: Q \Rightarrow P \rightarrow Q$
- $I_7: \neg(P \rightarrow Q) \Rightarrow P$
- $I_8: \neg(P \rightarrow Q) \Rightarrow \neg Q$
- $I_9: P, Q \Rightarrow P \wedge Q$ conjunction law.
- $I_{10}: \neg P, P \vee Q \Rightarrow Q$ Disjunction syllologism
- $I_{11}: P, P \rightarrow Q \Rightarrow Q$ modus ponens
- $I_{12}: \neg Q, P \rightarrow Q \Rightarrow \neg P$ Modus tollens
- $I_{13}: P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$ Transitive law
- $I_{14}: P \vee Q, P \rightarrow R, Q \rightarrow R \Rightarrow R$ Dillemma
- $I_{15}: P \rightarrow R, Q \rightarrow R \Rightarrow (P \vee Q) \rightarrow R$

Equivalence.

- $E_1: \neg(\neg P) \equiv P$ Double negation law
- $E_2: P \wedge Q \Leftrightarrow Q \wedge P$
- $E_3: P \vee Q \Leftrightarrow Q \vee P$

  } commutative laws

- $E_4: P \wedge (Q \wedge R) \Leftrightarrow (P \wedge Q) \wedge R$
- $E_5: P \vee (Q \vee R) \Leftrightarrow (P \vee Q) \vee R$

  } Associative laws

- $E_6: P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$
- $E_7: P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$

  } Distributive law

- $E_8: \neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$
- $E_9: \neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$

  } Demorgan's laws

- $E_{10}: P \vee P \Leftrightarrow P$
- $E_{11}: P \wedge P \Leftrightarrow P$
- $E_{12}: R \vee (P \wedge \neg P) \Leftrightarrow R \vee F \Leftrightarrow R$
- $E_{13}: R \wedge (P \vee \neg P) \Leftrightarrow R \wedge T \Leftrightarrow R$
- $E_{14}: R \vee (P \vee \neg P) \Leftrightarrow R \vee T \Leftrightarrow T$
- $E_{15}: R \wedge (P \wedge \neg P) \Leftrightarrow R \wedge T \Leftrightarrow F$
- $E_{16}: P \rightarrow Q \Leftrightarrow \neg P \vee Q$
- $E_{17}: \neg(P \rightarrow Q) \Leftrightarrow P \wedge \neg Q$
- $E_{18}: P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$
- $E_{19}: P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$
- $E_{20}: \neg(P \rightleftharpoons Q) \Leftrightarrow P \rightleftharpoons \neg Q$
- $E_{21}: P \rightleftharpoons Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$
- $E_{22}: P \rightleftharpoons Q \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$

Note: $P \vee \neg P \equiv T$

$P \wedge \neg P \equiv F$

Show that R is a valid inference from the premises $P \rightarrow Q$, $Q \rightarrow R$, and P

Premise position	steps	Given premise	Reason
$\{1\}$	(1)	P	Rule P
$\{2\}$	(2)	$P \rightarrow Q$	Rule P
$\{1,2\}$	(3)	Q	Rule T (1)(2) $\rightarrow P, P \rightarrow Q \Rightarrow Q$ (modus ponens)
$\{4\}$	(4)	$Q \rightarrow R$	Rule P
$\{1,2,4\}$	(5)	R	Rule T (3)(4) $\rightarrow P, P \rightarrow Q \Rightarrow Q$ (Modus ponens)

show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$

Given premises

$P \vee Q$, $P \to R$, $Q \to S$

{1} (1) $P \vee Q$ Rule P

{1} (2) $\neg P \to Q$ Rule T $\to \neg P \to Q \equiv \neg\neg(P) \vee Q$. $\neg P \to Q \equiv P \vee Q$

$\equiv P \vee Q$

{3} (3) $Q \to S$ Rule P

{1,3} (4) $\neg P \to S$ Rule T (2),(3) $P \to Q, Q \to R \Rightarrow P \to R$ (Transitive law)

{1,3} (5) $P \to \neg S$ Rule T (4) $\neg Q \to \neg P \equiv P \to Q$

{6} (6) $P \to R$ Rule P

{1,3,6} (7) $\neg S \to R$ Rule T (5)(6) $P \to Q, Q \to R \Rightarrow P \to R$ (Transitive Rule)

{1,3,6} (8) $S \vee R$ Rule T (7)

$\therefore$ $S \vee R$ logically followed from the given premises

show that $R \vee S$ follows logically from the premises $C \vee D$, $C \vee D \to \neg H$, $\neg H \to (A \wedge \neg B)$ and $(A \wedge \neg B) \to R \vee S$

{1} (1) $C \vee D$ Rule P

{2} (2) $C \vee D \to \neg H$ Rule P

{1,2} (3) $\neg H$ Rule T (1),(2) modus ponens $P, P \to Q = Q$

{4} (4) $\neg H \to A \wedge \neg B$ Rule P

{1,2,4} (5) $A \wedge \neg B$ Rule T (3),(4) modus ponens

{6} (6) $(A \wedge \neg B) \to R \vee S$ Rule P.

{1,2,4,6} (7) $R \vee S$.

$\therefore$ $R \vee S$ logically followed from the given premises

show that $R \wedge (P \vee Q)$ is a valid conclusion from the given premises

$P \vee Q$, $Q \to R$, $P \to M$, $\neg M$.

{1} (1) $\neg M$ Rule P

{2} (2) $P \to M$ Rule P.

{2} (3) $\neg M \to \neg P$ Rule T (contra-tve $= P \to Q \Leftrightarrow \neg Q \to \neg P$)

{1,2} (4) $\neg P$ Rule T (1),(3) modus ponens

{5} (5) $P \vee Q$ Rule P

Rule T (5) $P \to Q \Leftrightarrow \neg P \vee Q$

$\{8\}$ (8) $Q \to R$ Rule P

$\{1,2,5,8\}$ (9) R Rule T (7)(8)

$\{1,2,5,8\}$ (10) $R \wedge (P \vee Q)$ Rule T (9),(5) $\to P, Q \Rightarrow P \wedge Q$

$\therefore$ $R \wedge (P \vee Q)$ logically followed from the given premises

show that $\neg Q$, $P \to Q$ implies conclusion $\neg P$

$\{1\}$ (1) $\neg Q$ Rule P

$\{2\}$ (2) $P \to Q$ Rule P

$\{2\}$ (3) $\neg Q \to \neg P$ Rule T (2) $P \to Q \Leftrightarrow \neg Q \to \neg P$ (contrapostive)

$\{1,2\}$ (4) $\neg P$ Rule T (1),(3) modus ponens
$P, P \to Q \Rightarrow Q$.

$\therefore$ $\neg P$ logically followed from the given premises

show that $P \to Q, Q \to \neg R, R, P \vee (J \wedge S) \Rightarrow J \wedge S$

$\{1\}$ (1) $P \to Q$ Rule P

$\{2\}$ (2) $Q \to \neg R$ Rule P

$\{1,2\}$ (3) $P \to \neg R$ Rule T (1)(2) Transitive Rule
$P \to Q,\ Q \to R \Rightarrow P \to R$

$\{1,2\}$ (4) $R \to \neg P$ Rule T (3) contrapositive
$P \to Q \Leftrightarrow \neg Q \to \neg P$

$\{5\}$ (5) R Rule P

$\{1,2,5\}$ (6) $\neg P$ Rule P (4)(5) modusponens

$\{7\}$ (7) $P \vee (J \wedge S)$ Rule P

$\{7\}$ (8) $\neg P \to J \wedge S$ Rule T (7) $P \to Q \Leftrightarrow \neg P \vee Q$

$\{1,2,5,7\}$ (9) $J \wedge S$ Rule T (6),(8) modusponens $P, P \to Q \Rightarrow Q$

$\therefore$ $J \wedge S$ logically followed from the given premises.

Show that $\neg P$ follows logically from the premises $\neg(P \wedge \neg Q)$, $\neg Q \vee R$, $\neg R$

show that $\neg S$ is a valid argument from the premises $\neg(Q \vee R)$ $\neg R$ and
$\cancel{\neg(\neg P \wedge S)}$ $P \to Q$, $(\neg Q \vee R) \wedge \neg R$, $\neg(\neg P \wedge S)$

$\{1\}$ (1)

consistency of premises.

The premises $H_1, H_2 \ldots H_n$ of an argument said to be consistent. If their conjunction $H_1 \wedge H_2 \wedge \ldots H_n$ are true then In atleast one possible situation.

The premises $H_1, H_2 \ldots H_n$ of an argument are said to be "inconsistent". If their conjunction $H_1 \wedge H_2 \wedge \ldots H_n$ is false in all possible situations i.e $H_1 \wedge H_2 \wedge \ldots \wedge H_n$ then $\rightarrow F$

let the premises are $P \vee Q$ and $\neg P$ ($\neg P$ is consistent with the help of truth table.

P	Q	$P \vee Q$	$\neg P$	$(P \vee Q) \wedge (\neg P)$
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

The premises $(P \vee Q) \wedge (\neg P)$ are consistent

let the premises $P \wedge \neg P \wedge Q$ is in-consistent

P	Q	$\neg P$	$\neg P \wedge Q$	$(P) \wedge (\neg P \wedge Q)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

$\therefore (P) \wedge (\neg P \wedge Q)$ is inconsistent.

Show that $P \to Q, P \to R, Q \to \neg R$ and P are inconsistent without Truth Table.

{1} (1) $P \to Q$ Rule P

{2} (2) $Q \to \neg R$ Rule P

{1,2} (3) $P \to \neg R$ Rule T (1),(2) Transitive Rule $P \to Q, P \to R \Rightarrow Q \to R$.

{4} (4) P Rule P

{1,2,4} (5) $\neg R$ Rule T (3),(4) modus ponens

{6} (6) $P \to R$ Rule P

{6} (7) $\neg R \to \neg P$ Rule T (6) $P \to Q \equiv \neg R \to \neg Q$

{1,2,4,6} (8) $\neg P$ Rule T (5),(7) modus ponens

{1,2,4,6} (9) $P \wedge \neg P$ Rule T (4),(8) $P, Q \equiv P \wedge Q$.
F

Hence we get contradiction

$\therefore$ The given premises are inconsistent.

Show that the following premises are inconsistent.
If the contract is valid, then john is liable for penalty. If john is liable for penalty, He will go bank rupt. If the bank will loan him money, He will not go bank rupt. As a matter of act the contract is valid and the bank will loan him money.

Let the given statements are.

V: The contract is valid.

L: John is liable for penalty

M: Bank will loan him money

B: He will go bank rupt.

Let the given premises are.

$V \to L, L \to B, M \to \neg B$ $V \wedge M$

{1} (1) $V \to L$ Rule P

{2} (2) $L \to B$ Rule P

{1,2} (3) $V \to B$ Rule T (1),(2), Transitive Rule $P \to Q, Q \to R \Rightarrow P \to R$

{4} (6) M Rule T (4) simplification law $Q \wedge P \Rightarrow Q$

{1,2,4} (7) B Rule T (3),(5) modus ponens $P, P \to Q \Rightarrow Q$

{8} (8) $M \to \neg B$ Rule P

{1,2,4,8} (9) $\neg B$ Rule T (6),(8) modus ponens $P, P \to Q \Rightarrow Q$

{1,2,4,8} (10) $B \wedge \neg B$ Rule T (7)(9) $P \wedge Q = P \wedge Q$

F

∴ The given set of premises is inconsistent.

Show that the following premises are inconsistent

If Jack misses many classes through illness, then he fails high school. If Jack fails high school then he is uneducated. If Jack reads a lot of books, then he is not uneducated. Jack misses many classes through illness and reads a lot of books

M: Jack misses

H

B

E

$M \to H$, $H \to E$, $B \to \neg E$; $M \wedge B$

Rules C P (conditional proof) (Reduction Theorem)

Rule CP is generally used if the conclusion is in the form of $R \to S$.

In such case R is taken as additional premise and S is derived

Show that $R \to S$ can be derived from the given premises $P \to (Q \to S)$, $\neg R \vee P$ and Q

{1} (1) R Rule P (Additional premise)

{2} (2) $\neg R \vee P$ Rule P

{2} (3) $R \to P$ Rule T (2) $P \to Q \equiv \neg P \vee Q$

{1,2} (4) P Rule T (1),(3) modus ponens $P, P \to Q \Rightarrow Q$

{5} (5) $P \to (Q \to S)$ Rule P

{1,2,5} (6) $Q \to S$ Rule T (4)(5) modus ponens $P, P \to Q \Rightarrow Q$

{7} (7) Q Rule P

{1,2,5,7} (8) S Rule T (6),(7) modus ponens $P, P \to Q \Rightarrow Q$

{1,2,5,7} (9) $R \to S$ Rule CP (1)(8).

Show that $P \to S$ can be derived from the premises $\neg P \vee Q$, $\neg Q \vee R$, $R \to S$ $\quad P \to S$.

{1} (1) P Rule P (Additional premise)

{2} (2) $\neg P \vee Q$ Rule P

{2} (3) $P \to Q$ Rule T (2) $\neg P \vee Q \equiv \neg P \vee Q$

{1,2} (4) Q Rule T (1)(3) modus ponens $P, P \to Q \Rightarrow Q$

{5} (5) $\neg Q \vee R$ Rule P

{5} (6) $Q \to R$ Rule T (5) $\neg Q \vee R \equiv R \to Q$

{1,2,5} (7) R Rule T (4)(6) modus ponens $P, P \to Q \Rightarrow Q$

{8} (8) $R \to S$ Rule P

{1,2,5,8} (9) S Rule T (7)(8) modus ponens

{1,2,5,8} (10) $P \to S$ Rule CP (1),(9)

If there was a ball game, then travelling was difficult. If they arrived on time., then travelling was not difficult. They arrived on time. Therefore there was no ball game. Show that the statements constitute a valid argument.

R: They arrived on time.

Given premises are

$C; \neg P$

$P \rightarrow Q,\ R \rightarrow \neg Q,\ R$

{1} (1)	$P \rightarrow Q$	Rule P	
{2} (2)	$R \rightarrow \neg Q$	Rule P	
{3} (3)	R	Rule P	
{2,3} (4)	$\neg Q$	Rule T (2),(3) modus ponens	
{1,2,3} (5)	$\neg P$	Rule T (1)(4) modus tollens	$\neg Q, P \rightarrow Q \equiv \neg P$

In direct method of proof

The method which uses rule CP and concept of inconsistent at set of premises is called indirect method of proof

Procedure.

→ Assume that the conclusion C is false and consider as ¬C as an additional premise

→ From the given premises together with additional premise derive a contradiction

→ Thus C follows logically from the given premises

Note:

For a direct proof taking the given premises and derive the conclusion

For an indirect proof taking the negation of conclusion and using the given premises we derive the contradiction

By indirect proof prove that $\neg Q,\ P \rightarrow Q,\ P \vee R \rightarrow R$

Given conclusion R include ¬R as a new premise

{1} (1)	$P \vee R$	Rule P	
{1} (2)	$\neg P \rightarrow R$	Rule T (1), $P \rightarrow Q \equiv \neg P \vee Q$	
{1} (3)	$\neg R \rightarrow P$	Rule T (2) contrapositive	$\neg P \rightarrow \neg Q \equiv Q \rightarrow R$
{4} (4)	$\neg R$	Rule P (New premise)	
{1,4} (5)	P	Rule T (3),(4) modus ponens	
{6} (6)	$P \rightarrow Q$	Rule P	
{1,4,6} (7)	Q	Rule T (5),(6)	
{8} (8)	$\neg Q$	Rule P	

Here the given premise along with new premise leads to contradication

$\therefore \neg Q, P \rightarrow Q, P \vee R \Rightarrow R$

Using indirect method of proof show that $P \rightarrow Q, Q \rightarrow R, \neg(P \wedge R), P \vee R \Rightarrow R$.

show that $\neg(P \wedge Q)$ follows $\neg P \wedge \neg Q$ using Indirect proof.

$\neg(P \wedge Q)$.

Verify validity of the following

1. Every square is a rectangle
2. Every Rectangle is a parallelogram

∴ Every square is a parallelogram

Sol Let P: square

Q: Rectangle

R: parallelogram

$P \to Q$, $Q \to R$, $P \to R$

{1}	(1)	$P \to Q$	Rule P
{2}	(2)	$Q \to R$	Rule P
{1,2}	(3)	$P \to R$	Rule T (1)(2) Transitive Rule $P \to Q, Q \to R \equiv P \to R$

∴ $P \to R$ is a valid conclusion from the given premise

Test wheather the following is a valid argument

1. If sachin hits a centuary then he gets a car
2. sachin does not get a free car

∴ sachin ~~does~~ has not hit a centuary

$P \to Q$, $\neg Q$, $\neg P$

P: sachin hits a centuary then gets a car

Q: sachin does not get a free car

{1}	(1)	$P \to Q$	Rule P
{2}	(2)	$\neg Q$	Rule P
{1,2}	(3)	$\neg P$	Rule T (1)(2) Modus Tollens $\neg Q, P \to Q \equiv \neg P$

Predicate calculus.

Statement: A statement is a meaningful sentence. statements contain two parts

1. subject
2. predicate

EX: $\underbrace{\text{sekhar}}_{\text{subject}}$ $\underbrace{\text{is a good boy}}_{\text{predicate}}$

subject:

The part of a statement which contains noun or pronoun is known as subject

predicate:

The remaining part of the statement except subject is known as predicate.

Notation

G(s), where G is a predicate and s is a subject.

Predicate logic.

the logic based upon the analysis of predicates in any statement is called predicate logic.

M-place predicate

A predicate requiring m (m>0) names or objects is called M-place predicate.

EX: Amulya is a student.

Here the predicate is a one-place predicate because it is related to one object i.e Amulya. S(a)

Naveen is taller than raju

Here the predicate T is taller than in a two-place predicate because it is related to two objects naveen & Raju T(n,r)

A simple statement function.

A simple statement function of one variable is defined an expression consisting of a predicate symbol and an individual variable. such as a statement function becomes a statement when the variable is replaced by the name of any object. We can form a compound statement function.

EX : $G(x,y)$: x is greater than y.

If both x and y replaced by the names of objects, we get a statement. If $x=3$ and $y=2$ then $G(3,2)$: 3 is greater than 2

Quantifiers:

certain statements involve words that indicate quantity such as "all", "some", "none" or "one". These words help to determine the answer to the question. "Howmany". since these words indicate quantity, they are called Quantifiers.

Ex: Some mens are tails

All Birds have wings

No air balloon is perfectly round

Quantifiers are divided into two types

1. Universal Quantifier.
2. Existential Quantifier.

Universal Quantifier

The quantifiers "all" in the universal quantifier. We shall denoted by $\forall x$ (or) (x) and we get it as "For all x". The symbol for all x represents each of the following phrases which have the same meaning:

1. "For all x" 2. "For every x" 3. "For each x" ~~"For every x"~~ 4. "For every x is such that" 5. "For every each x is such that" 6. "All x are such that" 7. "for any x"

Ex:- For every real number x, $x^2 \geq 0$

symbolic form

$\forall x \; x \in R, x^2 \geq 0$

Note: A universal quantifier is generally followed the connective "$\rightarrow$"

Ex:- All apples are red

let $A(x)$: x is apple

$R(x)$: x is Red

symbolic form

For all x If x is a apple then x is red.

Existential Quantifier:

The quantifier "some" is the existential quantifier. It is denoted by $\exists x$. (There exist) The symbol there exist represent each of the following phrases which have the same meaning

1. For some x
2. Some x such that
3. There exists an x such that.
4. There is an x such that
5. There is atleast one x such that

Ex:- For some integers $x, x+3=0$

symbolic Form

$\exists x(x), x+3=0$

Note: The existential quantifiers generally followed by the connectives and '$\wedge$'

Ex: some parallelograms are square.

$P(x)$: x is a parallelogram

$S(x)$: x is a square

$\exists x[P(x) \wedge S(x)]$

(or)

$\exists x\, P(x) \wedge S(x)$

Negation of quantifiers:

$\neg(\forall x\, F(x)) = \exists x F(x)$

$\neg(\exists x\, F(x)) = \forall x \neg F(x)$

Write each of the following in symbolic form

a. All men are good
b. No men are good
c. some men are good.
d. some men are not good.

let $M(x)$: x is a man

$G(x)$: x is Good

a) For all x, if x is a man then x is good

$\forall x[M(x) \rightarrow G(x)]$

For some x x is a man and x is good.

$$\exists x\,[m(x) \wedge G(x)]$$

For some x x is a man and x is not good

$$\exists x\,[m(x) \wedge \neg G(x)]$$

Bound variables

A variable which has been bounded by the quantifiers is called bound variables.

Free variables

A variable which has not been bounded by the quantifiers is called free variable

Scope of the quantifiers

The smallest formula immediately following (For all x or there exist x) $\forall x$ (or) $\exists x$ is called scope of quantifiers

Ex: $\forall x\,[P(x,y)]$

Bound variable : x

Free variables ; x, y

scope of the variable $P(x,y)$

Inference theory of predicate calculus

We have 4 fundamental rules They are

1. Universal specification or Rule US
2. Universal generalization or Rule UG
3. Existensial specification or Rule ES
4. Existensial ~~specification~~ or Rule EG
   Generalization

Rule US .

If a statement is in the form of $\forall x\,P(x)$ is assumed to be true. Then the universal quantifier can be dropped to obtain $P(c)$ is true for abitary element c in the universe. These rule can be expressed as $\forall x\,P(x)$

$$\forall x\,P(x)$$

Rule UG

If a statement p(c) is true for every element of c in the universe then the universal quantifier will be prefixed to obtain $\forall x\, p(x)$ is True. These rule can be expressed as.

P(c), for all c

$\forall x\, p(x)$

Rule ES

If a statement is in the form of then [there exist x] $\exists x\, P(x)$ is assumed to be True. There exists an element c in the universe such that P(c) is true. These rule can be expressed as $\exists x\, P(x)$

P(c) for some c

Rule EG

If a statement p(c) is true for some c in the universe then $\exists x\, P(x)$ is true These rule can be expressed as P(c) for some c $\quad \exists x\, P(x)$

Note: 1) Elimination of quantifiers can be done by using rules of specification is called as Rule US & Rule ES

2) To prefix the correct quantifier we need the rules of generalization is called as Rule UG & Rule EG.

prove that the validity of the following arguments.

1. All mens are faliable.
2. All kings are men

∴ All kings are faliable.

let

m(x) : x is a men

F(x) : x is faliable

K(x) : x is King

Symbolic form of the given arguments

$\forall x[m(x) \to F(x)]$

$\forall x[K(x) \to m(x)]$

$\forall x\ [K(x) \to F(x)]$

$\{1\}$ (1) $\forall x\,(m(x) \to F(x))$ Rule P

$\{1\}$ (2) $(m(c) \to F(c))$ Rule US

$\{3\}$ (3) $\forall x\,(K(x) \to m(x))$ Rule P

$\{3\}$ (4) $K(c) \to m(c)$ Rule US

$\{1,3\}$ (5) $K(c) \to F(c)$ Rule T (2)(4) Transitive Rule

$P \to Q, Q \to R \equiv P \to R$

$\{1,3\}$ (6) $\forall x\,[K(x) \to F(x)]$ Rule UG

verify the validity of the following arguments

1. Tigers are dangerous animals
2. there are Tigers

$\therefore$ There are dangerous animals

let

$T(x)$: x is a Tiger

$D(x)$: x is a dangerous animal

symbolic form of given arguments

$$\forall x\,[T(x) \to D(x)]$$

$$\exists x\,(T(x))$$

$$\exists x\; D(x)$$

$\{1\}$ (1) $\forall x\,(T(x) \to D(x))$ Rule P

$\{1\}$ (2) $T(c) \to D(c)$ Rule US (1)

$\{3\}$ (3) $\exists x\; T(x)$ Rule P

$\{3\}$ (4) $T(c)$ Rule ES

$\{1,3\}$ (5) $D(c)$ Rule T (2), (4) modus ponens

$P, P \to Q \equiv Q$

$\{1,3\}$ (6) $\exists x\, D(x)$ Rule EG

Given an argument which will establish the validity of the following.

1. All integers are rational numbers
2. some integers are powers of three

$\therefore$ some rational numbers are powers of three.

let

$I(x)$: x is a integer
$R(x)$: x is a Rational number
$P(x)$: x is a power 3

symbolic form of given arguments

$$\forall x\,[I(x) \rightarrow R(x)]$$
$$\exists x\,[I(x) \wedge P(x)]$$
$$\exists x\,[R(x) \wedge P(x)]$$

$\{1\}$	(1)	$\forall x\,[I(x) \rightarrow R(x)]$	Rule P
$\{1\}$	(2)	$I(c) \rightarrow R(c)$	Rule US (1)
$\{3\}$	(3)	$\exists x\,[I(x) \wedge P(x)]$	Rule P
$\{3\}$	(4)	$I(c) \wedge P(c)$	Rule ES (3)
~~$\{5\}$~~	(5)	~~$\exists x\,[R(x) \wedge P(x)]$~~	Rule P
$\{5\}$	~~(6)~~	~~$R(c) \wedge P(c)$~~	Rule ES (5)
$\{3\}$	(5)	$I(c)$	Rule T(4) simplification law $P \cap Q \equiv P$
$\{3\}$	(6)	$P(c)$	Rule T(4) simplification law $Q \cap P \equiv Q$
$\{1,3\}$	(7)	$R(c)$	Rule T (2)(5) modus ponens
$\{1,3\}$	(8)	$R(c) \cap P(c)$	Rule T (6)(7) $P, Q \Rightarrow P \cap Q$
$\{1,3\}$	(9)	$\exists x\,(R(x) \wedge P(x)]$	Rule EG

Using CP or otherwise obtain the following implication for all
$\forall x(P(x) \rightarrow Q(x))$, $(x)(R(x) \rightarrow \neg Q(x)) \Rightarrow (x)(R(x) \Rightarrow \neg P(x))$

$\{1\}$	(1)	$(x)(P(x) \rightarrow Q(x))$	Rule P
$\{1\}$	(2)	$P(c) \rightarrow Q(c)$	Rule US
$\{3\}$	(3)	$(x)(P(x) \rightarrow \neg Q(x))$	Rule P
$\{3\}$	(4)	$P(c) \rightarrow \neg Q(c)$	Rule US
$\{3\}$	(5)	$Q(c) \rightarrow \neg R(c)$	Rule T (4) $P \rightarrow Q \equiv \neg Q \rightarrow \neg P$ contrapositive
$\{1,3\}$	(6)	$P(c) \rightarrow \neg R(c)$	Rule T (2)(5) A/C Transitive rule
$\{1,3\}$	(7)	$R(c) \rightarrow \neg P(c)$	Rule T(6) contrapositive
$\{1,3\}$	(8)	$\forall x\,(R(x) \rightarrow \neg P(x)]$	Rule UE

UNIT II
SET THEORY

Set: set is a collection of well-defined objects and it is represented by { } denoted by letters A, B ---- so on

EX: collection of students in the class

$S = \{1, 2, 3, 4 \ldots\ldots\}$

Representation: There are two ways to represent the sets.

1. Roster Form
2. set builder Form

Roster Form: Roster Form notation is a complete list of all the elements of the set.

EX: Find the list of even numbers between 2 to 40

$S = \{2, 4, 6, 8 \ldots\ldots 40\}$

Set builder Form: It is used to write in the form of variables

EX: $B = \{x / 2 \leq x \leq 40 \text{ and } x \text{ is even}\}$

The vertical bar is read as such that

Subset: If every element in a set A is also element of a set B then A is called subset of B. It is denoted by $A \subseteq B$

Note: The null set is a subset of every set and every set is a subset of itself

EX: If $A = \{1, 3, 4, 5, 8, 9\}$

$B = \{1, 2, 3, 5, 7\}$

$C = \{1, 5\}$ then $C \subseteq A$, $C \subseteq B$

Types of sets

1. **Null set:** The set with no elements is called an empty set or null set. A null set is denoted by ϕ. The null set is a subset of every set.

2. **Universal set:** All the sets are considered to be subsets of one particular set. This set is called the universal set. The universal set is denoted by $\mathcal{U}$ or U

EX:- $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{2, 4, 6, 8\}$

Disjointsets: Two sets are said to be disjoint if they have, common element in both sets

EX:- If $A = \{1,2,3\}$, $B = \{4,5,6\}$

$A \cap B = \phi$

cartesian product: (*)

let A and B are two sets the set of all ordered pair (a,b) where a belongs to A & b belongs to B is called cartesian product It is denoted by $A \times B$

EX: let $A = \{1,2,3\}$, $B = \{4,5\}$ Then

$A \times B = \{(1,4), (1,5), (2,4), (2,5), (3,4) (3,5)\}$

$B \times A = \{(4,1), (4,2), (4,3), (5,1) (5,2) (5,3)\}$

** Operations of sets

In set theory operations involve various ways of combinding, comparing and manipulating sets

1. UNION (U): The union of two sets A and B is a set containing all elements that are in either A, B or both (without duplicates)

combine both

Notation: $A \cup B$

EX:- If $A = \{1,2,3\}$, $B = \{3,4,5\}$ then $A \cup B = \{1,2,3,4,5\}$

2. Intersection (∩): The intersection of two sets A and B is a set containing only the elements that are in both A and B

Notation: $A \cap B$

EX:- If $A = \{1,2,3\}$, $B = \{3,4,5\}$ then $A \cap B$ $\{3\}$

3. Difference (-): The difference of sets A and B is the set of elements that are in A but not in B

Notation: $A - B$ (or) $A \setminus B$

EX: If $A = \{1,2,3\}$, $B = \{3,4,5\}$ then $A - B$ $\{1,2\}$

4. complement : $\bar{A}$, A', A^c The complement of a set A includes all elements in the universal set U that are not in A

Notation: $\bar{A}$

EX: If $U = \{1,2,3,4,5\}$ and $A = \{1,2,3\}$ then $\bar{A} = \{4,5\}$

5. Symmetric Difference: (Δ) The symmetric difference of A and B is the set of elements that are in either A or B but not in

EX: If $A = \{1,2,3\}$ and $B \{3,4,5\}$ and then $A \Delta B = \{1,2,4,5\}$

Principles of Inclusion and Exclusion:

The inclusion exclusion principles is counting technique which generalizes the familiar method of obtaining the no of elements in the union of two finite sets. symbollically expressed as

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

(or)

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Where

$n(A)$ = NO of elements in A

$n(B)$ = NO of elements in B

$n(A \cup B)$ = NO of elements in both A and B

$n(A \cap B)$ = NO of common elements in A and B

For any finite sets A, B, C we have $n(A \cup B \cup C) =$

$$n(A) + n(B) + n(C) - n(A \cap C) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C).$$

Find the no of mathematics students at a college taking atleast one of the following languages Telugu, Hindi and English given in the following data.

i) 65 study telugu, 20 study telugu & Hindi

ii) 45 study Hindi, 25 study telugu & english

iii) 42 study English, 15 study Hindi & English

iv) 8 study all three languages

Given

$n(T) = 65$

$n(H) = 45$

$n(E) = 42$

$n(T \cap H) = 20$

$n(T \cap E) = 25$

$n(H \cap E) = 15$

$n(T \cap H \cap E) = 8$

By inclusion & Exclusion principle

$$n(T \cup H \cup E) = n(T) + n(H) + n(E) - n(T \cap H) - n(T \cap E) - n(H \cap E) + n(T \cap H \cap E)$$

Formulas.

Description	Notation
NOT A	$\bar{A}$
one of A and B	$A \cup B$
Both A and B	$A \cap B$
Only A but not B	$A \cap \bar{B}$
Only B but not A	$\bar{A} \cap B$
Neither A nor B	$\bar{A} \cap \bar{B}$
Exactly one of A and B	$A \cap \bar{B}$ (or) $(\bar{A} \cap B)$
Atleast one A, B, & C	$A \cup B \cup C$
All of ABC	$A \cap B \cap C$
None of A, B, C	$\bar{A} \cap \bar{B} \cap \bar{C}$
Only A but not B and C	$A \cap \bar{B} \cap \bar{C}$

Formulae

$\bar{A} = |U| - |A|$

$|\bar{A} \cap \bar{B}| = |\overline{A \cup B}|$

$= |U| - |A \cup B|$

$|A \cap \bar{B}| = |A| - |A \cap B|$

$|\bar{A} \cap B| = |B| - |A \cap B|$

A sample of 80 people revealed that 25 like cinema and 60 like TV programs. Find the no. of people who like both cinema and TV programs.

Let A be the set of people who like cinema

B be the set of people who like TV programs

Given data

$n(A \cup B) = 80$

$n(A) = 25$

$n(B) = 60$

$n(A \cap B)$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$80 = 25 + 60 - n(A \cap B)$

$n(A \cap B) = 85 - 80$

$= 5$

The no of people watching cinema and TV programs are 5

A:

A certain computer center employee 100 computer programmers of this 47 can program in FORTAN languages 35 in PASCAL and 23 can program in both languages. How many can program in neither of these two languages.

Let A be the set of programmers in FORTAN
B be the set of programmers in PASCAL

Given that

$n(A) = 47$
$n(B) = 35$
$n(A \cap B) = 23$

$n(\bar{A} \cap \bar{B}) = n(\overline{A \cup B})$
$= |U| - n(A \cup B|$
$= |U| - n(A) + n(B) - n(A \cap B)$
$= 100 - 47 + 35 - 23$
$= 4$

Determine the no. of positive integers ≤ 100 which are divisible by 3 or 7.

Let A be the set of numbers divisible by 3
B be the set of numbers divisible by 7

We know that

$|A \cup B| = |A| + |B| - |A \cap B|$

$|A|$ = no. of integers divisible by $3 = \frac{100}{3} = 33$ $\quad 3\overline{)\,7}$

$|B|$ = no. of integers divisible by $7 = \frac{100}{7} = 14$

$|A \cap B|$ = no. of integers divisible by both (3 (or) 7 $= \frac{100}{3 \text{ or } 7}$

$= \frac{100}{21} = 4$

$|A \cup B| = |A| + |B| - |A \cap B|$
$= 33 + 14 - 4$
$= 43$

Relation

Let A and B be two sets then a subset of $A \times B$ is called a relation (or) Binary relation from $R: A \to B$ symbollically $R \subseteq A \times B$ and $(a, b) \in R$ where $a \in A$ & $b \in B$.

Domain of the Relation

It is denoted by Dom $R = \{a \mid a \in A$ and $(a,b) \in R$.

Range of Relation.

The set of all the second elements of the ordered pairs in a relation R is called the Range of Relation. It is denoted by $\text{Ran} R = \{b \mid b \in B$ and $(a,b) \in R$

** Operations on Relations

UNION of Relations: The union of two relations R and S sets A and B is the relation containing all pairs that are in either R (or) S

(combine two relations without duplicates)

$R \cup S$

Ex: $R = \{(1,2)\ (2,3)\}$ and $S = \{(2,3)\ (3,4)\}$

$R \cup S = \{(1,2), (2,3)\ (3,4)\}$

Intersection of Relations:

The intersection of two relations R and S on sets A and B is is the relation containing all pairs that are in both R and S (common elements in both relations)

Ex: If $R = \{(1,2)\ (2,3\}$ and $S = \{(2,3)\ (3,4)\}$

then $R \cap S = \{(2,3)\}$

Difference of Relations.

The difference between two relations R and S on sets A and B is the relation containing all pairs that are in R but not in S

Notation $R - S$

Ex: If $R = \{(1,2), (2,3)\}$ and $S = \{(2,3), (3,4)\}$

then $R - S = \{1,2\}$

Composition of Relations

The composition of two relations R on $A \times B$ and (A×C) S on $B \times C$ is a relation on $A \times C$

It contains pairs (a,c) such that there exists an element $b \in B$ where $(a,b) \in R$ and $(b,c) \in S$

Ex: If $R = \{(1,2)(2,3)\}$ and $S = \{(2,4)(3,5)\}$

$SOR = \{(1,4)(2,5)\}$

Compliment of a relation:

let "R" be a relation from A to B then the (relation) compliment of R is denoted by $\bar{R}$ (or R') and is defined as $\bar{R} = (A \times B) - R$

Ex: let $A = \{(a,b)\}$ $B = \{1,2,3\}$ and the relation R from A to B is given by $R = \{a,1)(a,3)(b,2)(b,3)\}$ then find R

$\bar{R} = (A \times B) - R$

$= \{(a,1)(a,2)(a,3)(b,1)(b,2)(b,3) - R$

$= \{(a,2)(b,1)\}$

Converse Relation:

let "R" be a relation from A to B then the converse of R is denoted by $\tilde{R}$ is a relation from B to A and is defined as

$\tilde{R} = \{(b,a)\ a \in A, b \in B \text{ and } (a,b) \in R\}$.

consider the sets $A = \{A B C\}, B = \{1 2 3\}$ and the relations $R = \{(a,1), (b,1), (c,2), (c,3)\}$ and $S = \{(a,1)(a,2)(b,1)(b,2)\}$ from A to B Determine $\bar{R}, \bar{S}$, RUS, RnS and converse of R, S

let $A \times B$

$= \{(A,1)(A,2)(A,3)(B,1)(B,2)(B,3)(C,1)(C,2)(C,3)\}$

Now $\bar{R} = A \times B - R$

$\{(A,2)(A,3)(B,2)(B,3)(C,1)\}$

$\bar{S} = A \times B - S$

$\{(A,3)(B,3)(C,1)(C,2)(C,3)\}$

$R \cup S = \{(a,1)(b,1)(c,2)(c,3)(a,2)(b,2)\}$

$\{(a,1)(b,1)(c,2)(c,3)(a,2)(b,2)\}$

$R \cap S = \{(a,1)(b,1)\}$

converse of R $\{(1,a)(1,b)(2,c)(3,c)\}$

converse of S $\{(1,a)(2,a)(1,b)(2,b)\}$

consider the sets $A = \{1,2,3\}$ and the relations $R = \{(1,1)(1,2)(2,3)(3,1)\}$ and $S = \{(3,1)(3,2)(3,3)(2,1)\}$ on A then compute $\bar{R}$ $\bar{S}$ RUS RnS and converse of S

let $A \times A$

$= \{(1,1)(1,2)(1,3)(2,1)(2,2)(2,3)(3,1)(3,2)(3,3)\}$

Now $\bar{R} = A \times A - R$

$\{(1,3)(2,1)(2,2)(3,2)(3,3)\}$

$\bar{S} = A \times A - S$

$\{(1,1)(1,2)(1,3)(2,2)(2,3)\}$

$R \cup S = \{(1,1)(1,2)(2,3)(3,1)(3,2)(3,3)(2,1)\}$

$R \cap S = \{(3,1)\}$

converse of R $\{(1,1)(2,1),(3,2)(1,3)\}$

converse of S $\{(1,3)(2,3)(3,3)(1,2)\}$

*** Properties of Relations

1. Reflexive Relation: A relation R on a set A is said to be Reflexive. If $(a,a) \in R, \forall a \in A$

Ex: If $A=\{1,2,3\}$ and $R=\{(1,1)(2,2)(3,3)(1,2)\}$ then R is Reflexive. because $(1,1)(2,2)(3,3)$ are all in R

2. Irreflexive Relation: A relation R on a set A is said to be irreflexive. If $(a,a) \notin R$ For any $a \in A$.

Ex: If $A=\{1,2,3\}$ and $R=\{(1,2)(1,3)(2,3)(3,1)\}$ is irreflexive

3. Symmetric: A relation R on a set A is said to be symmetric if $(a,b) \in R$ then $(b,a) \in R$

Ex: If $A=\{1,2,3\}$ and $R=\{(1,2)(2,1)(2,3)(3,2)\}$ then R is symmetric because whenever $(a,b) \in R, (b,a) \in R$ is

Ex: $(1,2) \in R$ Then $(2,1) \in R$

4. Anti-symmetric: A relation R on a set A is said to be anti-symmetric. If $a \neq b$ and $(a,b) \in R$ then $(b,a) \notin R$. For all $a,b \in A$.

(or)

If $(a,b) \in R$ then $(b,a) \in R$, $a=b$

Ex:- If $A=\{1,2,3\}$ and $R=\{(1,2)(1,3)(2,3)\}$ then R is anti symmetric because there are no pairs (a,b) and (b,a) in R where $a \neq b$.

If $R=\{(1,1)(2,2)(3,3)\}$ When $a=b$

5. Transitive: A relation R on a set A is said to be transitive if $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ $\forall a,b,c \in R$

Ex:- If $A=\{1,2,3\}$ and $R=\{(1,2)(2,3)(1,3)\}$ then R is transitive because $(1,2) \in R$ $(2,3) \in R$ then $(1,3) \in R$ $\forall (1,2,3) \in R$.

Representation of Relations

1. Relation of Matrix Represention
2. Di-graph representation

1. Relation of matrix Representation.

If $A=\{a_1,a_2,a_3 \cdots a_n\}$ and $B=\{b_1,b_2,b_3 \cdots b_n\}$ are finite sets containing m and n elements representing matrix. This is denoted by $M_R = [m_{ij}]_{m \times n}$

where $m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_i) \in R \\ 0 & \text{if } (a_i, b_i) \notin R \end{cases}$

EX: If $A = \{a_1, a_2, a_3, a_4\}$

$B = \{b_1, b_2, b_3, b_4\}$ then the relation R from A to B is given by $R = \{(a_1, b_1), (a_1, b_4), (a_2, b_2), (a_2, b_3)\ (a_3, b_1)\ (a_3, b_3)\}$

Now relation matrix.

$$M_R = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{cccc} b_1 & b_2 & b_3 & b_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array} = \begin{bmatrix} 1001 \\ 0110 \\ 1010 \end{bmatrix}_{3 \times 4}$$

Di-graph

A relation can be represented partially pictorial by drawing its Di-graph follows

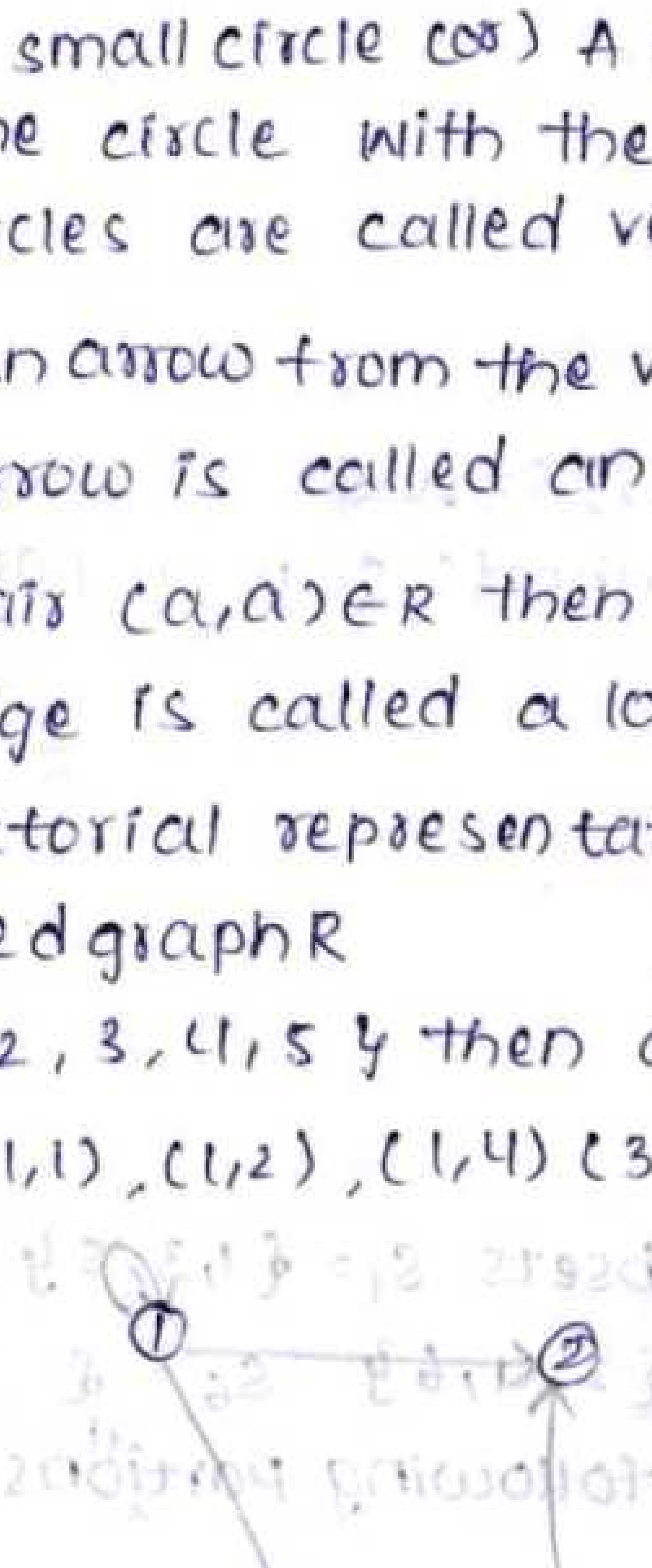
1. Draw a small circle (or) A bullet for each element of A. and table the circle with the corresponding element of A. These circles are called vertices or Nodes

2. Draw an arrow from the vertex a to b if and only if $(a, b) \in R$ these arrow is called an edge

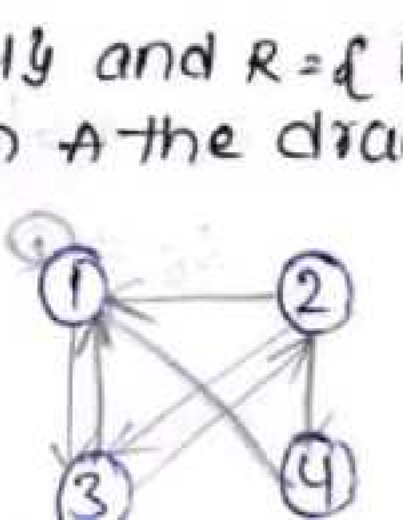
3. If pair $(a, a) \in R$ then draw a directed edge from a to a such edge is called a loop

4. This pictorial representation of R is called Di-graph (or) Directed graph R

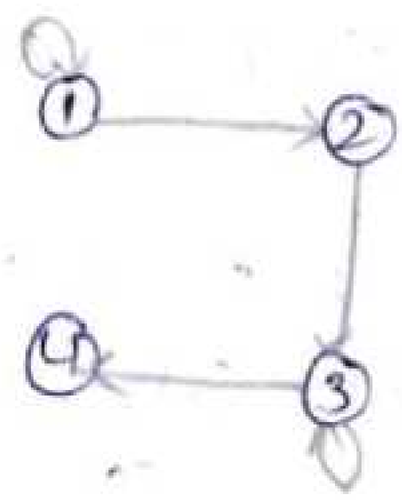
Ex: If $A = \{1, 2, 3, 4, 5\}$ then a relation R on A is given by $R = \{(1,1), (1,2), (1,4)\ (3,2)\}$. Draw the di-graph of R



Ex: If $A = \{1, 2, 3, 4\}$ and $R = \{(1,1)\ (1,3)\ (2,1)\ (2,3)\ (2,4)\ (3,1)\ (3,2)\ (4,1)\}$ is a relation R on A the draw the digraph R.



Let $R = \{(1,1)\ (1,2)\ (2,3)\ (3,3)\ (3,4)\}$ be a relation on $A = \{1,2,3,4\}$. Draw the di-graph of R. obtain R^2 and draw the di-graph of R^2

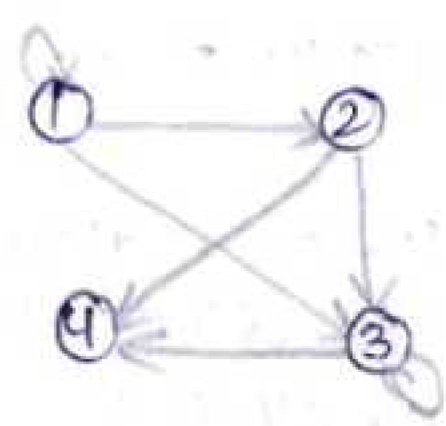


R^2 is the composite of R with itself and it consists of ordered pairs (a,c) such that $(a,b) \in R$, and $(b,c) \in R$

$R^2 = R O R$

$= \{(1,1)\ (1,2)\ (2,3)\ (3,3)\ (3,4)\} O \{(1,1)\ (1,2)\ (2,3)\ (3,3)\ (3,4)\}$

$= \{(1,1)\ (1,2)\ (1,3)\ (2,3)\ (2,4)\ (3,3)\ (3,4)\}$



Partition and covering

Let S be a non-empty set, S_1, S_2, S_3 and ---- S_n are the subsets of S the collection of subsets S_i is a partition of S If and only if

i) $S_i \neq \phi$ for each i

ii) $S_i \cap S_j = \phi$ for $i \neq j$

iii) $\bigcup_{i=1}^{m} S_i = S$ where $\bigcup_{i=1}^{m}$

Let $S = \{1,2,3,4,5,6\}$ and the subsets $S_1 = \{1,3,5\}$, $S_2 = \{2,4\}$, $S_3 = \{1,3\}$, $S_4 = \{3,5\}$, $S_5 = \{2,4,6\}$ $S_6 = \{1,2,3\}$, $S_7 = \{4,5\}$ $S_8 = \{6\}$ check wheather the following partitions are valid or not

$\pi_1 = \{S_6, S_7, S_8\}$ $\pi_2 = \{S_3, S_4, S_5\}$ $\pi_3 = \{S_1, S_2\}$

i) $S_i \neq \phi$

$S_6 = \{1,2,3\}$ $S_7 = \{4,5\}$ $S_8 = \{6\}$

$S_6 \neq \phi$ $S_7 \neq \phi$ $S_8 \neq \phi$

I^{st} condition satisfied.

$(S_6, S_8] = \{1,2,3\} \cap \{6\} = \phi$

$(S_7, S_8] = \{4,5\} \cap \{6\} = \phi$

IInd condition is also satisfied

iii $\bigcup_{i=1}^{m} S_i = S$

$[S_6, S_7, S_8] = S_6 \cup S_7 \cup S_8 = \{1,2,3\} \cup \{4,5\} \cup \{6\}$

$= \{1,2,3,4,5,6\} = S$

IIIrd condition is satisfied

$\therefore \pi_1 = S_6 S_7 S_8$ is a valid partition

$\pi_2 = \{S_3, S_4, S_5\}$

i) $S_i \neq \phi$ ii) $S_4 = \{3,5\}$ $S_5 = \{2,4,6\}$

$S_3 = \{1,3\}$ $S_4 \neq \phi$ $S_5 \neq \phi$

$S_3 \neq \phi$

Ist condition satisfied

ii) $S_i \cap S_j = \phi$

$[S_3 \cap S_4] = \{1,3\} \cap \{3,5\} = \{3\} \neq \phi$

$(S_3 \cap S_5] = \{1,3\} \cap \{2,4,6\} = \phi$

$[S_4 \cap S_5] = \{3,5\} \cap \{2,4,6\} = \phi$

IInd condition is not satisfied.

iii) $\bigcup_{i=1}^{m} S_i = S$

$[S_3, S_4, S_5] = S_3 \cup S_4 \cup S_5 = \{1,3\}, \{3,5\}, \{2,4,6\}$

$= \{1,3,5,2,4,6\}$

$= \{1,2,3,4,5,6\} = S$

$\therefore$ IIIrd condition is satisfied

$\therefore \pi_2 = S_3 S_4 S_5$ is not a valid partition

iii) $\pi_3 = \{S_1, S_2\}$

i) $S_i \neq \phi$

$S_1 = \{1,3,5\}$ $S_2 = \{2,4\}$

$S_1 \neq \phi$ $S_2 \neq \phi$

Ist condition is satisfied

ii) $S_i \cap S_j = \phi$

$(S_1 \cap S_2) = \{1,3,5\} \cap \{2,4\} = \phi$

IInd condition is satisfied

Covering of a set.

Let S be a given set and $A = \{a_1, a_2, a_3, \ldots a_n\}$ where each A_i

$\bigcup_{i=1}^{m} A_i = S$. A_i is a subset of S then set A is called a covering of sets and the subsets $A_1, A_2, A_3 \ldots\ldots A_m$ are said to be coverset of S

Ex: Let $S = \{A B C\}$ and consider the following collections of subsets $A = \{(a,b)\ (b,c)\}$ $B = \{(a)(a,c)\}$ $C = \{(a), (b,c)\}$ $D = \{(a,b,c)\}$ $E = \{(a),(b),(c)\}$ $F = \{(a), (a,b), (a,c)\}$ among them which subsets are covering of S and which are partitions.

consider $A = \{(a,b)\ (b,c)\}$ set A contains two subsets

$A_1 = \{a,b\}$

$A_2 = \{b,c\}$

$\bigcup_{i=1}^{m} \cancel{B}_i = S$

$A_1 \cup A_2 = \{a,b\} \cup \{b,c\}$

$= \{a,b,c\} = S$

$B = \{(a)\ (a,c)\}$

$B_1 = \{a\}$

$B_2 = \{a,c\}$

$B_1 \cup B_2 = \cancel{\{a\}\cup}\{a,c\} \neq S$

$\{a,c\} \neq S$

$C = \{(a), (b,c)\}$

$C_1 = \{a\}$

$C_2 = \{a,c\}$

$C_1 \cup C_2 = \{a\} \cup \{a,c\}$

$= \{a,c\} \neq S$

$D = \{(a,b,c\}$

$\{a,b,c\} = S$

$E = \{(a),(b),(c)\}$

$E_1 = \{a\}$

$E_2 = \{b\}$

$E_3 = \{c\}$

$F = \{(a),(a,b),(a,c)\}$

$F_1 = \{a\}$

$F_2 = \{a,b\}$

$F_3 = \{a,c\}$

$F_1 \cup F_2 \cup F_3 = \{a,b,c\} = S$

***Equivalence Relations

A Relation R on a set A is said to be equivalence relation on A. If it is satisfies reflexive, symmetric and transitive.

let $X = \{1,2,3,4\}$ and $R = \{(1,1)\ (1,4)\ (4,1)\ (4,4)\ (2,2)\ (2,3)\ (3,2)\ (3,3)\}$ check wheather equivalence is relation or not.

Given

$R = \{(1,1)\ (1,4)\ (4,1)\ (4,4)\ (2,2)\ (2,3)\ (3,2)\ (3,3)\}$

Reflexive relation

If $a \in X$ then $(a,a) \in R$

i.e $(1,1) \in R$, $(2,2) \in R$, $(3,3) \in R$, $(4,4) \in R$

$\therefore$ R is reflexive

symmetric relation

If $(a,b) \in R$ then $(b,a) \in R$

i.e $(1,4) \in R \rightarrow (4,1) \in R$

$(2,3) \in R \rightarrow (3,2) \in R$

$\therefore$ R is symmetric

Transitive relation

If $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$

i.e $(1,1) \in R\ (1,4) \in R \rightarrow (1,4) \in R$

$(1,4) \in R\ (4,1) \in R \rightarrow (1,1) \in R$

$(2,2) \in R\ (2,3) \in R \rightarrow (2,3) \in R$

$(2,3) \in R\ (3,2) \in R \rightarrow (2,2) \in R$

$(3,3) \in R\ (3,2) \in R \rightarrow (3,2) \in R$

$\therefore$ R is transitive.

$\therefore$ Given Relation R is equivalence Relation

2. verify the Relation $R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (3,1)\ (2,1)\}$ on the set $a = \{1,2,3,4\}$ is equivalence or not

Given

$R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (3,1)\ (2,1)\}$

Reflexive relation

i.e If $a \in X$ then $(a,a) \in R$

i.e $(1,1) \in R$, $(2,2) \in R$, $(3,3) \in R$, $(4,4) \in R$

Symmetric relation:

If $(a,b) \in R$ then $(b,a) \in R$

$\therefore$ R is not symmetric

Transitive Relation)

i.e If $(a,b) \in R, (b,c) \in R$ then $(a,c) \in R$

$(3,3) \in R \quad (3,1) \in R \rightarrow (3,1) \in R$

$(3,1) \in R \quad (1,1) \in R \rightarrow (3,1) \in R$

$(2,2) \in R \quad (2,1) \in R \rightarrow (2,1) \in R$

$(2,1) \in R \quad (1,1) \in R \rightarrow (2,1) \in R$

$(2,2) \in R$

$\therefore$ R is a transitive

$\therefore$ Given relation R is not equivalence Relation.

3. Let $X = \{1,2,3,\ldots 7\}$ and $R = \{(x,y) / x-y$ is divisible by $3\}$ show that R is equivalence relation.

Let
$R = \{(x,y)/ x-y$ is divisible by $3\}$ in other words $(x,y) \in R$ if there exists an integer K such that $x-y = 3K$

Reflexive relation

$\forall x \in X$, $(x-x) = 0$ is divisible by 3

$\rightarrow (x,x) \in R \quad \forall x \in x$

since 0 is divisible by 3 we have $(x,x) \in R \; \forall x \in X$

$\therefore$ R is Reflexive

Symmetric relation

A relation is symmetric $(x,y) \in R$ the $(y,x) \in R$ suppose $(x,y) \in R$ this means $x-y = 3K$ for some integer K

$(y-x) = -(x-y) = -3K$

$y-x$ is also divisible by 3 $(x,y) \in R$ then $x-y$ is divisible by 3 $-(y-x)$ is divisible by 3 $(y-x)$ also divisible by 3 $(y,x) \in R$

Transitive relation.

Let $(x,y) \in R$ and $(y,z) \in R$ then $(x,z) \in R$

We know that $x-y$ is divisible by 3

$y-z$ is divisible by 3

$x-y+y-z$ is divisible by 3

$x-z$ is divisible by 3

$(x-z) \in R$

$\therefore$ R is transitive.

Transitive closure.

Let x be a finite set containing n elements and R be a relation on x. Then the transitive closure Relation of ax is denoted by R^+ such that can be defined as

$$R^+ = R \cup R^2 \cup R^3 \cup \ldots \cup R^n$$

If $R = \{(1,2)\,(2,3)\,(3,3)\}$ be a relation on $A = \{1,2,3\}$. Find the transitive closure of R.

Given data

$A = \{1,2,3\}$ has 3 elements. So the transitive closure of R is $R^+ = R \cup R^2 \cup R^3$

$R^2 = R \circ R$

$= \{(1,2)\,(2,3)\,(3,3)\} \circ \{(1,2)\,(2,3)\,(3,3)\}$

$R^2 = \{(1,3)\,(2,3)\,(3,3)\}$

$R^3 = R \circ R^2 = \{(1,2)\,(2,3)\,(3,3)\} \circ (\,(1,3)\,(2,3)\,(3,3)\}$

$R^3 = \{(1,3)\,(2,3)\,(3,3)\}$

$R^+ = R \cup R^2 \cup R^3$

$= \{(1,2)(2,3)(3,3)\} \cup \{(1,3)\,(2,3)\,(3,3)\} \cup \{(1,3),(2,3)\,(3,3)\}$

$R^+ = \{(1,2)\,(1,3)\,(2,3)\,(3,3)\}$

Let $x = \{1,2,3,4\}$ and $R = \{(1,2)\,(2,3)\,(3,4)\}$ be a relation on x then find the transitive closure of R.

Given data

$R = \{(1,2)\,(2,3)\,(3,4)\}$

$R^2 = R \circ R$

$= \{(1,2)\ (2,3)\ (3,4)\} \circ \{(1,2)\ (2,3)\ (3,4)\}$

$R^2 = \{(1,3)\ (2,4)\}$

$R^3 = R \circ R^2$

$= \{(1,2)\ (2,3)\ (3,4)\} \circ \{(1,3)\ (2,4)\}$

$= \{\cancel{(1,3)}\ \cancel{(2,4)}\ (1,4)\}$

$R^4 = R \circ R^3$

$= \{(1,2)\ (2,3)\ (3,4)\} \circ \{(1,4)\}$

$= \{\phi\}$

The transitive closure of $R^+ = R \cup R^2 \cup R^3 \cup R^4$.

$= \{(1,2)\ (2,3)\ (3,4)\} \cup \{(1,3)\ (2,4)\} \cup \{(1,4)\} \cup \{\phi\}$

$= \{(1,2)\ (1,3)\ (1,4)\ (2,3)\ (2,4)\ (3,4)\}$

If $R = \{(a,b)\ (b,c)\ (c,d)\ (d,e)\}$ and $A = \{a,b,c,d,e\}$ then find the transitive closure of R.

Given data

$R = \{(a,b)\ (b,c)\ (c,d)\ (d,e)\}$

$A = \{a,b,c,d,e\}$ has 5 elements. so the transitive closure is

$R^+ = R \cup R^2 \cup R^3 \cup R^4 \cup R^5$

$R^2 = R \circ R$

$= \{(a,b)\ (b,c)\ (c,d)\ (d,e)\} \circ \{(a,b)\ (b,c)\ (c,d)\ (d,e)\}$

$= \{(a,c)\ (b,d)\ (c,e)\}$

$R^3 = R \circ R^2$

$= \{(a,b)\ (b,c)\ (c,d)\ (d,e)\} \circ \{(a,c)\ (b,d)\ (c,e)\}$

$= \{(a,d)\ (b,e)\}$

$R^4 = R \circ R^3$

$\{(a,b)\ (b,c)\ (c,d)\ (d,e)\} \circ \{(a,d)\ (b,e)\}$

$\{(a,e)\}$

$R^5 = R \circ R^4$

$\ldots (d,e)\} \circ \{(a,e)\} =$

The transitive closure of.

$R^+ = R \cup R^2 \cup R^3 \cup R^4 \cup R^5$

$= \{(a,b)\ (b,c)\ (c,d),\ (d,e)\} \cup$

$\{(a,c)\ (b,d)\ (c,e)\} \cup \{(a,d)\ (b,e)\}$

Compatability and partial order.

comp ata bility relations.

A relation R on a set A is said to be compatability relation if it satisfies reflexive property and symmetric property

A compatability relation is represented by $\approx$

Let A = {1, 2, 3} and let the relation R = {(1,1) (2,2) (3,3) (1,2) (2,3) (2,1) (3,2) (1,3) (3,1)} define the A. check wheather this relation is compatability relation or not?

let
$A = \{1, 2, 3\}$
$R = \{(1,1)(2,2)(3,3)(1,2)(2,3)(2,1)(3,2)(1,3)(3,1)\}$

i) Reflexive property:

If $a \in A$ then $(a,a) \in R$

$\{(1,1)(2,2)(3,3)\} \in R$

$\therefore$ R is Reflexive

R satisfies Reflexive property

ii) symmetric property

If $(a,b) \in R$ then $(b,a) \in R$

$(1,2) \in R$ then $(2,1) \in R$

$(2,3) \in R$ then $(3,2) \in R$

$(1,3) \in R$ then $(3,1) \in R$

$\therefore$ R satisfies symmetric property

In this given R satisfies Reflexive and symmetric property because R is compatability Relation.

- Partial ordering Relation.

A relation R on a set P is said to be partial ordering relation. If and only if If R is reflexive, anti symmetric and transitive

partial ordering Relation is represented by $\leq$

Partial ordered set (or) POSET ***

If partial ordering relation is a partial ordering on a set P. then the ordered pair $(P, \leq)$ is called a partial ordered set (or) POSET.

- Totally ordered Relation. ***

let $(P, \leq)$ is a partially ordered set. If for every two elements $a, b \in P$. we have either $a \leq b$ (or) $b \leq a$ (comparable). Then partial ordering relation is called a simple ordering on P and $(P, \leq)$ is

Ex:

If $P = \{1,2,3,4\}$ then $(P, \leq)$ is totally ordered set.

Pr'

Show that the relation "greater than or equal to" is a partially ordering relation on a set of Integer.

$Z = \{ - - - -3, -2, -1, 0, 1, 2, 3 - - - - \}$

A relation greater than or equal to is a partially ordering relation if it satisfies 3 properties.

(i) $\geq$ is Reflexive
(ii) $\geq$ is Anti-symmetric
(iii) $\geq$ is Transitive

Reflexive

For every integer a, $a \in Z$ then $(a,a) \in R$

i.e $1 \geq 1, 2 \geq 2, 3 \geq 3, \in R$

$\therefore \geq$ is reflexive.

Antisymmetric.

For any two Integers a, b if $(a,b) \in Z$

i) If $(a,b) \in R$, $(b,a) \in R$ then $a = b$
ii) If $(a,b) \in R$, $(b,a) \notin R$ then $a \neq b$

i.e if $a \geq b$, $b \geq a$ then $a = b$

$\geq$ is anti-symmetric

Transitive.

For any three integers a, b, c if $(a,b,c) \in Z$,

if $(a,b) \in R$ $(b,c) \in R$ then $(a,c) \in R$.

if $a \geq b$, $b \geq c$ then $a \geq c$

$\therefore \geq$ is transitive.

$\therefore$ The given relation $\geq$ is partially ordering relation.

If $P = \{1, 2, 3, 4, 6, 12\}$ $R = \{(x,y) \in R \,/\, x \text{ divides } y\}$ prove that given relation is partial ordering Relation or not.

Given data $P = \{1, 2, 3, 4, 6, 12\}$

$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,6)\ (1,12)\ (2,2)\ (2,4)\ (2,6)\ (2,12)\ (3,3)\ (3,6)\ (3,12)\ (4,4)\ (4,12)\ (6,6)\ (6,12)\ (12,12)\}$

i) Reflexive

∴ R satisfies Reflexive property.

Anti-symmetric

If $(a,b) \in R$ and $(b,a) \notin R$ then $a \neq b$

i.e. $(1,3) \in R \Rightarrow (3,1) \notin R$
$(1,2) \in R \Rightarrow (2,1) \notin R$
$(1,4) \in R \Rightarrow (4,1) \notin R$
$(1,6) \in R \Rightarrow (6,1) \notin R$
$(1,12) \in R \Rightarrow (12,1) \notin R$

∴ R satisfies anti-symmetric property

Transitive

If $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$

$(1,1) \in R$ $(1,2) \in R$ then $(1,2) \in R$
$(1,2) \in R$ $(2,2) \in R$ then $(1,2) \in R$
$(1,3) \in R$ $(3,3) \in R$ then $(1,3) \in R$
$(1,4) \in R$ $(4,4) \in R$ then $(1,4) \in R$
$(1,6) \in R$ $(6,6) \in R$ then $(1,6) \in R$
$(1,12) \in R$ $(12,12) \in R$ then $(1,12) \in R$
$(2,2) \in R$ $(2,4) \in R$ then $(2,4) \in R$
$(2,4) \in R$ $(4,12) \in R$ then $(2,12) \in R$
$(2,6) \in R$ $(6,12) \in R$ then $(2,12) \in R$
$(3,3) \in R$ $(3,6) \in R$ then $(3,6) \in R$

∴ R satisfies Transitive property

∴ R satisfies reflexive, Antisymmetric and Transitive property

∴ R is $\leq$

Hassie Diagram.

A partial ordering relation on a set P can be represented by means of a diagram is known as Hassie Diagram.

1. In such a diagram each element of P is represented by a(·) or o
2. If $x \leq y$ then x is drawn below y and if y covers x. then x and y are connected by a straight line (—)
3. If $x < y$ and y does not cover x then x and y are connected directly by a single line.

Note:-

1. In hassie diagram we don't draw loops
2. If there is an edge from a vertex to vertex b. and b to c

Rules.

1. Remove Reflexive
2. Remove Transitive
3. Arrange elements vertically
4. Draw lines omit arrows label the elements.

Let $P = \{1,2,3,4\}$ and $\leq$ be the relation "less than or equal to" or "$\leq$" then draw the hassie diagram.

Given

$P = \{1,2,3,4\}$

$R = \{(a,b) / a,b \in P \text{ and } a \leq b\}$

Here R is partially ordering relation because R is reflexive antisymmetric and Transitive. For hassie diagram

Remove Reflexive.

$R = \{ \cancel{(1,1)}\ (1,2)\ \cancel{(1,3)}\ \cancel{(1,4)}\ \cancel{(2,2)}\ (2,3)\ \cancel{(2,4)}\ \cancel{(3,3)}\ (3,4)\ (4,4) \}$

Remove Reflexive

Remove $\{(1,1)\ (2,2)\ (3,3)\ (4,4)\}$

Remove Transitive.

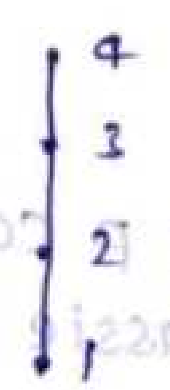
$(1,2) \in R\ (2,3) \in R \Rightarrow (1,3) \in R$

$(1,3) \in R\ (3,4) \in R \Rightarrow (1,4) \in R$

$(2,3) \in R\ (3,4) \in R \Rightarrow (2,4) \in R$

$P = \{(1,2), (2,3)\ (3,4)\}$

Hassie Diagram



Let $X = \{2,3,6,12,24,36\}$ and the relation $\leq$ and $x,y \in X$, $x \leq y$ and if x divides y draw the hassie diagram.

Given

$X = \{2,3,6,12,24,36\}$

$R = \{(x,y) / x,y \in X \text{ and } x/y\}$

$R = \{(2,2)\ (2,6)(2,12)\ (2,24)\ (2,36)\ (3,3)\ (3,6)\ (3,12)\ (3,24)\ (3,36)\ (6,6)\ (6,12)(6,24)\ (6,36)\ (12,12)\ (12,24)\ (12,36), (36,36)\}$

Remove Reflexive

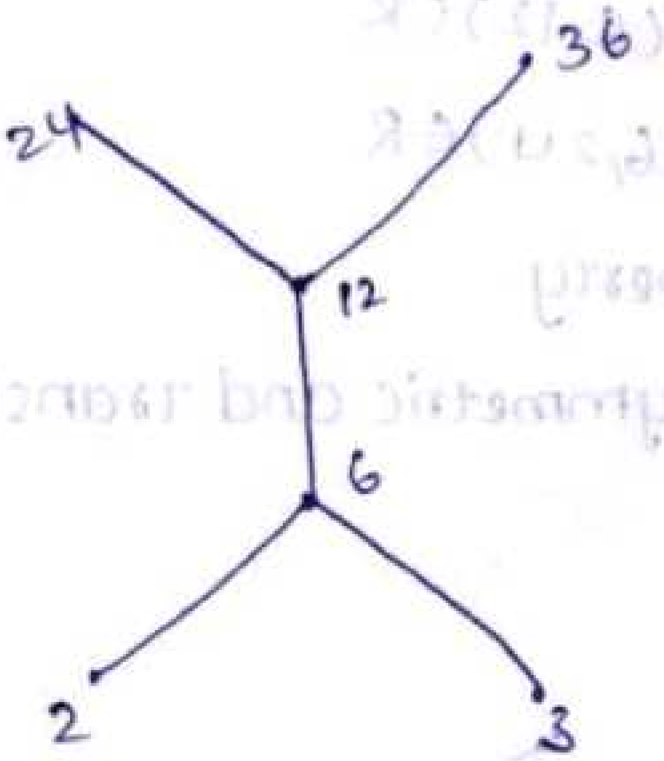
Remove. {(2,2) (3,3) (6,6) (12,12) (24,24) (36,36)}

Remove Transitive

(2,6) ∈ R (6,12) ∈ R → (2,12) ∈ R
(2,12) ∈ R (12,24) ∈ R → (2,24) ∈ R
(2,36) ∈ R (6,36) ∈ R → (2,36) ∈ R
(3,6) ∈ R (6,24) ∈ R → (3,24) ∈ R
(3,6) ∈ R (6,12) ∈ R → (3,12) ∈ R
(3,6) ∈ R (6,36) ∈ R → (3,36) ∈ R
(6,12) ∈ R (12,24) ∈ R → (6,24) ∈ R
(6,36) ∈ R (12,36) ∈ R → (6,36) ∈ R

x = {(2,6) (3,6) (6,12) (12,24) (12,36)}

Hassie diagram



Draw the hassie diagram p = {1,2,3,5,6,10,15,30}, /.

Let A = {1,2,3,6,12,24} on A Define the partial ordering relation R by a R b iff and only if a/b. draw hassie diagram.

Given

A = {1,2,3,6,12,24}

R = {(1,1) (1,2) (1,3) (1,6) (1,12) (1,24) (2,2) (2,6) (2,12) (2,24) (3,3) (3,6) (3,12) (3,24) (6,6) (6,12) (6,24) (12,12) (12,24) (24,24)}

Reflexive property

if A ∈ A then (a,a) ∈ R

i.e (1,1) ∈ R (2,2) ∈ R (3,3) ∈ R (6,6) ∈ R (12,12) ∈ R, (24,24) ∈ R

∴ R satisfies reflexive property.

Anti-symmetric property

if (a,b) ∈ R and (b,a) ∉ R then a ≠ b.

i.e (1,2) ∈ R ⇒ (2,1) ∉ R

$(1,12) \in R \Rightarrow (12,1) \notin R$

$(1,24) \in R \Rightarrow (24,1) \in R$

$\therefore$ R satisfies anti-symmetric property

Transitive property.

If $(a,b) \in R$ $(b,c) \in R$ then $(a,c) \in R$

$(1,1) \in R$ $(1,2) \in R$ then $(1,2) \in R$
$(1,3) \in R$ $(3,3) \in R$ then $(1,3) \in R$
$(1,6) \in R$ $(6,6) \in R$ then $(1,6) \in R$
$(1,12) \in R$ $(12,12) \in R$ then $(1,12) \in R$
$(1,24) \in R$ $(24,24) \in R$ then $(1,24) \in R$
$(2,2) \in R$ $(2,6) \in R$ then $(2,6) \in R$
$(2,6) \in R$ $(6,12) \in R$ then $(2,12) \in R$
$(2,12) \in R$ $(12,24) \in R$ then $(2,24) \in R$
$(3,3) \in R$ $(3,6) \in R$ then $(3,6) \in R$
$(3,12) \in R$ $(12,12) \in R$ then $(3,12) \in R$
$(3,24) \in R$ $(24,24) \in R$ then $(3,24) \in R$
$(6,6) \in R$ $(6,12) \in R$ then $(6,12) \in R$
$(6,24) \in R$ $(24,24) \in R$ then $(6,24) \in R$

$\therefore$ R satisfies Transitive property

$\therefore$ R satisfies Reflexive, Antisymmetric and Transitive property

$\therefore$ R is $\leq$

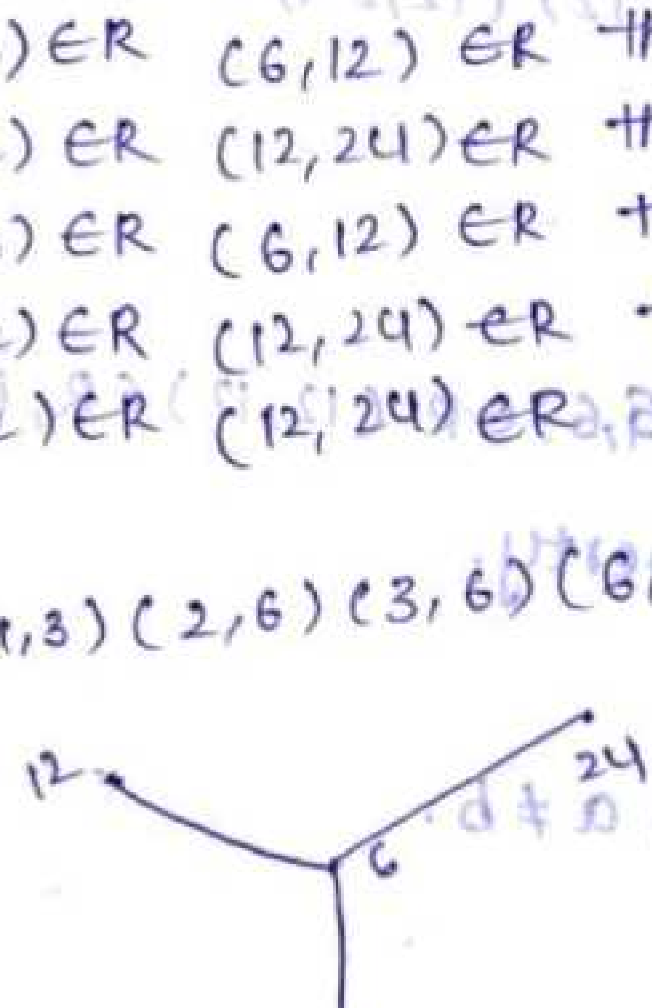
Remove Reflexive

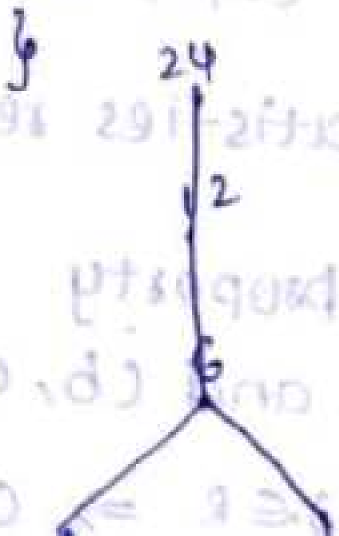
Remove $\{(1,1)(2,2)(3,3)(6,6)(12,12)(24,24)\}$

Remove Transitive

$(1,2) \in R$ $(2,6) \in R$ then $(1,6) \in R$
$(1,3) \in R$ $(3,12) \in R$ then $(1,12) \in R$
$(1,6) \in R$ $(6,24) \in R$ then $(1,24) \in R$
$(2,6) \in R$ $(6,12) \in R$ then $(2,12) \in R$
$(2,12) \in R$ $(12,24) \in R$ then $(2,24) \in R$
$(3,6) \in R$ $(6,12) \in R$ then $(3,12) \in R$
$(3,12) \in R$ $(12,24) \in R$ then $(3,24) \in R$
$(6,12) \in R$ $(12,24) \in R$ then $(6,24) \in R$

$A = \{(1,2)(1,3)(2,6)(3,6)(6,12)(12,24)\}$





Draw the hassie diagram $P\{1,2,3,5,6,10,15,30\}$. /.

Given

$P = \{1,2,3,5,6,10,15,30\}$

$R = \{(1,1)(1,2)(1,3)(1,5)(1,6)(1,10)(1,15)(1,30)(2,2)\cancel{(2,3)}\cancel{(2,5)}$
$(2,6)(2,10)\cancel{(2,15)}(2,30)(3,3)(3,6)(3,15)(3,30)(5,5)$
$(5,10)(5,15)(5,30),(6,30)(10,10)(10,30)(15,15)(15,30)(30,30)\}$
$(6,6)$

Remove Reflexive:

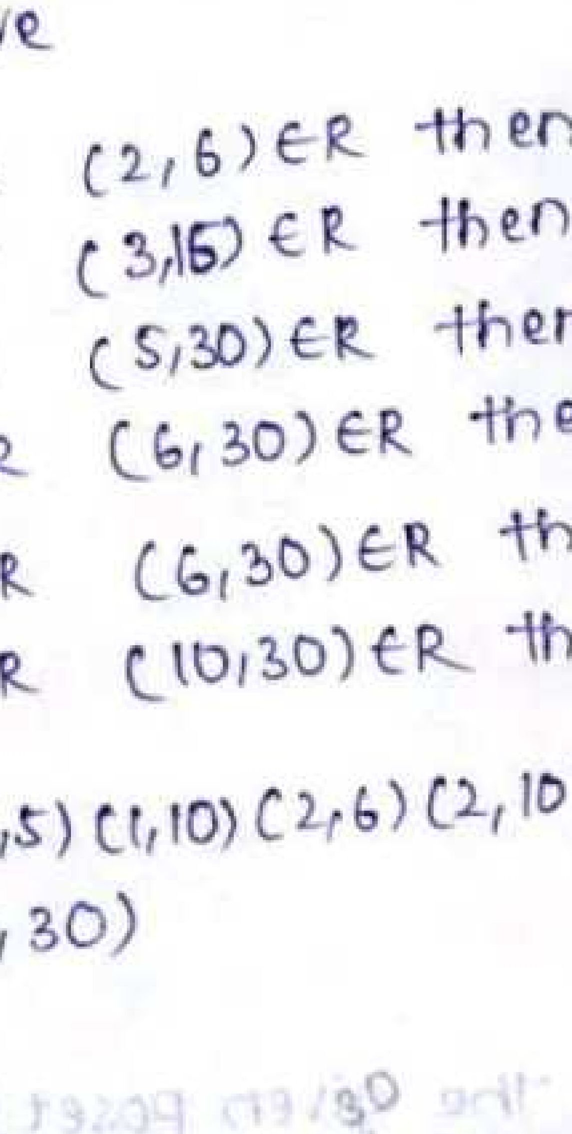
Remove: $\{(1,1)(2,2)(3,3)(5,5)(6,6)(10,10)(15,15)(30,30)\}$

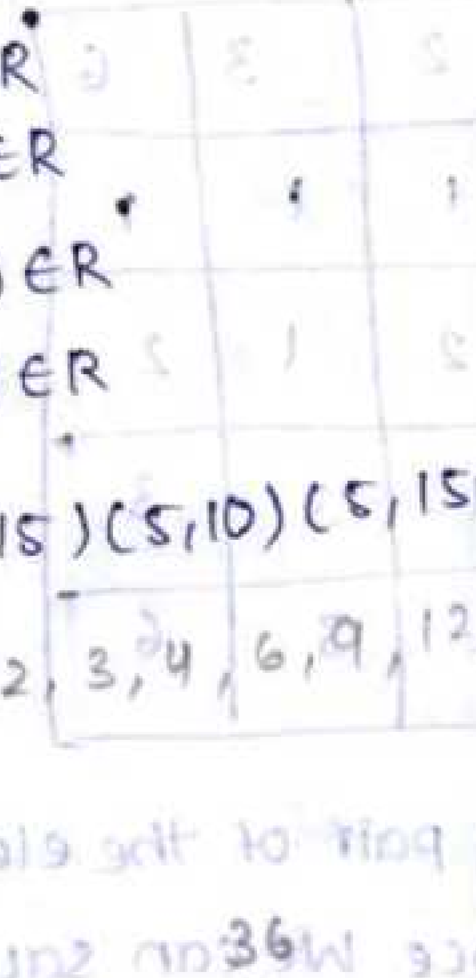
Remove Transitive

$(1,2)\in R$ $(2,6)\in R$ then $(1,6)\in R$
$(1,3)\in R$ $(3,15)\in R$ then $(1,15)\in R$
$(1,5)\in R$ $(5,30)\in R$ then $(1,30)\in R$
$(2,6)\in R$ $(6,30)\in R$ then $(2,30)\in R$
$(3,6)\in R$ $(6,30)\in R$ then $(3,30)\in R$
$(5,10)\in R$ $(10,30)\in R$ then $(5,30)\in R$

$P = \{(1,2)(1,3)(1,5)(1,10)(2,6)(2,10)(3,6)(3,15)(5,10)(5,15)(6,30)$
$(10,30)(15,30)$

1, 2, 3, 4, 6, 9, 12, 18, 36





lattice

A lattice is a partially ordered set $(L,\leq)$, in which every pair of elements such as $(a,b)\in L$ as a greatest lower bound (GLB) and a least upper bound (LUB)

LUB

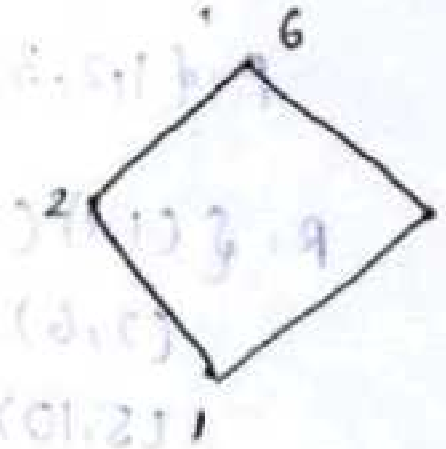
The LUB (supremum) of a subset $\{a,b\}\subseteq L$ is denoted by $a\vee b$ (or) $a\cup b$ (or) $a\oplus b$ is called the sum of a and b (or) joint.

GLB.

The GLB (infimum) of a subset $(a,b)\subseteq L$ is denoted by $a\wedge b$ (or) $a\wedge b$ (or) $a*b$ is called the product of a and b or

Check wheather the given poset of a hassie diagram is lattice or not

LUB Table.



LUB	1	2	3	6
1	1	2	3	6
2	2	2	6	6
3	3	6	3	6
6	6	6	6	6

GLB Table

GLB	1	2	3	6
1	1	1	1	1
2	1	2	1	2
3	1	1	3	3
6	1	2	3	6

∴ For every pair of the elements in the given poset both LUB and GLB exist. hence we can say that given poset is a lattice.

Let $L = \{1, 2, 3, 5, 30\}$ and R be the relation "is divisible defined on L show that L is laticee or not?

Given

$L = \{1, 2, 3, 5, 30\}$

R = is the divisibility relation defined on L oreo

$R = \{(1,1)\ (1,2)\ (1,3)\ (1,5)\ (1,30)\ (2,2)\ (2,30)\ (3,3)\ (3,30)(5,5)$
$(5,30)\ (30,30)\}$

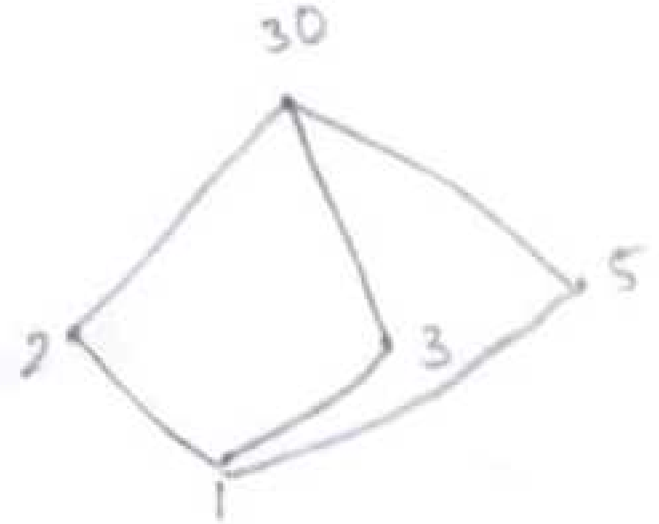
Remove Reflexive

Remove : $\{(1,1)(2,2)(3,3)(5,5)(30,30)\}$

Remove Transitive

$(1,2) \in R\ (2,30) \in R\ (1,30) \in R$

$\{(1,2)\ (1,3)\ (1,5)\ (2,30)\ (3,30)\ (5,30)\}$



GLB	1	2	3	5	30
1	1	1	1	1	1
2	1	2	1	1	2
3	1	1	3	1	3
5	1	1	1	5	5
30	1	2	3	5	30

LUB	1	2	3	5	30
1	1	2	3	5	30
2	2	2	30	30	30
3	3	30	3	30	30
5	5	30	30	30	30
30	30	30	30	30	30

For every pair of the elements in the given poset both LUB and GLB exist. hence we can say that given poset is a lattice.

Show that $(D_6, /)$ is a lattice or not (or)

Show that D_6, / that means factors of 6 is a lattice or not.

Factors of 6 : {1, 2, 3, 6}

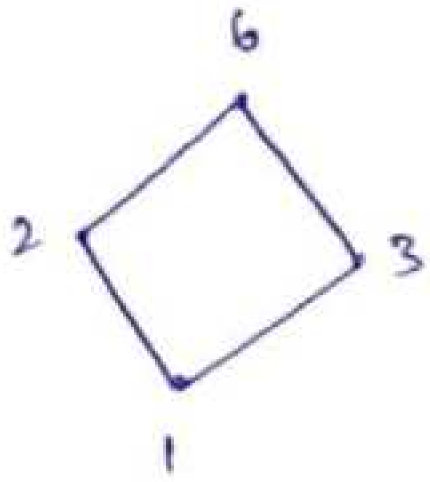
R = {(1,1) (1,2) (1,3) (1,6) (2,2) (2,6) (3,3) (3,6) (6,6)}

Remove Reflexive

Remove = {(1,1) (2,2) (3,3) (6,6)}

Remove Transitive:

(1,2) ∈ R (2,6) ∈ R (1,6) ∈ R



properties of lattice.

let $(L, \leq)$ be a lattice for $(a,b,c) \in L$, L satisfies following conditions:

1) Idempotant property.

$$a \wedge a = a \text{ and } a \vee a = a \quad \forall a \in L$$

2) commutative property

$$a \wedge b = b \wedge a \text{ and } a \vee b = b \vee a, \forall a, b \in L$$

3) Associative property

$$(a \wedge b) \wedge c = a \wedge (b \wedge c) \text{ and } (a \vee b) \vee c = a \vee (b \vee c) \quad \forall (a,b,c) \in L$$

4) Absorption property

$$a \wedge (a \vee b) = a \text{ and } a \vee (a \wedge b) = a \quad \forall a, b \in L$$

UNIT-III

Elementary combinatorics

Basics of counting.

The basics of counting are foundational principles in combinatorics that help determine the no. of ways events (or) outcomes can occur. These principles are used to solve problems involving arrangements, selections and combinations.

There are two fundamental rules (or) counting

1. sum rule (Disjunctive rule)
2. product rule (sequential rule)

sum rule.

If there are two tasks and the first task can be done in 'm' ways and the second task can be done in 'n' ways, and the tasks cannot be happen at the same time. then the total no. of ways one of the two tasks t_1 (or) t_2 can be performed in $m+n$ ways

EX1: If there are 5 boys and 4 girls in a class then there are $5+4=9$ ways of selecting one student as a class representative (either a boy or a girl)

EX2: suppose a hostel library has 12 books on maths, 10 books on physics, 16 books on computer science and 11 books on electronics suppose a student wishes to choose one of these books for study. The no. of ways in which he can choose a book is $12+10+16+11=49$ ways

product rule:

suppose that two task t_1 and t_2 are to be performed one after the other. If t_1 can be performed in n_1 different ways and for each of these ways. t_2 can be performed in n_2 difficult ways or different ways. Then both of tasks t_1 and t_2 can be performed simultaneously in $n_1 \cdot n_2$ ways

Ex: suppose a person has three shirts and five ties in how many ways a person can choose a shirt and a tie.

Sol: The no. of ways a person can choose a shirt and a tie is $3 \times 5 = 15$ ways

Explanation

let $S_1 S_2 S_3$ be a shirts

$T_1 T_2, T_3, T_4, T_5$ be 2 ties

$S_1 T_1$	$S_2 T_1$	$S_3 T_1$
$S_1 T_2$	$S_2 T_2$	$S_3 T_2$
$S_1 T_3$	$S_2 T_3$	$S_3 T_3$
		$S_3 T_4$

* In how many ways we construct sequence of 4 symbols (letters) in which the first two are english letters and next two are single digit numbers if

1. No digit (or) letter can be repeated
2. Repetition of letters and digits is allowed.

We have 26 letters in english and 10 digits (i.e 0 to 9). We need to construct sequence of 4 letters in which first two are english letters and next 2 are single digit numbers.

i) 26 25 10 9

A B 9 8

$26 \times 25 \times 10 \times 9$

If no digit or letter can be repeated then the no. of different ways $= 26 \times 25 \times 10 \times 9 = 58500$ ways

ii) A A 9 9

26 26 10 10

If repetition is allowed then the no. of different ways is

eq. $26 \times 26 \times 10 \times 10 = 67,600$ ways.

In how many ways can 3 different coins be placed in two different purses.

(you can put first coin in any of the 2 purses it gives you two opportunities

you can put ~~first~~ second coin any of the 2 purses it gives you two opportunities

you can put third coin any of the)

Note:- If you have n coins and 2 purses, then the no. of ways n coins be placed in 2 different purses $= 2^n$

combinations and permutations.

- combinatorics
  - permutations
    - linear permutation
      - without repetition
      - with repetion
    - circular permutation
  - combination
    - without repetition
    - with repetition

permutations:

1. selection
2. Arrangement < ordered / Distinguishable / Different

An ordered arrangement of a given set of objects taken some (or) all of them at a time is called a permutation of the objects.

→ The arrangement ~~(small co)~~ 'r' objects from a set of 'n' objects is called r-permutation

→ The no. of permutations of n distinct objects taken 'r' at a time without repetitions is denoted by $P(n,r) = \frac{n!}{(n-r)!}$

$$P(n,r) = \frac{n!}{(n-r)!}$$

How many different strings (sequences) of length 4 can be performed using the letters of the word "FLOWER"

The given word FLOWER has 6 letters all of which are distinct. Therefore the no. of strings of length 4 can be formed by using 6 different letters.

$$P(6,4) = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6\times5\times4\times3\times\cancel{2!}}{\cancel{2!}} = 360.$$

Find the no. of permutations of the letters in the WORD SUCCESS

The word success has 7 letters In this word we have 3 S's, 2 C's, 1U and 1E.

∴ The required no. of permutations $= \frac{7!}{\ldots} = \frac{7\times6\times5\times\overset{2}{\cancel{4}}\times\cancel{3!}}{\cancel{3!}\,\cancel{2!}}$

How many 9 letter words can be formed by using the letter of the word "DIFFICULT"

The given word DIFFICULT has 9 letters

D's = 1, I's = 2, F's = 2, C's = 1, U's = 1, L's = 1, T's = 1

∴ The no. of required 9 letter words = The no. of permutations of these letters

$$= \frac{9!}{1!\,2!\,2!\,1!\,1!\,1!\,1!}$$

$$= \frac{9\times8\times7\times6\times5\times\cancel{4}^{2}\times3\times2!}{1!\,\cancel{2}!\,\cancel{2}!\,1!\,1!\,1!\,1!}$$

$$= 90720$$

Find the no. of permutations of the letters of the word "MASSASAUGA" In how many of these all four A's are together? How many of them begin with S?

The given word MASSASUGA has 10 letters

The given word MASSASAUGA has 10 letters of which A's = 4, S's = 3, M's = 1, G's = 1, U's = 1

the required no. of permutations $= \frac{10!}{4!\,3!\,1!\,1!\,1!}$

$$= \frac{10\times9\times8\times7\times6\times5\times4!}{4!\,3!\,1!\,1!\,1!}$$

$$= 25200$$

If all A's are to be together we treat all A's as one single letter. Now we have to permute the letters (AAAA), SSS, M, U, G

1! 3! 1! 1! 1!

$$= \frac{7!}{1!\,3!\,1!\,1!\,1!}$$

$$= \frac{7\times6\times5\times4\times3!}{3!}$$

$$= 840$$

For permutation begin with S there are 9 places to fill with the remaining letters

$\therefore$ The no. of permutations begin with s $= \dfrac{9!}{4!\,2!\,1!\,1!\,1!}$

$$= \frac{9\times8\times7\times6\times5\times\cancel{4!}}{\cancel{4!}\,2!}$$ 7560

$$= 7560.$$

Find the value of n so that $P(n,2) = 90$?

$P(n,2) = 90$

$\dfrac{n!}{(n-2)!} = 90$

$\Rightarrow \dfrac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 90$

$\Rightarrow n^2 - n = 90$

$\Rightarrow n^2 - n - 90 = 0$

$\Rightarrow n^2 - 10n + 9n - 90 = 0$

$\Rightarrow n(n-10) + 9(n-10) = 0$

$\Rightarrow (n-10)(n+9) = 0$

$\Rightarrow n = 10, n = -9$

$\Rightarrow n = 10$

4. Recurrence Relations

Generating functions of sequence

Let $A = \{a_n\}_{n=0}^{\infty} = (a_0, a_1, \ldots, a_n \ldots)$ be the sequence of then its generating function is defined to be $A(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n + \ldots = \sum_{n=0}^{\infty} a_n x^n$

Note:

The sequence $a_n; n = 0$ to ∞ can also be denoted by $\{a_n\}$ or (a_n) or $\langle a_n \rangle$ or $\{a_r\}_{r=0}^{\infty}$ or $\{a_1, a_2, \ldots, a_n, \ldots\}$

Important Formulas

1. $\frac{1}{1-x} = (1-x)^{-1} = 1 + x + x^2 + \ldots \sum_{r=0}^{\infty} x^r$

2. $\frac{1}{1-ax} = (1-ax)^{-1} = 1 + ax + a^2x^2 + \ldots \sum_{r=0}^{\infty} a^r x^r$

3. $\frac{1}{1+x} = (1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots \sum_{r=0}^{\infty} (-1)^r (x)^r$

4. $\frac{1}{(1-x)^2} = (1-x)^{-2} = 1 + 2x + 3x^2 + \ldots \sum_{r=0}^{\infty} (r+1) x^r$

5. $\frac{1}{(1+x)^2} = (1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots = \sum_{r=0}^{\infty} (-1)^r (r+1) x^r$

*6. $(1-x)^{-n} = \sum_{r=0}^{\infty} c(n+r-1, r) x^r$

$(1+x)^{-n} = \sum_{r=0}^{\infty} (-1)^r c(n+r-1, r) x^r$

*7 $(1+x)^n = \sum_{r=0}^{n} c(n, r) x^r$

$(1-x)^n = \sum_{r=0}^{n} (-1)^r c(n, r) x^r$

8 $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

$1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

9. $a + ar + ar^2 + \ldots + ar^{n-1} = \begin{cases} \frac{a(1-r)^n}{(1-r)} & \text{if } r < 1 \\ \frac{a(r^n - 1)}{r-1} & \text{if } r > 1 \end{cases}$

$\ldots x^n \quad 1-x^{n+1}$

11. $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

1. Find the generating functions for the sequence $(1,1,1,1,1,1 \ldots)$

Given that

$a_0 = 1,\ a_1 = 1,\ a_2 = 1,\ a_3 = 1, \ldots$

$\therefore$ The generating function is

$A(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$

$= 1 + 1x + 1x^2 + 1x^3 + \cdots$

$= 1 + x + x^2 + x^3 + \cdots$

$= (1-x)^{-1}$

2. Find the generating function for the following sequences

(i) $1, 2, 3, 4 \ldots$

Given

$a_0 = 1,\ a_1 = 2,\ a_2 = 3,\ a_3 = 4 \ldots$

$\therefore$ The generating function is

$A(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$

$= 1 + 2x + 3x^2 + 4x^3 + \cdots$

$= (1-x)^{-2}$

(ii) $1, -2, 3, -4 \ldots$

Given

$a_0 = 1,\ a_1 = -2,\ a_2 = 3,\ a_3 = -4 \ldots$

$\therefore$ The generating function is

$A(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$

$= 1 - 2x + 3x^2 - 4x^3 + \cdots$

$= (1+x)^{-2}$

(iii) $0, 1, 2, 3$

Given

$a_0 = 0,\ a_1 = 1,\ a_2 = 2,\ a_3 = 3$

$\therefore$ The generating function is

$A(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$

$= 0 + 1x + 2x^2 + 3x^3 \cdots$

$= x + 2x^2 + 3x^3 \cdots$

iv $0, 1, -2, 3, -4 - - - -$

Given

$a_0 = 0, \; a_1 = 1, \; a_2 = -2, \; a_3 = 3, \; a_4 = -4$

$\therefore$ The generating function is

$$A(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 - - - -$$

$$= 0 + 1x - 2x^2 + 3x^3 - 4x^4 - - - - -$$

$$= 1x - 2x^2 + 3x^3 - 4x^4 - - - - -$$

$$= x(1 - 2x + 3x^2 - 4x^3 - -)$$

$$= x(1+x)^{-2}$$

v $\{a_n\} = \{4\}$

i.e $a_0 = a_1 = a_2 = a_3 = 4$

$\therefore$ The generating function is

$$A(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 - - - -$$

$$= 4 + 4x + 4x^2 + 4x^3 - - - -$$

$$= 4(1 + x + x^2 + x^3 - - - -)$$

$$= 4(1-x)^{-1}$$

· Find the generating function for the following sequences.

$1, 1, 0, 1, 1, 1 - - -$

Given

$a_0 = 1, \; a_1 = 1, \; a_2 = 0, \; a_3 = 1, \; a_4 = 1, \; a_5 = 1 - - -$

$\therefore$ The generating function is

$$A(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 - - - -$$

$$= 1 + x + 0x^2 + x^3 + - - - -$$

$$= 1 + x + 0 + x^3 + - - - -$$

$$= (1-x)^{-1} - x^2$$

** Find the generating function for the sequences

$a_n = 9.5^n \quad n \geq 0 - - -$

$$A(x) = \sum_{n=0}^{\alpha} a_n x^n$$

$$= \sum^{\alpha} 9.5^n x^n$$

$9\sum_{n=0}^{\alpha}(5x)^n$

$= 9(1+5x+(5x)^2+(5x)^3+\cdots)$

$= 9(1-5x)^{-1}$

$a_n = 4, n \geq 0$

$A(x) = \sum_{n=0}^{\alpha} a_n x^n$

$= \sum_{n=0}^{\alpha} 4, x^n$

$= 4\sum_{n=0}^{\alpha} x^n$

$= 4(1+x+x^2+x^3 \cdots)$

$= 4(1-x)^{-1}$

. Find the generating function $\{a_n\} = \{\frac{1}{n!}\}$

$\therefore$ The generating function is

$\sum_{n=0}^{\alpha} a^n x^n$

$\sum_{n=0}^{\alpha} \frac{1}{n} x^n$

$= 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$= e^x$

. Find the generating function $(1-\alpha x)^{-1}$

$1, \alpha, \alpha^2, \alpha^3 \cdots$

Given

$a_0 = 1, a_1 = \alpha, a_2 = \alpha^2, a_3 = \alpha^3$

$\therefore$ The generating function is

$A(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 \cdots$

$= 1 + \alpha x + \alpha^2 x^2 + \alpha^3 x^3 \cdots$

$= 1 + \alpha x + \alpha^2 x^2 + \alpha^3 x^3 \cdots$

$= 1 + \alpha x + \alpha x^2 + \alpha x^3 \cdots$

$= (1-\alpha x)^{-1}$

Find the sequences generated by the following functions

(i) $(2+x)^3$

$(x+y)^n = {}^nC_0\, y^0 x^n + {}^nC_1\, y^1 x^{n-1} + {}^nC_2\, y^2 x^{n-2} + \dots + {}^nC_n\, y^n x^0$

$A(x) = (\overset{y+x}{2+x})^3$

$= {}^3C_0\, 2^0 x^3 + {}^3C_1\, 2^1 x^{3-1} + {}^3C_2\, 2^2 x^{3-2} + {}^3C_3\, 2^3 x^{3-3}$

$= 1+x^3 + 3\,2.x^2 + 3.4\,x + 1.8.1$

$= x^3 + 6x^2 + 12x + 8$

$= 8 + 12x + 6x^2 + x^3$

$a_0 = 8,\ a_1 = 12\ \ a_2 = 6, a_3 = 1$

The sequence of the function $(2+x)^3 = 8, 12, 6, 1 \dots$

ii) $(2+x)^4$

$(x+y)^n = {}^nC_0\, y^0 x^n + {}^nC_1\, y^1 x^{n-1} + {}^nC_2\, y^2 x^{n-2} + \dots + {}^nC_n\, y^n x^0$

$A(x) = (2+x)^4$

$= {}^4C_0\, 2^0 x^4 + {}^4C_1\, 2^1 x^{4-1} + {}^4C_2\, 2^2 x^{4-2} + {}^4C_3\, 2^3 x^{4-3} +$

${}^4C_4\, 2^4 x^{4-4} \dots$

$= 1.x^4 + 4.2.x^3 + 6.4.x^2 + 4.8.x^1 + 1.16.1$

$= x^4 + 8x^3 + 24x^2 + 32x^1 + 16$

$= 16 + 32x + 24x^2 + 8x^3 + x^4$

$a_0 = 16,\ a_1 = 32,\ a_2 = 24,\ a_3 = 8,\ a_4 = 1$

The sequence of the function $(2+x)^4 = 16, 32, 24, 8, 1 \dots$

iii) $2x^2(1-x)^{-1}$

$2x^2(1 + x + x^2 + x^3 + \dots)$

$2x^2 + 2x^3 + 2x^4 + 2x^5 + \dots$

$a_0 = 0,\ a_1 = 0,\ a_2 = 2,\ a_3 = 2 \dots$

The sequence of the function is $0, 0, 2, 2, 2 \dots$

iv) $3x^3 + e^{2x}$

$$3x^3 + \left(1 + \frac{2x}{1!} + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \cdots\right)$$

$$1 + 2x + 2x^2 + \frac{(2x)^3}{3!} + \frac{3x^3}{1} + \cdots$$

$$1 + 2x + 2x^2 + \frac{\cancel{26}^{13} x^3}{\cancel{6}_3} + \cdots$$

$$a_0 = 1, a_1 = 2, a_2 = 2\ a_3 = \frac{13}{3} \cdots$$

Find the co-efficient of x^{27} in $(x^4 + x^5 + x^6 + \cdots)^5$

Given

$$(x^4 + x^5 + x^6 \cdots)^5$$

$$(x^4)^5 (1 + x + x^2 + \cdots)^5$$

$$x^{20} (1-x)^{-5}$$

$$x^{20} \sum_{r=0}^{\infty} c(n+r-1, r) x^r$$

$$x^{20} \sum_{r=0}^{\infty} c(5+r-1, r) x^r$$

$$\sum_{r=0}^{\infty} c(4+r, r) x^r x^{20}$$

$$\sum_{r=0}^{\infty} c(4+r, r) x^{20+r}$$

put $r = 7$ i.e x^{27}

$$c(4+7, 7) x^{27}$$

$\therefore$ The coefficient of x^{27} is

$$c(11, 7)$$

$$\Rightarrow \frac{11!}{(11-7)!\, 7!} \qquad \frac{n}{(n-r)!\, r!}$$

$$\Rightarrow \frac{11 \times 10 \times 9 \times 8 \times \cancel{7!}}{4!\, \cancel{7!}}$$

$$330$$

Find the co-efficient of x^{10} in $(1+x+x^2+\ldots\ldots)^2$

Given

$(1+x+x^2+\ldots)$

$(1-x)^{-2}$

$\sum_{r=0}^{\alpha} c(n+r-1, r)\, x^r$

$\sum_{r=0}^{\alpha} c(2+r-1, r)\, x^r$

$\sum_{r=0}^{\alpha} c(1+r, r)\, x^r$

$\sum_{r=0}^{\alpha}$ put $r = 10$

$c(1+10, 10)\, x^{10}$

The coefficient of x^{10} is $c(11,10) = \frac{11!}{(11-10)!\,10!} = \frac{11 \times \cancel{10!}}{1!\,\cancel{10!}}$

$= \frac{11}{1} = 11$

Find the co-efficient of x^{10} in $\frac{1}{(1-x)^3}$

Given

$\frac{1}{(1-x)^3}$

$= (1-x)^{-3}$

$\sum_{r=0}^{\alpha} c(n+r-1, r)\, x^r$

$\sum_{r=0}^{\alpha} c(3+r-1, r)\, x^r$

$\sum_{r=0}^{\alpha} c(2+r, r)\, x^r$

put $r = 10$

$c(2+10, 10)\, x^{10}$

The coefficient of x^{10} is $c(12,10) = \frac{12!}{(12-10)!\,10!} = \frac{12 \times 11 \times \cancel{10!}}{2!\,\cancel{10!}}$

Find the coefficient of x^{32} in $(1+x^5+x^9)^{10}$

Note:-

1. The coefficient of $x^{r_1} y^{r_2}$ in $(x+y)^n$ is $\frac{n!}{r_1! \cdot r_2!}$,

where $r_1+r_2=n$

2. The coefficient of $x_1^{r_1} x_2^{r_2} x_3^{r_3} \ldots x_t^{r_t}$ in $(x_1+x_2+\ldots+x_t)^n$

is $\frac{n!}{r_1! r_2! r_3! \ldots r_t!}$ $\quad r_1+r_2+r_3+r_4 \ldots r_t = n.$

sol The co-efficient of $1^{r_1}(x^5)^{r_2}(x^9)^{r_3}$ in $(1+x^5+x^9)^{10}$ is

$$\frac{10!}{r_1!+r_2!+r_3!} \quad \text{Where } r_1+r_2+r_3=10$$

To get co-efficient of x^{32} we take

$$x_1 = n-(r_2+r_3)$$

$$r_1 = 10-(2+3)$$

$$r_1 = 5$$

The coefficient of x^{32} in $(1+x^5+x^9)^{10} = \frac{10!}{5!\,2!\,3!}$

$$= \frac{10\times9\times8\times7\times\cancel{6}\times\cancel{5!}}{\cancel{5!}\,2!\,\cancel{3!}} = \frac{5040}{2}$$

$$= 2520$$

Find the coefficient of x^{25} in $(1+x^3+x^8)^{10}$

The co-efficient of $1^{r_1}(x^3)^{r_2}(x^8)^{r_3}$ in $(1+x^3+x^8)^{10}$ is

$$\frac{10!}{r_1!+r_2!+r_3!} \quad \text{Where } r_1+r_2+r_3=10$$

To get coefficient of x^{25} we take

$$r_1 = n-(r_2+r_3)$$

the co-efficient of x^{25} in

$$(1+x^3+x^8)^{10} = \frac{10!}{5!\,3!\,2!}$$

$$= \frac{10\times 9\times 8\times 7\times 6\times 5!}{5!\,3!\,2!}$$

$$= \frac{5040}{2}$$

$$= 2520.$$

Recurrance Relation

A recurrance relation is a formula that relates the n^{th} term of a sequence to one or more of its previous terms such as $a_0, a_1, a_2 \text{----} a_{n-1}$

(or)

A recurrance relation for the sequence $a_0, a_1, a_2, \text{---} a_{n-1}$ is an equation that express a_n in terms of one or more of the previous terms in the sequence namely $a_0, a_1, a_2 \text{--} a_{n-1}$ for all integers $n \geq 1$

Note: Recurrance Relation is also called difference equation

Ex1:- If a_n denotes n^{th} term of a ~~geometric~~ geometric progression with common ratio r. then $a_n = r(a_{n-1})$

Order of Recurrance Relation

The order of recurrance relation is the difference between largest and smallest subscript appear in the relation

Ex:- $a_n - a_{n-1} = 0$

$n-(n-1) = 0$

$\cancel{n}-\cancel{n}+1 = 0$

order $= 1$

Linear recurrance relation:

suppose n and k are non-negative integers. A relation for $n \geq k$

where $c_1(n), c_2(n), \ldots c_k(n)$ are functions of n and $f(n)$ also constants. is said to be linear recurrance relation.

1. If $c_0(n)$ and $c_k(n)$ are not equal to zero then the relation is said to be linear recurrance relation.

*2. If $f(n)=0$ in eq ① then the relation is said to be homogeneous linear reccurance relation

EX: $a_n - a_{n-1} + 2a_{n-1} = 0$

3. If $f(n) \neq 0$ in eq① then the relation is said to be non-linear homogeneous reccurance relation (or) non-homogeneous reccurance relation

EX: $a_n - a_{n-1} + 2a_{n-1} = 8$

Fibonacci Recurrance Relation.

The reccurrance relation $F_n = F_{n-1} + F_{n-2}$, $n \geq 2$ with initial conditions $f_0 = f_1 = 1$ is known as Fibonacci Reccurrance Relation.

*methods to solve Recurrance Relation.

There are 3 methods to solve the reccurrance relations they are

* 1. substitution method
2. Generating Function method.
3. characteristic roots method.

substitution method.

In this method the recurrance relation for a_n is used repeatedly to solve for a general expression for a_n in terms of n.

①. solve the recurrance relation $a_n = a_{n-1} + f(n)$ for $n \geq 1$ by using substitution method.

Given

$a_n = a_{n-1} + f(n) \rightarrow$ ①

put $n=1$ in eqn ① we get,

$a_1 = a_0 + f(1)$

put $n=2$ in eqn ① we get,

$a_2 = a_1 + f(2)$

put $n=3$ in eqn ① we get

$$a_3 = a_2 + f(3)$$

$$= a_0 + f(1) + f(2) + f(3)$$

Similarly

$$a_n = a_0 + f(1) + f(2) + f(3) + \cdots$$

$$a_n = a_0 + \sum_{r=1}^{\alpha} f(r)$$

2. solve the reccurrance relation $a_n = a_{n-1} + n^2$ where $a_0 = 7$ by using substitution method.

Given

$$a_n = a_{n-1} + n^2 \rightarrow ①$$

put $n=1$ in eqn ① we get

$$a_1 = a_0 + 1^2$$

$$= 7 + 1^2$$

put $n=2$ in eqn ① we get

$$a_2 = a_1 + 2^2$$

$$a_2 = 7 + 1^2 + 2^2$$

put $n=3$ in eqn ① we get

$$a_3 = a_2 + 3^2$$

$$= 7 + 1^2 + 2^2 + 3^2$$

Similarly

$$a_n = a_0 + 1^2 + 2^2 + 3^2 + \cdots$$

$$= 7 + 1^2 + 2^2 + 3^2 + \cdots$$

$$= 7 + \frac{n(n+1)(2n+1)}{6}$$

*** solve the recurrance relation $a_n = a_{n-1} + 3^n$ where $a_0 = 1$ by using substitution method

Given

$$a_0 = 1$$

$$a_n = a_{n-1} + 3^n \rightarrow ①$$

put $n=1$ in eqn ① we get

put $n=2$ in eq ① we get

$$a_2 = a_1 + 3^2$$
$$= 1 + 3^1 + 3^2$$

put $n=3$ in eq ① we get

$$a_3 = a_2 + 3^3$$
$$= 1 + 3^1 + 3^2 + 3^3$$

similarly

$$a_n = a_0 + 3^1 + 3^2 + 3^3 - - - -$$
$$= 1 + 3^1 + 3^2 + 3^3 \cdots\cdots$$
$$= \frac{1-3^{n+1}}{1-3}$$

solve the reccurrance relation $a_n = a_{n-1} + \frac{1}{n(n+1)}$ where $a_0 = 1$

Given

$$a_0 = 1$$

$$a_n = a_{n-1} + \frac{1}{n(n+1)} \rightarrow ①$$

put $n=1$ in eqn ① we get

$$a_1 = a_0 + \frac{1}{1(1+1)} = 1 + \frac{1}{2} = \frac{1}{1 \cdot 2}$$

put $n=2$ in eqn ①

$$a_2 = a_1 + \frac{1}{2(2+1)} \left(1 + \frac{1}{6}\right) = \frac{1}{2 \cdot 3} = a_0 + \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3}$$
$$= 1 + \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3}$$

put $n=3$ in eqn ③

$$a_3 = 1 + \frac{1}{3(3+1)} = \left(1 + \frac{1}{9}\right) = \frac{1}{3 \cdot 4}$$

$$a_3 = 1 + \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + - - - -$$

similarly

$$a_n = 1 + \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + - - - - + \frac{1}{n(n+1)}$$
$$= 1 + \frac{1}{1} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + - - - + \cancel{\frac{1}{n}} - \frac{1}{n+1}$$
$$= 2 - \frac{1}{n+1}$$

solve the recurrance relation $a_n = c(a_{n-1}) + f(n)$ for $n \geq 1$

$a_n = 1 + \sum_{r=1}^{n} r \cdot 3^r$

Given

$a_n = c(a_{n-1}) + f(n) \rightarrow ①$

put $n=1$

$a_1 = c(a_0) + f(1)$

put $n=2$

$a_2 = c(a_1) + f(2)$

$a_2 = c(c(a_0) + f(1)) + f(2)$

$= c^2 a_0 + cf(1) + f(2)$

$a_3 = c(a_2) + f(3)$

$= c(c^2 a_0 + cf(1) + f(2)) + f(3)$

$= c^3 a_0 + c^2 f(1) + cf(2) + f(3)$

$||^{ly}$ $a_n = c^n a_0 + c^{n-1} f(1) + c^{n-2} f(2) + \cdots + cf(n-1) + f(n)$

$\therefore a_n = c^n a_0 + \sum_{k=1}^{n} c^{n-r} f(r)$

solve the recurrance relation $a_n = a_{n-1} + n \cdot 3^n, a_0 = 1$

Given

$a_n = a_{n-1} + n \cdot 3^n \rightarrow ①$

put $n=1$

$a_1 = a_0 + (1) \cdot 3^1$

$1 + (1) \cdot 3^1$

put $n=2$

$a_2 = 1 + (1) \cdot 3^1 + (2) \cdot 3^2$

put $n=3$

$a_3 = 1 + 1 \cdot 3^1 + 2 \cdot 3^2 + 3 \cdot 3^3 \cdots$

similarly

$a_n = 1 + 1 \cdot 3^1 + 2 \cdot 3^2 + 3 \cdot 3^3 + \cdots + n \cdot 3^n$

$a_n = 1 + \sum_{r=1}^{n} r \cdot 3^r$

solve the recurrance relation $a_n = a_{n-1} + \frac{n(n+1)}{2}, n \geq 1$

Given

$a_n = a_{n-1} + \frac{n(n+1)}{2} \rightarrow ①$

put $n=1$

$$a_1 = a_0 + \frac{1(2)}{2}$$

put $n=2$

$$a_2 = a_1 + \frac{2(2+1)}{2}$$

$$= a_0 + \frac{1(2)}{2} + \frac{2(3)}{2}$$

put $n=3$

$$a_3 = a_2 + \frac{3(3+1)}{2}$$

$$= a_0 + \frac{1(2)}{2} + \frac{2(3)}{2} + \frac{3(4)}{2}$$

similarly

$$a_n = a_0 + \frac{1.2}{2} + \frac{2.3}{2} + \frac{3.4}{2}$$

$$a_n = a_0 + \sum_{r=1}^{n} \frac{r(r+1)}{2}$$

$a_n = a_{n-1} + n(n-1)$ where $a_0 = 1$ $\quad$ $a_0 + r(r-1)$

$a_n = 1 + \sum_{r=2}^{n} r.(r-1)$

$a_r = 2a_{r-1} + 1$ with $a_1 = 7$.

Given

$a_0 = 1$

$a_n = a_{n-1} + n(n-1) \rightarrow ①$

put $n=1$

$a_1 = a_0 + 1(0)$

put $n=2$

$a_2 = 1 + 1(0) + 2(1)$

put $n=3$

$a_3 = 1 + 1(0) + 2(1) + 3(2)$

similarly

$a_n = 1 + 1(0) + 2(1) + 3(2) + \text{---} \quad + n(n-1)$

$a_n = 1 + \sum_{r=2} r.(r-1)$

Given

$a_r = 2a_{r-1} + 1 \rightarrow$ ①

$a_1 = 7$

put $r = 2$

$a_2 = 2a_1 + 1$

$= 2(7) + 1$

put $r = 3$

$a_3 = 2a_2 + 1$

$= 2(2(7)+1) + 1$

$= 2^2(7) + 2 + 1$

$r = 4$

$a_4 = 2a_3 + 1$

$= 2(2^2(7) + 2 + 1) + 1$

$= 2^3(7) + 2^2 + 2 + 1$

$a_r = 2^{r-1}(7) + 2^{r-2} + 2^{r-3} + \cdots + 1$

$= 7 \cdot 2^{r-1} + 2^{r-1} - 1 \quad$ for $r > 1$

$a_r = (7)2^{r-1} + 2^{r-1} - 1$

$a_r = 2a_{r-1} + 1 \rightarrow$ ①

$a_1 = 7$

put $r = 2$

$a_2 = 2(a_1 + 1$

$= 2(7) + 1$

$r = 3$

$a_3 = 2a_2 + 1$

$= 2(2(7)+1) + 1$

$= 2^2(7) + 2 + 1$

$r = 4$

$a_4 = 2a_3 + 1$

$= 2(2^2(7) + 2 + 1) + 1$

$= 2^3(7) + 2^2 + 2 + 1$

$a_r = 2^{r-1}(7) + 2^{r-2} + 2^{r-3} + \cdots + 1$

$= 7 \cdot 2^{r-1}$

Characteristic Root method.

Let $a_n + c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k} = 0 \rightarrow$ ①, $n = k$

$c_k \neq 0$ be a linear reccurrance relation of degree k

1. Then the characteristic equation is $t^k + c_1 t^{k-1} + c_2 t^{k-2} + \cdots + c_k = 0 \rightarrow$ ② of ①
2. since eqn ② is of degree k find 'k' roots

Let $\alpha_1, \alpha_2, \alpha_3 \cdots \alpha_k$ are the roots of eq ② i.e characteristic roots are

① If $\alpha_1, \alpha_2 \cdots \alpha_k$ are distinct roots then $a_n = c_1\alpha_1^n + c_2\alpha_2^n + \cdots + c_k \alpha_k^n$ is the solution of eq ①

② If $\alpha_1, \alpha_2 \cdots \alpha_k$ are equal roots then $a_n = (c_1 + c_2 n + c_3 n^2 + \cdots + c_k \cdot n^{k-1})\alpha^n$ is the solution of eq ①

are complex roots then $a_n = r^n(c_1 \cos n\theta$

solve the reccurrance relation $a_n - 3a_{n-1} + 2a_{n-2} = 0$ for $n \geq 2$ using characteristic root method.

Given

$$a_n - 3a_{n-1} + 2a_{n-2} = 0 \rightarrow (1)$$

equation (1) is the second order homogeneous reccurrance relation

The characteristic equation is

$$t^2 - 3t + 2 = 0$$

$$t^2 - 2t - t + 2 = 0$$

$$t(t-2) - 1(t-2) = 0$$

$$t = 1, \; t = 2$$

The roots are real and distinct

$\therefore$ The general solution of homogeneous reccurrance relation is $\boxed{a_n = c_1 1^n + c_2 2^n}$

solve $a_n - 6a_{n-1} + 9a_{n-2} = 0$ –

Given

$$a_n - 6a_{n-1} + 9a_{n-2} = 0 \rightarrow (1)$$

eq (1) is the second order homogeneous reccurrance relation

The characteristic equation is

$$t^2 - 6t + 9 = 0$$

$$t^2 - 3t - 3t + 9 = 0$$

$$t(t-3) - 3(t-3) = 0$$

$$t = 3, \; t = 3$$

The roots are real and equal

$\therefore$ The general solution of homogeneous reccurrance relation is $\boxed{a_n = (c_1 + c_2 n) 3^n}$

solve the reccurrance relation $a_n - 7a_{n-1} + 10a_{n-2} = 0, \; n \geq 2$ with $a_0 = 10$, $a_1 = 41$ by using characteristic root method.

Given

$$a_n - 7a_{n-1} + 10a_{n-2} = 0 \longrightarrow (1)$$

eqn (1) is the second order homogeneous reccurrance relation

$t^2-5t-2t+10=0$

$t(t-5)-2(t-5)=0$

$t=5,\ t=2$

$\therefore$ The two roots are real and distinct

the general solution of homogenous reccurance relation is

$$\boxed{a_n=c_1 5^n+c_2 2^n} \longrightarrow ②$$

put $n=0$ in eqn ② we get

$a_0 = c_1+c_2=0$

$a_0 = c_1+c_2=10 \rightarrow ③$

put $n=1$ in eqn ② we get

$a_1 = 5c_1+2c_2 \Rightarrow 5c_1+2c_2=41 \rightarrow ④$

Solving eqn ③ and eqn ④

eqn ③ $\times 2 = 2c_1+\cancel{2c_2}=20$

④ $= 5c_1+\cancel{2c_2}=41$

$\quad(-)\quad(-)\quad(-)$

$3c_1 = \cancel{21}\ 7$

$c_1=7$

substitute c_1 values in eqn ② we get

$c_1+c_2=10$

$7+c_2=10$

$c_2=10-7$

$c_2=3$

$a_n=7(5)^n+3(2)^n$

solve the reccurrance relation $a_n=7a_{n-1}+16a_{n-2}-12a_{n-3}=0$ for $n\geq 3$ with the initial conditions $a_0=1,\ a_1=4\ \&\ a_2=8$

Given

$a_0=1,\ a_1=4,\ a_2=8$

$a_n=7a_{n-1}+16a_{n-2}-12a_{n-3}\rightarrow ①$

equation ① is the third order homogeneous reccurrance

$t^3-7t^2+16t-12=0$

$t=2$ $2^3-7(2)^2+16(2)-12$

$8-28+32-12 = -20+20 = 0$

$t=2$

Now $t=2$ is a root we can divide the cubic equation by $t=2$ using synthetic division

$$\begin{array}{c|cccc} 2 & 1 & -7 & 16 & -12 \\ & 0 & 2 & -10 & 12 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$t^2-5t+6=0$

$t^2-3t-2t+6=0$

$t(t-3)-2(t-3)=0$

$t=3, 2.$

The two roots $t=\cancel{3},2$ is equal and another root is 3 is distinct root

$$a_n=(c_1+c_2n)2^n+c_3 3^n \rightarrow ②$$

put $n=0$ in eqn ② we get

$a_0=c_1+c_3$

$c_1+c_3=1 \rightarrow ③$

put $n=1$ in eqn ② we get

$a_1=(c_1+c_2)2^1+c_3 3^1$

$a_1=2c_1+2c_2+3c_3 \Rightarrow 2c_1+2c_2+3c_3=4 \rightarrow ④$

put $n=2$ in eqn ② we get

$a_2=(c_1+2c_2)2^2+c_3 3^2$

$a_2=4c_1+8c_2+9c_3 \Rightarrow 4c_1+8c_2+9c_3=8 \rightarrow ⑤$

solving eqn ④ and ⑤

eqn ④ $\times 4 =$ $8c_1+\cancel{8c_2}+12c_3=16$

5 $\quad 4c_1+\cancel{8c_2}+9c_3=8$

$\quad (-)\quad (-)\quad (-)\quad (-)$

solving eqn (3) and (6)

eqn (3) $\times 3$: $3c_1 + 3\cancel{c_3} = 3$

(6) $4c_1 + 3\cancel{c_3} = 8$

$(-) \quad (-) \quad (-)$

$\cancel{-1}c_1 = \cancel{-}5$

$c_1 = 5$

substitute c_1 value in eqn (3)

$c_1 + c_3 = 1$

$5 + c_3 = 1$

$c_3 = -4$

substitute c_1 & c_3 values in eqn (4)

$2c_1 + 2c_2 + 3c_3 = 4$

$2(5) + 2c_2 + 3(-4) = 4$

$10 + 2c_2 \cancel{4} - 12 = 4$

$2c_2 - 2 = 4$

$2c_2 = 6$

$c_2 = \frac{\cancel{6}\,3}{\cancel{2}\,1}$

$c_2 = 3$

$(5+3)(2)^n + (-4)(3)^n$

solve $a_n = (2a_{n-1} - 2a_{n-2})$ for $n \geq 2$ with $a_0 = 1$, $a_1 = 2$

Given

$a_0 = 1$, $a_1 = 2$

$a_n = (2a_{n-1} - 2a_{n-2}) \rightarrow (1)$

$a_n - 2a_{n-1} + 2a_{n-2} = 0$

equation 1 is the second order homogenous reccurrance relation

The characteristic equation for eq (1) is

$t^2 - 2t + 2 = 0$

$-b \pm \sqrt{b^2 - 4ac}$

Here $a=1,\ b=-2,\ c=2$

$$\frac{2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= 2 \pm \sqrt{\frac{4-8}{2}}$$

$\sqrt{-4}$
$= \sqrt{(-1)(4)}$
$= i(2)$

$i = \sqrt{-1}$

$$= 2 \pm \sqrt{\frac{-4}{2}}$$

$$= \frac{2 \pm \sqrt{(-1)\,4}}{2}$$

$\frac{\cancel{2}(1 \pm i)}{\cancel{2}}$

$$= \frac{\cancel{2} \pm i \cdot \cancel{2}}{\cancel{2}} = 1 \pm i$$

He
$\therefore$ G.S is $a_n = r^n (c_1 \cos n\theta + c_2 \sin n\theta)$

$\theta = \tan^{-1}(y/x) = \tan^{-1}(1/1) = \pi/4$

where $r = |1 \pm i|$

$= \sqrt{1^2+1^2} = \sqrt{2}$

$\Rightarrow a_n = (\sqrt{2})^n \left(c_1 \cos \frac{n\pi}{4} + c_2 \sin \frac{n\pi}{4}\right) \rightarrow ②$

put $n=0$ in eqn ② we get

$a_0 = c_1 \cos 0 + c_2 \sin 0$

$c_1 + 0 = a_0$

$c_1 = 1$

put $n=1 \Rightarrow a_1 = (\sqrt{2})^1 \left(c_1 \cos \frac{\pi}{4} + c_2 \sin \frac{\pi}{4}\right)$

$= 2 = \sqrt{2}\left(c_1 \frac{1}{\sqrt{2}} + c_2 \frac{1}{\sqrt{2}}\right)$

$2 = \sqrt{2}\left(c_1 \frac{1}{\cancel{\sqrt{2}}} + c_2 \frac{1}{\cancel{\sqrt{2}}}\right)$ $\quad 2 = \cancel{\sqrt{2}} \cdot \frac{1}{\cancel{\sqrt{2}}}(c_1+c_2) = 2$

$\sqrt{2} = \frac{1}{\sqrt{2}}$

$\Rightarrow c_1 + c_2 = 2$

$c_2 = 2 - c_1 \Rightarrow 2 - 1 = 1$

$\therefore$ The General solution is $a_n = (\sqrt{2})^n \left(\cos \frac{n\pi}{4} + \sin \frac{n\pi}{4}\right)$

Non-homogeneous reccurrance relation.

The procedure to solve non-homogeneous linear reccurance relation by using characteristic roots method

The general form of non homogeneous linear reccurance relation is $c_0a_n + c_1a_{n-1} + \text{-------} c_na_{n-1} = f(n) \rightarrow (1)$

Where $f(n) \neq 0$

The general solution of non homogenous linear reccurrance relation is

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$a_n^{(h)}$ is the solution of non-homogeneous reccurance relation which can be obtained by taking $f(n)=0$ in eq (1)

The particular solution can be obtained on the nature of $f(n)$ to determine the particular solution we use the following cases

case 1: If $f(n)$ is in the form of $f(n) = g(n) \cdot a^n$ where

$g(n) = b_0 + b_1n + \text{------} + b_mn^m$ and if 'a' is not a root of a characteristic equation then the particular solution is in the form of $a_n^{(p)} = (A_0 + A_1n + A_2n^2 \text{---} + A_mn^m)a^n$

Note.

1. $f(n) = 2 \Rightarrow 2 \cdot 1^n$ Here $f(n) = g(n) \cdot a^n$

   Here $a=1$ is not a root and $g(n)$ is a constant so the particular solution is $a_n^{(p)} = A_0 - 1^n \Leftrightarrow a_n^{(p)} = A_0$

2. $f(n) = n \cdot 2^n$ Here $f(n)$ is in the form of $f(n) = g(n) \cdot a^n$ where $g(n) = n$ and the $a=2$, Here $a=2$ is not a characteristic root and $g(n)$ has upto n then the particular solution is in the form of $a_n^{(p)} = (A_0 + A^n)2^n$

case 2:

If $f(n)$ is in the form of $f(n) = g(n) \cdot a^n$ where $g(n) = b_0 + b_1n + \text{------} + b_mn^m$ and if 'a' is a root of a characteristic equation with multiplicity q then the particular solution is in the form of

$a_n^{(p)} = n^q(A_0 + A_1n + A_2n^2 + \text{---------} A_mn^m)a^n$

1. solve the reccurrance relation $a_n - 9a_{n-1} + 20a_{n-2} = 1$ by using characteristic roots method.

Given that

$$a_n - 9a_{n-1} + 20a_{n-2} \rightarrow ①$$

eqn① is a second order non-homogenious reccurrance relation.

$\therefore$ The general solution of non-homogenious reccurrance relation is

$$a_n = a_n^{(h)} + a_n^{(p)}$$

To find $a_n^{(h)}$

The homogeneous reccurrance relation of eqn① is

$$a_n - 9a_{n-1} + 20a_{n-1} = 0$$

the characteristic equation is

$$t^2 - 9t + 20 = 0 \rightarrow ②$$

$$t^2 - 5t - 4t + 20 = 0$$

$$t(t-5) - 4(t-5) = 0$$

$$t = 4, 5$$

$\therefore$ The roots are real and distinct.

The general solution of homogeneous reccurance relation is

$$a_n^{(h)} = c_1 4^n + c_2 5^n$$

To find $a_n^{(p)}$

Given

$$a_n - 9a_{n-1} + 20a_{n-2} = 1$$

Here $f(n) = 1$

$$f(n) = 1 \cdot 1^n$$

This is in the form of $f(n) = g(n) \cdot a^n$

where $g(n) = 1$

$$a = 1$$

Here since from eqn② $a=1$ is not a root of the characteristic equation and $g(n)$ is a constant. so that the particular substitute $a_n = A_0$ in eqn ①

$$A_0 - 9A_0 + 20A_0 = 1$$

$$12A_0 - 1$$

$$A_0 = 1/12$$

substitute A_0 value in particular solution

$$a_n^{(p)} = 1/12$$

∴ The general solution of non-homogeneous linear reccurrance relation is

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$a_n = c_1 4^n + c_2 5^n + 1/12$$

solve the reccurrance relation $a_n + 4a_{n-1} + 4a_{n-2} = 8$ for $n \geq 2$ given $a_0 = 1$, $a_1 = 2$

Given

$$a_n + 4a_{n-1} + 4a_{n-2} = 8 \rightarrow ①$$

eqn ① is a second order non homogeneous reccurrance relation

∴ The general solution is

$$a_n = a_n^{(h)} + a_n^{(P)}$$

To find $a_n^{(h)}$:

the homogeneous reccurance relation of eqn ① is

$$a_n + 4a_{n-1} + 4a_{n-2} = 0$$

The characteristic equation is

$$t^2 + 4t + 4 = 0 \rightarrow ②$$

$$t^2 + 2t + 2t + 4 = 0$$

$$t(t+2) + 2(t+2) = 0$$

$$t = -2, -2$$

∴ the two roots are equal

The general solution is

To find $a_n^{(P)}$

Given

$$a_n + 4a_{n-1} + 4a_{n-2} = 8$$

Here $f(n) = 8$

$$f(n) = 8 \cdot 1^n$$

Here since from eqn ① $\alpha = 1$ is not a root of a characteristic equation and $g(n)$ is a constant. so that the particular solution is

$$a_n^{(P)} = A_0 \cdot 1^n$$

$$a_n^{(P)} = A_0$$

substitue $a_n = A_0$ in eqn ①

$$A_0 + 4A_0 + 4A_0 = 8$$

$$9A_0 = 8$$

$$A_0 = 8/9$$

substitute A_0 value in particular solution

$$a_n^{(P)} = 8/9$$

$\therefore$ The general solution is

$$a_n = a_n^{(h)} + a_n^{(P)}$$

$$a_n = (c_1 + c_2 n)(-2)^n + 8/9 \rightarrow ③$$

Put $n=0$ in eqn 3

$$a_0 = (c_1 + c_2 n)(-2)^n + 8/9$$

$$a_0 = c_1 + 8/9$$

$$c_1 + 8/9 = 1$$

$$c_1 = 1 - \frac{8}{9} = \frac{1}{9}$$

Put $n=1$ in eqn ③

$$a_1 = (c_1 + c_2(1))(-2)^1 + 8/9$$

$$2 = -2c_1 - 2c_2 + 8/9$$

$$2 = (-2)\frac{1}{9} - 2c_2 + 8/9$$

$2c_2 = \frac{-2}{9} + \frac{8}{9} - 2$

$\frac{-2+8-18}{9}$

$= \frac{-12}{9}$

$2c_2 = \frac{-12}{9}$

$2c_2 = \frac{-\cancel{12}^{4}}{\cancel{9}_{3}} = \frac{\cancel{12}}{\cancel{18}}$

$c_2 = \frac{-\cancel{4}^{2}}{3 \times \cancel{2}_{1}} = \frac{-2}{3}$

substitute c_1 and c_2 in eqn 3

$$a_n = \left(\frac{1}{9} + \left(\frac{-2}{3}\right)n\right)(-2)^n + 8/9$$

solve the reccurrance relation $a_n - 5a_{n-1} + 6a_{n-2} = 1$

solve the reccurrance relation $a_n - a_{n-1} - 6a_{n-2} = -30$ with $a_0 = 20$, $a_1 = -5$

Given that

$a_n - 5a_{n-1} + 6a_{n-2} = 1 \rightarrow ①$

eqn ① is a second order non-homogenous reccurrance relation

$\therefore$ The g.s of n.h.r.r is

$a_n = a_n^{(h)} + a_n^{(p)}$

To find $a_n^{(h)}$

The H.r.r of eqn ① is

$a_n - 5a_{n-1} + 6a_{n-2} = 0$

The characteristic equation is

$t^2 - 5t + 6 = 0$

$t^2 - 3t - 2t + 6 = 0$

$t(t-3) - 2(t-3) = 0$

$t = 2, 3$

The roots are real and distinct

The g.s of H.R.R is

$a_n^{(h)} = c_1 2^n + c_2 3^n$

To find $a_n^{(P)}$

Given

$$a_n - 5a_{n-1} + 6a_{n-2} = 1$$

Here $f(n) = 1$

$f(n) = 1 \cdot 1^n$

This is the form of $f(n) = g(n) \cdot a^n$

Where $g(n) = 1$

$a = 1$

Here since from eqn ② $a = 1$ is not a root of the characteristic equation and $g(n)$ is a constant so that the particular solution is

$$a_n^{(P)} = A_0 \cdot 1^n$$

$$a_n^{(P)} = A_0$$

substitute $a_n = A_0$ in eqn ①

$$A_0 - 5A_0 + 6A_0 = 1$$

$$2A_0 = 1$$

$$A_0 = 1/2$$

substitute A_0 value in particular solution

$$a_n^{(P)} = 1/2$$

$\therefore$ The g.s of N.H.R.R

$$= a_n = a_n^{(h)} + a_n^{(P)}$$

$$a_n = c_1 2^n + c_2 3^n + 1/2$$

Given

$$a_n - a_{n-1} - 6a_{n-2} = -30 \rightarrow ①$$

eqn ① is a second order non homogeneous reccurrance relation

$\therefore$ The general solution is

$$a_n = a_n^{(h)} + a_n^{(P)}$$

To find $a_n^{(h)}$:

The H.r.r of eqn ① is

$$a_n - a_{n-1} - 6a_{n-2} = 0$$

$t^2 - t - 6 = 0$

$t^2 - 3t + 2t - 6 = 0$

$t(t-3) + 2(t-3) = 0$

$t = 3, -2$

The two roots are real and distinct.

The general solution is

$a_n^{(h)} = c_1 3^n + c_2 (-2)^n$

To find $a_n^{(P)}$

Given

$a_n = a_{n-1} - 6a_{n-2} = -30$

Here $f(n) = -30$

$f(n) = -30 \cdot 1^n$

Here since from eqn ① $a_0 = 20$ is not a root of a characteristic equation and $g(n)$ is a constant, so that the particular solution is,

$a_n^{(P)} = A_0 \cdot 1^n$

$a_n^{(P)} = A_0$

substitute $a_n = A_0$ in eqn ①

$A_0 - A_0 - 6A_0 = 30$

$-6A_0 = 30$

$A_0 = \frac{\cancel{30}\,5}{\cancel{-6}\,1}$

$A_0 = -5$

substitute A_0 value in particular solution

$a_n^{(P)} = -5$

$\therefore$ The general solution is

$a_n = a_n^{(h)} + a_n^{(P)}$

$a_n = c_1 3^n + c_2(-2)^n - 5 \rightarrow$ ③

put $n=0$ in eqn ③

$a_0 = c_1 + c_2(-2)^0 - 5$

$a_0 = c_1 - 5$

$c_1 - 5 = a_0$

$c_1 = 15$

put $n=1$ in eqn 3

$a_1 = c_1 3^1 + c_2(-2)^n = -5$

$a_1 = 3c_1 - 2c_2 - 5$

$-5 = 3c_1 - 2c_2 - 5$

$-5 = 3(15) - 2c_2 - 5$

$-5 = 45 - 2c_2 - 5$

$45 - 2c_2 - 5 = 5$

$2c_2 = 5 + 40$

$2c_2 = 45$

$c_2 = \frac{45}{2}$

substitute c_1 and c_2 in eq 3

$a_n = 15 + \frac{42}{2}(-2)^n - 5$

solve the reccurance relation $a_n - 7a_{n-1} + 10a_{n-2} = 4^n$

Given

$$a_n - 7a_{n-1} + 10a_{n-2} = 4^n \rightarrow (1)$$

eqn (1) is the secondorder non-homogeneous reccurrance relation

$\therefore$ The general solution of non homogeneous reccurrance relationis

$$a_n = a_n^{(h)} + a_n^{(P)}$$

To find $a_n^{(h)}$

From eqn (1) is a homogeneous reccurrance relation

$$a_n - 7a_{n-1} + 10a_{n-2} = 0$$

The characteristic equation is

$$t^2 - 7t + 10 = 0$$

$$t^2 - 5t - 2t + 10 = 0$$

$$t(t-2) - 5(t-2) = 0$$

$$t = 2, 5$$

the roots are real and distinct.

The general solution of the Homogeneous reccurrance relation is

$$a_n^{(h)} = c_1 2^n + c_2 5^n$$

To find $a_n^{(P)}$:

Given

$$a_n - 7a_{n-1} + 10a_{n-2} = 4^n$$

Here $f(n) = 4^n \Rightarrow f(n) = 1 \cdot 4^n$

This is in the form of $f(n) = g(n) \cdot a^n$

Here $a = 4 \quad g(n) = 1$

since $a = 4$ is not a root of characteristic equation $g(n)$ is constant. so that the particular solution is

$$a_n^{(P)} = A_0 4^n$$

substitute $a_n = A_0 4^n$ in eq (1)

$$A_0 4^n - 7A_0 4^{n-1} + 10A_0 4^{n-2} = 4^n$$

Divide through 4^{n-2}

$$\frac{a^m}{a^n} = a^{m-n}$$

$$A_0 4^{n-(n-2)} - 7A_0 4^{n-1-(n-2)} + 10A_0 4^{n-2-(n-2)} = 4^{n-(n-2)}$$

$$A_0 4^{\cancel{n}-\cancel{n}+2} - 7A_0 4^{\cancel{n}-1-\cancel{n}+2} + 10A_0 4^{\cancel{n}-\cancel{2}-\cancel{n}+\cancel{2}} = 4^{\cancel{n}-\cancel{n}+2}$$

$$= A_0 4^2 - 7A_0 4^1 + 10A_0 4^0 = 4^2$$

$$16A_0 - 28A_0 + 10A_0 = 16$$

$$-2A_0 = 16$$

$$A_0 = -8$$

substitute $A_0 = -8$ in particular solution

$$a_n^{(p)} = A_0 4^n$$

$$= -8 \cdot 4^n$$

$\therefore$ The general solution of nonhomogeneous reccurrance relation is

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$a_n = c_1 2^n + c_2 5^n - 8(4^n)$$

solve the reccurrance relation $S_K - 3S_{K-1} - 4S_{K-2} = 4^K$

Given

$$S_K - 3S_{K-1} - 4S_{K-2} = 4^K$$

$$\Rightarrow a_n - 3a_{n-1} - 4a_{n-2} = 4^n \rightarrow (1)$$

eqn (1) is the second order non-homogeneous reccurrance relation

$\therefore$ The general solution of non homogeneous reccurrance relation is

$$a_n = a_n^{(h)} + a_n^{(p)}$$

To find $a_n^{(h)}$

From eqn (1) is a homogeneous reccurance relation

$$a_n - 3a_{n-1} - 4a_{n-2} = 0$$

$t^2 - 3t - 4 = 0$

$t^2 - 4t + t - 4$

$t(t-4) + 1(t-4) = 0$

the characteristic equation is

$t^2 - 4t + t - 4 = 0$

$t(t-4) + 1(t-4) = 0$

$(t-4)(t-1) = 0$

$t = 4, 1$

The roots are real and distinct

The general solution of Homogeneous reccurrance relation is

$$a_n^{(h)} = c_1 4^n + c_2 1^n$$

To find $a_n^{(P)}$

Given

$$a_n - 3a_{n-1} - 4a_{n-2} = 4^n$$

Here $f(n) = 4^n \Rightarrow f(n) = 1 \cdot 4^n$

This is in the form of $f(n) = g(n) \cdot a^n$

Here $a = 4$ $\quad g(n) = 1$

since $a = 4$ is not a root of characteristic equation, $g(n)$ is constant, so that the particular solution is

$$a_n^{(P)} = A_0 4^n$$

substitute $a_n = A_0 4^n$

$$A_0 4^n - 3A_0 4^{n-1} - 4A_0 4^{n-2} = 4^n$$

Divide through 4^{n-2}

$$\frac{A_0 4^n}{4^{n-2}} - \frac{3A_0 4^{n-1}}{4^{n-2}} - \frac{4A_0 4^{n-2}}{4^{n-2}} = \frac{4^n}{4^{n-2}}$$

$$A_0 4^{\cancel{n}-(\cancel{n}-2)} - 3A_0 4^{\cancel{n}-1-(\cancel{n}-2)} - 4A_0 4^{\cancel{n}-2-(\cancel{n}-2)} = 4^{\cancel{n}-(\cancel{n}-2)}$$

$$A_0 4^2 - 3A_0 4^1 - 4A_0 4^0 = 4^2$$

$$16A_0 - 12A_0 - 4A_0 = 16$$

$$A_0 = 16$$

substitute $A_0 = 16$ in particular solution.

$$a_n^{(P)} = A_0 \cdot 4^n$$

$$= 16 \cdot 4^n$$

∴ The general solution of non-homogeneous reccurance relation

$$a_n = a_n^{(h)} + a_n^{(P)}$$

$$a_n = c_1 4^n + c_1 1^n + 16 \cdot 4^n$$

Solve the reccurrance relation $a_n - 6a_{n-1} + 8a_{n-2} = n.4^n$ where $a_0 = 5,\ a_1 = 22$

Given

$$a_n - 6a_{n-1} + 8a_{n-2} = n.4^n \rightarrow ①$$

eqn ① is a second order non homogeneous reccurrance relation.

The general solution for non homogeneous reccurrance relation

$$a_n = a_n^{(h)} + a_n^{(p)}$$

To find $a_n^{(h)}$:-

From eqn ① is a homogeneous reccurrance relation

$$a_n - 6a_{n-1} + 8a_{n-2} = 0$$

The characteristic equation is

$$t^2 - 6t + 8 = 0 \rightarrow ②$$

$$t^2 - 4t - 2t + 8 = 0$$

$$t(t-4) - 2(t-4) = 0$$

$$t = 2, 4$$

The roots are real and distinct

The general solution of the homogeneous reccurrance relation is

$$a_n^{(h)} = c_1 2^n + c_2 4^n$$

To find $a_n^{(p)}$:

Given

$$a_n - 6a_{n-1} + 8a_{n-2} = n.4^n \rightarrow ①$$

Here $f(n) = n.4^n$

This is in the form of $f(n) = g(n).a^n$

Here $g(n) = n \quad a = 4$

since from eqn ② $a = 4$ is a root of characteristic equation with multiplicity $q = 1$

$g(n)$ has upto n

The particular solution is

$$a_n^{(p)} = n^q (A_0 + A_1 n) 4^n$$

$$a_n^{(p)} = n^1 (A_0 + A_1 n) 4^n$$

substitute $a_n = (A_0 n + A_1 n^2)4^n$ in eqn ①

$$(A_0 n + A_1 n^2)4^n - 6(A_0(n-1) + A_1(n-1)^2)4^{n-1} + 8(A_0(n-2) + A_1(n-2)^2)4^{n-2} = n.4^n$$

Divide through 4^{n-2}

$$= \frac{(A_0 n + A_1 n^2)4^n}{4^{n-2}} - \frac{6(A_0(n-1) + A_1(n-1)^2)4^{n-1}}{4^{n-2}} + \frac{8(A_0(n-2) + A_1(n-2)^2)4^{n-2}}{4^{n-2}} = \frac{n.4^n}{4^{n-2}}$$

$$= (A_0 n + A_1 n^2)4^{\cancel{n}-(\cancel{n}-2)} - 6(A_0(n-1) + A_1(n^2-2n+1))4^{\cancel{n}-1-(\cancel{n}-2)} + 8[A_0 n - 2A_0 + A_1(n^2+4-4n)]4^{\cancel{n}-\cancel{2}-\cancel{n}+\cancel{2}} = n.4^{n-n+2}$$

$$= (A_0 n + A_1 n^2)4^2 - 6[A_0 n - A_0 + A_1 n^2 - 2A_1 n + A_1]4^1 + 8[A_0 n - 2A_0 + A_1 n^2 + 4A_1 - 4A_1 n]4^0 = n.4^2$$

$$= [16A_0 n + 16A_1 n^2 - 24A_0 n + 24A_0 - 24A_1 n^2 + 48A_1 n - 24A_1 + 8A_0 n - 16A_0 + 8A_1 n^2 + 32A_1 - 32A_1 n = 16n$$

$$= 8A_0 + 8A_1 + 16A_1 n = 16n$$

$$= 8(A_0 + A_1) + 16A_1 n = 16n$$

equating the coefficients

$$16A_1 = 16 \qquad 8(A_0 + A_1) = 0$$

$$A_1 = 1 \qquad A_0 + A_1 = 0$$

$$A_0 + 1 = 0 \Rightarrow A_0 = -1$$

$$A_0 = -1, A_1 = 1$$

substitute $A_0 = -1$ and $A_1 = 1$ in particular solution

$$a_n^{(P)} = (A_0 n + A_1 n^2)4^n$$

$$a_n^{(P)} = (-n + n^2)4^n$$

$$= n(n-1)4^n$$

$a_n = c_1 2^n + c_2 4^n + n(n-1)4^n \rightarrow$ ③

sub $n=0$ in eqn ③

$a_0 = c_1 2^0 + c_2 4^0 + 0(0\cdot1)4^0$

$a_0 = c_1 + c_2$

$c_1 + c_2 = 8 \rightarrow$ ④

sub $n=1$ in eqn ④

$a_1 = c_1(2)^1 + c_2(4)^1 + 1(1\cdot1)4^1$

$a_1 = 2c_1 + 4c_2$

$2c_1 + 4c_2 = 22 \rightarrow$ ⑤

solving ③ and 5

eqn $4 \times 2 \Rightarrow \cancel{2c_1} + 2c_2 = 16$

$5 \times 1 \Rightarrow \cancel{2c_1} + 4c_2 = 22$

$(-) \quad (-) \quad (-)$

$-2c_2 = -6$

$\therefore c_2 = \frac{-\cancel{6}\,3}{\cancel{2}}$

$c_2 = 3$

sub $c_2 = 3$ in eqn ④

$c_1 + c_2 = 8$

$c_1 + 3 = 8$

$c_1 = 5$ $\quad c_1 = 5,\ c_2 = 3$

substitue $c_1 = 5$ $c_2 = 3$ in eqn③ $a_n (5)(2)^n + (3)(4)^n + n(n-1)4^n$

solve the reccurrance relation $a_n - 5a_{n-1} + 6a_{n-2} = 3n^2 - 2n + 1$

Given

$a_n - 5a_{n-1} + 6a_{n-2} = 3n^2 - 2n + 1 \rightarrow$ ①

eq① is a second order non homogenous reccurrance relation

The general solution of non homogeneous reccurance relation is

$a_n = a_n^{(h)} + a_n^{(p)}$

To find $a_n^{(h)}$

From eqn ① is homogenous reccurrance relation

$a_n - 5a_{n-1} + 6a_{n-2} = 0$

The characteristic equation is

$t^2 - 5t + 6 = 0$

$t^2 - 3t - 2t + 6 = 0$

$t(t-3) - 2(t-3) = 0$

the general solution of homogeneous reccurrance is

$$a_n = c_1 2^n + c_2 3^n$$

To find $a_n^{(P)}$:

Given

$$a_n - 5a_{n-1} + 6a_{n-2} = 3n^2 - 2n + 1$$

Here $f(n) = 3n^2 - 2n + 1 \Rightarrow (f(n) = 3n^2 - 2n + 1 \cdot 1^n)$

This is in the form of $f(n) = g(n) \cdot a^n$

Here $g(n) = 3n^2 - 2n + 1$, $a^n = 1^n \Rightarrow a = 1$

The particular solution is

$$a_n^{(P)} = A_0 + A_1 n + A_2 n^2$$

substitute $a_n^{(p)}$ in eqn ①

$$A_0 + A_1 n + A_2 n^2 - 5[A_0 + A_1(n-1) + A_2(n-1)^2] + 6[A_0 A_1(n-2) + A_2(n-2)^2] = 3n^2 - 2n + 1$$

$$= A_0 + A_1 n + A_2 n^2 - 5[A_0 + A_1 n - A_1 + A_2(n^2 \cancel{+} 2n)] + 6[A_0 + A_1 n - 2A_1 + A_2(n^2 + 4 - 4n)] = 3n^2 - 2n + 1$$

$$= A_0 + A_1 n + A_2 n^2 - 5A_0 - 5A_1 n + 5A_1 - 5A_2 n^2 - 5A_2 + 10A_2 n + 6A_0 + 6A_1 n - 12A_1 + 6A_2 n^2 + 24A_2 - 24A_2 n = 3n^2 - 2n + 1$$

$$= 2A_0 + 2A_1 n + 2 \cdot A_2 n^2 - 7A_1 + 19A_2 - 14A_2 n = 3n^2 - 2n + 1$$

$$(2A_0 - 7A_1 + 19A_2) + 2A_1 n + 2A_2 n^2 - 14A_2 n = 3n^2 - 2n + 1$$

$$2A_0 - 7A_1 + 19A_2 + 2A_1 n + 2A_2 n^2 - 14A_2 n = 3n^2 - 2n + 1$$

$$2A_0 - 7A_1 + 19A_2 + (2A_1 - 14A_2)n + 2A_2 n^2 = 3n^2 - 2n + 1$$

Comparing the n^2 coefficients n^2 then

$$2A_2 \cancel{n^2} = 3\cancel{n^2}$$

$$2A_2 = 3$$

$$A_2 = 3/2$$

comparing the coefficients of n then

$$2A_1 - 14A_2 = -2$$

$$2A_1 - (\cancel{14}^{7})\left(\frac{3}{\cancel{2}_{1}}\right) = -2$$

$$2A_1 - 21 = -2$$

$A_1 = \frac{19}{2}$

Generating function method

Let the generating function be $A(x) = \sum_{n=0}^{\infty} a_n x^n$ (or)

$$a_0 + a_1 x + a_1 x^2 + \cdots$$

multiply each term of the sequence of reccurrance relation by x^n and taking the $\sum_{n}^{\infty}$

-> write each term in the form of $A(x)$

-> Find $A(x)$ decompose $A(x)$ into partial fractions. if necessary

-> Express $A(x)$ as a sum of series from this we can get A_n by equating the co-efficiente of x^n

Solve the reccurrance relation $a_n - 9a_{n-1} + 20a_{n-2} = 0$ with $a_0 = 3$ $a_1 = -10$.

Given

$$a_n - 9a_{n-1} + 20a_{n-2} = 0 \rightarrow (1)$$

let the generating function of $A(x) = \sum a_n x^n \rightarrow (2)$

multiply eqn① each term of eqn① by x^n and take the $\sum_{n=2}^{\alpha}$

f

$$\sum_{n=2}^{\alpha} a_n x^n - 9\sum_{n=2}^{\alpha} a_{n-1} x^n + 20 \sum_{n=2}^{\alpha} a_{n-2} x^n = 0 \longrightarrow ③$$

$$\sum_{n=2}^{\alpha} a_n x^n = a_2x^2 + a_3x^3 + a_4x^4 + \cdots\cdots$$

$$= (a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots\cdots) - a_0 - a_1x$$

$$= A(x) - a_0 - a_1x$$

$$\sum_{n=2}^{\alpha} a_{n-1} x^n = a_1x^2 + a_2x^3 + a_3x^4 + \cdots\cdots$$

$$= x(a_1x + a_2x^2 + a_3x^3 + \cdots)$$

$$= x\{(a_1x + a_2x^2 + a_3x^3 + \cdots) - a_0\}$$

$$= x(A(x)) - a_0x$$

$$\sum_{n=2}^{\alpha} a_{n-2} x^n = a_0x^2 + a_1x^3 + a_2x^4 + \cdots\cdots$$

$$= x^2(a_0 + a_1x + a_2x^2 + \cdots\cdots$$

$$= x^2(A(x))$$

$$A(x) - a_0 - a_1x - 9[xA(x) - a_0x] + 20[x^2 A(x)] = 0$$

$$A(x) - a_0 - a_1x - 9xA(x) + 9a_0x + 20x^2 \cdot A(x) = 0$$

$$A(x)[1 - 9x + 20x^2] - a_0 - a_1x + 9a_0x = 0$$

$$A(x)[1 - 9x + 20x^2] = a_0 + a_1x - 9a_0x = 0$$

$$A(x) = \frac{a_0 + a_1x - 9a_0x}{1 - 9x + 20x^2} \rightarrow A(x) = \frac{a_0(1-9x) + a_1x}{1 - 9x + 20x^2}$$

$$A(x) = \frac{a_0(1-9x) + a_1x}{(1-4x)(1-5x)}$$

$$A(x) = \frac{A}{1-4x} + \frac{B}{1-5x}$$

$$A(x) = A(1-4x)^{-1} + B(1-5x)^{-1}$$

$$\sum_{n=0}^{\alpha} a_n x^n = A\sum_{n=0}^{\alpha} 4^n x^n + B\sum_{n=0}^{\alpha} 5^n x^n$$

Equating the coefficients of x^n, $a_n = A4^n + B5^n \rightarrow (4)$

put $n=0$ in eqn 4

$a_0 = A(4)^0 + B(5)^0$

$a_0 = A + B$

$A+B = 3 \rightarrow (5)$

put $n=1$ in eqn 4

$a_1 = A(4)^1 + B(5)^1$

$4A_1 + 5B = -10 \rightarrow (6)$

solving 5 & 6 we get

eqn $5\times 4 = 4A + 4B = 12$

eqn $6 \times 1 = 4A + 5B = -10$

$(-) \quad (-) \quad (-)$

$-B = 22$

$B = -22$

sub $b = -22$ in eqn 5

$A + B = 3$

$A - 22 = 3$

$A = 3 + 22$

$A = 25$

$A = 25, \; B = -22$

sub A and B values in (4)

$a_n = 25(4)^n - 22(5)^n$

$$\sum_{n=0}^{\alpha} a_n x^n = 25\sum_{n=0}^{\alpha} 4^n x^n - 22\sum_{n=0}^{\alpha} 5^n x^n$$

Solve the reccurrance relation $a_n - 6a_{n-1} = 0$ for $n \geq 0$ and $a_0 = 1$ by using generating function method.

Given

$a_n - 6a_{n-1} = 0 \rightarrow (1)$

Let the generating function be $A(x) = \sum_{n=0}^{\alpha} a_n x^n \rightarrow (2)$

multiply eqn (1) each term by x^n and take the $\sum_{n=1}$

$$\sum_{n=1}^{\alpha} a_n x^n - 6\sum_{n=1}^{\alpha} a_{n-1} x^n = 0 \rightarrow (3)$$

NOW

$= (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots\dots) - a_0$

$= A(x) - a_0$

$\sum_{n=1}^{\alpha} a_{n-1} x^n = a_0 x + a_1 x^2 + a_2 x^3 + \dots\dots$

$= x(a_0 + a_1 x + a_2 x^2 + \dots)$

$= x(A(x))$

substitue these values in eqn ③

$A(x) = a_0 - 6(x A(x)) = 0$

$A(x) = a_0 - 6x A(x) = 0$

$A(x)(1-6x) = a_0$

$A(x) = \frac{a_0}{1-6x}$

$A(x) = \frac{A}{1-6x}$

$A(x) = A(1-6x)^{-1}$

$A(x) = A \sum_{i=0}^{\alpha} x^n 6^n$

$\sum_{n=0}^{\alpha} a_n x^n = A \sum_{n=0}^{\alpha} x^n 6^n$

$a_n = A \cdot 6^n \longrightarrow$ ④

put $n = 0$

$a_0 = A\, 6^0$

$a_0 = A.$

$a = 1$

sub $A = 1$ in eqn 4

$\sum_{n=0}^{\alpha} a_0 x^n = A \sum_{n=0}^{\alpha} x^n 6^n$

$\sum_{n=0}^{\alpha} a_0 x^n = 1 \sum_{n=0}^{\alpha} x^n 6^n$

solve the reccurrance relation $a_n - 7a_{n-1} + 10a_{n-2} = 0$ with $a_0 = 10$, $a_1 = 41$ by using generating function method

Given

let the generating function be $A(x) = \sum_{n=0}^{\alpha} a_n x^n \rightarrow ②$

multiply eqn① each term by x^n and take the $\sum_{n=2}^{\alpha}$ $n = 2$ to α

$$\sum_{n=2}^{\alpha} a_n x^n - 7\sum_{n=2}^{\alpha} a_{n-1} x^n + 10 \sum_{n=2}^{\alpha} a_{n-2} x^n = 0 \rightarrow ③$$

$$\sum_{n=2}^{\alpha} a_n x^n = a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots\cdots$$

$$= (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots\cdots) - a_0 - a_1 x$$

$$= A(x) - a_0 - a_1 x$$

$$\sum_{n=2}^{\alpha} a_{n-1} x^n = a_1 x^2 + a_2 x^3 + a_3 x^4 + \cdots\cdots$$

$$= x(a_1 x + a_2 x^2 + a_3 x^3 \cdots\cdots)$$

$$= x(a_1 x + a_2 x^2 + a_3 x^3 + \cdots\cdots) - a_0$$

$$= x(A(x)) - a_0 x$$

$$= x(A(x)) - x a_0$$

$$\sum_{n=2}^{\alpha} a_{n-2} x^n = a_0 x^2 + a_1 x^3 + a_2 x^4 + \cdots\cdots$$

$$= x^2 (A_0 + a_1 x + a_2 x^2 + \cdots\cdots)$$

$x^2 (A(x))$

subtitute these values in eqn 3

$$A(x) - a_0 - a_1 x - 7(xA(x) - a_0 x) + 10[x^2 A(x)] = 0$$

$$A(x) - a_0 - a_1 x - 7x A(x) + 7x a_0 + 10x^2 A(x)] = 0$$

$$A(x)(1 - 7x + 10x^2) = a_0 - 7x a_0 + a_1 x$$

$$A(x) = \frac{a_0 - 7x a_0 + a_1 x}{1 - 7x + 10x^2}$$

$$= \frac{a_0 - 7x a_0 + a_1 x}{(1-2x)(1-5x)}$$

$$= \frac{A}{1-2x} + \frac{B}{1-5x}$$

$$A(1-2x)^{-1} + B(1-5x)^{-1}$$

$$A(x) = A \sum_{n=0}^{\alpha} 2^n x^n + B \sum_{n=0}^{\alpha} 5^n x^n$$

$$\alpha \quad \alpha \quad n\ n \quad + B \sum^{\alpha} 5^n x^n$$

Equating the x^n co-efficients

$a_n = A\,2^n + B\,5^n \rightarrow$ (4)

put $n=0$ in eqn (4) we get

$a_n = A+B$

$a_0 = A+B$

$A+B = 10 \rightarrow$ (5)

put $n=1$ in eqn (4) we get

$a_1 = 2A+5B$

$2A+5B = 41 \rightarrow$ (6)

solve (5) and (6)

$$
\begin{aligned}
\text{eqn5} \times 5 &\rightarrow 5A + \cancel{5B} = 50\\
6 &\rightarrow 2A + \cancel{5B} = 41\\
&\quad (-)\quad (-)\quad (-)\\
\hline
&\quad 3A = 9
\end{aligned}
$$

$A = \frac{\cancel{9}\,3}{\cancel{3}\,1}$

$A = 3$

substitue $A = 3$ in eqn (5)

$3+B = 10$

$B = 10-3$

$B = 7$

$a_n = (3)(2)^n + (7)(5)^n$

solve the general solution for an using generating function

$a_n - 7a_{n-1} + 12a_{n-2} = 0$ with $a_0 = 10$ $a_1 = 41$

$a_n - 6a_{n-1} + 12a_{n-2} - 5a_{n-3} = 0$

Given

$a_n - 7a_{n-1} + 12a_{n-2} = 0 \rightarrow$ (1)

let the generating function be $A(x) = \sum_{n=0}^{\alpha} a_n x^n \rightarrow$ (2)

Multiply eqn (1) each term by x^n and take $n = 2$ to α $\left(\sum_{n=2}^{\alpha}\right.$

$$\sum_{n=2}^{\alpha} a_n x^n = (a_2x^2+a_3x^3+a_4x^4+\cdots$$
$$=(a_0+a_1x+a_2x^2+a_3x^3+\cdots)-a_0-a_1x$$
$$=A(x)-a_0-a_1x$$

$$\sum_{n=2}^{\alpha} a_{n-1}x^n = (a_1x^2+a_2x^3+a_3x^4+\cdots$$
$$=(a_0+a_2x^2+a_3x^3+\cdots)$$
$$=x(a_1x+a_2x^2+a_3x^3+\cdots)-a_0$$
$$=x(A(x))-a_0\cdot x$$
$$=x(A(x))-xa_0$$

$$\sum_{n=2}^{\alpha} a_{n-2}x^n = (a_1x^2+a_2x^3+a_3x^4+\cdots$$
$$=x^2(A_0+a_1x+a_2x^2+\cdots)$$
$$=x^2(A(x))$$

substitute thesevalues in eq 3

$$A(x)-a_0-a_1x-7(xA(x)-xa_0)+12(x^2(A(x))=0$$
$$A(x)-a_0-a_1x-7xA(x)-7xa_0+12x^2A(x)=0$$

$$A(x)(1-7x+12x^2)=a_0-7xa_0+a_1x$$

$$A(x)=\frac{a_0-7xa_0+a_1x}{(1-7x+12x^2)}$$

$A(x)=\sum_{n=0}^{\alpha} a_nx^n$

$$A(x)=\frac{a_0-7xa_0+a_1x}{(1-3x)(1-4x)}$$

$$=\frac{A}{1-3x}+\frac{B}{1-4x}$$

$$A(1-3x)^{-1}+B(1-4x)^{-1}$$

$$A(x)=A\sum_{n=0}^{\alpha}3^nx^n+B\sum_{n=0}^{\alpha}4^nx^n$$

$$\sum_{n=0}^{\alpha}a_nx^n=A\sum_{n=0}^{\alpha}3^nx^n+B\sum_{n=0}^{\alpha}4^nx^n$$

Equating the x^n co-efficients

$$a_n=A\,3^n+B\,4^n \rightarrow (4)$$

put $n=0$ in eqn (4) we get

$$a_0=A+B$$
$$A+B=10 \rightarrow (5)$$

$3A+4B=41 \rightarrow$ 6

solve eqn 5 and 6

eqn 5 $\times 4$ $\Rightarrow$ $4A+\cancel{4B}=40$

6 $\quad 3A+\cancel{4B}=41$

$\quad (-) \quad (-) \quad (-)$

$1A=-1$

$A=\frac{-1}{1}$

$=-1$

substitue $A=-1$ eqn 5

$-1+B=10$

$B=10+1$

$B=11$

substitute A and B values in eq 4

$a_n=(-1)(3)^n+(11)(4)^n$

Given

$a_n-6a_{n-1}+12a_{n-2}-5a_{n-3}=0 \rightarrow$ 1

let the generating function be $A(x)=\sum_{n=0}^{\infty} a_n x^n \rightarrow$ 2

multiply eq 1 each term by x^n and take $n=3$ to ∞ $\sum_{n=3}^{\infty}$

$$\sum_{n=3}^{\infty} a_n x^n - 6\sum_{n=3}^{\infty} a_{n-1} x^n + 12\sum_{n=3}^{\infty} a_{n-2} - 5\sum_{n=3}^{\infty} a_{n-3} = 0 \rightarrow 3$$

$$\sum_{n=3}^{\infty} a_n x^n = a_3x^3+a_4x^4+\cdots$$

$x(a_0+a_1x+a_2x^2+a_3x^3+\cdots$

$=(a_0+a_1x+a_2x^2+a_3x^3+\cdots)-a_0-a_1x-a_2x^2$

$=A(x)-a_0-a_1x-a_2x^2$

$$\sum_{n=3}^{\infty} a_{n-1}x^n = a_2x^3+a_3x^4+a_4x^5+\cdots$$

$=x(a_0+a_1x+a_2x^2+a_3x^3+\cdots)$

$=x(a_0+a_1x+a_2x^2+a_3x^3+\cdots)-a_0-a_1x$

$=x(A(x))-a_0x-a_1x$

$$\sum a_n x^n \; a_3x^3+a_4x^4+a_5x^5+\cdots$$

$= x^2(a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots) - a_0$

$= x^2(A(x)) - xa_0$

$\sum_{n=3}^{\alpha} a_{n-3}x^n = a_0x^3 + a_1x^4 + a_2x^5 + a_3x^6 + \cdots$

$= x^3(a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots)$

$= x^3(A(x))$

substitute these values in eq ③

$A(x) - a_0 - a_1x - a_2x^2 - 6(xA(x) - a_0x - a_1x) + 12(x^2(A(x) - xa_0 -$
$5(x^3A(x)) = 0$

$Ax - a_0 - a_1x - a_2x^2 - 7xA(x) - 7a_0x - 7a_1x + 12x^2A(x) - 12xa_0 - 5x^3A(x)$
$= 0$

$A(x)(1 - 7x + 12x^2 - 5x^3) = a_0 - a_1x - a_2x^2 + 7a_0x + 7a_1x + 12xa_0 = 0$

$A(x)(1 - 7x + 12x^2 - 5x^3) = a_0 - 8_1x - a_2x^2 + 19a_0x$

$$A(x) = \frac{a_0 - 8_1x - a_2x^2 + 19a_0x}{(1 - 7x + 12x^2 - 5x^3)}$$

$$\frac{A}{(1-3x)} + \frac{B}{(1-4x)} - \frac{C}{(1-7x)}$$

$A(1-3x)^{-1} + B(1-4x)^{-1} - C(1-7x)^{-1}$

$Ax = A\sum_{n=0}^{\alpha} 3^nx^n + B\sum_{n=0}^{\alpha} 4^nx^n - C\sum_{n=0}^{\alpha} 7^nx^n$

$\sum_{n=0}^{\alpha} a_nx^n = A\sum_{n=0}^{\alpha} 3^nx^n + B\sum_{n=0}^{\alpha} 4^nx^n - C\sum_{n=0}^{\alpha} 7^nx^n$

Solve the reccurrance relation $a_n = 3a_{n-1} + 2$ with $a_0 = 1$ by using generating functions method.

Given

$a_n - 3a_{n-1} = 2 \to ①$

let the generating function be $A(x) = \sum_{n=0}^{\alpha} a_nx^n \to ②$

multiply eq ① each term by x^n and take $n = 1$ to α $\sum_{n=1}^{\alpha}$

$\sum_{n=1}^{\alpha} a_nx^n - 3\sum_{n=1}^{\alpha} a_{n-1}x^n = 2\sum_{n=1}^{\alpha} x^n \to ③$

$= (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots) - a_0$

$= A(x) - a_0$

$\sum_{n=1}^{\alpha} a_{n-1} x^n = a_0 x + a_1 x^2 + a_2 x^3 + \cdots$

$= x(a_0 + a_1 x + a_2 x^2 + \cdots)$

$= x(A(x))$

$\sum_{n=1}^{\alpha} x^n = x + x^2 + x^3 + \cdots$

$= x(1 + x + x^2 + \cdots)$

$= x(1-x)^{-1}$

$= \frac{x}{1-x}$

substitute the values in eqn ②

$A(x) - a_0 - 3x(A(x)) = 2\cdot\left(\frac{x}{1-x}\right)$

$A(x) - a_0 - 3xA(x) = \frac{2x}{1-x}$

$A(x) = (1-3x) = \frac{2x}{1-x} + a_0$

$A(x)(1-3x) = \frac{2x}{1-x} + 1$

$A(x)(1-3x) = \frac{2x+1-x}{1-x}$

$= \frac{x+1}{1-x}$

$A(x) = \frac{x+1}{(1-x)(1-3x)}$ $\Rightarrow A(x) = \frac{A}{1-x} + \frac{B}{1-3x}$

$x+1 = A(1-3x) + B(1-x)$

$A(x) = \frac{A}{1-x} + \frac{B}{1-3x}$

$1 = A + B \rightarrow ④$

$1 = -3A - B \rightarrow ⑤$

$\therefore A = \lim_{x\to 1} \frac{1+x}{1-3x} = \frac{1+1}{1-3(1)} = \frac{2}{-2} = -1$

$2 = -2A$

$A = -1$

$\therefore B = \lim_{x\to 1/3} \frac{1+x}{1-x} = \frac{1+1/3}{1-1/3} = \frac{4/3}{2/3} = 2$

$1 = (-1) + B$

$B = 2$

$A(x) = \frac{-1}{1-x} + \frac{2}{1-3x}$

$= \sum_{n=0}^{\alpha} x^n + 2 \sum_{n=0}^{\alpha} (3x)^n$

$\sum_{n=1}^{\alpha} a_n x^n = -\sum_{n=0}^{\alpha} x^n + 2 \sum_{n=0}^{\alpha} 3^n x^n$

compare the coefficients of x^n

$= (-1+2 \cdot 3^n)$

$a_n - a_{n-1} = 3(n-1) \quad a_0 = 2$

Given

$a_n - 3a_{n-1} = 4 \rightarrow ①$

let the generating function be $A(x) = \sum_{n=0}^{\alpha} a_n x^n \rightarrow ②$

multiply eq① each term by x^n and take $n=1$ to α $\sum_{n=1}^{\alpha}$

$\sum_{n=1}^{\alpha} a_n x^n - 3 \sum_{n=1}^{\alpha} a_{n-1} x^n = 4 \sum_{n=1}^{\alpha} x^n \rightarrow ③$

$\sum_{n=1}^{\alpha} a_n x^n = a_1 x + a_2 x^2 + a_3 x^3 + \cdots$

$= (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots) - a_0$

$= A(x) - a_0$

$\sum_{n=1}^{\alpha} a_{n-1} x^n = a_0 x + a_1 x^2 + a_2 x^3 + \cdots$

$x(a_0 + a_1 x + a_2 x^2 + \cdots)$

$= x(A(x))$

$\sum_{n=1}^{\alpha} x^n = x + x^2 + x^3$

$x(1 + x + x^2 + \cdots)$

$x(1-x)^{-1}$

$= \frac{x}{1-x}$

substitute the values in eqn ②

$A(x) - a_0 - 3x(A(x)) = 4\left(\frac{x}{1-x}\right)$

$A(x) = (1-3x) = \frac{4x}{1-x} + a_0$

$A(x)\{1-3x) = \frac{4x}{1-x} + 1$

$A(x)(1-3x) = \frac{4x+1-x}{1-x}$

$$A(x) = \frac{3x+1}{(1-x)(1-3x)}$$

$$A(x) = \frac{A}{1-x} + \frac{B}{1-3x}$$

$$\therefore A \quad x \to 1 \quad \frac{3x+1}{1-3x} = \frac{3(1)+1}{1-3(1)} = \frac{\cancel{4}^{2}}{-\cancel{2}}$$

$$\therefore B \quad x \to 1/3 \quad \frac{1+3x}{1-x} = \frac{1+3(\frac{1}{3})}{1-1/3} = \frac{2}{2/3} = \frac{\cancel{2}}{1} \times \frac{3}{\cancel{2}} = 3$$

$$A(x) = \frac{-2}{1-x} + \frac{3}{1-3x}$$

$$A(x) = 2(1-x)^{-1} + 3(1-3x)^{-1}$$

$$2\sum_{n=0}^{\alpha} x^n + 3\sum_{n=0}^{\alpha} (3x)^n$$

$$\sum_{n=1}^{\alpha} a_n x^n = 2\sum_{n=0}^{\alpha} x^n + 3\sum_{n=0}^{\alpha} 3^n x^n$$

compare the co-efficients of x^n

$$(-2 1^n + 3 \cdot 3^n)$$

Given that

$$a_n - 9a_{n-1} + 20a_{n-2} = -20 \rightarrow (1)$$

$a_n - 9a_{n-1}$

eqn (1) is a second order non-homogeneous reccurronce relation:

$\therefore$ The general solution of non-homogeneous reccurrance relation is

$$a_n = a_n^{(h)} + a_n^{(P)}$$

To find $a_n^{(h)}$

The homogeneous reccurrance relation of eqn (1) is

$$a_n - 9a_{n-1} + 20a_{n-1} = 0$$

The characteristic equation is

$$t^2 - 9t + 20 = 0 \rightarrow (2)$$

$$t^2 - 5t - 4t + 20 = 0$$

$$t(t-5) - 4(t-5) = 0$$

$$t = 4, 5$$

the roots are real and distinct

the general solution of homogeneous reccurrance relation is

$$a_n^{(h)} = c_1 4^n + c_2 5^n.$$

To find $a_n^{(P)}$

Given

$$a_n - 9a_{n-1} + 20a_{n-1} = -20$$

Here $f(n) = -20$

$$f(n) = -20 \cdot 1^n$$

Here since from eqn ② $\alpha = 1$ is not a root of a characteristic equation and $g(n)$ is a constant, so that the particular solution is

$$a_n^{(P)} = A_0 \cdot 1^n$$

$$a_n^{(P)} = A_0 .$$

substitute $a_n = A_0$ in eqn ①

$$A_0 - 9A_0 + 20A_0 = -20$$

$$12A_0 = -20$$

$$A_0 = \frac{\cancel{-20}\ 10}{\cancel{12}\ 6} = \cancel{\frac{-10}{6}}\ \frac{5}{3}$$

substitute A_0 value in particular solution

$$a_n^{(P)} = -10/6$$

∴ The general solution of non-homogeneous linear reccurrance relation is

$$a_n = a_n^{(h)} + a_n^{(P)}$$

$$a_n = c_1 4^n + c_2 5^n + (-5/3)$$

$n=0$

$$a_0 = c_1 4^0 + c_2 5^0$$

$$a_0 = c_1 + c_2$$

$$c_1 + c_2 = 41 \quad 5\times5 \quad 5c_1 + \cancel{5c_2} = 205$$

$$4c_1 + \cancel{5c_2} = 10$$

$n=1$

$$a_1 = c_1 4^1 + c_2 5^1$$

$$1c_1 = 195$$

$$c_1 = 195$$

$$a_1 = 4c_1 + 5c_2$$

$$4c_1 + 5c_2 = 10$$

$$a_0 = 195 + c_2$$

$$41 = 195 + c_2$$

Given

$$a_n - 9a_{n-1} + 20a_{n-2} = 0$$

$$a_0 = 41, \quad a_1 = 10$$

let the generating function be $A(x) = \sum_{n=0}^{\alpha} a_n x^n \rightarrow ②$

multiply eqn ① each term by x^n and take the $\sum_{n=2}^{\alpha}$

$$\sum_{n=2}^{\alpha} a_n x^n - 9\sum_{n=2}^{\alpha} a_{n-1} x^n + 20\sum_{n=2}^{\alpha} a_{n-2} x^n = 0 \rightarrow ③$$

$$\sum_{n=2}^{\alpha} a_n x^n = a_2x^2 + a_3x^3 + a_4x^4 + \text{---}$$

$$(a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \text{---}) - a_0 - a_1x$$

$$= A(x) - a_0 - a_1x$$

$$\sum_{n=2}^{\alpha} a_{n-1} x^n = a_1x^2 + a_2x^3 + a_3x^4 \text{---}$$

$$(a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \text{---}) - a_0$$

$$x(a_1x + a_2x^2 + a_3x^3 + a_4x^4 \text{---}) - a_0$$

$$= x(A(x) - a_0$$

$$\sum_{n=2}^{\alpha} a_{n-2} x^n = a_0x^2 + a_1x^3 + a_2x^4 + a_3x^5 \text{---}$$

$$= x^2(a_0 + a_1x + a_2x^2 + a_3x^3 + \text{---})$$

$$= x^2(A(x))$$

$$A(x) - a_0 - a_1x - 9x(A(x) - a_0 + 20x^2A(x) = 0$$

$$A(x) - a_0 - a_1x - 9xA(x) + 9a_0x + 20x^2A(x) = 0$$

$$A(x)[1 - 9x + 20x^2] - a_0 - a_1x + 9a_0x = 0$$

$$A(x)[1 - 9x + 20x^2] = a_0 + a_1x - 9a_0x = 0$$

$$A(x) = \frac{a_0 + a_1x - 9a_0x}{1 - 9x + 20x^2}$$

$$A(x) = \frac{a_0(1 - 9x) + a_1x}{1 - 9x + 20x^2}$$

$$A(x) = \frac{a_0(1 - 9x) + a_1x}{(1 - 4x)(1 - 5x)}$$

$A(x) = A(1-4x)^{-1} + B(1-5x)^{-1}$

$A(x) = A\sum_{n=0}^{\alpha} 4^n x^n + B\sum_{n=0}^{\alpha} 5^n x^n$

$\sum_{n=0}^{\alpha} a_n x^n = A\sum_{n=0}^{\alpha} 4^n x^n + B\sum_{n=0}^{\alpha} 5^n x^n$

equating the coefficients of $x^n = a_n = A4^n + B5^n \rightarrow (4)$

put $n=0$ in eqn (4)

$a_0 = A(4)^0 + B(5)^0$

$a_0 = A + B$

$A + B = 41 \rightarrow (5)$

put $n=1$ in eqn (5)

$a_1 = A(4)^1 + B(5)^1$

$4A + 5B = 10 \rightarrow (6)$

solving 5 & 6 we get

$5\times5 \quad 5A + 5B = 205$

(6) $\quad 4A + 5B = 10$

$\quad\quad (-) \quad\quad (-)$

$1A = 195$

$A = 195$

substitue $A = 195$ in eqn (5)

$A + B = 41$

$195 + B = 41$

$B = 41 - 195$

$B = -154$

$t^2 - 9t + 20 = 0$

$t^2 - 5t - 4t + 20 = 0$

$t(t-5) - 4(t-5) = 0$

$t = 4, 5$.

Graph:- A graph G is defined as a pair of sets (V, E). It is denoted by $G=(V,E)$ or $G(V,E)$ or G

where V or $V(G)$ = set of all vertices in G

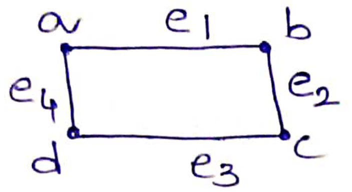
E or $E(G)$ = set of all edges connecting the vertices in G

$|V|$ = number of vertices

$|E|$ = number of edges.

Note:- Vertices are also called nodes. or points.

Eg:-

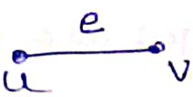


Here $V(G)=\{a,b,c,d\}$

$E(G)=\{e_1,e_2,e_3,e_4\}$ or $\{\{a,b\},\{b,c\},\{c,d\},\{a,d\}\}$

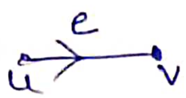
Adjacent vertices:- Two vertices u and v are said to be adjacent if there is an edge between u and v

Eg:- 1)



Here u, v are adjacent

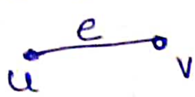
2)



Here u is adjacent to v and v is adjacent from u

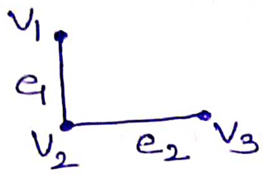
Incident edge:- The edge 'e' that joins the vertices u and v is said to be incident on each of its end points u and v.

Eg:-



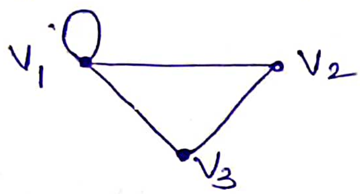
Here 'e' is called incident edge.

Adjacent edges :- Two non-parallel edges are said to be adjacent if they are incident on a common vertex.



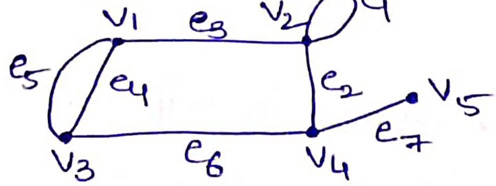
Here v_2 is a common vertex of e_1, e_2.
So, e_1, e_2 are adjacent edges

Self loop or loop :- Loop is an edge drawn from a vertex to itself. (or) An edge associated with a vertex pair (v_i, v_i) is called a loop or self loop.



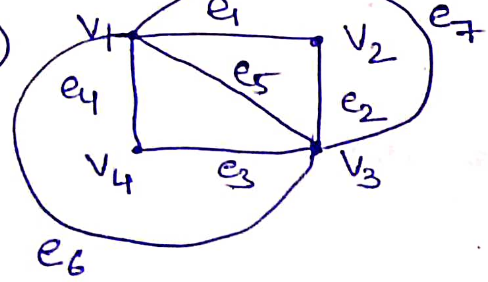
Parallel edges or multiple edges :- If two edges have same end points then the edges are called parallel edges (or) If there are more than one edge associated with a given pair of vertices, then these edges are called parallel edges or multiple edges.

Eg:- 1.)



Here e_4, e_5 are parallel edges

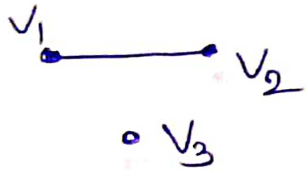
2.)



Here e_5, e_6, e_7 are multiple or parallel edges.

Isolated vertex:- A vertex having no edge incident on it is said to be an isolated vertex.

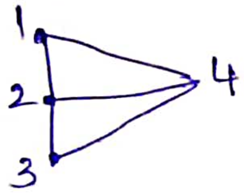
Eg:-



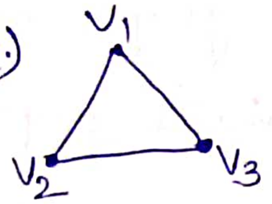
Here V_3 is an isolated vertex.

Simple graph:- A graph which has neither self loops nor parallel edges is called a simple graph.

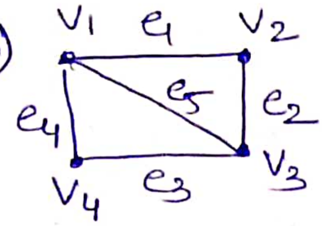
Eg:- 1.)



2.)



3.)



Note:- 1.) Maximum number of edges possible in a simple graph with 'n' vertices $= \frac{n(n-1)}{2}$

2.) Number of simple graphs possible with n vertices $= 2^{\frac{n(n-1)}{2}}$

3.) In a simple graph with 'n' vertices, degree of each vertex is $\leq (n-1)$.

Eg:- How many simple graphs are possible with 5 vertices and 3 edges.

Sol:- Given edges $= 3$

no. of vertices $(n) = 5$

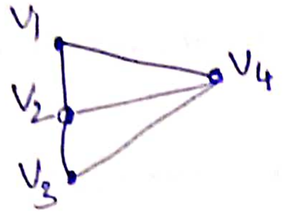
$\therefore$ Possible max. no. of edges $= \frac{n(n-1)}{2} = \frac{5(5-1)}{2} = 10$

$\therefore$ Required no. of simple graphs $= {}^{10}C_3 = 120$

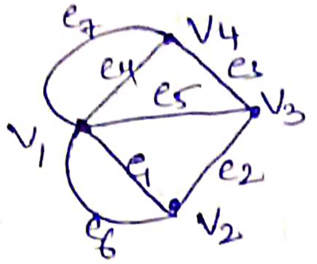
($\because$ If vertices & edges are given, the no. of simple graphs $= C$(Max. no. of edges, no. of edges))

Finite graph :- A graph G with a finite number of vertices and a finite number of edges is called a finite graph.

Eg:- 1.)

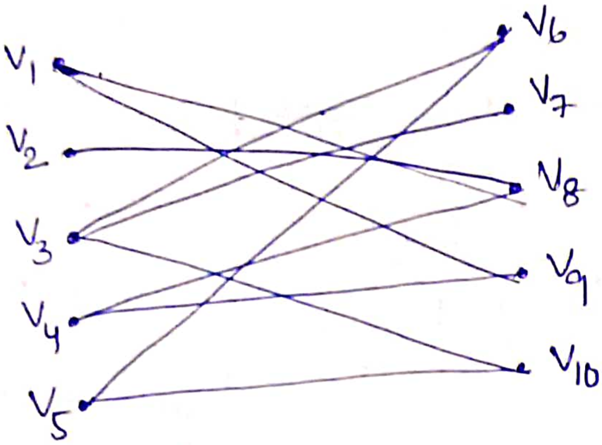


2.)



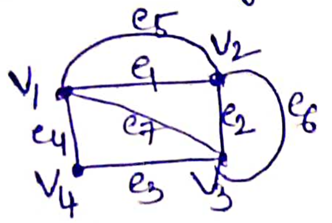
Infinite graph:- A graph G that is not a finite graph is said to be an infinite graph.

Eg:-

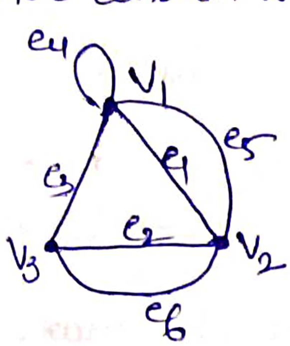


Multi graph:- A graph which contains some parallel edges is called multigraph. Multigraph may or may not contain self loops.

Eg:- 1.)

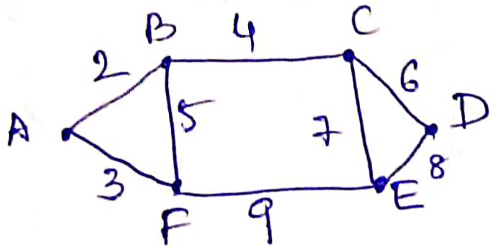


2.)



Weighted graph:- A graph in which weights or costs are assigned to every edge of the graph is called weighted graph.

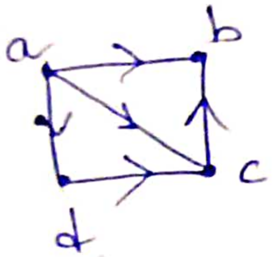
Eg:-



In this graph 2,3,4,5,6,7, 8,9 are weights of edges.

Directed graph or digraph:- A graph in which every edge is directed is called directed graph or digraph.
A directed graph contains an ordered pair of vertices denoted by (u,v).
The first element 'u' is called initial node or starting vertex.
The second element 'v' is called terminal node or ending vertex.

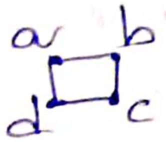
Eg:-



Vertex set, V = {a,b,c,d}
Edge set, E = {(a,b), (a,c), (a,d), (c,b), (d,c)}

Undirected graph or Non-directed graph:- A graph in which every edge is undirected is called undirected graph i.e., there is no specific direction to represent the edges.
Undirected graph contains unordered pair of vertices denoted as {u,v}

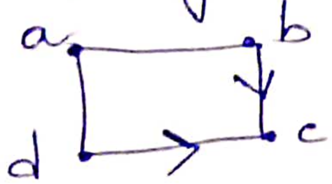
Eg:-



Vertex set, V = {a,b,c,d}
Edge set, E = {{a,b}, {b,c}, {c,d}, {d,a}}

Mixed graph:- A graph in which some edges are directed and some edges are undirected is called a mixed graph.

Eg:-



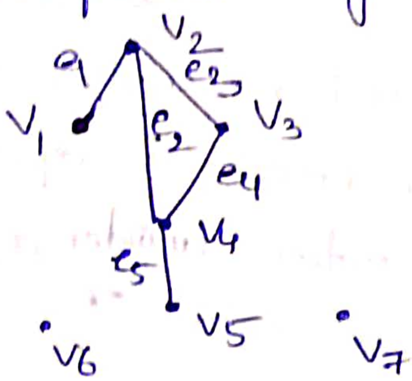
Degree of a vertex in undirected graph:- The number of edges incident on a vertex V and with loops counted twice is called degree of vertex V and is denoted by $deg_G(v)$ or $deg(V)$ or $d(V)$

Eg:-

$d(V_1) = 4$
$d(V_2) = 3$
$d(V_3) = 3$
$d(V_4) = 2$

Pendent vertex:- A vertex of degree one is called a pendent vertex and an edge incident on a pendent vertex is called a pendent edge.

Eg:-



Here the vertices V_6 & V_7 are isolated vertices
V_1, V_5 are pendent vertices
e_1, e_5 are pendent edges.

Note:- A vertex of degree zero is called an isolated vertex.

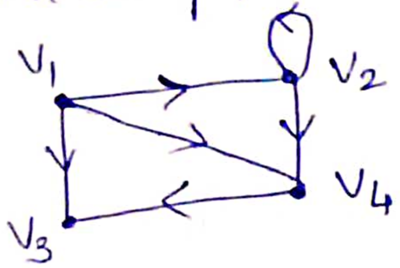
Indegree and outdegree of vertex for a directed graph:-

Indegree of a vertex:- The indegree of a vertex V in a directed graph is the number of edges which are coming into the vertex V i.e., number of Incoming edges. Indegree of a vertex V is denoted by $deg^{-}(V)$ or $d^{-}(V)$ or $Indeg(V)$.

Outdegree of a vertex:- The outdegree of a vertex V in a directed graph is the number of edges which are going out from the vertex V i.e, number of outgoing edges. Outdegree of a vertex V is denoted by $deg^{+}(V)$ or $d^{+}(V)$ or $outdeg(V)$.

Note: For a loop, we can take indegree = 1 & outdegree = 1

Eg:-



Vertex	Indegree	Outdegree	Total degree
V_1	0	3	3
V_2	2	2	4
V_3	2	0	2
V_4	2	1	3

Matrix representation of a graph:- A graph can be represented by using a matrix. There are two ways to represent the graph by a matrix.

1) Adjacency matrix
2) Incidence matrix

Eg:-

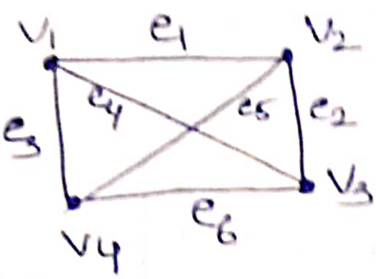
$$A_G = \begin{array}{c} \\ a \\ b \\ c \end{array}\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

2.) Incidence matrix:-

i.) For non-directed graph:- Let G be graph with n vertices and m edges. Then the incidence matrix is denoted by I_G is defined by matrix.

$$I(G) = [a_{ij}]_{n\times m}, \text{ where } a_{ij} = \begin{cases} 1, & \text{when } v_i \text{ is incident with } e_j \\ 0, & \text{otherwise} \end{cases}$$

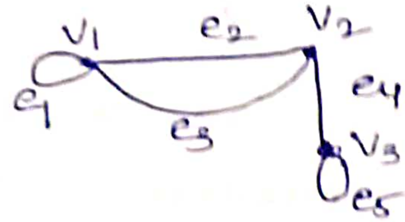
Eg:-1)



then

$$I(G) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}\begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

2)



$$I_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array}\begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

ii.) For digraph or directed graph:- Let $G = (V, E)$ be a graph with n vertices and m edges without loops, then the incidence matrix I_G is defined by $I(G) = (b_{ij})_{n\times m}$ defined by

$$b_{ij} = \begin{cases} 1, & \text{if } v_i \text{ is the initial vertex of the edge } e_j \\ -1, & \text{if } v_j \text{ is the final vertex of } e_j \\ 0, & \text{otherwise} \end{cases}$$

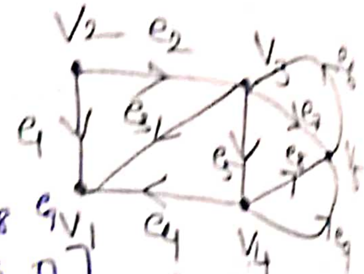
Eg:- write the incidence matrix for

$$I(G) = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

1) find the incidence matrix.

$$I(G) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

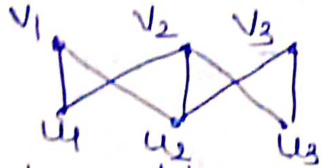
2) find the incidence matrix for digraph



$$I(G) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 \end{array} \\ \begin{bmatrix} -1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & -1 \end{bmatrix} \end{array}$$

Bipartite graph:- A graph $G=(V,E)$ is said to be bipartite graph if the vertex set V can be partitioned into two disjoint subsets V_1 and V_2 suchthat every edge in E connects a vertex in V_1 and a vertex in V_2.

Eg:-

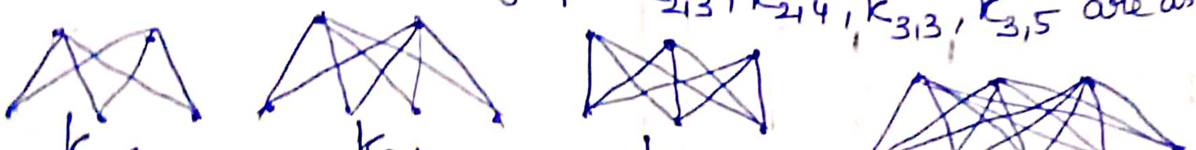


$V_1 = \{v_1, v_2, v_3\}$

$V_2 = \{u_1, u_2, u_3\}$

Complete bipartite graph:- The complete bipartite graph on m and n vertices denoted by $K_{m,n}$ is the graph whose vertex set is partitioned into two sets V_1 with 'm' vertices and V_2 with 'n' vertices in which there is an edge between each pair of vertices v_1 & v_2, where v_1 is in V_1 & v_2 is in V_2.

The complete bipartite graphs $K_{2,3}$, $K_{2,4}$, $K_{3,3}$, $K_{3,5}$ are as follows



$K_{2,3}$ $K_{2,4}$ $K_{3,3}$ $K_{3,5}$

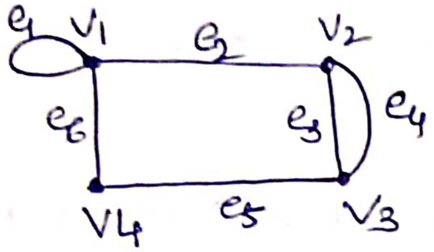
Fundamental theorem of graph theory (or) Handshaking theorem:-

The $G=(V,E)$ be an undirected graph with 'e' edges then

$\sum_{v_i \in V} deg(v_i) = 2e$ (or)

The sum of the degrees of all vertices in an undirected graph is even.

Proof:- Verify the above theorem by the following graph



$V = \{v_1, v_2, v_3, v_4\}$

$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$

$deg(v_1) = 4$

$deg(v_2) = 3$

$deg(v_3) = 3$

$deg(v_4) = 2$

$\therefore \sum_{i=1}^{4} deg(v_i) = 4+3+3+2 = 12$

$2|E| = 2 \times$ number of edges $= 2 \times 6 = 12$

1) How many vertices are needed to construct a graph with 7 edges in which each vertex is of degree 2.

Sol: Let the required no. of vertices be 'n'

Given edges, $e = 7$

Each vertex is of degree $= 2$

By Handshaking theorem, $\sum_{i=1}^{n} deg(v_i) = 2e$

$\Rightarrow d(v_1) + d(v_2) + \cdots + d(v_n) = 2 \times 7$

$\Rightarrow 2 + 2 + \cdots + 2$ (by n times) $= 14$

$\Rightarrow 2n = 14 \Rightarrow n = 7$

2.) How many edges are there in a graph with 10 vertices of each of degree 6.

Sol:- Let e be the number of edges.

The given graph contain 10 vertices of each of degree 6.

By Handshaking theorem, $\sum_{i=1}^{10} d(v_i) = 2e$

$$\Rightarrow d(v_1) + d(v_2) + \cdots + d(v_{10}) = 2e$$

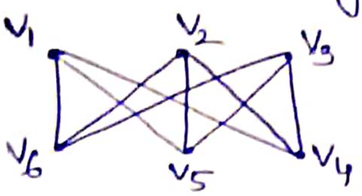
$$\Rightarrow 6 + 6 + \cdots + 6 \text{ (10 times)} = 2e$$

$$\Rightarrow 6 \times 10 = 2e$$

$$\Rightarrow e = 30.$$

Regular graph:- A graph in which all vertices are of same degree is called a regular graph & simply regular.

Eg:-

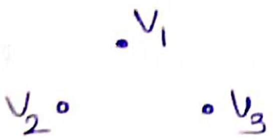


Here degree of each vertex is 3.

Null graph:- A graph $G=(V,E)$ is said to be a null graph if $E=\phi$ or

A graph G contains no edges is called null graph.

Eg:-



is a null graph since there is no edges between the vertices.

Note:- 1.) The maximum degree of any vertex in a simple graph is $(n-1)$

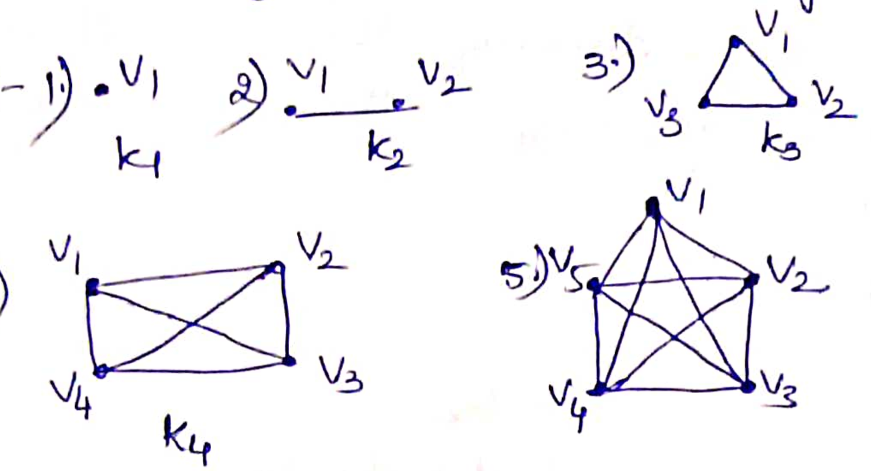
2.) The maximum number of edges in a simple graph with 'n' vertices is $\frac{n(n-1)}{2}$

Complete graph:- A simple graph G is said to be complete graph, if every vertex of G is connected with every other vertex of G i.e., if G contains only one edge between each pair of distinct vertices.

The complete graph with 'n' vertices is denoted by K_n.

A complete graph K_n has exactly $\frac{n(n-1)}{2}$ edges.

Eg:-



* **Isomorphism:-** Two graphs G and G' are said to be isomorphic to eachother if there is a one-one correspondence between their vertices and between their edges suchthat the ~~incidence~~ adjacence relationship must be preserved.

Verification of isomorphism between the graphs.

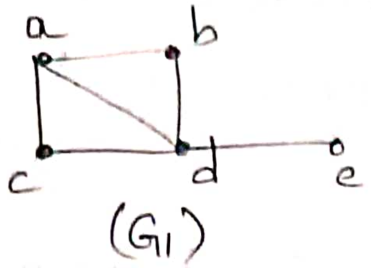
1.) The same number of vertices

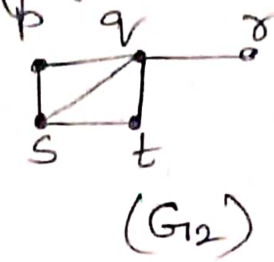
2.) The same number of edges

3.) An equal number of vertices with a given degree.

If any of the above criteria fails, the graphs are not isomorphic.

1.) show that the following graphs are isomorphic





Sol:- $G_1 = \{a, b, c, d, e\}$ $\qquad$ $G_2 = \{p, q, r, s, t\}$

$|V_1| = 5$ = no. of vertices in G_1 $\qquad$ $|V_2| = 5$ = no. of vertices in G_2

$|E_1| = 6$ = no. of edges in G_1 $\qquad$ $|E_2| = 6$ = " edges in G_2

i.) $|V_1| = |V_2|$

ii.) $|E_1| = |E_2|$

iii.) In G_1 $\qquad$ In G_2

Vertex	degree
a	3
b	2
c	2
d	4
e	1

Vertex	degree
p	2
q	4
r	1
s	3
t	2

We should have $a \to s$, $d \to q$ and $e \to r$

we take $b \to p$ and $c \to t$

Thus,

V	a	b	c	d	e
f(V)	s	p	t	q	r

Adjacency matrix of G1, $A_{G_1} = \begin{array}{c|c} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$

Adjacency matrix of G2,

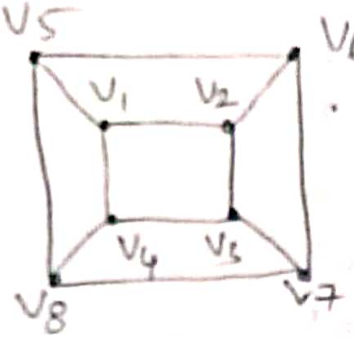
$A_{G_2} = \begin{array}{c|c} & \begin{array}{ccccc} p & q & r & s & t \end{array} \\ \begin{array}{c} p \\ q \\ r \\ s \\ t \end{array} & \begin{bmatrix} \cancel{0} & \cancel{1} & \cancel{0} & \cancel{1} & \cancel{0} \\ \cancel{1} & \cancel{0} & \cancel{1} & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ & & & & \\ & & & & \end{bmatrix} \end{array}$ $\qquad$ $A_{G_2} = \begin{array}{c|c} & \begin{array}{ccccc} s & p & t & q & r \end{array} \\ \begin{array}{c} s \\ p \\ t \\ q \\ r \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$

$\Rightarrow A_{G_1} = A_{G_2}$

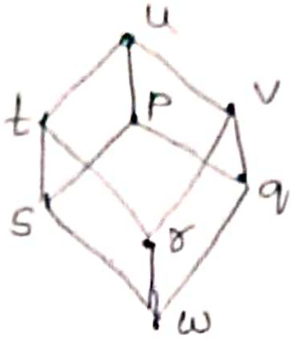
$\Rightarrow f$ is one-one, onto and also preserves adjacency.

$\therefore G_1$ and G_2 are isomorphic.

2) show that G_1, G_2 are Isomorphic



G_1



G_2

sol:- i) Number of vertices

$|V_1| = 8$, $|V_2| = 8$

$\therefore |V_1| = |V_2|$

ii) No. of edges

$|E_1| = 12$, $|E_2| = 12$

$\therefore |E_1| = |E_2|$

$A_{G_1} =$

iii) In G_1, $d(v_1) = 3$
$d(V_2) = 3$
$d(V_3) = 3$
$d(V_4) = 3$
$d(V_5) = 3$
$d(V_6) = 3$
$d(V_7) = 3$
$d(V_8) = 3$

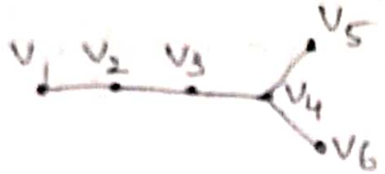
In G_2, $d(p) = 3$
$d(q) = 3$
$d(r) = 3$
$d(s) = 3$
$d(t) = 3$
$d(u) = 3$
$d(v) = 3$
$d(w) = 3$

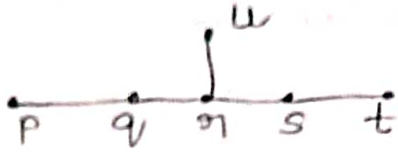
We should have $V_1 \to p$, $V_2 \to q$, $V_3 \to w$, $V_4 \to s$, $V_5 \to u$, $V_6 \to v$, $V_7 \to r$, $V_8 \to t$

Thus,

V	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
$f(V)$	p	q	w	s	u	v	r	t

3.) check the isomorphism for the following graphs





Sol: $d(v_1)=1$
$d(v_2)=2$
$d(v_3)=2$
$d(v_4)=3$
$d(v_5)=1$
$d(v_6)=1$

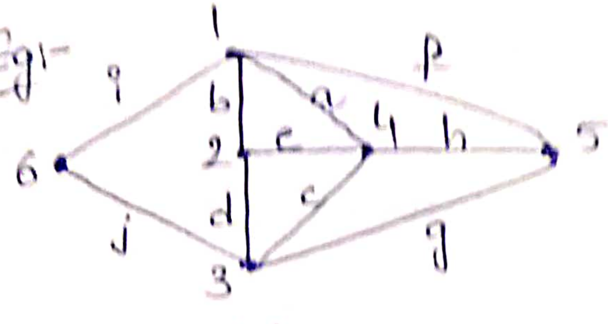
$d(p)=1$
$d(q)=2$
$d(r)=3$
$d(s)=2$
$d(t)=1$
$d(u)=1$

We conclude that there are 2 pendent vertices adjacent to v_4 and there is only one pendent vertex adjacent to r. So, a contradiction exists.

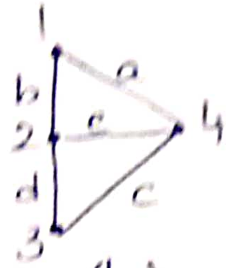
$\therefore$ The two graphs are not isomorphic.

Subgraph:- A graph H is said to be a subgraph of a graph G, if all the vertices and all the edges of H are in G and each edge of H has the same end vertices in H as in G. It is denoted by $H \subseteq G$

Eg:-



(a)



(b)

∴ Graph b is a subgraph to graph a.

Note:- 1.) Every graph is a subgraph of itself.

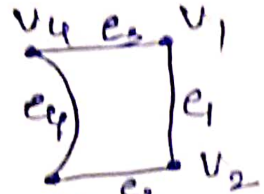
2.) A subgraph of a subgraph of G is a subgraph of G.

3.) A single vertex in a graph G is a subgraph of G.

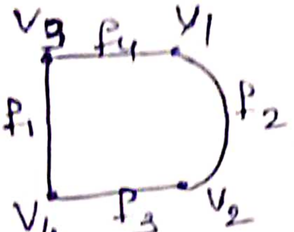
4.) A single edge in G, together with its end vertices is a subgraph of G.

5.) Two subgraphs g_1 and g_2 of a graph 'G' are said to be edge-disjoint if g_1 and g_2 do not have any edges in common.

Eg:-



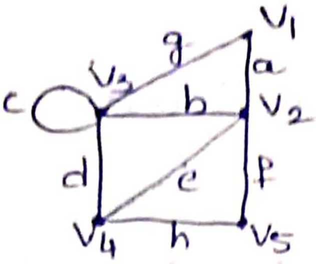
(g_1)



(g_2)

Walk:- A finite alternating sequence of vertices and edges (no repetition of edge allowed) beginning and ending with vertices such that each edge is incident with the vertices preceding and following it, is called a walk or edge train or chain. i.e., no edge appears more than once in a walk.

Eg:-



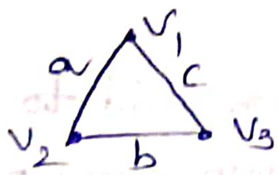
(G)

Here $V_1 a V_2 b V_3 d V_4 e V_2 f V_5$ is a walk

Note:- 1.) the vertices with which a walk begins and ends are called terminal vertices or terminal points.

2.) A walk is said to be closed walk if the terminal points are same.

Eg:-



$V_1 c V_3 b V_2 a V_1$ is a closed walk

3.) A walk which is not closed is called an open walk.

In the above Eg $V_1 c V_3 b V_2$ is a open walk.

Path:- An open walk in which no vertex appears more than once is called a path or simple path or elementary path.

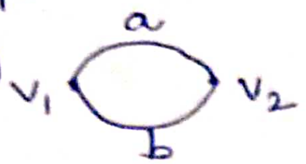
Note:- 1.) In the graph (G) the path $V_1 a V_2 b V_3 d V_4$ is of length 3.

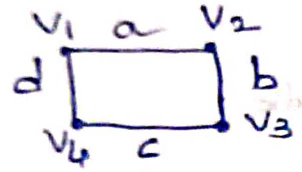
2.) An edge that is not a self-loop is a path of length 1.

3.) A self loop is a walk but not a path.

Circuit:- A path of length greater-than 1 with no repeated edges and whose end vertices are equal is called a circuit. A circuit may have repeated vertices other-than the end vertices.

Cycle:- A cycle is a circuit with no other repeated vertices except its end vertices.

Eg:-1)  $v_1 b v_2 a v_1$ is a circuit

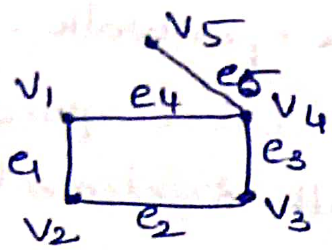
2)  $v_1 a v_2 b v_3 c v_4 d v_1$ is a circuit

Every vertex is a circuit is of degree 2

Every self loop is a circuit but every circuit is not a self loop.

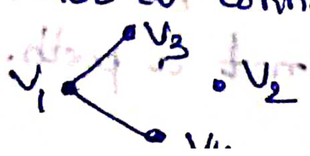
Connected graph:- A graph G is said to be connected if there is atleast one edge between every pair of vertices in G.

Eg:-



$v_1 e_1 v_2 e_2 v_3 e_3 v_4 e_5 v_5$ is a path

Disconnected graph:- A graph G is said to be disconnected graph if it is not a connected graph.



formation of components :- If G is a connected graph, then G contains only one connected component and it is equal to G. If G is a disconnected graph. Consider a vertex v in G which is not joined by any path. The vertex v and all the vertices of G that are joined by some paths to v together with all the edges incident on them form a component G_1. similarly consider the vertex u which is not in G_1 and incident with the edges on them to form G_2. Continue this procedure to find the components.

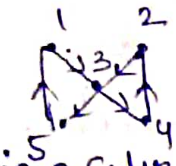
Note:- 1.) If G is connected, then G is the connected component.

2.) A disconnected graph G consists of two or more connected components.

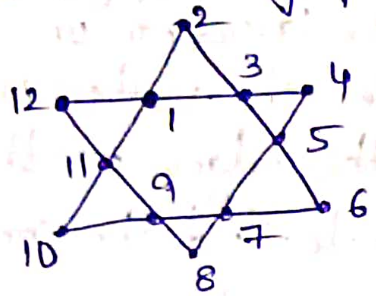
Euler line:- Let G be a graph. A closed walk running through every edge of the graph G exactly once is called an Euler line. or Euler circuit

Euler graph:- A graph 'G' that contains Euler line is called an Euler graph.

Eg:-

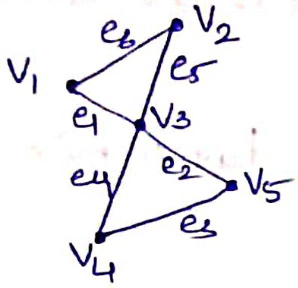


is a Euler graph



1 2 3 4 5 6 7 8 9 10 11 12 1 is Euler line

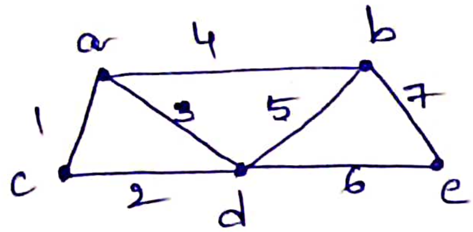
is not Euler graph



$V_1 e_1 V_3 e_2 V_5 e_3 V_4 e_4 V_3 e_5 V_2 e_6 V_1$ is ~~not~~ a Euler line

Unicursal line:- Let G be a graph then an open walk running through all edges of the graph 'G' exactly once is called an unicursal line

Eg:-

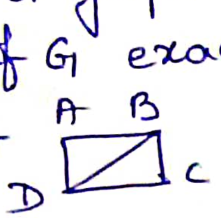


a1c2d3a4b5d6e7b

Unicursal graph:- A connected graph that contains unicursal line is called a unicursal graph.

Euler path:- A path of graph G is called Euler path if it includes each edge of G exactly once & it contains every edge of G

Eg:-



Euler path b/n B and D is B-D-C-B-A-D.

Euler theorem:-

A given connected graph G is an Euler graph iff if all vertices of G are of even degree.

Proof:- i) Suppose G is an Euler graph

∴ G contains an Euler line

In tracing this walk everytime we meet a vertex V it goes through 'two' new edges.

By using one edge which we entered, another edge which we are going to leave.

From the first vertex, we "exited" and we can "enter" into that vertex at the last.

∴ Every vertex in a graph G is of even degree

ii) Suppose all vertices of G are of even degree.

Construct a Euler line starting from vertex V.

∵ Every vertex is of even degree,

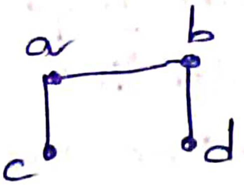
we can enter into every vertex and exit from the same vertex.

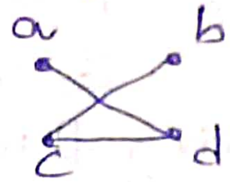
Tracing continues until reaching the vertex V.

∴ G contains an Euler line.

Operations on graphs:-

Complement of a graph:- The complement of a simple graph $G=(V,E)$ is a graph $\bar{G}=(V,\bar{E})$, where $\bar{E}=\{\{v_i,v_j\} / \{v_i,v_j\} \notin E\}$

Eg:- If $G =$ 

then $\bar{G} =$ 

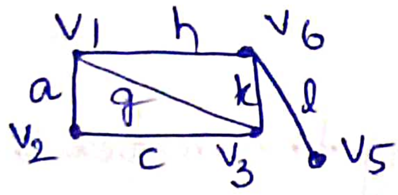
Note:- 1) If G is a graph with 'n' vertices then $\bar{G}$ is obtained from k_n by simply deleting the edges of G.

2) If G has 'n' vertices and 'p' edges then $\bar{G}$ has 'n' vertices and $\frac{n(n-1)}{2} - p$ edges

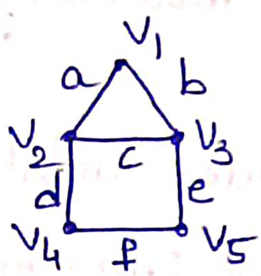
3) Let G_1 and G_2 be simple graphs, then G_1 and G_2 are isomorphic iff $\bar{G_1}$ and $\bar{G_2}$ are isomorphic.

Union of graphs:- Let $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ be two graphs. Then the union of G_1 and G_2 is defined as $G_1 \cup G_2 = (V,E)$ where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$.

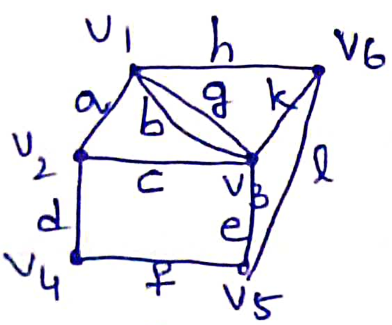
Eg:-



G1



G2

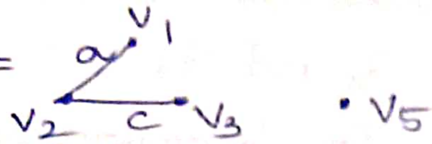


G

clearly $G = G_1 \cup G_2$

Intersection of graphs:- Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. Then the intersection of G_1 and G_2 is defined as $G_1 \cap G_2 = (V, E)$ where $V = V_1 \cap V_2$ & $E = E_1 \cap E_2$.

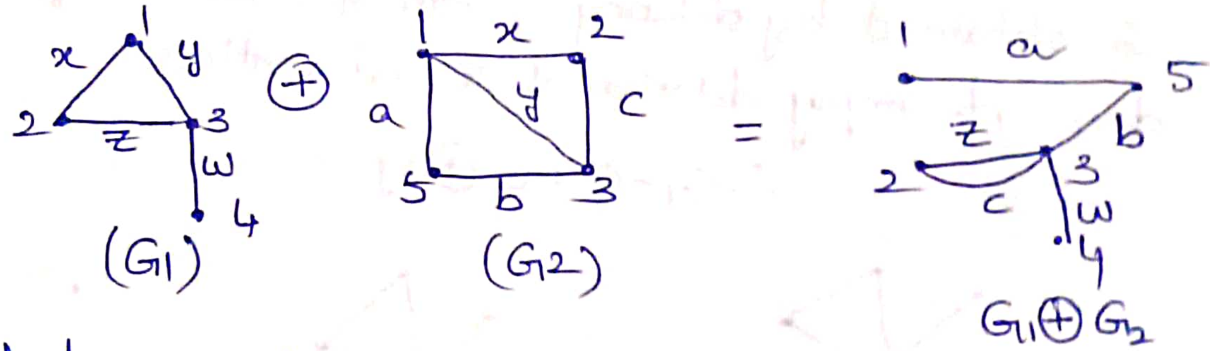
Eg:- for the above example, $G_1 \cap G_2 =$



Ring sum of graphs:- Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. Then the ring sum of two graphs G_1 and G_2 is defined as $G_1 \oplus G_2 = (V, E)$, where $V = V_1 \cup V_2$ and $E = (E_1 \cup E_2) - (E_1 \cap E_2)$

= edges that either in G_1 or G_2 but not both.

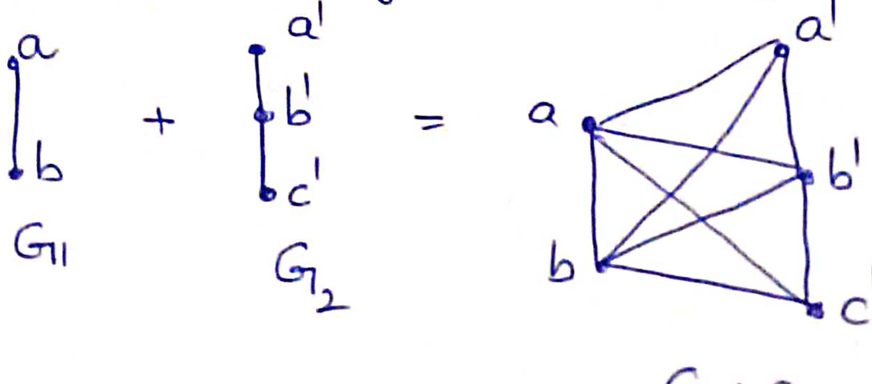
Eg:-



Sum of two graphs:-

$V(G_1 + G_2) = V(G_1) + V(G_2)$

$E(G_1 + G_2) =$ Edges which are in G_1 and G_2 are obtained by joining each vertex of G_1 to each vertex of G_2



$G_1 + G_2$

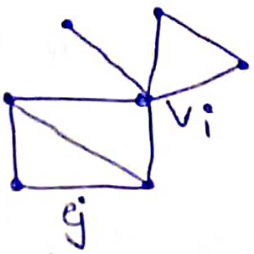
Decomposition:- A graph G is said to have decomposed into two subgraphs G_1 and G_2 if $G_1 \cup G_2 = G$ and $G_1 \cap G_2 = \phi$ i.e., some of the vertices may occur both in G_1 and G_2.

In decomposition, the isolated vertices are disregarded.

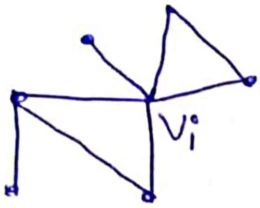
Deletion:- If v_i is a vertex in graph G. Then $G - v_i$ denotes a subgraph of G obtained by deleting v_i from G. Deletion of a vertex always implies the deletion of all edges incident on that vertex.

If e_j is an edge in G, then $G - e_j$ is a subgraph of G obtained by deleting e_j from G. Deletion of an edge does not imply deletion of its end vertices

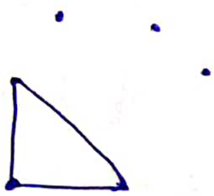
$$\therefore G - e_j = G \oplus e_j$$



(G)

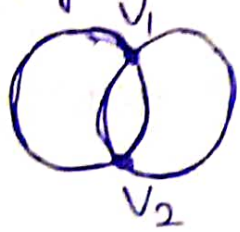


$(G - e_j)$

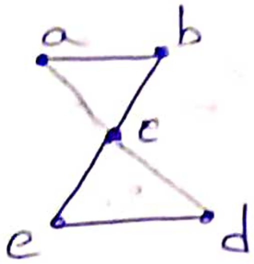


$(G - v_i)$

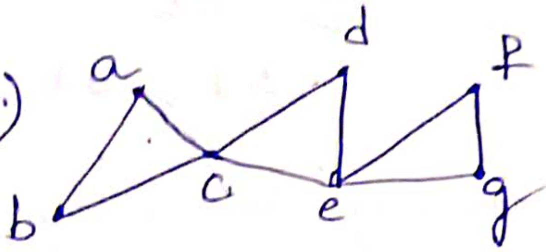
Arbitrarily traceable curve:- Let G be a graph and v be a vertex. The graph is said to be arbitrarily traceable graph from the vertex v if an Euler line is always obtained when one follows any walk from the vertex v according to the single rule that whenever one arrives at a vertex one shall select any edge. i.e, if every walk with initial vertex v can be extended to an Euler line of G.
& Every cycle of a graph must pass through the vertex v

Eg:-1) 

is arbitrarily traceable graph from V_1 & V_2

2.) 

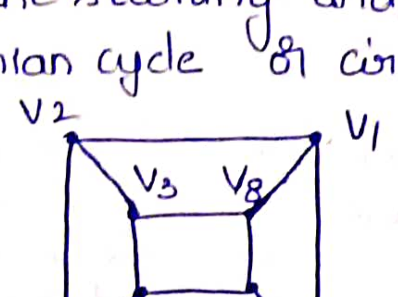
is arbitrarily traceable graph from c.

3.) 

is not arbitrarily traceable from any vertex.
since from any vertex every cycle does not have a common vertex.

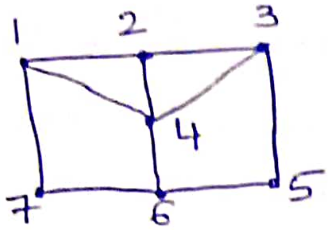
Note:- An Euler graph G is arbitrarily traceable from vertex V in G iff every circuit in G contains V. (passes through)

* **Hamiltonian cycle**:- In a connected graph a closed walk that visits every vertex of the graph G exactly once except the starting and ending vertices. It is called Hamiltonian cycle or circuit.

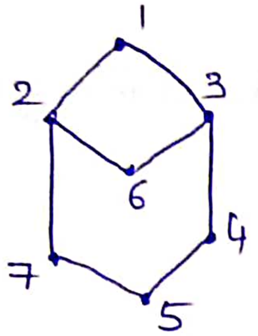


$V_1V_2V_3V_4V_5V_6V_7V_8V_1$

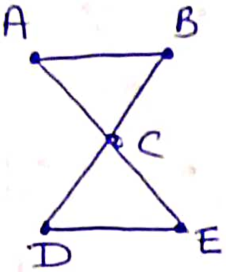
* **Hamiltonian graph**:- A graph containing a Hamiltonian circuits is called a Hamiltonian graph.



* **Hamiltonian Path**:- In a connected graph an open walk that visits every vertex of the graph G exactly once. But it contains an open walk.



1-2-6-3-4-5-7

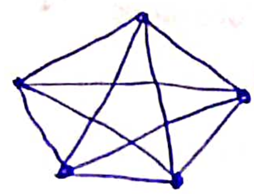


A-B-C-D-E

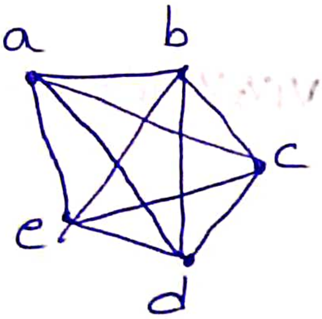
(or) If we remove any one edge from a Hamilton circuit then we left with a path. This path is called a Hamiltonian path.

Problem

1.) Identify five different Hamiltonian cycles of the following undirected graph



Sol:-



five different Hamiltonian cycles are
a-b-c-d-e-a
a-c-d-b-e-a
a-e-d-c-b-a
a-d-c-e-b-a
a-d-e-c-b-a

Note

1.) Clearly by definition, a Hamiltonian path in a graph G traverses through every vertex of G.

2.) If a graph G that contains a Hamiltonian circuit then G contains a Hamiltonian path.

3.) There exists graph with Hamiltonian paths that have no Hamiltonian circuits

4.) The length of a Hamiltonian path in connected graph of n vertices is 'n-1'.

5.) In a Hamiltonian circuit or path every vertex appears exactly once. Hence Hamiltonian circuit cannot include a self-loop or a set of parallel edges.

Traveling salesman problem :- This problem is related to Hamiltonian circuit.

Problem: A salesman required to visit a number of cities during his trip. Given the distances between the cities, In what order should the salesman travels so as to visit every city precisely once and return to his home city, with the minimum mileage traveled?

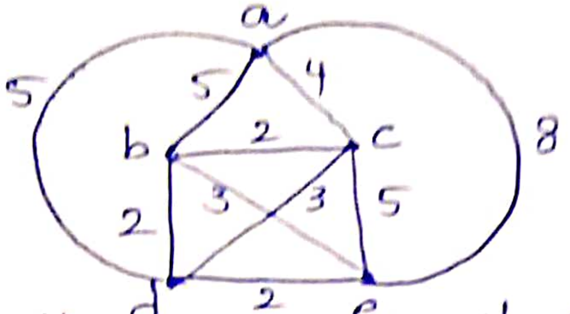
Solution:- Represent the cities by vertices and the roads between them by edges. Then we get a graph. i) In this graph for every edge 'e' there corresponds a real number $w(e)$. Such graph is called a weighted graph. Here $w(e)$ is called as the weight of the edge 'e'.

ii) If each of the cities has a road to every other city, we have a complete weighted graph. The graph has numerous Hamiltonian circuits and we are to select the Hamiltonian circuit that has the smallest sum of distances.

iii) The no. of different Hamiltonian circuits in a complete graph of 'n' vertices is $\frac{(n-1)!}{2}$

iv.) First we list all the $\frac{(n-1)!}{2}$, Hamiltonian circuits that are possible in the given graph. Next calculate the distance traveled on each of these Hamiltonian circuits. Then select the Hamiltonian circuit with the least distance. This provid the solution for the travelling salesman problem.

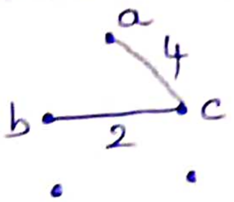
1.) A travelling salesman wants to visit each 'n' cities exactly once and returns to its starting point with minimum distance starting at vertex 'a' and 'd'



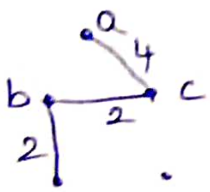
Consider Hamiltonian circuit acbdea

<u>Sol</u>:- <u>step①</u> Select minimum distance edge path {a,c}

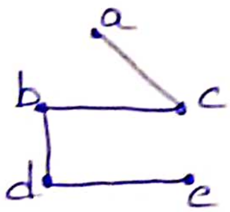
<u>step②</u> Select minimum distance from 'c' path = {a,c,b}



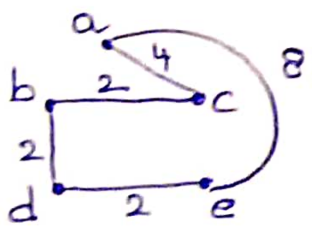
<u>step③</u> Path = {a,c,b}



<u>step④</u> Path = {a,c,b,d,e}



<u>step⑤</u> Path = {a,c,b,d,e,a}



∴ Total ~~minimum~~ distance = 18

Spanning Tree

A spanning tree is a tree that connects all the vertices of a graph with the minimum possible no.of edges. Thus a spanning tree is always defined for a graph and it is always a subset of that graph.

Ex

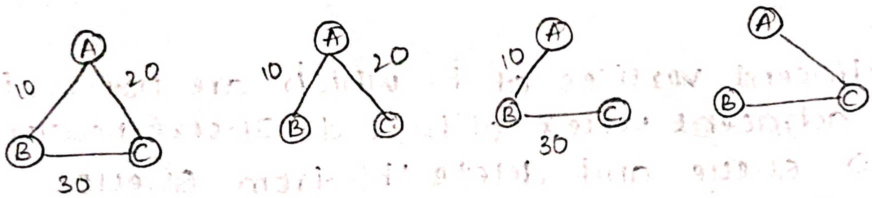
connected graph G

spanning Tree

Minimum spanning Tree

A minimum spanning tree is a subset of edges of a connected weighted undirected graph that connects all the vertices together with the minimum possible Total edge weight

Ex



BFS (Breadth First search)

BFS is used for searching spanning tree of the graph. We use queue data structure with maximum size of total number of vertices in the graph to implement BFS

BFS Algorithm

step 1: Define a Queue with maximum size of total number of vertices in the graph

step 2: select any vertex as starting vertex for traversal. Visit that vertex and insert it into the queue

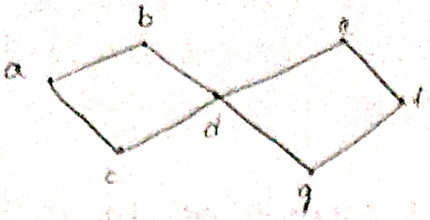
step 3: visit all the unvisited adjacent vertices of the vertex which is at front of the queue and insert them into the queue

step 4: when there is no new vertex to visit from the vertex at front of the queue then delete that vertex from queue

step 5: Repeat step 3 and step 4 until queue becomes empty

step 6: when queue becomes empty, final we get spanning tree by removing unused edges from the graph

EX:



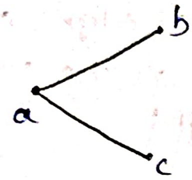
By using a BFS algorithm to find a spanning tree of the following graph G

Take a queue with size = no. of vertices = 7

step1: select the vertex 'a' as the starting vertex and insert 'a' into queue

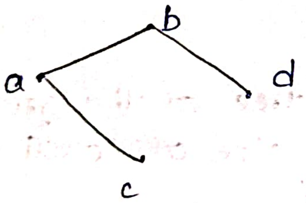
a						

step2: visit all adjacent vertices of 'a' which are not visited. Here the adjacent vertices of 'a' are b and c. Insert newly visited vertices b and c into queue and delete 'a' from queue



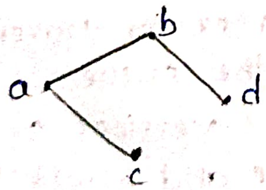
$\cancel{a}$	b	c				

step3: visit all adjacent vertices of b which are not visited. Here the unvisited adjacent vertex of b is d. Insert newly visited vertex d into queue and delete 'b' from queue



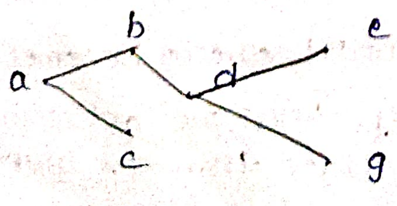
$\cancel{a}$	$\cancel{b}$	c	d			

step4: visit all unvisited adjacent vertices of c. Here unvisited vertex of c is d. If we visit d from c then cycle will be formed, so we cannot visit d from c and also delete c from queue



$\cancel{a}$	$\cancel{b}$	$\cancel{c}$	d			

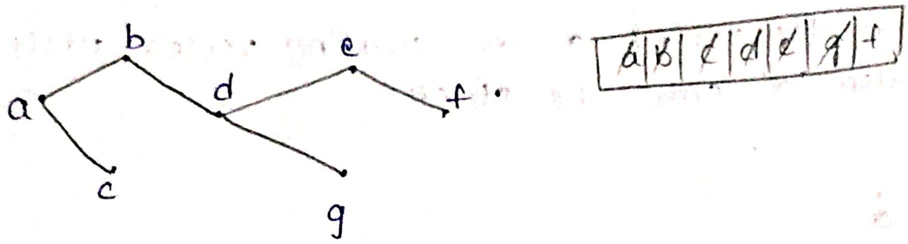
step5: visit all unvisited vertices of d. Here unvisited vertices of d are e and g. Insert e and g into queue and delete d from queue



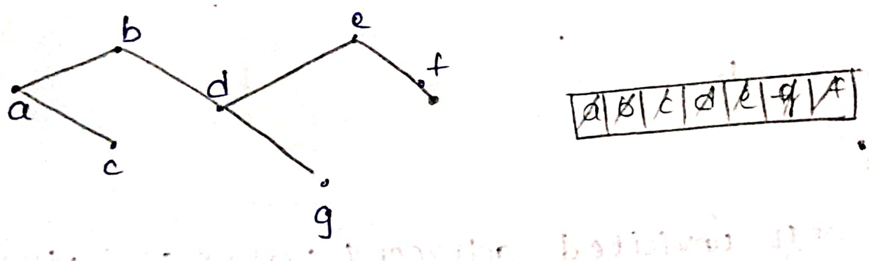
$\cancel{a}$	$\cancel{b}$	$\cancel{c}$	$\cancel{d}$	e	g	

step6: Visit all unvisited adjacent vertices of e. Here unvisited vertex of e is f, insert newly visited vertex f into queue and delete e from Queue

step7: Visit all unvisited adjacent vertices of g. Here unvisited adjacent vertex of g is f. If we visit f from g then cycle will be formed. so we cannot visit f from g and also delete g from Queue



step8: visit all unvisited adjacent vertices of f. Here no unvisited vertex of f, so we delete f from Queue



DFS (Depth First search)

DFS is used for finding spanning of the graph. We use stack data structure with maximum size of total number of vertices in the graph to implement DFS

DFS algorithm

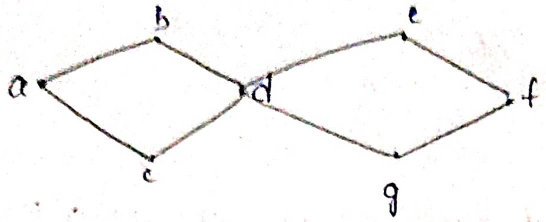
step1: Define a stack with maximum size of total number of vertices of the graph

step2: select any vertex as starting point for traversal. visit that vertex and push it on to stack

step3: Visit any one of the unvisited adjacent vertex of the vertex which is at top of the stack and push it on to the stack

step4: Repeat step 3, until there is no new vertex to visit from the vertex which is at top of the stack step5: When there is no new vertex to visit then use back tracking and pop(delete) one vertex from the stack

step 6: Repeat steps 3, 4, and 5 until stack becomes empty

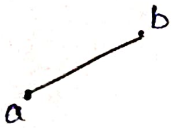


By using DFS algorithm to find a spanning tree of the following graph

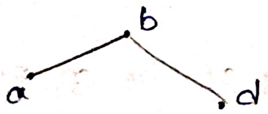
Take stack with size = no. of vertices = 7

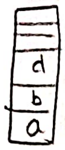
step 1: select the vertex a as starting vertex. visit the vertex 'a' and push 'a' onto the stack

step 2: visit any unvisited adjacent vertex of 'a'. Here the adjacent vertices of 'a' are b and c. We have to visit either b or c visit b and push b onto the stack

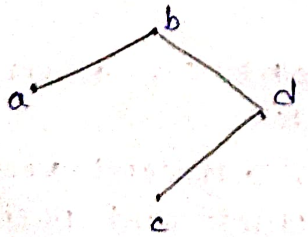


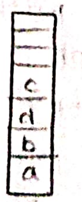
step3: visit any unvisited adjacent vertex of b. Here the unvisited adjacent vertex of b is d visit d and push d onto the stack



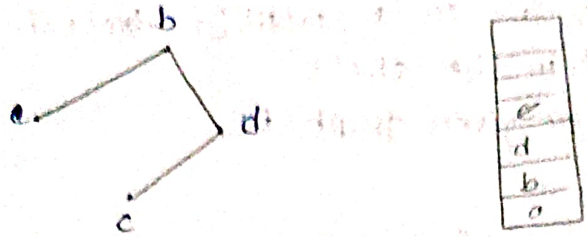


step 4: visit any unvisited adjacent vertex of d. Here unvisited adjacent vertices of d are c, e, g. We visit either g or c or e visit c and push c onto the stack

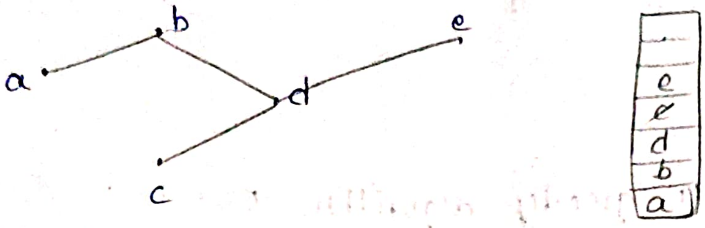




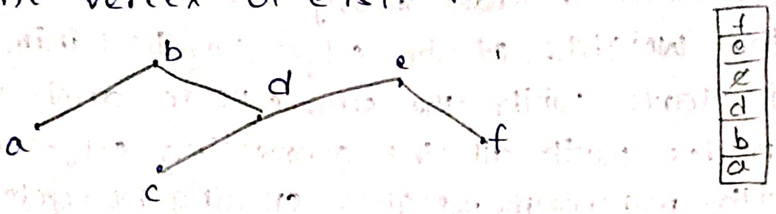
step5: visit any unvisited adjacent vertex of c. Here unvisited adjacent vertex of c is 'a'. If we visit 'a' from c then cycle will be formed. so we cannot visit a from c Now c will be deleted from stack and we use back track to d.



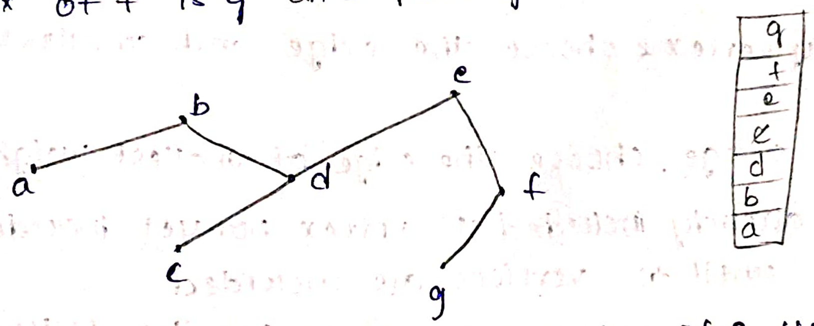
step 6: After back-track to d, visit any unvisited adjacent vertex of d. Here unvisited adjacent vertex of d are g and e. We visit either g or e. visit e and push e onto the stack



step 7: visit any unvisited adjacent vertex of e. Here unvisited adjacent vertex of e is f. visit f and push f onto the stack



step 8: visit any unvisited adjacent vertex of f. Here unvisited vertex of f is g and push g onto the stack



step 9 : visit any unvisited adjacent vertex of g. Here unvisited adjacent vertex of g is d. If we visit d from g then cycle will be formed, so we cannot visit d from g. Now g will be deleted from stack

Now back track to f. There is no any unvisited adjacent vertex of f, so f will be deleted from stack

Now back track to e. There is no any unvisited adjacent vertex of e, so e will be deleted from stack

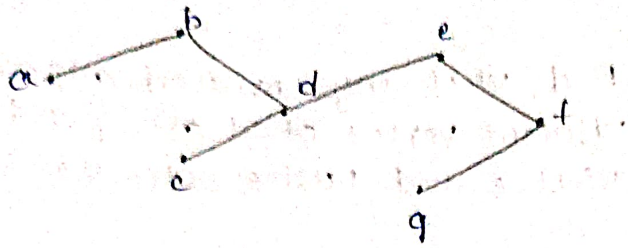
Now back track to d. Here unvisited vertex of d is g. If we visit g from d then cycle will be formed. so we cannot visit g. Now d will be deleted from stack

Now backtrack to b. There is no unvisited adjacent vertex of b, so b will be deleted from the stack

Now backtrack to 'a' Here unvisited adjacent vertex of 'a' is c. If we visit c from 'a' then cycle will be formed.

so we cannot visit c from 'a'. so 'a' will be deleted from the stack
All the vertices are deleted in the stack

∴ The spanning tree of given graph is



Prim's algorithm

Prim's algorithm is a greedy algorithm that is used to find the minimum spanning tree from a graph. Prim's algorithm finds the subset of edges that includes every vertex of the graph, such that the sum of the weights of the edges can be minimized.

Prim's algorithm starts with the single node and explores all the adjacent nodes with all the connecting edges at every step. The edges with minimum weights crossing no cycles in the graph get selected.

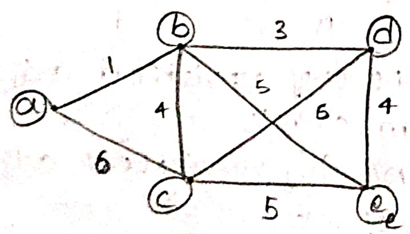
Algorithm

Step1: select any vertex & choose the edge and smallest weight from G

Step2 : At each stage, choose the edge of smallest weight joining a vertex already included to vertex not yet included

Step 3: Continue until all vertices are included

To find the minimal spanning tree for the following weighted using prims algorithm



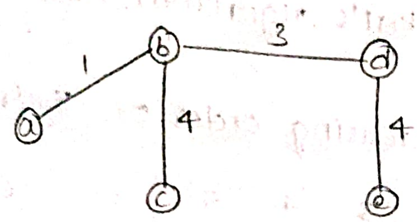
Step1: select any vertex from the given graph. Now, consider the vertex a

Step 2: From the vertex a consider the adjacent vertex with the minimum weighted edge. such that they do not form any loops or cycle

Now draw an edge from a to b

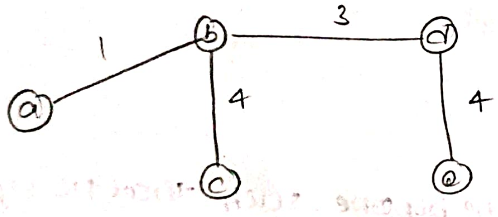
step3: Now, next consider the minimum weighted edge from the vertex a and b and the next minimum weighted edge is b to d

step 4. Next, consider the minimum weighted edge from the vertex a, b and d the next minimum weighted edge is b to c and c to d..



step5. Next, consider the minimum weighted edge from the verted i.e c-e if we add this edge it forms a loop so we cannot add this edge.

By using the prim's algorithm the minimum spanning tree is



The weight of the spanning tree is

$1+3+4+4=12$

Kruskal's algorithm

Kruskal's algorithm is the concept that is introduced in the Graph theory of discrete mathematics It is used to discover the shortest path between two points in a connected weighted graph

Algorithm

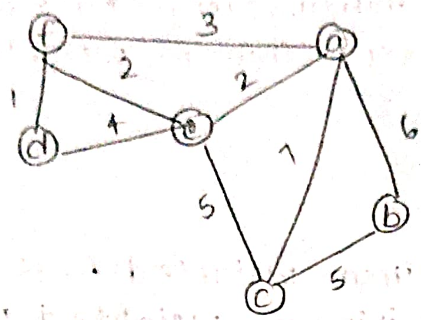
step1: Remove all loops and parallel edges

step2: Arrange the edges of G in increasing order of their weights

step3: choose an edge of minimum weight If there is more than one edge of minimum weight then choose arbitrarily one of these edges

step 4: At each stage choose the not selected edge of minimum weight whose inclusion will not create a cycle

step 5: If G has n vertices then stop after n-1 edges have been choosen. otherwise repeat step 4

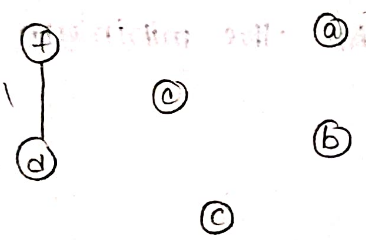


By the above graph using kruskal's algorithm find the minimum spanning tree

step 1 List the edges in increasing order of their weight

Weight	1	2	2	3	4	5	5	6	7
Edge	f-d	f-e	e-a	f-a	d-e	e-c	c-b	a-b	a-c

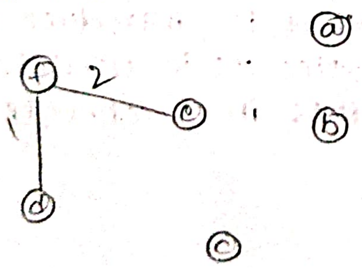
step 2 select the minimum weighted edge f-d



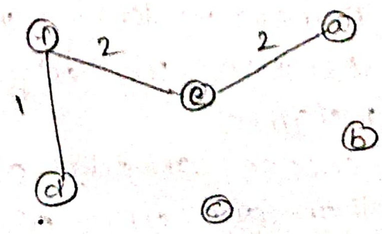
Next add the above edges one by one such that no cycle formed

step 3: Add the next minimum weighted edge. Here two edges having the weight 2. so we can add them arbitarily such that they do not form a cycle

Add f-e

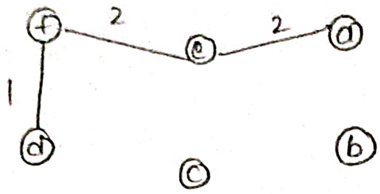


Now add an edge e-a



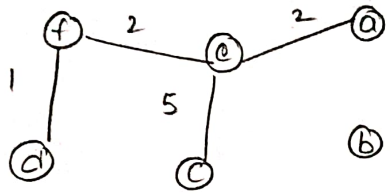
step 4: Next minimum weighted edge is f-a. But if we add this edge then cycle will be formed. so we cannot add an edge f-a

step5: Next minimum weighted edge is d-e But if we add this edge then cycle will be formed so we cannot add an edge d-e

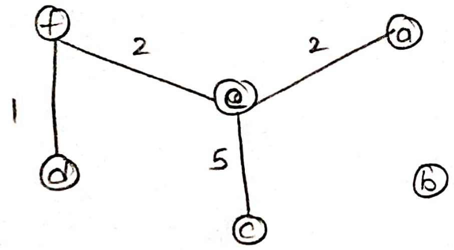


step6: Next minimum weighted edges are e-c and c-b. Add these edges one after another such that they do not form a cycle

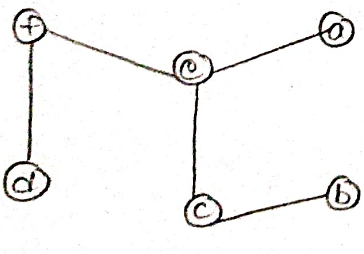
Now add an edge e-c



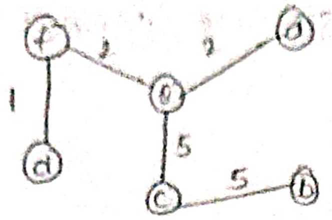
Now add an edge c-b



step7: Add next minimum weighted edge is a-b we cannot add this edge because cycle will be formed



step8: Add next minimum weighted edge a-c we cannot add this edge because cycle will be formed



this is minimum spanning tree weight of minimum spanning tree . $1+2+2+5+5=15$