Analog Circuits

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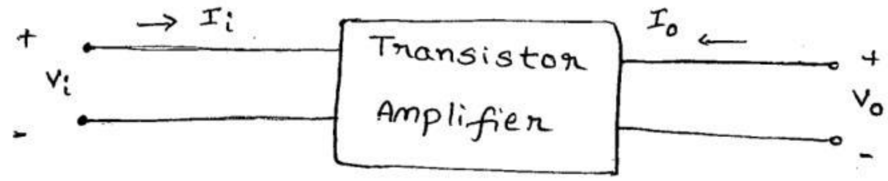
BJT AMPLIFIERS

H-Parameter Representation of a Transistor

A transistor can be treated as a two-port Network



Here I_i = Input current to the Amplifier

V_i = Input voltage to the Amplifier

I_o = output current of the Amplifier

V_o = output voltage of the Amplifier

Transistor is a current operated device.

Here input voltage V_i and output current I_o are the dependent variables.

Input current I_i and output voltage V_o are Independent variables.

$$V_i = f_1(I_1, V_o)$$

$$I_o = f_2(I_i, V_o)$$

This can be written in the equation form as follows

$$V_i = h_{11} I_i + h_{12} V_o$$

$$I_o = h_{21} I_i + h_{22} V_o$$

The above equation can also be written using alphabetic Notations

$$V_i = h_i I_i + h_r V_o$$

$$I_o = h_f I_i + h_o V_o$$

Definitions of h-parameter:

The parameters in the above equation are defined as follows

$h_{11} = h_i = \left.\frac{V_i}{I_i}\right|_{V_o=0}$ = Input resistance with output short circuited.

$h_{12} = h_r = \left.\frac{V_i}{I_o}\right|_{I_i=0}$ = Reverse voltage transfer ratio with input open circuited.

$h_{21} = h_f = \left.\frac{I_o}{I_i}\right|_{V_o=0}$ = short circuit Forward current gain with output short circuited.

$h_{22} = h_o = \left.\frac{I_o}{V_o}\right|_{I_i=0}$ = output Admittance with input open circuited.

BJT H-parameter Model:

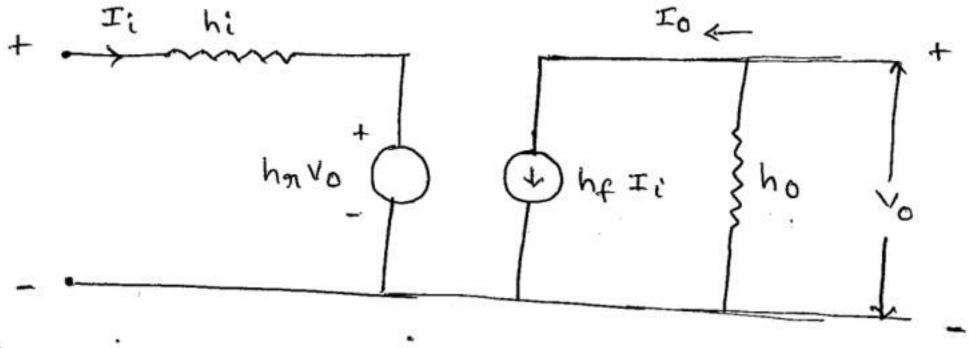
Based on the definition of hybrid parameters the mathematical model for two port networks known as h-parameter model (Hybrid parameter model) can be developed.

The two equations of a transistor is given by

$$V_i = h_i I_i + h_r V_o$$

$$I_o = h_f I_i + h_o V_o$$

Based on above two equations the equivalent circuit or Hybrid Model for transistor can be drawn.



Advantages (or) Benifits of h-parameters

1) Real numbers at audio frequencies
2) Easy to measure
3) Can be obtained from the transistor static characteristic curves.
4) Convinient to use in circuit analysis and design.
5) Easily convertable from one configuration to other
6) Most of the transistor manufacturers sepecify the h-parameters.

H parameter Model for CE Configuration

Let us consider the common emitter configuration shown in figure below. The Variables I_b, I_c, V_b and V_c represent total instantaneous corrents and Voltages.

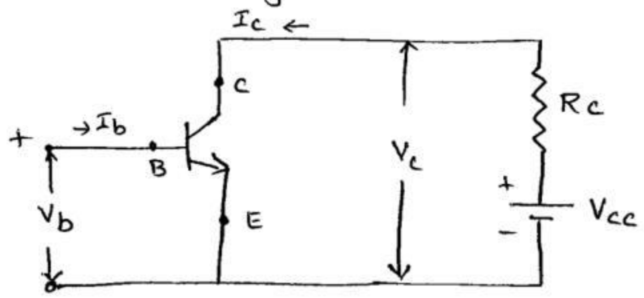
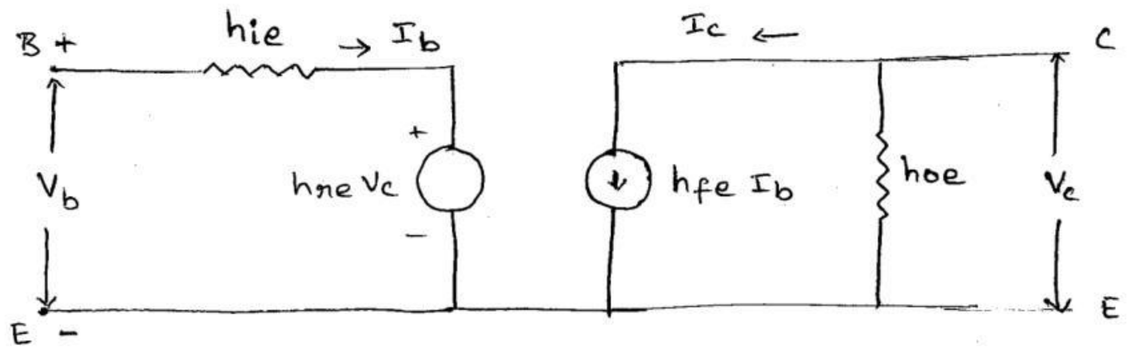


Fig: simple common emitter configuration

Here I_b - Input current V_b - Input voltage

I_c - output current V_c - output voltage

h-parameter model for common emitter configuration is shown in figure below.



$$V_b = h_{ie} I_b + h_{re} V_c$$

$$I_c = h_{fe} I_b + h_{oe} V_c$$

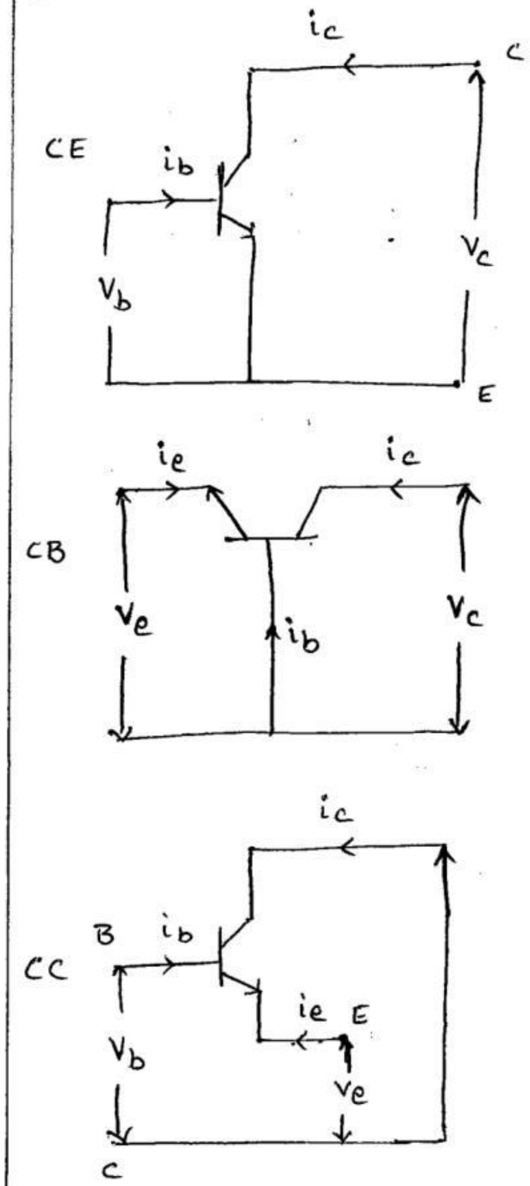
where
$$h_{ie} = \left.\frac{\Delta V_B}{\Delta I_B}\right|_{V_c = \text{constant}} = \left.\frac{V_b}{I_b}\right|_{V_c = \text{constant}}$$

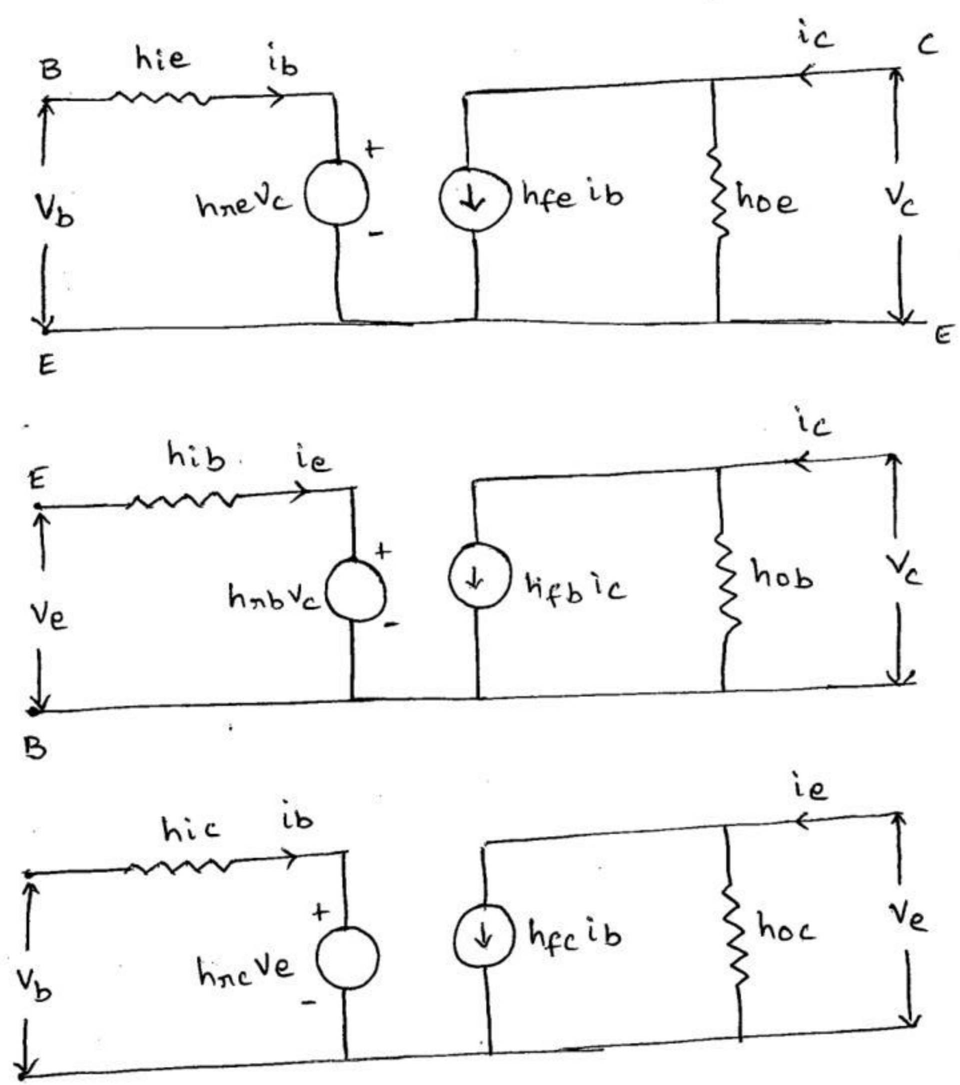
$$h_{re} = \left.\frac{\Delta V_B}{\Delta V_c}\right|_{I_B = \text{constant}} = \left.\frac{V_b}{V_c}\right|_{I_b = \text{constant}}$$

$$h_{fe} = \left.\frac{\Delta I_c}{\Delta I_B}\right|_{V_c = \text{constant}} = \left.\frac{i_c}{i_b}\right|_{V_c = \text{constant}}$$

$$h_{oe} = \left.\frac{\Delta I_c}{\Delta V_c}\right|_{I_B = \text{constant}} = \left.\frac{i_c}{V_c}\right|_{I_b = \text{constant}}$$

Hybrid model for the transistor in three different configurations





$V_b = h_{ie} i_b + h_{re} V_c$

$i_c = h_{fe} i_b + h_{oe} i_c$

$V_e = h_{ib} i_e + h_{rb} V_c$

$i_c = h_{fb} i_e + h_{ob} i_c$

$V_b = h_{ic} i_b + h_{rc} V_e$

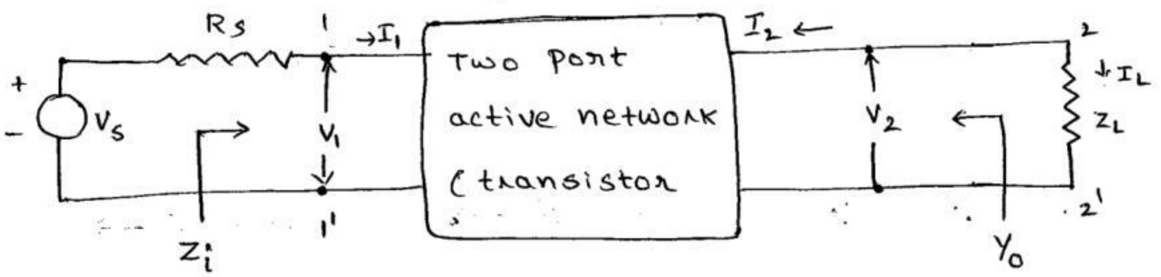
$i_e = h_{fc} i_b + h_{oc} V_e$

Typical h-parameter values for a transistor

Parameter	CE	CC	CB
h_i	1100Ω	1100Ω	22Ω
h_r	2.5×10^{-4}	1	3×10^{-4}
h_f	50	-51	-0.98
h_o	$25\mu A/V$	$25\mu A/V$	$0.49\mu A/V$

Analysis of a transistor amplifier circuit using h-parameter model.

A transistor amplifier can be constructed by connecting an external load and signal source as indicated in figure below. and biasing the transistor properly.



The hybrid parameter model for above network is shown in figure below.

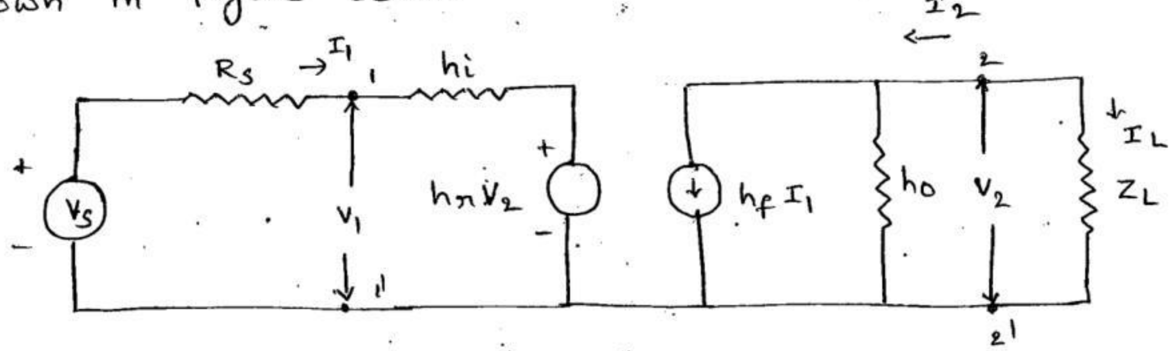


Fig: Transistor hybrid parameter model.

1) **Current Gain (or) Current Amplification A_i :**

For a transistor amplifier the current gain A_I is defined as the ratio of output current to input current.

$$A_I = \frac{I_L}{I_1} = \frac{-I_2}{I_1}$$

From the circuit $I_2 = h_f I_1 + h_o V_2 \longrightarrow (1)$

$$V_2 = I_L Z_L = -I_2 Z_L \longrightarrow (2)$$

Sub (2) in (1)

$$I_2 = h_f I_1 - I_2 Z_L h_o$$

$$I_2 + I_2 Z_L h_o = h_f I_1$$

$$I_2 (1 + Z_L h_o) = h_f I_1 \Rightarrow \frac{I_2}{I_1} = \frac{h_f}{1 + Z_L h_o}$$

$$\boxed{A_I = \frac{-I_2}{I_1} = \frac{-h_f}{1 + Z_L h_o}}$$

	CE	CB	CC
A_I	$\frac{-h_{fe}}{1 + Z_L h_{oe}}$	$\frac{-h_{fb}}{1 + Z_L h_{ob}}$	$\frac{-h_{fc}}{1 + Z_L h_{oc}}$

2) **Input Impedance Z_i**

In the circuit R_s is the signal source resistance the impedance seen when looking in to the amplifier terminals $(1, 1')$ is the amplifier input impedance Z_i

$$Z_i = \frac{V_1}{I_1}$$

From figure $V_1 = h_i I_1 + h_r V_2$

So $$Z_i = \frac{h_i I_1 + h_r V_2}{I_1} = h_i + h_r \frac{V_2}{I_1} \longrightarrow (1)$$

$$V_2 = -I_2 Z_L = A_I I_1 Z_L \qquad \left(\because A_I = \frac{-I_2}{I_1}\right)$$

$$(1) \Rightarrow \quad Z_i = h_i + h_r \frac{A_I I_1 Z_L}{I_1}$$

$$Z_i = h_i + h_r A_I Z_L$$

$$Z_i = h_i - h_r Z_L \frac{h_f}{1 + h_o Z_L} \qquad \left[\because A_I = \frac{-h_f}{1 + h_o Z_L}\right]$$

$$Z_i = h_i - \frac{h_f h_r}{\frac{1}{Z_L} + h_o}$$

$$Z_i = h_i - \frac{h_f h_r}{Y_L + h_o} \qquad \left[\because Y_L = \frac{1}{Z_L}\right]$$

	CE	CB	CC
Z_i	$h_{ie} - \frac{h_{fe} h_{re}}{Y_L + h_{oe}}$	$h_{ib} - \frac{h_{fb} h_{rb}}{Y_L + h_{ob}}$	$h_{ic} - \frac{h_{fc} h_{rc}}{Y_L + h_{oc}}$

3) **Voltage gain (A_V):**

The ratio of output voltage V_2 to input voltage gives the voltage gain of the transistor

$$A_V = \frac{V_2}{V_1}$$

Substituting $V_2 = -I_2 Z_L = A_I I_1 Z_L$

$$\Rightarrow \quad A_V = \frac{A_I I_1 Z_L}{V_1} = \frac{A_I Z_L}{V_1 / I_1} = \frac{A_I Z_L}{Z_i}$$

	CE	CB	CC
A_V	$\frac{A_I Z_L}{Z_i}$	$\frac{A_I Z_L}{Z_i}$	$\frac{A_I Z_L}{Z_i}$

4) Output Admittance (Y_0) :

$$Y_0 = \frac{I_2}{V_2} \quad \text{with} \quad V_s = 0 \quad \text{and} \quad R_L = \infty$$

From the circuit $I_2 = h_f I_1 + h_o V_2$

Dividing by V_2, $\frac{I_2}{V_2} = h_f \frac{I_1}{V_2} + h_o \longrightarrow (1)$

With $V_s = 0$, by KVL in input circuit

$$R_s I_1 + h_i I_1 + h_r V_2 = 0$$

$$I_1 (R_s + h_i) + h_r V_2 = 0$$

Hence $\frac{I_1}{V_2} = \frac{-h_r}{R_s + h_i}$

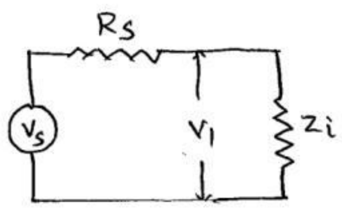
Now Eq (1) $\Rightarrow \frac{I_2}{V_2} = \frac{-h_f h_r}{R_s + h_i} + h_o$

$$\Rightarrow Y_0 = h_o - \frac{h_f h_r}{R_s + h_i}$$

	CE	CB	CC
Y_0	$h_{oe} - \frac{h_{fe} h_{re}}{R_s + h_{ie}}$	$h_{ob} - \frac{h_{fb} h_{rb}}{R_s + h_{ib}}$	$h_{oc} - \frac{h_{fc} h_{rc}}{R_s + h_{ic}}$

5) Voltage gain (A_{vs}) (Including source) :

$$A_{vs} = \frac{V_2}{V_s} = \frac{V_2}{V_1} \frac{V_1}{V_s} \Rightarrow A_{vs} = A_v \frac{V_1}{V_s}$$



$$V_1 = \frac{V_s Z_i}{R_s + Z_i} \Rightarrow \frac{V_1}{V_s} = \frac{Z_i}{R_s + Z_i}$$

Now $A_{vs} = \frac{A_v Z_i}{R_s + Z_i}$

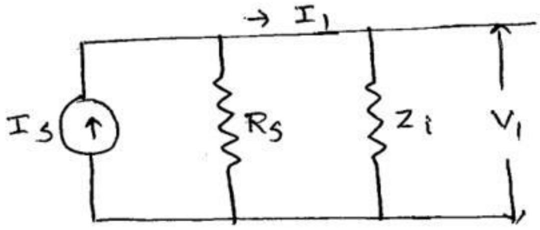
$$A_{VS} = \frac{A_I R_L}{Z_i} \times \frac{Z_i}{R_S + Z_i} = \frac{A_I R_L}{R_S + Z_i}$$

If $R_S = 0$ then $A_{VS} = \frac{A_I R_L}{Z_i} = A_V$.

6) Current Amplification (A_{IS})

$$A_{IS} = \frac{-I_2}{I_S} = \frac{-I_2}{I_1} \cdot \frac{I_1}{I_S} = A_I \frac{I_1}{I_S}$$

The modified input circuit using Norton's equivalent circuit for the source for the calculation of A_{IS}



$$A_{IS} = A_I \frac{R_S}{R_S + Z_i}$$

$$A_{VS} = \frac{A_{IS} Z_L}{R_S}$$

⇒ In CE configuration

Current Gain $A_I = \frac{-h_{fe}}{1 + h_{oe} Z_L}$ $[Z_L = R_L]$

Input Impedance $Z_i = h_{ie} - \frac{h_{fe} h_{re}}{Y_L + h_{oe}}$ $[Y_L = \frac{1}{Z_L} = \frac{1}{R_L}]$

Voltage Gain $A_V = A_I \frac{Z_L}{Z_i}$

Output Admittance $Y_o = h_{oe} - \frac{h_{fe} h_{re}}{h_{ie} + R_S}$

⇒ In CB configuration

Current gain $A_I = \frac{-h_{fb}}{1 + h_{ob} Z_L}$

Input Impedance $Z_i = h_{ib} - \frac{h_{fb} h_{rb}}{Y_L + h_{ob}}$

Voltage gain $A_V = A_I \frac{Z_L}{Z_i}$

Output Admittance $Y_o = h_{ob} - \frac{h_{fb} h_{rb}}{h_{ib} + R_S}$

⇒ In CC configuration

Current gain $A_I = \dfrac{-h_{fc}}{1 + h_{oc} Z_L}$

Input Impedance $Z_i = h_{ic} - \dfrac{h_{fc} h_{rc}}{Y_L + h_{oc}}$

Voltage gain $A_V = \dfrac{A_I Z_L}{Z_i}$

Output Admittance $Y_o = h_{oc} - \dfrac{h_{fc} h_{rc}}{h_{ic} + R_s}$

Conversion formulae for hybrid parameters

CB	CC
$h_{ib} = \dfrac{h_{ie}}{1 + h_{fe}}$	$h_{ic} = h_{ie}$
$h_{rb} = \dfrac{h_{ie} h_{oe}}{1 + h_{fe}} - h_{re}$	$h_{rc} = 1$
$h_{fb} = \dfrac{-h_{fe}}{1 + h_{fe}}$	$h_{fc} = -(1 + h_{fe})$
$h_{ob} = \dfrac{h_{oe}}{1 + h_{fe}}$	$h_{oc} = h_{oe}$

1) **Characteristics of common emitter Amplifier**

1) Current gain A_I is high for $R_L < 10 K\Omega$

2) The voltage gain is high for normal values of Load resistance R_L

3) The input resistance R_i is medium

4) The output resistance R_o is moderately high

Applications of common emitter amplifier:

1. of the three configurations CE amplifier alone is capable of providing both voltage gain and current gain.
2. The output resistance R_o and input resistance R_i are moderately high
3. CE amplifier is widely used for Amplification purpose

2) Characteristics of common Base Amplifier:

1. Current gain is less than unity and its magnitude decreases with the increase of load resistance R_L
2. voltage gain A_v is high for normal values of R_L
3. The input resistance R_i is the lowest of all the three configurations.
4. The output resistance R_o is the highest of all the three configurations.

Applications of common base Amplifier

The CB Amplifier is not commonly used for Amplification purpose. It is used for

1) Matching a very low impedance source.
2) As a non inverting amplifier with voltage gain exceeding unity
3) For driving a high impedance load
4) As a constant current source.

3) Characteristics of common collector Amplifier

1. For low value of R_L ($< 10 k\Omega$) the current gain A_I is high and almost equal to that of a CE amplifier

2. The voltage gain A_V is less than unity.

3. The input resistance is the highest of all the three configurations.

4. The output resistance is the lowest of all the three configurations.

Applications of common collector Amplifier:

1. The CC Amplifier is widely used as a buffer stage between a high impedance source and low impedance load. (CC Amplifier is called emitter follower)

Comparison of Transistor Amplifier Configurations.

The characteristics of three configurations are summarized in table below. Here the quantities A_I, A_V, R_i, R_o and A_p (Power gain) are calculated for $R_L = R_S = 3\,k\Omega$

Quantity	CB	CC	CE
A_I	0.98	47.5	-46.5
A_V	131	0.989	-131
A_p	128.38	46.98	6091.5
R_i	22.6 Ω	144 kΩ	1065 Ω
R_o	1.72 MΩ	80.5 Ω	45.5 kΩ

Simplified CE Hybrid model (or) Approximate CE Hybrid model (Approximate Analysis):

As the h parameters themselves vary widely for the same type of transistor, It is justified to make approximations and simplify the expressions for A_I, A_V, A_P, R_i and R_o.

The behaviour of the transistor circuit can be obtained by using the simplified hybrid model. The h-parameter equivalent circuit of the transistor in the CE configuration is shown in figure below.

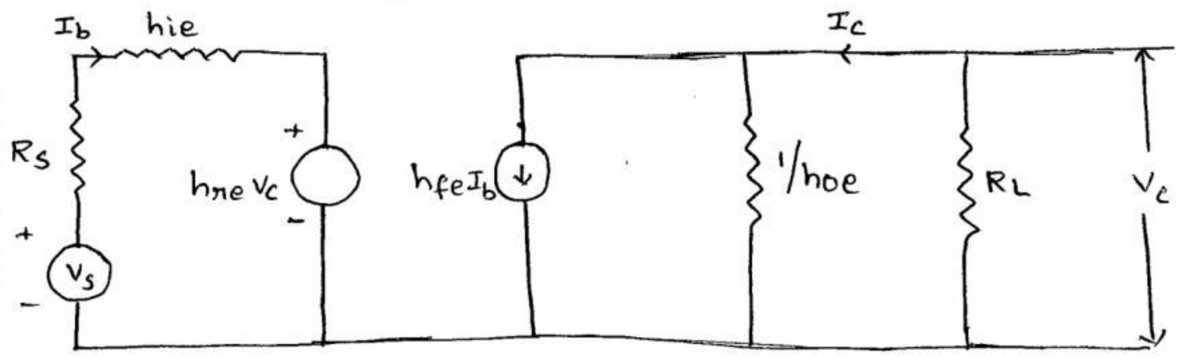


Fig: Exact CE Hybrid model.

Here $\frac{1}{h_{oe}}$ is in parallel with R_L

The parallel combination of two unequal impedances is approximately equal to the lower value ie R_L. Hence if $\frac{1}{h_{oe}} >> R_L$, then the term h_{oe} may be neglected provided that $h_{oe} R_L << 1$

If h_{oe} is omitted, the collector current I_c is given by

$$I_c = h_{fe} I_b .$$

under this condition the magnitude of voltage generated in the emitter circuit is

$$h_{re} |V_c| = h_{re} I_c R_L = h_{re} h_{fe} I_b R_L$$

since $h_{re} h_{fe} \approx 0.01$, this voltage may be neglected in comparison with the voltage drop across h_{ie}. ie $h_{ie} I_b$ provided that R_L is not too large. ie if the load resistance R_L is small it is possible to neglect the parameter h_{re} and h_{oe} and the approximate equivalent circuit is obtained as shown in figure below.

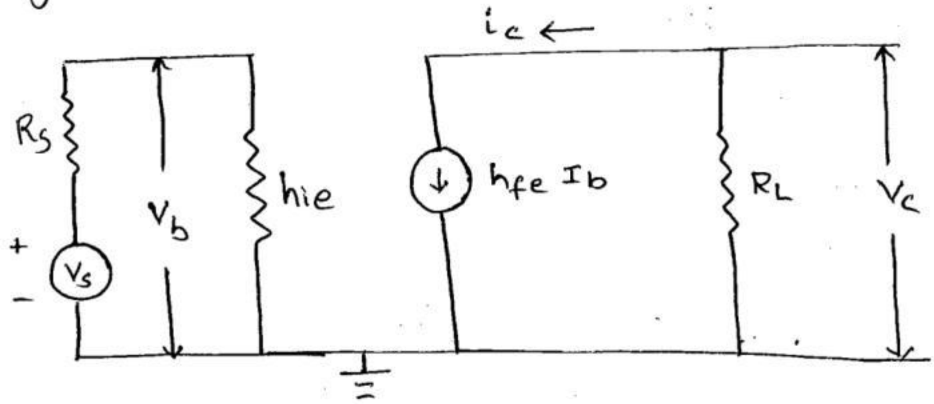


Fig: Approximate CE Hybrid model.

1) Current Gain (A_I):

The current gain for CE configuration is

$$A_I = \frac{-h_{fe}}{1 + h_{oe} R_L} \quad , \quad \text{If } h_{oe} R_L < 0.1$$

$$\boxed{A_I = -h_{fe}}$$

2) Input Impedance (Z_i):

By exact analysis $Z_i = R_i = \frac{V_1}{I_1}$

$V_1 = h_{ie} I_1 + h_{re} V_2$

$$Z_i = \frac{h_{ie} I_1 + h_{re} V_2}{I_1} = h_{ie} + h_{re} \frac{V_2}{I_1}$$

$$V_2 = -I_2 Z_L = -I_2 R_L = A_I I_1 R_L \quad \left[\because A_I = \frac{-I_2}{I_1}\right]$$

$$\Rightarrow Z_i = h_{ie} + \frac{h_{re} A_I I_1 R_L}{I_1} \quad \left[\because V_2 = A_I I_1 R_L\right]$$

$$R_i = \left[h_{ie} + h_{re} A_I R_L\right]$$

$$R_i = h_{ie}\left[1 + \frac{h_{re} A_I R_L}{h_{ie}}\right]$$

$$R_i = h_{ie}\left[1 + \frac{h_{re} A_I R_L}{h_{ie}} \times \frac{h_{fe} h_{oe}}{h_{fe} h_{oe}}\right]$$

using the typical values for the h-parameters

$$\frac{h_{re} h_{fe}}{h_{ie} h_{oe}} \simeq 0.5$$

$$\Rightarrow R_i = h_{ie}\left[1 + \frac{0.5 A_I R_L h_{oe}}{h_{fe}}\right]$$

we know that $A_I = \dfrac{-h_{fe}}{1 + h_{oe} R_L}$ If $h_{oe} R_L < 0.1$

then $A_I = -h_{fe}$

$$\Rightarrow R_i = h_{ie}\left[1 - \frac{0.5 h_{fe} R_L h_{oe}}{h_{fe}}\right]$$

$$\Rightarrow R_i = h_{ie}\left[1 - 0.5 h_{oe} R_L\right]$$

If $h_{oe} R_L < 0.1$

then $\boxed{R_i \simeq h_{ie}}$ $\left[R_i = Z_i\right]$

Voltage gain: $A_V = A_I \frac{R_L}{R_i} = \frac{-h_{fe} R_L}{h_{ie}}$

Output Impedance:

It is the ratio of V_c to I_c with $V_s = 0$ and R_L excluded. The simplified circuit has infinite output impedance because with $V_s = 0$ and external voltage source applied at output, it is found that $I_b = 0$ and hence $I_c = 0$

$$R_o = \frac{V_c}{I_c} = \infty \quad [\because I_c = 0]$$

Approximate analysis of CE Amplifier

current gain $A_I = -h_{fe}$

Input resistance $R_i = h_{ie}$

Voltage gain $A_V = \frac{-h_{fe} R_L}{h_{ie}}$

output resistance $R_o = \infty$

Analysis of CC Amplifier Using the approximate Model:

Figure shows the equivalent circuit of CC Amplifier using the approximate model with the collector grounded, input signal applied between base and ground and load connected between emitter and ground.

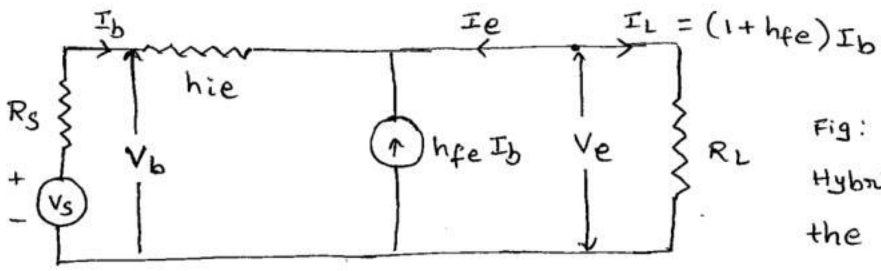


Fig: Simplified Hybrid model for the CC circuit

1) current gain :-

$$A_I = \frac{I_L}{I_b} = \frac{(1+h_{fe})\, I_b}{I_b} = (1+h_{fe})$$

2) Input resistance

$$V_b = I_b h_{ie} + (1+h_{fe})\, I_b R_L$$

$$R_i = \frac{V_b}{I_b} = h_{ie} + (1+h_{fe})\, R_L$$

3) Voltage gain

$$A_V = \frac{V_e}{V_b} = \frac{(1+h_{fe})\, I_b R_L}{\left[h_{ie} I_b + (1+h_{fe})\, I_b R_L\right]}$$

$$A_V = \frac{(1+h_{fe})\, R_L}{h_{ie} + (1+h_{fe})\, R_L} = \frac{h_{ie} + (1+h_{fe})\, R_L - h_{ie}}{h_{ie} + (1+h_{fe})\, R_L}$$

$$A_V = 1 - \frac{h_{ie}}{h_{ie} + (1+h_{fe})\, R_L}$$

$$A_V = 1 - \frac{h_{ie}}{R_i} \qquad \left[\because\ R_i = h_{ie} + (1+h_{fe})\, R_L\right]$$

4) Output Impedance :-

$$\text{output admittance } (Y_o) = \frac{\text{short circuit current in o/p terminals}}{\text{open circuit voltage b/n o/p terminals}}$$

short circuit current in output terminals $= (1+h_{fe})\, I_b = (1+h_{fe}) \dfrac{V_s}{R_s + h_{ie}}$

open circuit voltage b/n output terminals $= V_s$

$$\therefore \quad Y_o = \frac{1+h_{fe}}{R_s + h_{ie}} \Rightarrow R_o = \frac{h_{ie} + R_s}{1 + h_{fe}}$$

output impedance including R_L ie $R_o' = R_o \| R_L$

Analysis of CB Amplifier using the approximate model

Figure shows the equivalent circuit of CB amplifier using the approximate model, with the base grounded, input signal is applied between emitter and base and load connected between collector and base

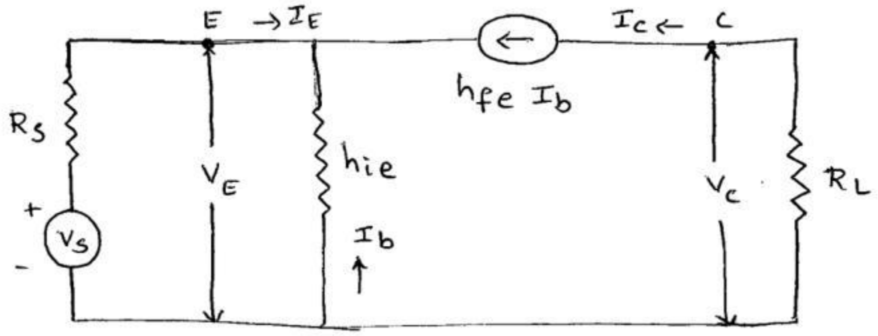


Fig: Simplified Hybrid model for the CB circuit

1) Current gain:

From the figure above $A_I = \frac{-I_c}{I_e} = \frac{-h_{fe} I_b}{I_e}$

$$I_e = -(I_b + I_c)$$

$$I_e = -(I_b + h_{fe} I_b) = -(1 + h_{fe}) I_b$$

$$\therefore A_I = \frac{-h_{fe} I_b}{-(1+h_{fe}) I_b} = \frac{h_{fe}}{1 + h_{fe}} = -h_{fb}$$

2) Input Resistance:

Input Resistance $R_i = \frac{V_e}{I_e}$

From figure $V_e = -I_b h_{ie}$, $I_e = -(1+h_{fe}) I_b$

$$R_i = \frac{h_{ie}}{1 + h_{fe}} = h_{ib}$$

3) Voltage gain:

$$A_V = \frac{V_c}{V_e}$$

$$V_c = -I_c R_L = -h_{fe} I_b R_L$$

$$V_e = -I_b h_{ie}$$

$$A_V = \frac{h_{fe} R_L}{h_{ie}}$$

Output Impedance

$$R_o = \frac{V_c}{I_c} \quad \text{with} \quad V_s = 0, \quad R_L = \infty$$

with $V_s = 0$, $I_e = 0$ and $I_b = 0$ hence $I_c = 0$

$$\therefore \quad R_o = \frac{V_c}{0} = \infty$$

Approximate Analysis of CB Amplifier

1) Current gain $A_I = \dfrac{h_{fe}}{1 + h_{fe}} = -h_{fb}$

2) Input Resistance $R_i = \dfrac{h_{ie}}{1 + h_{fe}} = h_{ib}$

3) Voltage gain $A_V = \dfrac{h_{fe} R_L}{h_{ie}}$

4) Output Resistance $R_o = \infty$

Approximate Analysis of CC Amplifier

1) Current gain $A_I = (1 + h_{fe})$

2) Input resistance $R_i = h_{ie} + (1 + h_{fe}) R_L$

3) Voltage gain $A_V = 1 - \dfrac{h_{ie}}{R_i}$

4) Output Resistance $R_o = \dfrac{h_{ie} + R_s}{1 + h_{fe}}$

Problem: A CE Amplifier is drawn by a voltage source of Internal resistance $r_s = 800\Omega$ and the load impedance is a resistance $R_L = 1000\Omega$. The h parameters are $h_{ie} = 1K\Omega$, $h_{re} = 2\times10^{-4}$, $h_{fe} = 50$ and $h_{oe} = 25\mu A/V$. compote the current gain A_I, input resistance R_i, voltage gain A_V, and output resistance R_o using exact analysis and approximate analysis.

Solution: Given data

$r_s = 800\Omega$, $R_L = 1000\Omega$, $h_{ie} = 1K\Omega$, $h_{re} = 2\times10^{-4}$, $h_{fe} = 50$, and $h_{oe} = 25\mu A/V$

Exact Analysis:-

Current Gain $A_I = \dfrac{-h_{fe}}{1 + h_{oe}R_L} = -48.78$

Input Resistance $R_i = h_{ie} - \dfrac{h_{fe}h_{re}}{h_{oe} + \frac{1}{R_L}} = 990.24\Omega$

Voltage gain $A_V = A_I \dfrac{R_L}{R_i} = -49.26$

Output Resistance

$$Y_o = h_{oe} - \frac{h_{fe}h_{re}}{h_{ie} + R_s} = 194\times10^{-5}\ \text{mho}$$

$$R_o = \frac{1}{Y_o} = 51.42\ K\Omega$$

Approximate Analysis:

$$A_I = -h_{fe} = -50$$

$$R_i = h_{ie} = 1\ K\Omega$$

$$A_V = \frac{-h_{fe} R_L}{h_{ie}} = \frac{-50 \times 1000}{1000} = -50$$

$$R_O = \infty$$

Problem: A voltage source of internal resistance $R_S = 900\,\Omega$ drives a CC amplifier using load resistance $R_L = 2000\,\Omega$. The CE h-parameters are $h_{ie} = 1200\,\Omega$, $h_{re} = 2 \times 10^{-4}$, $h_{fe} = 60$ and $h_{oe} = 25\,\mu A/V$. Compute the current gain A_I, input resistance R_i, voltage gain A_V, and output resistance R_O using exact analysis and approximate analysis.

Sol Conversion formulae:

$$h_{ic} = h_{ie} = 1200\,\Omega$$

$$h_{fc} = -(1 + h_{fe}) = -(1 + 60) = -61$$

$$h_{rc} = 1$$

$$h_{oc} = h_{oe} = 25\,\mu A/V$$

Exact Analysis:

$$A_I = \frac{-h_{fc}}{1 + h_{oc} R_L} = 58.095$$

$$R_i = h_{ic} - \frac{h_{fc} h_{rc}}{Y_L + h_{oe}} = 117.39\,K\Omega$$

$$A_V = \frac{A_I R_L}{R_i} = 0.9897$$

Output Admittance

$$Y_0 = h_{oc} - \frac{h_{fc}\, h_{rc}}{h_{ic} + R_s}$$

$$\Rightarrow R_O = \frac{1}{Y_0} = 34.396\,\Omega$$

Approximate Analysis

$$A_I = 1 + h_{fe} = 1 + 60 = 61$$

$$R_i = h_{ie} + (1 + h_{fe}) R_L = 123.2\,K\Omega$$

$$A_V = 1 - \frac{h_{ie}}{R_i} = 0.99$$

$$R_o = \frac{h_{ie} + R_s}{1 + h_{fe}} = 34.43\,\Omega$$

Problem:

For a CB transistor Amplifier driven by a voltage source of internal resistance $R_s = 1200\Omega$, the load impedance is a resistor $R_L = 1000\Omega$. The h-parameters are $h_{ib} = 22\Omega$, $h_{rb} = 3\times10^{-4}$, $h_{fb} = -0.98$, $h_{ob} = 0.5\,\mu A/V$. Compute the current gain A_I, Input impedance R_i, voltage gain A_V, overall voltage gain A_{VS}, over all current gain A_{IS}, output impedance R_o and power gain A_p using exact and approximate analysis.

Solution:

$$\text{Current gain } A_I = \frac{-h_{fb}}{1 + h_{ob} R_L} = 0.98$$

$$\text{Input Impedance } R_i = h_{ib} - \frac{h_{fb}\, h_{rb}}{Y_L + h_{ob}} = 22.3\Omega$$

Coupling of the amplifier

- **RC coupling**
- **Transformer Coupling**
- **Direct Coupling**

Purpose of coupling device:

The basic purposes of a coupling device are

- To transfer the AC from the output of one stage to the input of next stage.
- To block the DC to pass from the output of one stage to the input of next stage, which means to isolate the DC conditions.

Types of Coupling

Joining one amplifier stage with the other in cascade, using coupling devices form a **Multi-stage amplifier circuit**. There are **four** basic methods of coupling, using these coupling devices such as resistors, capacitors, transformers etc. Let us have an idea about them.

Resistance-Capacitance Coupling

This is the mostly used method of coupling, formed using simple **resistor-capacitor** combination. The capacitor which allows AC and blocks DC is the main coupling element used here.

The coupling capacitor passes the AC from the output of one stage to the input of its next stage. While blocking the DC components from DC bias voltages to effect the next stage. Let us get into the details of this method of coupling in the coming chapters.

Impedance Coupling

The coupling network that uses **inductance** and **capacitance** as coupling elements can be called as Impedance coupling network.

In this impedance coupling method, the impedance of coupling coil depends on its inductance and signal frequency which is **jwL**. This method is not so popular and is seldom employed.

Transformer Coupling

The coupling method that uses a **transformer as the coupling** device can be called as Transformer coupling. There is no capacitor used in this method of coupling because the transformer itself conveys the AC component directly to the base of second stage.

The secondary winding of the transformer provides a base return path and hence there is no need of base resistance. This coupling is popular for its efficiency and its impedance matching and hence it is mostly used.

Direct Coupling

If the previous amplifier stage is connected to the next amplifier stage directly, it is called as **direct coupling**. The individual amplifier stage bias conditions are so designed that the stages can be directly connected without DC isolation.

The direct coupling method is mostly used when the load is connected in series, with the output terminal of the active circuit element. For example, head-phones, loud speakers etc.

Role of Capacitors in Amplifiers

Other than the coupling purpose, there are other purposes for which few capacitors are especially employed in amplifiers. To understand this, let us know about the role of capacitors in Amplifiers.

The Input Capacitor C_{in}

The input capacitor C_{in} present at the initial stage of the amplifier, couples AC signal to the base of the transistor. This capacitor C_{in} if not present, the signal source will be in parallel to resistor R_2 and the bias voltage of the transistor base will be changed.

Hence C_{in} allows, the AC signal from source to flow into input circuit, without affecting the bias conditions.

The Emitter By-pass Capacitor C_e

The emitter by-pass capacitor C_e is connected in parallel to the emitter resistor. It offers a low reactance path to the amplified AC signal.

In the absence of this capacitor, the voltage developed across R_E will feedback to the input side thereby reducing the output voltage. Thus in the presence of C_e the amplified AC will pass through this.

Coupling Capacitor C_C

The capacitor C_C is the coupling capacitor that connects two stages and prevents DC interference between the stages and controls the operating point from shifting. This is also called as **blocking capacitor** because it does not allow the DC voltage to pass through it.

In the absence of this capacitor, R_C will come in parallel with the resistance R_1 of the biasing network of the next stage and thereby changing the biasing conditions of the next stage.

Amplifier Consideration

For an amplifier circuit, the overall gain of the amplifier is an important consideration. To achieve maximum voltage gain, let us find the most suitable transistor configuration for cascading.

CC Amplifier

- Its voltage gain is less than unity.
- It is not suitable for intermediate stages.

CB Amplifier

- Its voltage gain is less than unity.
- Hence not suitable for cascading.

CE Amplifier

- Its voltage gain is greater than unity.
- Voltage gain is further increased by cascading.

The characteristics of CE amplifier are such that, this configuration is very suitable for cascading in amplifier circuits. Hence most of the amplifier circuits use CE configuration.

In the subsequent chapters of this tutorial, we will explain the types of coupling amplifiers.

- From the above graph, it is understood that the frequency rolls off or decreases for the frequencies below 50Hz and for the frequencies above 20 KHz. whereas the voltage gain for the range of frequncies between 50Hz and 20 KHz is constant.

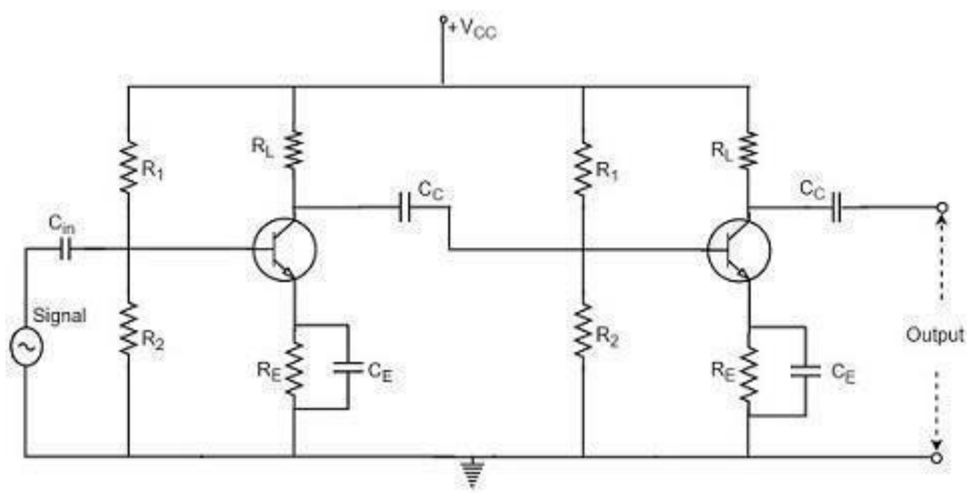
RC Coupling Amplifier

The resistance-capacitance coupling is, in short termed as RC coupling. This is the mostly used coupling technique in amplifiers.

Construction of a Two-stage RC Coupled Amplifier

The constructional details of a two-stage RC coupled transistor amplifier circuit are as follows. The two stage amplifier circuit has two transistors, connected in CE configuration and a common power supply V_{CC} is used. The potential divider network R_1 and R_2 and the resistor R_e form the biasing and stabilization network. The emitter by-pass capacitor C_e offers a low reactance path to the signal.

The resistor R_L is used as a load impedance. The input capacitor C_{in} present at the initial stage of the amplifier couples AC signal to the base of the transistor. The capacitor C_C is the coupling capacitor that connects two stages and prevents DC interference between the stages and controls the shift of operating point. The figure below shows the circuit diagram of RC coupled amplifier.



Operation of RC Coupled Amplifier

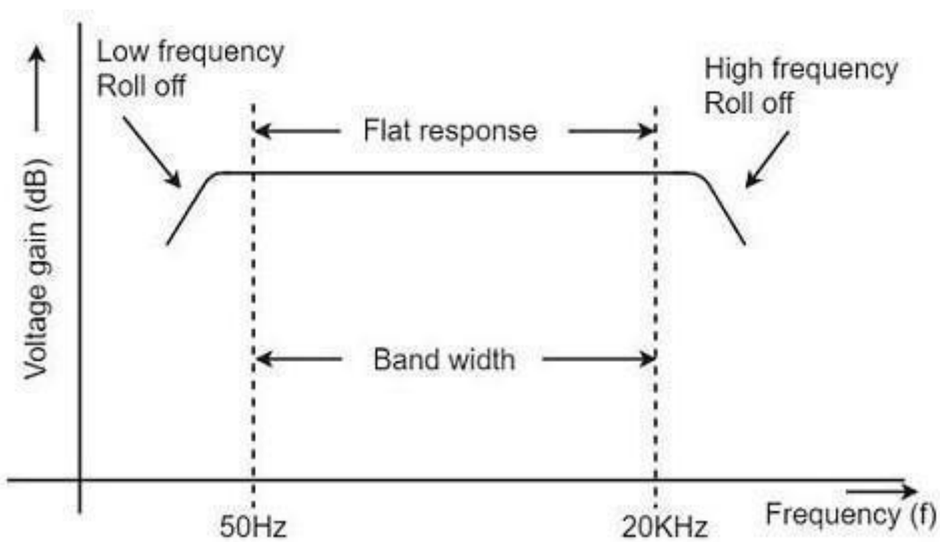
When an AC input signal is applied to the base of first transistor, it gets amplified and appears at the collector load R_L which is then passed through the coupling capacitor C_C to the next stage. This becomes the input of the next stage, whose amplified output again appears across its collector load. Thus the signal is amplified in stage by stage action.

The important point that has to be noted here is that the total gain is less than the product of the gains of individual stages. This is because when a second stage is made to follow the first stage, the **effective load resistance** of the first stage is reduced due to the shunting effect of the input resistance of the second stage. Hence, in a multistage amplifier, only the gain of the last stage remains unchanged.

As we consider a two stage amplifier here, the output phase is same as input. Because the phase reversal is done two times by the two stage CE configured amplifier circuit.

Frequency Response of RC Coupled Amplifier

Frequency response curve is a graph that indicates the relationship between voltage gain and function of frequency. The frequency response of a RC coupled amplifier is as shown in the following graph.



From the above graph, it is understood that the frequency rolls off or decreases for the frequencies below 50Hz and for the frequencies above 20 KHz. whereas the voltage gain for the range of frequencies between 50Hz and 20 KHz is constant.

We know that,

$$X_C=12\pi f_c XC=12\pi fc$$

It means that the capacitive reactance is inversely proportional to the frequency.

At Low frequencies (i.e. below 50 Hz)

The capacitive reactance is inversely proportional to the frequency. At low frequencies, the reactance is quite high. The reactance of input capacitor C_{in} and the coupling capacitor C_c are so high that only small part of the input signal is allowed. The reactance of the emitter by pass capacitor C_E is also very high during low frequencies. Hence it cannot shunt the emitter resistance effectively. With all these factors, the voltage gain rolls off at low frequencies.

At High frequencies (i.e. above 20 KHz)

Again considering the same point, we know that the capacitive reactance is low at high frequencies. So, a capacitor behaves as a short circuit, at high frequencies. As a result of this, the loading effect of the next stage increases, which reduces the voltage gain. Along with this, as the capacitance of emitter diode decreases, it increases the base current of the transistor due to which the current gain (β) reduces. Hence the voltage gain rolls off at high frequencies.

At Mid-frequencies (i.e. 50 Hz to 20 KHz)

The voltage gain of the capacitors is maintained constant in this range of frequencies, as shown in figure. If the frequency increases, the reactance of the capacitor C_C decreases which tends to increase the gain. But this lower capacitance reactive increases the loading effect of the next stage by which there is a reduction in gain.

Due to these two factors, the gain is maintained constant.

Advantages of RC Coupled Amplifier

The following are the advantages of RC coupled amplifier.

- The frequency response of RC amplifier provides constant gain over a wide frequency range, hence most suitable for audio applications.
- The circuit is simple and has lower cost because it employs resistors and capacitors which are cheap.
- It becomes more compact with the upgrading technology.

Disadvantages of RC Coupled Amplifier

The following are the disadvantages of RC coupled amplifier.

- The voltage and power gain are low because of the effective load resistance.
- They become noisy with age.
- Due to poor impedance matching, power transfer will be low.

Applications of RC Coupled Amplifier

The following are the applications of RC coupled amplifier.

- They have excellent audio fidelity over a wide range of frequency.
- Widely used as Voltage amplifiers
- Due to poor impedance matching, RC coupling is rarely used in the final stages.

Transformer Coupled Amplifier

We have observed that the main drawback of RC coupled amplifier is that the effective load resistance gets reduced. This is because, the input impedance of an amplifier is low, while its output impedance is high.

When they are coupled to make a multistage amplifier, the high output impedance of one stage comes in parallel with the low input impedance of next stage. Hence, effective load resistance is decreased. This problem can be overcome by a **transformer coupled amplifier**.

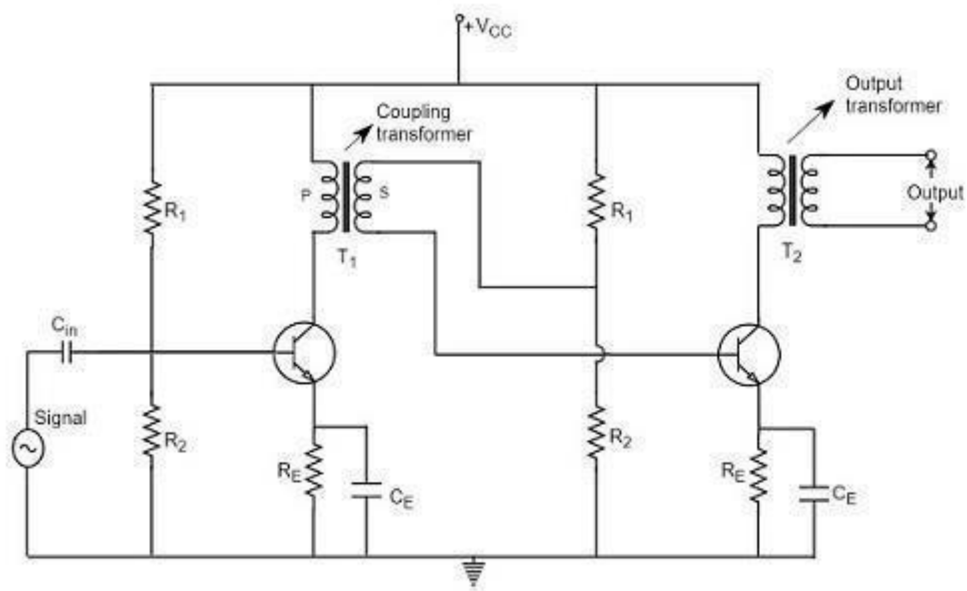
In a transformer-coupled amplifier, the stages of amplifier are coupled using a transformer. Let us go into the constructional and operational details of a transformer coupled amplifier.

Construction of Transformer Coupled Amplifier

The amplifier circuit in which, the previous stage is connected to the next stage using a coupling transformer, is called as Transformer coupled amplifier.

The coupling transformer T_1 is used to feed the output of 1st stage to the input of 2nd stage. The collector load is replaced by the primary winding of the transformer. The secondary winding is connected between the potential divider and the base of 2nd stage, which provides the input to the 2nd stage. Instead of coupling capacitor like in RC coupled amplifier, a transformer is used for coupling any two stages, in the transformer coupled amplifier circuit.

The figure below shows the circuit diagram of transformer coupled amplifier.



The potential divider network R_1 and R_2 and the resistor R_e together form the biasing and stabilization network. The emitter by-pass capacitor C_e offers a low reactance path to the signal. The resistor R_L is used as a load impedance. The input capacitor C_{in} present at the initial stage of the amplifier couples AC signal to the base of the transistor. The capacitor C_C is the coupling capacitor that connects two stages and prevents DC interference between the stages and controls the shift of operating point.

Operation of Transformer Coupled Amplifier

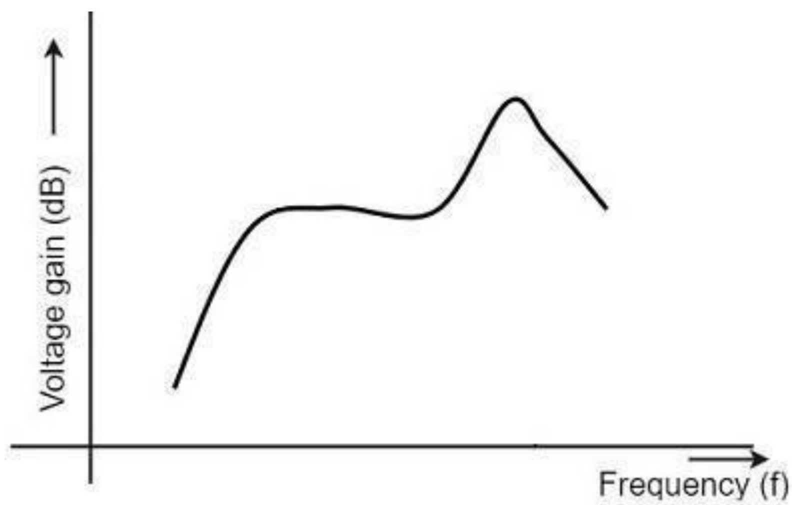
When an AC signal is applied to the input of the base of the first transistor then it gets amplified by the transistor and appears at the collector to which the primary of the transformer is connected.

The transformer which is used as a coupling device in this circuit has the property of impedance changing, which means the low resistance of a stage (or load) can be reflected as a high load resistance to the previous stage. Hence the voltage at the primary is transferred according to the turns ratio of the secondary winding of the transformer.

This transformer coupling provides good impedance matching between the stages of amplifier. The transformer coupled amplifier is generally used for power amplification.

Frequency Response of Transformer Coupled Amplifier

The figure below shows the frequency response of a transformer coupled amplifier. The gain of the amplifier is constant only for a small range of frequencies. The output voltage is equal to the collector current multiplied by the reactance of primary.



At low frequencies, the reactance of primary begins to fall, resulting in decreased gain. At high frequencies, the capacitance between turns of windings acts as a bypass condenser to reduce the output voltage and hence gain.

So, the amplification of audio signals will not be proportionate and some distortion will also get introduced, which is called as **Frequency distortion**.

Advantages of Transformer Coupled Amplifier

The following are the advantages of a transformer coupled amplifier −

- An excellent impedance matching is provided.
- Gain achieved is higher.
- There will be no power loss in collector and base resistors.
- Efficient in operation.

Disadvantages of Transformer Coupled Amplifier

The following are the disadvantages of a transformer coupled amplifier −

- Though the gain is high, it varies considerably with frequency. Hence a poor frequency response.

- Frequency distortion is higher.
- Transformers tend to produce hum noise.
- Transformers are bulky and costly.

Applications

The following are the applications of a transformer coupled amplifier −

- Mostly used for impedance matching purposes.
- Used for Power amplification.
- Used in applications where maximum power transfer is needed.

Direct Coupled Amplifier

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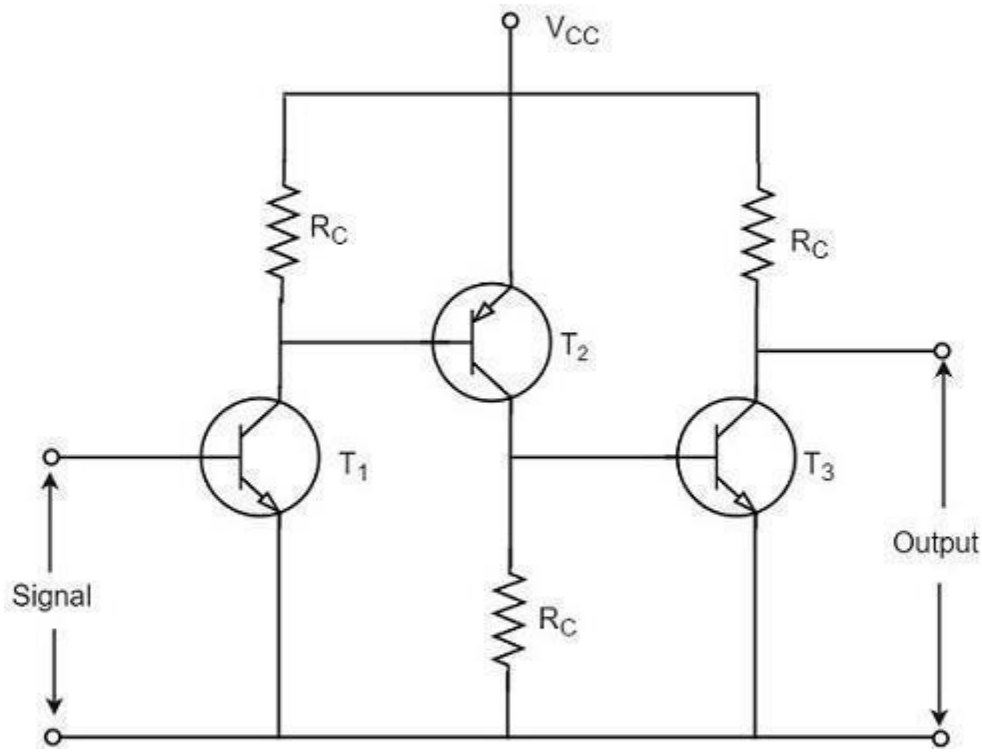
The other type of coupling amplifier is the direct coupled amplifier, which is especially used to amplify lower frequencies, such as amplifying photo-electric current or thermo-couple current or so.

Direct Coupled Amplifier

As no coupling devices are used, the coupling of the amplifier stages is done directly and hence called as **Direct coupled amplifier**.

Construction

The figure below indicates the three stage direct coupled transistor amplifier. The output of first stage transistor T_1 is connected to the input of second stage transistor T_2.



The transistor in the first stage will be an NPN transistor, while the transistor in the next stage will be a PNP transistor and so on. This is because, the variations in one transistor tend to cancel the variations in the other. The rise in the collector current and the variation in β of one transistor gets cancelled by the decrease in the other.

Operation

The input signal when applied at the base of transistor T_1, it gets amplified due to the transistor action and the amplified output appears at the collector resistor R_c of transistor T_1. This output is applied to the base of transistor T_2 which further amplifies the signal. In this way, a signal is amplified in a direct coupled amplifier circuit.

Advantages

The advantages of direct coupled amplifier are as follows.

- The circuit arrangement is simple because of minimum use of resistors.
- The circuit is of low cost because of the absence of expensive coupling devices.

Disadvantages

The disadvantages of direct coupled amplifier are as follows.

- It cannot be used for amplifying high frequencies.
- The operating point is shifted due to temperature variations.

Applications

The applications of direct coupled amplifier are as follows.

- Low frequency amplifications.
- Low current amplifications.

Comparisons

Let us try to compare the characteristics of different types of coupling methods discussed till now.

S.No	Particular	RC Coupling	Transformer Coupling	Direct Coupling
1	Frequency response	Excellent in audio frequency range	Poor	Best
2	Cost	Less	More	Least
3	Space and Weight	Less	More	Least
4	Impedance matching	Not good	Excellent	Good
5	Use	For voltage amplification	For Power amplification	For amplifying extremely low frequencies

Feedback Amplifiers

Introduction :

Feedback plays a very important role in electronic circuits and the basic parameters such as input impedance, output impedance, current gain, voltage gain and bandwidth may be altered considerably by the use of feedback for a given amplifier.

In large signal amplifiers and electronic measuring instruments, the major problem of distortion should be avoided by feedback.

Feedback is also used to maintain the gain independent of external factors.

A portion of the output signal is ~~altered~~ taken ~~out~~ from the output of the amplifier and is combined with the normal input signal and thereby the feedback is accomplished.

Basic concept of feed back :

A block diagram of an amplifier with feedback is shown in figure below. The output quantity is sampled by a suitable sampler and fed to the feedback network. The output of feedback network which has a fraction of the output signal is combined with external source signal. ϕ_s through a comparator and fed to the basic amplifier.

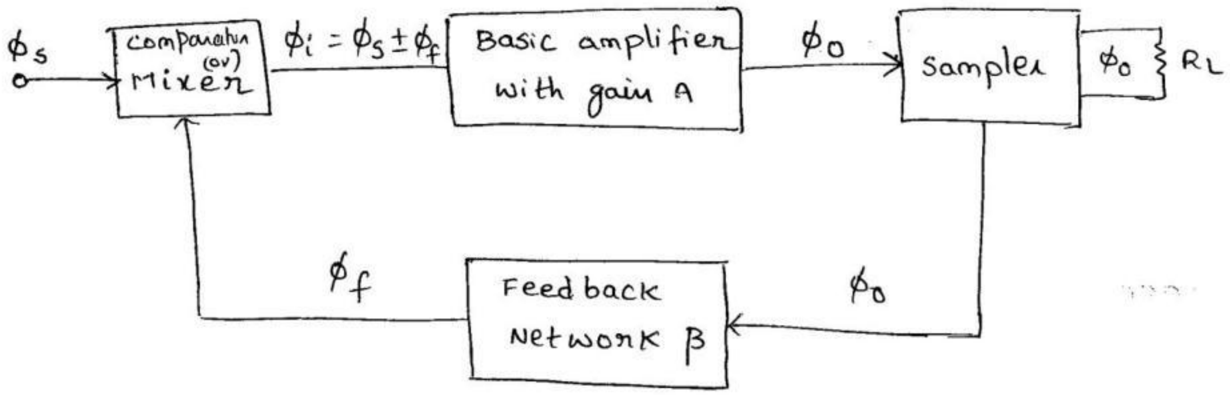


Fig: Block diagram of an amplifier with feedback

Here $A \rightarrow$ gain of the basic amplifier $= \dfrac{\phi_o}{\phi_i}$

$\beta \rightarrow$ feedback ratio $= \dfrac{\phi_f}{\phi_o}$

$A_f \rightarrow$ gain of the feedback amplifier $= \dfrac{\phi_o}{\phi_s}$

$\phi_s \rightarrow$ a.c input signal

$\phi_f \rightarrow$ feedback signal.

There are two types of feedback

(i) positive feedback (ii) Negative feedback.

1) Positive feedback :-

If the feedback signal ϕ_f is inphase with the input signal ϕ_s, then the net effect of the feedback will increase the input signal given to the amplifier ie $\phi_i = \phi_s + \phi_f$. Hence the input voltage applied to the basic amplifier is increased which further increases ϕ_o. This type of feedback is said to be positive or regenerative feedback. Gain of the amplifier with positive feedback is

$$A_f = \frac{\phi_o}{\phi_s} = \frac{\phi_o}{\phi_i - \phi_f} = \frac{1}{\frac{\phi_i}{\phi_o} - \frac{\phi_f}{\phi_o}}$$

$$A_f = \frac{1}{\frac{1}{A} - \beta} = \frac{A}{1 - A\beta}$$

Here $|A_f| > |A|$. The product of the open loop gain and feedback factor is called the loop gain $(A\beta)$. If $|A\beta| = 1$, then $A_f = \infty$. Hence the gain of the amplifier with positive feedback is infinite and the amplifier gives an a.c output without a.c input signal. Thus the amplifier acts as an oscillator. The positive feedback increases the instability of an amplifier, reduces the bandwidth and increases the distortion and noise. The property of the positive feedback is utilised in oscillators.

2. Negative feedback :-

If the feedback signal ϕ_f is out of phase with the input signal ϕ_s, then $\phi_i = \phi_s - \phi_f$. So the input voltage applied to the basic amplifier is decreased and corresponding the output is decreased. Hence, the voltage gain is reduced. This type of feedback is known as negative or degenerative feedback. Gain of the amplifier with negative feedback is

$$A_f = \frac{\phi_o}{\phi_s} = \frac{\phi_o}{\phi_i + \phi_f} = \frac{1}{\frac{\phi_i}{\phi_o} + \frac{\phi_f}{\phi_o}} = \frac{1}{\frac{1}{A} + \beta}$$

$$A_f = \frac{A}{1 + A\beta}$$

Here $|A_f| < |A|$.

If $|A\beta| >> 1$, $|A| >> \frac{1}{\beta}$

$\frac{1}{\beta} << |A|$, then $A_f \simeq \frac{1}{\beta}$, where β = feedback ratio

so the gain depends entirely on the feedback network.

If the feedback network contains only stable passive elements, the gain of the amplifier using negative feed back is also stable.

Negative feedback is used to improve the performance of electronic amplifiers.

Negative feedback helps to increase the bandwidth, decreased distortion and noise, modify input and output resistances as desired.

All the above advantages are obtained at the expense of reduction in voltage gain.

Effects of Negative feedback on Amplifier characteristics

1. Stabilisation of Gain :-

the gain of the amplifier with negative feedback is

$$A_f = \frac{A}{1 + A\beta}$$

differentiating the above equation with respect to A,

$$\frac{dA_f}{dA} = \frac{1}{1+A\beta} + A\frac{-1}{(1+A\beta)^2}\cdot\beta$$

$$\frac{dA_f}{dA} = \frac{1+A\beta - A\beta}{(1+A\beta)^2} = \frac{1}{(1+A\beta)^2}$$

$$\frac{dA_f}{dA} = \frac{1}{(1+A\beta)}\frac{1}{(1+A\beta)} = \frac{A_f}{A}\frac{1}{(1+A\beta)}$$

$$\frac{dA_f}{A_f} = \frac{dA}{A}\frac{1}{(1+A\beta)}$$

The term $\frac{dA_f}{A_f}$ represents the fractional change in amplifier voltage gain with feedback and $\frac{dA}{A}$ denotes the fractional change in voltage gain without feedback the term $\frac{1}{1+A\beta}$ is called Sensitivity.

Therefore the sensitivity is defined as the ratio of percentage change in voltage gain with feedback to the percentage change in voltage gain without feedback.

$$\text{sensitivity} = \frac{\frac{dA_f}{A_f}}{\frac{dA}{A}} = \frac{1}{1+A\beta}$$

The reciprocal of the term sensitivity is called desensitivity

$$\text{desensitivity} = 1+A\beta.$$

2) Increase of Bandwidth

The bandwidth of an amplifier is the difference between the upper cut-off frequency f_2 and the lower cut-off frequency

The product of voltage gain and bandwidth of an amplifier without feedback and with feedback remains the same. ie $A_f \times BW_f = A \times BW$.

As the voltage gain of a feedback amplifier reduces by the factor $\frac{1}{1+A\beta}$, its bandwidth would be increased by $(1+A\beta)$.

ie $A_f \times BW_f = A \times BW \Rightarrow \frac{A_f}{A} \times BW_f = BW$

$$\frac{1}{1+A\beta} \times BW_f = BW \Rightarrow BW_f = BW(1+A\beta)$$

where A is the mid band gain without feedback.

Due to the negative feedback in the amplifier, the upper cut-off frequency (f_{2f}) is increased by the factor $(1+A\beta)$ and the lower cut-off frequency f_{1f} is decreased by the same factor $(1+A\beta)$.

These upper and lower 3 dB frequencies of an amplifier with negative feedback are given by the relations

$$f_{2f} = f_2(1+A\beta) \quad \text{and} \quad f_{1f} = f_1 \cdot \frac{1}{1+A\beta}.$$

3. Decreased Distortion:-

consider an Amplifier with a open loop voltage gain and a total harmonic distortion D. Then, with the introduction of negative feedback with the feed-back ratio (β). the distortion will reduce to

$$D_f = \frac{D}{1+A\beta}$$

4. Decreased Noise:-

there are many sources of noise in an Amplifier depending upon the active device used. with the use of negative feedback with the feedback ratio β, the noise N can be reduced by a factor of $\frac{1}{1+A\beta}$. thus the noise with feedback is given by

$$N_f = \frac{N}{1+A\beta}$$

Problem: An Amplifier has an openloop gain of 1000 and a feedback ratio of 0.04. If the open loop gain changes by 10% due to temperature, find the percentage change in gain of the amplifier with feedback.

solution:- Given $A=1000$, $\beta=0.04$ and $\frac{dA}{A}=10$

we know that the percentage change in gain of the amplifier with feedback is

$$\frac{dA_f}{A_f} = \frac{dA}{A} \frac{1}{(1+A\beta)} = 10 \times \frac{1}{1+(1000 \times 0.04)} = 0.25\%$$

<u>Problem</u>: An Amplifier has voltage gain with feedback is 100. If the gain without feedback changes by 20% and the gain with feedback should not vary more than 2%, determine the values of open loop gain A and feedback ratio β.

<u>Solution</u>: Given $A_f = 100$, $\frac{dA_f}{A_f} = 2\% = 0.02$ and $\frac{dA}{A} = 20\% = 0.2$

we know that

$$\frac{dA_f}{A_f} = \frac{dA}{A} \frac{1}{1+A\beta} \Rightarrow 2 = 20 \frac{1}{1+A\beta}$$

$$\Rightarrow 1+A\beta = 10$$

the gain with feedback is $A_f = \frac{A}{1+A\beta}$

$$\Rightarrow 100 = \frac{A}{10} \Rightarrow A = 1000$$

$$1+A\beta = 10 \Rightarrow \beta = \frac{9}{1000} = 0.009.$$

<u>Problem</u>: An Amplifier has a midband gain of 125 and a bandwidth of 250 kHz (a) If 4% negative feedback is introduced. find the new band width and gain.
(b) If the bandwidth is to be restricted to 1 MHz, find the feedback ratio.

<u>Solution</u>: Given $A = 125$, $BW = 250\ kHz$ and $\beta = 4\% = 0.04$

we know that $B.W_f = (1+A\beta)\, B.W$

$$B.W_f = \left(1 + (125 \times 0.04)\right) \times 250 \times 10^3$$

$$BW_f = 1.5\,MHz$$

Gain with feedback $A_f = \frac{A}{1+A\beta} = \frac{125}{1+(125\times 0.04)} = 20.83$

(b) $$BW_f = (1+A\beta)\,BW$$

$$1\times 10^6 = \left(1 + 125\,\beta'\right) \times 250 \times 10^3$$

$$1+125\,\beta' = 4 \Rightarrow \beta' = \frac{3}{125} = 0.024 = 2.4\%$$

Problem: An Amplifier has a voltage gain of 400, $f_1 = 50\,Hz$, $f_2 = 200\,KHz$, and a distortion of 10% without feedback. Determine the amplifier voltage gain f_{1f}, f_{2f} and D_f when a negative feedback is applied with feedback ratio of 0.01.

Solution: Given $A = 400$, $f_1 = 50\,Hz$, $f_2 = 200\,KHz$, $D = 10\%$ and $\beta = 0.01$

we know that voltage gain with feedback

$$A_f = \frac{A}{1+A\beta} = \frac{400}{1+(400\times 0.01)} = 80$$

New lower 3dB frequency

$$f_{1f} = \frac{f_1}{1+A\beta} = \frac{50}{1+(400\times 0.01)} = 10\,Hz$$

New upper 3dB frequency

$$f_{2f} = (1+A\beta)\times f_2$$

$$f_{2f} = (1+400\times 0.01)\times 200\times 10^3 = 1\,MHz$$

Distortion with feedback

$$D_f = \frac{D}{1+A\beta} \Rightarrow D_f = \frac{10}{5} = 2\%$$

Types of Negative feedback connections :

there are four different combinations in which negative feedback may be accomplished.

1) voltage series feedback
2) voltage shunt feed back
3) current series feedback
4) current shunt feedback.

Series feedback connection increase the i/p resistance
shunt feedback connection decrease the i/p resistance
Voltage " " decrease the o/p "
current " " increase " " "

1) voltage series feed back increases input impedance
decreases output "
2) voltage shunt feed back decreases the input resistance
decreases the output "
3) current series feed back increases the input resistance
increases the output "
4) current shunt feed back decreases the input "
increases the output "

It is desirable that most cascade Amplifiers need to have higher input ~~impedance~~ resistance and Lower output resistance.

The voltage series type of feedback has the high input impedance and low output resistance, but it ~~decreases~~ suffers the highest degree in voltage gain.

on the other hand, current shunt feedback has the least desirable effects since it decreases input resistance and increases output resistance.

1) <u>voltage</u> <u>series</u> <u>feedback</u> :

A block diagram of voltage series feedback is illustrated in figure below. Here the input to the feedback network is in parallel with the output of the amplifier. A fraction of the output voltage through the feedback n/w is applied in series with the input voltage of the amplifier. The shunt connection at the o/p reduces the output resistance R_0. The series connection at the input increases the input resistance.

In this case, the amplifier is a true voltage amplifier. The voltage feedback factor is given by

$$\beta = \frac{V_f}{V_o}$$

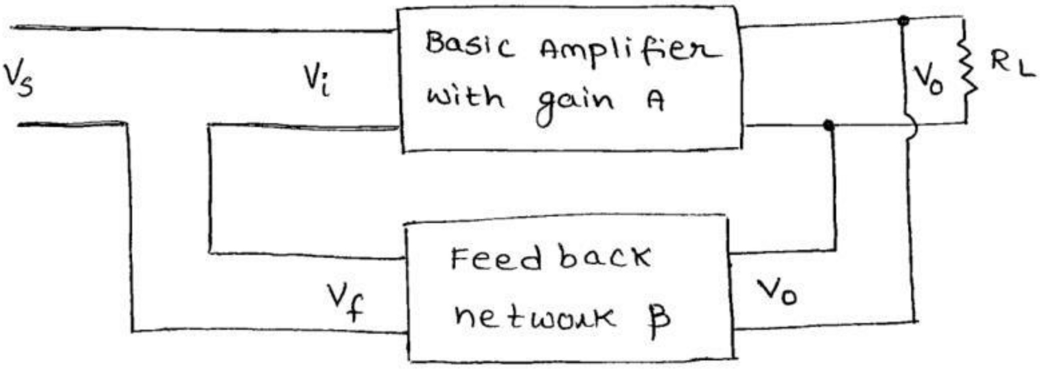


Fig: Block diagram of the voltage series feedback.

<u>Input</u> <u>and</u> <u>output</u> <u>resistances</u> :-

Figure below shows the voltage series feedback circuit used to calculate input and output resistances

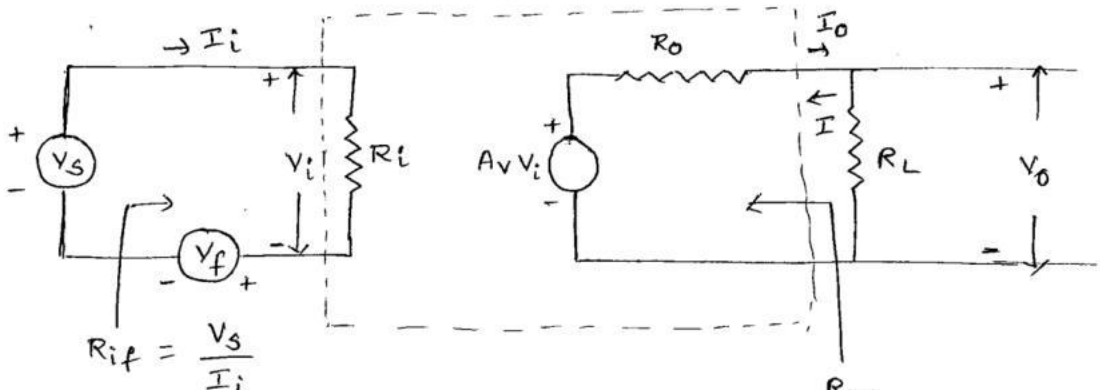


Fig: voltage series feedback circuit for the calculation of input and output resistances.

Here $V_s = V_i + V_f$

$$V_s = I_i R_i + \beta V_o$$

$$V_s = I_i R_i + \beta A I_i R_i$$

$$A = \frac{V_o}{V_i} \Rightarrow \frac{V_o}{I_i R_i}$$

$$V_o = A I_i R_i$$

Therefore $R_{if} = \frac{V_s}{I_i} = \frac{I_i R_i + \beta A_i R_i}{I_i} = (1 + A\beta) R_i$

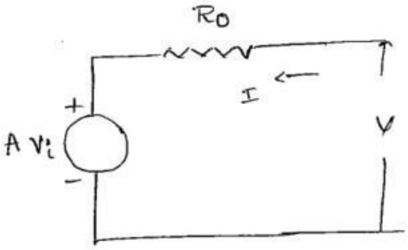
Hence the input resistance of a voltage series feedback amplifier is given by

$$R_{if} = (1 + A\beta) R_i$$

where R_i is the input resistance of the amplifier without feedback.

For measuring the output resistance, R_L is disconnected and V_s is set to zero. Then external voltage V is applied across the output terminals and the current I delivered by V is calculated. Then $R_{of} = \frac{V}{I}$

Here $I = \frac{V - AV_i}{R_o}$



Due to feedback, input voltage V_f reduces output voltage AV_i which opposes V. Therefore

$$I = \frac{V - AV_i}{R_o} = \frac{V + \beta A V}{R_o}$$

therefore $R_{of} = \frac{R_o}{1 + A\beta}$

Hence the output resistance of a Voltage series feedback amplifier is given by

$$R_{of} = \frac{R_o}{1 + A\beta}$$

where R_o is the output resistance of the amplifier without feedback.

Problem: A voltage - series negative feedback amplifier has a voltage gain without feedback of $A = 500$, input resistance $R_i = 3K\Omega$, output resistance $R_o = 20k\Omega$ and feedback ratio $\beta = 0.01$. calculate the voltage gain A_f, input resistance R_{if} and output resistance R_{of} of the amplifier with feedback.

solution: $A = 500$, $R_i = 3k\Omega$, $R_o = 20k\Omega$ and $\beta = 0.01$

voltage gain $A_f = \frac{A}{1+A\beta} = \frac{500}{1 + (500 \times 0.01)} = 83.3$

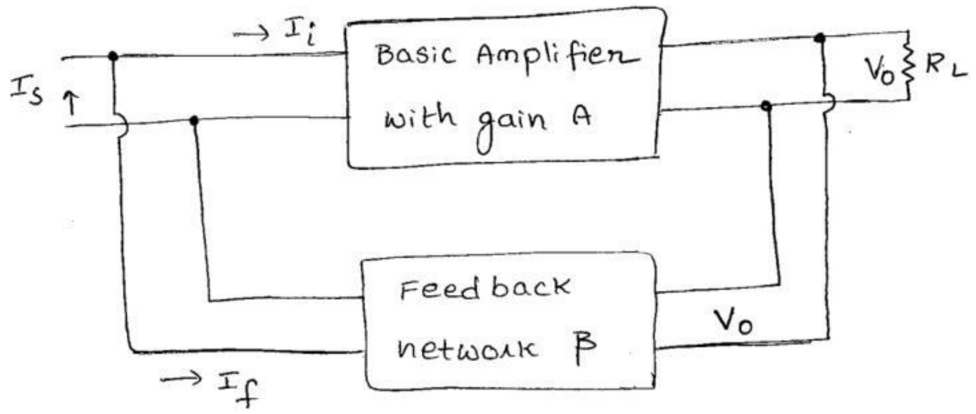
Input resistance $R_{if} = (1 + A\beta) R_i = (1 + 500 \times 0.01) \times 10^3$
$= 18k\Omega$.

output resistance $R_{of} = \frac{R_o}{1 + A\beta} = \frac{20 \times 10^3}{1 + 500 \times 0.01} = 3.33 k\Omega$

2) Voltage shunt feedback :-

A Voltage shunt feedback is illustrated in figure below. Here a fraction of the output voltage is supplied in parallel with the input voltage through the feedback network. The feedback signal I_f is proportional to the output voltage V_o. Therefore the feedback factor is given by $\beta = \frac{I_f}{V_o}$. This type of amplifier is called a trans resistance amplifier. The voltage - shunt feedback provides a stabilised overall gain and decreases both input and output resistances by a factor $(1 + A\beta)$

$$R_{if} = \frac{R_1}{1+A\beta} \quad \text{and} \quad R_{of} = \frac{R_0}{1+A\beta}$$



Input and output resistances :-

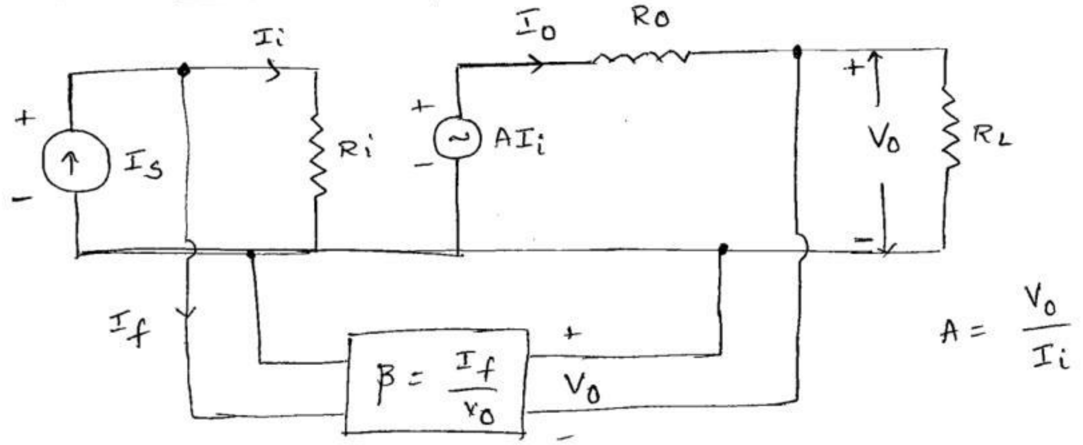


Fig : Equivalent circuit of voltage shunt feedback.

$$R_{if} = \frac{V_i}{I_s} = \frac{V_i}{I_i + I_f} = \frac{V_i}{I_i + \beta V_0} = \frac{V_i / I_i}{1 + \beta \frac{V_0}{I_i}}$$

$$R_{if} = \frac{R_i}{1 + A\beta}$$

thus the input impedance gets reduced by a factor $(1 + A\beta)$

output Impedance :

~~$-AI_i + I_0 R_0 + V_0 = 0$~~

$V_0 =$

$$V_0 = I_0 R_0 + A I_i$$

For $I_s = 0$ $I_i = -I_f$, then

$$V_o = I_o R_o - A I_f$$

$$V_o = I_o R_o - A \beta V_o$$

$$V_o (1 + A\beta) = I_o R_o$$

$$R_{of} = \frac{V_o}{I_o} = \frac{R_o}{1 + A\beta}$$

3. Current series feedback Amplifier

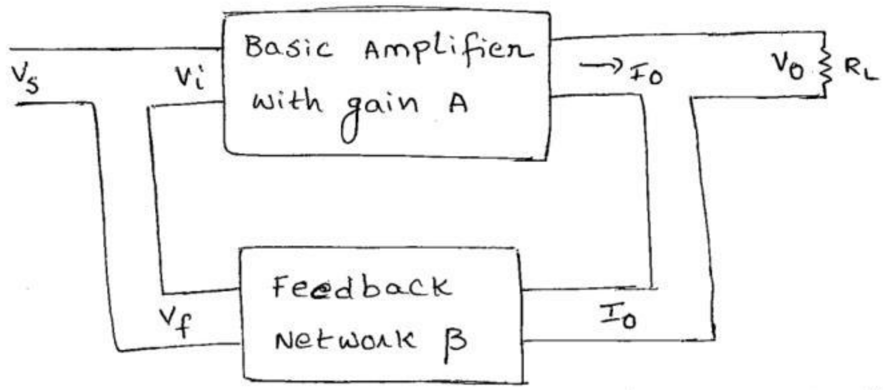
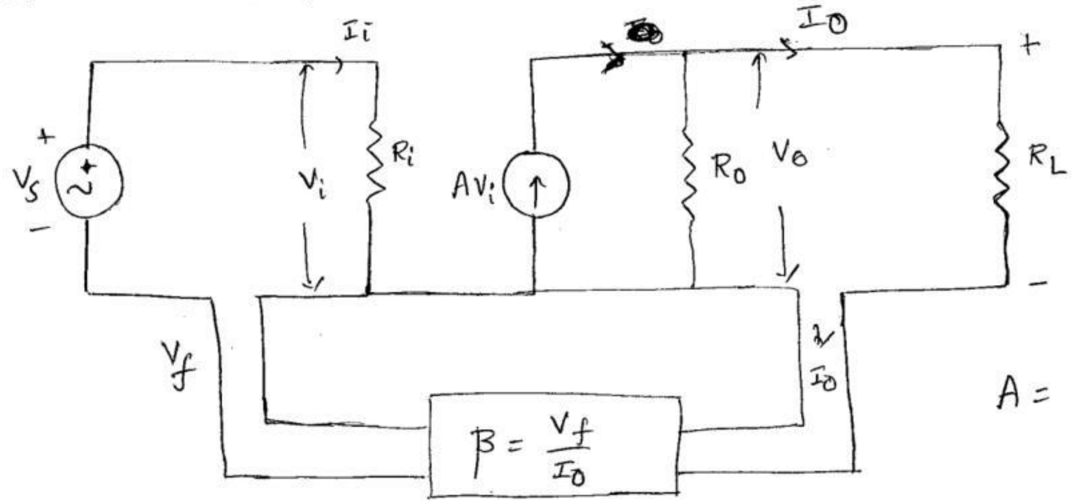


Fig: Block diagram of the current series feedback.

A block diagram of a current series feedback is illustrated in figure above. In current series feedback, a voltage is developed which is proportional to the output current. This is called current feedback even though it is a voltage that subtracts from the input voltage. Because of the series connection at the input and output, the input and output resistances get increased. This type of amplifier is called transconductance amplifier. The transconductance feedback factor β is given by $\beta = V_f / I_o$

Input and output resistances :



$$A = \frac{I_o}{V_i}$$

$$V_s = V_i + V_f = I_i R_i + \beta I_o$$

$$V_s = I_i R_i + A\beta V_i$$

$$V_s = I_i R_i + A\beta \; I_i R_i = (1 + A\beta) I_i R_i$$

$$\frac{V_s}{I_i} = R_i (1 + A\beta) \Rightarrow \boxed{R_{if} = R_i (1 + A\beta)}$$

To obtain the output impedance assume that source voltage is transfered to output terminals, with V_s short circuited ie $V_s = 0$, resulting in a current I_o in to the circuit.

$$V_s = V_i + V_f \qquad \text{if } V_s = 0 \text{ then } V_i = -V_f$$

$$I_o = AV_i + \frac{V_o}{R_o} = -A V_f + \frac{V_o}{R_o}$$

$$I_o = -A\beta I_o + \frac{V_o}{R_o}$$

$$\frac{V_o}{R_o} = I_o (1 + A\beta) \Rightarrow \frac{V_o}{I_o} = (1 + A\beta) R_o$$

$$\Rightarrow R_{of} = (1 + A\beta) R_o$$

4) Current - shunt feedback :

The block diagram of current shunt feedback is shown in figure below

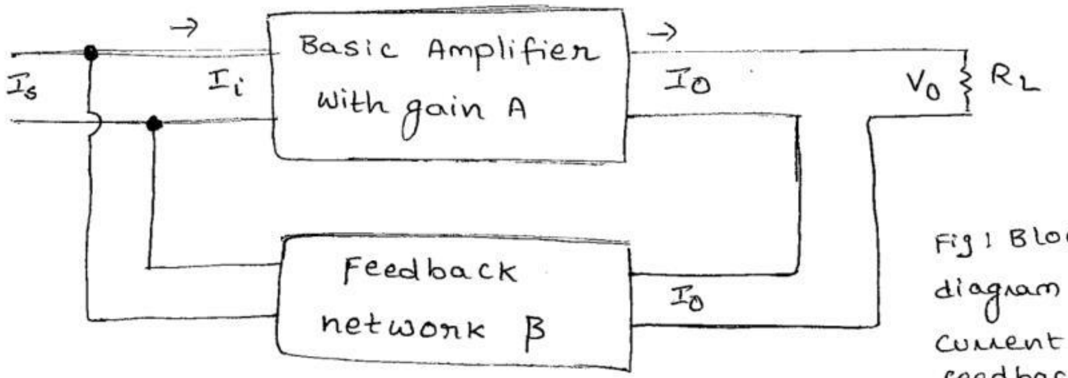


Fig 1 Block diagram of current shunt feedback

The shunt connection at the input reduces the input resistance and the series connection at the output increases the output resistance. This is a true current amplifier. The current feedback factor is given by $\beta = \frac{I_f}{I_o}$

Here Amplifier gain $A = \frac{I_o}{I_i}$

Input and output resistances :

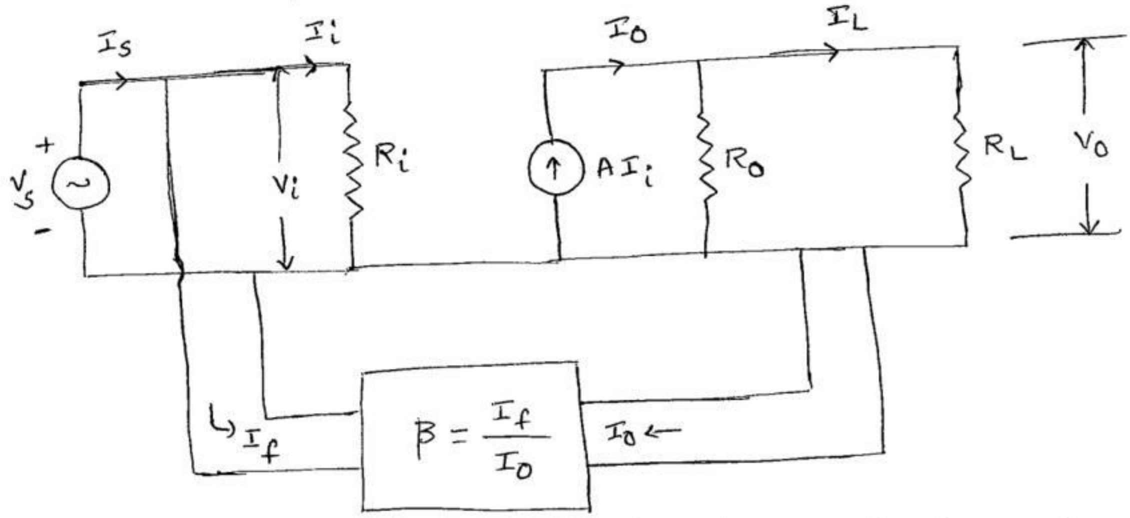


Fig: Equivalent circuit of current shunt feedback Connection

Input Resistance

From the Equivalent circuit

$$I_s = I_i + I_f$$

$$I_s = \frac{V_i}{R_i} + \beta I_o \qquad \left[\because \beta = \frac{I_f}{I_o}\right]$$

$$I_s = \frac{V_i}{R_i} + \beta A I_i \qquad \left[\because A = \frac{I_o}{I_i}\right]$$

$$I_s = \frac{V_i}{R_i} + \beta A \frac{V_i}{R_i} \qquad \left[\because I_i = \frac{V_i}{R_i}\right]$$

$$I_s = \frac{V_i}{R_i}\left[1 + A\beta\right]$$

$$R_{if} = \frac{V_i}{I_s} = \frac{R_i}{1 + A\beta}$$

Output resistance:

we know that

$$I_s = I_i + I_f$$

For $I_s = 0$, $I_i = -I_f$

From the equivalent circuit $I_o = A I_i + \frac{V_o}{R_o}$

$$I_o = \frac{V_o}{R_o} - A I_f = \frac{V_o}{R_o} - A\beta I_o$$

$$\frac{V_o}{R_o} = I_o + A\beta I_o \Rightarrow \frac{V_o}{I_o} = (1 + A\beta) R_o$$

$$R_{of} = \frac{V_o}{R_o} = (1 + A\beta) R_o$$

$$\therefore \boxed{R_{of} = (1 + A\beta) R_o}$$

Problem:

When a negative feedback is applied to an Amplifier of gain 100, the overall gain falls to 50. calculate (i) the feedback factor β (ii) If the same feedback factor maintained, the value of the amplifier gains required if the overall gain is to be 75

Solution: (i) $A = 100$, $A_f = 50$

$$A_f = \frac{A}{1 + A\beta} \Rightarrow \beta = 0.01$$

(ii) $A_f = 75$

$$A_f = \frac{A}{1 + A\beta} \Rightarrow 75 = \frac{A}{1 + 0.01A}$$

$$\Rightarrow A = 300.$$

Problem:

The gain of the amplifier without feedback is 50 whereas with -ve feedback it falls to 25. If due to ageing, the amplifier gain falls to 40. Find the percentage reduction in gain

i) without feedback (ii) with negative feedback.

Solution: $A_f = \frac{A}{1 + A\beta}$ Given $A_f = 25$, $A = 50$

then $\beta = 0.02$

(i) without feedback % reduction in gain $= \frac{50-40}{50} \times 100$

$= 20\%$

when the gain without feedback was 50, the gain with feedback, was 50. Now the gain with out feedback falls to 40

then $A_f = \frac{A}{1+A\beta} = \frac{40}{1+0.02\times 40} = 22.2$ | % reduction in gain $= \frac{25-22.2}{25} \times 100 = 11.2\%$

OSCILLATORS

Any circuit which is used to generate an a.c voltage without an a.c input signal is called oscillator.

To generate a.c voltage, the circuit is supplied energy from a d.c source.

Positive feedback is used to generate oscillations of desired frequency.

Classification of oscillators:

Oscillators are classified in to the following different ways

① According to the waveform generated

a) Sinusoidal oscillator: An electronic device that generate sinusoidal oscillations of desired frequency is known as a sinusoidal oscillator.

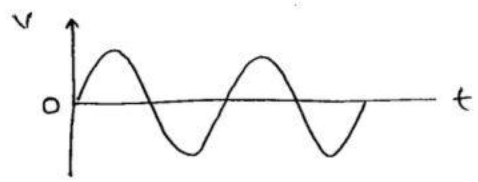


fig: Sinusoidal wave form.

b) Relaxation (or) Non Sinusoidal oscillators: The oscillators which produce square waves, triangular waves, sawtooth waves are known as Relaxation oscillators

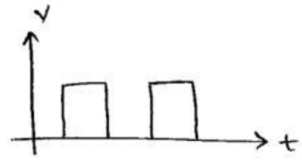


Fig: square

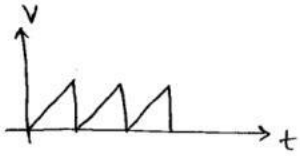


Fig: Sawtooth

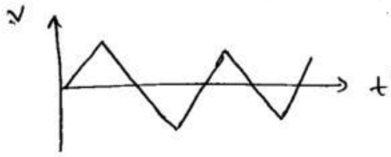


Fig: Triangular

② According to the fundamental mechanisms involved

a) Negative resistance oscillators: Negative resistance oscillators uses negative resistance of the amplifying device to neutralize the positive resistance of the oscillator

b) Feedback oscillators: these oscillators uses positive feedback in the feedback amplifier to satisfy the barkhausen criterion.

③ According to the frequency generated

a) Audio frequency oscillators (upto 20KHz)

b) Radio frequency oscillators (20KHz to 30MHz)

c) Very high frequency oscillator (30 MHz to 300 MHz)

d) ultra high frequency oscillator (300MHz to 3GHz)

e) Microwave frequency oscillator (above 3GHz)

④ According to the type of circuit used

a) LC tuned oscillator

b) RC oscillators.

<u>Basic theory of oscillators</u>:

The feedback is a property which allows to connect the part of the output to the same circuit

As the phase of the feedback signal is same as that of the input applied, the feedback is called positive feedback.

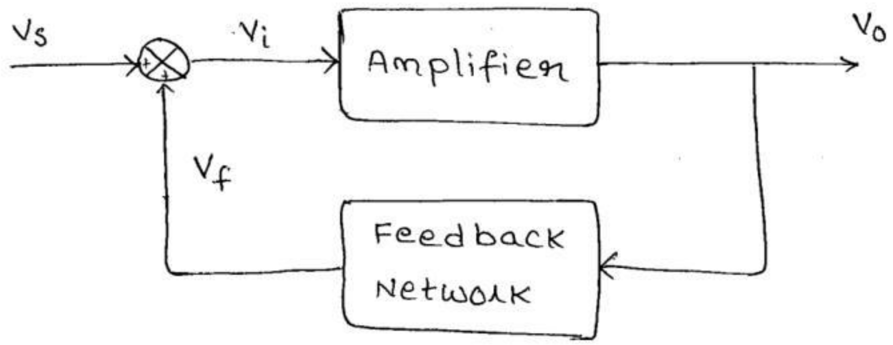


Fig: concept of positive feed back.

Here Amplifier gain called open loop gain (gain without feedback) given by

$$A = \frac{V_o}{V_i} \longrightarrow (1)$$

the gain with feedback (closed loop gain or over all gain) denoted by A_f

$$A_f = \frac{V_o}{V_s} \longrightarrow (2)$$

the feedback is positive and voltage V_f is added to V_s to generate V_i

$$\therefore \quad V_i = V_s + V_f \longrightarrow (3)$$

the feedback voltage V_f depends on the feedback gain β given by

$$\beta = \frac{V_f}{V_o} \longrightarrow (4)$$

$\therefore$ Eq (3) $\Rightarrow$ $V_i = V_s + \beta V_o$ $\left[\because V_f = \beta V_o \text{ from } (4) \right]$

$$\Rightarrow V_i = V_s + \beta A V_i \quad [\because V_o = AV_i \text{ from } (1)]$$

$$\Rightarrow V_s = V_i(1 - A\beta)$$

But $$A_f = \frac{V_o}{V_s} = \frac{V_o}{V_i(1-A\beta)} = \frac{A}{1-A\beta} \quad \left[\because A = \frac{V_o}{V_i}\right]$$

Here $|A_f| > |A|$.

The product of openloop gain and feedback factor is called loop gain ($A\beta$)

If $|A\beta| = 1$ then $A_f = \infty = \frac{V_o}{V_s}$

$$\Rightarrow V_s = 0$$

Hence the gain of the amplifier with positive feedback is infinite and the amplifier gives an a.c output without a.c input signal. Thus the amplifier acts as an oscillator.

Barkhausen criterion (Conditions for oscillations):

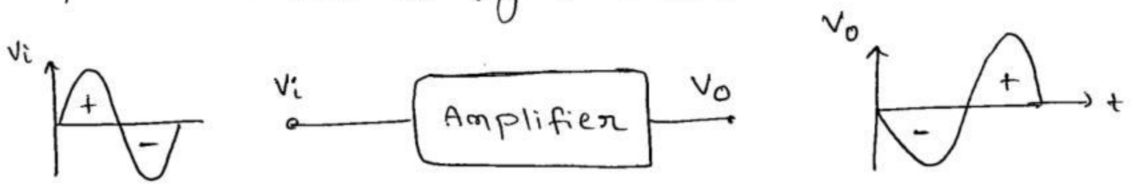
Mechanism for start of oscillations:

The oscillator circuit is set in to oscillations by a random variation caused in the base current due to noise component or a small variation in the dc power supply. The noise components ie small random electrical voltages and currents are always present in any conductor, tube or transistor. Even when no electrical signal is applied, the ever present noise will cause some small signal at the output of the amplifier.

If a small fraction β of the output signal is fed back to the input with proper phase relation, then this feedback signal will be amplified by the amplifier.

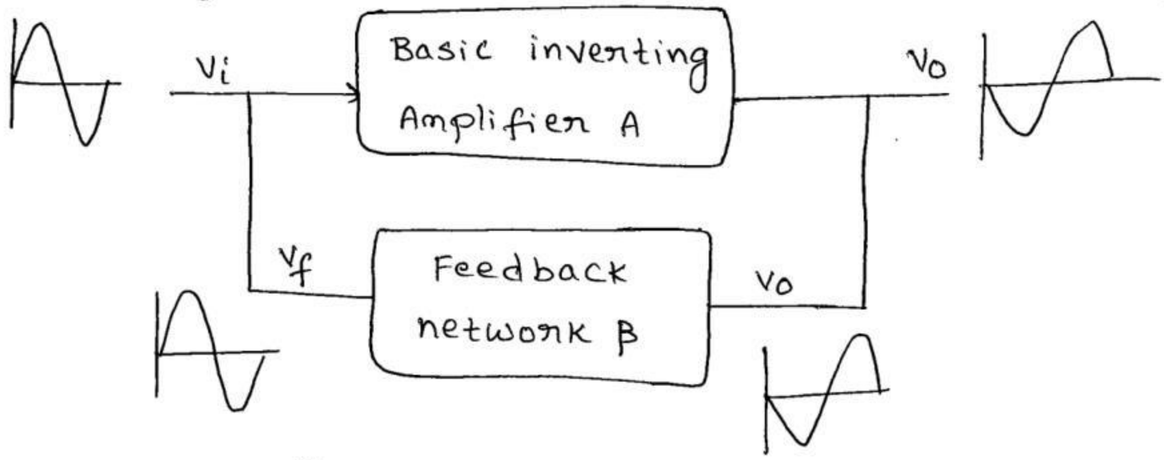
If $A > \frac{1}{\beta}$ (ie $A\beta > 1$), then the output increases and thereby the feedback signal becomes larger. this process continues and the output goes on increasing. But as the signal level increases, the gain of the amplifier decreases and at a particular value of output, the gain of the amplifier is reduced exactly equal to $1/\beta$ (ie $A = \frac{1}{\beta} \Rightarrow A\beta = 1$). then the output voltage remains constant at frequency f_r called frequency of oscillation.

consider a basic inverting amplifier with an open loop gain A. As basic amplifier is inverting, it produces a phase shift of 180° between input and output as shown in figure below.



Now the input Vi applied to the amplifier is to be derived from its output Vo using feed back n/w. but the feedback must be positive ie the voltage derived from output using feedback network must be inphase with Vi. Thus the feedback network must introduce a phase shift of 180° while feeding back the voltage from output to input. This ensures positive feedback.

The arrangement is shown in figure below



Here $A = \frac{V_o}{V_i} \Rightarrow V_o = A V_i \longrightarrow (1)$

$\beta = \frac{V_f}{V_o} \Rightarrow V_f = \beta V_o \longrightarrow (2)$

$\therefore \quad V_f = A\beta V_i \longrightarrow (3)$

For the oscillator, we want that feedback should drive the amplifier and hence V_f must act as V_i from Equation (3), we can write that, V_f is sufficient to act as V_i when $|A\beta| = 1$

And the phase of V_f is same as V_i ie feedback network should introduce 180° phase shift in addition to 180° phase shift introduced by inverting amplifier. This ensures positive feedback. So total phase shift around a loop is 360°.

In this condition V_f drives the circuit and without external input, circuit works as an oscillator.

The two conditions which are required for the circuit to work as an oscillator are called

The Barkhausen criterion states that

1. The total phase shift around a loop, as the signal proceeds from input through amplifier, feedback network to input again, completing a loop is precisely 0° or 360°.

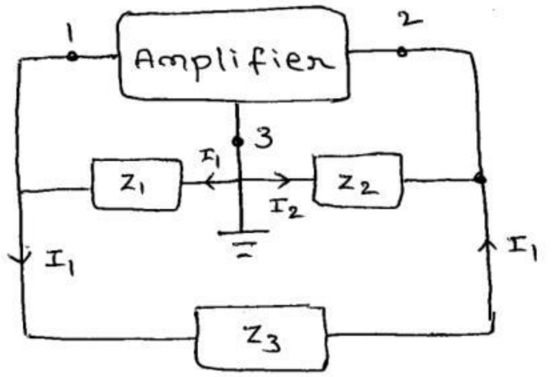
2. The magnitude of the product of the open loop gain of the amplifier and the magnitude of the feedback factor β is unity ie $|A\beta| = 1$

Differences between Alternator and oscillator:

Alternator	oscillator
1. It is a rotating device which has rotating parts	1. It is not a rotating device.
2. It creates Energy	2. It doesn't create energy. only converts dc energy to ac energy.
3. It has fixed frequency	3. The frequency of the oscillator ranges from few Hz to MHz.

LC oscillators:

General form of LC oscillators:



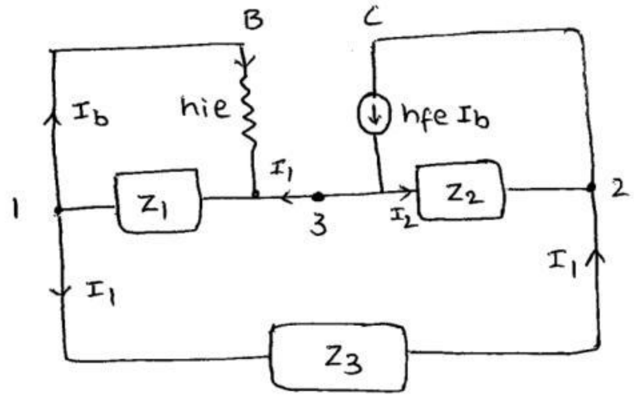


Fig a: General form of an oscillator   Fig(b): It's Equivalent ckt

In the general form of oscillator shown in fig above, any of the active devices such as transistor, FET, and operational amplifier may be used in the amplifier section.

Z_1, Z_2 and Z_3 are the reactive elements constituting the feedback tank circuit which determines the frequency of oscillation. Here Z_1 and Z_2 serve as a.c voltage divider for the output voltage and feedback signal. Therefore the voltage across Z_1 is the feedback signal. The frequency of oscillation of LC oscillator is

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

The inductive and capacitive reactances are represented by Z_1, Z_2 and Z_3. In fig(a) the output terminals are 2 and 3, and input terminals are 1 and 3. Fig (b) gives the equivalent circuit of fig(a).

Load Impedance :

Since z_1 and the input resistance hie of the transistor are in parallel, their equivalent impedance z' is given by $z' = z_1 || hie$

$$\frac{1}{z'} = \frac{1}{z_1} + \frac{1}{hie} \Rightarrow z' = \frac{z_1 hie}{z_1 + hie} \longrightarrow (1)$$

Now the load impedance z_L between the output terminals 2 and 3 is the equivalent impedance of z_2 in parallel with the series combination of z' and z_3.

Therefore

$$z_L = z_2 || (z' + z_3)$$

$$\frac{1}{z_L} = \frac{1}{z_2} + \frac{1}{z' + z_3}$$

$$\frac{1}{z_L} = \frac{1}{z_2} + \frac{1}{\frac{z_1 hie}{z_1 + hie} + z_3}$$

$$\frac{1}{z_L} = \frac{1}{z_2} + \frac{z_1 + hie}{hie(z_1 + z_3) + z_3 hie}$$

$$\frac{1}{z_L} = \frac{1}{z_2} + \frac{z_1 + hie}{}$$

$$\frac{1}{z_L} = \frac{hie(z_1 + z_3) + z_1 z_3 + z_1 z_2 + hie\, z_2}{z_2 [hie(z_1 + z_3) + z_1 z_3]}$$

$$\frac{1}{z_L} = \frac{hie(z_1 + z_2 + z_3) + z_1 z_2 + z_1 z_3}{z_2 [hie(z_1 + z_3) + z_1 z_3]}$$

$$z_L = \frac{z_2 [hie(z_1 + z_3) + z_1 z_3]}{hie(z_1 + z_2 + z_3) + z_1 z_2 + z_1 z_3} \longrightarrow (2)$$

Voltage gain without feedback is given by

$$A_V = \frac{-h_{fe} Z_L}{h_{ie}} \longrightarrow (3)$$

Feedback fraction β

The output voltage between the terminals 3 and 2 in terms of the current I_1 is given by

$$V_o = -I_1(z' + z_3) = -I_1\left[\frac{z_1 h_{ie}}{z_1 + h_{ie}} + z_3\right]$$

$$V_o = -I_1\left(\frac{z_1 h_{ie} + z_1 z_3 + z_3 h_{ie}}{z_1 + h_{ie}}\right)$$

$$V_o = -I_1\left[\frac{h_{ie}(z_1 + z_3) + z_1 z_3}{z_1 + h_{ie}}\right] \longrightarrow (4)$$

The voltage feed back to the input terminals 1 and 3 is given by

$$V_{fb} = -I_1 z' = -I_1\left[\frac{z_1 h_{ie}}{z_1 + h_{ie}}\right] \longrightarrow (5)$$

Therefore the feedback ratio β is given by

$$\beta = \frac{V_{fb}}{V_o} = I_1\left[\frac{z_1 h_{ie}}{z_1 + h_{ie}}\right]\left[\frac{z_1 h_{ie}}{h_{ie}(z_1 + z_3) + z_1 z_3}\right]\frac{1}{I_1}$$

$$\beta = \frac{z_1 h_{ie}}{h_{ie}(z_1 + z_3) + z_1 z_3} \longrightarrow (6)$$

Equation for the oscillator

For oscillation we must have

$$A_V \beta = 1$$

Substituting the values of A_v and β, we get

$$\left(\frac{-h_{fe}\, Z_L}{h_{ie}}\right)\left(\frac{Z_1 h_{ie}}{h_{ie}(Z_1+Z_3)+Z_1 Z_3}\right) = 1$$

$$\frac{h_{fe}\, Z_2\left[h_{ie}(Z_1+Z_3)+Z_1 Z_3\right]}{\left[h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2+Z_1Z_3\right]}\left(\frac{Z_1}{h_{ie}(Z_1+Z_3)+Z_1Z_3}\right) = -1$$

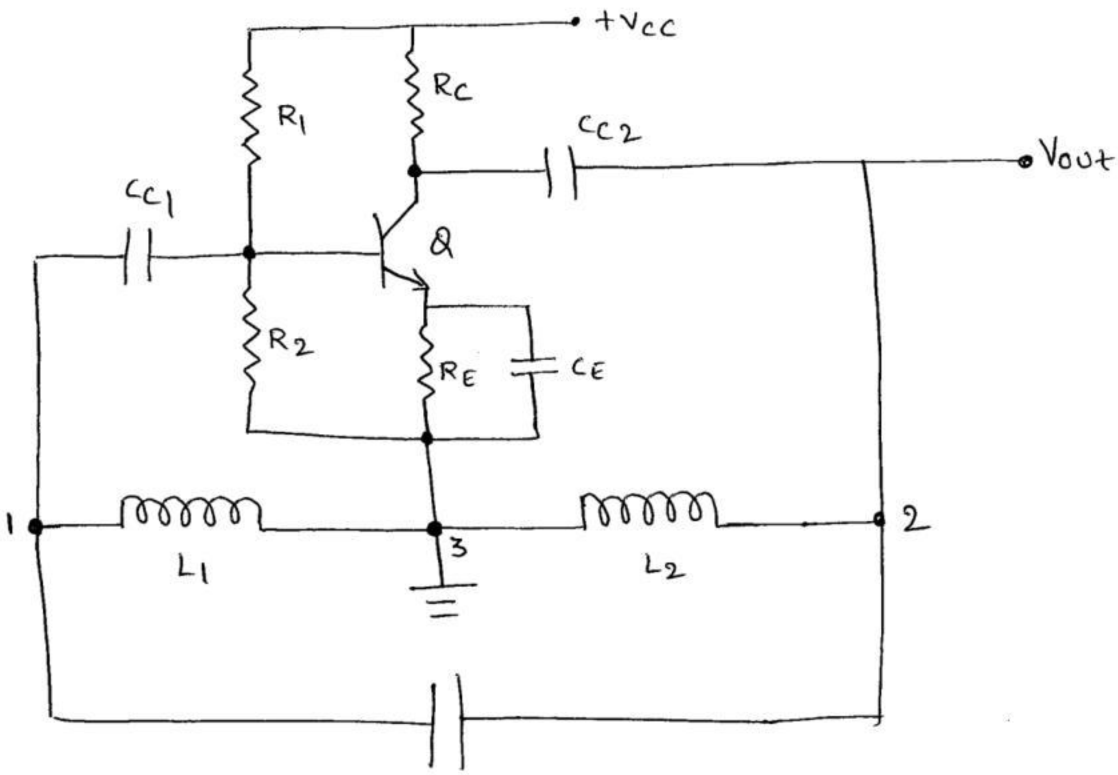
$$\frac{h_{fe}\, Z_1 Z_2}{h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2+Z_1Z_3} = -1$$

$$h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2+Z_1Z_3+h_{fe}Z_1Z_2 = 0$$

$$h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2(1+h_{fe})+Z_1Z_3 = 0$$

This is the general equation for the oscillator.

Hartley oscillator:-



In the Hartley oscillator shown in figure above Z_1 and Z_2 are inductors and Z_3 is a capacitor. Resistors R_1, R_2 and R_E provides the necessary dc bias to the transistor. C_E is a bypass capacitor. C_{C_1} and C_{C_2} are coupling capacitors. The feedback network ~~constitu-~~ ~~-ting~~ consisting of inductors L_1 and L_2, and capacitor C determines the frequency of the oscillator

When the supply voltage $+V_{CC}$ is switched on a transient current is produced in the tank circuit and consequently harmonic oscillations are setup in the circuit.

The oscillatory current in the tank circuit produces a-c voltages across L_1 and across L_2. As terminal 3 is grounded, it will be at zero potential. If terminal 1 is at positive potential with respect to 3 at any instant, terminal 2 will be a negative potential with respect to 3 at the same instant. Thus the phase difference between the terminals 1 and 2 is always 180°.

In the CE mode, the transistors provides the phase difference between the input and output. Therefore the total phase shift is 360°. Thus, at the frequency determinant for the tank circuit, the necessary condition for sustained oscillations is satisfied. If the feedback is adjusted, so that the loop gain $A\beta = 1$, the circuit acts as an oscillator.

The frequency of oscillation is $f_r = \frac{1}{2\pi\sqrt{LC}}$ where $L = L_1 + L_2 + 2M$, where M is the co-efficient of mutual inductance between coils L_1 and L_2. The condition for sustained oscillation is

$$h_{fe} \geq \frac{L_1 + M}{L_2 + M}$$

<u>Analysis</u>:

The general equation for LC oscillator is

$$h_{ie}(z_1 + z_2 + z_3) + (1 + h_{fe}) z_1 z_2 + z_1 z_3 = 0 \longrightarrow (1)$$

$$z_1 = j\omega L_1 + j\omega M$$

$$z_2 = j\omega L_2 + j\omega M$$

$$z_3 = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

Substituting these values in Eq (1)

$$\Rightarrow h_{ie}\left(j\omega L_1 + j\omega M + j\omega L_2 + j\omega M - \frac{j}{\omega C}\right) + (1 + h_{fe})(j\omega L_1 + j\omega M)(j\omega L_2 + j\omega M) + (j\omega L_1 + j\omega M)\left(\frac{-j}{\omega C}\right) = 0$$

$$\Rightarrow j\omega h_{ie}\left(L_1 + L_2 + 2M - \frac{1}{\omega^2 C}\right) + (1 + h_{fe})\, j^2\omega^2\left[L_1 L_2 + M(L_1 + L_2) + M^2\right] - j^2\left[\frac{L_1}{C} + \frac{M}{C}\right] = 0 \longrightarrow (2)$$

The frequency of oscillation ω_r can be determined by equating imaginary part of Eq (2) to zero

$$\omega_r h_{ie}\left(L_1 + L_2 + 2M - \frac{1}{\omega_r^2 C}\right) = 0$$

$$L_1 + L_2 + 2M = \frac{1}{\omega_r^2 C}$$

$$\omega_r^2 = \frac{1}{(L_1 + L_2 + 2M)C} \longrightarrow (3)$$

Let $L_{eq} = L_1 + L_2 + 2M$

$$\omega_r^2 = \frac{1}{L_{eq} C} \Rightarrow \omega_r = \frac{1}{\sqrt{L_{eq} C}} \Rightarrow f_r = \frac{1}{2\pi\sqrt{L_{eq} C}}$$

The condition for maintainence of oscillation is obtained by substituting Eq (3) in Eq (2)

$$(3) \Rightarrow \frac{1}{\omega_r^2 C} = (L_1 + L_2 + 2M)$$

$$(2) \Rightarrow j\omega h_{ie}(L_1 + L_2 + 2M - L_1 - L_2 - 2M) + (1 + h_{fe})$$

$$j^2\omega^2\left[L_1L_2 + M(L_1+L_2) + M^2\right] + \frac{L_1}{C} + \frac{M}{C} = 0$$

$$\Rightarrow -(1+h_{fe})\frac{1}{(L_1 + L_2 + 2M)C}\left[L_1L_2 + M(L_1+L_2) + M^2\right] + \frac{L_1 + M}{C} = 0$$

$$\Rightarrow \frac{(1+h_{fe})(L_1L_2 + M(L_1+L_2) + M^2)}{(L_1 + L_2 + 2M)\cancel{C}} = \frac{L_1 + M}{\cancel{C}}$$

$$\Rightarrow (1+h_{fe})(L_1L_2 + M(L_1+L_2) + M^2) = L_1^2 + L_1M + L_1L_2 + L_2M + 2L_1M + 2M^2$$

$$\Rightarrow (1+h_{fe})(L_1L_2 + M(L_1+L_2) + M^2) - L_1L_2 - M(L_1+L_2) - M^2 = L_1^2 + 2L_1M + M^2$$

$$\Rightarrow \left[L_1L_2 + M(L_1+L_2) + M^2\right]\left(\cancel{1} + h_{fe} - \cancel{1}\right) = L_1 + 2L_1M + M^2$$

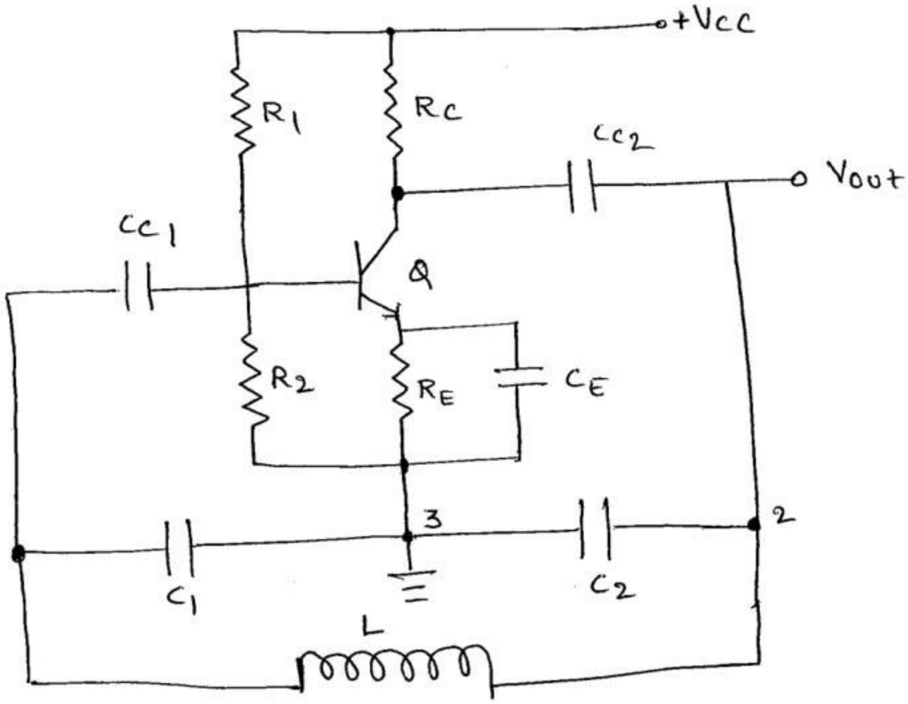
$$h_{fe} = \frac{L_1 + 2L_1M + M^2}{L_1L_2 + M(L_1+L_2) + M^2}$$

$$h_{fe} = \frac{(L_1+M)^2}{(L_1+M)(L_2+M)} = \frac{L_1+M}{L_2+M}$$

the minimum value of h_{fe} required to maintain sustain oscillation is

$$\boxed{h_{fe} \geq \frac{L_1+M}{L_2+M}}$$

COLPITTS OSCILLATOR :



In the colpitts oscillator shown in figure above z_1 and z_2 are capacitors and z_3 is an inductor. The Resistors R_1, R_2 and R_E provide the necessary dc bias to the transistor. C_E is a bypass capacitor

C_{c1} and C_{c2} are coupling capacitors. The feedback network consisting of capacitors C_1 and C_2 and inductor L determines the frequency of the oscillator.

when the supply voltage $+V_{cc}$ is switched a transient current is produced in the tank circuit and consequently oscillations are set up in the circuit. The oscillatory current in the tank circuit produces a.c voltages across C_1 and C_2. If terminal 1 is at a positive potential with respect to 3 at any instant, terminal 2 will be a negative potential with respect to 3 at the same instant. Thus the phase difference between the terminals 1 and 2 is always 180°. In CE mode transistor provides 180°, then the total phase shift is 360°. Thus at the frequency determinant for the tank circuit, the necessary condition for sustained oscillation is satisfied.

If the feedback is adjusted so that the loop gain $A\beta = 1$, the circuit acts as an oscillator. The frequency of oscillation is

$$f_r = \frac{1}{2\pi\sqrt{L C_{eq}}}$$

where $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

Analysis: For colpitts oscillator

$$z_1 = \frac{1}{j\omega C_1} = \frac{-j}{\omega C_1} \; ; \quad z_2 = \frac{1}{j\omega C_2} = \frac{-j}{\omega C_2} \; ; \quad z_3 = j\omega L$$

the general equation for the oscillator

$$h_{ie}(z_1 + z_2 + z_3) + z_1 z_2 (1 + h_{fe}) + z_1 z_3 = 0 \longrightarrow (1)$$

$$h_{ie}\left(\frac{-j}{\omega C_1} - \frac{j}{\omega C_2} + j\omega L\right) + \frac{-j}{\omega C_1}\,\frac{-j}{\omega C_2}\,(1 + h_{fe}) + \frac{-j}{\omega C_1} \cdot j\omega L = 0$$

$$-j\,h_{ie}\left(\frac{1}{\omega C_1} + \frac{1}{\omega C_2} - \omega L\right) + \left(\frac{L}{C_1} - \frac{(1 + h_{fe})}{\omega^2 C_1 C_2}\right) = 0 \longrightarrow (2)$$

The frequency of oscillation f_r is found by equating the imaginary part of eq (1) to zero. Thus we get

$$\frac{1}{\omega_r C_1} + \frac{1}{\omega_r C_2} - \omega_r L = 0$$

$$\frac{1}{\omega_r}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \omega_r L$$

$$\omega_r^2 = \frac{C_1 + C_2}{L\, C_1 C_2} \Rightarrow \omega_r^2 = \frac{1}{L \frac{C_1 C_2}{C_1 + C_2}}$$

$$\omega_r^2 = \frac{1}{L\, C_{eq}} \quad \text{where} \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

~~$f_r = \frac{1}{2\pi\sqrt{L\,C_{eq}}}$~~

$$\omega_r = \frac{1}{\sqrt{L\,C_{eq}}} \longrightarrow (3) \Rightarrow \boxed{f_r = \frac{1}{2\pi\sqrt{L C_{eq}}}}$$

The condition for maintenance of oscillation is determined by substituting Eq (3) in Eq (2). After substituting Eq (3) in (2), the imaginary part of Eq (2) becomes zero, then the real part

$$\frac{L}{C_1} - \frac{(1+h_{fe})}{\omega_n^2 C_1 C_2} = 0$$

$$\frac{L}{C_1} = \frac{(1+h_{fe})}{\frac{1}{L C_{eq}} C_1 C_2} = 0$$

$$\frac{L}{C_1} = \frac{(1+h_{fe})}{\frac{1}{L \frac{C_1 C_2}{C_1+C_2}} \times C_1 C_2}$$

$$1+h_{fe} = \frac{L}{C_1} \frac{C_1+C_2}{L} \Rightarrow 1+h_{fe} = \frac{C_1+C_2}{C_1}$$

$$\Rightarrow 1+h_{fe} = 1+\frac{C_2}{C_1}$$

$$\Rightarrow \boxed{h_{fe} = \frac{C_2}{C_1}}$$

The minimum value of h_{fe} required to obtain sustained oscillation is

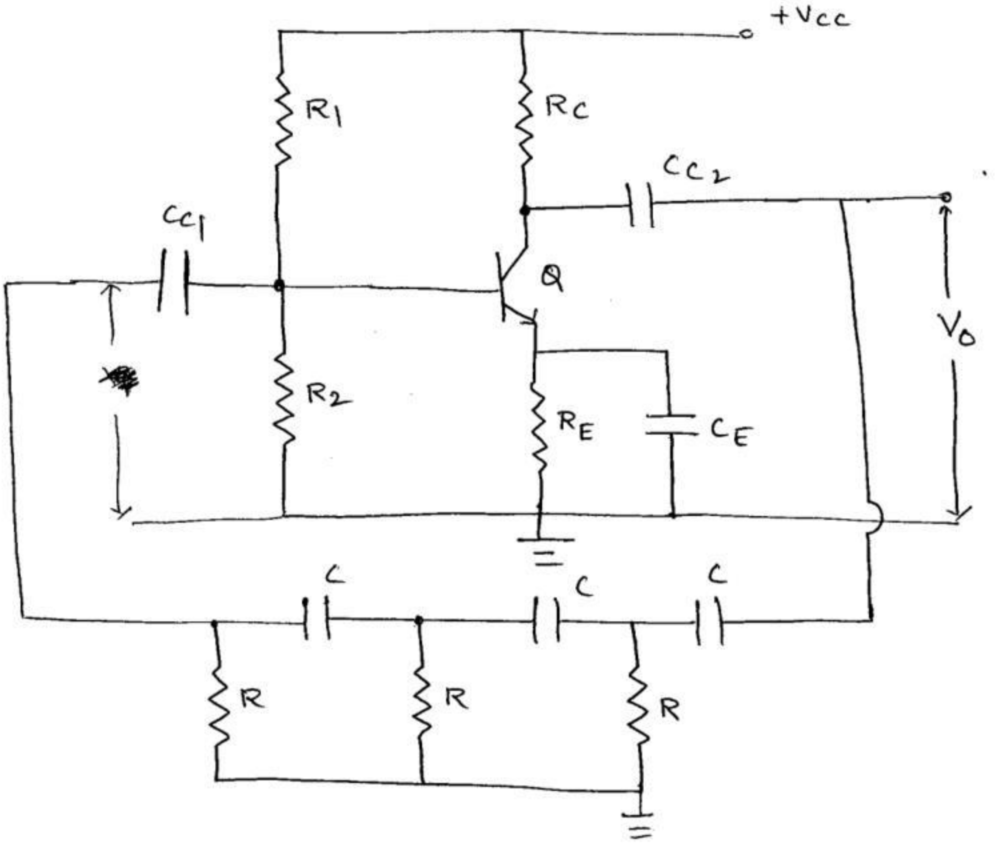
$$\boxed{h_{fe} \geq \frac{C_2}{C_1}}$$

RC OSCILLATORS

All the oscillators using LC circuits operate well at high frequencies. At low frequencies, as the inductors and capacitors required for the time circuit would be very bulky. RC oscillators are found to be more suitable. Two important RC oscillators are

1) RC phase shift oscillator
2) Wien Bridge oscillator.

RC phase shift oscillator:



In this oscillator the required phase shift of 180° in the feedback loop from output to input is obtained by using R and C components instead of tank circuit.

Here a common emitter amplifier is followed by three RC sections of RC phase shift n/w, the output of the last section being returned to the input.

The phase shift ϕ, given by each RC section is $\phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$. The value of R is adjusted such that ϕ becomes 60°. Therefore 3 RC sections produce a total phase shift of 180° between its input and output voltages for only the given frequency. therefore at the specific frequency f_r, the total phase shift from the base of the transistor around the circuit and back to the base will be exactly 360° or 0°, thereby satisfying Barkhausen condition for oscillation.

the frequency of oscillation is given by

$$f_r = \frac{1}{2\pi RC\sqrt{6+4K}} \quad \text{where } K = \frac{R_C}{R}$$

<u>Analysis</u>: the equivalent circuit of RC phase shift oscillator using h-parameter model is shown in figure below.

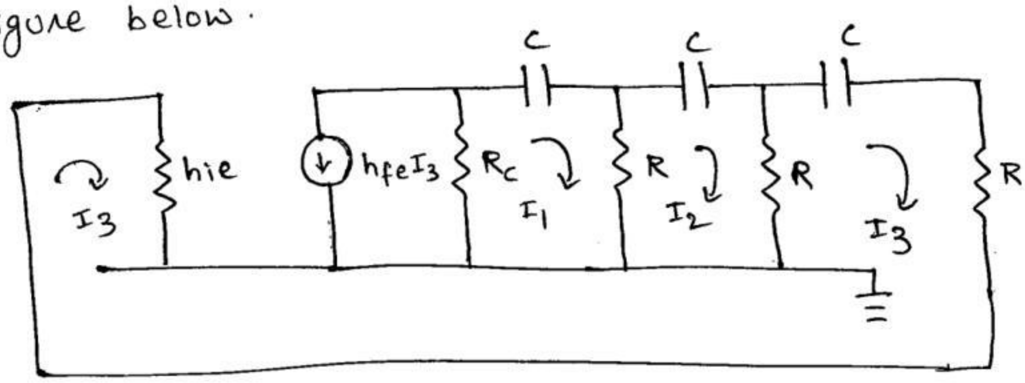


Fig: Equivalent circuit using h-parameter model.

The modified equivalent circuit is shown in figure below.

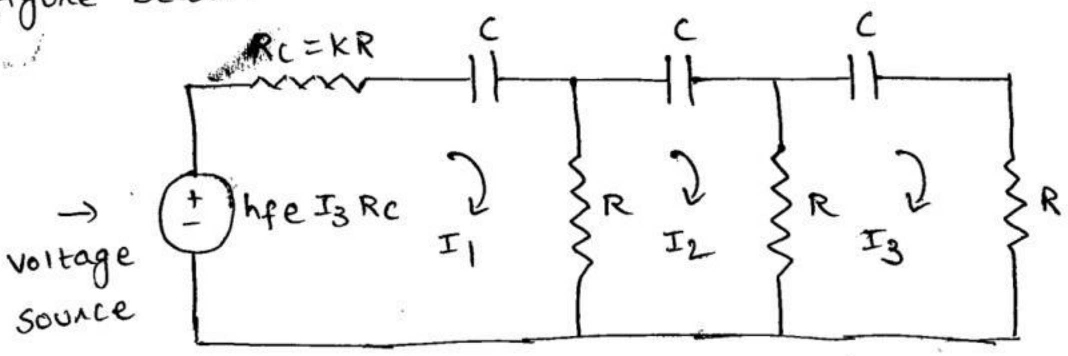


Fig: Modified Equivalent circuit.

Applying KVL for the Various loops in the modified Equivalent circuit

Loop 1:

$$-h_{fe} I_3 R_C = I_1 R_C + \frac{I_1}{j\omega C} + (I_1 - I_2) R$$

$$-h_{fe} I_3 K R = I_1 K R + \frac{I_1}{j\omega C} + I_1 R - I_2 R$$

$$-h_{fe} I_3 K R = I_1 \left(KR + R + \frac{1}{j\omega C} \right) - I_2 R$$

$$I_1 \left[(K+1) R + \frac{1}{j\omega C} \right] - I_2 R + I_3 h_{fe} K R = 0 \longrightarrow (1)$$

Loop 2:

$$\frac{I_2}{j\omega C} + (I_2 - I_3) R + (I_2 - I_1) R = 0$$

$$-I_1 R + I_2 \left(2R + \frac{1}{j\omega C} \right) - I_3 R = 0 \longrightarrow (2)$$

Loop 3:

$$\frac{I_3}{j\omega C} + I_3 R + (I_3 - I_2) R = 0$$

$$-I_2 R + I_3 \left(2R + \frac{1}{j\omega C} \right) = 0 \longrightarrow (3)$$

Solving the Equations (1), (2) and (3)

$$\begin{vmatrix} (K+1) R + \frac{1}{j\omega C} & -R & h_{fe} K R \\ -R & 2R + \frac{1}{j\omega C} & -R \\ 0 & -R & 2R + \frac{1}{j\omega C} \end{vmatrix} = 0$$

$$\Rightarrow \left[(K+1)R + \frac{1}{j\omega C}\right]\left[\left(2R + \frac{1}{j\omega C}\right)^2 - R^2\right] + R\left[-R\left(2R + \frac{1}{j\omega C}\right)\right] +$$

$$h_{fe}\, KR(R^2) = 0$$

$$\Rightarrow \left[(K+1)R + \frac{1}{j\omega C}\right]\left(4R^2 - \frac{1}{\omega^2 C^2} + \frac{4R}{j\omega C} - R^2\right) - 2R^3 - \frac{R^2}{j\omega C} +$$

$$h_{fe}\, KR^3 = 0$$

$$\Rightarrow 3R^3(K+1) + \frac{3R^2}{j\omega C} - \frac{(K+1)R}{\omega^2 C^2} - \frac{1}{j\omega^3 C^3} + \frac{4R^2(K+1)}{j\omega C}$$

$$- \frac{4R}{\omega^2 C^2} - 2R^3 - \frac{R^2}{j\omega C} + h_{fe} KR^3 = 0$$

$$\Rightarrow R^3(3K + 1 + h_{fe}K) - \left(\frac{(K+1)R + 4R}{\omega^2 C^2}\right) + j\left[\frac{-4(K+1)R^2}{\omega C}\right.$$

$$\left. - \frac{2R^2}{\omega C} + \frac{1}{\omega^3 C^3}\right] = 0 \longrightarrow (4)$$

To get frequency of oscillation f_r, the imaginary part is made equal to zero

$$\frac{-4(K+1)R^2}{\omega_r C} - \frac{2R^2}{\omega_r C} + \frac{1}{\omega_r^3 C^3} = 0$$

$$-4KR^2\omega_r^2 C^2 - 4R^2\omega_r^2 C^2 - 2R^2\omega_r^2 C^2 + 1 = 0$$

$$1 = 6R^2\omega_r^2 C^2 + 4KR^2\omega_r^2 C^2$$

$$\frac{1}{\omega_r^2} = (4K+6)R^2 C^2$$

$$\omega_r = \frac{1}{RC\sqrt{6+4K}} \Rightarrow \boxed{f_r = \frac{1}{2\pi RC\sqrt{6+4K}}}$$

where $K = R_C / R$

The condition for maintainence of oscillation is obtained by equating real part to zero

$$R^3(3K+1+h_{fe}K) - \left(\frac{(K+1)R+4R}{\omega_r^2 C^2}\right) = 0$$

$$R^3(3K+1+h_{fe}K)\,\omega_r^2 C^2 = KR - 5R = 0$$

$$R^3(3K+1+h_{fe}K)\frac{1}{(6+4K)R^2C^2} \times C^2 = (K+5)R$$

$$3K+1+h_{fe}K = (6+4K)(5+K)$$

$$3K+1+h_{fe}K = 30+26K+4K^2$$

$$4K^2+23K+29 = h_{fe}K$$

$$h_{fe} = 4K+23+\frac{29}{K}$$

If $K=1$, then $\boxed{h_{fe}=56}$

To find the minimum value of h_{fe} for the oscillator

$$\frac{dh_{fe}}{dK} = 0$$

$$\frac{d}{dK}\left[4K+23+\frac{29}{K}\right] = 0$$

$$4 - \frac{29}{K^2} = 0 \implies K = 2.69$$

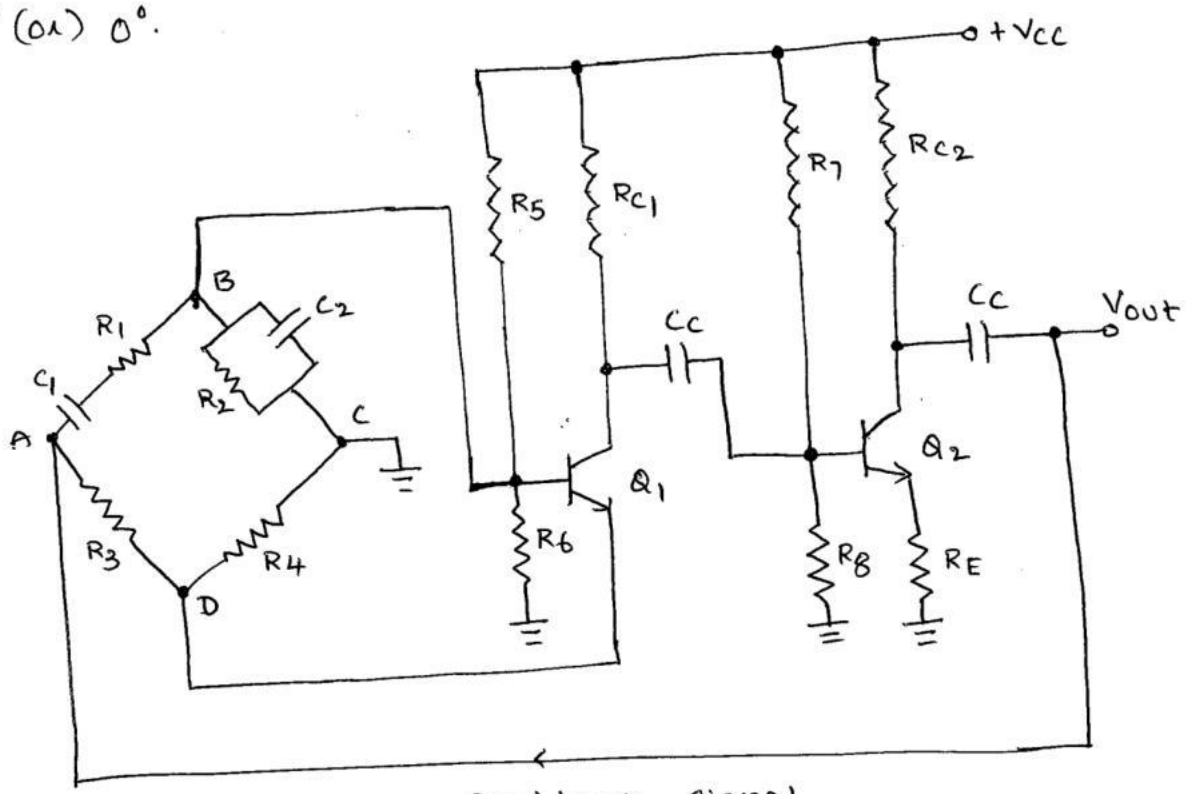
$$h_{fe(min)} = 4(2.69)+23+\frac{29}{2.69} = 44.54.$$

Thus for the circuit to oscillate we must select a transistor whose hfe should be greater than 44.54

$\therefore$ $\boxed{h_{fe} \geq 44.54}$

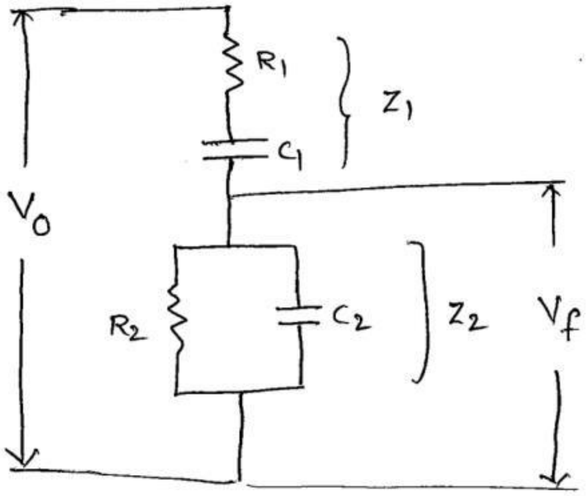
Wien Bridge oscillator:

Figure shows the circuit of a wien-bridge oscillator. The circuit consists of two-stage RC coupled amplifier which provides a phase shift of 360° (or) 0°.



feed back signal

A balanced bridge is used as the feed back network which has no need to provide additional phase shift. The feedback network consists of a lead-lag network (R_1-C_1 and R_2-C_2) and a voltage divider ($R_3 - R_4$). The lead - lag network provides a positive feedback to the input of the first stage and the voltage divider provides a negative feedback to the emitter of Q_1.



From the circuit

$$z_1 = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1} \longrightarrow (1)$$

$$z_2 = R_2 \,||\, \frac{1}{j\omega C_2} = \frac{R_2}{1 + j\omega C_2 R_2} \longrightarrow (2)$$

$$\beta = \frac{V_f}{V_o}$$

From the above circuit

$$V_f = V_o \frac{z_2}{z_1 + z_2}$$

$$\beta = \frac{V_f}{V_o} = \frac{z_2}{z_1 + z_2} = \frac{\dfrac{R_2}{1 + j\omega C_2 R_2}}{\dfrac{1 + j\omega R_1 C_1}{j\omega C_1} + \dfrac{R_2}{1 + j\omega C_2 R_2}}$$

$$\beta = \frac{\dfrac{R_2}{1+j\omega C_2 R_2}}{\dfrac{(1+j\omega R_1 C_1)(1+j\omega R_2 C_2)+j\omega R_2 C_1}{(j\omega C_1)(1+j\omega C_2 R_2)}}$$

$$\beta = \frac{j\omega R_2 C_1}{1+j\omega R_1 C_1 + j\omega R_2 C_2 - \omega^2 R_1 R_2 C_1 C_2 + j\omega R_2 C_1}$$

$$\Rightarrow \beta = \frac{j\omega R_2 C_1}{(1-\omega^2 R_1 R_2 C_1 C_2) + j(\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)} \rightarrow (3)$$

$$\beta = \frac{j\omega R_2 C_1}{(1-\omega^2 R_1 R_2 C_1 C_2) + j(\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)} \times \frac{(1-\omega^2 R_1 R_2 C_1 C_2) - j(\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)}{(1-\omega^2 R_1 R_2 C_1 C_2) - j(\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)}$$

$$\beta = \frac{j(\omega R_2 C_1 - \omega^3 R_1 R_2^2 C_1^2 C_2) + \omega R_2 C_1(\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)}{(1-\omega^2 R_1 R_2 C_1 C_2)^2 + (\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)^2}$$

To find the frequency of oscillator make imaginary part is zero.

$$\frac{\omega R_2 C_1 - \omega^3 R_1 R_2^2 C_1^2 C_2}{(1-\omega^2 R_1 R_2 C_1 C_2)^2 + (\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)^2} = 0$$

$$\omega_r R_2 C_1 = \omega_r^3 R_1 R_2^2 C_1^2 C_2$$

$$1 = \omega_r^2 R_1 R_2 C_1 C_2$$

$$\omega_r^2 = \frac{1}{R_1 R_2 C_1 C_2} \longrightarrow (4)$$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \rightarrow (5)$$

$$f_n = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \longrightarrow (6)$$

If $R_1 = R_2 = R$; $C_1 = C_2 = C$;

$$f_n = \frac{1}{2\pi\sqrt{R^2 C^2}} = \frac{1}{2\pi RC} \longrightarrow (7)$$

$\omega_n = \frac{1}{RC}$, substitute in (3)

$$\beta = \frac{j \frac{1}{RC} R_2 C_1}{(1-1) + j\left(\frac{1}{RC} R_1 C_1 + \frac{1}{RC} R_2 C_2 + \frac{1}{RC} R_2 C_1\right)}$$

Her $R_1 = R_2 = R$; $C_1 = C_2 = C$

$$\beta = \frac{j}{j(1+1+1)} = \frac{1}{3}$$

~~the~~ For sustained oscillations

$$A\beta = 1$$

$$A = 3$$

The minimum value of voltage gain required for sustained oscillation is 3

$$\therefore \boxed{A \geq 3}$$

Frequency stability:

The measure of ability of an oscillator to maintain desire frequency as precisely as possible for a long time is called frequency stability of an oscillator.

The factors which affect the frequency stability

1) Temperature changes → L and C values in feedback circuit changes Hence frequency changes.
2) If temperature changes, the parameters of BJT, FET changes Hence frequency changes.
3) changes in power supply causes change in frequency.
4) changes in atmospheric conditions, due to aging.
5) changes in Load connected, the effective resistance in feedback circuit changes, Hence frequency changes.
6) collector base junction is in reverse bias condition. So there will be internal capacitance. The capacitance effect the capacitance in feedback circuit. Hence frequency changes.

The variation of frequency with temperature is given by a factor

$$S_\omega = \frac{\Delta\omega / \Delta\omega_r}{\Delta T / T_r}$$

where $\omega_r \rightarrow$ desired frequency

$T_r \rightarrow$ operating Temperature

$\Delta\omega \rightarrow$ change in frequency

$\Delta T \rightarrow$ change in Temp.

The frequency stability is defined as

$$S_\omega = \frac{d\theta}{d\omega}$$

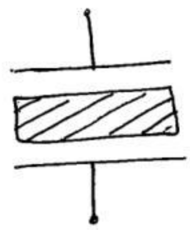
$d\theta$ = phase shift introduced for a small change in desired frequency.

Frequency stability can be improved by following modifications

1) Enclosing the oscillator circuit in a constant temperature chamber.
2) maintaining the constant voltage by using zener diodes.

__Crystal oscillator:__

__construction:__



In nature, crystal is in the shape of hexagonal prism.

For practical use the prism is cut in to a rectangular slab, which is mounted on parallel metal plates.

Crystal materials : Quartz , Rochelle salt etc.

Crystal Exhibits a property called piezo Electric Effect.

1) when a mechanical pressure is applied on the crystal, the crystal tends to vibrate and develop a.c voltages across the opposite faces of the crystal.

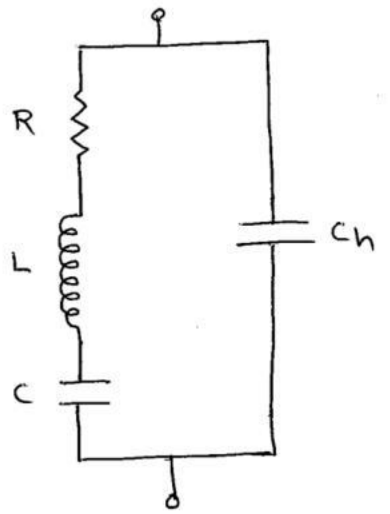
2) If we apply a.c voltages across the two faces of the crystal It vibrates causing mechanical distortion in the crystal state.

→ The crystal has the greater stability in holding the constant frequency (crystal oscillator is a stable oscillator

→ Generally we prefer Quartz as Crystal because of less expensive and Quartz is usually available in nature.

Figure shows a crystal controlled oscillator circuit. Here it is a colpitts oscillator in which the inductor is replaced by the crystal. In this type, a piezo electric crystal, usually quartz, is used as a resonant circuit replacing an LC circuit.

The AC Equivalent circuit of a piezo electric crystal is shown in figure below.



when the crystal is not Vibrating it is equivalent to capacitance C_M, because it has two metal plates separated by a dielectric. This capacitance is Known as mounting capacitance.

when a crystal vibrates it is equivalent to RLC series circuit.

R → internal frictional losses

L → mass of the crystal, indication of inertia

C → stiffness.

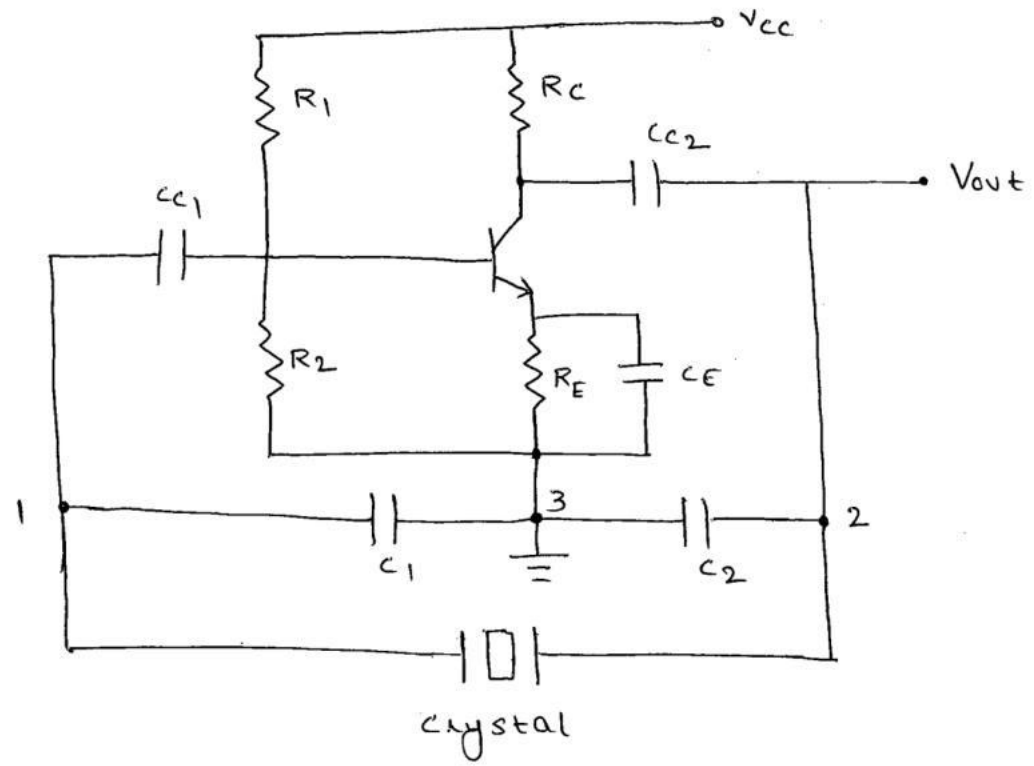


Fig: crystal oscillator.

Here the resonant frequency f_r is given by

$$f_r = \frac{1}{2\pi\sqrt{LC}\sqrt{\frac{Q^2}{1+Q^2}}}$$

$Q \rightarrow$ Quality factor $= \frac{X_L}{R} = \frac{\omega L}{R}$

In general Q-value is very high

If $Q^2 >> 1$, then f_r becomes

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Series Resonance:

$$X_L = X_C \quad (X_L - X_C = 0)$$

$$\omega_s L = \frac{1}{\omega_s C}$$

$$\omega_s^2 = \frac{1}{LC} \Rightarrow \omega_s = \frac{1}{\sqrt{LC}}$$

$$f_s = \frac{1}{2\pi\sqrt{LC}}.$$

Parallel Resonance :-

$$X_{series} = X_{CM}$$

$$(X_L - X_C) = X_{CM}$$

$$\omega_p L - \frac{1}{\omega_p C} = \frac{1}{\omega_p C_M}$$

$$\omega_p^2 LC - 1 = \frac{C}{C_M}$$

$$\omega_p^2 LC = 1 + \frac{C}{C_M} = \frac{C + C_M}{C_M}$$

$$\omega_p^2 = \frac{C + C_M}{L C C_M} \Rightarrow \omega_p^2 = \frac{1}{L C_{eq}}$$

where $C_{eq} = \frac{C C_M}{C + C_M}$

$$\therefore \omega_p = \frac{1}{\sqrt{L C_{eq}}} \Rightarrow f_p = \frac{1}{2\pi\sqrt{L C_{eq}}}$$

Problem: In hartley oscillator calculate L_2 with $L_1 = 15\,mH$, $C = 50\,PF$ and $M = 5\mu H$ and the frequency of oscillator is 168 KHz.

Solution:

$$f_r = \frac{1}{2\pi\sqrt{L_{eq}\, C}}$$

where $L_{eq} = L_1 + L_2 + 2M$

$f_r = 168\,KHz$, $C = 50\,PF$, $L_1 = 15\,mH$, $M = 5\mu H$

$$\therefore L_2 = 2.94\,mH.$$

pb) In transistorized Hartley oscillator the two inductors are 2mH and 20 μH. while the frequency is change from 950 KHz to 2050 KHz. Calculate the range over which the capacitor is to be varied.

Solution: Given data

$L_1 = 2\,mH$, $L_2 = 20\,\mu H$, $M = 0$ | $f_{r_1} = 950\,KHz$, $f_{r_2} = 2050\,KHz$

$$L_{eq} = L_1 + L_2 = 2.02 \times 10^{-3}$$

$$f_{r_1} = \frac{1}{2\pi\sqrt{L_{eq} C_1}} \Rightarrow C_1 = 0.139\,\mu F.$$

$$f_{r_2} = \frac{1}{2\pi\sqrt{L_{eq} C_2}} \Rightarrow C_2 = 2.98\,PF.$$

problem: In a Hartley oscillator the value of capacitor in tuned circuit is 500 PF and two sections of coil have inductances 38 μH and 12 μH Find the frequency of oscillator and feed back factor β.

Solution:

$$f_r = \frac{1}{2\pi\sqrt{L_{eq} C}}$$

$L_{eq} = L_1 + L_2 + 2M$

$M = 0$

$L_{eq} = 0.5\,\mu H$.

$$f_r = \frac{1}{2\pi\sqrt{0.5 \times 10^{-6} \times 500 \times 10^{-12}}} = 1\,MHz$$

feed back factor $\beta = \frac{V_f}{V_o}$

Feed back voltage V_f is proportional to X_{L_1}

output voltage V_o is proportional to X_{L_2}

$$\frac{V_f}{V_o} = \frac{X_{L1}}{X_{L2}} = \frac{j\omega L_1}{j\omega L_2} = \frac{L_1}{L_2}$$

$$\beta = 3.166.$$

Problem: In colpitts oscillator $C_1 = 0.2\,\mu F$, $C_2 = 0.02\,\mu F$ If the frequency of oscillator is 10 KHz. Find the value of inductor. and also find the required gain for oscillation.

Solution: $C_1 = 0.2\,\mu F$, $C_2 = 0.02\,\mu F$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 0.01\,\mu F$$

$$f_r = \frac{1}{2\pi\sqrt{L\,C_{eq}}} \Rightarrow L = 14.07\,mH.$$

$$A_V = \frac{V_o}{V_i} = \frac{V_o}{V_f}$$

V_o is proportional to $X_{C2} = \frac{1}{j\omega C_2}$

V_f is proportional to $X_{C1} = \frac{1}{j\omega C_1}$

$$A = \frac{1/j\omega C_2}{1/j\omega C_1} = \frac{C_1}{C_2} \Rightarrow A = \frac{0.2\,\mu F}{0.02\,\mu F} = 10$$

$$A \geq \frac{C_1}{C_2} \Rightarrow A \geq 10$$

Problem: colpitts oscillator is designed with $C_1 = 7500\,PF$, $C_2 = 100\,PF$, the inductance is variable. Determine the range of inductance values if the frequencies of oscillation is to vary between 950 KHz to 2050 KHz.

Solution: $f_{r_1} = \dfrac{1}{2\pi\sqrt{L_1 C_{eq}}}$

$C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2} = 9.86 \times 10^{-11}$, $f_{r_1} = 950\,KHz$

$\Rightarrow L_1 = 0.28\,mH$.

Ily $f_{r_2} = \dfrac{1}{2\pi\sqrt{L_2 C_{eq}}}$, $f_{r_2} = 2050\,KHz$

$\Rightarrow L_2 = 0.06\,mH$.

Problem: The frequency sensitive arms of wien bridge oscillator $C_1 = C_2 = 0.001\,PF$, $R_1 = 10\,K\Omega$, R_2 is kept variable. The frequency is varied from 10 KHz to 50 KHz. By varying R_2 find the minimum and maximum values of R_2.

Solution: $f = \dfrac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$

$f_1 = 10\,KHz$, $R_1 = 10\,K\Omega$, $C_1 = C_2 = 0.001\,PF$

$R_2 = 2.53 \times 10^{16}\,\Omega$

$f_2 = 50\,KHz$, $R_1 = 10\,K\Omega$, $C_1 = C_2 = 0.001\,PF$

$R_2 = 1.013 \times 10^{15}\,\Omega$.

Problem: A crystal oscillator has $L = 0.4\,H$, $C = 0.085\,PF$, $C_M = 1\,PF$, $R = 5\,K\Omega$. Find series and parallel resonating frequencies. By what percent does the parallel resonating frequency exceeds series resonating frequency and also find quality factor of the crystal.

Solution:

Series resonating frequency $f_s = \frac{1}{2\pi\sqrt{LC}}$

$$\therefore f_s = 863.13 \text{ KHz}$$

Parallel resonating frequency $f_p = \frac{1}{2\pi\sqrt{L C_{eq}}}$

$$C_{eq} = \frac{C C_M}{C + C_M} = 7.83 \times 10^{-14}$$

$$f_p = \frac{1}{2\pi\sqrt{L C_{eq}}} = \frac{1}{2\pi\sqrt{0.4 \times 7.83 \times 10^{-14}}}$$

$$f_p = 899.3 \text{ KHz}$$

$$\text{percentage} = \frac{f_p - f_s}{f_s} \times 100 = 4.19\,\%$$

Quality factor $Q = \frac{X_s}{R}$

$$Q = \frac{\omega_s L}{R} = \frac{2\pi f_s L}{R} = 433.8.$$

UNIT 4

Amplifier Classifications:

- In small-signal amplifiers, the main factors are usually amplification linearity and magnitude of gain.
- Large-signal or power amplifiers, on the other hand, primarily provide sufficient power to an output load to drive a speaker or other power device, typically a few watts to tens of watts.
- The main features of a large-signal amplifier are the circuit's power efficiency, the maximum amount of power that the circuit is capable of handling, and the impedance matching to the output device.

Amplifier classes represent the amount the output signal varies over one cycle of operation for a full cycle of input signal

Power Amplifier Classes:

1. **Class A**: The output signal varies for a full 360° of the input signal.Bias at the half of the supply
2. **Class B**: provides an output signal varying over one-half the input signal cycle, or for 180° of signal. Bias at the zero level

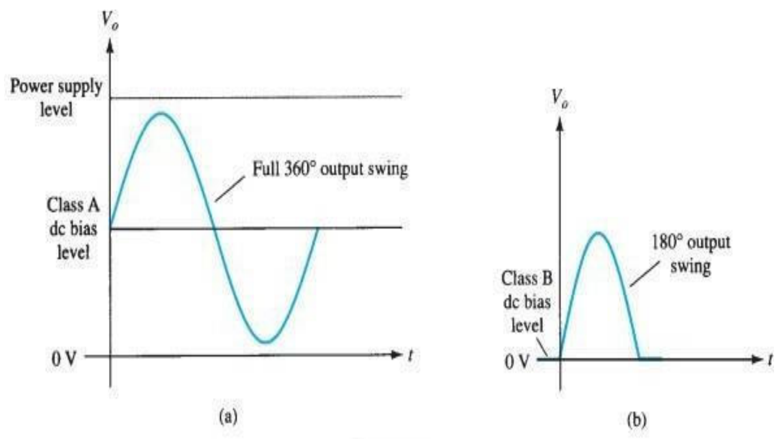


FIG. 12.1
Amplifier operating classes.

SERIES-FED CLASS A AMPLIFIER

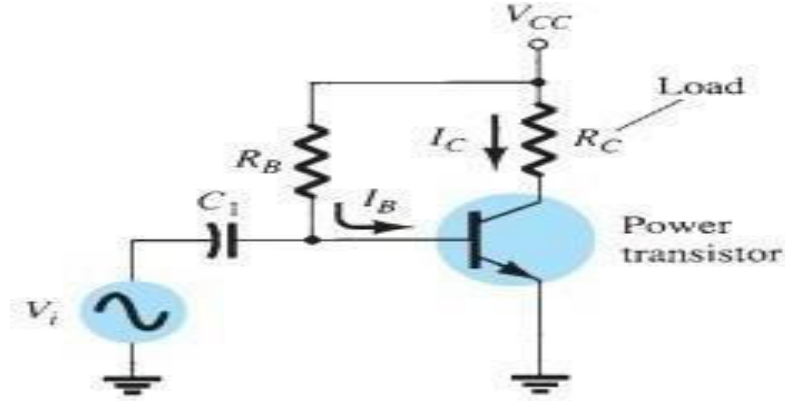


FIG. 12.2
Series-fed class A large-signal amplifier.

AC OPERATION

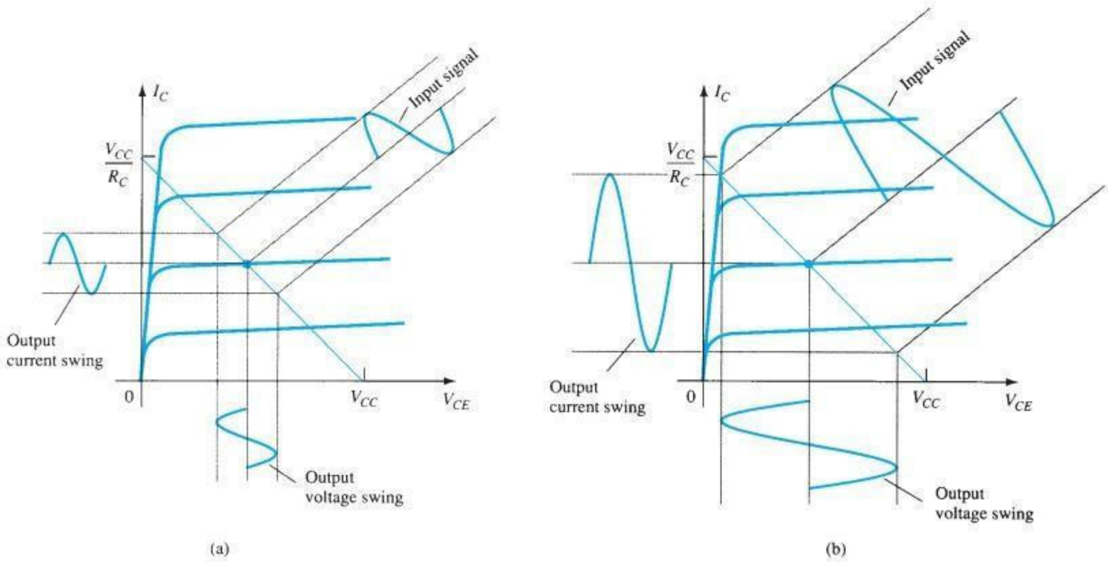


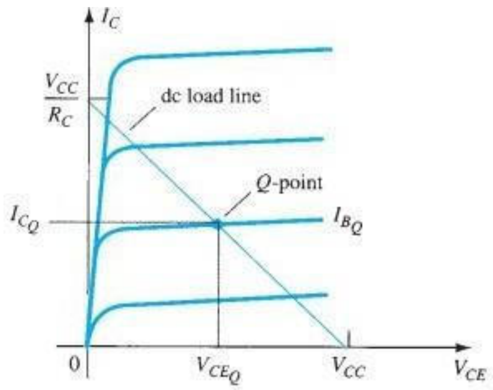
FIG. 12.4
Amplifier input and output signal variation.

DC Bias Operation

$$I_B = \frac{V_{CC} - 0.7\text{ V}}{R_B}$$

$$I_C = \beta I_B$$

$$V_{CE} = V_{CC} - I_C R_C$$



PowerConsideration

The power drawn from the supply is $$P_i(\text{dc}) = V_{CC} I_{C_Q}$$

Output Power

$$P_o(\text{ac}) = V_{CE}(\text{rms}) I_C(\text{rms})$$

$$P_o(\text{ac}) = I_C^2(\text{rms}) R_C$$

$$P_o(\text{ac}) = \frac{V_C^2(\text{rms})}{R_C}$$

Efficiency:

$$\%\,\eta = \frac{P_o(\text{ac})}{P_i(\text{dc})} \times 100\%$$

Maximum Efficiency

$$\text{maximum } V_{CE}(\text{p-p}) = V_{CC}$$

$$\text{maximum } I_C(\text{p-p}) = \frac{V_{CC}}{R_C}$$

$$\text{maximum } P_o(\text{ac}) = \frac{V_{CC}(V_{CC}/R_C)}{8} = \frac{V_{CC}^2}{8R_C}$$

$$\text{maximum } P_i(\text{dc}) = V_{CC}(\text{maximum } I_C) = V_{CC}\frac{V_{CC}/R_C}{2} = \frac{V_{CC}^2}{2R_C}$$

$$\text{maximum } \% \, \eta = \frac{\text{maximum } P_o(\text{ac})}{\text{maximum } P_i(\text{dc})} \times 100\% = \frac{V_{CC}^2/8R_C}{V_{CC}^2/2R_C} \times 100\% = 25\%$$

TRANSFORMER-COUPLED CLASS A AMPLIFIER

TransformerAction

- A transformer can increase or decrease voltage or current levels according to its turns ratio **a=N_1:N_2**
- The impedance connected to one side of a transformer can be made to appear either larger or smaller (step up or step down) at the other side of the transformer.

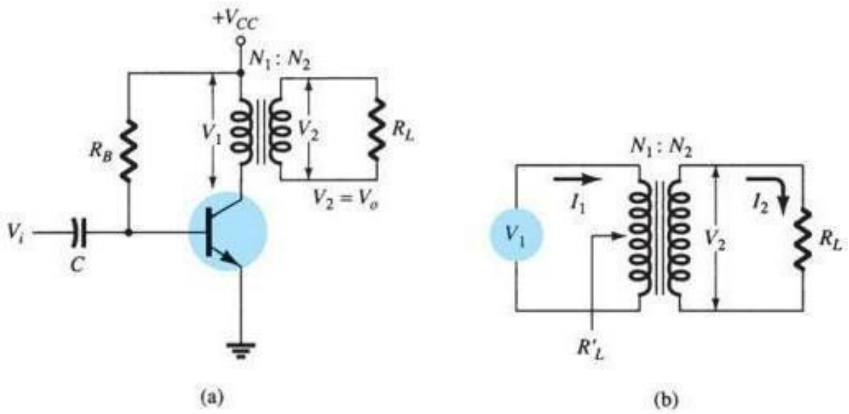
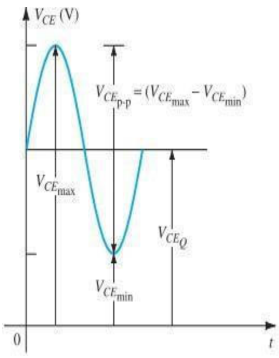
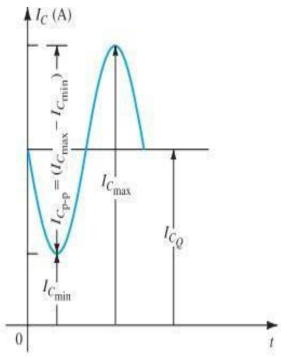
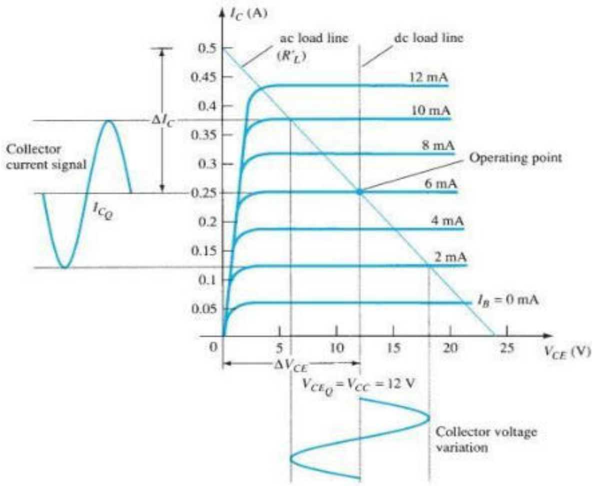


FIG. 12.6
Transformer-coupled audio power amplifier.

OperaationofAmplifierStage







$$V_{CE}(\text{p-p}) = V_{CE_{max}} - V_{CE_{min}}$$
$$I_C(\text{p-p}) = I_{C_{max}} - I_{C_{min}}$$

$$P_o(\text{ac}) = \frac{(V_{CE_{max}} - V_{CE_{min}})(I_{C_{max}} - I_{C_{min}})}{8}$$

Efficiency

$$P_i(\text{dc}) = V_{CC} I_{C_Q}$$

$$\%\,\eta = \frac{P_o(\text{ac})}{P_i(\text{dc})} \times 100\%$$

$$\%\,\eta = 50\left(\frac{V_{CE_{max}} - V_{CE_{min}}}{V_{CE_{max}} + V_{CE_{min}}}\right)^2 \%$$

PUSH PULL AMPLIFIER

- Class B operation is provided when the dc bias leaves the transistor biased just off, the transistor turning on when the ac signal is applied.

- This is essentially no bias, and the transistor conducts current for only one-half of the signal cycle.

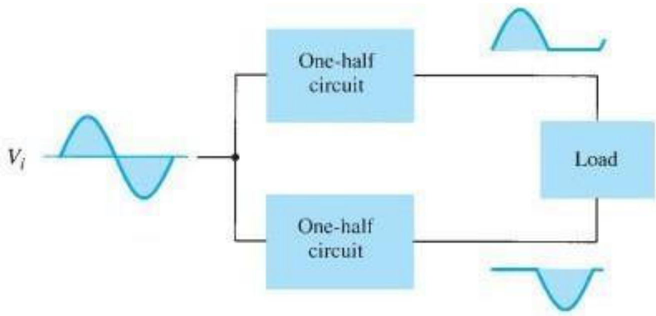


FIG. 12.12
Block representation of push–pull operation.

Efficiency & Power Consideration

$$P_o(\text{ac}) = \frac{V_L^2(\text{rms})}{R_L}$$

$$P_o(\text{ac}) = \frac{V_L^2(\text{p-p})}{8R_L} = \frac{V_L^2(\text{p})}{2R_L}$$

Efficiency

$$\%\,\eta = \frac{P_o(\text{ac})}{P_i(\text{dc})} \times 100\%$$

$$\%\,\eta = \frac{P_o(\text{ac})}{P_i(\text{dc})} \times 100\% = \frac{V_L^2(\text{p})/2R_L}{V_{CC}[(2/\pi)I(\text{p})]} \times 100\% = \frac{\pi}{4}\frac{V_L(\text{p})}{V_{CC}} \times 100\%$$

$V_L(\text{p}) = V_{CC}$, $\quad$ $\text{maximum efficiency} = \frac{\pi}{4} \times 100\% = 78.5\%$

Maximum Power Considerations.

$$\text{maximum } P_o(\text{ac}) = \frac{V_{CC}^2}{2R_L}$$

$$\text{maximum } I_{dc} = \frac{2}{\pi}I(\text{p}) = \frac{2V_{CC}}{\pi R_L}$$

$$\text{maximum } P_i(\text{dc}) = V_{CC}(\text{maximum } I_{dc}) = V_{CC}\left(\frac{2V_{CC}}{\pi R_L}\right) = \frac{2V_{CC}^2}{\pi R_L}$$

$$\text{maximum } \% \eta = \frac{P_o(\text{ac})}{P_i(\text{dc})} \times 100\% = \frac{V_{CC}^2/2R_L}{V_{CC}[(2/\pi)(V_{CC}/R_L)]} \times 100\%$$

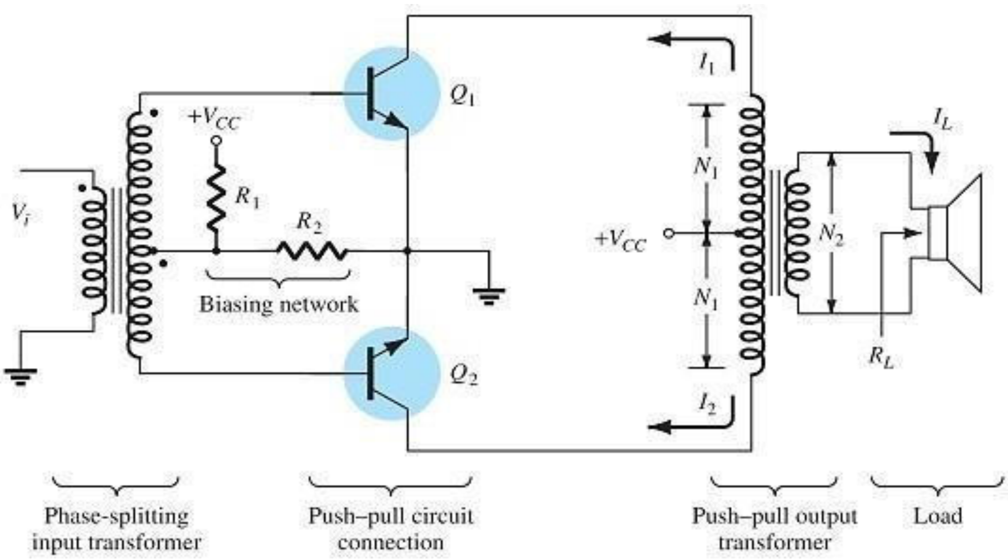
$$= \frac{\pi}{4} \times 100\% = \mathbf{78.54\%}$$

$$V_L(\text{p}) = 0.636V_{CC} \quad \left(= \frac{2}{\pi}V_{CC}\right)$$

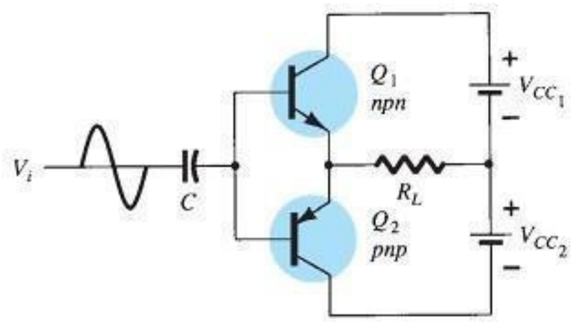
$$\text{maximum } P_{2Q} = \frac{2V_{CC}^2}{\pi^2 R_L}$$

ClassB AmplifierCircuit

Transformer-Coupled Push–Pull Circuits

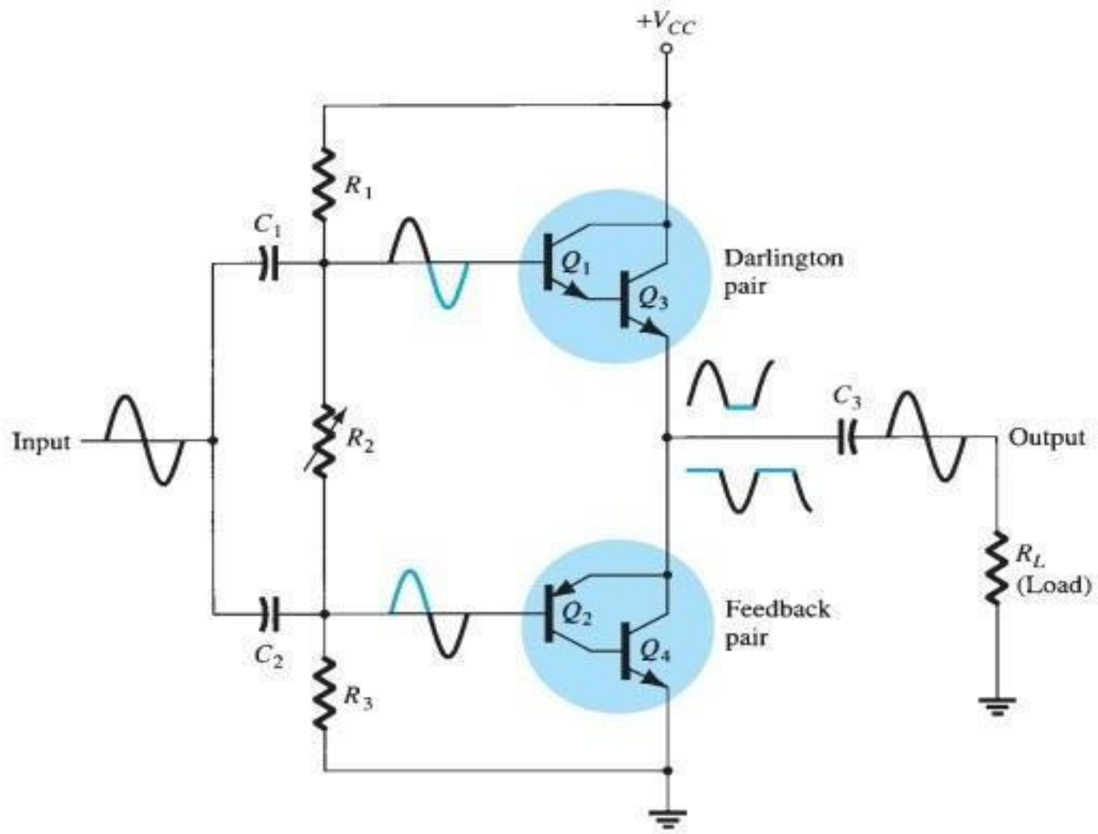


Complementary-Symmetry Circuits



Quasi-Complementary Push–Pull Amplifier

The push–pull operation is achieved by using complementary transistors (Q 1 and Q2) before the matched npn output transistors (Q3 and Q4)



UNIT – 5

LINEAR WAVESHAPING

....

High pass, low pass RC circuits, their response for sinusoidal, step, pulse, square and ramp inputs. RC network as differentiator and integrator, attenuators, its applications in CRO probe, RL and RLC circuits and their response for step input, Ringing circuit.

....

A linear network is a network made up of linear elements only. A linear network can be described by linear differential equations. The principle of superposition and the principle of homogeneity hold good for linear networks. In pulse circuitry, there are a number of waveforms, which appear very frequently. The most important of these are sinusoidal, step, pulse, square wave, ramp, and exponential waveforms. The response of *RC, RL,* and *RLC* circuits to these signals is described in this chapter. Out of these signals, the sinusoidal signal has a unique characteristic that it preserves its shape when it is transmitted through a linear network, i.e. under steady state, the output will be a precise reproduction of the input sinusoidal signal. There will only be a change in the amplitude of the signal and there may be a phase shift between the input and the output waveforms. The influence of the circuit on the signal may then be completely specified by the ratio of the output to the input amplitude and by the phase angle between the output and the input. No other periodic waveform preserves its shape precisely when transmitted through a linear network, and in many cases the output signal may bear very little resemblance to the input signal.

The process whereby the form of a non-sinusoidal signal is altered by transmission through a linear network is called linear wave shaping.

THE LOW-PASS *RC* CIRCUIT

Figure 1.1 shows a low-pass *RC* circuit. A low-pass circuit is a circuit, which transmits only low-frequency signals and attenuates or stops high-frequency signals.

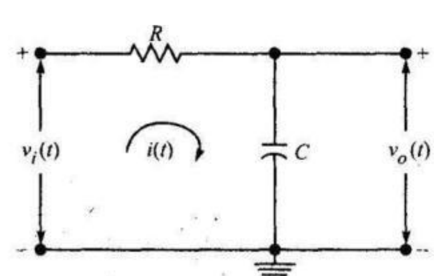


Figure 1.1 The low-pass *RC* circuit.

At zero frequency, the reactance of the capacitor is infinity (i.e. the capacitor acts as an open circuit) so the entire input appears at the output, i.e. the input is transmitted to the output with zero attenuation. So the output is the same as the input, i.e. the gain is unity. As the frequency increases the capacitive reactance decreases and so the output decreases. At very high frequencies the capacitor virtually acts as a short-circuit and the output falls to zero.

Sinusoidal Input

The Laplace transformed low-pass *RC* circuit is shown in Figure 1.2(a). The gain versus frequency curve of a low-pass circuit excited by a sinusoidal input is shown in Figure 1.2(b). This curve is obtained by keeping the amplitude of the input sinusoidal signal constant and varying its frequency and noting the output at each frequency. At low frequencies the output is equal to the input and hence the gain is unity. As the frequency increases, the output decreases and hence the gain decreases. The frequency at which the gain is $1/\sqrt{2}$ (= 0.707) of its maximum value is called the cut-off frequency. For a low-pass circuit, there is no lower cut-off frequency. It is zero itself. The upper cut-off frequency is the frequency (in the high-frequency range) at which the gain is $1/\sqrt{2}$. i-e- 70.7%, of its maximum value. The bandwidth of the low-pass circuit is equal to the upper cut-off frequency f_2 itself.

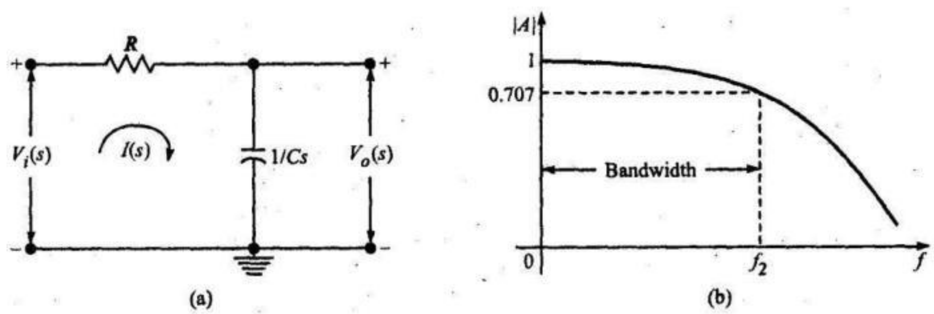


Figure 1.2 (a) Laplace transformed low-pass *RC* circuit and (b) its frequency response.

For the network shown in Figure 1.2(a), the magnitude of the steady-state gain A is given by

$$A = \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{1}{1 + RCs} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j2\pi fRC}$$

$$\therefore \quad |A| = \frac{1}{\sqrt{1 + (2\pi fRC)^2}}$$

At the upper cut-off frequency f_2, $|A| = \frac{1}{\sqrt{2}}$

$$\therefore \quad \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + (2\pi f_2 RC)^2}}$$

Squaring both sides and equating the denominators,

$$2 = 1 + (2\pi f_2 RC)^2$$

$\therefore$ The upper cut-off frequency, $f_2 = \dfrac{1}{2\pi RC}$.

So $$A = \frac{1}{1 + j\dfrac{f}{f_2}} \quad \text{and} \quad |A| = \frac{1}{\sqrt{1 + \left(\dfrac{f}{f_2}\right)^2}}$$

The angle θ by which the output leads the input is given by

$$\theta = \tan^{-1}\frac{f}{f_2}$$

Step-Voltage Input

A step signal is one which maintains the value zero for all times $t < 0$, and maintains the value V for all times t > 0. The transition between the two voltage levels takes place at $t = 0$ and is accomplished in an arbitrarily small time interval. Thus, in Figure 1.3(a), $v_i = 0$ immediately before $t = 0$ (to be referred to as time $t = 0^-$) and $v_i = V$, immediately after t= 0 (to be referred to as time $t = 0^+$). In the low-pass RC circuit shown in Figure 1.1, if the capacitor is initially uncharged, when a step input is applied, since the voltage across the capacitor cannot change instantaneously, the output will be zero at $t = 0$, and then, as the capacitor charges, the output voltage rises exponentially towards the steady-state value V with a time constant RC as shown in Figure 1.3(b).

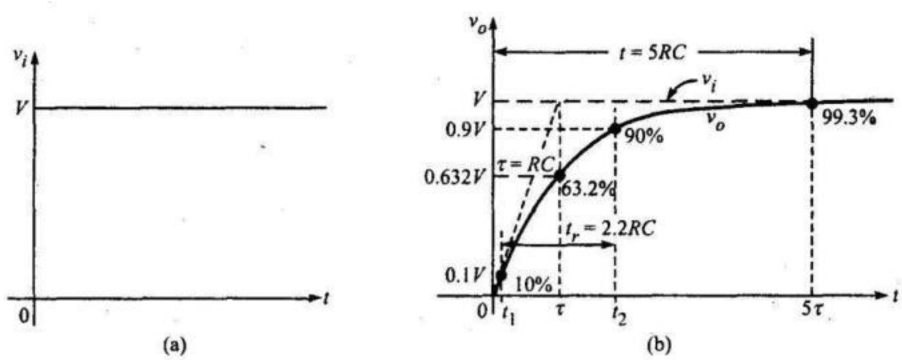


Figure 1.3 (a) Step input and (b) step response of the low-pass *RC* circuit.

Let V' be the initial voltage across the capacitor. Writine KVL around the IOOD in Fieure 1.1.

$$v_i(t) = Ri(t) + \frac{1}{C}\int i(t)\,dt$$

Differentiating this equation,

$$\frac{dv_i(t)}{dt} = R\frac{di(t)}{dt} + \frac{1}{C}i(t)$$

Since $$v_i(t) = V, \quad \frac{dv_i(t)}{dt} = 0$$

$\therefore$ $$0 = R\,\frac{di(t)}{dt} + \frac{1}{C}i(t)$$

Taking the Laplace transform on both sides,

$$0 = R\,[sI(s) - I(0^+)] + \frac{1}{C}I(s)$$

$\therefore$ $$I(0^+) = I(s)\left(s + \frac{1}{RC}\right)$$

The initial current $I(0^+)$ is given by

$$I(0^+) = \frac{V - V'}{R}$$

$\therefore$ $$I(s) = \frac{I(0^+)}{s + \frac{1}{RC}} = \frac{V - V'}{R\left(s + \frac{1}{RC}\right)}$$

and $$V_o(s) = V_i(s) - I(s)R = \frac{V}{s} - \frac{(V - V')R}{R\left(s + \frac{1}{RC}\right)} = \frac{V}{s} - \frac{V - V'}{s + \frac{1}{RC}}$$

Taking the inverse Laplace transform on both sides,

$$v_o(t) = V - (V - V')e^{-t/RC}$$

where V' is the initial voltage across the capacitor (V_{initial}) and V is the final voltage (V_{final}) to which the capacitor can charge.

So, the expression for the voltage across the capacitor of an RC circuit excited by a step input is given by

$$v_o(t) = V_{\text{final}} - (V_{\text{final}} - V_{\text{initial}})e^{-t/RC}$$

If the capacitor is initially uncharged, then $v_o(t) = V(1 - e^{-t/RC})$

Expression for rise time

When a step signal is applied, the rise time t_r is defined as the time taken by the output voltage waveform to rise from 10% to 90% of its final value: It gives an indication of how fast the circuit can respond to a discontinuity in voltage. Assuming that the capacitor in Figure 1.1 is initially uncharged, the output voltage shown in Figure 1.3(b) at any instant of time is given by

$$v_o(t) = V(1 - e^{-t/RC})$$

At $t = t_1$, $v_o(t) \neq 10\%$ of $V = 0.1\,\text{V}$

$$\therefore \quad 0.1V = V(1 - e^{-t_1/RC})$$

$$\therefore \quad e^{-t_1/RC} = 0.9 \quad \text{or} \quad e^{t_1/RC} = \frac{1}{0.9} = 1.11$$

$$\therefore \quad t_1 = RC \ln (1.11) = 0.1RC$$

At $t = t_2$, $v_o(t) = 90\%$ of $V = 0.9\,\text{V}$

$$\therefore \quad 0.9V = V(1 - e^{-t_2/RC})$$

$$\therefore \quad e^{-t_2/RC} = 0.1 \quad \text{or} \quad e^{t_2/RC} = \frac{1}{0.1} = 10$$

$$\therefore \quad t_2 = RC \ln 10 = 2.3RC$$

$$\therefore \quad \text{Rise time, } t_r = t_2 - t_1 = 2.2RC$$

This indicates that the rise time t_r is proportional to the time constant *RC* of the circuit. The larger the time constant, the slower the capacitor charges, and the smaller the time constant, the faster the capacitor charges.

Relation between rise time and upper 3-dB frequency

We know that the upper 3-dB frequency (same as bandwidth) of a low-pass circuit is

$$f_2 = \frac{1}{2\pi RC} \quad \text{or} \quad RC = \frac{1}{2\pi f_2}$$

$$\therefore \quad \text{Rise time, } t_r = 2.2RC = \frac{2.2}{2\pi f_2} = \frac{0.35}{f_2} = \frac{0.35}{\text{BW}}$$

Thus, the rise time is inversely proportional to the upper 3-dB frequency.

The *time constant* ($\tau = RC$) of a circuit is defined as the time taken by the output to rise to 63.2% of the amplitude of the input step. It is same as the time taken by the output to rise to 100% of the amplitude of the input step, if the initial slope of rise is maintained. See Figure 1.3(b). The Greek letter τ is also employed as the symbol for the time constant.

Pulse Input

The pulse shown in Figure 1.4(a) is equivalent to a positive step followed by a delayed negative step as shown in Figure 1 .4(b). So, the response of the low-pass *RC* circuit to a pulse for times less than the pulse width t_p is the same as that for a step input and is given by $v_0(t) = V(1 - e^{-t/RC})$. The responses of the low-pass *RC* circuit for time constant $RC \gg t_p$, *RC* smaller than t_p and *RC* very small compared to t_p are shown in Figures 1.5(a), 1.5(b), and 1.5(c) respectively.

If the time constant *RC* of the circuit is very large, at the end of the pulse, the output voltage will be $V_p(t) = V(1 - e^{-t_p/RC})$, and the output will decrease to zero from this value with a time constant RC as shown in Figure 1.5(a). Observe that the pulse waveform is distorted when it is passed through a linear network. The output will always extend beyond the pulse width t_p, because whatever charge has accumulated across the capacitor C during the pulse cannot leak off instantaneously.

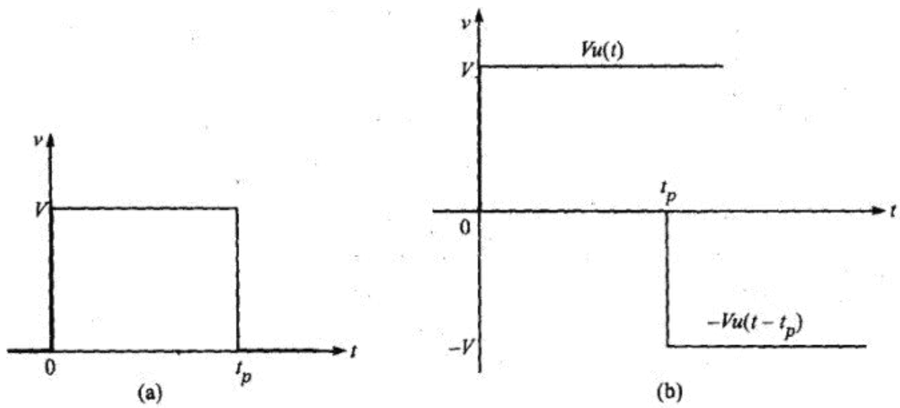


Figure 1.4 (a) A pulse and (b) a pulse in terms of steps.

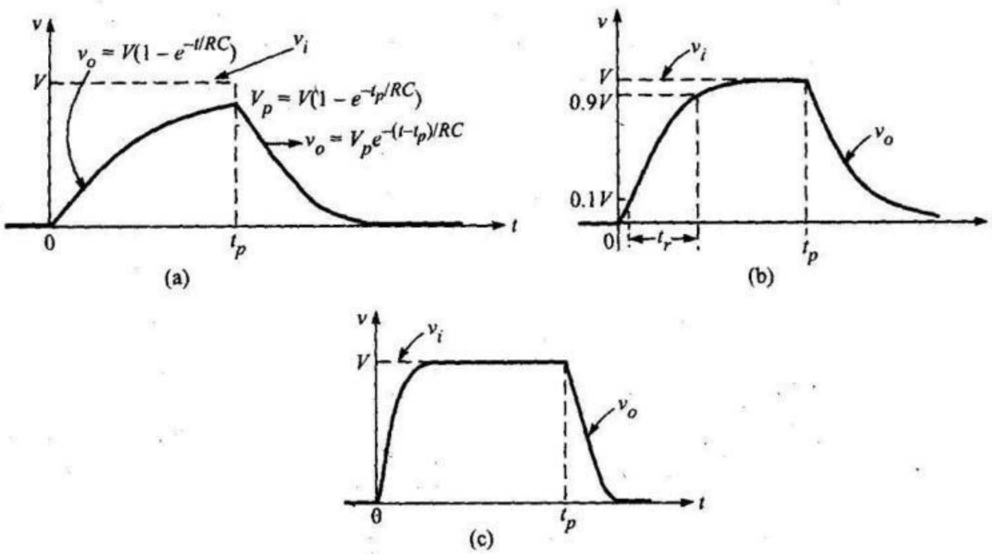


Figure 1.5 Pulse response for (a) $RC >> t_p$, (b) $RC < t_p$, and (c) $RC << t_p$.

If the time constant RC of the circuit is very small, the capacitor charges and discharges very quickly and the rise time t_r will be small and so the distortion in the wave shape is small. For minimum distortion (i.e. for preservation of wave shape), the rise time must be small compared to the pulse width t_p. If the upper 3-dB frequency /2 is chosen equal to the reciprocal of the pulse width t_p, i.e. if $f_2 = 1/t_p$ then $t_r = 0.35t_p$ and the output is as shown in Figure 1.5(b), which for many applications is a reasonable reproduction of the input. As a rule of thumb, we can say:

A pulse shape will be preserved if the 3-dB frequency is approximately equal to the reciprocal of the pulse width.

Thus to pass a 0.25 μ.s pulse reasonably well requires a circuit with an upper cut-off frequency of the order of 4 MHz.

Square-Wave Input

A square wave is a periodic waveform which maintains itself at one constant level *V'* with respect to ground for a time T_1 and then changes abruptly to another level *V"*, and remains constant at that level for a time T_2, and repeats itself at regular intervals of $T = T_1 + T_2$. A square wave may be treated as a series of positive and negative steps. The shape of the output waveform for a square wave input depends on the time constant of the circuit. If the time constant is very small, the rise time will also be small and a reasonable reproduction of the input may be obtained.

For the square wave shown in Figure 1.6(a), the output waveform will be as shown in Figure 1.6(b) if the time constant *RC* of the circuit is small compared to the period of the input waveform. In this case, the wave shape is preserved. If the time constant is comparable with the period of the input square wave, the output will be as shown id Figure 1.6(c). The output rises and falls exponentially. If the time constant is very large compared to the period of the input waveform, the output consists of exponential sections, which are essentially linear as indicated in Figure 1.6(d). Since the average voltage across *R* is zero, the dc voltage at the output is the same as that of the input. This average value is indicated as *V&.* in all the waveforms of Figure 1.6.

In Figure 1.6(c), the equation for the rising portion is

$$v_{01} = V' - (V' - V_2)e^{-t/RC}$$

where V_2 is the voltage across the capacitor at $t = 0$, and V' is the level to which the capacitor can charge.

The equation for the falling portion is

$$v_{02} = V'' - (V'' - V_1)e^{-(t - T_1)/RC}$$

where V_1 is the voltage across the capacitor at $t = T_1$ and V'' is the level to which the capacitor can discharge.

Setting $v_{01} = V_1$ at $t = T_1$,

$$V_1 = V' - (V' - V_2)e^{-T_1/RC} = V'(1 - e^{-T_1/RC}) + V_2e^{-T_1/RC}$$

Setting $v_{02} = V_2$ at $t = T_1 + T_2$,

$$V_2 = V'' - (V'' - V_1)e^{-(T_1+T_2-T_1)/RC} = V''(1 - e^{-T_2/RC}) + V_1e^{-T_2/RC}$$

Substituting this value of V_2 in the expression for V_1,

$$V_1 = V'(1 - e^{-T_1/RC}) + [V''(1 - e^{-T_2/RC}) + V_1 e^{-T_2/RC}]e^{-T_1/RC}$$

i.e.
$$V_1 = \frac{V'(1-e^{-T_1/RC}) + V''(1-e^{-T_2/RC})e^{-T_1/RC}}{1-e^{-(T_1+T_2)/RC}}$$

Similarly substituting the value of V_1 in the expression for V_2,

$$V_2 = \frac{V''(1 - e^{-T_2/RC}) + V'(1 - e^{-T_1/RC})\, e^{-T_2/RC}}{1 - e^{-(T_1+T_2)/RC}}$$

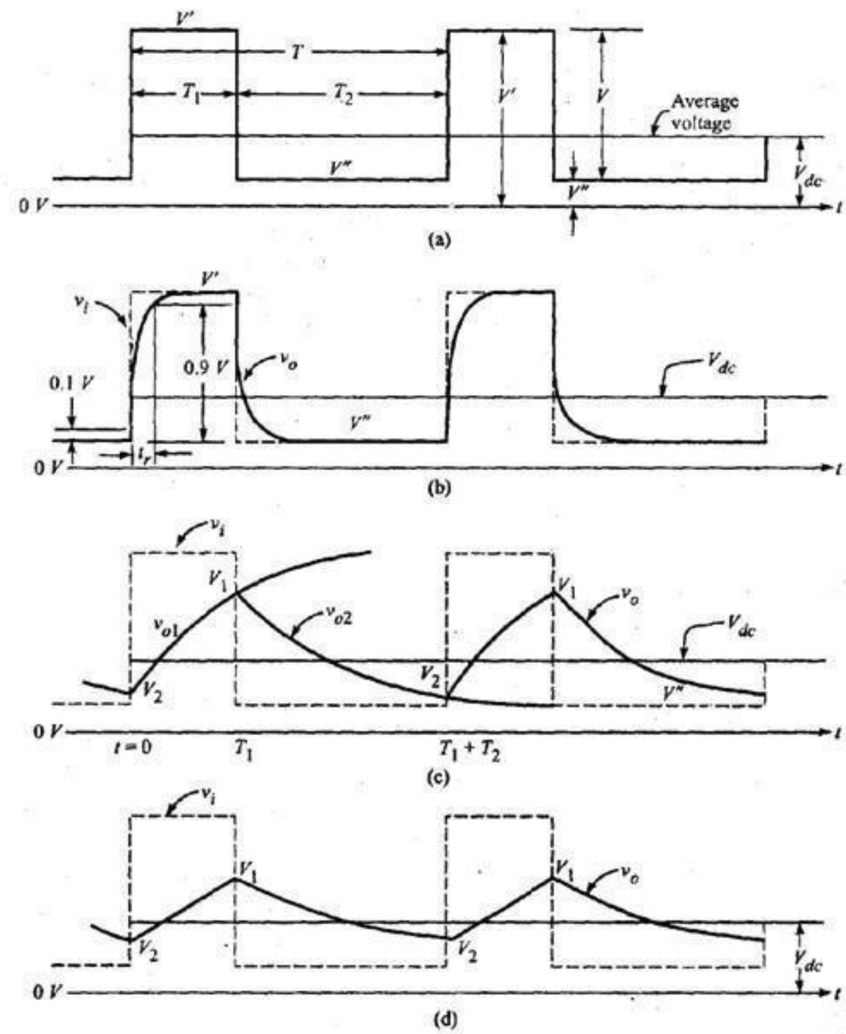


Figure 1.6 Response of a low-pass RC circuit to a square wave input: (a) square-wave input wave form, (b) output waveform for $RC \ll T$, (c) output waveform for $RC = T$, and (d) output waveform for $RC \gg T$.

For a symmetrical square wave with zero average value,

$$T_1 = T_2 = \frac{T}{2} \quad \text{and} \quad V' = -V'' = \frac{V}{2}. \text{ So, } V_2 \text{ will be equal to } -V_1$$

$$\therefore \qquad V_1 = \frac{\frac{V}{2}(1 - e^{-T/2RC}) - \frac{V}{2}(1 - e^{-T/2RC})\, e^{-T/2RC}}{1 - e^{-T/RC}}$$

$$= \frac{V}{2} \frac{1 - e^{-T/2RC} - e^{-T/2RC} + e^{-T/RC}}{1 - e^{-T/RC}}$$

$$= \frac{V}{2} \frac{(1 - e^{-T/2RC})^2}{(1 + e^{-T/2RC})(1 - e^{-T/2RC})}$$

$$= \frac{V}{2} \left(\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}} \right)$$

$$= \frac{V}{2} \left(\frac{e^{T/2RC} - 1}{e^{T/2RC} + 1} \right)$$

$$= \frac{V}{2} \left(\frac{e^{2x} - 1}{e^{2x} + 1} \right) = \frac{V}{2} \tanh x$$

where $x = \dfrac{T}{4RC}$ and T is the period of the square wave.

Now, $$V_2 = -V_1 = -\frac{V}{2}\left(\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}} \right) = \frac{V}{2} \left(\frac{1 - e^{T/2RC}}{1 + e^{T/2RC}} \right)$$

1.1.5 Ramp Input

When a low-pass RC circuit shown in Figure 1.1 is excited by a ramp input, i.e.

$$v_i(t) = \alpha t, \text{ where } \alpha \text{ is the slope of the ramp}$$

we have, $$V_i(s) = \frac{\alpha}{s^2}$$

From the frequency domain circuit of Figure 1.2(a), the output is given by

$$V_o(s) = V_i(s) \frac{\dfrac{1}{Cs}}{R + \dfrac{1}{Cs}} = \frac{\alpha}{s^2} \cdot \frac{1}{1 + RCs} = \frac{\alpha}{RC} \frac{1}{s^2\left(s + \dfrac{1}{RC} \right)}$$

$$= \frac{\alpha}{RC} \left[\frac{-(RC)^2}{s} + \frac{RC}{s^2} + \frac{(RC)^2}{s + \dfrac{1}{RC}} \right]$$

i.e. $$V_o(s) = \frac{-\alpha RC}{s} + \frac{\alpha}{s^2} + \frac{\alpha RC}{s + \dfrac{1}{RC}}$$

Taking the inverse Laplace transform on both sides,

$$v_o(t) = -\alpha RC + \alpha t + \alpha RCe^{-t/RC}$$
$$= \alpha(t - RC) + \alpha RCe^{-t/RC}$$

If the time constant RC is very small, $e^{-t/RC} \approx 0$

$$\therefore \qquad v_o(t) = \alpha(t - RC)$$

When the time constant is very small relative to the total ramp time T, the ramp will be transmitted with minimum distortion. The output follows the input but is delayed by one time constant RC from the input (except near the origin where there is distortion) as shown in Figure 1.7(a). If the time constant is large compared with the sweep duration, i.e. if $RC/T \gg 1$, the output will be highly distorted as shown in Figure 1.7(b).

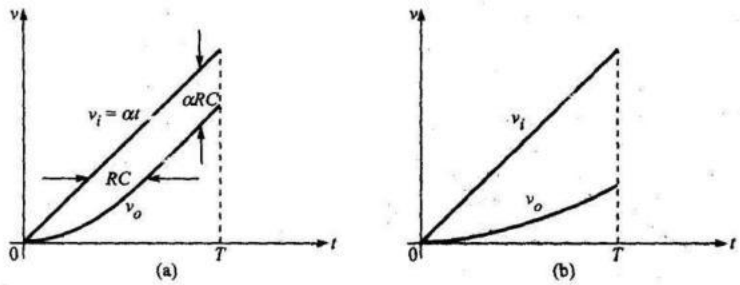


Figure 1.7 Response of a low-pass RC circuit for a ramp input for (a) $RC/T \ll 1$ and (b) $RC/T \gg 1$.

Expanding $e^{-t/RC}$ in to an infinite series in t/RC in the above equation for $v_o(t)$,

$$v_o(t) = \alpha(t - RC) + \alpha RC\left(1 - \frac{t}{RC} + \left(\frac{t}{RC}\right)^2 \frac{1}{2!} - \left(\frac{t}{RC}\right)^3 \frac{1}{3!} + \ldots\right)$$
$$= \alpha t - \alpha RC + \alpha RC - \alpha t + \frac{\alpha t^2}{2RC} - \ldots$$
$$\approx \frac{\alpha t^2}{2RC} \approx \frac{\alpha}{RC}\left(\frac{t^2}{2}\right)$$

This shows that a quadratic response is obtained for a linear input and hence the circuit acts as an integrator for $RC/T \gg 1$.

The transmission error e_t for a ramp input is defined as the difference between the input and the output divided by the input at the end of the ramp, i.e. at $t = T$.

For $RC/T \ll 1$,

$$e_t = \left.\frac{\alpha t - (\alpha t - \alpha RC)}{\alpha t}\right|_{t=T}$$
$$= \frac{\alpha RC}{\alpha T} = \frac{RC}{T} = \frac{1}{2\pi f_2 T}$$

where f_2 is the upper 3-dB frequency. For example, if we desire to pass a 2 ms pulse with less than 0.1% error, the above equation yields f_2 > 80 kHz and $RC < 2$ µ.s.

THE LOW-PASS *RC* CIRCUIT AS AN INTEGRATOR

If the time constant of an *RC* low-pass circuit is very large, the capacitor charges very slowly and so almost all the input voltage appears across the resistor for small values of time. Then, the current in the circuit is *vffylR* and the output signal across *C* is

$$v_o(t) = \frac{1}{C}\int i(t)\,dt = \frac{1}{C}\int \frac{v_i(t)}{R}\,dt = \frac{1}{RC}\int v_i(t)\,dt$$

Hence the output is the integral of the input, i.e. if $v_i(t) = \alpha t$, then

$$v_o(t) = \frac{\alpha t^2}{2RC}$$

As time increases, the voltage drop across *C* does not remain negligible compared with that across *R* and the output will not remain the integral of the input. The output will change from a quadratic to a linear function of time. ***If the time constant of an RC low-pass circuit is very large in comparison with the. time required for the input signal to make an appreciable change, the circuit acts as an integrator.*** A criterion for good integration in terms of steady-state analysis is as follows: The low-pass circuit acts as an integrator provided the time constant of the circuit $RC > 15T$, where *T* is the period of the input sine wave. When $RC > 15T$, the input sinusoid will be shifted at least by 89.4° (instead of the ideal 90° shift required for integration) when it is transmitted through the network.

An *RC* integrator converts a square wave into a triangular wave. Integrators are almost invariably preferred over differentiators in analog computer applications for the following reasons:

1. It is easier to stabilize an integrator than a differentiator because the gain of an integrator decreases with frequency whereas the gain of a differentiator increases with frequency.
2. An integrator is less sensitive to noise voltages than a differentiator because of its limited bandwidth.
3. The amplifier of a differentiator may overload if the input waveform changes very rapidly.
4. It is more convenient to introduce initial conditions in an integrator.

EXAMPLE 1.1 A pulse generator with an output resistance $R_S = 500\ \Omega$ is connected to an oscilloscope with an input capacitance of $C_i = 30$ pF. Determine the fastest rise time that can be displayed.

Solution: The circuit works as a low-pass filter shown in Figure 1.1 with a time constant

$$R_S C_i = 500\ \Omega \times 30\ \text{pF} = 15\ \text{ns}$$

$\therefore$ Fastest rise time, $t_r = 2.2RC = 2.2 \times 15\ \text{ns} = 33\ \text{ns}$

EXAMPLE 1.2 A 10 V step is switched on to a 50 kΩ resistor in series with a 500 pF capacitor. Calculate the rise time of the capacitor voltage, the time for the capacitor to charge to 63.2% of its maximum voltage, and the time for the capacitor to be completely charged.

Solution: The circuit acts as a low-pass filter shown in Figure 1.1.

(a) The rise time of the capacitor voltage is

$$t_r = 2.2RC = 2.2 \times 50\ \text{k}\Omega \times 500\ \text{pF} = 55\ \mu\text{s}$$

(b) The time for the capacitor to charge to 63 2% of the maximum voltage is

$$\tau = RC = 50\ \text{k}\Omega \times 500\ \text{pF} = 25\ \mu\text{s}$$

(c) The time for the capacitor to be completely charged (99% value) is

$$5\tau = 5RC = 5 \times 25\ \mu\text{s} = 125\ \mu\text{s}$$

THE HIGH-PASS *RC* CIRCUIT

Figure 1.30 shows a high-pass *RC* circuit. At zero frequency the reactance of the capacitor is infinity and so it blocks the input and hence the output is zero. Hence, this capacitor is called the *blocking capacitor* and this circuit, also called the *capacitive coupling circuit,* is used to provide dc isolation between the input and the output. As the frequency increases,'the reactance of the capacitor decreases and hence the output and gain increase. At very high frequencies, the capacitive reactance is very small so a very small voltage appears, across *C* and, so the output is almost equal to the input and the gain is equal to 1. Since this circuit attenuates low-frequency signals and allows transmission of high-frequency signals with little or no attenuation, it is called a high-pass circuit.

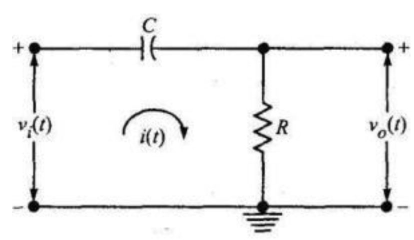


Figure 1.30 The high-pass *RC* circuit.

Sinusoidal Input

Figure 1.31 (a) shows the Laplace transformed high-pass *RC* circuit. The gain versus frequency curve of a high-pass circuit excited by a sinusoidal input is shown in Figure 1.31(b). For a sinusoidal input v,, t;he output signal *vO* increases in amplitude with increasing frequency. The frequency at which the gain is 1/V2 of its maximum value is called the lower cut-off or lower 3-dB frequency. For a high-pass circuit, there is no upper cut-off frequency because all high frequency signals are transmitted with zero attenuation. Therefore, $f_2 - f_1$. Hence bandwidth B.W= f_2 - fi =∞

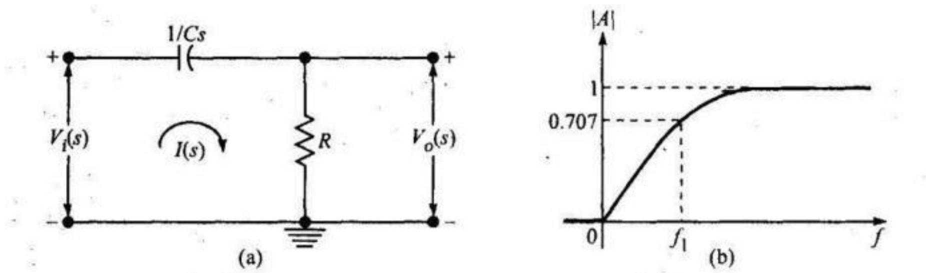


Figure 1.31 (a) Laplace transformed high-pass circuit and (b) gain versus frequency plot.

Expression for the lower cut-off frequency

For the high-pass *RC* circuit shown in Figure 1.31 (a), the magnitude of the steady-state gain *A,* and the angle θ by which the output leads the input are given by

$$A = \frac{V_o(s)}{V_i(s)} = \frac{R}{R + \frac{1}{Cs}} = \frac{1}{1 + \frac{1}{RCs}}$$

Putting $s = j\omega$, $A = \frac{1}{1 - j\frac{1}{\omega RC}} = \frac{1}{1 - j\frac{1}{2\pi fRC}}$

$\therefore$ $|A| = \frac{1}{\sqrt{1 + \left(\frac{1}{2\pi fRC}\right)^2}}$ and $\theta = -\tan^{-1}\frac{1}{2\pi fRC}$

At the lower cut-off frequency f_1, $|A| = 1/\sqrt{2}$

$$\therefore \quad \frac{1}{\sqrt{1 + \left(\frac{1}{2\pi f_1 RC}\right)^2}} = \frac{1}{\sqrt{2}}$$

Squaring and equating the denominators,

$$\frac{1}{2\pi f_1 RC} = 1 \quad \text{i.e.} \quad f_1 = \frac{1}{2\pi RC}$$

This is the expression for the lower cut-off frequency of a high-pass circuit.

Relation between f1 and tilt

The lower cut-off frequency of a high-pass circuit is /i = V2*nRC*. The lower cut-off frequency produces a tilt.For a 10% change in capacitor voltage, the time or pulse width involved is

$$t = 0.1RC = \text{PW}$$

$$\therefore \quad \frac{\text{PW}}{RC} = 0.1 = \text{Fractional tilt}$$

$$\therefore \quad \text{Fractional tilt} = \frac{\text{PW}}{RC} = 2\pi f_1 \cdot \text{PW}$$

This equation applies only when the tilt is 10% or less. When the tilt exceeds 10%, the voltage should be treated as exponential instead of linear and the equation $V_o = V_f - (V_f - V_i)e^{-t/RC}$ should be applied.

Step Input

When a step signal of amplitude *V* volts shown in Figure 1.32(a) is applied to the high-pass *RC* circuit of Figure 1.30, since the voltage across the capacitor cannot change instantaneously the output will be just equal to the input at *t* = 0 (for f < 0, v,- = 0 and v_a = 0). Later when the capacitor charges exponentially, the output reduces exponentially with the same time constant *RC*. The expression for the output voltage for *t* > 0 is given by

$v_o(t) = V_f - (V_f - V_{in})e^{-t/RC} = 0 - (0 - V)e^{-t/RC} = Ve^{-t/RC}$ Figure 1.32(b) shows the response of the circuit for large, small, and very small time constants.For *t* > 5r, the output will reach more than 99% of its final value. Hence although the steady state is approached asymptotically, for most applications we may assume that the final value has been reached after 5f. If the initial slope of the exponential is maintained, the output falls to zero in a time *t* = T.

The voltage across a capacitor can change instantaneously only when an infinite current passes through it, because for any finite current *i(t)* through the capacitor, the instantaneous change in voltage across the capacitor is given by $\frac{1}{C}\int_0^0 i(t)\,dt = 0$.

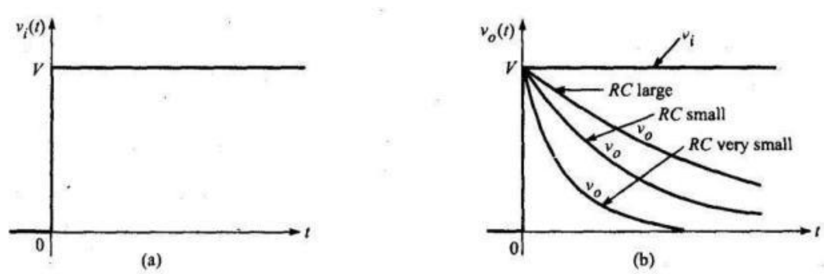


Figure 1.32 (a) Step input and (b) step response for different time constants.

Pulse Input

A pulse of amplitude V and duration t_p shown in Figure 1.4(a) is nothing but the sum of a positive step of amplitude V starting at $t = 0$ and a negative step of amplitude V starting at t_p as shown in Figure 1.4(b). So, the response of the circuit for $0 < t < t$,, for the pulse input is the same as that for a step input and is given by $v_0(t) = Ve^{-t/RC}$. At $t = t_p$, $v_o(t) = V = Ve^{-t/RC}$. At $t = t_p$, since the input falls by V volts suddenly and since the voltage across the capacitor cannot change instantaneously, the output also falls suddenly by V volts to $V_p - V$. Hence at $t = t^+$, $v_a(t) = Ve^{-t_p/RC} - V$. Since $V_p < V$, $V_p - V$ is negative. So there is an undershoot at $t = t_p$ and hence for $t > t_p$, the output is negative. For $t > t_p$, the output rises exponentially towards zero with a time constant RC according to the expression $(Ve^{-t_p/RC} - V)e^{-(t-t_p)/RC}$.

The output waveforms for $RC \gg t_p$, RC comparable to t_p and $RC \ll t_p$ are shown in Figures 1.33(a), (b), and (c) respectively. There is distortion in the outputs and the distortion is the least when the time constant is very large. Observe that there is positive area and negative area in the output waveforms. The negative area will always be equal to the positive area. So if the time constant is very large the tilt (the almost linear decrease in the output voltage) will be small and hence the undershoot will be very small, and for $t > t_p$, the output rises towards the zero level very very slowly. If the time constant is very small compared to the pulse width (i.e. $RC/t_p \ll T$), the output consists of a positive spike or pip of amplitude V volts at the beginning of the pulse and a negative spike of the same amplitude at the end of the pulse. Hence a high-pass circuit with a very small time constant is called a *peaking circuit* and this process of converting pulses into pips by means of a circuit of short time constant is called peaking.

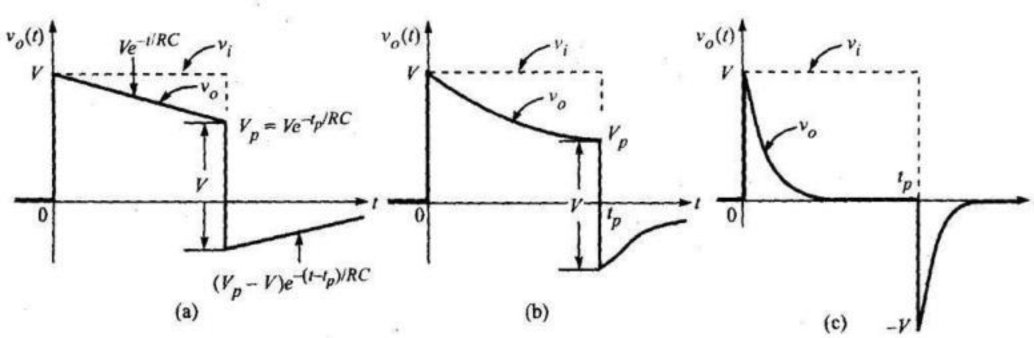


Figure 1.33 Pulse response for (a) $RC \gg t_p$, (b) RC comparable to t_p and (c) $RC \ll t_p$.

Square-Wave Input

A square wave shown in Figure 1.34(a) is a periodic waveform, which maintains itself at one constant level V with respect to ground for a time T_1 and then changes abruptly to another level V'' and remains constant at that level for a time T_2, and then repeats itself at regular intervals of $T = T_1 + T_2$. A square wave may be treated as a series of positive and negative steps. The shape of the output depends on the time constant of the circuit. Figures 1.34(b), 1.34(c), 1.34(d), and 1.34(e) show the output waveforms of the high-pass RC circuit under .steady-state conditions for the cases (a) $RC \gg T$, (b) $RC > T$, (c) $RC - T$, and (d) $RC \ll T$ respectively. When the time constant is arbitrarily large (i.e. RC/T_1 and RC/T_2 *are* very very large in comparison to unity) the output is same as the input but with zero dc level. When $RC > T$, the output is in the form of a tilt. When RC is comparable to T, the output rises and falls exponentially. When $RC \ll T$ (i.e. RC/T_1 and RC/T_2 *are very* small in comparison to unity), the output consists of alternate positive and negative spikes. In this case the peak-to-peak amplitude of the output is twice the peak-to-peak value of the input.In fact, for any periodic input waveform under steady-state conditions, the average level of the output waveform from the high-pass circuit of Figure 1.30 is always zero independently of the dc level of the input. The proof is as follows: Writing KVL around the loop of Figure 1.30,

$$v_i(t) = \frac{1}{C}\int i(t)\,dt + v_o(t)$$

$$= \frac{1}{RC}\int v_o(t)\,dt + v_o(t) \qquad \left(\because\ i(t) = \frac{v_o(t)}{R}\right)$$

Differentiating,

$$\frac{dv_i(t)}{dt} = \frac{v_o(t)}{RC} + \frac{dv_o(t)}{dt}$$

Multiplying by dt and integrating this equation over one period T,

$$\int_{t=0}^{t=T} dv_i(t) = \int_{t=0}^{t=T} \frac{v_o(t)\,dt}{RC} + \int_{t=0}^{t=T} dv_o(t)$$

i.e.

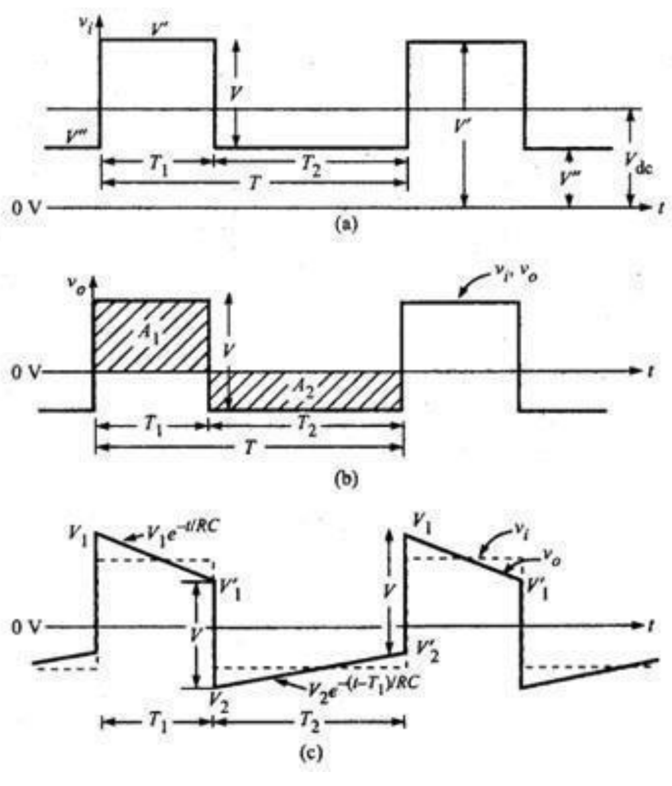
$$v_i(T) - v_i(0) = \frac{1}{RC}\int_0^T v_o(t)\,dt + v_o(T) - v_o(0)$$

Under steady-state conditions, the output waveform (as well as the input signal) is repetitive with a period T so that $v_o(T) = v_o(0)$ and $v_i(T) = v_i(0)$.

Under steady-state conditions, the output waveform (as well as the input signal) is repetitive with a period T so that $v_0(T) = v_0(0)$ and $v_i(T) = v_i(0)$.

Hence $\int_0^T v_o(t)\,dt = 0.$ Since this integral represents the area under the output waveform over one cycle, we can say that the average level of the steady-state output signal is always zero.This can also be proved based on frequency domain analysis as follows.The periodic input signal may be resolved into a Fourier series consisting of a constant term and an infinite number of sinusoidal components whose frequencies are multiples of / = 1/T. Since the blocking capacitor presents infinite impedance to the dc input voltage, none of these dc components reach the output under steady-state conditions. Hence the output signal is a sum of sinusoids whose frequencies are multiples of/. This waveform is therefore periodic with a fundamental period *T* but without a dc component.With respect to the high-pass circuit of Figure 1.30, we can say that:

1. The average level of the output signal is always zero, .independently of the average level of the input. The output must consequently extend in both negative and positive directions with respect to the zero voltage axis and the area of the part of the waveform above the zero axis must equal the area which is below the zero axis.
2. When the input changes abruptly by an amount *V,* the output also changes abruptly by an equal amount and in the same direction.
3. During any finite time interval when the input maintains a constant level, the output decays exponentially towards zero voltage.



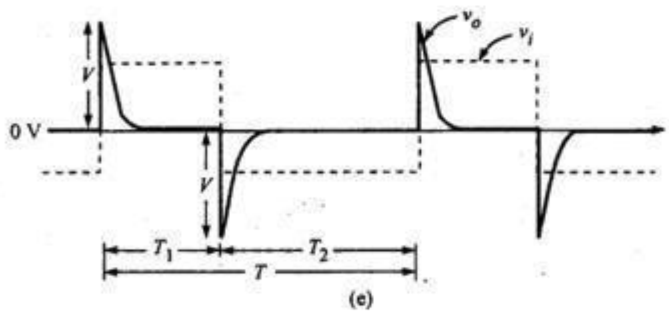


Figure 1.34 (a) A square wave input, (b) output when RC is arbitrarily large, (c) output when $RC > T$, (d) output when RC is comparable to T, and (e) output when $RC << T$.

Under steady-state conditions, the capacitor charges and discharges to the same voltage levels in each cycle. So the shape of the output waveform is fixed.

For $0 < t < T_1$, the output is given by $v_{o1} = V_1 e^{-t/RC}$

At $t = T_1$, $v_{o1} = V_1' = V_1 e^{-T_1/RC}$

For $T_1 < t < T_1 + T_2$, the output is $v_{o2} = V_2 e^{-(t-T_1)/RC}$

At $t = T_1 + T_2$, $v_{o2} = V_2' = V_2 e^{-T_2/RC}$

Also $V_1' - V_2 = V$ and $V_1 - V_2' = V$

From these relations V_1, V_1', V_2 and V_2' can be computed.

Expression for the percentage tilt

We will derive an expression for the percentage tilt when the time constant RC of the circuit is very large compared to the period of the input waveform, i.e. $RC \gg T$. For a symmetrical square wave with zero average value

$$V_1 = -V_2, \text{ i.e. } V_1 = |V_2|, \; V_1' = -V_2', \text{ i.e. } V_1' = |V_2'|, \text{ and } T_1 = T_2 = \frac{T}{2}$$

The output waveform for $RC >> T$ is shown in Figure 1.35. Here,

$$V_1' = V_1 e^{-T/2RC} \quad \text{and} \quad V_2' = V_2 e^{-T/2RC}$$

$$V_1 - V_2' = V$$

i.e.

$$V_1 - V_2 e^{-T/2RC} = V_1 + V_1 e^{-T/2RC} = V$$

$\therefore$

$$V_1 = \frac{V}{1 + e^{-T/2RC}} \quad \text{or} \quad V = V_1(1 + e^{-T/2RC})$$

$$\%\text{ tilt, } P = \frac{V_1 - V_1'}{\frac{V}{2}} \times 100\% = \frac{V_1 - V_1 e^{-T/2RC}}{V_1(1 + e^{-T/2RC})} \times 200\% = \frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}} \times 200\%$$

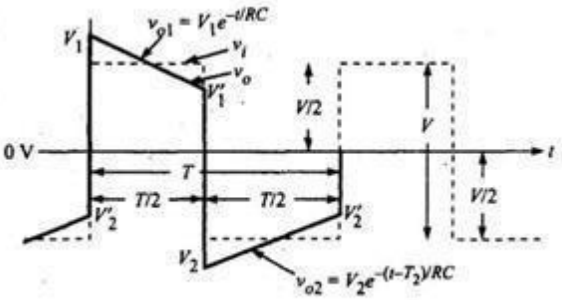


Figure 1.35 Linear tilt of a symmetrical square wave when $RC >> T$.

When the time constant is very large, i.e. $\frac{T}{RC} << 1$

$$P = \frac{1-\left[1+\left(\frac{-T}{2RC}\right)+\left(\frac{-T}{2RC}\right)^2\frac{1}{2!}+\ldots\right]}{1+1+\left(\frac{-T}{2RC}\right)+\left(\frac{-T}{2RC}\right)^2\frac{1}{2!}+\ldots} \times 200\%$$

$$= \frac{\frac{T}{2RC}}{2} \times 200\%$$

$$= \frac{T}{2RC} \times 100\%$$

$$= \frac{\pi f_1}{f} \times 100\%$$

where $f_1 = \frac{1}{2\pi RC}$ is the lower cut-off frequency of the high-pass circuit.

Ramp Input

A waveform which is zero for $t < 0$ and which increases linearly with time for / > 0 is called a ramp or sweep voltage.

When the high-pass *RC* circuit of Figure 1.30 is excited by a ramp input v/(r) = *at,* where *a* is the slope of the ramp, then

$$V_i(s) = \frac{\alpha}{s^2}$$

From the Laplace transformed circuit of Figure 1.31(a),

$$V_o(s) = V_i(s)\frac{R}{R+\frac{1}{Cs}} = \frac{\alpha}{s^2}\frac{RCs}{1+RCs}$$

$$= \frac{\alpha}{s\left(s+\frac{1}{RC}\right)} = \alpha RC\left(\frac{1}{s}-\frac{1}{s+\frac{1}{RC}}\right)$$

Taking the inverse Laplace transform on both sides,

$$v_o(t) = \alpha RC(1 - e^{-t/RC})$$

For times t which are very small in comparison with RC, we have

$$v_o(t) = \alpha RC\left[1-\left\{1+\left(\frac{-t}{RC}\right)+\left(\frac{-t}{RC}\right)^2\frac{1}{2!}+\left(\frac{-t}{RC}\right)^3\frac{1}{3!}+\dots\right\}\right]$$

$$= \alpha RC\left[\frac{t}{RC}-\frac{t^2}{2(RC)^2}+\dots\right]$$

$$= \alpha t-\frac{\alpha t^2}{2RC} = \alpha t\left(1-\frac{t}{2RC}\right)$$

Figure 1.36 shows the response of the high-pass circuit for a ramp input when (a) $RC \gg T$, and (b) $RC \ll T$, where T is the duration of the ramp. For small values of T, the output signal falls away slightly from the input as shown in the Figure 1.36(a).

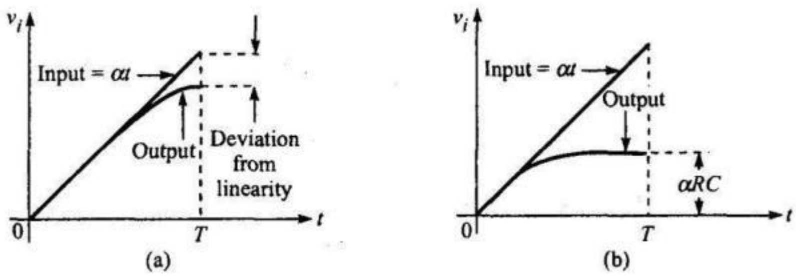


Figure 1.36 Response of the high-pass circuit for a ramp input when (a) $RC \gg T$ and (b) $RC \ll T$.

When a ramp signal is transmitted through a linear network, the output departs from the input. A measure of the departure from linearity expressed as the transmission error e, is defined as the difference between the input and the output divided by the input. The transmission error at a time

$$e_t = \left.\frac{v_i - v_o}{v_i}\right|_{t=T} \approx \left.\frac{\alpha t-\alpha t\left(1-\frac{t}{2RC}\right)}{\alpha t}\right|_{t=T} \approx \frac{T}{2RC} = \pi f_1 T$$

where $f_1 = \dfrac{1}{2\pi RC}$ is the lower 3-dB frequency of the high-pass circuit.

$t = T$ is then

For large values of t in comparison with RC, the output approaches the constant value aRC as indicated in Figure 1.36(b).

THE HIGH-PASS *RC* CIRCUIT AS A DIFFERENTIATOR

When the time constant of the high-pass RC circuit is very very small, the capacitor charges very quickly; so almost all the input v,(0 appears across the capacitor and the voltage across the resistor will be negligible compared to the voltage across the capacitor. Hence the current is determined entirely by the capacit nce. Then the current

$$i(t) = C\frac{dv_i(t)}{dt}$$

and the output signal across R is

$$v_o(t) = RC\frac{dv_i(t)}{dt}$$

Thus we see that the output is proportional to the derivative of the input.***The high-pass RC circuit acts as a differentiator provided the RC time, constant of the circuit is very small in comparison with the time required for the input signal to make an appreciable change.*** The

derivative of a step signal is an impulse of infinite amplitude at the occurrence of the discontinuity of step. The derivative of an ideal pulse is a positive impulse followed by a delayed negative impulse, each of infinite amplitude and occurring at the points of discontinuity. The derivative of a square wave is a waveform which is uniformly zero except, at the points of discontinuity. At these points, precise differentiation would yield impulses of infinite amplitude, zero width and alternating polarity. For a square wave input, an *RC* high-pass circuit with very small time constant will produce an output, which is zero except at the points of discontinuity. At these points of discontinuity, there will be peaks of finite amplitude *V*. This is because the voltage across *R* is not negligible compared with that across *C*.An *RC* differentiator converts a triangular wave into a square wave.For the ramp $v_i = at$, the value of $RC(dv/dt) = aRC$. This is true except near the origin. The output approaches the proper derivative value only after a lapse of time corresponding to several time constants. The error near $\theta = 0$ is again due to the fact that in this region the voltage across *R* is not negligible compared with that across *C*.

If we assume that the leading edge of a pulse can be approximated by a ramp, then we can measure the rate of rise of the pulse by using a differentiator. The peak output is measured on an oscilloscope, and from the equation = *aRC,* we see that this voltage divided by the product *RC* gives the slope *a.A criteria for good differentiation in terms of steady-state sinusoidal analysis is,* that if a sine wave is applied to the high-pass *RC* circuit, the output will be a sine

$$\tan\theta = \frac{X_C}{R} = \frac{1}{\omega RC}$$

wave shifted by a leading angle θ such that: with the output being proportional to $sin(a>t + \theta)$. In order to have true differentiation, we must obtain cos ωt. In other words,θ must equal 90°. This result can be obtained only if *R* =,0 or C = 0.

However, if ωRC =0.01, then $1/\omega RC$ = 100 and θ = 89.4°, which is sufficiently close to 90° for most purposes. If ωRC = 0.1, then *90* - 84.3° and for some applications this may be close enough to 90°.If the peak value of input is V_m, the output is and if $\omega RC \ll 1$, then the output is approximately $V_m \omega RC$ cos *(at.* This result agrees with the expected value $RC(dv_i/dt)$. If ωRC = 0.01 , then the output amplitude is 0.01 times the input amplitude.

$$v_o = \frac{V_m R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \sin(\omega t + \theta)$$

ATTENUATORS

Attenuators are resistive networks, which are used to reduce the amplitude of the input signal. The simple resistor combination of Figure 1.61 (a) would multiply the input signal by the ratio $a = R_2/(R_1 + R_2)$ independently of the frequency. If the output of the attenuator is feeding a stage of amplification, the input capacitance C_2 of the amplifier will be the stray capacitance shunting the resistor R_2 of the attenuator and the attenuator will be as shown in Figure 1.61(b), and the attenuation now is not independent of frequency. Using Thevenin's theorem, the circuit in Figure 1.61(b) may be replaced by its equivalent circuit shown in Figure 1,6l(c), in which R is equal to the parallel combination of R_1 and R_2.

Normally R_1 and R_2 must be large so that the nominal input impendence of the attenuator is large enough to prevent loading down the input signal. But if R_1 and R_2 are large, the rise time $t_r = 2.2[(R_1||R_2)*C_2]$ will be large and a large rise time is normally unacceptable. The attenuator may be compensated by shunting *R* by a capacitor *C* as shown in Figure 1.61(d), so that its attenuation is once again independent of frequency. The circuit has been drawn in Figure 1.61(e) to suggest that the two resistors and the two capacitors may be viewed as the four arms of a bridge. If $R_1C_1 = R_2C_2$, the bridge will be balanced and no current will flow in the branch connecting the point *X* to the point *Y*. For the purpose of computing the output, the branch *X-Y* may be omitted and the output will again be equal to c_M, independent of the frequency. In practice, C_1 will ordinarily have to be made adjustable.

Suppose a step signal of amplitude *V* volts is applied to the circuit. As the input changes abruptly by *V* volts at $t = 0$, the voltages across Ci and C_2 will also change abruptly. This happens because at / = 0, the capacitors act as short-circuits and a very large (ideally infinite) current flows though the capacitors for an irifinitesimally small time so that a finite charge $q = \int_{0-}^{0+} i(t)\,dt$ is delivered to each capacitor. The initial output voltage is determined by the capacitors.

Since the same current flows through the capacitors C_1 and C_2, we have

$$\text{Charge accumulated in capacitor } C_1 = \int_{0^-}^{0^+} i(t)\,dt = q$$

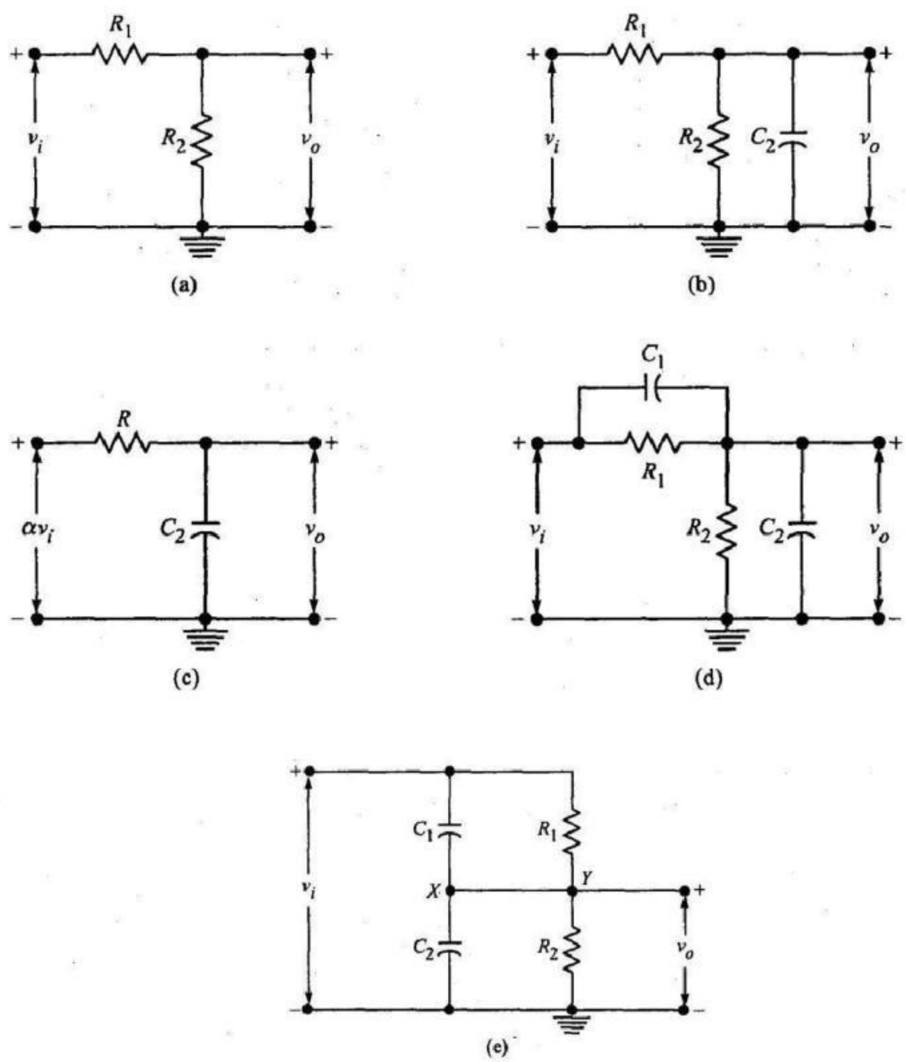


Figure 1.61 An attenuator: (a) ideal circuit, (b) actual circuit, (c) equivalent circuit, (d) compensated attenuator, and (e) compensated attenuator redrawn as a bridge.

$$\therefore \qquad \text{Initial voltage across } C_1 = \frac{q}{C_1} = V_1$$

Charge accumulated in capacitor $C_2 = \int_{0-}^{0+} i(t)\,dt = q$

Initial voltage across $C_2 = \dfrac{q}{C_2} = V_2 = v_o(0^+)$

Input signal, $V = V_1 + V_2 = \dfrac{q}{C_1} + \dfrac{q}{C_2} = q\left(\dfrac{C_1 + C_2}{C_1 C_2}\right)$

$$\frac{v_o(0+)}{V} = \frac{\dfrac{q}{C_2}}{q\left(\dfrac{C_1 + C_2}{C_1 C_2}\right)} = \frac{C_1}{C_1 + C_2}$$

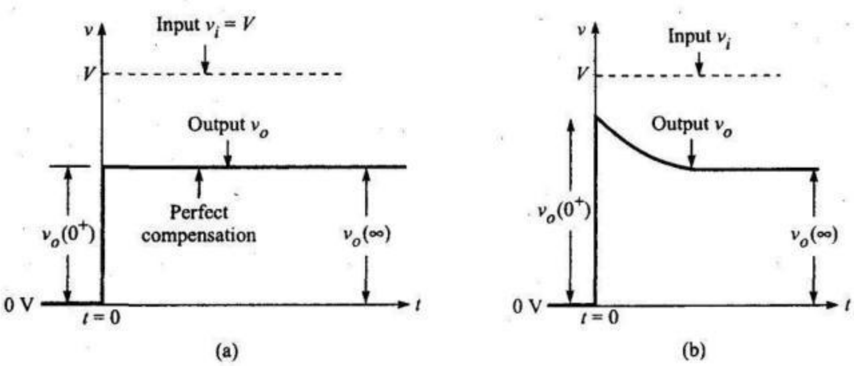
Or

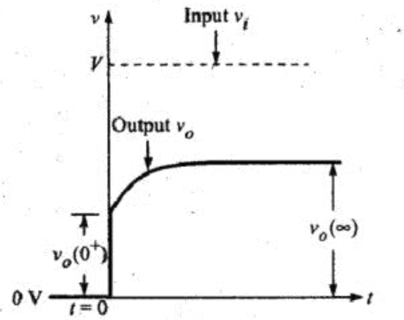
$$v_o(0+) = V\frac{C_1}{C_1 + C_2} = v_i\frac{C_1}{C_1 + C_2}$$

The final output voltage is determined by the resistors *R* and *R2,* because the capacitors C[and *C2* act as open circuits for the applied dc voltage under steady-state conditions. Hence

$$v_o(\infty) = V\frac{R_2}{R_1 + R_2} = v_i\frac{R_2}{R_1 + R_2}$$

Looking back from the output terminals (with the input short circuited) we see a resistor *R* = *R{R2/(Ri + R2)* in parallel with C = *C1* + *C2*. Hence the decay or rise of the output (when the attenuator is not perfectly compensated) from the initial to the final value takes place exponentially with a time constant *r* = *RC*. The responses of an attenuator for Cj equal to, greater than, and less than *R2C2/R{* are indicated in Figure 1.62.





(c)

(a) perfect compensation means
$v_o(0^+) = v_o(\infty)$; $\therefore C_1 = \dfrac{R_2 C_2}{R_1}$ i.e. $R_1 C_1 = R_2 C_2$

(b) over compensation means
$v_o(0^+) > v_o(\infty)$; $\therefore C_1 > R_2 C_2 / R_1$

(c) under compensation means
$v_o(0^+) < v_o(\infty)$; $\therefore C_1 < R_2 C_2 / R_1$

Figure 1.62 Response of compensated attenuator: (a) perfect compensation, (b) over compensation, and (c) under compensation.

Perfect compensation is obtained if $v_o(0^+) = v_o(\infty)$, that is, if the rise time $t_r = 0$

$$\therefore \qquad V \frac{C_1}{C_1 + C_2} = V \frac{R_2}{R_1 + R_2}$$

i.e.
$$R_1 C_1 + R_2 C_1 = R_2 C_1 + R_2 C_2$$

i.e.
$$R_1 C_1 = R_2 C_2 \quad \text{or} \quad C_1 = \frac{R_2 C_2}{R_1}$$

This is the balanced bridge condition. The extreme values of $v_0(0^+)$ are 0 for $C_1 = 0$ In the.above analysis we have assumed that an infinite current flows through the capacitors at $t = 0^+$ and hence the capacitors get charged instantaneously. This is valid only if the generator resistance is zero. In general, the output resistance of the generator is not zero but is of some finite value. Hence the impulse response is physically impossible. So, even though the attenuator is compensated, the ideal step response can never be obtained. Neverthless, an improvement in rise time does result if a compensated attenuator is used. For example, if the output is one-tenth of the input, then the rise time of the output using the attenuator is one-tenth of what it would be without the attenuator.

The compensated attenuator will reproduce faithfully the signal, which appears at its input terminals. However, if the output impedance of the generator driving the attenuator is not zero, the signal will be distorted right at the input to the attenuator. This situation is illustrated in Figure 1.63(a) in which a generator of step voltage V and of source resistance R_s is connected to the attenuator. Since the lead which joins the point X and point Y may be open circuited, the circuit may be redrawn as in Figure 1.63(b). Usually $R_s \ll R_1 + R_2$, so the input to the attenuator will be an exponential of time constant $R_s C'$, where C' is the capacitance of the series combination of Cj and C2> i.e. $C' = C_1\ C/(C_1 + C_2)$. It is this exponential waveform rather than the step, which the attenuator will transmit faithfully. If the generator terminals were connected directly to the terminals to which the attenuator output is connected, the generator would see a capacitance C_2. In this case the waveform at these terminals would be an exponential with time constant $T = R_S C_2$.

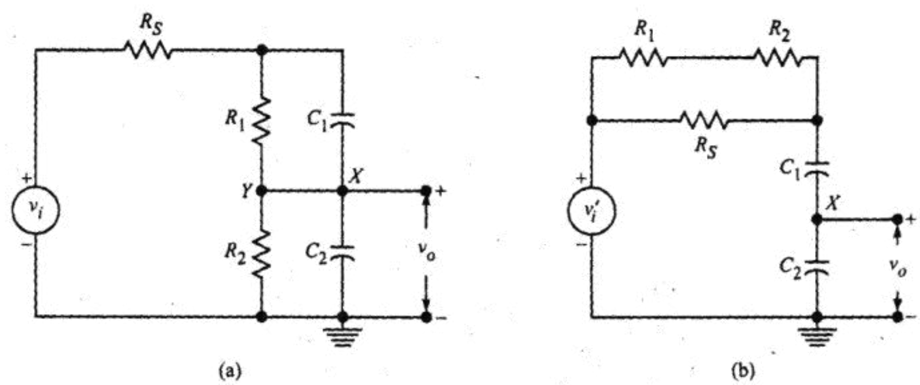


Figure 1.63 (a) Compensated attenuator including source resistance R_S and (b) its equivalent circuit with $v_i' = V(R_1 + R_2)/(R_S + R_1 + R_2)$.

When the attenuator is used^ the time constant is T' = *R$C^f^*. $\frac{\tau'}{\tau} = \frac{C'}{C_2} = \frac{C_1}{C_1 + C_2} = a$ an improvement in waveform results. For example, if the attenuation is equal to 10(a = 1/10), then the rise time of the waveform would be divided by a factor 10.-

RL CIRCUITS

In previous session we discussed the behaviour of *RC* low-pass and high-pass circuits for various types of input waveforms. Suppose the capacitor *C* and-resistor *R* in those circuits are replaced by a resistor *R'* and an inductor *L* respectively, then, if the time constant *LIR'* equals the time constant *RC,* all the preceding results remain unchanged.

When a large time constant is required, the inductor is rarely used because a large value of inductance can be obtained only with an iron-core inductor which is physically large, heavy and expensive relative to the cost of a capacitor for a similar application. Such an iron-cored inductor will be shunted with a large amount of stray distributed capacitance. Also the nonlinear properties of the iron cause distortion, which may be undesirable. If it is required to pass very low frequencies through a circuit in which *L* is" a shunt element, then the inductor may become prohibitively large. Of course in circuits where a small value of /?' is tolerable, a more reasonable value of inductance may be used. In low time constant applications, a small inexpensive air-cored inductor may be used.

Figure 1.73(a) shows the *RL* low-pass circuit. At very low frequencies the reactance of the inductor is small, so the output across the resistor /?' is almost equal to the input. As the frequency increases, the reactance of the inductor increases and so the signal is attenuated. At very high frequencies the output is almost equal to zero. So the circuit in Figure 1.73(a) acts as a low-pass filter.The circuit of Figure 1.73(b) acts as a high-pass circuit because at low frequencies, since the reactance of the inductor is small, the output across the inductor is small and the output increases as the frequency increases because the reactance of the inductor increases as the frequency increases and at high frequencies the output is almost equal to the input.

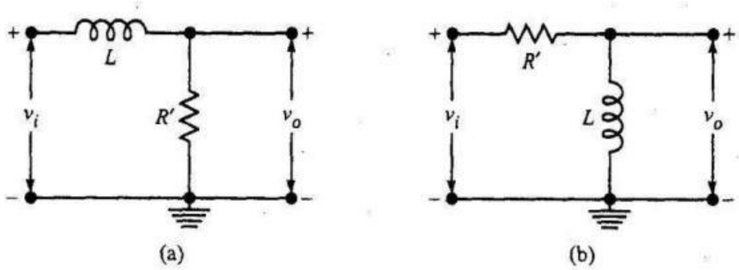


Figure 1.73 (a) *RL* low-pass circuit and (b) *RL* high-pass circuit.

RLC CIRCUITS

RLC Series Circuit

Consider a series *RLC* circuit shown in Figure 1.75.

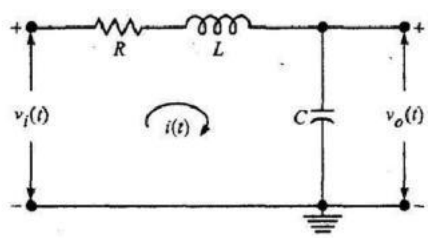


Figure 1.75 A series *RLC* circuit.

Writing the KVL around the loop, we obtain

$$v_i(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt$$

Taking the Laplace transform on both sides,

$$V_i(s) = I(s)\left[R + Ls + \frac{1}{Cs}\right] = \frac{I(s)}{Cs}[LCs^2 + RCs + 1]$$

$$V_o(s) = I(s)\frac{1}{Cs}$$

The transfer function of the circuit of Figure 1.75 is

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{LC\left[s^2 + \frac{R}{L}s + \frac{1}{LC}\right]}$$

The roots of the characteristic equation s_1 and s_2 are the values of s satisfying the equation

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$\therefore \qquad s_1, s_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

If $(R/2L)^2 > 1/LC$, i.e. $R > 2\sqrt{L/C}$, both the roots are real and different. The circuit is overdamped and there are no oscillations in the output. If $(R/2L)^2 = 1/LC$, i.e. $R = 2\sqrt{L/C}$, both the roots are real and equal. The circuit is critically damped. If $(R/2L)^2 < 1/LC$, i.e. $R < 2\sqrt{L/C}$, the roots are complex conjugate of each other. The circuit is underdamped and there will be oscillations in the output. The output is a sinusoid whose amplitude decays with time.

The term $\sqrt{L/C}$ is known as the *characteristic impedance* of the circuit.

For a step input of amplitude V, $V_i(s) = \dfrac{V}{s}$

$$\therefore \qquad V_o(s) = \left(\frac{V}{LC}\right)\left(\frac{1}{s\left[s^2 + \frac{R}{L}s + \frac{1}{LC}\right]}\right)$$

and

$$I(s) = \frac{V}{L\left[s^2 + \frac{R}{L}s + \frac{1}{LC}\right]}$$

The current response is:

Case (a): overdamped circuit, $R > 2\sqrt{L/C}$

$$i(t) = \frac{V}{2AL}\left[e^{-s_1 t} - e^{-s_2 t}\right], \quad \text{here } s_1 > s_2$$

where $A = \sqrt{\left(\frac{R}{2}\right)^2 - \frac{1}{LC}}$

Case (b): critically damped circuit, $R = 2\sqrt{L/C}$

$$i(t) = \frac{Vt}{L}e^{-Rt/2L}$$

Case (c): underdamped circuit, $R < 2\sqrt{L/C}$

$$i(t) = \frac{V}{BL}e^{-Rt/2L} \sin Bt \quad \text{where } B = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

The response of *i(t)* and the response of *v0(t)* for the above three cases are shown in Figures 1.76(a) and 1.76(b) respectively.

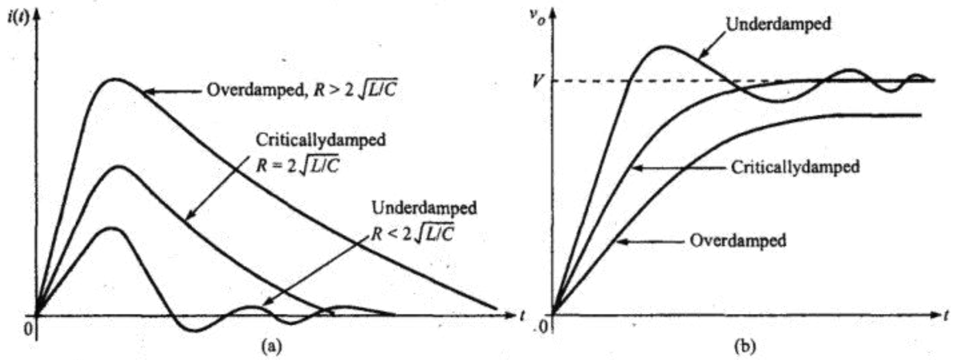


Figure 1.76 (a) Current response and (b) voltage response of series *RLC* circuit to a step voltage.

RLC Parallel Circuit

In the *RL* circuit shown in Figure 1.73(b), to include the effect of coil winding capacitance, output capacitance and stray capacitance to ground, a capacitor is added across the output. So, the *RLC* circuit shown in Figure 1.77(a) results. In terms of a current source, the equivalent circuit shown in Figure 1.77(b) results.

The transfer function of the network of Figure 1.77(a) is

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{RC}\left(\frac{s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}\right)$$

The roots of the characteristic equation are

$$s_1, s_2 = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

These are also the characteristic roots of the network in Figure 1.77(b).

The circuit is overdamped if $R < \frac{1}{2}\sqrt{L/C}$, critically damped if $R = \frac{1}{2}\sqrt{L/C}$ and underdamped if $R > \frac{1}{2}\sqrt{L/C}$. The response to the voltage across the *RLC* parallel circuit is similar to that to the current through the *RLC* series circuit with the difference that the input to the *RLC* parallel circuit is a step current.

In the series *RLC* network, the current response to a step input voltage ultimately dies to zero because of the capacitor in series. In the parallel *RLC* circuit the voltage across the *RLC* network is zero because of the inductance.

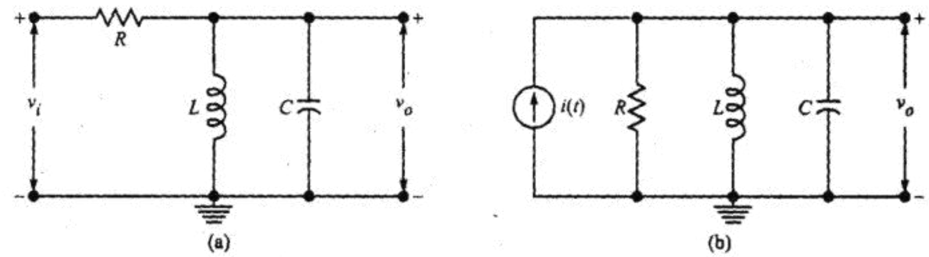


Figure 1.77 (a) v_i is applied through R to a parallel LC circuit and (b) parallel RLC circuit driven by a current source.

UNIT – II

NON LINEAR WAVESHAPING

………

Diode clippers, Transistor clippers, clipping at two independent levels, Transfer characteristics of clippers, Emitter coupled clipper, Comparators, applications of voltage comparators, clamping operation, clamping circuits using diode with different inputs, Clamping circuit theorem, practical clamping circuits, effect of diode characteristics on clamping voltage, Transfer characteristics of clampers.

………

In the previous chapter we discussed about linear wave shaping. We saw how a change of wave shape was brought about when a non-sinusoidal signal is transmitted through a linear network like *RC* low pass and high pass circuit. In this chapter, we discuss some aspects of nonlinear wave shaping like clipping and clamping. The circuits for which the outputs are non-sinusoidal for sinusoidal inputs are called nonlinear wave shaping circuits, for example clipping circuits and clamping circuits.

Clipping means cutting and removing a part. A clipping circuit is a circuit which removes the undesired part of the waveform and transmits only the desired part of the signal which is above or below some particular reference level, i.e. it is used to select for transmission that part of an arbitrary waveform which lies above or below some particular reference. Clipping circuits are also called *voltage* (or current) *limiters, amplitude selectors* or *slicers.*

Nonlinear wave shaping circuits may be classified as clipping circuits and clamping circuits. Clipping circuits may be single level clippers or two level clippers.

Single level clippers may be series diode clippers with and without reference or shunt diode clippers with and without reference. Clipping circuits may use diodes or transistors.

Clamping circuits may be negative clampers (positive peak clampers) with and without reference or positive clampers (negative peak clampers) with and without reference.

CLIPPING CIRCUITS

In general, there are three basic configurations of clipping circuits.

1. A series combination of a diode, a resistor and a reference voltage.
2. A network consisting of many diodes, resistors and reference voltages.
3. Two emitter coupled transistors operating as a differential amplifier.

Diode Clippers

Figure 2.1(a) shows the v-i characteristic of a practical diode. Figures 2.1(b), (c), (d), and (e) show the *v-i* characteristics of an idealized diode approximated by a curve which is piece-wise linear and continuous. The break point occurs at V_r, where V_r = 0.2 V for Ge and V_r = 0.6 V for Si. Usually V_r is very small compared to the reference voltage V_R and can be neglected.

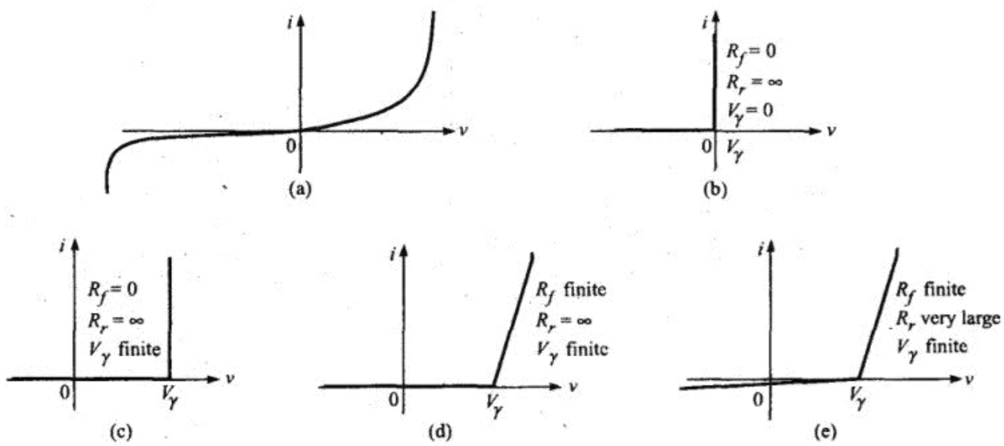


Figure 2.1 *v–i* characteristics of a diode.

Shunt Clippers

Clipping above reference level

Using the ideal diode characteristic of Figure 2.2(a), the clipping circuit shown in Figure 2.2(b), has the transmission characteristic shown in Figure 2.2(c). The transmission characteristic which is a plot of the output voltage v_0 as a function of the input voltage v, also exhibits piece-wise linear discontinuity. The break point occurs at the reference voltage V_R. To the left of the break point i.e. for $v_t < V_R$ the diode is reverse biased (OFF) and the equivalent circuit shown in Figure 2.2(d) results. In this region the signal v, may be transmitted directly to the output, since there is no load across the output to cause a drop across the series resistor /?. To the right of the break point i.e. for v(> V_R the diode is forward biased (ON) and the equivalent circuit shown in Figure 2,2(e) results and increments in the inputs are totally attenuated and the output is fixed at V_R. Figure 2.2(c) shows a sinusoidal input signal of amplitude large enough so that the signal makes excursions past the break point. The corresponding output exhibits a suppression of the positive peak of the signal. The output will appear as if the positive peak had been *clipped off* or *sliced off*

Clipping below reference level

If this clipping circuit of Figure 2.2(b), is modified by reversing the diode as shown in Figure 2.3(a), the corresponding piece-wise linear transfer characteristic and the output for a sinusoidal input will be as shown in Figure 2.3(b). In this circuit, the portion of the waveform more positive than V_R is transmitted without any attenuation but the portion of the waveform less positive than V_R is totally suppressed. For $V_j < V_R$, the diode conducts and acts as a short circuit and the equivalent circuit shown in Figure 2.3(c) results and the output is fixed at V_R. For v, > V_R, the diode is reverse biased and acts as an open circuit and the equivalent circuit shown in Figure 2.3(d) results and the output is the same as the input.

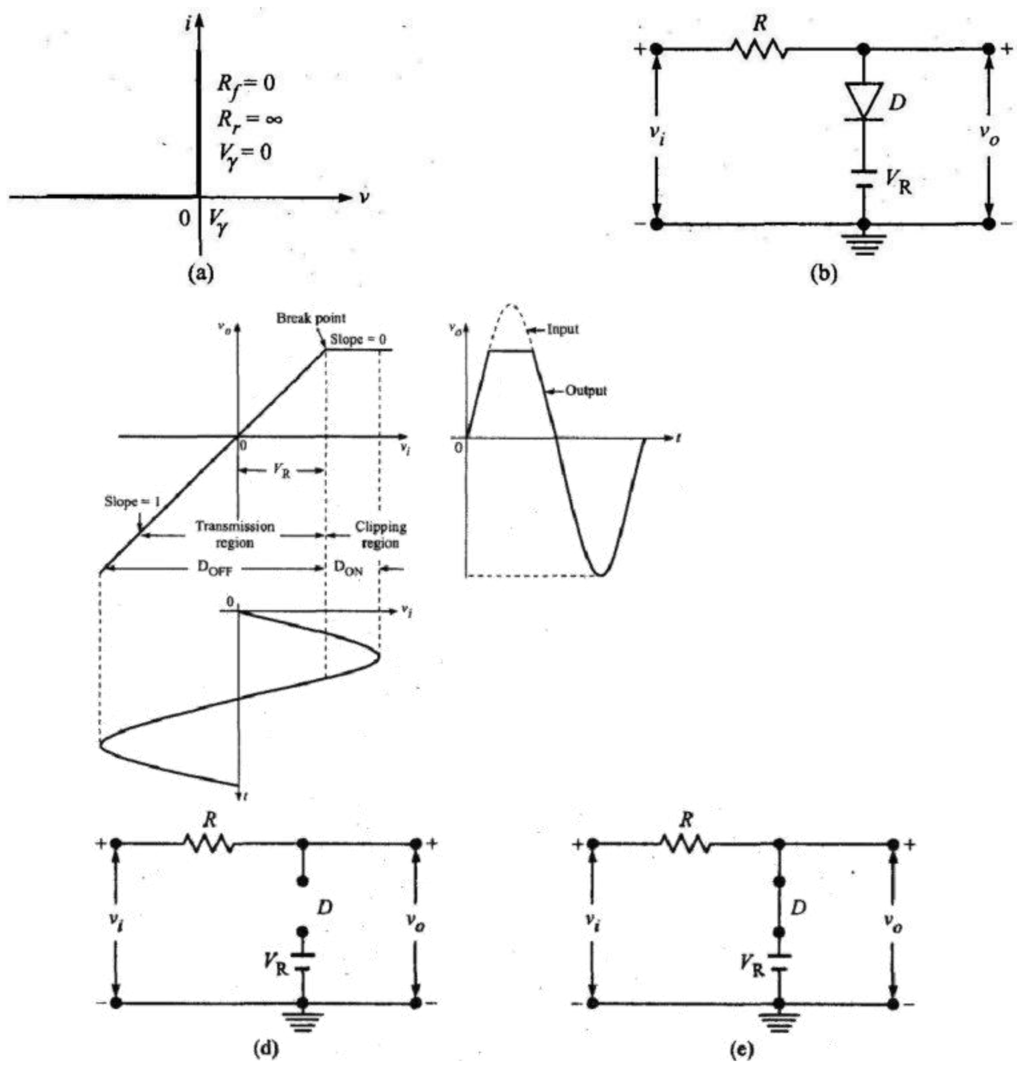


Figure 2.2 (a) *v-i* characteristic of an ideal diode, (b) diode clipping circuit, which removes that part of the waveform that is more positive than V_R, (c) the piece-wise linear transmission characteristic of the circuit, a sinusoidal input and the clipped output, (d) equivalent circuit for $v_(< V_R$, and (e) equivalent circuit for v, > V_R.

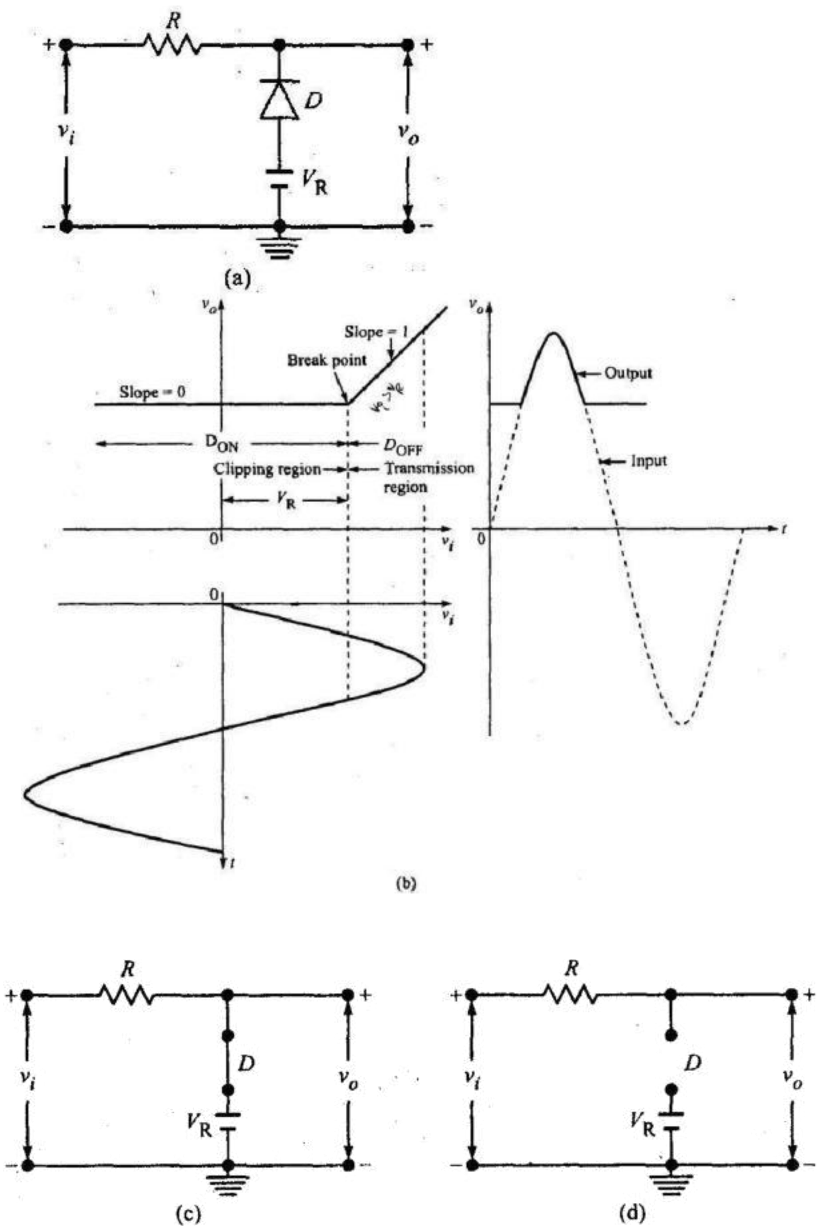


Figure 2.3 (a) A diode clipping circuit, which transmits that part of the sine wave that is more positive than V_R, (b) the piece-wise linear transmission characteristic, a sinusoidal input and the clipped output, (c) equivalent circuit for $v_i < V_R$, and (d) equivalent circuit for $v_i > V_R$.

In Figures 2.1(b) and 2.2(a), we assumed that $R_r = \infty$ and $R_f = 0$. If this condition does not apply, the transmission characteristic must be modified. The portions of those curves which are indicated as having unity slope must instead be considered as having a slope of $R_r/(R_r + R)$, and those, having zero slope as having a slope of $R_f/(R_f + R)$. In the transmission region of a diode clipping circuit, it is required that $R_r \gg R$, i.e. $R_r = kR$, where k is a large number, and in the attenuation region, it is required that $R \gg R_f$. From these equations we can deduce that $R = J\ R_r x R^\wedge$, i.e. the external resistance R is to be selected as the geometric mean of Rf and /?,. The ratio $R_r IRf$ serves as a figure of merit for the diodes used in these applications. A zener diode may also be used in combination with a *p-n* junction diode to obtain single-ended clipping, i.e. one-level clipping.

Series Clippers

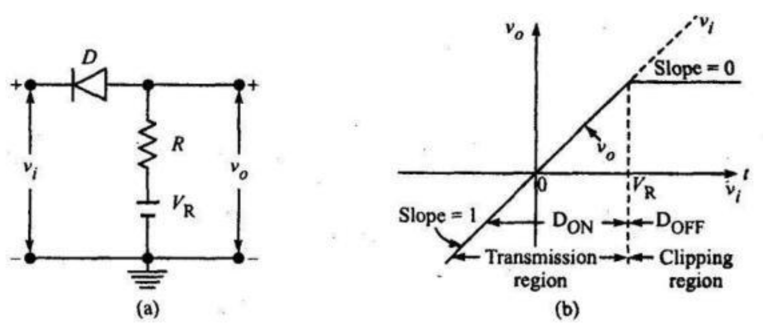
Clipping above the reference voltage V_n

Figure 2.4(a) shows a series clipper circuit using a p-n junction diode. V_R is the reference voltage source. The diode is assumed to be ideal (/?/ = 0, R_r = °°, V_y= 0) so that it acts as a short circuit when it is ON and as a open circuit when it is OFF. Since the diode is in the series path connecting the input and the output it is called a series clipper. The v_0 versus v, characteristic called the *transfer characteristic* is shown in Figure 2.4(b). The output for a sinusoidal input is shown in Figure 2.4(c).

The circuit works as follows:

For v, < V_R, the diode Dj is forward biased because its anode is at a higher potential than its cathode. It conducts and acts as a short circuit and the equivalent circuit shown in Figure 2.4(d) results. The difference voltage between the input v,- and the reference voltage V_R i.e. (V_R

– v_i) is dropped across . Therefore v_0 = v_i and the slope of the transfer characteristic for v_i < V_R is 1. Since the input signal is transmitted to the output without any change, this region is called the transmission region.



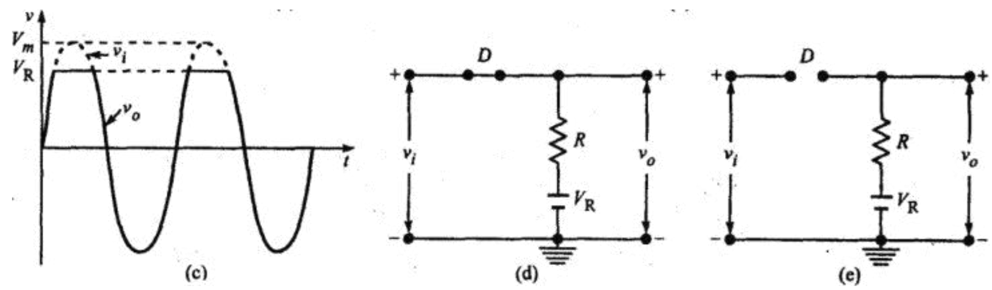
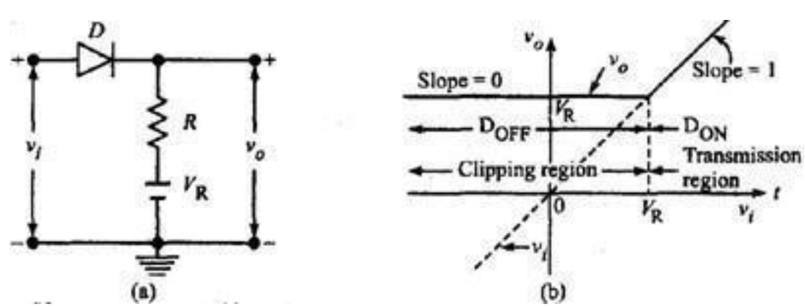


Figure 2.4 (a) Diode series clipper circuit diagram, (b) transfer characteristic, (c) output waveform for a sinusoidal input, (d) equivalent circuit for v; < V_R, and (e) equivalent circuit for v(> V_R.

For v, > V_R, the diode is reverse biased because its cathode is at a higher potential than its anode, it does not conduct and acts as an open circuit and the equivalent circuit shown in Figure 2.4(e) results. No current flows through *R* and so no voltage drop across it. So the output voltage v_0 = V_R and the slope of the transfer characteristic is zero. Since the input signal above V_R is clipped OFF for v, > V_R, this region is called the *clipping region.* The equations V_0=V_i for V_i < V_R and V_0= V_R for V_i > V_R are called the transfer characteristic equations.

Clipping below the reference voltage V_B

Figure 2.5(a) shows a series clipper circuit using a p-n junction diode and a reference voltage source V_R. The diode is assumed to be ideal (R_f = 0, R_r = °°, V_y = 0) so that it acts as a short circuit when it is ON and as a open circuit when it is OFF. Since the diode is in the series path connecting the input and the output it is called a series clipper. The transfer characteristic is shown in Figure 2.5(b). The output for a sinusoidal input is shown in Figure 2.5(c).



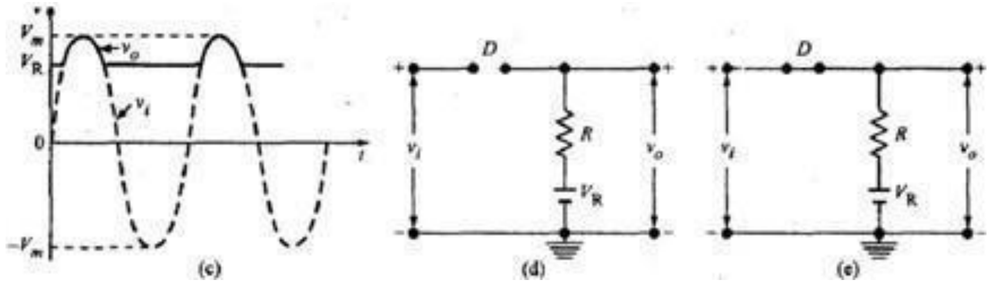


Figure 2.5 (a) Diode series clipper circuit diagram, (b). transfer characteristics, (c) output for a sinusoidal input, (d) equivalent circuit for v_i- < V_R, and (e) equivalent circuit for v_i- > V_R.

The circuit works as follows:

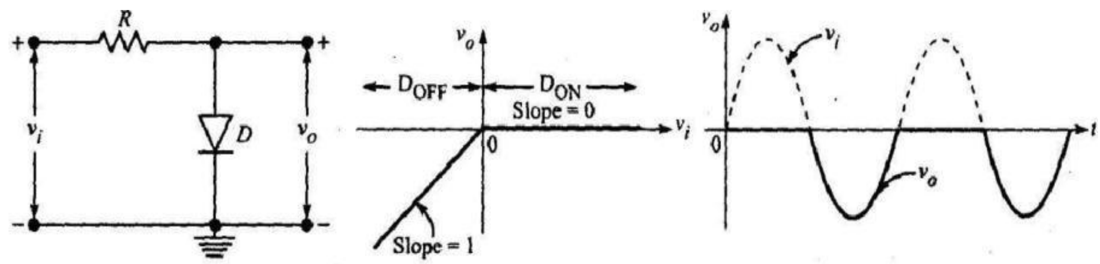
For $v_i < V_R$, D is reversed biased because its anode is at a lower potential than its cathode. The diode does not conduct and acts as an open circuit and the equivalent circuit shown in Figure 2.5(d) results. No current flows through *R* and hence no voltage drop across *R* and hence v_o = VR- So the slope of the transfer characteristic is zero for v, < V_R. Since the input is clipped off for v, < V_R, this region is called the clipping region.

For v, > VR, the diode is forward biased because its anode is at a higher potential than its cathode. The diode conducts and acts as a short circuit and the equivalent circuit shown in Figure 2.5(e) results. Current flows through /? and the difference voltage between the input and the output voltages v, - VR drops across /? and the output v_0 = v_i. The slope of the transfer characteristic for v, > VR is unity. Since the input is transmitted to the output for v; > V_R, this region is called the transmission region. The equations are called the transfer characteristic equations.

$$v_o = V_R \text{ for } v_i < V_R$$
$$v_o = v_i \text{ for } v_i > V_R$$

Some single-ended diode clipping circuits, their transfer characteristics and the output waveforms for sinusoidal inputs are shown below (Figure 2.6).

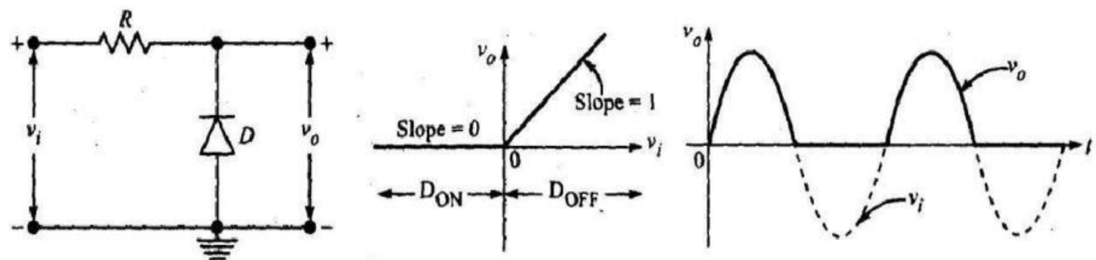
Some single-ended clipping circuits



For $v_i < 0$, D is OFF $\therefore v_o = v_i$

For $v_i > 0$, D is ON $\therefore v_o = 0$

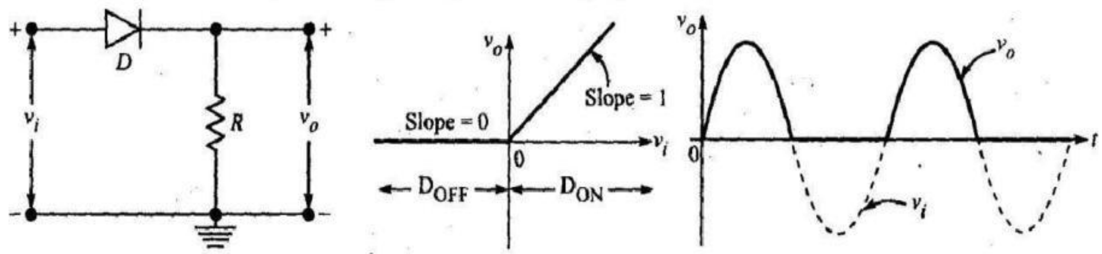
(a) Shunt clipper, positive peak clipping without reference



For $v_i < 0$, D is ON $\therefore v_o = 0$

For $v_i > 0$, D is OFF $\therefore v_o = v_i$

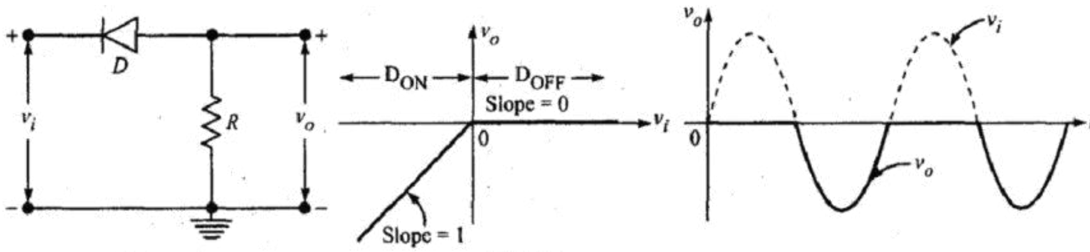
(b) Shunt clipper, negative peak clipping without reference



For $v_i < 0$, D is OFF $\therefore v_o = 0$

For $v_i > 0$, D is ON $\therefore v_o = v_i$

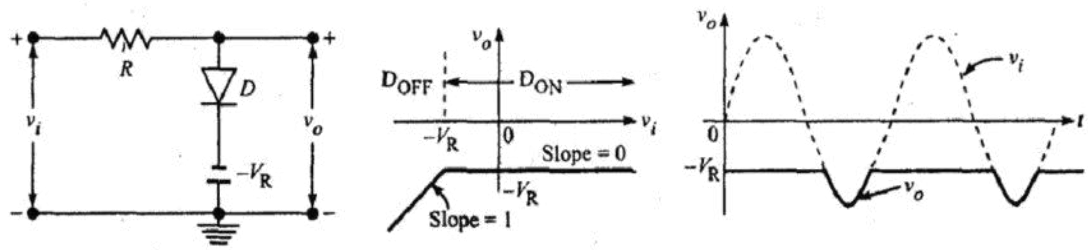
(c) Series clipper, negative peak clipping without reference



For $v_i < 0$, D is ON $\therefore v_o = v_i$

For $v_i > 0$, D is OFF $\therefore v_o = 0$

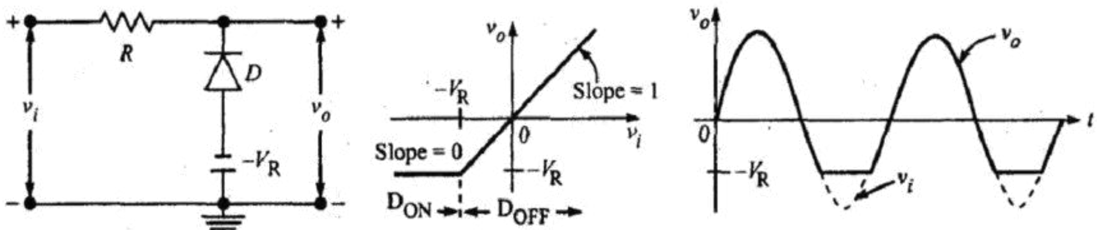
(d) Series clipper, positive peak clipping without reference



For $v_i < -V_R$, D is OFF $\therefore v_o = v_i$

For $v_i > -V_R$, D is ON $\therefore v_o = -v_i$

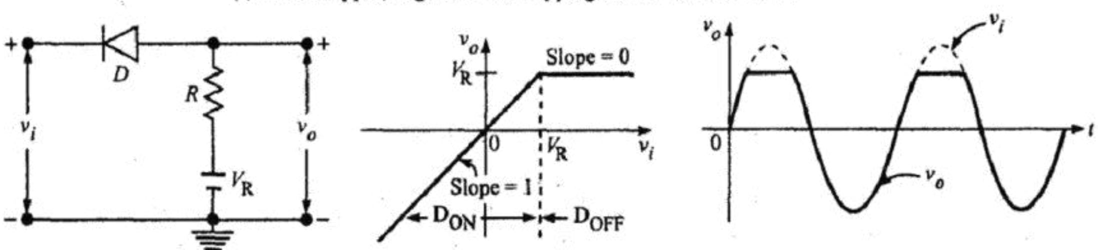
(e) Shunt clipper, negative base clipping above reference level



For $v_i < -V_R$, D is ON $\therefore v_o = -V_R$

For $v_i > -V_R$, D is OFF $\therefore v_o = v_i$

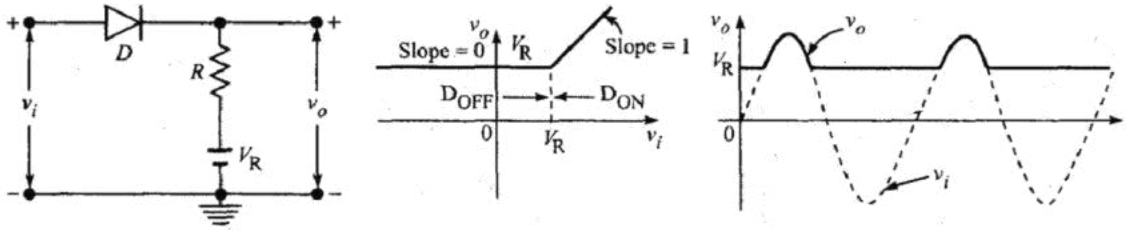
(f) Shunt clipper, negative base clipping below reference level



For $v_i < V_R$, D is ON $\therefore v_o = v_i$

For $v_i > V_R$, D is OFF $\therefore v_o = V_R$

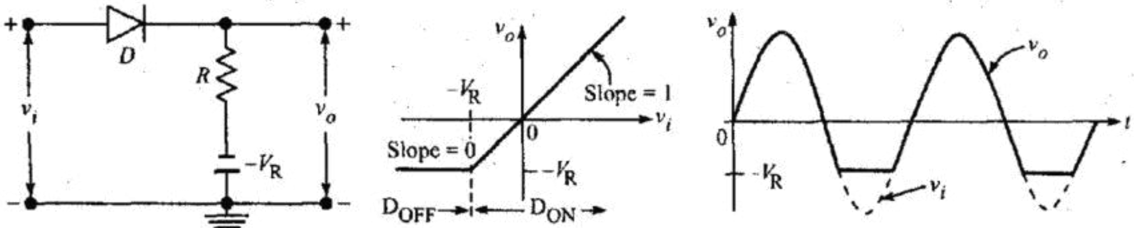
(g) Series clipper, positive base clipping above reference level



For $v_i < V_R$, D is OFF $\therefore v_o = V_R$

For $v_i > V_R$, D is ON $\therefore v_o = v_i$

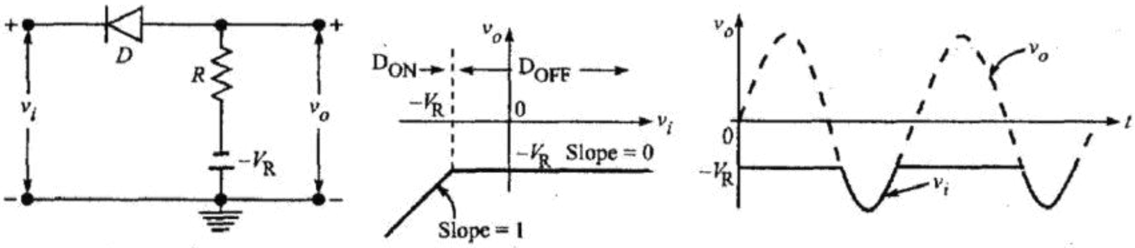
(h) Series clipper, positive base clipping before reference level



For $v_i < -V_R$, D is OFF $\therefore v_o = -V_R$

For $v_i > -V_R$, D is ON $\therefore v_o = v_i$

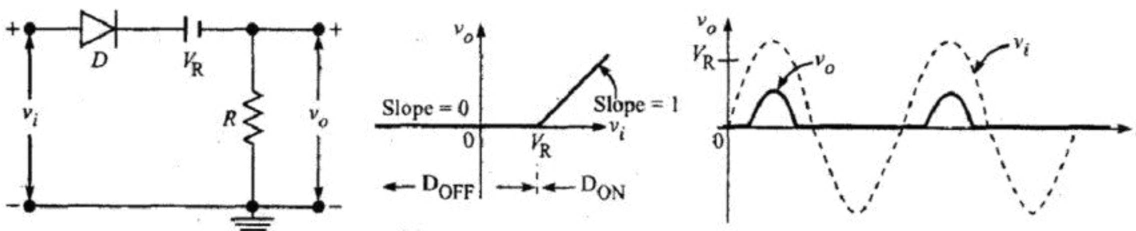
(i) Series clipper, negative base clipping above reference level



For $v_i < -V_R$, D is ON $\therefore v_o = v_i$

For $v_i > -V_R$, D is OFF $\therefore v_o = -V_R$

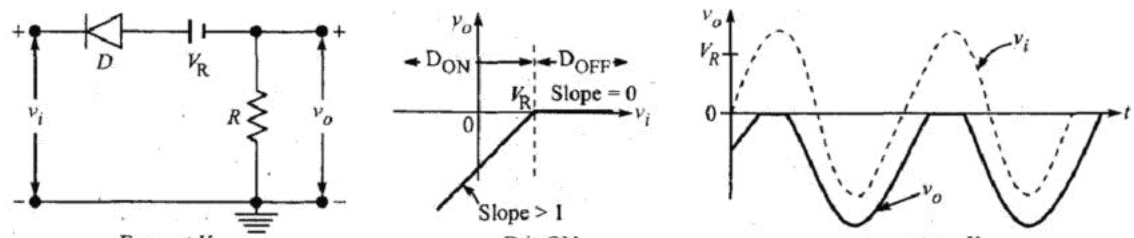
(j) Series clipper, negative base clipping above reference level



For $v_i < V_R$, D is OFF $\therefore v_o = 0$

For $v_i > V_R$, D is ON $\therefore v_o = v_i - V_R$

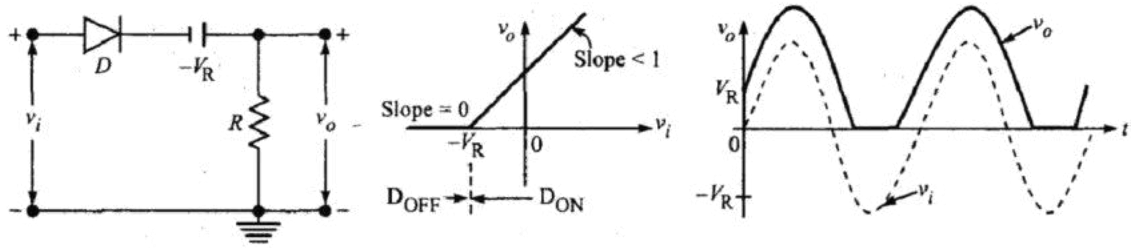
(k) Series clipper (Biased)



For $v_i < V_R$, D is ON $\therefore v_o = v_i - V_R$

For $v_i > V_R$, D is OFF $\therefore v_o = 0$

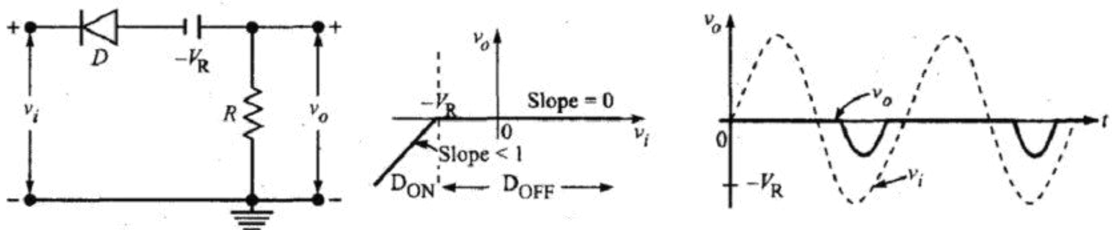
(l) Series clipper (Biased)



For $v_i < -V_R$, D is OFF $\therefore v_o = 0$

For $v_i > -V_R$, D is ON $\therefore v_o = v_i + V_R$

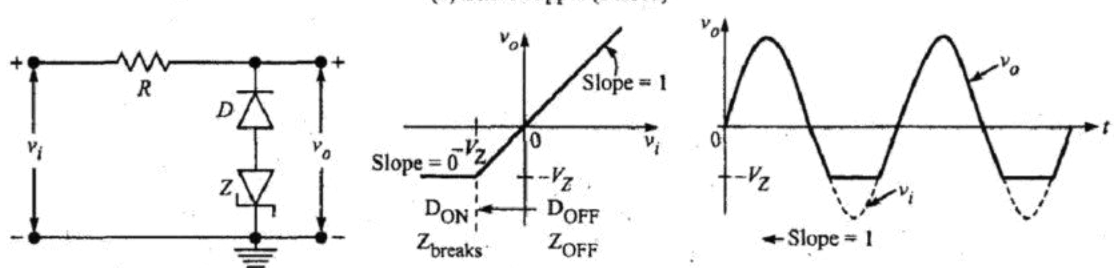
(m) Series clipper (Biased)



For $v_i < -V_R$, D is ON $\therefore v_o = v_i + V_R$

For $v_i > -V_R$, D is OFF $\therefore v_o = 0$

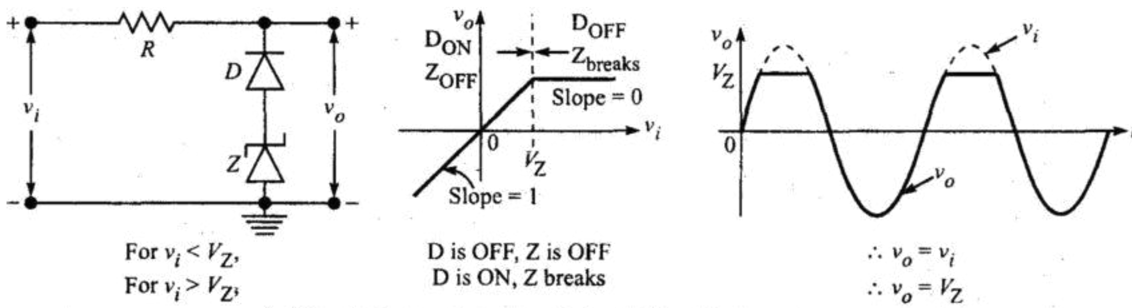
(n) Series clipper (Biased)



For $v_i < -V_Z$, D is ON, Z breaks $\therefore v_o = -V_Z$

For $v_i > -V_Z$, D is OFF, Z is OFF $\therefore v_o = v_i$

(o) Shunt clipper using zener diode, negative clipping



(p) Shunt clipper using zener diode, positive clipping

Figure 2.6 Examples of single-ended clipping circuits.

In the clipping circuits, the diode may appear as a series element or as a shunt element. The use of the diode as a series element has the disadvantage that when the diode is OFF and it is intended that there be no transmission, fast signals or high frequency waveforms may be transmitted to the output through the diode capacitance. The use of the diode as a shunt element has the disadvantage that when the diode is open and it is intended that there be transmission, the diode capacitance together with all other capacitances in shunt with the output terminals will round off the sharp edges of the input waveforms and attenuate, the high frequency signals.

Clipping at Two Independent Levels

A parallel, a series, or a series-parallel arrangement may be used in double-ended limiting at two independent levels. A parallel arrangement is shown in Figure 2.7. Figure 2.8 shows the transfer characteristic and the output for a sinusoidal input. The input-output characteristic has two breakpoints, one at v_0 = v, = V_{R1} and the second at v_0 = v, = -V_{R2} and has the following characteristics.

Input v_i	*Output* v_o	*Diode status*
$v_i > V_{R1}$	$v_o = V_{R1}$	D_1 ON, D_2 OFF
$-V_{R2} < v_i < V_{R1}$	$v_o = v_i$	D_1 OFF, D_2 OFF
$v_i < -V_{R2}$	$v_o = -V_{R2}$	D_1 OFF, D_2 ON

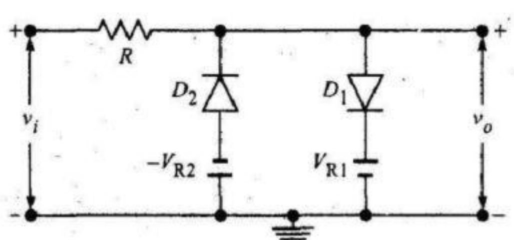


Figure 2.7 A diode clipper which limits at two independent levels.

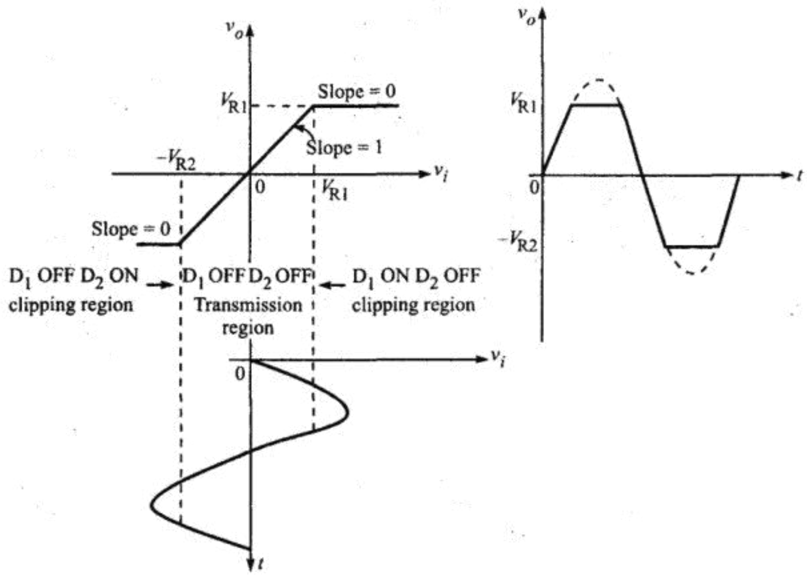


Figure 2.8 The piece-wise linear transfer curve, the input sinusoidal waveform and the corresponding output for the clipper of Figure 2.7.

The two level diode clipper shown in Figure 2.8 works as follows. For v, > V_{R1}, DI is ON and D_2 is OFF and the equivalent circuit shown in Figure 2.9(a) results. So the output v_0 = V_{R1} and the slope of the transfer characteristic is zero.

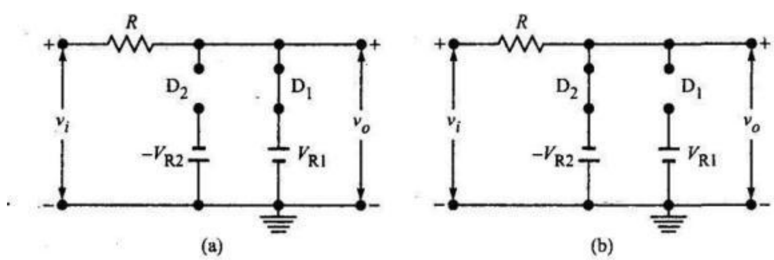


Figure 2.9 (a) Equivalent circuit for $v_i > V_{R1}$ and (b) equivalent circuit for $v_i < - V_{R2}$.

For v, < - V_{R2}, DI is OFF and D_2 is ON and the equivalent circuit shown in Figure 2.9(b) results. So the output v_0 = - V_{R2} and the slope of the transfer characteristic is zero. For-V_{R2} < v, < VRI, D! is OFF and D_2 is OFF and the equivalent circuit shown in Figure 2.10 results. So the output v_0 = v/ and the slope of the transfer characteristic is one.

The circuit of Figure 2.7 is called a slicer because the output contains a slice of the input between two reference levels $V_{R!}$ and V_{R2}. Looking at the input and output waveforms, we observe that this circuit may be used to convert a sine wave into a square wave, if V_{DI} = Vm. and if the amplitude of the input signal is very large compared with the difference in the reference levels, the output will be a symmetrical square wave. Two zener diodes in series opposing may also be used to form a double-ended clipper.

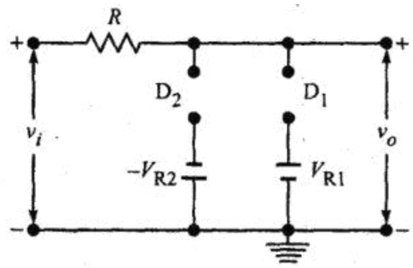
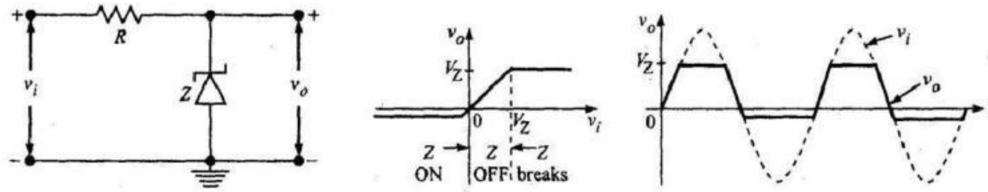


Figure 2.10 Equivalent circuit for $-V_{R2} < v_i < V_{R1}$.

If the diodes have identical characteristics, then, a symmetrical limiter is obtained. Some double-ended clippers, their transfer characteristics and the outputs for sine wave inputs are shown in Figure 2.11.

Some double-ended clipping circuits

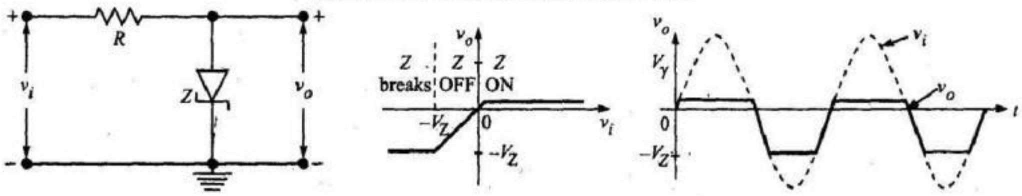


For $v_i < -V_\gamma$, Z is ON $\therefore v_o = -V_\gamma$

For $-V_\gamma < v_i < V_Z$, Z is OFF $\therefore v_o = v_i$

For $v_i > V_Z$, Z breaks $\therefore v_o = V_Z$

(a) Two level clipping using a zener diode

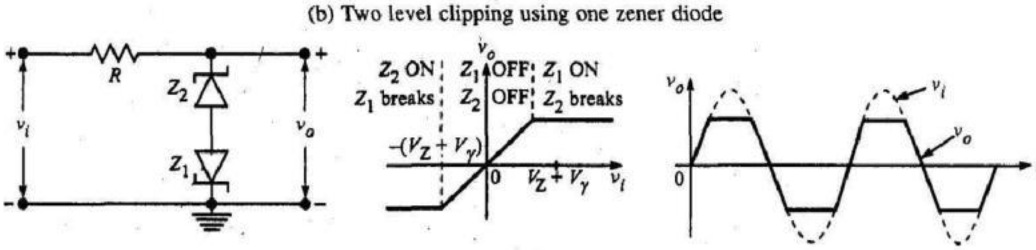


For $v_i < -V_Z$, Z breaks $\therefore v_o = -V_Z$

For $-V_Z < v_i < V_\gamma$, Z is OFF $\therefore v_o = v_i$

For $v_i > V_\gamma$, Z is ON $\therefore v_o = V_\gamma$

(b) Two level clipping using one zener diode

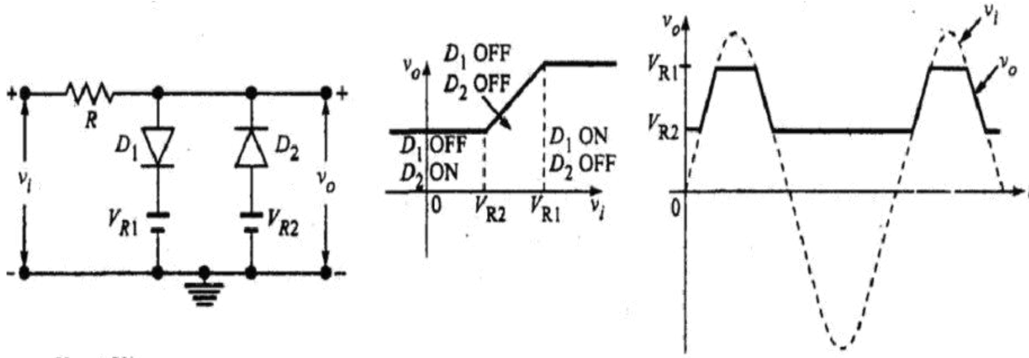


For $v_i < -(V_Z + V_\gamma)$, Z_1 breaks, Z_2 is on $\therefore v_o = -(V_Z + V_\gamma)$

For $-(V_Z + V_\gamma) < v_i < (V_Z + V_\gamma)$, Z_1 is OFF, Z_2 is OFF $\therefore v_o = v_i$

For $v_i > (V_Z + V_\gamma)$, Z_1 is ON, Z_2 breaks $\therefore v_o = V_Z + V_\gamma$

(c) Two level clipping using two zener diodes



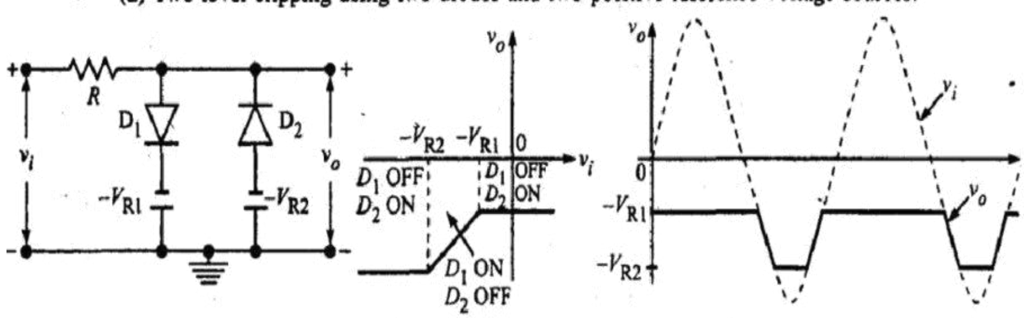
$V_{R2} < V_{R1}$

For $v_i < V_{R2}$, D$_1$ is OFF, D$_2$ is ON $\therefore v_o = V_{R2}$

For $V_{R2} < v_i < V_{R1}$, D$_1$ is OFF, D$_2$ is OFF $\therefore v_o = v_i$

For $v_i > V_{R1}$, D$_1$ is ON, D$_2$ is OFF $\therefore v_o = V_{R1}$

(d) Two level clipping using two diodes and two positive reference voltage sources.



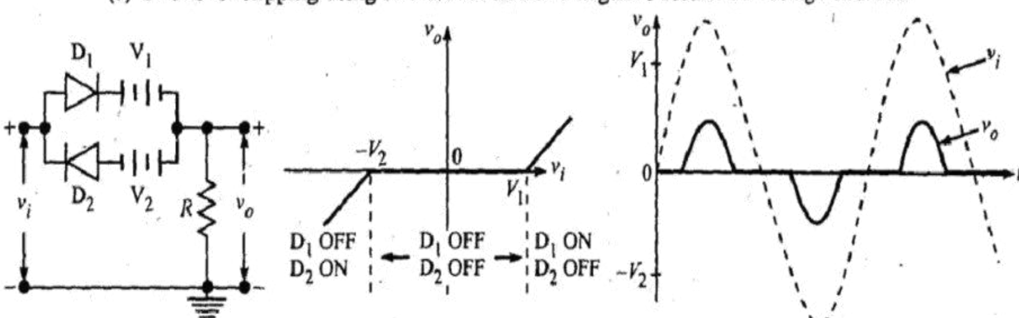
$-V_{R2} < -V_{R1}$

For $v_i < -V_{R2}$, D$_1$ is OFF, D$_2$ is ON $\therefore v_o = -V_{R2}$

For $-V_{R2} < v_i < -V_{R1}$, D$_1$ is OFF, D$_2$ is OFF $\therefore v_o = v_i$

For $v_i > -V_{R1}$ D$_1$ is ON, D$_2$ is OFF $\therefore v_o = -V_{R1}$

(e) Two level clipping using two diodes and two negative reference voltage sources.



For $v_i < -V_2$, D$_1$ is OFF, D$_2$ is ON $\therefore v_o = v_i + V_2$

For $-V_2 < v_i < V_1$, D$_1$ is OFF, D$_2$ is OFF $\therefore v_o = 0$

For $v_i > V_1$, D$_1$ is ON, D$_2$ is OFF $\therefore v_o = v_i - V_1$

(f) Double ended series biased clipper

Figure 2.11 Examples of double-ended clippers.

Transistor Clippers

A nonlinear device is required for clipping purposes. A diode exhibits a nonlinearity, which occurs when it goes from OFF to ON. On the other hand, the transistor has two pronounced nonlinearities, which may be used for clipping purposes. One occurs when the transistor crosses from the cut-in region into the active region and the second occurs when the transistor crosses from the active region into the saturation region. Therefore, if the peak-to-peak value of the input waveform is such that it can carry the transistor across the boundary between the cut-in and active regions, or across the boundary between the active and saturation regions, a portion of the input waveform will be clipped. Normally, it is required that the portion of the input waveform, which keeps the transistor in the active region shall appear at the output without distortion. In that case, it is required that the input current rather than the input voltage be the waveform of the signal of interest. The reason for this requirement is that over a large signal excursion in the active region, the transistor output current responds nominally linearly to the input current but is related in a quite nonlinear manner to the input voltage. So, in transistor clippers a current drive needs to be used.

A transistor clipper is shown in Figure 2.19. The resistor *R* which represents either the signal source impedance or a resistor deliberately introduced must be large compared with the input resistance of the transistor in the active region. Under these circumstances, the input base current will very nearly have the waveform of the input voltage, because the base current is given by $i_B = (v_i - V_\gamma)/R$ where V_r is the base-to-emitter cut-in voltage. V_y » 0.1 V for Ge and V_y ~ 0.5 V for Si.

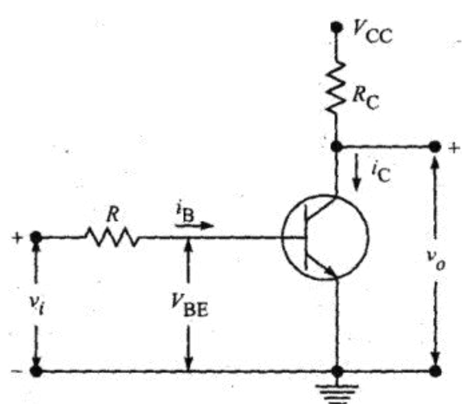


Figure 2.19 A transistor clipper.

If a ramp input signal v_i which starts at a voltage below cut-off and carries the transistor into saturation is applied, the base voltage, the base current, and the collector current waveforms of the transistor clipper will be as shown in Figure 2.20.

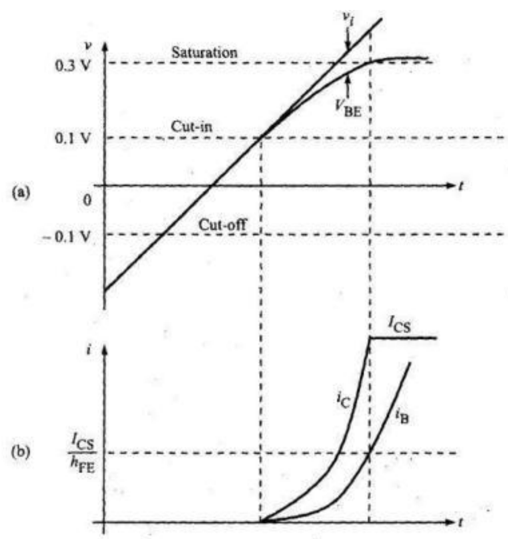


Figure 2.20 Waveforms of the transistor clipper of Figure 2.19: (a) voltage V_{BE} which results when a ramp input drives the transistor from cut-off into saturation, and (b) the base and collector currents.

The waveforms which result when a sinusoidal voltage v, carries the transistor from cut-off to saturation are shown in Figure 2.21. The base circuit is biased so that cut-in occurs when v_{BE} reaches the voltage *V*.

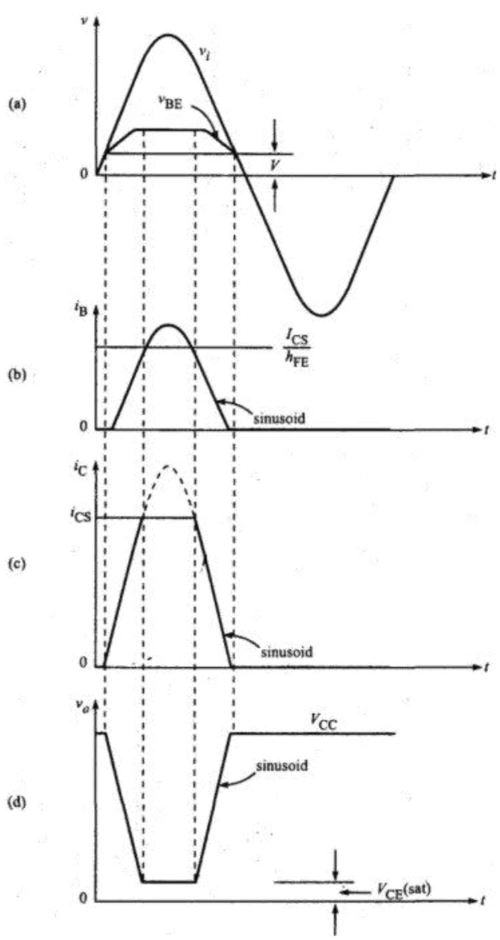


Figure 2.21 Waveforms for the transistor clipper of figure 2.19: (a) in put voltage and the base – to-emitter voltage (b) the base current (c) the collector current (d) the out put voltage

Emitter-Coupled Clipper

An emitter-coupled clipper is shown in Figure 2.22. It is a two-level clipper using transistors. The base of Q_2 is fixed at a voltage V_{BB2}, and the input is applied to B_1. If initially the input is negative, Q_1 is OFF and only Q_2 carries the current. Assume that V_{BB2} has been adjusted so that Q_2 operates in its active region. Let us assume that the current / in the emitter resistance is constant. This is valid if $IV_{BE2}I$ is small compared to $V_{BB2} + V_{EE}$ When v_t is below the cut-off point of Q_1, all the current 7 flows through Q_2. As v, increases, Q_1 will eventually come out of cut-off, both the transistors will be carrying currents but the current in Q_2 decreases while the current in Q_1 increases, the sum of the currents in the two transistors remaining constant and equal to 7. The input signal appears at the output, amplified but not inverted. As v_1 continues to increase, the common emitter will follow the base of Q_1. Since the base of Q_2 is fixed, a point will be reached when the rising emitter voltage cuts off Q_2. Thus, the input signal is amplified but twice limited, once by the cutoff of Q_1 and once by the onset of cut-off in Q_2. The total range Av_0, over which the output can follow the input is V_E and is constant and therefore adjustable through an adjustment of 7. The absolute voltage of the portion of the input waveform selected for transmission may be selected through an adjustment of a biasing voltage on which v, is superimposed or through an adjustment of V_{BB2}- The total range of input voltage Av, between the clipping limits is Av_0/A, where A is the gain of the amplifier stage. Figure 2.23 shows the transfer characteristic of an emitter-coupled clipper.

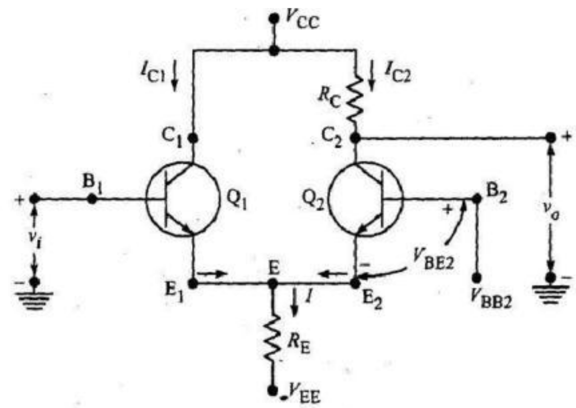


Figure 2.22 An emitter-coupled clipper.

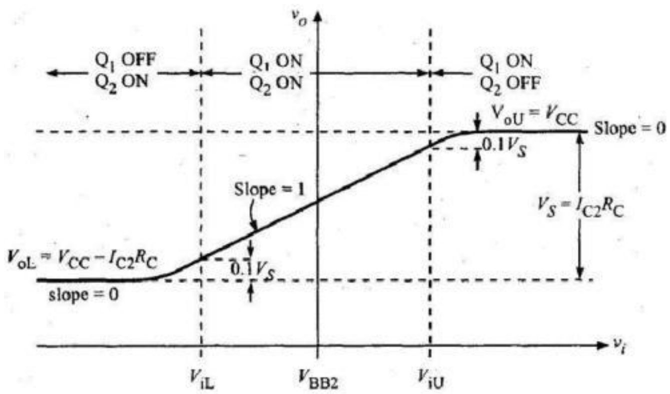


Figure 2.23 The transfer characteristic of the emitter-coupled clipper.

Comparators

A comparator circuit is one, which may be used to mark the instant when an arbitrary waveform attains some particular reference level. The nonlinear circuits, which can be used to perform the operation of clipping may also be used to perform the operation of comparison. In fact, the clipping circuits become elements of a comparator system and are Usually simply referred to as comparators. The distinction between comparator circuits and the clipping circuits is that, in a comparator there is 'no interest in reproducing any part of the signal waveform, whereas in a clipping circuit, part of the signal waveform is needed to be reproduced without any distortion.

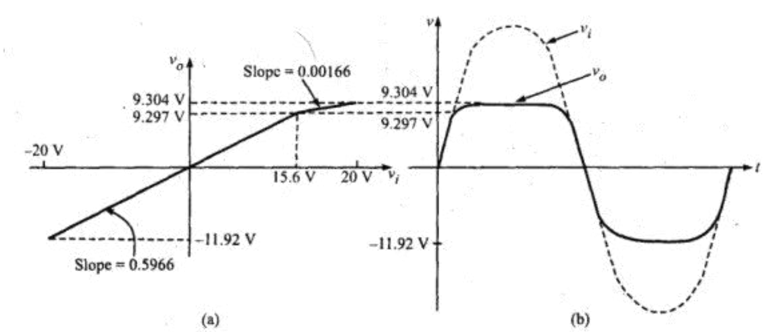


Figure 2.69 Example 2.19: (a) transfer characteristic and (b) output waveform in the presence of R_L.

Figure 2.70 shows the circuit diagram of a diode comparator. As long as the input voltage v, is less than the reference voltage V_R, the diode D is ON and the output is fixed at V_R. When v, > V_R, the diode is OFF and hence v_0 = v,. The break occurs at v, = V_R at time t = t\. So, this circuit can be used to mark the instant at which the input voltage reaches a particular reference level V_R.

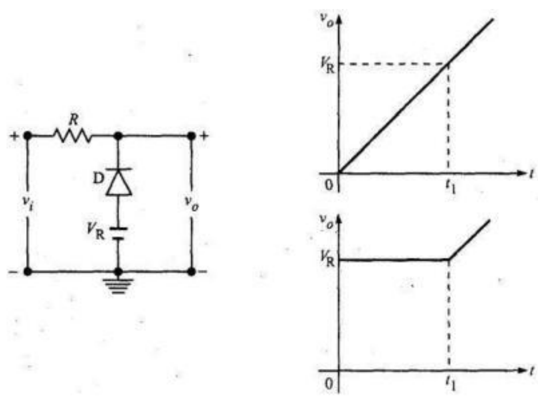


Figure 2.70 Diode comparator.

Comparators may be non-regenerative or regenerative. Clipping circuits fall into the category of non-regenerative comparators. In regenerative comparators, positive feedback is employed to obtain an infinite forward gain (unity loop gain). The Schmitt trigger and the blocking oscillator are examples of regenerative comparators. The Schmitt trigger comparator generates approximately a step input. The blocking oscillator comparator generates a pulse rather than a step output waveform. Most applications of comparators make use of the step or pulse natures of the input. Operational amplifiers and tunnel diodes may also be used as comparators.

Applications of voltage comparators

Voltage comparators may be used:

1. In accurate time measurements
2. In pulse time modulation
3. As timing markers generated from a sine wave.
4. In phase meters
5. In amplitude distribution analyzers
6. To obtain square wave from a sine wave
7. In analog-to-digital converters.

CLAMPING CIRCUITS

Clamping circuits are circuits, which are used to clamp or fix the extremity of a periodic waveform to some constant reference level *V.R*. Under steady-state conditions, these circuits restrain the extremity of the waveform from going beyond V_R. Clamping circuits may be one-way clamps or two-way clamps. When only one diode is used and a voltage change in only one direction is restrained, the circuits are called one-way clamps. When two diodes are used and the voltage change in both the directions is restrained, the circuits are called two-way clamps.

The Clamping Operation

When a signal is transmitted through a capacitive coupling network *(RC* high-pass circuit), it looses its dc component, and a clamping circuit may be used to introduce a dc component by fixing the positive or negative extremity of that waveform to some reference level. For this reason, the clamping circuit is often referred to as *dc restorer* or *dc reinserter.* In fact, it should be called a *dc inserter,* because the dc component introduced may be different from the dc component lost during transmission. The clamping circuit only changes the dc level of the input signal. It does not affect its shape

Classification of clamping circuits

Basically clamping circuits are of two types: (1) positive-voltage clamping circuits and (2) negative-voltage clamping circuits.

In positive clamping, the negative extremity of the waveform is fixed at the reference level and the entire waveform appears above the reference level, i.e. the output waveform is positively clamped with reference to the reference level. In negative clamping, the positive extremity of the waveform is fixed at the reference level and the entire waveform appears below the reference, i.e. the output waveform is negatively clamped with respect to the reference level. The capacitors are essential in clamping circuits. The difference between the clipping and clamping circuits is that while the clipper clipps off an unwanted portion of the input waveform, the clamper simply clamps the maximum positive or negative peak of the waveform to a desired level. There will be no distortion of waveform.

Negative Clamper

Figure 3.1 (a) shows the circuit diagram of a basic negative clamper. It is also termed a positive peak clamper since the circuit clamps the positive peak of a signal to zero level. Assume that the signal source has negligible output impedance and that the diode" is ideal, Rf= 0 n and V_y = 0 V in that, it exhibits an arbitrarily sharp break at 0 V, and that its input signal shown in Figure 2.71(b) is a sinusoid which begins at t = 0. Let the capacitor C be uncharged at t = 0.

During the first quarter cycle, the input signal rises from zero to the maximum value. The diode conducts during this time and since we have assumed an ideal diode, the voltage across it is zero. The capacitor *C* is charged through the series combination of the signal source and the diode and the voltage across C rises sinusoidally. At -the end of the first quarter cycle, the voltage across the capacitor, $v_C = V_m$.When, after the first quarter cycle, the peak has been passed and the input signal begins to fall, the voltage v_C across the capacitor is no longer able to follow the input, because there is no path for the capacitor to discharge. Hence, the voltage across the capacitor remains constant at $v_C = V_m$, and the charged capacitor acts as a voltage source of V volts and after the first quarter cycle, the output is given by v_0 = v, - V_m. During the succeeding cycles, the positive extremity of the signal will be *clamped* or *restored* to zero and the output

for $v_i = 0$, $v_o = -V_m$.
for $v_i = V_m$, $v_o = 0$,
for $v_i = -V_m$, $v_o = -2V_m$.

waveform shown in Figure 2.7 l(c) results. Therefore

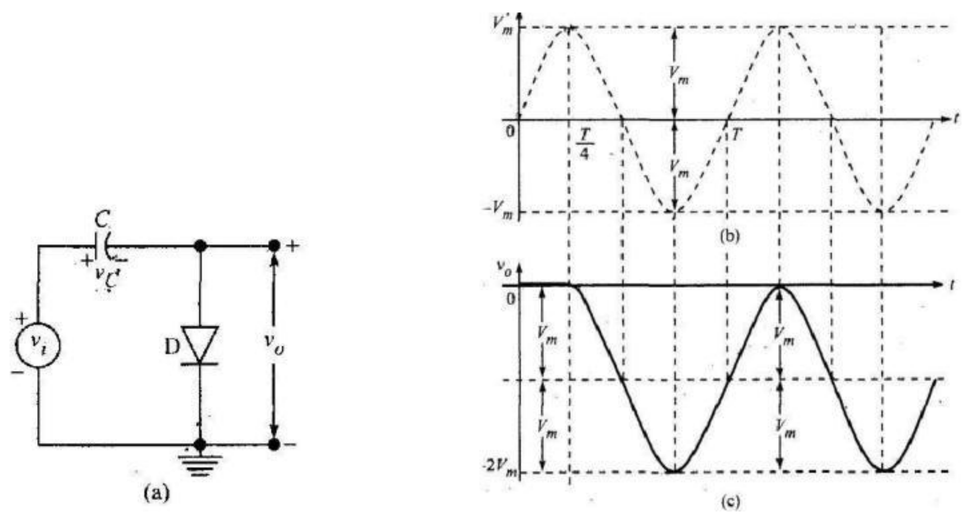


Figure 2.71 (a) A negative clamping circuit, (b) a sinusoidal input, and (c) a steady-state clamped output.

Suppose that after the steady-state condition has been reached, the amplitude of the input signal is increased, then the diode will again conduct for at most one quarter cycle and the dc voltage across the capacitor would rise to the new peak value, and the positive excursions of the signal would be again restored to zero.

Suppose the amplitude of the input signal is decreased after the steady-state condition has been reached. There is no path for the capacitor to discharge. To permit the voltage across the capacitor to decrease, it is necessary to shunt a resistor across *C,* or equivalently to shunt a resistor across D. In the latter case, the capacitor will discharge through the series combination of the resistor R across the diode and the resistance of the source, and in a few cycles the positive extremity would be again clamped at zero as shown in Figure 2.72(b). A circuit with such a resistor *'R* is shown in Figure 2.72(a).

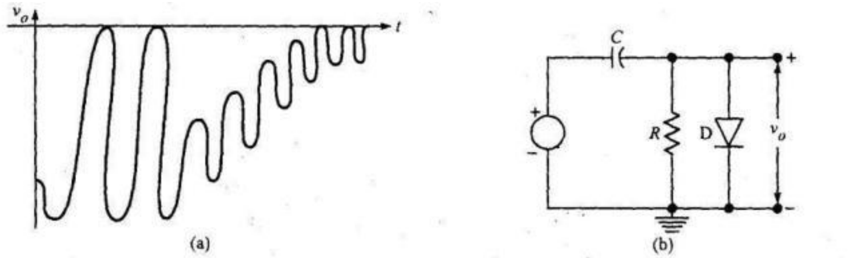


Figure 2.72 (a) Clamping circuit with a resistor R across the diode D and (b) output during transient period.

Positive Clamper

Figure 2.73(a) shows a positive clamper. This is also termed as negative peak clamper since this circuit clamps the negative peaks of a signal to zero level. The negative peak clamper, i.e. the positive clamper introduces a positive dc.

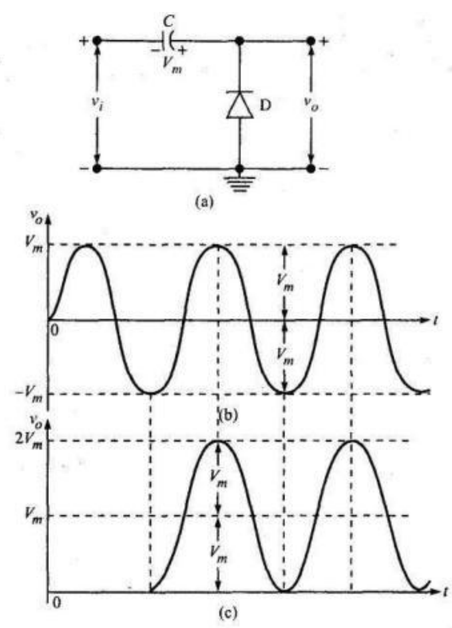


Figure 2.73 (a) A positive clamping circuit, (b) a sinusoidal input, and (c) a steady-state clamped output.

Let the input voltage be $v_i = V_m$ sin *(ot* as shown in Figure 2.73(b). When v, goes negative, the diode gets forward biased and conducts and in a few cycles the capacitor gets charged to V_m with the polarity shown in Figure 2.73(a). Under steady-state conditions, the capacitor acts as a constant voltage source and the output is $v_o = v_i - (-V_m) = v_i + V_m$.

Based on the above relation between v_0 and v,, the output voltage waveform is plotted. As seen in Figure 2.73(c) the negative peaks of the input signal are clamped to zero level. Peak-to-peak value of output voltage = peak-to-peak value of input voltage = $2V_m$. There is no distortion of waveform. To accommodate for variations in amplitude of input, the diode D is shunted with a resistor as shown in Figure 2.74(a). When the amplitude of the input waveform is reduced, the output will adjust to its new value as shown in Figure 2.74(b).

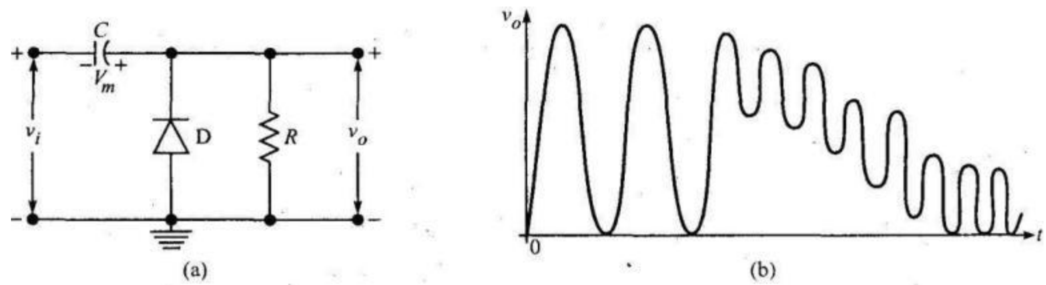


Figure 2.74 (a) Clamping circuit with a resistor R across D and (b) output during transient period.

Biased Clamping

If a voltage source of V_R volts is connected in series with the diode of a clamping circuit, the input waveform will be clamped with reference to V_R. Depending on the position of the diode, the input waveform may be positively clamped with reference to V_R, or negatively clamped with reference to V_R.

Clamping Circuit Taking Source and Diode Resistances Into Account

In the discussion of the clamping circuit of Figure 2.71, we neglected the output resistance of the source as well as the diode forward resistance. Many times these resistances cannot be neglected. Figure 2.79 shows a more realistic clamping circuit taking into consideration the output resistance of the source R_s, which may be negligible or may range up to many thousands of ohms depending on the source, and the diode forward resistance R_f which may range from tens to hundreds of ohms. Assume that the diode break point V_γ occurs at zero voltage.

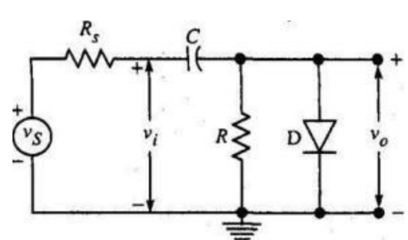


Figure 2.79 Clamping circuit considering the source resistance and the diode forward resistance.

The precision of operation of the circuit depends on the condition that $R \gg R_f$, and $R_r \gg R$. When the input is positive, the diode is ON and the equivalent circuit shown in Figure 2.80(a) results. When the input is negative, the diode is *OFF* and the equivalent circuit shown in Figure 2.80(b) results.

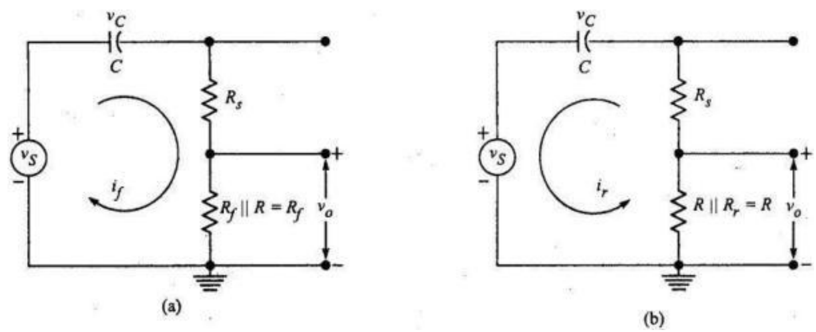


Figure 2.80 (a) Equivalent circuit when the diode is conducting and (b) the equivalent circuit when the diode is not conducting.

The transient waveform

When a signal is suddenly applied to the circuit shown in Figure 2.79 the capacitor charges (transient period) and gradually the steady-state condition is reached in which the positive peaks will be clamped to zero. The equivalent circuits shown in Figures 2.80(a) and 2.80(b) may be used to calculate the transient response.

Relation between tilts in forward and reverse directions

The steady-state output waveform for a square wave input. Consider that the square wave input shown in Figure 2.82(a) is applied to the clamping circuit shown in Figure 2.79. The general form of the output waveform would be as shown in Figure 2.82(b), extending in both positive and negative directions and is determined by the voltages V_1, V_2, V_1^1, and V'_2. These voltages may be calculated as discussed below.

In the interval $0 < t < T$, the input is at its higher level; so the diode is ON and the capacitor charges with a time constant $(R_S + R_f)C$, and the output decays towards zero with the same time constant. Hence, $V'_1 = V_1 e^{-T_1/(R_f+R_S)C}$ ---------- (i)

In the interval $T_1 < t < T_1 + T_2$, the input is at its lower level; so the diode is OFF and the capacitor discharges with a time constant $(R + R_S)C$, and the output rises towards zero with the same time constant. Hence $V'_2 = V_2 e^{-T_2/(R+R_S)C}$ ---------(ii)

Considering the conditions at t= 0. At $t = 0$~, $v_S = V''$, $v_O = V_2$, the diode D is OFF and the equivalent circuit of Figure 2.80(b) results. The voltage across the capacitor is given by

$$v_C = V'' - \frac{V_2'}{R}(R + R_S) \quad \text{-----------(iii)}$$

At $t = 0^+$, the input signal jumps to V, the output jumps to V_t, the diode conducts and the equivalent circuit of Figure 2.80(a) results. The voltage across the capacitor is given by

$$v_C = V' - \frac{V_1}{R_f}(R_f + R_S) \quad \text{------------(iv)}$$

Since the voltage across the capacitor cannot change instantaneously, equating equations (iii) and (iv), we have

$$V' - \frac{V_1(R_f + R_S)}{R_f} = V'' - \frac{V_2'(R + R_S)}{R} \quad \text{------------(v)}$$

$$V' - V'' = V = \frac{V_1(R_f + R_S)}{R_f} - \frac{V_2'(R + R_S)}{R}$$

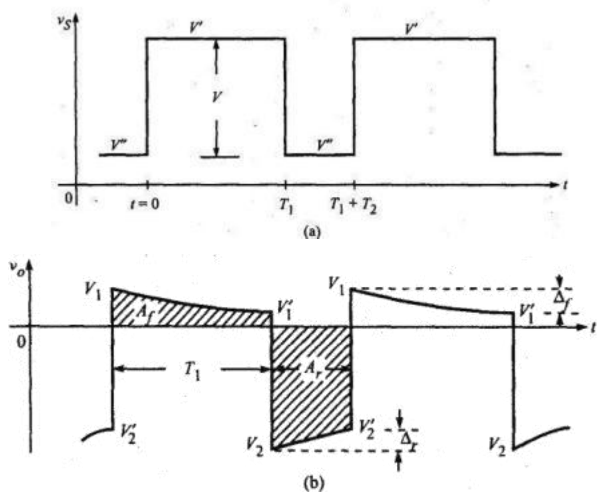


Figure 2.82 (a) A square wave input signal of peak-to-peak amplitude *V,* (b) the general form of the steady-state output of a clamping circuit with;' the input as in (a).

***Considering the conditions at t=* T_r.** At $t = T_r$, $v_S = V$, $v_0 = V_1$, the diode D is ON, and the equivalent circuit of Figure 2,80(a) results. The voltage across the capacitor is given by

$$v_C = V' - \frac{V_1'}{R_f}(R_f + R_S) \quad \text{-------------(vi)}$$

At $t = T_r$, , $v_S = V'' = v_0 = V_2$, the diode D is OFF, and the equivalent circuit of Figure 2.80(b) results.

The voltage across the capacitor is given by

$$v_C = V'' - \frac{V_2}{R}(R + R_S) \quad \text{------------(vii)}$$

Since the voltage across the capacitor cannot change instantaneously, equating equations (vi) and (vii), we get

$$V' - \frac{V_1'(R_f + R_S)}{R_f} = V'' - \frac{V_2(R + R_S)}{R}$$

$$V' - V'' = V = \frac{V_1'(R_f + R_S)}{R_f} - \frac{V_2(R + R_S)}{R} \quad \text{-----------(viii)}$$

From equations (i), (ii), (v) and (viii), the values V_1, V', V_2 and V_2' can be computed and the output waveform determined.

If the source impedance is taken into account, the output voltage jumps are smaller than the abrupt discontinuity V in the input. Only if $R_s = 0$, are the jumps in input and output voltages equal. Thus, when $R_s = 0$, $V_1 - V_2' = V_1' - V_2 = V$ Observe that the response is independent of the absolute levels V' and V'' of the input signal and is determined only by the amplitude V. It is possible, for example, for V'' to be negative or even for both V and V'' to be negative.
The average level of the input plays no role in determining the steady-state output waveform.

Under steady-state conditions, there is a tilt in the output waveform in both positive and negative directions. The relation between the tilts can be obtained by subtracting Eq. (viii) from Eq. (v), i.e.

$$\frac{R_f + R_S}{R_f}(V_1 - V_1') - \frac{R + R_S}{R}(V_2' - V_2) = 0$$

Where,

$V_1 - V_1' = \Delta_f$ = tilt in the forward direction
$V_2' - V_2 = \Delta_r$ = tilt in the reverse direction

$$\therefore \quad \Delta_f = \frac{R_f}{R_f + R_S} \times \frac{R + R_S}{R} \Delta_r$$

Since R_s is usually much smaller than R, then, the tilt in the forward direction Ay is almost always less than the tilt A_r in the reverse direction. Only when R_s « R_f, are the two tilts almost equal.

Clamping Circuit Theorem

Under steady-state conditions, for any input waveform, the shape of the output waveform of a clamping circuit is fixed and also the area in the forward direction (when the diode conducts) and the area in the reverse direction (when the diode does not conduct) are related.

The clamping circuit theorem states that, for any input waveform under steady-state conditions, the ratio of the area *Af* ***under the output voltage curve in the forward direction to that in the reverse direction*** A_r ***is equal to the ratio R//R-***

This theorem applies quite generally independent of the input waveform and the magnitude of the source resistance. The proof is as follows:

Consider the clamping circuit of Figure 2.79, the equivalent circuits in Figures 2.80(a) and 2.80(b), and the input and output waveforms of Figures 2.82(a) and 2.82(b) respectively.

In the interval $0 < t < T_1$ the input is at its upper level, the diode is ON, and the equivalent circuit of Figure 2.80(a) results. If $v_f(t)$ is the output waveform in the forward direction, then the capacitor charging current is $i_f(t) = \frac{v_f(t)}{R_f}$ Therefore, -the charge gained by the capacitor during the forward interval is

$$Q_g = \int_0^{T_1} i_f(t)dt = \frac{1}{R_f}\int_0^{T_1} v_f(t)dt = \frac{A_f}{R_f}$$

In the interval $T_1 < t < T_1 + T_2$, the input is at its lower level, the diode is OFF, and the equivalent circuit of Figure 2.80(b) results. If $v_r(t)$ is the output voltage in the reverse direction, then the current which discharges the capacitor is $i_r(t) = \frac{v_r(t)}{R}$

Therefore, the charge lost by the capacitor during the reverse interval is

$$Q_l = \int_{T_1}^{T_1+T_2} i_r(t)dt = \frac{1}{R}\int_{T_1}^{T_1+T_2} v_r(t)dt = \frac{A_r}{R}$$

Under steady-state conditions, the net charge acquired by the capacitor over one cycle must be equal to zero. Therefore, the charge gained in the interval $0 < t < T_1$, will be equal to the charge lost in the interval $T_1 < t < T_1 + T_2$, i.e. $Q_g = Q_l$

$$\frac{A_f}{R_f} = \frac{A_r}{R} \quad \text{i.e.} \quad \frac{A_f}{A_r} = \frac{R_f}{R}$$