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Rajampet, Annamayya District, A.P – 516126, INDIA

CIVIL ENGINEERING

Lecture Notes
on
Design of Pre-stressed Concrete Members

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CIVIL ENGINEERING

Design of Pre-stressed Concrete Members

UNIT-1

Prestressed concrete :- Unit - 1

Code books:-

IS 1343 : 2012

IS 456 : 2000

IS 456 : 2025 (Draft version)

IS 875 : (part 1-5)

SP 24 : 1983

SP 34 : 1987 (Hand book on concrete reinforcement detailing)

IRC 112 : 2020 - used for Roads, Bridges

IRC 18 : 200 - Design criteria for prestressed concrete, road bridges.

IRC SP 65 : 2018 - Guidelines for design and construction for precast and prestressed.

IRC concrete Bridge rules 1947 - (3rd report 2014).

Miscellaneous Codes:-

IS 2090 - 1983 : (high tensile steel bars) used in concrete

IS 2193 - 1986 : (specification for precast prestressed concrete steel lighting)

IS 3370 - 2021 : Code of practice for concrete structures for storage of liquids part-3 for prestressed concrete.

IS 6003 - 2010 : specification for indente prestressed concrete.

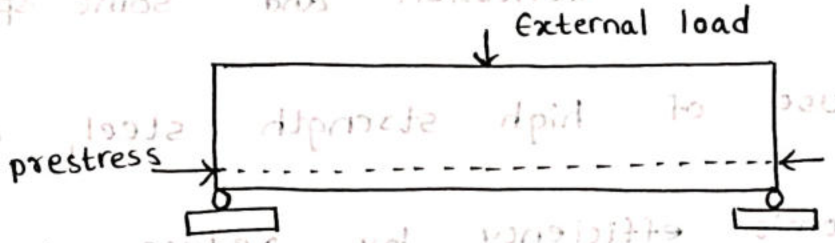
IS 6006 - 2014 : specification for the uncoated, stress Relieved stand for prestressed concrete.

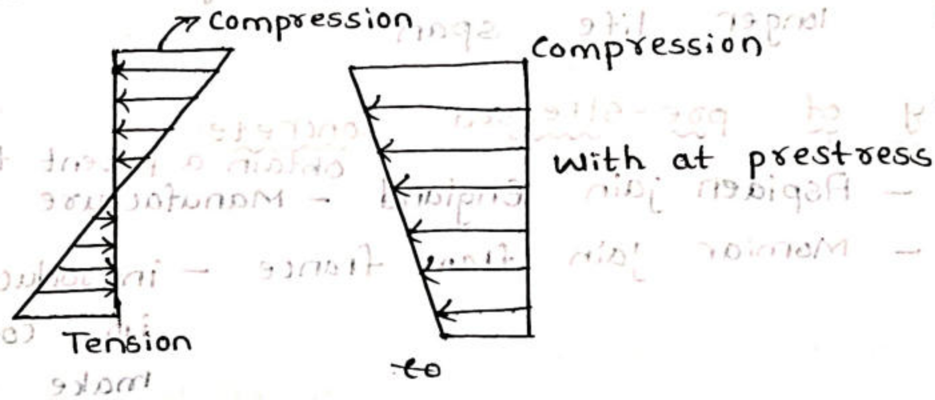
History, Advantages, Applications of prestressed:-

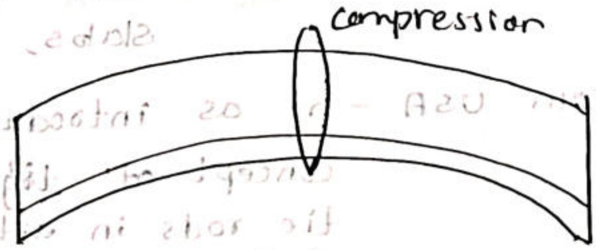
History:-

Prestressed concrete:-

prestressing is a process of preloading the structures before application of design loads in such a way so has to improve it's general perfomance.







Concepts of pre-stressing :-

Using a compressive force they their, support their own weight plus significant support imposed forces represented by the book on top

Objectives of pre-stressing :-

1. Control or Eliminates tensile stresses in the concrete (cracking) atleast upto service load levels
2. control or Eliminate deflection and some specific load level
3. Allow the use of high strength steel and Concrete
4. Achieve economic efficiency by reduce maitenance and longer life span

History of pre-stressed concrete:.

1824 – Joseph Aspiden jain England – obtain a patent for the Manufacture of OPC

1857 – Morniar jain fram france – introduce steel wires in concrete to make flower pots, Slabs, Archs, pipes

1886 – Jackson Ph. from USA – he as introdued the concept of tighting steel tie rods in ~~art~~ concret arches and artificial stone.

1925 - Dill Rh foom USA – use high strength and unbonded steel rod the rods were tensioned and anchored after hardening of the concrete.

1926 - E. Freyssinet from France – Father of pre-stressed concrete

– use high tensile steel wires

$f_y = 1725$ mpa.

he developed conical wedges for end anchorage

he develop double acting jacks

1928 — E. Floyer from germany develope long line pre tension ing method.

1940 - Magnel. G from belgium he develop anchoring hangering systems for post tensioning from flat wedges.

1897 - 1988 — Ull-rich frenchter walder from germany he develop balanced cantilever construction method for bridges. These balanced cantilever construction this commonly used in construction method.

prestressed concrete:

Bridge type:-

1. Assam - pholorp babarma in (pezapur)
2. Rameshwaram
3. chinab bridge
4. Okha to bedcolara bridge

prestage concrete box glandar bridge sybortic design -usiging segmental construction when precast statements are Joint.

(5) span configration

multiple span according river using pre-cast plastring gladers to resist heavy flood forces and sesmic activitys.

Need for prestressing

Principle of pre-stressing:-

1. pre-stressing is a method in which compression force is applied to Concrete
2. The effect of pre-stressing is to reduce the tensile stress in the section to the point till the tensile stress below cracking stress Thus, the concrete does not crack.
3. It is then possible to treate Concrete as a elastic material.
4. The concrete can be visualize to have two compressive force
   a) Internal prestressing force
   b) External forces (Dead load & live load etc...)
5. These two forces must countract each other.

Advantages of pre-stressed concrete:- in compared with RCC:-

1. It Needs about 1/3 Quantity of steel & 1/4 Quantity of concrete. as compared to the RCC.
2. lighter and slender member can be used
3. Factory made members are possible in pre tensioning
4. Members like Railway sleepers, electric poles, boundary pillars, Contry girders can be made.
5. long span structures are possible so that saveing of weight is significant in thus become economical.
6. pre-stressed members are tested before use
7. Dead loads are get counter balanced by eccentric pre-stresseing
8. Cracks can be eliminated in tension zone
9. It has high fattiague resistance
10. It has high ability to resist impact &
11. It has high live load capacity carrying
12. Use the entire section to resist the load
13. It is free from cracks from service loads.

and enables entire section to take part in resisting moment.

14. It take full advantages of high strength concrete and high strength steel.

15. It Needs less materials

16. It it In smaller & lighter section

17. It as better corrosion resistance

18. It is good for water tanks and Neucler plants.

19. It is very effective for deflection control.

20. It as better shear resistance.

21. pre-stressed concrete bridges can be designed without any tensile stress under service loads

Disadvantages of pre-stressing concrete:-

1. Initial cost of equipment is very high
2. Required skilled supervision
3. Very long slender members are difficult to transport.
4. Requires high tensile steel which is 2.5 to 3.5 times costler than mild steel
5. Pre-stressed concrete is less fiber resistance
6. Require skilled builders and experienced engineers
7. It required high strength concrete
8. It Need higher quality material
9. More complex technically
10. It is more expensive
12. It is harder to recycle.
13. Availability of experienced engineer are less.
14. Required complicated formwork

Applications:-

1. Bridges
2. Slabs in building
3. Water tanks
4. Concrete piples
5. Thinshell structures
6. Offshour platforms
7. Neuclear powerplants
8. Repair & Rehabilitation

Pre-stressing steel:-

Steel to be use on pre-stressing must have high tensile strength and good surface condition and good bonding with concrete.

The steel used for prestressing are available in 3 forms:-

1. Single wires (Tendons)
2. Group of wires also termed as [strands (or) Cables]
3. Alloy steel round bars

1. Single wires (Tendons):-

These are hot drawn high tensile steel wire of diameter rangeing from [1.5 mm to 8 mm] and having tensile steel and other properties as specified in following projects may be use.

2. Wire strands (cables):-

These are harddrawn steel wires may be used in the form of cables.

The diameter of strand cable varies [7mm to 17

3. Round bars:-

These are high tensile which ar used in pre-stressing system it is available in [10 mm - 32 mm] diameter.

Ultimate tensile strength

Colddraw high tensile steel wires used for pr shall conform to the specification shown below

Dia in (mm)	1.5 mm	2. mm	2.5	3	4	5	7	8
min Ultimate strength	2350 N/mm^2	2200 N/mm^2	2050 N/mm^2	1900 N/mm^2	1750 N/mm^2	1600 N/mm^2	1500 N/mm^2	1400 N/mm^2

Pre-stressing equipments:-

The equipments required for pre-stressing are

1. Tensioning equipment is required - Hydraulic

2. Temporary griping device – Double cone, Wedge.
3. Releasing device – For gradual and unifore release
4. End enchrage – strong enough to hold stress.

Methods of pre-stressing:-

1. pre-tensioning steel is tensioned before the casting of concrete
2. post-tensioning steel is tensioned after the casting of concrete

pre-tensioning method is best suitable for Factory production where as post-tensioning method is suitable for both castining and precast

Systems of pre-stress:-

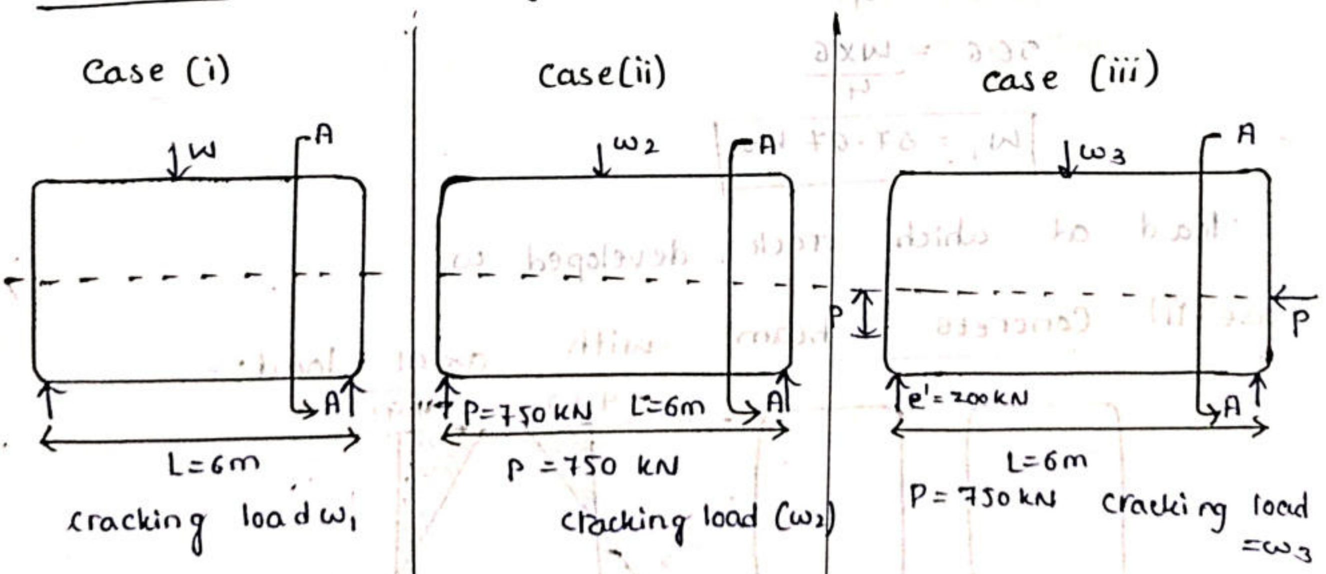
systems depending upon their paytents and end anchorage system in th

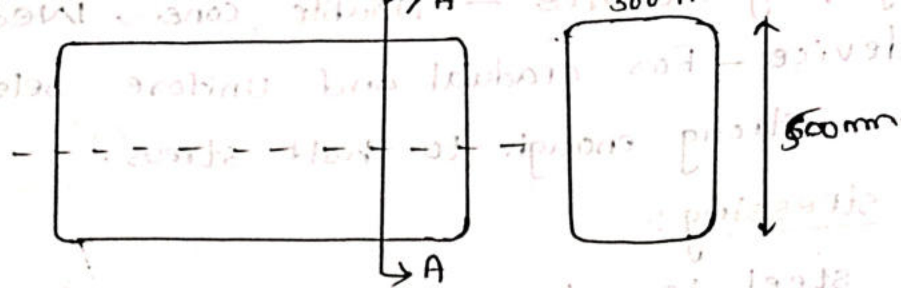
a) Frayssinet system
b) Magnal blaton system
c) lemc call system

~~Principle of prestressing~~:-

~~1. pre-stressing is a method in which~~

Need for pre-casting:-





$f_{ck} = 0.7 \sqrt{f_{ck}}$

$= 0.7 \sqrt{1.25 \times f_c - 1.65\,\sigma}$

$= 0.7 \sqrt{1.25 \times 40 - 1.65 \times 5.0}$

$\boxed{f_{ck} = 4.52 \text{ Mpa}}$

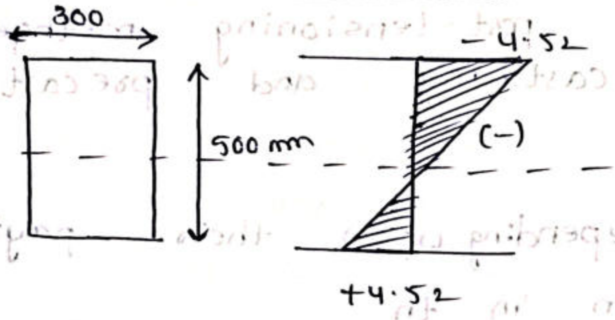
Case-②

$f_{cr} = 0.7 \times \sqrt{f_{ck}}$

$= 0.7 \times \sqrt{1.25 \times f_{ck}' - 1.65\ldots}$

$= 0.7 \times \sqrt{1.25 \times 40 - 1.65 \times 5}$

$\boxed{f_{ck} = 4.52 \text{ Mpa}}$



As per body equation

$$\frac{\sigma}{y} = \frac{M}{I} \Rightarrow M = \sigma \times Z$$

$$= 4.52 \times 7$$

$$= 4.5 \times \frac{300 \times 500^2 \times 10^{-6}}{6}$$

$$M = 56.5 \text{ kN}$$

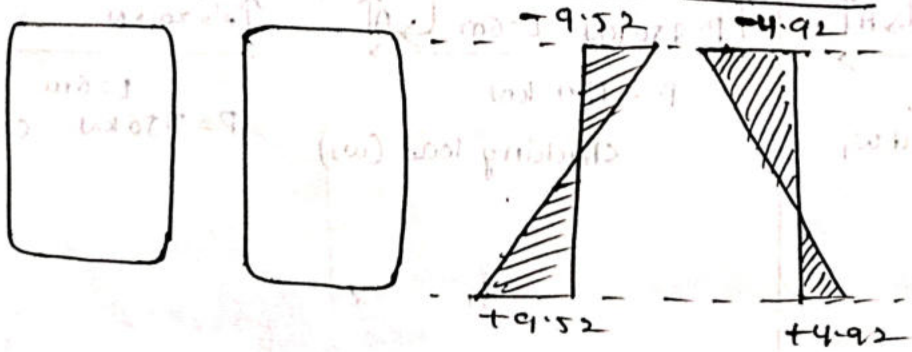
$$M = \frac{WL}{4}$$

$$56.6 = \frac{W \times 6}{4}$$

$$\boxed{W_1 = 37.67 \text{ kN}}$$

(load at which crack developed ω_1

Case (ii) Concrete beam with axial load :-



$p_r \quad \frac{P}{A} = \frac{750 \times 10^3}{300 \times 500} = 5 \text{ Mpa}$

1. $\boxed{f_{cr} = 4.52 \text{ Mpa}}$ same as (case i)

2.

3. $\frac{\sigma}{y} = \frac{M}{I} = M = \sigma \times Z$

$$M_{crack} = \frac{9.52 \times 300 \times 500^2 \times 10^{-6}}{6}$$

$$\boxed{M_{crack} = 119 \text{ kN}}$$

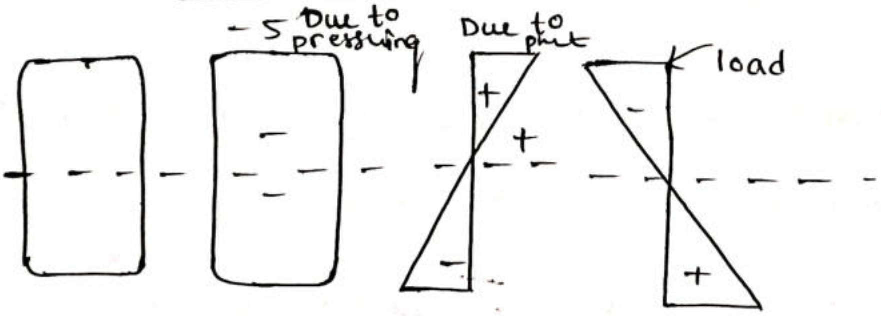
$m = \frac{WL}{4}$

$119 = \frac{W \times 6}{4}$

$\boxed{W_2 = 79.33 \text{ kN}}$ cracking load

[2.10 times] as increase as compare to case-1

Case (iii) :- concrete beam with eccentric pre-stressing :-



$\frac{P}{A} = 5 \text{ mpa.}$

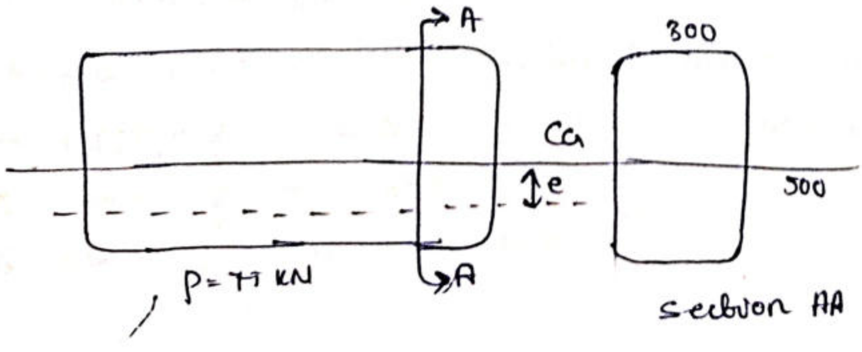
$M_p = p \times e$

$= 750 \times 0.2$

$= 150 \text{ kN}$

$= 5 - 72 + x = 4.52$

$\boxed{x = 21.52 \text{ mpa}}$



$$\frac{\sigma}{y} = \frac{M}{I} \Rightarrow M = \sigma \times Z$$

$$= \frac{300 \times 500^2 \times 10^{-6}}{6}$$

$M_{crack} = 269$ kN

$$m = \frac{\omega L}{4}$$

$$269 = \frac{\omega \times 6}{4}$$

$\boxed{\omega_3 = 179.33 \text{ kN}}$ 4.76 times increase as compare to Case-1 procedure followed to the pre-tensioning:-

1. Anchoring the tendons against the end abutments.
2. placing of Jacks
3. Apply the tension to the tendons
4. Cutting of concrete
5. Cutting of tendons.

post-tensioning:-

In post-tension the tendons are tensioned after the concrete had harden. commonly metal (or) plastic ducts are plased inside the concrete before Casting. After the concrete hardened before casting, After the tendons are placed inside the -
The ducts, stress are achored again concrete. Grout may be injected into the duct later.
These can be done either as pre-cast (or) Cast in place

Suitability (or) Advantages of post-tensioning methods:-

1. These is suitable for cast in situ (or) pre-cast members
2. loss of pre-stress is less [15 to 18%]
3. There is no limit of casting as the method can be applied at site also.

Disadvantages of post-tensioning method:-

1. These method is costly because of sheeting and gouting
2. Methods of post-tensioning:

1. casting of concrete
2. placement of tendons
3. placement of the anchoage blanks and jacks.
4. Applying tension to the tendons
5. Skeeting of the wedges.
6. Cutting the tendons

Differentiate between pre-stressing ... method:-

pre-stressing	post-tensioning
1. These method is best suitable for factory production under controled condition.	1. These method is suitable for both cast in situ and precast members.
2. loss of pre-stress is more (18 - 20%)	2. loss of pre-stress is less (15 - 18%)
3. size of member is restricted because large members are more difficult to transfer.	3. size of member is unrestricted, ∴ any size of member can be casted.
4. These method is economical.	4. These method is costly because of use of sheeting and grouting.
5. minimum grade of concrete to be used is M40	5. Minimum grade of concrete can to be used is M30.

Systems of pre-stressing:-

For pre-tensioning the following system are adopted:- Hoyer's system, Gifford udall system.

For post tensioning the following system are adopted

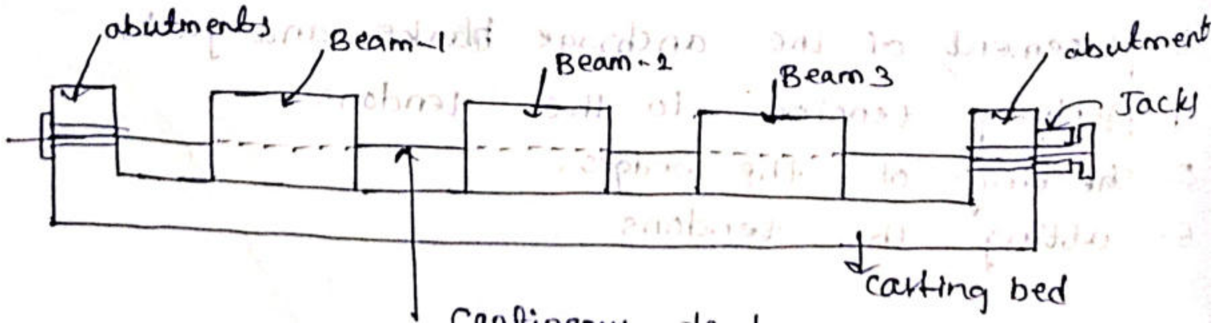
a) Frayssinet system

b) Magnal balton system

c) lemcall system

pre-tensioning system:-

Hoyer's system:-



hoyer's system (or) long line method is often in pre-tensioning.

1. It is a lock scale production
2. Two bulk mates (or) abutments independently anchor to the ground (or) provided several meters apart. (100 m)
3. Wires are streched between the bulk head, moulds.
4. Moulds are placed enclosing the wires
5. The concrete is Now poure so that a no. of beams can be produced in one line
6. After the concrete can harden the wires are release from bulk heads and are cut offs.
7. The pre-stress is transfer to the bond between tendons and concrete.

<u>Hoyer's effect:</u>-

1) After streching the tendons the diameter reduced from the original value due to the poison's effect.

2) when the pre-stress transfered after prestress of concrete the ends of the tendons sink in the concrete. The pre-stress at the ends of the tendons is zero.

3) The dia of the tendons regains its original value towards the ends over the transmission length.

4) The change of dia of the from the original value (at the end to the reduced value) after the transmission length, creates a width, effect in concrete.

5) This helps in the tranfer at pre-stress in trondons to concrete.

6) This is known as hoyer's effet.

Post-<u>tensioning</u> <u>system</u>:-

a) Frayssinet system:- It was introduced by a french engineer. frayssinet and it was the first method to be introduced. high strength steel wires 5 mm (or) 7mm diameter about 12 no's group into a cable. With the helical spring inside. spring keep proper spacing for the wires enters provides a channel which can be cement grouted it further resist to transfer the reaction to the concrete.

Cable is incerted in the duct

1. Anchrage device consist concrete cylinder with a concentric conical hole and poragation on its surface under conical plug carrying grooves on it's surface.

2. These cylinder's are keept in proper position and the conical plug are pushed into holes. after cables are tighten. The central whole passing actually. cement grout to be injected through

Advantages:

1. Securing the wires is not expansive
2. Desired streching force is applied quickly
3. The plugs may be left in concrete and they do not project beyond the ends of the member.

Disadvantages:

1. Stresses in wires may not be exactly same (All the wires are streached together)
2. Jacks used are heavy and expensive
3. The greatest streaching force available is 250 kN 500 kN which is not sufficients.

2) Magnal bolton system:

1) These method was introduced by famous engineer proffeson magnal of benzium

2) If fraysssinet system several wires are streached at a time. In magnal bolton system 2 meth are Streached at a time.

3. Cable ~~wire~~ passed through a rectangular section which contains of wire layers of [5 to 8 mm dia]

4. A cable consist of wires in multiple of 8 wires cable's with maximum of 64 wire's are also used under special conditions.

5. wires in 2-adjacent layer are seperated with Clearence of 4 mm

6. wires are maintained in the form by providing spaces at regular interval through out the length of cable.

7. wires are anchred by wedging through at a time in sandwich plate.

8. These plates are 25 mm thich and are provided with 2 widge shape groves on its two faces.

8. The wires are taken two in each grove and tighten with the help of hydraulic Jack.

9. A steel wedge is given between the tightened wires anchorage there agains the places.

Advantages:

1. These method saves the cost of sheeting as ducts formed by rubber pores.
2. wires are placed in layer with proper horizontal and vertical spacing by provideing spaces
3. only two-wires are streched at a-time unifore stress induced in every wire.

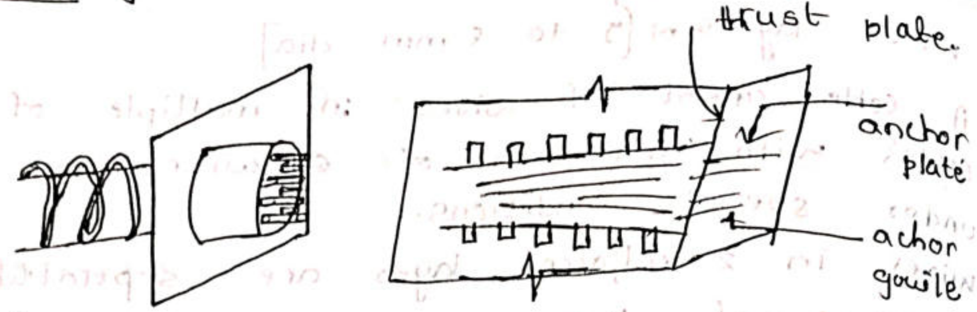
3) lemccall system:

These system originated in Britten and is widely used in india. These is a single wire system. each wire is stresses indlpendently using a double acting job.

Any no. of wires can be group together to form a cable in these system. Their are two types of anchrage device in these system

1. Tube anchorages
2. plate anchorages

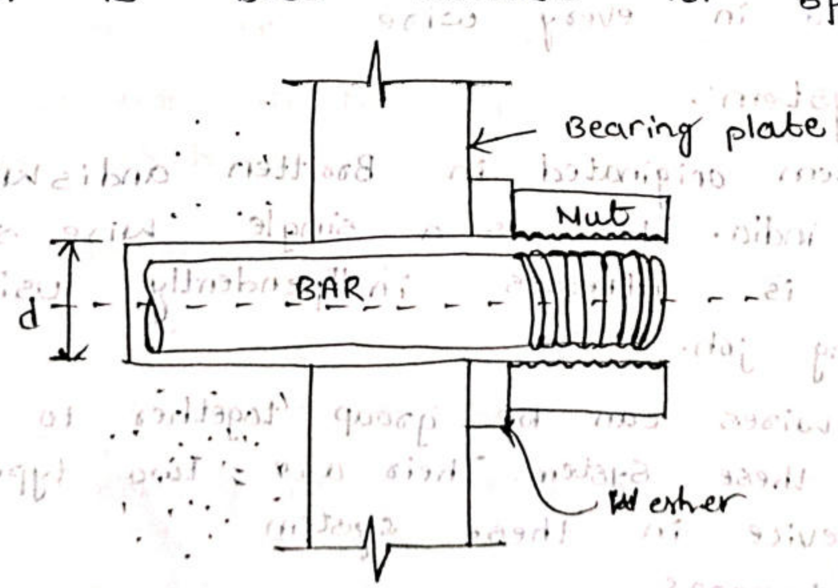
① Tube anchorages:-



Tube anchorage consist of a brearing plate, anchor plate, anchon anchor plate may be squar (or) circular and have 8 – 12 tapped hol to accomodate The individual presstresing wire in addition grout entry hole is also provid for grouted. Anchor wedges are split cone wedg carrying serrations on its flat plate (or) flat surface. There is a tube unit which is fabric component (steel) in corporating a trust plate, A steel tube with their surrounding helex. These unit is attached to end shutter and form an efficient cast in component of the anchor

③ lemccall system:-

In these method high tensile allowed steel bars. (silica megnisia steel) are instead of wires with tensile strength varying for minimum (950 N/mm² to 2100N/ max) These steel rods are provided in 22 mm, 25 mm, 28 mm and 30 mm diameters and length upto 20 m. The anchoring of the bar is done by screwing, special threaded units, these System is best suitable for span (12–15m)



CIVIL ENGINEERING

Design of Pre-stressed Concrete Members

UNIT-2

02/07/25

Losses of pre-stress Unit - 2

Notations:-

Geometrical properties:-

The commonly used geometric properties of a pre-stressed member are defined as follows.

A_c = Area of concrete section.

= Net cross-sectional area of concrete excluding the

A_p = area of pre-stressing steel

= Total cross-sectional area of the tendons

A = Area of pre-stressed members

= Gross cross sectional area of pre-stressed members

= $(A_c + A_p)$

A_t = A Transformed area of pre-stressed members

= Area of member when steel is substituted equalent area of concrete

$$A_t = A_c + m \cdot A_p$$

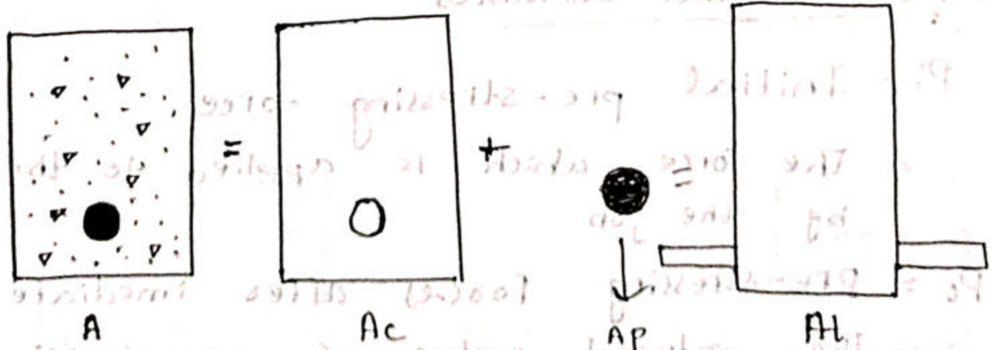
Where, $m = \dfrac{E_s \text{ (or) } E_p}{E_c}$

M = modular ratio

E_c = short term elastic modulus of concrete

E_p (or) E_s = elastic modulus of steel.

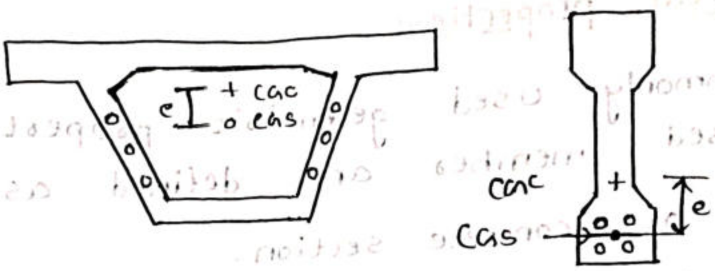
The following fig shows the commonly used areas of the prestressed members.



Areas of pre-stressed members

CGC = centroid of concrete

= centroid of crosssection



CGC = centroid of crosssection

= It may lie out side the concrete

CGS = centroid of pre-stressing steel

= centroid of tendons

= The CGS may lie out side of concrete

I = Moment of inertia of members

i.e, second moment area of gross-section about the CGC

I_t = Moment of inertia of transformed section

= second moment of area of transformed secti about the centroid of section

e = eccentricity of CGS with respect to CGC

Vertical distance b/w CGC and CGS

If CGS below CGC 'e' will be consider positive (or) vice-versa.

CGS, CGC, e of showing for pre-stressed concrete members.

Pg = load member variables

P_i = Initial pre-stressing force

= The force which is applied to the tend by the job

P_0 = pre-stressing forces after imediate loss

= The reduced value of pre-stressing forc after elastic shortening anchorage and loss due to fiction.

Pe = effective pre-stressing force after time dependent losses

= The final losses of pre-stressing force after the occurance of griding, shrinkage and Relaxation.

Introduction

In pre-stressed concrete applications the most important variable is pre-stressing force. In the early days it was absorved that the pre-stresing force doesn't stay constant but reduces with time. even during pre-stressing of the tendons and the transfer of pre-stress to the concrete member. There is a drop of pre-stressing force from the recorded value in the job gauge. The varies reductions of the pre-stressing force are termed as losses in pre-stress

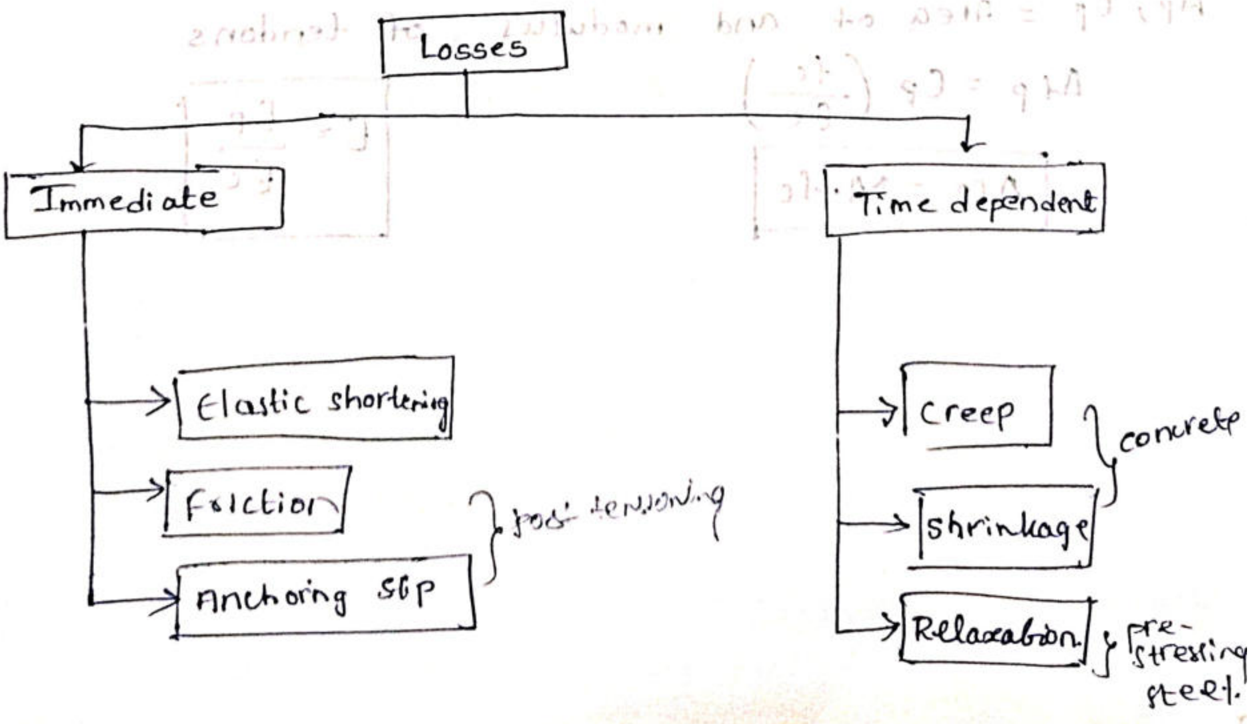
1) Over estimation of losses:

over estimation of losses leads to high pre-stressing force cause excessive camber and tensile stress on (compression side) und

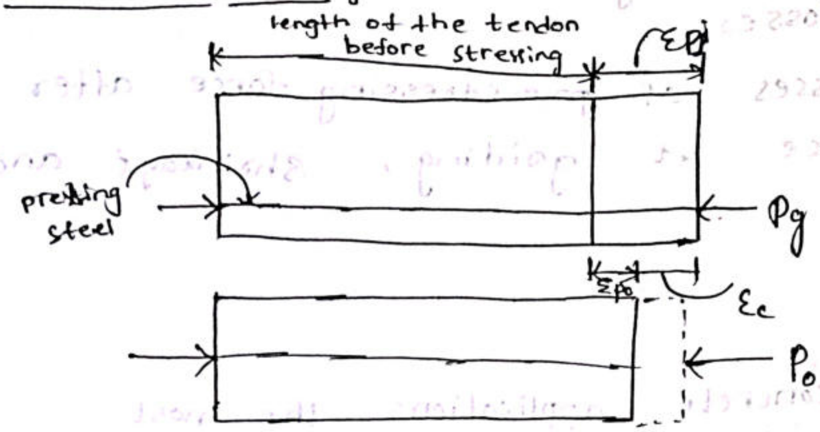
2) Under estimation of losses:

leads to little pre-stressing force, thus not using the system to its full capacity

The excact determination of the pre-stressing loss is not fesiable all the times some times it is resonable to loss of estimation.



Elastic shortening :-



P_j = Initial pre-stressing force in the tendons

ε_{pj} = Initial strain in the pre-stressing tendons

P_o = pre-stressing force in the tendons after elastic shortening losses.

ε_{po} = Strain in tendons after elastic shortening losses

ε_c = strain in concrete due to elastic shortening of tendons.

$$\varepsilon_{pj} = \frac{P_i}{A_p E_p} = \frac{\sigma_i}{E_p}$$

$$\varepsilon_c = \varepsilon_{pj} - \varepsilon_{po}$$

$$\Delta f_p = \varepsilon_p \Delta \varepsilon_p$$

$$\Delta f_p = E_p \varepsilon_c$$

$$\varepsilon_{po} = \frac{P_o}{A_p \cdot E_p}$$

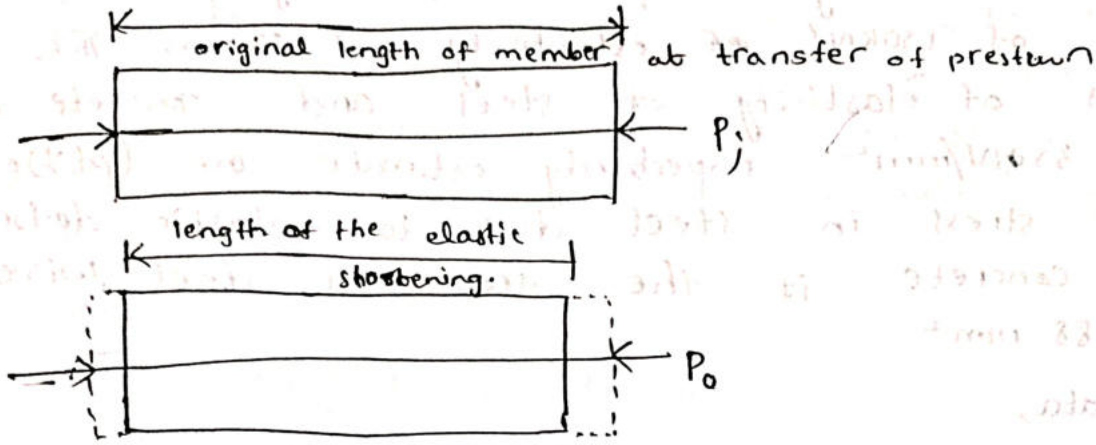
A_p, E_p = Area of and modulus of tendons

$$\Delta f_p = E_p \left(\frac{f_c}{E_c}\right)$$

$$\boxed{\Delta f_p = M \cdot f_c}$$

$$\boxed{E = \frac{E_p}{E_c}}$$

Pre-tension Axial members:-



$$\Delta f_p = m \cdot f_c \rightarrow \frac{P_o}{A_c} = \frac{P_j}{A_t}$$

$$= m \cdot \left(\frac{P_o}{A_c}\right)$$

$$= m\left(\frac{P_j}{A_t}\right) \simeq m\left(\frac{P_j}{A}\right) \text{ tendon c/s area}$$

$$\boxed{\Delta f_p = m\frac{P_j}{A}}$$

$$\varepsilon_c = \varepsilon_{pj} - \varepsilon_{po} \Rightarrow \frac{P_o}{A_c \cdot E_c} = \frac{P_j}{A_p \cdot E_p} - \frac{P_o}{A_p E_p}$$

$$\Rightarrow P_o\left(\frac{1 E_p}{A_c E_c} + \frac{1 \cancel{E_p}}{A_p \cancel{E_p}}\right) = \frac{P_j \cancel{E_p}}{A_p \cdot \cancel{E_p}}$$

$$= P_o\left(\frac{m}{A_c} + \frac{1}{A_p}\right) = \frac{P_j}{A_p}$$

$$\frac{P_o}{A_c} = \frac{P_j}{m \cdot A_p + A_c} \Rightarrow \frac{P_o}{A_c} = \frac{P_j}{A_t} = \frac{P_j}{A}$$

1. An I-section pre-stressed concrete beam - 100 mm X 300 mm is pre-tension by straight wires carrying an Inial force of 150KN/ of effenticity of 50 mm the modulus of elasticity of steel and concrete are 210 & 35KN/mm² respectively estimate the (pt%) of loss of stress in steel due to elastic deformation of concrete if the area of steel wires is 188 mm²

given data,

Initial force (P) = 150 kN

Eccentricity (e) = 50 mm

Area of concrete section = 100X300 = 3X10⁴ mm²

Area of steel wires = 188 mm²

$$\therefore \text{Stress in wires} = \frac{(150\times10^3)}{188} = 800\ N/mm^2$$

$$M\cdot I = I = \frac{bd^3}{12} = \frac{(100\times(300)^3)}{12}$$

$$= 2.25\times10^8\ mm^4$$

$$m = \frac{E_s}{E_c} = \frac{210}{35} = 6$$

stress in concrete at the level of tendon

$$= \left(\frac{P_o}{A}\right) + \left(\frac{M.e}{I}\right)\times y$$

$$= \left(\frac{150\times10^3}{30{,}000}\right) + \left(\frac{150\times10^3\times50}{2.25\times10^8}\right)\times50$$

$$= 6.667\ N/mm^2$$

loss of pre-stress $= 6\times6.667$

$= 40.002\ N/mm^2$

pecentage of loss of stress in steel

$$= \frac{40.002}{800}\times100$$

An I-section pre-stressed concrete beam 500×1000 mm is pre-tensioned using ten 12.7 mm stand stress to 0.75 × 1860 N/mm² the stranks are s with 100 mm eccentricity from the bottom and clear cover is 50 mm estimate the pecentage loss of pre-strett due to elastic shortening given modular ratio m = 7

given data,

size = 500 × 1000 mm

pre-tensioned using (12.7 mm) tendons

Initial stress in steel = 0.75 × 1860 = 1395 N/mm²

strand centroid eccentricity from bottom fibre = 100 − 50 = 50 mm

percentage loss of prestress due to elastic shortening = ?

Modular ratio (m) = 7

Initial force (P) = stress × Area

$$= 1395 \times \frac{\pi}{4} \times (12.7)^2 = 176.71 \times 10^3 \text{ N}$$

stress in concreret at the level of tendon

$$f_c = \left(\frac{P_o}{A}\right) + \left(\frac{M \cdot e}{I}\right) \times y$$

Moment of inertia $(I) = \frac{BD^3}{12} = \frac{500 \times (1000)^3}{12} = 4.16 \times 10^{10} \text{ mm}^4$

$$f_c = \left(\frac{176.71 \times 10^3}{126.67}\right) + \left(\frac{17671 \times 10^3 \times 50}{4.16 \times 10^{10}}\right) \times 50$$

$$f_c = 1395 \text{ N/mm}^2$$

loss due to pre stress = $m \cdot f_c$

$= 7 \times 1395$

$= 9765 \text{ N/mm}^2$

Percentage of loss of stress in steel

$$= \frac{9765}{1395} \times 100$$

$$\sigma_c = \frac{f_{pi}}{m} = \frac{1395}{7} = 199.3$$

percentage loss $\text{Loss \%} = \frac{m \cdot \sigma_c}{f_{pi}} \times 100 = \frac{7 \times 199.3}{1395} = 100\%$.

for pretensioning beams, average percentage loss due to elastic shortening is taken as = 50% of stress in concrete at steel level (transfer stage)

$$\text{loss} = \frac{m \cdot \sigma_c}{2 \times f_{pi}} \times 100 = \frac{7 \times 199.3}{2 \times 1395} \times 100 = 50\%.$$

Anchorage slip:

1. when the tendon force is transferred from the jack to the anchoring ends the friction wedges slip over a small distance
2. Anchorage block also moves before it settens on concrete.
3. losses of prestress is due to the consequent reduction in the length of the tendon
4. Certain quantity of pre-stress is release due to these slip of wire through the anchorage
5. Amount of slip depends on type of wedge and stress in the wire
6. The magnitude of slip can be known from the test or patens and of the anchorage system loss of stress is caused by a adefinite total of shortening.
7. percentage loss is higher for
8. Due to setting of anchorage bar as per tendons shortens, level of their reverse.
9. Effect of anchorage slip is present upto a certain length is called as setting length.
10. Anchorage loss can be accounted at the site by over extending the tendon during pre-stress operation by the amount of drawn in before anchoring
11. loss of pre-stress due to slip can be calculated

$$\frac{P}{A} = \frac{E_s \Delta}{L}$$

where,

Δ = slip of anchorage

L = length of cable

A = C/S area of cable

E_s = modulus of elasticity of steell

P = prestressing force in the cable

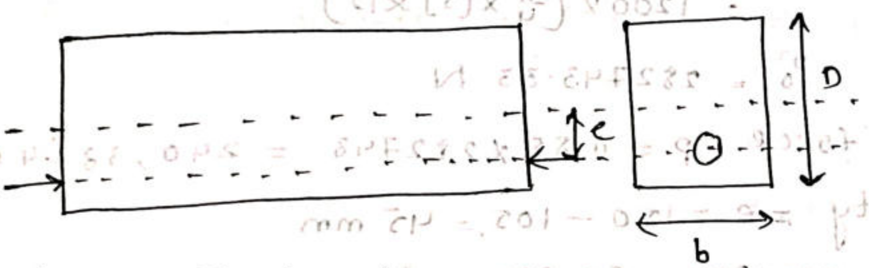
A concrete beam is post-tensioned by a cable carrying an Initial stress on $1000 \, N/mm^2$. The slip and the blocking end was absorbed to be in 5 mm the modulus of elasticity of beam is $210 kN/mm^2$ estimate the procedure loss of stress due to anchorage strip If the length of the beam is 30 m

loss of due to anchorage slip $= \dfrac{E_s \Delta}{L}$

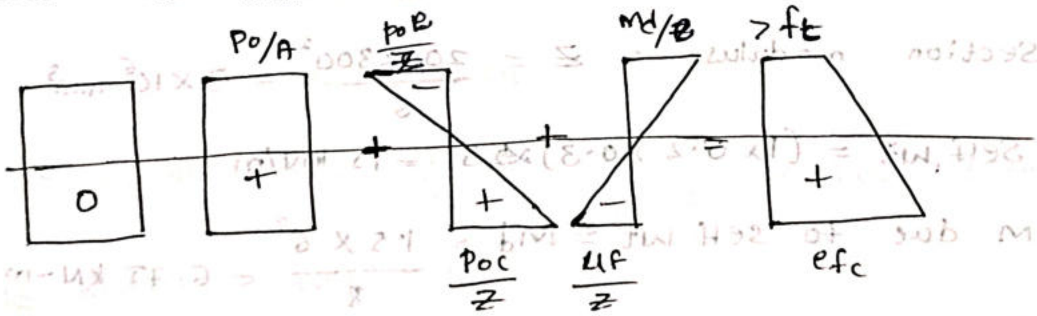
$$= \frac{210 \times 10^3 \times 5}{30000} = 35$$

$$= \frac{35}{1000} \times 100 = 3.5\,\%$$

Analysis of Section:-



Strain J transfer



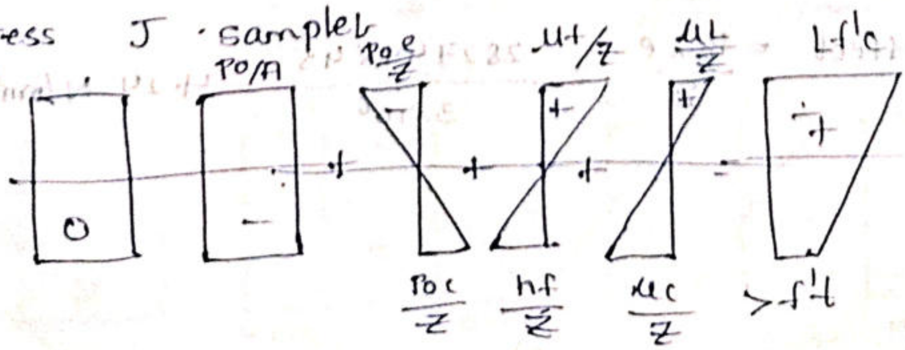
Strain at tranfer at top

$$\frac{P}{A} + \frac{Poe}{Z} + \frac{Md}{Z} = \geq f_t \rightarrow ①$$

Strain transfer at Bottom

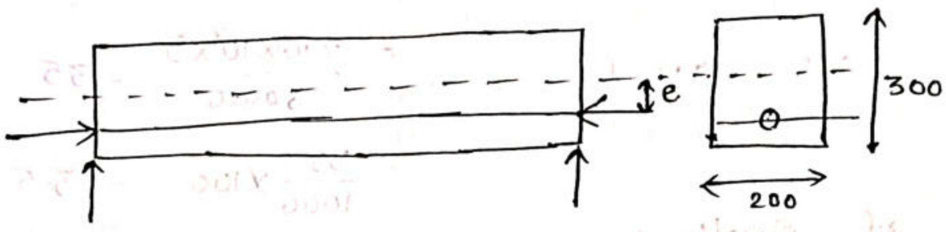
$$\frac{P_o}{A} + \frac{Poe}{Z} - \frac{Mf}{Z} = \leq -f_c \rightarrow ②$$

Stress J sampler



A Concrete beam 200 x 300 mm is p stressed to 1200 N/mm²
tension wires of 5 mm dia from Soffit of
The wires are located at 105 mm is supporting a Super
beam. The S.S.B of 6m span 15% losses where
imposed load of 2.5 kN/m assume at the mid span
required estimate the stress condition
of the beam the following

1) pre-stress + self wt
2) pre-stress + live load



Initial pressing force = stress x Area

$= 1200 \times \left(\frac{\pi}{4} \times (5)^2 \times 12\right)$

$P_0 = 282743.33$ N

Final prestressing force $P = 0.85 \times 282743 = 240,3824$

eccentricity $= e = 150 - 105 = 45$ mm

Area $= 200 \times 300 = 60,000$ mm²

Section modulus $= Z = \frac{200 \times 300^2}{6} = 3 \times 10^6$ mm³

Self wt $= (1 \times 0.2 \times 0.3) \times 25 = 1.5$ kN/m

BM due to self wt $= M_d = \frac{1.5 \times 6^2}{8} = 6.75$ kN-m

BM due to live load $= M_L = \frac{2.5 \times 6^2}{8} = 11.25$ kN-m

(II) Stress at transfer:-

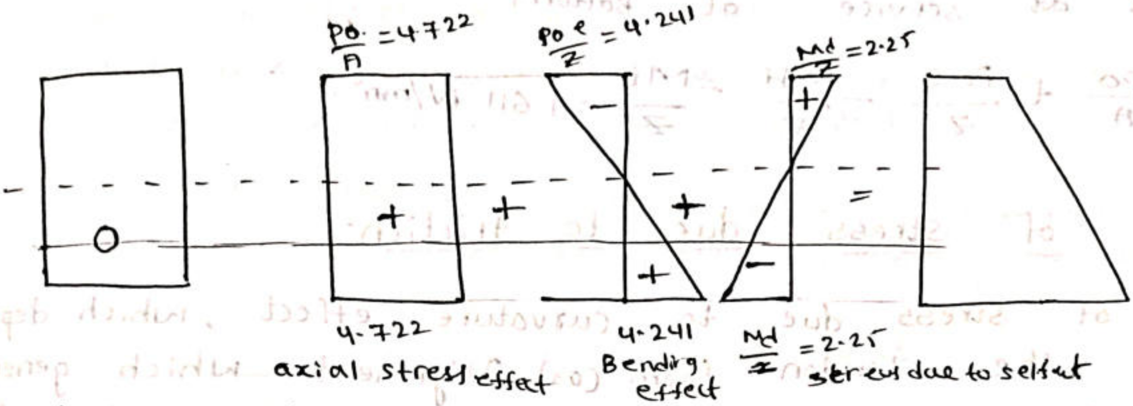
(i) stress due to pre-stress

a) axial effect, $\frac{P_0}{A} = \frac{282744}{60000} = 4.712$ N/mm²

b) Bending effect $= \frac{P_0 \times e}{Z} = \frac{282744 \times 45}{3 \times 10^6} = 4.24$ N/mm²

(iii) stress due to self wt :

$$\frac{M_d}{Z} = \frac{6.75\times10^6}{3\times10^6} = 2.25 \text{ N/mm}^2$$



stress in transfer at top $= \frac{P_o}{A} - \frac{P_o e}{Z} + \frac{M_d}{Z} =$

$= 4.722 - 4.241 + 2.25 = 2.731 \text{ N/mm}^2$

stress in transfer at Bottom $= \frac{P_o}{A} + \frac{P_o e}{Z} - \frac{M_d}{Z}$

$= 4.722 + 4.241 - 2.25$

$= 6.713 \text{ N/mm}^2$

③ Stress due to service Loads:

stress due to prestress:-

ⓐ axial effect $= \frac{P_o}{A} = \frac{240332.4}{60,000} = 4.005 \text{ N/mm}^2$

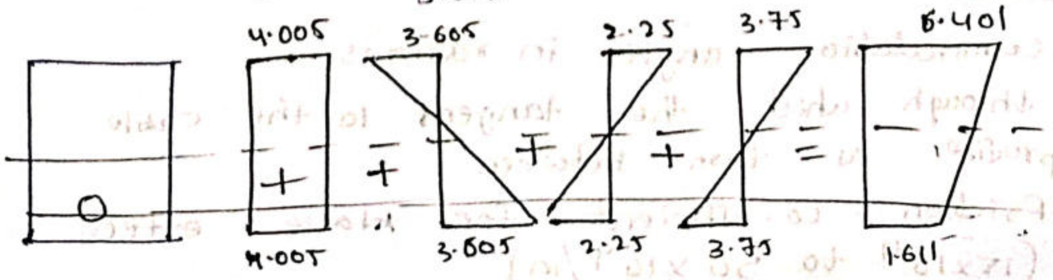
ⓑ Bending effect $= \frac{P_o \times e}{Z} = \frac{240,332.4\times45}{3\times10^6} = 3.605 \text{ N/mm}^2$

(ii) Stress due to self weight

$$\frac{M_d}{Z} = \frac{6.75\times10^6}{3\times10^6} = 2.25 \text{ N/mm}^2$$

(iii) stress due to super imposed load:-

$$\frac{M_L}{Z} = \frac{11.25\times10^6}{3\times10^6} = \cancel{0.833}\ 3.75 \text{ N/mm}^2$$



Stress at service at top

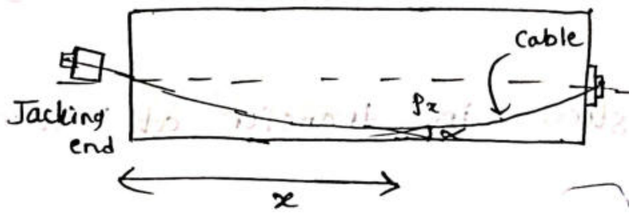
$$\frac{Po}{A} - \frac{Po\,e}{Z} + \frac{Md}{Z} + \frac{ML}{Z} = 6.401 \ N/mm^2$$

Stress at service at Bottom

$$\frac{Po}{A} + \frac{Po.e}{Z} - \frac{Md}{Z} - \frac{ML}{Z} = 1.611 \ N/mm^2$$

losses of stress due to friction:

a) loss of stress due to curvature effect, which dep upon the tendon form (or) Alignment which gene follows a curve profile along the length of the beam



b) loss of stress due to wobble effect which depends upon the local deviation in the alignmen for the cable. The wobble (or) wave effect is t result of accidental (or) unavoidable misalignment, since duct's are sheth cannot be perfectly located to follow a predetermine profile thro out the length of the beam by refering the above fig the magnitude of the prestress force 'P_x at a distance x' from the tensio ends follows an exponential function of the t

$$\boxed{P_x = P_0 e^{-(\mu\alpha + kx)}}$$

e = exponential function
Po = pre-stressing force at the jacking end
μ = co-efficient friction at jack and duct (0.21, 0.23, 0

$\boxed{e = 2.7183}$

α = cummulative angle in radians through which the tangent to the cable profile as turn between

k = Friction co-efficient for wave effect (15×10^{-4} to 50×10^{-4}/m)

μ = values of co-efficient of friction

(0.55 for steel moving on smooth concrete)

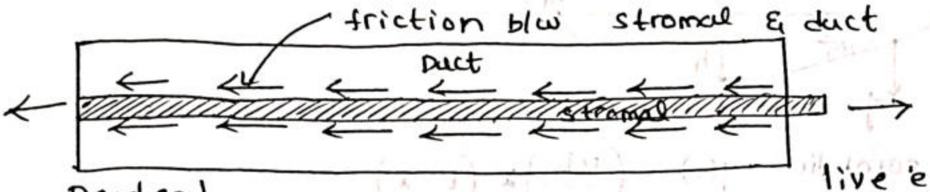
(0.35 for steel moving on steel fixed to duct)

(0.25 for steel moving on steel fixed to concrete)

(0.25 for steel moving on lead)

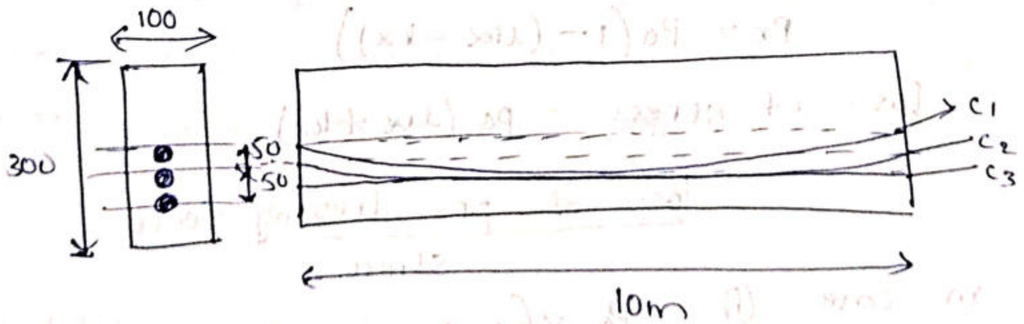
(0.18 − 0.30 for multilayer wire rope cables in regid rectangular steel sheath).

(0.15 − 0.25 for multilayer wire rope cables) with spacer places providing lateral seperation.



Dead end face < Live end face

1) A concrete beam of 10 m span 100 mm wide and 300 mm deep is pre-stressed by 3-cables the area of each caple is 200 mm² the initial stress in the cable is 1200 N/mm² cable -① is parabolic with an eccentricity of 50 mm above the centroid and the supports and 50 mm below at the centre of span. cable -② is also parabolic with 0 eccentricity at support's and 50 mm below at centroid at centre of the span. Cable -③ is straight with uniform eccentricity of 50 mm below the centroid if the cables are tensioned from one end only estimate the percentage loss of loss of stress in each cable due to friction curvature effect and wobble effect. assume $\mu = 0.35$ and $k = 0.0015$ /m.



Given data,

width of beam = 100 mm

Depth of beam = 300 mm

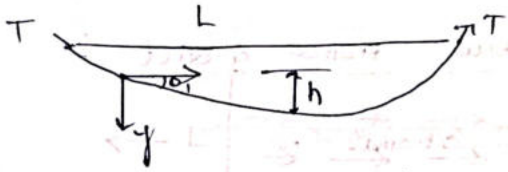
Area of each cable $(A_{cs}) = 200\ mm^2$

initial stress $(\sigma_{case}) = 1200 N/mm^2$

pre-stressing force in each case

$$= A_{case} \times \sigma_{case} = (2.4 \times 10^5) N$$

$$\mu = 0.35, \quad k = 0.0015 /m$$



equation of parabolic $y(1) = \left(\frac{4h}{L^2}\right) x (L-x)$

$@ x = 0$

$$\frac{dh}{dx} = \frac{4h}{L}$$

Cable - ① $h = 100$ mm

slope @ end $\frac{4 \times 100}{10000} = 0.04$

Cummulative angle b/w tayout, $\alpha = 2 \times 0.04$
$= 0.08$ radians.

Initial pre-stressing force in each case

$$P_0 = 200 \times 1200$$

$$P_0 = 2,40,000 N$$

prestressing force in the cable at any point

$$P_x = P_0 \cdot e^{-(\mu\alpha + kx)}$$

for small values of $(\mu\alpha + kx)$

$$P_x = P_0 (1 - (\mu\alpha - kx))$$

loss of stress $= P_0 (\mu\alpha + kx)$

loss of pre-stressing force
stress.

in Case - ① $= P_0 \times (0.35 \times 0.08 + 0.0015 \times 10)$

$= 0.043\ P_0$

in case - 2 $= P_o \times (0.35 + 0.04 + 0.0015 \times 10)$

$= 0.029\, P_o$

in case - 3 $= P_o \times (0 + 0.0015 \times 10) = 0.015\, P_o$ Total $= \underline{0.087\, P_o}$

% loss $= \left(\frac{0.087 \times P_o}{P_o}\right) \times 100 = 8.7\%$.

loss of shrinkage of Concrete:-

For pretensioning $= 0.0003$

⇒ post tensioning $= \frac{0.0002}{\log_{10}(t+2)}$

where,

t is the age of concrete at transparent

loss due to creep of Concrete:-

$= \theta \times m \times f_c$

θ = creep co-efficient

m = modulur ratio

f_c = original pre-stress in concrete

Pre tension loss is greather than post tension loss

loss due to shrinkage $= 3 \times 10^{-4} \times E_s$

loss due to relaxation of steel $= 1\%$ to 8%.

$0.5 f_p = 0$

$0.6 f_p = 3.0$

$0.7 f_p = 5.0$

$0.8 f_p = 8.0$

2) **Ultimate creep co-efficient (or) simply co-efficient:-**

The ratio of the ultimate creep strain to the elastic creep strain is define as Ultimate creep co-efficient (or) creep co-efficient (θ)

ε (or) cH $= \theta \cdot \varepsilon \cdot \theta l$

1) **ultimate creep strain method:-**

The loss of stress in steel due to creep of concrete.

$= \varepsilon_{cc} \times f_c \times E_s$

where, ε_{cc} = Ultimate creep strain for a sustain unit stress

f_c = compressive stress in concrete at the level of steel

E_s = modulus of elasticity of steel

(2) Creep co-efficient method:

$$\text{creep co-efficient} = \frac{\text{strain creep}}{\text{strain elasticity}} = \frac{\varepsilon_c}{\varepsilon_s}$$

$$\text{loss of stress in steel} = \varepsilon_c \cdot \varepsilon_s = \phi\, \varepsilon_c \cdot E_s$$

$$= \phi \left[\frac{f_c}{E_c} \right] E_s$$

$$= \phi \cdot f_c \cdot \alpha_e$$

where, ϕ = creep co-efficient

ε_c = creep strain

ε_s = elastic strain

$\alpha_e = \dfrac{E_s}{E_c}$

f_c = stress in concrete

E_c = elasticity of concrete.

Then E_s = modulus of elasticity of steel

$$\text{loss in pre-stress} = E_p \theta \cdot \varepsilon_{e1}$$

$$= E_p\, \theta \left(\frac{f_c}{E_c} \right)$$

$$= m \cdot \theta \cdot f_c$$

$$\text{loss in pre-stress } (\Delta f_p) = E_p \cdot \varepsilon_{cr, ult} = E_p \cdot \theta \cdot \varepsilon_{el}$$

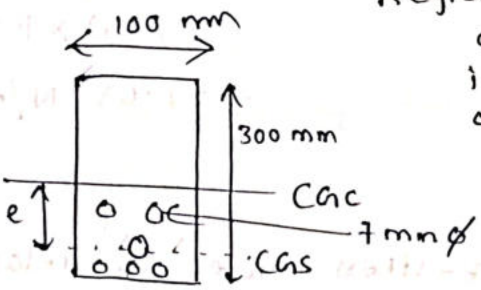
① A concrete beam of dimensions 100 X 300 mm is post-tensioned with 5 straight wires of 7 mm ϕ dia meters the anchorage average pre-stress after shooterm loss is 0.7 fpk = 1200 N/mm² and age of loading is given as 28 days. given that find out the loss of pre-stree due to creep, shrinkage & Relaxation. neglect the wt of beam in the constit: or of stress.

given that,

$0.7 f_{pk} = 1200\ N/mm^2$

$E_p = 200 \times 10^3$ mpa

$E_c = 35000$ mpa.



Area of concrete $= 100 \times 300$

$= 30000\ mm^2$

Moment of inertia $(I) = \frac{BD^3}{12} = \frac{100 \times 300^3}{12} = 225 \times 10^6\ mm^4$

Area of tendons $(A_p) = 5 \times \frac{\pi}{4} \times (7)^2 = 192.42\ mm^2$

pre-stressing force after shooter loss

$P_o = A_p \cdot f_{po}$

$= 192.42 \times 1200$

$= 230.904 \times 10^3\ N$

Modular ratio $(m) = \frac{E_p}{E_c} = \frac{200 \times 10^3}{35000} = 5.71$

stress in concrete at the level of CGS

$f_c = \frac{-P_o}{A} - \frac{P_o \cdot e}{I} \times e$

$= -\frac{230.904 \times 10^3}{3 \times 10^4} - \frac{230.904 \times 10^3 \times (50)^2}{225 \times 10^6}$

$f_c = -10.26\ N/mm^2$

<u>loss of pre-stress due to creep</u>:

$= m \cdot \theta \cdot f_c$

$= 5.71 \times 1.6 \times (-10.26)$

$\Delta f_{pc} = 93.64\ N/mm^2$

Table 2C.1 ↓ ⑥ pg no $\theta = 1.6$ for 600 mm loss of stress in shrinkage :-

Strain in shrinkage $= \frac{0.0002}{\log_{10}(t+2)} = \frac{0.0002}{\log_{10}(28+2)} = 1.354$

loss of prestress due to shrinkage

$(\Delta f_p)_{sh} = E_p \cdot \varepsilon_{sh}$

$= 2 \times 10^5 \times 1.354 \times 10^{-4}$

$= 27.08 \ N/mm^2$

loss of pre-stress due to relaxation :-

$(\Delta f_p)_m = 70 \ N/mm^2$

loss of prestressing force creep $= \Delta f_p \cdot A_p$

$= 93.64 \times 192.42$

$= 18018 \ N$

loss of prestressing force shrinkage $= (\Delta f_p)_{sh} \times A_p$

$= 27.08 \times 192.42$

$= 5210.73 \ N$

loss of pre-stressing force Relaxation $= (\Delta f_p)_m \times A_p$

$= 70 \times 192.42$

$= 13469.4 \ N$

Total long term prestressing force (Neglecting the the intraction of losses and pre-stressing force)

$= 18018 + 5210.73 + 13469.4$

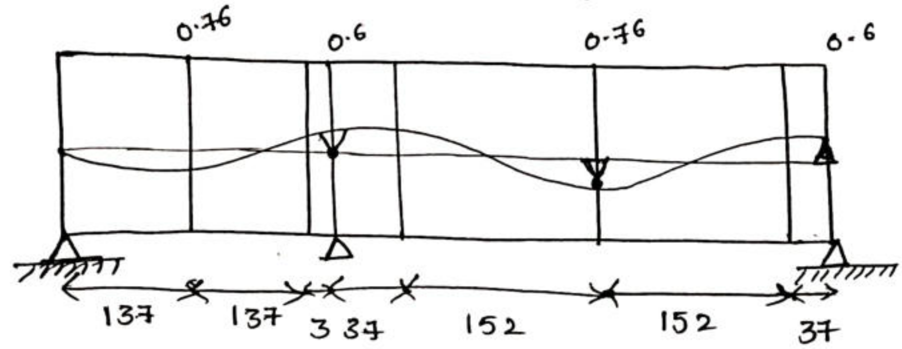
$= 36698 \ N$

% loss of pre-stress $= \frac{36698}{230988} \times 100$

$= 15.97\%$

A 4 span continuous bridge girder is post tensioned with a tendon consisting 20 strands with fpk = 1860 N/mm^2 half - half the girder is shown fig below.
The symmetrical tendon is simultaneously stress upto 75 fpk from both ends and then anchored. The tendon properties are $A_p = 2800\ mm^2$ and ep = 195000 mpa, $\mu = 0.20$, $k = 0.0020/m$ the anchorage slip $\Delta s = 6\ mm$ Calculate ? A

a) The expected elongation of tendons after stritching
b) The force variation diagram along the tendon before and after anchorage.



Given data,

continuous bridge girder (4 span) post-tensioned tendons consist of 20 strands

Ultimate tensile strength of strand fpk = 1860 N/mm^2

Stressing level 0.75 fpk = 0.75 × 1860 = 1395 N/mm^2

Area of tendons (A_p) = 2800 mm^2

modulus of elasticity of tendon (E_p) = 195000 Mpa

friction co-efficient $(\mu) = 0.20$

wobble co-efficient $k = 0.0020/m$

Anchorage slip $(\Delta s) = 6\ mm$

Spans = 13.7 + 13.7 + 3 + 3.7 + 15.2 + 15.2 + 3.7

= 68.2 m   [peak eccentricities (e) = 0.76, 0.60, 0.76, 0.60 m

① Initial jacking force :-

$$P_j = \sigma_j \cdot A_p = (1395) \times (2800) = 3.906 \times 10^6\ N = 3906\ kN$$

② Force at a distance x along the tendon

$$P(x) = P_j\, e^{-(\mu\theta + kx)}$$

Friction / curvature parameters:-

for a parabolic tendon in a span L_s with midspan eccentricity e_{max}, the total change in angle over the span is

$$\Delta \Theta_s \simeq \frac{8 e_{max}}{L_s} \text{ (radians)}$$

$$\Theta_{total} = \sum \frac{8 e_{max}}{L_s} \sim$$

CIVIL ENGINEERING

Design of Pre-stressed Concrete Members

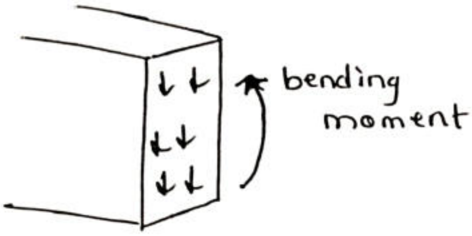
UNIT-3

3. Unit :-

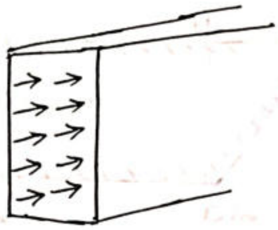
When the load is applied on to the beam, it would deform by bending. This generates internal stresses which can be represented by a SF(V) and BM(M).

SF is the resultant of Vertical shear stresses which acts parallel to cross section. BM is the resultant of normal stresses which acts normal to the cross section

shear stresses

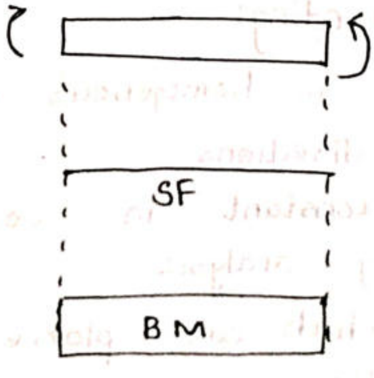


normal stresses

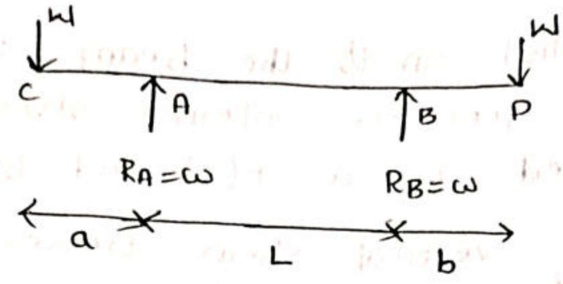


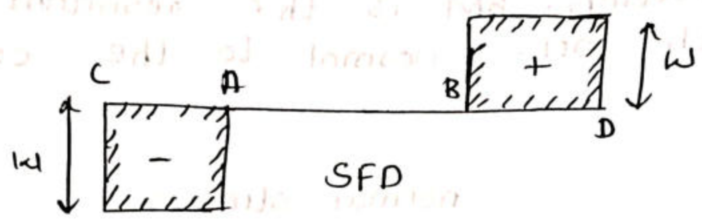
Pure bending or Simple bending :-

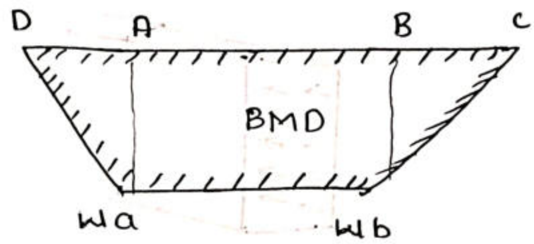
- If the length of the beam is subjected to a constant bending moment and no shear force (i.e, zero shear force) then the stresses will be setup in the length of the beam due to B.M only and that length of the beam is said to be in pure bending or simple bending
- The stress set up in that length of the beam are known as bending stresses.

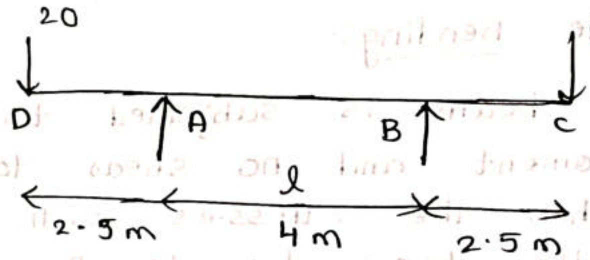


Pure bending:-









$$20 \times 9 + R_A \times 6.5 \cancel{R_B \times 2.5} - 30 = 0$$

$R_A = 23.07$

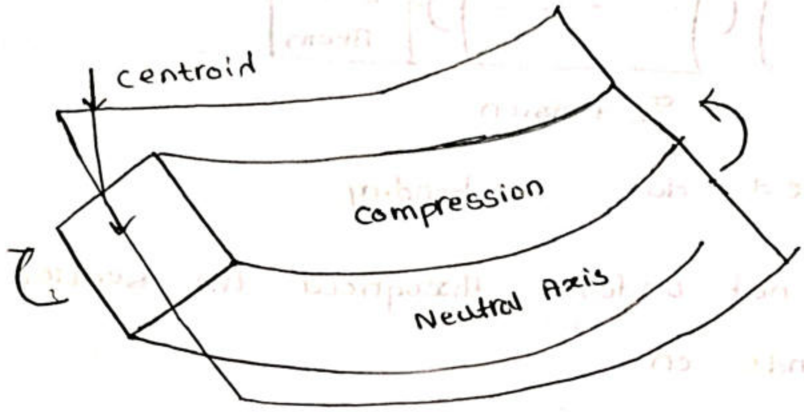
$R_B = 26.93$

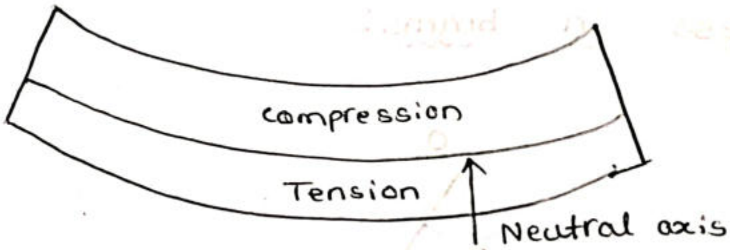
Theory of simple bending:-

1. Material of the beam is homogenou's and isotropic ⇒ constant E in all directions
2. young's modulus is constant in compression and tension ⇒ to simplify analysis
3. Transverse section which are plane before bending remains plain after bending ⇒ eliminate effects of strain in other direction (next side)
4. Beam is initially straight and all longitudinal filaments bend in circular arc's ⇒ simply calculations.
5. Radius of curvature is large compared with dimensions of cross section ⇒ simplify calculation
6. Each layer of the beam is fr...

to (or) contract ⇒ otherwise they will additional internal stresses.

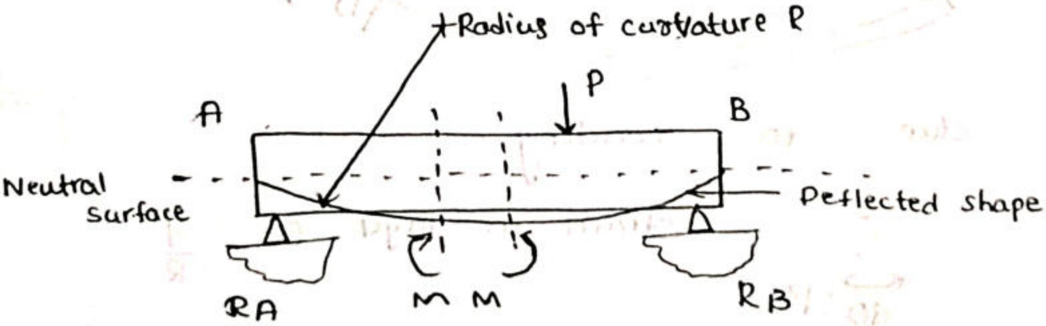
No change in the Neutral Axis:-

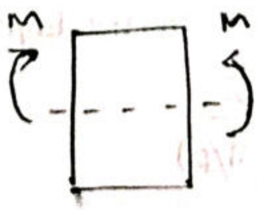




Bending stress in beams:-

consider the S.S.B below

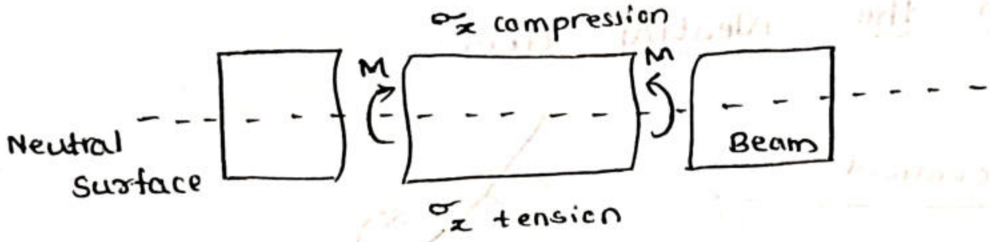




what stress are generated within due to bending?

Axial stress due to bending

M = bending moment



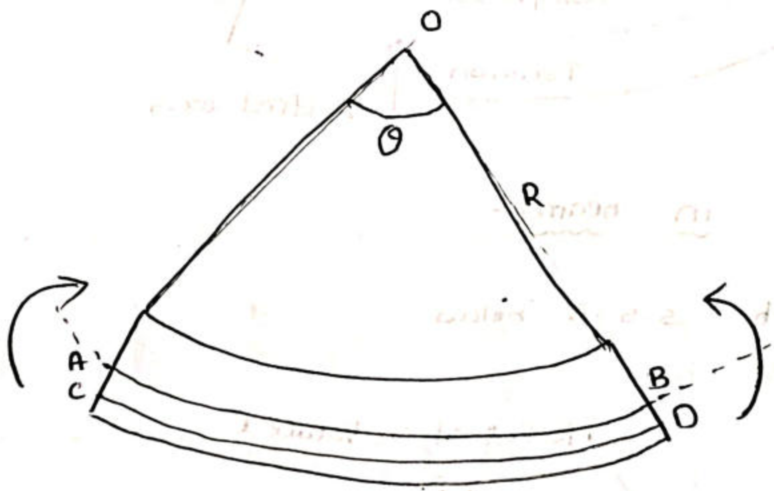
stress generated due to bending

→ σ_x is not uniform throughout the section depth

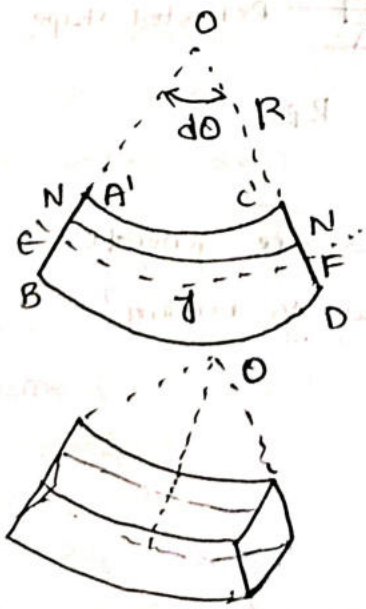
→ σ_x depends on

1. Bending moment M
2. Geometry of cross-section.

Bending stress in beams:



Stresses due to bending:-



strain in layer EF $= \frac{y}{R}$

$$E = \frac{\text{stress in the layer EF}}{\text{strain in the layer EF}}$$

$$E = \frac{\sigma}{(y/R)}$$

$$\frac{\sigma}{y} = \frac{E}{R} \Rightarrow \sigma = \frac{E}{R} y$$

Analysis of member's under flexure:-

principle of mechanics:-

① equilibrium of internal forces with the external loads the compression in concrete (C) is equal to the tension in the tendon (T). The couple of C & T are equal to the moment due to external loads

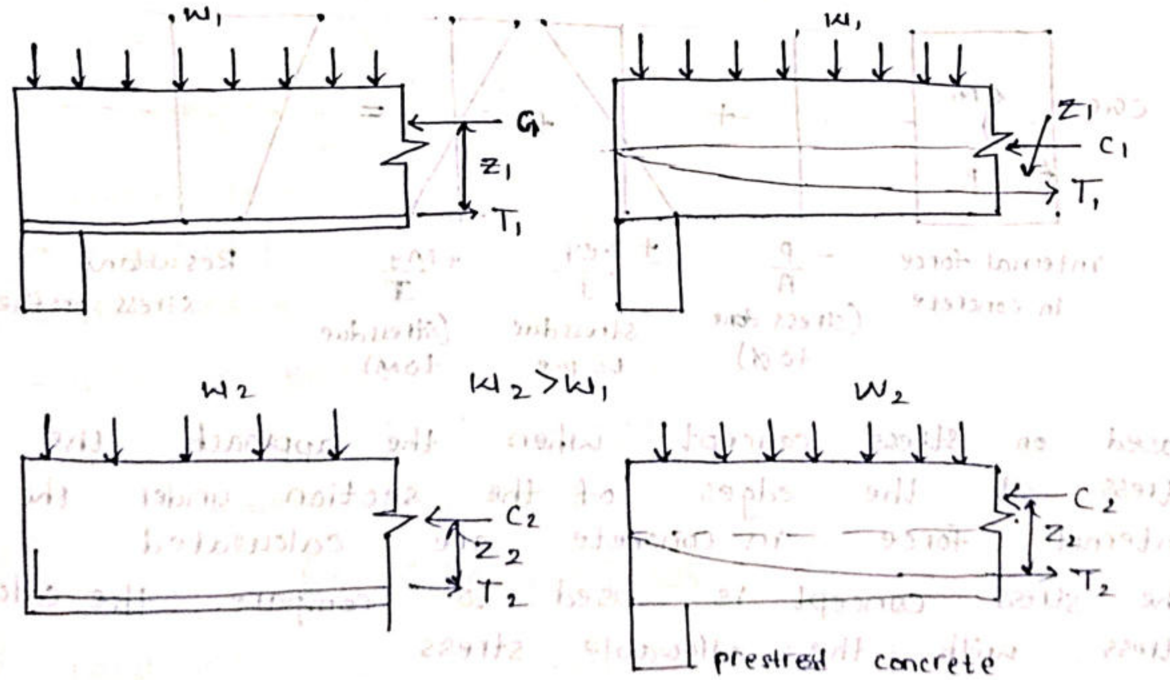
② Comparibility of the strains in concrete and steel for bonded tendons. The formulation also involves the first assumption of plane section remains plane after bending for unbonded tendons that compability is in terms of deformation.

③ Constitutive relationship relationg the stresses and the strain in the materials.

Variation of mechanical internal forces:-

In reinforced concrete member's under flexure the values at comp in concrete (C) tension in steel (T) Increase with increasing external loads - the change in leverArem(Z) is not large.

In prestressed concrete members, under flexure, at transfer of prestress (C) is located close to (T). The couple of C & T balance only the self weight at service loads C shifts and the levearm (Z) get's large The variation of 'C & T' are not appreciable.



Reinforced concrete

$C_2 > C_1$, $Z_2 \simeq Z_1$

prestressed concrete

$C_2 \simeq C_1$, $Z_2 > Z_1$

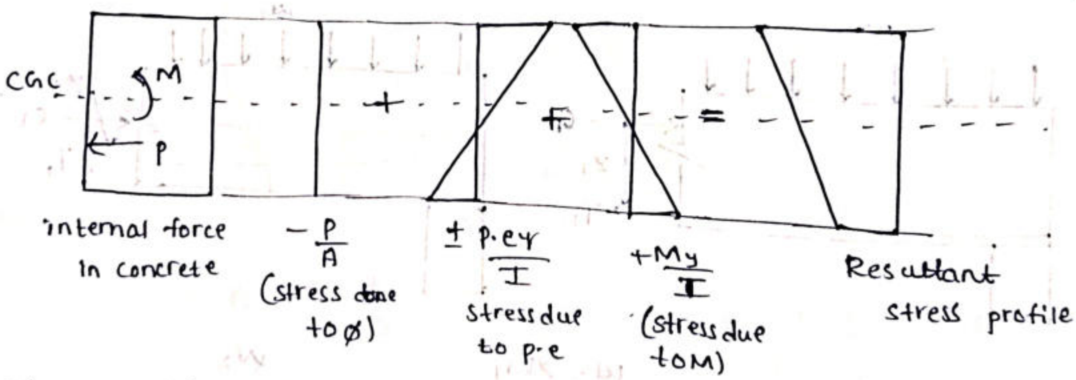
C_1, T_1 = Compression and tension at transfer due to self wt
C_2, T_2 = compression and tension under service loads
W_1 = self wt
W_2 = service loads.
Z_1 = learver arm at transfer
Z_2 = leverarm under service loads.

Analysis at transfer and service loads:.

The analysis at transfer and under service loads are similar. Hence they are presented togethes. A prestoessed members usually remains uncrack. under service loads the concrete and steel are treated as elastic materials the principles of supesposition is applied the increase in stress in pre-stressing steel due to bending is neglected. There are three approaches to analize a prestressed member and transfer and under service loads.

a) Based on stress concept
b) Based force concept
c) Based on load ~~beha~~ balancing concept.

a) Based on stress concept:-



Based on stress concept when the approach the stress at the edges of the section under the internal force in concrete are calculated. The stress concept is used to compare the calculated stress with the allowable stress.
The following fig shows a S.S.B cas under a UDL and pre-stressed with constant eccentricity along it's length.
The first stress profile is due to compression (P)

The second profile is due to the eccenticity
The third profile is due to the moment

At transfer the moment is due to self weight. The at service the moment is due to service loads.

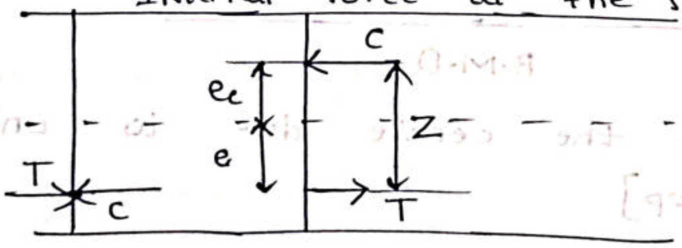
stress profiles at a section due to internal forces the resultant stress at a distance 'y' from CGC is given by the principle of superposition

$$f = \frac{-P}{A} \pm \frac{P.e.y}{I} + \frac{My}{I} \rightarrow ①$$

B) Based on force concept:-

The approach based on force concept is enaloges to the study of reinforced concrete the tension in prestressing steel (T) the resultant compression in concrete (C) are consider to balance the external loads. these approach is used to determine. the dimensions of a section and to check the service load capacity. The stress at concrete Calculated by these approach are same as those based on stress concept the stress at the extreme edges are compared with the allowable stress.

Internal force at the section.



Internal force at prestressing (neglecting self wt)

Internal force after loading

The equilibrium equations are follows

$$C = T \rightarrow ②$$

$$\text{moment } (M) = C \cdot Z$$

$$M = C(e_c + e) \rightarrow ③$$

The resultant stress in concrete at distance 'y' from the CGC is given as follows

$$f = \frac{-C}{A} + \frac{C \cdot e_c \cdot y}{I} \rightarrow ④$$

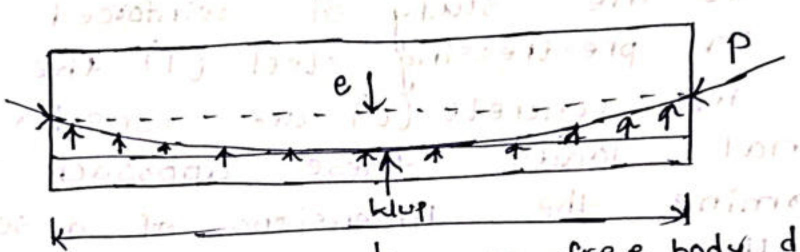
substituting $C = P$ and

$$C \cdot e_c = M - P_e \rightarrow (5)$$

$$f = \frac{-P}{A} + \frac{P \cdot e y}{I} \pm \frac{M y}{I} \rightarrow (5)$$

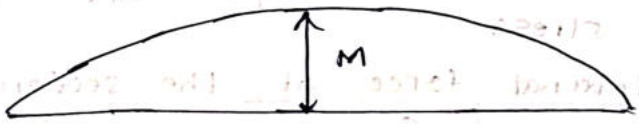
c) Based on load Balancing concepti-

The approach based on load bal concept is used for a member is curved (or) harp tendons and the analysis of indeterminate condition. The moment upward crest and upward deflection member (chamber) due to prestress in th tendons are calculated. The upward crest balan part of the superimposed load



free body diagram of concrete

The expressions for three profiles of tendon's in S.S.B are given for a parabolic tendon



B.M.D

The moment at the centre due to uniform upward crest [Wup]

$$M = \frac{W_{up} L^2}{8} \rightarrow (6)$$

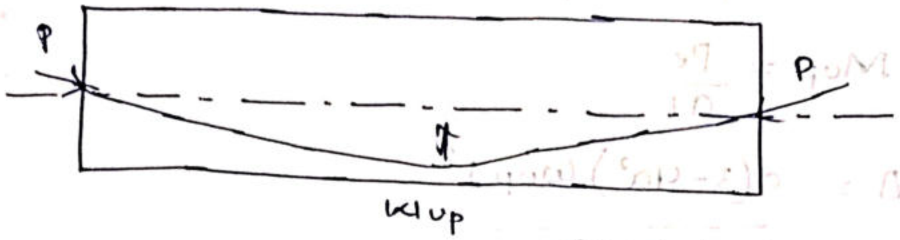
The moment at the centre from the pre-stressing force is given as $M = P_e$

The expression of Wup is calculated by equating the two expressions the upward dection (Δ) can be calculated based on elastic

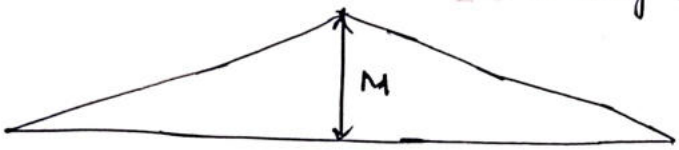
$$W_{up} = \frac{8 \cdot P_e}{L^2}$$

$$\Delta = \frac{5 W_{up} L^2}{384 EI} \rightarrow (7)$$

For single harped tendon



free body diagram for concrete



Bending moment diagram

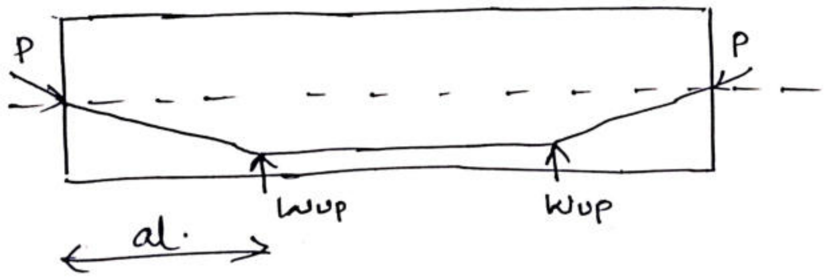
Moment at the centre due to upward trust is given by the following equation it equate for due to the eccentricity of tendon as before, the upward trust can be calculated

$$M = \frac{W_{up} L}{4} = P.e$$

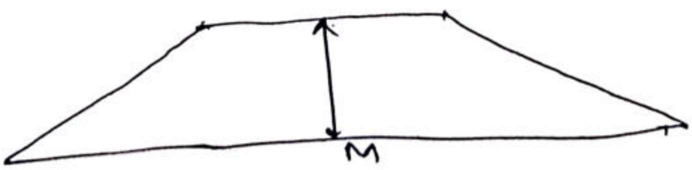
$$W_{up} = \frac{4Pe}{L}$$

$$\Delta = \frac{W_{up} L^2}{48EI} \rightarrow (8)$$

for, Doubly harped tendon's :-



free body diagram for concrete



Bending moment diagram

The moment at the centre due to the upward trust (W_{up}) it is equated to the moment due to eccentricity of tendon as before

$$M = w_{up}\, a \cdot L = P \cdot e$$

$$M_{up} = \frac{Pe}{aL}$$

$$\Delta = \frac{a(3-4a^2)\, w_{up}\, L^3}{24\, EI}$$

Analysis for shear:-

The Analysis of reinforced concrete and pre-stressed concrete member for shear more difficult compared to the Analysis of axial load (or) friction.

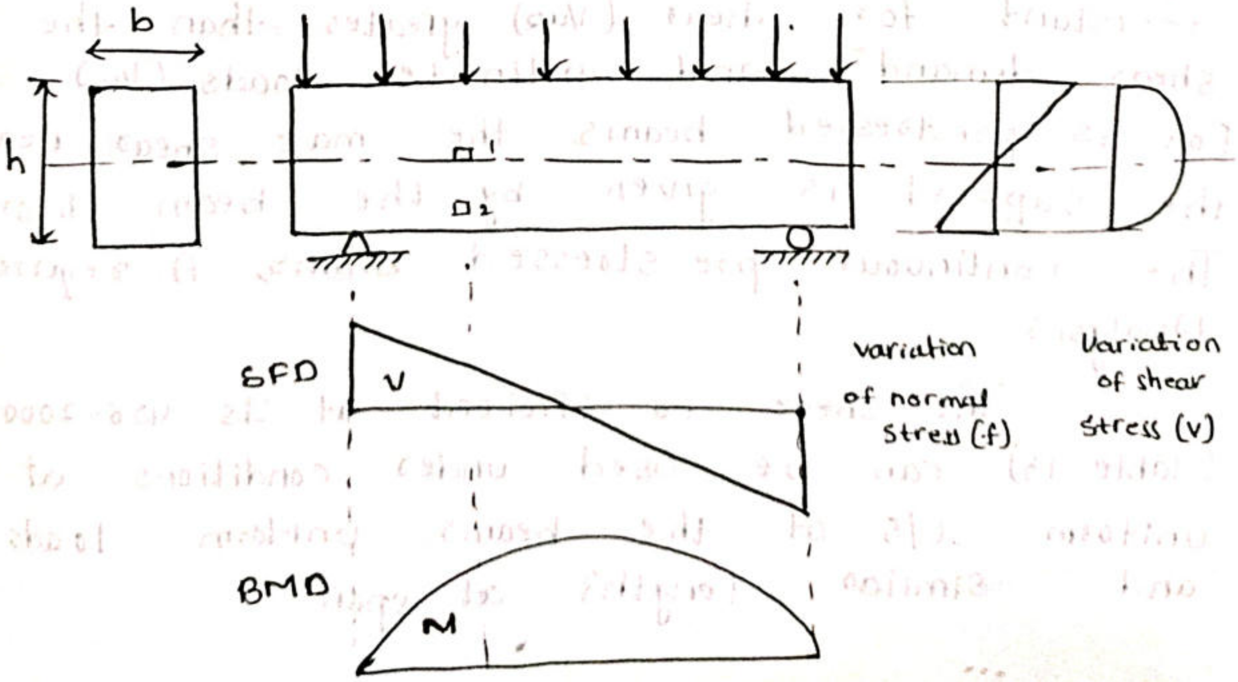
The Analysis of Axial load (or) friction are based on the . of mechanics

1. Equalibrium of internal and External forces
2. and comparability of strains in concrete. ~~consti~~
3. constitute relations of materials

The conventional analysis of shear is based on equalibrium forces by a simple equation the compatibility of strain not the constitute relationship (relating stress and strain of the materials, concrete (and) steel. are not used

The strength of each material corresponds to the ultimate strength the stg of concrete under shear Although based on test results is eperical in nature shear stress generated in beams due to bending (or) Twisting. The two types of shear stress are called flexural shear stress and tortional shear stress respectively

Stress in an uncracked beam:-



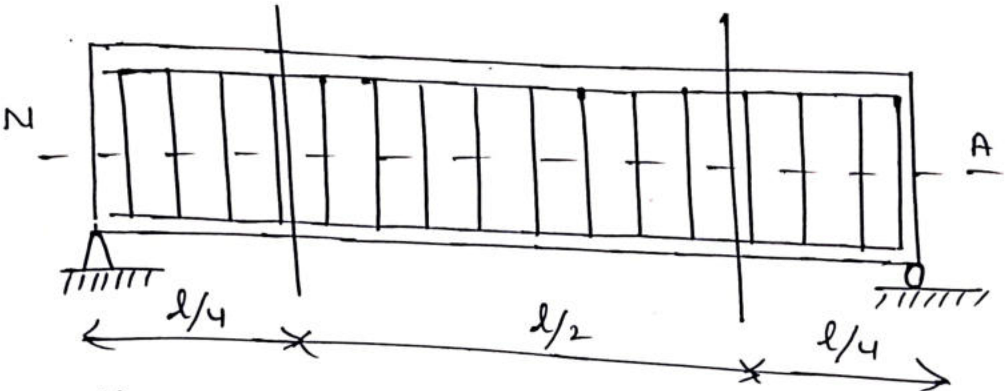
The fig shows the Variations of shear and moments along to span of a SSB under a UDL, then the variations at normal and shear stress along the depth at the section at a beam also shown

under a general loading the shear force and the moment vary along the length, then normal stress and shear stress as well as along the depth. The combination at the normal & shear stress generate a two dimensional stress field at a point at any point the beam, the state at two dimensional stresses can be expressed in terms at the principal stress the more circle at the stress is helpful to understand the state at stress. Before cracking the carried by steel is neglegable when the principle tensile stress exceeds the cracking stress the concrete cracks and their is re-distribution at stress between concrete and steel. for a point on the Neutral axis element -1, the shear stress is max and normal stress is 'zero'. The principle tensile stress (σ_1) is inclined at 45° to the neutral axis.

Calculation at shear demand the Objective at design is to provide ultimate resistant for shear (V_{uR}) greater than the shear demand and ultimate loads. (V_u). For SS prestressed beams the max shear near the support is given by the beam theory. The Continuous pre-stressed beams, A regarest Analysis.

The shear co-efficient at IS 456-2000 (Table 13) can be used under conditions at uniform C/S at the beams, uni-form loads and similar length's at span

Design the stirrups the design is done for the critical section. It critical section because 22.6.2 (page 36) in general cause the phase at support is considered as a critical section when the reaction at the support introduce comp. at the end of the beam. The critical section can be iselected at the distance eff depth from the phase at the support The eff depth is selected as the greater at depth of C.G.S from the extreame compre fibre (or) depth at centroid at non-presstred steel.



Since the CGS is at a higher location near the support the eff depth will be equal to depth at centroid at non prestressed steel (Rcc)

To very the spacing of stirrups along the span, other section be may be selected for design. Usually the following shcrease is selected for beams under a uniform load

Spacing for water at the span adjustment to the support - for half half at the span at the middle

For large beams more kerlational spacng may be selected.

CIVIL ENGINEERING

Design of Pre-stressed Concrete Members

UNIT-4

Date: 06/10/2025

Unit - 4

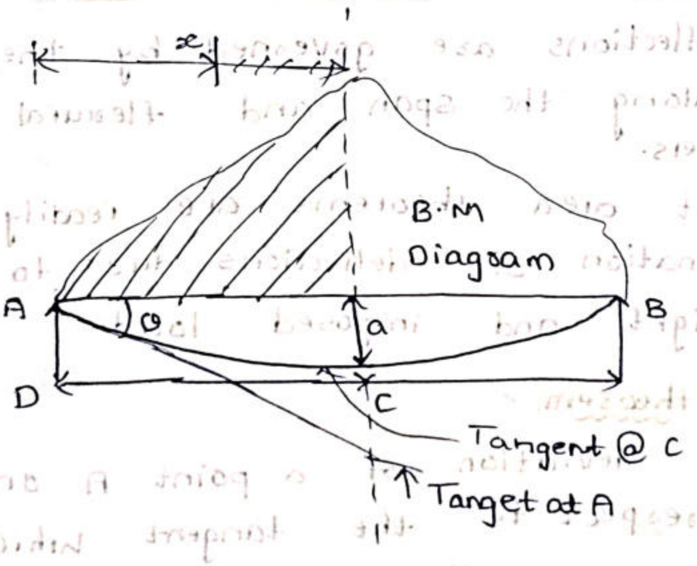
Deflection of unit concrete members

Factor's influencing deflection:-

Deflection of pre-stressed concrete members are influenced by the following factors.

1. Imposed load and self weight
2. Magnitude of the pre-stressing force
3. Cable profile
4. Second moment of area of crosssection
5. Modulus of elasticity of concrete
6. Shrinkage, creep and relaxation of steel stress
7. Span of the member
8. fixity conditions

Shortterm deflections of uncrack members:-



where,

θ = slope of the elastic curve at A

AD = Intercept between the tangent at C and the vertical at A

a = deflection at the centre.

A = Area of B.M.D b/w A & C

x = Distance of the centroid of the B.M.D b/w A & C from the left support.

$$\theta = \frac{\text{Area of BMD}}{\text{flexural rigidity}} = \left(\frac{A}{EI}\right) \qquad a = \frac{\text{moment of area of BMD}}{\text{flexural rigidity}} = \left(\frac{Ax}{EI}\right)$$

Computation of deflections:-

Precracking stage:-

Gross MI

Called as short term or instantaneous deflection Mohr's theorem

Post-cracking stage:-

Effective MI of cracked section

Moment - curvature relationships (section, properties of cracked beam).

- In both cases the effect of creep and shrinkage of concrete is to increase.
- The long term deflections under sustained loads, which is estimated by using empirical methods that involve the use of effective (long term) modulus of elasticity or by multiplying short-term

Short term deflections of uncracked members:-

- Short term deflections are governed by the BM distribution along the span and flexural rigidity of the members.
- Mohr's moment area theorems are readily applicable for the estimation of deflections due to prestressing force, self weight and imposed loads.

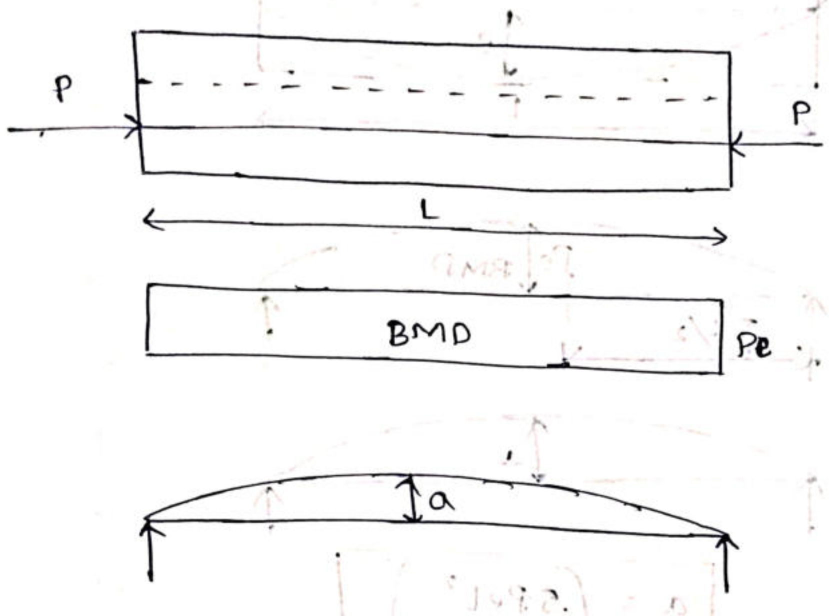
Moment area theorem:-

- The vertical deviation of a point A on an elastic curve with respect to the tangent which is extended from another point B equals the moment of area under (M/EI) diagram b/w those two points (A and B). This moment is computed about point A where the deviation from B to A is to be determined.

$$\boxed{y_B = \int_A^B \frac{M}{EI} x\,dx}$$

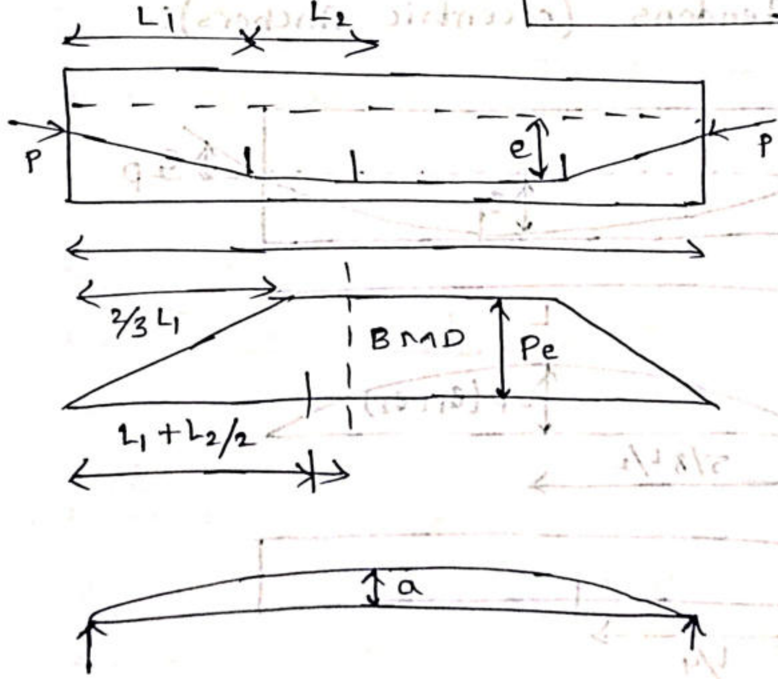
Effect of tendon profile on deflection:

① straight tendon:



$$\boxed{\Rightarrow a = \frac{Pe}{EI} \times l}$$

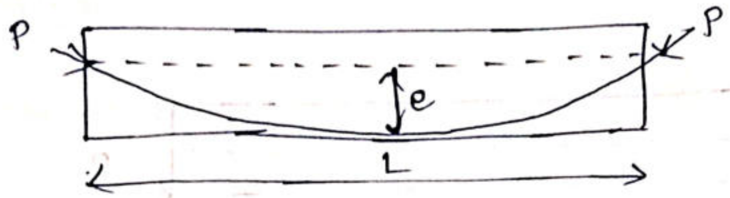
② Trapezoidal tendons:

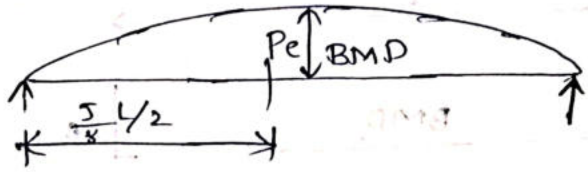


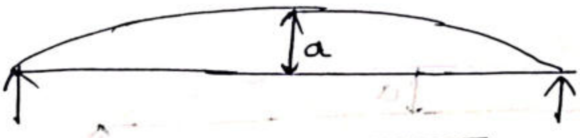
$$a = \frac{Pe}{EI}\left(l_2(l_1 + l_2/2) + (l_1/2)(2/3\, l_1)\right)$$

$$= \frac{Pe}{6EI}\left(2l_1^2 + 6l_1 l_2 + 3l_2^2\right)$$

③ parabolic Tendons: (Concentric Anchors)

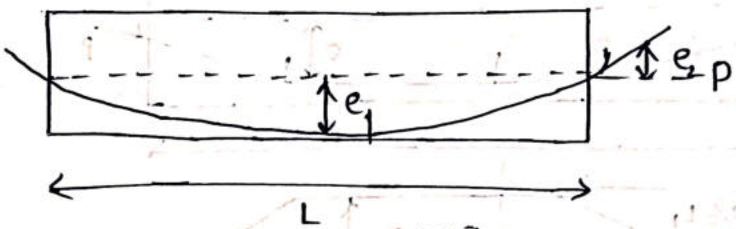


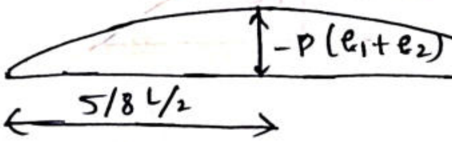


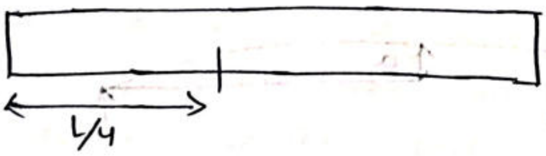


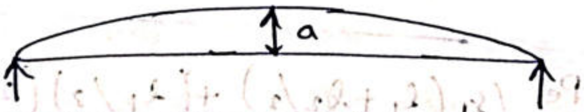
$$\boxed{a = -\left(\frac{5PeL^2}{48EI}\right)}$$

④ parabolic tendons (eccentric Anchors):-





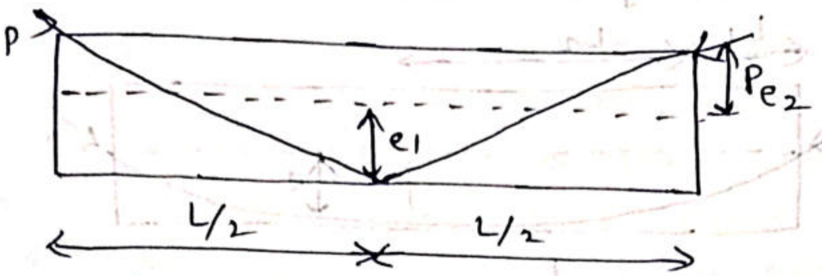


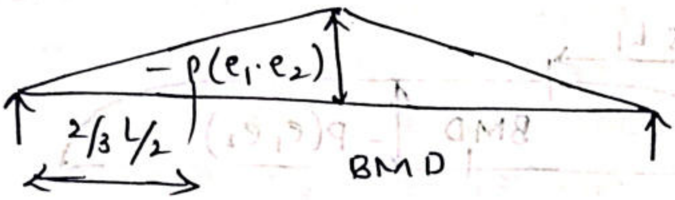


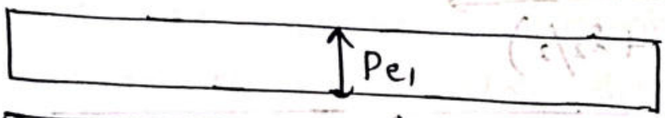
$$a = -\left(\frac{5PL^2}{48EI}(e_1+e_2)\right) + \left(\frac{Pe_2L^2}{8EI}\right)$$

$$= +\frac{PL^2}{48EI}(-5e_1+e_2)$$

5. Sloping Tendons (eccentric Anchors):-



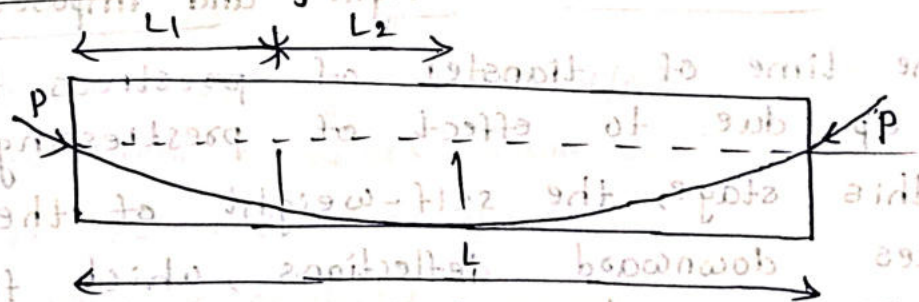


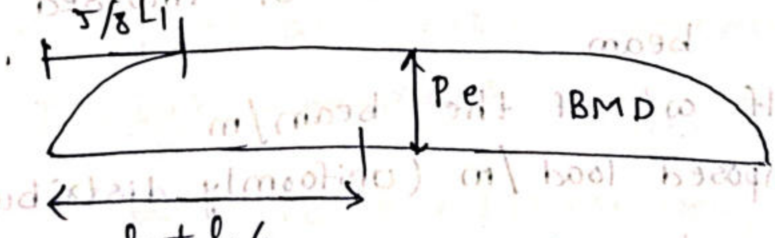


$$a = -\left(\frac{PL^2}{12EI}(e_1+e_2)\right) + \left(\frac{Pe_2L^2}{8EI}\right)$$

$$= \frac{PL^2}{24EI}(-2e_1+e_2)$$

⑥ parabolic and straight tendons:-

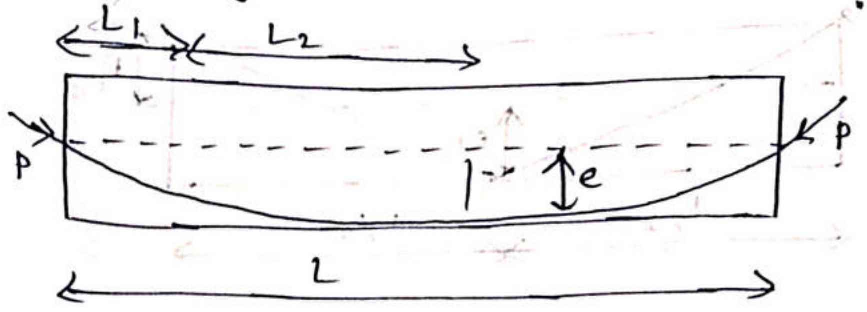


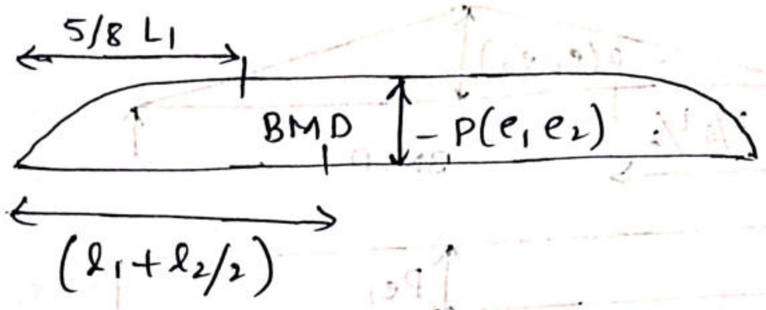


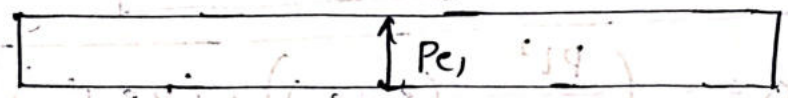
$$a = \frac{-Pe}{EI}\left(\frac{2}{3}l_1\left(5/8\ l_1\right) + l_2\left(l_1 + l_2/2\right)\right)$$

$$= -\frac{Pe}{EI}\left(\left(5l_1^2 + 12l_1l_2 + 6l_2^2\right)\right)$$

(f) parabolic and straight tendons (eccentric Anchors) :-







$$\boxed{a = -\frac{P(e_1+e_2)}{EI}\left(5l_1^2 + 12 l_1 l_2 + 6l_2^2\right) + \left(\frac{Pe_2 L^2}{8EI}\right)}$$

Deflections due to self-weight and imposed load:

→ At the time of transfer of prestress the beam hogs up due to effect of prestressing

→ At this stage, the self-weight of the beam induces downward deflections, which further increase due to effect of imposed load on the beam

If, g = self wt of the beam/m

q = Imposed load/m (uniformly distributed)

The downward deflection is computed as

$$\boxed{a = \frac{5(g+q)\,wl^4}{384EI}}$$

Problem:-

(1) A deck of pre-stressed concrete curvert made up of a slab 500 mm thick the slab is spaning over 10.4 m and supports a UDL comprising the dead and live loads of 23.5 kN/m^2 the modulus of elasticity of concrete is 38 kN/mm^2. The concrete slab is prestressed by straight cables each containing 12 high tensile wires of 7mm dia stressed to 1200 N/mm^2 at constant eccentricity of 195 mm. The cables are spaced at 328 mm intervals the transverse direction estimate the instaneous (or) short term deflection of the slab at centre of spans under pre-stressed and the imposed loads.

given data,

Slab thickness = 500 mm

Sab spanning = 10.4 m

Total load = 23.5 kN/m^2

modulus of elasticity (E) = 38 kN/mm^2

12 high tensile wires = 7mm dia

wire stressed = 1200 N/mm^2

constant eccentricity = 195 mm

cables spaced at = 328 mm intervals

$$I = \frac{bd^3}{12} = \frac{1000 \times (500)^3}{12} = 1.041 \times 10^{10} \ mm^4$$

Force in each cable $= \left(\frac{12 \times 38 \times 1200}{1000}\right) = 547$ kN

spacing of cable intransverse direction = 328 mm

hence the prestressing force per meter width of the slab $P = \left(\frac{1000 \times 547}{328}\right) = 1667.68$

$e = 195$ mm

$$a_p = -\frac{PeL^2}{8EI} = \frac{-1667.68 \times 195 \times (10.4 \times 10^3)^2}{8 \times 38 \times 1.041 \times 10^{10}}$$

$a_p = -11.11$ mm ↑ (upward).

$$a_w = \frac{5WL^4}{384EI} = 12.90 \text{ mm}$$

Resultant deflection $= (-11.25 + 12.90 \text{ mm})$

$= 1.795 \text{mm} \downarrow$ (downward)

② A prestressed concrete beam of rectangular section 120mm wide and 300 mm deep and span over 6m the beam is prestressed by straight cable carrying an effective force 200 kN at an eccentricity 50 mm the modulus of elasticity of 38 kN/mm² compute the deflection at centre of span for the following cases in that

① Deflection under prestressed + self wt

② Find the magnitude of uniformity distributed live load which will nolify the deflection due to prestress and self wt

given data,

wide = 120 mm

deep = 300mm

span = 6m

effective force $P' = 200$ kN

eccentricity = 50 mm

$E = 38 \text{ kN/m}^2$

$$I = \frac{bd^3}{12} = \frac{120 \times 300^3}{12} = 270 \times 10^6 \text{ mm}^4$$

$$W = 0.120 \times 0.300 \times 24 = 0.86 \text{ N/mm}$$

$$a_p = \frac{-PeL^2}{8EI} = \frac{200 \times 50 \times (6000)^2}{8 \times 38 \times 27 \times 10^7} = -4.38 \text{ mm} \uparrow$$

$$a_w = \frac{5wL^4}{384EI} = \frac{5 \times 0.86 \times 10^{-3} \times (6000)^4}{384 \times 38 \times 27 \times 10^7} = 1.41 \text{ mm}$$

$= -4.38 + 1.41$

$= -2.96$ mm $\uparrow$ (upward)

If Q = live load on the beam where neutralized the deflection due to Self wt and prestress this magnitude is calculated.

$$Q_v = \frac{Q \times 384 EI}{5 \times L^4}$$

$$= -\frac{2.96 \times 384 \times 38 \times 27 \times 10^7}{5 \times (6000)^4}$$

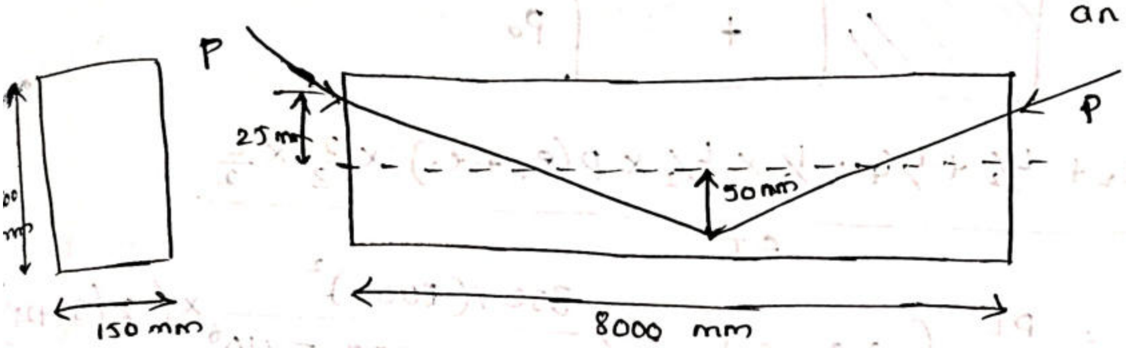
$$= -1.79 \times 10^{-3} \text{ kN/mm}$$

$$Q_v = 1.79 \text{ N/mm}$$

A concrete beam is prestressed by a having eccentricity as shown in fig the force in the cable is ~~3~~350 kN's the beam supports ! concentrated loads of 200 kN at centre of span if $E = 38$ kN/mm² loss ratio 0.85 and the creep co-efficient = 1.6

a) compute

1) Shortterm deflection under presstress and self wt.
2) long term deflection under presstress and self wt and live load



given data,

$$I = \frac{150 \times 300^3}{12} = 337.5 \times 10^6 \text{ mm}^4$$

$P = 350$ kN $= 350 \times 10^{3}$ N

$W_0 = 0.15 \times 0.3 \times 1 \times 24 = 1.08$ kN/m

$L = 8$ m

$W = 20$ kN $= 20 \times 10^{3}$ N

(eta) $\eta = 0.8$, creep co-efficient = 1.6

Deflection due to dead load

$$y_D = \frac{5 W_0 L^4}{384 EI}$$

$$= \frac{5 \times 1.08 \times 10^{-3} \times (8000)^4}{384 \times 38 \times 337.5 \times 10^6}$$

$$\left(= \frac{\cancel{5 \times 1.08 \times 10^{-3} \times (8)^4}}{\cancel{384 \times 38 \times 10^{6} \times \frac{150 \times (300)^3}{12}}} \right)$$

$y_D = 4.49$ mm ↓

Deflection due to live load

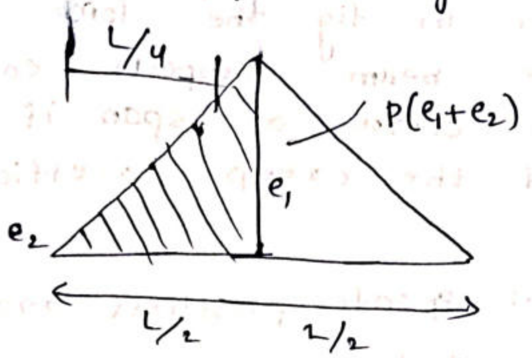
$$y_L = \frac{WL^4}{48 EI}$$

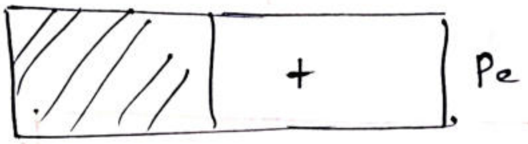
$$\cancel{\frac{20 \times (8)^4}{48 \times 38 \times 10^{-6} \times 0.150 \times (300)^3}}$$

$y_L = \cancel{1663}$ m
166.34 mm

$$y_L = \frac{20 \times (8000)^3}{48 \times 38 \times 337.5 \times 10^6}$$

Deflection due to prestressing force:-





$$y_P = \frac{Pe_2 + L/2 + L/4 - \frac{1}{2} \times L/2 \times P(e_1+e_2) \times \frac{2}{3} \times \frac{L}{2}}{EI}$$

$$y_P = \frac{PL^2}{24EI}\left(-2e_1 - e_2\right) \Rightarrow \frac{350 \times (8000)^2}{24 \times 38 \times 337.5 \times 10^6} \times (-2(50+25$$

$$= \frac{350 \times 10^3 \times (8000)^2}{24 \times 38} \quad \cancel{\frac{350 \times (8)^2}{24 \times 38 \times 10^{-6} \times \frac{0.15 \times (0.3)^3}{12}}} \times (\cancel{-2 \times 0.05} - 0.025)$$

$$\boxed{y_P = -0.00545 \text{ m}} \simeq -5.45 \text{ mm}$$

Shortterm deflection due to prestress and self wt

$= y_P + y_D$

$= -5.45 + 4.49$

$= -0.96$ m ↑ upwards

long-term deflection

$= (1+\phi)(y_D + y_L + \eta\, y_P)$

$= (1+1.6)(4.49 + 16.63 - 0.8 \times 5.45)$

$\Rightarrow 43.56$ mm

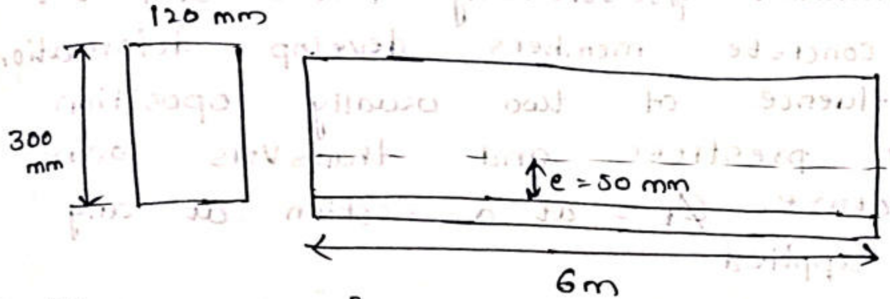
A prestressed concrete beam of rectangular section 120 mm and 300 mm deep having a span of 6m the beam is prestressed by a straight cable. Carrying a effective force 180 kN at an eccentricity of 50 mm it supports a supper imposed load of 4kN/m compute the deflection following stages check weather they compare

a) prestress + self wt

b) pre-stress + self wt + Super imposed laad including the effect creep and shrinkage.

Assume, creep co-efficient = 1.8

$e_c = 36\ kN/mm^2$



$p = 180\ kN = 180 \times 10^3 N$

$W_0 = 0.12 \times 0.3 \times 1 \times 24 = 0.86\ kN/m$

$W_L = 4\ kN/m$

$\phi = 1.8\ ,\ e = 50\ mm$

$I = 270 \times 10^6\ mm^4$

$E = 36 \times 10^3\ N/mm^2$

$L = 6m = 6000\ mm$

Deflection due to dead load

$$y_D = \frac{5 W_0 L^4}{384 EI} = \frac{5 \times 0.86 \times 10^{-3} \times (6000)^4}{384 \times 36 \times 270 \times 10^6} \qquad y_D = 1.5\ mm\ (\downarrow)$$

Deflection due to live load

$$y_L = \frac{5 W_L L^4}{384 EI} = \frac{5 \times 4 \times (6000)^4}{384 \times 36 \times 27 \times 10^7}$$

$y_L = 67.4\ mm\ (\downarrow)$

Deflection due to prestressing force:-

$$y_P = \frac{P \times e \times L^2}{8 EI} = \frac{180 \times 10^3 \times 50 \times (6000)^2}{8 \times 36 \times 10^3 \times 27 \times 10^7}$$

$y_P = 4.16\ mm \downarrow$

Prediction of long term Deflection:-

The deformation of pre-stressed members Change with time as a result of creep and Shrinkage at concrete and relaxation of stress in steel. The deflection of pre-stressed members can be computed relative to ex a given data. If the magnitude and longitudinal distribution of curvatures for the beam span (or) ohm –

are known for that instant waste on the load history which includes prestressing forces and L-L The prestressed concrete members develop, deformation under the influence of two usually oposition effect which are prestress and transvers loads and net curvature ϕ_t = at a section at any given stage is applied

$$\phi_t = \phi_{mt} + \phi_{pt}$$

ϕ_{pt} = Change of curvature caused by transverse loads
ϕ_{mt} = Change of curvature caused by prestress
ϕ_t = Net curvature

under the section of the compressive stress distribution in the Concrete Changes its time

In practical cases the change of stress being small it may be assumed that the concrete creep under constant stresses, the creep strain due to transverse loads is directly computed under the tension of creep Co-efficient. So that the change of curvature can be estimated by the expression:

$$\phi_{mt} = (1+\phi)\phi_{i}$$

where

ϕ = creep co-efficient

ϕ_i = Initia curvature immediately after the application of transverse loads

loss of pre-stressing force due to relaxation, Shrinkage creep

$Lp = (p_i - p_t)$

fore 'e' = eccentricity of prestressing force at the section. , EI = flexural rigidity

The curvature due to prestress after time 't' can be expressed as $\phi_{pt} = \frac{-P_i e}{EI}\left[\left[1 - \frac{Lp}{2p_i}\right]\phi + 1 - \frac{Lp}{P_i}\right]$

a_{i1} = Initial deflection due to transverse load

a_{ip} = Initial deflection due to tr

Total long term deflection after time (t) is obtained to the expression by

$$a_f = a_{i1}(1+\phi) - a_{ip}\left[\left(1 - \frac{Lp}{P_i}\right) + \left(1 - \frac{Lp}{2p_i}\right)\phi\right]$$

In these expression. the negative sign refers the deflection the upward direction

Final longterm deflection

$$a_f = \left[a_{i1} - a_{ip} \times \frac{P_e}{P_i}\right](1+\phi)$$

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CIVIL ENGINEERING

Design of Pre-stressed Concrete Members

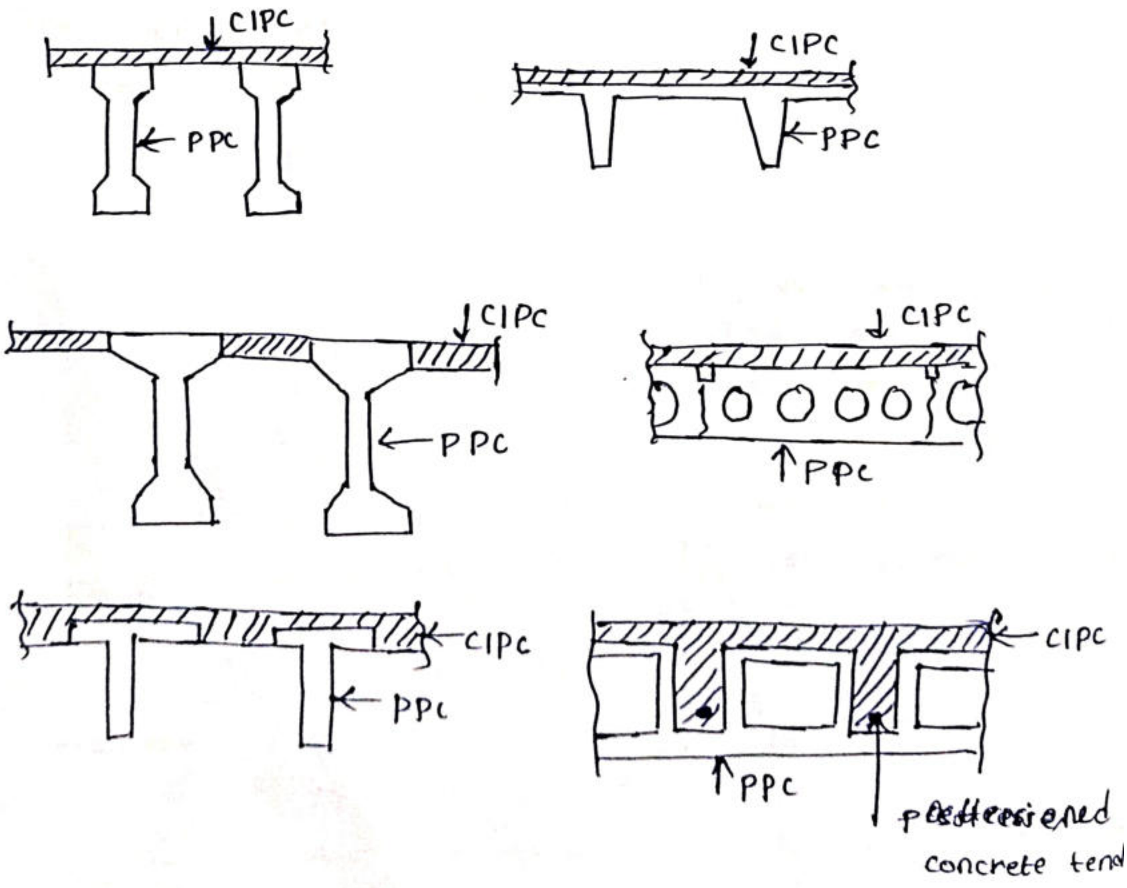
UNIT-5

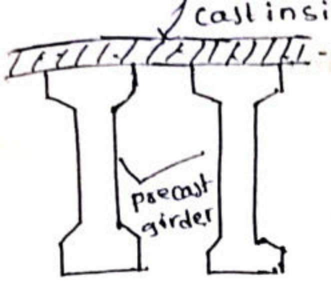
Unit - 5

Composite beam:

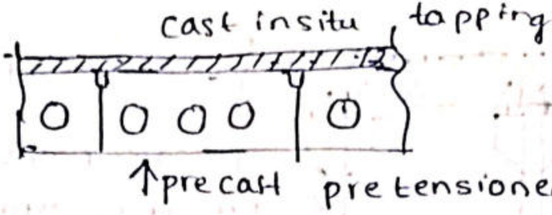
A composite beam is one whose cross-section consists of two or more elements of different materials, acting together while carrying some or all of the loads.

- Composite prestressed concrete consists of precast prestressed beams and cast insitu concrete.
- The insitu portion is not usually prestressed and therefore after consist of lower grade concrete provided with ordinary reinforcement.
- After the insitu concrete has hardened, the two elements perfom as one.
- Depending on the stiffness, the precast member can be designed to carry the weight of the in situ concrete or can be propped, so that it carries only its self wt during casting.
- In latter case the props are removed when the concrete has hardened and the weight of insitu topping is then carried by the composite action.

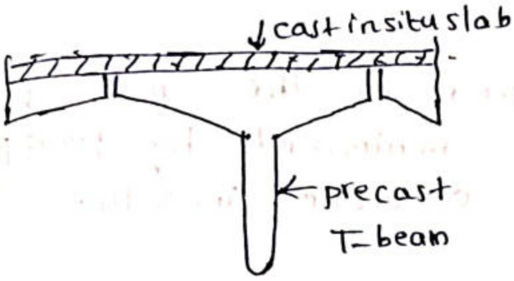




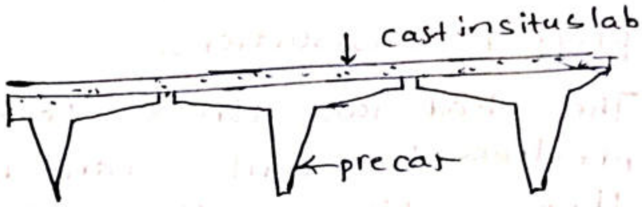
(a) slab and girder



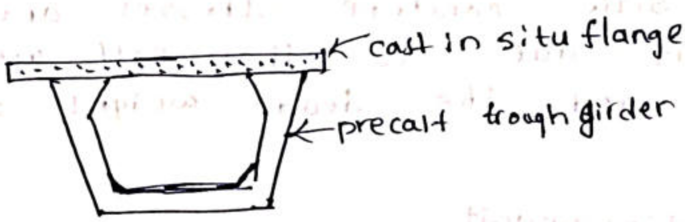
(b) pretensioned plank plus topping



(c) single T-section



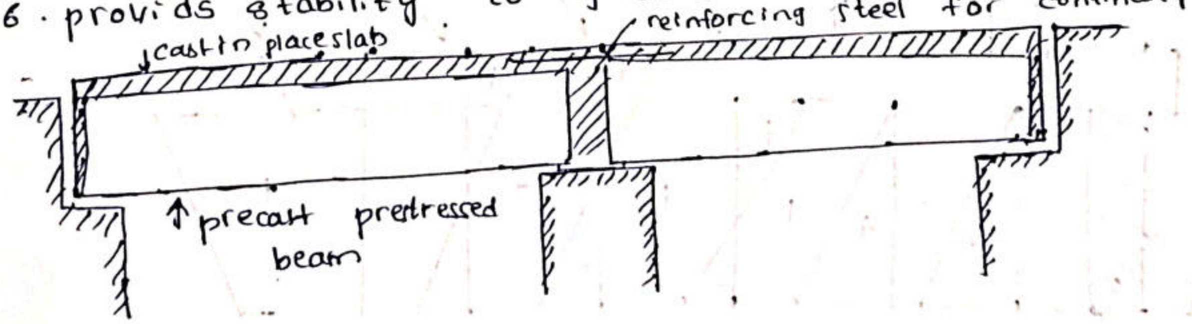
(d) Double T-section

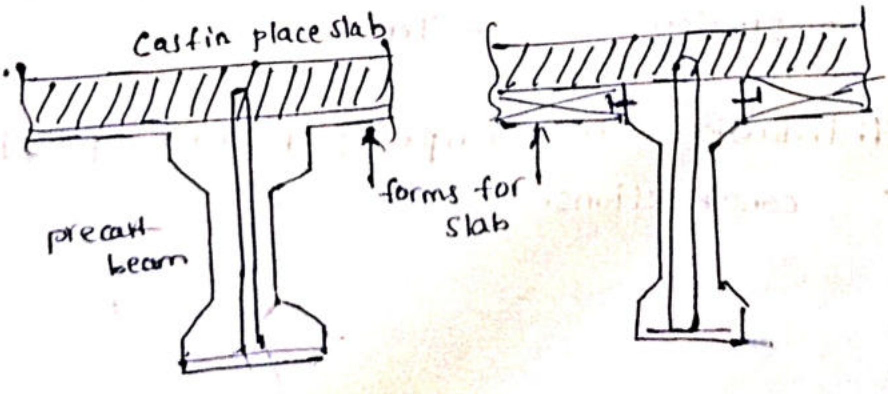


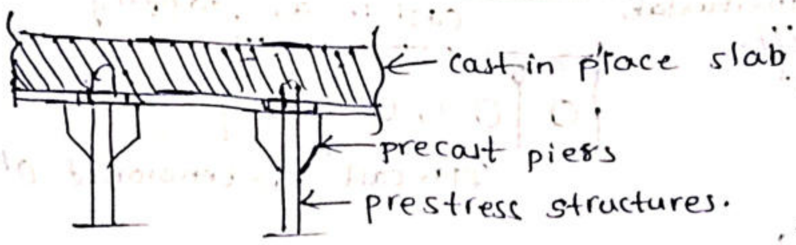
(e) Trough girder.

Advantages:-

1. economical
2. less time consuming
3. Reduction in the false work and shoring cost.
4. No need of formwork and scaffoldings
5. CIPC slab provides continuity at the ends of elements over adjacent spans.
6. provids stability to girders.







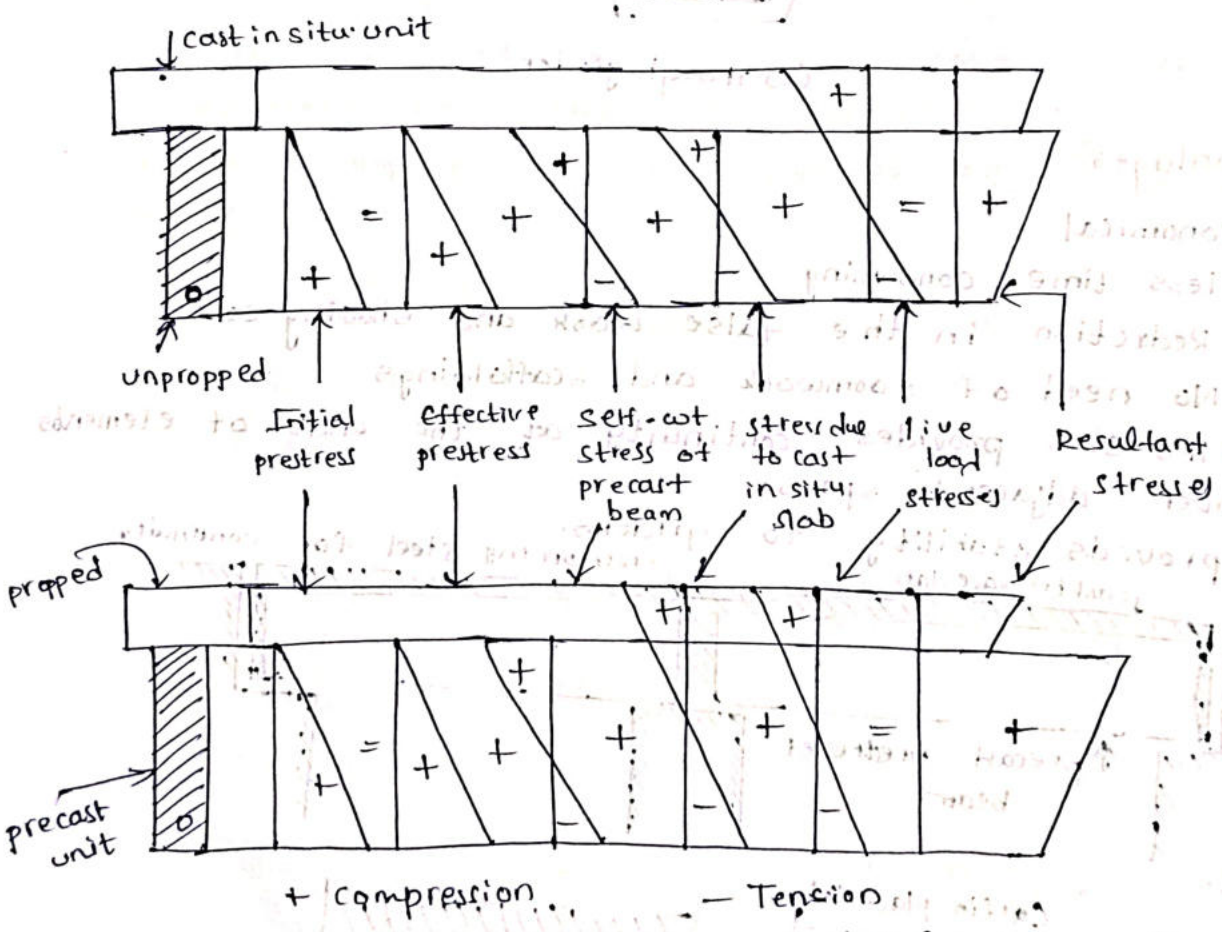
Types of Composite constructions:

1. **propped constructions:**

The dead load stress developed in the precast prestressed units can be minimized by propping them while casting the concrete in situ.

2. **Unpropped Construction:**

If the precast units are not propped while placing the in situ concrete stresses are developed in the unit due to the self weight of the member and the dead weight of the in situ concrete.



Stress distribution in unpropped and propped Composite constructions.

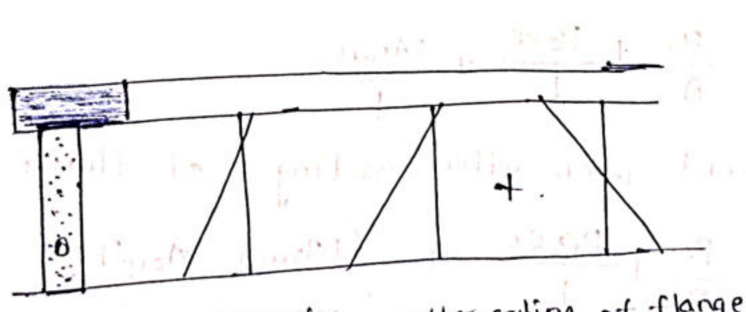
Analysis of Composite section:-

Advantages:

pretensioning in plant is more cost effective than post tensioning on site. Because the precast prestressed concrete element is factory-produced and contains the bulk of reinforcement, rigorous quality control and higher mechanical properties can be achieved at relatively low cost. The cast in situ concrete slab does not need to have high mechanical properties and thus is suitable to field conditions.

The analysis of composite section depends upon the type of ~~construction refers to whether the precast member~~ composite section. the stages of prestressing the type of construction and the loads. The type of construction refers to whether the precast member is propped or unpropped during the casting of the CIP portion. If the precast member is supported by props along

The following diagram are for a composite section with precast web and cast in place flange. The web is prestressed before the flange is cast.
At transfer and after casting of the flange (before the section behaves like a composite section) the following are the stress profiles for the precast web



Section | At transfer due to $P_o + M_{sw}$ | after casting of flange due to $P_e + M_{sw}$ | M_{CIP}.

Stress profiles for the precast web

here

P_o = prestress at transfer after short term losses

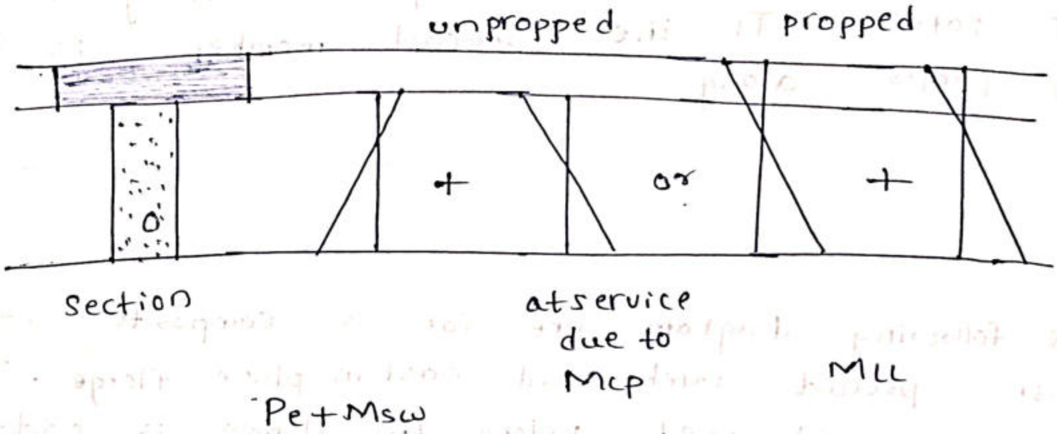
P_e = effective prestress during casting of flange after long term losses.

M_{sw} = Moment due to self weight of precast web.

At transfer the loads acting on the precast web are P_1 and M_{sw}. By the time the flange is cast, the prestress reduces to P_e due to long term losses. In addition to P_e and M_{sw}.

the web also carries M_{cp}. The width of the flange is calculated based on the concept of effective flange width as per clause 23.1.2 IS 456-2000.

At service (after the section behaves like a composite section) the following are the stress profiles for the full depth of the composite section.



Stress profiles for the composite section.

Stress in precast web at transfer:-

$$f = \frac{P_o}{A} \pm \frac{P_o e c}{I} \pm \frac{M_{sw} c}{I}$$

stress in precast web after casting of flange

$$f = -\frac{P_e}{A} \pm \frac{P_e e c}{I} \pm \frac{(M_{sw} + M_{cip}) c}{I}$$

Stress at precast web at service:

i) For unpropped construction.

$$f = -\frac{P_e}{A} \pm \frac{P_e e c}{I} \pm \frac{(M_{sw} + M_{cIP}) c}{I} \pm \frac{M_{LL} c'}{I'}$$

(b) propped construction:-

$$f = -\frac{P_e}{A} \pm \frac{P_e e c}{I} \pm \frac{M_{sw} C}{I} \pm \frac{(M_{cp}+M_{LL}) c'}{I'}$$

where,

A = Area of precast web

c = Distance of edge from CGC of precast web

c' = distance of edge from CGC of composite section

e = eccentricity of CGS

I = moment of inertia of the precast web

I' = moment of inertia of the composite section.

From the analysis for ultimate strength the ultimate moment capacity is calculated This is compared with the demand under factored loads.