ANNAMACHARYA UNIVERSITY

EXCELLENCE IN EDUCATION; SERVICE TO SOCIETY
(ESTD UNDER AP PRIVATE UNIVERSITIES (ESTABLISHMENT AND REGULATION) ACT, 2016)
RAJAMPET-516126:A.P; INDIA

DEPARTMENT OF MECHANICAL ENGINEERING

LECTURE NOTES

THEORY OF MACHINES
[24AMEC43T]

Prepared by
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ANNAMACHARYA UNIVERSITY

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Title of the Course: Theory of Machines
Category: PCC
Semester: IV Semester
Course Code: 24AMEC43T
Branch/es: Mechanical Engineering

Lecture Hours	Tutorial Hours	Practice Hours	Credits
3	0	0	3

Course Objectives:
1. To know the various basics related to simple mechanisms and their inversions.
2. To understand the velocity and acceleration in simple mechanisms and also effects of gyroscopic couple.
3. To familiarize the different gears and gear trains.
4. To apply the different balancing methods for rotating and reciprocating masses.
5. To understand the different vibrations and its natural frequencies for single degree of freedom system.

Course Outcomes:
At the end of the course, the student will be able to
1. Analyze different mechanisms, inversions of different kinematic chains and mobility of mechanisms
2. Analyze the velocity and acceleration diagrams of simple plane mechanisms by using relative velocity method and understand the effects of gyroscopic principle.
3. Analyze the phenomenon of interference in gears and velocity ratio of gear trains.
4. Estimate the balancing masses for rotating and reciprocating members in automotive applications.
5. Analyze the natural frequencies of mechanical systems based on governing equations.

Unit 1 Simple Mechanisms 09
Kinematic Link-Types of Links-Types of constraint motions-Classification of Kinematic Pairs – Kinematic Chain-Degree of freedom – Grashof's Law-Inversions of four bar chain, single and double slider crank mechanisms- Description of straight-line mechanisms – Peacellier and Harts mechanism- Steering gear mechanism: Ackerman, Davis steering mechanisms.

Unit 2 Velocity and Acceleration analysis, Gyroscope 10
Velocity and Acceleration analysis: Velocity analysis of simple mechanisms using relative velocity method-rubbing velocity– Acceleration analysis of simple mechanisms – four bar chain – single slider crank chain - Coriolis component of acceleration
Gyroscope: Principle of gyroscope-gyroscopic couple- gyroscopic effect in an aero plane, ship- simple Problems.

Unit 3 Gears & Gear trains 09
Gears: Gear terminology-classification of toothed wheels -Involute and cycloidal gear profiles- law of gearing – interference in involute gears -Length of path of contact- arc of contact- contact ratio.
Gear Trains: Types of gear trains –epicyclic gear trains-simple problems.

Unit 4 Balancing of Rotating masses & Balancing of Reciprocating masses 10
Balancing of Rotating masses: Need for balancing, balancing of single mass and several masses indifferent planes, using analytical and graphical methods.
Balancing of Reciprocating masses: Primary and Secondary balancing of reciprocating masses – graphical Method – balancing of locomotives – variation of tractive force, swaying couple, hammer blow.

Unit 5 Vibrations **09**

Introduction-Types of vibratory motion-Types of free vibrations-Natural frequency of free longitudinal vibrations-equilibrium method and energy method –Transverse vibrations –Dunkerly's method-Whirling speed of shafts- simple systems (Cantilever and Simply supported beams). Torsional vibrations - Natural frequency of torsional vibration- Single rotor, and Two-rotor system.

Prescribed Textbooks:

1. P.L. Ballaney, Theory of Machines & Mechanisms, 25/e, Khanna Publishers.
2. S.S. Rattan, Theory of Machines, 4/e, Tata McGraw Hill.

Reference Books:

1. J.E. Shigley, Theory of Machines and Mechanisms, 4/e, Oxford.
2. R.S.Khurmi &J.K.Gupta, Theory of Machines, S. Chand Publications.
3. Thomas Bevan, Theory of Machines, 3rd edition, CBS Publishers & Distributors, .
4. Jagadishlal, Theory of Mechanisms and Machines, Metropolitan Company Pvt Ltd.
5. R.K.Bansal, Theory of Machines, Lakshmi Publications.

Web Resources:

1. NPTEL :: Mechanical Engineering - Theory Of Mechanisms
2. Theory of Machine 02 | Simple Mechanism Part-2 - Question Practice Series | Abhyas | ME | GATE
3. Lecture 26: Analytical Velocity Analysis – III
4. Lecture 42 : Gears: Basic Concepts
5. Module 4 Lecture 1 Balancing of Single Slider Machines
6. NPTEL :: Mechanical Engineering - Mechanical Vibrations

CO-PO Mapping:

Course Outcomes	Engineering Knowledge	Problem Analysis	Design/Development of solutions	Conduct investigations of complex problems	Engineering Tool usage	The Engineer and The World	Ethics	Individual and Collaborative team work	Communication	Project management and finance	Life-long learning	PSO 1	PSO 2
24AMEC43T.1	3	3	2	2	-	-	1	-	1	-	1	1	2
24AMEC43T.2	3	3	2	2	-	-	1	-	1	-	1	3	2
24AMEC43T.3	3	3	2	2	-	-	1	-	1	-	1	2	1
24AMEC43T.4	3	3	2	2	-	-	1	-	1	-	1	2	3
24AMEC43T.5	3	3	2	2	-	-	1	-	1	-	1	1	2

UNIT-I.

The subject theory of machines may be defined as that branch of Engineering-Science, which deals with the study of relative motion between the various parts of machine, and forces which act on them.

Machine:- A machine is a device which receives energy in some available form and utilises it to do some particular type of work.

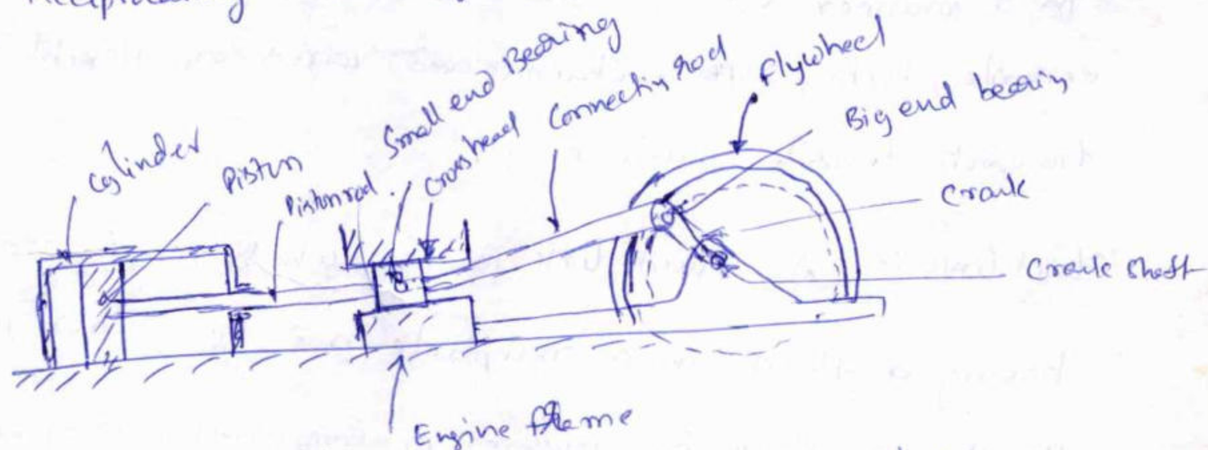
(or)

A machine is a device which receives energy and transforms it into some useful work.

Kinematic Link (or) Element:- Each part of the machine, which moves relative to some other part, is known as a Kinematic Link.

A Link may consists of several parts, which are rigidly fastened together, so that they do not move relative to one another.

Example:- Reciprocating Steam Engine



1st link :- Piston, Piston rod, and cross head constitutes one link.

2nd link :- connecting rod, small end bearing, Big end bearing.

3rd link :- Crank, Crank shaft and fly wheel.

4th link :- Cylinder, Engine frame and Main Bearings.

A Link (or) element need not to be a rigid body, but it must be a resistant body. A body is said to be resistant body if it is capable of transmitting the required forces with neglegible deformation.

The characteristics of link :-
(or)
Properties

(i) It should have relative motion.
(ii) It must be a resistant body.

TYPES OF LINKS

In order to transmit motion, the driver and follower may be connected by the following three types of Links.

1. Rigid Link :- A rigid link is one which does not undergo any deformation while transmitting motion. Strictly speaking rigid links donot exist.

2. Flexible link :- A flexible link is one which is partly deformed by in a manner not to affect the transmission of motion. For example, belts, ropes, chains and wires are flexible links and transmit tensile forces only.

3. Fluid link :- A fluid link is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only as in the case of hydraulic presses, jacks and brakes.

STRUCTURE :- It is an assemblage of a number of resistant bodies (Known as members) having no relative motion between them and meant for carring loads having straining action.

Machine	Structure
(1) The parts of machine move relative one another	(1) The parts of structure doesnot move relative to one another
(2) The machine transforms the available energy in to work	(2) The Structure donot transforms the energy in to work
(3) The machine transmits the motion and power	(3) The structure transmits only forces
(4) The parts of machine are called links	(4) The parts of structure are called the members
(5) The Machine constitutes the Mechanism	(5) The Structure is not having the mechanism
(6) Examples of machines are ;- Lathe, Milling Mach, Shaping Machn etc.	(6) Examples of structures are! - Bridges, Trusses, frames, Girders etc

Kinematic Pair :- The two links or elements of a machine, when in contact with each other are said to form a pair. If the relative motion between them is completely or succesfully constrained (i.e in a definite direcn), the pair is Known as Kinematic pair.

Types of Constraint Motion

(1) Completely Constraint Motion (2) Incompletely constraint Motion
(3) Succesfully constraint Motion

Kinematic chain:- When the Kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion (i.e completely or successfully constrained motion), it is called a Kinematic Chain.

(or)

In other words, a Kinematic chain may be defined as a Combination of Kinematic pairs, joined in such a way that each link forms a part of two pairs and the relative motion between the links or elements is completely or successfully constrained.

Mechanism:- When one of the links of a Kinematic chain is fixed, then the chain is known as mechanism.

If the Kinematic chain contains four links then it is called Simple Kinematic chain and the machine is called the Simple machine.

If the Kinematic chain contains more than four links, then it is called Compound Kinematic chain and the machine is called the Compound machine.

Classification of Kinematic Pairs

(1) According to the type of relative motion between the elements

(a) Sliding pair (b) Turning Pair (c) Rolling pair (d) Screw pair (e) Spherical pair

(2) According to the type of contact between the elements

(a) Lower pair (b) Higher pair

(3) According to the type of closure

(a) Self closed pair (b) Force closed pair

movability (or) Number of degrees of freedom

In the design or analysis of a mechanism, one of the most important concern is the number of degrees of freedom of the mechanism. It is defined as the number of input parameters (variables) which must be independently controlled in order to bring the mechanism into useful engineering purpose.

Now let us consider a plane mechanism with 'l' number of links. Since in a mechanism, one of the links is to be fixed, therefore the number of movable links will be $(l-1)$ and the total number of degrees of freedom will be $3(l-1)$ before they connected to any other link.

In general a mechanism with 'l' number of links connected by 'j' number of binary joints or lower pairs (having single degree of freedom) and 'h' number of higher pairs (having two degree of freedom), then the movabity (or) of the mechanism is given by the equation is

$$\boxed{\text{no. of DOF } (n) \Rightarrow \quad n = 3(l-1) - 2j - h}$$

for plane mechanism $h=0$

The above equation is called "KUTZ BATCH" criterion for movability of a mechanism having plane motion.

where l = no. of links

j = no. of Binary joints

h = no. of Higher pairs.

if $n=0$ it is called structure; $n=+1, \ldots$ etc it is a mechanism with no. of DOF.

$n=-1$ or negative it is called statistically indeterminant structure.

Grubler's Criterion for plane Mechanisms

The Grubler's criterion applies to mechanism with only single degree of freedom joints where the overall movability of the mechanism is Unity. Substituting $n=1$ and $h=0$ in Kutzbach equation ($n = 3(l-1)-2j-h$)

we have

$$1 = (3l-1) - 2j$$

$$\Rightarrow 1 = 3l - 3 - 2j$$

$$\Rightarrow \boxed{3l - 2j - 4 = 0}$$ → This equation is called the Grubler's criterion for plane mechanism

* Note:- A plane mechanism with a movability of 1 and only single degree of freedom joints can't have odd number of links. The simplest ~~form~~ possible mechanism of this type are a 4 bar mechanism and a slider crank mechanism in which $l=4$ and $j=4$.

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(1) The Relation between the number of links and number of kinetic pairs is given by the equation

$$\boxed{l = 2p - 4}$$

(2) The Relation between the number of joints and the number of links is given by the equation

$$\boxed{j = \frac{3}{2}l - 2}$$

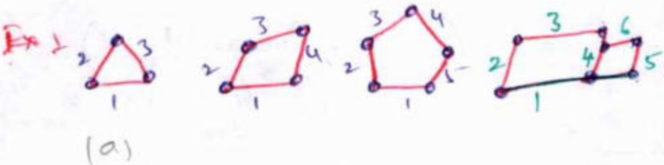
Note:- The above two equations are used for lower pairs to decide the Kinematic Chain or not.

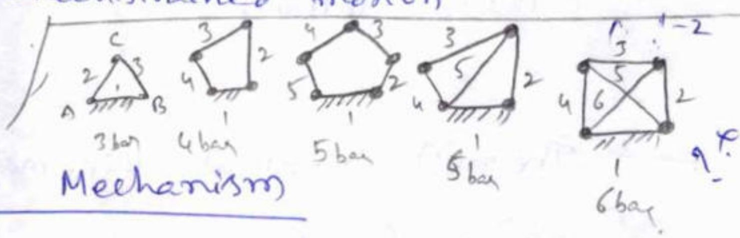
if (1) L.H.S > R.H.S → it is called Structure (or) Locked chain

(2) L.H.S = R.H.S → it is called a Kinematic chain having contrained motion

(3) L.H.S <= R.H.S → it is called a chain with Unconstrained motion

Ex:-





Inversion of Mechanism

Def. The method of obtaining different mechanisms by fixing different links in a kinematic chain is known as 'inversion of the Mechanism'.

It may be noted that the relative motions between the various links is not changed in any manner through the process of inversion, but their absolute motions (those measured with respect to fixed link) may be changed drastically.

Note:- The part (link or element) of mechanism which is initially moves with respect to the frame or fixed link is called driver and the part of mechanism to which motion is transmitted is called follower. Most of mechanisms are reversible i.e the same link can play the role of a driver and follower at different times.

Ex:- Reciprocating Steam engine → piston is a driver and flywheel is follower.

Reciprocating Compressor → piston is a follower and flywheel is driver.

TYPES OF KINEMATIC CHAINS

- Simple Kinematic Chain (Having '4' links)
- Complex (or) Compound Kinematic chain (Having more than '4' links)

The most important Kinematic chains are those which consists of four lower pairs, each pair being a sliding pair or turning pair. The following three types of kinematic chains with four lower pairs are important from the subject point of view.

(1) Fourbar chain (or) Quadric cycle chain.

(2) Single Slider Crank Mechanism.

(3) Double Slider Crank Mechanism.

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* *

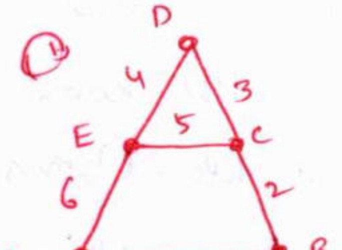
To Determine Nature of Chain (Structure, Kinematic chain, Unconstraint chain)

A.W. Klein is given $j + \frac{h}{2} = \frac{3}{2} l - 2$

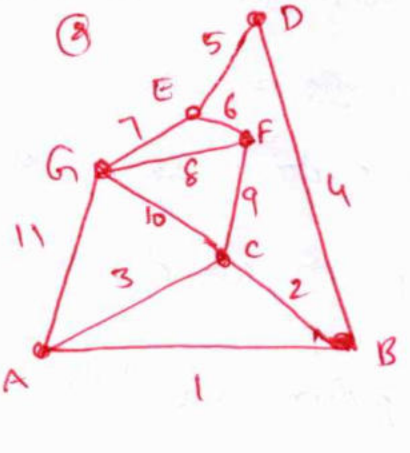
- j = no. of binary joint
- h = No. of higher pairs
- l = no. of links

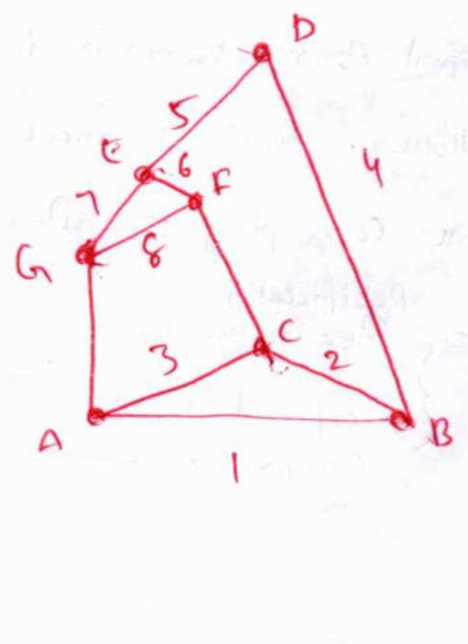
Note:- for Lower pairs $h = 0$

(1)



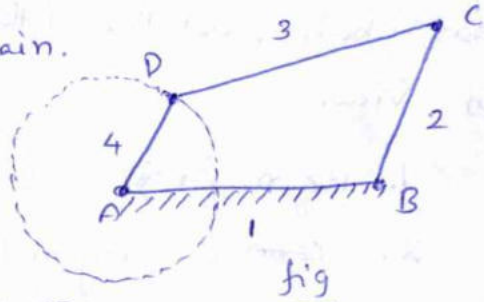
(2)





FOUR BAR CHAIN (or) Quadric Cycle Chain

The simplest and the basic kinematic chain is a four bar chain or a quadric cycle chain. It consists of a four links each of them forms a turning pairs at A, B, C and D. The four links may be of different lengths.



fig

According to the Grashof's law for four bar mechanism, the sum of shortest and longest link lengths should not be greater than the sum of the remaining two link lengths, if there is to be continuous relative motion between the two links.

A very important consideration in designing a mechanism is to ensure that the input crank makes a complete revolution relative to the other links. The mechanism in which no link makes a complete revolution will not be usefull. In four bar chain, one of the links in particular the shorter link will make a complete revolution relative to the other three links, if it satisfies the Grashof's law. Such link is known as a crank or driver(4). In the fig the link 4 is AD ~~the~~ called the crank. The link 'BC' makes a partial rotation or oscillates is known as lever(2) or rocker or follower. The link 'CD' which connects the crank and lever is called connecting rod(3) or coupler. The fixed link AB(1) is known as frame(1) of the mechanism.

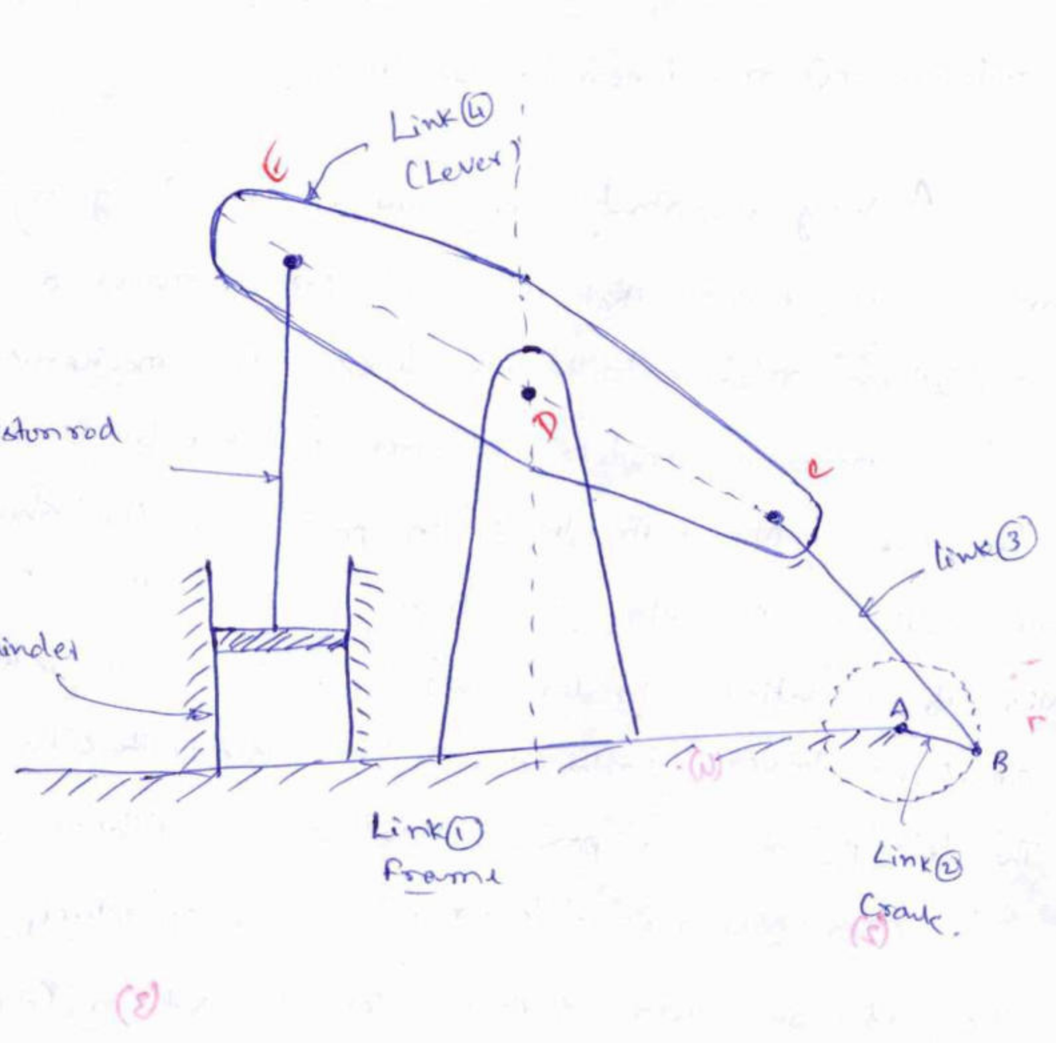
When the crank (AD) is a driver, the mechanism is transforming rotary motion into oscillating motion.

Inversions of four Bar Chain

Though there are many inversions of the four bar chain, yet the following are important from the subject point of view.

1. Beam Engine (crank and lever Mechanism)
2. ~~com~~ Coupling rod of a locomotive (Double crank mechanism)
3. Watt's indicator mechanism (Double lever mechanism)

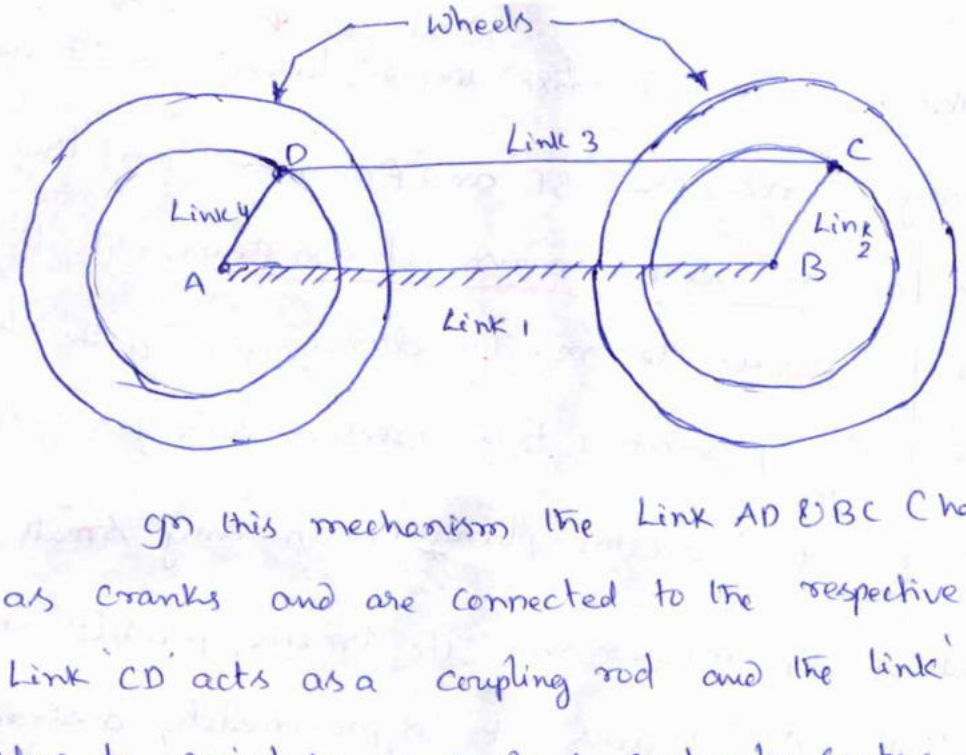
1. Bean Engine



A part of the mechanism of beam engine which consists of four links a is shown in fig. In this mechanism when the crank rotates about the fixed centre A, and the lever oscillates about a fixed Centre D. The End 'E' of the lever 'CDE' is connected to a piston rod which reciprocates due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion in to reciprocating motion.

2. Coupling Rod of a Locomotive

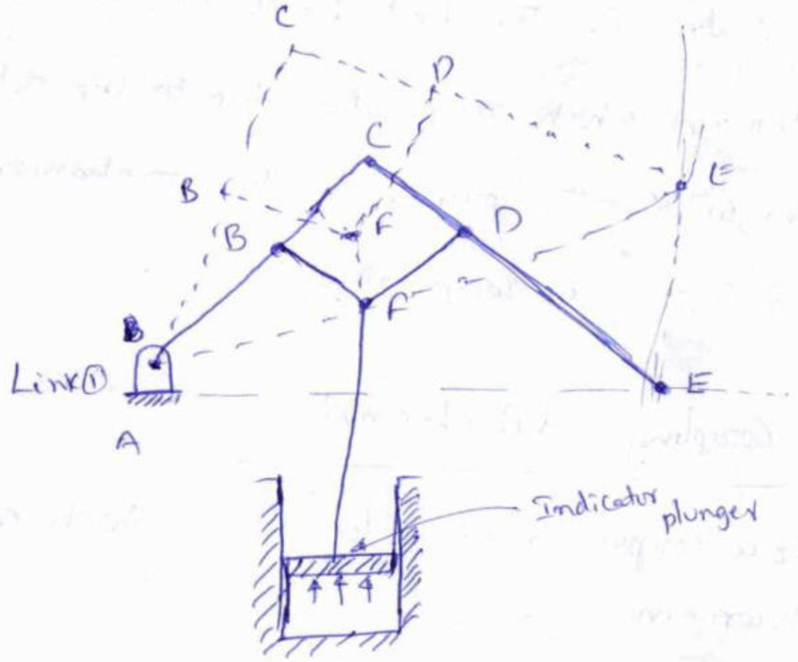
The mechanism of a coupling rod of a locomotive which consists of 'Four' links is shown in fig.



In this mechanism the Link AD & BC (having equal length) acts as cranks and are connected to the respective wheels. The Link 'CD' acts as a coupling rod and the link 'AB' is fixed in order to maintain a constant center to center distance between them. This mechanism is meant for transmitting rotary motion from one wheel to other wheel.

3. Watt's indicator Mechanism

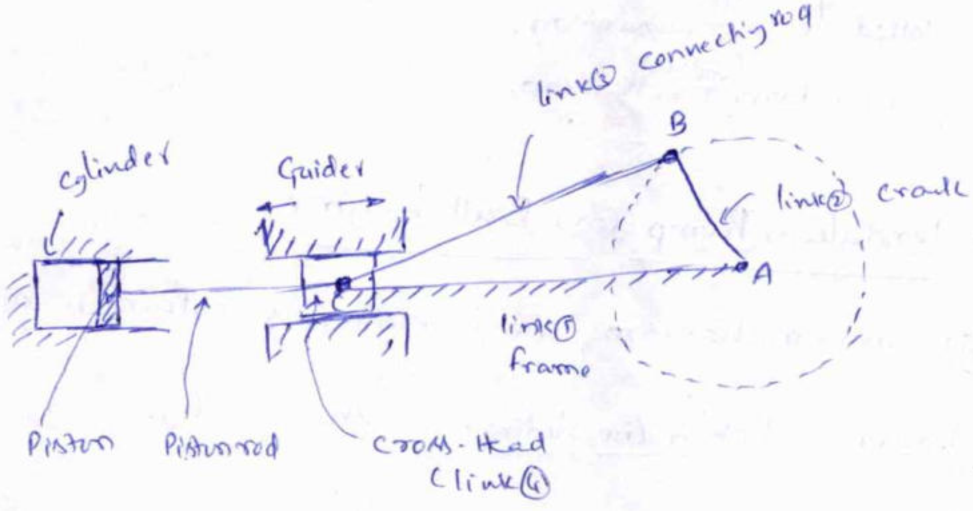
A watt's indicator mechanism which consists of '4' links is shown in fig. This mechanism also called as watt's straight line mechanism.



The four links are fixed link at A, link AC, link CE and link BFD. It may be noted that 'BF' and 'FD' forms one link because these two parts have no relative motion between them. The links CE and BFD acts as levers. The displacement of the link BFD is directly proportional to the pressure of the gas or steam which acts on the indicator plunger. On any small displacement of the mechanism, the tracing point E at the end of the link 'CE' traces out approximately a straight line.

Single Slider Crank Chain

A single slider crank chain is a modification of the basic four bar chain. It consists of one sliding pair and three turning pairs. It is usually found in reciprocating steam engine mechanism. This type of mechanism converts rotary motion into reciprocating motion and vice-versa.



Single Slider Crank Chain

In a slider crank chain as shown in fig. the links 1&2, links 2&3, links 3&4 form the three turning pairs, while the link 4&1 forms a sliding pair.

The link ① corresponds to the frame of the engine, which is fixed, The link ② corresponds to the crank, The link ③ corresponds to the connecting rod and the link ④ corresponds to cross head. As the crank rotates, the cross head reciprocates in the guides and thus the piston reciprocates in the cylinder.

Inversions of Single Slider Crank Chain

The following are the important inversions of the mechanism found.

(1) Pendulum Pump (or) Bull engine.

(2) Oscillating cylinder engine.

(3) Rotary internal Combustion engine (or) Gnome engine.

(4) Crank and Slotted lever quick return motion mechanism.

(5) Whit-worth quick return motion mechanism.

(1) Pendulum Pump (or) Bull Engine

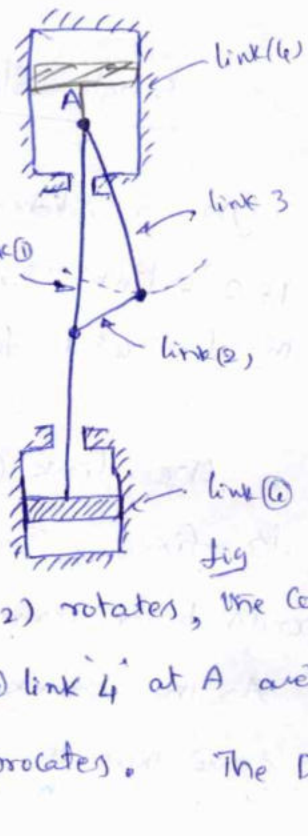
In this mechanism, the inversion is obtained by fixing the cylinder or link '4' (i.e sliding pair) shown in fig.

Link ① → Piston Rod

Link ② → Crank

Link ③ → Connecting Rod

Link ④ → Cylinder



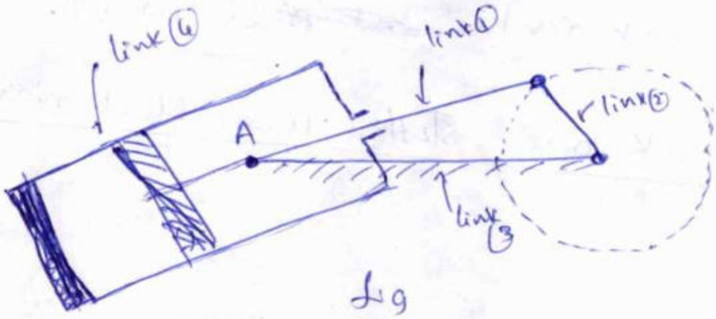
fig

In this case, when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to a fixed link '4' at A and the piston attached to the piston rod (link 1) reciprocates. The Duplex pump which is used to supply feed water to boilers have two pistons attached to link ① as shown in above fig.

(2) Oscillating Cylinder Engine

The arrangement of oscillating cylinder engine mechanism as shown in fig. is used to convert reciprocating motion into rotary motion.

① → Piston rod.
② → Crank
③ → Connecting Rod.
④ → Cylinder.

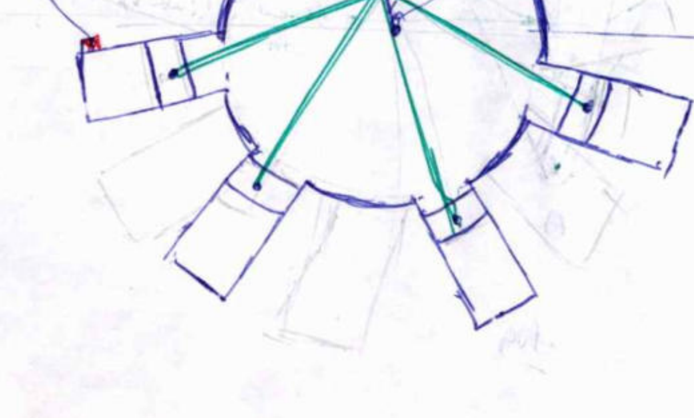


fig

In this mechanism the link ③ forming the turning pair is fixed. The link ③ corresponds to the connecting rod of reciprocating steam engine mechanism. When the crank (link 2) rotates, the piston attached to the piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at A.

(3) Rotary Internal Combustion Engine (or) Gnome Engine

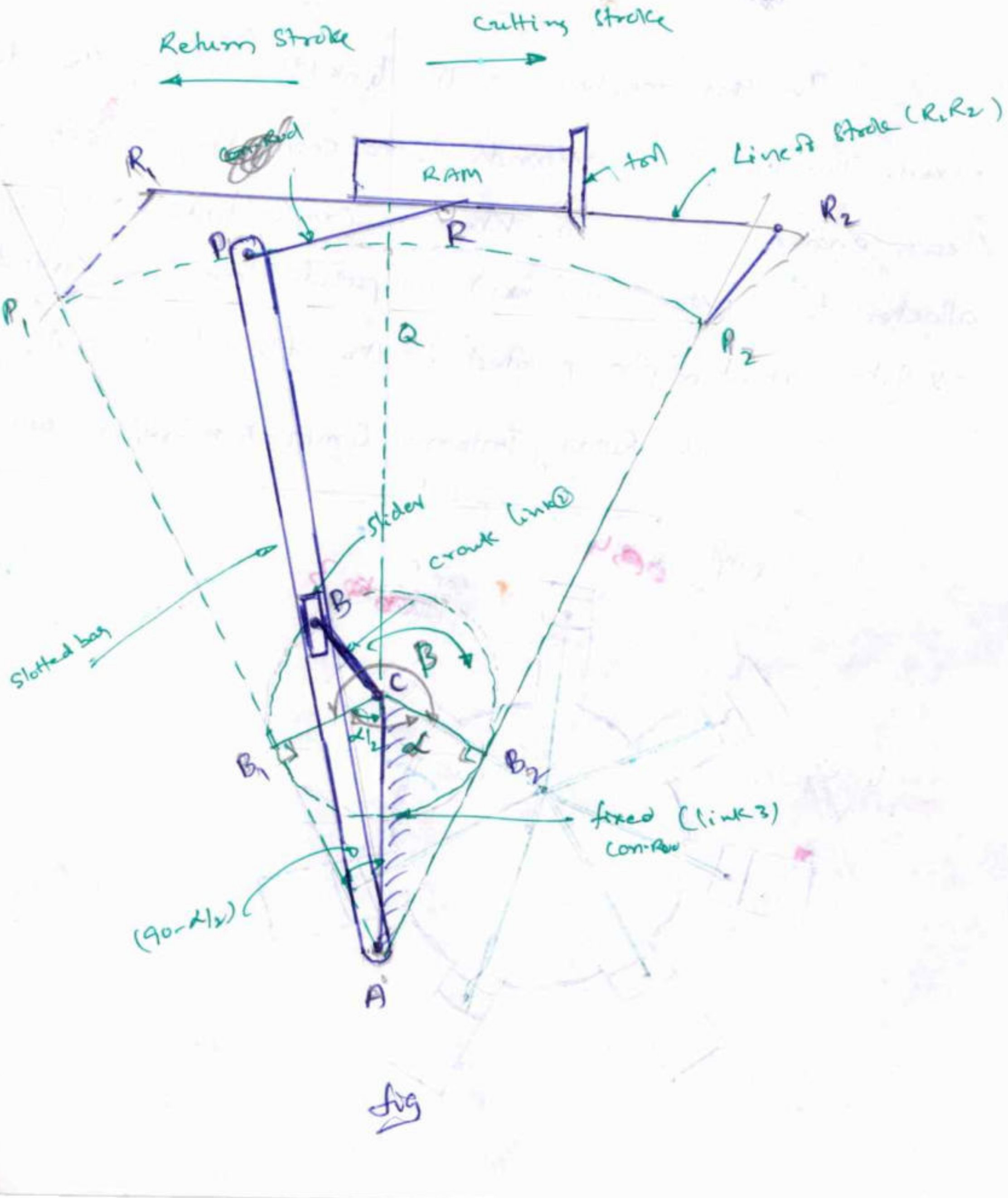
link ② crank fixed



Sometimes back, rotary I.C-engines wer Used in aviation.

But now a days gas turbines are Used in its place. It Consists of Seven cylinders in one plane and all revolves about fixed center 'D'. as shown in fig. while the Crank (link 2) is fixed. In this mechanism when the connectin rod (link 3) rotates, the piston (link 4) reciprocates inside the cylinder which revolves ~~about itself~~ (link 1).

(iii) Crank and Slotted lever Mechanism



fig

The Crank and Slotted lever mechanism mostly Used in Shaping Machines, Slotting machines and in rotary I.C. engines.

In this mechanism the link 'AC' [corresponds the com. rod of reciprocating steam engin] (i.e link 3) forming the turning pair is fixed and shown in fig. The driving crank (CB) (link 2) revolves with Uniform angular speed about the fixed center C. A Sliding block attached to the Crank pin at B which slides along the Slotted bar 'AP' and Thus Causes 'AP' to oscillate about the pivot point 'A'. A short link 'PR' transmits the motion from 'AP' to the 'ram' which carries the tool and reciprocates along the line of stroke R_1R_2. The line of stroke of the ram (i.e R_1R_2) is perpendicular to 'AC' produced.

In the extreeme position AP_1 and AP_2 are tangential to the circle and the cutting tool is at the end of stroke. The forward or cutting stroke occurs when the crank rotates from the position CB_1 to CB_2 through an angle 'β'. in clockwise direction. The return stroke occurs when the crank rotates from the position CB_2 to CB_1 (through an angle α) in the clockwise direction. Since the crank has Uniform angular speed, therefore

$$\frac{\text{Time of cutting Stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360-\beta} \text{ (or) } \frac{360-\alpha}{\alpha}$$

Since the tool travels a distance of R_1R_2 during the Cutting and Return stroke, therefore travel of tool (or) length of stroke. $= R_1R_2 = P_1P_2 = 2P_1Q$.

from fig. $2P_1Q = 2 \times \sin(90-\alpha/_2)\, AP_1 = 2AP_1 \cos\alpha/_2 = 2AP \times \cos\alpha/_2$

$[\because AP_1 = AP_2 = AP]$

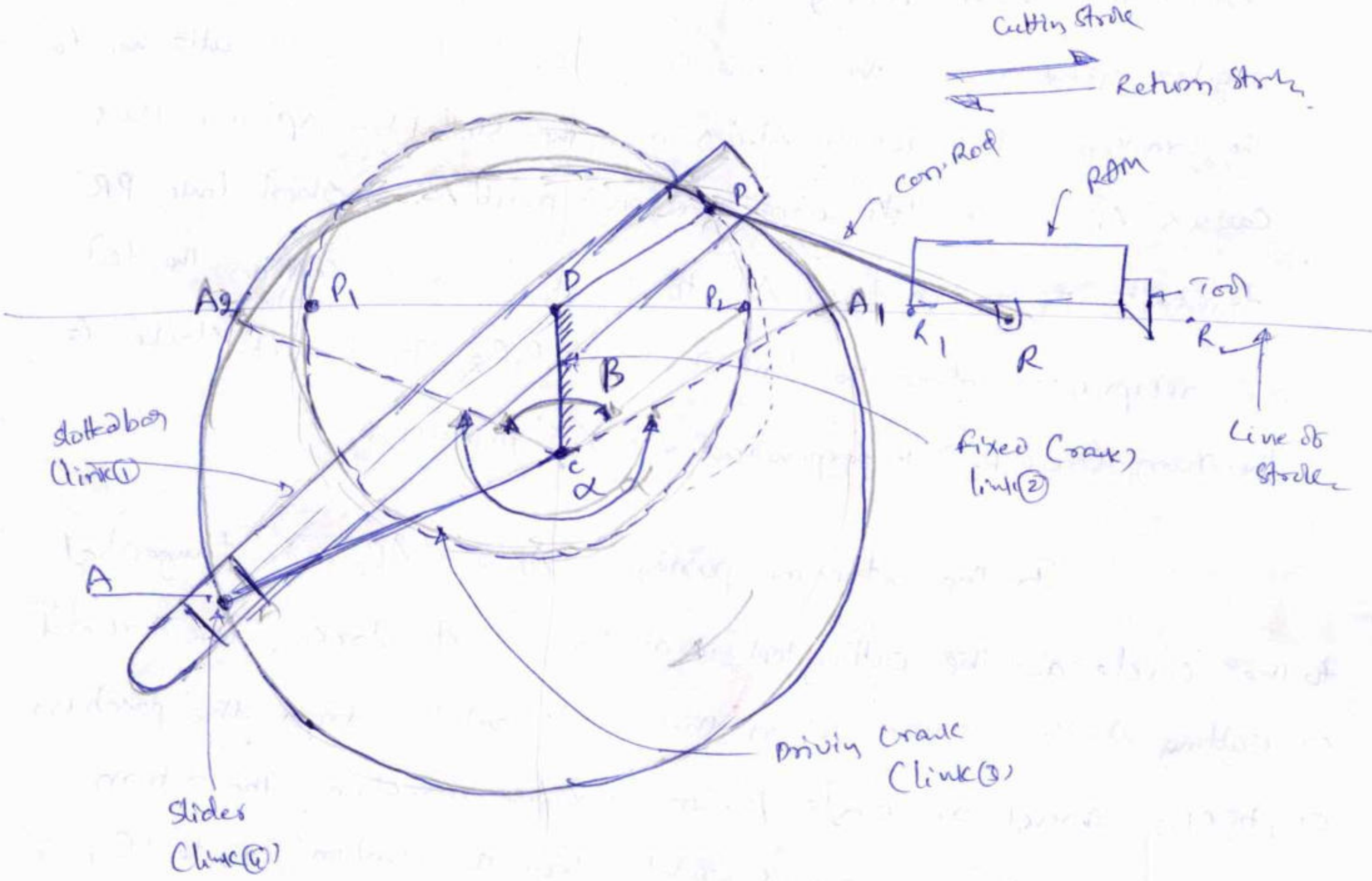
$\Rightarrow R_1R_2 = 2P_1Q = 2AP \times \frac{CB_1}{AC}$

$$\Rightarrow \boxed{R_1R_2 = 2AP \times \frac{CB}{AC}}$$

WHITWORTH QUICK RETURN MOTION MECHANISM

This mechanism is mostly used in Shaping and slotting machin.

The mechanism is shown in fig.



In this mechanism the link `CD' (link 2) forming the turning pair is fixed, The link '2' corresponds to crank in reciprocating steam engine. The driving crank CA' (link 3) rotates at angular speed. The slider (link 4) attached to the crank pin at 'A' slides along the slotted bar PA (link 1) which oscillates at a pivot point D. The connecting rod PR carries the Ram at `R' to which a cutting tool is fixed. The motion of a tool is contrained along the line 'RD' produced i.e along a line passing through 'D' and perpendicular to 'CD'.

When the Driving Crank 'CA' moves from CA_1 to CA_2 (or the link 'DP' from the position DP_1 to DP_2) through an angle 'α' in the clockwise direction, the tool moves from the left hand end of its stroke to the right hand end through a distance '2PD'.

Now when the driving crank moves from the position CA_2 to CA_1 (or the link 'DP' from DP_2 to DP_1) through an angle 'β' in the clockwise, the tool moves back from the right hand end of its stroke to the left hand end.

A little consideration will show that the time taken during the left to right movement of the ram will be equal to the time taken by the driving crank to move from CA_1 to CA_2.
Similarly, the time taken during the right to left movement of the ram will be equal to the time taken by the driving crank move from CA_2 to CA_1.

Since the Crank link 'CA' rotates at Uniform angular velocity therefore time taken during the cutting stroke is more than the time taken during the return stroke.

$$\therefore \frac{\text{Time of Cutting Stroke}}{\text{Time of Return Stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360-\alpha} = \frac{360-\beta}{\beta}$$

The length of effective cutting stroke $R_1R_2 = P_1R_1 = P_2R_2 = PR = \underline{2 \times PD}$

$$\boxed{R_1R_2 = 2PD}$$

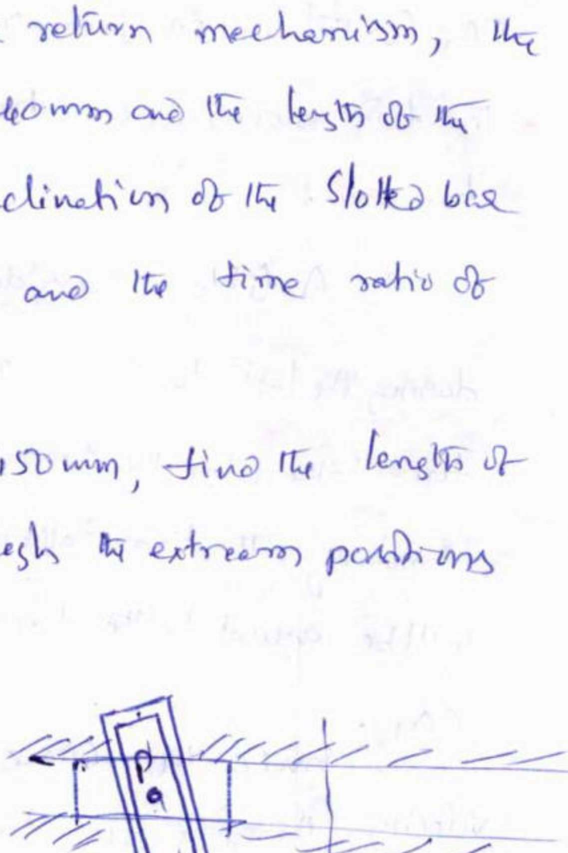
Problems

① A crank and slotted lever mechanism used in a shaper has a center distance of 300 mm between the center of oscillation of the slotted lever and the center of rotation of the crank. The radius of the crank is 120 mm. find the ratio of the time of cutting to the time of return stroke

② In a crank and slotted lever quick return mechanism, the distance between the fixed centers is 240 mm and the length of the driving crank is 120 mm. Find the inclination of the slotted bar with the vertical in extreme position and the time ratio of cutting stroke to return stroke.

If the length of slotted bar is 450 mm, find the length of stroke, if the length of stroke passes through the extreme positions of the free end of the lever,

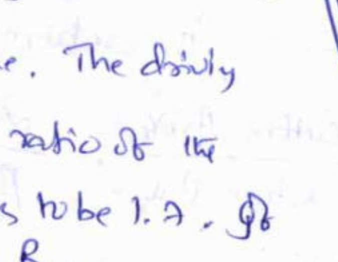
③ The fig shows the lay out of a quick return mechanism of the oscillating link type, for a special purpose machine. The driving crank 'BC' is 30 mm long and the ratio of the working stroke to the return stroke is to be 1.7. If the length of the working stroke of (R_1R_2) is 120 mm, determine the length of the Dimensions of AC and AP



(Ans :- $\alpha = 133.3°$

$AC = 75.7$ mm

$AP = 151.4$ mm

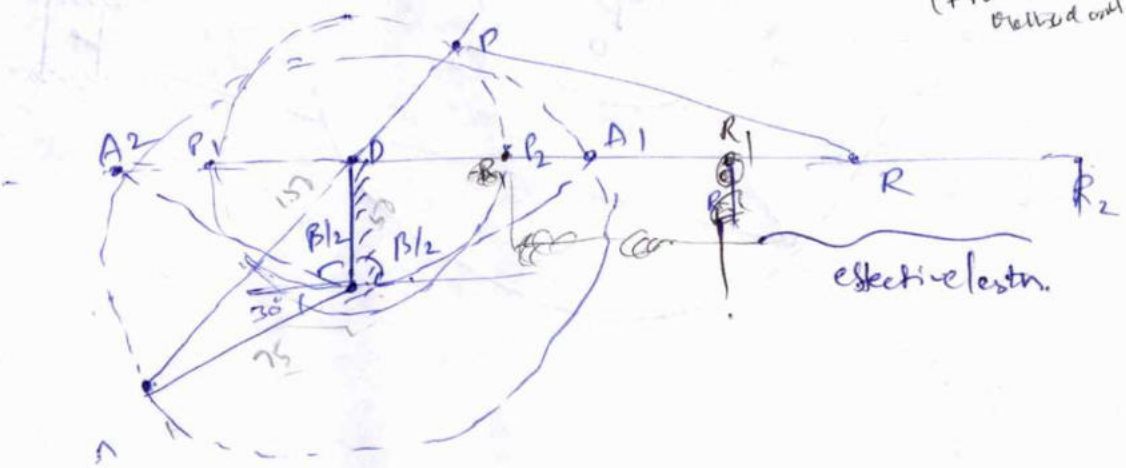


(4) In a whitworth quick return motion mechanism as shown in fig the distance between the fixed centers is 50mm and the length of the driving crank is 75mm. The length of slotted lever is 150mm and the length of connecting rod is 135mm. find the ratio of the time of cutting stroke to the time of return stroke and also the effective stroke.

$(\beta = 96.4°)$

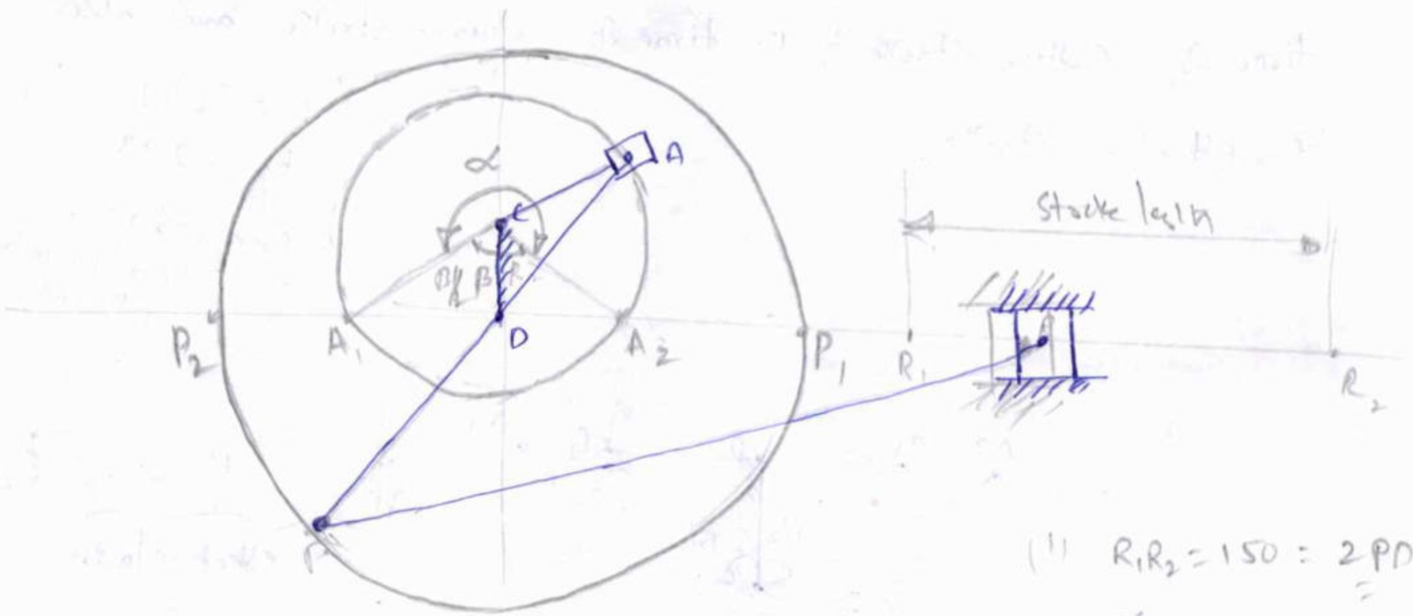
$\frac{t_f}{t_R} = 2.735$

$(R_1R_2 = 87.5$ mm, (from Geometrical method))



(5) The whitworth quick return motion mechanism has the driving crank 150 mm long. The distance between the fixed centers is 100mm. the line of stroke of the Ram passes through the centre of rotation of the slotted lever whose free end is connected to the Ram by a connecting link. find the ratio of time of cutting to time of return (Ans: 2.735)

⑥ A whitworth quick return motion mechanism, as shown in fig has the following particulars. Length of stroke = 150mm; Driving crank length = 40mm; Ratio of cutting time to return time = 2; find the lengths of 'CD' and 'PD'. Also determine the angles α and β.

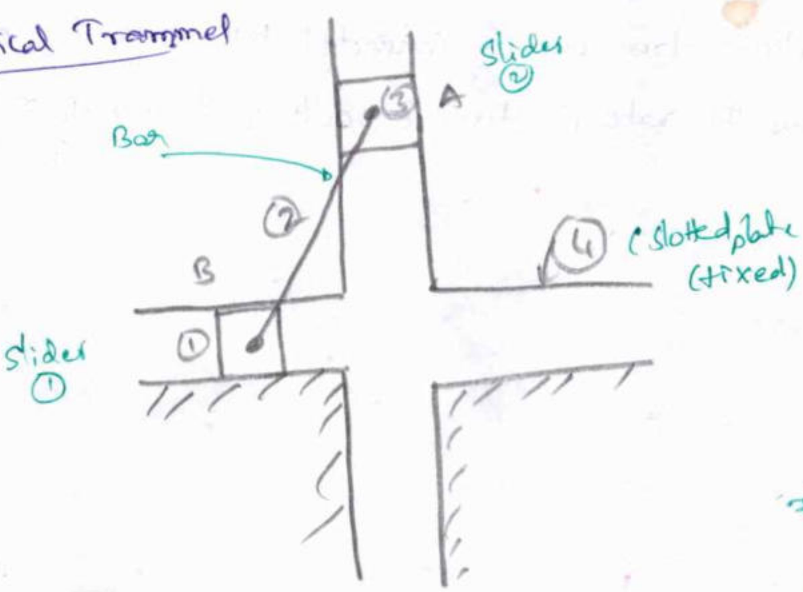


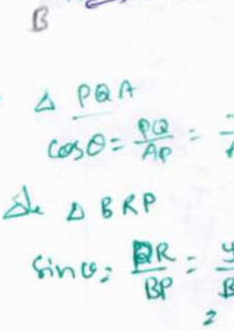
(1) $R_1R_2 = 150 = 2PD$

(2) $\frac{\alpha}{\beta} = 2$

(3) $CA = 40$

(1) Elliptical Trammel





from Δ PQA

$\cos\theta = \frac{PQ}{AP} = \frac{x}{AP}$

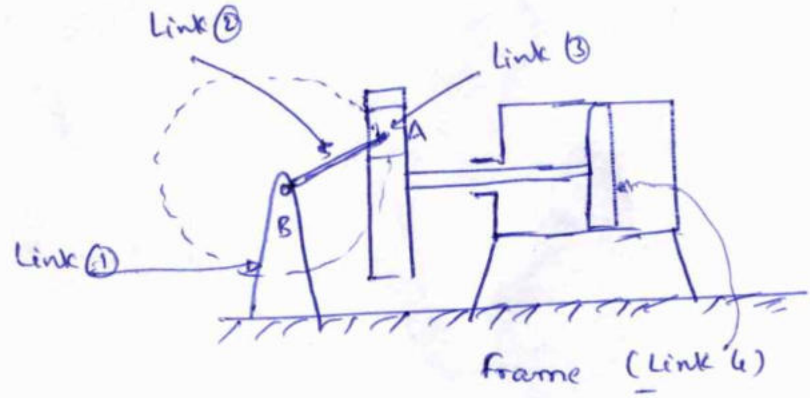
from the Δ BRP

$\sin\theta = \frac{BR}{BP} = \frac{y}{BP}$

$\sin^2\theta + \cos^2\theta = 1$

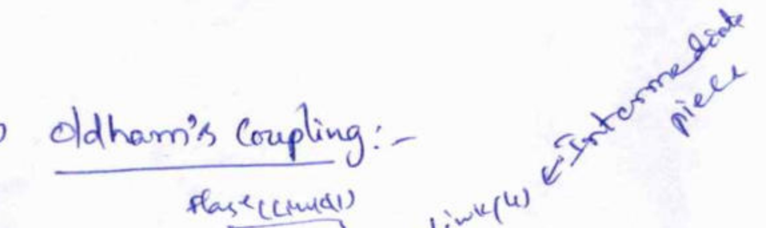
$$\boxed{\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = 1}$$

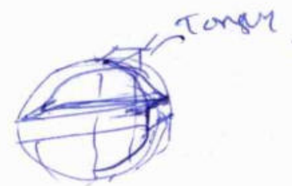
(2) Scotch yoke mechanism



This mechanism is used for converting rotary motion into reciprocating motion. The inversion is obtained by fixing either the Link (1) or link (3). In the fig the link (1) is fixed. In this mechanism when the link 2 rotates about center B, the link 4 (which corresponds to a frame) reciprocates. The fixed link 1 guides the frame.

(3) Oldham's Coupling:-





∴ maximum sliding speed of each tounge (m/sc)

$$v = \omega . r$$

ω = Angular speed of each shaft

r = Distance between the two axes of shaft

Types of joints in a chain

1. Binary Joint :- When

Unit II: (Mechanisms with Lower Pairs)

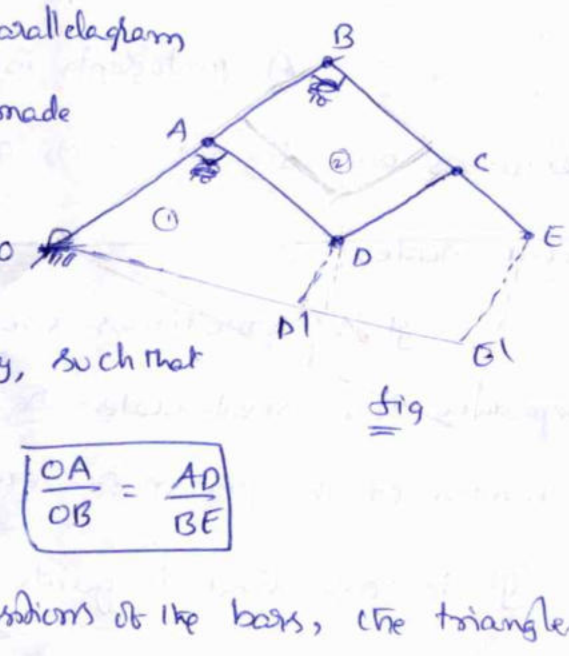
§ STRAIGHT LINE MOTION MECHANISMS

When the two elements of a pair have a surface contact and the relative motion takes place, the surface of one element slides over the surface of the other, the pair formed is known as lower pair.

PANTOGRAPH

A pantograph is an instrument used to produce to an enlarged or a reduced scale and as exactly as possible the paths described by a given point.

It consists of a jointed parallelogram 'ABCD' as shown in fig. It is made up of bars connected by turning pairs. The bars 'BA' and 'BC' are extended to 'O' and 'E' respectively, such that



fig

$$\frac{OA}{AB} = \frac{OB}{BE} \Rightarrow \boxed{\frac{OA}{OB} = \frac{AD}{BE}}$$

Thus for all relative positions of the bars, the triangles OAD, and OBE are similar and the points 'O, D, E' are in one straight line. It may be proved that point 'E' traces out the same path as described by the point 'D'.

From similar triangles 'OAD' and 'OBE', we find that

$$\boxed{\frac{OD}{OE} = \frac{AD}{BE}}$$

Let point 'O' be fixed and the points 'D' and 'E' move to some new position D' & E'. Then

$$\frac{OD}{OE} = \frac{DD'}{EE'}$$

A little consideration will show that the straight line DD' is parallel to the to the straight line EE'. Hence if 'O' is fixed to the frame of machine by means of turning pair and 'D' is attached to a point in the machine, which has rectilinear motion relative to the frame, then 'E' will also trace out a straight line path. Similarly, if 'E' is constrained to move in a straight line, then 'D' will trace out a straight line parallel to the former.

A pantograph is mostly used for the reproduction of plane areas and figures such as maps, plans etc. on enlarged or reduced scales.

It is sometimes used as an indicator rig in order to reproduce to a small scale the displacement of the cross-head and therefore of the piston of reciprocating steam engine.

It is also used to guide cutting tools.

Straight line Motion Mechanisms

One of the most common forms of the constraint mechanisms is that it permits only relative motion of an oscillatory nature along a straight line. The mechanisms used for this purpose are called straight line mechanisms.

TYPES

The straight line motion mechanisms are two types

(1) The mechanisms in which only turning pairs are used.

(2) The mechanisms in which one sliding pair is used.

The above two mechanisms may produce exact straight line motion (or) approximate straight line motion.

(i) Exact Straight Line Motion Mechanisms with Turning Pairs

The principle adopted for mathematically correct or exactly straight line motion is described in fig.

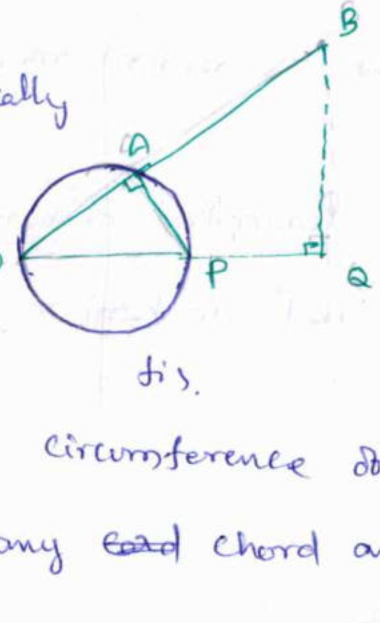


fig.

Let 'O' be the ~~fixed~~ point on the circumference of a circle of diameter 'OP'. Let 'OA' be any ~~rod~~ chord and 'B' is a point on 'OA' produced, such that

$$OA \times OB = \text{constant}.$$

then the locus of a point 'B' straight line perpendicular to the diameter 'OP'. This may be proved as follows

Draw 'BQ' perpendicular to 'OP' produced, join AP. The triangles OAP and OQB are similar

$$\frac{OA}{OP} = \frac{OQ}{OB}$$

(or) $OA \times OB = OQ \times OP$

(or) $OQ = \dfrac{OA \times OB}{OP}$

But 'OP' is constant as it is the diameter of a circle, therefore, if $OA \times OB$ is constant, then 'OQ' will be constant. Hence the point 'B' moves along the straight path 'BQ', which is perpendicular to OP.

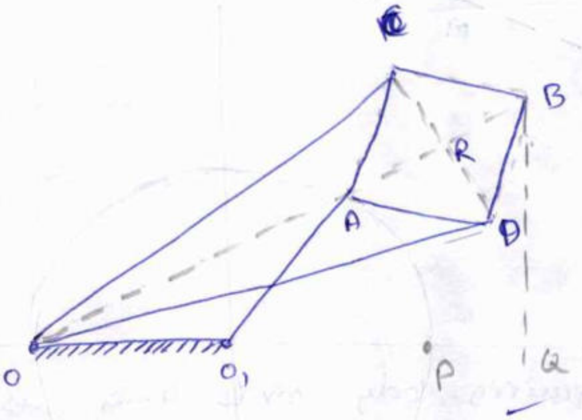
The following are the two well known types of exact straight line motion mechanisms made up of turning pairs.

(1) Peaucellier mechanism

(2) Hart's Mechanism

Peaucellier Mechanism

This mechanism contains eight links (8). It consists of a fixed link OO_1 and the other straight links O_1A, OC, OD, AD, CA, DB, BC are connected by turning pairs at their intersections as shown in fig. (ABCD → is forms a Rhombus)



The pin at 'A' is constrained to move along the circumference of a circle with the fixed diameter 'OP' by means of a link O_1A.

from fig. $AC = CB = BD = DA$; $OC = OD$; and $OO_1 = O_1A$

It may be proved that the product $OA \times OB$ remains constant, when the link O_1A rotates. Join CD to bisect AB at R. Now from right angled triangles ORC and BRC

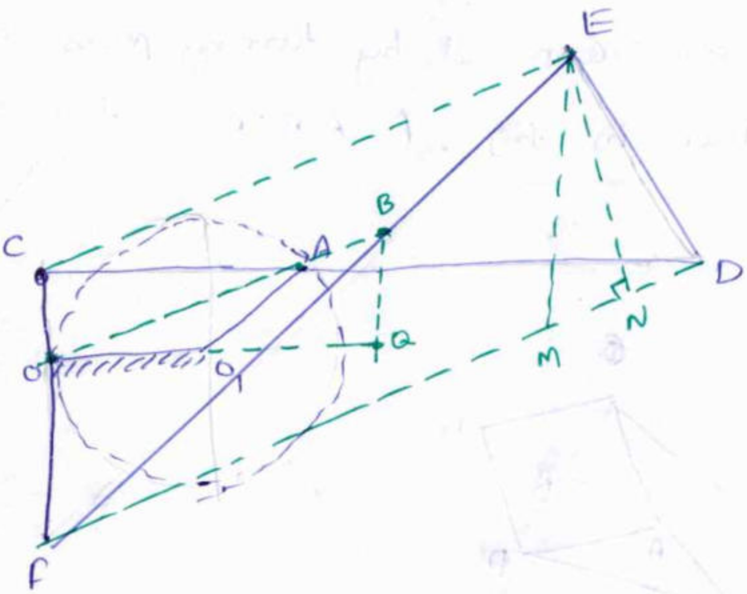
$$OC^2 = OR^2 + RC^2 \quad \text{—(i)}$$
$$CB^2 = BR^2 + RC^2 \quad \text{—(ii)}$$

(i) − (ii) ⇒ $OC^2 - CB^2 = OR^2 - BR^2$ $\quad [a^2 - b^2 = (a+b)(a-b)$

⇒ $OC^2 - CB^2 = (OR + BR)(OR - BR) = OB \times OA$ $\quad [\because BR = AR]$

⇒ $\therefore$ $OA \times O.B =$ Constant $\quad \{\because$ OC & CB are links}

Hart's Mechanism



This mechanism requires only six (6) links as compared with eight(8) links required by the Peaucellier mechanism.

It consists of a fixed link OO_1 and other straight links O_1A, FC, CD, DE and EF are connected by turning pairs at their points of intersection as shown in fig. The links FC and DE are equal in length and the lengths of links CD and EF are also equal. The points O. A. and B devide the links FC, CD and EF in the same ratio. A little consideration will show that BOCE is a trapezium and OA and OB are respectively parallel to FD and CE.

Hence OAB is a straight line. It may be proved now that the product OA×OB is constant.

from similar triangles CFE and OFB

$$\frac{CE}{FC} = \frac{OB}{OF} \quad \text{or} \quad \boxed{OB = OF \times \frac{CE}{FC}} \quad \text{— (i)}$$

from similar triangles FCD and OCA.

$$\frac{OA}{OC} = \frac{FD}{CF} \text{ or } \frac{FD}{FC}$$

$$\Rightarrow \boxed{OA = OC \times \frac{FD}{FC}} \quad \text{—— (ii)}$$

multiplying (i) x (ii) $\Rightarrow \quad OA \times OB = OC \times \frac{FD}{FC} \times OF \times \frac{CE}{FC}$

$$\Rightarrow \quad OA \times OB = \frac{OC \times OF}{FC^2} \times FD \times CE$$

But $\frac{OC \times OF}{FC^2}$ = constant

$$\text{then} \Rightarrow \boxed{OA \times OB = FD \times CE}$$

Now drawing a parallel EM from point E to CF and EN perpendicular to FD. Therefore.

$$FD \times CE = (FN + ND) \times FM \qquad (\because CE = FM)$$

$$= (FN + ND) \times (FN - NM)$$

$$= (FN + ND) \times (FN - ND) \quad [\because NM = ND]$$

$$\boxed{FD \times CE = FN^2 - ND^2}$$

⇒ ~~But from right angle triangles FNM & FND~~ $\left\{ \cancel{FM^2 = FN^2 + NM^2} \atop \cancel{FD^2 = FN^2 + ND^2 \,(NM)} \right.$ $\left[\because NM = ND\right]$

$\therefore \cancel{FD \times CE = (FD^2 - NM^2) - (FM^2 - NM^2) = FD^2 - FM^2}$

from right angle triangle FNE & END

$$\left.\begin{array}{l} FE^2 = FN^2 + EN^2 \\ \Rightarrow FN^2 = FE^2 - EN^2 \end{array}\right| \begin{array}{l} ED^2 = EN^2 + ND^2 \\ ND^2 = ED^2 - EN^2 \end{array}$$

$$\therefore \quad FD \times CE = FN^2 - ND^2 = (FE^2 - EN^2) - (ED^2 - EN^2)$$

$$\Rightarrow \quad FD \times CE = FE^2 - ED^2 = \text{constant}$$

$\therefore OA \times OB =$ constant.

It therefore follows that if the mechanism is pivoted about 'O' as fixed point and the point 'A' is constrained to move on a circle with centre O_1, then the point 'B' will trace a straight line perpendicular to the diameter OP produced.

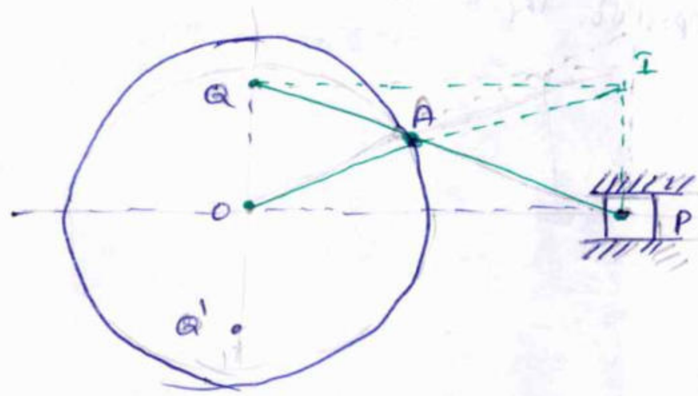
Note:- This mechanism has a great practical disadvantage that even when the path of 'B' is short, a large amount of space is taken up by the mechanism.

Exact Straight Line Motion Consisting of One Sliding Pair

This Mechanism contains one sliding Pair and remain turning Pairs.

Example of this type Mechanism is:- Scott-Russell's Mechanism

Scott-Russell's Mechanism



The Scott-Russell's Mechanism Consists of a fixed member and moving member 'P' of a sliding pair as shown in fig.

The straight link 'PAQ' is connected by turning pairs to the link 'OA' and the link (slider) P. The link 'OA' rotates about 'O'. A little Consideration will show that the mechanism 'OAP' is same as that of the reciprocating engine mechanism in which 'OA' is the Crank and 'PA' is the Connecting rod. In This mechanism, the straight line motion is not generated but it is merely Copied.

'A' is the middle point of 'PQ' and OA = AP = AQ. The instantaneous Center for the link PAQ lies at 'I' in 'OA' produced and is such that 'IP' is perpendicular to 'OP'. Joint IQ. Then 'Q' moves along the perpendicular to IQ.

Since 'OPIQ' is a rectangle and 'IQ' is ~~rectangle~~ perpendicular to OQ, therefore 'Q' moves along the vertical line 'OQ' for all positions of QP. Hence 'Q' traces the straight line OQ'. If 'OA' makes one complete revolution, then P' will oscillate along the line OP' through a distance '2OA' on each side of 'O' and 'Q' will oscillate along 'OQ'' through the same distance '2OA' above and below O.

Thus, the locus of Q is a copy of the locus of P.

Approximate Straight Line motion Mechanisms

The approximate straight line motion mechanisms are the modifications of the 4-Bar Chain mechanism. The following are the important from subject point of view.

1. Watt's Mechanism.
2. Modified Scott - Russel Mechanism
3. Grasshopper Mechanism
4. Tchebicheff's mechanism.
5. Roberts Mechanism.

1. Watt's Mechanism

It is a crossed four bar chain mechanism and was used by 'Watt' for his early steam engines to guide the piston rod in a cylinder to have an approximate straight line motion.

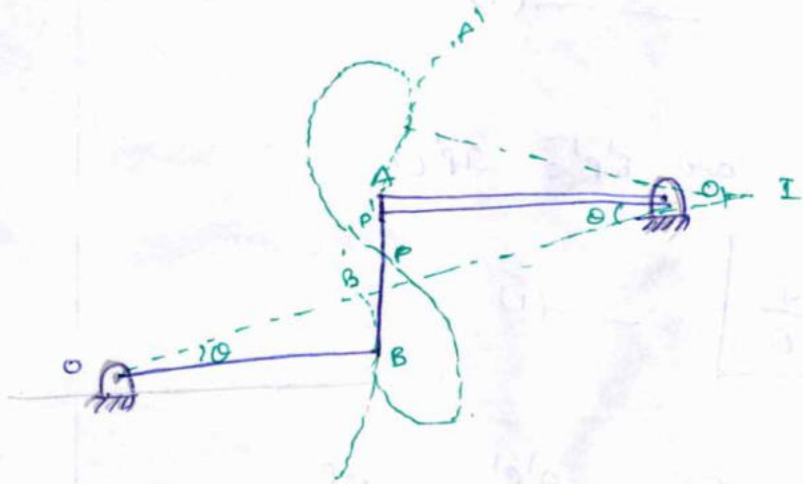


fig.

In fig 'OBAO₁' is a crossed four bar chain in which 'O' and 'O₁' are fixed. In the mean position of the mechanism, OB and O₁A are parallel and coupling rod 'AB' is perpendicular to

O_1A and OB. The tracing point 'P' traces out an approximate straightline over certain positions of its movement, if

$\frac{PB}{PA} = \frac{O_1A}{OB}$. this may be proved as follows:

A little consideration will show that in the initial mean position of the mechanism, the instantaneous centers of the link 'BA' lies at infinity. Therefore the motion of the point 'P' is along the vertical line BA. Let $OB'A'O_1$ be the new position of the mechanism after the links 'OB' and 'O_1A' are displaced through an angle θ and ϕ respectively. The instantaneous Center now lies at I. Since the angle 'θ' and 'ϕ' are very small, therefore

$$\text{Arc } BB' = \text{Arc } AA'$$

(or)

$$\Rightarrow \theta \times OB = O_1A \times \phi$$

$$\Rightarrow \frac{OB}{O_1A} = \frac{\phi}{\theta} \text{ ——— (i)}$$

But Also $A'P' = IP'\phi$ and $B'P' = IP'\theta$

$$\Rightarrow \boxed{\frac{A'P'}{B'P'} = \frac{\phi}{\theta}} \text{ ——— (ii)}$$

From (i) & (ii) $\Rightarrow \frac{OB}{O_1A} = \frac{A'P'}{B'P'} = \frac{AP}{BP}$

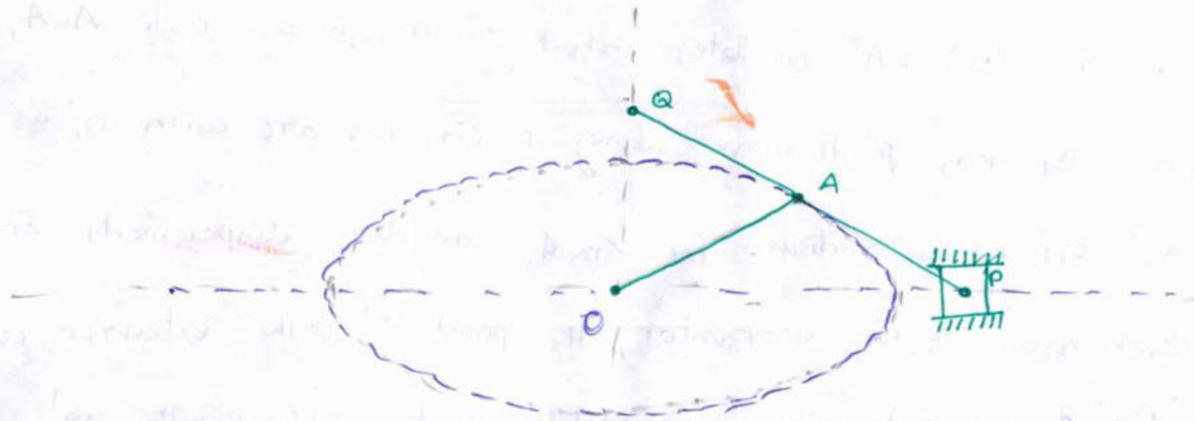
(or) $\boxed{\frac{O_1A}{OB} = \frac{PB}{PA}}$

Thus the point P divides the link AB in to two parts whose lengths are inversely proportional to the lengths of the adjacent links.

2. Modified Scott - Russel Mechanism

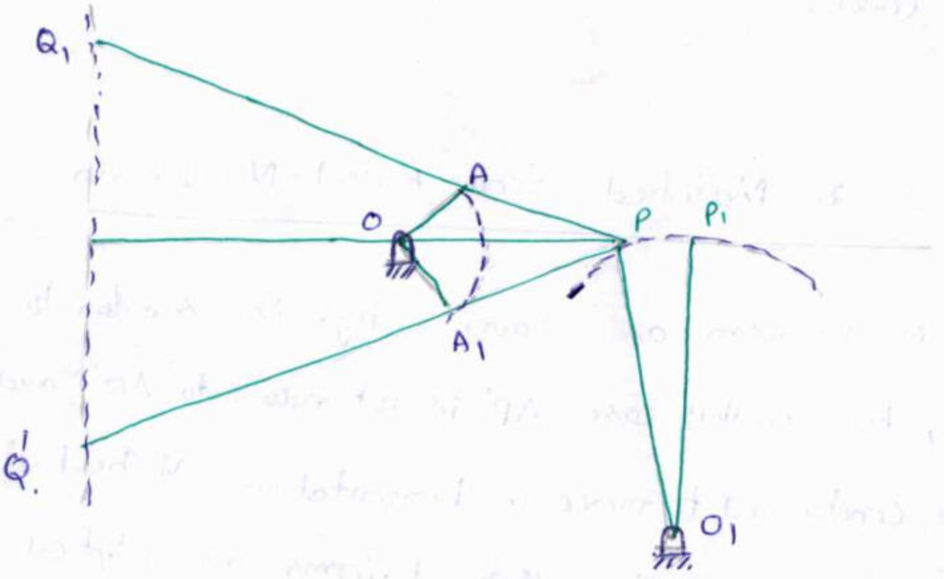
This Mechanism as shown in fig. is similar to Scott-Russel mechanism, but in this case 'AP' is not equal to 'AQ' and the points 'P' and 'Q' are constrained to move in horizontal and vertical directions. A little consideration will show that it forms an elliptical trammel, So that any point 'A' on 'PQ' traces an ellipse with semi-major axis 'AQ' and semi minor axis 'AP'.

If the point 'A' moves in a circle, then for point 'Q' to move along an approximate straight line, the length 'OA' must be equal to $\frac{(AP)^2}{AQ}$. This is limited to only small displacement of P.



Modified Scott - Russel Mechanism

3. Grasshopper mechanism



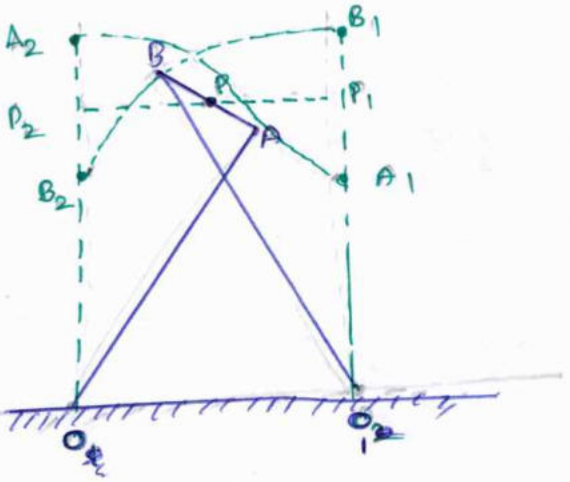
This mechanism is a modification of modified-Scott Russel's mechanism with the difference that the point 'p' does not slide along a straight line, but moves in a circular arc with centre 'O_1'

It is a four bar mechanism and all the pairs are turning pairs as shown in fig. In this mechanism, the centeres 'O' and 'O_1' are fixed. The link 'OA' oscillates about 'O' through an angle AOA, which causes the Pin 'P' to move along a circular arc with 'O_1' as a center and O_1P as a radius. for small angular displacements of 'OP' on each side of the horizontal, the point 'Q' on the extension of the link 'PA' traces out an approximately a straight line path QQ'.

If the lengths are such that $OA = \dfrac{(AP)^2}{AQ}$

4. Tchebicheff's Mechanism

It is a four bar mechanism in which the crossed links OA and O_1B are of equal lengths as shown in fig. The point 'P', which is the mid point of 'AB' traces out an approximately straight line parallel to OO_1. The

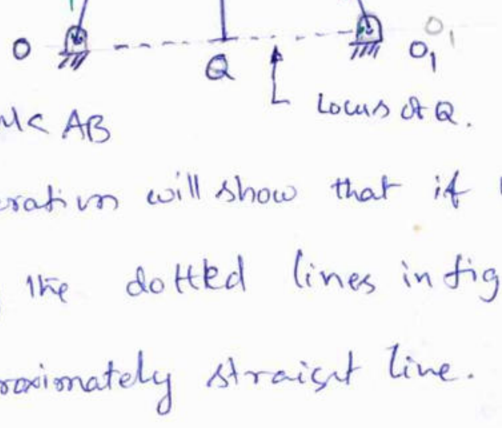


fig

proportions of the links are, usually, such that the point 'P' is exactly above 'O' or 'O_1' in the extreme positions of the mechanism i.e when 'BA' lies along 'OA' or when BA lies along O_1B. It may be noted that the point 'P' will be lie on a straight line parallel to OO_1, in the two extreme positions and in the mid position, if the lengths are such that 'OA' $= (AP)^2/AQ$.

5. Roberts mechanism

It is also a four bar chain mechanism, which in its mean position, has the form of a trapezium. The link 'OA' and 'O_1B' are of equal length and OO_1 is fixed. A bar 'PQ' is rigidly attached to the link AB at its middle point 'P'. A little consideration will show that if the mechanism is displaced as shown by the dotted lines in fig. The point 'Q' will trace out an approximately straight line.



UNIT - III

STEERING Mechanisms

The steering gear mechanism is used for changing the direction of two or more of the wheel axles with reference to the chassis, so as to move the automobile in any desired path. Usually the two back wheels have a common axis, which is fixed in direction with reference to the chassis and the steering is done by means of the front wheels.

In automobiles, the front wheels are placed over the front axles, which are pivoted at the points 'A' and 'B'. as shown in fig. These points are fixed to the chassis. The back wheels are placed over the back axle, at the two ends of the differential tube. When the vehicle takes a turn, the front wheels along with the respective axles turn about the respective pivoted points. The back wheels remain straight ~~&~~ and do not turn. Therefore the steering is done by means of front wheels only

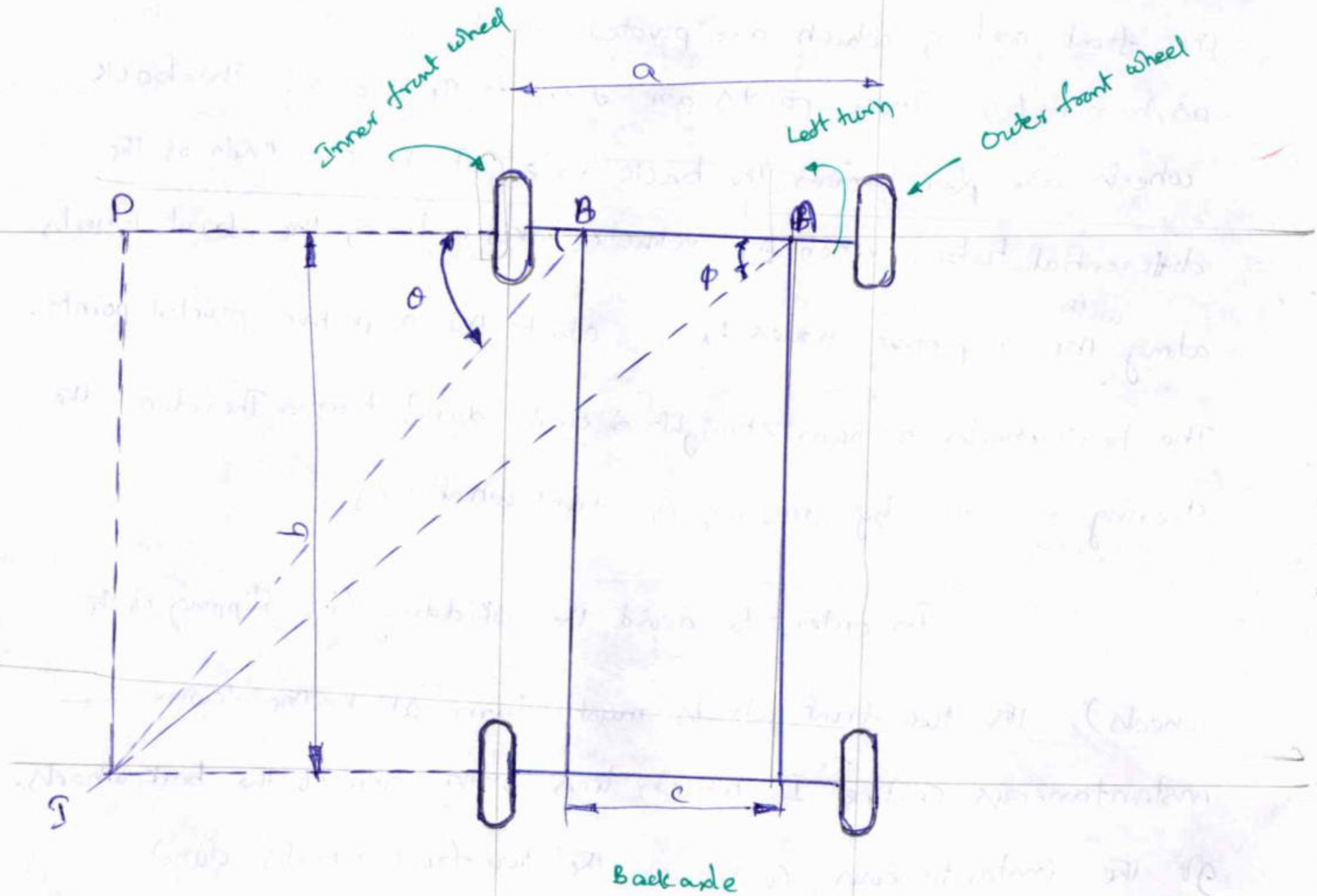
In order to avoid the skidding (i.e slipping of the wheels), the two front wheels must turn about the same instantaneous centre 'I' which lies on the axis of the back wheels. If the instantaneous center of the two front wheels do not coincide with the instantaneous centre of the back wheels, the skidding on the front or back wheels will definitely take place, which will cause more wear and tear of the tyres.

Thus, the condition for correct steering is that all the four wheels must turn about the same instantaneous centre. The axis of the inner wheels makes a larger turning angle θ than the angle ϕ subtended by the axis of the outer wheel.

Let a = wheel track

b = wheel base, and

c = Distance between the pivots A and B of the front axle.



Now from Triangle IPB
$$\cot\theta = \frac{BP}{IP}$$

from Triangle IPA
$$\cot\phi = \frac{AP}{IP} = \frac{BP + AB}{IP} = \frac{BP}{IP} + \frac{AB}{IP}$$

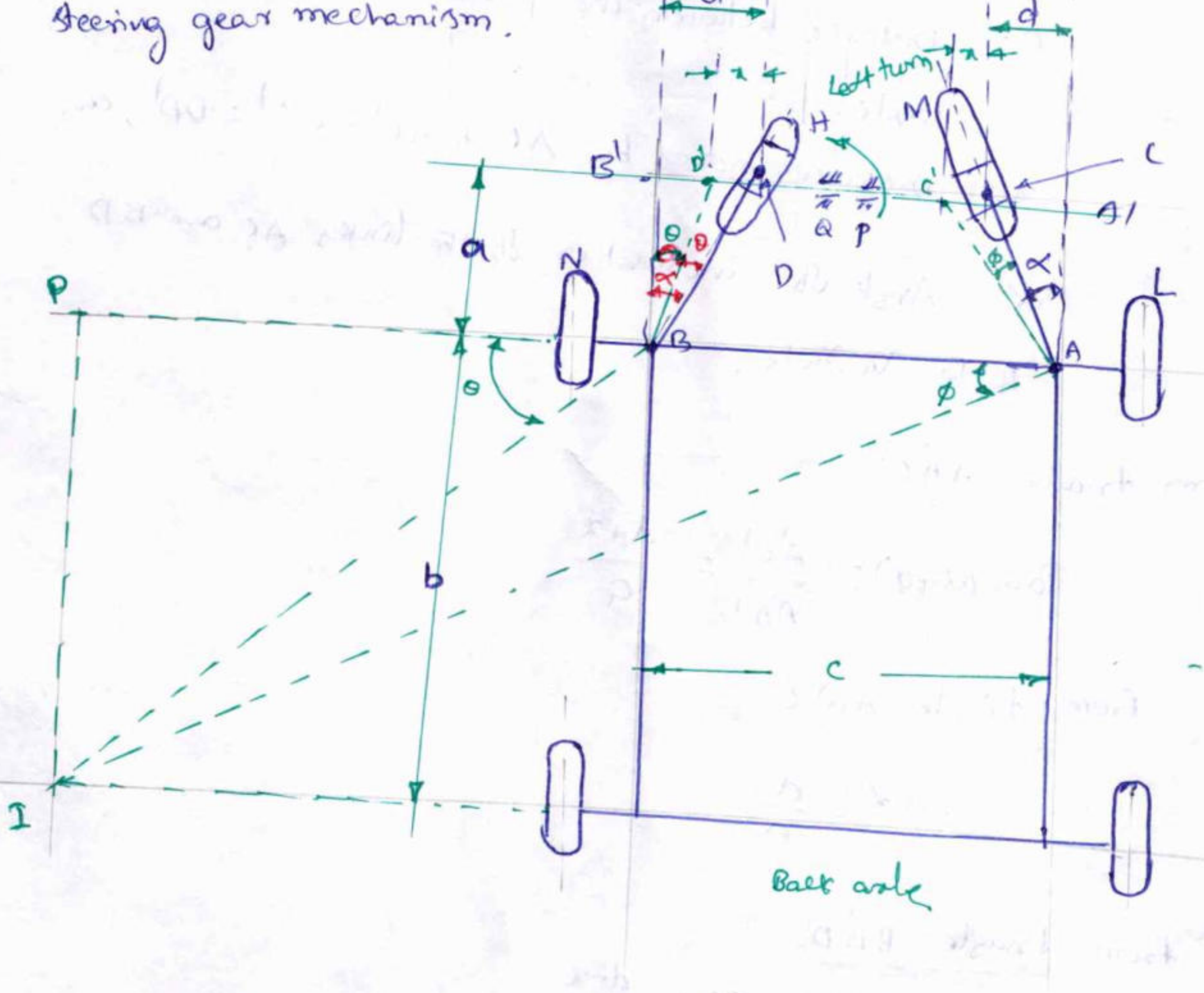
$$\Rightarrow \quad \cot\phi = \cot\theta + \frac{c}{b}$$

$$\Rightarrow \quad \boxed{\cot\phi - \cot\theta = \frac{c}{b}}$$

The above equation is the fundamental equation for correct equation.

DAVIS STEERING GEAR

The Davis Steering Gear is shown in fig. It is an exact steering gear mechanism.



The slotted links AM and BH are attached to the front wheel axle, which turn on pivots 'A' and 'B' respectively. The rod 'CD' is constrained constrained to move in the direction of its length, by the sliding members at 'P' and 'Q'. These constraints are connected to the slotted link 'AM' and 'BH' by a sliding and a turning pair at each end. The steering is affected by moving 'CD' to the right or left of its normal position. C'D' shows the position of 'CD' for turning to the left.

Let a = Vertical distance between AB and CD

b = Wheel base

d = horizontal distance between the AC and BD.

c = Distance between the pivots 'A' and 'B' of the front axle.

x = Distance moved by AC to $AC' = CC' = DD'$, and

α = Angle of inclination of the links AC and BD to the vertical.

from triagle $AA'C'$

$$\tan(\alpha+\phi) = \frac{A'C'}{AA'} = \frac{d+x}{a}$$

from triangle $AA'C$

$$\tan\alpha = \frac{d}{a}$$

from triagle $BB'D'$

$$\tan(\alpha-\phi) = \frac{d-x}{a}$$

we know that $\tan(\alpha+\phi) = \dfrac{\tan\alpha + \tan\phi}{1-\tan\alpha \times \tan\phi}$

$$\Rightarrow \frac{d+x}{a} = \frac{\frac{d}{a} + \tan\phi}{1-\frac{d}{a}\times\tan\phi}$$

$$\Rightarrow \frac{d+x}{a} = \frac{d+a\tan\phi}{a-d\tan\phi}$$

$$\Rightarrow (d+x)(a-d\tan\phi) = a(d+a\tan\phi)$$

$$\Rightarrow \cancel{ad} - d^2\tan\phi + xa - xd\tan\phi = \cancel{ad} + a^2\tan\phi$$

$$\Rightarrow a^2\tan\phi + d^2\tan\phi + xd\tan\phi = xa$$

$$\Rightarrow (a^2+d^2+xd)\times\tan\phi = xa$$

$$\Rightarrow \tan\phi = \frac{xa}{(a^2+d^2+xd)}$$

lly from $\tan(\alpha-\phi) = \dfrac{\tan\alpha - \tan\phi}{1+\tan\alpha\tan\phi}$

$$\Rightarrow \tan\theta = \frac{xa}{a^2+d^2-xd}$$

$\therefore$ from the condition of correct steering

$$\cot\phi - \cot\theta = \frac{c}{b} \Rightarrow \frac{1}{\tan\phi} - \frac{1}{\tan\theta} = \frac{c}{b}$$

$$\Rightarrow \frac{a^2+d^2+xd}{xa} - \frac{a^2+d^2-xd}{xa} = \frac{c}{b}$$

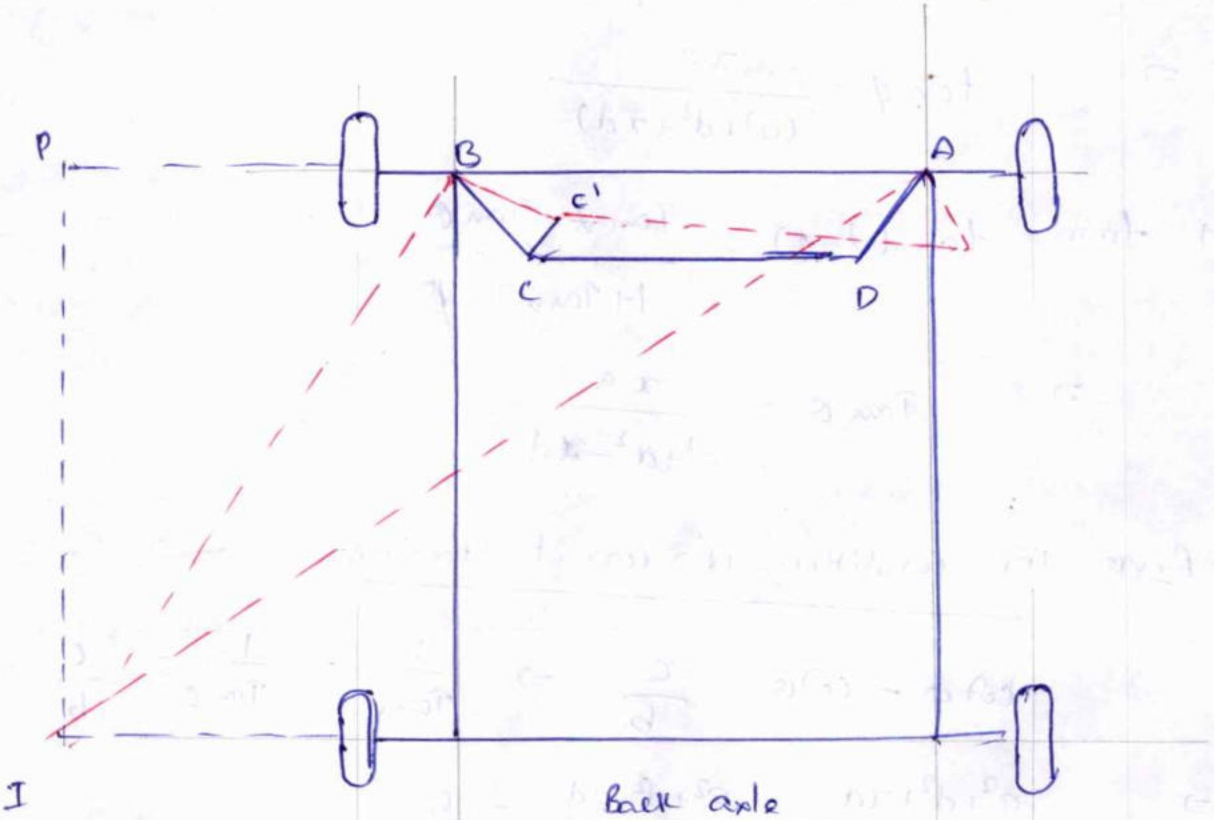
$$\Rightarrow \frac{(a^2+d^2+xd)-(a^2+d^2-xd)}{xa} = \frac{c}{b}$$

$$\Rightarrow \frac{2xd}{xa} = \frac{c}{b} \Rightarrow \boxed{\frac{d}{a} = \frac{c}{2b}}$$

$$\therefore \boxed{\tan\alpha = \frac{c}{2b}}$$

Note:- Though the gear is theoritically correct, but due to the presence of more sliding members, the wear will be increased which produces slackness between the sliding surfaces, thus eliminating the original accuracy. Hence Davis steering gear is not in common use.

Ackerman Steering Gear



The Ackerman steering gear mechanism is much simpler than Davis gear. The difference between the Ackerman and Davis steering gears are:

1. The whole mechanism of the Ackerman steering gear is on the back of the front wheels. where as in Davis steering gear, it is in front of the wheels.

2. The Ackerman steering gear consists of turning pairs, where as the davis steering gear consists of sliding members.

In Ackerman steering gear, the mechanism ABCD is a four bar crank chain as shown in fig. The shorter links BC and AD are of equal lengths and are connected by hinge joints with front wheel axles. The longer links AB and CD are of unequal length. The following are the only three positions for correct steering.

1. when the vehicle moves along a straight path, the longer links AB and CD are parallel and the shorter links BC and AD are equally inclined to the longitudinal axis of the vehicle, as shown by Thick (firm) lines in fig.

2. when the vehicle is steering to the left, The position of the gear is shown by dotted lines in fig. In this position, the lines of the front wheel axle intersect on the back wheel axle at J, for correct steering.

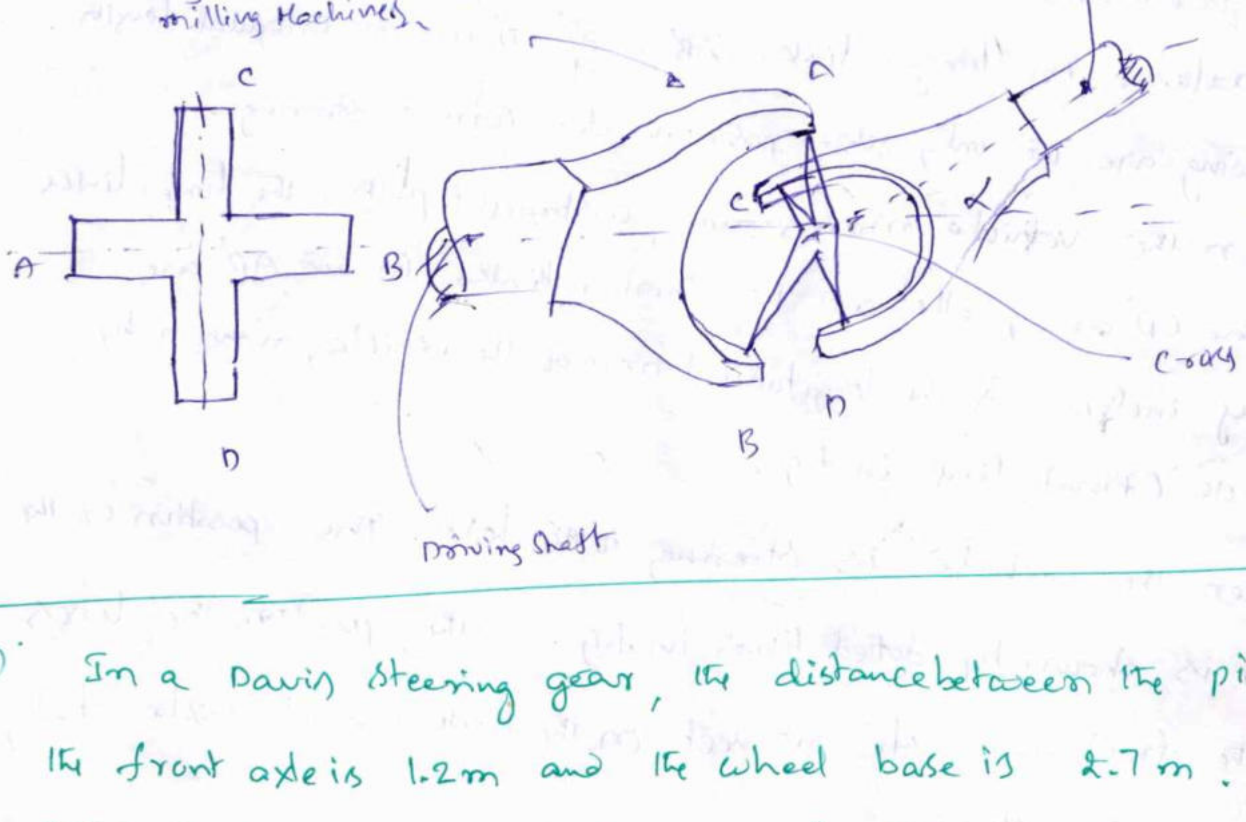
3. when the vehicle is steering to the right, the similar position may be obtained.

In order to satisfy the fundamental equation for correct steering the links AD and DC are suitably proportioned.

Universal (or) Hooke's joint

A Hooke's joint is used to connect two shaft, which are intersecting at a small angle, as shown in fig. The end of each shaft is forked to U-type and each fork is provided with two bearings for the arms of a cross. The arms of the cross is perpendicular to each other. The motion is transmitted from the driving shaft to the driven shaft through a cross. The inclination of the two shafts may be constant, but in actual practice it varies, when the motion is transmitted.

Application: (1) Transmission from GearBox to Differential (Automobile) [Double Hook joint is used.]
(2) Transm of power to different spindles of multiple drilling Machine
(3) Knee joint in milling Machines.



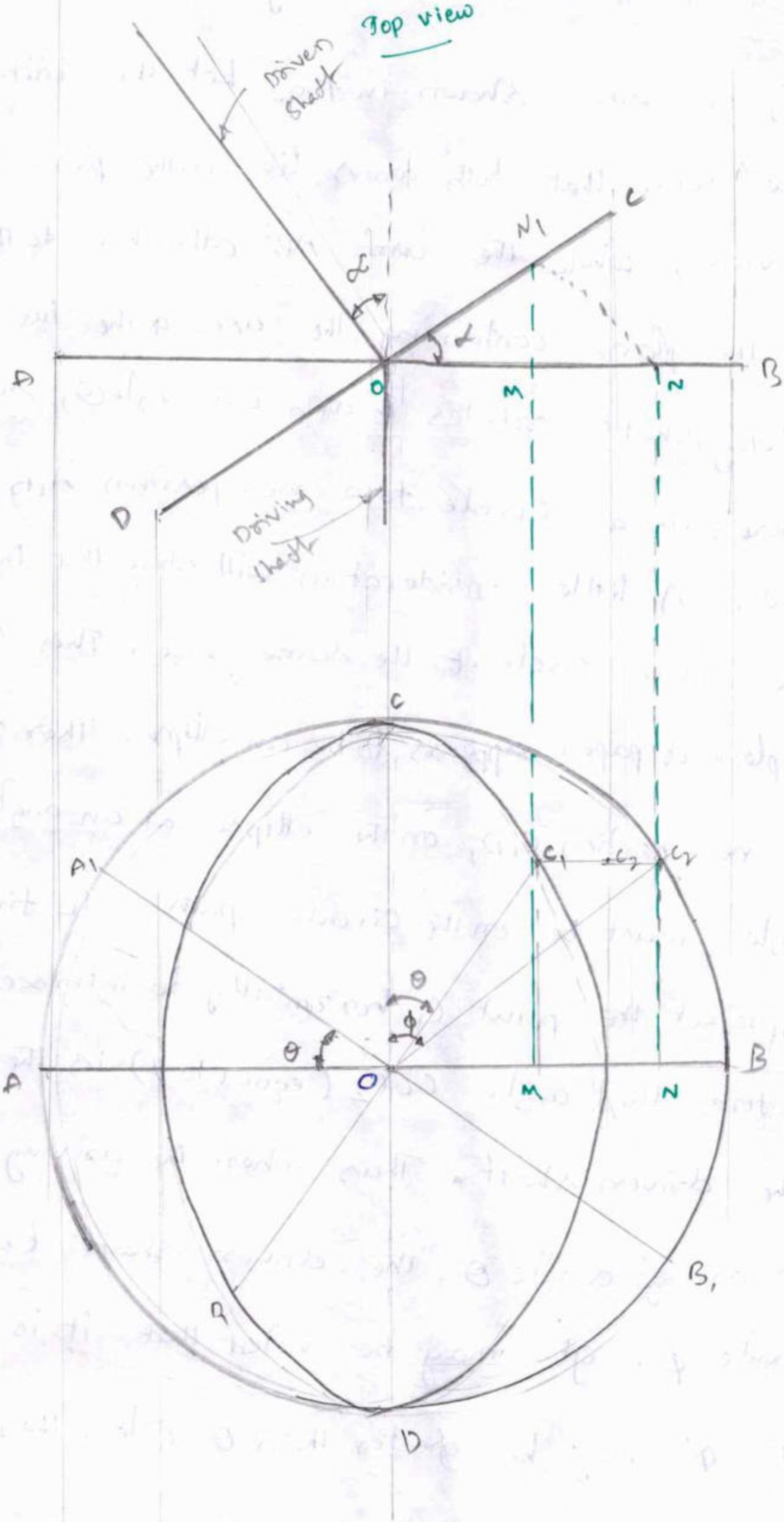
(P) In a Davis steering gear, the distance between the pivots of the front axle is 1.2 m and the wheel base is 2.7 m. Find the inclination of the track arms to the longitudinal axis of the car, when it is moving along a straight path.

(Sol: $\tan\alpha = \frac{c}{2b} = \frac{1.2}{2\times2.7} = 0.222$; $\alpha = 12.5^\circ$)

Ratio of the Shaft Velocities

2, 11, 12, 15, 17

Top view



Front view

The top and front views connecting the two shafts by a Universal joint are shown in fig. Let the initial position of the cross be such that both arms lie in the plane of paper in the front view, while the arm AB attached to the driving shaft lies in the plane containing the axes of the two shaft. Let the driving shaft rotates through an angle θ, so that the arm AB moves in a circle to a new position A_1B_1 as shown in front view. A little consideration will show that the arm CD will also move in a circle of the same size. This circle, when projected in the plane of paper appears to be an ellipse. Therefore the arm CD takes new position C_1D_1 on the ellipse at an angle θ. But the true angle must be on the circular path. To find the true angle, project the point C_1 horizontally to intersect the circle at C_2. Therefore the angle COC_2 (equal to ϕ) is the true angle turned by the driven shaft. Thus when the driving shaft turns through an ~~of~~ angle θ, the driven shaft turns through an angle ϕ. It may be noted that it is not necessary that 'ϕ' may be greater than 'θ' or less than θ. At a particular point, it may be equal to θ.

from $\angle OC_1M \Rightarrow \quad \tan\theta = \dfrac{OM}{C_1M} \quad$ — (i)

from $\angle OC_2N \Rightarrow \quad \tan\phi = \dfrac{ON}{C_2N} \quad$ — (ii)

from topview $\quad ON = ON_1 \Rightarrow$ llly $\quad OM = ON_1 \cos\alpha = ON\cos\alpha$

$$\Rightarrow \frac{(i)}{(ii)} \Rightarrow \frac{\tan\theta}{\tan\phi} = \frac{\frac{OM}{C_1M}}{\frac{ON}{C_2N}} = \frac{OM}{ON} \quad [\because C_1M = C_2N]$$

$$\Rightarrow \frac{\tan\theta}{\tan\phi} = \frac{\cancel{ON}\cos\alpha}{\cancel{ON}}$$

$$\Rightarrow \boxed{\tan\theta = \tan\phi\cos\alpha}$$

Differentiating the above equation w.r.t. t both sides

$$\sec^2\theta \times \frac{d\theta}{dt} = \cos\alpha \times \sec^2\phi \times \frac{d\phi}{dt} \quad — (A)$$

Let assume θ is the angular displacement of driving shaft

ω = Angular speed of driving shaft $= \dfrac{d\theta}{dt}$

llly $\quad \phi$ is the angular displacement of driven shaft

ω_1 = Angular speed of driven shaft $= \dfrac{d\phi}{dt}$

$$\sec^2\theta \times \omega = \cos\alpha \times \sec^2\phi \cdot \omega_1$$

$$\frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\cos\alpha \times \sec^2\phi} \quad — (B)$$

from fundamentals

$$\sec^2\phi = 1 + \tan^2\phi = 1 + \left(\frac{\tan\theta}{\cos\alpha}\right)^2 = 1 + \frac{\sin^2\theta}{\cos^2\theta \cdot \cos^2\alpha}$$

$$\Rightarrow \sec^2\phi = \frac{\cos^2\theta \cdot \cos^2\alpha + \sin^2\theta}{\cos^2\theta\cos^2\alpha} = \frac{(1-\sin^2\alpha)\cos^2\theta + \sin^2\theta}{\cos^2\theta\cos^2\alpha}$$

$$\Rightarrow \sec^2\phi = \frac{\cos^2\theta - \cos^2\theta\sin^2\alpha + \sin^2\theta}{\cos^2\theta\cos^2\alpha} = \frac{1 - \cos^2\theta\sin^2\alpha}{\cos^2\theta\cos^2\alpha}.$$

$\Rightarrow$

$$\Rightarrow \frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\cos\alpha \sec^2\phi} = \frac{\cos^2\theta \cos^2\alpha}{\cos^2\theta \times \cos\alpha \,(1-\cos^2\theta \sin^2\alpha)}$$

$$\Rightarrow \boxed{\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1-\cos^2\theta \sin^2\alpha}}$$

(or)

$$\boxed{\frac{N_1}{N} = \frac{\cos\alpha}{1-\cos^2\theta \sin^2\alpha}}$$

where N_1 = Speed of Driving shaft in r.p.m

N = Speed of Driven shaft in r.p.m

Maximum and Minimum Speeds of Driven shaft

from the Ratio of shaft speeds.

$$\frac{N_1}{N} = \frac{\cos\alpha}{1-\cos^2\theta \sin^2\alpha}$$

N_1 is maximum when $(1-\cos^2\theta \sin^2\alpha)$ is minimum

it is possible when $\theta = 0°$ or $180°$ or $360°$.

$$\therefore \frac{N_1}{N} = \frac{\cos\alpha}{1-\cos^2 0° \sin^2\alpha} = \frac{\cos\alpha}{1-\sin^2\alpha} = \frac{\cos\alpha}{\cos^2\alpha}$$

$$\Rightarrow \frac{N_1}{N} = \frac{1}{\cos\alpha}$$

$$\Rightarrow \boxed{N_{1(max)} = \frac{N}{\cos\alpha}}$$

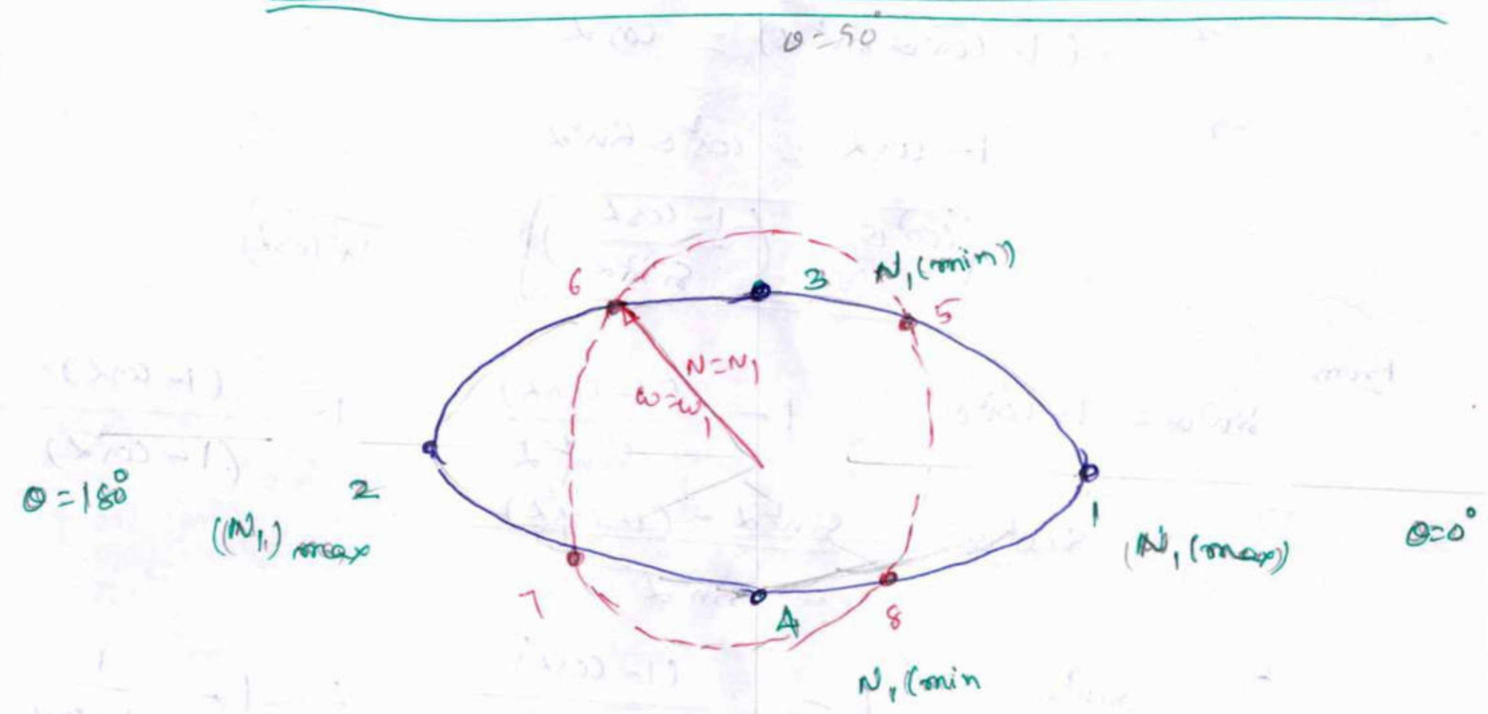
$|||^{ly}$ N_1 is minimum when $(1-\cos^2\theta \sin^2\alpha)$ is minimum

it is possible when $\theta = 90^\circ, 270^\circ$ etc.

$$\therefore \frac{N_1}{N} = \frac{\cos\alpha}{1-\cos^2 90 \sin^2\alpha} = \frac{\cos\alpha}{1-0\times\sin^2\alpha} = \cos\alpha$$

$$\therefore \boxed{N_{1(min)} = N\cos\alpha}$$

Polar Diagram - Salient features of Driven shaft speed



at 1 & 2 $\rightarrow$ N_1 is maximum

at 3 & 4 $\rightarrow$ N_1 is minimum

$\theta = 270$

at 5, 6, 7, 8 $\rightarrow$ N_1 is equal to N

Condition for equal Speeds of the Driving & Driven Shaft

From Ratio of Speeds of Shafts

$$\frac{\omega_1}{\omega} = \frac{N_1}{N} = \frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha}$$

If the Speed of Driving and Driven Shaft is same

$N_1 = N \Rightarrow 1 = \dfrac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha}$

$$\Rightarrow (1-\cos^2\theta\sin^2\alpha) = \cos\alpha$$

$$\Rightarrow 1-\cos\alpha = \cos^2\theta\sin^2\alpha$$

$$\Rightarrow \boxed{\cos^2\theta = \left(\frac{1-\cos\alpha}{\sin^2\alpha}\right)} = \frac{1}{(1+\cos\alpha)}$$

From

$$\sin^2\theta = 1-\cos^2\theta = 1-\frac{(1-\cos\alpha)}{\sin^2\alpha} = 1-\frac{(1-\cos\alpha)}{(1-\cos^2\alpha)}$$

$$\Rightarrow \sin^2\theta = \frac{\sin^2\alpha-(1-\cos\alpha)}{\sin^2\alpha} =$$

$$\Rightarrow \sin^2\theta = 1-\frac{(1-\cos\alpha)}{(1+\cos\alpha)\cancel{(1-\cos\alpha)}} = 1-\frac{1}{1+\cos\alpha}$$

$$\Rightarrow \sin^2\theta = \frac{1+\cos\alpha-1}{1+\cos\alpha} = \frac{\cos\alpha}{1+\cos\alpha}$$

$$\therefore \boxed{\sin^2\theta = \frac{\cos\alpha}{1+\cos\alpha}}$$

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} = \frac{\dfrac{\cos\alpha}{1+\cos\alpha}}{\dfrac{(1-\cos\alpha)}{\sin^2\alpha}} = \frac{\cos\alpha}{(1+\cos\alpha)}\times\frac{\sin^2\alpha}{(1-\cos\alpha)}$$

$$\Rightarrow \tan^2\theta = \frac{\cos\alpha \times \sin^2\alpha}{1-\cos^2\alpha} = \frac{\cos\alpha \times \cancel{\sin^2\alpha}}{\cancel{\sin^2\alpha}}$$

$$\Rightarrow \tan^2\theta = \cos\alpha$$

$$\Rightarrow \boxed{\tan\theta = \pm\sqrt{\cos\alpha}}$$

There are two values of θ corresponding to positive sign and two values corresponding to negative sign. Hence, there are four values of θ, at which the speeds of the driving and driven shafts are same. This is shown by points 5, 6, 7 and 8 in polar diagram,

---

Maximum fluctuation of Speed

We know that the maximum speed of the driven shaft

$$\boxed{N_{1(max)} = \frac{N}{\cos\alpha}}$$

and minimum speed of the driven shaft

$$\boxed{N_{1(min)} = N\cos\alpha}$$

The maximum fluctuating speed of driven shaft (q) is equal to the difference between the maximum and minimum speeds of the driven shaft.

$$\therefore q = N_{1(max)} - N_{1(min)} = \frac{N}{\cos\alpha} - N\cos\alpha$$

$$\Rightarrow q = N\left(\frac{1}{\cos\alpha} - \cos\alpha\right)$$

$$\Rightarrow q = N_p \frac{1-\cos^2\alpha}{\cos\alpha} = N_p \frac{\sin^2\alpha}{\cos\alpha}$$

$$\Rightarrow \boxed{q = N \tan\alpha \times \sin\alpha}$$

(or)

$$\boxed{q = \omega \tan\alpha \sin\alpha}$$

Since 'α' is a small angle therefor substitution $\cos\alpha = 1$ and $\sin\alpha = $ 'α' Radians

$$\Rightarrow q = \omega \times \frac{\sin\alpha}{\cos\alpha} \times \sin\alpha = \omega \frac{\alpha \times \alpha}{1}$$

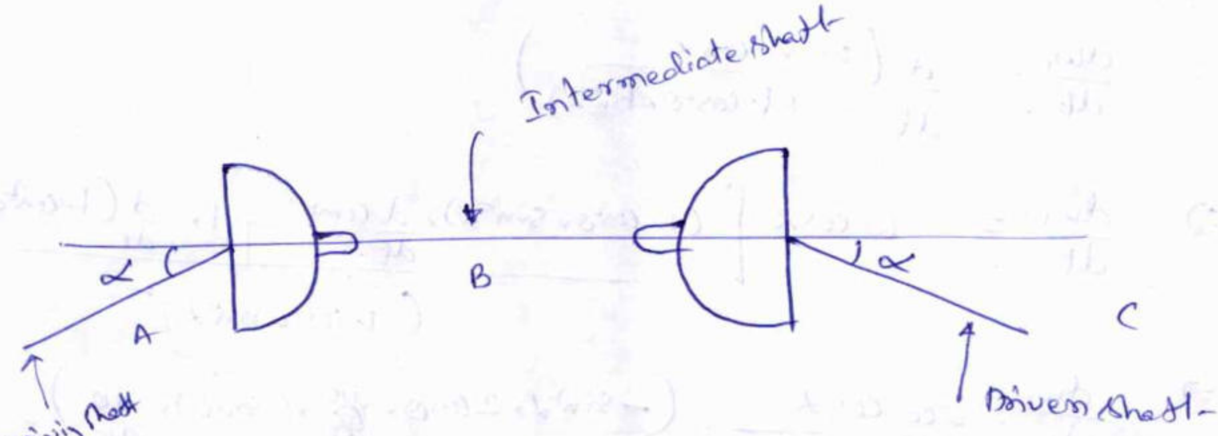
$$\Rightarrow \boxed{q = \omega \times \alpha^2 \text{ or } N\alpha^2}$$

$\therefore$ Maximum fluctuation Speed $\boxed{q_{(max)} = \omega \alpha^2}$

Hence, the maximum fluctuation of speed of the driven shaft approximately varies as the square of the angle between the two shaft.

Double Hooke's joint

We know that the velocity of driven shaft is not constant, but varies from maximum to minimum values. In order to have a constant velocity ratio of driving and driven shafts, an intermediate shaft with a Hooke's joint at each end as shown in fig. is used. This type of joint is known as double Hooke's joint.



Let the driving, intermediate and driven shafts in the same time, rotate through an angles θ, ϕ and γ from the position

Now for shafts A and B $\quad \tan\theta = \tan\phi \cos\alpha \quad$ -(i)

for shafts B and C $\quad \tan\gamma = \tan\phi \cos\alpha \quad$ -(ii)

from equations (i) and (ii) we see that $\theta = \gamma$; $N_A = N_C$

this shows that the speed of the driving and driven shaft is constant. In other words, this joint gives a velocity ratio equal to Unity, if

1. The axes of the driving and driven shafts are in the same plane
2. The driving and driven shaft makes equal angles with the intermediate shaft.

Angular Acceleration of the Driven Shaft

From $\frac{\omega_1}{\omega} = \frac{N_1}{N} = \frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha}$

$$\omega_1 = \omega\frac{(\cos\alpha)}{(1-\cos^2\theta\sin^2\alpha)}$$

Differentiating the above expression, w.r.t. t; we obtain the angular acceleration.

$$\frac{d\omega_1}{dt} = \frac{d}{dt}\left(\omega \times \frac{\cos\alpha}{(1-\cos^2\theta\times\sin^2\alpha)}\right)$$

$$\Rightarrow \frac{d\omega_1}{dt} = \omega\cos\alpha\left[\frac{(1-\cos^2\theta\times\sin^2\alpha)\times\frac{d}{dt}(\text{const}) - 1\times\frac{d}{dt}(1-\cos^2\theta\times\sin^2\alpha)}{(1-\cos^2\theta\sin^2\alpha)^2}\right]$$

$$\Rightarrow \frac{d\omega_1}{dt} = \frac{-\omega\cos\alpha}{(1-\cos^2\theta\sin^2\alpha)^2}\left(-\sin^2\alpha\times 2\cos\theta\times\cancel{\frac{d\theta}{dt}}\times(-\sin\theta)\times\frac{d\theta}{dt}\right)$$

$$\Rightarrow \boxed{\frac{d\omega_1}{dt} = \frac{-\omega^2\cos\alpha\,(\sin 2\theta\times\sin^2\alpha\times\cancel{\omega})}{(1-\cos^2\theta\sin^2\alpha)^2}}$$

[$\because$ – ve indicates the Retardation]

For angular acceleration to be maximum, differentiate $\frac{d\omega_1}{dt}$ with respect to θ and equate to zero. The result is approximated as

$$\boxed{\cos 2\theta = \frac{\sin^2\alpha\,(2-\cos^2 2\theta)}{2-\sin^2\alpha}}$$

Note:- If the value of α is less than 30°, then $\cos 2\theta$ may approximately be written as

$$\boxed{\cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha}}$$

Condition for Max Angular accel

(P) Two shafts with an included angle of 160° are connected by a Hooke's joint. The driving shaft runs at a Uniform speed of 1500 r.p.m. The driven shaft carries a flywheel of mass 12 kg and 100 mm radius of gyration. Find the maximum angular acceleration of the driven shaft and the maximum torque required. $(T_{max} = I\alpha_{max})$ $= I\frac{d\omega}{dt}$ $I = mk^2$ (Ans:- $\theta = 41.65°$, $\frac{d\omega_1}{dt} = 3090$ rad/s² (max) ; $T = 371$ N-m) $I = 0.12$ kg-m²

$\cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha}$
$\theta = \checkmark$

put a value in
= max (+ve value)

(P). The angle between the axes of two shafts connected by Hooke's joint is 18°. Determine the angle turned through by the driving shaft when the velocity ratio is maximum and Unity. (Ans: $\theta = 0°, 180°$; $\theta = 44.3°$ (or) $135.7°$) ($\because \cos\theta = \pm 0.7159$)

(i) $\left(\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha}\right.$ (ii) $1 = \frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha}$

(P). Two shafts are connected by a Hooke's joint. The driving shaft revolves Uniformly at 500 r.p.m. If the total permissible variation in speed of driven shaft is not to exceed ±6% of the mean speed (ω), find the greatest permissible angle between the centre lines of the shafts. (Ans $\alpha = 19.64°$)

$q = 0.12\omega$

$q = \omega\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$ ✓
$\cos\alpha = 0.94$
$\alpha = 19.64°$
$\left(x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\right.$

(P) Two shafts are connected by a Universal joint. The driving shaft rotates at a Uniform speed of 1200 rpm. Determine the greatest permissible angle between the shaft axes so that the total fluctuation of speed does not exceed 100 r.p.m. Also calculate the maximum and minimum speeds of the driven shaft. (Ans $\alpha = 16.4°$; $N_{1(max)} = 1251$ r.p.m ; $N_{1(min)} = 1151$ r.p.m)

$q = N\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$

$q = 100$; $N = 1200$; $\alpha = 16.4°$

$N_{1(max)} = \frac{N}{\cos\alpha}$; $N_{1(min)} = N\cos\alpha$

(P) The driving shaft of a Hooke's joint runs at a uniform speed of 240 r.p.m and the angle 'α' between the shafts is 20°. The driven shaft with attached masses has a mass of 55 kg at a radius of gyration of 150 mm.

1. If a steady torque of 200 N-m resists rotation of the driven shaft, find the torque required at the driving shaft when $\theta = 45^\circ$.

2. At what value of 'α' will the total fluctuation of speed of the driven shaft be limited to 24 r.p.m.

(Ans $T' = 102.6$ N-m ; $\alpha = 18.2^\circ$)

(P) A Double Universal joint is used to connect two shafts in the same plane. The intermediate shaft is inclined at an angle of 20° to the driving shaft as well as the driven shaft. Find the maximum and minimum speed of the intermediate shaft and the driven shaft if the driving shaft has a constant speed of 500 r.p.m. ($N_{B(max)} = 532.1$ rpm; $N_{B(min)} = 469.85$ rpm)

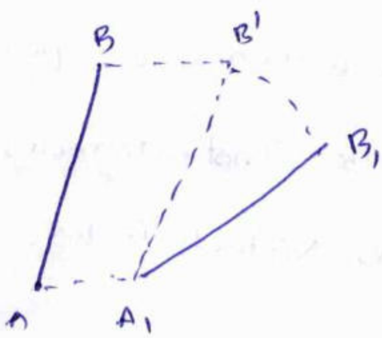
$N_{C(max)} = 566.25$ rpm.

$N_{C(min)} = 441.5$ rpm.

UNIT-IV (Kinematics, Analysis of Mechanisms and plane motion of Body)

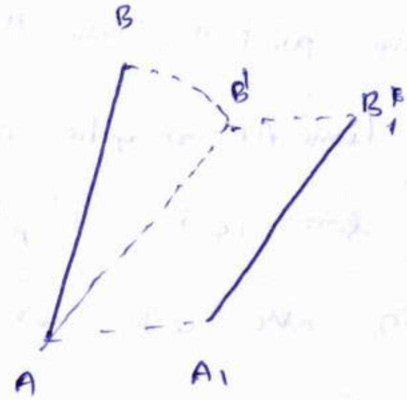
Plane Motion of Body

Sometimes, a body has simultaneously a motion of rotation as well as translation, such as wheel of a car, a sphere rolling on a ground (without slipping). Such a motion will have the combined effect of rotation and translation.



(a)

(or)



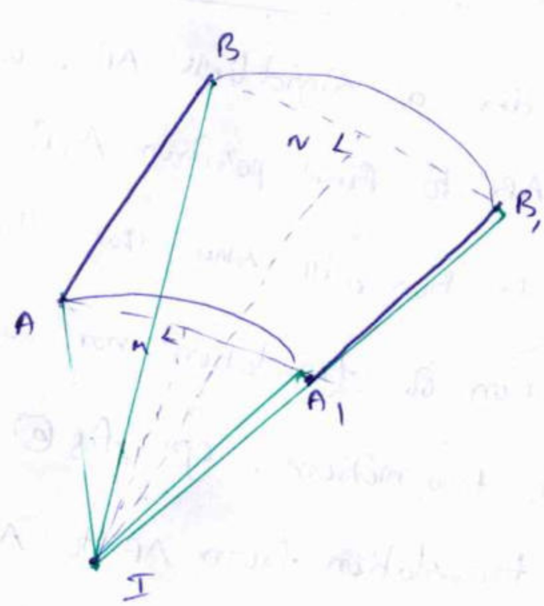
(b)

Motion of link

Consider a rigid link AB, which moves from its initial position AB to final position A_1B_1 as shown in fig. (a). A little consideration will show that the link neither has wholly a motion of translation nor wholly rotational, but a combination of two motions. In fig (a), the link has first the motion of translation from AB to A_1B' and then the motion of rotation about A_1, till it occupies the final position A_1B_1. Similarly in fig (b), the link AB has first the motion of rotation from AB to A_1B' about A and then the motion of translation from AB' to A_1B_1. Such a motion of link AB to A_1B_1 is an example of combined motion of rotation and translation, it being immaterial whether the motion of rotation takes first or the motion of translation.

In actual practice, the motion of link AB is so gradual that it is difficult to see the two separate motions. But we see the two seperate motions, though the point B moves faster then the point A. Thus this combined motion of rotation and translation of the link AB may be assumed to be a motion of pure rotation about some center I, known as the Instantaneous Center of rotation also called as Centro (or) Virtual Center. The

How to Locate an Instantaneous Center



Since the points A and B of the link has moved to A_1 and B_1 respectively under the motion of rotation, therefore the position of center of rotation must lie on the intersection of the right bisectors of chords AA_1 and BB_1. Let these bisector intersect at I as shown in fig, which is the instantaneous center of rotation or virtual center of the link AB

From the above we see that the position of the link AB goes on changing, therefore the center about which the motion is assumed to take place (i.e, instantaneous centre of rotation) also goes on changing. Thus the instantaneous center of moving body may be defined as that centre which goes on changing from one instant to another. The locus of all such instantaneous centers is known as Centrode. A line drawn through an instantaneous center and perpendicular to the plane of motion is called instantaneous axis. The locus of this axis is known as axode.

Space & Body Centrode

A Rigid body in plane motion relative to second rigid body, supposed fixed in space, may be assumed to be rotating about an instantaneous centre at that particular moment. In otherwords, the instantaneous center is a point in the body which may be considered fixed at any particular moment. " The locus of the instantaneous center in space during a definite motion of the body is called the 'SPACE CENTRODE' and the locus of the instantaneous centre relative to the body itself is called the 'BODY CENTRODE'. These two centrodes have the instantaneous center as a common point at any instant and during the motion of the body, the body centrode rolls without slipping over the space centrod.

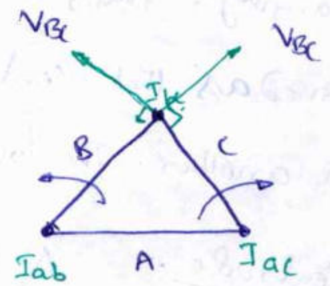
Aronhold Kennedy (or Three Centres in Line) Theorem

'The Aronhold Kennedy's theorem states that if three bodies move relatively to each other, they have Three instantaneous centres and lie on a straight line.'

Consider three kinematic links A, B and C having relative plane motion. The Number of instantaneous centers (N) is given by

$$N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$

where n = number of links = 3



The two instantaneous centres at the pin joints of B with A, and C with A are (I_{ab} & I_{ac}) the permanent instantaneous Centers, Accordiy to the Kennedy's Theorem the third instantaneous Centre I_{bc} must lie on the line joining I_{ab} and I_{ac}. In order to prove this, let us Consider that the instantaneous centre I_{bc} lies outside the line joining I_{ab} and I_{ac} as shown in fig.. The point I_{bc} b

Properties of the Instantaneous Centre

1. A rigid link rotates instantaneously relative to another link at the instantaneous centre for the configuration of the mechanism considered.

2. The two rigid links have no linear velocity relative to each other at the instantaneous center. At this point, the two rigid links have the same linear velocity relative to the third rigid link.

Numbers of Instantaneous centers in a Mechanism

The number of instantaneous centers in a constrained kinematic chain is equal to the number of possible combinations of two links. Mathematically the number of instantaneous centers (N)

$$\Rightarrow \quad N = \frac{n(n-1)}{2}, \quad \text{where } n = \text{number of links.}$$

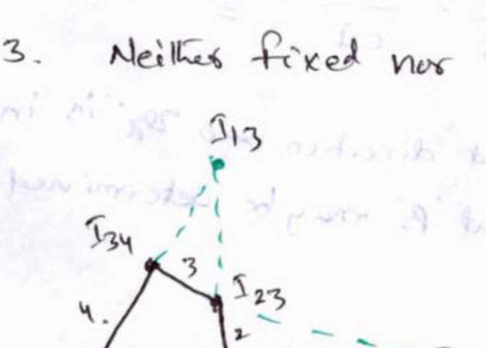
Types of instantaneous Centers

The instantaneous centers for a mechanism are of the following 3 types

1. Fixed Instantaneous Centers
2. Permanent Instantaneous Centers

} Primary Instantaneous Center

3. Neither fixed nor Permanent instantaneous Center → Secondary Instantaneous Centers.



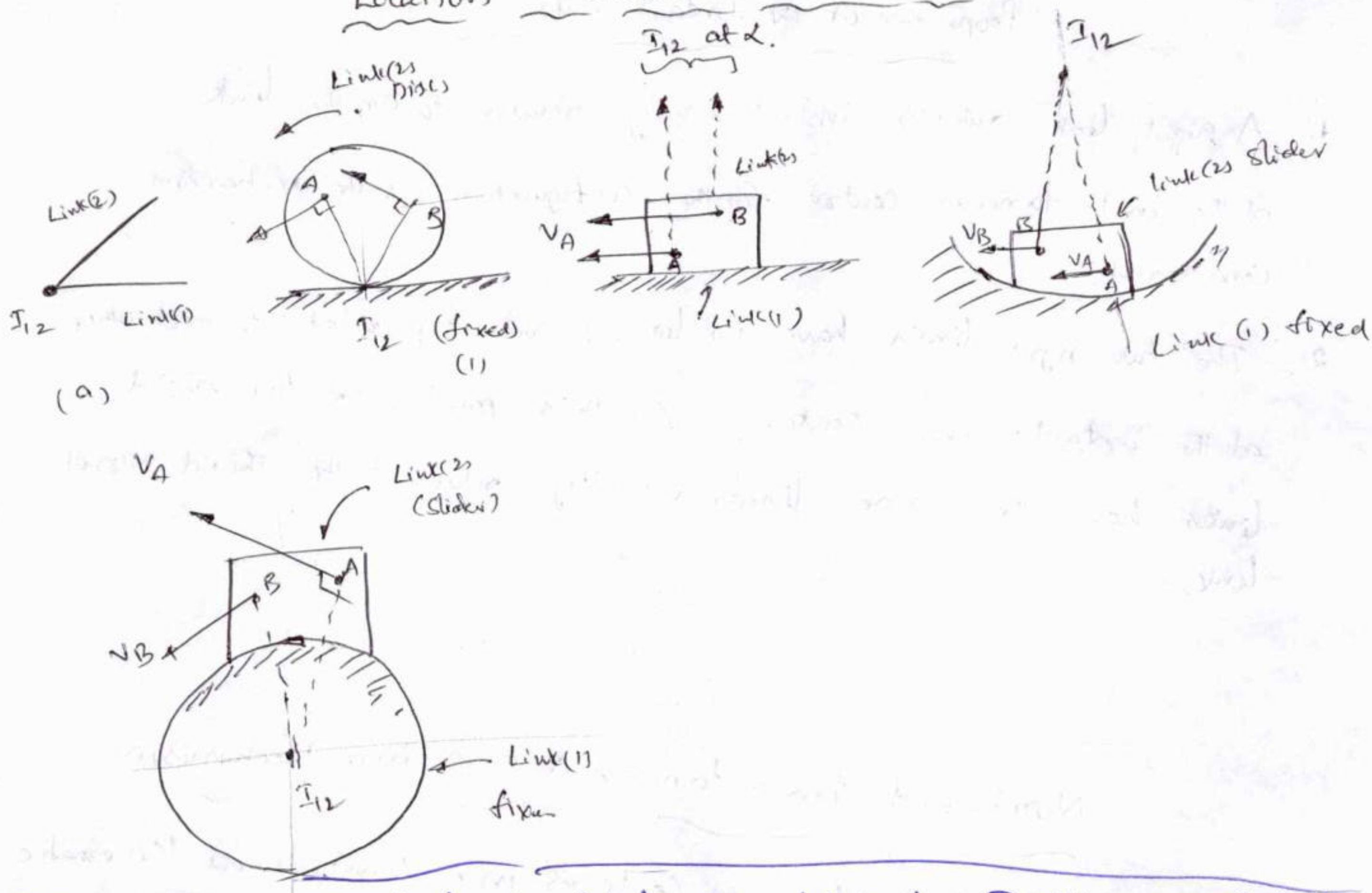
I_{12}	I_{23}	I_{34}
I_{13}	I_{24}	
I_{14}		

I_{12} & I_{14} → fixed

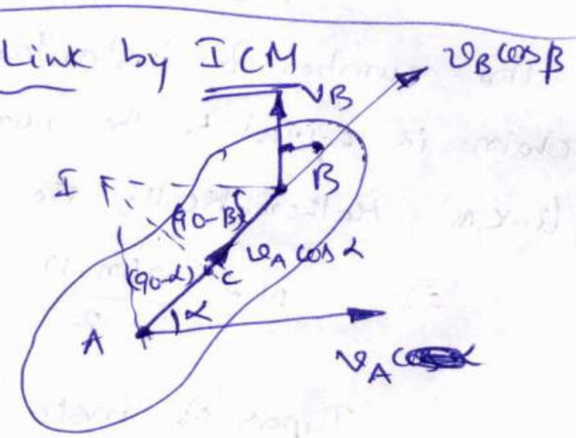
I_{34} & I_{23} → Permanent

I_{13}, I_{24} → neither fixed nor Permanent

Location of Instantaneous Centres



Velocity of a Point on a Link by ICM



$$v_A \cos\alpha = v_B \cos\beta$$

$$\frac{v_A}{v_B} = \frac{\cos\beta}{\cos\alpha} = \frac{\sin(90-\beta)}{\sin(9-\alpha)}.$$

(ABI) From Lami's theorem

$$\frac{AI}{\sin(90-\beta)} = \frac{BI}{\sin(90-\alpha)}$$

$$\Rightarrow \quad \frac{\sin(90-\beta)}{\sin(90-\alpha)} = \frac{AI}{BI}$$

$$\therefore \quad \frac{v_A}{v_B} = \frac{AI}{BI} \qquad (or) \qquad \frac{v_A}{AI} = \frac{v_B}{BI} = \frac{v_C}{CI} = \omega$$

From the above:-

1. If 'v_A' is known in magnitude and direction and 'v_B' is in direction only, then the velocity of a point 'B' may be determined in magnitude and direction.

Method of Locating Instantaneous Centres in a Mechanism

(1) First of all, Determine the number of instantaneous centres (N) by using the relation

$$N = \frac{n(n-1)}{2}$$, where n: Number of links

(2) Mark the list of all instantaneous Centers in a mechanism.

Draw the table chart for this

Ex:- 4 links → $N = 6$ ($\because \frac{4(4-1)}{2} = 6$) (Four bar chain)

Number of links	1	2	3	4	.	..
No. of Instantaneous Centers	I_{12}	I_{23}	I_{34}	–	.	
	I_{13}	I_{24}	–	–		
	I_{14}	–	–	.		

(3) Locate the fixed and Permanent Instantaneous Centres by inspection.

(4) Locate the remaining neither fixed nor permanent instantaneous Centres (Secondary Centres) by Kennedy's theorem. This is done by circle diagram as shown in example fig.

Ex:- Four bar Chain

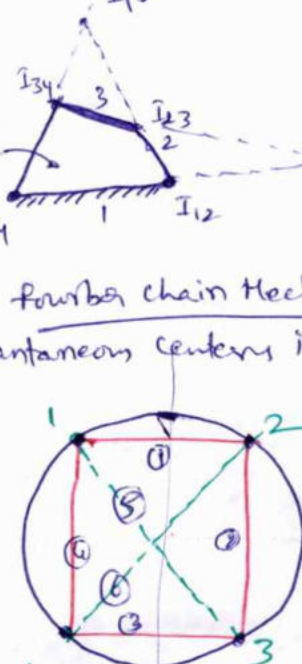
Four bar chain Mechanism

(5) Draw Circle and do the steps 3 & 4. i.e

(i) mark the points on a circle equal to the number of links in a mechanism

Ex:- (4) in this example. and locate the primary Instantaneous centers i.e $I_{14}, I_{12}, I_{23}, I_{34}$.

(ii) Locate the Secondary ~~pair~~ instantaneous centers I_{13} & I_{24}

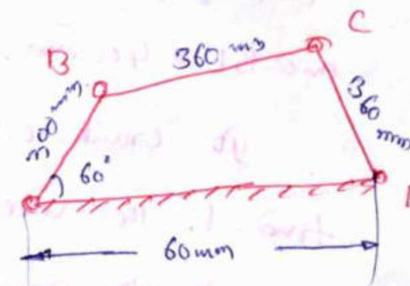


(P) In a pin joined four bar chain mechanism as shown in fig.

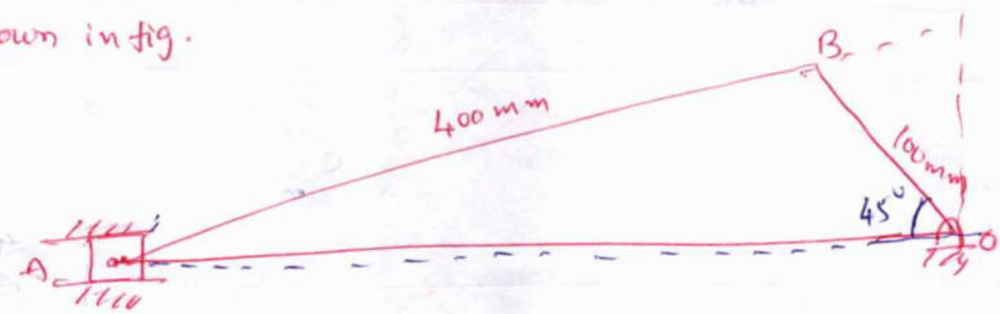
AB = 300mm, BC = CD = 360mm and AD = 600mm.

The angle BAD = 60°. The Crank AB rotates uniformly at 100 r.p.m. Locate the all instantaneous centers and find the angular velocity of the link BC. (Ans:- $v_B = \omega_{AB} \times AB = \omega_{BC} \cdot I_{13} \times B = 6.282$ rad/sec)

$I_{13}B = 500mm = 0.5m$



(P) Locate the all instantaneous centres of the slider crank mechanism as shown in fig.



~~Note~~ The lengths of crank OB and connecting rod AB are 100mm and 400mm respectively. If the crank rotates clockwise with an angular velocity of 10 rad/sec. Find 1. Velocity of the slider A, and 2. Angular velocity of the connecting rod AB.

(Ans. $\frac{v_A}{I_{13}A} = \frac{v_B}{I_{13}B} \Rightarrow v_A = 0.82$ m/sec)

$\omega_{AB} = \frac{v_A}{I_{13}A} = \frac{v_B}{I_{13}B} = 1.78$ rad/s.)

(D) A mechanism as shown in fig has the following dimensions.

OA = 200mm ; AB = 1.5m = 1500mm ; BC = 600mm ; CD = 500mm

and BE = 400mm. Locate the all instantaneous centers.

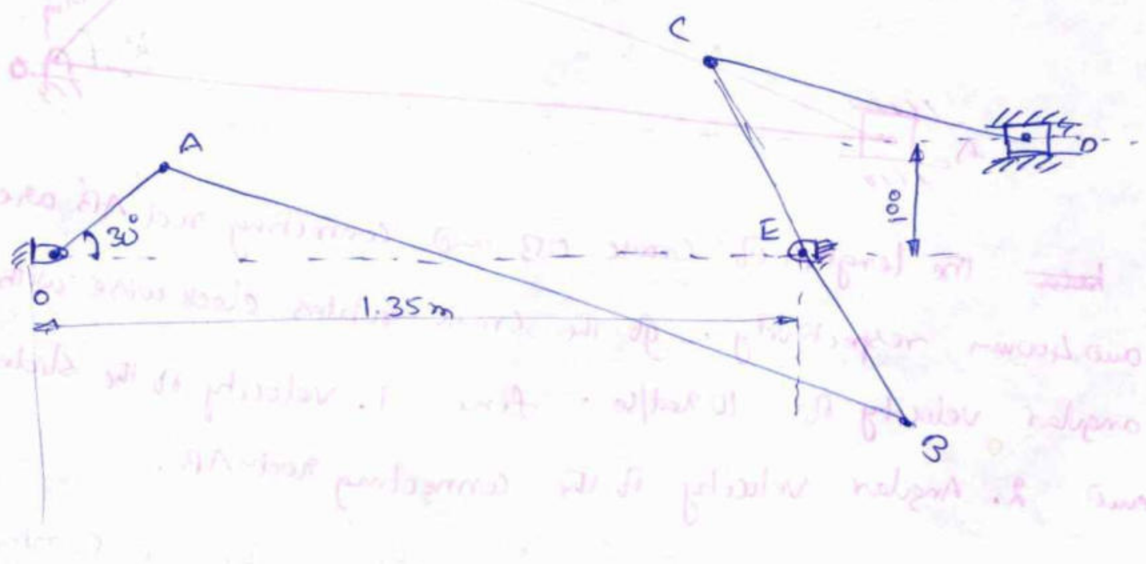
If crank OA rotates uniformly at 120 r.p.m. clockwise,

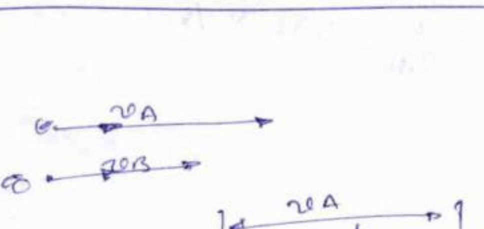
find 1. the velocity of B, C and D

2. the angular velocity of the links AB, BC, and CD

(Ans:- $v_B = 3.2$ m/sec $v_C = 1.6$ m/sec $v_D = 1.05$ m/sec

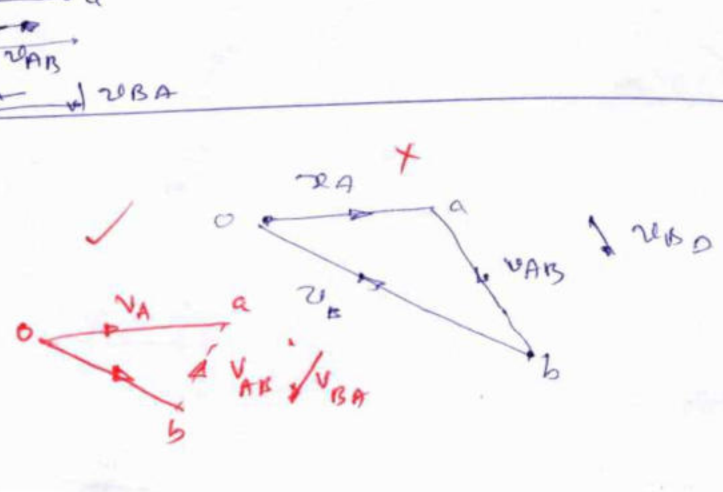
$\omega_{AB} = 2.99$ rad/sec $\omega_{BC} = 8$ rad/sec $\omega_{CD} = 2.16$ rad/sec





$v_{AB} = \bar{v}_A - \bar{v}_B \Rightarrow \bar{ba} = \bar{oa} - \bar{ob}$

$v_{BA} = \bar{v}_B - \bar{v}_A \Rightarrow \bar{ab} = \bar{ob} - \bar{oa}$

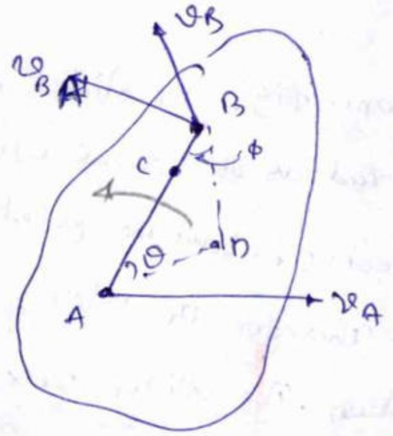


Velocity in Mechanism

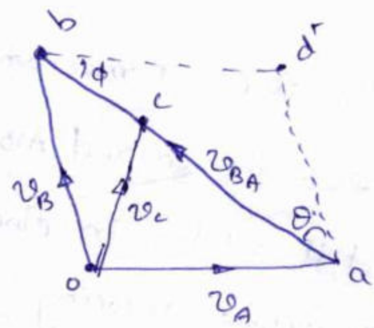
(1) Relative Velocity of Two Bodies moving in straight Lines

(2) Relative velocity of Two Bodies moving in an inclined path.

(3) Determining the velocity of a point on a Link by Relative velocity method.



Link

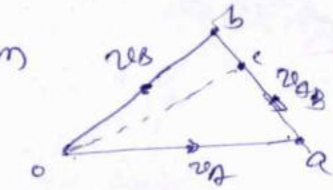


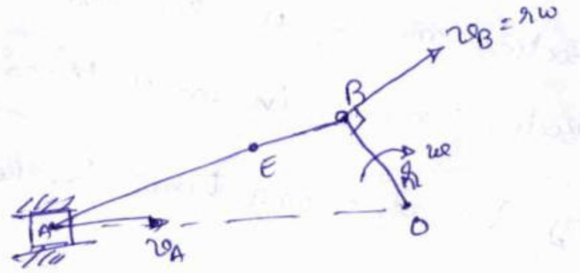
Velocity diagram.

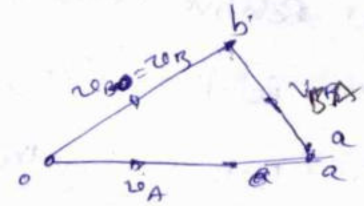
The relative velocity method is based on a link as shown in fig. Let the Absolute velocity of the point A i.e v_A is known in magnitude and direction and the Absolute velocity of a point B i.e (v_B) is known in direction only. Then the velocity of B is determined by drawing the velocity diagram. The procedure for drawing velocity diagram is as follows.

1. take a convenient point o, known as pole
2. Through 'o' draw 'oa' line parallel and equal to v_A, to some suitable scale
3. Through 'a', draw a line perpendicular to AB, this line represents the velocity of B with respect to A i.e v_{BA}.
4. Through o, draw a line parallel to v_B intersecting the line of v_{BA} at b.
5. Measure 'ob', which gives the required velocity of point B (v_B) to the scale.

Velocity in Slider Crank mechanism







(a) Slider Crank Mechanism velocity diagram

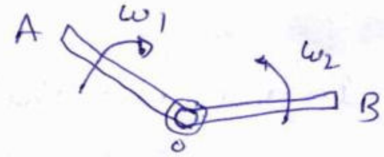
A slider ~~a~~ crank mechanism is shown in fig. the slider 'A' is attached to the Conrod AB. Let the radius of crank 'OB' be 'r' and let it rotates in clockwise direction, about the point 'O' with uniform angular velocity ω rad/sec. Therefor the velocity of B, is known in ~~mag~~ magnatude and direction. The slider reciprocates along the line of stroke AO. the velocity of slider may be determined by relative Nelocity method. as discussed below.

1. from any point 'o', draw vector ob parallel to the direction of v_B (or perpendicular to OB). Such that $ob = v_B = \omega r$, to some suitable scale.

2. Since AB is a rigid link, therefore the velocity of 'A' relative to 'B' is perpendicular to AB. Now draw the vector ba perpendicular to AB to represent the velocity of 'A' with respect to 'B', i.e v_{AB}.
3. From point 'o', draw vector 'oa' parallel to the path of motion of the slider A. The vectors ba and oa intersect at a, No 'oa' represents the velocity of slider 'A' ie v_A to scale.

∴ the angular velocity of con. Rod is determined as follows $\omega_{AB} = \dfrac{v_{BA}}{AB} = \dfrac{ab}{AB}$.

Rubbing velocity at a pin point



The Rubbing velocity at point O

$$v_o = (\omega_1 \pm \omega_2) \times r$$

take +ve if the links move in opposite direction

take −ve if the links move in same direction

r = Radius of pin

The Rubbing velocity is defined as

the algebraic sum of the angular velocity of two links which are connected by pin joints, multiplied by the radius of pin.

(1) The fig. shows the structure of Whitworth Q.R. mechanism used in Reciprocating machine tools. The various dimensions of the tool are as follows, OQ = 100 mm; OP = 200 mm; RQ = 150 mm; RS = 500mm. The crank makes an angle of 60° with the vertical. Determine the velocity of slider S (cutting tool) when crank rotates at 120 rpm clockwise. Find also the angular velocity of link RS and velocity of sliding block T on slotted lever QT,

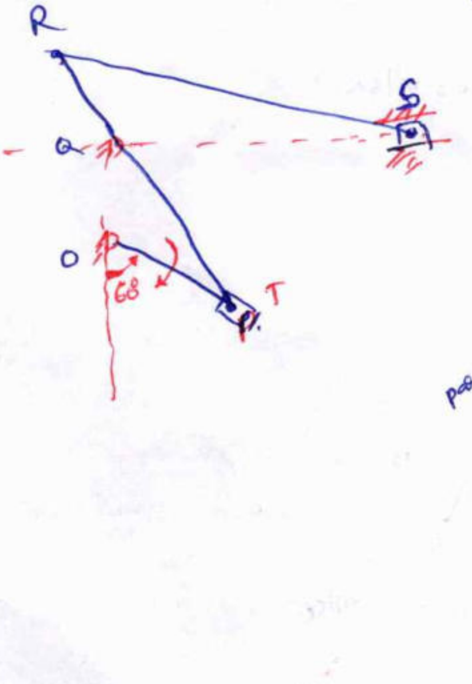
Sol: N_{PO} = 120 rpm; ω_{PO} = 12.57 rad/s; $v_{PO} = v_P$ = 2.514 m/sec

(Ans: $v_s \approx$ 0.8 m/s = os;

$v_{SR} = rs \approx$ 0.96 m/sec

ω_{RS} = 0.92 rad/sec

$v_{tp} = pt \approx$ 0.85 m/sec



Problems

① In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 r.p.m. clockwise, while the link CD = 80 mm oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when the angle BAD = 60°.

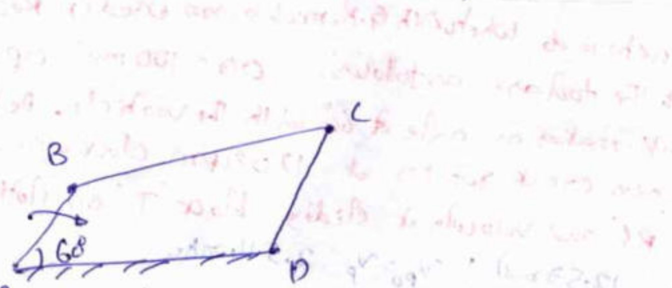
Sol:- Given Data:

$N_{AB} = 120\ rpm \Rightarrow \omega_{AB} = 12.568\ rad/s$,

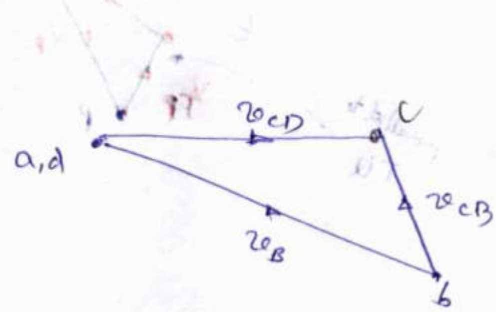
$AB = 40mm = 0.04m$

$BC = AD = 150\ mm$; $CD = 80mm = 0.08m$.

$\angle BAD = 60°$;



$v_B = \omega_{AB} \cdot AB = 12.568 \times 0.04 = 0.503\ m/sec$



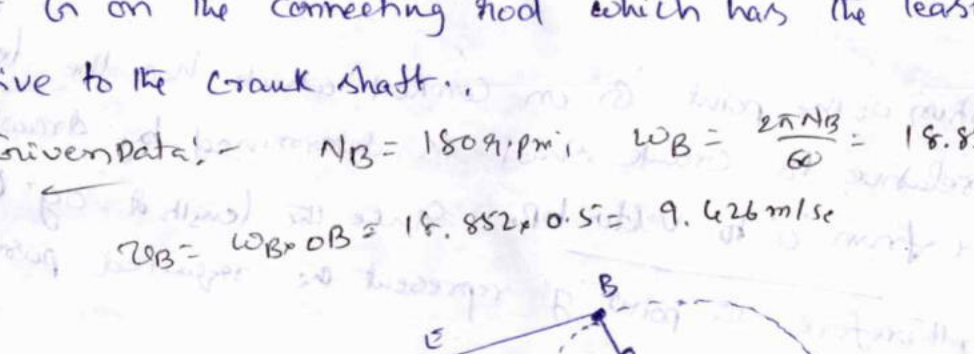
Calculate $v_{CD} = v_C = \text{vector } dc = 0.385\ m/sec$

Angular velocity of CD link $\omega_{CD} = \frac{v_C}{CD} = \frac{0.385}{0.08} = 4.8\ rad/sec$

(2) The crank and connecting rod of the theoretical steam engine are 0.5m and 2m long respectively. The crank makes 180 r.p.m. in clock wise direction. When it has turned 45° from the inner dead centre position, determine 1. velocity of the piston 2. angular velocity of the connecting rod 3. velocity of the point E on the connecting rod 1.5 m from the gudgeon pin.
4. velocity of rubbing at pins of the crank shaft, crank and cross head when the diameters of their pins are 50mm, 60mm and 30mm respectively. 5. Position and linear velocity of any point G on the connecting rod which has the least velocity relative to the crank shaft.

Sol:- Given Data:- $N_B = 180$ r.p.m, $\omega_B = \frac{2\pi N_B}{60} = 18.852$ rad/sec.

$v_B = \omega_B \times OB = 18.852 \times 0.5 = 9.426$ m/sec



Velocity diagram

Sol:- $v_p = 8.15$ m/sec , $v_{PB} = 6.8$ m/sec ;

$\omega_{PB} = \frac{v_{PB}}{PB} = \frac{6.8}{2} = 3.4$

$\frac{BE}{BP} = \frac{be}{bp} \Rightarrow be = \frac{BE}{BP} \times bp \Rightarrow v_E = oe$ -

(h) Velocity at Rubbing :-

$d_O = 50mm$

$d_B = 60 mm$

$d_P = 30 mm$

(i) Rubbing velocity at Crank shaft $= \frac{d_O}{2}(\omega_{BO} + 0.)$

(ii) Rubbing velocity at crank pin $= \frac{d_B}{2}(\omega_{OB} + \omega_{BP})$

(Clock + Anticlock

(iii) Rubbing velocity at Crosshead $= \frac{d_P}{2}(\omega_{BP} + 0)$

(5). The Position of the point 'G' on Con.Rod which has the least velocity relative to crank shaft is determined by drawing a perpendicular from 'o' to vector bp. Since the length of 'og' will be the least, therefore the point 'g' represent the required position of 'G' on the Con.Rod.

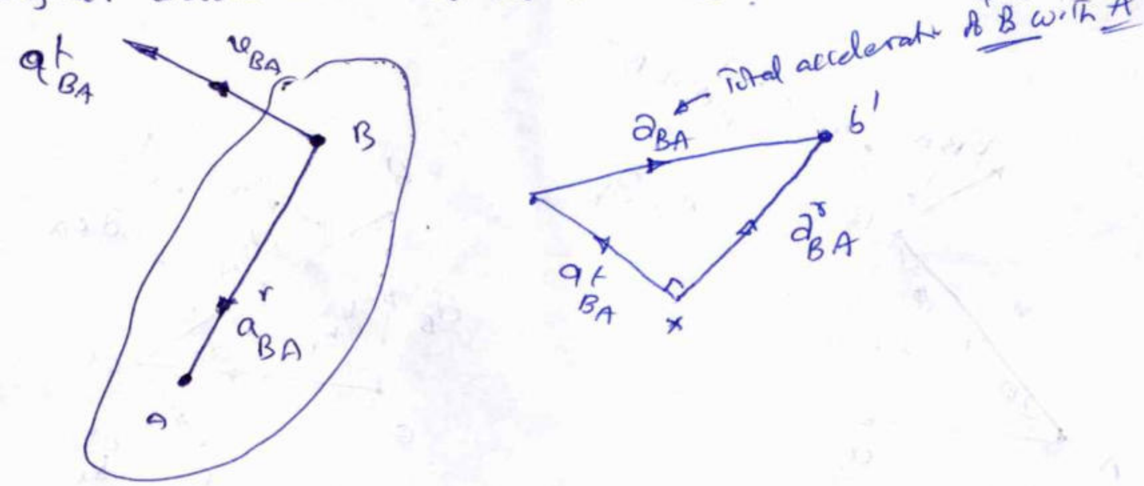
$\therefore$ $v_G = og \simeq 8 m/s$

$$\frac{bg}{bp} = \frac{BG}{BP} \Rightarrow BG = \underline{\underline{1.47 m}}$$

Acceleration in Mechanisms

Acceleration diagram for a link

Consider two points 'A' and 'B' on a rigid link as shown in fig. Let the point 'B' moves with respect to A, with an angular velocity of 'ω' rad/sec and let 'α' rad/sec² be the angular acceleration of the link AB.



We have already discussed that acceleration of a particle whose velocity changes both in magnitude and direction at any instant, has the following two components.

(1) The centripetal or radial component :- which is perpendicular to the velocity of the particle at a given instant.

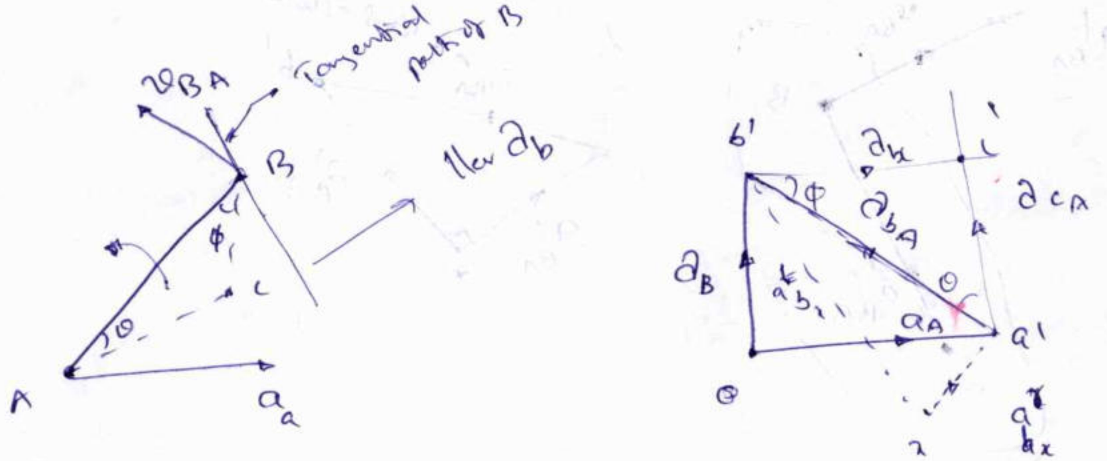
(2) The tangential component, which is parallel to the velocity of the particle at the given instant,

Thus for a link 'AB', the velocity of point 'B' with respect to 'A' (v_{BA}) is perpendicular to the link AB as shown in fig. Since the point 'B' moves with respect to A with an angular velocity of 'ω' rad/sec, therefore centripetal or radial component of the acceleration of 'B' with respect to A.

$a^r_{BA} = \omega^2 \times$ Length of Link AB $= \omega^2 \times AB = \frac{v^2_{(BA)}}{AB}$

$a^t_{BA} = \alpha \times AB$

Acceleration of a point on a Link



$a^r_{bx} = \frac{(v_{AB})^2}{AB}$

$\boxed{a^t_{bx} = \alpha \, AB}$

a_a = Absolute acceleration of point A in both magnitude and direction

a = Absolute acceleration of point B in ~~both~~ only direction

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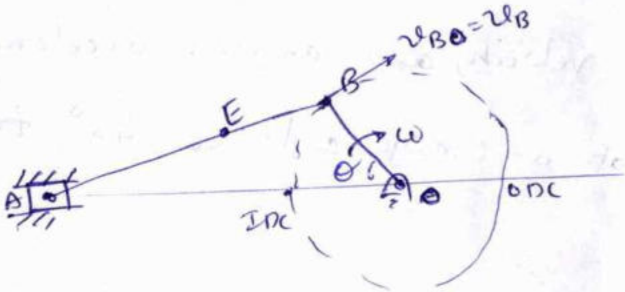
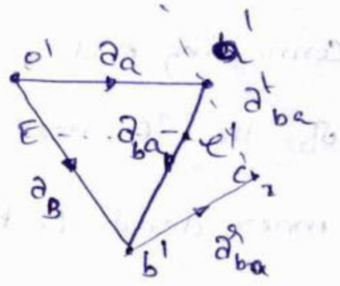
Note: the angular Acceleration of link AB is calculated by a^t_{bx}.

Acceleration in a Slider Crank Mechanism





Acceleration Diagram

(1) $a^r_{BO} = \frac{(v_{BO})^2}{OB} = a_B$

(2) $a^r_{ba} = \frac{(v_{BA})^2}{BA} = a^r_{ba}$

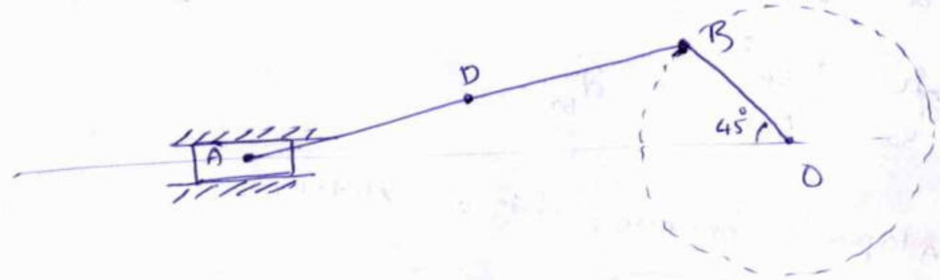
$Ee' = a_e$

(3) a_A is parallel becaus it is a slider.

(4) $\boxed{\alpha_{AB} = \frac{a^t_{ba}}{AB}}$

(P) The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine
1. Linear velocity and acceleration of the midpoint of the connecting rod 2. Angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.

Sol:-



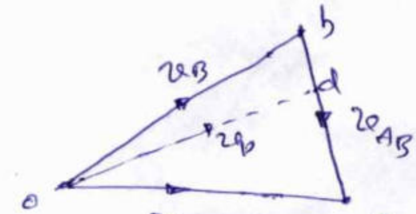
OB = 150 mm ; AB = 600 mm.

N_{OB} = 300 r.p.m ; $\omega_{BO} = \frac{2\pi N_{OB}}{60}$ = 31.42 rad/sec.

$v_{BO} = \omega_{BO} \times OB = 4.713$ m/s

By drawing velocity diagram

v_{BA} = 3.4 m/sec
v_A = 4 m/sec
v_D = 4.1 m/s

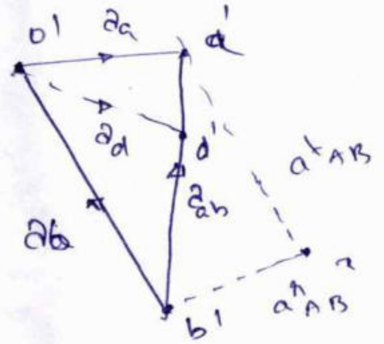


velocity diagram

Acceleration;

$$a^r_{BO} = a_b = \frac{(v_{BO})^2}{OB} = 148.1 \, m/sec^2$$

$$a^r_{AB} = \frac{(v_{AB})^2}{AB} = 19.3 \, m/sec^2$$



Acceleration Diagram

$a_d = 117 \, m/sec^2$

$\omega_{AB} = 5.67 \quad \left(\frac{v_{AB}}{BA} \right)$

$a^t_{AB} = 103 \, m/sec^2$; $\alpha_{AB} = \frac{a^t_{AB}}{BA} = 171.67 \, rad/sec^2$

Selection of a Belt Drive

The following are the various important factors upon which the selection of a belt drive depends.

1. Speed of the driving and driven shaft.
2. Speed reduction ratio.
3. Power to be transmitted,
4. Centre distance between the shafts
5. Positive drive requirements
6. shafts layout
7. Space available
8. Service conditions

Materials

1. Leather 2. Cotton or fabric 3. Rubber belt 4. Balata

Power Transmission (Flat Belt Drives)

① Amount of Power transmitted depends up on the following factors

* The velocity of the belt.
* The Tension under which the belt is placed on the pulley
* The arc of contact between the belt and the smaller pulley.
* The conditions under which the belt is used.

Types of Belt Drives

(1) Light Drives	(2) Meadium Drives	(3) Heavy Drives
(upto 10m/sec)	(10 to 22 m/sec)	(> 22m/sec)
(Agriculture M/c & small machine tools)	(Machine Tools)	(compressors & generators)

Types of Belts

(1) Flat Belts	(2) . Ve Belts	(3) Circular belt or Rope.
(upto 8 m)	(small distances)	(more than 8 m)
(Factories & workshops with moderate power	(same ")	(Factories & work shops for great amount of power.

Types of Flat Belt Drives

(1) Open Belt Drive

(2) Crossed Belt drive (or) Twist belt Drive (to avoid Rubbing the distance between shafts is 20b)

(3) Quarter turn belt Drive with guid pulley (or Right angle belt drive) (Pulley width ≥ 1.4b)

(4) Belt drive with idler pulley or pulleys

(5) Compound Belt Drive.

(6) Stepped or Cone pulley drive

7. Fast and loose pulley drive.

Velocity Ratio of Belt Drive

It is the ratio between the velocities of the driver and the follower (or) driven pulley.

proof:- Let d_1 = diameter of driver

d_2 = diameter of driven.

N_1 = Speed of driver

N_2 = Speed of driven.

Length of belt passes over the driver, in one minute $= \pi d_1 N_1$

Length of belt passes over the follower, in one minute $= \pi d_2 N_2$

$\therefore$ the Length of belt passes over the driver in one minute = Length of belt passes over the follower in one minute

$$\Rightarrow \quad \pi d_1 N_1 = \pi d_2 N_2$$

$$\Rightarrow \quad \frac{N_1}{N_2} = \frac{d_2}{d_1} \quad (\text{or}) \quad \boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}}$$

If the thickness of belt is considered

$$\text{then} \quad \boxed{\frac{N_1}{N_2} = \frac{d_2+t}{d_1+t}} \quad (\text{or}) \quad \boxed{\frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}}$$

(or)

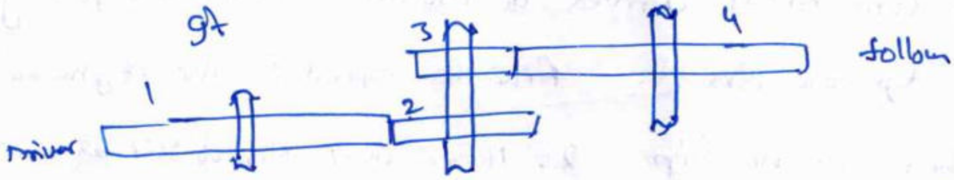
$$v_1 = \frac{\pi d_1 N_1}{60}, \quad v_2 = \frac{\pi d_2 N_2}{60}$$

$$\Rightarrow \quad v_1 = v_2 \Rightarrow \boxed{d_1 N_1 = d_2 N_2}$$

$$\Rightarrow \boxed{\frac{d_1}{d_2} = \frac{N_2}{N_1}}$$

Velocity Ratio of a Compound Belt Drive

$$\text{Velocity Ratio} \Rightarrow \frac{\text{Speed of last driven pulley}}{\text{Speed of first driver.}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameter of followers}}$$



$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \Rightarrow \quad \frac{N_4}{N_3} = \frac{d_3}{d_4}$$

But $\quad N_3 = N_2 \Rightarrow$

$$\Rightarrow \quad N_4 = N_3 \times \frac{d_3}{d_4} = N_4 = N_1 \times \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

$$\Rightarrow \quad \boxed{\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}}$$

Slip of the Belt

πd_1

$\cancel{\frac{N_2}{N_1} = \frac{d_1}{d_2}} \quad \cancel{\frac{\pi d_1 N_1}{} = v -}$

Velocity of belt passing over the driver per second

slip of driver

velocity of belt $\quad v = \frac{\pi d_1 N_1}{60} - \frac{\pi d_1 N_1}{60} \times \frac{S_1}{100} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right)$ — ①

velocity of the belt passing over the follower per second.

slip of follower

$$\frac{\pi d_2 N_2}{60} = v - v \times \frac{S_2}{100} = v\left(1 - \frac{S_2}{100}\right) \quad - ②$$

$\therefore$ Sub ① in ②

$$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right)\left(1 - \frac{S_2}{100}\right)$$

$$\Rightarrow \quad \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S_2}{100} - \frac{S_1}{100} + \frac{S_1 S_2}{100}\right)$$

$$\Rightarrow \quad \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S_1 + S_2}{100}\right) \quad \Bigg| \Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S}{100}\right)$$

S = total % of S = $\%S_1 + \%S_2$

(1) An engine running at 150 r.p.m, drives a lineshaft by means of belt. The engine pulley is 750 mm diameter and the pulley on the lineshaft being 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Find the speed of the dynamo shaft, when 1. There is no slip 2. there is a slip of 2% at each drive

(1500 rpm) ($S_1 = 2$; $S_2 = 2$; (1440 rpm

Creep of Belt

When the belt passes from the slack side to tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side. Due to ~~the~~ these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as creep. The total effect of creep is to reduce slightly the speed of the driven pulley or follower. Considering the creep the velocity ratio is given by

$$\boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(\frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}\right)}$$

where: $\sigma_1 = \sigma_2$ = Stress in the belt on the tight and slack side respectively

E = Young's modulus of the belt material.

(P) The power is transmitted from a pulley 1000 mm diameter running at 200 r.p.m to a pulley 2250 mm diameter by means of a belt. Find the speed lost by the driven pulley as a result of creep, if the stress on the tight and slack side of the belt is 1.4 MPa and 0.5 MPa respectively. The young's modulus of the material of belt is 100 MPa.

consider creep

$$\left(\left(N_2^{(1)} = N_1 \frac{d_1}{d_2} \right) ; \quad N_2^{(2)} = N_1 \frac{d_1}{d_2} \frac{E+\sqrt{\sigma_2}}{E+\sqrt{\sigma_1}} \right)$$

(neglect creep)

$$N_2^{(1)} - N_2^{(2)} = 0.2 \text{ rpm}$$

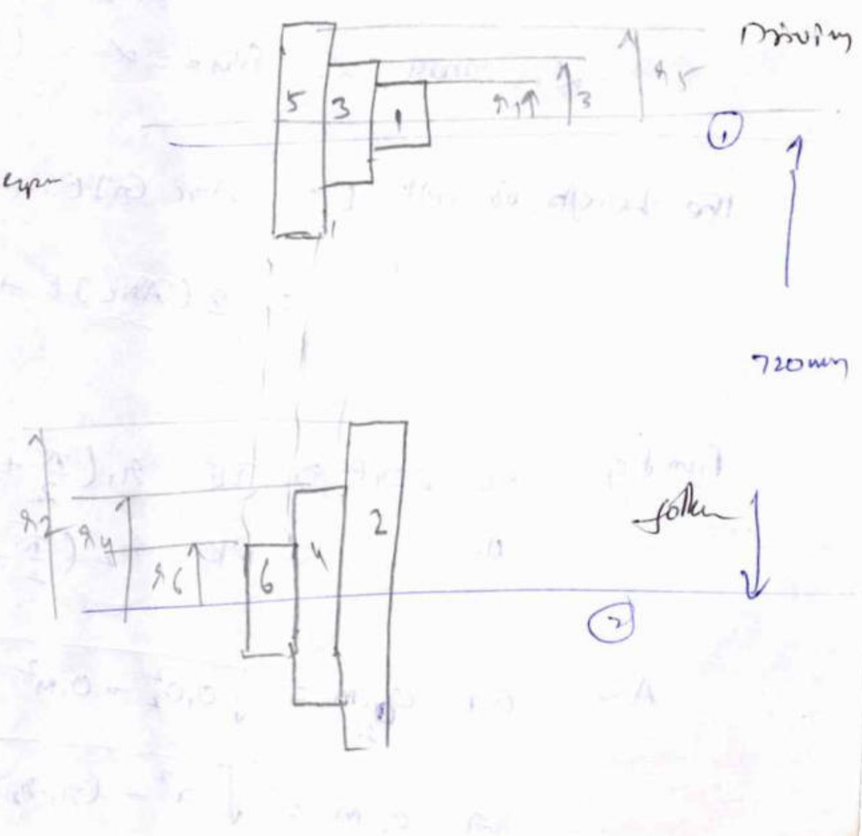
(P) A shaft which rotates at a constant speed of 160 r.p.m is connected by belting to a parallel shaft 720 mm apart, which has to run at 60, 80, and 100 r.p.m. The smallest pulley on the driving shaft is 40 mm in radius. Determine the remaining radii of the stepped pulleys for 1. a crossed belt and 2. an open belt. Neglect belt thickness and slip.

$N_1 = N_3 = N_5 = 160$ r.p.m

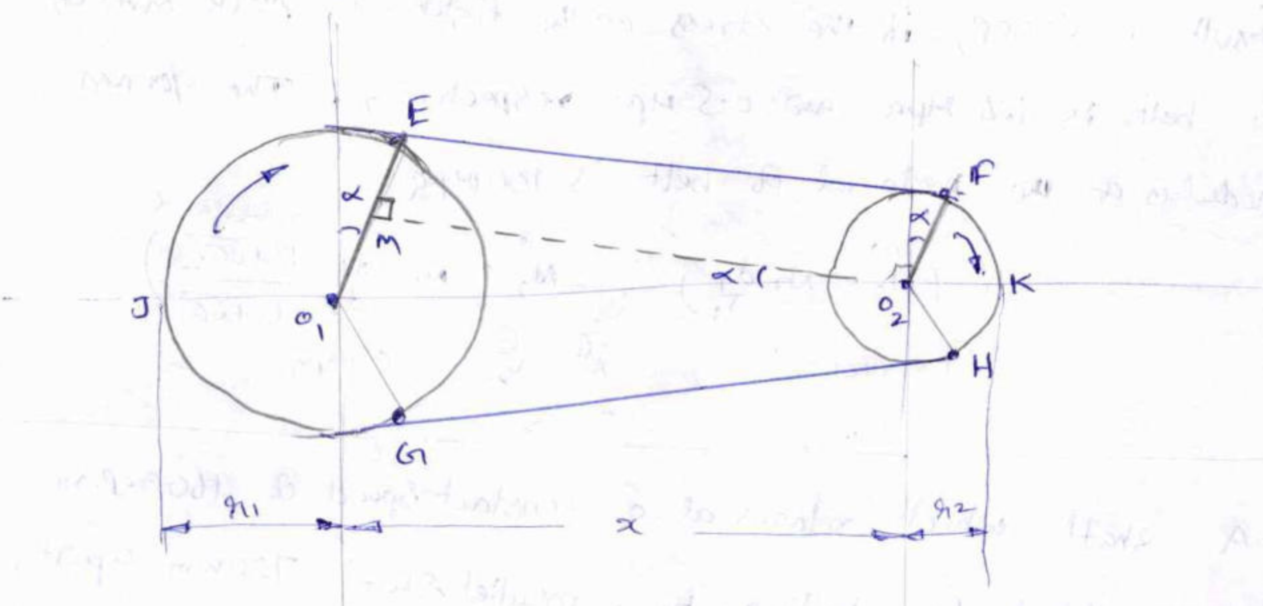
$x = 720$ mm

$N_2 = 60$; $N_4 = 80$; $N_6 = 100$ rpm

$r_1 = 40$ mm



Length of Open Belt Drive



Let $r_1 = r_2 =$ Radii of driver and follower pulleys respectively.

$x =$ Distance between the centres of two pulleys $(O_1 O_2)$

$L =$ Length of the belt.

from $\triangle O_1MO_2$: $\sin\alpha = \dfrac{O_1E - O_1ME}{O_1O_2} = \dfrac{(r_1 - r_2)}{x}$

α is small $\therefore \sin\alpha = \alpha = \dfrac{(r_1 - r_2)}{x}$

The Length of belt $L = \text{Arc } GJE + LF + \text{Arc } FKH + GH$

$= 2(\text{Arc } JE + EF + \text{Arc } FK)$ $\left[\begin{array}{l} \because EF = GH \\ JG = JE \ \& \\ FK = KH \end{array}\right]$

from fig Arc $JO_1E \Rightarrow JE = r_1\left(\frac{\pi}{2} + \alpha\right)$

$\text{||}^{ly} \Rightarrow FK = r_2\left(\frac{\pi}{2} - \alpha\right)$

And $EF = O_2M = \sqrt{O_1O_2^2 - O_1M^2}$

$\Rightarrow O_2M = \sqrt{x^2 - (r_1 - r_2)^2} = x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$

By using Bi-nomial theorem ~~of~~ form $= x\sqrt{1-\left(\frac{1}{x^2}\right)}$

$$= x\left[1-\frac{1}{2}\left(\frac{1}{x^2}\right)+\cdots\right]$$

$$\cancel{= x} \{$$

$$\therefore\ x\sqrt{1-\left(\frac{r_1-r_2}{x}\right)^2} = x\left[1-\frac{1}{2}\frac{(r_1-r_2)^2}{x^2}\right]$$

$$= \boxed{x-\frac{(r_1-r_2)^2}{2x}} = O_2M = EF$$

$\therefore$ The length of belt $L = 2\left[r_1\left(\frac{\pi}{2}+\alpha\right)+x-\frac{(r_1-r_2)^2}{2x}+r_2\left(\frac{\pi}{2}-\alpha\right)\right]$

$$\Rightarrow L = 2\left[\frac{\pi r_1}{2}+\alpha r_1+x-\frac{(r_1-r_2)^2}{2x}+\frac{\pi r_2}{2}-\alpha r_2\right]$$

$$\Rightarrow L = 2\left[\frac{\pi(r_1+r_2)}{2}+x-\frac{(r_1-r_2)^2}{2x}+\alpha(r_1-r_2)\right]$$

$$\Rightarrow L = \pi(r_1+r_2)+2x-\frac{(r_1-r_2)^2}{x}+\frac{(r_1-r_2)}{x}$$

$$\Rightarrow L = 2\left[\frac{\pi(r_1+r_2)}{2}+\alpha(r_1-r_2)+x-\frac{(r_1-r_2)^2}{2x}\right]$$

$$\Rightarrow L = \left[\pi(r_1+r_2)+2\alpha(r_1-r_2)+2x-\frac{(r_1-r_2)^2}{x}\right]$$

$$\Rightarrow L = \left[\pi(r_1+r_2)+2\frac{(r_1-r_2)}{x}(r_1-r_2)+2x-\frac{(r_1-r_2)^2}{x}\right]$$

$$\Rightarrow L = \left[\pi(r_1+r_2)+\frac{2(r_1-r_2)^2}{x}+2x-\frac{(r_1-r_2)^2}{x}\right]$$

$$\Rightarrow \boxed{L = \pi(r_1+r_2)+2x+\frac{(r_1-r_2)^2}{x}}$$

$$\Rightarrow$$

$$\boxed{L = \frac{\pi}{2}(d_1+d_2)+2x+\frac{(d_1-d_2)^2}{4x}}$$

Length of cross-belt Drive

$$L = \pi(r_1+r_2) + 2x + \frac{(r_1+r_2)^2}{x}$$

(or)

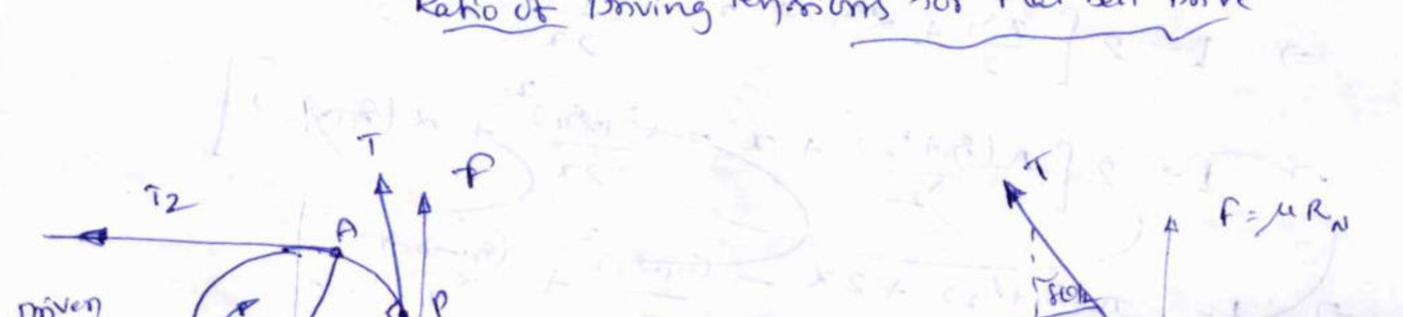
$$\boxed{L = \frac{\pi(d_1+d_2)}{2} + 2x + \frac{(d_1+d_2)^2}{4x}}$$

Power Transmitted by Belt Drive

Power $P = (T_1 - T_2) \times v$ (watt)

Length of –

Ratio of Driving Tensions for flat belt Drive



Let T_1 = Tension in the belt on the tight side

T_2 = Tension in the belt on the Slack side

θ = Angle of Contact in (Radian) (the angle subtend by the arc AB, along which the belt touches the pulley at the center)

Now consider a small portion of the belt PQ, subtended an angle $\delta\theta$ at the centre of the pulley. The belt PQ is in equilibrium under the following forces.

1. Tension 'T' in the belt at P,

2. Tension $(T + \delta T)$ in the belt at Q.

3. Normal reaction R_N and

4. frictional force $f = \mu R_N$

μ = co-efficient of friction between belt & pulley.

Resolving the forces horizontally.

$$T \sin(\delta\theta/2) + (T + \delta T)\sin(\delta\theta/2) = R_N$$

sin $\sin(\delta\theta/2) \approx \delta\theta/2$ [$\because$ $\delta\theta/2$ is small)

$$\Rightarrow T \times \frac{\delta\theta}{2} + T \times \frac{\delta\theta}{2} + \delta T \frac{\delta\theta}{2} = R_N$$

$$\Rightarrow R_N = 2T\frac{\delta\theta}{2} \quad [\because \frac{\delta T \cdot \delta\theta}{2} \text{ is small})$$

$$\Rightarrow \boxed{R_N = T\,\delta\theta} \quad -(1)$$

Resolving the forces vertically

$$T\cos\left(\frac{\delta\theta}{2}\right) + \mu R_N = T + \delta T \cos\frac{\delta\theta}{2}$$

for small angles $\cos\left(\frac{\delta\theta}{2}\right) = 1$

$$\therefore \quad T + \mu R_N = T + \delta T$$

$$\Rightarrow \boxed{R_N = \frac{\delta T}{\mu}} \quad -(2)$$

(1) = (2) $\Rightarrow$ $T\,\delta\theta = \frac{\delta T}{\mu}$

$$\mu\,\delta\theta = \frac{\delta T}{T}$$

Integrating the both sides w.r.t.

$$\Rightarrow \int_0^{\theta} \mu \, d\theta = \int_{T_2}^{T_1} \frac{dT}{T}$$

$$\Rightarrow \mu [\theta]_0^{\theta} = [\log T]_{T_2}^{T_1}$$

$$\Rightarrow \mu (\theta - 0) = \log_e T_1 - \log_e T_2$$

$$\Rightarrow \mu\theta = \log_e (T_1/T_2)$$

$$\Rightarrow \boxed{\frac{T_1}{T_2} = e^{\mu\theta}}$$

Note:- If the logarithm expressed in terms of base 10

$$\boxed{2.3 \log (T_1/T_2) = \mu\theta}$$

Determination of Angle of contact

when the two pulleys of different diameters are connected by means of an open belt as shown in fig. then the angle of contact & lap (θ) at smaller pulley b must be taken into consideration

Let r_1 = Radius of ~~smaller~~ larger pulley

r_2 = Radius of smaller pulley

x - Distance between the centers of two pulley.

for open Belt Drive $\theta = (180 - 2\alpha) \frac{\pi}{180^\circ}$

where $\alpha = \sin^{-1} \left(\frac{r_1 - r_2}{x} \right)$

for cross Belt Drive $\theta = (180 + 2\alpha) \frac{\pi}{180^\circ}$

where $\alpha = \sin^{-1} \left(\frac{r_1 + r_2}{x} \right)$

(P) A casting weighing 9KN hangs freely from a rope which makes 2.5 turns round a drum of 300 mm diameter revolving at 20r.p.m. The other end of the rope is pulled by a man. The co-efficient of friction is 0.25. Determine 1. The force required by the man (T_2) (2) Power to raise the casting

$$\left(\theta = 2.5 \times 2\pi = 5\pi \text{ rad}; \quad T_2 = 176.47\ \text{N}. \quad P = 2.772\ \text{kW} \right)$$

(P) Two pulleys, one 450mm diameter and other 200mm diameter are on parallel shafts 1.95 m apart and are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley.

What power can be transmitted by the belt when the larger pulley rotates at 200rev/min, if the maximum permissible tension in the belt is 1KN, and the co-efficient of friction between the belt and pulley is 0.25?

(P) A shaft rotating at 200 rpm, drives another shaft at 300r.p.m and transmits 6KW through a belt. The belt is 100 mm wide and 10 mm thick. The distance between the shafts is 4 m. The smaller pulley is 0.5 m in diameter. Calculate the stress in the belt, if it is 1. An open belt drive 2. a cross-belt drive. Take $\mu = 0.3$

(P) Find the Power transmitted by a belt running over a pulley of 600mm diameter at 200 r.p.m. The co-efficient of friction between the belt and the pulley is 0.25, angle of lap 160° and the maximum Tension in the belt is 2500 N.

(Ans P = 7690W = 7.69kW

(P) A leather belt is required to transmit 7.5 KW from a pulley 1.2 m in diameter, running at 250 r.p.m. The angle embraced is 165° and the co-efficient of friction between the belt and the pulley is 0.3. If the safe working stress for the leather belt is 1.5 MPa, density of leather is 1 Mg/m3 and Thickness of belt is 10mm, Determine the width of the belt taking centrifugal tension into account. ($T_{max} = T_1 + T_C$; $T_m = \sigma \times b \times t$)

$T_C = m v^2$ $\boxed{m = \rho \times A \times l}$

$\boxed{b = 65.8\ mm}$

$T_C = 24.68b$ $T_1 = 824.6$ $\boxed{m = 10b}$

$T_{max} = 1500b$

(P) Determine the width of a 9.75mm thick leather belt required to transmit 15 KW from a motor running at 900 r.p.m. The diameter of the driving pulley of the motor is 300mm. The driven pulley runs at 300 r.p.m and the distance between the centre of two pulleys is 3 meters. The density of leather is 1000 Kg/m3. The maximum allowable stress in the leather is 2.5 MPa. The co-efficient of friction between the leather and pulley is 0.3. Assume open belt drive and neglect the sag and slip of the belt. (b = 80mm)

($\frac{N_2}{N_1} = \frac{d_1}{d_2}$)

$d_2 = 0.9\ m$

$\theta = 180 - 2\alpha$ $\alpha = \sin^{-1}\left(\frac{r_2 - r_1}{x}\right)$

Max Tension in belt

$T =$ max stress $\times$ Area

$T = \sigma \times b \times t$

$T_G = T = T_1 + T_C$

(b) An open belt drive connects two pulleys 1.2 m and 0.5 m diameter, on parallel shafts 4 meters apart. The mass of the belt is 0.9 kg/m length and the maximum tension is not to exceed 2000 N. The co-efficient of friction is 0.3. The 1.2 m pulley, which is the driver, runs at 200 r.p.m. Calculate the torque on each of the two shafts, the power transmitted, $P(T_1-T_2)v$ (13.78 kW) and power lost in friction. What is the efficiency of the drive. (η)

$P_I - P_o$ (0.87 kW)

$$\left[P_I = \frac{2\pi N_1 T_D}{60} \; (13.78\,kW) ; \quad P_o = \frac{2\pi N_2 T_{follo}}{60} \; (12.91\,kW) \right]$$

$$\eta = \frac{P_I - P_o}{P_I} \times 100 = 93.7\%$$

$T_D = (T_1 - T_2)\, r_d$ (657.6 N-m)

$T_f = (T_1 - T_2)\, r_f$ (274 N-m)

Initial Tension

(P) In a flat belt drive, the initial tension is 2000 N. The co-efficient of friction between the belt and the pulley is 0.3 and the angle of lap on the smaller pulley is 150°. The smaller pulley has a radius of 200 mm and rotates at 500 r.p.m. Find the power in kW transmitted by the belt.
An (15.7 kW)

$T_0 = \frac{T_1 + T_2}{2}$

$\frac{T_1}{T_2} = e^{\mu\theta}$

$T_1 - T_2)v$

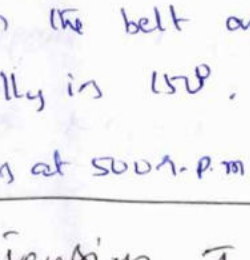
Max tension $T_m = T_{t_1} + T_C$

Centrifugal Tension

$m_b = m r\, d\theta$

$$F_C = \frac{m_b v^2}{r} = m r\, d\theta \cdot \frac{v^2}{r}$$

$$F_C = m\, d\theta\, v^2$$



$$F_C = T_C \sin\frac{d\theta}{2} + T_C \sin\frac{d\theta}{2}$$

$$m\, d\theta\, v^2 = 2T_C \frac{d\theta}{2} \Rightarrow \boxed{T_C = mv^2}$$

m = mass per unit length (kg)

(T_C acts on both sides)

$v =$

$r =$

$T_C =$ on both.

length of belt $PQ = r\, d\theta$

mass of belt $m = m r\, d\theta$.

$F_C = \frac{mv^2}{r}$

$\boxed{T_{max} = T_1 + T_C}$

Condition for Max Power Transmission

$$P = (T_1 - T_2)\, v = T_1\left(1 - \frac{1}{e^{\mu\theta}}\right) v$$

$\frac{dP}{dv} = 0$,

$$P = T_1 C\, v = (T_{m} - T_C)\, C\, v$$

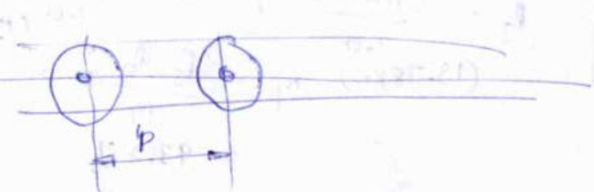
$$P = (T_m - mv^2)\, C\, v\, ;$$

$\boxed{T_{max} = \frac{T_C}{3}}$

CHAIN DRIVES

Terms Used in the Chain Drive

(1) Pitch of the chain:- It is the distance between the hinge Centre of a link and the corresponding hinge centre of the adjacent link. It is usually denoted by 'p'.



(2) Pitch circle diameter of the chain sprocket:- It is the diameter of the circle on which the hinge centres of the chain lie, when the chain is wrapped round a sprocket as shown in fig.

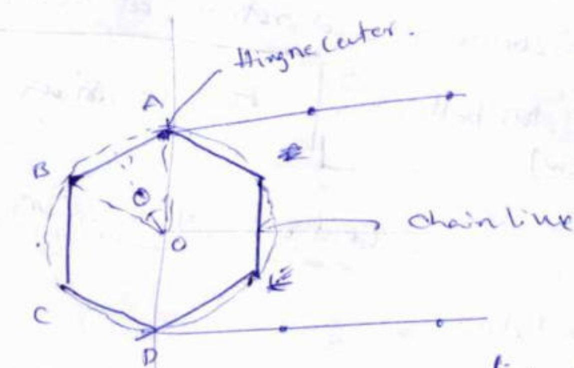


fig ①

~~From~~ the points A, B, C, D are the hinge centers of ~~Sm~~ the chain and the circle drawn through these centers is called pitch circle and its diameter is known as pitch circle diameter.

Relation between the Pitch and pitch circle diameter

from the fig(1) the pitch length is a chord AB. consider the pitch length 'AB' of the chain subtending an angle 'θ' at the center of the sprocket.

Let d = Diameter of the pitch circle

T = Number of teeth on the sprocket

from fig:

$$\sin \theta/2 = \left(\frac{AB}{2}\right)/OA$$

$$\Rightarrow \quad AB = 2\,OA \sin(\theta/2)$$

$$\therefore \quad \text{Pitch } \boxed{(p) = d \times \sin \theta/2} \qquad \left[\because OA = r\ ;\ 2OA = d\right]$$

(or)

$$\text{Pitch circle diameter } \boxed{d = p \times \operatorname{cosec}(\theta/2)}$$

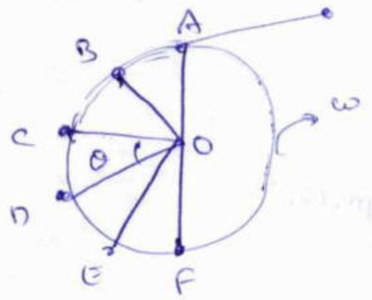
if T = No. of Teeth on the sprocket

$$\therefore \ ~~at~~ \text{ angle } \boxed{\theta = \frac{360^\circ}{T}}$$

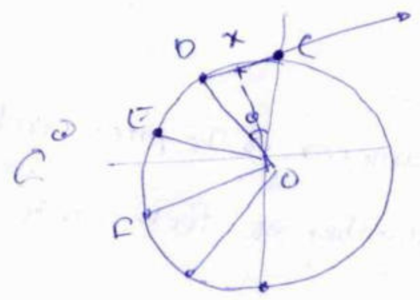
$$\therefore \quad p = d \sin\left(\frac{360}{2T}\right) \Rightarrow \boxed{p = d \sin\left(\frac{180}{T}\right)} \ *$$

$$\text{(h)} \quad \boxed{d = p \times \operatorname{cosec}\left(\frac{180}{T}\right)}$$

Relation between the Chain Speed and Angular Velocity of Sprocket



(a)



(b)

$\cos\theta/2 = \frac{OX}{OC}$

For angular position of sprocket as shown in fig (a)

$\boxed{v = \omega \cdot OA}$

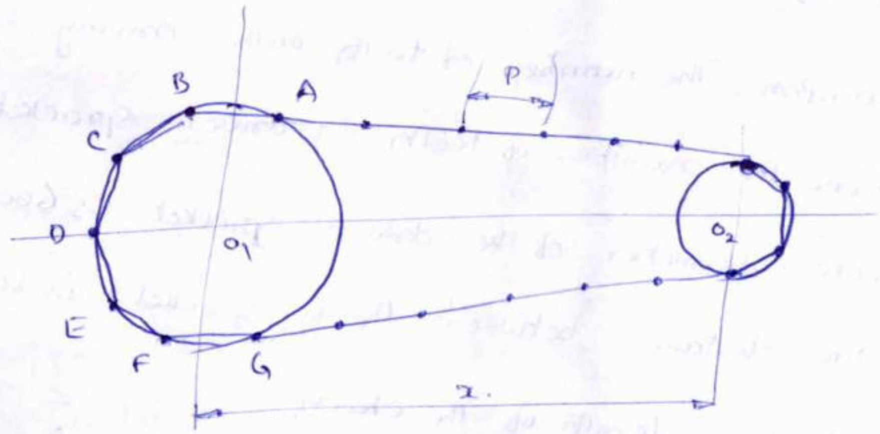
For angular position of sprocket as shown in fig (b)

$v = \omega \times OX \Rightarrow v = \omega \times OC \times \cos\theta/2$

[$\because OA = OC$]

$\Rightarrow \boxed{v = \omega \times OA \cos(\theta/2)}$

Length of chain



from :- Length of open belt drive formula

$$L = \pi(r_1+r_2) + 2x + \frac{(r_1-r_2)^2}{x}$$

from $p_1 = d_1 \sin\left(\frac{180}{T_1}\right)$ | $p_2 = p_1 = d_2 \sin\left(\frac{180}{T_2}\right)$

$r_1 = \frac{p}{2}\operatorname{cosec}\left(\frac{180}{T_1}\right)$ | $r_2 = \frac{p}{2}\operatorname{cosec}\left(\frac{180}{T_2}\right)$

$$\therefore L = \pi\left[\frac{p}{2}\operatorname{cosec}\left(\frac{180}{T_1}\right) + \frac{p}{2}\operatorname{cosec}\left(\frac{180}{T_2}\right)\right] + 2x + \frac{\left[\left(\frac{p}{2}\operatorname{cosec}\frac{180}{T_1}\right) - \left(\frac{p}{2}\operatorname{cosec}\frac{180}{T_2}\right)\right]^2}{x}$$

$$\Rightarrow L = \left[\frac{p}{2}(T_1+T_2) + 2x + \frac{p^2}{4x}\left(\operatorname{cosec}\left(\frac{180}{T_1}\right) - \operatorname{cosec}\left(\frac{180}{2}\right)\right)^2\right]$$

$$\Rightarrow L = p\left[\frac{(T_1+T_2)}{2} + \frac{2x}{p} + \frac{p\left(\operatorname{cosec}\left(\frac{180}{T_1}\right) - \operatorname{cosec}\left(\frac{180}{T_2}\right)\right)^2}{4x}\right]$$

$\Rightarrow$ $\boxed{\text{If } x = mp \ \& \ \frac{x}{p} = m}$

then
$$L = p\left[\frac{(T_1+T_2)}{2} + 2m + \frac{\left(\operatorname{cosec}\left(\frac{180}{T_1}\right) - \operatorname{cosec}\left(\frac{180}{T_2}\right)\right)^2}{4m}\right]$$

$$\boxed{L = p \cdot K}$$

where K = Multiplying factor $= \left[\left(\frac{T_1+T_2}{2}\right) + 2m + \frac{\left(\operatorname{cosec}\left(\frac{180}{T_1}\right) - \operatorname{cosec}\left(\frac{180}{T_2}\right)\right)^2}{4m}\right]$

(P) A chain drive is used for reduction of speed from 240 r.p.m to 120 r.p.m. The number of teeth on the driving sprocket is 20. Find the number of teeth on driven sprocket. If the pitch circle diameter of the driven sprocket is 600 mm and center to centre distance between the two sprockets is 800 mm determine the pitch and length of the chain.

A (Hint:- $\frac{N_1}{N_2} = \frac{T_2}{T_1}$, $d_1 = P_1 \operatorname{cosec}\left(\frac{180}{T_1}\right)$)

$T_2 = 60$; $p = 47.1\,mm$

$\boxed{L = 3.0615\ m\ ;\quad K = 64.86 \simeq 65}$

-V- Belt

$$\frac{T_1}{T_2} = e^{\frac{\mu\theta}{\sin\beta}}$$

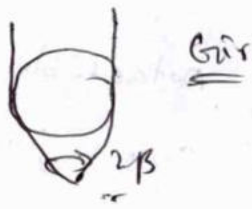
(P) A compressor, requiring 90 kW is to run at about 250 r.p.m. The drive is by -V- belts from an electric motor running at 750 r.p.m. The diameter of the pulley on the compressor shaft must not be greater than 1 meter while the centre distance between the pulleys is limited to 1.75 meter. The belt speed should not exceed 1600 m/min.

Determine the number of -V- belts required to transmit the power if each belt has a cross-sectional area of 375 mm², density 1000 kg/m³ and an allowable tensile stress of 2.5 Mpa. The groove angle of the pulley is 35°. The co-efficient of friction between the belt and the pulley is 0.25. Calculate also the length required of each belt.

(no. of belts $n = 5.6 \simeq 6$; $L_0 = 5.664\,m$

Rope Drives

Types of Rope drives



(1) fibre ropes (upto 60 m) (2) wire Ropes (upto 150 m)

Ratio of belt Tension $\frac{T_1}{T_2} = e^{\frac{\mu\theta}{\sin\beta}}$

2β = Groove angle

β = Semigroove angle.

(P). A rope drive transmitts 600 kW from a pulley of effective diameter 4 m, which runs at a speed of 90 r.p.m. The angle of lap is 160. the angle of groove is 45°. the co-efficient of friction 0.28. the mass of rope 1.5 kg/m and the allowable tension in each rope (T_{max}) 2400 N. find the number of ropes required.

$(v = \frac{\pi d N}{60} = 18.85$ m/s ; $T_c = 533$ N

$T_1 = T_{max} - T_c$

no. of ropes = $\frac{\text{Total Power of } \cancel{\text{each rope}}}{\text{Power of each rope}} = \frac{600}{30.69} \approx 19.58 \approx 20$

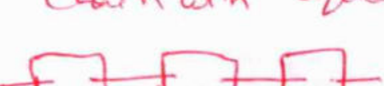
Classification of chains

1. Hoisting and hauling chains (or Crane chains)
2. Conveyor Chains (or Tractive ~~tra~~ chains)
3. Power Transmitting chains (or Driving chains)

(1) Hoisting and hauling chains

chain with oval link

Chain with Square links



Conveyor chains

(1) Detachable & hook joint type chain

(2) Closed joint type chain

Power Transmitting Chains

1. Block chain
2. Bush Roller chain
3. Inverted tooth & Silent chain

Additional Problems

(P) A pulley is driven by a flat belt, the angle of lap being 120°. The belt is 100 mm wide by 6mm thick and density 1000 kg/m³. If the co-efficient of friction is 0.3 and the maximum stress in the belt is not to exceed 2MPa, find the greatest power which the belt can transmit and the corresponding speed of the belt.

[Ans: $T_{max} = 1200N$; $m = 0.6 kg/m$; $v = 25.82 m/sec$; $T_c = \frac{T_{max}}{3} = 400N$
$T_1 = T_{max} - T_c = 800N$; $T_2 = 425.5 N$; $P = (T_1 - T_2) \times v = 9.67 kW$]

Coriolli's Component of Acceleration

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism, then the coriolis component of acceleration must be calculated.

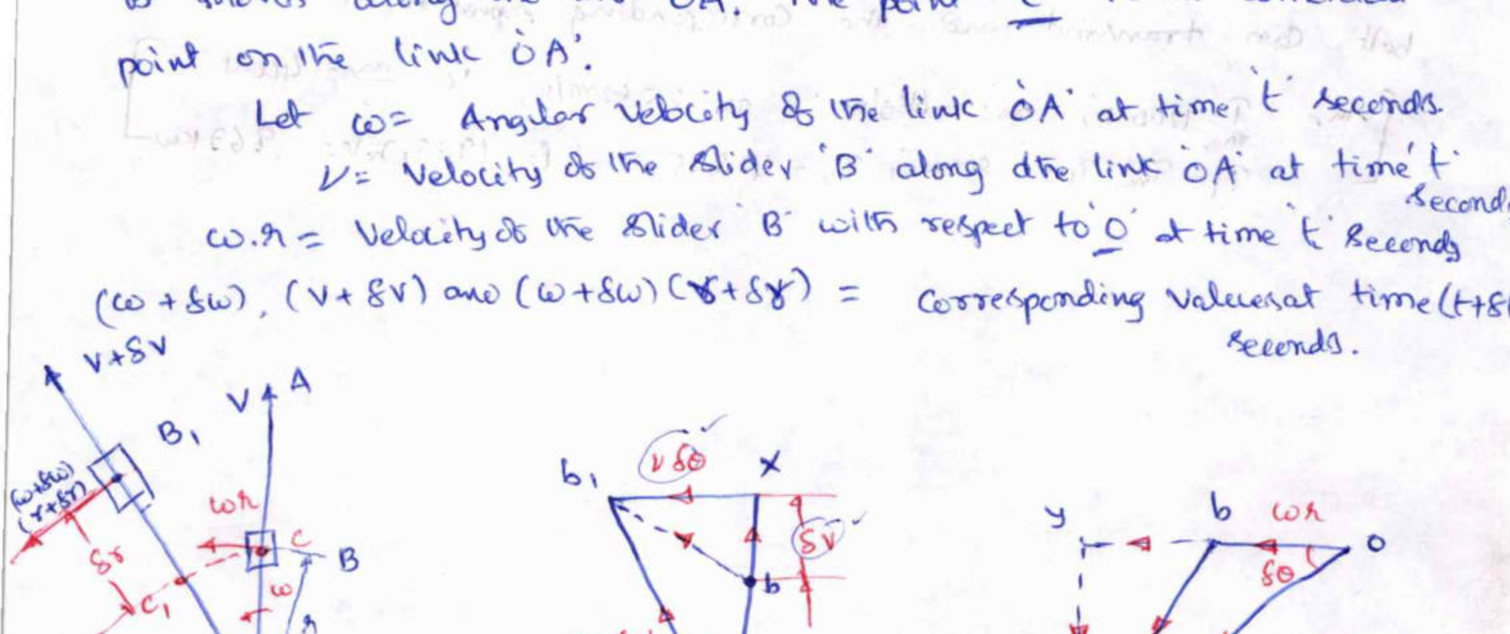
Consider a link 'OA' and a slider 'B' as shown in fig. The slider 'B' moves along the link 'OA'. The point 'C' is the coincident point on the link 'OA'.

Let ω = Angular velocity of the link 'OA' at time 't' seconds.

V = Velocity of the slider 'B' along the link 'OA' at time 't' seconds

$\omega . r$ = Velocity of the slider 'B' with respect to 'O' at time 't' seconds

$(\omega+\delta\omega)$, $(V+\delta V)$ and $(\omega+\delta\omega)(r+\delta r)$ = Corresponding values at time $(t+\delta t)$ seconds.



(a) (b) (c)

Fig: Coriolis Component of Acceleration.

Let us now find out the acceleration of the slider 'B' with respect to O and with respect to its coincide point 'C' lying on the link OA.

from Fig (b): the vector 'bb_1' represents the change in velocity in time δt sec. the vector bx represents the component of change of velocity bb_1 ~~in the~~ along direction ~~perpendicular~~ OA (Radial directn) and the vector 'xb_1' its component in the direction perpendicular to OA (i.e Tangential direction).

$\therefore\ bx = ox - ob = (v+\delta v)\cos\delta\theta - (v) \approx$ (Acting radially outward)

lly $= (v+\delta v) - (v) = \delta v$ ($\because \cos\delta\theta = 1$ small angle)

lly $\therefore\ xb_1 = (v+\delta v)\sin\delta\theta = (v+\delta v)\delta\theta$ [$\because$ small angle $\sin\theta = \theta$]

$xb_1 = v\delta\theta + \delta v\delta\theta \approx v\delta\theta$ [$\because$ product of small x small = very small its value] from fig (a) $\quad yb_1 = (\omega+\delta\omega)(r+\delta r)\sin\delta\theta \downarrow$

$= [\omega r + \omega\delta r + r\delta\omega + \delta r\delta\omega] \times \delta\theta$ [small ∠ $\sin\delta\theta = \delta\theta$]

$= \omega r\delta\theta + \omega\delta r\delta\theta + r\delta\omega\delta\theta + \delta r\delta\omega\delta\theta \downarrow$

$= \omega r\delta\theta \downarrow$ [∵ product of small quantities is neglected] [Acting radially inward]

$by = oy - ob = (\omega+\delta\omega)(r+\delta r)\cos\delta\theta - \omega r$ [small ∠ $\cos\delta\theta = 1$]

$= \cancel{\omega r} + \omega\delta r + r\delta\omega + \delta\omega\delta r - \cancel{\omega r}$

$= r\delta\omega + \omega\delta r$

∴ Total component of change of velocity along Radial direction

$= bx - yb_1 = (\delta v - \omega r\delta\theta)\uparrow$

∴ Radial component of the Acceleration of slider 'B' with respect to 'O' on the link 'OA' acting Radially outward from 'O' to 'A'.

$$a^r_{BO} = \lim_{\delta t\to 0}\frac{(\delta v - \omega r\delta\theta)}{\delta t} = \frac{dv}{dt} - \omega r\frac{d\theta}{dt} = \frac{dv}{dt} - \omega^2 r \uparrow$$

∴ Also, the Total component of change of velocity along Tangential direction

$= xb_1 + by = v\delta\theta + r\delta\omega + \omega\delta r$

∴ Tangential component of the acceleration of the slider 'B' with respect 'O' on the link 'OA' acting perpendicular to 'OA' and towards left.

$$a^t_{BO} = \lim_{\delta t\to 0}\frac{(v\delta\theta + r\delta\omega + \omega\delta r)}{\delta t} = v\frac{d\theta}{dt} + r\frac{d\omega}{dt} + \omega\frac{dr}{dt} = v\omega + r\alpha + \omega v$$

$$\Rightarrow \boxed{a^t_{BO} = 2v\omega + r\alpha}$$

Now:- the Radial component of acceleration of the coincident point 'C' with respect to 'O' acting in the direction from 'C' to 'O'

$$a^r_{CO} = \omega^2 r \downarrow$$

the Tangential Component of the Slider 'B' with respect to 'O', acting in the direction perpendicular to 'CO' and towards left

$$a^t_{CO} = \overleftarrow{\alpha r}$$

∴ The Radial component of the Slider 'B' with respect to the coincident point 'C' on the link 'OA' acting radially outwards

$$a^r_{BC} = a^r_{BO} + a^r_{CO} = \left(\frac{dv}{dt} - \omega^2 r\right) + (\omega^2 r)$$ [(−)ve Resultant is opposite]

$$\boxed{a^r_{BC} = \frac{dv}{dt} \uparrow}$$

∴ The Tangential Component of the Slider 'B' with respect to the coincident point 'C' on the link 'OA' acting in the direction perpendicular to 'OA' and towards left.

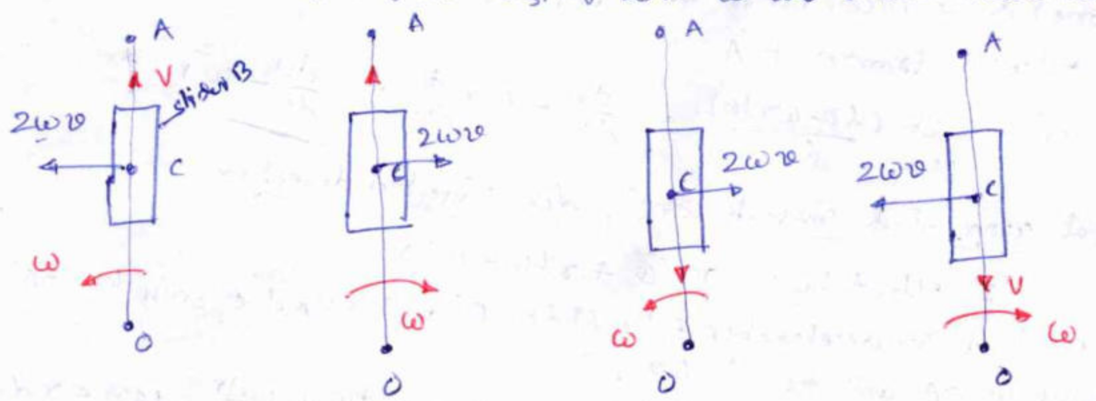
$$a^t_{BC} = a^t_{BO} - a^t_{CO} = (2\omega v + \alpha r) - \alpha r = \overleftarrow{2\omega v}$$

∴ This Tangential Component of acceleration of the slider 'B' with respect to the coincident point 'C' on the link is known as "Coriollis Component of Acceleration" and is always perpendicular to the link.

∴ Coriollis component of the acceleration of 'B' with respect to 'C'

$$\boxed{a^c_{BC} = a^t_{BC} = 2\omega v}$$

Note: In the above discussion, the anticlockwise direction for ω and radially outward direction for 'v' are taken as positive. It may be noted that the direction of coriollis component of acceleration changes sign, if either 'ω' or 'v' is reversed in the direction. But if the coriollis component component of acceleration will not be changed if the sign of both 'ω' and 'v' are reversed in the direction.



Precessional Motion and Gyroscopic Couple

When a body moves along a curved path with a uniform linear velocity, a force in the direction of centripetal acceleration (known as centripetal force) has to be applied externally over the body, so that it moves along the required curved path. This external force applied is known as active force.

When a body itself is moving with uniform linear velocity along a curved path, it is subjected to the centrifugal force radially outwards. This centrifugal force is called reactive force. The action of reactive & centrifugal force is to tilt & move the body along radially outward direction.

* Note:- Centrifugal force is equal in magnitude to centripetal force but opposite in direction.

Precessional Angular motion

We know that the angular acceleration is the rate of change of angular velocity with respect to time. It is a vector quantity and may be represented by drawing a vector diagram with the help of a right hand screw rule.

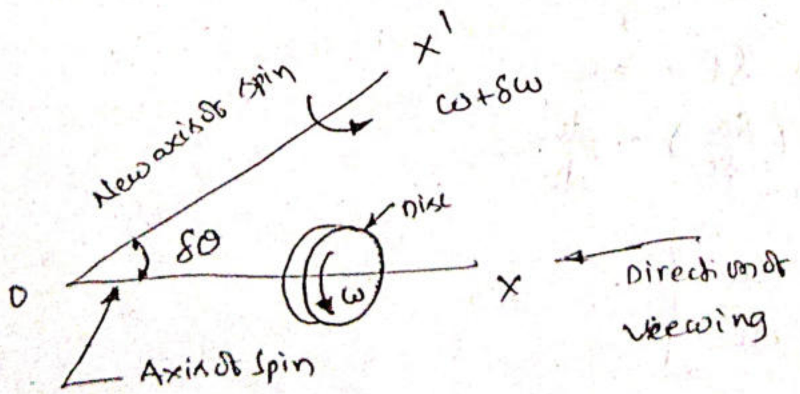


fig (a)

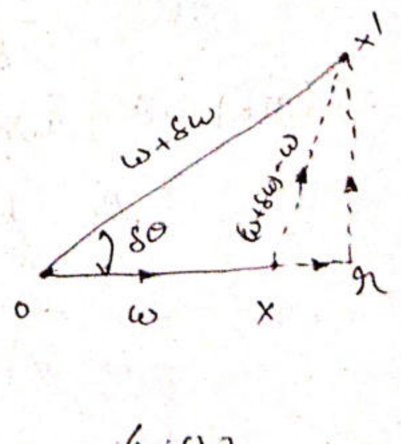


fig (b)

Consider a disc as shown in Fig (a). revolving & spinning about the axis $\underline{OX}$' (known as axis of spin) in anti clockwise when seen from the front, with an angular velocity ω' in plane at right angles to the paper.

After a short interval of time 'δt', let the disc be spinning, about the new axis of spin OX' (at an angle $\delta\theta$) with an angular velocity $(\omega + \delta\omega)$. Using the right hand screw rule, the initial angular velocity of the disc (ω) is represented by vector ox; and the final angular velocity of the disc $(\omega+\delta\omega)$ is represented by vector ox' as shown in fig. (b). The vector 'xx'' represents the change of angular velocity in time 'δt', that is the angular acceleration of the disc. This may be resolved into two components, one is parallel to 'ox' and the other is perpendicular to the 'ox'.

⇒ Component of angular acceleration in the direction of 'ox',

$$\alpha_t = \frac{xr}{\delta t} = \frac{or - ox}{\delta t} = \frac{ox' \cos\delta\theta - ox}{\delta t}$$

$$\alpha_t = \frac{(\omega+\delta\omega)\cos\delta\theta - \omega}{\delta t}$$

Since '$\delta\theta$' is very small, $\therefore \cos\delta\theta = 1$ then

$$\alpha_t = \frac{(\omega+\delta\omega) - \omega}{\delta t} = \frac{\delta\omega}{\delta t}$$

in the limit, when $\delta t \to 0$

$$\alpha_t = \underset{\delta t \to 0}{Lt}\left(\frac{\delta\omega}{\delta t}\right) = \frac{d\omega}{dt}$$

$$\boxed{\therefore \alpha_t = \frac{d\omega}{dt}}$$

Component of angular acceleration in the perpendicular to $\underline{ox'}$.

$$\alpha_c = \frac{xx'}{\delta t} = \frac{ox' \sin\delta\theta}{\delta t} = \frac{(\omega+\delta\omega)\sin\delta\theta}{\delta t}$$

Sinse $\delta\theta$ is very small $\sin\delta\theta = \delta\theta$

$$\therefore \alpha_c = \frac{(\omega+\delta\omega)\delta\theta}{\delta t} = \frac{\omega \times \delta\theta + \delta\omega\delta\theta}{\delta t}$$

product of $\delta\omega \times \delta\theta$ is very small and it is neglected

then $\alpha_c = \dfrac{\omega \times \delta\theta}{\delta t}$

In the Limit when $\delta t \to 0$; $\alpha_t = \lim_{\delta t \to 0} \dfrac{\omega \times \delta\theta}{\delta t} = \omega \times \dfrac{d\theta}{dt}$

$$\Rightarrow \boxed{\alpha_c = \omega \times \frac{d\theta}{dt}}$$

$$= \boxed{\alpha_c = \omega \times \omega_p}$$

where $\omega_p = \dfrac{d\theta}{dt}$ = Angular velocity of precession

Total Angular Acceleration of the disc = Vector xx'

$\therefore$

α = Vector sum of $\alpha_t + \alpha_c$

$$\Rightarrow \boxed{\alpha = \frac{d\omega}{dt} + \omega \times \frac{d\theta}{dt} = \frac{d\omega}{dt} + \omega\omega_p}$$

the $\dfrac{d\theta}{dt}$ is called angular velocity of precission and it is the angular velocity of axis of spin about a certain axis, which is perpendicular to the plane in which the axis of spin is going to rotate. The axis about which the axis of spin is to turn, is known as axis of precession.

"The Angular motion of the axis of spin about the axis of precession is known as precissional angular motion"

* The angular acceleration α_c' is known as gyroscopic Acceleration

radial compont of

Gyroscopic Couple

Consider a disc spinning with an angular velocity 'ω' rad/sec in anticlockwise direction when about the axis of spin OX, in anticlockwise direction when seen from the front as shown in fig (a). Since the plane in which the disc is rotating is parallel to the plane 'YOZ', therefore it is called 'plane of spinning'. The plane 'XOZ' is a horizontal plane and the axis of spin rotates in a plane parallel to the horizontal plane about an axis 'OY'. In other words, the axis of spin is said to be rotating or pressing about an axis OY. In other words, the axis of spin is said to be rotating or processing about an axis 'OY' (which is perpendicular to both the axes 'OX' and 'OZ') at an angular velocity ω_p rad/sec. This horizontal plane 'XOZ' is called plane of precession and OY is called the axis of precession.

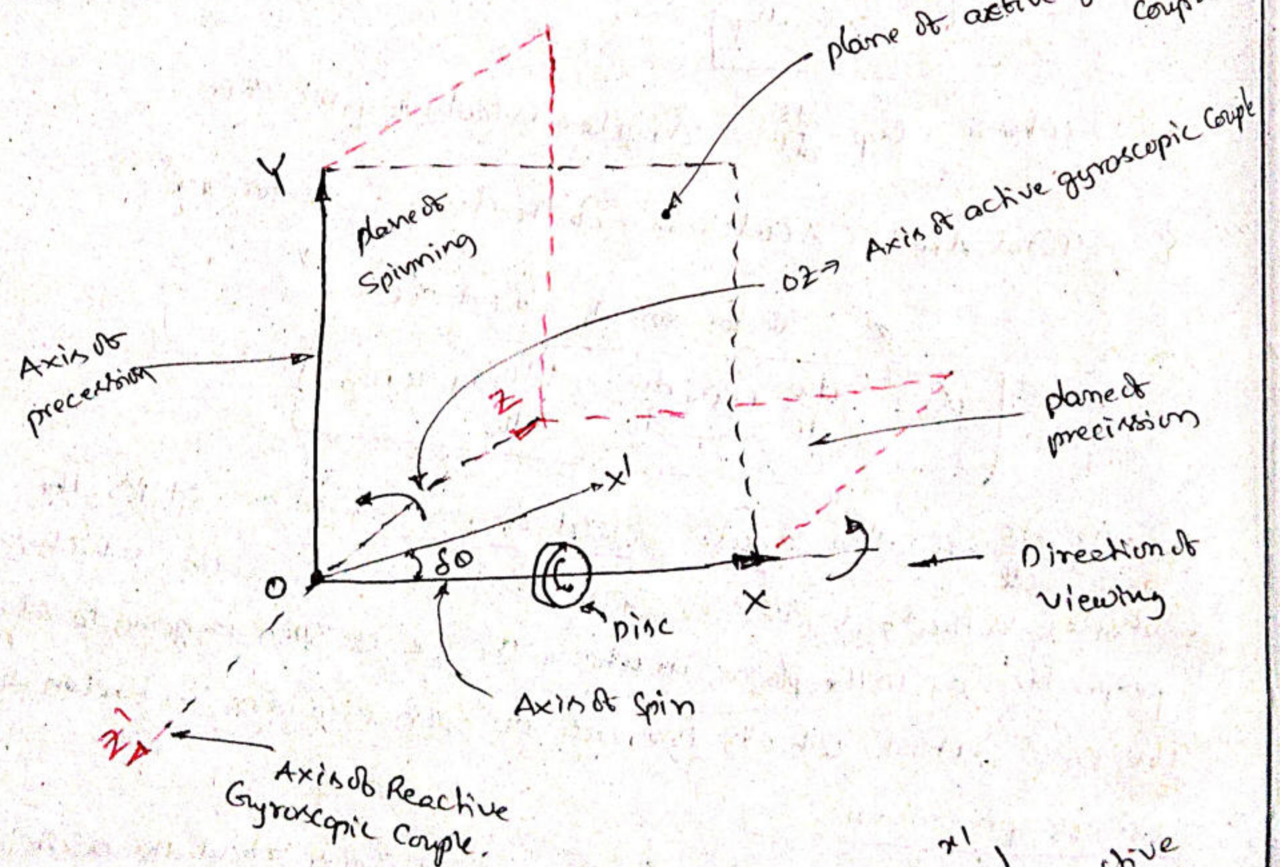


fig (a)

(Gyroscopic couple)

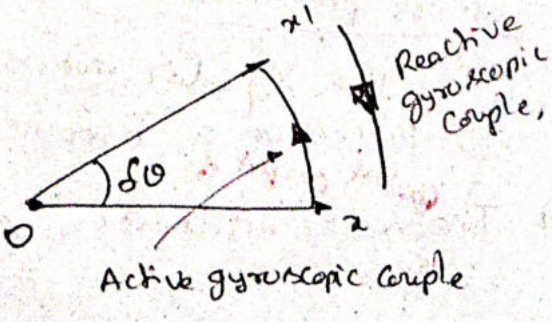


fig (b)

Let I = Mass moment of Inertia of the disc about OX;

ω = Angular velocity of the disc.

$\therefore$ Angular momentum of the disc $= \underline{\underline{I\omega}}$

Since the angular momentum is a vector quantity, therefore it may be represented by the vector $\overrightarrow{ox}$ as shown in fig (b).
The axis of spin 'OX' is also rotating anticlockwise when seen from the top about the axis 'OY'. Let the axis 'OX' is turned in the plane 'XOZ' through a small angle '$\delta\theta$' radians to the position OX', in time 'δt' seconds. Assuming the angular velocity 'ω' to be constant, the angular momentum will now be represented by vector ox'.

$$\therefore \text{ Change in angular momentum } = \overrightarrow{ox'} - \overrightarrow{ox} = \overrightarrow{xx'}$$

$$= \overrightarrow{ox} \cdot \delta\theta$$

$$= I \cdot \omega \cdot \delta\theta$$

$$\text{Rate of change of angular momentum } = I \cdot \omega \cdot \frac{\delta\theta}{dt}$$

Since the rate of change of angular momentum will result by the application of a couple to the disc, therefore the couple applied to the disc causing precession,

$$C = \lim_{\delta t \to 0} I.\omega.\frac{\delta\theta}{\delta t} = I.\omega.\frac{d\theta}{dt} = I.\omega.\omega_p$$

$$\therefore \boxed{C = I.\omega.\omega_p} \; . \; \underline{\underline{N\text{-}m}}$$

where:- I = mass moment of Inertia of disc $kg\text{-}m^2$

ω = Angular velocity of disc (in rad)

$\omega_p = \frac{d\theta}{dt}$ = Angular velocity of precession.

Notes

1) The Couple $I.\omega.\omega_p$ in the direction of the vector $\underline{xx'}$ is
the 'active gyroscopic couple'. The plane 'XOY' is called
the plane of 'active gyroscopic Couple'. The axis 'OZ'
perpendicular to the plane 'XOY' is called axis of 'active
gyroscopic couple'.

2) when the axis of spin itself moves with angular velocity 'ω_p',
the disc is subjected to 'reactive couple' whose magnitude is
same (i.e $I\omega\omega_p$) but opposite in direction to that of active
couple. This reactive couple to which the disc is subjected
when the axis of spin rotates about the axis of precession is
known as 'reactive gyroscopic couple. The axis of the reactive
gyroscopic couple is represented by $\underline{OZ'}$.

3. The gyroscopic couple is usually applied through the bearings
which supports the shaft. The bearings will resist equal
and opposite couple.

4. The gyroscopic principle is used in an instrument & toy known as
'Gyroscope'.

5. The gyroscopes are installed in ships in order to minimize the
rolling and pitching effects of waves.

6. Gyroscopes are also used in aeroplanes, monorail cars,
gyrocompasses etc.

① A Uniform disc of diameter 300mm and of mass 5kg is mounted on one end of an arm of length 600mm. The other end of the arm is free to rotate in a Universal bearing. If the disc rotates about the arm with a speed of 300 r.p.m clockwise, looking from the front, with what speed will it precess about the vertical axis.

$I = mk^2$

$\left(K = \frac{r}{\sqrt{2}}\right)$

Sol:- Given Data:- dia of disc $d = 300mm = 0.3m$

Radius $r = 150\ mm = 0.15m$

mass of disc $m = 5kg$

length of arm $l = 600\ mm = 0.6m$

Speed of disc $N = 300 rpm$.

Angular velocity of disc $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 31.42\ rad/sec.$

mass moment of Inertia of a circular disc $I = \frac{mr^2}{2}$

$\Rightarrow I = \frac{1}{2} \times 5 \times (0.15)^2 = 0.056\ kg\text{-}m^2$

Couple due to mass of the disc $C = mgl$

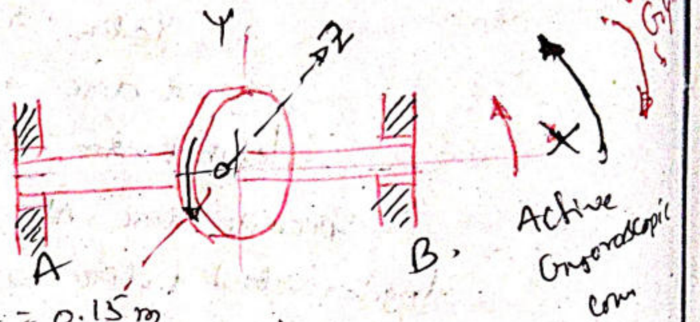
$\Rightarrow C = 5 \times 9.81 \times 0.6 = 29.43\ N\text{-}m$

Let ω_p = Speed of precession

from $C = I.\omega.\omega_p$

$\therefore \omega_p = \frac{C}{I\omega} = \frac{29.43}{0.056 \times 31.42} = 16.7\ rad/sec$

① A Uniform disc of 150 mm diameter has a mass of 5 kg. It is mounted Centrally in bearings which maintain its axle in a horizontal plane. The disc spins about its axle with a constant speed of 1000 r.p.m. while the axle precesses uniformly about the vertical at 60 r.p.m. The directions of rotations are as shown in fig. If the distance between the bearings is 100 mm, find the resultant reaction at each bearing due to the mass and gyroscopic effects.



Solution:- Dia of Disc $d = 150\,mm = 0.15\,m$

radius $r = 0.075\,m$

mass of disc $m = 5\,Kg$

Speed of disc $N = 1000\,r.p.m$

Angular velocity of disc $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1000}{60} = 104.7\,rad/sec.$

Speed of precession $N_p = 60\,r.p.m$

Ang. velocity of precession $\omega_p = \frac{2\pi N_p}{60} = \frac{2\pi \times 60}{60} = 6.284\,rad/sec.$

Distance between the Bearing $x = 100\,mm = 0.1\,m.$

mass moment of Inertia of disc $I = \frac{mr^2}{2} = \frac{5 \times (0.075)^2}{2} = 0.014\,kg\text{-}m^2$

Gyroscopic Couple acting on disc $C = I\omega\omega_p$

$\Rightarrow C = 0.014 \times 104.7 \times 6.284 = 9.2\,N\text{-}m$

$C = 9.2\,N\text{-}m$

Let 'F' be the force at each bearing due to the gyroscopic couple $\Rightarrow F = \frac{C}{x} = \frac{9.2}{0.1} = 92\,N.$

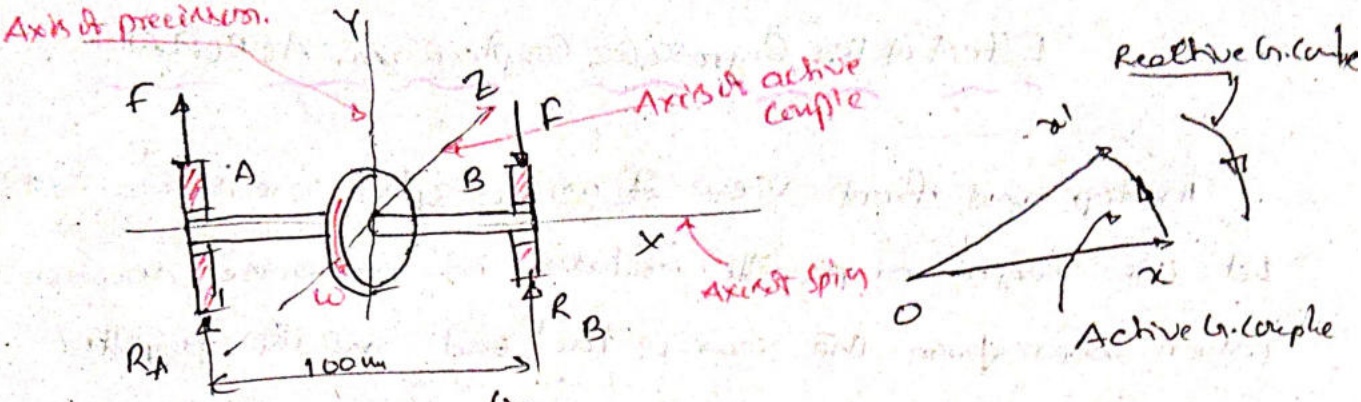
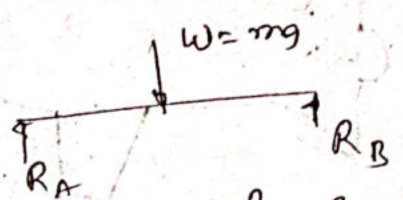


fig.

The force 'F' will act in opposite directions at the bearings as shown in fig. Now let R_A and R_B be the reaction at the bearing 'A' and 'B' respectively due to weight of the disc. Since the Disc mounted centrally on the bearing shaft.

from BM Concept (MOS Subject).



$R_A = R_B = \frac{W}{2}$

∴ the Reactions at 'A' & 'B' are given by,

$$R_A = R_B = \frac{W}{2} = \frac{mg}{2} = \frac{5\times 9.8}{2} = \underline{\underline{24.5\,N}}$$

Resultant reactions at each bearing

Let R_{A_1} & R_{B_1} = Resultant reaction at the bearings 'A' & 'B' respectively.

Since the reactive Gyroscopic Couple acts in clockwise direction when seen from the front, therefore its effect is to increase the reaction on the Left hand side bearing (A) and to decrease the reaction on the right hand side bearing (B).

∴ ~~$R_{A_1} = R_A + F = 24.5 + 92 = 116.5\,N$ (upwards)~~

~~&~~

~~$R_{B_1} = R_A - F$~~

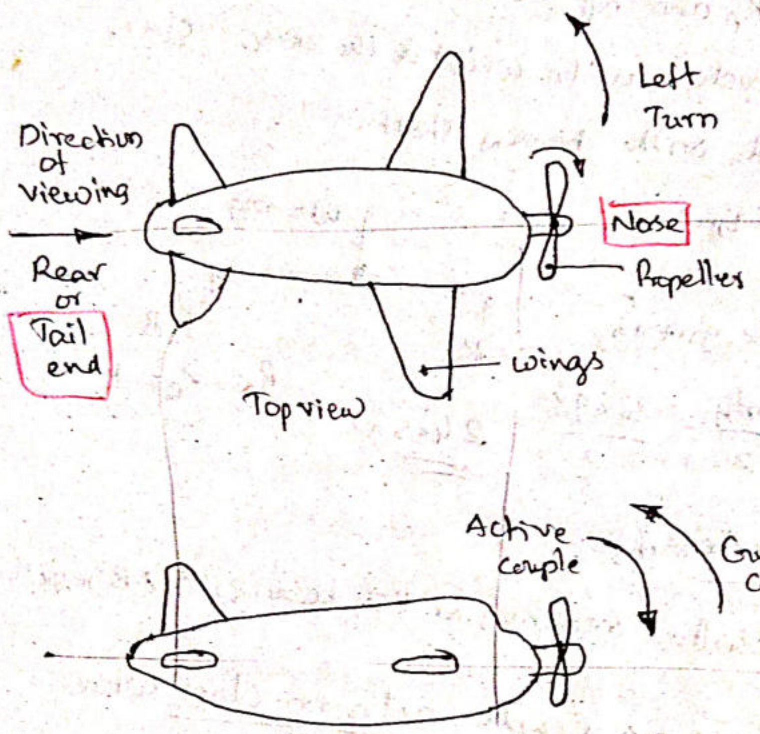
$$R_{A_1} = F + R_A = 92 + 24.5 = 116.5\,N \text{ (upwards)}$$

&

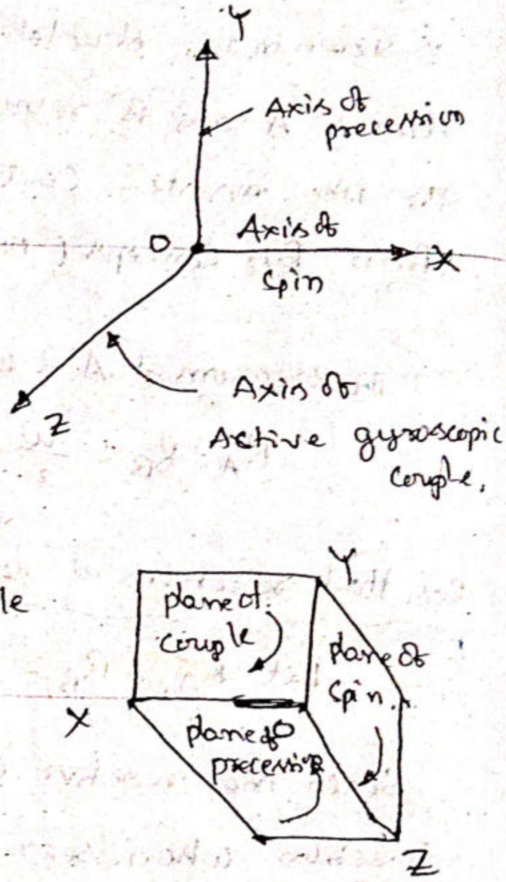
$$R_{B_1} = F - R_B = 92 - 24.5 = 67.5\,N \text{ (downwards)}$$

Effect of the Gyroscopic Couple on An AEROPLANE

The top and front view of an aeroplane are shown in fig. Let the engine or propeller rotates in clockwise direction when seen from the rear or tail end and the propeller takes a turn to left.



1. (a)



1. (b)

Aeroplane taking a left turn.

Let:- ω = Angular velocity of the engine (rad/sec)

m = mass of engine and propeller (Kg)

k = Radius of Gyration (m)

I = mass moment of Inertia of engine & Propeller (Kg-m^2)

$= mk^2$

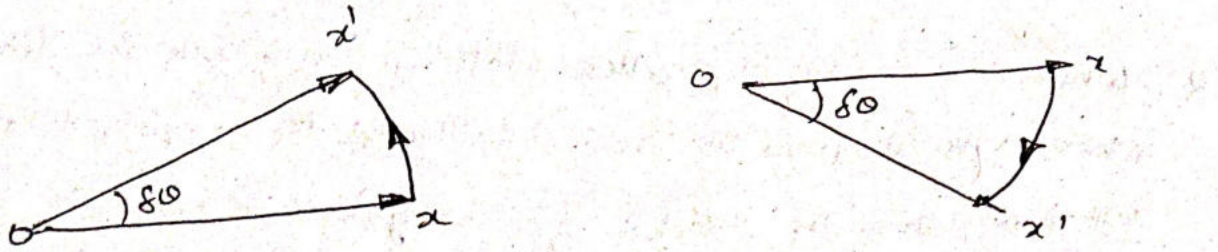
v = Linear velocity of the aeroplane (m/sec)

R = Radius of Curvature (m)

$\boxed{\omega_p = \text{Angular velocity of precession} = \frac{v}{R}}$ rad/sec

$$\boxed{\omega_p = \frac{v}{R}}$$

$\therefore$ Gyroscopic Couple acting on the areoplane $\boxed{C = I\omega\omega_p}$



(a) Aeroplane taking left turn (b) Aeroplane taking right turn.

fig:- Effect of gyroscopic couple on an aeroplane

Before taking the left turn, the angular momentum vector is represented by 'ox', when it takes left turn, the active gyroscopic couple will change the direction of the angular momentum vector from ox to ox' as shown in fig (a). The vector xx' in the limit represents the change of angular momentum & the active gyroscopic couple and is perpendicular to 'ox'. Thus the plane of active gyroscopic couple 'XOY' will be perpendicular to 'xx'', we find that the direction of active gyroscopic couple is clockwise as shown in the front view of fig 1.(a). In other words for left hand turning, the active gyroscopic couple on the aeroplane in the axis 'OZ' will be clockwise in fig 1.(b). The reactive gyroscopic couple (equal in magnitude of active gyroscope couple) will acts in the opposite direction (i.e in anticlockwise direction) and the effect of this couple is, therefore to raise the Nose and Dip the tail of the Aeroplane.

Notes . effect of Gyroscopic couple

1. when the aeroplane takes a right turn for the above conditions, the effect of reactive gyroscopic couple will be 'dip the Nose' and Raise the Tail of the aeroplane.

2. When the engine or propeller rotates in anticlockwise direction, when viewed from the rear or tail and the aeroplane takes a left turn, then the effect of reactive gyroscopic couple is 'dip the Nose and Raise the tail of aeroplane.

3. when the aeroplane takes the right turn of a conditions at Note(2) then the effect of reactive gyroscopic couple is 'Raise the Nose' and 'Dip the tail' of the aeroplane.

4. When the engine or propeller rotates in clockwise direction when viewed from the front and the aeroplane takes a left turn, the effect of reactive gyroscopic couple is dip the Nose and Raise the tail' of the aeroplane.

5. when the aeroplane takes right turn under the Note(4) conditions, then the effect of reactive gyroscopic couple is Raise the Nose and Dip the tail of the aeroplane.

Viewing from Rear	Clockwise	Left Turn	Nose Up & Tail Down
		Right Turn	Nose Down & Tail Up
	Anti-Clock wise	Left Turn	Nose Down & Tail Up
		Right Turn	Nose Up & Tail Down
Viewing from Front	clockwise ~~Anticlock~~ wise	Left Turn	Nose down & Tail Up
		Right Turn	Nose Up & Tail Down
	Anti-Clock wise	Left Turn	Nose Up & Tail Down
		Right Turn	Nose down & Tail Up

(P) An aeroplane makes a complete half circle of 50 meters radius, towards left, when flying at 200 Km/hr. The rotary engine and the propeller of the plane has a mass of 400 kg and a radius of gyration of 0.3 m. The engine rotates at 2400 r.p.m. clockwise when viewed from the rear. find the gyroscopic couple on the aircraft and state its effect on it.

Sol:- Given Data:- Radius of curvature $R = 50$ meters

Left Turn ; clockwise: Viewed from Rear

Speed $V = 200\,Km/hr = 200 \times \frac{1000}{3600} = 55.6\,m/sec$

mass of Craft $m = 400\,kg$

Radius of gyration $k = 0.3\,m$

engine speed $N = 2400\,r.p.m$ (~~clo~~)

Angular velocity $\omega = \frac{2\pi N}{60} = \frac{2\pi \cdot 2400}{60} = 251\,rad/sec.$

Mass moment of Inertia $I = mk^2 = 400 \times (0.3)^2$

$\Rightarrow I = 36\ kg\text{-}m^2$

~~Coupl.~~ precession angular velocity $\omega_p = \frac{V}{R} = \frac{55.6}{50} = 1.11\,rad/sec$

Gyroscopic Couple $C = I\omega\omega_p = 36 \times 251 \times 1.11 = 10046\,N\text{-}m$

or $C = 10.046\,N\text{-}m$

State of effect:- Nose upwards and Tail downwards

Naval Ship

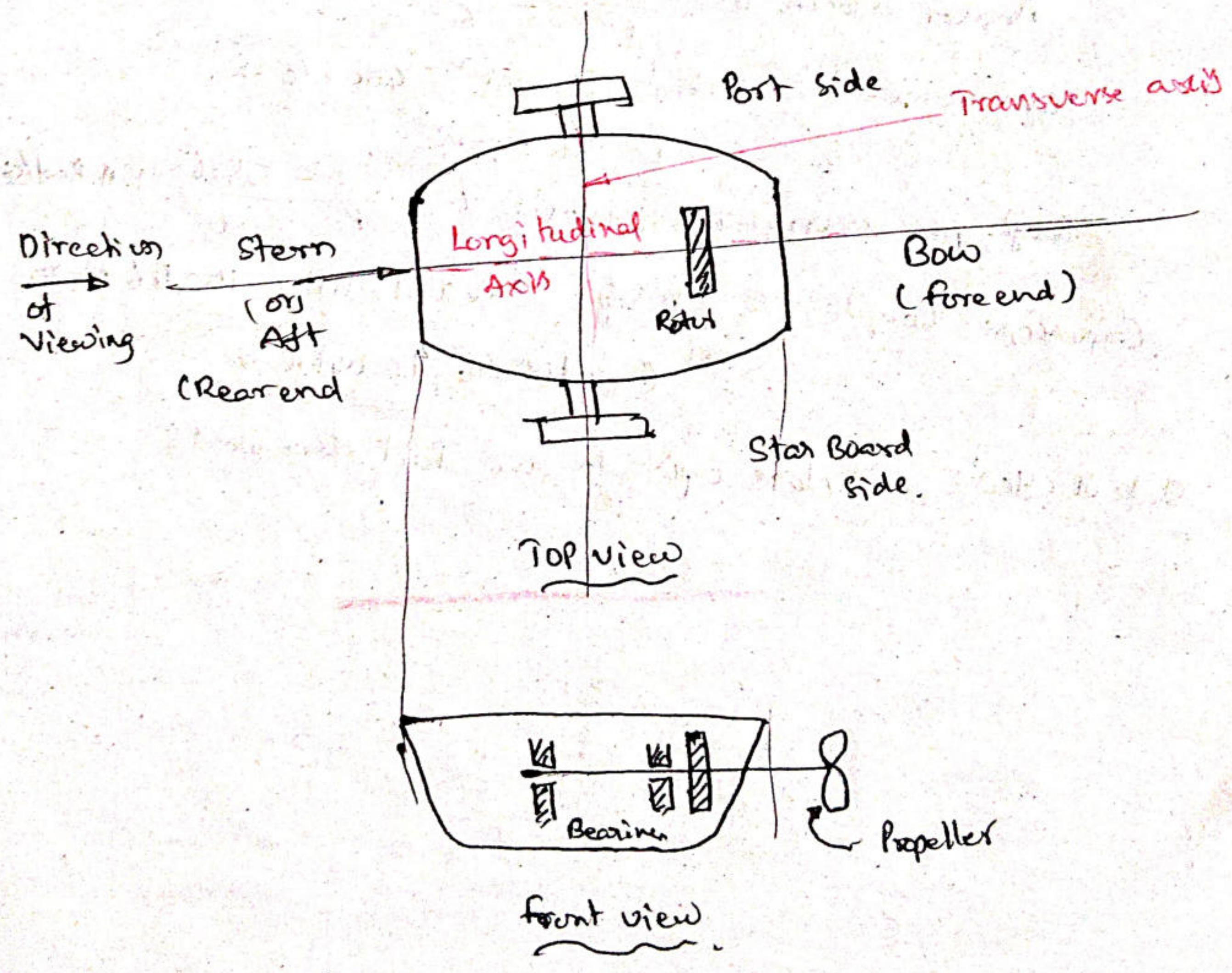
Terminology of Naval Ship

The Top and front Views of a naval ship are shown in fig. The fore end of the ship is called 'Bow' and the rear end is known as 'stern' or 'Aft'.

The left hand and right hand sides of the ship, when viewed from the 'stern' are called as 'PORT' and 'STAR-BOARD' respectively.

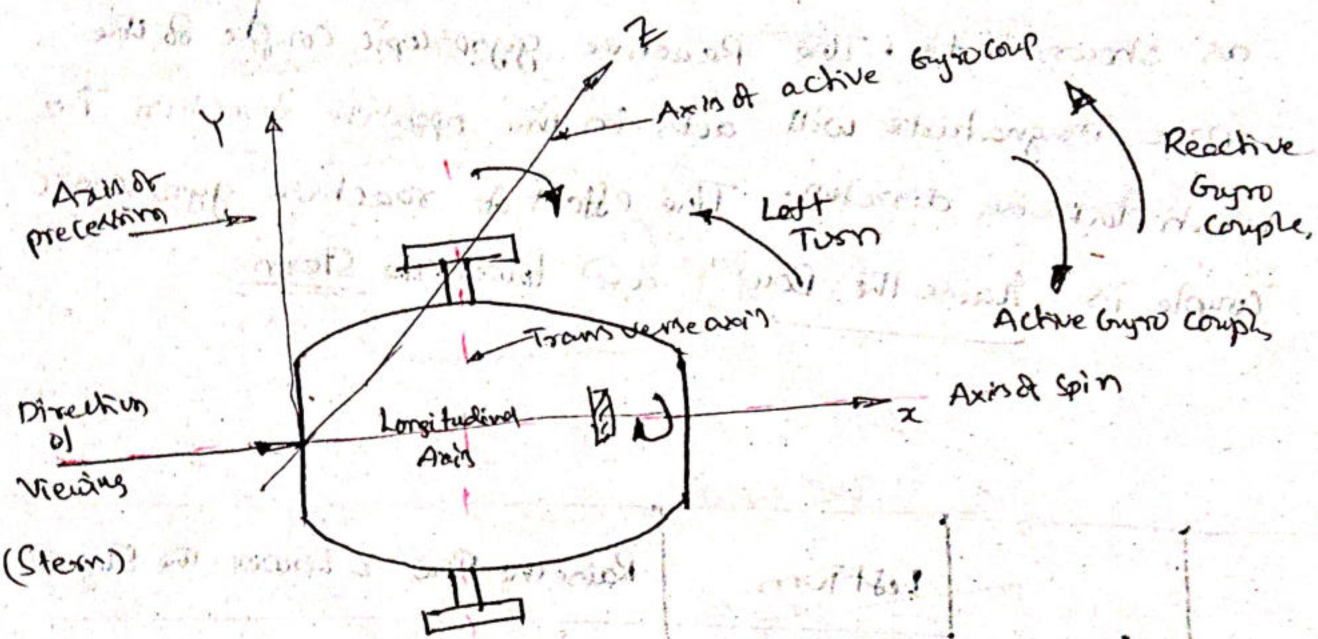
The effect of Gyroscopic couple on the Naval ship are studied in the following cases:

1. Steering 2. Pitching 3. Rolling



Effect of Gyroscopic couple on a Naval Ship during STEERING

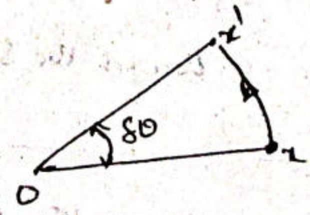
Steering is the turning of a complete ship in a curve towards left or right, while it moves forward. Consider the ship taking a left turn, and rotor rotates in the clockwise direction when viewed from the stern as shown in fig.



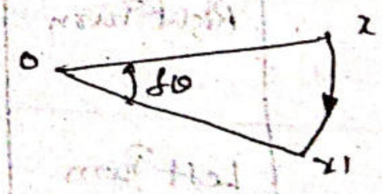
Naval Ship taking a left turn

The effect of gyroscopic couple on a naval ship during steering taking a left or Right turn may be obtained in the similar way as for an aeroplane as discussed early.

When the rotor of the ship rotates in the clockwise direction when viewed from the stern, it will have its angular momentum vector in the direction ox in fig.(a)



(a) steering to Left



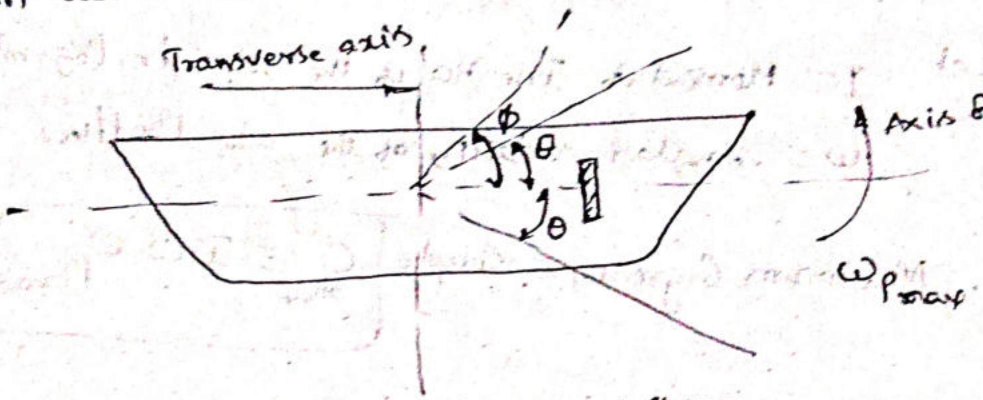
(b) Steering to Right

As the ship steers to the left, the active gyroscopic couple will change the angular momentum vector 'ox' to 'ox''. The vector 'xx'' now represents the active gyroscopic couple and it is perpendicular to 'ox'. Thus the plane of active gyroscopic couple is perpendicular to xx' and its direction is in the axis 'OZ' for the left hand turn is clockwise as shown in fig. The Reactive gyroscopic couple of the same magnitude will acts in the opposite direction i.e anticlockwise direction. The effect of reactive gyroscopic couple is 'Raise the Bow' and lower the Stern.

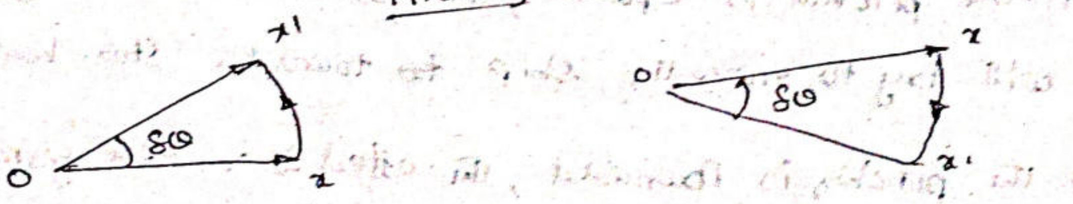
Viewing from Rear	Clockwise	Left Turn	Raise the Bow & Lower the Stern
		Right Turn	Lower the Bow & Raise the Stern
	Anticlockwise	Left Turn	Lower the Bow & Raise the Stern
		Right Turn	Raise the Bow & Lower the Stern.
Viewing from front	Clockwise	Left Turn	Lower the Bow & Raise the Stern
		Right Turn	Raise the Bow & Lower the Stern
	Anti clockwise	Left Turn	Raise the Bow & Lower the Stern
		Right Turn.	Lower the Bow & Raise the Stern.

Effect of Gyroscopic Couple on Naval Ship during Pitching

Pitching is the movement of a complete ship up and down in a vertical plane about transverse axis as shown in fig. In this case, the transverse axis is the axis of precession. The pitching of the ship is assumed to take place with simple harmonic motion (SHM) i.e. the motion of the axis of spin about transverse axis is simple harmonic.



Pitching of Naval Ship



Pitching upwards   Pitching downward

$\therefore$ Angular displacement of the axis of spin from mean position after time 't' seconds

$$\therefore \theta = \phi \sin \omega_1 t$$

where:- ϕ = Amplitude of swing i.e maximum angle turned from the mean position (in radians)

ω_1 = Angular velocity of S.H.M

$$= \frac{2\pi}{\text{Time period of S.H.M in seconds}} = \frac{2\pi}{t_p} \text{ rad/sec}$$

$$\boxed{\therefore \omega_1 = \frac{2\pi}{t_p}}$$

Angular velocity of precession, $\omega_p = \frac{d\theta}{dt}$

$$\Rightarrow \omega_p = \frac{d}{dt}(\phi \sin\omega_1 t) = \phi\omega_1 \cos\omega_1 t$$

If the $\cos(\omega_1 t) = 1$ then the angular velocity of precession is maximum.

$$\therefore \boxed{\omega_{p(max)} = \phi\omega_1} \Rightarrow \boxed{\omega_{p_{max}} = \phi \times \frac{2\pi}{t_p}}$$

Let I = Moment of Inertia of the rotor in ($kg\text{-}m^2$)

ω = Angular velocity of the rotor (rad/sec)

$\therefore$ Maximum Gyroscopic Couple $\boxed{C_{max} = I.\omega.\omega_{p(max)}}$

- When the pitching is upward, the effect of reactive gyroscope couple will try to move the ship ~~to~~ towards star-board.
- When the pitching is Downward, the effect of reactive gyroscope couple is to turn the ship towards port side.

(i) The Angular Acceleration during pitching.

$$\alpha = \frac{d^2\theta}{dt^2} = -\phi\omega_1^2 \sin\omega_1 t$$

The angular acceleration is maximum when $\sin\omega_1 t = 1$

$$\boxed{\therefore \alpha_{max} = -\phi\omega_1^2}$$

Effect of Gyroscopic Couple on a Naval Ship during Rolling

We know that, for the effect of gyroscopic couple to occur, the axis of precession should always be perpendicular to the axis of spin. If however the axis of precession becomes ~~paral~~ parallel to the axis of spin, there will be no effect of the gyroscopic couple acting on the body of the ship.

In case of rolling of a ship, the axis of precession (longitudinal axis) is always parallel to the axis of spin for all positions. Hence, there is no effect of the gyroscopic couple acting on the body of a ship.

(1) The turbine rotor of a ship has a mass of 8 tonnes and a radius of gyration 0.6 m. It rotates at 1800 r.p.m, clockwise, when looking from the stern. Determine the gyroscopic couple, if the ship travels at 100 km/hr and steer to the left in a curve of 75 m radius.

Sol. Given Data:- mass of ship $m = 8$ tonne $= 8\times10^3$ kg

Radius of gyration $k = 0.6$ m;

Speed $N = 1800$ r.p.m; angular speed $\omega = \frac{2\pi N}{60} = \frac{2\pi\times1800}{60} = 188.5$ rad/sec

Velocity of ship $v = 100$ km/hr $= 100\times\frac{5}{18} = 27.8$ m/sec

Radius of curvature $R = 75$ m

mass moment of inertia $I = mk^2 = 8\times10^3(0.6)^2 = 2880$ kg-m^2

Angular velocity of precession $\omega_p = \frac{v}{R} = \frac{27.8}{75} = 0.37$ rad/sec.

Gyroscopic Couple $C = I\omega\omega_p = 2880\times188.5\times0.37 = 200866$ N-m

$\Rightarrow C = 200.866$ KN-m

Effect:- When rotor rotates in clockwise and viewing from stern, (Rear) and the ship takes left turn, then the effect of gyroscopic couple is Raise the BOW and Lowers the STERN

(1) The turbine rotor of a ship has a mass of 3500 kg. It has a radius of gyration of 0.45 m and a speed of 3000 r.p.m clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship:

1. when the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/hr.
2. when the ship is pitching SHM, the bow falling with its maximum velocity. The period of pitching is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees
3. when ship is rolling

Sol:- Given data:- mass of ship $m = 3500\,kg$,

Radius of gyration $K = 0.45\,m$

Speed $N = 3000$ r.p.m;

Angular speed $\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 3000}{60} = 314.2$ rad/sec.

1. When the ship is steering to the left

Given:- Radius of curvature $R = 100\,m$

Velocity of ship $v = 36\,km/hr = 36 \times \frac{5}{18} = 10\,m/sec.$

$\therefore$ mass moment of Inertia $I = mk^2 = 3500 \times (0.45)^2$

$\Rightarrow I = 708.75\,kg\text{-}m^2$

Angular velocity of precession $\omega_p = \frac{v}{R} = \frac{10}{100} = 0.1$ rad/sec.

Gyroscopic couple $C = I\omega\omega_p = 708.75 \times 314.2 \times 0.1 = 22270\,N\text{-}m$

$\Rightarrow \underline{C = 22.270\,KN\text{-}m}$

Gyroeffect is:- Raise the BOW and lower the STERN.

2. When the ship is pitching with the bow falling (Downward)

Given:- Time period $t_p = 40 sec.$

Total angular displacement $\Rightarrow 2\phi = 12^0$

$\phi = \pm 6^0 = \frac{6 \times \pi}{180} = 0.105$ rad

Angular velocity of SHM $\Rightarrow \omega_1 = \frac{2\pi}{t_p} = \frac{2\pi}{40} = 0.157$ rad/sec

Maximum angular velocity of precession $\omega_p = \phi\omega_1 = 0.105 \times 0.157$

$\Rightarrow \omega_p = 0.0165$ rad/sec.

$\therefore$ Gyroscopic Couple $C = I\omega\omega_p = 708.75 \times 314.2 \times 0.0165$

$\Rightarrow C = 3675$ N-m (or) 3.675 kN-m

effect:- When the bow is falling (pitching Downward); the effect of reactive Gyroscopic Couple is to move the ship towards PORT side.

3. When ship is rolling:- No-effect because AS is parallel to AP

(P) The mass of the turbine rotor of a ship is 20 tonnes and has a radius of gyration of 0.60m. Its speed is 2000 r.p.m. The ship pitches 6° above and 6° below the horizontal position. A complete oscillation takes 30 seconds and the motion is SHM. Determine the following.

1. Maximum Gyroscopic Couple 2. Maximum angular acceleration during pitching 3. The direction in which the bow will tend to turn when rising, if the rotation of the rotor is clockwise when looking from the left.

Sol:- Given Data:- mass $m = 20$ tonnes $= 20 \times 10^3$ Kg

Radius of Gyration $K = 0.6$ m

Speed $N = 2000$ r.p.m : Angular Speed $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2000}{60} = 209.5$ rad/sec

Amplitude $\phi = 6^0 = 6 \times \frac{\pi}{180} = 0.105$ rad.

Time period $t_p = 30$ sec.

Mass moment of Inertia $I = mk^2 = 2000(0.6)^2 = 7200 \text{ kg-m}^2$

Angular velocity of SHM $\omega_1 = \frac{2\pi}{t_p} = \frac{2\pi}{30} = 0.21$ rad/sec.

Maximum angular velocity of precession $\omega_{p\,max} = \phi\omega_1 = 0.105 \times 0.21$

$$\omega_{p\,max} = 0.022 \text{ rad/sec.}$$

∴ Maximum Gyroscopic Couple $C_{max} = I\omega\omega_{p(max)}$

$$C_{max} = 7200 \times 0.21 \times 0.022 = 33185 \text{ N-m}$$

$$\underline{C_{max} = 33.185 \text{ KN-m}}$$

2. Maximum Angular acceleration during pitching

$$\alpha_{max} = \phi(\omega_1)^2 = 0.105 \times (0.21)^2 = 0.0046 \text{ rad/sec}^2$$

3. <u>Effect</u>:- pitching upward and rotating clockwise when looking from Stern (Rear) is the ship turns towards Star Board i.e turning Right ward.

Unit VI ~~Hair~~ Higher Pairs

(Toothed wheels)

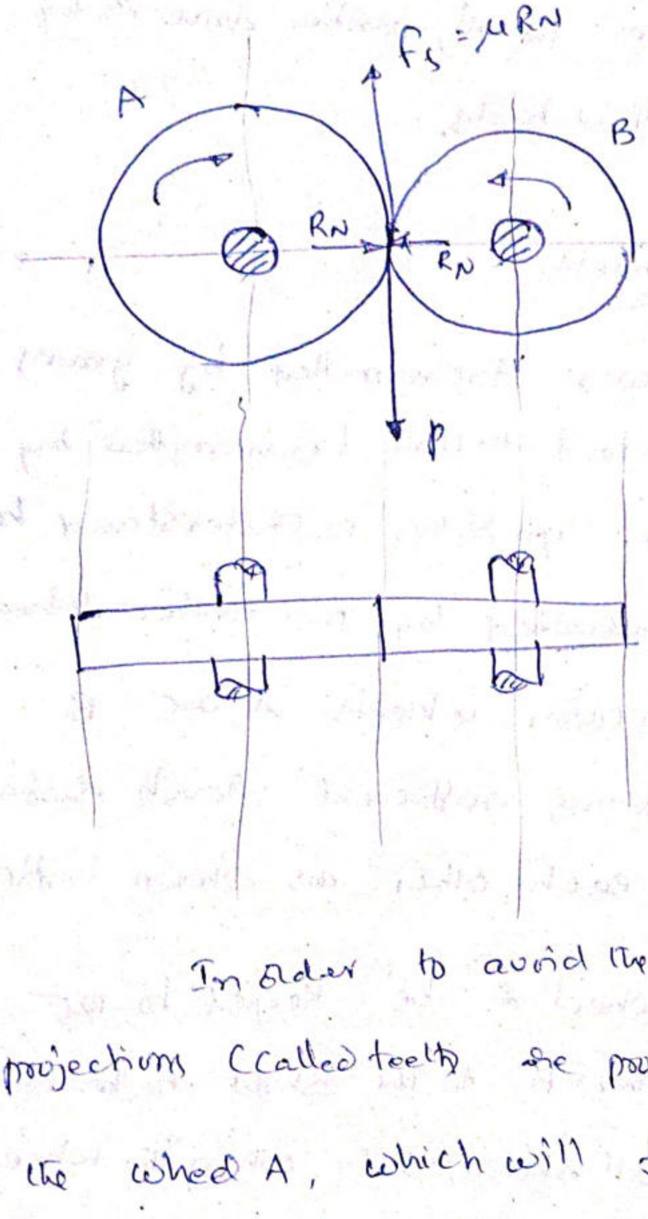
3)2 The Distance between the shaft is small the gear drive ~~the belts~~ is recommended. the gear drive is a positive drive. In precission machines, in which a definite velocity ratio is of importance, the only positive drive is by means of gears or toothed wheels.

friction wheels

The motion and power transmitted by gears is kinematically equivalent to that transmitted by friction wheels (or) discs. In order to understand how the motion can be transmitted by two toothed wheels, consider two plain circular wheels A and B mounted on shafts, having sufficient rough surfaces and pressing against each other as shown in fig.

Let the wheel A be keyed to the rotating shaft and the wheel B to the shaft to be rotated. A little consideration will show, that when the wheel A is rotated by a rotating shaft, it will rotate the wheel B in the opposite direction. as shown in fig.

The wheel B will be rotated as long as the tangential force exerted by the wheel A doesn't exceed the maximum frictional resistance between the two wheels. But when the tangential force (P) exceeds the frictional resistance (F), slipping will take place between the two wheels. Thus the friction drive is not a positive drive.



In order to avoid the slipping, the number of projections (called teeth) are provided on the periphery of the wheel A, which will fit into the corresponding recesses on the periphery of the wheel B. A friction wheel with the teeth cut on it is known as toothed wheel (or) gear.

Types (or) classification of Toothed wheels

(I) According to the position of axes of the shafts

(a) Parallel (b) Intersecting (c) Non-intersecting and Non-parallel.

(II) According to the peripheral velocity of the gears

(a) Low velocity (< 3 m/sec) (b) Medium velocity (> 3 m/sec < 15 m/sec) (c) High velocity (> 15 m/sec)

(III) According to the type of gearing

(a) External gearing (b) Internal gearing

(c) Rack and pinion.

(IV) According to the position of teeth on the gear surface

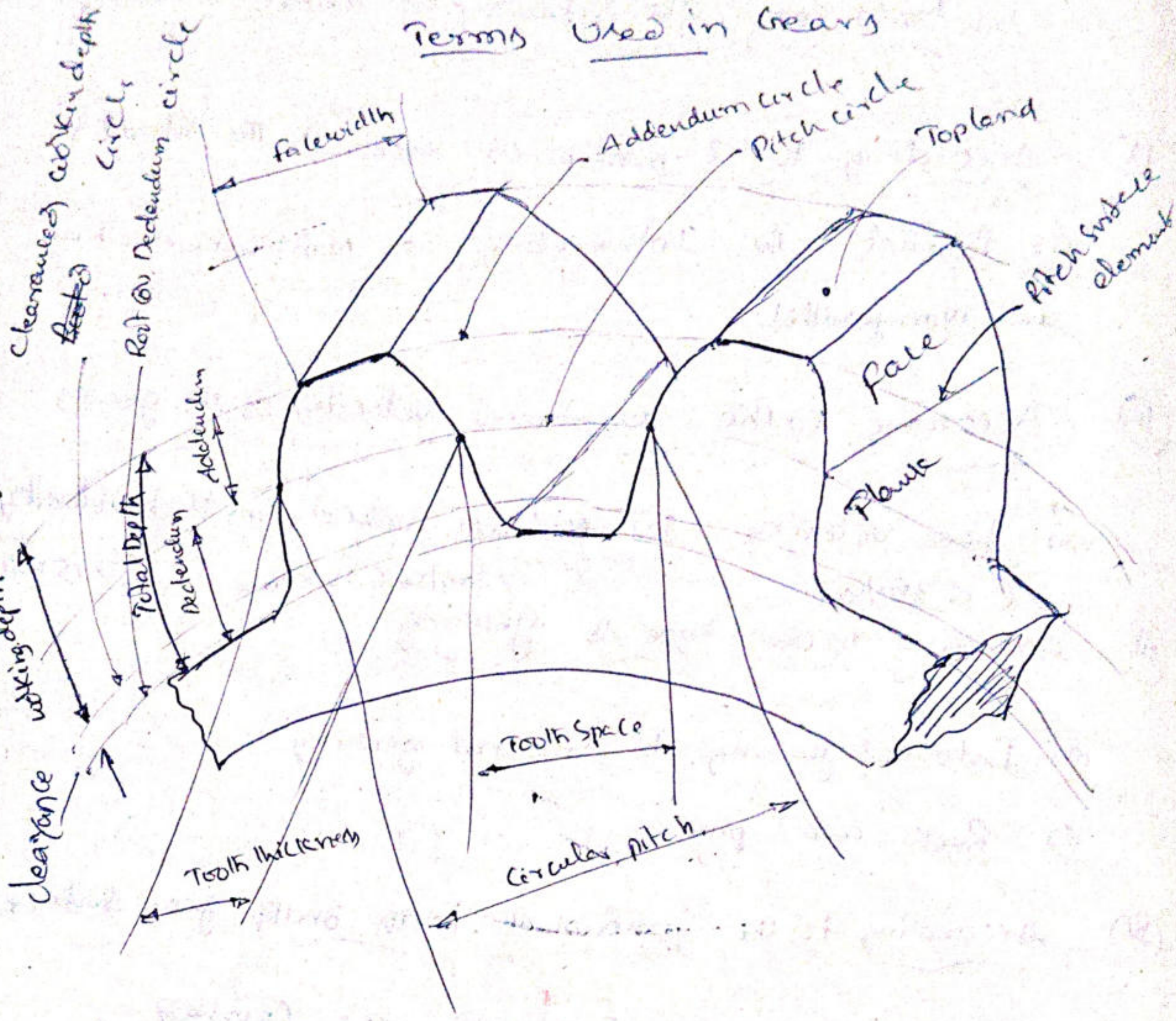
(a) Straight (Spur) (b) Inclined (Helical) (c) Curved. (Spiral)

For: (I) (a) Ex Spur Gear, Helical Gear → Single Helical Gear / Double Helical Gear (Herringbone Helical Gear)

(I) (b) Bevel Gearing. Helical Bevel Gearing

(c) Spiral Gearing & Skew Bevel gearing.

Terms Used in Gears



(1) Pitch circle :- It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

(2) Pitch circle diameter - It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as pitch diameter.

(3) Pitch point :- It is the common point of contact between the two pitch circles.

(4) Pitch surface :- It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

(5) Pressure angle (or Angle of obliquity) :- It is the angle between the common normal to the two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ϕ. The standard pressure angles are $14\frac{1}{2}°$ and 20.

(6) Addendum :- It is the radial distance of a tooth from pitch circle to the top of the tooth.

(7) Dedendum :- It is the radial distance of a tooth from pitch circle to the bottom of the tooth.

(8) Addendum circle :- It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

(9) Dedendum circle :- It is the circle drawn through the bottom of the teeth. It is also called as root circle.

(10) Circular pitch :- It is the distance measured on the circumference of the pitch circle from one point on one tooth to the corresponding point on the next tooth. It is usually denoted by p_c.

$\therefore$ Circular pitch $p_c = \frac{\pi D}{T}$

Note: If the two gears have same circular pitch

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \Rightarrow \boxed{\frac{D_1}{T_1} = \frac{D_2}{T_2}} \text{ or } \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

(11) Diameteral pitch :- It is the ratio of number of teeth to the pitch circle diameter in millimeters. It is denoted by (P_d) :

$$\therefore \boxed{P_d = \frac{T}{D} = \frac{\pi}{P_c}}$$

(12) Module :- (m) :- It is the ratio of pitch circle diameter in millimeter to the number of teeth. It is usually denoted by (m).

$$\therefore \text{ module } \boxed{m = \frac{D}{T}}$$

(13) Clearance :- It is the radial distance from the top of tooth to the bottom of tooth in meshing gear. The circle passing through the top of the meshing gear is known as clearance circle.

(14) Total Depth :- It is the Radial Distance between the adendum and dedendum circles of the gear. It is equal to the sum of Adendum and Dedendum.

(15) Working Depth :- It is the radial distance from the adendum circle to the clearance circle. It equal to the sum of the adendum of the two meshing gears

(16) Tooth thickness :- It is the width of the tooth measured along the pitch circle.

(17) Tooth Space :- It is the width of space between the two adjacent teeth measured along the pitch circle.

* (18) Back lash :- It is the difference between the tooth space and tooth thickness measured along the pitch circle. Theoretically back lash is zero. But actual practice to avoid the jamming of gears, the back lash must be allowed.

(19) Face of Tooth :- It is the surface of the gear tooth above the pitch surface.

(20) Flank of tooth :- It is the surface of the gear tooth below the pitch surface.

(21) Topland :- It is the surface of the top of the tooth.

(22) Face width :- It is the width of the gear tooth measured parallel to the axis.

(23) Profile :- It is the curve formed by the face and flank of the tooth.

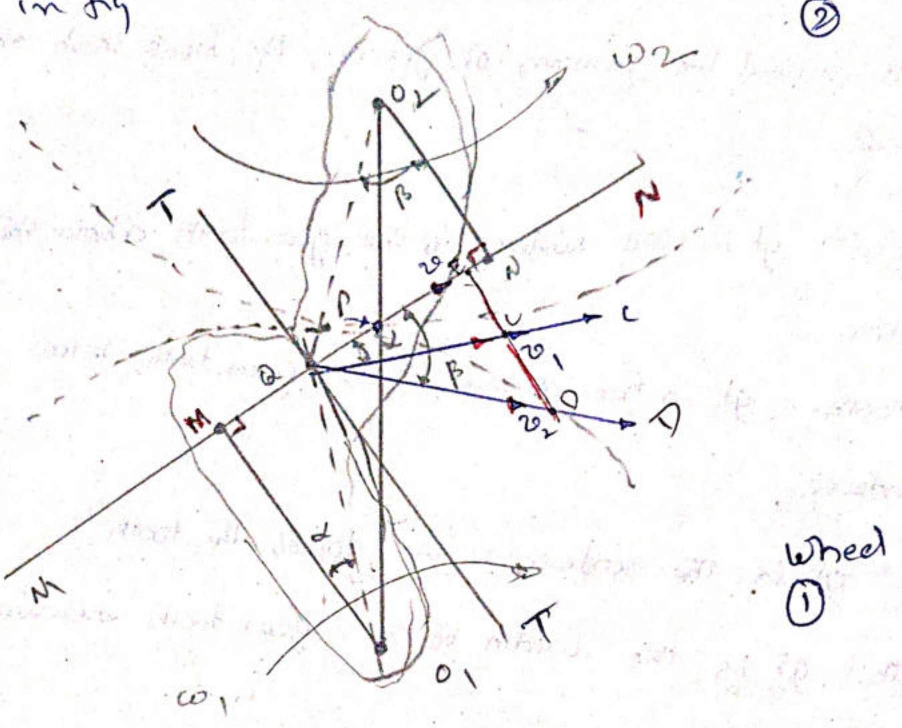
(24) Fillet Radius :- It is the radius that connects the root circle to the profile of the tooth.

Gear Materials

Metals	Non-Metal
Cast-Iron,	Wood,
Steel,	Rawhide,
phosphor Bronze	compressed paper.

Law of Gearing (or) Condition for Constant Velocity Ratio of toothed wheels

Consider the positions of the two teeth, one on the wheel ① (Pinion) and other on the wheel ② as shown in fig



Let the two teeth come in contact at point 'Q', and the wheels rotate in the direction as shown in fig.

Let TT' be the common tangent and MN be the common Normal to the Curves at the point of contact Q. From the Centres O_1 and O_2, draw O_1M and O_2N perpendicular to MN. A little consideration will show that the point 'Q' moves in the direction QC, when considered as a point on wheel ①, and in the direction QD when considered as a point on wheel ②.

Let v_1 and v_2 be the velocities of the point Q on the wheels 1 and 2, respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal.

$$\therefore \quad v_1 \cos\alpha = v_2 \cos\beta$$

$$\text{or} \quad v_1 \times \frac{O_1M}{O_1Q} = v_2 \times \frac{O_2N}{O_2Q}$$

$$\text{or} \quad (\omega_1 \times O_1Q) \times \frac{O_1M}{O_1Q} = (\omega_2 \times O_2Q) \times \frac{O_2N}{O_2Q}$$

$$\therefore \quad \omega_1 \, O_1M = \omega_2 \, O_2N$$

$$\therefore \quad \boxed{\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}} \quad \text{(1)}$$

From similar triangles O_1MP and O_2NP

$$\boxed{\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}} \quad \text{(2)}$$

(1) & (2) :

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P}} = \frac{D_2}{D_1} = \frac{T_2}{T_1} = \frac{PN}{MP}$$

From the above, we see that the angular velocity ratio is inversely proportional to the ratio of the distances of the point P from the centres O_1 and O_2, or the common normal to the two surfaces at the point of contact Q intersects the line of centres at point P which divides the centre distance inversely as the ratio of angular velocities.

Therefore in order to have a constant angular velocity ratio for all positions of the wheels, the point 'P' must be the fixed point (called pitch point) for the two wheels.

(In other words, the common normal at the point of contact between a pair of teeth must always pass through the pitch point.)

This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as 'law of gearing'.

Path of contact:- It is the path traced by a point of contact of two teeth from the beginning to end of engagement.

Length of path of contact:- It is the length of common normal cut-off by the addendum circles of the wheel and pinion (path of approach & path of Recess)

Arc of contact:- It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth.

(i) Arc of Approach:- It is the portion of the arc of the path of contact from the beginning of engagement to the pitch point.

(ii) Arc of Recess:- It is the portion of the arc of the path of contact from the pitch point to the end of engagement.

Velocity of Sliding of Teeth

The sliding between a pair of teeth in contact at Q occurs along the tangent T-T to the tooth curves shown in fig.

"The velocity of sliding is the velocity of one tooth relative to its mating tooth along the common tangent at the point of contact".

The velocity of point Q, considered as a point on wheel 1, along the common tangent TT is represented by EC. From similar triangles QEC and O_1MQ.

$$\frac{EC}{MQ} = \frac{v_1}{O_1Q} = \omega_1 \quad (\text{or})$$

$$\Rightarrow \boxed{EC = \omega_1 MQ}$$

Similarly, the velocity of point Q, considered as a point on wheel 2, along the common tangent TT is represented by ED, from similar triangles QED and O_2NQ.

$$\frac{ED}{QN} = \frac{v_2}{O_2Q} = \omega_2 \quad (\text{or})$$

$$\boxed{ED = \omega_2 QN}$$

Let V_s = Velocity of sliding at Q.

$$\therefore V_s = ED - EC = \omega_2 QN - \omega_1 MQ$$

$$\Rightarrow V_s = \omega_2 (QP + PN) - \omega_1 (MP - PQ)$$

$\Rightarrow V_s = (\omega_1 + \omega_2)\, PQ + PN\omega_2 - MP\omega_1$

$\therefore$ From $\frac{\omega_1}{\omega_2} = \frac{NP}{MP} = \frac{O_2P}{O_1P}$ (From Similarity O_1MP & O_2NP)

$$\boxed{\therefore \text{Velocity of Sliding } V_s = (\omega_1 + \omega_2) \cdot QP}$$

The velocity of sliding is proportional to the distance of the point of contact from the pitch point.

Forms of tooth

(1) cycloidal tooth

(2) Involute tooth

* cycloidal tooth are most stronger than involute tooth
* There is no interference in the cycloidal tooth
* The Involute teeth can made by a ~~sin.~~ single profile (or) curve
* The most advantage of involute gears is that the centre distance for a pair of involute gears can be varied with in the limits without changing the velocity ratio.

* The pressure angle is constant in involute gears from starting of engagement to end of engagement,

But in cycloidal gears the pressure angle is maximum at the engagement and reduces to zero at the pitch point and again becomes maximum at the end of engagement.

- In cycloidal gears contact takes place between convex & concave surfaces, this leads less wear in cycloidal gears, where as it more in Involute due to two convex surfaces are in contact.

Systems of Gear teeth

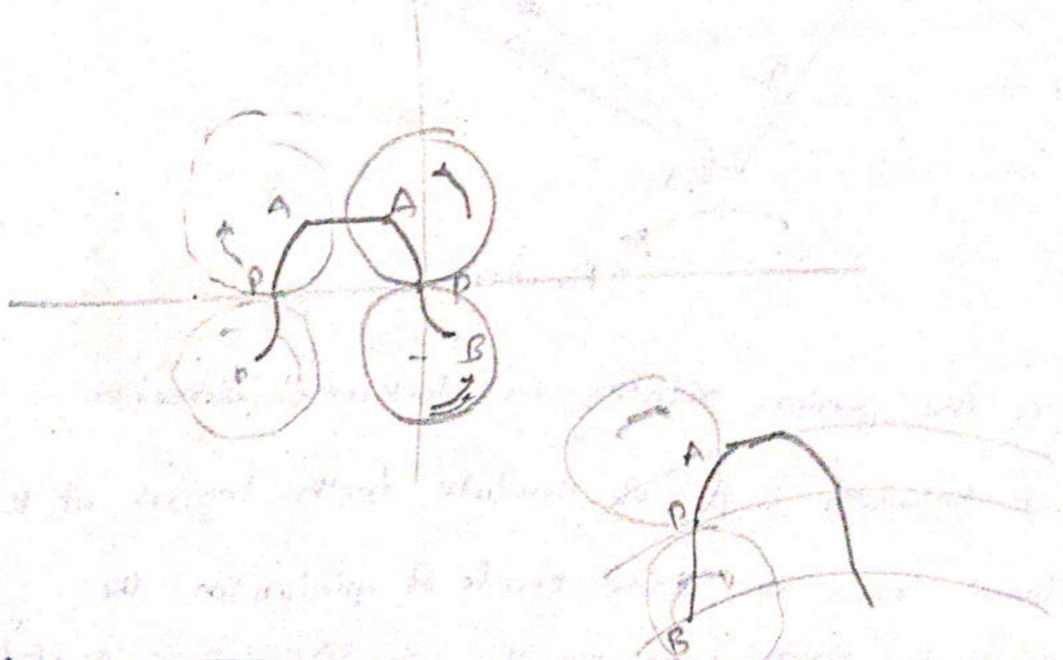
The following are four systems of gear teeth commonly used in practice.

(1) 14½° composite system (General purpose system)

2) 14½° full depth involute system (for Spur & Helical Gears)

(3) 20° full depth involute system (To become more stronger tooth)

(4) 20° Stub involute system → (for Heavy loads)



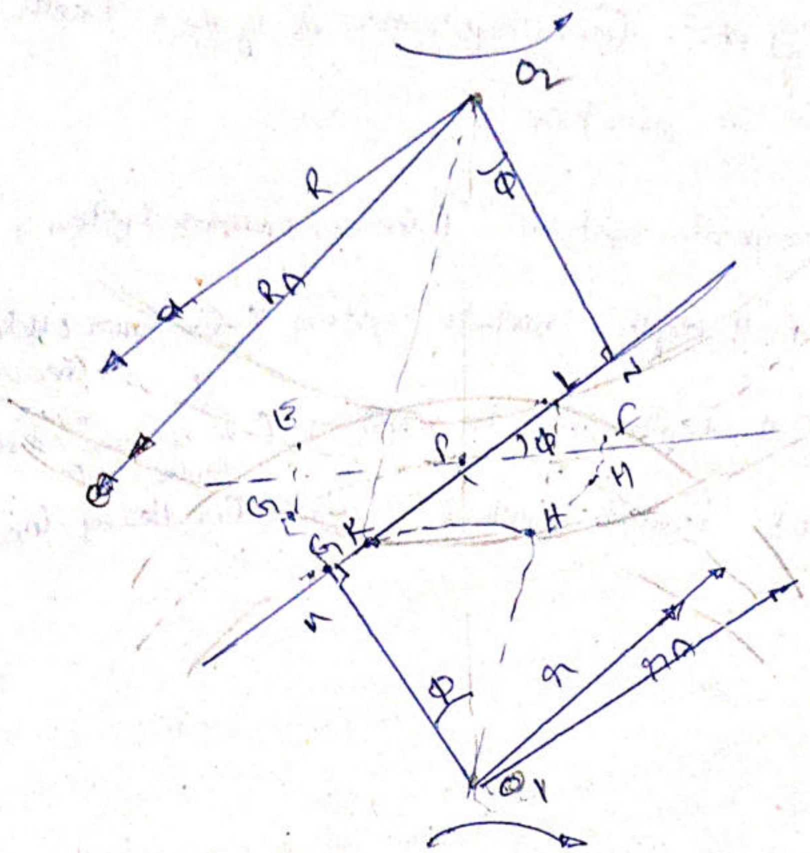
Conjugate Teeth

Two curves of any shape that fulfill the law of gearing can be as the profiles of teeth. In other words, An arbitrary shape of one of the mating teeth can be taken and applying the law of gearing, the shape of the other can be determined. Such gear are said to be have conjugate teeth and the action between them, is called conjugate action.

Length of path of contact

Consider a pinion driving the wheel as shown in fig.

wheel



Pinion

When the pinion rotates in clockwise direction, the contact between a pair of involute teeth begins at K (on the flank near the base circle of pinion (or) the outer end of the tooth face on the wheel) and ends at L (on the outer end of the tooth face on the pinion (on the flank end near the base circle of the wheel).

The MN is the common Normal at the point of contacts and it is the common tangent to the base circles.

The point 'K' is the intersection of the addendum circle of wheel and the common tangent. The point 'L' is the intersection of the addendum circle of pinion and common tangent.

we have discussed that the length of path of contact is the length of common normal cut-off by addendum circles of the wheel and pinion. Thus the length of path of contact is 'KL' which is the sum of the parts of the path of contacts KP and PL. The part of the path of contacts 'KP' ~~and PL~~ is known as path of approach and the part of the path of contact 'PL' is known as path of recess.

Let $r_A = O_1 L$ = Radius of addendum of pinion

$r = O_1P$ = Radius of pitch circle of pinion

$R_A = O_2 K$ = Radius of addendum of wheel

$R = O_2P$ = Radius of pitch circle of wheel

ϕ = pressure angle

from fig: $O_1M = O_1P\cos\phi \Rightarrow r\cos\phi$

$O_2N = O_2P\cos\phi = R\cos\phi$

from $\angle O_2KN$: $O_2K^2 = KN^2 + O_2N^2$

$\Rightarrow KN = \sqrt{(O_2K)^2 - O_2N^2}$

$\Rightarrow KN = \sqrt{R_A^2 - R^2\cos^2\phi}$

from $\angle O_2PN$

$PN = R\sin\phi$

$\therefore KP = KN - PN = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$

From $\lfloor O_1ML$

$$ML = \sqrt{O_1L^2 - O_1M^2}$$

$$\Rightarrow ML = \sqrt{r_A^2 - r^2\cos^2\phi}$$

From fig.

$$PL = ML - MP = \underline{\sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi}$$

KP = Path of Approach

PL = Path of Recess

∴ Length of path of contact $KL = KP + PL$

$$\boxed{KL = \sqrt{R_A^2 - R^2\cos^2\phi} + \sqrt{r_A^2 - r^2\cos^2\phi} - (R+r)\sin\phi}$$

Length of Arc of contact

Def:- The arc of contact is the path traced by a point on the pitch circle from the begining to the end of the engagement of a given pair of teeth.

the arc of contact is GPH & EPF

GF is also decides in to two parts

Consider the arc of contact GPH,

GP is called Arc of approach and PH is called Arc of recess, From fig :

Length of arc of approach GP = $\frac{\text{Length of path of approach}}{\cos\phi}$

or $GP = \frac{KP}{\cos\phi}$

Ily The length of arc of recess $PH = \frac{PL}{\cos\phi}$

$$\therefore \boxed{\text{The Total Length of arc of Contact } (GH) = \frac{KL}{\cos\phi}}$$

Contact Ratio (or) Number of pairs of teeth in contact

It is defined as the ratio of Length of Arc of Contact to the Circular pitch.

$$\therefore \text{contact Ratio} = \frac{\text{Length of Arc of Contact}}{\text{Circular pitch}}$$

$$= \frac{GH}{p_c} = \frac{GH}{\pi m}$$

UNDER CUTTING

The fig. shows a pinion. A portion of its deddendum falls inside the base circle. The profile of the tooth inside the base circle is radial. If the addendum of its mating gear is more than the limiting value, it interferes with the dedendum of the pinion and two gears are locked.

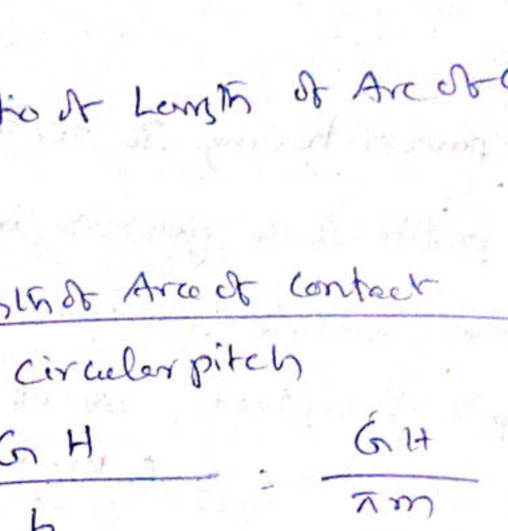
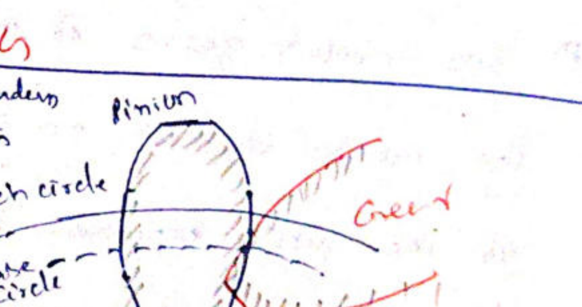


fig (a)

However, if a cutting rake having similar teeth is used to cut the teeth in the pinion, it will remove the portion of the pinion tooth which would have interferred with the gear as shown in fig (b). A Gear having its material removed in this manner is said to be "Undercut" and the process is called Under cutting.



Undercut

fig (b)

* Under cutting will not take place if the teeth are designed to avoid an Interference.

Interference in involute Gears

the tip of tooth on the pinion will then Undercut the tooth on the wheel at the root and remove the part of the involute profile of tooth on the wheel. This effect is known as interference, and occurs when the teeth are being cut, In brief, the phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.

Therefore the interference may only be prevented, if the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency.

when the interference is just prevented the maximum length of path of contact is 'MN', when the maximum addendum circles for pinion and wheel pass through the points of tangency N and M respectively.

the Maximum length of path of approach $MP = r\sin\phi$

the Maximum length of path of recess $PN = R\sin\phi$

∴ The Maximum length of path of contact $MN = (r+R)\sin\phi$

the Maximum length of arc of contact $= (r+R)\tan\phi$

Note: In case of the addenda on a pinion and wheel is such that the path of approach and path of recess are half of their maximum possible values

(1) ∴ The path of approach (KP)

$$KP = \frac{1}{2} MP$$

$$\Rightarrow \boxed{\sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi = \frac{1}{2} r\sin\phi}$$

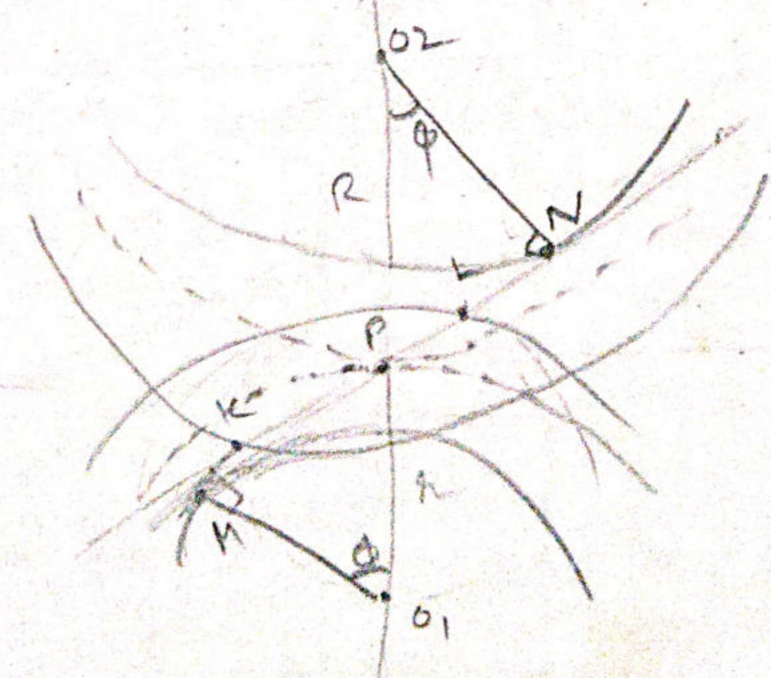
(2) ∴ The path of recess (PL)

$$PL = \frac{1}{2} PN$$

$$\Rightarrow \boxed{\sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi = \frac{R\sin\phi}{2}}$$

(3) The Length of path of contact (KL) $= \frac{1}{2} MN$

$$\Rightarrow \sqrt{R_A^2 - R^2\cos^2\phi} + \sqrt{r_A^2 - r^2\cos^2\phi} - (R+r)\sin\phi = \frac{(R+r)\sin\phi}{2}$$



Unit VII GEAR TRAINS

Some times, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called gear train (or) train of toothed wheels.

The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shaft. A gear train may consist of spur, bevel or spiral gears.

Types of Gear Trains

The following are the different types of gear trains depending upon the arrangement of wheels.

1. Simple gear train 2. Compound gear train
3. Reverted gear train 4. Epicyclic gear train.

In the first three types of gear trains the axes of the shafts over which the gears are mounted are fixed relative to each other. But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis,

Simple Gear Train

When there is only one gear on each shaft, as shown in fig.(1) it is known as "simple gear train". The gears are represented by their pitch circles.

When the distance between the two shafts is small, the two gear 1 and 2 are made to mesh with each other to transmit motion from one shaft to other shaft.

$$\frac{N_1}{N_3} = \frac{T_3}{T_1}$$

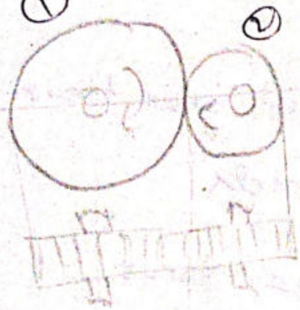
$\frac{N_1}{N_2} = \frac{T_2}{T_1}$ $\quad \frac{N_2}{N_3} =$

$\frac{N_1}{\cancel{N_2}} \times \frac{\cancel{N_2}}{N_3} = \frac{\cancel{T_2}}{T_1} \times \frac{T_3}{\cancel{T_2}}$

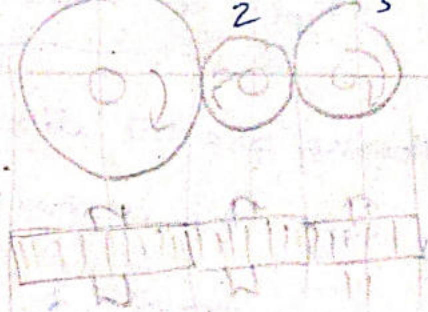
$\frac{N_1}{N_3} = \frac{T_3}{T_1}$

Speed Ratio =

(1) Driver follower (2)



1 2 3



<u>Speed Ratio (velocity Ratio)</u>:- It is defined as the ratio of speed of the driver ~~follower~~ to the speed of drivers (follower)

$$\text{Speed Ratio} = \frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{T_2}{T_1}$$

<u>Train value</u>:- It is the Reciprocal of Speed Ratio i.e the Ratio of speed of follower to the speed of driver

$$\text{Train value} = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{T_1}{T_2}$$

Note:- If the distance between the two shafts is large, then it is obtained by the following two methods

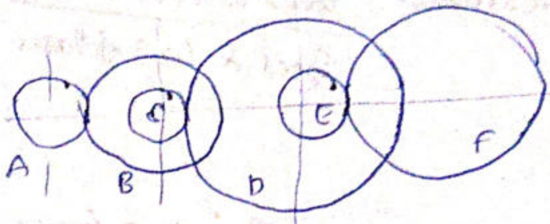
(i) By providing the large size Gear

(ii) By providing one or more intermediate gears, (idler gears)

~~Speed Ratio = Speed of First driver / Speed of last follower = Product of No of teeth on drivens (followers) / Product of No of teeth on drivers~~

(11) The gearing of a machine tool is shown in fig. The motor shaft is connected to gear 'A' and rotates at 975 r.p.m. The gear wheels B, C, D and E are fixed to parallel shafts rotating together. The final gear 'F' is fixed ~~is~~ on the output shaft. What is the speed of gear 'F'? The number of teeth on each gear are as given below. ($N_F = 52$ r.p.m)

Gear :-	A	B	C	D	E	F
No. of Teeth :-	20	50	25	75	26	65



(12) Two parallel shafts, about 600 mm apart are to be connected by spur gears. One shaft is to run at 360 r.p.m and the other at 120 r.p.m. Design the gears if the circular pitch is to be 25 mm. ($T_1 = 38$; $T_2 = 3T_1 = 114$; ($T_1 = \frac{\pi d_1}{P_c}$; $T_2 = \frac{\pi d_2}{P_c}$. New $d_1' = 302.36$ m; $d_2' = 907.1$ m, $x' = 604.73$ m

---

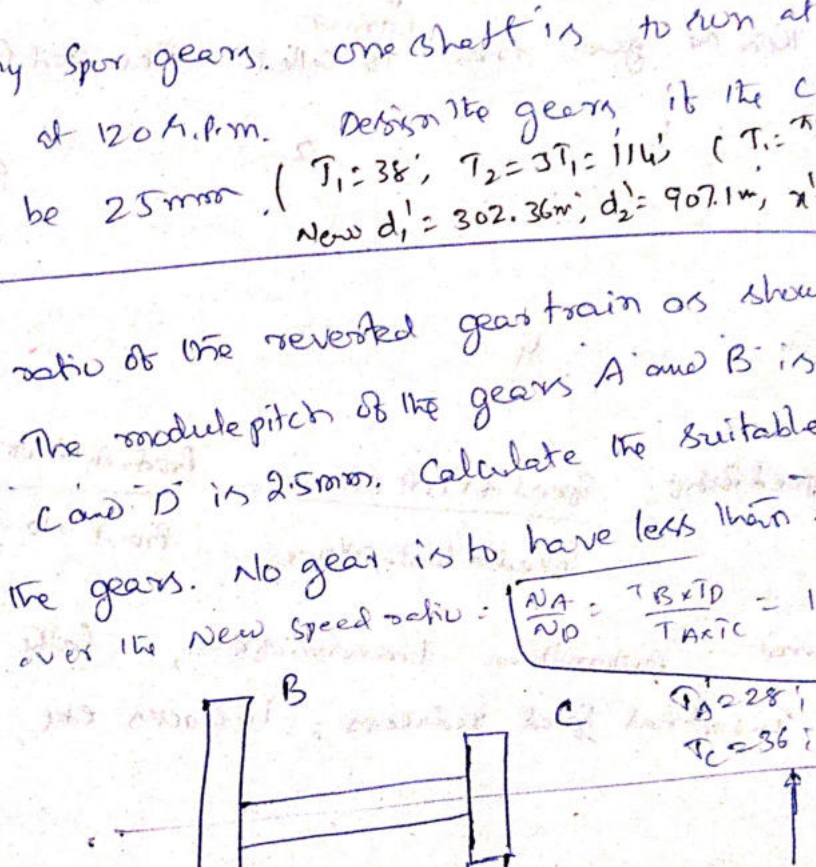
(13) The speed ratio of the reverted gear train as shown in fig. is to be 12. The module pitch of the gears A and B is 3.125 mm and of gears C and D is 2.5 mm. Calculate the suitable number of teeth for the gears. No gear is to have less than 24 teeth. [After problem over the New speed ratio : $\boxed{\frac{N_A}{N_D} = \frac{T_B \times T_D}{T_A \times T_C} = 12.3}$

$T_A = 28$; $T_B = 100$
$T_C = 36$; $T_D = 124$

Hint

$\frac{N_A}{N_B} \times \frac{N_C}{N_D} = \sqrt{12} \times \sqrt{12}$

$\boxed{\frac{T_B}{T_A} = \frac{T_D}{T_C} = \sqrt{12}}$



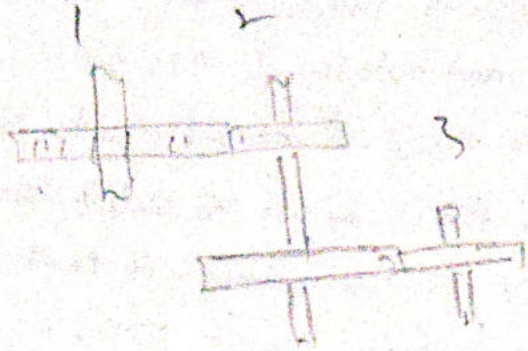
$m = \frac{2r_A}{T_A} = \frac{2r_B}{T_B}$

[Hinting

$\frac{N_A}{N_D} = 12$; $m_A = m_B = 3.125$; $m_C = m_D = 2.5$

$r_A + r_B = r_C + r_D = 200$

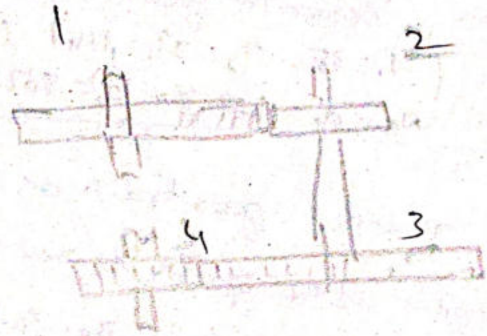
Compound Gear



Speed Ratio:- $\dfrac{\text{Speed of first driver}}{\text{Speed of Last follower}} = \dfrac{\text{Prod. of No. of teeth on followers}}{\text{Prod. of No. of teeth on drivers}}$

Reverted Gear Train

When the axes of the first gear and last gear (follower) are co-axial, then the gear train is called Reverted Gear train
(driver)



$r_1 + r_2 = r_3 + r_4$

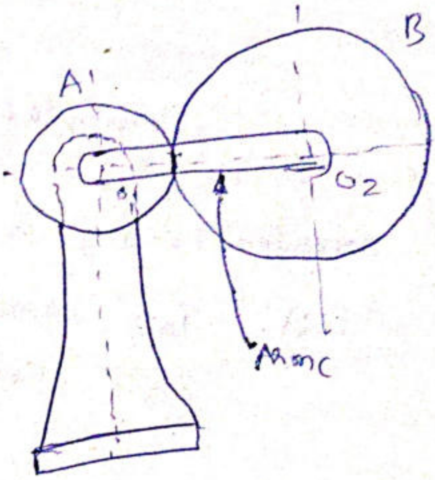
$T_1 + T_2 = T_3 + T_4$

Speed Ratio = $\dfrac{\text{Speed of first driver}}{\text{Speed of last follower}} = \dfrac{\text{Prod. of No. of teeth on followers}}{\text{Pro. of No. of teeth on drivers}}$

Applications:- Automotive transmission, Lathe Back gears, Industrial Speed reducers, in clocks etc.

Epicyclic Gear Train

In an epicyclic geartrain, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in fig.



Where a gear 'A' and the arm 'C' have a common axis at O_1 about which they can rotate. The Gear 'B' meshes with gear 'A' and has its axis on the arm at O_2, about which the gear 'B' can rotate. If the arm is fixed, the gear train is simple and gear 'A' can drive gear B or vice-versa, but if gear 'A' is fixed and the arm is rotated about the axis of gear A (i.e O_1), then the gear 'B' is forced to rotate upon and around gear 'A'. Such a motion is called epicyclic and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as epicyclic gear trains ('epi' means upon and 'cyclic' means around). The epicyclic gear trains may be simple and compound.

Velocity Ratio of Epicyclic Gear Train

The following two methods may be used for finding out the velocity ratio of an epicyclic gear train.

1. Tabular Method 2. Algebraic Method.

1. Tabular Method

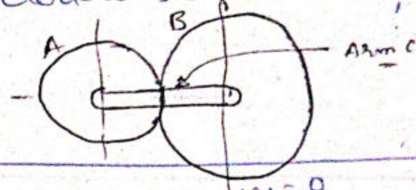
Consider the fig. of simple epicyclic gear train

Let T_A = Number of teeth on gear A.

T_B = Number of teeth on gear B.

Step No.	Conditions of motion	Revolutions of elements: Arm C (N_1)	Gear A (N_A)	Gear B (N_B)
1.	Arm fixed - gear A rotates through +1 revolution i.e 1 rev anticlockwise	0	+1	$-\frac{T_A}{T_B}$
2.	Arm fixed - gear A rotates through +x revolutions	0	+x	$-x \times \frac{T_A}{T_B}$
3.	Add +y revolutions to all elements	+y	+y	+y
4.	Total motion	+y	x+y	$y - x \times \frac{T_A}{T_B}$

① In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m in the anti clockwise direction about the centre of the gear A which is fixed, Determine the speed of gear B. If the gear A instead of being fixed, makes 300 rpm. in the clockwise direction, what will be the speed of gear B.?



Sol:- Given Data $T_A = 36$; $T_B = 45$; (1) Arm $N_C = +150$ rpm $N_B = ?$

(2) $N_A = -300$ rpm ; $N_B = ?$

SNO	Condition of motion	Revolution of Elements: Arm C	Gear A (N_A)	Gear B (N_B)
1.	Arm C is fixed and give +1 rev to Gear A	0	$N_A = +1$	$N_B = -\frac{T_A}{T_B}$
2	Arm C is fixed and give +x rev to Gear A	0	$N_A = +x$	$N_B = -x \cdot \frac{T_A}{T_B}$
3.	Add +y rev to all elements and the total motion	+y	$N_A = x + y$	$N_B = y - x\frac{T_A}{T_B}$

①

$N_C = +150$ rpm $= +y$ ∴ $y = 150$ rpm

N_A is fixed $N_A = 0$; $x + y = 0$; $x = -150$ rpm

∴ $N_B = y - x\frac{T_A}{T_B} = 150 - (-150) \cdot \frac{36}{45} = +270$ rpm

(2). $N_A = -300$ rpm $\qquad N_B = ?$

$y = 150$ rpm

$N_A = x + y$; $\quad -300 = x + y$

$x = -300 - y = -450$ rpm

$\therefore N_B = y - x\left(\frac{T_A}{T_B}\right) = 150 - (-450) \times \frac{36}{45}$

$= +510$ rpm

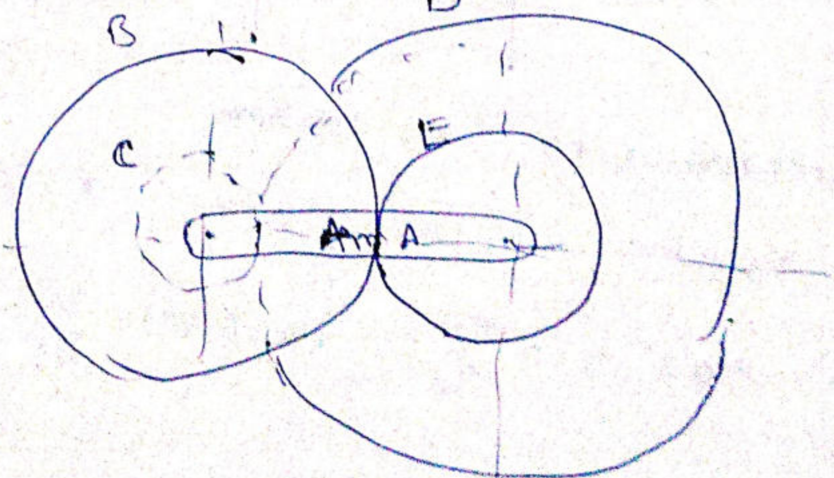
---

(1) In a reverted epicyclic gear train, the arm 'A' carries two gears 'B' and 'C' and a compound gear D-E. The Gear 'B' meshes with gear 'E' and gear 'C' meshes with Gear 'D'. The Number of teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed and direction of gear C when the Gear B is fixed and the arm makes 100 r.p.m. clockwise.

Sol. Given Data

$T_B = 75$; $T_C = 30$; $T_D = 90$

$T_E = ?$



S.No	Condition of motion	revolution of elements			
		Arm A	Compound Gear D-E	Gear B	Gear C
1	Arm A fixed and +1 rev to D-E ($N_D = N_E = +1$)	0	$N_D = N_E = +1$	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm A fixed and +x rev to DE	0	$+x$	$-x\frac{T_E}{T_B}$	$-x\frac{T_D}{T_C}$
3	Add +y rev to all elem & total motion	$+y$	$x+y$	$y - x\frac{T_E}{T_B}$	$y - x\frac{T_D}{T_C}$
(4)	~~Total motion~~				

B

$$T_B + T_E = T_C + T_D$$

$$T_E = 30 + 90 - 75 = \underline{45}$$

when the Gear 'B' is fixed $N_B = 0$; Arm A, $N_A = -100$ rpm

$N_C = ?$

$$\therefore N_A = +y \Rightarrow \underline{y = -100 \text{ rpm}}$$

$$N_B = 0 = y - x\frac{T_E}{T_B}$$

$$\Rightarrow x\frac{T_E}{T_B} = y \Rightarrow x = y \times \frac{T_B}{T_E}$$

$$\Rightarrow x = -100 \times \frac{75}{45} = \underline{-166.67 \text{ rpm}}$$

$$\therefore N_C = y - x\frac{T_D}{T_C} = -100 - (-166.67) \times \frac{90}{30}$$

$$= +400 \text{ r.p.m.}$$

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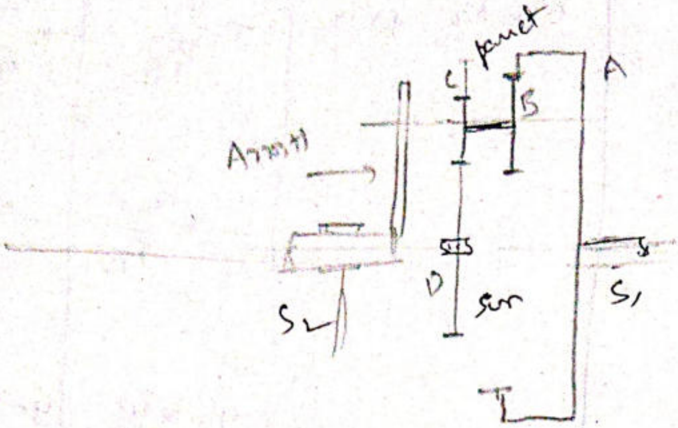
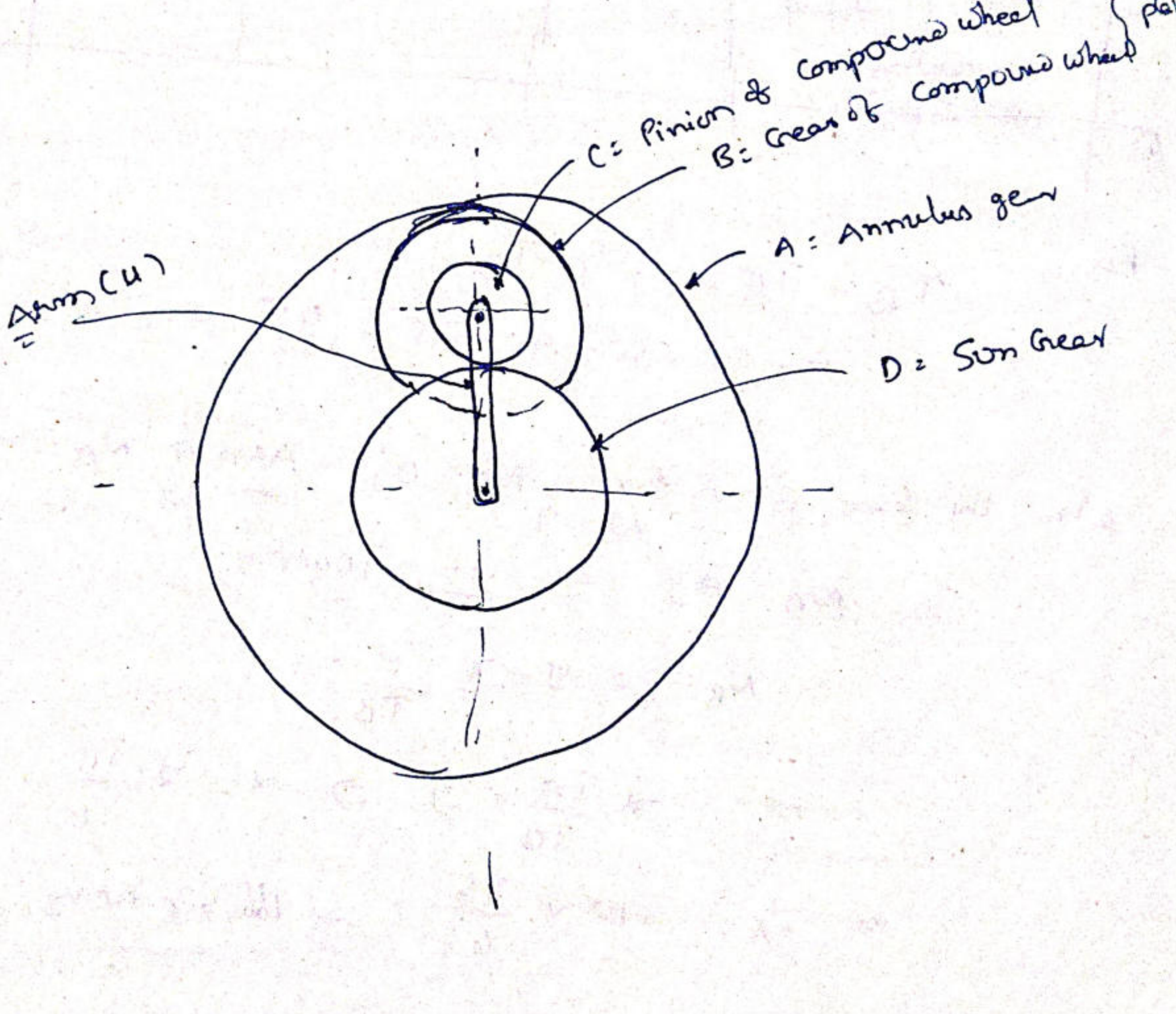
Compound - epicyclic Gear-Train

(Sun & Planet Gear)





S.No	Condition of motion	Arm (H)	Gear 'D' (Driver)	Compound Gear C&B	Annular Gear A
		N_H	N_D	$N_C = N_B$	N_A
1.	Arm 'H' is fixed. Gear 'D' rotates with +1 rev. in anticlockwise	0	+1	$= -\frac{T_D}{T_C}$	$= -\frac{T_D}{T_C} \times \frac{T_B}{T_A}$
2.	Arm 'H' is fixed. Gear 'D' rotates with +x rev in anticlockwise	0	$+x$	$= -x \times \frac{T_D}{T_C}$	$= -x \frac{T_D}{T_C} \times \frac{T_B}{T_A}$
3.	Add +y rev all the elements & the total motion	+y	+y	$= y - x \frac{T_D}{T_C}$	$= y - x \frac{T_D}{T_C} \times \frac{T_B}{T_A}$
4.					

HW

An epicyclic train is shown in fig. is composed of a fixed annular wheel 'A', having 150 teeth. A wheel 'B' is meshing with 'A', and it drives wheel 'D', through an idle whee 'C', and 'D' being concentric with 'A'. The wheels 'B' and 'C' are carried on an arm which revolves at 100 r.p.m., about the axis of 'A' or 'D'. If the wheels 'B' and 'D' are having 25 teeth and 40 teeth respectively, find the Number of teeth of 'C' and the speed and sence of rotation of 'C'.

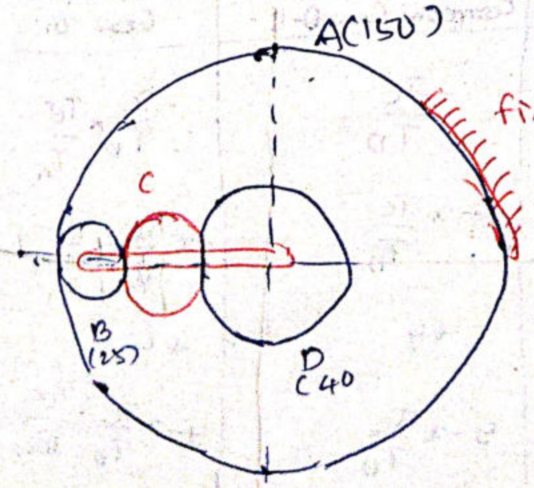
Ans:

$T_A = 2T_B + 2T_C + T_D$

$\therefore T_C = 30$

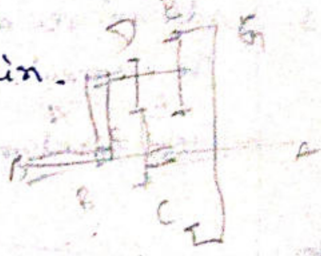
$y = -100$; $x = 100$

$N_C = -600$ rpm (clockwise)



(1) Two shafts 'A' and 'B' are co-axial. A gear C (50 teeth) is rigidly mounted on shaft 'A'. A compound Gear D-E gears with 'C' and an internal gear G. The gear 'D' has 20 teeth and gears with 'C' and 'E' has 35 teeth and gears with gear 'G'. The gear G is fixed and is Concentric with the shaft axis. The compound gear D-E is mounted on a pin which projects from an arm keyed to the shaft 'B'. Sketch the arrangement and find the number of teeth on internal gear G assuming that all gears have the same module. If the shaft 'A' rotates at 110 r.p.m (N_C) find the speed of the shaft 'B'.

Assume $T_D = 20$ $T_E = 35$



Sol:- sketch- Same like sun & planet gear train.

Number of teeth on internal gear G.

$$\frac{d_G}{2} = \frac{d_C}{2} + \frac{d_D}{2} + \frac{d_E}{2}$$

(or) $d_G = d_C + d_D + d_E$

(or)

$$T_G = T_C + T_D + T_E = \underline{105}$$

		Revolution of elements			
S.No	condition of motion	Arm (Shaft)	Gear 'C'	Compound Gear D-E	Gear 'G'
(1)		0	+1	$-\frac{T_C}{T_D}$	$-\frac{T_C}{T_D} \times \frac{T_E}{T_G}$
		0	$+x$	$-x\frac{T_C}{T_D}$	$-x\frac{T_C}{T_D} \cdot \frac{T_E}{T_G}$
		$+y$	$+y$	$+y$	$+y$
		y	$x+y$	$y - x\frac{T_C}{T_D}$	$y - x\frac{T_C}{T_D} \cdot \frac{T_E}{T_G}$

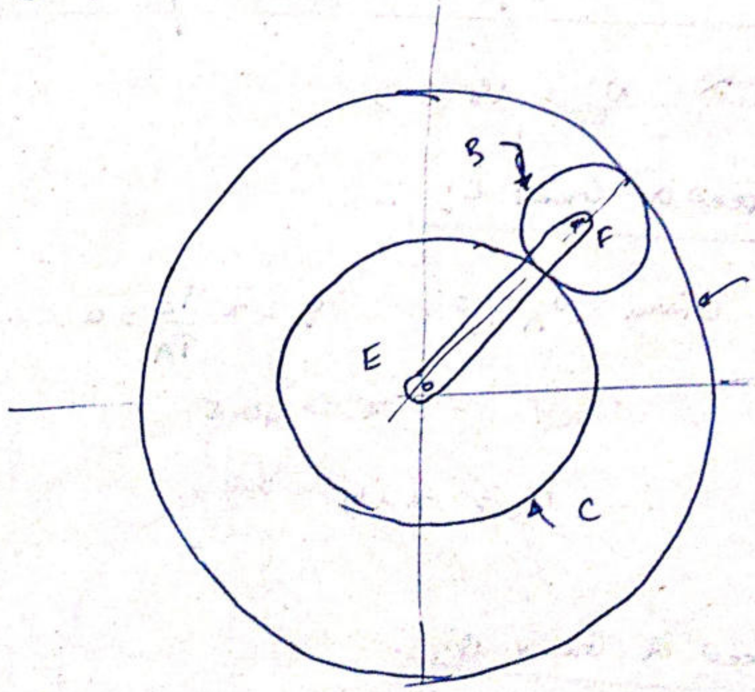
(i) $N_G = 0 \Rightarrow \quad y - x \frac{T_C}{T_D} \cdot \frac{T_E}{T_G} = 0; \Rightarrow y - \frac{5}{6}x = 0$ — (i)

(ii) $N_C = 110$ rpm $\Rightarrow \quad x + y = 100$ — (ii)

$x = 60$; $y = 50$

$\therefore$ Shaft B $= +y = +50$ anticlockwise speed ans.

(P) An epicyclic gear consists of three gears A, B, and C as shown in fig. The gear 'A' has 72 internal teeth and gear 'C' has 32 external teeth. The gear 'B' meshes with both 'A' and 'C' and is carried by an arm EF which rotates about the center of 'A' at 18 r.p.m. If the gear 'A' is fixed, determine the speed of gears 'B' and 'C'.



Sol:-

$$\frac{T_A}{2} = \frac{T_C}{2} + T_B \Rightarrow \boxed{T_A = T_C + 2T_B}$$

S.No	condition of motion	Revolution of elements			
		Arm EF	Gear C	Gear B	Gear A
1.	Arm fixed, Gear 'C' rotates +1 rev (Anticlo	0	+1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B}\times\frac{T_B}{T_A} = -\frac{T_C}{T_A}$
2.	Arm fixed, Gear 'C' rotate +x Rev	0	+x	$-x\frac{T_C}{T_B}$	$= -x\frac{T_C}{T_A}$
3.	Add +y Rev to all elements	+y	+y	+y	+y
4.	Total motion	+y	x+y	$y - x\frac{T_C}{T_B}$	$y - x\frac{T_C}{T_A}$

$N_{EF} = 18 = y$

(1) Speed of Gear C :-

Given $N_A = 0$; $y - x\frac{T_C}{T_A} = 0$

$\Rightarrow x = 40.5$

$\therefore N_C = x+y = 40.5+18 = 58.5$ in the direction of Arm (Anticlo

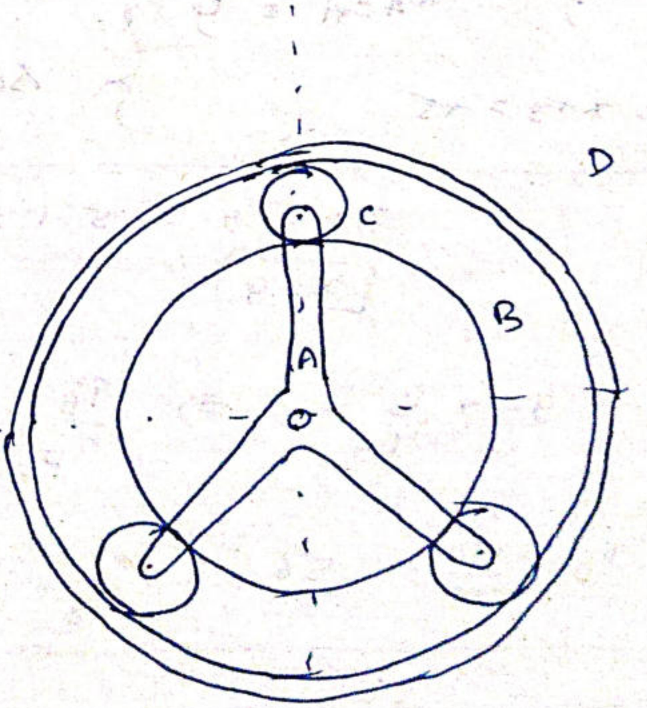
(2) Speed of Gear B :-

$N_B = -46.8$ rpm.

(opposite direct to Arm clockwise)

(P) In an epicyclic gear of the 'sun and planet' type shown in fig. The pitch circle diameter of the internally toothed ring is to be 224 mm and the module 4 mm. When the ring 'D' is stationary, the spider A, which carries three planet wheels 'C' of equal size, is to make one revolution in the same sence as the sun wheel 'B' for every five revolutions of the driving spindle carrying the sun wheel B. Determine suitable numbers of teeth for all the wheels.

Sol:- $d_D = 224mm$; $m = 4mm$; $\frac{N_A}{N_B} = \frac{1}{5}$



S.No	Condition of motion	Revolution of elements: Spider 'A'	Sunwheel B	Planet C	Internal Gear D
1.	Spider 'A' fixed, Sunwheel 'B' rotates +1 Rev (Anticlock)	0	+1	$-\frac{T_B}{T_C}$	$-\frac{T_B}{T_C}\times\frac{T_C}{T_D} = -\frac{T_B}{T_D}$
2.	Spider 'A' fixed, Sunwheel B rotates +x Rev (Anticlock)	0	+x	$-x\frac{T_B}{T_C}$	$= -x\frac{T_B}{T_D}$
3.	Add +y Rev to all elements	+y	+y	+y	+y
4.	Total motion	+y	x+y	$y-x\frac{T_B}{T_C}$	$y-x\frac{T_B}{T_D}$

$N_B = 5$; $N_A = +1 = y$

$N_B = x+y = +5$ } Same sense

$x = 5-y = 5-1 = 4$

$\boxed{x=4}$

$N_D = 0$; $y - x\frac{T_B}{T_D} = 0 \Rightarrow \frac{T_B}{T_D} = \frac{1}{4}$

But $T_D = \frac{d_D}{m} = 56$; $T_B = 14$

from Diag $T_D = T_B + 2T_C$

$\Rightarrow T_C = 21$