ANNAMACHARYA INSTITUTE OF TECHNOLOGY AND SCIENCES:: RAJAMPET
(An Autonomous Institution)

DEPARTMENT OF MECHANICAL ENGINEERING

LECTURE NOTES

DESIGN OF MACHINE MEMBERS
[23A0362T]

Prepared by
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(An Autonomous Institution)

Title of the Course:	Design of Machine Members	
Category:	Professional Core	
Couse Code:	23A0362T	
Branch/es:	Mechanical Engineering	
Semester:	III-II	

Lecture Hours	Tutorial Hours	Practice Hours	Credits
3	0	0	3

Course Objectives: The objectives of the course are to

1. Understanding of the mechanical engineering design process for static and dynamic loads under various loading conditions.
2. Explore the design principles and analysis of bolted and welded joints and butt welds under various loading conditions.
3. Study the design principles of power transmission shafts and couplings, focusing on the analysis of shafts subjected loads, as well as the design of various types of couplings.
4. Explain the design principles of friction clutches, brakes, and springs, design of different brakes, clutches, helical and leaf springs under various loading conditions.
5. Demonstrate the design principles of sliding and rolling contact bearings, gears, spur gears, beam strength, and load considerations.

Course Outcomes:
At the end of the course, the student will be able to
1. Apply design principles for components subjected to static and dynamic loads, analyze and design for fatigue failure using relevant criteria
2. Design and analyze bolted and welded joints, considering factors such as different types of loads, including eccentric loading scenarios.
3. Design power transmission shafts and couplings for fluctuating loads, and selecting appropriate couplings such as flange, bushed pin, and universal couplings.
4. Design friction clutches, brakes, and springs, applying the various theories and analyze the working for mechanical applications.
5. Design and analyze the sliding and rolling contact bearings, spur gears, considering beam strength, dynamic, and wear load factors.

Unit 1	**Introduction, Design for Static and Dynamic loads**	**10 Hours**

Mechanical Engineering Design: Design process, design considerations, codes and standards of designation of materials, selection of materials.
Design for Static Loads: Modes of failure, design of components subjected to axial, bending, torsional and impact loads. Theories of failure for static loads.
Design for Dynamic Loads: Endurance limit, fatigue strength under axial, bending and torsion, stress concentration, notch sensitivity. Types of fluctuating loads, fatigue design for infinite life. Soderberg, Goodman and modified Goodman criterion for fatigue failure. Fatigue design under combined stresses.

Unit 2	**Design of Bolted and Welded Joints**	**09 Hours**

Design of Bolted Joints: Threaded fasteners, preload of bolts, various stresses induced in the bolts. Torque requirement for bolt tightening, gasketed joints and eccentrically loaded bolted joints.
Welded Joints: Strength of lap and butt welds, Joints subjected to bending and torsion. Eccentrically loaded welded joints.

Unit 3	Power transmission shafts and Couplings	10 Hours
Power Transmission Shafts: Design of shafts subjected to bending, torsion and axial loading. Shafts subjected to fluctuating loads using shock factors. **Couplings:** Design of flange and bushed pin couplings, universal coupling.		

Unit 4	Design of Clutches, Brakes and Springs	10 Hours
Friction Clutches: Torque transmitting capacity of disc and centrifugal clutches. Uniform wear theory and uniform pressure theory. **Brakes:** Different types of brakes. Concept of self-energizing and self-locking of brake. Band and block brakes, disc brakes. **Springs:** Design of helical compression, tension, torsion and leaf springs.		

Unit 5	Design of Bearings and Gears	10 Hours
Design of Sliding Contact Bearings: Lubrication modes, bearing modulus, McKee's equations, design of journal bearing. Bearing Failures. **Design of Rolling Contact Bearings:** Static and dynamic load capacity, Stribeck's Equation, equivalent bearing load, load-life relationships, load factor, selection of bearings from manufacturer's catalogue. **Design of Gears:** Spur gears, beam strength, Lewis equation, design for dynamic and wear loads.		

Textbooks:
1. Design of Machine Elements, V.B.Bhandari, 3/e, Tata McGraw Hill, 2010.
2. Machine Design an Integrated approach, R.L. Norton, 2/e, Pearson Education, 2004.

Reference Books:
1. Machine Design, R.K. Jain, Khanna Publications, 1978.
2. Mechanical Engineering Design, J.E. Shigley, 2/e, Tata McGraw Hill, 1986.
3. Design data handbook, K. Mahadevan &K.Balaveera Reddy, CBS Publications, 4/e, 2018.
4. Machine design, Dr. N. C. Pandya &Dr. C. S. Shah, 17/e, Charotar Publishing House Pvt. Ltd, 2009.

CO-PO Mapping:

Course Outcomes	Engineering Knowledge	Problem Analysis	Design/Development of solutions	Conduct investigations of complex problems	Modern tool usage	The engineer and society	Environment and sustainability	Ethics	Individual and team work	Communication	Project management and finance	Life-long learning	PSO1	PSO2
23A0362T1	3	2	2	2	-	-	-	-	-	-	-	-	2	3
23A0362T2	3	3	2	3	-	-	-	-	-	-	-	-	2	2
23A0362T3	3	3	3	3	-	-	-	-	-	-	-	-	2	2
23A0362T4	3	3	3	3	-	-	-	-	-	-	-	-	3	2
23A0362T5	3	3	3	3	-	-	-	-	-	-	-	-	2	1

Introduction

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

Classifications of Machine Design

The machine design may be classified as follows:

1. ***Adaptive design.*** In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.

2. ***Development design.*** This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. ***New design.*** This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design. The designs, depending upon the methods used, may be classified as follows:

(*a*) ***Rational design.*** This type of design depends upon mathematical formulae of principle of mechanics.

(*b*) ***Empirical design.*** This type of design depends upon empirical formulae based on the practice and past experience.

(*c*) ***Industrial design.*** This type of design depends upon the production aspects to manufacture any machine component in the industry.

(*d*) ***Optimum design.*** It is the best design for the given objective function under the specified constraints. It may be achieved by minimising the undesirable effects.

(*e*) ***System design.*** It is the design of any complex mechanical system like a motor car.

(*f*) ***Element design.*** It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.

(*g*) ***Computer aided design.*** This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimisation of a design.

General Considerations in Machine Design

Following are the general considerations in designing a machine component:

1. ***Type of load and stresses caused by the load.*** The load, on a machine component, may act in several ways due to which the internal stresses are set up. The various types of load and stresses are discussed later.

2. ***Motion of the parts or kinematics of the machine.*** The successful operation of any machine depends largely upon the simplest arrangement of the parts which will give the motion required.

The motion of the parts may be:

(*a*) Rectilinear motion which includes unidirectional and reciprocating motions.

(*b*) Curvilinear motion which includes rotary, oscillatory and simple harmonic.

(*c*) Constant velocity.

(*d*) Constant or variable acceleration.

3. ***Selection of materials.*** It is essential that a designer should have a thorough knowledge of the properties of the materials and their behaviour under working conditions. Some of the important characteristics of materials are: strength, durability, flexibility, weight, resistance to heat and corrosion, ability to cast, welded or hardened, machinability, electrical conductivity, etc. The various types of engineering materials and their properties are discussed later.

4. ***Form and size of the parts.*** The form and size are based on judgment. The smallest practicable cross-section may be used, but it may be checked that the stresses induced in the designed cross-section are reasonably safe. In order to design any machine part for form and size, it is necessary to know the forces which the part must sustain. It is also important to anticipate any suddenly applied or impact load which may cause failure.

5. ***Frictional resistance and lubrication.*** There is always a loss of power due to frictional resistance and it should be noted that the friction of starting is higher than that of running friction. It is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.

6. ***Convenient and economical features.*** In designing, the operating features of the machine should be carefully studied. The starting, controlling and stopping levers should be located on the basis of convenient handling. The adjustment for wear must be provided employing the various take up devices and arranging them so that the alignment of parts is preserved. If parts are to be changed for different products or replaced on account of wear or breakage, easy access should be provided and the necessity of removing other parts to accomplish this should be avoided if possible. The economical operation of a machine which is to be used for production or for the processing of material should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

7. ***Use of standard parts.*** The use of standard parts is closely related to cost, because the cost of standard or stock parts is only a fraction of the cost of similar parts made to order. The standard or stock parts should be used whenever possible; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts and pins. Bolts and studs should be as few as possible to avoid the delay caused by changing drills, reamers and taps and also to decrease the number of wrenches required.

8. ***Safety of operation.*** Some machines are dangerous to operate, especially those which are speeded up to insure production at a maximum rate. Therefore, any moving part of a machine which is within the zone of a worker is considered an accident hazard and may be the cause of an injury. It is, therefore, necessary that a designer should always provide safety devices for the safety of the operator. The safety appliances should in no way interfere with operation of the machine.

9. ***Workshop facilities.*** A design engineer should be familiar with the limitations of this employer's workshop, in order to avoid the necessity of having work done in some other workshop. It is sometimes necessary to plan and supervise the workshop operations and to draft methods for casting, handling and machining special parts.

10. ***Number of machines to be manufactured.*** The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured. If only a few articles are to be made, extra expenses are not justified unless the machine is large or of some special design. An order calling for small number of the product will not permit any undue expense in the workshop processes, so that the designer should restrict his specification to standard parts as much as possible.

11. ***Cost of construction.*** The cost of construction of an article is the most important consideration involved in design. In some cases, it is quite possible that the high cost of an article may immediately bar it from further considerations. If an article has been invented and tests of handmade samples have shown that it has commercial value, it is then possible to justify the expenditure of a considerable sum of money in the design and development of automatic machines to produce the article, especially if it can be sold in large numbers. The aim of design engineer under all conditions should be to reduce the manufacturing cost to the minimum.

12. ***Assembling.*** Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested and then taken to be transported to their place of service. The final location of any machine is important and the design engineer must anticipate the exact location and the local facilities for erection.

General Procedure in Machine Design

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows:

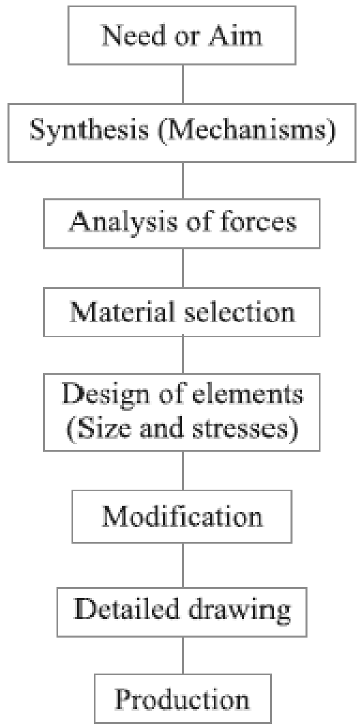


Fig.1. General Machine Design Procedure

1. ***Recognition of need.*** First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

2. ***Synthesis*** **(*Mechanisms*).** Select the possible mechanism or group of mechanisms which will give the desired motion.

3. ***Analysis of forces.*** Find the forces acting on each member of the machine and the energy transmitted by each member.

4. ***Material selection.*** Select the material best suited for each member of the machine.

5. ***Design of elements*** **(*Size and Stresses*).** Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than the permissible limit.

6. *Modification.* Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

7. *Detailed drawing.* Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

8. *Production.* The component, as per the drawing, is manufactured in the workshop.
The flow chart for the general procedure in machine design is shown in Fig.

Note: When there are number of components in the market having the same qualities of efficiency, durability and cost, then the customer will naturally attract towards the most appealing product. The aesthetic and ergonomics are very important features which gives grace and lustre to product and dominates the market.

Engineering materials and their properties

The knowledge of materials and their properties is of great significance for a design engineer. The machine elements should be made of such a material which has properties suitable for the conditions of operation. In addition to this, a design engineer must be familiar with the effects which the manufacturing processes and heat treatment have on the properties of the materials. Now, we shall discuss the commonly used engineering materials and their properties in Machine Design.

Classification of Engineering Materials

The engineering materials are mainly classified as:
1. Metals and their alloys, such as iron, steel, copper, aluminum, etc.
2. Non-metals, such as glass, rubber, plastic, etc.
The metals may be further classified as:
(a) Ferrous metals and (*b*) Non-ferrous metals.
The ****ferrous metals*** are those which have the iron as their main constituent, such as cast iron, wrought iron and steel.

The ***non-ferrous*** metals are those which have a metal other than iron as their main constituent, such as copper, aluminum, brass, tin, zinc, etc.

Selection of Materials for Engineering Purposes

The selection of a proper material, for engineering purposes, is one of the most difficult problems for the designer. The best material is one which serves the desired objective at the minimum cost. The following factors should be considered while selecting the material:

1. Availability of the materials,

2. Suitability of the materials for the working conditions in service, and

3. The cost of the materials.

The important properties, which determine the utility of the material, are physical, chemical and mechanical properties. We shall now discuss the physical and mechanical properties of the material in the following articles.

Physical Properties of Metals

The physical properties of the metals include luster, colour, size and shape, density, electric and thermal conductivity, and melting point. The following table shows the important physical properties of some pure metals.

Mechanical Properties of Metals

The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness. We shall now discuss these properties as follows:

1. Strength. It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called stress.

2. Stiffness. It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.

3. Elasticity. It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.

4. Plasticity. It is property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

5. Ductility. It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminium, nickel, zinc, tin and lead.

6. Brittleness. It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads snap off without giving any sensible elongation. Cast iron is a brittle material.

7. Malleability. It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminium.

8. Toughness. It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed upto the point of fracture. This property is desirable in parts subjected to shock and impact loads.

9. Machinability. It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material. It may be noted that brass can be easily machined than steel.

10. Resilience. It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for spring materials.

11. Creep. When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called **creep.** This property is considered in designing internal combustion engines, boilers and turbines.

12. Fatigue. When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as ***fatigue**. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.

13. Hardness. It is a very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal. The hardness is usually expressed in numbers which are dependent on the method of making the test. The hardness of a metal may be determined by the following tests:

(a) Brinell hardness test,

(b) Rockwell hardness test,

(c) Vickers hardness (also called Diamond Pyramid) test, and

(d) Shore scleroscope.

Steel

It is an alloy of iron and carbon, with carbon content up to a maximum of 1.5%. The carbon occurs in the form of iron carbide, because of its ability to increase the hardness and strength of the steel. Other elements *e.g.* silicon, sulphur, phosphorus and manganese are also present to greater or lesser amount to impart certain desired properties to it. Most of the steel produced now-a-days is ***plain carbon steel*** or simply ***carbon steel.*** A carbon steel is defined as a steel which has its properties mainly due to its carbon content and does not contain more than 0.5% of silicon and 1.5% of manganese.

The plain carbon steels varying from 0.06% carbon to 1.5% carbon are divided into the following types depending upon the carbon content.

1. Dead mild steel — up to 0.15% carbon

2. Low carbon or mild steel — 0.15% to 0.45% carbon

3. Medium carbon steel — 0.45% to 0.8% carbon

4. High carbon steel — 0.8% to 1.5% carbon

According to Indian standard *[IS: 1762 (Part-I)–1974], a new system of designating the steel is recommended. According to this standard, steels are designated on the following two basis: (*a*) On the basis of mechanical properties, and (*b*) On the basis of chemical composition. We shall now discuss, in detail, the designation of steel on the above two basis, in the following pages.

Steels Designated on the Basis of Mechanical Properties

These steels are carbon and low alloy steels where the main criterion in the selection and inspection of steel is the tensile strength or yield stress. According to Indian standard IS: 1570 (Part–I)- 1978 (Reaffirmed 1993), these steels are designated by a symbol 'Fe' or 'Fe E' depending on whether the steel has been specified on the basis of minimum tensile strength or yield strength, followed by the figure indicating the minimum tensile strength or yield stress in N/mm2. For example 'Fe 290' means a steel having minimum tensile strength of 290 N/mm2 and 'Fe E 220' means a steel having yield strength of 220 N/mm2.

Steels Designated on the Basis of Chemical Composition

According to Indian standard, IS : 1570 (Part II/Sec I)-1979 (Reaffirmed 1991), the carbon steels are designated in the following order :

(*a*) Figure indicating 100 times the average percentage of carbon content,

(*b*) Letter 'C', and

(*c*) Figure indicating 10 times the average percentage of manganese content. The figure after multiplying shall be rounded off to the nearest integer.

For example 20C8 means a carbon steel containing 0.15 to 0.25 per cent (0.2 per cent on average) carbon and 0.60 to 0.90 per cent (0.75 per cent rounded off to 0.8 per cent on an average) manganese.

Effect of Impurities on Steel

The following are the effects of impurities like silicon, sulphur, manganese and phosphorus on steel.

1. *Silicon.* The amount of silicon in the finished steel usually ranges from 0.05 to 0.30%. Silicon is added in low carbon steels to prevent them from becoming porous. It removes the gases and oxides, prevent blow holes and thereby makes the steel tougher and harder.

2. *Sulphur.* It occurs in steel either as iron sulphide or manganese sulphide. Iron sulphide because of its low melting point produces red shortness, whereas manganese sulphide does not affect so much. Therefore, manganese sulphide is less objectionable in steel than iron sulphide.

3. *Manganese.* It serves as a valuable deoxidising and purifying agent in steel. Manganese also combines with sulphur and thereby decreases the harmful effect of this element remaining in the steel. When used in ordinary low carbon steels, manganese makes the metal ductile and of good bending qualities. In high speed steels, it is used to toughen the metal and to increase its critical temperature.

4. *Phosphorus.* It makes the steel brittle. It also produces cold shortness in steel. In low carbon steels, it raises the yield point and improves the resistance to atmospheric corrosion. The sum of carbon and phosphorus usually does not exceed 0.25%.

Manufacturing considerations in Machine design

Manufacturing Processes

The knowledge of manufacturing processes is of great importance for a design engineer. The following are the various manufacturing processes used in Mechanical Engineering.

1. ***Primary shaping processes.*** The processes used for the preliminary shaping of the machine component are known as primary shaping processes. The common operations used for this process are casting, forging, extruding, rolling, drawing, bending, shearing, spinning, powder metal forming, squeezing, etc.

2. ***Machining processes.*** The processes used for giving final shape to the machine component, according to planned dimensions are known as machining processes. The common operations used for this process are turning, planning, shaping, drilling, boring, reaming, sawing, broaching, milling, grinding, hobbing, etc.

3. ***Surface finishing processes.*** The processes used to provide a good surface finish for the machine component are known as surface finishing processes. The common operations used for this process are polishing, buffing, honing, lapping, abrasive belt grinding, barrel tumbling, electroplating, super finishing, sheradizing, etc.

4. ***Joining processes.*** The processes used for joining machine components are known as joining processes. The common operations used for this process are welding, riveting, soldering, brazing, screw fastening, pressing, sintering, etc.

5. ***Processes effecting change in properties.*** These processes are used to impart certain specific properties to the machine components so as to make them suitable for particular operations or uses. Such processes are heat treatment, hot-working, cold-working and shot peening.

Other considerations in Machine design

1. Workshop facilities.
2. Number of machines to be manufactured
3. Cost of construction

Stress

When some external system of forces or loads acts on a body, the internal forces (equal and opposite) are set up at various sections of the body, which resist the external forces. This internal force per unit area at any section of the body is known as ***unit stress*** or simply a ***stress***. It is denoted by a Greek letter sigma (σ). Mathematically,

$$\text{Stress, } \sigma = P/A$$

Where P = Force or load acting on a body, and

A = Cross-sectional area of the body.

In S.I. units, the stress is usually expressed in Pascal (Pa) such that 1 Pa = 1 N/m^2. In actual practice, we use bigger units of stress *i.e.* megapascal (MPa) and gigapascal (GPa), such that

$$1 \text{ MPa} = 1 \times 10^6 \text{ N/m}^2 = 1 \text{ N/mm}^2$$

And $$1 \text{ GPa} = 1 \times 10^9 \text{ N/m}^2 = 1 \text{ kN/mm}^2$$

Strain

When a system of forces or loads act on a body, it undergoes some deformation. This deformation per unit length is known as ***unit strain*** or simply a ***strain.*** It is denoted by a Greek letter epsilon (ε). Mathematically,

$$\text{Strain, } \varepsilon = \delta l / l \quad \text{or} \quad \delta l = \varepsilon . l$$

Where δl = Change in length of the body, and

l= Original length of the body.

Tensile Stress and Strain

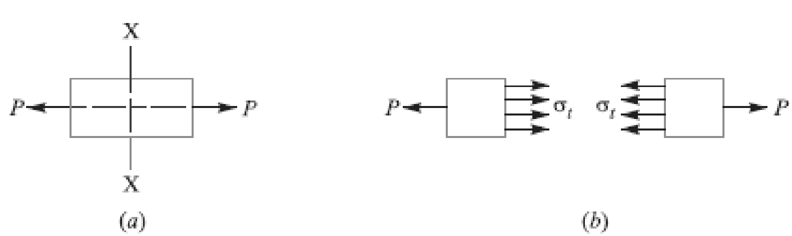


Fig. Tensile stress and strain

When a body is subjected to two equal and opposite axial pulls P (also called tensile load) as shown in Fig. (*a*), then the stress induced at any section of the body is known as ***tensile stress*** as shown in Fig. (*b*). A little consideration will show that due to the tensile load, there will be a decrease in cross-sectional area and an increase in length of the body. The ratio of the increase in length to the original length is known as ***tensile strain.***

Let P = Axial tensile force acting on the body,

A = Cross-sectional area of the body,

l = Original length, and

δl = Increase in length.

Then Tensile stress, $\sigma_t = P/A$

and tensile strain, $\varepsilon_t = \delta l / l$

Young's Modulus or Modulus of Elasticity

Hooke's law* states that when a material is loaded within elastic limit, the stress is directly proportional to strain, *i.e.*

$$\sigma \propto \varepsilon \qquad \text{or} \qquad \sigma = E.\varepsilon$$

$$E = \frac{\sigma}{\varepsilon} = \frac{P \times l}{A \times \delta l}$$

where E is a constant of proportionality known as ***Young's modulus*** or ***modulus of elasticity.*** In S.I. units, it is usually expressed in GPa *i.e.* GN/m^2 or kN/mm^2. It may be noted that Hooke's law holds good for tension as well as compression.

The following table shows the values of modulus of elasticity or Young's modulus (E) for the materials commonly used in engineering practice.

Values of E for the commonly used engineering materials.

Material	*Modulus of elasticity (E) in GPai.e. GN/m^2 for kN/mm^2*
Steel and Nickel	200 to 220
Wrought iron	190 to 200
Cast iron	100 to 160
Copper	90 to 110
Brass	80 to 90
Aluminium	60 to 80
Timber	10

Shear Stress and Strain

When a body is subjected to two equal and opposite forces acting tangentially across the resisting section, as a result of which the body tends to shear off the section, then the stress induced is called ***shear stress.***

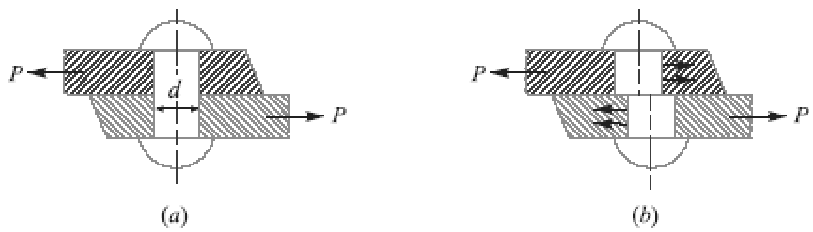


Fig. Single shearing of a riveted joint.

The corresponding strain is known as ***shear strain*** and it is measured by the angular deformation accompanying the shear stress. The shear stress and shear strain are denoted by the Greek letters tau (τ) and phi (φ) respectively. Mathematically,

$$\text{Shear stress, } \tau = \frac{\text{Tangential force}}{\text{Resisting area}}$$

Consider a body consisting of two plates connected by a rivet as shown in Fig. (*a*). In this case, the tangential force *P* tends to shear off the rivet at one cross-section as shown in Fig. (*b*). It may be noted that when the tangential force is resisted by one cross-section of the rivet (or when shearing takes place at one cross-section of the rivet), then the rivets are said to be in ***single shear.*** In such a case, the area resisting the shear off the rivet,

$$A = \frac{\pi}{4} \times d^2$$

And shear stress on the rivet cross-section

$$\tau = \frac{P}{A} = \frac{P}{\frac{\pi}{4} \times d^2} = \frac{4P}{\pi d^2}$$

Now let us consider two plates connected by the two cover plates as shown in Fig. *(a).* In this case, the tangential force *P* tends to shear off the rivet at two cross-sections as shown in Fig. (b). It may be noted that when the tangential force is resisted by two cross-sections of the rivet (or when the shearing takes place at Two cross-sections of the rivet), then the rivets are said to be in *double shear.* In such a case, the area resisting the shear off the rivet,

$$A = 2 \times \frac{\pi}{4} \times d^2 \qquad \text{(For double shear)}$$

and shear stress on the rivet cross-section.

$$\tau = \frac{P}{A} = \frac{P}{2 \times \frac{\pi}{4} \times d^2} = \frac{2P}{\pi d^2}$$

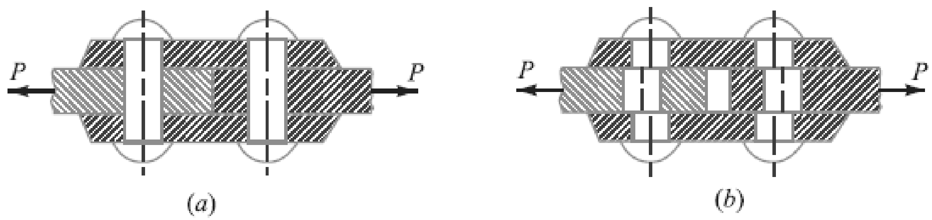


Fig. Double shearing of a riveted joint.

Notes:

1. All lap joints and single cover butt joints are in single shear, while the butt joints with double cover plates are in double shear.

2. In case of shear, the area involved is parallel to the external force applied.

3. When the holes are to be punched or drilled in the metal plates, then the tools used to perform the operations must overcome the ultimate shearing resistance of the material to be cut. If a hole of diameter '*d*' is to be punched in a metal plate of thickness '*t*', then the area to be sheared,

$$A = \pi d \times t$$

And the maximum shear resistance of the tool or the force required to punch a hole,

$$P = A \times \tau_u = \pi\, d \times t \times \tau_u$$

Where σ_u= Ultimate shear strength of the material of the plate.

Shear Modulus or Modulus of Rigidity

It has been found experimentally that within the elastic limit, the shear stress is directly proportional to shear strain. Mathematically

$$\tau \propto \phi \quad \text{or} \quad \tau = C \,.\, \phi \quad \text{or} \quad \tau / \phi = C$$

Where τ= Shear stress,

φ= Shear strain, and

C = Constant of proportionality, known as shear modulus or modulus of rigidity. It is also denoted by *N* or *G*.

The following table shows the values of modulus of rigidity (*C*) for the materials in every day use:

Values of C for the commonly used materials

Material	*Modulus of rigidity (C) in GPa i.e. GN/m^2 or $kNmm^2$*
Steel	80 to 100
Wrought iron	80 to 90
Cast iron	40 to 50
Copper	30 to 50
Brass	30 to 50
Timber	10

Linear and Lateral Strain

Consider a circular bar of diameter *d* and length *l*, subjected to a tensile force *P* as shown in Fig. (*a*).

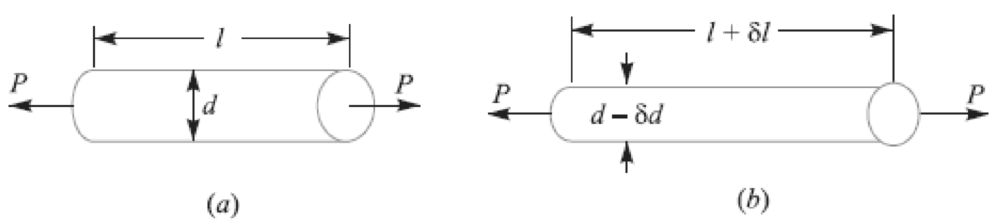


Fig. Linear and lateral strain.

A little consideration will show that due to tensile force, the length of the bar increases by an amount δl and the diameter decreases by an amount δd, as shown in Fig. (*b*). similarly, if the bar is subjected to a compressive force, the length of bar will decrease which will be followed by increase in diameter.

It is thus obvious, that every direct stress is accompanied by a strain in its own direction which is known as ***linear strain*** and an opposite kind of strain in every direction, at right angles to it, is known as ***lateral strain.***

4.18 Poisson's Ratio

It has been found experimentally that when a body is stressed within elastic limit, the lateral strain bears a constant ratio to the linear strain, Mathematically,

$$\frac{\text{LateralStrain}}{\text{LinearStrain}} = \text{Constant}$$

This constant is known as **Poisson's ratio** and is denoted by $1/m$ or μ.

Following are the values of Poisson's ratio for some of the materials commonly used in engineering practice.

Values of Poisson's ratio for commonly used materials

S.No.	*Material*	*Poisson 's ratio (1/m or μ)*
1	**Steel**	0.25 **to** 0.33
2	Cast iron	0.23 **to** 0.27
3	Copper	0.31 to 0.34
4	Brass	0.32 **to** 0.42
5	Aluminium	0.32 to 0.36
6	Concrete	0.08 to 0.18
7	Rubber	0.45 to 0.50

Volumetric Strain

When a body is subjected to a system of forces, it undergoes some changes in its dimensions. In other words, the volume of the body is changed. The ratio of the change in volume to the original volume is known as ***volumetric strain.*** Mathematically, volumetric strain,

$$\varepsilon_v = \delta V / V$$

Where δV = Change in volume, and V = Original volume

Notes : 1. Volumetric strain of a rectangular body subjected to an axial force is given as

$$\varepsilon_v = \frac{\delta V}{V} = \varepsilon\left(1 - \frac{2}{m}\right); \text{ where } \varepsilon = \text{Linear strain.}$$

2. Volumetric strain of a rectangular body subjected to three mutually perpendicular forces is given by

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

where ε_x, ε_y and ε_z are the strains in the directions x-axis, y-axis and z-axis respectively.

Bulk Modulus

When a body is subjected to three mutually perpendicular stresses, of equal intensity, then the ratio of the direct stress to the corresponding volumetric strain is known as ***bulk modulus.*** It is usually denoted by K. Mathematically, bulk modulus,

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\delta V / V}$$

Relation Between Bulk Modulus and Young's Modulus

The bulk modulus (K) and Young's modulus (E) are related by the following relation,

$$K = \frac{m.E}{3\,(m-2)} = \frac{E}{3\,(1-2\mu)}$$

Relation between Young's Modulus and Modulus of Rigidity

The Young's modulus (E) and modulus of rigidity (G) are related by the following relation,

$$G = \frac{m.E}{2\,(m+1)} = \frac{E}{2\,(1+\mu)}$$

<u>Factor of Safety</u>

It is defined, in general, as the **ratio of the maximum stress to the working stress.** Mathematically,

Factor of safety = Maximum stress/ Working or design stress

In case of ductile materials *e.g.* mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases,

Factor of safety = Yield point stress/ Working or design stress

In case of brittle materials *e.g.* cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress.

Factor of safety = Ultimate stress/ Working or design stress

This relation may also be used for ductile materials.

The above relations for factor of safety are for static loading.

Problem:

A steel bar 2.4 m long and 30 mm square is elongated by a load of 500 kN. If poisson's ratio is 0.25, find the increase in volume. Take E = 0.2 × 10^6 N/mm^2.

Solution. Given : l = 2.4 m = 2400 mm ; A = 30 × 30 = 900 mm^2 ; P = 500 kN = 500 × 10^3 N ; $1/m$ = 0.25 ; E = 0.2 × 10^6 N/mm^2

Let δV = Increase in volume.

We know that volume of the rod,

$$V = \text{Area} \times \text{length} = 900 \times 2400 = 2160 \times 10^3 \text{ mm}^3$$

and Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{P/A}{\varepsilon}$$

$$\therefore \quad \varepsilon = \frac{P}{A.E} = \frac{500 \times 10^3}{900 \times 0.2 \times 10^6} = 2.8 \times 10^{-3}$$

We know that volumetric strain,

$$\frac{\delta V}{V} = \varepsilon\left(1 - \frac{2}{m}\right) = 2.8 \times 10^{-3}\ (1 - 2 \times 0.25) = 1.4 \times 10^3$$

$$\therefore \quad \delta V = V \times 1.4 \times 10^{-3} = 2160 \times 10^3 \times 1.4 \times 10^{-3} = 3024 \text{ mm}^3 \textbf{ Ans.}$$

Stresses due to Change in Temperature—Thermal Stresses

Whenever there is some increase or decrease in the temperature of a body, it causes the body to expand or contract. A little consideration will show that if the body is allowed to expand or contract freely, with the rise or fall of the temperature, no stresses are induced in the body. But, if the deformation of the body is prevented, some stresses are induced in the body. Such stresses are known as ***thermal stresses.***

Let l = Original length of the body,

t = Rise or fall of temperature, and

α = Coefficient of thermal expansion,

δl Increase or decrease in length,

$$\delta l = l.\,\alpha.t$$

If the ends of the body are fixed to rigid supports, so that its expansion is prevented, then compressive strain induced in the body,

$$\varepsilon_c = \frac{\delta l}{l} = \frac{l.\alpha.t}{l} = \alpha.t$$

Thermal stress,

$$\sigma_{th} = \varepsilon_c.E = \alpha.t.E$$

1. When a body is composed of two or different materials having different coefficient of thermal expansions, then due to the rise in temperature, the material with higher coefficient of thermal expansion will be subjected to compressive stress whereas the material with low coefficient of expansion will be subjected to tensile stress.

2. When a thin tyre is shrunk on to a wheel of diameter D, its internal diameter d is a little less than the wheel diameter. When the type is heated, its circumference $\pi\, d$ will increase to $\pi\, D$. In this condition, it is slipped on to the wheel. When it cools, it wants to return to its original circumference $\pi\, d$, but the wheel if it is assumed to be rigid, prevents it from doing so.

$$\text{Strain, } \varepsilon = \frac{\pi D - \pi d}{\pi d} = \frac{D - d}{d}$$

This strain is known as ***circumferential*** or ***hoop strain.***

Therefore, Circumferential or hoop stress,

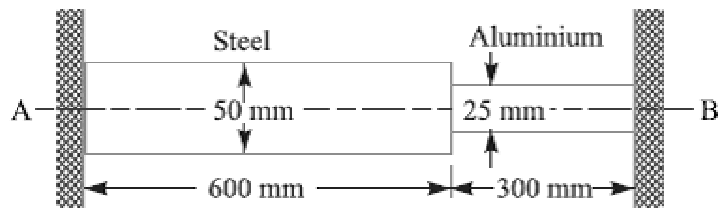
$$\sigma = E.\varepsilon = \frac{E\,(D - d)}{d}$$

Problem:

A composite bar made of aluminum and steel is held between the supports as shown in Fig. The bars are stress free at a temperature of 37°C. What will be the stress in the two bars when the temperature is 20°C, if (a) the supports are unyielding; and (b) the supports yield and come nearer to each other by 0.10 mm?

It can be assumed that the change of temperature is uniform all along the length of the bar.

Take Es = 210 GPa ; Ea = 74 GPa ; $\alpha_s = 11.7 \times 10^{-6}$ / °C ; and $\alpha_a = 23.4 \times 10^{-6}$ / °C.



Solution. Given : $t_1 = 37°C$; $t_2 = 20°C$; $E_s = 210$ GPa $= 210 \times 10^9$ N/m² ; $E_a = 74$ GPa $= 74 \times 10^9$ N/m² ; $\alpha_s = 11.7 \times 10^{-6}$/ °C ; $\alpha_a = 23.4 \times 10^{-6}$/ °C , $d_s = 50$ mm $= 0.05$ m ; $d_a = 25$ mm $= 0.025$ m ; $l_s = 600$ mm $= 0.6$ m ; $l_a = 300$ mm $= 0.3$ m

Let us assume that the right support at B is removed and the bar is allowed to contract freely due to the fall in temperature. We know that the fall in temperature,

$$t = t_1 - t_2 = 37 - 20 = 17°C$$

∴ Contraction in steel bar

$$= \alpha_s \,.\, l_s \,.\, t = 11.7 \times 10^{-6} \times 600 \times 17 = 0.12 \text{ mm}$$

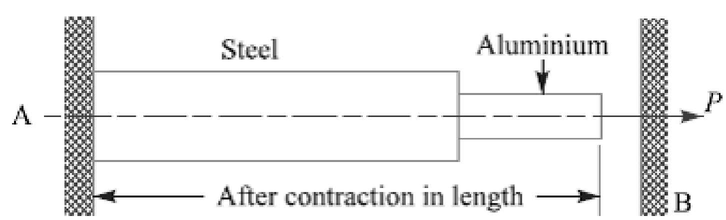
and contraction in aluminium bar

$$= \alpha_a \,.\, l_a \,.\, t = 23.4 \times 10^{-6} \times 300 \times 17 = 0.12 \text{ mm}$$

$$\text{Total contraction } = 0.12 + 0.12 = 0.24 \text{ mm} = 0.24 \times 10^{-3} \text{ m}$$

It may be noted that even after this contraction (*i.e.* 0.24 mm) in length, the bar is still stress free as the right hand end was assumed free.

Let an axial force P is applied to the right end till this end is brought in contact with the right hand support at B, as shown in Fig.



We know that cross-sectional area of the steel bar,

$$A_s = \frac{\pi}{4}(d_s)^2 = \frac{\pi}{4}(0.05)^2 = 1.964 \times 10^{-3} \text{ m}^2$$

and cross-sectional area of the aluminium bar,

$$A_a = \frac{\pi}{4}(d_a)^2 = \frac{\pi}{4}(0.025)^2 = 0.491 \times 10^{-3} \text{ m}^2$$

We know that elongation of the steel bar,

$$\delta l_s = \frac{P \times l_s}{A_s \times E_s} = \frac{P \times 0.6}{1.964 \times 10^{-3} \times 210 \times 10^9} = \frac{0.6P}{412.44 \times 10^6} \text{ m}$$

$$= 1.455 \times 10^{-9} P \text{ m}$$

and elongation of the aluminium bar,

$$\delta l_a = \frac{P \times l_a}{A_a \times E_a} = \frac{P \times 0.3}{0.491 \times 10^{-3} \times 74 \times 10^9} = \frac{0.3P}{36.334 \times 10^6} \text{ m}$$

$$= 8.257 \times 10^{-9} P \text{ m}$$

∴ Total elongation, $\delta l = \delta l_s + \delta l_a$

$$= 1.455 \times 10^{-9} P + 8.257 \times 10^{-9} P = 9.712 \times 10^{-9} P \text{ m}$$

Let σ_s = Stress in the steel bar, and

σ_a = Stress in the aluminium bar.

(*a*) ***When the supports are unyielding***

When the supports are unyielding, the total contraction is equated to the total elongation, *i.e.*

$$0.24 \times 10^{-3} = 9.712 \times 10^{-9} P \quad \text{or} \quad P = 24\,712 \text{ N}$$

∴ Stress in the steel bar,

$$\sigma_s = P/A_s = 24\,712 / (1.964 \times 10^{-3}) = 12\,582 \times 10^3 \text{ N/m}^2$$

$$= 12.582 \text{ MPa} \quad \textbf{Ans.}$$

and stress in the aluminium bar,

$$\sigma_a = P/A_a = 24\,712 / (0.491 \times 10^{-3}) = 50\,328 \times 10^3 \text{ N/m}^2$$

$$= 50.328 \text{ MPa} \quad \textbf{Ans.}$$

(*b*) ***When the supports yield by 0.1 mm***

When the supports yield and come nearer to each other by 0.10 mm, the net contraction in length

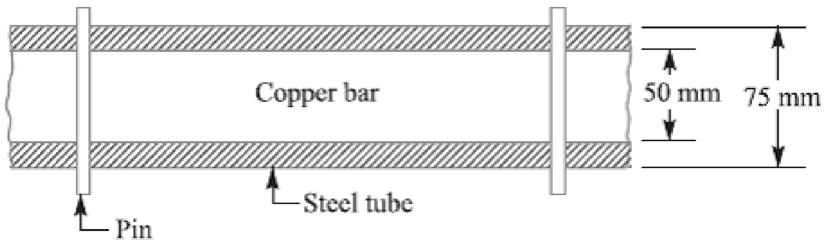
$$= 0.24 - 0.1 = 0.14 \text{ mm} = 0.14 \times 10^{-3} \text{ m}$$

Problem:

A copper bar 50 mm in diameter is placed within a steel tube 75 mm external diameter and 50 mm internal diameter of exactly the same length. The two pieces are rigidly fixed together by two pins 18 mm in diameter, one at each end passing through the bar and tube. Calculate the stress induced in the copper bar, steel tube and pins if the temperature of the combination is raised by 50°C. Take E_s = 210 GN/m^2 ; E_c = 105 GN/m^2 ; α_s = 11.5 × 10^{-6}/°C and α_c = 17 × 10^{-6}/°C.

Solution. Given: d_c = 50 mm ; d_{se} = 75 mm ; d_{si} = 50 mm ; d_p = 18 mm = 0.018 m ; t = 50°C; E_s = 210 GN/m^2 = 210 × 10^9 N/m^2 ; E_c = 105 GN/m^2 = 105 × 10^9 N/m^2 ; α_s = 11.5 × 10^{-6}/°C ; α_c = 17 × 10^{-6}/°C

The copper bar in a steel tube is shown in Fig. 4.18.



We know that cross-sectional area of the copper bar,

$$A_c = \frac{\pi}{4}(d_c)^2 = \frac{\pi}{4}(50)^2 = 1964 \text{ mm}^2 = 1964 \times 10^{-6} \text{ m}^2$$

and cross-sectional area of the steel tube,

$$A_s = \frac{\pi}{4}\left[(d_{se})^2 - (d_{si})^2\right] = \frac{\pi}{4}\left[(75)^2 - (50)^2\right] = 2455 \text{ mm}^2$$

$$= 2455 \times 10^{-6} \text{ m}^2$$

Let l = Length of the copper bar and steel tube.

We know that free expansion of copper bar

$$= \alpha_c \,.\, l \,.\, t = 17 \times 10^{-6} \times l \times 50 = 850 \times 10^{-6}\, l$$

and free expansion of steel tube

$$= \alpha_s \,.\, l \,.\, t = 11.5 \times 10^{-6} \times l \times 50 = 575 \times 10^{-6}\, l$$

∴ Difference in free expansion

$$= 850 \times 10^{-6}\, l - 575 \times 10^{-6}\, l = 275 \times 10^{-6}\, l \qquad ...(i)$$

Since the free expansion of the copper bar is more than the free expansion of the steel tube, therefore the copper bar is subjected to a compressive stress, while the steel tube is subjected to a tensile stress. Let a compressive force P newton on the copper bar opposes the extra expansion of the copper bar and an equal tensile force P on the steel tube pulls the steel tube so that the net effect of reduction in length of copper bar and the increase in length of steel tube equalizes the difference in free expansion of the two.

Therefore, Reduction in length of copper bar due to force P

$$= \frac{P \,.\, l}{A_c \,.\, E_c}$$

$$= \frac{P.l}{1964 \times 10^{-6} \times 105 \times 10^{9}} = \frac{P.l}{206.22 \times 10^{6}} \text{ m}$$

and increase in length of steel bar due to force P

$$= \frac{P.l}{A_s . E_s} = \frac{P.l}{2455 \times 10^{-6} \times 210 \times 10^{9}} = \frac{P.l}{515.55 \times 10^{6}} \text{ m}$$

$$\therefore \text{Net effect in length} = \frac{P.l}{206.22 \times 10^{6}} + \frac{P.l}{515.55 \times 10^{6}}$$

$$= 4.85 \times 10^{-9} \, P.l + 1.94 \times 10^{-9} \, P.l = 6.79 \times 10^{-9} \, P.l$$

Equating this net effect in length to the difference in free expansion, we have

$$6.79 \times 10^{-9} \, P.l = 275 \times 10^{-6} \, l \quad \text{or} \quad P = 40\,500 \text{ N}$$

Stress induced in the copper bar, steel tube and pins

We know that stress induced in the copper bar,

$$\sigma_c = P / A_c = 40\,500 / (1964 \times 10^{-6}) = 20.62 \times 10^{6} \text{ N/m}^2 = 20.62 \text{ MPa } \textbf{Ans.}$$

Stress induced in the steel tube,

$$\sigma_s = P / A_s = 40\,500 / (2455 \times 10^{-6}) = 16.5 \times 10^{6} \text{ N / m}^2 = 16.5 \text{ MPa } \textbf{Ans.}$$

and shear stress induced in the pins,

$$\tau_p = \frac{P}{2\,A_p} = \frac{40\,500}{2 \times \frac{\pi}{4} (0.018)^2} = 79.57 \times 10^{6} \text{ N/m}^2 = 79.57 \text{ MPa } \textbf{Ans.}$$

...($\because$ The pin is in double shear)

Impact Stress

Sometimes, machine members are subjected to the load with impact. The stress produced in the member due to the falling load is known as ***impact stress.*** Consider a bar carrying a load *W* at a height *h* and falling on the collar provided at the lower end, as shown in Fig.

Let *A* = Cross-sectional area of the bar,

E = Young's modulus of the material of the bar,

l = Length of the bar,

δl = Deformation of the bar,

P = Force at which the deflection δl is produced,

σ_i = Stress induced in the bar due to the application of impact load, and

h = Height through which the load falls.

We know that energy gained by the system in the form of strain energy

$$= \frac{1}{2} \times P \times \delta l$$

And potential energy lost by the weight

$$= W(h + \delta l)$$

Since the energy gained by the system is equal to the potential energy lost by the weight, therefore

$$\frac{1}{2} \times P \times \delta l = W\ (h + \delta l)$$

$$\frac{1}{2}\sigma_i \times A \times \frac{\sigma_i \times l}{E} = W\left(h + \frac{\sigma_i \times l}{E}\right) \qquad \ldots\left[\because P = \sigma_i \times A, \text{ and } \delta l = \frac{\sigma_i \times l}{E}\right]$$

$$\therefore \qquad \frac{Al}{2E}(\sigma_i)^2 - \frac{Wl}{E}(\sigma_i) - Wh = 0$$

From this quadratic equation, we find that

$$\sigma_i = \frac{W}{A}\left(1 + \sqrt{1 + \frac{2hAE}{Wl}}\right) \qquad \ldots \text{[Taking +ve sign for maximum value]}$$

When $h = 0$, then $\sigma_i = 2W/A$. This means that the stress in the bar when the load in applied suddenly is double of the stress induced due to gradually applied load.

Problem:

An unknown weight falls through 10 mm on a collar rigidly attached to the lower end of a vertical bar 3 m long and 600 mm2 in section. If the maximum instantaneous extension is known to be 2 mm, what is the corresponding stress and the value of unknown weight? Take E = 200 kN/mm^2.

Solution. Given : $h = 10$ mm ; $l = 3$ m $= 3000$ mm ; $A = 600$ mm^2 ; $\delta l = 2$ mm ; $E = 200$ kN/mm$^2 = 200 \times 10^3$ N/mm^2

Stress in the bar

Let σ = Stress in the bar.

We know that Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\sigma . l}{\delta l}$$

$$\therefore \quad \sigma = \frac{E . \delta l}{l} = \frac{200 \times 10^3 \times 2}{3000} = \frac{400}{3} = 133.3 \text{ N/mm}^2 \text{ Ans.}$$

Value of the unknown weight

Let W = Value of the unknown weight.

We know that

$$\sigma = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hAE}{Wl}}\right]$$

$$\frac{400}{3} = \frac{W}{600}\left[1 + \sqrt{1 + \frac{2 \times 10 \times 600 \times 200 \times 10^3}{W \times 3000}}\right]$$

$$\frac{400 \times 600}{3W} = 1 + \sqrt{1 + \frac{800\,000}{W}}$$

$$\frac{80\,000}{W} - 1 = \sqrt{1 + \frac{800\,000}{W}}$$

Squaring both sides,

$$\frac{6400 \times 10^6}{W^2} + 1 - \frac{160\,000}{W} = 1 + \frac{800\,000}{W}$$

$$\frac{6400 \times 10^2}{W} - 16 = 80 \quad \text{or} \quad \frac{6400 \times 10^2}{W} = 96$$

$$\therefore \quad W = 6400 \times 10^2 / 96 = 6666.7 \text{ N} \text{ Ans.}$$

Resilience

When a body is loaded within elastic limit, it changes its dimensions and on the removal of the load, it regains its original dimensions. So long as it remains loaded, it has stored energy in itself. On removing the load, the energy stored is given off as in the case of a spring. This energy, which is absorbed in a body when strained within elastic limit, is known as ***strain energy.*** The strain energy is always capable of doing some work.

The strain energy stored in a body due to external loading, within elastic limit, is known as ***resilience*** and the maximum energy which can be stored in a body up to the elastic limit is called ***proof resilience***. The proof resilience per unit volume of a material is known as

modulus of resilience. It is an important property of a material and gives capacity of the material to bear impact or shocks. Mathematically, strain energy stored in a body due to tensile or compressive load or resilience,

$$U = \frac{\sigma^2 \times V}{2E}$$

And Modulus of resilience

$$= \frac{\sigma^2}{2E}$$

Where σ = Tensile or compressive stress,

V = Volume of the body, and

E = Young's modulus of the material of the body.

When a body is subjected to a shear load, then modulus of resilience (shear)

$$= \frac{\tau^2}{2C}$$

Where τ = Shear stress, and

C = Modulus of rigidity.

When the body is subjected to torsion, then modulus of resilience

$$= \frac{\tau^2}{4C}$$

Problem:

A wrought iron bar 50 mm in diameter and 2.5 m long transmits shock energy of 100 N-m. Find the maximum instantaneous stress and the elongation. Take E = 200 GN/m^2.

Solution. Given : $d = 50$ mm ; $l = 2.5$ m $= 2500$ mm ; $U = 100$ N-m $= 100 \times 10^3$ N-mm ; $E = 200\ GN/m^2 = 200 \times 10^3\ N/mm^2$

Maximum instantaneous stress

Let σ = Maximum instantaneous stress.

We know that volume of the bar,

$$V = \frac{\pi}{4} \times d^2 \times l = \frac{\pi}{4}(50)^2 \times 2500 = 4.9 \times 10^6\ mm^3$$

We also know that shock or strain energy stored in the body (U),

$$100 \times 10^3 = \frac{\sigma^2 \times V}{2E} = \frac{\sigma^2 \times 4.9 \times 10^6}{2 \times 200 \times 10^3} = 12.25\ \sigma^2$$

$$\therefore \quad \sigma^2 = 100 \times 10^3 / 12.25 = 8163 \quad \text{or} \quad \sigma = 90.3\ N/mm^2 \ \textbf{Ans.}$$

Elongation produced

Let δl = Elongation produced.

We know that Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\delta l / l}$$

$$\therefore \quad \delta l = \frac{\sigma \times l}{E} = \frac{90.3 \times 2500}{200 \times 10^3} = 1.13 \text{ mm}$$ **Ans.**

Torsional Shear Stress

When a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine member is said to be subjected to ***torsion***. The stress set up by torsion is known as ***torsional shear stress***. It is zero at the centroidal axis and maximum at the outer surface. Consider a shaft fixed at one end and subjected to a torque (*T*) at the other end as shown in Fig. As a result of this torque, every cross-section of the shaft is subjected to torsional shear stress. We have discussed above that the torsional shear stress is zero at the centroidal axis and maximum at the outer surface. The maximum torsional shear stress at the outer surface of the shaft may be obtained from the following equation:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{C \cdot \theta}{l} \quad \text{-------- (i)}$$

Where τ = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress,

r = Radius of the shaft,

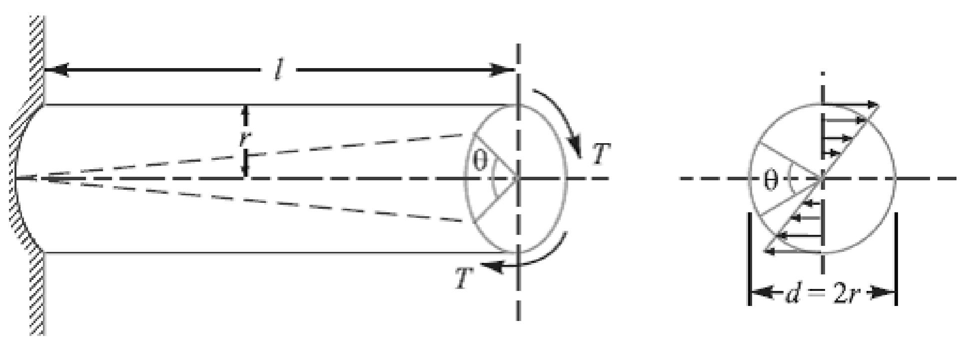
T = Torque or twisting moment,

J = Second moment of area of the section about its polar axis or polar moment of inertia,

C = Modulus of rigidity for the shaft material,

l = Length of the shaft, and

θ = Angle of twist in radians on a length *l*.



The above equation **is** known as ***torsion equation***. It is based on the following assumptions:

1. The material of the shaft is uniform throughout.

2. The twist along the length of the shaft is uniform.

3. The normal cross-sections of the shaft, which were plane and circular before twist, remain plane and circular after twist.

4. All diameters of the normal cross-section which were straight before twist, remain straight with their magnitude unchanged, after twist.

5. The maximum shear stress induced in the shaft due to the twisting moment does not exceed its elastic limit value.

Note: 1. Since the torsional shear stress on any cross-section normal to the axis is directly proportional to the distance from the centre of the axis, therefore the torsional shear stress at a distance *x* from the centre of the shaft is given by

$$\frac{\tau_x}{x} = \frac{\tau}{r}$$

2. From equation (*i*), we know that

$$\frac{T}{J} = \frac{\tau}{r} \quad \text{or} \quad T = \tau \times \frac{J}{r}$$

For a solid shaft of diameter (*d*), the polar moment of inertia,

$$J - I_{XX} + I_{YY} - \frac{\pi}{64} \times d^4 + \frac{\pi}{64} \times d^4 = \frac{\pi}{32} \times d^4$$

Therefore,

$$T = \tau \times \frac{\pi}{32} \times d^4 \times \frac{2}{d} = \frac{\pi}{16} \times \tau \times d^3$$

In case of a hollow shaft with external diameter (d_o) and internal diameter (d_i), the polar moment of inertia,

$$J - \frac{\pi}{32}\,[(d_o)^4 \quad (d_i)^4] \text{ and } r - \frac{d_o}{2}$$

$$T = \tau \times \frac{\pi}{32}[(d_o)^4 - (d_4)^4] \times \frac{2}{d_o} = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right]$$

$$= \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \qquad \ldots\left(\text{Substituting, } k = \frac{d_i}{d_o}\right)$$

3. The expression ($C \times J$) is called ***torsional rigidity*** of the shaft.

4. The strength of the shaft means the maximum torque transmitted by it. Therefore, in order to design a shaft for strength, the above equations are used. The power transmitted by the shaft (in watts) is given by

$$P = \frac{2\,\pi\,N\,.\,T}{60} = T\,.\omega \qquad \ldots\left(\because\ \omega = \frac{2\,\pi\,N}{60}\right)$$

Where T = Torque transmitted in N-m, and

ω = Angular speed in rad/s.

Problem:

A shaft is transmitting 100 kW at 160 r.p.m. Find a suitable diameter for the shaft, if the maximum torque transmitted exceeds the mean by 25%. Take maximum allowable shear stress as 70 MPa.

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 160$ r.p.m ; $T_{max} = 1.25\ T_{mean}$; $\tau = 70$ MPa $= 70$ N/mm^2

Let T_{mean} = Mean torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted (P),

$$100 \times 10^3 = \frac{2\pi N . T_{mean}}{60} = \frac{2\pi \times 160 \times T_{mean}}{60} = 16.76\ T_{mean}$$

$$\therefore \quad T_{mean} = 100 \times 10^3/16.76 = 5966.6 \text{ N-m}$$

and maximum torque transmitted,

$$T_{max} = 1.25 \times 5966.6 = 7458 \text{ N-m} = 7458 \times 10^3 \text{ N-mm}$$

We know that maximum torque (T_{max}),

$$7458 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 70 \times d^3 = 13.75\ d^3$$

$$\therefore \quad d^3 = 7458 \times 10^3/13.75 = 542.4 \times 10^3 \text{ or } d = 81.5 \text{ mm } \textbf{Ans.}$$

Bending Stress

In engineering practice, the machine parts of structural members may be subjected to static or dynamic loads which cause bending stress in the sections besides other types of stresses such as tensile, compressive and shearing stresses. Consider a straight beam subjected to a bending moment M as shown in Fig.

The following assumptions are usually made while deriving the bending formula.

1. The material of the beam is perfectly homogeneous (*i.e.* of the same material throughout) and isotropic (*i.e.* of equal elastic properties in all directions).

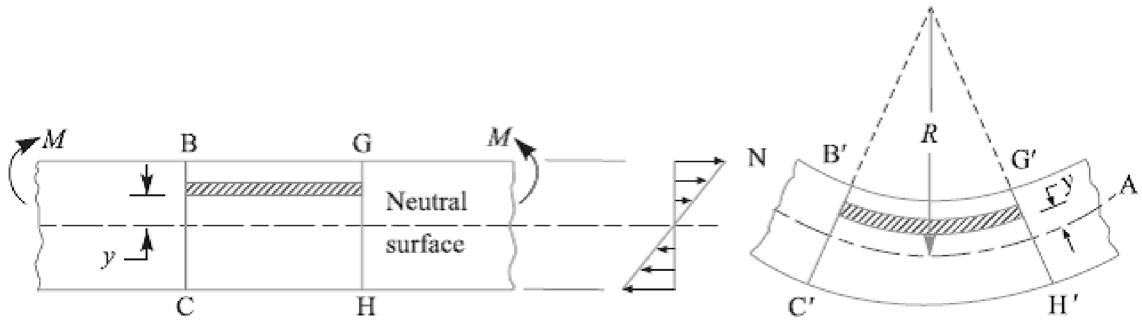
2. The material of the beam obeys Hooke's law.

3. The transverse sections (*i.e.* BC or GH) which were plane before bending remain plane after bending also.

4. Each layer of the beam is free to expand or contract, independently, of the layer, above or below it.

5. The Young's modulus (E) is the same in tension and compression.

6. The loads are applied in the plane of bending.



A little consideration will show that when a beam is subjected to the bending moment, the fibres on the upper side of the beam will be shortened due to compression and those on the lower side will be elongated due to tension. It may be seen that somewhere between the top and bottom fibres there is a surface at which the fibres are neither shortened nor lengthened. Such a surface is called ***neutral surface*.** The intersection of the neutral surface with any normal cross-section of the beam is known *as* ***neutral axis*.** The stress distribution of a beam is shown in Fig. The bending equation is given by

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Where M = Bending moment acting at the given section,

σ = Bending stress,

I = Moment of inertia of the cross-section about the neutral axis,

y = Distance from the neutral axis to the extreme fibre,

E = Young's modulus of the material of the beam, and

R = Radius of curvature of the beam.

From the above equation, the bending stress is given by

$$\sigma = y \times \frac{E}{R}$$

Since E and R are constant, therefore within elastic limit, the stress at any point is directly proportional to y, *i.e.* the distance of the point from the neutral axis.

Also from the above equation, the bending stress,

$$\sigma = \frac{M}{I} \times y = \frac{M}{I / y} = \frac{M}{Z}$$

The ratio I/y is known as ***section modulus*** and is denoted by Z.

Notes: 1. the neutral axis of a section always passes through its centroid.

2. In case of symmetrical sections such as circular, square or rectangular, the neutral axis passes through its geometrical centre and the distance of extreme fibre from the neutral axis is $y = d / 2$, where d is the diameter in case of circular section or depth in case of square or rectangular section.

3. In case of unsymmetrical sections such as L-section or T-section, the neutral axis does not pass through its geometrical centre. In such cases, first of all the centroid of the section is calculated and then the distance of the extreme fibres for both lower and upper side of the section is obtained. Out of these two values, the bigger value is used in bending equation.

Problem:

A beam of uniform rectangular cross-section is fixed at one end and carries an electric motor weighing 400 N at a distance of 300 mm from the fixed end. The maximum bending stress in the beam is 40 MPa. Find the width and depth of the beam, if depth is twice that of width.

Solution. Given: $W = 400$ N ; $L = 300$ mm ; $\sigma_b = 40$ MPa $= 40$ N/mm^2 ; $h = 2b$

The beam is shown in Fig. 5.7.

Let b = Width of the beam in mm, and

h = Depth of the beam in mm.

∴ Section modulus,

$$Z = \frac{b \cdot h^2}{6} = \frac{b\,(2b)^2}{6} = \frac{2\,b^3}{3} \text{ mm}^3$$

Maximum bending moment (at the fixed end),

$$M = W.L = 400 \times 300 = 120 \times 10^3 \text{ N-mm}$$

We know that bending stress (σ_b),

$$40 = \frac{M}{Z} = \frac{120 \times 10^3 \times 3}{2\,b^3} = \frac{180 \times 10^3}{b^3}$$

∴ $b^3 = 180 \times 10^3/40 = 4.5 \times 10^3$ or $b = 16.5$ mm **Ans.**

and $h = 2b = 2 \times 16.5 = 33$ mm **Ans.**

Problem:

A cast iron pulley transmits 10 kW at 400 r.p.m. The diameter of the pulley is 1.2 metre and it has four straight arms of elliptical cross-section, in which the major axis is twice the minor axis. Determine the dimensions of the arm if the allowable bending stress is 15 MPa.

Solution. Given : $P = 10$ kW $= 10 \times 10^3$ W ; $N = 400$ r.p.m ; $D = 1.2$ m $= 1200$ mm or $R = 600$ mm ; $\sigma_b = 15$ MPa $= 15$ N/mm^2

Let T = Torque transmitted by the pulley.

We know that the power transmitted by the pulley (P),

$$10 \times 10^3 = \frac{2\pi N . T}{60} = \frac{2\pi \times 400 \times T}{60} = 42\,T$$

$$\therefore \quad T = 10 \times 10^3/42 = 238 \text{ N-m} = 238 \times 10^3 \text{ N-mm}$$

Since the torque transmitted is the product of the tangential load and the radius of the pulley, therefore tangential load acting on the pulley

$$= \frac{T}{R} = \frac{238 \times 10^3}{600} = 396.7 \text{ N}$$

Since the pulley has four arms, therefore tangential load on each arm,

$$W = 396.7/4 = 99.2 \text{ N}$$

and maximum bending moment on the arm,

$$M = W \times R = 99.2 \times 600 = 59\,520 \text{ N-mm}$$

Let $2b$ = Minor axis in mm, and

$2a$ = Major axis in mm = $2 \times 2b = 4b$ ...(Given)

$\therefore$ Section modulus for an elliptical cross-section,

$$Z = \frac{\pi}{4} \times a^2 b = \frac{\pi}{4} (2b)^2 \times b = \pi\, b^3 \text{ mm}^3$$

We know that bending stress (σ_b),

$$15 = \frac{M}{Z} = \frac{59\,520}{\pi\, b^3} = \frac{18\,943}{b^3}$$

or $b^3 = 18\,943/15 = 1263$ or $b = 10.8$ mm

$\therefore$ Minor axis, $2b = 2 \times 10.8 = 21.6$ mm **Ans.**

and major axis, $2a = 2 \times 2b = 4 \times 10.8 = 43.2$ mm **Ans.**

Stress Concentration:

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighborhood of the discontinuity is different. His irregularity in the stress distribution caused by abrupt changes of form is called ***stress concentration.*** It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc. In order to understand fully the idea of stress concentration, consider a member with different cross-section under a tensile load as shown in Fig. A little consideration will show that the nominal stress in the right and left hand sides will be uniform but in the region where the cross-section is changing, a re-distribution of the force within the member must take place. The material near the edges is stressed considerably higher than the average value. The maximum stress occurs at some point on the fillet and is directed parallel to the boundary at that point.

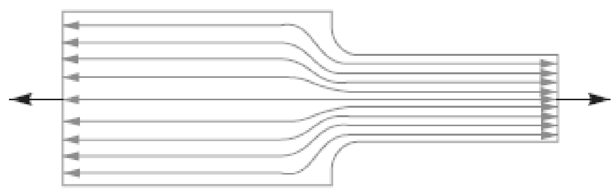


Fig. Stress concentration

Theoretical or Form Stress Concentration Factor

The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area. Mathematically, theoretical or form stress concentration factor,

$$K_t = \text{Maximum stress/ Nominal stress}$$

The value of K_t depends upon the material and geometry of the part. In static loading, stress concentration in ductile materials is not so serious as in brittle materials, because in ductile materials local deformation or yielding takes place which reduces the concentration. In brittle materials, cracks may appear at these local concentrations of stress which will increase the stress over the rest of the section. It is, therefore, necessary that in designing parts of brittle materials such as castings, care should be taken. In order to avoid failure due to stress concentration, fillets at the changes of section must be provided.

In cyclic loading, stress concentration in ductile materials is always serious because the ductility of the material is not effective in relieving the concentration of stress caused by cracks, flaws, surface roughness, or any sharp discontinuity in the geometrical form of the member. If the stress at any point in a member is above the endurance limit of the material, a crack may develop under the action of repeated load and the crack will lead to failure of the member.

Stress Concentration due to Holes and Notches

Consider a plate with transverse elliptical hole and subjected to a tensile load as shown in Fig.1(a). We see from the stress-distribution that the stress at the point away from the hole is practically uniform and the maximum stress will be induced at the edge of the hole. The maximum stress is given by

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{b}\right)$$

And the theoretical stress concentration factor,

$$K_t = \frac{\sigma_{max}}{\sigma} = \left(1 + \frac{2a}{r}\right)$$

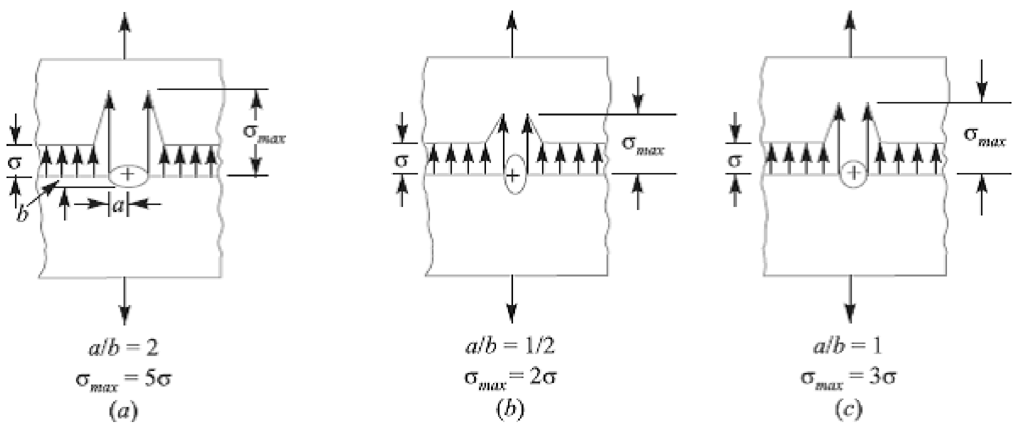


Fig.1. Stress concentration due to holes.

The stress concentration in the notched tension member, as shown in Fig. 2, is influenced by the depth a of the notch and radius r at the bottom of the notch. The maximum stress, which applies to members having notches that are small in comparison with the width of the plate, may be obtained by the following equation,

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{r}\right)$$

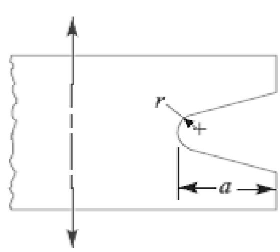


Fig.2. Stress concentration due to notches.

Methods of Reducing Stress Concentration

Whenever there is a change in cross-section, such as shoulders, holes, notches or keyways and where there is an interference fit between a hub or bearing race and a shaft, then stress concentration results. The presence of stress concentration can not be totally eliminated but it may be reduced to some extent. A device or concept that is useful in assisting a design engineer to visualize the presence of stress concentration and how it may be mitigated is that of stress flow lines, as shown in Fig.3. The mitigation of stress concentration means that the stress flow lines shall maintain their spacing as far as possible.

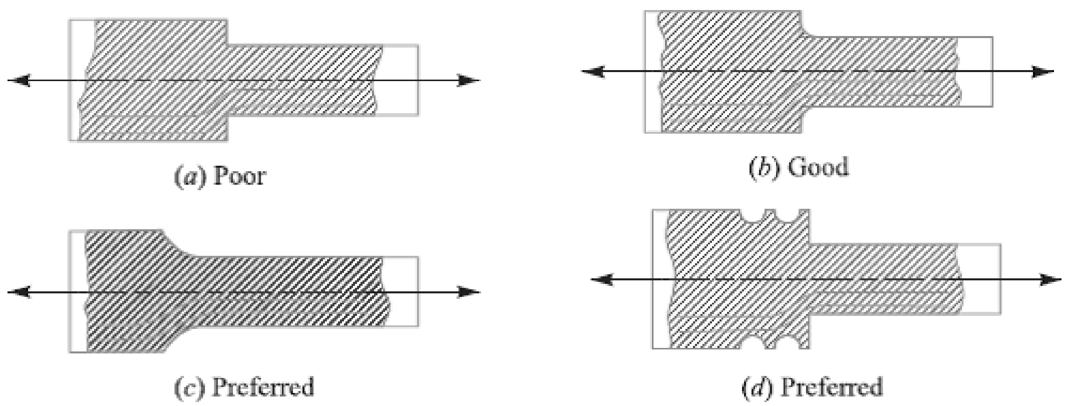


Fig.3

In Fig. 3 (a) we see that stress lines tend to bunch up and cut very close to the sharp re-entrant corner. In order to improve the situation, fillets may be provided, as shown in Fig. 3 (b) and (c) to give more equally spaced flow lines.

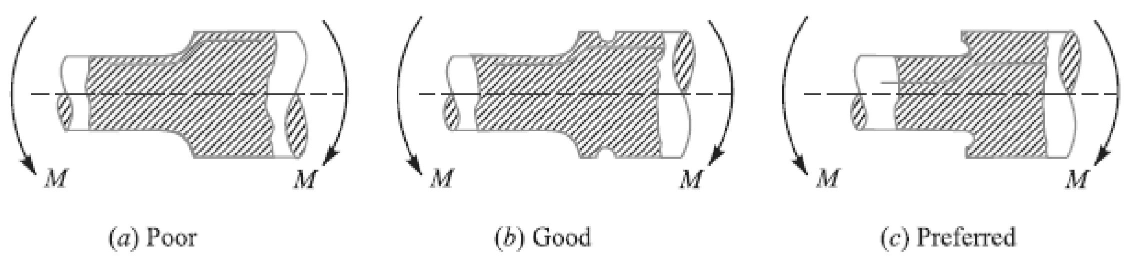


Fig. reducing stress concentration in cylindrical members with shoulders

Fig. Reducing stress concentration in cylindrical members with holes.

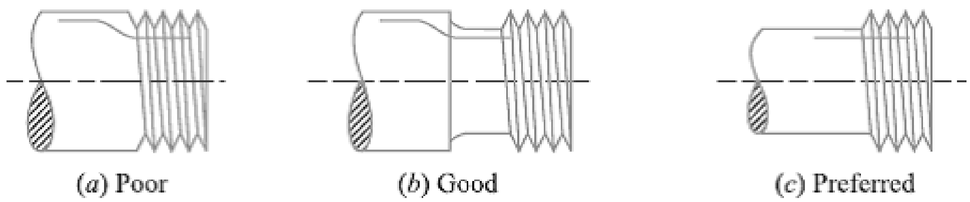


Fig. Reducing stress concentration in cylindrical members with holes

Completely Reversed or Cyclic Stresses

Consider a rotating beam of circular cross-section and carrying a load *W*, as shown in Fig1. This load induces stresses in the beam which are cyclic in nature. *A* little consideration will show that the upper fibres of the beam (*i.e*. at point *A*) are under compressive stress and the lower fibres (*i.e*. at point *B*) are under tensile stress. After half a revolution, the point *B* occupies the position of point *A* and the point *A* occupies the position of point *B*. Thus the point *B* is now under compressive stress and the point *A* under tensile stress. The speed of variation of these stresses depends upon the speed of the beam.

From above we see that for each revolution of the beam, the stresses are reversed from compressive to tensile. The stresses which vary from one value of compressive to the same value of tensile or ***vice versa*,** are known as ***completely reversed*** or ***cyclic stresses.*** The stresses which vary from a minimum value to a maximum value of the same nature, (*i.e*. tensile or compressive) are called ***fluctuating stresses.*** The stresses which vary from zero to a certain maximum value are called ***repeated stresses.*** The stresses which vary from a minimum value to a maximum value of the opposite nature (*i.e*. from a certain minimum compressive to a certain maximum tensile or from a minimum tensile to a maximum compressive) are called ***alternating stresses.***

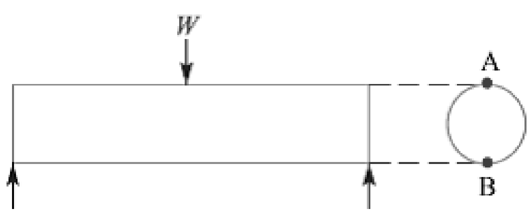


Fig.1. Shaft subjected to cyclic load

Fatigue and Endurance Limit

It has been found experimentally that when a material is subjected to repeated stresses; it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue.** The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The failure may occur even without any prior indication. The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.

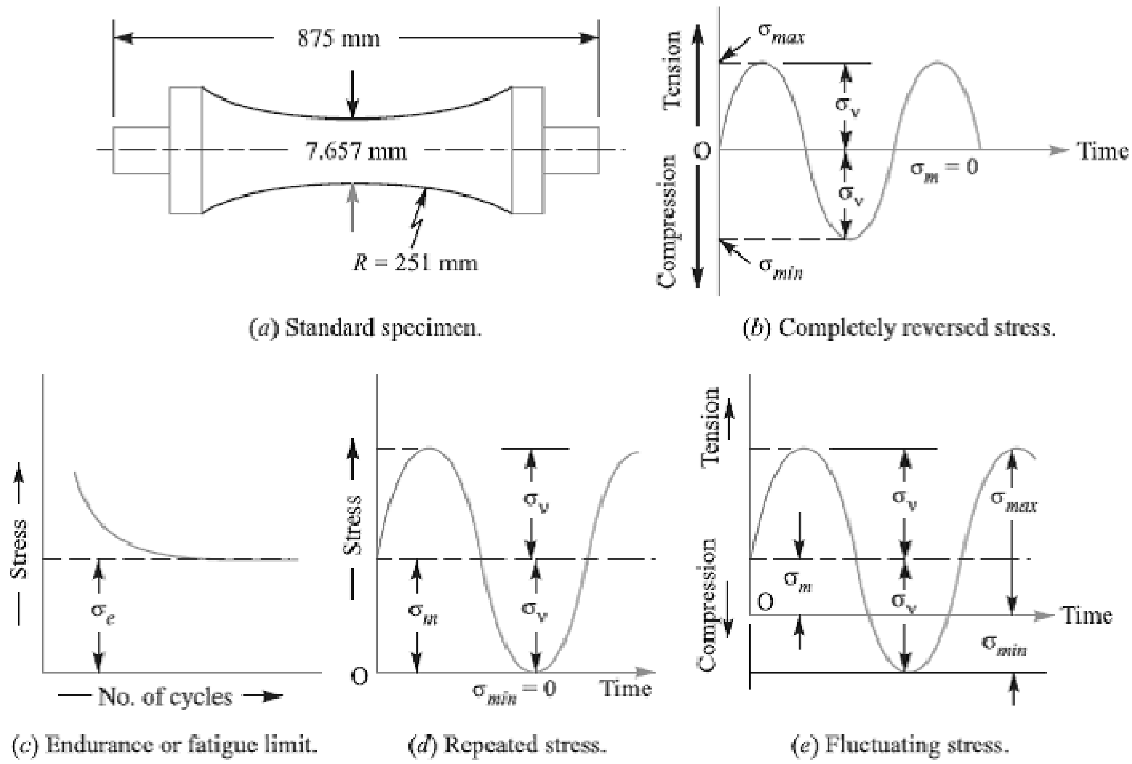


Fig.2. Time-stress diagrams.

In order to study the effect of fatigue of a material, a rotating mirror beam method is used. In this method, a standard mirror polished specimen, as shown in Fig.2 (*a*), is rotated in a fatigue testing machine while the specimen is loaded in bending. As the specimen rotates, the bending stress at the upper fibres varies from maximum compressive to maximum tensile while the bending stress at the lower fibres varies from maximum tensile to maximum compressive. In other words, the specimen is subjected to a completely reversed stress cycle.
This is represented by a time-stress diagram as shown in Fig.2 (*b*). A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve as shown in Fig.2 (*c*). A little consideration will show that if the stress is kept below a certain value as shown by dotted line in Fig.2 (*c*), the material will not fail whatever may be the number of cycles. This stress, as represented by dotted line, is known as ***endurance*** or ***fatigue limit*** (σe). It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 107 cycles).

It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term ***endurance strength*** may be used when referring the fatigue strength of the material. It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions.

We have seen that when a machine member is subjected to a completely reversed stress, the maximum stress in tension is equal to the maximum stress in compression as shown in Fig.2 (*b*). In actual practice, many machine members undergo different range of stress than the completely reversed stress. The stress ***verses*** time diagram for fluctuating stress having values σ_{min} and σ_{max} is shown in Fig.2 (*e*). The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component σv. The following relations are derived from Fig. 2 (*e*):

1. Mean or average stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

2. Reversed stress component or alternating or variable stress,

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

For repeated loading, the stress varies from maximum to zero (*i.e.* $\sigma min = 0$) in each cycle as shown in Fig.2 (*d*).

$$\sigma_m = \sigma_v = \frac{\sigma_{max}}{2}$$

3. Stress ratio, $R = \sigma_{max}/\sigma_{min}$. For completely reversed stresses, $R = -1$ and for repeated stresses, $R = 0$. It may be noted that R cannot be greater than unity.

4. The following relation between endurance limit and stress ratio may be used

$$\sigma'_e = \frac{3\sigma_e}{2 - R}$$

Effect of Loading on Endurance Limit—Load Factor

The endurance limit (σe) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

Let K_b = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.

K_a = Load correction factor for the reversed axial load. Its value may be taken as 0.8.

K_s = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

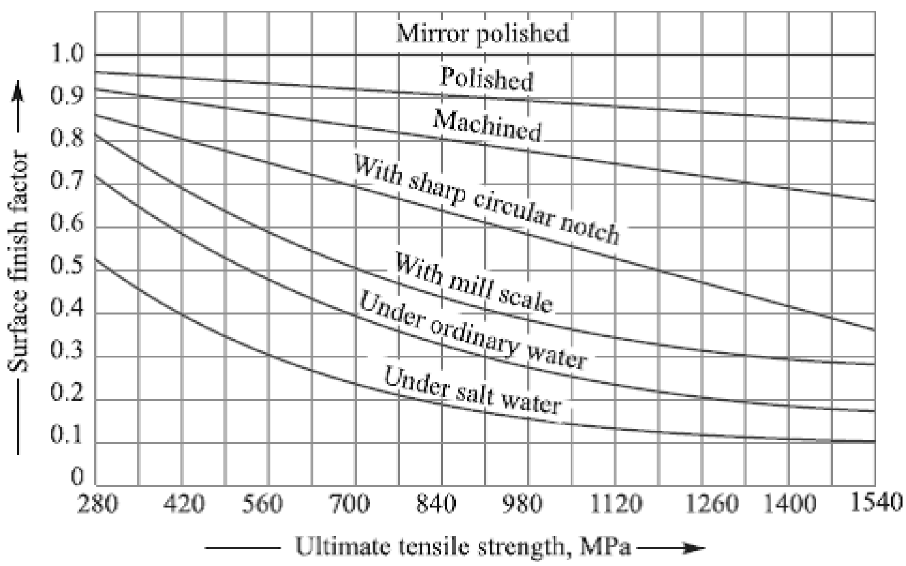
∴ Endurance limit for reversed bending load, $\sigma_{eb} = \sigma_e K_b = \sigma_e$

Endurance limit for reversed axial load, $\sigma_{ea} = \sigma_e K_a$

and endurance limit for reversed torsional or shear load, $\tau_e = \sigma_e K_s$

Effect of Surface Finish on Endurance Limit—Surface Finish Factor

When a machine member is subjected to variable loads, the endurance limit of the material for that member depends upon the surface conditions. Fig. shows the values of surface finish factor for the various surface conditions and ultimate tensile strength.



When the surface finish factor is known, then the endurance limit for the material of the machine member may be obtained by multiplying the endurance limit and the surface finish factor. We see that for a mirror polished material, the surface finish factor is unity. In other words, the endurance limit for mirror polished material is maximum and it goes on reducing due to surface condition.

Let K_{sur} = Surface finish factor.

Then, Endurance limit,

$$\sigma_{e1} = \sigma_{eb}.K_{sur} = \sigma_e.K_b.K_{sur} = \sigma_e.K_{sur} \quad ...(\because K_b = 1)$$

...(For reversed bending load)

$$= \sigma_{ea}.K_{sur} = \sigma_e.K_a.K_{sur} \quad ...(\text{For reversed axial load})$$

$$= \tau_e.K_{sur} = \sigma_e.K_s.K_{sur} \quad ...(\text{For reversed torsional or shear load})$$

Effect of Size on Endurance Limit—Size Factor

A little consideration will show that if the size of the standard specimen as shown in Fig.2 (*a*) is increased, then the endurance limit of the material will decrease. This is due to the fact that a longer specimen will have more defects than a smaller one.

Let K_{sz} = Size factor.

Then, Endurance limit,

$$\sigma_{e2} = \sigma_{e1} \times K_{sz} \quad \text{...(Considering surface finish factor also)}$$
$$= \sigma_{eb}.K_{sur}.K_{sz} = \sigma_e.K_b.K_{sur}.K_{sz} = \sigma_e.K_{sur}.K_{sz} \quad (\because K_b = 1)$$
$$= \sigma_{ea}.K_{sur}.K_{sz} = \sigma_e.K_a.K_{sur}.K_{sz} \quad \text{...(For reversed axial load)}$$
$$= \tau_e.K_{sur}.K_{sz} = \sigma_e.K_s.K_{sur}.K_{sz} \quad \text{... (For reversed torsional or shear load)}$$

The value of size factor is taken as unity for the standard specimen having nominal diameter of 7.657 mm. When the nominal diameter of the specimen is more than 7.657 mm but less than 50 mm, the value of size factor may be taken as 0.85. When the nominal diameter of the specimen is more than 50 mm, then the value of size factor may be taken as 0.75.

Effect of Miscellaneous Factors on Endurance Limit

In addition to the surface finish factor (K_{sur}), size factor (K_{sz}) and load factors K_b, K_a and K_s, there are many other factors such as reliability factor (K_r), temperature factor (K_t), impact factor (K_i) etc. which has effect on the endurance limit of a material. Considering all these factors, the endurance limit may be determined by using the following expressions:

1. For the reversed bending load, endurance limit,

$$\sigma'_e = \sigma_{eb}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

2. For the reversed axial load, endurance limit,

$$\sigma'_e = \sigma_{ea}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

3. For the reversed torsional or shear load, endurance limit,

$$\sigma'_e = \tau_e.K_{sur}.K_{sz}.K_r.K_t.K_i$$

In solving problems, if the value of any of the above factors is not known, it may be taken as unity.

Relation between Endurance Limit and Ultimate Tensile Strength

It has been found experimentally that endurance limit (σ_e) of a material subjected to fatigue loading is a function of ultimate tensile strength (σ_u).

For steel, $\sigma_e = 0.5\ \sigma_u$;
For cast steel, $\sigma_e = 0.4\ \sigma_u$;
For cast iron, $\sigma_e = 0.35\ \sigma_u$;
For non-ferrous metals and alloys, $\sigma_e = 0.3\ \sigma_u$

Factor of Safety for Fatigue Loading

When a component is subjected to fatigue loading, the endurance limit is the criterion for failure. Therefore, the factor of safety should be based on endurance limit. Mathematically,

$$\text{Factor of safety } (F.S.) = \frac{\text{Endurance limit stress}}{\text{Design or working stress}} = \frac{\sigma_e}{\sigma_d}$$

For steel, $\sigma_e = 0.8 \text{ to } 0.9\ \sigma_y$

σ_e = Endurance limit stress for completely reversed stress cycle, and

σ_y = Yield point stress.

Fatigue Stress Concentration Factor

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor. Since the determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as

Fatigue stress concentration factor,

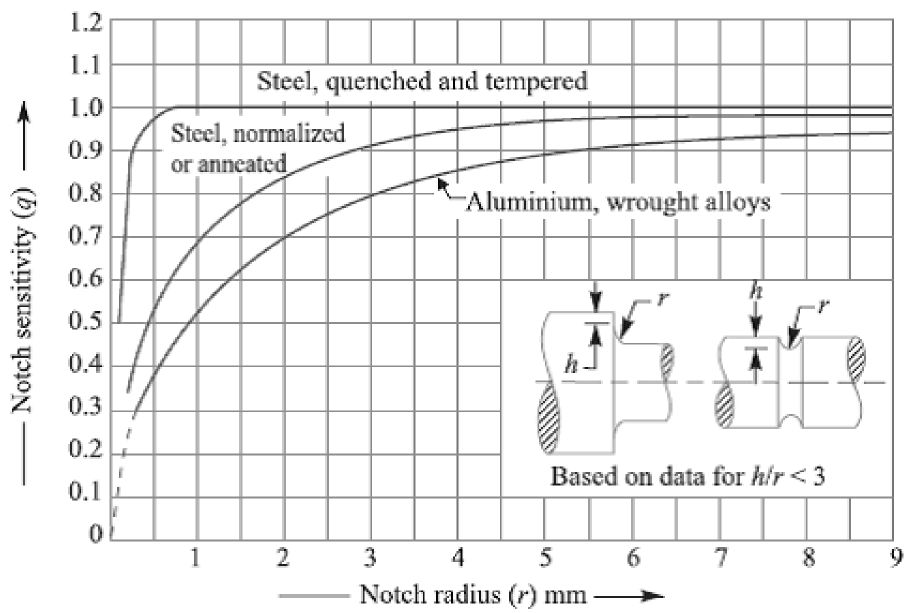
$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

Notch Sensitivity

In cyclic loading, the effect of the notch or the fillet is usually less than predicted by the use of the theoretical factors as discussed before. The difference depends upon the stress gradient in the region of the stress concentration and on the hardness of the material. The term ***notch sensitivity*** is applied to this behaviour. It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material. Since the extensive data for estimating the notch sensitivity factor (*q*) is not available, therefore the curves, as shown in Fig., may be used for determining the values of *q* for two steals. When the notch sensitivity factor *q* is used in cyclic loading, then fatigue stress concentration factor may be obtained from the following relations:

$$q = \frac{K_f - 1}{K_t - 1}$$

Or



$$K_f = 1 + q\,(K_t - 1) \qquad \text{...[For tensile or bending stress]}$$

And

$$K_{fs} = 1 + q\,(K_{ts} - 1) \qquad \text{...[For shear stress]}$$

Where K_t = Theoretical stress concentration factor for axial or bending loading, and

K_{ts} = Theoretical stress concentration factor for torsional or shear loading.

Problem: Determine the thickness of a 120 mm wide uniform plate for safe continuous operation if the plate is to be subjected to a tensile load that has a maximum value of 250 kN and a minimum value of 100 kN. The properties of the plate material are as follows: Endurance limit stress = 225 MPa, and Yield point stress = 300 MPa. The factor of safety based on yield point may be taken as 1.5.

Let t = Thickness of the plate in mm.

$\therefore$ Area, $A = b \times t = 120\, t$ mm²

We know that mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{250 + 100}{2} = 175 \text{ kN} = 175 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{W_m}{A} = \frac{175 \times 10^3}{120t} \text{ N/mm}^2$$

$$\text{Variable load, } W_v = \frac{W_{max} - W_{min}}{2} = \frac{250 - 100}{2} = 75 \text{ kN} = 75 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Variable stress, } \sigma_v = \frac{W_v}{A} = \frac{75 \times 10^3}{120t} \text{ N/mm}^2$$

According to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{1.5} = \frac{175 \times 10^3}{120t \times 300} + \frac{75 \times 10^3}{120t \times 225} = \frac{4.86}{t} + \frac{2.78}{t} = \frac{7.64}{t}$$

$\therefore$ $t = 7.64 \times 1.5 = 11.46$ say 11.5 mm **Ans.**

Problem:

Determine the diameter of a circular rod made of ductile material with a fatigue strength (complete stress reversal), σ_e = 265 MPa and a tensile yield strength of 350 MPa. The member is subjected to a varying axial load from $W_{min} = -300 \times 10^3$ N to $W_{max} = 700 \times 10^3$ N and has a stress concentration factor = 1.8. Use factor of safety as 2.0.

Let d = Diameter of the circular rod in mm.

$$\therefore \quad \text{Area, } A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2 \text{ mm}^2$$

We know that the mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{700 \times 10^3 + (-300 \times 10^3)}{2} = 200 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{W_m}{A} = \frac{200 \times 10^3}{0.7854\, d^2} = \frac{254.6 \times 10^3}{d^2} \text{ N/mm}^2$$

Variable load, $W_v = \frac{W_{max} - W_{min}}{2} = \frac{700 \times 10^3 - (-300 \times 10^3)}{2} = 500 \times 10^3$ N

$\therefore$ Variable stress, $\sigma_v = \frac{W_v}{A} = \frac{500 \times 10^3}{0.7854\, d^2} = \frac{636.5 \times 10^3}{d^2}$ N/mm^2

We know that according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e}$$

$$\frac{1}{2} = \frac{254.6 \times 10^3}{d^2 \times 350} + \frac{636.5 \times 10^3 \times 1.8}{d^2 \times 265} = \frac{727}{d^2} + \frac{4323}{d^2} = \frac{5050}{d^2}$$

$\therefore$ $d^2 = 5050 \times 2 = 10\ 100$ or $d = 100.5$ mm **Ans.**

Problem:

A circular bar of 500 mm length is supported freely at its two ends. It is acted upon by a central concentrated cyclic load having a minimum value of 20 kN and a maximum value of 50 kN. Determine the diameter of bar by taking a factor of safety of 1.5, size effect of 0.85, surface finish factor of 0.9. The material properties of bar are given by: ultimate strength of 650 MPa, yield strength of 500 MPa and endurance strength of 350 MPa.

Solution. Given : l = 500 mm ; W_{min} = 20 kN = 20 × 10^3 N ; W_{max} = 50 kN = 50 × 10^3 N ; $F.S.$ = 1.5 ; K_{sz} = 0.85 ; K_{sur} = 0.9 ; σ_u = 650 MPa = 650 N/mm^2 ; σ_y = 500 MPa = 500 N/mm^2 ; σ_e = 350 MPa = 350 N/mm^2

Let d = Diameter of the bar in mm.

We know that the maximum bending moment,

$$M_{max} = \frac{W_{max} \times l}{4} = \frac{50 \times 10^3 \times 500}{4} = 6250 \times 10^3 \text{ N-mm}$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times l}{4} = \frac{20 \times 10^3 \times 500}{4} = 2550 \times 10^3 \text{ N-mm}$$

$\therefore$ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{6250 \times 10^3 + 2500 \times 10^3}{2} = 4375 \times 10^3 \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{6250 \times 10^3 - 2500 \times 10^3}{2} = 1875 \times 10^3 \text{ N-mm}$$

Section modulus of the bar,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$$

$\therefore$ Mean or average bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{4375 \times 10^3}{0.0982\, d^3} = \frac{44.5 \times 10^6}{d^3} \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{1875 \times 10^3}{0.0982\, d^3} = \frac{19.1 \times 10^6}{d^3} \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 650} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \qquad \text{...(Taking } K_f = 1\text{)}$$

$$= \frac{68 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{139 \times 10^3}{d^3}$$

$\therefore$ $d^3 = 139 \times 10^3 \times 1.5 = 209 \times 10^3$ or $d = 59.3$ mm

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 500} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \qquad \text{...(Taking } K_f = 1\text{)}$$

$$= \frac{89 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{160 \times 10^3}{d^3}$$

$\therefore$ $d^3 = 160 \times 10^3 \times 1.5 = 240 \times 10^3$ or $d = 62.1$ mm

Taking larger of the two values, we have $d = 62.1$ mm **Ans.**

Problem:

A 50 mm diameter shaft is made from carbon steel having ultimate tensile strength of 630 MPa. It is subjected to a torque which fluctuates between 2000 N-m to – 800 N-m. Using Soderberg method, calculate the factor of safety. Assume suitable values for any other data needed.

Solution. Given : $d = 50$ mm ; $\sigma_u = 630$ MPa $= 630$ N/mm^2 ; $T_{max} = 2000$ N-m ; $T_{min} = -800$ N-m

We know that the mean or average torque,

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{2000 + (-800)}{2} = 600 \text{ N-m} = 600 \times 10^3 \text{ N-mm}$$

∴ Mean or average shear stress,

$$\tau_m = \frac{16\, T_m}{\pi d^3} = \frac{16 \times 600 \times 10^3}{\pi (50)^3} = 24.4 \text{ N/mm}^2 \qquad \ldots\left(\because\ T = \frac{\pi}{16} \times \tau \times d^3\right)$$

Variable torque,

$$T_v = \frac{T_{max} - T_{min}}{2} = \frac{2000 - (-800)}{2} = 1400 \text{ N-m} = 1400 \times 10^3 \text{ N-mm}$$

∴ Variable shear stress, $\tau_v = \dfrac{16\, T_v}{\pi d^3} = \dfrac{16 \times 1400 \times 10^3}{\pi (50)^3} = 57 \text{ N/mm}^2$

Since the endurance limit in reversed bending (σ_e) is taken as one-half the ultimate tensile strength (*i.e.* $\sigma_e = 0.5\, \sigma_u$) and the endurance limit in shear (τ_e) is taken as $0.55\, \sigma_e$, therefore

$$\tau_e = 0.55\, \sigma_e = 0.55 \times 0.5\, \sigma_u = 0.275\, \sigma_u$$
$$= 0.275 \times 630 = 173.25 \text{ N/mm}^2$$

Assume the yield stress (σ_y) for carbon steel in reversed bending as 510 N/mm^2, surface finish factor (K_{sur}) as 0.87, size factor (K_{sz}) as 0.85 and fatigue stress concentration factor (K_{fs}) as 1.

Since the yield stress in shear (τ_y) for shear loading is taken as one-half the yield stress in reversed bending (σ_y), therefore

$$\tau_y = 0.5\, \sigma_y = 0.5 \times 510 = 255 \text{ N/mm}^2$$

Let $F.S.$ = Factor of safety.

We know that according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} = \frac{24.4}{255} + \frac{57 \times 1}{173.25 \times 0.87 \times 0.85}$$
$$= 0.096 + 0.445 = 0.541$$

∴ $F.S. = 1 / 0.541 = 1.85$ **Ans.**

Problem:

A simply supported beam has a concentrated load at the centre which fluctuates from a value of P to 4 P. The span of the beam is 500 mm and its cross-section is circular with a diameter of 60 mm. Taking for the beam material an ultimate stress of 700 MPa, a yield stress of 500 MPa, endurance limit of 330 MPa for reversed bending, and a factor of safety of 1.3, calculate the maximum value of P. Take a size factor of 0.85 and a surface finish factor of 0.9.

Solution. Given : $W_{min} = P$; $W_{max} = 4P$; $L = 500$ mm; $d = 60$ mm ; $\sigma_u = 700$ MPa $= 700$ N/mm^2 ; $\sigma_y = 500$ MPa $= 500$ N/mm^2 ; $\sigma_e = 330$ MPa $= 330$ N/mm^2 ; $F.S. = 1.3$; $K_{sz} = 0.85$; $K_{sur} = 0.9$

We know that maximum bending moment,

$$M_{max} = \frac{W_{max} \times L}{4} = \frac{4P \times 500}{4} = 500P \text{ N-mm}$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times L}{4} = \frac{P \times 500}{4} = 125P \text{ N mm}$$

∴ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{500P + 125P}{2} = 312.5P \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{500P - 125P}{2} = 187.5P \text{ N-mm}$$

Section modulus, $$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}(60)^3 = 21.21 \times 10^3 \text{ mm}^3$$

∴ Mean bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{312.5P}{21.21 \times 10^3} = 0.0147P \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{187.5P}{21.21 \times 10^3} = 0.0088P \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.3} = \frac{0.0147P}{700} + \frac{0.0088P \times 1}{330 \times 0.9 \times 0.85} \quad \text{...(Taking } K_f = 1)$$

$$= \frac{21P}{10^6} + \frac{34.8P}{10^6} = \frac{55.8P}{10^6}$$

∴ $$P = \frac{1}{1.3} \times \frac{10^6}{55.8} = 13\,785 \text{ N} = 13.785 \text{ kN}$$

Introduction to Screwed Joints:

A screw thread is formed by cutting a continuous helical groove on a cylindrical surface. A screw made by cutting a single helical groove on the cylinder is known as **single threaded** (or single-start) screw and if a second thread is cut in the space between the grooves of the first, a **double threaded** (or double-start) screw is formed. Similarly, triple and quadruple (i.e. multiple-start) threads may be formed. The helical grooves may be cut either **right hand** or **left hand.**

A screwed joint is mainly composed of two elements i.e. a bolt and nut. The screwed joints are widely used where the machine parts are required to be readily connected or disconnected without damage to the machine or the fastening. This may be for the purpose of holding or adjustment in assembly or service inspection, repair, or replacement or it may be for the manufacturing or assembly reasons. The parts may be rigidly connected or provisions may be made for predetermined relative motion.

Advantages and Disadvantages of Screwed Joints

Following are the advantages and disadvantages of the screwed joints.

Advantages

1. Screwed joints are highly reliable in operation.

2. Screwed joints are convenient to assemble and disassemble.

3. A wide range of screwed joints may be adapted to various operating conditions.

4. Screws are relatively cheap to produce due to standardization and highly efficient manufacturing processes.

Disadvantages

The main disadvantage of the screwed joints is the stress concentration in the threaded portions which are vulnerable points under variable load conditions.

Note : The strength of the screwed joints is not comparable with that of riveted or welded joints.

Important Terms Used in Screw Threads

The following terms used in screw threads, as shown in Fig. 1, are important from the subject point of view:

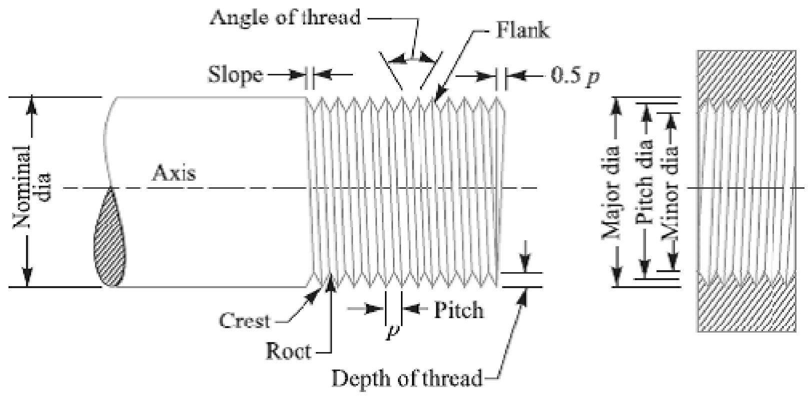


Fig.1 Terms used in screw threads

1. Major diameter. It is the largest diameter of an external or internal screw thread. The screw is specified by this diameter. It is also known as **outside** or **nominal diameter.**

2. Minor diameter. It is the smallest diameter of an external or internal screw thread. It is also known as **core** or **root diameter.**

3. Pitch diameter. It is the diameter of an imaginary cylinder, on a cylindrical screw thread, the surface of which would pass through the thread at such points as to make equal the width of the thread and the width of the spaces between the threads. It is also called an **effective diameter.** In a nut and bolt assembly, it is the diameter at which the ridges on the bolt are in complete touch with the ridges of the corresponding nut.

4. Pitch. It is the distance from a point on one thread to the corresponding point on the next. This is measured in an axial direction between corresponding points in the same axial plane. Mathematically,

$$\text{Pitch} = \frac{1}{\text{No. of threads per unit length of screw}}$$

5. Lead. It is the distance between two corresponding points on the same helix. It may also be defined as the distance which a screw thread advances axially in one rotation of the nut. Lead is equal to the pitch in case of single start threads, it is twice the pitch in double start, thrice the pitch in triple start and so on.

6. Crest. It is the top surface of the thread.

7. Root. It is the bottom surface created by the two adjacent flanks of the thread.

8. Depth of thread. It is the perpendicular distance between the crest and root.

9. Flank. It is the surface joining the crest and root.

10. Angle of thread. It is the angle included by the flanks of the thread.

11. Slope. It is half the pitch of the thread.

Contents: Design of bolted joints under eccentric loading-1

Eccentric Load Acting Parallel to the Axis of Bolts

Consider a bracket having a rectangular base bolted to a wall by means of four bolts as shown in Fig.1. A little consideration will show that each bolt is subjected to a direct tensile load of

$$W_{t1} = \frac{W}{n}, \text{ where } n \text{ is the number of bolts.}$$

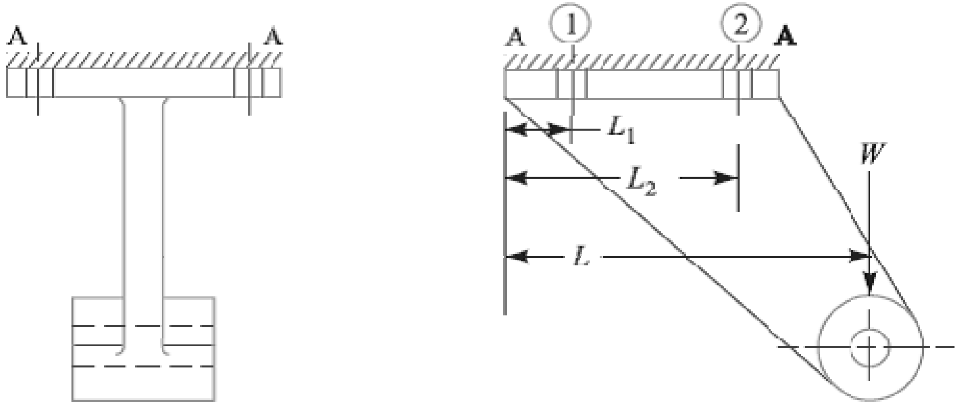


Fig.1. Eccentric load acting parallel to the axis of bolts.

Further the load *W* tends to rotate the bracket about the edge *A-A*. Due to this, each bolt is stretched by an amount that depends upon its distance from the tilting edge. Since the stress is a function of elongation, therefore each bolt will experience a different load which also depends upon the distance from the tilting edge. For convenience, all the bolts are made of same size. In case the flange is heavy, it may be considered as a rigid body.

Let w be the load in a bolt per unit distance due to the turning effect of the bracket and let W_1 and W_2 be the loads on each of the bolts at distances L_1 and L_2 from the tilting edge.

Load on each bolt at distance L_1,

$$W_1 = w.L_1$$

And moment of this load about the tilting edge

$$= w.L_1 \times L_1 = w\,(L_1)^2$$

Similarly, load on each bolt at distance L_2,

$$W_2 = w.L_2$$

And moment of this load about the tilting edge

$$= w.L_2 \times L_2 = w\,(L_2)^2$$

So, Total moment of the load on the bolts about the tilting edge

$$= 2w\,(L_1)^2 + 2w\,(L_2)^2 \quad \textbf{...(\textit{i})}$$

... (Since, there are two bolts each at distance of L_1 and L_2)

Also the moment due to load W about the tilting edge

$$= W.L \quad \textbf{... (ii)}$$

From equations **(*i*)** and **(*ii*)**, we have

$$W.L = 2w\,(L_1)^2 + 2w(L_2)^2 \qquad \text{or} \qquad w = \frac{W.L}{2\,[(L_1)^2 + (L_2)^2]} \quad \text{...(iii)}$$

It may be noted that the most heavily loaded bolts are those which are situated at the greatest distance from the tilting edge. In the case discussed above, the bolts at distance *L*2 are heavily loaded.

So, Tensile load on each bolt at distance L_2,

$$W_{t2} = W_2 = w.L_2 = \frac{W.L.L_2}{2[(L_1)^2 + (L_2)^2]} \quad \text{... [From equation (iii)]}$$

And the total tensile load on the most heavily loaded bolt,

$$W_t = W_{t1} + W_{t2} \quad \textbf{... (iv)}$$

If d_c is the core diameter of the bolt and σt is the tensile stress for the bolt material, then total tensile load,

$$W_t = \frac{\pi}{4}\,(d_c)^2\,\sigma_t \quad \text{...(v)}$$

From equations **(*iv*)** and **(*v*),** the value of *dc* may be obtained.

Problem:

A bracket, as shown in Fig.1, supports a load of 30 kN. Determine the size of bolts, if the maximum allowable tensile stress in the bolt material is 60 MPa. The distances are: L_1 = 80 mm, L_2 = 250 mm, and L = 500 mm.

Solution. Given : W = 30 kN ; σ_t = 60 MPa = 60 N/mm^2 ; L_1 = 80 mm ; L_2 = 250 mm ; L = 500 mm

We know that the direct tensile load carried by each bolt,

$$W_{t1} = \frac{W}{n} = \frac{30}{4} = 7.5 \text{ kN}$$

and load in a bolt per unit distance,

$$w = \frac{W.L}{2\,[(L_1)^2 + (L_2)^2]} = \frac{30 \times 500}{2\,[(80)^2 + (250)^2]} = 0.109 \text{ kN/mm}$$

Since the heavily loaded bolt is at a distance of L_2 mm from the tilting edge, therefore load on the heavily loaded bolt,

$$W_{t2} = w.L_2 = 0.109 \times 250 = 27.25 \text{ kN}$$

∴ Maximum tensile load on the heavily loaded bolt,

$$W_t = W_{t1} + W_{t2} = 7.5 + 27.25 = 34.75 \text{ kN} = 34\ 750 \text{ N}$$

Let d_c = Core diameter of the bolts.

We know that the maximum tensile load on the bolt (W_t),

$$34\ 750 = \frac{\pi}{4}(d_c)^2\ \sigma_t = \frac{\pi}{4}(d_c)^2\ 60 = 47\ (d_c)^2$$

$$\therefore \quad (d_c)^2 = 34\ 750 / 47 = 740$$

or $$d_c = 27.2 \text{ mm}$$

From DDB (coarse series), we find that the standard core diameter of the bolt is 28.706 mm and the corresponding size of the bolt is M 33. **Ans.**

Eccentric Load Acting Perpendicular to the Axis of Bolts

A wall bracket carrying an eccentric load perpendicular to the axis of the bolts is shown in Fig.2.

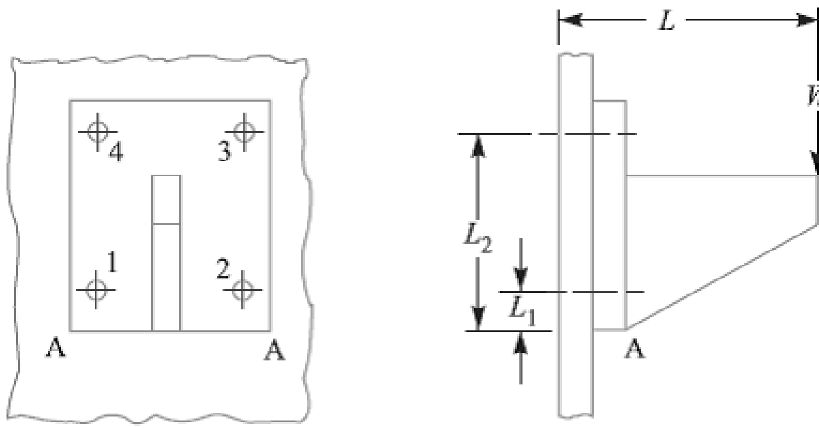


Fig. 2. Eccentric load perpendicular to the axis of bolts.

In this case, the bolts are subjected to direct shearing load which is equally shared by all the bolts. Therefore direct shear load on each bolts,

$W_s = W/n$, where n is number of bolts.

A little consideration will show that the eccentric load W will try to tilt the bracket in the clockwise direction about the edge A-A. As discussed earlier, the bolts will be subjected to tensile stress due to the turning moment. The maximum tensile load on a heavily loaded bolt (W_t) may be obtained in the similar manner as discussed in the previous article. In this case, bolts 3 and 4 are heavily loaded.

Maximum tensile load on bolt 3 or 4,

$$W_{t2} = W_t = \frac{W.L.L_2}{2\left[(L_1)^2 + (L_2)^2\right]}$$

When the bolts are subjected to shear as well as tensile loads, then the equivalent loads may be determined by the following relations:

Equivalent tensile load,

$$W_{te} = \frac{1}{2}\left[W_t + \sqrt{(W_t)^2 + 4(W_s)^2}\right]$$

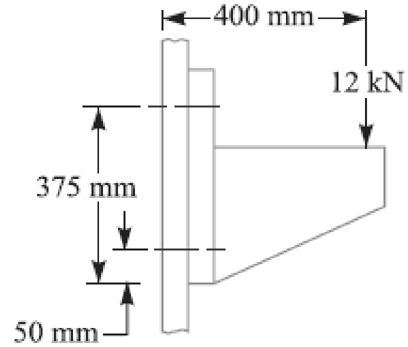
And equivalent shear load,

$$W_{se} = \frac{1}{2}\left[\sqrt{(W_t)^2 + 4(W_s)^2}\right]$$

Knowing the value of equivalent loads, the size of the bolt may be determined for the given allowable stresses.

Problem:

For supporting the travelling crane in a workshop, the brackets are fixed on steel columns as shown in Fig. The maximum load that comes on the bracket is 12 kN acting vertically at a distance of 400 mm from the face of the column. The vertical face of the bracket is secured to a column by four bolts, in two rows (two in each row) at a distance of 50 mm from the lower edge of the bracket. Determine the size of the bolts if the permissible value of the tensile stress for the bolt material is 84 MPa. Also find the cross-section of the arm of the bracket which is rectangular.



Solution. Given : $W = 12$ kN $= 12 \times 10^3$ N ; $L = 400$ mm ; $L_1 = 50$ mm ; $L_2 = 375$ mm ; $\sigma_t = 84$ MPa $= 84$ N/mm^2 ; $n = 4$

We know that direct shear load on each bolt,

$$W_s = \frac{W}{n} = \frac{12}{4} = 3 \text{ kN}$$

Since the load W will try to tilt the bracket in the clockwise direction about the lower edge, therefore the bolts will be subjected to tensile load due to turning moment. The maximum loaded bolts are 3 and 4 (See Fig.1), because they lie at the greatest distance from the tilting edge A–A (*i.e.* lower edge).

We know that maximum tensile load carried by bolts 3 and 4,

$$W_t = \frac{W.L.L_2}{2\,[(L_1)^2 + (L_2)^2]} = \frac{12 \times 400 \times 375}{2\,[(50)^2 + (375)^2]} = 6.29 \text{ kN}$$

Since the bolts are subjected to shear load as well as tensile load, therefore equivalent tensile load,

$$W_{te} = \frac{1}{2}\left[W_t + \sqrt{(W_t)^2 + 4(W_s)^2}\right] = \frac{1}{2}\left[6.29 + \sqrt{(6.29)^2 + 4 \times 3^2}\right] \text{kN}$$

$$= \frac{1}{2}(6.29 + 8.69) = 7.49 \text{ kN} = 7490 \text{ N}$$

Size of the bolt

Let d_c = Core diameter of the bolt.

We know that the equivalent tensile load (W_{te}),

$$7490 = \frac{\pi}{4}(d_c)^2\,\sigma_t = \frac{\pi}{4}(d_c)^2\,84 = 66\,(d_c)^2$$

$\therefore$ $(d_c)^2 = 7490 / 66 = 113.5$ or $d_c = 10.65$ mm

From Table 11.1 (coarse series), the standard core diameter is 11.546 mm and the corresponding size of the bolt is M 14. **Ans.**

Cross-section of the arm of the bracket

Let t and b = Thickness and depth of arm of the bracket respectively.

$\therefore$ Section modulus,

$$Z = \frac{1}{6}\,t\,b^2$$

Assume that the arm of the bracket extends upto the face of the steel column. This assumption gives stronger section for the arm of the bracket.

$\therefore$ Maximum bending moment on the bracket,

$$M = 12 \times 10^3 \times 400 = 4.8 \times 10^6 \text{ N-mm}$$

We know that the bending (tensile) stress (σ_t),

$$84 = \frac{M}{Z} = \frac{4.8 \times 10^6 \times 6}{t.b^2} = \frac{28.8 \times 10^6}{t.b^2}$$

$\therefore$ $t.b^2 = 28.8 \times 10^6 / 84 = 343 \times 10^3$ or $t = 343 \times 10^3 / b^2$

Assuming depth of arm of the bracket, $b = 250$ mm, we have

$$t = 343 \times 10^3 / (250)^2 = 5.5 \text{ mm } \textbf{Ans.}$$

Eccentric Load Acting in the Plane Containing the Bolts

When the eccentric load acts in the plane containing the bolts, as shown in Fig.1, then the same procedure may be followed as discussed for eccentric loaded riveted joints.

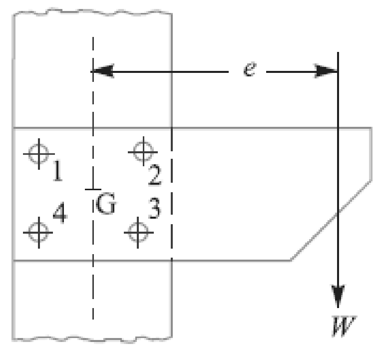


Fig. 1. Eccentric load in the plane containing the bolts.

Problem:

Fig.2 shows a solid forged bracket to carry a vertical load of 13.5 kN applied through the centre of hole. The square flange is secured to the flat side of a vertical stanchion through four bolts. Calculate suitable diameter D and d for the arms of the bracket, if the permissible stresses are 110 MPa in tenstion and 65 MPa in shear. Estimate also the tensile load on each top bolt and the maximum shearing force on each bolt.

Solution. Given : $W = 13.5$ kN = 13 500 N ; $\sigma_t = 110$ MPa = 110 N/mm^2 ; $\tau = 65$ MPa = 65 N/mm^2

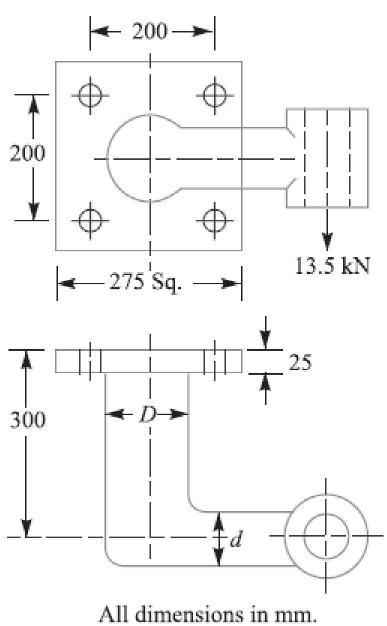


Fig.2

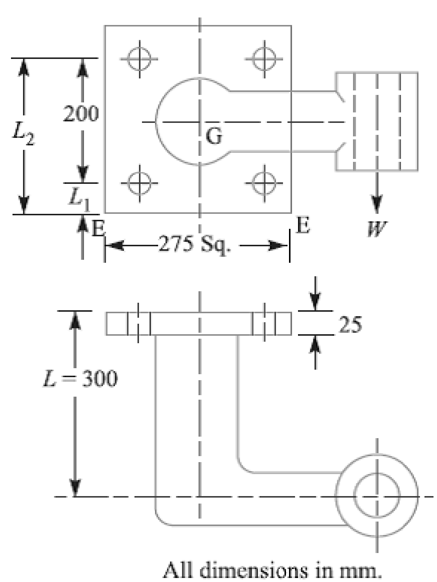


Fig.3

Diameter D for the arm of the bracket

The section of the arm having D as the diameter is subjected to bending moment as well as twisting moment. We know that bending moment,

$$M = 13\ 500 \times (300 - 25) = 3712.5 \times 10^3 \text{ N-mm}$$

and twisting moment, $T = 13\ 500 \times 250 = 3375 \times 10^3$ N-mm

$\therefore$ Equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3712.5 \times 10^3)^2 + (3375 \times 10^3)^2} \text{ N-mm}$$
$$= 5017 \times 10^3 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$5017 \times 10^3 = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 65 \times D^3 = 12.76\, D^3$$

$\therefore$ $D^3 = 5017 \times 10^3 / 12.76 = 393 \times 10^3$

or $D = 73.24$ say 75 mm **Ans.**

Diameter (d) for the arm of the bracket

The section of the arm having d as the diameter is subjected to bending moment only. We know that bending moment,

$$M = 13\ 500 \left(250 - \frac{75}{2}\right) = 2868.8 \times 10^3 \text{ N-mm}$$

and section modulus, $Z = \dfrac{\pi}{32} \times d^3 = 0.0982\, d^3$

We know that bending (tensile) stress (σ_t),

$$110 = \frac{M}{Z} = \frac{2868.8 \times 10^3}{0.0982\, d^3} = \frac{29.2 \times 10^6}{d^3}$$

$\therefore$ $d^3 = 29.2 \times 10^6 / 110 = 265.5 \times 10^3$ or $d = 64.3$ say 65 mm **Ans.**

Tensile load on each top bolt

Due to the eccentric load W, the bracket has a tendency to tilt about the edge E–E, as shown in Fig. 11.46.

Let w = Load on each bolt per mm distance from the tilting edge due to the tilting effect of the bracket.

Since there are two bolts each at distance L_1 and L_2 as shown in Fig. 11.46, therefore total moment of the load on the bolts about the tilting edge E–E

$$= 2\,(w.L_1)\,L_1 + 2(w.L_2)\,L_2 = 2w\,[(L_1)^2 + (L_2)^2]$$
$$= 2w\,[(37.5)^2 + (237.5)^2] = 115\ 625\, w \text{ N-mm} \quad ...(i)$$

...($\because$ $L_1 = 37.5$ mm and $L_2 = 237.5$ mm)

and turning moment of the load about the tilting edge

$$= W.L = 13\ 500 \times 300 = 4050 \times 10^3 \text{ N-mm} \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$w = 4050 \times 10^3 / 115\ 625 = 35.03 \text{ N/mm}$$

$\therefore$ Tensile load on each top bolt

$$= w.L_2 = 35.03 \times 237.5 = 8320 \text{ N } \textbf{Ans.}$$

Maximum shearing force on each bolt

We know that primary shear load on each bolt acting vertically downwards,

$$W_{s1} = \frac{W}{n} = \frac{13\,500}{4} = 3375 \text{ N} \qquad \text{...(}\because \text{ No. of bolts, } n = 4\text{)}$$

Since all the bolts are at equal distances from the centre of gravity of the four bolts (G), therefore the secondary shear load on each bolt is same.

Distance of each bolt from the centre of gravity (G) of the bolts,

$$l_1 = l_2 = l_3 = l_4 = \sqrt{(100)^2 + (100)^2} = 141.4 \text{ mm}$$

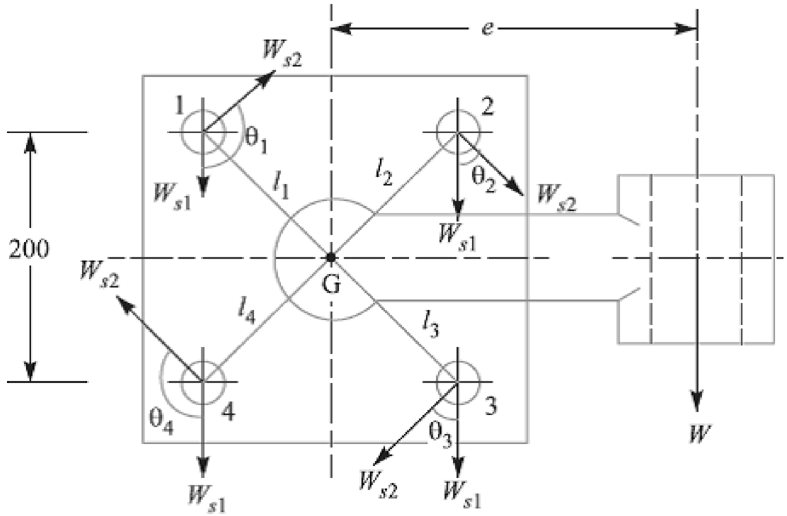


Fig.4

$\therefore$ Secondary shear load on each bolt,

$$W_{s2} = \frac{W.e.l_1}{(l_1)^2 + (l_2)^2 + (l_3)^3 + (l_4)^2} = \frac{13\,500 \times 250 \times 141.4}{4\,(141\;4)^2} = 5967 \text{ N}$$

Since the secondary shear load acts at right angles to the line joining the centre of gravity of the bolt group to the centre of the bolt as shown in Fig. 4, therefore the resultant of the primary and secondary shear load on each bolt gives the maximum shearing force on each bolt. From the geometry of the Fig. 4, we find that

$$\theta 1 = \theta 4 = 135°, \text{ and } \theta 2 = \theta 3 = 45°$$

Maximum shearing force on the bolts 1 and 4

$$= \sqrt{(W_{s1})^2 + (W_{s2})^2 + 2\,W_{s1} \times W_{s2} \times \cos 135°}$$

$$= \sqrt{(3375)^2 + (5967)^2 - 2 \times 3375 \times 5967 \times 0.7071} = 4303 \text{ N } \textbf{Ans.}$$

And maximum shearing force on the bolts 2 and 3

$$= \sqrt{(W_{s1})^2 + (W_{s2})^2 + 2\, W_{s1} \times W_{s2} \times \cos 45°}$$

$$= \sqrt{(3375)^2 + (5967)^2 + 2 \times 3375 \times 5967 \times 0.7071} = 8687 \text{ N}$$ **Ans.**

Introduction to Welded Joints

Introduction

A welded joint is a permanent joint which is obtained by the fusion of the edges of the two parts to be joined together, with or without the application of pressure and a filler material. The heat required for the fusion of the material may be obtained by burning of gas (in case of gas welding) or by an electric arc (in case of electric arc welding). The latter method is extensively used because of greater speed of welding. Welding is extensively used in fabrication as an alternative method for casting or forging and as a replacement for bolted and riveted joints. It is also used as a repair medium *e.g.* to reunite metal at a crack, to build up a small part that has broken off such as gear tooth or to repair a worn surface such as a bearing surface.

Advantages and Disadvantages of Welded Joints over Riveted Joints

Following are the advantages and disadvantages of welded joints over riveted joints.

Advantages

1. The welded structures are usually lighter than riveted structures. This is due to the reason, that in welding, gussets or other connecting components are not used.

2. The welded joints provide maximum efficiency (may be 100%) which is not possible in case of riveted joints.

3. Alterations and additions can be easily made in the existing structures.

4. As the welded structure is smooth in appearance, therefore it looks pleasing.

5. In welded connections, the tension members are not weakened as in the case of riveted joints.

6. A welded joint has a great strength. Often a welded joint has the strength of the parent metal itself.

7. Sometimes, the members are of such a shape (*i.e.* circular steel pipes) that they afford difficulty for riveting. But they can be easily welded.

8. The welding provides very rigid joints. This is in line with the modern trend of providing rigid frames.

9. It is possible to weld any part of a structure at any point. But riveting requires enough clearance.

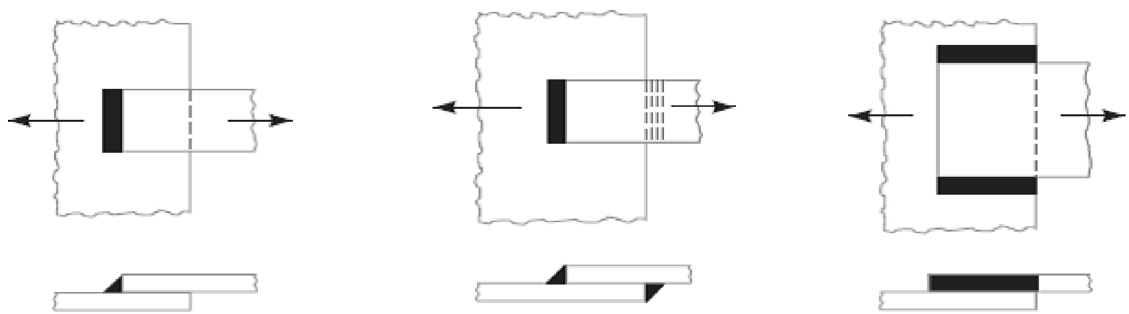
10. The process of welding takes less time than the riveting.

Disadvantages

1. Since there is an uneven heating and cooling during fabrication, therefore the members may get distorted or additional stresses may develop.
2. It requires a highly skilled labour and supervision.
3. Since no provision is kept for expansion and contraction in the frame, therefore there is a possibility of cracks developing in it.
4. The inspection of welding work is more difficult than riveting work.

<u>Types of Welded Joints</u>

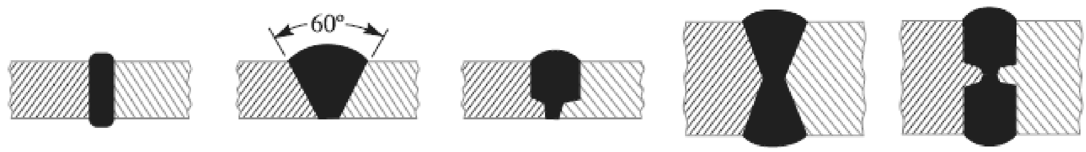
Following two types of welded joints are important from the subject point of view:
1. Lap joint or fillet joint, and **2.** Butt joint.



(*a*) Single transverse. (*b*) Double transverse. (*c*) Parallel fillet.

Fig.1. Types of Lab and Butt Joints

<u>Lap Joint</u>
The lap joint or the fillet joint is obtained by overlapping the plates and then welding the edges of the plates. The cross-section of the fillet is approximately triangular. The fillet joints may be
1. Single transverse fillet, **2.** Double transverse fillet and **3.** Parallel fillet joints.
The fillet joints are shown in Fig.1. A single transverse fillet joint has the disadvantage that the edge of the plate which is not welded can buckle or warp out of shape.
<u>Butt Joint</u>
The butt joint is obtained by placing the plates edge to edge as shown in Fig.2. In butt welds, the plate edges do not require beveling if the thickness of plate is less than 5 mm. On the other hand, if the plate thickness is 5 mm to 12.5 mm, the edges should be beveled to V or U-groove on both sides.



(a) Square butt joint. (b) Single V-butt joint. (c) Single U-butt joint. (d) Double V-butt joint. (e) Double U-butt joint.

Fig. 2. Types of Butt joints

The butt joints may be

1. Square butt joint, **2.** Single V-butt joint **3.** Single U-butt joint,

4. Double V-butt joint, and **5.** Double U-butt joint.

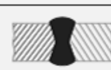
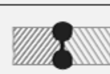
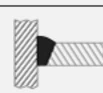
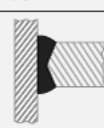
These joints are shown in Fig. 2.

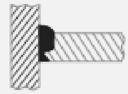
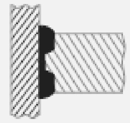
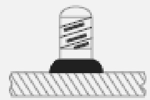
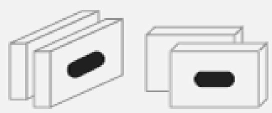
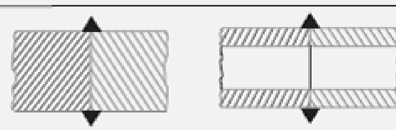
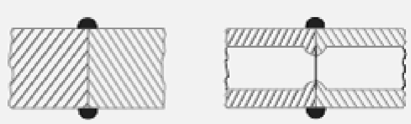
The other type of welded joints are corner joint, edge joint and T-joint as shown in Fig. 3.

(a) Corner joint. (b) Edge joint. (c) T-joint.

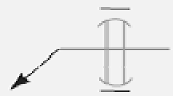
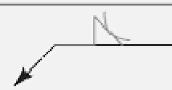
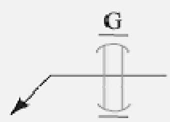
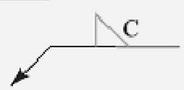
Fig. 3. Other types of Joints

Basic Weld Symbols

S. No.	Form of weld	Sectional representation	Symbol
1.	Fillet		
2.	Square butt		
3.	Single-V butt		
4.	Double-V butt		
5.	Single-U butt		
6.	Double-U butt		
7.	Single bevel butt		
8.	Double bevel butt		

S. No.	*Form of weld*	*Sectional representation*	*Symbol*
9.	Single-*J* butt		
10.	Double-*J* butt		
11.	Bead (edge or seal)		
12.	Stud		⊥
13.	Sealing run		○
14.	Spot		
15.	Seam		
16.	Mashed seam		
17.	Plug		
18.	Backing strip		=
19.	Stitch		
20.	Projection		△
21.	Flash		
22.	Butt resistance or pressure (upset)		|

Supplementary Weld Symbols

S. No.	*Particulars*	*Drawing representation*	*Symbol*
1.	Weld all round		○
2.	Field weld		●
3.	Flush contour		—
4.	Convex contour		
5.	Concave contour		
6.	Grinding finish		G
7.	Machining finish		M
8.	Chipping finish		C

Elements of a welding symbol

Elements of a Welding Symbol

A welding symbol consists of the following eight elements:

1. Reference line, **2.** Arrow,

3. Basic weld symbols, **4.** Dimensions and other data,

5. Supplementary symbols, **6.** Finish symbols,

7. Tail, and **8.** Specification, process or other references.

Standard Location of Elements of a Welding Symbol

The arrow points to the location of weld, the basic symbols with dimensions are located on one or both sides of reference line. The specification if any is placed in the tail of arrow. Fig. 1. shows the standard locations of welding symbols represented on drawing.

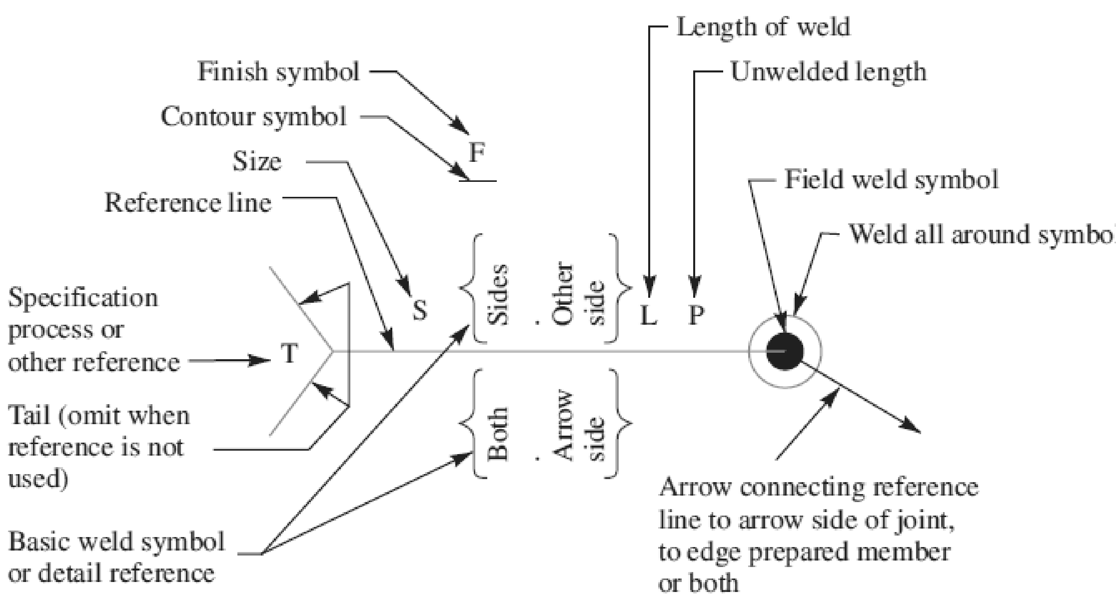
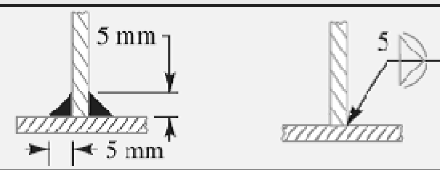
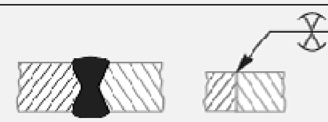
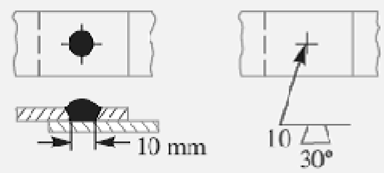
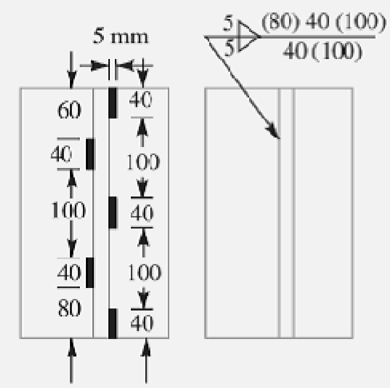


Fig.1 Standard location of weld symbols.

Some of the examples of welding symbols represented on drawing are shown in the following table.

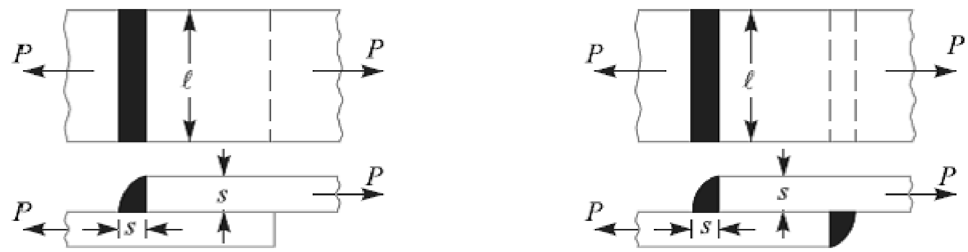
Representation of welding symbols.

S. No.	*Desired weld*	*Representation on drawing*
1.	Fillet-weld each side of Tee- convex contour	
2.	Single V-butt weld -machining finish	
3.	Double V- butt weld	
4.	Plug weld - 30° Groove-angle-flush contour	
5.	Staggered intermittent fillet welds	

Contents: Design of Welded Joints

Strength of Transverse Fillet Welded Joints

We have already discussed that the fillet or lap joint is obtained by overlapping the plates and then welding the edges of the plates. The transverse fillet welds are designed for tensile strength. Let us consider a single and double transverse fillet welds as shown in Fig. 1(*a*) and (*b*) respectively.



(*a*) Single transverse fillet weld. (*b*) Double transverse fillet weld.

Fig.1 Transverse fillet welds.

The length of each side is known as ***leg*** or ***size of the weld*** and the perpendicular distance of the hypotenuse from the intersection of legs (*i.e.* *BD*) is known as ***throat thickness.*** The minimum area of the weld is obtained at the throat *BD*, which is given by the product of the throat thickness and length of weld.

Let t = Throat thickness (*BD*),

s = Leg or size of weld,

= Thickness of plate, and

l = Length of weld,

From Fig.2, we find that the throat thickness,

$$t = s \times \sin 45° = 0.707\ s$$

Therefore, Minimum area of the weld or throat area,

$$A = \text{Throat thickness} \times \text{Length of weld}$$

$$= t \times l = 0.707\ s \times l$$

If σ_t is the allowable tensile stress for the weld metal, then the tensile strength of the joint for single fillet weld,

$$P = \text{Throat area} \times \text{Allowable tensile stress} = 0.707\ s \times l \times \sigma_t$$

And tensile strength of the joint for double fillet weld,

$$P = 2 \times 0.707\ s \times l \times \sigma_t = 1.414\ s \times l \times \sigma_t$$

Note: Since the weld is weaker than the plate due to slag and blow holes, therefore the weld is given a reinforcement which may be taken as 10% of the plate thickness.

Strength of Parallel Fillet Welded Joints

The parallel fillet welded joints are designed for shear strength. Consider a double parallel fillet welded joint as shown in Fig.3 (*a*). We have already discussed in the previous article, that the minimum area of weld or the throat area,

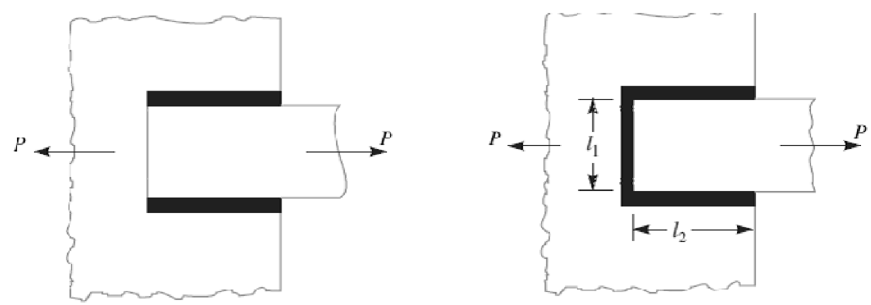
$$A = 0.707\, s \times l$$

If τ is the allowable shear stress for the weld metal, then the shear strength of the joint for single parallel fillet weld,

$$P = \text{Throat area} \times \text{Allowable shear stress} = 0.707\, s \times l \times \tau$$

And shear strength of the joint for double parallel fillet weld,

$$P = 2 \times 0.707 \times s \times l \times \tau = 1.414\, s \times l \times \tau$$



(*a*) Double parallel fillet weld.

(*b*) Combination of transverse and parallel fillet weld.

Fig.3

Notes: 1. If there is a combination of single transverse and double parallel fillet welds as shown in Fig. (*b*), then the strength of the joint is given by the sum of strengths of single transverse and double parallel fillet welds. Mathematically,

$$P = 0.707 s \times l_1 \times \sigma_t + 1.414\, s \times l_2 \times \tau$$

Where l_1 is normally the width of the plate.

2. In order to allow for starting and stopping of the bead, 12.5 mm should be added to the length of each weld obtained by the above expression.

3. For reinforced fillet welds, the throat dimension may be taken as 0.85 *t*.

Problem:

A plate 100 mm wide and 10 mm thick is to be welded to another plate by means of double parallel fillets. The plates are subjected to a static load of 80 kN. Find the length of weld if the permissible shear stress in the weld does not exceed 55 MPa.

Solution. Given: *Width = 100 mm ;
Thickness = 10 mm ; P = 80 kN = 80×10^3 N ;
τ = 55 MPa = 55 N/mm^2

Let l = Length of weld, and

s = Size of weld = Plate thickness = 10 mm ... (Given)

We know that maximum load which the plates can carry for double parallel fillet weld (P),

$$80 \times 10^3 = 1.414 \times s \times l \times \tau = 1.414 \times 10 \times l \times 55 = 778\, l$$

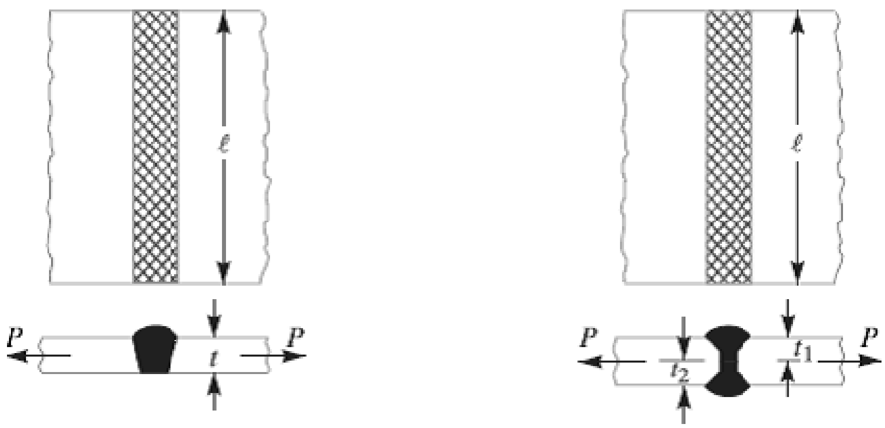
$$\therefore \quad l = 80 \times 10^3 / 778 = 103 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l = 103 + 12.5 = 115.5 \text{ mm} \quad \textbf{Ans.}$$

Strength of Butt Joints

The butt joints are designed for tension or compression. Consider a single V-butt joint as shown in Fig. 4(*a*).



(*a*) Single *V*-butt joint. (*b*) Double *V*-butt joint.

Fig.4. Butt Joints

In case of butt joint, the length of leg or size of weld is equal to the throat thickness which is equal to thickness of plates. Therefore, Tensile strength of the butt joint (single-*V* or square butt joint),

$$P = t \times l \times \sigma_1$$

Where l = Length of weld. It is generally equal to the width of plate. And tensile strength for double-*V* butt joint as shown in Fig. 4(*b*) is given by

$$P = (t_1 + t_2)\, l \times \sigma_t$$

Where t_1 = Throat thickness at the top, and
t_2 = Throat thickness at the bottom.

It may be noted that size of the weld should be greater than the thickness of the plate, but it may be less. The following table shows recommended minimum size of the welds.

Stresses for Welded Joints

The stresses in welded joints are difficult to determine because of the variable and unpredictable parameters like homogenuity of the weld metal, thermal stresses in the welds, changes of physical properties due to high rate of cooling etc. The stresses are obtained, on the following assumptions:

1. The load is distributed uniformly along the entire length of the weld, and
2. The stress is spread uniformly over its effective section.

The following table shows the stresses for welded joints for joining ferrous metals with mild steel electrode under steady and fatigue or reversed load.

Stress Concentration Factor for Welded Joints

The reinforcement provided to the weld produces stress concentration at the junction of the weld and the parent metal. When the parts are subjected to fatigue loading, the stress concentration factors should be taken into account.

Problem:

A plate 100 mm wide and 12.5 mm thick is to be welded to another plate by means of parallel fillet welds. The plates are subjected to a load of 50 kN. Find the length of the weld so that the maximum stress does not exceed 56 MPa. Consider the joint first under static loading and then under fatigue loading.

Solution. Given: *Width = 100 mm ; Thickness = 12.5 mm ; $P = 50$ kN $= 50 \times 10^3$ N ; $\tau = 56$ MPa $= 56$ N/mm^2

Length of weld for static loading

Let l = Length of weld, and

s = Size of weld = Plate thickness

= 12.5 mm ... (Given)

We know that the maximum load which the plates can carry for double parallel fillet welds (P),

$$50 \times 10^3 = 1.414\, s \times l \times \tau$$

$$= 1.414 \times 12.5 \times l \times 56 = 990\, l$$

$$\therefore \quad l = 50 \times 10^3 / 990 = 50.5 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l = 50.5 + 12.5 = 63 \text{ mm}$$ **Ans.**

Length of weld for fatigue loading

From Table 10.6, we find that the stress concentration factor for parallel fillet welding is 2.7.

$\therefore$ Permissible shear stress,

$$\tau = 56 / 2.7 = 20.74 \text{ N/mm}^2$$

We know that the maximum load which the plates can carry for double parallel fillet welds (P),

$$50 \times 10^3 = 1.414\, s \times l \times \tau = 1.414 \times 12.5 \times l \times 20.74 = 367\, l$$

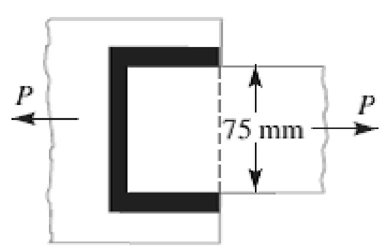
$$\therefore \quad l = 50 \times 10^3 / 367 = 136.2 \text{ mm}$$

Adding 12.5 for starting and stopping of weld run, we have

$$l = 136.2 + 12.5 = 148.7 \text{ mm}$$ **Ans.**

Problem:

A plate 75 mm wide and 12.5 mm thick is joined with another plate by a single transverse weld and a double parallel fillet weld as shown in Fig. The maximum tensile and shear stresses are 70 MPa and 56 MPa respectively. Find the length of each parallel fillet weld, if the joint is subjected to both static and fatigue loading.



Solution. Given : Width = 75 mm ; Thickness = 12.5 mm ;
$\sigma_\tau = 70$ MPa = 70 N/mm^2 ; $\tau = 56$ MPa = 56 N/mm^2.

The effective length of weld (l_1) for the transverse weld may be obtained by subtracting 12.5 mm from the width of the plate.

$$\therefore \quad l_1 = 75 - 12.5 = 62.5 \text{ mm}$$

Length of each parallel fillet for static loading

Let $\quad l_2$ = Length of each parallel fillet.

We know that the maximum load which the plate can carry is

$$P = \text{Area} \times \text{Stress} = 75 \times 12.5 \times 70 = 65\ 625 \text{ N}$$

Load carried by single transverse weld,

$$P_1 = 0.707\, s \times l_1 \times \sigma_t = 0.707 \times 12.5 \times 62.5 \times 70 = 38\ 664 \text{ N}$$

and the load carried by double parallel fillet weld,

$$P_2 = 1.414\, s \times l_2 \times \tau = 1.414 \times 12.5 \times l_2 \times 56 = 990\, l_2 \text{ N}$$

$\therefore$ Load carried by the joint (P),

$$65\ 625 = P_1 + P_2 = 38\ 664 + 990\, l_2 \quad \text{or} \quad l_2 = 27.2 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l_2 = 27.2 + 12.5 = 39.7 \text{ say } 40 \text{ mm} \quad \textbf{Ans.}$$

Length of each parallel fillet for fatigue loading

From Table 10.6, we find that the stress concentration factor for transverse welds is 1.5 and for parallel fillet welds is 2.7.

$\therefore$ Permissible tensile stress,

$$\sigma_t = 70 / 1.5 = 46.7 \text{ N/mm}^2$$

and permissible shear stress,

$$\tau = 56 / 2.7 = 20.74 \text{ N/mm}^2$$

Load carried by single transverse weld,

$$P_1 = 0.707\, s \times l_1 \times \sigma_t = 0.707 \times 12.5 \times 62.5 \times 46.7 = 25\ 795 \text{ N}$$

and load carried by double parallel fillet weld,

$$P_2 = 1.414\, s \times l_2 \times \tau = 1.414 \times 12.5\, l_2 \times 20.74 = 366\, l_2 \text{ N}$$

$\therefore$ Load carried by the joint (P),

$$65\ 625 = P_1 + P_2 = 25\ 795 + 366\, l_2 \quad \text{or} \quad l_2 = 108.8 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l_2 = 108.8 + 12.5 = 121.3 \text{ mm} \quad \textbf{Ans.}$$

Special Cases of Fillet Welded Joints

The following cases of fillet welded joints are important from the subject point of view.

1. ***Circular fillet weld subjected to torsion.*** Consider a circular rod connected to a rigid plate by a fillet weld as shown in Fig. 1.

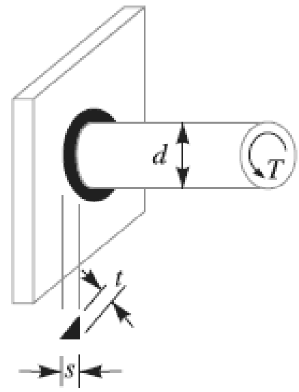


Fig. 1. Circular fillet weld subjected to torsion.

Let d = Diameter of rod,

r = Radius of rod,

T = Torque acting on the rod,

s = Size (or leg) of weld,

t = Throat thickness,

J = Polar moment of inertia of the

$$\text{weld section} = \frac{\pi t d^3}{4}$$

We know that shear stress for the material,

$$\tau = \frac{T.r}{J} = \frac{T \times d/2}{J}$$

$$= \frac{T \times d/2}{\pi t d^3/4} = \frac{2T}{\pi t d^2}$$

This shear stress occurs in a horizontal plane along a leg of the fillet weld. The maximum shear occurs on the throat of weld which is inclined at 45° to the horizontal plane.

Length of throat, $t = s \sin 45° = 0.707\, s$ and maximum shear stress,

$$\tau_{max} = \frac{2T}{\pi \times 0.707\, s \times d^2} = \frac{2.83\, T}{\pi\, s\, d^2}$$

2. Circular fillet weld subjected to bending moment. Consider a circular rod connected to a rigid plate by a fillet weld as shown in Fig.2.

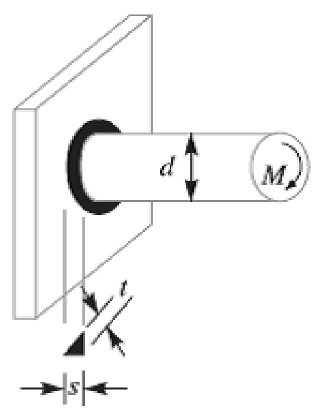


Fig.2.Circular fillet weld subjected to Bending moment.

Let d = Diameter of rod,

M = Bending moment acting on the rod,

s = Size (or leg) of weld,

t = Throat thickness,

Z = Section modulus of the weld section

$$= \frac{\pi t d^2}{4}$$

We know that the bending stress

$$\sigma_b = \frac{M}{Z} = \frac{M}{\pi t d^2 / 4} = \frac{4M}{\pi t d^2}$$

This bending stress occurs in a horizontal plane along a leg of the fillet weld. The maximum bending stress occurs on the throat of the weld which is inclined at 45° to the horizontal plane.

Length of throat, t = s sin 45° = 0.707 s and maximum bending stress,

$$\sigma_{b(max)} = \frac{4\,M}{\pi \times 0.707\,s \times d^2} = \frac{5.66\,M}{\pi\,s\,d^2}$$

3. Long fillet weld subjected to torsion. Consider a vertical plate attached to a horizontal plate by two identical fillet welds as shown in Fig.3.

Let T = Torque acting on the vertical plate,

l = Length of weld,

s = Size (or leg) of weld,

t = Throat thickness, and

J = Polar moment of inertia of the weld section

$$= 2 \times \frac{t \times l^3}{12} = \frac{t \times l^3}{6} \ \ldots$$

It may be noted that the effect of the applied torque is to rotate the vertical plate about the Z-axis through its mid point. This rotation is resisted by shearing stresses developed between two fillet welds and the horizontal plate. It is assumed that these horizontal shearing stresses vary from zero at the Z-axis and maximum at the ends of the plate. This variation of shearing stress is analogous to the variation of normal stress over the depth (l) of a beam subjected to pure bending.

Therefore, Shear stress,

$$\tau = \frac{T \times l/2}{t \times l^3/6} = \frac{3\,T}{t \times l^2}$$

The maximum shear stress occurs at the throat and is given by

$$\tau_{max} = \frac{3T}{0.707\, s \times l^2} = \frac{4.242\, T}{s \times l^2}$$

Eccentrically Loaded Welded Joints

An eccentric load may be imposed on welded joints in many ways. The stresses induced on the joint may be of different nature or of the same nature. The induced stresses are combined depending upon the nature of stresses. When the shear and bending stresses are simultaneously present in a joint (see case 1), then maximum stresses are as follows:

Maximum normal stress,

$$\sigma_{t(max)} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

And Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Where σ_b = Bending stress, and

τ = Shear stress.

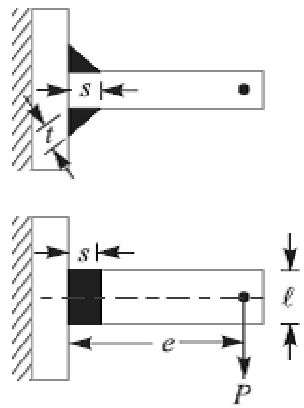


Fig.1. Eccentrically loaded welded joint

When the stresses are of the same nature, these may be combined vectorially (see case 2).

We shall now discuss the two cases of eccentric loading as follows:

Case 1

Consider a *T*-joint fixed at one end and subjected to an eccentric load *P* at a distance *e* as shown in Fig. 1

Let s = Size of weld,

l = Length of weld, and

t = Throat thickness.

The joint will be subjected to the following two types of stresses:

1. Direct shear stress due to the shear force *P* acting at the welds, and

2. Bending stress due to the bending moment $P \times e$.

We know that area at the throat,

$$A = \text{Throat thickness} \times \text{Length of weld}$$

$$= t \times l \times 2 = 2\,t \times l \text{ ... (For double fillet weld)}$$

$$= 2 \times 0.707\,s \times l = 1.414\,s \times l \text{ ... (since, } t = s\cos 45° = 0.707\,s)$$

Shear stress in the weld (assuming uniformly distributed),

$$\tau = \frac{P}{A} = \frac{P}{1.414\,s \times l}$$

Section modulus of the weld metal through the throat,

$$Z = \frac{t \times l^2}{6} \times 2 \qquad \text{...(For both sides weld)}$$

$$= \frac{0.707\, s \times l^2}{6} \times 2 = \frac{s \times l^2}{4.242}$$

Bending moment, $M = P \times e$

$$\therefore \text{Bending stress, } \sigma_b = \frac{M}{Z} = \frac{P \times e \times 4.242}{s \times l^2} = \frac{4.242\, P \times e}{s \times l^2}$$

We know that the maximum normal stress,

$$\sigma_{t(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2}$$

And maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2}$$

Case 2

When a welded joint is loaded eccentrically as shown in Fig.2, the following two types of the stresses are induced:

1. Direct or primary shear stress, and

2. Shear stress due to turning moment.

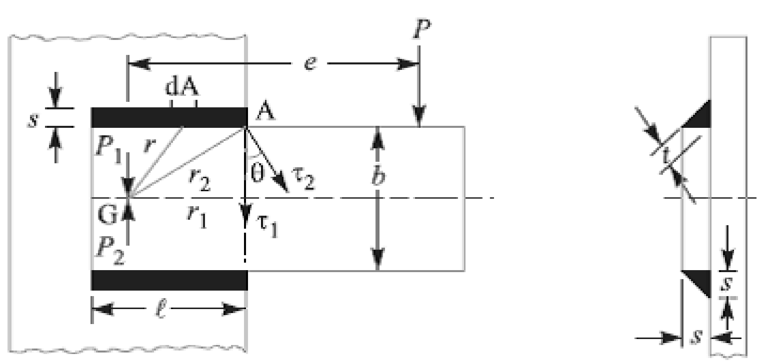


Fig.2 eccentrically loaded welded joint.

Let P = Eccentric load,

e = Eccentricity i.e. perpendicular distance between the line of action of load and centre of gravity (G) of the throat section or fillets,

l = Length of single weld,

s = Size or leg of weld, and

t = Throat thickness.

Let two loads P_1 and P_2 (each equal to P) are introduced at the centre of gravity 'G' of the weld system. The effect of load P_1 = P is to produce direct shear stress which is assumed to be uniform over the entire weld length. The effect of load P_2 = P is to produce a turning moment of magnitude P × e which tends of rotate the joint about the centre of gravity 'G' of the weld system. Due to the turning moment, secondary shear stress is induced.

We know that the direct or primary shear stress,

$$\tau_1 = \frac{\text{Load}}{\text{Throat area}} = \frac{P}{A} = \frac{P}{2\,t \times l}$$

$$= \frac{P}{2 \times 0.707\,s \times l} = \frac{P}{1.414\,s \times l}$$

Since the shear stress produced due to the turning moment (T = P × e) at any section is proportional to its radial distance from G, therefore stress due to P × e at the point A is proportional to AG (r2) and is in a direction at right angles to AG. In other words,

$$\frac{\tau_2}{r_2} = \frac{\tau}{r} = \text{Constant}$$

$$\tau = \frac{\tau_2}{r_2} \times r \qquad \text{...}(i)$$

Where τ_2 is the shear stress at the maximum distance (r_2) and τ is the shear stress at any distance r. Consider a small section of the weld having area dA at a distance r from G.

Shear force on this small section

$$= \tau \times dA$$

And turning moment of this shear force about G,

$$dT = \tau \times dA \times r = \frac{\tau_2}{r_2} \times dA \times r^2 \qquad \text{... [From equation } (i)]$$

Total turning moment over the whole weld area,

$$T = P \times e = \int \frac{\tau_2}{r_2} \times dA \times r^2 = \frac{\tau_2}{r_2} \int dA \times r^2$$

$$= \frac{\tau_2}{r_2} \times J \qquad \left(\because J = \int dA \times r^2\right)$$

Where J = Polar moment of inertia of the throat area about G.

Shear stress due to the turning moment i.e. secondary shear stress,

$$\tau_2 = \frac{T \times r_2}{J} = \frac{P \times e \times r_2}{J}$$

In order to find the resultant stress, the primary and secondary shear stresses are combined vectorially.

Resultant shear stress at A,

$$\tau_A = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\tau_1 \times \tau_2 \times \cos\theta}$$

θ = Angle between τ_1 and τ_2, and

$\cos\theta = r_1 / r_2$

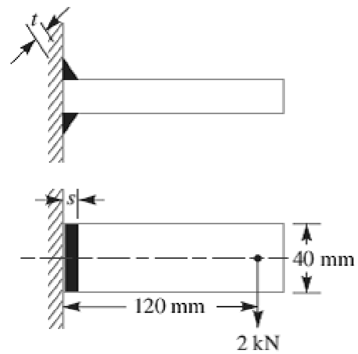
Problem:

A welded joint as shown in Fig. 10.24, is subjected to an eccentric load of 2 kN. Find the size of weld, if the maximum shear stress in the weld is 25 MPa.

Solution. Given: $P = 2\text{kN} = 2000$ N ; $e = 120$ mm ; $l = 40$ mm ; $\tau_{max} = 25$ MPa $= 25$ N/mm²

Let s = Size of weld in mm, and

t = Throat thickness.



The joint, as shown in Fig. 10.24, will be subjected to direct shear stress due to the shear force, $P = 2000$ N and bending stress due to the bending moment of $P \times e$.

We know that area at the throat,

$$A = 2t \times l = 2 \times 0.707\, s \times l$$
$$= 1.414\, s \times l$$
$$= 1.414\, s \times 40 = 56.56 \times s \text{ mm}^2$$

$\therefore$ Shear stress,
$$\tau = \frac{P}{A} = \frac{2000}{56.56 \times s} = \frac{35.4}{s} \text{ N/mm}^2$$

Bending moment, $M = P \times e = 2000 \times 120 = 240 \times 10^3$ N-mm

Section modulus of the weld through the throat,

$$Z = \frac{s \times l^2}{4.242} = \frac{s\,(40)^2}{4.242} = 377 \times s \text{ mm}^3$$

$\therefore$ Bending stress,
$$\sigma_b = \frac{M}{Z} = \frac{240 \times 10^3}{377 \times s} = \frac{636.6}{s} \text{ N/mm}^2$$

We know that maximum shear stress (τ_{max}),

$$25 = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} = \frac{1}{2}\sqrt{\left(\frac{636.6}{s}\right)^2 + 4\left(\frac{35.4}{s}\right)^2} = \frac{320.3}{s}$$

$\therefore$ $s = 320.3 / 25 = 12.8$ mm **Ans.**

Problem:

A bracket carrying a load of 15 kN is to be welded as shown in Fig. Find the size of weld required if the allowable shear stress is not to exceed 80 MPa.

Solution. Given : $P = 15$ kN $= 15 \times 10^3$ N ; $\tau = 80$ MPa $= 80$ N/mm²; $b = 80$ mm ; $l = 50$ mm; $e = 125$ mm

Let s = Size of weld in mm, and

t = Throat thickness.

We know that the throat area,

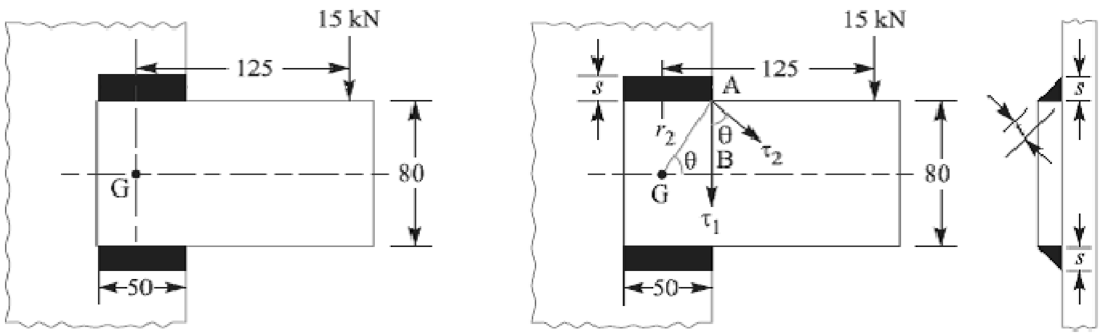
$$A = 2 \times t \times l = 2 \times 0.707\, s \times l$$
$$= 1.414\, s \times l = 1.414 \times s \times 50 = 70.7\, s \text{ mm}^2$$

∴ Direct or primary shear stress,

$$\tau_1 = \frac{P}{A} = \frac{15 \times 10^3}{70.7\, s} = \frac{212}{s} \text{ N/mm}^2$$

$$J = \frac{t.l\,(3b^2 + l^2)}{6} = \frac{0.707\, s \times 50\,[3\,(80)^2 + (50)^2]}{6} \text{ mm}^4$$

$$= 127\,850\, s \text{ mm}^4 \qquad \ldots (\because\ t = 0.707\, s)$$



All dimensions in mm.

∴ Maximum radius of the weld,

$$r_2 = \sqrt{(AB)^2 + (BG)^2} = \sqrt{(40)^2 + (25)^2} = 47 \text{ mm}$$

Shear stress due to the turning moment *i.e.* secondary shear stress,

$$\tau_2 = \frac{P \times e \times r_2}{J} = \frac{15 \times 10^3 \times 125 \times 47}{127\,850\, s} = \frac{689.3}{s} \text{ N/mm}^2$$

and

$$\cos\theta = \frac{r_1}{r_2} = \frac{25}{47} = 0.532$$

We know that resultant shear stress,

$$\tau = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\,\tau_1 \times \tau_2 \cos\theta}$$

$$80 = \sqrt{\left(\frac{212}{s}\right)^2 + \left(\frac{689.3}{s}\right)^2 + 2 \times \frac{212}{s} \times \frac{689.3}{s} \times 0.532} = \frac{822}{s}$$

$$\therefore \quad s = 822 / 80 = 10.3 \text{ mm} \quad \textbf{Ans.}$$

Shafts:

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending.

In other words, we may say that a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines. The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used. An ***axle*****,** though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave. A ***spindle*** is a short shaft that imparts motion either to a cutting tool (*e.g*. drill press spindles) or to a work piece (*e.g*. lathe spindles).

Types of Shafts

The following two types of shafts are important from the subject point of view:

1. ***Transmission shafts*****.** These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2. ***Machine shafts*****.** These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

Stresses in Shafts

The following stresses are induced in the shafts:

1. Shear stresses due to the transmission of torque (*i.e.* due to torsional load).

2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.

3. Stresses due to combined torsional and bending loads.

Design of Shafts

The shafts may be designed on the basis of

1. Strength, and **2.** Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered:

(*a*) Shafts subjected to twisting moment or torque only,

(*b*) Shafts subjected to bending moment only,

(*c*) Shafts subjected to combined twisting and bending moments, and

(*d*) Shafts subjected to axial loads in addition to combined torsional and bending loads.

Shafts Subjected to Twisting Moment Only

a) Solid shaft:

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that

$$\frac{T}{J} = \frac{\tau}{r}$$

Where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

r = Distance from neutral axis to the outer most fibre

= d / 2; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} d^4$$

Then we get, $$T = \frac{\pi d^3}{16} \tau$$

From this equation, diameter of the solid shaft (d) may be obtained.

b) Hollow Shaft:

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_0)^4 - (d_i)^4\right]$$

Where d_o and d_i = Outside and inside diameter of the shaft, and $r = d_0 / 2$.

Substituting these values in equation **(*i*)**, we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right]$$

Let k = Ratio of inside diameter and outside diameter of the shaft = d_i / d_o

Now the equation **(*iii*)** may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau \, (d_o)^3 \; (1 - k^4)$$

From the equations, the outside and inside diameter of a hollow shaft may be determined.

It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft. When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \qquad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3 \, (1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation:

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

Where T = Twisting moment in N-m, and

N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$T = (T_1 - T_2)\, R$

Where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and R = Radius of the pulley.

Shafts Subjected to Bending Moment Only

a) Solid Shaft:

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

Where M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.

b) Hollow Shaft:

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4 (1 - k^4) \qquad \ldots(\text{where } k = d_i / d_o)$$

And $y = d_0 / 2$

Again substituting these values in equation, we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4 (1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

From this equation, the outside diameter of the shaft (do) may be obtained.

Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view:

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.

2. Maximum normal stress theory or Rankine’s theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

a) Solid Shaft:

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of σ_b and τ

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[\sqrt{M^2 + T^2}\right]$$

$$\text{or} \qquad \frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2}$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation **(*i*)** may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3$$

From this expression, diameter of the shaft (d*)* may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}(M + \sqrt{M^2 + T^2})\right]$$

$$\text{or} \qquad \frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$$

The expression $\frac{1}{2}[M + \sqrt{M^2 + T^2}]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment. By limiting the maximum normal stress [$\sigma_b(max)$] equal to the allowable bending stress (σb), then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this expression, diameter of the shaft (d*)* may be evaluated.

b) Hollow shaft:

In case of a hollow shaft, the equations ***(ii)*** and ***(v)*** may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4)$$

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) - \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

Problem:

A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another pulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and μ = 0.24. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.

Solution. Given : AB = 800 mm ; $\alpha_C = 20°$; D_C = 600 mm or R_C = 300 mm ; AC = 200 mm ; D_D = 700 mm or R_D = 350 mm ; DB = 250 mm ; $\theta = 180° = \pi$ rad ; W = 2000 N ; T_1 = 3000 N ; $T_1/T_2 = 3$; τ = 40 MPa = 40 N/mm^2

The space diagram of the shaft is shown in Fig (*a*).

We know that the torque acting on the shaft at D,

$$T = (T_1 - T_2)\, R_D = T_1\left(1 - \frac{T_2}{T_1}\right) R_D$$

$$= 3000\left(1 - \frac{1}{3}\right) 350 = 700 \times 10^3 \text{ N-mm} \qquad ...(\because T_1/T_2 = 3)$$

The torque diagram is shown in Fig. (*b*).

Assuming that the torque at D is equal to the torque at C, therefore the tangential force acting on the gear C,

$$F_{tc} = \frac{T}{R_C} = \frac{700 \times 10^3}{300} = 2333 \text{ N}$$

and the normal load acting on the tooth of gear C,

$$W_C = \frac{F_{tc}}{\cos \alpha_C} = \frac{2333}{\cos 20°} = \frac{2333}{0.9397} = 2483 \text{ N}$$

The normal load acts at 20° to the vertical as shown in Fig. Resolving the normal load vertically and horizontally, we get

Vertical component of W_C *i.e.* the vertical load acting on the shaft at C,

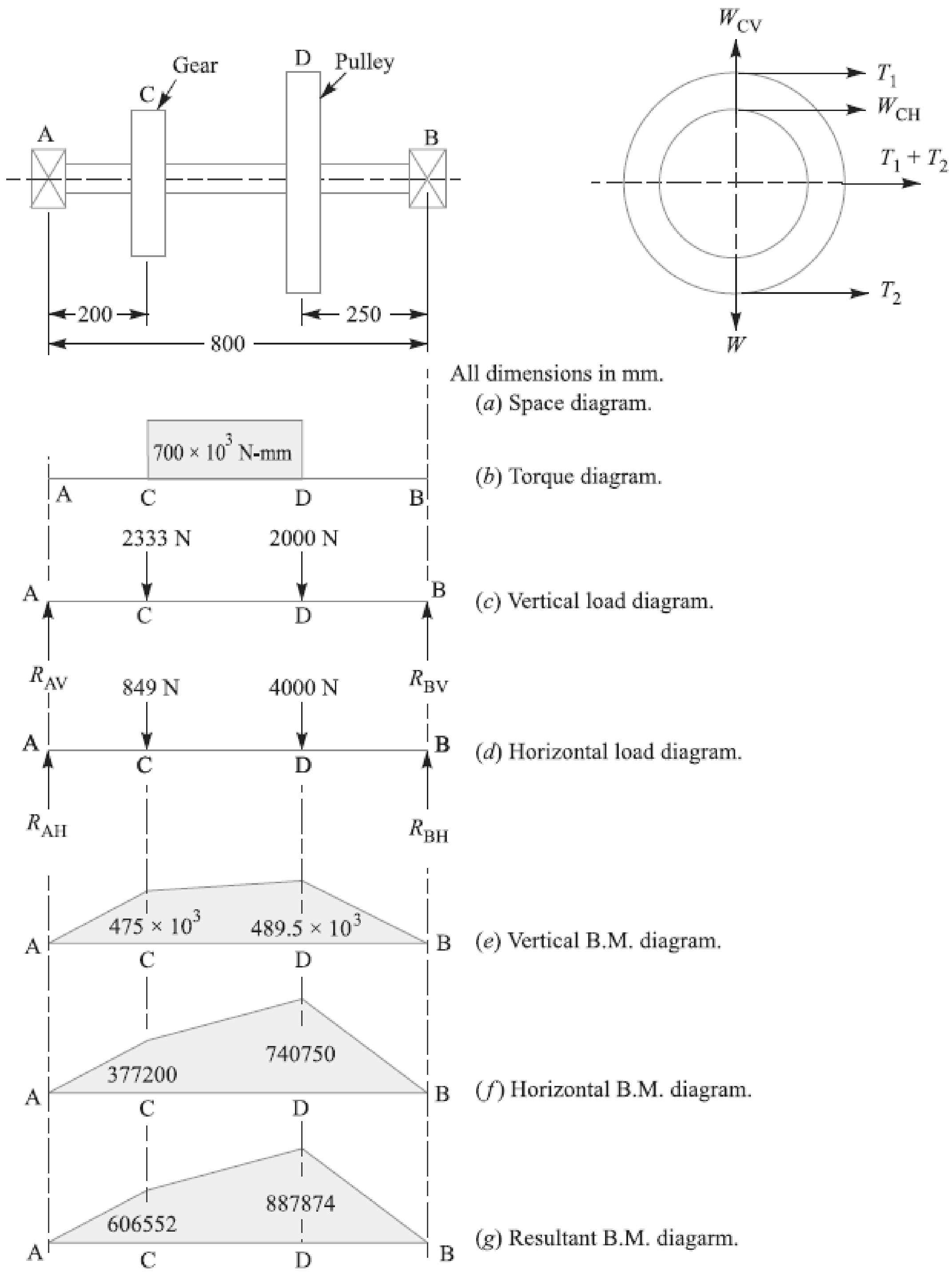
$$W_{CV} = W_C \cos 20°$$
$$= 2483 \times 0.9397 = 2333 \text{ N}$$

and horizontal component of W_C *i.e.* the horizontal load acting on the shaft at C,

$$W_{CH} = W_C \sin 20°$$
$$= 2483 \times 0.342 = 849 \text{ N}$$

Since $T_1 / T_2 = 3$ and $T_1 = 3000$ N, therefore

$$T_2 = T_1 / 3 = 3000 / 3 = 1000 \text{ N}$$



(*a*) Space diagram.

(*b*) Torque diagram.

(*c*) Vertical load diagram.

(*d*) Horizontal load diagram.

(*e*) Vertical B.M. diagram.

(*f*) Horizontal B.M. diagram.

(*g*) Resultant B.M. diagarm.

∴ Horizontal load acting on the shaft at *D*,

$$W_{DH} = T_1 + T_2 = 3000 + 1000 = 4000 \text{ N}$$

and vertical load acting on the shaft at *D*,

$$W_{DV} = W = 2000 \text{ N}$$

The vertical and horizontal load diagram at C and D is shown in Fig. 14.6 (*c*) and (*d*) respectively.

Now let us find the maximum bending moment for vertical and horizontal loading.

First of all considering the vertical loading at C and D. Let R_{AV} and R_{BV} be the reactions at the bearings A and B respectively. We know that

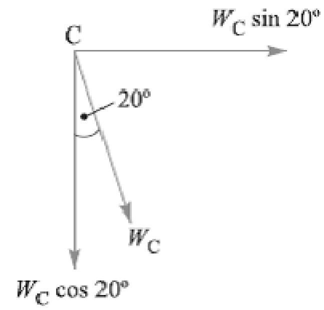


Fig. 14.7

$$R_{AV} + R_{BV} = 2333 + 2000 = 4333 \text{ N}$$

Taking moments about A, we get

$$R_{BV} \times 800 = 2000\,(800 - 250) + 2333 \times 200 = 1\,566\,600$$

$\therefore$ $$R_{BV} = 1\,566\,600 / 800 = 1958 \text{ N}$$

and $$R_{AV} = 4333 - 1958 = 2375 \text{ N}$$

We know that B.M. at A and B,

$$M_{AV} = M_{BV} = 0$$

B.M. at C, $$M_{CV} = R_{AV} \times 200 = 2375 \times 200 = 475 \times 10^3 \text{ N-mm}$$

B.M. at D, $$M_{DV} = R_{BV} \times 250 = 1958 \times 250 = 489.5 \times 10^3 \text{ N-mm}$$

The bending moment diagram for vertical loading is shown in Fig. 14.6 (*e*).

Now consider the horizontal loading at C and D. Let R_{AH} and R_{BH} be the reactions at the bearings A and B respectively. We know that

$$R_{AH} + R_{BH} = 849 + 4000 = 4849 \text{ N}$$

Taking moments about A, we get

$$R_{BH} \times 800 = 4000\,(800 - 250) + 849 \times 200 = 2\,369\,800$$

$\therefore$ $$R_{BH} = 2\,369\,800 / 800 = 2963 \text{ N}$$

and $$R_{AH} = 4849 - 2963 = 1886 \text{ N}$$

We know that B.M. at A and B,

$$M_{AH} = M_{BH} = 0$$

B.M. at C, $$M_{CH} = R_{AH} \times 200 = 1886 \times 200 = 377\,200 \text{ N-mm}$$

B.M. at D, $$M_{DH} = R_{BH} \times 250 = 2963 \times 250 = 740\,750 \text{ N-mm}$$

The bending moment diagram for horizontal loading is shown in Fig. 14.6 (*f*).

We know that resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(475 \times 10^3)^2 + (377\,200)^2} = 606\,552 \text{ N-mm}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(489.5 \times 10^3)^2 + (740\,750)^2} = 887\,874 \text{ N-mm}$$

Maximum bending moment

The resultant B.M. diagram is shown in Fig. 14.6 (*g*). We see that the bending moment is maximum at D, therefore

Maximum B.M., $M = M_D = 887\,874$ N-mm **Ans.**

Diameter of the shaft

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(887\,874)^2 + (700 \times 10^3)^2} = 1131 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1131 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 1131 \times 10^3 / 7.86 = 144 \times 10^3$ or $d = 52.4$ say 55 mm **Ans.**

Problem:

A steel solid shaft transmitting 15 kW at 200 r.p.m. is supported on two bearings 750 mm apart and has two gears keyed to it. The pinion having 30 teeth of 5 mm module is located 100 mm to the left of the right hand bearing and delivers power horizontally to the right. The gear having 100 teeth of 5 mm module is located 150 mm to the right of the left hand bearing and receives power in a vertical direction from below. Using an allowable stress of 54 MPa in shear, determine the diameter of the shaft.

Solution. Given : $P = 15$ kW $= 15 \times 10^3$ W ; $N = 200$ r.p.m. ; $AB = 750$ mm ; $T_D = 30$; $m_D = 5$ mm ; $BD = 100$ mm ; $T_C = 100$; $m_C = 5$ mm ; $AC = 150$ mm ; $\tau = 54$ MPa $= 54$ N/mm^2

The space diagram of the shaft is shown in Fig. 14.8 (*a*).

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 200} = 716 \text{ N-m} = 716 \times 10^3 \text{ N-mm}$$

The torque diagram is shown in Fig. 14.8 (*b*).

We know that diameter of gear

= No. of teeth on the gear × module

$\therefore$ Radius of gear C,

$$R_C = \frac{T_C \times m_C}{2} = \frac{100 \times 5}{2} = 250 \text{ mm}$$

and radius of pinion D,

$$R_D = \frac{T_D \times m_D}{2} = \frac{30 \times 5}{2} = 75 \text{ mm}$$

Assuming that the torque at C and D is same (*i.e.* 716×10^3 N-mm), therefore tangential force on the gear C, acting downward,

$$F_{tC} = \frac{T}{R_C} = \frac{716 \times 10^3}{250} = 2870 \text{ N}$$

and tangential force on the pinion D, acting horizontally,

$$F_{tD} = \frac{T}{R_D} = \frac{716 \times 10^3}{75} = 9550 \text{ N}$$

The vertical and horizontal load diagram is shown in Fig. 14.8 (*c*) and (*d*) respectively.

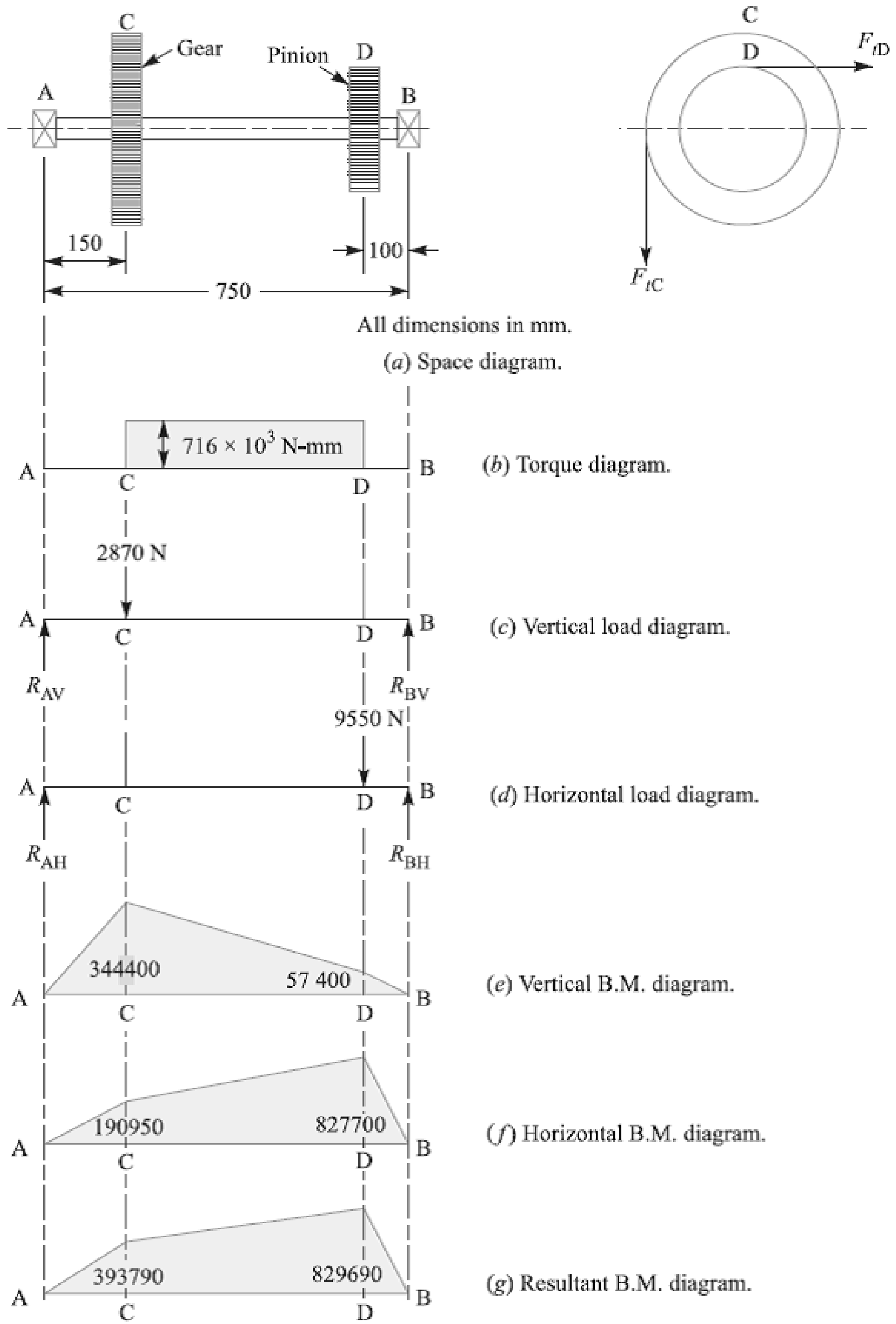
Now let us find the maximum bending moment for vertical and horizontal loading.

First of all, considering the vertical loading at *C*. Let R_{AV} and R_{BV} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AV} + R_{BV} = 2870 \text{ N}$$

Taking moments about *A*, we get

$$R_{BV} \times 750 = 2870 \times 150$$



(*a*) Space diagram.

(*b*) Torque diagram.

(*c*) Vertical load diagram.

(*d*) Horizontal load diagram.

(*e*) Vertical B.M. diagram.

(*f*) Horizontal B.M. diagram.

(*g*) Resultant B.M. diagram.

$\therefore \quad R_{BV} = 2870 \times 150 / 750 = 574$ N

and $R_{AV} = 2870 - 574 = 2296$ N

We know that B.M. at A and B,

$$M_{AV} = M_{BV} = 0$$

B.M. at C, $\quad M_{CV} = R_{AV} \times 150 = 2296 \times 150 = 344\ 400$ N-mm

B.M. at D, $\quad M_{DV} = R_{BV} \times 100 = 574 \times 100 = 57\ 400$ N-mm

The B.M. diagram for vertical loading is shown in Fig. 14.8 (*e*).

Now considering horizontal loading at D. Let R_{AH} and R_{BH} be the reactions at the bearings A and B respectively. We know that

$$R_{AH} + R_{BH} = 9550 \text{ N}$$

Taking moments about A, we get

$$R_{BH} \times 750 = 9550\,(750 - 100) = 9550 \times 650$$

$\therefore \quad R_{BH} = 9550 \times 650 / 750 = 8277$ N

and $\quad R_{AH} = 9550 - 8277 = 1273$ N

We know that B.M. at A and B,

$$M_{AH} = M_{BH} = 0$$

B.M. at C, $\quad M_{CH} = R_{AH} \times 150 = 1273 \times 150 = 190\ 950$ N-mm

B.M. at D, $\quad M_{DH} = R_{BH} \times 100 = 8277 \times 100 = 827\ 700$ N-mm

The B.M. diagram for horizontal loading is shown in Fig. 14.8 (*f*).

We know that resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(344\ 400)^2 + (190\ 950)^2}$$
$$= 393\ 790 \text{ N-mm}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(57\ 400)^2 + (827\ 700)^2}$$
$$= 829\ 690 \text{ N-mm}$$

The resultant B.M. diagram is shown in Fig. 14.8 (*g*). We see that the bending moment is maximum at D.

$\therefore$ Maximum bending moment,

$$M = M_D = 829\ 690 \text{ N-mm}$$

Let $\quad d$ = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(829\ 690)^2 + (716 \times 10^3)^2} = 1096 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1096 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 54 \times d^3 = 10.6\ d^3$$

$\therefore \quad d^3 = 1096 \times 10^3 / 10.6 = 103.4 \times 10^3$

or $\quad d = 47$ say 50 mm **Ans.**

Shafts Subjected to Axial Load in addition to Combined Torsion and Bending Loads:

When the shaft is subjected to an axial load (F) in addition to torsion and bending loads as in propeller shafts of ships and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (□b). We know that bending equation is

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{or} \quad \sigma_b = \frac{M \cdot y}{I} = \frac{M \times d/2}{\frac{\pi}{64} \times d^4} = \frac{32M}{\pi d^3}$$

And stress due to axial load

$$= \frac{F}{\frac{\pi}{4} \times d^2} = \frac{4F}{\pi d^2} \qquad \text{...(For round solid shaft)}$$

$$= \frac{F}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]} = \frac{4F}{\pi\left[(d_o)^2 - (d_i)^2\right]} \qquad \text{...(For hollow shaft)}$$

$$= \frac{F}{\pi\,(d_o)^2\,(1 - k^2)} \qquad \text{... }(\because k = d_i/d_o)$$

Resultant stress (tensile or compressive) for solid shaft,

$$\sigma_1 = \frac{32M}{\pi d^3} + \frac{4F}{\pi d^2} = \frac{32}{\pi d^3}\left(M + \frac{F \times d}{8}\right) \qquad \text{...}(i)$$

$$= \frac{32M_1}{\pi d^3} \qquad \text{...}\left(\text{Substituting } M_1 = M + \frac{F \times d}{8}\right)$$

In case of a hollow shaft, the resultant stress,

$$\sigma_1 = \frac{32M}{\pi\,(d_o)^3\,(1 - k^4)} + \frac{4F}{\pi\,(d_o)^2\,(1 - k^2)}$$

$$= \frac{32}{\pi\,(d_o)^3\,(1 - k^4)}\left[M + \frac{F\,d_o\,(1 + k^2)}{8}\right] = \frac{32M_1}{\pi(d_o)^3\,(1 - k^4)}$$

In case of long shafts (slender shafts) subjected to compressive loads, a factor known as ***column factor*** (α) must be introduced to take the column effect into account.

Therefore, Stress due to the compressive load,

$$\sigma_c = \frac{\alpha \times 4F}{\pi d^2}$$

or

$$= \frac{\alpha \times 4F}{\pi (d_o)^2 \; (1 - k^2)}$$

The value of column factor (α) for compressive loads* may be obtained from the following relation :

Column factor,

$$\alpha - \frac{1}{1 - 0.0044 \; (L / K)}$$

This expression is used when the slenderness ratio (L / K) is less than 115. When the slenderness ratio (L / K) is more than 115, then the value of column factor may be obtained from the following relation:

Column factor, α

$$\alpha - \frac{\sigma_y \, (L / K)^2}{C \pi^2 \, E}$$

Where L = Length of shaft between the bearings,

K = Least radius of gyration,

σ_y = Compressive yield point stress of shaft material, and

C = Coefficient in Euler's formula depending upon the end conditions.

The following are the different values of C depending upon the end conditions.

C =1, for hinged ends,

= 2.25, for fixed ends,

= 1.6, for ends that are partly restrained as in bearings.

In general, for a hollow shaft subjected to fluctuating torsional and bending load, along with an axial load, the equations for equivalent twisting moment (T_e) and equivalent bending moment (M_e) may be written as

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha \, F \, d_o \, (1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \frac{\pi}{16} \times \tau \, (d_o)^3 \; (1 - k^4)$$

$$M_e = \frac{1}{2}\left[K_m \times M + \frac{\alpha F \, d_o \; (1 + k^2)}{8} + \sqrt{\left\{K_m \times M + \frac{\alpha F d_o \; (1 + k^2)}{8}\right\}^2 + (K_t \times T)^2}\right]$$

$$= \frac{\pi}{32} \times \sigma_b \, (d_o)^3 \; (1 - k^4)$$

It may be noted that for a solid shaft, k = 0 and d_0 = d. When the shaft carries no axial load, then F = 0 and when the shaft carries axial tensile load, then α = 1.

Problem:

A hollow shaft is subjected to a maximum torque of 1.5 kN-m and a maximum bending moment of 3 kN-m. It is subjected, at the same time, to an axial load of 10 kN. Assume that the load is applied gradually and the ratio of the inner diameter to the outer diameter is 0.5. If the outer diameter of the shaft is 80 mm, find the shear stress induced in the shaft.

Solution. Given: $T = 1.5$ kN-m $= 1.5 \times 10^3$ N-m ; $M = 3$ kN-m $= 3 \times 10^3$ N-m ;

$F = 10$ kN $= 10 \times 10^3$ N ; $k = d_i / d_o = 0.5$; $d_o = 80$ mm $= 0.08$ m

Let τ = Shear stress induced in the shaft.

Since the load is applied gradually, therefore from DDB, we find that $K_m = 1.5$; and $K_t = 1.0$

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o (1 + k^2)^2}{8}\right] + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 3 \times 10^3 + \frac{1 \times 10 \times 10^3 \times 0.08 (1 + 0.5^2)^2}{8}\right] + (1 \times 1.5 \times 10^3)^2}$$

$$= \sqrt{(4500 + 125)^2 + (1500)^2} = 4862 \text{ N-m} = 4862 \times 10^3 \text{ N-mm}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$4862 \times 10^3 = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4) = \frac{\pi}{16} \times \tau (80)^3 (1 - 0.5^4) = 94\,260\,\tau$$

$$\therefore \quad \tau = 4862 \times 10^3 / 94\,260 = 51.6 \text{ N/mm}^2 = 51.6 \text{ MPa} \textbf{ Ans.}$$

Problem:

A hollow shaft of 0.5 m outside diameter and 0.3 m inside diameter is used to drive a propeller of a marine vessel. The shaft is mounted on bearings 6 metre apart and it transmits 5600 kW at 150 r.p.m. The maximum axial propeller thrust is 500 kN and the shaft weighs 70 kN.

Determine:

1. The maximum shear stress developed in the shaft, and
2. The angular twist between the bearings.

Solution. Given : $d_o = 0.5$ m ; $d_i = 0.3$ m ; $P = 5600$ kW $= 5600 \times 10^3$ W ; $L = 6$ m ; $N = 150$ r.p.m. ; $F = 500$ kN $= 500 \times 10^3$ N ; $W = 70$ kN $= 70 \times 10^3$ N

1. ***Maximum shear stress developed in the shaft***

Let τ = Maximum shear stress developed in the shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{5600 \times 10^3 \times 60}{2\pi \times 150} = 356\ 460 \text{ N-m}$$

and the maximum bending moment,

$$M = \frac{W \times L}{8} = \frac{70 \times 10^3 \times 6}{8} = 52\ 500 \text{ N-m}$$

Now let us find out the column factor α. We know that least radius of gyration,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right]}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]}}$$

$$= \sqrt{\frac{[(d_o)^2 + (d_i)^2][(d_o)^2 - (d_i)^2]}{16\,[(d_o)^2 - (d_i)^2]}}$$

$$= \frac{1}{4}\sqrt{(d_o)^2 + (d_i)^2} = \frac{1}{4}\sqrt{(0.5)^2 + (0.3)^2} = 0.1458 \text{ m}$$

$\therefore$ Slenderness ratio,

$$L / K = 6 / 0.1458 = 41.15$$

and column factor,

$$\alpha = \frac{1}{1 - 0.0044\left(\dfrac{L}{K}\right)} \qquad \ldots\left(\because \frac{L}{K} < 115\right)$$

$$= \frac{1}{1 - 0.0044 \times 41.15} = \frac{1}{1 - 0.18} = 1.22$$

Assuming that the load is applied gradually, therefore from Table 14.2, we find that

$$K_m = 1.5 \text{ and } K_t = 1.0$$

Also $k = d_i / d_o = 0.3 / 0.5 = 0.6$

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o\ (1 + k^2)}{8}\right]^2 \quad (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 52\ 500 + \frac{1.22 \times 500 \times 10^3 \times 0.5\ (1 + 0.6^2)}{8}\right]^2 + (1 \times 356\ 460)^2}$$

$$= \sqrt{(78\ 750 + 51\ 850)^2 + (356\ 460)^2} = 380 \times 10^3 \text{ N-m}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$380 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_o)^3\ (1 - k^4) = \frac{\pi}{16} \times \tau\,(0.5)^3\ [1 - (0.6)^4] = 0.02\,\tau$$

$\therefore$ $\tau = 380 \times 10^3 / 0.02 = 19 \times 10^6 \text{ N/m}^2 = 19$ MPa **Ans.**

2. ***Angular twist between the bearings***

Let θ = Angular twist between the bearings in radians.

We know that the polar moment of inertia for a hollow shaft,

$$J = \frac{\pi}{32}[(d_o)^4 - (d_i)^4] = \frac{\pi}{32}[(0.5)^4 - (0.3)^4] = 0.005\,34 \text{ m}^4$$

From the torsion equation,

$$\frac{T}{J} = \frac{G \times \theta}{L}, \text{ we have}$$

$$\theta = \frac{T \times L}{G \times J} = \frac{356\,460 \times 6}{84 \times 10^9 \times 0.00\,534} = 0.0048 \text{ rad}$$

... (Taking G = 84 GPa = 84×10^9 N/m^2)

$$= 0.0048 \times \frac{180}{\pi} = 0.275° \textbf{ Ans.}$$

Design of Shafts on the basis of Rigidity

Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity.

1. Torsional rigidity. The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be affected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft. The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G.\theta}{L} \quad \text{or} \quad \theta = \frac{T.L}{J.G}$$

where θ = Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

2. Lateral rigidity. It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, *i.e.*

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

BIS codes of Shafts

The standard sizes of transmission shafts are:

25 mm to 60 mm with 5 mm steps; 60 mm to 110 mm with 10 mm steps ; 110 mm to 140 mm with 15 mm steps ; and 140 mm to 500 mm with 20 mm steps. The standard length of the shafts are 5 m, 6 m and 7 m.

:

Problems:

Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diameter of the hollow shaft being half the external diameter. Both the shafts have the same material and length.

Solution. Given : $d_o = d$; $d_i = d_o / 2$ or $k = d_i / d_o = 1 / 2 = 0.5$

Comparison of weight

We know that weight of a hollow shaft,

$$W_H = \text{Cross-sectional area} \times \text{Length} \times \text{Density}$$

$$= \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right] \times \text{Length} \times \text{Density} \quad ...(i)$$

and weight of the solid shaft,

$$W_S = \frac{\pi}{4} \times d^2 \times \text{Length} \times \text{Density} \quad ...(ii)$$

Since both the shafts have the same material and length, therefore by dividing equation (*i*) by equation (*ii*), we get

$$\frac{W_H}{W_S} = \frac{(d_o)^2 - (d_i)^2}{d^2} = \frac{(d_o)^2 - (d_i)^2}{(d_o)^2} \quad ...(\because d = d_o)$$

$$= 1 - \frac{(d_i)^2}{(d_o)_2} = 1 - k^2 = 1 - (0.5)^2 = 0.75 \textbf{ Ans.}$$

Comparison of strength

We know that strength of the hollow shaft,

$$T_H = \frac{\pi}{16} \times \tau\, (d_o)^3\, (1 - k^4) \quad ...(iii)$$

and strength of the solid shaft,

$$T_S = \frac{\pi}{16} \times \tau \times d^3 \quad ...(iv)$$

Dividing equation (*iii*) by equation (*iv*), we get

$$\frac{T_H}{T_S} = \frac{(d_o)^3\,(1 - k^4)}{d^3} = \frac{(d_o)^3\,(1 - k^4)}{(d_o)^3} = 1 - k^4 \quad ...(\because d = d_o)$$

$$= 1 - (0.5)^4 = 0.9375 \textbf{ Ans.}$$

Comparison of stiffness

We know that stiffness

$$= \frac{T}{\theta} = \frac{G \times J}{L}$$

$\therefore$ Stiffness of a hollow shaft,

$$S_H = \frac{G}{L} \times \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \quad ...(v)$$

and stiffness of a solid shaft,

$$S_S = \frac{G}{L} \times \frac{\pi}{32} \times d^4 \qquad ...(vi)$$

Dividing equation (*v*) by equation (*vi*), we get

$$\frac{S_H}{S_S} = \frac{(d_o)^4 - (d_i)^4}{d^4} = \frac{(d_o)^4 - (d_i)^4}{(d_o)^4} = 1 - \frac{(d_i)^4}{(d_o)^4} \qquad ...(\because d = d_o)$$

$$= 1 - k^4 = 1 - (0.5)^4 = 0.9375 \textbf{ Ans.}$$

Shaft Coupling

Shafts are usually available up to 7 meters length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

Shaft couplings are used in machinery for several purposes, the most common of which are the following:

1. To provide for the connection of shafts of units those are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.

2. To provide for misalignment of the shafts or to introduce mechanical flexibility.

3. To reduce the transmission of shock loads from one shaft to another.

4. To introduce protection against overloads.

5. It should have no projecting parts.

Types of Shafts Couplings

Shaft couplings are divided into two main groups as follows:

1. Rigid coupling. It is used to connect two shafts which are perfectly aligned. Following types of rigid coupling are important from the subject point of view:

(*a*) Sleeve or muff coupling.

(*b*) Clamp or split-muff or compression coupling, and

(*c*) Flange coupling.

2. Flexible coupling. It is used to connect two shafts having both lateral and angular misalignment. Following types of flexible coupling are important from the subject point of view:

(*a*) Bushed pin type coupling,

(*b*) Universal coupling, and

(*c*) Oldham coupling.

Sleeve or Muff-coupling

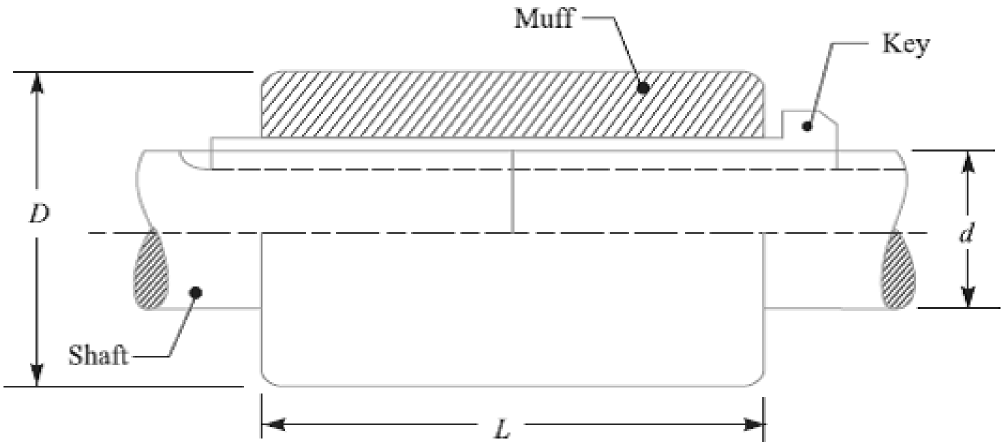
It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shafts by means of a gib head key, as shown in Fig. The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. It is, therefore, necessary that all the elements must be strong enough to transmit the torque. The usual proportions of a cast iron sleeve coupling are as follows:

Outer diameter of the sleeve, $D = 2d + 13$ mm

And length of the sleeve, L = 3.5 d

Where d is the diameter of the shaft.

In designing a sleeve or muff-coupling, the following procedure may be adopted.



1. Design for sleeve

The sleeve is designed by considering it as a hollow shaft

Let T = Torque to be transmitted by the coupling, and

τ_c = Permissible shear stress for the material of the sleeve which is cast iron.

The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \times D^3 (1 - k^4) \qquad \ldots (\because k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.

2. Design for key

The key for the coupling may be designed in the similar way as discussed in Unit-5. The width and thickness of the coupling key is obtained from the proportions. The length of the coupling key is at least equal to the length of the sleeve (i.e. 3.5 d). The coupling key is usually made into two parts so that the length of the key in each shaft,

$$l = \frac{L}{2} = \frac{3.5\,d}{2}$$

After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

$$T = l \times w \times \tau \times \frac{d}{2} \qquad \ldots \text{(Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \ldots \text{(Considering crushing of the key)}$$

Note: The depth of the keyway in each of the shafts to be connected should be exactly the same and the diameters should also be same. If these conditions are not satisfied, then the key will be bedded on one shaft while in the other it will be loose. In order to prevent this, the key is made in two parts which may be driven from the same end for each shaft or they may be driven from opposite ends.

Problem: Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.

Solution.

Given: $P = 40$ kW $= 40 \times 10^3$ W; $N = 350$ r.p.m.; $\tau_s = 40$ MPa $= 40$ N/mm2; $\sigma_{cs} = 80$ MPa $= 80$ N/mm^2; $\sigma_c = 15$ MPa $= 15$ N/mm^2.

$$T - \frac{P \times 60}{2\,\pi N} - \frac{40 \times 10^3 \times 60}{2\,\pi \times 350} - 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 1100 \times 10^3/7.86 = 140 \times 10^3 \text{ or } d = 52 \text{ say } 55 \text{ mm Ans.}$$

2. Design for sleeve

We know that outer diameter of the muff,

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm } \textbf{Ans.}$$

and length of the muff,

$$L = 3.5\, d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shear stress in the muff. Let τ_c be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft, therefore the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125}\right]$$

$$= 370 \times 103\, \tau_c$$

$$\therefore \quad \tau_c = 1100 \times 10^3/370 \times 10^3 = 2.97 \text{ N/mm}^2$$

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm2, therefore the design of muff is safe.

3. Design for key

From Design data Book, we find that for a shaft of 55 mm diameter,

Width of key, w = 18 mm **Ans.**

Clamp or Compression Coupling or split muff coupling

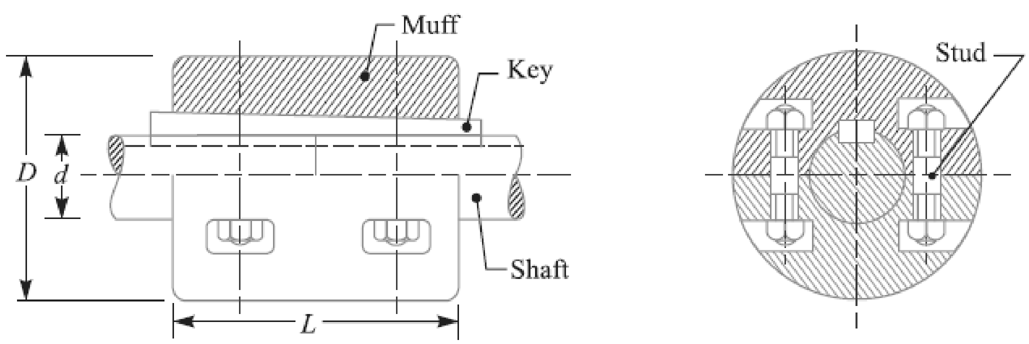
It is also known as **split muff coupling.** In this case, the muff or sleeve is made into two halves and are bolted together as shown in Fig. The halves of the muff are made of cast iron. The shaft ends are made to a butt each other and a single key is fitted directly in the keyways of both the shafts. One-half of the muff is fixed from below and the other half is placed from above. Both the halves are held together by means of mild steel studs or bolts and nuts. The number of bolts may be two, four or six. The nuts are recessed into the bodies of the muff castings. This coupling may be used for heavy duty and moderate speeds. The advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the coupling. The usual proportions of the muff for the clamp or compression coupling are:

Diameter of the muff or sleeve, $D = 2d + 13$ mm

Length of the muff or sleeve, $L = 3.5\ d$

Where d = Diameter of the shaft.

In the clamp or compression coupling, the power is transmitted from one shaft to the other by means of key and the friction between the muff and shaft. In designing this type of coupling, the following procedure may be adopted.



1. Design of muff and key

The muff and key are designed in the similar way as discussed in muff coupling.

2. Design of clamping bolts

Let T = Torque transmitted by the shaft,

d = Diameter of shaft,

d_b = Root or effective diameter of bolt,

n = Number of bolts,

σ_t = Permissible tensile stress for bolt material,

μ = Coefficient of friction between the muff and shaft, and

L = Length of muff.

We know that the force exerted by each bolt

$$= \frac{\pi}{4}(d_b)^2 \sigma_t$$

Then, Force exerted by the bolts on each side of the shaft

$$= \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}$$

Let p be the pressure on the shaft and the muff surface due to the force, then for uniform pressure distribution over the surface,

$$p = \frac{\text{Force}}{\text{Projected area}} = \frac{\frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d}$$

Then, Frictional force between each shaft and muff,

$$F = \mu \times \text{pressure} \times \text{area} = \mu \times p \times \frac{1}{2} \times \pi d \times L$$

$$= \mu \times \frac{\frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d} \times \frac{1}{2} \pi d \times L$$

$$= \mu \times \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2} \times \pi = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n$$

And the torque that can be transmitted by the coupling,

$$T = F \times \frac{d}{2} = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n \times \frac{d}{2} = \frac{\pi^2}{16} \times \mu \, (d_b)^2 \sigma_t \times n \times d$$

From this relation, the root diameter of the bolt (d_b) may be evaluated.

Flange Coupling

A flange coupling usually applies to a coupling having two separate cast iron flanges. Each flange is mounted on the shaft end and keyed to it. The faces are turned up at right angle to the axis of the shaft. One of the flanges has a projected portion and the other flange has a corresponding recess. This helps to bring the shafts into line and to maintain alignment. The two flanges are coupled together by means of bolts and nuts. The flange coupling is adapted to heavy loads and hence it is used on large shafting.

The flange couplings are of the following three types:

1. Unprotected type flange coupling. In an unprotected type flange coupling, as shown in Fig.1, each shaft is keyed to the boss of a flange with a counter sunk key and the flanges are coupled together by means of bolts. Generally, three, four or six bolts are used. The keys are staggered at right angle along the circumference of the shafts in order to divide the weakening effect caused by keyways.

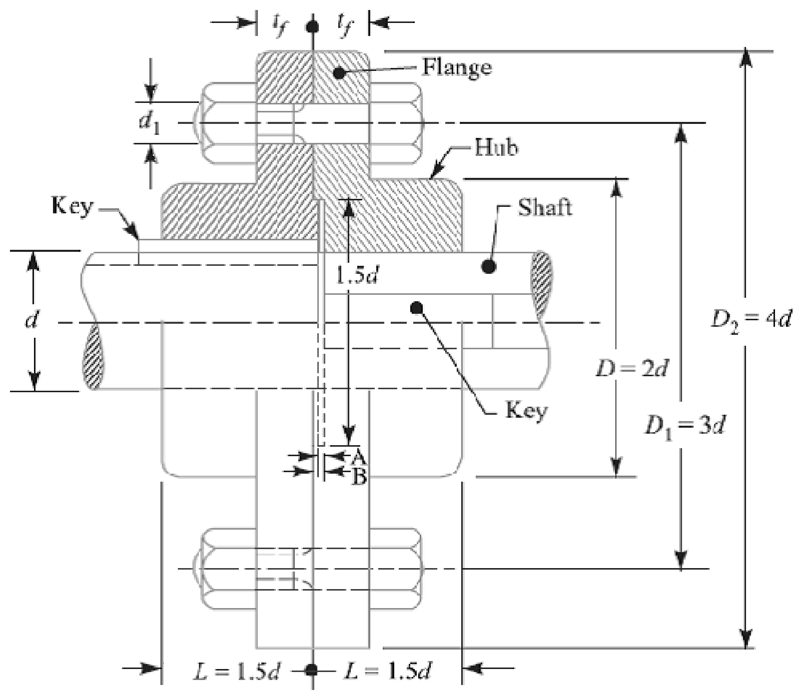


Fig.1 Unprotected Type Flange Coupling.

The usual proportions for an unprotected type cast iron flange couplings, as shown in Fig.1, are as follows:

If d is the diameter of the shaft or inner diameter of the hub, then Outside diameter of hub,

$D = 2\ d$

Length of hub, $L = 1.5\ d$

Pitch circle diameter of bolts, $D_1 = 3d$

Outside diameter of flange,

$D_2 = D_1 + (D_1 - D) = 2\ D_1 - D = 4\ d$

Thickness of flange, $t_f = 0.5\ d$

Number of bolts = 3, for d upto 40 mm

= 4, for d upto 100 mm

= 6, for d upto 180 mm

2. Protected type flange coupling. In a protected type flange coupling, as shown in Fig.2, the protruding bolts and nuts are protected by flanges on the two halves of the coupling, in order to avoid danger to the workman. The thickness of the protective circumferential flange (t_p) is taken as 0.25 d. The other proportions of the coupling are same as for unprotected type flange coupling.

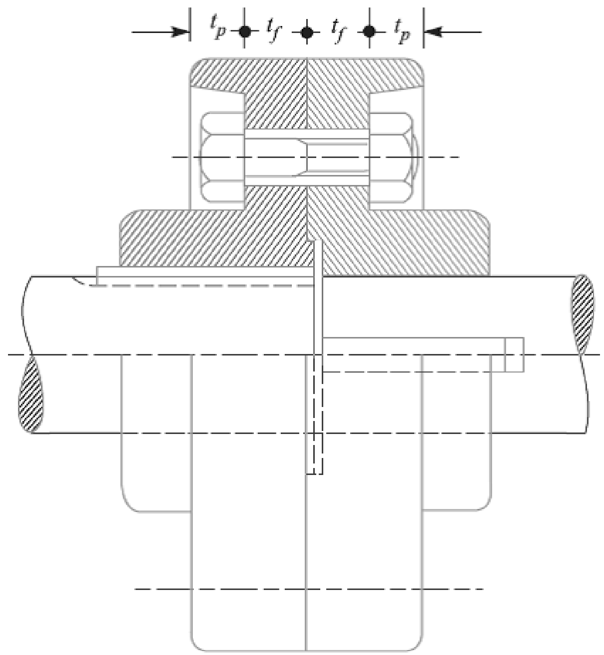


Fig.2. Protected Type Flange Coupling.

3. Marine type flange coupling. In a marine type flange coupling, the flanges are forged integral with the shafts as shown in Fig.3.

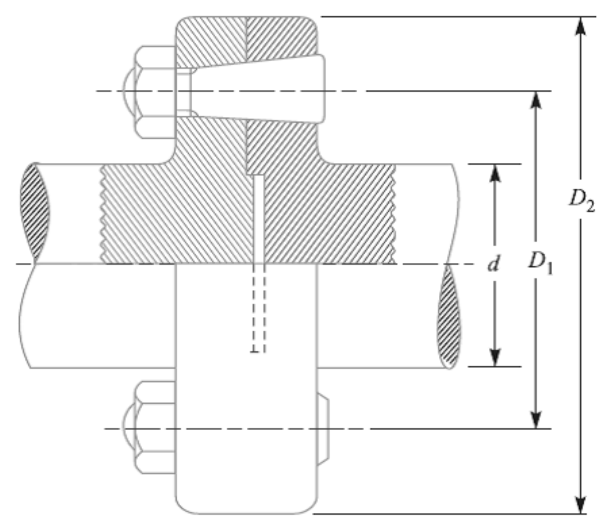


Fig.3. Solid Flange Coupling or Marine Type flange coupling.

The flanges are held together by means of tapered headless bolts, numbering from four to twelve depending upon the diameter of shaft. The other proportions for the marine type flange coupling are taken as follows:

Thickness of flange = d / 3

Taper of bolt = 1 in 20 to 1 in 40

Pitch circle diameter of bolts, D_1 = 1.6 d

Outside diameter of flange, D_2 = 2.2 d

Design of Flange Coupling

Consider a flange coupling as shown in Fig.1 and Fig.2.

Let d = Diameter of shaft or inner diameter of hub,

D = Outer diameter of hub,

D_1 = Nominal or outside diameter of bolt,

D_1 = Diameter of bolt circle,

n = Number of bolts,

t_f = Thickness of flange,

τ_s, τ_b and τ_k = Allowable shear stress for shaft, bolt and key material respectively

τ_c = Allowable shear stress for the flange material i.e. cast iron,

σ_{cb}, and σ_{ck} = Allowable crushing stress for bolt and key material respectively.

The flange coupling is designed as discussed below:

1. Design for hub

The hub is designed by considering it as a hollow shaft, transmitting the same torque (T) as that of a solid shaft.

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right)$$

The outer diameter of hub is usually taken as twice the diameter of shaft. Therefore from the above relation, the induced shearing stress in the hub may be checked.

The length of hub (L) is taken as 1.5 d.

2. Design for key

The key is designed with usual proportions and then checked for shearing and crushing stresses. The material of key is usually the same as that of shaft. The length of key is taken equal to the length of hub.

3. Design for flange

The flange at the junction of the hub is under shear while transmitting the torque. Therefore, the torque transmitted,

T = Circumference of hub × Thickness of flange × Shear stress of flange × Radius of hub

$$= \pi D \times t_f \times \tau_c \times \frac{D}{2} = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

The thickness of flange is usually taken as half the diameter of shaft. Therefore from the above relation, the induced shearing stress in the flange may be checked.

4. Design for bolts

The bolts are subjected to shear stress due to the torque transmitted. The number of bolts (n) depends upon the diameter of shaft and the pitch circle diameter of bolts (D_1) is taken as 3 d.

We know that

Load on each bolt

$$= \frac{\pi}{4} (d_1)^2 \ \tau_b$$

Then, Total load on all the bolts

$$= \frac{\pi}{4} (d_1)^2 \ \tau_b \times n$$

And torque transmitted,

$$T = \frac{\pi}{4} (d_1)^2 \ \tau_b \times n \times \frac{D_1}{2}$$

From this equation, the diameter of bolt (d_1) may be obtained. Now the diameter of bolt may be checked in crushing.

We know that area resisting crushing of all the bolts $= n \times d_1 \times t_f$

And crushing strength of all the bolts $= (n \times d_1 \times t_f)\ \sigma_{cb}$

Torque,

$$T = (n \times d_1 \times t_f \times \sigma_{cb}) \ \frac{D_1}{2}$$

From this equation, the induced crushing stress in the bolts may be checked.

Problem: Design a cast iron protective type flange coupling to transmit 15 kW at 900 r.p.m. from an electric motor to a compressor. The service factor may be assumed as 1.35. The following permissible stresses may be used :

Shear stress for shaft, bolt and key material = 40 MPa

Crushing stress for bolt and key = 80 MPa

Shear stress for cast iron = 8 MPa

Draw a neat sketch of the coupling.

Solution. Given: P = 15 kW = 15 × 103 W; N = 900 r.p.m. ; Service factor = 1.35 ; $\tau_s = \tau_b = \tau_k$ = 40 MPa = 40 N/mm^2 ; $\sigma_{cb} = \sigma_{ck}$ = 80 MPa = 80 N/mm^2 ; τ_c = 8 MPa = 8 N/mm^2.

The protective type flange coupling is designed as discussed below:

1. Design for hub

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 900} = 159.13 \text{ N-m}$$

Since the service factor is 1.35, therefore the maximum torque transmitted by the shaft, T_{max} = 1.35 × 159.13 = 215 N-m = 215 × 103 N-mm

We know that the torque transmitted by the shaft (T),

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$d^3 = 215 \times 10^3 / 7.86 = 27.4 \times 10^3 \quad \text{or} \quad d = 30.1 \text{ say } 35 \text{ mm Ans.}$$

We know that outer diameter of the hub,

D = 2d = 2 × 35 = 70 mm Ans.

And length of hub, L = 1.5 d = 1.5 × 35 = 52.5 mm Ans.

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}).

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(70)^4 - (35)^4}{70}\right] = 63\,147\, \tau_c$$

Then, τ_c = 215 × 103/63 147 = 3.4 N/mm2 = 3.4 MPa

Since the induced shear stress for the hub material (i.e. cast iron) is less than the permissible value of 8 MPa, therefore the design of hub is safe.

2. Design for key

Since the crushing stress for the key material is twice its shear stress (i.e. $\sigma_{ck} = 2\tau_k$), therefore a square key may be used. From DDB, we find that for a shaft of 35 mm diameter,

Width of key, w = 12 mm Ans.

And thickness of key, t = w = 12 mm Ans.

The length of key (l) is taken equal to the length of hub.

Then, l = L = 52.5 mm Ans.

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 - l \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} - 11\ 025\ \tau_k$$

Then, τ_k = 215 × 103/11 025 = 19.5 N/mm2 = 19.5 MPa

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 52.5 \times \frac{12}{2} \times \sigma_{ck} \times \frac{35}{2} = 5512.5\ \sigma_{ck}$$

Σ_{ck} = 215 × 103/ 5512.5 = 39 N/mm^2 = 39 MPa.

Since the induced shear and crushing stresses in the key are less than the permissible stresses, therefore the design for key is safe.

3. Design for flange

The thickness of flange (t_f) is taken as 0.5 d.

Then, t_f = 0.5 d = 0.5 × 35 = 17.5 mm Ans.

Let us now check the induced shearing stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (70)^2}{2} \times \tau_c \times 17.5 = 134\ 713\ \tau_c$$

τ_c = 215 × 103/134 713 = 1.6 N/mm2 = 1.6 MPa

Since the induced shear stress in the flange is less than 8 MPa, therefore the design of flange is safe.

4. Design for bolts

Let d_1 = Nominal diameter of bolts.

Since the diameter of the shaft is 35 mm, therefore let us take the number of bolts,

n = 3 and pitch circle diameter of bolts,

D_1 = 3d = 3 × 35 = 105 mm

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi}{4}(d_1)^2 \ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2 \ 40 \times 3 \times \frac{105}{2} = 4950\ (d_1)^2$$

$$(d_1)^2 = 215 \times 103/4950 = 43.43 \text{ or } d_1 = 6.6 \text{ mm}$$

Assuming coarse threads, the nearest standard size of bolt is M 8. Ans.

Other proportions of the flange are taken as follows:

Outer diameter of the flange,

$D_2 = 4\ d = 4 \times 35 = 140$ mm Ans.

Thickness of the protective circumferential flange,

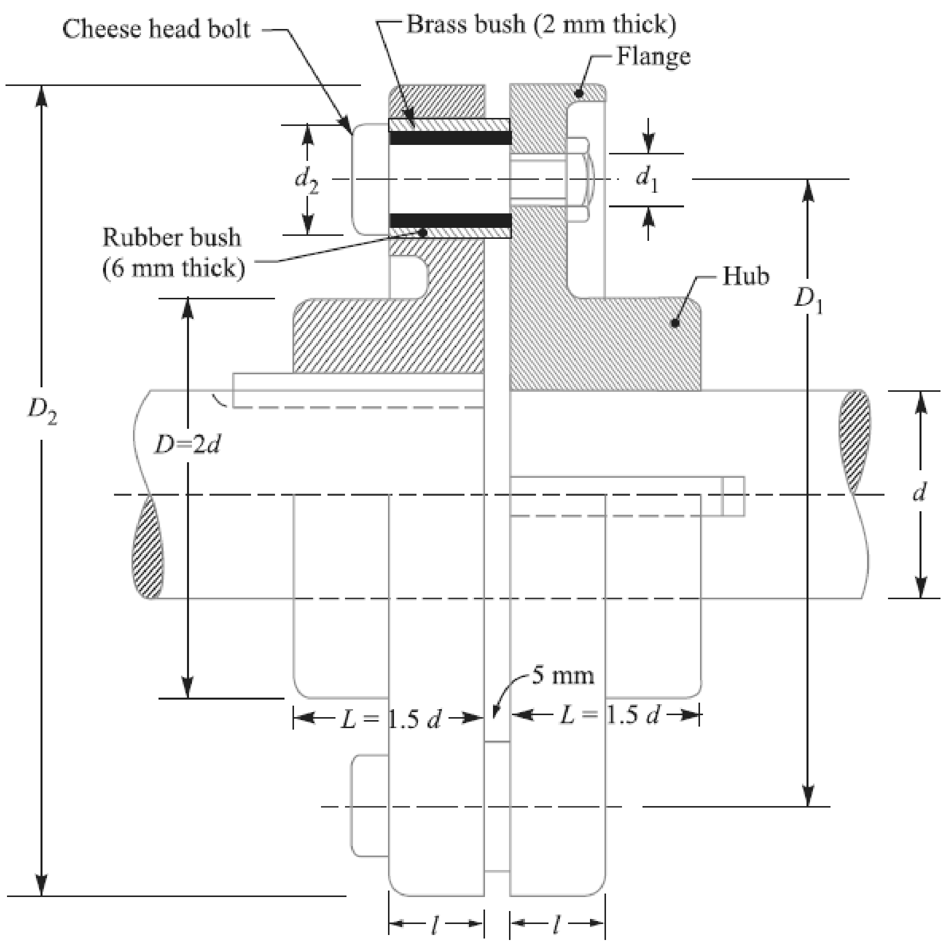
$t_p = 0.25\ d = 0.25 \times 35 = 8.75$ say 10 mm Ans.

Flexible Coupling:

We have already discussed that a flexible coupling is used to join the abutting ends of shafts. when they are not in exact alignment. In the case of a direct coupled drive from a prime mover to an electric generator, we should have four bearings at a comparatively close distance. In such a case and in many others, as in a direct electric drive from an electric motor to a machine tool, a flexible coupling is used so as to permit an axial misalignemnt of the shaft without undue absorption of the power which the shaft are transmitting.

Bushed-pin Flexible Coupling

A bushed-pin flexible coupling, as shown in Fig., is a modification of the rigid type of flange coupling. The coupling bolts are known as pins.



The rubber or leather bushes are used over the pins. The two halves of the coupling are dissimilar in construction. A clearance of 5 mm is left between the face of the two halves of the coupling. There is no rigid connection between them and the drive takes place through the medium of the compressible rubber or leather bushes.

In designing the bushed-pin flexible coupling, the proportions of the rigid type flange coupling are modified. The main modification is to reduce the bearing pressure on the rubber or leather bushes and it should not exceed 0.5 N/mm2. In order to keep the low bearing pressure, the pitch circle diameter and the pin size is increased.

Let l = Length of bush in the flange,

D_2 = Diameter of bush,

P_b = Bearing pressure on the bush or pin,

n = Number of pins, and

D_1 = Diameter of pitch circle of the pins.

We know that bearing load acting on each pin,

$W = p_b \times d_2 \times l$

Then, Total bearing load on the bush or pins

$= W \times n = p_b \times d_2 \times l \times n$

And the torque transmitted by the coupling,

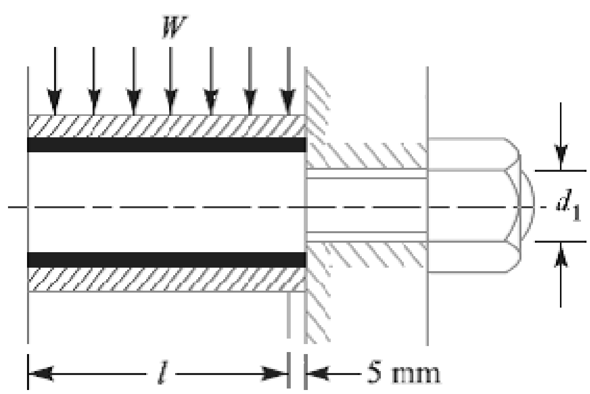
$$T = W \times n\left(\frac{D_1}{2}\right) = p_b \times d_2 \times l \times n\left(\frac{D_1}{2}\right)$$

The threaded portion of the pin in the right hand flange should be a tapping fit in the coupling hole to avoid bending stresses.

The threaded length of the pin should be as small as possible so that the direct shear stress can be taken by the unthreaded neck.

Direct shear stress due to pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4}(d_1)^2}$$



Since the pin and the rubber or leather bush is not rigidly held in the left hand flange, therefore the tangential load (W) at the enlarged portion will exert a bending action on the pin as shown in Fig. The bush portion of the pin acts as a cantilever beam of length l. Assuming a uniform distribution of the load W along the bush, the maximum bending moment on the pin,

$$M = W\left(\frac{l}{2} + 5 \text{ mm}\right)$$

We know that bending stress,

$$\sigma = \frac{M}{Z} = \frac{W\left(\frac{l}{2} + 5 \text{ mm}\right)}{\frac{\pi}{32}(d_1)^3}$$

Since the pin is subjected to bending and shear stresses, therefore the design must be checked either for the maximum principal stress or maximum shear stress by the following relations:

Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

and the maximum shear stress on the pin

$$= \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

The value of maximum principal stress varies from 28 to 42 MPa.

Note: After designing the pins and rubber bush, the hub, key and flange may be designed in the similar way as discussed for flange coupling.

Problem:

Design a bushed-pin type of flexible coupling to connect a pump shaft to a motor shaft transmitting 32 kW at 960 r.p.m. The overall torque is 20 percent more than mean torque. The material properties are as follows:

(a) The allowable shear and crushing stress for shaft and key material is 40 MPa and 80 MPa respectively.

(b) The allowable shear stress for cast iron is 15 MPa.

(c) The allowable bearing pressure for rubber bush is 0.8 N/mm2.

(d) The material of the pin is same as that of shaft and key.

Draw neat sketch of the coupling.

Solution. Given: P = 32 kW = 32×10^3 W; N = 960 r.p.m. ; $T_{max} = 1.2\ T_{mean}$; $\tau_s = \tau_k = 40$ MPa = 40 N/mm^2 ; $\sigma_{cs} = \sigma_{ck} = 80$ MPa = 80 N/mm^2 ; $\tau_c = 15$ MPa = 15 N/mm^2 ; $p_b = 0.8$ N/mm^2.

1. Design for pins and rubber bush

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{32 \times 10^3 \times 60}{2\pi \times 960} = 318.3 \text{ N-m}$$

$$T_{max} = 1.2\ T_{mean} = 1.2 \times 318.3 = 382 \text{ N-m} = 382 \times 10^3 \text{ N-mm}$$

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$d^3 = 382 \times 10^3 / 7.86 = 48.6 \times 10^3 \text{ or } d = 36.5 \text{ say } 40 \text{ mm}$$

$$d_1 = \frac{0.5\, d}{\sqrt{n}} = \frac{0.5 \times 40}{\sqrt{6}} = 8.2 \text{ mm}$$

In order to allow for the bending stress induced due to the compressibility of the rubber bush, the diameter of the pin (d_1) may be taken as 20 mm. Ans.

The length of the pin of least diameter i.e. d_1 = 20 mm is threaded and secured in the right hand coupling half by a standard nut and washer. The enlarged portion of the pin which is in the left hand coupling half is made of 24 mm diameter. On the enlarged portion, a brass bush of thickness 2 mm is pressed. A brass bush carries a rubber bush. Assume the thickness of rubber bush as 6 mm.

So, Overall diameter of rubber bush,

$d_2 = 24 + 2 \times 2 + 2 \times 6 = 40$ mm Ans.

and diameter of the pitch circle of the pins,

$D_1 = 2\,d + d_2 + 2 \times 6 = 2 \times 40 + 40 + 12 = 132$ mm Ans.

Let l = Length of the bush in the flange.

We know that the bearing load acting on each pin,

$W = p_b \times d_2 \times l = 0.8 \times 40 \times l = 32\, l$ N

And the maximum torque transmitted by the coupling (T_{max}),

$$382 \times 10^3 = W \times n \times \frac{D_1}{2} - 32\, l \times 6 \times \frac{132}{2} = 12\,672\, l$$

$l = 382 \times 103/12\,672 = 30.1$ say 32 mm

And $W = 32\, l = 32 \times 32 = 1024$ N

So, Direct stress due to pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4}(d_1)^2} = \frac{1024}{\frac{\pi}{4}(20)^2} = 3.26 \text{ N/mm}^2$$

Since the pin and the rubber bush are not rigidly held in the left hand flange, therefore the tangential load (W) at the enlarged portion will exert a bending action on the pin. Assuming a uniform distribution of load (W) along the bush, the maximum bending moment on the pin,

$$M = W\left(\frac{l}{2} + 5\right) = 1024\left(\frac{32}{2} + 5\right) = 21\,504 \text{ N-mm}$$

$$Z = \frac{\pi}{32}(d_1)^3 = \frac{\pi}{32}(20)^3 = 785.5 \text{ mm}^3$$

$$\sigma = \frac{M}{Z} = \frac{21\,504}{785.5} = 27.4 \text{ N/mm}^2$$

Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[27.4 + \sqrt{(27.4)^2 + 4\,(3.26)^2}\right]$$

$$= 13.7 + 14.1 = 27.8 \text{ N/mm}^2$$

And maximum shear stress

$$= \frac{1}{2}\left[\sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(27.4)^2 + 4\,(3.26)^2}\right] = 14.1 \text{ N/mm}^2$$

Since the maximum principal stress and maximum shear stress are within limits, therefore the design is safe.

2. Design for hub

We know that the outer diameter of the hub,

$D = 2\ d = 2 \times 40 = 80$ mm

And length of hub, $L = 1.5\ d = 1.5 \times 40 = 60$ mm

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(80)^4 - (40)^4}{80}\right] = 94.26 \times 10^3\ \tau_c$$

$\tau_c = 382 \times 103 / 94.26 \times 103 = 4.05$ N/mm^2 = 4.05 MPa

Since the induced shear stress for the hub material (i.e. cast iron) is less than the permissible value of 15 MPa, therefore the design of hub is safe.

3. Design for key

Since the crushing stress for the key material is twice its shear stress (i.e. $\sigma_{ck} = 2\ \tau_k$), therefore a square key may be used. From Table 13.1, we find that for a shaft of 40 mm diameter,

Width of key, w = 14 mm Ans.

and thickness of key,t = w = 14 mm Ans.

The length of key (L) is taken equal to the length of hub, i.e.

$L = 1.5\ d = 1.5 \times 40 = 60$ mm

Let us now check the induced stresses in the key by considering it in shearing and crushing. Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times w \times \tau_k \times \frac{d}{2} = 60 \times 14 \times \tau_k \times \frac{40}{2} = 16\,800\ \tau_k$$

$$\tau_k = 382 \times 10^3/16\,800 = 22.74 \text{ N/mm}^2 = 22.74 \text{ MPa}$$

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 60 \times \frac{14}{2} \times \sigma_{ck} \times \frac{40}{2} = 8400\ \sigma_{ck}$$

$\sigma_{ck} = 382 \times 103/8400 = 45.48$ N/mm2 = 45.48 MPa

Since the induced shear and crushing stress in the key are less than the permissible stresses of 40 MPa and 80 MPa respectively, therefore the design for key is safe.

4. Design for flange

The thickness of flange (t_f) is taken as 0.5 d.

$t_f = 0.5\ d = 0.5 \times 40 = 20$ mm

Let us now check the induced shear stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (80)^2}{2} \times \tau_c \times 20 = 201 \times 10^3 \tau_c$$

τ_c =382 × 103 / 201 × 103 = 1.9 N/mm2 = 1.9 MPa

Since the induced shear stress in the flange of cast iron is less than 15 MPa, therefore the design of flange is safe.

Problem:

Design a cast iron protective type flange coupling to transmit 15 kW at 900 r.p.m. from an electric motor to a compressor. The service factor may be assumed as 1.35. The following permissible stresses may be used:

Shear stress for shaft, bolt and key material = 40 MPa

Crushing stress for bolt and key = 80 MPa

Shear stress for cast iron = 8 MPa

Draw a neat sketch of the coupling.

Solution. Given: P = 15 kW = 15 × 103 W; N = 900 r.p.m. ; Service factor = 1.35 ; $\tau_s = \tau_b = \tau_k$ = 40 MPa = 40 N/mm^2 ; $\sigma_{cb} = \sigma_{ck}$ = 80 MPa = 80 N/mm^2 ; τ_c = 8 MPa = 8 N/mm^2.

The protective type flange coupling is designed as discussed below:

1. Design for hub

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 900} = 159.13 \text{ N-m}$$

Since the service factor is 1.35, therefore the maximum torque transmitted by the shaft, T_{max} = 1.35 × 159.13 = 215 N-m = 215 × 103 N-mm

We know that the torque transmitted by the shaft (T),

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$d^3 = 215 \times 10^3 / 7.86 = 27.4 \times 10^3 \quad \text{or} \quad d = 30.1 \text{ say } 35 \text{ mm } \textbf{Ans.}$$

We know that outer diameter of the hub,

D = 2d = 2 × 35 = 70 mm Ans.

And length of hub, L = 1.5 d = 1.5 × 35 = 52.5 mm Ans.

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}).

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(70)^4 - (35)^4}{70}\right] = 63\,147\, \tau_c$$

Then, τ_c = 215 × 103/63 147 = 3.4 N/mm2 = 3.4 MPa

Since the induced shear stress for the hub material (i.e. cast iron) is less than the permissible value of 8 MPa, therefore the design of hub is safe.

2. Design for key

Since the crushing stress for the key material is twice its shear stress (i.e. $\sigma_{ck} = 2\tau_k$), therefore a square key may be used. From DDB, we find that for a shaft of 35 mm diameter,

Width of key, w = 12 mm Ans.

And thickness of key, t = w = 12 mm Ans.

The length of key (l) is taken equal to the length of hub.

Then, l = L = 52.5 mm Ans.

Let us now check the induced stresses in the key by considering it in shearing and crushing. Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\ 025\ \tau_k$$

Then, τ_k = 215 × 103/11 025 = 19.5 N/mm2 = 19.5 MPa

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 52.5 \times \frac{12}{2} \times \sigma_{ck} \times \frac{35}{2} = 5512.5\ \sigma_{ck}$$

Σ_{ck} = 215 × 103/ 5512.5 = 39 N/mm^2 = 39 MPa.

Since the induced shear and crushing stresses in the key are less than the permissible stresses, therefore the design for key is safe.

3. Design for flange

The thickness of flange (t_f) is taken as 0.5 d.

Then, t_f = 0.5 d = 0.5 × 35 = 17.5 mm Ans.

Let us now check the induced shearing stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (70)^2}{2} \times \tau_c \times 17.5 = 134\ 713\ \tau_c$$

τ_c = 215 × 103/134 713 = 1.6 N/mm2 = 1.6 MPa

Since the induced shear stress in the flange is less than 8 MPa, therefore the design of flange is safe.

4. Design for bolts

Let d_1 = Nominal diameter of bolts.

Since the diameter of the shaft is 35 mm, therefore let us take the number of bolts,

n = 3 and pitch circle diameter of bolts,

$D_1 = 3d = 3 \times 35 = 105$ mm

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi}{4}(d_1)^2 \ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2 \ 40 \times 3 \times \frac{105}{2} = 4950\ (d_1)^2$$

$$(d_1)^2 = 215 \times 103/4950 = 43.43 \text{ or } d_1 = 6.6 \text{ mm}$$

Assuming coarse threads, the nearest standard size of bolt is M 8. Ans.

Other proportions of the flange are taken as follows:

Outer diameter of the flange,

$D_2 = 4\ d = 4 \times 35 = 140$ mm Ans.

Thickness of the protective circumferential flange,

$t_p = 0.25\ d = 0.25 \times 35 = 8.75$ say 10 mm Ans.

Problem:

Two 35 mm shafts are connected by a flanged coupling. The flanges are fitted with 6 bolts on 125 mm bolt circle. The shafts transmit a torque of 800 N-m at 350 r.p.m. For the safe stresses mentioned below, calculate 1. Diameter of bolts; 2. Thickness of flanges; 3. Key dimensions ; 4. Hub length; and 5. Power transmitted. Safe shear stress for shaft material = 63 MPa Safe stress for bolt material = 56 MPa Safe stress for cast iron coupling = 10 MPa Safe stress for key material = 46 MPa

Solution. Given: d = 35 mm; n = 6; D1 = 125 mm; T = 800 N-m = 800 × 103 N-mm; N = 350 r.p.m.; τs = 63 MPa = 63 N/mm^2; τ_b = 56 MPa = 56 N/mm^2 ; τ_c = 10 MPa = 10 N/mm^2 ; τ_k = 46 MPa = 46 N/mm^2.

1. Diameter of bolts

Let d_1 = Nominal or outside diameter of bolt. We know that the torque transmitted (T),

$$800 \times 10^3 = \frac{\pi}{4}(d_1)^2 \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2\, 56 \times 6 \times \frac{125}{2} = 16\,495\,(d_1)^2$$

$(d_1)^2 = 800 \times 10^3 / 16\,495 = 48.5$ or $d_1 = 6.96$ say 8 mm **Ans.**

2. Thickness of flanges

Let t_f = Thickness of flanges.

We know that the torque transmitted (T),

$$800 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (2 \times 35)^2}{2} \times 10 \times t_f = 76\,980\, t_f \quad \ldots (\because D = 2d)$$

t_f = 800 × 103 / 76 980 = 10.4 say 12 mm **Ans.**

3. Key dimensions

From Table 13.1, we find that the proportions of key for a 35 mm diameter shaft are:

Width of key, w = 12 mm **Ans.**

And thickness of key, t = 8 mm**Ans.**

The length of key (l) is taken equal to the length of hub (L).

l = L = 1.5 d = 1.5 × 35 = 52.5 mm

Let us now check the induced shear stress in the key. We know that the torque transmitted (T),

$$800 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} \times = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\ 025\ \tau_k$$

τ_k = 800 × 103 / 11 025 = 72.5 N/mm2

Since the induced shear stress in the key is more than the given safe stress (46 MPa), therefore let us find the length of key by substituting the value of τk = 46 MPa in the above equation, i.e.

$$800 \times 10^3 = l \times 12 \times 46 \times \frac{35}{2} = 9660\ l$$

l = 800 × 103 / 9660 = 82.8 say 85 mm **Ans.**

4. Hub length

Since the length of key is taken equal to the length of hub, therefore we shall take hub length,

L = l = 85 mm **Ans.**

5. Power transmitted

We know that the power transmitted,

$$P = \frac{T \times 2\pi N}{60} = \frac{800 \times 2\pi \times 350}{60} = 29\ 325\ \text{W} = 29.325\ \text{kW Ans.}$$

Problem:

The shaft and the flange of a marine engine are to be designed for flange coupling, in which the flange is forged on the end of the shaft. The following particulars are to considered in the design:

Power of the engine = 3 MW

Speed of the engine = 100 r.p.m.

Permissible shear stress in bolts and shaft = 60 MPa

Number of bolts used = 8

Pitch circle diameter of bolts = 1.6 × Diameter of shaft

Find: 1. diameter of shaft; 2. diameter of bolts; 3. thickness of flange; and 4. diameter of flange.

Solution. Given: P = 3 MW = 3 × 106 W; N = 100 r.p.m.; τb = τs = 60 MPa = 60 N/mm^2; n = 8; D1 = 1.6 d

1. Diameter of shaft

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{3 \times 10^6 \times 60}{2\pi \times 100} = 286 \times 10^3 \text{ N-m} = 286 \times 10^6 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$286 \times 10^6 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\, d^3$$

$$d_3 = 286 \times 10^6 / 11.78 = 24.3 \times 10^6$$

or $d = 2.89 \times 10^2 = 289$ say 300 mm **Ans.**

2. Diameter of bolts

Let d_1 = Nominal diameter of bolts.

The bolts are subjected to shear stress due to the torque transmitted. We know that torque transmitted (T),

$$286 \times 10^6 = \frac{\pi}{4} (d_1)^2 \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} \times (d_1)^2\, 60 \times 8 \times \frac{1.6 \times 300}{2}$$

$= 90\,490\, (d_1)^2$ ... (Since $D_1 = 1.6$ d)

So, $(d_1)^2 = 286 \times 10^6 / 90\,490 = 3160$ or $d_1 = 56.2$ mm

Assuming coarse threads, the standard diameter of the bolt is 60 mm (M 60). The taper on the bolt may be taken from 1 in 20 to 1 in 40. **Ans.**

3. Thickness of flange

The thickness of flange (t_f) is taken as d / 3.

So, tf = d / 3 = 300/3 = 100 mm **Ans.**

Let us now check the induced shear stress in the flange by considering the flange at the junction of the shaft in shear. We know that the torque transmitted (T),

$$286 \times 10^6 = \frac{\pi d^2}{2} \times \tau_s \times t_f = \frac{\pi (300)^2}{2} \times \tau_s \times 100 = 14.14 \times 10^6\, \tau_s$$

$\tau_s = 286 \times 10^6 / 14.14 \times 10^6 = 20.2$ N/mm^2 = 20.2 MPa

Since the induced shear stress in the *flange is less than the permissible shear stress of 60 MPa, therefore the thickness of flange (tf = 100 mm) is safe.

4. Diameter of flange

The diameter of flange (D_2) is taken as 2.2 d.

So, $D_2 = 2.2\, d = 2.2 \times 300 = 660$ mm **Ans.**

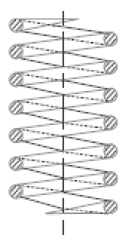
Introduction

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are as follows:

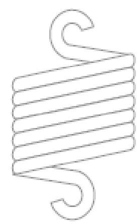
1. To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.
2. To apply forces, as in brakes, clutches and spring loaded valves.
3. To control motion by maintaining contact between two elements as in cams and followers.
4. To measure forces, as in spring balances and engine indicators.
5. To store energy, as in watches, toys, etc.

Types of springs:

1. Helical springs. The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads.



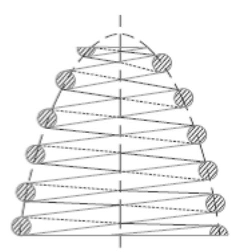
(*a*) Compression helical spring.



(*b*) Tension helical spring.

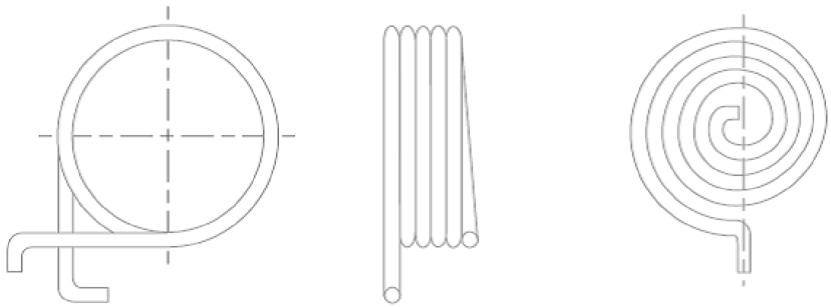
2. Conical and volute springs. The conical and volute springs, as shown in Fig. 23.2, are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired

(*a*) Conical spring.



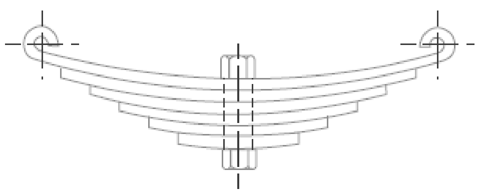
(*b*) Volute spring.

3. Torsion springs. These springs may be of **helical** or **spiral** type as shown in Fig. The **helical type** may be used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms.

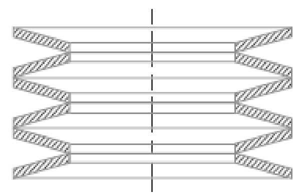


(a) Helical torsion spring. (b) Spiral torsion spring.

4. Laminated or leaf springs. The laminated or leaf spring (also known as **flat spring** or **carriage spring**) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts.



Laminated or leaf springs.



Disc or bellevile springs.

5. Disc or bellevile springs. These springs consist of a number of conical discs held together against slipping by a central bolt or tube.

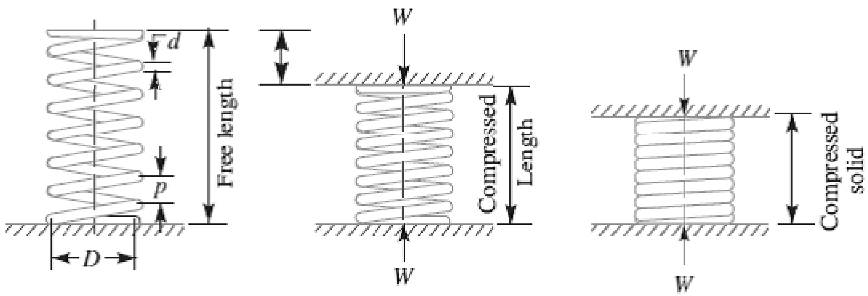
6. Special purpose springs. These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or liquid) can behave as a compression spring. These springs are used for special types of application only.

<u>Terms used in Compression Springs</u>

1. Solid length. When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be **solid.**

Solid length of the spring, $L_s = n'.d$ where n' = Total number of coils, and d = Diameter of the wire.

2. Free length. The free length of a compression spring, as shown in Fig., is the length of the spring in the free or unloaded condition.



Free length of the spring,

L_F = Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance)

$= n'.d + \delta_{max} + 0.15\ \delta_{max}$

3. Spring index. The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Spring index, C = D / d where D = Mean diameter of the coil, and d = Diameter of the wire.

4. Spring rate. The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically, Spring rate, k = W / δ where W = Load, and δ = Deflection of the spring.

5. Pitch. The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically, Pitch of the coil,

$$p = \frac{\text{Free Length}}{n' - 1}$$

<u>**Stresses in Helical Springs of Circular Wire**</u>

Consider a helical compression spring made of circular wire and subjected to an axial load *W*, as shown in Fig.(*a*).

Let *D* = Mean diameter of the spring coil,

d = Diameter of the spring wire,

n = Number of active coils,

G = Modulus of rigidity for the spring material,

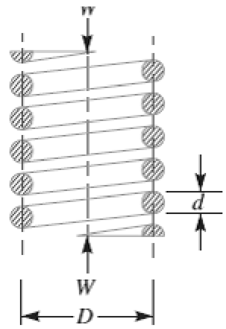
W = Axial load on the spring,

τ = Maximum shear stress induced in the wire,

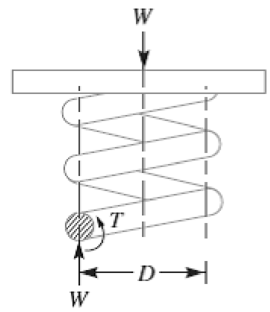
C = Spring index = *D*/*d*,

p = Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load *W*.



(*a*) Axially loaded helical spring.



(*b*) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Now consider a part of the compression spring as shown in Fig. (*b*). The load *W* tends to rotate the wire due to the twisting moment (*T*) set up in the wire. Thus torsional shear stress is induced in the wire.

A little consideration will show that part of the spring, as shown in Fig.(*b*), is in equilibrium under the action of two forces *W* and the twisting moment *T*. We know that the twisting moment,

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\tau_1 = \frac{8 W.D}{\pi \, d^3} \qquad \ldots(i)$$

The torsional shear stress diagram is shown in Fig. (*a*).

In addition to the torsional shear stress (τ_1) induced in the wire, the following stresses also act on the wire:

1. Direct shear stress due to the load *W*, and

2. Stress due to curvature of wire.

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8 W.D}{\pi \, d^3} \pm \frac{4W}{\pi \, d^2}$$

Maximum shear stress induced in the wire,

= Torsional shear stress + Direct shear stress

$$= \frac{8 W.D}{\pi \, d^3} + \frac{4W}{\pi \, d^2} = \frac{8 W.D}{\pi \, d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8\,W.D}{\pi\,d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi\,d^3} \qquad \ldots(iii)$$

... (Substituting $D/d = C$)

where K_S = Shear stress factor $= 1 + \dfrac{1}{2C}$

∴ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} = K \times \frac{8\,W.C}{\pi\,d^2} \qquad \ldots(iv)$$

where $K = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$

Deflection of Helical Springs of Circular Wire

Total active length of the wire,

$$l = \text{Length of one coil} \times \text{No. of active coils} = \pi\,D \times n$$

Let θ = Angular deflection of the wire when acted upon by the torque T.

∴ Axial deflection of the spring,

$$\delta = \theta \times D/2 \qquad \ldots(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \quad \theta = \frac{T.l}{J.G} \qquad \ldots\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where J = Polar moment of inertia of the spring wire

$= \dfrac{\pi}{32} \times d^4$, d being the diameter of spring wire.

and G = Modulus of rigidity for the material of the spring wire.

Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T.l}{J.G} = \frac{\left(W \times \frac{D}{2}\right)\pi\,D.n}{\frac{\pi}{32} \times d^4\,G} = \frac{16 W.D^2.n}{G.d^4} \qquad \ldots(ii)$$

Substituting this value of θ in equation (i), we have

$$\delta = \frac{16 W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\,W.D^3.n}{G.d^4} = \frac{8\,W.C^3.n}{G.d} \qquad \ldots(\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\,D^3.n} = \frac{G.d}{8\,C^3.n} = \text{constant}$$

Buckling of Compression Springs

It has been found experimentally that when the free length of the spring (L_F) is more than four times the mean or pitch diameter (D), then the spring behaves like a column and may fail by buckling at a comparatively low load.

$$W_{cr} = k \times K_B \times L_F$$

where k = Spring rate or stiffness of the spring = W/δ,

L_F = Free length of the spring, and

K_B = Buckling factor depending upon the ratio L_F / D.

Surge in springs

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end.

$$f_n = \frac{d}{2\pi D^2 . n} \sqrt{\frac{6\, G . g}{\rho}} \text{ cycles/s}$$

Where d = Diameter of the wire,

D = Mean diameter of the spring,

n = Number of active turns,

G = Modulus of rigidity,

g = Acceleration due to gravity, and

ρ = Density of the material of the spring.

Problem: A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm^2, find the axial load which the spring can carry and the deflection per active turn.

Solution. Given : $d = 6$ mm ; $D_o = 75$ mm ; $\tau = 350$ MPa $= 350$ N/mm^2 ; $G = 84$ kN/mm^2 $= 84 \times 10^3$ N/mm^2

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

$\therefore$ Spring index, $$C = \frac{D}{d} = \frac{69}{6} = 11.5$$

Let W = Axial load, and

δ / n = Deflection per active turn.

1. *Neglecting the effect of curvature*

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$350 = K_S \times \frac{8\,W.D}{\pi\,d^3} = 1.043 \times \frac{8\,W \times 69}{\pi \times 6^3} = 0.848\,W$$

$\therefore$ $W = 350 / 0.848 = 412.7$ N **Ans.**

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 412.7\,(69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm } \textbf{Ans.}$$

2. *Considering the effect of curvature*

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire (τ),

$$350 = K \times \frac{8W.C}{\pi\,d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\,W$$

$\therefore$ $W = 350 / 0.913 = 383.4$ N **Ans.**

and deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 383.4\,(69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm } \textbf{Ans.}$$

Problem: Design a spring for a balance to measure 0 to 1000 N over a scale of length 80 mm. The spring is to be enclosed in a casing of 25 mm diameter. The approximate number of turns is 30. The modulus of rigidity is 85 kN/mm^2. Also calculate the maximum shear stress induced.

Solution:

Design of spring

Let D = Mean diameter of the spring coil,

d = Diameter of the spring wire, and

C = Spring index = D/d.

Since the spring is to be enclosed in a casing of 25 mm diameter, therefore the outer diameter of the spring coil ($D_o = D + d$) should be less than 25 mm.

We know that deflection of the spring (δ),

$$80 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 1000 \times C^3 \times 30}{85 \times 10^3 \times d} = \frac{240\,C^3}{85\,d}$$

$$\therefore \quad \frac{C^3}{d} = \frac{80 \times 85}{240} = 28.3$$

Let us assume that d = 4 mm. Therefore

$$C^3 = 28.3\,d = 28.3 \times 4 = 113.2 \quad \text{or} \quad C = 4.84$$

and $D = C.d = 4.84 \times 4 = 19.36$ mm **Ans.**

We know that outer diameter of the spring coil,

$$D_o = D + d = 19.36 + 4 = 23.36 \text{ mm } \textbf{Ans.}$$

Since the value of D_o = 23.36 mm is less than the casing diameter of 25 mm, therefore the assumed dimension, d = 4 mm is correct.

Maximum shear stress induced

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 4.84 - 1}{4 \times 4.84 - 4} + \frac{0.615}{4.84} = 1.322$$

∴ Maximum shear stress induced,

$$\tau = K \times \frac{8\,W.C}{\pi\,d^2} = 1.322 \times \frac{8 \times 1000 \times 4.84}{\pi \times 4^2}$$

$$= 1018.2 \text{ N/mm}^2 = 1018.2 \text{ MPa } \textbf{Ans.}$$

References:

1. Machine Design - V.Bandari .

2. Machine Design – R.S. Khurmi

3. Design Data hand Book - S MD Jalaludin.

Problem: Design a helical compression spring for a maximum load of 1000 N for a deflection of 25 mm using the value of spring index as 5. The maximum permissible shear stress for spring wire is 420 MPa and modulus of rigidity is 84 kN/mm^2.

Take Wahl's factor, $K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$

Solution. Given : W = 1000 N ; δ = 25 mm ; $C = D/d = 5$; τ = 420 MPa = 420 N/mm^2 ; G = 84 kN/mm^2 = 84 × 10^3 N/mm^2

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

and maximum shear stress (τ),

$$420 = K \times \frac{8\,W.C}{\pi\, d^2} = 1.31 \times \frac{8 \times 1000 \times 5}{\pi\, d^2} = \frac{16\;677}{d^2}$$

$$\therefore \quad d^2 = 16\;677 / 420 = 39.7 \quad \text{or} \quad d = 6.3 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 3 having diameter (d) = 6.401 mm.

$\therefore$ Mean diameter of the spring coil,

$$D = C.d = 5\,d = 5 \times 6.401 = 32.005 \text{ mm } \textbf{Ans.} \qquad \ldots (\because C = D/d = 5)$$

and outer diameter of the spring coil,

$$D_o = D + d = 32.005 + 6.401 = 38.406 \text{ mm } \textbf{Ans.}$$

2. *Number of turns of the coils*

Let n = Number of active turns of the coils.

We know that compression of the spring (δ),

$$25 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 1000\,(5)^3 n}{84 \times 10^3 \times 6.401} = 1.86\,n$$

$$\therefore \quad n = 25 / 1.86 = 13.44 \text{ say } 14 \textbf{ Ans.}$$

For squared and ground ends, the total number of turns,

$$n' = n + 2 = 14 + 2 = 16 \textbf{ Ans.}$$

3. *Free length of the spring*

We know that free length of the spring

$$= n'.d + \delta + 0.15\,\delta = 16 \times 6.401 + 25 + 0.15 \times 25$$

$$= 131.2 \text{ mm } \textbf{Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{131.2}{16 - 1} = 8.75 \text{ mm } \textbf{Ans.}$$

Problem: Design a close coiled helical compression spring for a service load ranging from 2250 N to 2750 N. The axial deflection of the spring for the load range is 6 mm. Assume a spring index of 5. The permissible shear stress intensity is 420 MPa and modulus of rigidity, G = 84 kN/mm^2. Neglect the effect of stress concentration. Draw a fully dimensioned sketch of the spring, showing details of the finish of the end coils.

Solution. Given : $W_1 = 2250$ N ; $W_2 = 2750$ N ; $\delta = 6$ mm ; $C = D/d = 5$; $\tau = 420$ MPa $= 420$ N/mm^2 ; $G = 84$ kN/mm^2 $= 84 \times 10^3$ N/mm^2

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil for a maximum load of W_2 = 2750 N, and

d = Diameter of the spring wire.

We know that twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2750 \times \frac{5d}{2} = 6875\ d \qquad \ldots\left(\because C = \frac{D}{d} = 5\right)$$

We also know that twisting moment (T),

$$6875\ d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 420 \times d^3 = 82.48\ d^3$$

$$\therefore \quad d^2 = 6875 / 82.48 = 83.35 \quad \text{or} \quad d = 9.13 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 3/0 having diameter (d) = 9.49 mm.

∴ Mean diameter of the spring coil,

$$D = 5d = 5 \times 9.49 = 47.45 \text{ mm } \textbf{Ans.}$$

We know that outer diameter of the spring coil,

$$D_o = D + d = 47.45 + 9.49 = 56.94 \text{ mm } \textbf{Ans.}$$

and inner diameter of the spring coil,

$$D_i = D - d = 47.45 - 9.49 = 37.96 \text{ mm } \textbf{Ans.}$$

2. *Number of turns of the spring coil*

Let n = Number of active turns.

It is given that the axial deflection (δ) for the load range from 2250 N to 2750 N (*i.e.* for W = 500 N) is 6 mm.

We know that the deflection of the spring (δ),

$$6 = \frac{8\ W.C^3.n}{G.d} = \frac{8 \times 500\ (5)^3\ n}{84 \times 10^3 \times 9.49} = 0.63\ n$$

$$\therefore \quad n = 6 / 0.63 = 9.5 \text{ say } 10 \textbf{ Ans.}$$

For squared and ground ends, the total number of turns,

$$n' = 10 + 2 = 12 \textbf{ Ans.}$$

3. *Free length of the spring*

Since the compression produced under 500 N is 6 mm, therefore maximum compression produced under the maximum load of 2750 N is

$$\delta_{max} = \frac{6}{500} \times 2750 = 33 \text{ mm}$$

We know that free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\ \delta_{max}$$
$$= 12 \times 9.49 + 33 + 0.15 \times 33$$
$$= 151.83 \text{ say } 152 \text{ mm } \textbf{Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{152}{12 - 1} = 13.73 \text{ say } 13.8 \text{ mm } \textbf{Ans.}$$

Energy Stored in Helical Springs of Circular Wire

We know that the springs are used for storing energy which is equal to the work done on it by some external load.

Let W = Load applied on the spring, and

δ = Deflection produced in the spring due to the load W.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W.\delta$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} \text{ or } W = \frac{\pi\,d^3.\tau}{8K.D}$$

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times \pi\,d^3.\tau}{8K.D} \times \frac{D^3.n}{G.d^4} = \frac{\pi\,\tau.D^2.n}{K.d.G}$$

Substituting the values of W and δ in equation ***(i)***, we have

$$U = \frac{1}{2} \times \frac{\pi\,d^3.\tau}{8\,K.D} \times \frac{\pi\,\tau.D^2.n}{K.d.G}$$

$$= \frac{\tau^2}{4\,K^2.G}(\pi\,D.n)\left(\frac{\pi}{4} \times d^2\right) = \frac{\tau^2}{4\,K^2.G} \times V$$

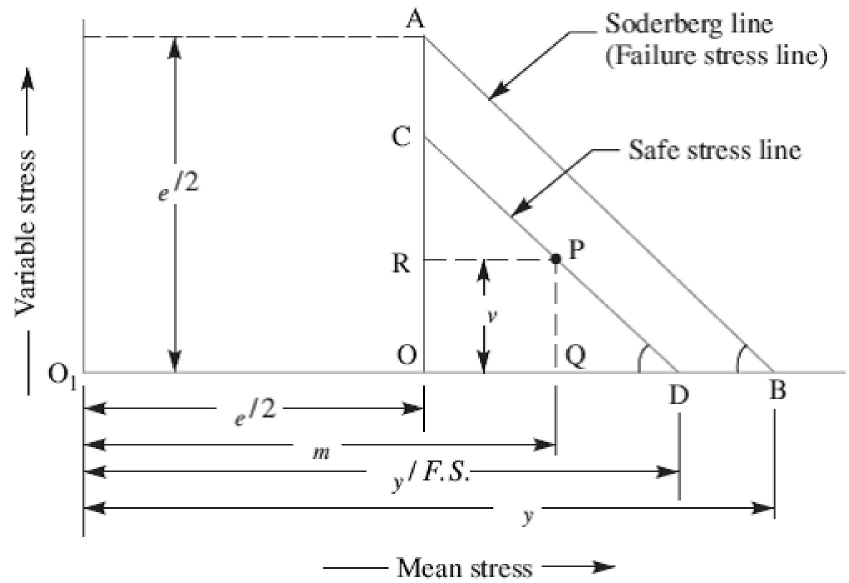
Where V = Volume of the spring wire

= Length of spring wire × Cross-sectional area of spring wire

Helical Springs Subjected to Fatigue Loading

The helical springs subjected to fatigue loading are designed by using the Soderberg line method. The spring materials are usually tested for torsional endurance strength under a repeated stress that varies from zero to a maximum. Since the springs are ordinarily loaded in one direction only (the load in springs is never reversed in nature), therefore a modified Soderberg diagram is used for springs, as shown in Fig.

The endurance limit for reversed loading is shown at point A where the mean shear stress is equal to τe / 2 and the variable shear stress is also equal to τe / 2. A line drawn from A to B (the yield point in shear, τy) gives the Soderberg's failure stress line. If a suitable factor of safety (F.S.) is applied to the yield strength (τy), a safe stress line CD may be drawn parallel to the line AB, as shown in Fig. Consider a design point P on the line CD. Now the value of factor of safety may be obtained as discussed below:



From similar triangles PQD and AOB, we have

$$\frac{PQ}{QD} = \frac{OA}{OB} \quad \text{or} \quad \frac{PQ}{O_1D - O_1Q} = \frac{OA}{O_1B - O_1O}$$

$$\frac{\tau_v}{\dfrac{\tau_y}{F.S.} - \tau_m} = \frac{\tau_e / 2}{\tau_y - \dfrac{\tau_e}{2}} = \frac{\tau_e}{2\tau_y - \tau_e}$$

or $$2\,\tau_v \,.\, \tau_y - \tau_v . \,\tau_e = \frac{\tau_e \,.\, \tau_y}{F.S.} - \tau_m . \tau_e$$

$$\therefore \qquad \frac{\tau_e \,.\, \tau_y}{F.S.} = 2\,\tau_v . \tau_y - \tau_v . \tau_e + \tau_m . \tau_e$$

Dividing both sides by $\tau_e . \tau_y$ and rearranging, we have

$$\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\tau_v}{\tau_e}$$

Springs in Series

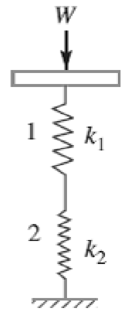
Total deflection of the springs,

$$\delta - \delta_1 + \delta_2$$

$$\frac{W}{k} = \frac{W}{k_1} + \frac{W}{k_2}$$

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

k = Combined stiffness of the springs.



Springs in Parallel

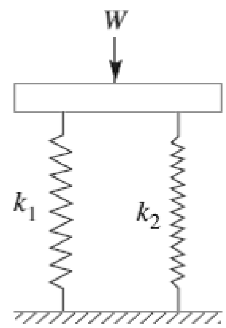
$$W = W_1 + W_2$$

$$\delta.k = \delta.k_1 + \delta.k_2$$

$$k = k_1 + k_2$$

k = Combined stiffness of the springs, and

δ = Deflection produced.



Surge in Springs or finding natural frequency of a helical spring:

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end.

This wave of compression travels along the spring indefinitely. If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur. This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon is called ***surge.***

It has been found that the natural frequency of spring should be at least twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies up to twentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 . n} \sqrt{\frac{6\,G.g}{\rho}} \text{ cycles/s}$$

Where d = Diameter of the wire,

D = Mean diameter of the spring,

n = Number of active turns,

G = Modulus of rigidity,

g = Acceleration due to gravity, and

ρ = Density of the material of the spring.

The surge in springs may be eliminated by using the following methods:

1. By using friction dampers on the centre coils so that the wave propagation dies out.

2. By using springs of high natural frequency.

3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

Energy Stored in Helical Springs of Circular Wire

We know that the springs are used for storing energy which is equal to the work done on it by some external load.

Let W = Load applied on the spring, and

δ = Deflection produced in the spring due to the load W.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W.\delta$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8 W.D}{\pi d^3} \text{ or } W = \frac{\pi d^3.\tau}{8 K.D}$$

We know that deflection of the spring,

$$\delta = \frac{8 W.D^3.n}{G.d^4} = \frac{8 \times \pi d^3.\tau}{8 K.D} \times \frac{D^3.n}{G.d^4} = \frac{\pi \tau.D^2.n}{K.d.G}$$

Substituting the values of W and δ in equation ***(i)***, we have

$$U = \frac{1}{2} \times \frac{\pi d^3.\tau}{8 K.D} \times \frac{\pi \tau.D^2.n}{K.d.G}$$

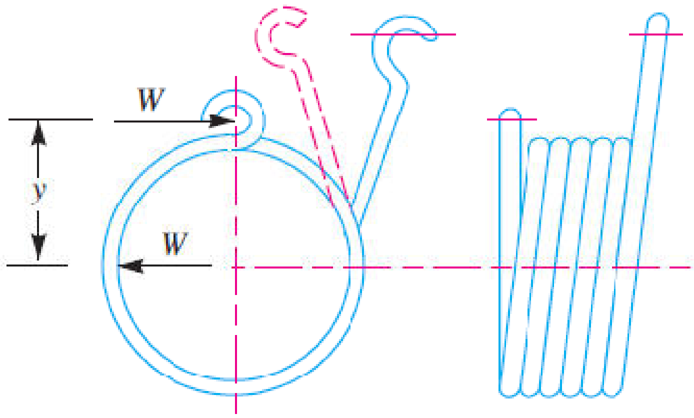
$$= \frac{\tau^2}{4 K^2.G} (\pi D.n)\left(\frac{\pi}{4} \times d^2\right) = \frac{\tau^2}{4 K^2.G} \times V$$

Where V = Volume of the spring wire

= Length of spring wire × Cross-sectional area of spring wire

Helical Torsion Springs

The helical torsion springs as shown in Fig., may be made from round, rectangular or square wire. These are wound in a similar manner as helical compression or tension springs but the ends are shaped to transmit torque. The primary stress in helical torsion springs is bending stress whereas in compression or tension springs, the stresses are torsional shear stresses. The helical torsion springs are widely used for transmitting small torques as in door hinges, brush holders in electric motors, automobile starters etc. A little consideration will show that the radius of curvature of the coils changes when the twisting moment is applied to the spring. Thus, the wire is under pure bending. According to A.M. Wahl, the bending stress in a helical torsion spring made of round wire is



$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = K \times \frac{32\,W.y}{\pi\,d^3}$$

Where K = Wahl's stress factor $= = \frac{4C^2 - C - 1}{4C^2 - 4C}$

C = Spring index,

M = Bending moment = W × y,

W = Load acting on the spring,

y = Distance of load from the spring axis, and

d = Diameter of spring wire.

And

Total angle of twist or angular deflection,

$$*\theta = \frac{M.l}{E.I} = \frac{M \times \pi D.n}{E \times \pi\,d^4/64} = \frac{64\,M.D.n}{E.d^4}$$

Where l = Length of the wire = π.D.n,

E = Young's modulus, $= \frac{\pi}{64} d^4$

D = Diameter of the spring, and

n = Number of turns.

And deflection,

$$\delta = \theta \times y = \frac{64\,M.D.n}{E.d^4} \times y$$

When the spring is made of rectangular wire having width b and thickness t, then

$$\sigma_b = K \times \frac{6\,M}{t.b^2} = K \times \frac{6\,W \times y}{t.b^2}$$

Where

$$K = \frac{3C^2 \quad C \quad 0.8}{3C^2 - 3C}$$

Angular deflection,

$$\theta = \frac{12\,\pi M.D.n}{E.t.b^3}; \quad \text{and} \quad \delta = \theta.y = \frac{12\,\pi M.D.n}{E.t.b^3} \times y$$

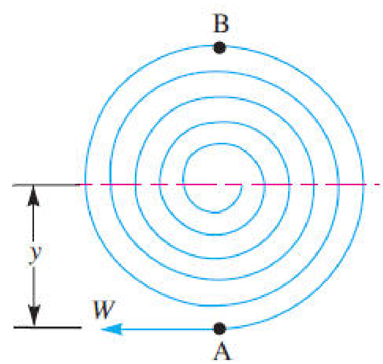
In case the spring is made of square wire with each side equal to b, then substituting t = b, in the above relation, we have

$$\sigma_b = K \times \frac{6\,M}{b^3} = K \times \frac{6W \times y}{b^3}$$

$$\theta = \frac{12\,\pi M.D.n}{E.b^4}; \quad \text{and} \quad \delta = \frac{12\,\pi M.D.n}{E.b^4} \times y$$

Flat Spiral Spring

A flat spring is a long thin strip of elastic material wound like a spiral as shown in Fig. These springs are frequently used in watches and gramophones etc. When the outer or inner end of this type of spring is wound up in such a way that there is a tendency in the increase of number of spirals of the spring, the strain energy is stored into its spirals. This energy is utilised in any useful way while the spirals open out slowly. Usually the inner end of spring is clamped to an arbor while the outer end may be pinned or clamped. Since the radius of curvature of every spiral decreases when the spring is wound up, therefore the material of the spring is in a state of pure bending.



Let W = Force applied at the outer end A of the spring,

y = Distance of centre of gravity of the spring from A,

l = Length of strip forming the spring,

b = Width of strip,

t = Thickness of strip,

I = Moment of inertia of the spring section = $b.t^3/12$, and

Z = Section modulus of the spring section = $b.t^2/6$

When the end A of the spring is pulled up by a force W, then the bending moment on the spring, at a distance y from the line of action of W is given by

$$M = W \times y$$

The greatest bending moment occurs in the spring at *B* which is at a maximum distance from the application of *W*.

Bending moment at *B*,

$$M_B = M_{max} = W \times 2y = 2W.y = 2M$$

Maximum bending stress induced in the spring material,

$$\sigma_b = \frac{M_{max}}{Z} = \frac{2W \times y}{b.t^2/6} = \frac{12W.y}{b.t^2} = \frac{12M}{b.t^2}$$

Assuming that both ends of the spring are clamped, the angular deflection (in radians) of the spring is given by

$$\theta = \frac{M.l}{E.I} = \frac{12\ M.l}{E.b.t^3}$$

And the deflection,

$$\delta = \theta \times y = \frac{M.l.y}{E.I}$$

$$= \frac{12\ M.l.y}{E.b.t^3} = \frac{12W.y^2.l}{E.b.t^3} = \frac{\sigma_b.y.l}{E.t}$$

The strain energy stored in the spring

$$= \frac{1}{2} M.\theta = \frac{1}{2} M \times \frac{M.l}{E.I} = \frac{1}{2} \times \frac{M^2.l}{E.I}$$

$$= \frac{1}{2} \times \frac{W^2.y^2.l}{E \times bt^3/12} = \frac{6\ W^2.y^2.l}{E.b.t^3}$$

$$= \frac{6\ W^2.y^2.l}{E.b.t^3} \times \frac{24bt}{24bt} = \frac{144\ W^2 y^2}{Eb^2 t^4} \times \frac{b\,t\,l}{24}$$

... (Multiplying the numerator and denominator by 24 *bt*)

$$= \frac{(\sigma_b)^2}{24\ E} \times b\,t\,l = \frac{(\sigma_b)^2}{24\ E} \times \text{Volume of the spring}$$

Problem: A helical torsion spring of mean diameter 60 mm is made of a round wire of 6 mm diameter. If a torque of 6 N-m is applied on the spring, find the bending stress induced and the angular deflection of the spring in degrees. The spring index is 10 and modulus of elasticity for the spring material is 200 kN/mm^2. The number of effective turns may be taken as 5.5.

Solution. Given : $D = 60$ mm ; $d = 6$ mm ; $M = 6$ N-m = 6000 N-mm ; $C = 10$; $E = 200$ kN/mm^2 = 200×10^3 N/mm^2 ; $n = 5.5$

Bending stress induced

We know that Wahl's stress factor for a spring made of round wire,

$$K = \frac{4C^2 - C - 1}{4C^2 - 4C} = \frac{4 \times 10^2 - 10 - 1}{4 \times 10^2 - 4 \times 10} = 1.08$$

∴ Bending stress induced,

$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = 1.08 \times \frac{32 \times 6000}{\pi \times 6^3} = 305.5 \text{ N/mm}^2 \text{ or MPa} \quad \textbf{Ans.}$$

Angular deflection of the spring

We know that the angular deflection of the spring (in radians),

$$\theta = \frac{64\,M.D.n}{E.d^4} = \frac{64 \times 6000 \times 60 \times 5.5}{200 \times 10^3 \times 6^4} = 0.49 \text{ rad}$$

$$= 0.49 \times \frac{180}{\pi} = 28° \quad \textbf{Ans.}$$

Problem: A spiral spring is made of a flat strip 6 mm wide and 0.25 mm thick. The length of the strip is 2.5 metres. Assuming the maximum stress of 800 MPa to occur at the point of greatest bending moment, calculate the bending moment, the number of turns to wind up the spring and the strain energy stored in the spring. Take E = 200 kN/mm^2.

Bending moment in the spring

Let M = Bending moment in the spring.

We know that the maximum bending stress in the spring material (σ_b),

$$800 = \frac{12\,M}{b.t^2} = \frac{12\,M}{8\,(0.25)^2} = 32\,M$$

∴ $M = 800 / 32 = 25$ N-mm **Ans.**

Number of turns to wind up the spring

We know that the angular deflection of the spring,

$$\theta = \frac{12\,M.l}{E.b.t^3} = \frac{12 \times 25 \times 2500}{200 \times 10^3 \times 6\,(0.25)^3} = 40 \text{ rad}$$

Since one turn of the spring is equal to 2π radians, therefore number of turns to wind up the spring

$$= 40 / 2\pi = 6.36 \text{ turns } \textbf{Ans.}$$

Strain energy stored in the spring

We know that strain energy stored in the spring

$$= \frac{1}{2} M.\theta = \frac{1}{2} \times 24 \times 40 = 480 \text{ N-mm} \;\; \textbf{Ans.}$$

Concentric or Composite Springs or coaxial springs or nested springs

A concentric or composite spring is used for one of the following purposes:

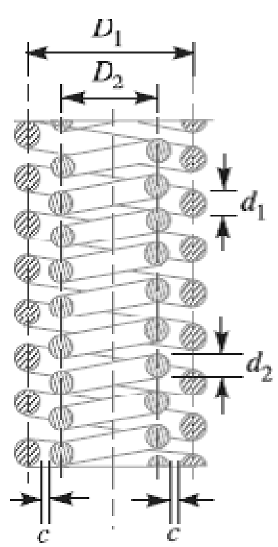
1. To obtain greater spring force within a given space.

2. To insure the operation of a mechanism in the event of failure of one of the springs.

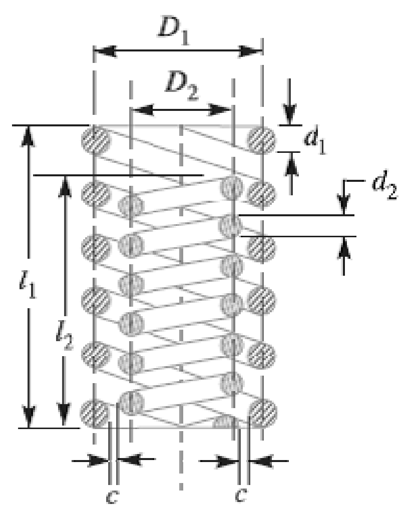
The concentric springs for the above two purposes may have two or more springs and have the same free lengths as shown in Fig. (*a*) And are compressed equally.

Such springs are used in automobile clutches; valve springs in aircraft, heavy duty diesel engines and rail-road car suspension systems. Sometimes concentric springs are used to obtain a spring force which does not increase in a direct relation to the deflection but increases faster. Such springs are made of different lengths as shown in Fig. (*b*). The shorter spring begins to act only after the longer spring is compressed to a certain amount. These springs are used in governors of variable speed engines to take care of the variable centrifugal force. The adjacent coils of the concentric spring are wound in opposite directions to eliminate any tendency to bind.

If the same material is used, the concentric springs are designed for the same stress. In order to get the same stress factor (*K*), it is desirable to have the same spring index (*C*).



(*a*)



(*b*)

Consider a concentric spring as shown in Fig. (*a*).

Let W = Axial load,

W_1 = Load shared by outer spring,

W_2 = Load shared by inner spring,

d_1= Diameter of spring wire of outer spring,

d_2 = Diameter of spring wire of inner spring,

D_1 = Mean diameter of outer spring,

D_2 = Mean diameter of inner spring,

δ_1 = Deflection of outer spring,

δ_2 = Deflection of inner spring,

n_1 = Number of active turns of outer spring, and

n_2 = Number of active turns of inner spring.

Assuming that both the springs are made of same material, then the maximum shear stress induced in both the springs is approximately same, *i.e.*

$$\tau_1 = \tau_2$$

$$\frac{8\,W_1 . D_1 . K_1}{\pi\,(d_1)^3} = \frac{8\,W_2 . D_2 . K_2}{\pi\,(d_2)^3}$$

When stress factor, $K_1 = K_2$, then

$$\frac{W_1 . D_1}{(d_1)^3} = \frac{W_2 . D_2}{(d_2)^3}.$$

If both the springs are effective throughout their working range, then their free length and deflection are equal, *i.e.*

$$\delta_1 = \delta_2$$

$$\frac{8W_1\,(D_1)^3\,n_1}{(d_1)^4\,G} = \frac{8W_2\,(D_2)^3\,n_2}{(d_2)^4\,G} \quad \text{or} \quad \frac{W_1\,(D_1)^3\,n_1}{(d_1)^4} = \frac{W_2\,(D_2)^3\,n_2}{(d_2)^4} \qquad ...(ii)$$

When both the springs are compressed until the adjacent coils meet, then the solid length of both the springs is equal, *i.e.*

$n_1 . d_1 = n_2 . d_2$

The equation ***(ii)*** may be written as

$$\frac{W_1\,(D_1)^3}{(d_1)^5} = \frac{W_2\,(D_2)^3}{(d_2)^5} \qquad ...(iii)$$

Now dividing equation ***(iii)*** by equation ***(i)***, we have

$$\frac{(D_1)^2}{(d_1)^2} = \frac{(D_2)^2}{(d_2)^2} \quad \text{or} \quad \frac{D_1}{d_1} = \frac{D_2}{d_2} = C, \text{ the spring index} \qquad ...(iv)$$

i.e. the springs should be designed in such a way that the spring index for both the springs is same. From equations ***(i)*** and ***(iv)***, we have

$$\frac{W_1}{(d_1)^2} = \frac{W_2}{(d_2)^2} \quad \text{or} \quad \frac{W_1}{W_2} = \frac{(d_1)^2}{(d_2)^2} \qquad ..(v)$$

From Fig. 23.22 (*a*), we find that the radial clearance between the two springs,

$$*c = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right)$$

Usually, the radial clearance between the two springs is taken as

$$\frac{d_1 - d_2}{2}$$

$$\therefore \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right) = \frac{d_1 - d_2}{2}$$

or $$\frac{D_1 - D_2}{2} = d_1 \qquad \text{------ (vi)}$$

From equation ***(iv)***, we find that

$$D_1 = C.d_1, \text{ and } D_2 = C.d_2$$

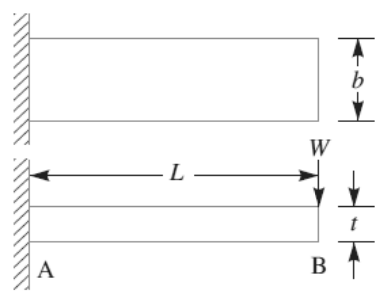
Substituting the values of D_1 and D_2 in equation ***(vi)***, we have

$$\frac{C.d_1 - C.d_2}{2} = d_1 \quad \text{or} \quad C.d_1 - 2\,d_1 = C.d_2$$

$$d_1\,(C-2) = C.d_2 \quad \text{or} \quad \frac{d_1}{d_2} = \frac{C}{C-2}$$

Leaf Springs

Leaf springs (also known as **flat springs**) are made out of flat plates. The advantage of leaf spring over helical spring is that the ends of the spring may be guided along a definite path as it deflects to act as a structural member in addition to energy absorbing device. Thus the leaf springs may carry lateral loads, brake torque, driving torque etc., in addition to shocks. Consider a single plate fixed at one end and loaded at the other end as shown in Fig. This plate may be used as a flat spring.



W

Let t = Thickness of plate,

b = Width of plate, and

L = Length of plate or distance of the load from the cantilever end.

We know that the maximum bending moment at the cantilever end A,

$$M = W.L$$

And section modulus,

$$Z = \frac{I}{y} = \frac{b\,t^3/12}{t/2} = \frac{1}{6} \times b.t^2$$

Bending stress in such a spring,

$$\sigma = \frac{M}{Z} = \frac{W.L}{\frac{1}{6} \times b.t^2} = \frac{6\,W.L}{b.t^2}$$

We know that the maximum deflection for a cantilever with concentrated load at the free end is given by

$$\delta = \frac{W.L^3}{3E.I} = \frac{W.L^3}{3E \times b.t^3 / 12} = \frac{4\,W.L^3}{E.b.t^3}$$

$$= \frac{2\,\sigma.L^2}{3\,E.t}$$

If the spring is not of cantilever type but it is like a simply supported beam, with length 2*L* and load 2*W* in the centre, as shown in Fig. then Maximum bending moment in the centre,

$M = W.L$

Section modulus, $Z = b.t2 / 6$

Bending stress,

$$\sigma = \frac{M}{Z} = \frac{W.L}{b.t^2 / 6}$$

$$= \frac{6\,W.L}{b.t^2}$$

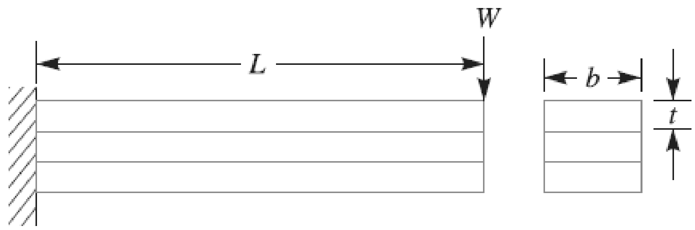
We know that maximum deflection of a simply supported beam loaded in the centre is given by

$$\delta = \frac{W_1\,(L_1)^3}{48\,E.I} = \frac{(2W)\,(2L)^3}{48\,E.I} = \frac{W.L^3}{3\,E.I}$$

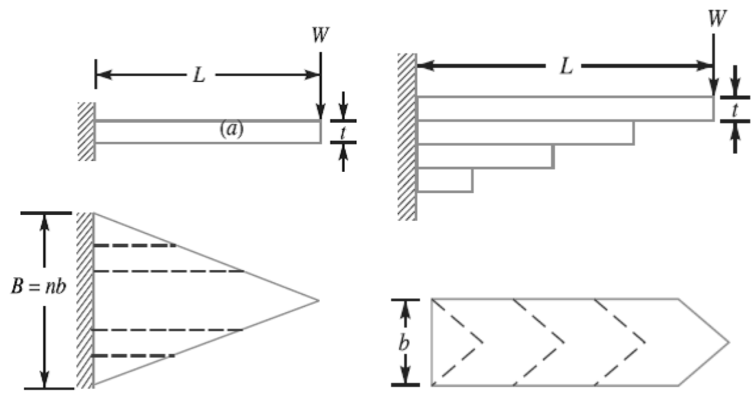
From above we see that a spring such as automobile spring (semi-elliptical spring) with length 2*L* and loaded in the centre by a load 2*W*, may be treated as a double cantilever. If the plate of cantilever is cut into a series of *n* strips of width *b* and these are placed as shown in Fig., then equations ***(i)*** and ***(ii)*** may be written as

$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad \ldots(iii)$$

And $$\delta = \frac{4\,W.L^3}{n.E.b.t^3} = \frac{2\,\sigma.L^2}{3\,E.t} \qquad \ldots(iv)$$



The above relations give the stress and deflection of a leaf spring of uniform cross section. The stress at such a spring is maximum at the support.



If a triangular plate is used as shown in Fig., the stress will be uniform throughout. If this triangular plate is cut into strips of uniform width and placed one below the other, as shown in Fig. to form a graduated or laminated leaf spring, then

$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad \ldots(v)$$

$$\delta = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{\sigma.L^2}{E.t} \qquad \ldots(vi)$$

where n = Number of graduated leaves.

A little consideration will show that by the above arrangement, the spring becomes compact so that the space occupied by the spring is considerably reduced.

When bending stress alone is considered, the graduated leaves may have zero width at the loaded end. But sufficient metal must be provided to support the shear. Therefore, it becomes necessary to have one or more leaves of uniform cross-section extending clear to the end. We see from equations ***(iv)*** and ***(vi)*** that for the same deflection, the stress in the uniform cross-section leaves (*i.e.* full length leaves) is 50% greater than in the graduated leaves, assuming that each spring element deflects according to its own elastic curve. If the suffixes

F and G are used to indicate the full length (or uniform cross section) and graduated leaves, then

$$\sigma_F = \frac{3}{2}\sigma_G$$

$$\frac{6 W_F . L}{n_F . b . t^2} = \frac{3}{2}\left[\frac{6\, W_G . L}{n_G . b . t^2}\right] \quad \text{or} \quad \frac{W_F}{n_F} = \frac{3}{2} \times \frac{W_G}{n_G}$$

$$\frac{W_F}{W_G} = \frac{3\, n_F}{2\, n_G} \qquad \ldots(vii)$$

Adding 1 to both sides, we have

$$\frac{W_F}{W_G} + 1 = \frac{3\, n_F}{2\, n_G} + 1 \quad \text{or} \quad \frac{W_F + W_G}{W_G} = \frac{3\, n_F + 2\, n_G}{2\, n_G}$$

$$W_G = \left(\frac{2\, n_G}{3\, n_F + 2\, n_G}\right)(W_F + W_G) = \left(\frac{2\, n_G}{3\, n_F + 2\, n_G}\right) W \qquad \ldots(viii)$$

where W = Total load on the spring = $W_G + W_F$

W_G = Load taken up by graduated leaves, and

W_F = Load taken up by full length leaves.

From equation **(*vii*)**, we may write

$$\frac{W_G}{W_F} = \frac{2\, n_G}{3\, n_F}$$

or

$$\frac{W_G}{W_F} + 1 = \frac{2\, n_G}{3\, n_F} + 1$$

$$\frac{W_G + W_F}{W_F} = \frac{2\, n_G + 3\, n_F}{3\, n_F}$$

$$\therefore \quad W_F = \left(\frac{3\, n_F}{2\, n_G + 3\, n_F}\right)(W_G + W_F) = \left(\frac{3\, n_F}{2\, n_G + 3\, n_F}\right) W \qquad \ldots(ix)$$

Bending stress for full length leaves,

$$\sigma_F = \frac{6\, W_F . L}{n_F . b\, t^2} = \frac{6\, L}{n_F . b . t^2}\left(\frac{3\, n_F}{2\, n_G + 3\, n_F}\right) W = \frac{18\, W . L}{b . t^2\, (2\, n_G + 3\, n_F)}$$

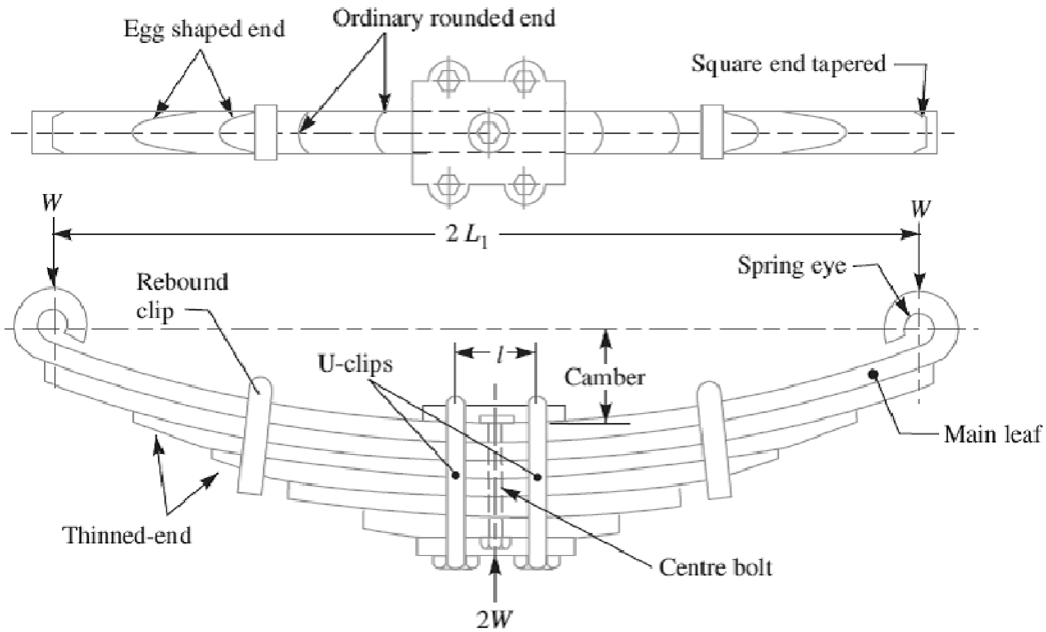
Since

$$\sigma_F = \frac{3}{2}\sigma_G, \text{ therefore}$$

$$\sigma_G = \frac{2}{3}\sigma_F = \frac{2}{3} \times \frac{18\, W . L}{b . t^2\, (2\, n_G + 3\, n_F)} = \frac{12\, W . L}{b . t^2\, (2\, n_G + 3\, n_F)}$$

The deflection in full length and graduated leaves is given by equation **(*iv*)**, *i.e.*

$$\delta = \frac{2\,\sigma_F \times L^2}{3\,E.t} = \frac{2\,L^2}{3\,E.t}\left[\frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}\right] = \frac{12\,W.L^3}{E.b.t^3\,(2\,n_G + 3\,n_F)}$$



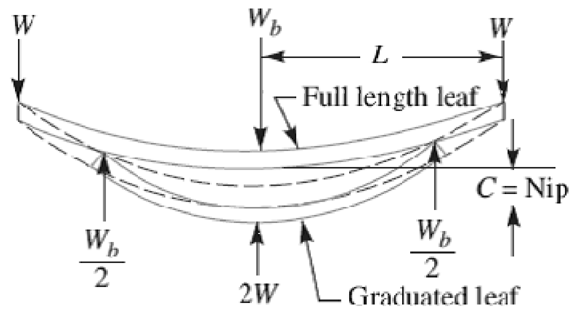
Equalised Stress in Spring Leaves (Nipping)

We have already discussed that the stress in the full length leaves is 50% greater than the stress in the graduated leaves. In order to utilise the material to the best advantage, all the leaves should be equally stressed.

This condition may be obtained in the following two ways:

1. By making the full length leaves of smaller thickness than the graduated leaves. In this way, the full length leaves will induce smaller bending stress due to small distance from the neutral axis to the edge of the leaf.

2. By giving a greater radius of curvature to the full length leaves than graduated leaves, as shown in Fig. before the leaves are assembled to form a spring. By doing so, a gap or clearance will be left between the leaves. This initial gap, as shown by *C* in Fig, is called ***nip.***



Consider that under maximum load conditions, the stress in all the leaves is equal. Then at maximum load, the total deflection of the graduated leaves will exceed the deflection of the full length leaves by an amount equal to the initial gap *C*. In other words,

$$\delta_G = \delta_F + C$$

$$C = \delta_G - \delta_F = \frac{6\,W_G\,.\,L^3}{n_G\,E.b.t^3} - \frac{4\,W_F.L^3}{n_F.E.b.t^3} \qquad \ldots(i)$$

Since the stresses are equal, therefore

$$\sigma_G = \sigma_F$$

$$\frac{6\,W_G.L}{n_G\,b.t^2} = \frac{6\,W_F\,L}{n_F\,b.t^2} \quad \text{or} \quad \frac{W_G}{n_G} - \frac{W_F}{n_F}$$

$$\therefore \qquad W_G = \frac{n_G}{n_F} \times W_F = \frac{n_G}{n} \times W$$

$$W_F = \frac{n_F}{n_G} \times W_G = \frac{n_F}{n} \times W$$

Substituting the values of *W*G and *W*F in equation **(*i*)**, we have

$$C = \frac{6W.L^3}{n.E.b.t^3} - \frac{4W.L^3}{n.E.b.t^3} = \frac{2W.L^3}{n.E.b.t^3} \qquad \ldots(ii)$$

The load on the clip bolts (W_b) required to close the gap is determined by the fact that the gap is equal to the initial deflections of full length and graduated leaves.

$$\therefore \qquad C = \delta_F + \delta_G$$

$$\frac{2W.L^3}{n.E.b.t^3} = \frac{4L^3}{n_F.E.b.t^3} \times \frac{W_b}{2} + \frac{6\,L^3}{n_G.E.b.t^3} \times \frac{W_b}{2}$$

Or

Problem: Design a leaf spring for the following specifications:

Total load = 140 kN ; Number of springs supporting the load = 4 ; Maximum number of leaves = 10; Span of the spring = 1000 mm ; Permissible deflection = 80 mm.

Take Young's modulus, E = 200 kN/mm2 and allowable stress in spring material as 600 MPa.

Solution. Given : Total load = 140 kN ; No. of springs = 4; n = 10 ; $2L$ = 1000 mm or L = 500 mm ; δ = 80 mm ; E = 200 kN/mm^2 = 200 × 10^3 N/mm^2 ; σ = 600 MPa = 600 N/mm^2

We know that load on each spring,

$$2W = \frac{\text{Total load}}{\text{No. of springs}} = \frac{140}{4} = 35 \text{ kN}$$

$$\therefore \quad W = 35 / 2 = 17.5 \text{ kN} = 17\,500 \text{ N}$$

Let t = Thickness of the leaves, and

b = Width of the leaves.

We know that bending stress (σ),

$$600 = \frac{6\,W.L}{n.b.t^2} = \frac{6 \times 17\,500 \times 500}{n.b.t^2} = \frac{52.5 \times 10^6}{n.b.t^2}$$

$$\therefore \quad n.b.t^2 = 52.5 \times 10^6 / 600 = 87.5 \times 10^3 \qquad ...(i)$$

and deflection of the spring (δ),

$$80 = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{6 \times 17\,500\,(500)^3}{n \times 200 \times 10^3 \times b \times t^3} = \frac{65.6 \times 10^6}{n.b.t^3}$$

$$\therefore \quad n.b.t^3 = 65.6 \times 10^6 / 80 = 0.82 \times 10^6 \qquad ...(ii)$$

Dividing equation (*ii*) by equation (*i*), we have

$$\frac{n.b.t^3}{n.b.t^2} = \frac{0.82 \times 10^6}{87.5 \times 10^3} \quad \text{or} \quad t = 9.37 \text{ say } 10 \text{ mm}$$ **Ans.**

Now from equation (*i*), we have

$$b = \frac{87.5 \times 10^3}{n.t^2} = \frac{87.5 \times 10^3}{10\,(10)^2} = 87.5 \text{ mm}$$

and from equation (*ii*), we have

$$b = \frac{0.82 \times 10^6}{n.t^3} = \frac{0.82 \times 10^6}{10\,(10)^3} = 82 \text{ mm}$$

Taking larger of the two values, we have width of leaves,

b = 87.5 say 90 mm **Ans.**

Problem:

A truck spring has 12 number of leaves, two of which are full length leaves.

The spring supports are 1.05 m apart and the central band is 85 mm wide. The central load is to be 5.4 kN with a permissible stress of 280 MPa. Determine the thickness and width of the steel spring leaves. The ratio of the total depth to the width of the spring is 3. Also determine the deflection of the spring.

Solution. Given : $n = 12$; $n_F = 2$; $2L_1 = 1.05$ m $= 1050$ mm ; $l = 85$ mm ; $2W = 5.4$ kN $= 5400$ N or $W = 2700$ N ; σ_F 280 MPa = 280 N/mm^2

Thickness and width of the spring leaves

Let t = Thickness of the leaves, and

b = Width of the leaves.

Since it is given that the ratio of the total depth of the spring ($n \times t$) and width of the spring (b) is 3, therefore

$$\frac{n \times t}{b} = 3 \quad \text{or} \quad b = n \times t / 3 = 12 \times t / 3 = 4\,t$$

We know that the effective length of the spring,

$$2L = 2L_1 - l = 1050 - 85 = 965 \text{ mm}$$

$$\therefore \quad L = 965 / 2 = 482.5 \text{ mm}$$

and number of graduated leaves,

$$n_G = n - n_F = 12 - 2 = 10$$

Assuming that the leaves are not initially stressed, therefore maximum stress or bending stress for full length leaves (σ_F),

$$280 = \frac{18\,W.L}{b.t^2\ (2n_G + 3n_F)} = \frac{18 \times 2700 \times 482.5}{4\,t \times t^2\,(2 \times 10 + 3 \times 2)} = \frac{225\,476}{t^3}$$

$$\therefore \quad t^3 = 225\,476 / 280 = 805.3 \quad \text{or} \quad t = 9.3 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

and $\quad b = 4\,t = 4 \times 10 = 40$ mm **Ans.**

Deflection of the spring

We know that deflection of the spring,

$$\delta = \frac{12\,W.L^3}{E.b.t^3\ (2n_G + 3n_F)}$$

$$= \frac{12 \times 2700 \times (482.5)^3}{210 \times 10^3 \times 40 \times 10^3\ (2 \times 10 + 3 \times 2)} \text{ mm}$$

$= 16.7$ mm **Ans.** ... (Taking $E = 210 \times 10^3$ N/mm^2)