



ANNAMACHARYA UNIVERSITY

EXCELLENCE IN EDUCATION; SERVICE TO SOCIETY
(ESTD UNDER AP PRIVATE UNIVERSITIES (ESTABLISHMENT AND REGULATION) ACT, 2016)
RAJAMPET-516126:A.P; INDIA

DEPARTMENT OF MECHANICAL ENGINEERING

LECTURE NOTES

COURSE TITLE	: BASIC THERMODYNAMICS
COURSE CODE	: 24AMEC31T
REGULATION	: AU-24
YEAR & SEMESTER	: II YEAR I SEM
CREDITS	: 3
ACADEMIC YEAR	: 2026-27
PREPARED BY	: Mr.C.Ramanjaneyulu



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(UNIVERSITY LISTED IN UGC AS PER THE SECTION 2(f) OF THE UGC ACT, 1956)

RAJAMPET, Annamayya District, AP – 516126, INDIA

Title of the Course: Basic Thermodynamics
Category: ES
Semester: III Semester
Course Code: 24AMEC31T
Branch/es: Mechanical Engineering

Lecture Hours

3

Tutorial Hours

-

Practice Hours

-

Credits

3

Course Objectives:

1. To impart the awareness on laws of thermodynamics.
2. To enable the students to understand second law of thermodynamics and its applications to various systems.
3. To familiarize steam properties to understand working of steam power plants.
4. To acquire knowledge on equations of state and properties of gas mixtures.
5. To understand the inter-relationship between various air standard cycles used in gas power cycles.

Course Outcomes:

At the end of the course, the student will be able to

1. Apply the first law of thermodynamics for various energy systems
2. Analyze the performance of thermal engineering devices by using Second law of Thermodynamics.
3. Solve the properties of pure substances by using steam tables.
4. Calculate the P-V-T properties of gases and mixtures.
5. Analyze the Air standard cycles by using the thermodynamic principles

Unit 1 Basic Concepts & First Law of Thermodynamics

12

System, Control Volume, Surrounding, Boundaries, Universe, Types of Systems, Macroscopic and Microscopic viewpoints, Concept of Continuum, Thermodynamic Equilibrium, State, Property, Process, Cycle – Reversibility, Quasi – static Process, Irreversible Process, Causes of Irreversibility – Energy in State and in Transition - Types, Work and Heat, Point and Path function.

Zeroth Law of Thermodynamics – Ideal Gas Scale – PMM I - Joule's Experiments – First law of Thermodynamics – Corollaries – First law applied to a Process – applied to a flow system – Steady Flow Energy Equation

Unit 2 Second Law of Thermodynamics

12

Limitations of the First Law – Thermal Reservoir, Heat Engine, Heat pump, Parameters of performance, Second Law of Thermodynamics, Kelvin-Planck and Clausius Statements and their Equivalence / Corollaries, PMM of Second kind, Carnot's principle, Carnot cycle and its specialties, Thermodynamic scale of Temperature.

Clausius Inequality, Entropy, Principle of Entropy Increase – Energy Equation, Availability and Irreversibility Elementary Treatment of the Third Law of Thermodynamics.

Unit 3 Properties of Pure Substances

10

Pure Substances-, P-V-T- surfaces, T-S and h-s diagrams, Mollier Charts, Phase Transformations – Triple point at critical state properties during change of phase, Dryness Fraction – Clausius – Clapeyron Equation, Constructional use of Property tables. Mollier charts – Various Thermodynamic processes and energy Transfer – Steam Calorimetry.

Thermodynamic relations: -Thermodynamic Potentials, Gibbs and Helmholtz Functions, Maxwell Relations.

Unit 4 Perfect Gas Laws & Mixture Of Perfect Gases**13**

Equation of State, specific and Universal Gas constants – various Non-flow processes, properties, end states, Heat and Work Transfer, changes in Internal Energy – Throttling and Free Expansion Processes – Flow processes – Deviations from perfect Gas Model – Vander Waals Equation of State – Compressibility charts – variable specific Heats – Gas Tables.

Mole Fraction, Mass fraction Gravimetric and volumetric Analysis – Dalton's Law of partial pressure, Avogadro's Laws of additive volumes – Mole fraction, Volume fraction and partial pressure, Equivalent Gas constant and Molecular Internal Energy, Enthalpy, specific heats and Entropy of Mixture of perfect Gases and Vapour.

Unit 5 Air Standard Cycles**06**

Otto, Diesel and Dual cycles, P-V and T-S diagrams - Description and Efficiencies- Mean effective pressures. Comparison of Otto, Diesel and Dual cycles.

Prescribed Textbooks:

1. Engineering Thermodynamics. PK Nag, TMH, 6th edition, 2017.
2. Thermodynamics – An Engineering Approach. Yunus Cengel & Boles, TMH. McGraw Higher Education 9th edition, 2019.

Reference Books:

1. Fundamentals of Thermodynamics. Sonntag, Borgnakke and Van Wylen, John Wiley & sons (ASIA) Pvt Ltd. Publisher: Wiley; 8 edition (December 26, 2012).
2. Thermodynamics. Mc Graw Hill J. P. Holman, McGraw-Hill College; 4th edition (January 1, 1988).
3. An introduction to Thermodynamics. YVC Rao, Universities Press, 3rd edition 2004.
4. Engineering Thermodynamics, Jones & Dugan, PHI INDIA (2011).
5. Basic Engineering Thermodynamics. A. Venkatesh, Universities Press; First edition (2007).

Web resources:

1. <https://www.youtube.com/watch?v=9GMBpZZtjXM&list=PLD8E646BAB3366BC8>
2. https://www.youtube.com/watch?v=pMmHdWvN_Fl&list=PLyqSpQzTE6M_QOKxVxZ5nQ48gOkzg7zWP&index=1
3. <https://www.youtube.com/watch?v=2LPQX4F-GoA&list=PLwdnzlV3ogoWV-n1YItO933MxgPXfEiM>
4. <https://www.youtube.com/watch?v=ZXHbqJj5mk4&list=PLjA7ulhseb0-tML7eIONaelFTts6xoG5B>

CO-PO Mapping:

Course Outcomes	Engineering Knowledge	Problem Analysis	Design/Development of solutions	Conduct investigations of complex problems	Engineering tool usage	The Engineer and the World	Ethics	Individual and Collaborative Team work	Communication	Project management and finance	Life-long learning	PSO1	PSO2
24AMEC31T .1	3	2	1	1	-	-	-	-	1	-	1	2	2
24AMEC31T .2	3	3	1	1	-	-	-	-	1	-	1	2	3
24AMEC31T .3	3	2	1	1	-	-	-	-	1	-	1	2	1
24AMEC31T .4	3	2	1	1	-	-	-	-	1	-	1	2	1
24AMEC31T .5	3	3	1	1	-	-	-	-	1	-	1	2	2

THERMODYNAMICS :-

- It is the science which deals with Energy.
- Energy can be viewed as the ability to cause changes.
- Thermodynamics includes the aspects of energy & energy transformation, energy conversions, including power generation, refrigeration & energy savings.
- It studies the relationship b/w the properties of system.
- It is based upon the observation of common experience which have been formulated into thermodynamic laws.

Thermodynamics
(Greek word) ┌→ Therme (heat)
└→ Dynamics (power or force)

⊛ Thermodynamics is based on four laws namely zeroth, first, second & third law of thermodynamics.

- Zeroth law builds the knowledge of thermal equilibrium and establishes the concept of temperature.
- First law introduces the concept of internal energy.
- Second law introduces the concept of Entropy.
- Third law enables the evaluation of absolute entropy.

Applications of Thermodynamics :-

All activities in nature involve some interaction

b/w energy & matter.

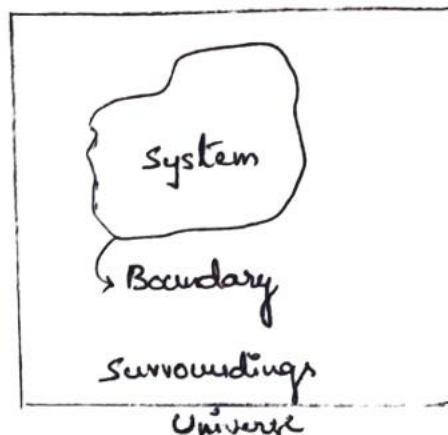
- Human body, where heart is pumping the blood to all parts of human body. Various energy conversions occur in body cells & the heat generated by the body is constantly rejected to the environment.
- Many house hold appliances are designed by using the principles of thermodynamics. Some of the examples are
 - Heating & air conditioning systems
 - Refrigerators, humidifiers & pressure cookers
 - Water heater, shower & the iron
 - Even the computer & T.V.
- Thermodynamics plays a major part in the design & analysis of
 - Automotive engines
 - Rockets & jet engines
 - Conventional or nuclear power plants & solar collectors
 - Design of vehicles from car to aeroplanes.
 - Design of turbines
 - In chemical process plants & many industrial applications etc.

SYSTEM

A system is defined as a quantity of matter or a region in space chosen for study.

Surroundings :-

The mass or region outside the system is called surroundings.



Boundary :- The real or imaginary

surface that separates the system from its surroundings is called the boundary. It may be fixed or movable.

Boundary has zero thickness, and thus it can neither contain any mass nor occupy any volume in space.

Universe :-

System & surroundings together comprises as Universe.

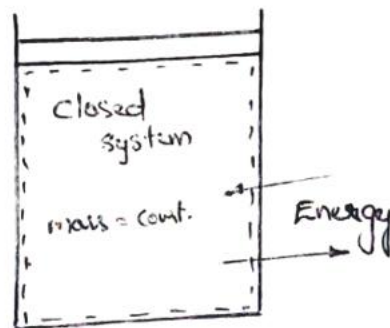
$$\text{Universe} = \text{system} + \text{surroundings}.$$

* In order to understand this concept, best example is piston & cylinder arrangement. It has 3 fixed & one movable boundary.

Types of systems :-

- ① closed system ② open system ③ Isolated system.

① Closed system :- In this system, mass remains constant within the boundary of the system & energy



may transfer across its boundary.

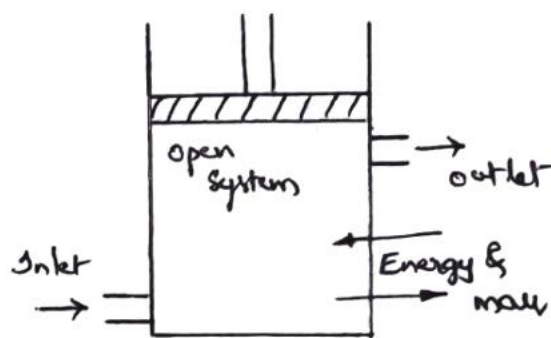
Ex:- Gas contained in piston cylinder arrangement with non-flow process.

In this case when the gas is heated, no mass transfers across the boundary, but the volume may get changed. So energy transfer takes place.

② Open System:- (control volume)

Both mass & energy transfer across the boundary.

Ex:- Compressor, Gas turbines, engines etc.



③ Isolated System:-

Neither mass nor energy get transfer across its boundaries.

Ex:- A fluid enclosed in a perfectly insulated closed vessel. (thermos flask).

A system may be classified as

* Homogeneous

* Heterogeneous

(i) Homogeneous system:- If the system consists of single phase it is called as homogeneous system. It may be either solid, liquid or gaseous state.

Ex:- Mixture of gases, water + nitric acid

(ii) Heterogeneous system:- If the system consists of more than one phase it is called heterogeneous system.

Ex:- water & steam, ice & water, water & oil etc.

Properties of a System:-

Any measurable characteristics which describes a physical condition or state of a system or state of a body is known as a property. Some of the observable characteristics such as pressure, temp, volume etc.

Types of Properties:-

- Intensive properties
- Extensive properties.

(i) Intensive properties:- These properties are those that are independent of the mass of a system such as temperature, pressure & density.

(ii) Extensive properties:- These properties depends on mass of the system. such as total volume, total mass & total momentum; enthalpy, entropy etc.

m
V
T
P
ρ



$\frac{1}{2} m$	$\frac{1}{2} m$
$\frac{1}{2} V$	$\frac{1}{2} V$
T	T
P	P
ρ	ρ

} Extensive properties

} Intensive properties.

An easy way to determine whether a property is intensive or extensive is to divide the system into two equal parts with an imaginary partition. Each part will have the same value of intensive properties as the original system, but half the value of the extensive properties.

Note:- Extensive properties per unit mass are called specific properties
ex:- $v = V/m$ etc.

Difference b/w closed system & Open system:

closed system

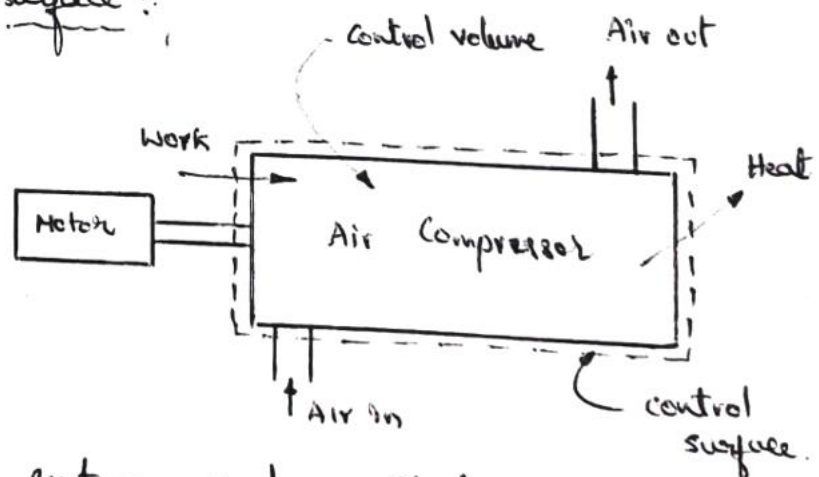
- ① Only energy crosses the boundary.
- ② System contains fixed or constant amount of mass.
- ③ It is not very complicated.
- ④ Boundary of the system does not allow the matter to flow in or out of the system.
- ⑤ Oftenly used in analysis.
- ⑥ Ex: Piston cylinder arrangement without valves.

Open system

- ① Both mass & energy crosses the boundary.
- ② System contains varying amount of mass.
- ③ It is complicated due to the movement of fluids.
- ④ Boundary of the system allows the matter to flow in or out of the system.
- ⑤ Used in most of applications.
- ⑥ Ex:- Engine, compressor, turbines, heaters, power plants etc.

Control volume & Control surface:

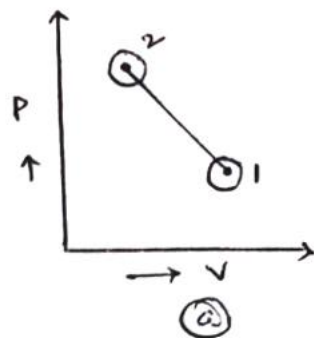
For thermodynamic analysis an open system (compressor) is considered.



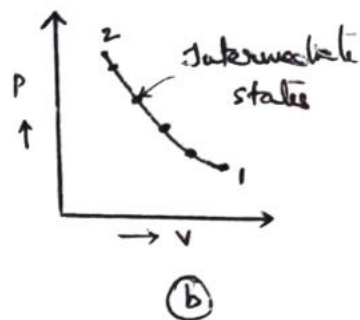
If the volume of a system under study remains constant then that volume is called control volume. The control volume is surrounded by a control surface. Matter as well as energy crosses the control surface.

State :- State is the condition of a system at an instant of time described by its properties such as pressure, temp, density etc.

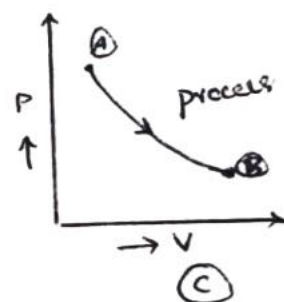
Points ① & ② in the figure shows two different states of a system.



Path :- A thermodynamic system passing through a series of states constitutes a path. (or) It is a line through which system passes when it undergoes a change in state.



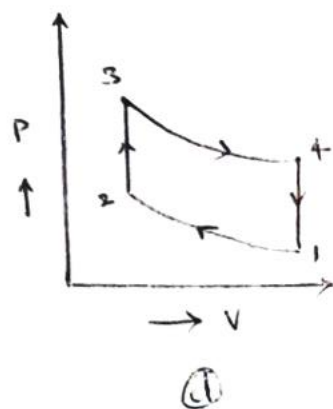
Process :- A process is defined as a transition in which a system changes from initial state to a final state.



Ex:- For example state A is the initial state of the system. Due to expansion of the system the final state is B. The line AB is the process that has taken place. as shown in the figure (c)

Thermodynamic Cycling (or) Cyclic process :-

If a system undergoes a number of different processes and finally reaches to its initial state by forming a complete cycle then it is known as cyclic process.



Thermodynamic Equilibrium :-

A system is said to be in a state of thermodynamic equilibrium if there is no change in any property at all points in a system.

A system will be in a state of thermodynamic equilibrium if the following equilibria are satisfied.

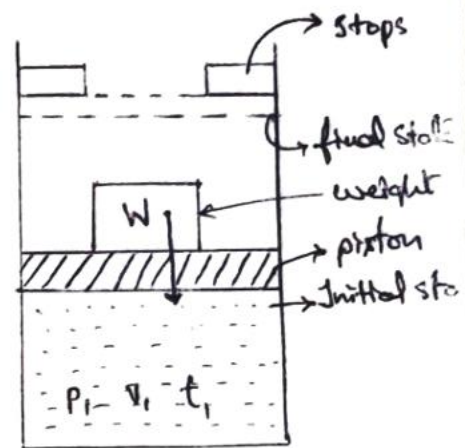
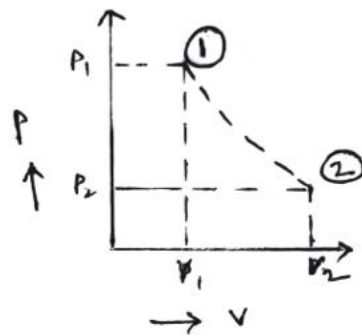
- ① Mechanical Equilibrium :- A system is said to be in a state of mechanical equilibrium if there are no unbalanced forces within the system and/or b/w system & surroundings. [pressure should be constant]
- ② Chemical Equilibrium :- There should not be any chemical reaction within the system and b/w system & surroundings.
- ③ Thermal Equilibrium :- If there exists a uniformity of temperature throughout system or b/w the system & surroundings. (or) (Equality of temperature)
- ④ Electrical Equilibrium :- If there exists a uniformity of electrical potential throughout the system.

Quasi-Static Process :- (Quasi-Equilibrium process)

Let us consider a system of gas contained in a cylinder. The system initially is in an equilibrium state, represented by the properties P, V, T . The weight on the piston just balances the upward force exerted by the gas. If the weight is removed, there will be an unbalanced force b/w the system & surroundings and under gas pressure, the piston will move up till it hits the stops. The system again comes to an equilibrium

state, being described by the properties P_2, V_2, T_2 . But the intermediate states passed through by the system are non-equilibrium states which cannot be described by thermodynamic coordinates. Figure shows point ① & ② as the initial & final equilibrium states joined by a dotted line, which has got no meaning otherwise.

Such a process, which is but a locus of all the equilibrium points passed through by the system, is



known as a quasi-static process, quasi means 'almost'. Infinite slowness is the characteristic feature of a quasi-static process.

A quasi-static process is thus a succession of equilibrium states. A quasi-static process is also called as reversible process.

MICROSCOPIC & MACROSCOPIC APPROACHES:-

Let us consider a gas filled in the container which is containing number of molecules. These molecules will have certain characteristics like position, momentum, velocity, pressure etc.

The analysis of thermodynamic systems can be explained in two ways.

- Microscopic view
- Macroscopic view.

Microscopic Approach :-

This approach considers a system being composed of large no. of particles known as molecules. Each molecule is having different energy & velocity.

These values constantly changes with time as the molecules collide with one another. So the microscopic approach is considered with structure of matter hence it is also called as Statistical thermodynamics.

Ex:- velocity, K.E, momentum etc.

Macroscopic Approach :-

A macroscopic approach is concerned with certain quantity of matter by taking into account which is going at gross level or overall level behaviour. Hence it is referred as classical thermodynamics.

Ex:- pressure, volume, temperature etc.

If a pressure gauge is fitted to the gas filled container the pressure can be measured at any instant.

It is very difficult to measure the physical parameters such as pressure, temperature, volume, impulse etc., with microscopic approach.

MICROSCOPIC VIEW

- ① Every molecule in the matter has its own properties & they move with independent velocities.
- ② Large no. of variables are required for describing a system.
- ③ Measurement of variable is very difficult.
- ④ Time required for measuring the variable is more.
- ⑤ Both mathematical & statistical methods are required for describing a system.
- ⑥ It explains the structure of matter.
- ⑦ It is limited to few applications of engineering problems.
- ⑧ It is also known as Statistical thermodynamics.

MACROSCOPIC VIEW

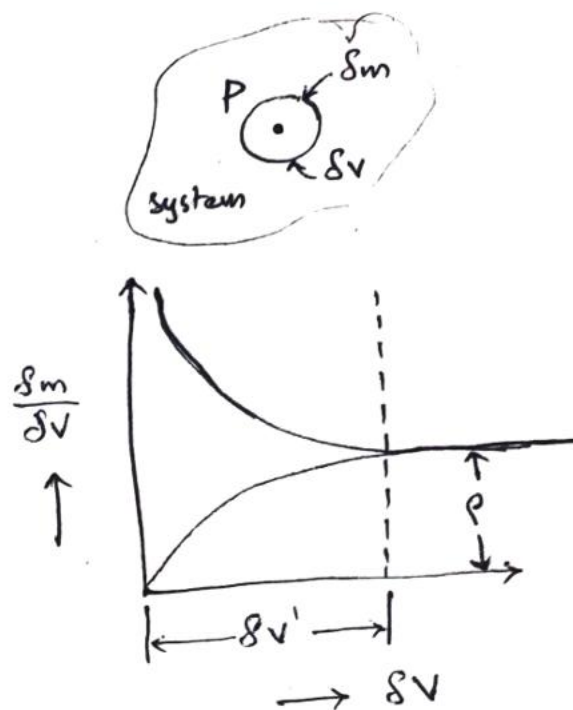
- ① It is assumed that all the molecules behave in the same manner in the system.
- ② Only few variables are required to describe the system.
- ③ The properties are easily measured at any point of time with the help of gauges.
- ④ Time required for measuring the variables is less.
- ⑤ Simple mathematical formulae are sufficient for analysis.
- ⑥ It doesn't explain the structure of matter.
- ⑦ It provides an easy way to the solution of engineering applications.
- ⑧ It is also known as Classical thermodynamics.

Concept of Continuum:-

It is very much useful in classical thermodynamics. From the continuum point of view, the matter is seen as being distributed through space & not as in the particle view, localised. Matter exists in big chunks having mass, energy & momentum as do particles, but it also possesses some additional continuum characteristics such as volume, density & temp etc.

In other words, continuum idealisation treats the substance as being continuous disregarding the action of individual molecules.

Let us consider the mass δm in a volume δv surrounding the point P . The ratio $\delta m / \delta v$ is the avg mass density of the system within the volume δv . We suppose that at first δv is rather large, and is subsequently shrunk about the point P . If we plot $\delta m / \delta v$ against δv , the avg density tends to approach an asymptote as δv increases. However when δv becomes so small as to contain relatively few molecules, ~~pass~~ into & out the avg density ^{fluctuates} ^{substantially} with time as molecules pass into & out of the volume in random motion & so it is impossible to speak of a definite value of $\delta m / \delta v$. The smallest volume which may be regarded as continuous is $\delta v'$.



Mathematically, density at a point may be expressed as,

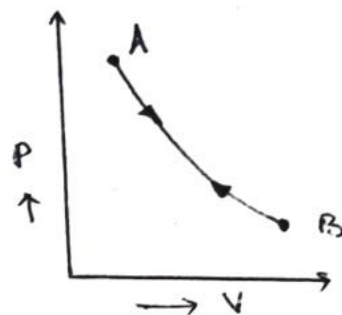
$$\rho = \lim_{\delta v \rightarrow \delta v'} \frac{\delta m}{\delta v} \quad \text{Kg/m}^3$$

where $\delta v'$ is the smallest volume about point P in which the fluid can be called in continuum.

REVERSIBILITY (OR) IDEAL PROCESS :-

A reversible process is one which is performed in such a way that at conclusion of the process, both the system & surroundings may be restored to their initial states, without producing any changes in the rest of the universe.

Let the state of a system be represented by A & let the system be taken to state B by following the path A-B. If the system & also the surroundings are restored to their initial states & no change in the universe is produced, then the process A-B will be a reversible process.



A reversible process is carried out infinitely slowly with an infinitesimal gradient, so that every state passed through by the system is an equilibrium state. So a reversible process coincides with a quasi-static process.

Ex:-

- Frictionless relative motion
- Electrolysis process
- Polytropic expansion or compression of a fluid.
- Isothermal expansion or compression.

Irreversible Process (Irreversibility) :-

Any natural process carried out with a finite gradient is an irreversible process. A reversible process which consists

of a succession of equilibrium states, is an idealized hypothetical process, approached only as a limit. It is said to be an asymptote to reality. All spontaneous processes are irreversible. Time has an important effect on reversibility. If the time allowed for a process to occur is infinitely large, even though the gradient is finite, the process becomes reversible. However, if this time is squeezed to a finite value, the finite gradient makes the process irreversible.

Ex:-

- Fluid flow with friction
- combustion of air & fuel.
- Plastic deformation
- Heat transfer ... etc.

Causes of Irreversibility :-

The irreversibility of a process may be due to either one or both of the following.

- Lack of equilibrium during the process.
- Involvement of dissipative effects.

Irreversibility due to lack of equilibrium :-

The lack of equilibrium (mechanical, thermal or chemical) b/w the system & its surroundings, or b/w two systems or two parts of the same system, causes a spontaneous change which is irreversible.

The following are specific examples in this regard:

- (a) Heat transfer through a finite temp difference.
- (b) Lack of pressure equilibrium within the interior of the system or b/w the system & the surroundings.
- (c) Free expansion.

Irreversibility due to Dissipative Effects:-

The irreversibility of a process may be due to the dissipative effects in which work is done without producing an equivalent increase in the kinetic or potential energy of any system. The transformation of work into molecular internal energy either of the system or of the reservoir takes place through the agency of such phenomena as friction, viscosity, inelasticity, electrical resistance & magnetic hysteresis. These effects are known as dissipative effects & work is said to be dissipated.

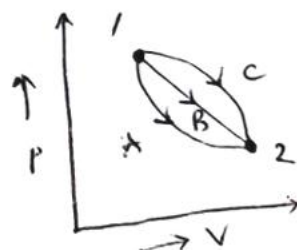
Ex:- (a) Friction

(b) Paddle-wheel work transfer

(c) Transfer of electricity through a resistor.

Point functions & path functions:-

→ Thermodynamic properties are the point functions for a given state where there is one & only one value for each property. This can be located on a graph with a point. This is also known as state functions. These are exact differentials.



Ex: pressure, vol, temp etc.

$$\int_1^2 dp = P_2 - P_1 ; \int_1^2 dV = V_2 - V_1 ; \int_1^2 dT = T_2 - T_1$$

→ Path functions are certain quantities which cannot be located on a graph by a point. These are generally given by area under the curve. These are inexact or partial differentials.

Ex:- Heat, work, energy etc.

$$\int_1^2 \delta Q \neq Q_2 - Q_1, \quad : \quad \int_1^2 \delta W \neq W_2 - W_1$$

Point function	Path function
① change in function depends only on initial & final state of a process i.e., point functions ② Function does not depend on path followed by the system. ③ Differentials are exact & perfect. ④ pressure, vol, temp etc.	① The change in a function depends on a path followed by a system i.e., path functions. ② Function depends on path followed by the system. ③ Differentials are inexact & partial ④ Ex: work, energy etc.

Work :-

Work is the energy transferred (with transfer of mass) because of a property difference (other than temp) that exists b/w the system & surroundings.

$$\text{work} = \text{force} \times \text{displacement} \quad \text{N-m (or) Joules.}$$

Energy :-

Energy is the capacity to do work. In other words, energy is defined as capacity to exert a force through a distance.

It can be classified into

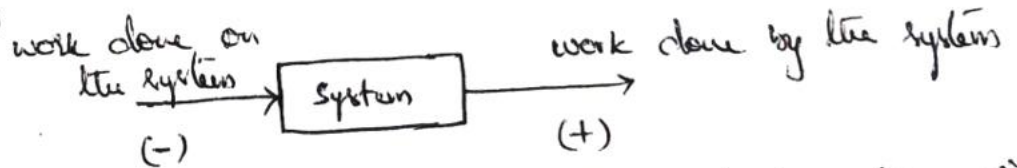
- stored energy (PE, KE, IE etc)
- Energy in transit. (Heat, work, electricity etc)

Energy possessed by a system when it is static is called energy in state i.e., stored energy. It is also known as internal energy.

Energy in transition are mainly

- (a) Work transfer
- (b) Heat transfer

Work transfer :-



Work transfer from the system to the outside is positive

i.e., power producing systems.

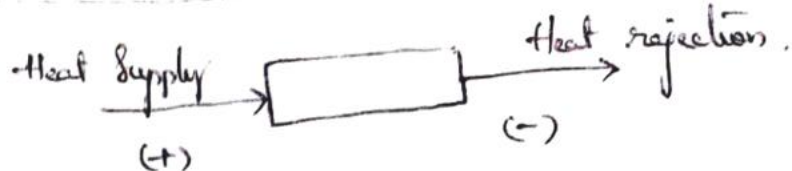
Ex:- Power plants, automobiles, ships, aeroplanes, rockets etc.

Work transfer from outside to the system is negative i.e.

power consuming systems.

Ex:- pumps, motors, iron box, freezer, air conditioner, compressors etc.

Heat transfer:-



Heat transfer from the system to outside is negative.

Ex:- Air conditioners, refrigerators etc.

Heat transfer from outside to system is positive.

Ex:- Heat given to a boiler drum, heat given inside the engine of an automobile etc.



Energy Vs Heat:-

Energy

- ① It is the capacity to do work (or) exert the force through a distance.
- ② It is available in different forms. It changes from one form to another. ex: Mechanical to electrical.
- ③ Energy is divided into two types.
 - (a) Internal energy
 - (b) Energy in transition
- ④ Energy is present in the system & at the boundary of the system.

Heat

- ① Heat is one form of energy that exists due to temperature difference b/w system & surroundings.
- ② It transfers from high temp to low temp systems without mass transfer.
- ③ Heat belongs to energy in transition.
- ④ Heat is present only at the boundary of the system.

Type of TD Process :-

<u>Process</u>	<u>law</u>	<u>Relation</u>
① Const. pressure process	$P = c$	$P_1 = P_2 = P_3$ at all state points
② Const. vol. process	$V = c$	$V_1 = V_2 = V_3$, at all state points
③ Const. Temp process	$T = c ; PV = c$	$P_1 V_1 = P_2 V_2 \dots \dots$
④ Adiabatic process	$P V^\gamma = c$	$P_1 V_1^\gamma = P_2 V_2^\gamma = P_3 V_3^\gamma \dots \dots$
⑤ Polytropic process	$P V^n = c$	$P_1 V_1^n = P_2 V_2^n \dots \dots$

Workdone is given by $W = \int P \cdot dV$

Derivation for work :

① Const. Pressure process (Isobaric) :-

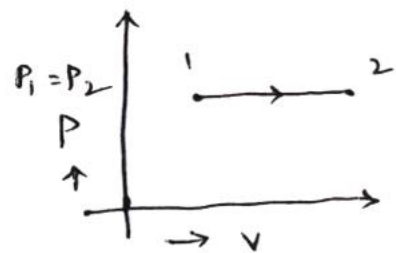
$$P_1 = P_2$$

$$W = \int P \cdot dV$$

$$W_{1-2} = \int_1^2 P \cdot dV$$

$$= P \int_1^2 dV = P [V]_1^2 = P(V_2 - V_1)$$

$$W = P_1 (V_2 - V_1)$$



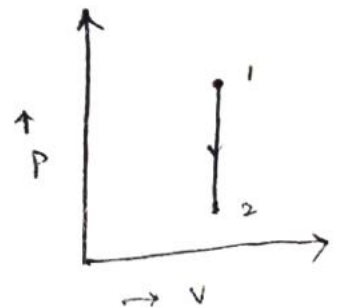
② constant volume process (Isochoric) :-

$$V_1 = V_2$$

$$W = \int P \cdot dV$$

$$W_{1-2} = \int_1^2 P \cdot dV = P \int_1^2 dV = P [V]_1^2$$

$$= P(V_2 - V_1) = P(V - V) = 0$$



$W = 0$ [work done by a closed system in a constant vol. process is zero]

③ Constant temperature process (Isothermal) :- $[T_1 = T_2 = T_3 = C]$

$$P_1 V_1 = P_2 V_2 \dots\dots = C$$

$$PV = C \quad (\text{or}) \quad P = \frac{C}{V}$$

$$\log = \log \frac{10^x}{10^y} \\ \log = \log 10^x - \log 10^y$$

$$W = \int_1^2 p \cdot dv$$

$$= P \int_1^2 \frac{C}{V} dv = C \int_1^2 \frac{1}{V} \cdot dv$$

$$= C [\log V]_1^2 = C [\log V_2 - \log V_1]$$

$$= C \log \left(\frac{V_2}{V_1} \right)$$

$$P_1 V_1 = P_2 V_2$$

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

$$W = PV \log \left(\frac{V_2}{V_1} \right)$$

$$(\text{or}) \quad W = PV \log \left(\frac{P_1}{P_2} \right)$$

① A mass of 1.5 kg of air is compressed in a quasi-static process from 0.1 MPa to 0.7 MPa for which $PV = C$ i.e., isothermal process. The initial density of air is 1.16 kg/m^3 . Find the work done by the piston to compress the air.

Sol. Given: $m = 1.5 \text{ kg}$

$$P_1 = 0.1 \text{ MPa}$$

$$P_2 = 0.7 \text{ MPa}$$

$$\rho_1 = 1.16 \text{ kg/m}^3$$

req.: work done = ?

$$PV = C$$

$$\text{Density} = \frac{m}{V} \Rightarrow \rho_1 = \frac{m}{V_1} \Rightarrow 1.16 = \frac{1.5}{V_1}$$

$$V_1 = 1.293 \text{ m}^3$$

$$\therefore P_1 V_1 = P_2 V_2$$

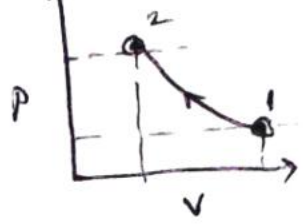
$$V_2 = \frac{P_1 V_1}{P_2} = \frac{0.1 \times 1.293}{0.7}$$

$$W = P_1 V_1 \log \left(\frac{P_1}{P_2} \right) = 0.1 \times 1.293 \log \left(\frac{0.1}{0.7} \right) \times 10^3$$

$$= -252.38 \text{ kJ}$$

Compression process:-

In compression process pressure increases & volume decreases. Movement of piston in upward direction increases the pressure of gas acting on piston.



In compression process $\frac{V_2}{V_1} < 1$

$\therefore$ work = Negative i.e., workdone on the system.

Expansion process:-

In this process pressure decreases if volume increases.

In expansion process $\frac{V_2}{V_1} > 1$

$\therefore$ work = Positive i.e., workdone by the system.

② The properties of closed system change following relation b/w pressure & vol. as $pV = 3$, where P is in bar & V is in m^3 . Calculate the workdone when pressure increases from 1.5 bar to 7.5 bar.

Q.1 Given, $P_1 = 1.5$ bar
 $P_2 = 7.5$ bar

$$P_1 V_1 = 3$$

$$1.5 V_1 = 3 \Rightarrow V_1 = 2 \text{ m}^3$$

$$P_2 V_2 = 3$$

$$7.5 V_2 = 3 \Rightarrow V_2 = 0.4 \text{ m}^3$$

$$W = \int_{V_1}^{V_2} p \cdot dV = \int_2^{0.4} \frac{3}{V} dV$$
$$= 3 \left[\ln(0.4) - \ln(2) \right]$$

$$= -4.828 \times 10^3 \text{ bar m}^3$$

$$= -4.828 \times 100 \text{ kPa m}^3$$

$$= -483 \text{ kJ}$$

$$\int \frac{1}{V} dV = \ln V$$

- ③ A gas undergoes reversible non flow process according to the relation $P = -3V + 15$ where V is the volume in m^3 and P is the pressure in bar. Determine the workdone when the volume changes from $3 m^3$ to $6 m^3$.

Sol:- $P = -3V + 15$

$$W = \int_{V_1}^{V_2} P \cdot dV = \int_3^6 (-3V + 15) dV$$

$$= \left[-\frac{3V^2}{2} + 15V \right]_3^6$$

$$= \left[-\frac{3(6)^2}{2} + 15(6) \right] - \left[-\frac{3(3)^2}{2} + 15(3) \right]$$

$$= 4.5 \text{ bar} \Rightarrow 4.5 \times 100 \text{ kJ}$$

$$= 450 \text{ kJ}$$

- ④ During non flow reversible process a gas enclosed in a cylinder piston assembly expands from $2 m^3$ to $4 m^3$ with the relation $P = V^2 + \frac{6}{3}$ where P is the pressure in bar. Determine the work.

Sol:- $W = \int_{V_1}^{V_2} P dV$

$$= \int_2^4 \left(V^2 + \frac{6}{3} \right) dV = \left[\frac{V^3}{3} + \frac{6V}{3} \right]_2^4$$

$$= \left[\frac{(4)^3}{3} + \frac{6(4)}{3} \right] - \left[\frac{(2)^3}{3} + \frac{6(2)}{3} \right] = 18.66 + 4 = 22.66 \text{ bar} \cdot m^3$$

$$= 2266 \text{ kJ}$$

- ⑤ A gas contained in a piston cylinder arrangement expands from $0.75 m^3$ to $1.35 m^3$ volume while the pressure remains constant at 200 kPa . If the gas system receives 30 kJ of work from a wheel, determine the net workdone by the system.

$$V_1 = 0.75 \text{ m}^3$$

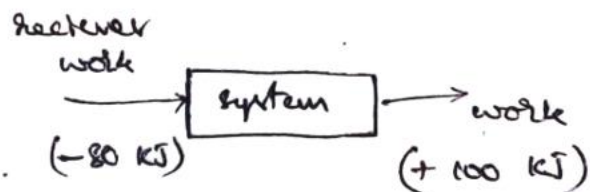
$$P = 200 \text{ kPa}$$

$$V_2 = 1.25 \text{ m}^3$$

pressure remains constant

$$W = P_1 [V_2 - V_1]$$

$$= 200 [1.25 - 0.75] = 100 \text{ kJ}$$



Since it is receiving 80 kJ of work from outside.

$$\therefore \text{Net workdone} = 100 - 80 = \underline{20 \text{ kJ}}$$

Q. Air at pressure of 50 bar & 0.2 m^3 is expanded at const. pressure until the volume is doubled, it is then expanded according to the polytropic process $PV^{1.3} = \text{const.}$, until the volume is 0.8 m^3 . Calculate the workdone in each process. Indicate the process on

P-v diagram.

$$P_1 = 50 \text{ bar} \times 100$$

$$V_1 = 0.2 \text{ m}^3$$

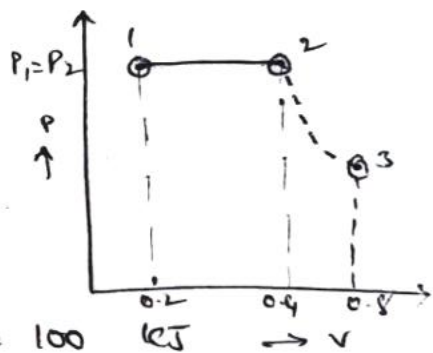
$$V_2 = 0.4 \text{ m}^3$$

$$V_3 = 0.8 \text{ m}^3$$

$$PV^{1.3} = \text{const.}$$

$$W = P_1 (V_2 - V_1)$$

$$= 500 (0.4 - 0.2) = 100 \text{ kJ}$$



(1-2) is constant

(2-3) is polytropic

Polytropic process

$$W = \frac{P_2 V_2 - P_3 V_3}{n-1}$$

$$P_1 V_1^{1.3} = P_2 V_2^{1.3} \Rightarrow P_2 =$$

$$P_2 V_2^{1.3} = P_3 V_3^{1.3} \Rightarrow P_3 = 2.3 \times 10^5 \text{ kJ}$$

$$W = 1251.7 \text{ kJ}$$

Q. A gas undergoes two processes that are in series. The 1st process is an expansion that is carried out according to the law $PV = \text{const.}$ Second process is a constant pressure process that returns the gas to the initial volume of the 1st process.

The starting of the 1st process is at 400 kPa & 0.025 m³ with the expansion to a pressure of 200 kPa. sketch the process on p-v diagram & determine work of combined process.

Sol: 2 process (1-2) → p-v constant
(2-3) → pressure constant.

$$P_1 = 400 \text{ kPa} \quad V_1 = 0.025 \text{ m}^3$$

$$P_2 = 200 \text{ kPa}$$

$$P_1 V_1 = P_2 V_2 \Rightarrow \frac{P_1 V_1}{P_2} = V_2 = 0.05 \text{ m}^3$$

$$P_2 = P_3 = 200 \text{ kPa}$$

$$V_3 = V_1 = 0.025 \text{ m}^3$$

workdone in 1st process

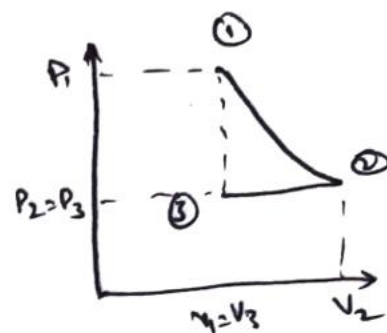
$$W = P_1 V_1 \log \left[\frac{P_1}{P_2} \right] = 6.93 \text{ KJ}$$

2nd process

$$W = P_2 (V_3 - V_2)$$

$$= 200 (0.025 - 0.05) = -5 \text{ KJ}$$

$$\therefore \text{Total workdone} = 6.93 - 5 = \underline{1.93 \text{ KJ}}$$



8) A fluid at a pressure of 3 bar in the sp. volume 0.18 m³/kg contained in a cylinder behind piston expand reversibly to a pressure of 0.6 bar if follows the law $p v^2 = \text{const}$. calculate the workdone by the fluid on the piston.

Sol:

$$P_1 = 3 \text{ bar}$$

$$P_2 = 0.6 \text{ bar}$$

$$V_1 = 0.18 \text{ m}^3/\text{kg}$$

$$p v^2 = C$$

$$P = \frac{C}{v^2}$$

$$W = \int_1^2 P \cdot dv$$

$$= \int_1^2 \frac{C}{v^2} dv$$

$$= \int_1^2 C v^{-2} dv$$

$$= C \left[\frac{v^{-2+1}}{-2+1} \right]_1^2$$

$$= C \left(\frac{v^{-1}}{-1} \right)_1^2$$

$$= \int_1^2 -C v^{-1} dv$$

$$= -C [V_2^{-1} - V_1^{-1}]$$

(or)

$$\frac{P_1 V_1 - P_2 V_2}{2-1}$$

$$W = C \left[\frac{1}{V_1} - \frac{1}{V_2} \right]$$

$$P_1 V_1^2 = C$$

$$3 \times (0.18)^2 = C \Rightarrow C = 0.0972 \text{ bar} \cdot \text{m}^3/\text{kg} \\ = 9.72 \text{ KJ}$$

$$P_1 V_1^2 = P_2 V_2^2$$

$$V_2^2 = \frac{P_1 V_1^2}{P_2} = \sqrt{0.162} = 0.4024 \text{ m}^3$$

$$W = C \left[\frac{1}{V_1} - \frac{1}{V_2} \right]$$

$$= 0.2984 \text{ bar} \cdot \text{m}^3/\text{kg} \Rightarrow 0.298 \times 100 \text{ KPa} \cdot \text{m}^3/\text{kg} \\ = 29.8 \frac{\text{KJ}}{\text{kg}} \times \frac{\text{m}^3}{\text{kg}} \quad [\text{NM} = \text{J}] \\ = 29.8 \text{ KJ/kg}$$

Relations b/w volume, pressure & temperature :-

① Constant pressure process :-

$$\boxed{\frac{V_1}{V_2} = \frac{T_1}{T_2}}$$

where P is constant.

② constant volume process :-

$$\boxed{\frac{P_1}{P_2} = \frac{T_1}{T_2}}$$

where V is constant.

③ Constant temperature process (Isothermic) :-

$$\frac{P_1 V_1}{P_2 V_2} = \frac{m R T_1}{m R T_2} \Rightarrow \boxed{P_1 V_1 = P_2 V_2}$$

$$P V^{\gamma} = C$$

④ Adiabatic process :-

$$\frac{P_1 V_1}{P_2 V_2} = \frac{m R T_1}{m R T_2} \Rightarrow \frac{T_1}{T_2} = \left(\frac{P_1}{P_2} \right)^{\left(\frac{V_1}{V_2} \right)} \rightarrow \textcircled{1}$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\frac{P_1}{P_2} = \left[\frac{V_2}{V_1} \right]^\gamma \rightarrow \textcircled{2} \quad \text{substitute in eq ①}$$

$$\therefore \frac{T_1}{T_2} = \left[\frac{V_2}{V_1} \right]^\gamma \left[\frac{V_2}{V_1} \right]^{-1} = \left[\frac{V_2}{V_1} \right]^{\gamma-1}$$

$$\boxed{\frac{T_1}{T_2} = \left[\frac{V_2}{V_1} \right]^{\gamma-1}}$$

⑤ Polytropic process :- similar to the above derivation.

$$\boxed{\frac{T_1}{T_2} = \left[\frac{V_2}{V_1} \right]^{n-1}}$$

④ A quantity of air having a volume of 0.03 m^3 at a temp of 200°C & a pressure of 150 N/cm^2 is expanded at a constant pressure, a volume of 0.06 m^3 , it is then expanded adiabatically to a volume of 0.12 m^3 . Calculate workdone during each process, temp & pressure at the end of adiabatic process. Assume γ is equal to 1.41 , also draw the process on $p-v$ diagram.

Sol:-

$$V_1 = 0.03 \text{ m}^3$$

$$T_1 = 200^\circ\text{C} + 273 = 473 \text{ K}$$

$$P_1 = 150 \text{ N/cm}^2$$

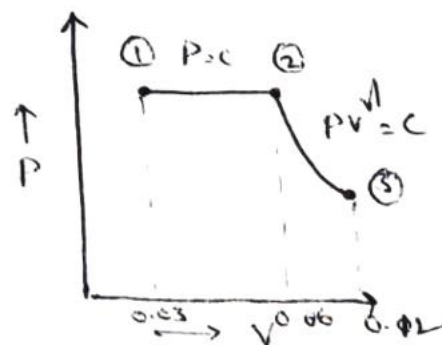
$$= 1500 \text{ kPa}$$

$$V_2 = 0.06 \text{ m}^3$$

$$V_3 = 0.12 \text{ m}^3$$

$$\gamma = 1.41$$

$$\therefore PV^\gamma = C$$



Process (1-2):

$$W = P_1 (V_2 - V_1)$$

$$= 1500 (0.06 - 0.03)$$

$$W_{(1,2)} = 45 \text{ kPa}$$

process (2-3): $PV^\gamma = C$

$$W_{(2-3)} = \frac{P_2 V_2 - P_3 V_3}{\gamma - 1}$$

$$= \frac{1500 \times 0.06 - 564 \times 0.12}{1.41 - 1}$$

$$= 54.4 \text{ kJ}$$

$$\frac{T_1}{T_2} = \frac{V_1}{V_2} \Rightarrow T_2 = \frac{V_2 T_1}{V_1} = 946 \text{ K}$$

$$\frac{T_3}{T_2} = \left[\frac{V_2}{V_3} \right]^{\gamma-1} \Rightarrow T_3 = 946 \left[\frac{0.06}{0.12} \right]^{(1.41-1)}$$

$$T_3 = 711.98 \text{ K}$$

workdone at const. pressure $W = 45 \text{ kPa}$

pressure at final process $P_3 = 564 \text{ kPa}$

Temperature at final process $T_3 = 711.98 \text{ K}$

Adiabatic process:-

In adiabatic process heat transfer is zero.

The condition for adiabatic process is $PV^\gamma = C \Rightarrow P = \frac{C}{V^\gamma}$

$$W = \int P \cdot dV$$

$$W_{1-2} = \int_1^2 P \cdot dV = \int_1^2 \frac{C}{V^\gamma} dV$$

$$= C \int_1^2 \frac{1}{V^\gamma} dV = C \int_1^2 V^{-\gamma} dV = C \left[\frac{V^{-\gamma+1}}{-\gamma+1} \right]_1^2$$

$$W = C \left[\frac{V_2^{-\gamma+1}}{-\gamma+1} - \frac{V_1^{-\gamma+1}}{-\gamma+1} \right]$$

$$= \frac{(P_2 V_2^\gamma) V_2^{-\gamma+1}}{-\gamma+1} - \frac{(P_1 V_1^\gamma) V_1^{-\gamma+1}}{-\gamma+1} = \frac{P_2 V_2 - P_1 V_1}{-\gamma+1}$$

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma-1}$$

$$\int x^n = \frac{x^{n+1}}{n+1}$$

Polytropic Process:-

In this process the condition is $PV^n = C$
where n is known as polytropic index.

$$PV^n = C \Rightarrow P = \frac{C}{V^n}$$

$$W = \int P dV \Rightarrow W_{1-2} = \int_1^2 P \cdot dV$$

$$= \int_1^2 \frac{C}{V^n} dV = C \int_1^2 V^{-n} dV$$

$$= C \left[\frac{V^{-n+1}}{-n+1} \right]_1^2 = C \left[\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} \right]$$

$$= \frac{(P_2 V_2^n) V_2^{-n+1} - (P_1 V_1^n) V_1^{-n+1}}{n-1}$$

$$= \frac{P_2 V_2 - P_1 V_1}{-n+1} \Rightarrow \boxed{W = \frac{P_1 V_1 - P_2 V_2}{n-1}}$$

Imp *

State or show that work is a path function & not a point function:-

In the figures shown, the system is at state ①. To move the system point ① & ②, it can follow any path such as A (or) B (or) C etc. The area under the curve represents the work done.

In the diagrams, the area under each curve is not equal even though initial & final points are same.

Hence work is a path function and not a point function.



fig (A)

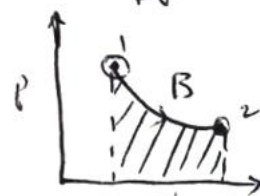


fig (B)

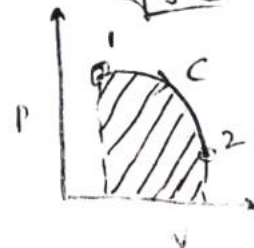


fig (C)

PdV - work (or) Displacement Work:

Let P_1, V_1 are initial press & vol

& P_2, V_2 are final " "

When the piston moves an infinitesimal distance dl , & if 'a', be the area of piston, the force F acting on piston,

$$F = P \cdot a$$

And the infinitesimal amount of work done by the gas on the piston

$$\delta W = F \cdot dl = P a dl = P dV$$

with change in vol from V_1 to V_2 ,

Amount of wo by the system

$$W_{1-2} = \int_{V_1}^{V_2} P dV$$

The ~~infinite~~ integration $\int P dV$ can be performed only on a quasi-static path.

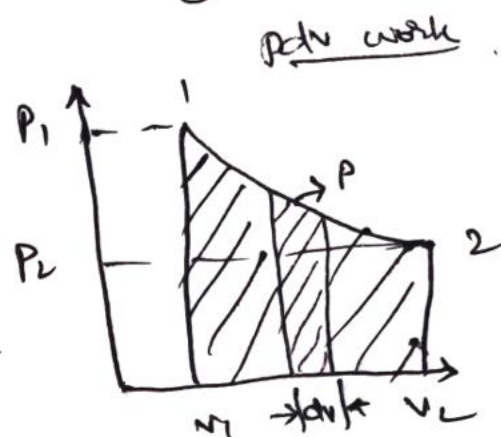
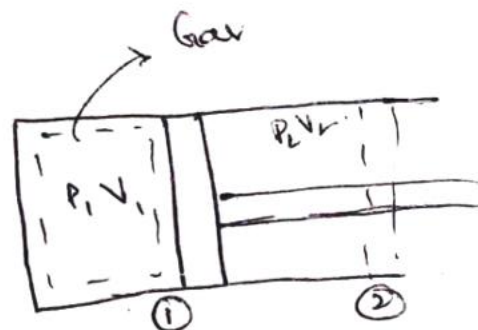
$$1 \text{ Pa} = 1 \text{ N/m}^2$$

$$1 \text{ bar} = 10^5 \text{ Pa} = 100 \text{ kPa}$$

$$1 \text{ atm} = 101.325 \text{ kPa} = 1.01325 \text{ bar}$$

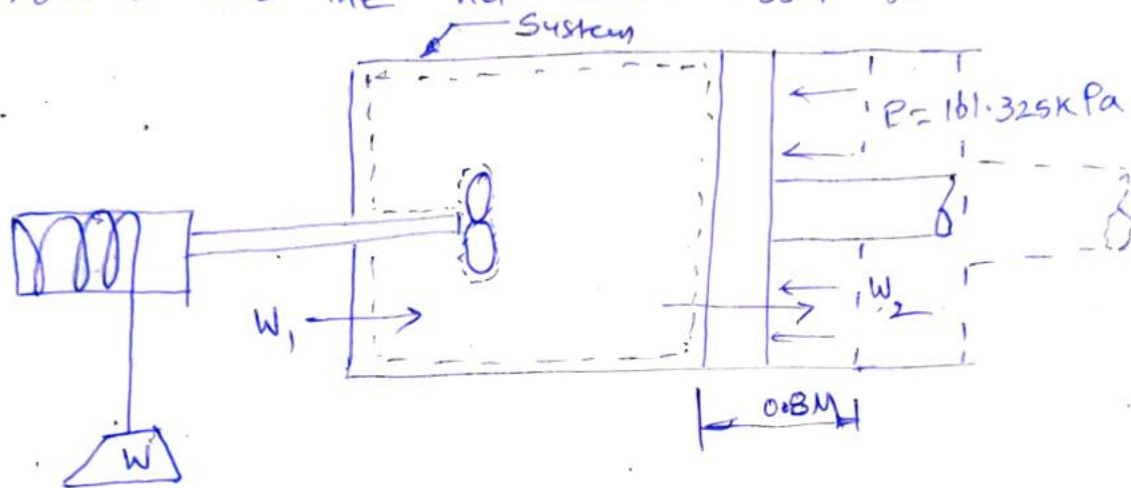
$$1 \text{ Joule} = 1 \text{ Nm}$$

$$\text{watt} = \text{J/s}$$



where $dV = a dl$
= infinitesimal
dis. vol.

P) A Piston and Cylinder machine containing a fluid system has a stirring device on the cylinder. The piston is frictionless, and is held down against the fluid due to atm. pressure of 101.325 kPa . The stirring device is turning 10000 revolutions with an average torque against the fluid of $1.275 \text{ N}\cdot\text{m}$. Meanwhile the piston of 0.6 m diameter moves out 0.8 m . Find the net work transfer for the system.



Work done by the stirring device upon system

$$W_1 = 2\pi NT$$

$$= 2\pi \times 10000 \times 1.275 = 80 \text{ kJ}$$

This is the Negative work for system.

Work done by the system upon surroundings.

$$W_2 = (PA) \cdot L$$

$$= 101.325 \times \frac{\pi}{4} (0.6)^2 \times 0.8 = 22.9 \text{ kJ}$$

This is positive work for the system.

Hence the net work transfer for the system

$$W = W_1 + W_2 = -80 + 22.9 = \underline{\underline{-57.1 \text{ kJ}}}$$

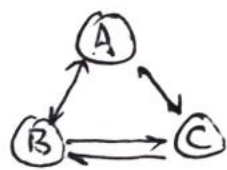
ZEROTH LAW OF THERMODYNAMICS :: $T_A = T_B$ $T_B = T_C$ $\therefore T_A = T_C$ $T_A = T_B$

When two bodies are in thermal equilibrium with a third body, they are also in thermal equilibrium with each other.

This can be stated as, when a body A is in thermal equilibrium with a body B & when B is separately in thermal equilibrium with a body C, then A & C will also be in thermal equilibrium with each other, i.e., A & C will not undergo any physical changes when ^{they} brought in contact.

$$\therefore A = B$$

$$B = C \text{ then } A = C$$



Note:- Zeroth law is the basis of temperature measurement.

Equality of Temperature:-

Temperature is generally defined as the degree of hotness or intensity of heat. The physical meaning of temperature is that it describes whether the body is hot or cold.

Let two bodies, one hot (at temp θ_1) & one cold (at temp θ_2) be placed in contact with each other ($\theta_1 > \theta_2$) while is isolated from all other bodies. After some time, we observe that hot body becomes colder whereas the colder one becomes hotter. It indicates that both the bodies have experienced a change in one or more of their properties. We also find that both the bodies are now at same temp θ_m . ($\theta_m < \theta_1$; $\theta_m > \theta_2$) The state, that these two bodies have attained same temp. is a state of thermal equilibrium.

The condition of equality of temp is stated as follows:

"Two systems have equal temperatures if there are no changes in their properties when they are brought in thermal contact with each other."

Thermometry :- It is the ^{science of} measurement of temperature.

Principle of Thermometry :-

In order to obtain a quantitative measure of temp, a reference body is used. Besides this a certain physical characteristic is to be selected, which changes with temp. This reference body is called thermometer. & its selected characteristic is called thermodynamic property. The Principle of thermometer lies in the thermometric property of fluid used in it. A very common thermometer consists of a small amount of mercury in an evacuated capillary tube. The expansion of mercury in the tube when it comes in contact with hot body, is used as thermometric property.

The following properties are generally used.

- 1) Pressure exerted by gases.
- 2) Volume expansion of solids, liquids & gases
- 3) Vapour pressure of liquids
- 4) Electrical resistance of solids
- 5) Thermo electricity.

Reference Points:-

Reference points are the fixed points of fixed temp's., marked on the thermometer. Universally adopted reference points are (i) Ice point
(ii) Steam point.

The ice point is the equilibrium temp of ice & air-saturated water under a pressure of one atmosphere. In other words it refers to melting of ice at atmospheric pressure.

The steam point is the equilibrium temp of water & water vapour under a pressure of one atmosphere. In brief, it refers to boiling point (saturation temp) of water at atm. pressure.

The thermometer is graduated into equal no. of divisions in b/w these reference points. The number assigned to the reference points are arbitrary. Following table shows the most popular reference points on different scales.

S. NO	SCALE	ICE POINT	STEAM POINT	NO. OF DIVISIONS
1)	CELSIUS	0°C	100°C	100
2)	FAHRENHEIT	32°F	212°C	180
3)	REAUMUR	0°R	80°R	80

Constant Volume Gas Thermometer:-

The figure below shows constant vol gas thermometer. (platinum-iridium) Bulb (B) contains a small amount of gas. (hydrogen) The bulb is connected to one end of U-tube mercury manometer (M) b.

means of a glass capillary tube (C). The other end of the manometer is open to atmosphere. This end can be moved vertically becoz of flexible tubing (F).

The thermometric property that is made use of, is pressure in the bulb. The gas in the bulb is kept at const. vol by keeping the mercury level in the tube at mark A. This is done by raising or lowering the left limb of the tube. The difference 'h' in mercury columns in two limbs gives the gauge pressure (P) as

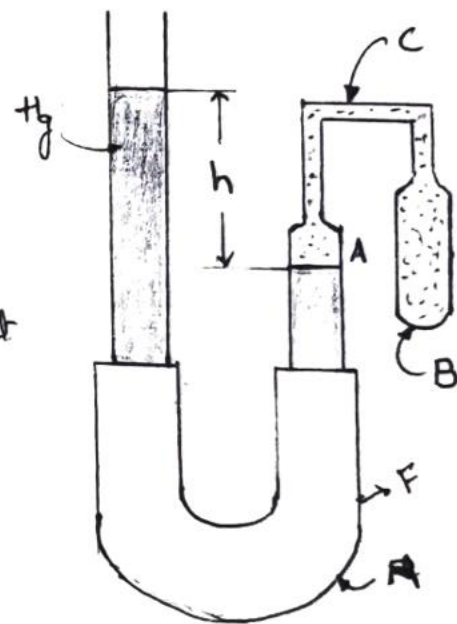
$$P = P_{atm} + \rho_m g h \rightarrow (1)$$

where P_{atm} = atmospheric pressure

ρ_m = density of mercury.

When the bulb is brought in contact with the system whose temp is to be measured, the bulb in course of time, comes in thermal equilibrium with the system. The gas in the bulb expands

when it absorbs heat of given system, pushing down mercury column in right limb. The flexible limb of manometer is then adjusted so that mercury again touches mark A. The difference in mercury levels 'h' is recorded & pressure 'P' of the gas in the bulb is estimated using eq (1).



The temperature can be calculated by using the ideal gas equation

$$PV = nRT$$

Scales of temperature :-

To give a numerical value to the temp of the body we have to define a scale of temperature. The earlier method is to choose two fixed point temp's, one the melting point of ice (ice point) & the other boiling point of water (steam point) at 1 atm pressure as fixed temp points. We assign arbitrarily t_i to ice point & t_s to steam point. Correspondingly let the length of Hg column are l_1 & l_2 .

The temp corresponding to any length ' l ' may be defined assuming linear relation.,

$$t = al + b \rightarrow \textcircled{1} [a, b \text{ are constants}]$$



Thus a change of one degree in temp means a change of $\frac{l_2 - l_1}{t_2 - t_1}$ in the length of mercury column. Thus we can graduate scale.

Different scales of temperature :-

① Centigrade scale ($^{\circ}\text{C}$)

→ It is also known as Celsius scale.

→ Melting of ice is marked as 0°C & boiling point of water as 100°C under a pressure of 760 mm of Hg.

② Absolute (or) Kelving Scale ($^{\circ}\text{K}$)

→ Melting point of ice is 273 K & boiling point is 373 K

③ Fahrenheit scale ($^{\circ}\text{F}$)

→ Melting point of ice is 32°F & boiling point 212°F .

④ Rankine scale ($^{\circ}\text{R}$)

→ Melting point of ice is 0°R & boiling of water as 80°R .

Note :- Relation b/w $^{\circ}\text{C}$, $^{\circ}\text{F}$ & $^{\circ}\text{R}$

$$\frac{C}{100} = \frac{F - 32}{180} = \frac{R}{80}$$

Types of thermometers.

Thermometer

Thermometric property

Range

1) Hg in glass thermometer	length	-40 to 525°C
2) const. vol gas thermometer	pressure	-200 to 1500°C
3) const. pressure gas thermometer	volume	-200 to 1500°C
4) Electrical resistance thermometer	Resistance	-200 to 120°C
5) Thermocouple	emf	0 to 1800°C
6) Radiation pyrometer	wave length	$> 400^{\circ}\text{C}$

FIRST LAW OF THERMODYNAMICS :-

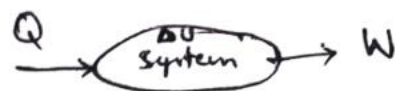
This is also known as law of conservation of energy.

It states that energy can neither be created nor destroyed but can be convertible from one form to another form. It means, two forms of energy, heat & work are mutually convertible.

Whenever heat is absorbed by a system it increases its internal energy ~~plus~~ to do some external work.

This can be represented as follows.

$$Q = \Delta U + W$$



where Q is heat energy entering a system

ΔU is internal energy,

W is external work.

First law can also be stated as,

If a system undergoes a cyclic change (the end state being precisely the same as the initial state), then the algebraic sum of the work delivered to the surroundings is proportional to the algebraic sum of heat taken from the surroundings. It is expressed as

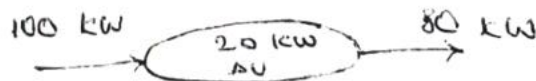
$$\oint dQ = \oint dW \quad (\text{or}) \quad \Sigma Q = \Sigma W$$

Internal Energy:-

Energy stored in a system which is neither heat nor work is called internal energy.

Ex:- when a gas is heated it expands producing some work but the amount of work produced will be less than heat supply. This difference b/w heat & work is known as internal energy which is stored in the system.

$$\Delta U = Q - W$$



Ideal gas temp scale :-

Assume that the bulb of a constant vol. gas thermometer contains certain amount of gas such that when the bulb is surrounded by water at its triple point, the pressure P_t is 1000 mm of mercury. (Triple point is the temp at which substance exists in all three states.) Keeping the vol. of gas V constant let the following procedure be carried out.

$$T = C P$$

where $C = \frac{273.16}{P_t}$

(i) Bulb of the thermometer is surrounded by steam condensing at 1 atm & pressure P_1 is determined. Then calculate the temp as

$$\theta_1 = 273.16 \frac{P_1}{P_t} = 273.16 \frac{P_1}{1000}$$

(ii) Some amount of gas is removed from the bulb so that when it is surrounded by water at its triple point, the pressure P_t is 500 mm of Hg. At this instant, pressure is determined say P_2 . Then θ_2 for steam condensing at 1 atm is calculated.

$$\theta_2 = 273.16 \frac{P_2}{500}$$

Similarly $\theta_3, \theta_4, \dots, \theta_n$ are found from the experimentally determined values of $P = P_3, P_4, \dots, P_n$ by going on reducing the vol. of gas against $P_t = 250, 100, \dots$ mm Hg.

Note that $\theta_t = 273.16 \frac{P}{P_t}$

θ vs P_t is plotted & extrapolate the curve to the axis where $P_t = 0$. Read from the graph $\lim_{P_t \rightarrow 0} \theta$

[The temp of triple point of water is 273.16 K]

It may be shown that all the gases indicate the same temp as P_t is lowered & made to approach zero.

A similar series of tests can be conducted with a constant pressure gas thermometer.

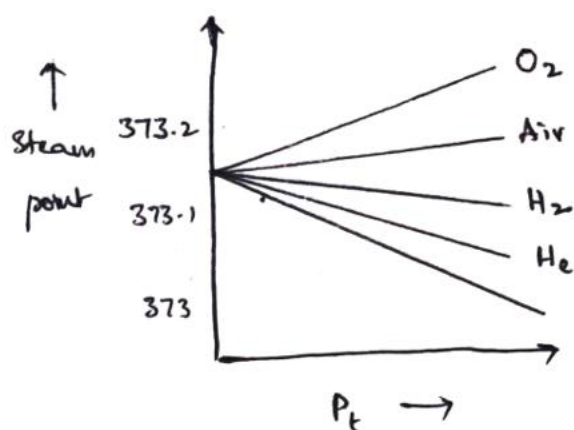
$$\therefore \theta = 273.16 \frac{V}{V_t}$$

θ vs P is plotted.

Replacing θ by T , the ideal gas temp is given by

$$T = 273.16 \lim_{P_t \rightarrow 0} \frac{P}{P_t}$$

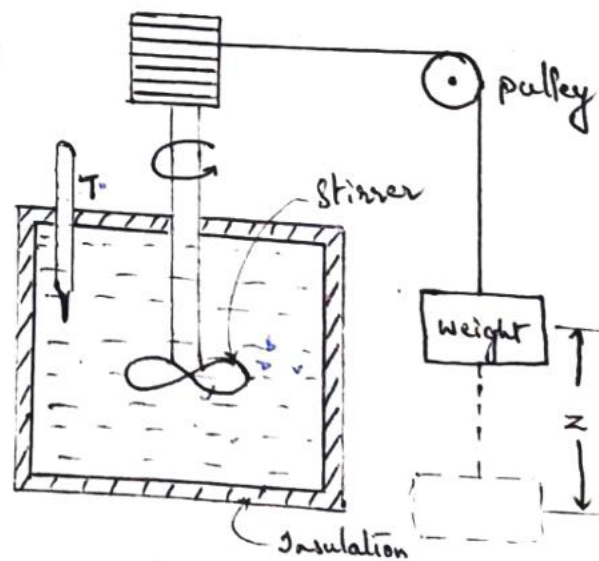
$$(or) T = 273.16 \lim_{V_t \rightarrow 0} \frac{V}{V_t}$$



JOULE'S EXPERIMENT :

A quantity of liquid is taken in an insulated vessel. Work is done on the fluid by stirring the paddle wheel. This work is put to the fluid causes a rise in temp of fluid. The amount of work done is calculated by the product of weight & the vertical height of weight displaced.

Next by removing the insulation the system is immersed in a water bath, such that heat is transferred from the fluid till the original state of fluid is reached which will be indicated by temp & pressure.



In this way system undergoes a complete cycle. The amount of heat rejected by the fluid is equal to the increase of energy of water bath.

Conclusion: The net work input W was directly proportional to the net heat Q transferred from the system regardless of the kind of work done, the rate at which the work was done & the scheme used for transferring it into heat.

Then for a closed system undergoing cyclic process

$$\oint \delta Q = \oint \delta W$$

$$Q_{1-2} + Q_{2-1} = W_{1-2} + W_{2-1}$$

Perpetual Motion Machine I: PMM-I [Perpetual $\rightarrow$ continuing forever]

PMM-I refers to the Perpetual Motion machine of first kind. It is a hypothetical machine that will continuously churn out work but without absorbing heat from its surroundings. (theoretical)

But such a machine is not feasible from a practical point of view, for it violates law of conservation of energy (1st law of TD).

$$Q=0 \rightarrow \boxed{\text{PMM I}} \rightarrow W$$

The reverse of PMM I is also not true. It is a hypothetical machine which is not feasible as it violates the 1st law of TD.

$$W=0 \rightarrow \boxed{\text{PMM I (reverse)}} \rightarrow Q$$

Corollaries of First law: - (Corollary: practical consequence that follows naturally)

Cor. 1: - There exists a property of a closed system, such that a change in its value during any change of state is given by the difference b/w the heat supplied & work done. And this property is called 'internal energy' of the system. Mathematically, [it increases or decreases with change of state]

$$\Delta U = \Delta Q - \Delta W$$

This energy equation is called Non-flow energy equation (NFEE)

Cor. 2 :-

"The energy of an isolated system is constant." By isolated system we mean a system which exchanges neither heat nor work with its surroundings. In other words both Q & W are zero and $\Delta U = 0$

Alternatively, the second corollary may be stated as, that the "internal energy of a closed system remains unchanged if the system is isolated from the surroundings."

In a way this is Law of Conservation of Energy.

Cor. 3 :-

"A perpetual machine of 1st kind is impossible". It is a machine that produces & supplies 'work' by itself without absorbing any energy from the surroundings.

Internal Energy :-

Energy stored in a system which is neither heat nor work is called internal energy.

Ex:- when a gas is heated it expands producing some work, but amount of work produced will be less than heat supply. This difference b/w heat & work is known as internal energy which is stored in a system.

$$\Delta E = Q - W$$

$$Q - W$$

$$\Delta U = Q - W$$

5 In general compression process 2 kJ of mechanical work is supplied to 5 kg of working substance. 800 J of heat is rejected to the cooling water jacket. Calculate change in specific internal energy.

$$Q = \Delta E + W$$

$$\Delta E = Q - W$$

$$W = 2 \text{ kJ} = 2000 \text{ J/5 kg}$$

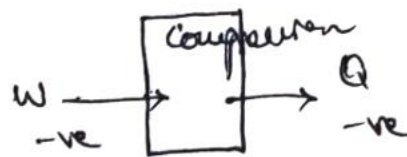
$$= -400 \text{ J/kg}$$

$$Q = 800 \text{ J/5 kg} = -160 \text{ J/kg}$$

$$\therefore \Delta E = Q - W$$

$$= -160 + 400$$

$$\Delta E = 240 \text{ J/kg}$$



Piston & cylinder m/c contains a fluid system which passes through complete cycle of 4 process. During the cycle sum of all heat transfers is -170 kJ. The system completes 100 cycles per minute. Complete following table showing method for each item. Calculate the net work op in kW.

Process	$\sum Q$ (kJ/m)	$\sum W$ (kJ/m)	ΔE (kJ/m)
(a-b)	0	2170	<u>-2170</u>
(b-c)	21000	0	<u>21000</u>
(c-d)	-2100	<u>34800</u>	-36,600
(d-a)	<u>-35,900</u>	<u>-53,670</u>	<u>17770</u>

$$\sum Q = -170 \text{ kJ/m} \times 100 = -17000$$

$$\sum Q = Q_{a-b} + Q_{b-c} + Q_{c-d} + Q_{d-a}$$

$$Q_{d-a} = \sum Q - Q_{a-b} - Q_{b-c} - Q_{c-d}$$

$$= -17000 - 0 - 21000 + 2100 = -35900 \text{ kJ/min}$$

for $Q_{a-b} \Rightarrow Q = \Delta E + W$

$$\Delta E = Q - W$$

$$\Delta E = 0 - 2170 = -2170 \text{ kJ/min}$$

For $Q_{b-c} \Rightarrow \Delta E = Q - W$

$$\Delta E = 21000 - 0 = 21000 \text{ kJ/min}$$

For $Q_{c-d} \Rightarrow \Delta E = Q - W$

$$Q = \Delta E + W \Rightarrow W = Q - \Delta E$$

$$= -2100 + 36600 = 34500 \text{ kJ/min}$$

$\Sigma W = \Sigma Q$ for cyclic process.

$$\Sigma W = W_{a-b} + W_{b-c} + W_{c-d} + W_{d-a}$$

$$W_{d-a} = \Sigma W - W_{a-b} - W_{b-c} - W_{c-d} = -53670 \text{ kJ/min}$$

For $W_{d-a} \Rightarrow \Delta E = Q - W = -35900 + 53670$

$$\Delta E = 17770 \text{ kJ/min}$$

Acc to 1st law of TD $\Sigma Q = \Sigma W$

$$\Delta E_{\text{cycle}} = 0$$

Total work $\Rightarrow -17000 \text{ kJ/min}$

$$= \frac{-17000}{60} = -283.33 \text{ kW}$$

④ A gas undergoes a thermodynamic cycle consisting of 3 processes of beginning at initial stage where $P_1 = 1 \text{ bar}$, $V_1 = 1.5 \text{ m}^3$ & $U_1 = 512 \text{ kJ}$. The processes are as follows.

(a) process (1-2) $\Rightarrow$ Compression with $PV = \text{const}$, $P_2 = 2 \text{ bar}$
 $Q_2 = 690 \text{ kJ}$

(b) process (2-3) $\Rightarrow W_{2-3} = 0$, $Q_{2-3} = -150 \text{ kJ}$

(c) process (3-1) $\Rightarrow W_{3-1} = 50 \text{ kJ}$

Neglecting KE & PE changes, determine Q values, (Q_{1-2} , Q_{3-1})

④ $P_1 = \text{const}$

$$\Rightarrow 1 \times 1.5 = 2 \times V_2$$

$$P_1 V_1 = P_2 V_2$$

$$V_2 = 0.75$$

$$Q_{1-2} = \Delta E_{1-2} + W_{1-2}$$

$$= (690 - 512) + (-103.972)$$

$$Q_{1-2} = 74.03 \text{ kJ}$$

$$W_{1-2} = P_1 V_1 \ln \left[\frac{P_1}{P_2} \right]$$

$$= 100 \times 1.5 \ln \left(\frac{1}{2} \right)$$

$$W_{1-2} = -103.972 \text{ kJ}$$

⑥ process (2-3) $\Rightarrow W_{2-3} = 0$

$$Q_{2-3} = -150 \text{ kJ (given)}$$

$$= -150 \text{ kJ}$$

$$\Delta U_{2-3} = Q_{2-3} - W_{2-3} = -150 - 0 = -150 \text{ kJ}$$

$$U_3 - U_2 = -150 \text{ kJ}$$

$$U_3 - 690 = -150 \Rightarrow U_3 = 540 \text{ kJ}$$

⑦ $W_{3-1} = 50 \text{ kJ}$

$$Q_{3-1} = \Delta U_{3-1} + W_{3-1}$$

$$= U_1 - U_3 + W_{3-1}$$

$$= 512 - 540 + 50 \Rightarrow Q_{3-1} = 22 \text{ kJ}$$

⑧ A system receives 200 kJ of heat at const. vol. process & rejects 220 kJ of heat at const. pressure & 40 kJ of work is done on the system. The system is brought to its initial condition by adiabatic. Calculate adiabatic work.

If initial internal energy is 240 kJ then what is I.E. at all points. Represent cycle on p-v diagram.

1-2 $Q_{1-2} = 200 \text{ kJ}$

2-3 $Q_{2-3} = -220 \text{ kJ}$

3-1

$$U_1 = 240 \text{ kJ}$$

① 1-2 $\therefore W_{1-2} = 0$ [∵ const vol. process]

$$Q_{1-2} = 200 \text{ kJ}$$

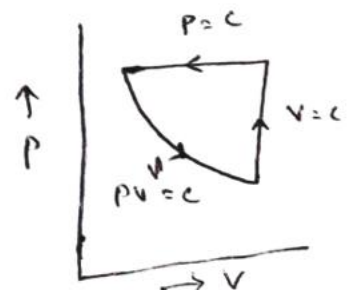
$$Q = \Delta U_{1-2} + W$$

$$U_2 - U_1 = 200$$

$$U_2 = 200 + U_1$$

$$U_2 = 200 + 240 = 440 \text{ kJ}$$

$$U_3 =$$



② 2-3 :- Cont. process.

$$Q_{2-3} = -220 \text{ kJ} ; W_{2-3} = -40 \text{ kJ}$$

$$Q = \Delta U_{2-3} + W$$

$$-220 + 40 = U_3 - U_2 \Rightarrow U_3 = -220 + 40 + U_2$$

$$= -220 + 40 + 440$$

$$U_3 = 260 \text{ kJ}$$

③ 3-1 :- Adiabatic process.

$$PV^\gamma = C, \text{ For cyclic process } \Sigma Q = \Sigma W \text{ \& } \Sigma E \text{ or } \Sigma U = 0$$

$$Q_{3-1} = \Delta U_{3-1} + W_{3-1}$$

$$= U_1 - U_3 + W_{3-1}$$

In adiabatic process Heat transfer = 0

$$\therefore Q_{3-1} = 0 \quad \therefore U_1 - U_3 = -W_{3-1}$$

$$W_{3-1} = 20 \text{ kJ}$$

$$\Sigma Q = -220 + 200$$

$$= -20 \text{ k}$$

$$\Sigma W = 0 + -40 + W_{3-1}$$

$$W_{3-1} = -40 + 20$$

$$= -20 \text{ kJ}$$

$$\therefore \text{Adiabatic process, } W_{3-1} = 20 \text{ kJ}$$

$$\therefore U_2 = 440 \text{ kJ}, U_3 = 260 \text{ kJ.}$$

④ The system is composed of 2 kg of fluid expands in a piston cylinder machine from initial state of 1 MPa, 100°C to final temp of 30°C. If there is no heat transfer find out work for process. Assume $R = 0.287 \text{ kJ/kg K}$.

Sol: $m = 2 \text{ kg}$ $T_1 = 100^\circ \text{C}$ $R = 0.287 \text{ kJ/kg K}$
 $P = 1 \text{ MPa}$ $T_2 = 30^\circ \text{C}$

Adiabatic process :- (Heat transfer is zero)

$$PV^\gamma = C$$

$$PV = mRT$$

$$P_1 V_1 = 2 \times 0.287 \times 100^\circ \text{C}$$

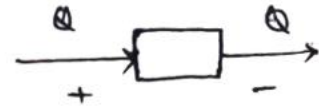
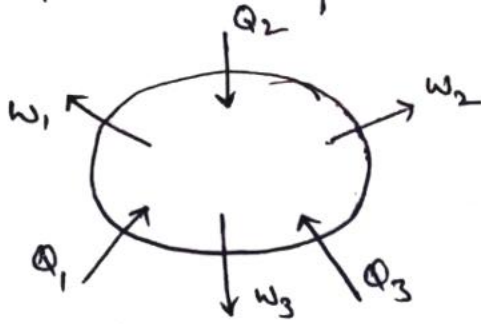
$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{mRT_1 - mRT_2}{\gamma - 1}$$

$$= \frac{mR(T_1 - T_2)}{\gamma - 1} \quad \left[\gamma = 1.4 \right]$$

$$= \frac{2 \times 0.287 (100 - 30)}{1.4 - 1}$$

$$= 120.45 \text{ kJ}$$

① Write first law of TD equation for the following systems.



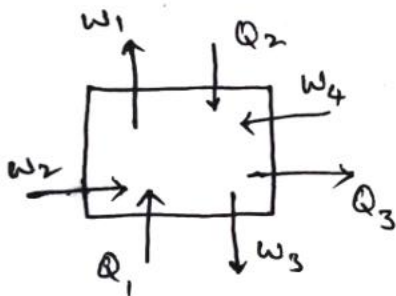
$$Q = \Delta U + W$$

$$\Delta U = \frac{W - Q}{Q - W}$$

$$\Rightarrow \Delta U = (W_1 + W_2 + W_3) - (Q_1 + Q_2 + Q_3)$$

$$\therefore (Q_1 + Q_2 + Q_3) = \Delta U + (W_1 + W_2 + W_3)$$

② Give the relation b/w heat energy, work & internal energy for the following system of multiple energy conversion.



$$\text{Sol: } Q_1 + Q_2 - Q_3 = \Delta U + (W_1 - W_2 + W_3 - W_4)$$

③ The system undergoes a cyclic process composed of 4 processes. The heat transfers in each process are 400 KJ, -365 KJ, -200 KJ, 250 KJ. The respective work transfers are 140 KJ, 0 KJ, -55 KJ & 0. Is the above data consistent with 1st law of TD?

$$\text{Sol: For cyclic process, } \sum Q = \sum W$$

$$\sum Q = 400 - 365 - 200 + 250 = 85 \text{ KJ}$$

$$\sum W = 140 + 0 - 55 + 0 = 85 \text{ KJ}$$

$$\therefore \sum Q = \sum W = 85 \text{ KJ}$$

The given data is consistent of the cyclic process.

& follows 1st law of TD.

⑨ A closed system undergoes a TD cycle consisting of 4 separate processes. The heat & work transfers are given in the following table. Show that the data is consistent with 1st law of TD. Calculate net work output in kW & change in internal energy.

process	Heat (kJ/m)	work (kJ/m)
1-2	20000	0
2-3	-10000	30000
3-4	0	20000
4-1	15000	-25000

$$\Sigma Q = 20000 - 10000 + 0 + 15000 = 25000 \text{ kJ/m}$$

$$\Sigma W = 0 + 30000 + 20000 - 25000 = 25000 \text{ kJ/m}$$

$$\therefore \Sigma Q = \Sigma W$$

The data is consistent. It follows 1st law of TD.

$$\begin{aligned} \text{Net work} &= 25000 \text{ kJ/m} \\ &= 416.66 \text{ kW} \end{aligned}$$

change in ΔU ,

$$\text{process (1-2)} : Q_{1-2} = \Delta U_{1-2} + W_{1-2}$$

$$\Delta U_{(1-2)} = 20000 - 0 = 20000 \text{ kJ/m} \rightarrow \textcircled{1}$$

$$\text{process (2-3)} \quad \Delta U_{(2-3)} = -10000 - 30000 = -40000 \text{ kJ/m} \rightarrow \textcircled{2}$$

$$\text{process (3-4)} \quad \Delta U_{(3-4)} = 0 - 20000 = -20000 \text{ kJ/m} \rightarrow \textcircled{3}$$

$$\text{process (4-1)} \quad \Delta U_{(4-1)} = 15000 + 25000 = 40000 \text{ kJ/m} \rightarrow \textcircled{4}$$

$$\Delta U_{(1-2-3-4-1)} = \Sigma Q - \Sigma W = 25000 - 25000 = 0$$

First law applied to flow process: (Open System)

In ^{an} open system, we know that energy (heat & work) as well as matter cross the boundary. The matter may take internal, kinetic, potential, chemical & magnetic energies along with it while crossing the boundary. However, chemical & magnetic energies are generally neglected.

In open system there are 2 types of flow process, namely.

1) Steady flow process

2) Unsteady flow process

1) In steady flow process the mass flowing into the system is equal to the mass flowing out of the system. In a given time & there is no change in the stored energy of the system.

Ex:-

Imagine a bath tub with a closed drain & turn on water which fills the tub & starts overflowing.

2) In unsteady flow process, the mass flowing into the system is not equal to the mass flowing out from the system in a given time & there is change in stored energy of the system i.e., the rate of net flow of matter is not constant.

Ex:- Imagine a bath tub with a closed drain at the bottom or with an open drain & turn on water. As the quantity of matter in the tub is not constant, it is an unsteady process.

Joules law - Internal Energy of perfect gas:-

Joules law states that internal energy of a perfect gas is independent of vol & pressure. It only depends on temperature. Thus $dU = nC_v dT$ is applicable for any

change of state b/w 1 & 2 irrespective of the process
since internal energy is a point function.

From above definition we can express the sp. heat
at constant vol. as $C_v = \frac{\partial u}{\partial T}$

where ∂u is specific change in internal energy in KJ/kg

Heat Transfer in various non-flow process:-

First law for a closed system is given by

$$Q_{1-2} - W_{1-2} = \Delta U$$

For an ideal gas internal energy is a function of temp alone.

$$\therefore \Delta u = m c_v (T_2 - T_1) \rightarrow \text{eq.}$$

This eq. is applicable for any process since it is a
point function & does not depend on path of the process
and depends only on the state temp's.

Let us apply this eq. to various possible process of a closed system
which changes from one state to another to develop relation
for heat transfer.

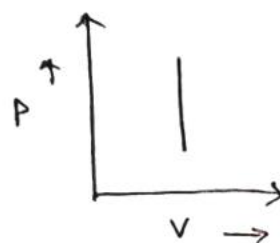
① Constant volume process.

Applying 1st law,

$$Q_{1-2} - W_{1-2} = \Delta U$$

$$\therefore W_{1-2} = 0$$

$$Q_{1-2} = \Delta U = m c_v (T_2 - T_1)$$



It indicates that the total amount of energy supplied in the form of heat is stored by the system as internal energy & thus

$$(\dot{Q}_{1-2})_v = m c_v (T_2 - T_1)$$

⑥ Constant Pressure process:-

Applying 1st law,

$$\dot{Q}_{1-2} - W_{1-2} = \Delta U$$

$$W_{1-2} = P_1 (V_2 - V_1) ; \Delta U = m c_v (T_2 - T_1)$$

$$\therefore \dot{Q}_{1-2} = m c_v (T_2 - T_1) + P_1 (V_2 - V_1)$$

$$= m c_v (T_2 - T_1) + (P_2 V_2 - P_1 V_1) \quad [\because P_1 = P_2]$$

$$= m c_v (T_2 - T_1) + m R (T_2 - T_1) \quad [\because P_1 V_1 = m R T_1]$$

$$m c_p (T_2 - T_1) = m (T_2 - T_1) (c_v + R) \quad [\because \dot{Q}_{1-2} = m c_p (T_2 - T_1)]$$

$$c_p = c_v + R$$

$$\boxed{c_p - c_v = R} \rightarrow \text{eq.}$$

This indicates that c_p is always greater than c_v & the amount of heat required in a const. press. process is greater than the const. vol. process to bring the same change in temp.

⑦ Isothermal process:- $[PV = C]$

$$\dot{Q}_{1-2} - W_{1-2} = \Delta U$$

$$\Delta U = m c_v (T_2 - T_1) \quad [\because \text{Temp} = \text{const}]$$

$$\Delta U = 0$$

$$\therefore Q_{1-2} = W_{1-2}$$

$$Q_{1-2} = P_1 V_1 \ln \frac{V_2}{V_1}$$

i.e., In isothermal process the system will not store any energy & total amount of heat supplied will convert into work.

Though the temp is constant in isothermal process $Q \neq 0$, moreover it is equal to W .

① Adiabatic Process:- (Reversible)

$$Q_{1-2} - W_{1-2} = \Delta U$$

$$\Delta U = m C_V (T_2 - T_1) ; W_{1-2} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{m R (T_1 - T_2)}{\gamma - 1}$$

$$W_{1-2} = m C_V (T_1 - T_2) \left[\because \frac{R}{\gamma - 1} = C_V \right]$$

$$\therefore Q_{1-2} = \Delta U + W_{1-2}$$

$$= m C_V (T_2 - T_1) + m C_V (T_1 - T_2)$$

from $C_p - C_V = R$
 $C_p / C_V = \gamma$

$$Q_{1-2} = 0$$

In adiabatic process heat transfer is zero i.e., the system does work without any heat interaction but at the expense of its own internal energy. So the temp changes.

"In adiabatic process heat transfer is zero but temp changes."

② Polytropic process :-

$$Q_{1-2} = W_{1-2} + \Delta U$$

$$= \frac{P_1 V_1 - P_2 V_2}{n-1} + m c_v (T_2 - T_1)$$

$$= \frac{m R (T_1 - T_2)}{n-1} + \frac{m R}{\gamma-1} (T_2 - T_1) \quad \left[\because c_v = \frac{R}{\gamma-1} \right]$$

$$= m R (T_1 - T_2) \left[\frac{1}{n-1} - \frac{1}{\gamma-1} \right]$$

$$= m R (T_1 - T_2) \left[\frac{(\gamma-1) - (n-1)}{(n-1)(\gamma-1)} \right]$$

$$= \frac{m R (T_1 - T_2)}{n-1} \left[\frac{\gamma-n}{\gamma-1} \right]$$

$$Q_{1-2} = \frac{\gamma-n}{\gamma-1} \times \text{work done}$$

$$= \frac{m R}{\gamma-1} \cdot \frac{\gamma-n}{n-1} (T_1 - T_2)$$

$$= m c_v \frac{n-\gamma}{n-1} (T_2 - T_1)$$

$$Q_{1-2} = m c_N (T_2 - T_1)$$

$$\left[c_N = c_v \left[\frac{n-\gamma}{n-1} \right] \text{ is polytropic specific heat} \right]$$

$$\boxed{Q = m c_N \Delta T}$$

First law for an isolated system :

In an isolated system $dQ=0$ & $dW=0$ & hence $\Delta E=0$

Thus $E = \text{constant}$. Thus it states that in an isolated system energy remains constant. For example universe is an isolated system for which the energy remains constant.

- ⑩ A domestic refrigerator is loaded with vegetables, food, extra etc and then door is closed, during a certain period, the m/c consumed 1 kw-hr of energy & I.E of a system drops by 5000 KJ. Find the net heat transfer for the system?

Given Internal energy $\Delta E = -5000 \text{ KJ}$
 $W = 1 \text{ kw-hr}$
 $= -3600 \text{ KJ}$

$$\Delta Q = \Delta E + W$$

$$= -5000 - 3600 = -8600 \text{ KJ}$$

- ⑪ An air reservoir of vol. 5.5 m^3 contains air at 16 bar & 42°C , a valve is opened & some quantity of air escapes to atmosphere. The pressure of air in the reservoir reduced to 12 bar, when the valve is closed. Calculate the mass of the air left from the reservoir. Assume adiabatic process.

Given:
 $V_1 = 5.5 \text{ m}^3$
 $P_1 = 16 \text{ bar} = 1600 \text{ kPa}$
 $P_2 = 12 \text{ bar} = 1200 \text{ kPa}$
 $T_1 = 42 + 273 = 315 \text{ K}$

$$PV^\gamma = \text{const.}$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$P_1 V_1 = P_2 V_2$$

$$P_1 V_1 = nRT_1$$

$$\frac{T_1}{T_2} = \frac{P_1}{P_2} \times \frac{V_1^{1-\gamma}}{V_2^{1-\gamma}}$$

$$\frac{T_1}{T_2} = \frac{P_1}{P_2} \times \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= \frac{P_1}{P_2} \times \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}$$

$$= \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma} + 1}$$

① $m_1 = \frac{P_1 V_1}{R T_1} = \frac{1600 \times 5.5}{0.287 \times 315} = 97.34 \text{ kg.}$

② $P_1 V_1^\gamma = P_2 V_2^\gamma$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^\gamma \Rightarrow \left(\frac{V_2}{V_1}\right) = \left(\frac{P_1}{P_2}\right)^{1/\gamma}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \therefore \frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \frac{315}{T_2} = \left[\frac{1600}{1200}\right]^{\frac{1.4-1}{1.4}} \Rightarrow T_2 = 291.6 \text{ K}$$

$$V_1 = V_2 = 5.5 \text{ m}^3.$$

$$P_2 V_2 = m_2 R T_2$$

$$m_2 = \frac{1200 \times 5.5}{0.287 \times 291} = 79.02 \text{ kg.}$$

$$m = m_1 - m_2 = 18.3 \text{ kg (left mass)}$$

Steady Flow Energy Equation :-

Consider a control vol of a thermodynamic open system through which working fluid is flowing at a steady state.

The fluid enters with an internal energy (u_1), pressure (p_1), vol (v_1), with a velocity (C_1) & at a height of (z_1) from the datum. Due to its velocity it possesses K.E & due to its height it possesses P.E. Q is heat supplied & W is the work produced. If fluid leaves system with u_2, p_2, v_2 & C_2 at a height z_2 , A/c to 1st law of TD energy i/p must be equal to energy o/p. Assume $m = 1 \text{ kg}$ of fluid.

<u>Energy</u>	<u>input</u>	<u>output</u>
① Internal energy	u_1	u_2
② K.E	$\frac{1}{2} C_1^2$	$\frac{1}{2} C_2^2$
③ P.E	$g z_1$	$g z_2$
④ flow work	$p_1 v_1$	$p_2 v_2$

Total energy entering = Total energy leaving

$$Q + \left[u_1 + p_1 v_1 + \frac{1}{2} C_1^2 + g z_1 \right] = \left[u_2 + p_2 v_2 + \frac{1}{2} C_2^2 + g z_2 \right] + W$$

$$\left[h_1 + \frac{1}{2} C_1^2 + g z_1 \right] + Q = \left[h_2 + \frac{1}{2} C_2^2 + g z_2 \right] + W \quad \left[\text{where } h = u + p v \right. \\ \left. \text{enthalpy} \right]$$

Enthalpy is defined as the sum of IE & flow work.

$$h = u + p v$$

Neglecting changes in K.E & P.E. i.e., $C_1 = C_2$ $Z_1 = Z_2$ we get

$$\therefore h_1 + Q = h_2 + W$$

$$Q = (h_2 - h_1) + W$$

$$\boxed{Q = \Delta H + W}$$

Applications of SFEE :-

Most of the engineering applications are steady flow process. All devices can be divided into 3 categories.

① Work producing devices [$W = +ve$]

→ Automobiles, gas turbines, steam engines, turbines, rockets etc.

② Work consuming devices [$W = -ve$]

→ Pumps, Air compressor, motors, fridge, A.C, fans etc.

③ No work interacted devices [$W = 0$]. These are also known as neutral devices mainly used heat exchanger etc.,

→ Boilers, condensers, radiators, cooling towers, nozzles, throttle valve etc.

Many Energy Analysis are derived steady energy flow equation for following devices :-

(1) Turbine (2) Compressor (3) Heat exchangers

(4) Boiler (or) evaporator (or) steam generator

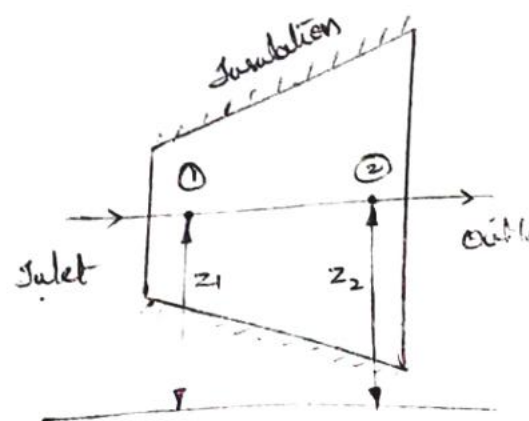
(4) Nozzle

(5) Condenser

(5) Throttle valve.

Turbine :-

Turbine is a rotary device used for producing mechanical energy. Considering one kg of fluid & applying steady flow energy equation.



$$\left[h_1 + \frac{1}{2} c_1^2 + g z_1 \right] + Q = \left[h_2 + \frac{1}{2} c_2^2 + g z_2 \right] + W$$

Assume that change in velocity is very less & hence K.E changes will be neglected.

From diagram $z_1 = z_2$

$$\therefore h_1 + Q = h_2 + W \Rightarrow Q = (h_2 - h_1) + W$$

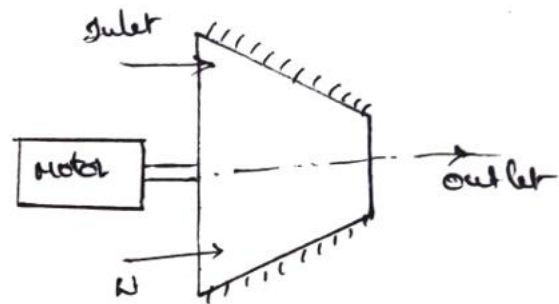
Since system is insulated heat transfer is zero.

$$Q = 0$$

$$\boxed{W = h_1 - h_2}$$

② Compressor :-

Compressor is a device which is used to increase the pressure of air, considering 1 kg of fluid & applying SFEE.



$$\left(h_1 + \frac{1}{2} c_1^2 + g z_1 \right) + Q = \left(h_2 + \frac{1}{2} c_2^2 + g z_2 \right) - W$$

Neglecting K.E & P.E changes, then

$$h_1 + Q = h_2 - W$$

[$\because$ Bcoz of insulation $Q = 0$]

$$\therefore \boxed{W = h_2 - h_1}$$

③ Heat Exchangers :-

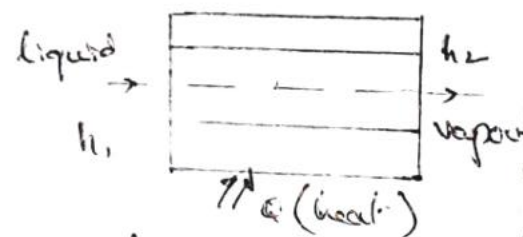
① Boiler or Evaporator

Boiler is a device which is used to convert liquid into vapour convection i.e., to evaporate liquid (or) to produce steam.

$$\text{Steam} \cdot \left[h_1 + \frac{1}{2} c_1^2 + g z_1 \right] + Q = \left[h_2 + \frac{1}{2} c_2^2 + g z_2 \right] + W$$

Neglecting KE & PE changes then

$$h_1 + Q = h_2 + W$$



Boiler is a no-work interaction device. Hence workdone $W = 0$.

$$\therefore \boxed{Q = h_2 - h_1}$$

③ Condenser:-

$$\left[h_1 + \frac{1}{2} c_1^2 + g z_1 \right] - Q = \left(h_2 + \frac{1}{2} c_2^2 + g z_2 \right) + W$$

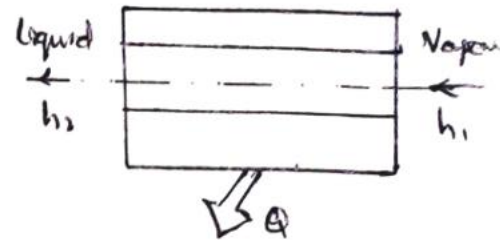
Neglecting KE & PE,

$$h_1 - Q = h_2 + W$$

$$\therefore Q = h_2 - h_1$$

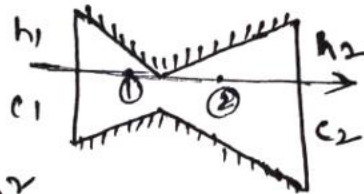
$$Q = h_1 - h_2$$

[$W=0$, since NO work interaction device]



Condenser is a device to convert vapour to liquid i.e., heat rejection process.

④ Nozzle:-



$$\frac{1}{2} c_1^2 + h_1 + Q = h_2 + W + \frac{1}{2} c_2^2$$

$$W=0 \quad \text{NO WORK}$$

The function of nozzle is to increase velocity of fluid i.e. K.E considering 1 kg of fluid.

and $Q=0$ since it is insulated heat transfer is '0'

$$\frac{1}{2} c_1^2 + Q = h_2 - h_1 + \frac{1}{2} c_2^2$$

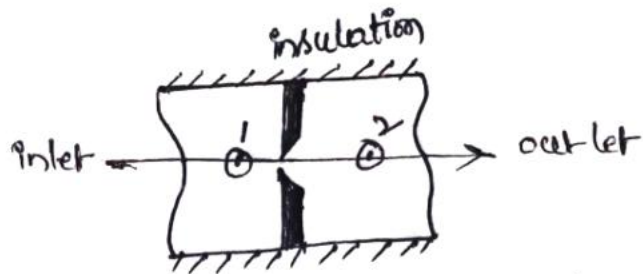
$$\frac{1}{2} c_1^2 + h_1 = \frac{1}{2} c_2^2 + h_2$$

$$h_1 - h_2 = \frac{1}{2} (c_2^2 - c_1^2)$$

$$c_1^2 + 2(h_1 - h_2) = c_2^2$$

$$c_2 = \sqrt{2(h_1 - h_2) + c_1^2}$$

⑤ Throttle valves:-



This is mainly used in Refrigerator and A.C's. throttle valve is used to reduce temp of fluid.

When fluid flows through a very small passage like orifice, there is decreasing pressure & flow is said to be "throttled."

Consider in 1 kg of fluid & applying energy equation.

$$[h_1 + \frac{1}{2}C_1^2 + g z_1] + Q = [h_2 + \frac{1}{2}C_2^2 + g z_2] + W$$

$$Q = 0 \quad [\because \text{insulation}]$$

$$W = 0$$

$$\therefore h_1 = h_2$$

Throttling process is also known as constant enthalpy process (Isenthalpy process)

- ① Steam enters the steam condenser with an enthalpy of 2090 kJ/kg & velocity 510 m/s & after the condensation process ... at enthalpy of 209 kJ/kg & with a velocity of 10 m/s. Determine heat rejected to cooling water per kg of steam ~~condensed~~.

$$[h_1 + \frac{1}{2}C_1^2 + g z_1] + Q = [h_2 + \frac{1}{2}C_2^2 + g z_2] + W$$

$$Q = -ve$$

$$h_1 = 2090 \text{ kJ/kg}$$

$$h_2 = 209 \text{ kJ/kg}$$

$$C_1 = 510 \text{ m/s}$$

$$C_2 = 10 \text{ m/s}$$

$$2090 + \frac{1}{2}(510)^2 + Q = 209 + \frac{1}{2}(10)^2 + W$$

$$[W = 0]$$

$$Q = 2011 \text{ kJ/kg}$$

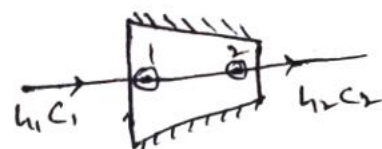
$$Q = -2011 \text{ kJ/kg}$$

$$\frac{1}{2} \times \frac{\text{kg} \cdot \text{m}^2}{\text{sec}^2} = \frac{\text{kg} \cdot \text{m}}{\text{sec}^2} = \text{N}$$

$$\frac{1}{2} \times m v^2 = \frac{1}{2} \times \text{kg} \cdot \frac{\text{m}^2}{\text{sec}^2} = \text{kg} \cdot \frac{\text{m}}{\text{sec}^2} = \text{N}$$

- ② A gas leaving turbojet engine flows into jet pipe with an enthalpy of 960 kJ/kg & velocity of 250 m/s. The exit from pipe is at an enthalpy of 800 kJ/kg. The exhaust is at same pressure as intake. Neglecting heat losses from system, determine velocity of gas leaving the pipe.

sol: $h_1 = 960 \text{ kJ/kg}$ $C_1 = 250 \text{ m/s}$
 $h_2 = 800 \text{ kJ/kg}$ $C_2 = ?$



$$\left[h_1 + \frac{1}{2} C_1^2 + g z_1 \right] + Q = \left[h_2 + \frac{1}{2} C_2^2 + g z_2 \right] + W$$

$Q = 0 \Rightarrow$ insulated
 $W = 0 \Rightarrow$ no work interaction device

$$C_2 = \sqrt{C_1^2 + 2(h_1 - h_2) \times 1000}$$

$$= \sqrt{(250)^2 + 2(960 - 800) \times 10^3}$$

$C_2 = 618.46 \text{ m/s}$ 618.4 m/s

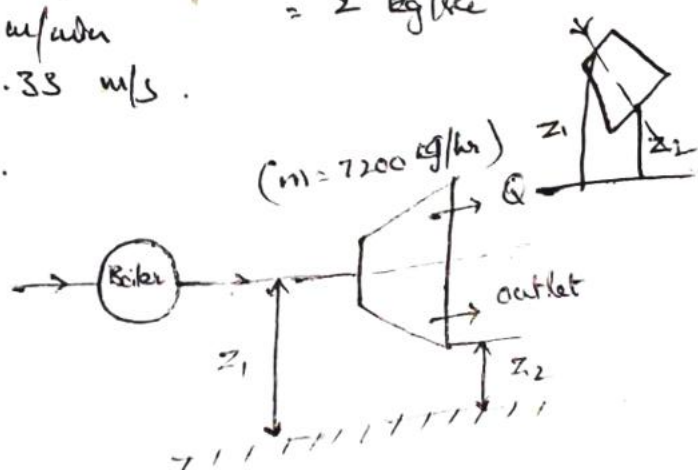
- ③ A steam turbine operates under steady flow conditions. It receives 7200 kg/hr of steam from boiler. Steam enters turbine at an enthalpy of 2800 kJ/kg. The velocity of steam is 4000 m/min at an elevation of 4 m. The velocity of steam leaving turbine is 8000 m/min at an enthalpy of 2000 kJ/kg with a velocity 8000 m/min at an elevation of 1 m. Due to radiation heat losses from the turbine to the surrounding is 1580 kJ/hr. Calculate o/p of turbine.

sol: $h_1 = 2800 \text{ kJ/kg}$ $h_2 = 2000 \text{ kJ/kg}$ $m = 7200 \text{ kg/hr}$
 $C_1 = 4000 \text{ m/min}$ $C_2 = 8000 \text{ m/min}$ $= 2 \text{ kg/sec}$
 $= 66.67 \text{ m/s}$ $= 133.33 \text{ m/s}$
 $Z_1 = 4 \text{ m}$ $Z_2 = 1 \text{ m}$

$$Q = -1580 \text{ kJ/hr}$$

$$= \frac{-1580}{3600} \text{ kJ/sec}$$

$$= -0.438 \text{ kJ/sec}$$



$$N = ?$$

$$\left[h_1 + \frac{1}{2} C_1^2 + g z_1 \right] + Q = \left[h_2 + \frac{1}{2} C_2^2 + g z_2 \right] + W$$

$$\therefore W = \left[(h_1 - h_2) + \frac{1}{2} \left(\frac{C_1^2 - C_2^2}{1000} \right) + \frac{g(z_1 - z_2)}{1000} \right] + Q$$

$$\text{Heat loss per kg of steam} = \frac{Q}{m} = \frac{-0.438}{2} = -0.219 \text{ kJ/kg}$$

$$W = 800 + \frac{1}{2} (C_1^2 - C_2^2)$$

$$W = 793.6 \text{ kJ/kg} - 5832 \text{ kJ/kg}$$

$$\text{Output of the turbine } W = 793.6 \times 2 \text{ kg/sec}$$

$$W = 1587.2 \text{ kW.}$$

Feb

- ③ Refrigerant vapour enters condenser of refrigeration plant with enthalpy of 223.75 kJ/kg & leaves with an enthalpy of 64.6 kJ/kg. Cooling water enters at 15°C & leaves at 20°C. Calculate mass flow rate of water per unit flow rate of refrigerant.

$$h_1 = 223.75 \text{ kJ/kg}$$

$$h_2 = 64.6 \text{ kJ/kg}$$

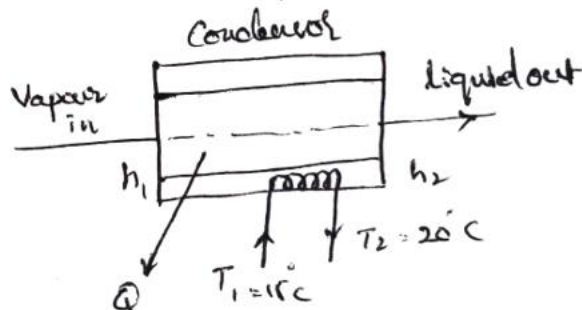
$$T_1 = 15 + 273 = 288 \text{ K}; T_2 = 20 + 273 = 293 \text{ K}$$

A/c to 1st law heat loss by refrigerant = heat gained by water.

$$m_r (h_1 - h_2) = m_w c_{pw} (T_2 - T_1)$$

$$1 (64.6 - 223.75) = m_w (4.187) (293 - 288)$$

$$m_w = 7.602 \text{ kg/sec}$$



$$\left[\begin{array}{l} m_r = 1 \\ c_{pw} = 4.187 \text{ kJ/kg K} \\ c_{pav} = 1.005 \text{ kJ/kg K} \end{array} \right]$$

- ④ Show that I.E. of a system is a point function & thermodynamic property of a system?

⑤ Internal Energy: Energy stored in the body is called I.E. It is neither heat nor work. It is associated with

motion of molecules, atoms & sub-atomic particles.

Consider a closed system from ① to ② through path A & returned to its initial state through B. The cycle is 1-A-2-B-1



Acc to 1st law of TD for cyclic process

$$\Sigma Q = \Sigma W$$

$$Q_A + Q_B = W_A + W_B \rightarrow \textcircled{1}$$

Applying 1st law for states A & B

$$Q_A = \Delta E_A + W_A \rightarrow \textcircled{2}$$

$$Q_B = \Delta E_B + W_B \rightarrow \textcircled{3}$$

$$Q_A + Q_B = (\Delta E_A + \Delta E_B) + (W_A + W_B)$$

$$W_A + W_B = (\Delta E_A + \Delta E_B) + (W_A + W_B)$$

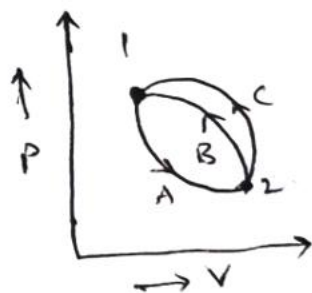
$$\Delta E_A + \Delta E_B = 0$$

$$\boxed{\Delta E_A = -\Delta E_B} \rightarrow \textcircled{4}$$

Change in I.E of system of path A & B is same.

(-ve) sign is due to change in direction).

checking for one more path or process



considering a system returning to its initial state through path C. Now

the cycle is 1-A-2-C-1.

$$\Sigma Q = \Sigma W$$

$$Q_A + Q_C = W_A + W_C \rightarrow \textcircled{5}$$

$$Q_A = \Delta E_A + W_A \rightarrow \textcircled{6}$$

$$Q_C = \Delta E_C + W_C \rightarrow \textcircled{7}$$

$$Q_A + Q_C = (\Delta E_A + \Delta E_C) + (W_A + W_C)$$

$$\boxed{\Delta E_A = -\Delta E_C} \rightarrow \textcircled{8}$$

From eq. (7) it is evident that change in I.E. is same for path A & path C.

From eq. (4) & eq. (8), we can write as follows

$$\boxed{\Delta E_A = \Delta E_B = \Delta E_C} \quad (\text{neglecting sign})$$

Hence I.E. is a point function. From above I.E. is same b/w two fixed state point & it is not depend on path (or) process. Hence the I.E. of system is point function & properties of system.

- ④ A cylinder contains 0.115 m^3 of gas at 1 bar & 90°C . The gas is compressed to a volume of 0.0288 m^3 , the final pressure being 5.67 bar. Calculate (a) the mass of gas. (b) the value of index of compression (c) the increase in internal energy of gas (d) the heat transfer during the compression.

If after the above compression, the gas is to be cooled at constant pressure to its original temp of 90°C , find the further work of compression required, assume $\gamma = 1.4$ & $R = 0.3 \text{ kJ/kg K}$.

Sol:

$$V_1 = 0.115 \text{ m}^3$$

$$P_1 = 1 \times 10^2 \text{ kPa}$$

$$T_1 = 90^\circ \text{C} = 363 \text{ K}$$

$$V_2 = 0.0288 \text{ m}^3$$

$$P_2 = 5.67 \text{ kPa}$$

$$\gamma = 1.4$$

$$R = 0.3 \text{ kJ/kg K}$$

find $m = ?$

$$n = ?$$

$$\Delta U_{1-2} = ?$$

$$Q_{1-2} = ?$$

$$\text{If } P_3 = P_2$$

$$T_3 = T_1 = 363 \text{ K}$$

$$W_{2-3} = ?$$

Limitations of First law of TD :-

- (i) According to 1st law of TD, heat & work are mutually convertible. This is not true in real practice.
- (ii) There is no restriction on the direction of flow of work & heat which is not true.
- (iii) In general heat is not completely converted into work.
- (iv) High pressure gas expands to low pressure, but the reverse is not possible in general i.e., some external energy is required which violates the 1st law of TD.
- (v) In an automobile, if brakes are applied the brake work is converted into heat & dissipated into the atmosphere. If reverse process is considered i.e., by giving the heat to brake wheel, it should rotate, which is not possible.
- (vi) A/c to Joule's experiment work can be completely converted into heat but in general complete conversion of heat into work is not possible becoz of losses.

$$W \xrightarrow{=} Q$$

$$Q \xrightarrow{>} W$$

Thermal Reservoir :-

→ It is a reservoir having infinite amount of heat quantity in it such that when we add or remove finite amount of heat from it its temp will not change.

→ Thermal reservoir is a body of infinite heat capacity,

which is capable of absorbing or rejecting unlimited qty of heat without affecting its temperature.

→ All the process associated with a thermal reservoir are assumed to be quasi-static. [ex:- ocean water, atm. air]

Heat Source:

The thermal reservoir from which heat is transferred to a system during a cycle is called 'heat source'.

Ex: Boiler furnace, Sun, Nuclear reactors, combustion chamber of an engine or turbine etc.

Heat Sink:-

The thermal reservoir to which heat is rejected from the system during a cycle is called as 'heat sink'.

Ex: Atmospheric air, river, sea (or) ocean water etc.

Heat Engine:-

A heat engine is a device which converts heat energy (low grade) into mechanical work (high grade energy) [ex:- Steam power plant]

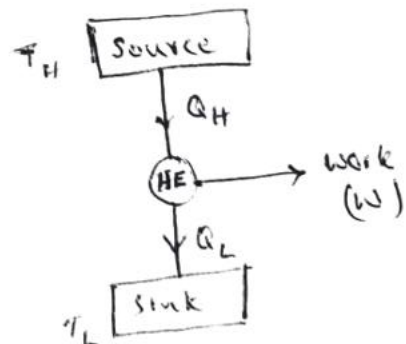
Thermal efficiency $\eta_{th} = \frac{Q/p}{1/p} = \frac{\text{Net workdone}}{\text{Heat supplied.}}$

$Q_H \rightarrow$ heat supply from source

$Q_L \rightarrow$ heat rejected to sink

$T_H \rightarrow$ highest (or) hot body temp

$T_L \rightarrow$ lowest (or) cold body temp



$$\eta = \frac{Q/p}{1/p} = \frac{W}{Q}$$

where $W = Q_H - Q_L$

$Q = Q_H$

$$\eta = \frac{Q_H - Q_L}{Q_H}$$

(*)

$$\eta = \frac{T_H - T_L}{T_H}$$

Refrigerator :-

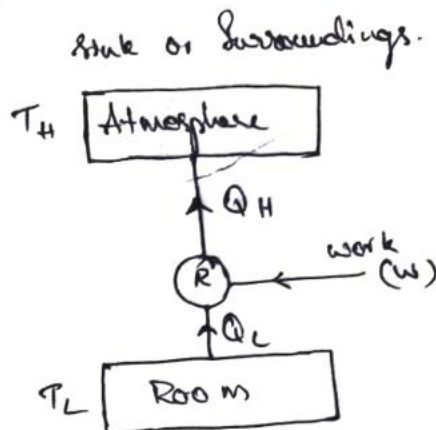
Refrigerator is a device which is used to produce cooling effect & maintaining at low temp.

The efficiency of refrigerator is generally expressed in terms COP (Coefficient of performance).

$$\text{Efficiency } \eta = \frac{O/P}{I/P}$$

$$= \frac{\text{cooling effect}}{\text{work}} = \frac{Q_L}{W}$$

$$\text{COP} = \frac{Q_L}{Q_H - Q_L} \quad \left[\begin{array}{l} Q_L + W = Q_H \\ \therefore W = Q_H - Q_L \end{array} \right]$$



$$(or) \text{ COP} = \frac{T_L}{T_H - T_L}$$

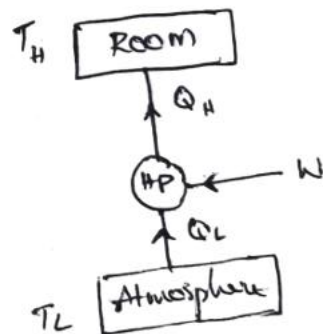
Ex: Refrigerators, A.C.'s etc.,

Heat Pump :-

Heat pump is a device which delivers heat energy from low temp to high temp while working in a cycle.

COP of heat pump can be represented as follows.

$$\text{COP}_{HP} = \frac{O/P}{I/P} = \frac{\text{heating effect}}{\text{work}} = \frac{Q_H}{W}$$



$$Q_H = Q_L + W$$

$$W = Q_H - Q_L$$

$$\boxed{\text{COP}_{HP} = \frac{Q_H}{Q_H - Q_L}}$$

(or)

$$\boxed{(\text{COP})_{HP} = \frac{T_H}{T_H - T_L}}$$

Note :- Derive the relation b/w COP of heat pump & COP of Refrigerator.

$$\frac{Q_H}{Q_H - Q_L} - \frac{Q_L}{Q_H - Q_L} = 1$$

$$\boxed{(\text{COP})_{HP} - (\text{COP})_R = 1}$$

① A heat engine, one heat pump & a refrigerator are working b/w two heat reservoirs one at 600°K , the other at 300°K . calculate

① Efficiency of heat engine

② COP_{HP}

③ COP_R

Sol: ① Efficiency of HE = $\frac{T_H - T_L}{T_H} = \frac{600 - 300}{600} = \frac{3}{6} = 0.5 = 50\%$

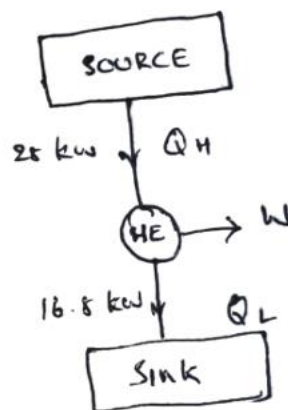
② $\text{COP}_{\text{HP}} = \frac{T_H}{T_H - T_L} = \frac{600}{600 - 300} = 2$

③ $\text{COP}_R = \frac{T_L}{T_H - T_L} = \frac{300}{600 - 300} = 1$

② A heat engine receives heat at the rate of 1500 kJ/min & gives an o/p of 8.2 kW . Determine

④ Thermal efficiency

⑥ Rate of heat rejection



Sol:

$Q_H = 1500 \text{ kJ/min}$

$Q_H = \frac{1500}{60} \text{ kJ/sec} = 25 \text{ kW}$

$W = 8.2 \text{ kW}$

$\eta = \frac{Q_H - Q_L}{Q_H} = \frac{W}{Q_H} = \frac{8.2}{25} = 32.8\%$

$W = Q_H - Q_L \Rightarrow -Q_L = -16.8 \text{ kW}$

$Q_L = 16.8 \text{ kW}$

③ A H-E, H-P & refrigerator receives 500 kJ of heat each but they reject 250 kJ , 400 kJ & 600 kJ of heat respectively.

Determine ① η of HE ② COP_R ③ COP_{HP}

Represent these values with the help of diagrams.

Sol: Efficiency HE, (η) :-

$$\eta = \frac{O/P}{I/P} = \frac{Q_H - Q_L}{Q_H}$$

$$= \frac{500 - 250}{500} = 0.5$$

$$\eta = 50\%$$

$$\textcircled{2} \text{ COP}_R = \frac{O/P}{I/P} = \frac{Q_L}{Q_H - Q_L} = \frac{500}{100} = 5$$

$$\textcircled{3} \text{ COP}_{HP} = \frac{O/P}{I/P} = \frac{Q_H}{Q_H - Q_L} = \frac{600}{100} = 6$$

- ④ Calculate COP & Heat transfer to condenser of a refrigerator in kJ/hr which has refrigeration capacity 12000 kJ/hr when power input is 0.75 kW.

Sol: $Q_L = 12000 \text{ kJ/hr}$
 $= 200 \text{ kJ/min}$

$$W = 0.75 \text{ kW}$$

$$= 0.75 \times 3600 = 2700 \text{ kJ/hr}$$

$$Q_H = Q_L + W$$

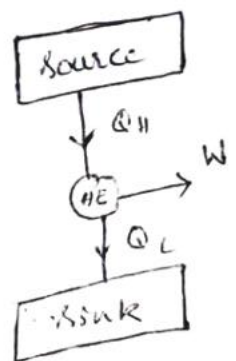
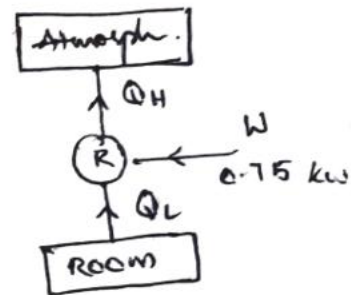
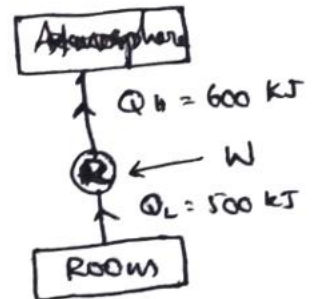
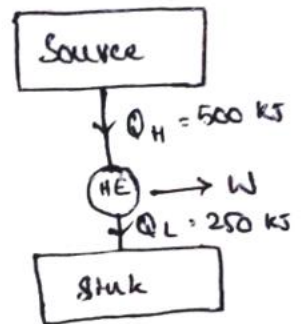
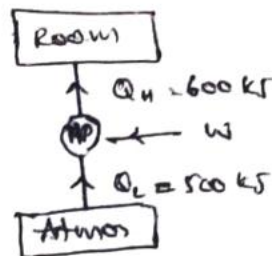
$$= 12000 + 2700 = 14700 \text{ kJ/hr}$$

$$\text{COP} = \frac{Q_L}{Q_H - Q_L} = \frac{12000}{2700} = 4.44$$

- ⑤ Reversible H.E operates b/w 2 reservoirs at 420 K & 280 K. If o/p from engine is 2.5 kJ. Determine efficiency of engine and its heat interactions with reservoirs. Subsequently engine is made to operate as a heat pump b/w same reservoir. Calculate COP_{HP} and power input required when heat transfer rate from 280 K reservoir is 5 kW.

Sol: $\eta = \frac{T_H - T_L}{T_H} = \frac{420 - 280}{420} = 0.33$

$$\eta = 33.3\%$$



$$W = 2.5 \text{ kJ}$$

$$Q_H = Q_L + W$$

$$\eta = \frac{O/P}{I/P}$$

$$0.333 = \frac{W}{Q_H} = \frac{2.5}{Q_H}$$

$$\therefore Q_H = 7.5 \text{ kJ}$$

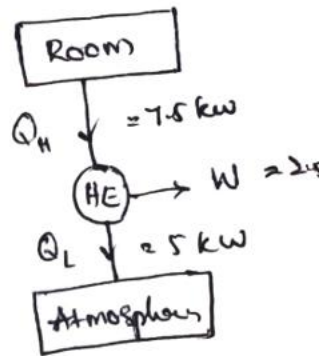
$$W + Q_L = Q_H \Rightarrow Q_L = Q_H - W = 5 \text{ kJ}$$

$$\text{COP}_{HP} = \frac{420}{420 - 280} = 3$$

$$3 = \frac{Q_H}{Q_H - Q_L} \Rightarrow Q_H = 3Q_H - 3Q_L$$

$$2Q_H = 3Q_L$$

$$Q_H = \frac{3}{2} \times 5 = 7.5 \text{ kW}$$



- ⑥ A reversible heat engine operating b/w 900 K & 300 K is used to a reversible refrigerator for which temp limits are 300 K & 250 K. The engine absorbs 1800 kJ of energy as heat from reservoir at 900 K. The net output from engine & refrigerator system is 360 kJ. Make calculation for heat extracted from refrigerator cabinet & total heat rejected at 300 K.

$$\eta = \frac{T_H - T_L}{T_H} = \frac{900 - 300}{900}$$

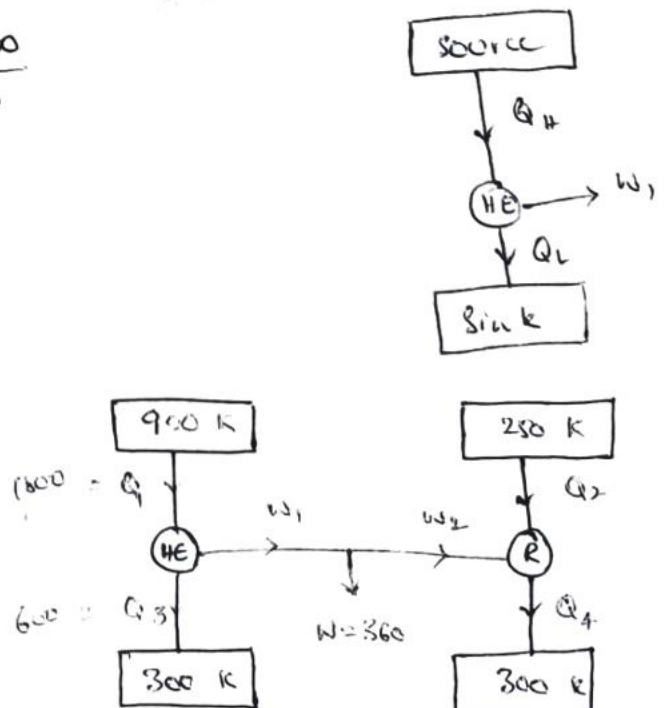
$$\eta = 66.67\%$$

$$0.667 = \frac{1800 - Q_L}{1800}$$

$$Q_L = 600 \text{ kJ}$$

$$\text{COP}_R = \frac{T_L}{T_H - T_L} = \frac{250}{300 - 250}$$

$$\text{COP}_R = 5$$



$$\text{COP}_R = \frac{Q_L}{Q_H - Q_L} \Rightarrow 5 = \frac{Q_2}{Q_4 - Q_2} \rightarrow (1)$$

$$W_1 = Q_1 - Q_3$$

$$W_1 = 1800 - 600 = \underline{1200 \text{ KJ}}$$

$$\text{Given, } W = W_1 - W_2$$

$$W_2 = 360 - 1200 = -840$$

$$W_2 = \underline{840 \text{ KJ}}$$

$$\therefore Q_4 - Q_2 = W_2 = \underline{840 \text{ KJ}}$$

$$\therefore 5 = \frac{Q_2}{Q_4 - Q_2} \Rightarrow 5 = \frac{Q_2}{840} \Rightarrow Q_2 = 4200 \text{ KJ} \rightarrow (2)$$

$$\therefore Q_4 - Q_2 = 840$$

$$Q_4 = 840 + 4200 = 5040 \text{ KJ} \rightarrow (3)$$

Heat extracted from refrigerator $Q_2 = 4200 \text{ KJ} \rightarrow (4)$

Total heat rejected at 300 K reservoir,

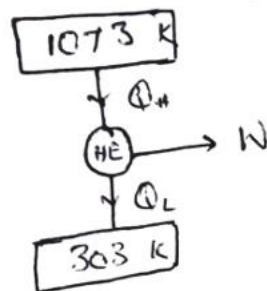
$$\underset{\text{HE}}{Q_3} + \underset{\text{R}}{Q_4} = 5040 + 600 = 5640 \text{ KJ} \rightarrow (5)$$

(7) Cyclic H.E operates b/w source temp of 800°C & sink temp of 30°C , what is the least rate of heat rejection per kW, net o/p of engine.

$$\eta = \frac{\text{o/p}}{\text{i/p}} = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H}$$

$$= \frac{T_H - T_L}{T_H} = \frac{1073 - 303}{1073}$$

$$\eta = 71.76\%$$



$$\eta = \frac{W}{Q_H} \Rightarrow 0.7176 = \frac{W}{Q_H} \Rightarrow Q_H = 1.3927$$

$$W = Q_H - Q_L \Rightarrow Q_L = 1 - 1.3927$$

$$Q_L = 0.3927 \text{ kW}$$

SECOND LAW OF TD :-

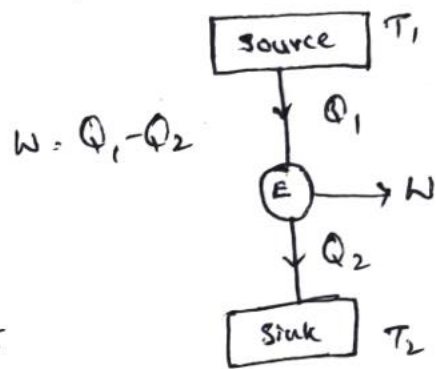
It is stated in two ways i.e., through

- (i) Kelvin-Planck Statement
- (ii) Clausius Statement

Kelvin-Planck's Statement :-

"It is impossible to construct an engine working on a cyclic process, whose sole purpose is to convert all the heat supplied to it into equivalent amount of work."

If $Q_2 = 0$ (i.e., $W_{\text{net}} = Q_1$
or $\eta = 100\%$),



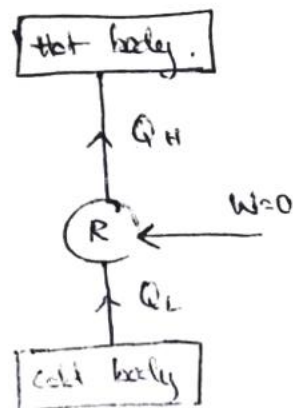
the heat engine will produce net work in a complete cycle by exchanging heat with only one reservoir, thus violating the Kelvin-Planck statement.

Such a heat engine is called a Perpetual motion machine of the second kind (PMM2). A PMM2 is impossible.

Clausius Statement :-

"It is impossible for the heat to flow from a body at a lower temp to a body at higher temp without aid of external agency". (C1)

"Heat flows from a hot body to a cold body."



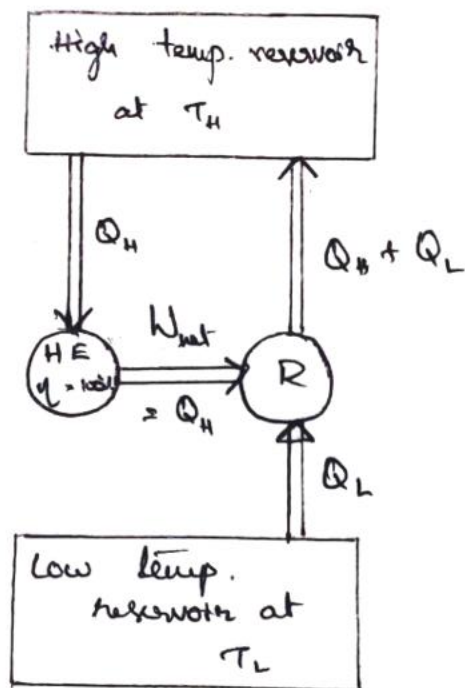
Corollaries of Second law:-

- ① No engine, working on a cyclic process is more efficient than (Carnot's) reversible engine when working b/w the same two limits of temperature.
- ② All reversible engines have the same efficiency when working b/w the same two reservoirs of heat.
- ③ Whenever a system executes a complete cyclic process the $\oint dQ/T$ around the cycle is less than zero or in the limit is equal to zero. This shows the inequality of Clausius.
- ④ $\oint dQ/T = 0$ for any reversible cyclic process & consequently for any reversible process b/w reference states ① & ② is a property of the system in state ②. This property is called 'entropy'.
- ⑤ A true absolute TD temp scale may be defined which is independent of the characteristics of the thermodynamic medium.

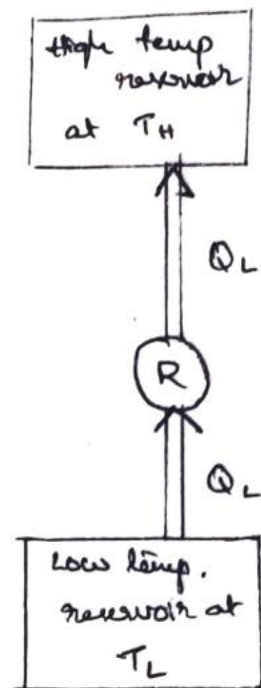
Equivalence of the two statements:-

The Kelvin Planck & the Clausius statements are equivalent in their consequence, & either statement can be used as the expression of II law of TD. Any device that violates the Kelvin-Planck statement also violates the Clausius statement & vice versa.

Consider the heat engine - refrigerator combination as shown in fig., operating b/w the same two reservoirs. The heat engine is assumed to have, in violation of Kelvin-Planck statement, a thermal η of 100%. & therefore it converts



(a) A refrigerator that is powered by a 100% efficient heat engine.



(b) The equivalent refrigerator

all the heat Q_H it receives, to work W . This work is now supplied to a refrigerator that removes heat in the amount of Q_L from the low temp reservoir & rejects heat in the amount of $Q_L + Q_H$ to the high temp reservoir. During this process, the high temp reservoir receives a net amount of heat Q_L (the diff b/w $Q_L + Q_H$ and Q_H). Thus, the combination of these two devices can be viewed as a refrigerator, that transfers heat in an amount of Q_L from a cooler body to a warmer one without requiring any input from outside. This is clearly a violation of the Clausius statement.

It can also be shown in a similar manner that a violation of the Clausius statement leads to the violation Kelvin-Planck statement. Therefore, both are two equivalent expressions of II law of TD.

Perpetual Motion Machine of Second Kind (PMM II) :-

It is impossible to construct a PMM II. It is a hypothetical m/c that continuously supply heat from low temp reservoir to high temp without taking any input or work from surroundings.

But such a m/c is not feasible from practical point of view. It violates II law of TD. (Clausius statement)

Carnot Cycle/Engine :-

It is a reversible cycle which consists of all reversible process. i.e.,

- Reversible isothermal heat addition
- Reversible adiabatic expansion
- Reversible isothermal heat rejection
- Reversible adiabatic compression.

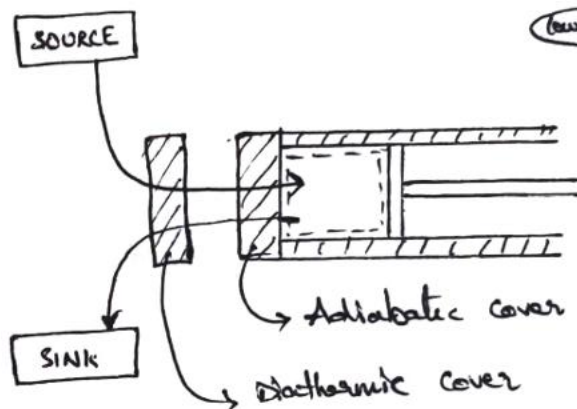
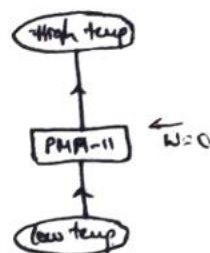


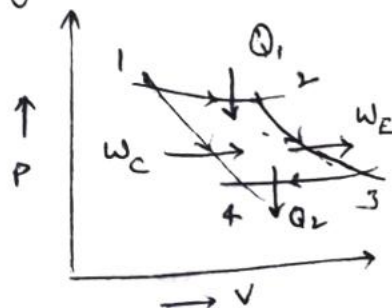
fig: CARNOT ENGINE



Carnot has proposed a piston cylinder arrangement with working medium & two covers as shown in fig.

Process (1-2) : $T_H = C$

By replacing diathermic cover an amount of heat Q_1 is supplied from the source at constant temp T_H .



Process (2-3) :- The diathermic cover is replaced by an adiabatic cover & the medium with its stored energy pushes the piston forward by developing work W_E adiabatically & reversibly & the temp of medium decreases from T_H to T_L .

Process (3-4) :- ^(T₁ = C) The adiabatic cover is replaced by diathermic cover & Q₂ amount of heat is made to leave the system at T₂ to a sink at T₂ reversibly.

Process (4-1) :- ^(T_L to T_H) The diathermic cover is again replaced by adiabatic cover & pump work W_p is done on the medium reversibly & adiabatically till the temp rises to T₁ & system returns to its initial state, as shown in PV diagram.

For a steady flow process,

A/c to I law of TD for a cycle,

$$\sum Q = \sum W$$

$$Q_H - Q_L = W_E - W_C$$

$$\therefore \text{Efficiency of Carnot engine} = \frac{\text{Net work output}}{\text{Heat supply}}$$

$$\eta_{\text{Carnot}} = \frac{W_E - W_C}{Q_H} = \frac{Q_H - Q_L}{Q_H}$$

$$\therefore \eta_{\text{Carnot}} = \frac{T_H - T_L}{T_H}$$

Reversible Carnot Heat Engine :-

Since all the process in the Carnot cycle are reversible, it is possible to reverse the engine. Let individually all the process are reversed then

Process (1-2) → Isothermal heat rejection

(2-3) → Adiabatic pumping

(3-4) → Isothermal heat addition

(4-1) → Adiabatic expansion.

Thus all the energy transfers associated with the process are reversed in direction but their magnitude remains the same.

Thus the W_{net} is negative & heat is transferred from lower temp reservoir to higher temp reservoir thus acts as a heat pump or refrigerator.

⊕ Development of Carnot cycle is practically impossible becoz.

- All process in nature are irreversible.
- It is not possible to replace the covers diathermic & adiabatic continuously.
- Isothermal heat addition & rejection requires infinite time i.e., piston has to move infinitely slow where adiabatic process ^(expansion & compression) must be completed very quickly. Two kinds of motion can't be obtained.
- Friction less motion of piston is not possible.
- The curves isothermal & adiabatic when drawn on P-V diagram due to the small diff. in their slope they will not converge easily & requires a large stroke. Thus V_s will be very high with less enclosed area i.e., work development.

Thermodynamic Temp Scale :-

we know that η of reversible Carnot heat engine is a function of temp only.

The absolute temp scale is also known as Kelvin scale. It depends on heat absorbed & heat rejected & is independent of peculiar characteristics of working substance. In defining Kelvin scale the triple point of water ($0^\circ\text{C} = 273.16\text{ K}$) is taken as standard reference point.

$$\Rightarrow \frac{Q_2}{Q_1} = \frac{T_2}{T_1} \quad (or) \quad T = 273.16 \frac{Q}{Q_0} \cdot \left[\frac{Q}{Q_0} + \frac{T}{T_0} \right]$$

Thus the absolute TD temp scale has a definite zero point of water which can be attained by imaging a series of reversible engines extending from source T_1 to a lower temp. If enough engines are placed in series to make the total work output equal to Q_1 , then the heat rejected by last engine is zero. However 2nd law do not allow to develop such an engine.

When the heat rejected approaches zero the temp of heat rejection also approaches a zero limit. Thus it appears a definite zero point exists an absolute temp scale but the point can't be reached without violation of second law.

- ① An engine operating on Carnot cycle works within temp limit of 600 K & 300 K. If engine receives 2000 KJ of heat. Calculate workdone & thermal efficiency of engine.

Sol:-

$$T_1 = 600 \text{ K}$$

$$T_2 = 300 \text{ K}$$

$$Q_H = 2000 \text{ KJ}$$

$$\eta_{\text{Carnot}} = \frac{T_H - T_L}{T_H} = \frac{W}{Q_H}$$

$$\Rightarrow \frac{600 - 300}{600} = \frac{W}{2000}$$

$$Q_H - Q_L = W$$

$$2000 - Q_L = 1000$$

$$Q_L = 1000 \text{ KJ}$$

$$W = 1000 \text{ KJ}$$

$$\eta_{th} = 0.5 = 50\%$$

- ② Determine power required to run refrigerator that transfers 2000 KJ/min heat from cooled space at 0°C to the surrounding atmosphere at 27°C . The refrigerator operates on reverse Carnot cycle.

$$\text{COP}_{\text{ref}} = \frac{T_L}{T_H - T_L} = \frac{Q_L}{W}$$

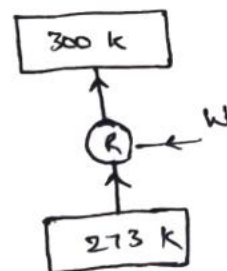
$$\text{COP} = \frac{273}{300 - 273} = 10.111$$

$$10.111 = \frac{Q_L}{Q_H - Q_L} \quad [\because Q_L = 2000 \text{ kJ/min}]$$

$$Q_H = 2197.8 \text{ kJ/min}$$

$$W = Q_H - Q_L = 2197.8 - 2000 = 197.8 \text{ kJ/min}$$

$$W = 3.297 \text{ kW}$$



- ③ HE_1, HE_2, HE_3 are connected in series as shown. They are working with same thermal efficiency. Calculate W_1, W_2, W_3 . (Carnot engine).

Q4. Given

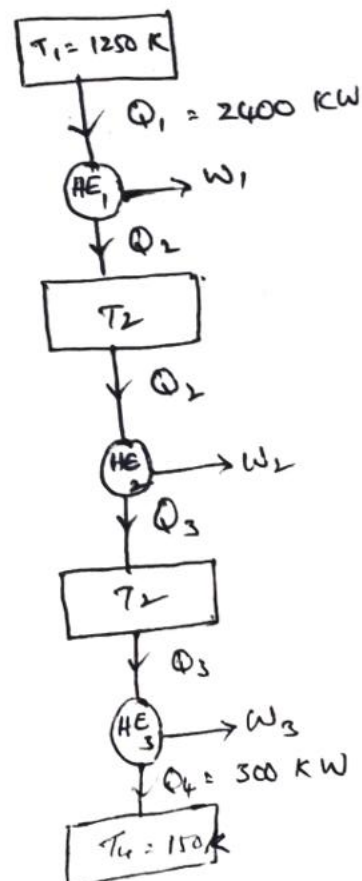
$$\eta_1 = \eta_2 = \eta_3$$

$$\begin{aligned} \frac{HE_1}{\eta_1} &= \frac{Q_H - Q_L}{Q_H} = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1} \\ &= 1 - \frac{Q_2}{Q_1} \end{aligned}$$

$$1 - \eta = \frac{Q_L}{Q_1} \rightarrow (1)$$

$$\frac{HE_2}{\eta_2} = \frac{Q_2 - Q_3}{Q_2} = 1 - \frac{Q_3}{Q_2}$$

$$1 - \eta_2 = \frac{Q_3}{Q_2} \rightarrow (2)$$



HE₃ :-

$$\eta_3 = \frac{Q_3 - Q_4}{Q_3} = 1 - \frac{Q_4}{Q_3}$$

$$1 - \eta_3 = \frac{Q_4}{Q_3} \rightarrow (3)$$

Multiplying (1), (2) & (3), we get.

$$(1 - \eta)^3 = \frac{Q_2}{Q_1} \times \frac{Q_3}{Q_2} \times \frac{Q_4}{Q_3} = \frac{Q_4}{Q_1}$$

$$(1 - \eta)^3 = \frac{300}{2400}$$

$$1 - \eta = 0.5$$

$$\eta = 0.5 = 50\%$$

HE₁ :-

$$\eta_1 = \frac{Q_1 - Q_2}{Q_1} = \frac{W_1}{Q_1}$$

$$W_1 = \eta_1 \times Q_1 = 2400 \times 0.5 = \underline{1200 \text{ kW}}$$

HE₂ :-

$$\eta_2 = \frac{W_2}{Q_2}$$

$$\frac{Q_1 - Q_2}{Q_1} = 0.5$$

$$1 - \frac{Q_2}{Q_1} = 0.5 \Rightarrow 1 - \frac{Q_2}{2400} = 0.5$$

$$\therefore W_2 = \eta_2 \times Q_2$$
$$= 0.5 \times 1200$$

$$Q_2 = \underline{1200 \text{ kW}}$$

$$W_2 = \underline{600 \text{ kW}}$$

HE₃ :-

$$\eta_3 = \frac{W_3}{Q_3}$$

$$1 - \frac{Q_4}{Q_3} = 0.5$$

$$1 - \frac{300}{Q_3} = 0.5 \Rightarrow Q_3 = \underline{600 \text{ kW}}$$

$$W_3 = \eta_3 \times Q_3$$
$$= 0.5 \times 600$$

$$W_3 = \underline{300 \text{ kW}}$$

- ④ 2 Reversible heat engines A & B are connected in series with 'A' rejecting heat directly to 'B' through an intermediate reservoir. Engine 'A' receives 200 kJ of heat from a reservoir at 421°C & engine 'B' is in thermal communication with a sink at 4.4°C . If work output of 'A' is twice of 'B', calculate
- (1) Intermediate temp b/w A & B.
 - (2) η of each engine.
 - (3) Heat rejected to cold sink.
- Draw the operating system for the above data.

sol: Given

$$W_A = 2 \times W_B$$

$$\text{HE}_1 :- \eta_A = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\frac{T_1 - T_2}{T_1} = \frac{W_A}{Q_1}$$

$$W_A = Q_1 \left[\frac{T_1 - T_2}{T_1} \right] \rightarrow (1)$$

$$\text{HE}_2 :- \eta_B = \frac{T_2 - T_3}{T_2} = \frac{Q_2 - Q_3}{Q_2}$$

$$W_B = Q_2 \left[\frac{T_2 - T_3}{T_2} \right] \rightarrow (2)$$

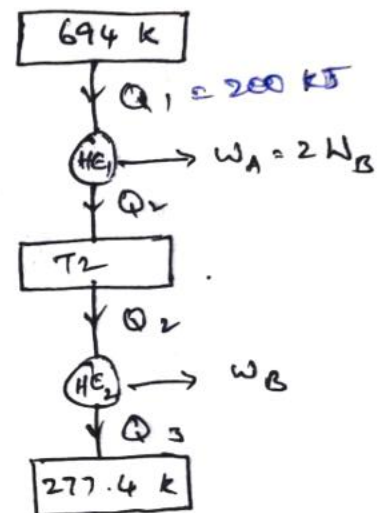
we know that $W_A = 2W_B$

$$Q_1 \left[\frac{T_1 - T_2}{T_1} \right] = 2 Q_2 \left[\frac{T_2 - T_3}{T_2} \right]$$

$$\frac{Q_1}{T_1} \times \frac{T_1 - T_2}{T_1} = 2 \frac{Q_2}{T_2} \left(\frac{T_2 - T_3}{T_2} \right)$$

$$T_1 - T_2 = 2T_2 - 2T_3$$

$$T_1 + 2T_3 = 3T_2$$



$$Q_1 = T_1 ; Q_2 = T_2$$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$Q_1 = \frac{Q_2 T_1}{T_2}$$

$$694 + 554.8 = T_2$$

$$T_2 = 416.26 \text{ K}$$

$$\therefore \eta = \frac{T_1 - T_2}{T_1}$$

HEA:-

$$\eta_A = \frac{694 - 416.26}{694} = 40\%$$

$$\eta_A = \frac{W_A}{Q_1} \Rightarrow W_A = 200 \times 0.4 = 80 \text{ KJ}$$

HEB:-

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$Q_2 = \frac{Q_1}{T_1} \times T_2 = \frac{200}{694} \times 416.26$$

$$Q_2 = 119.96 \text{ KW}$$

$$\eta_B = \frac{Q_2 - Q_3}{Q_2} = \frac{T_2 - T_3}{T_2} = \frac{416.26 - 277.4}{416.26} = 33.3\%$$

$$\eta_B = \frac{W_B}{Q_2} \Rightarrow W_B = \eta_B \times Q_2 = 0.333 \times 119.96 = 39.946 \text{ KJ}$$

$$\therefore Q_3 = Q_2 - W_B$$

$$Q_3 = 120 - 39.6 = 80.4 \text{ KJ}$$

⑤ Consider 3 hypothetical heat engines A, B & C, each operating b/w 1000 K & 300 K. When each engine involves itself with a heat interaction of 1000 KJ with high temp reservoir, it is claimed that while A develops a work of 600 KJ, B & C develop 700 & 800 KJ. Use Carnot statement & identify the engines A, B, & C are reversible, irreversible or impossible.

sol:	$T_1 = 1000 \text{ K}$	$W_A = 600 \text{ KJ}$
	$T_2 = 300 \text{ K}$	$W_B = 700 \text{ KJ}$
	$Q_1 = 1000 \text{ KJ}$	$W_C = 800 \text{ KJ}$

All the engines are operating b/w same two temp reservoirs,

$$\therefore \eta_{\text{max}} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{1000} = 0.7$$

Consider engine (A)

$$\eta_A = \frac{W_A}{Q_1} = \frac{600}{1000} = 0.6$$

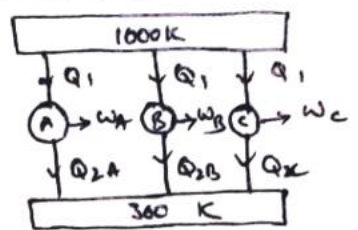
As $\eta_A < \eta_{\text{max}}$, the heat engine is irreversible.

engine (B) :- $\eta_B = \frac{W_B}{Q_1} = \frac{700}{1000} = 0.7$

$$\eta_{\text{max}} = \eta_B = 0.7, \text{ so it is reversible.}$$

engine (C) :- $\eta_C = \frac{W_C}{Q_1} = \frac{800}{1000} = 0.8$

$$\eta_{\text{max}} < \eta_C, \text{ so it is impossible.}$$



- ⑤ A direct heat engine A & a reversed heat engine B operate b/w 177°C & 27°C . The COP of B as a heat pump is 2.5. A driver B. The magnitude of heat interaction of A & B with the reservoir at 27°C are 200 & 50 KJ. The combined work output of A & B is 20 KJ. Identify whether the heat engine A is reversible or irreversible.

Sol: $T_1 = 450 \text{ K}$

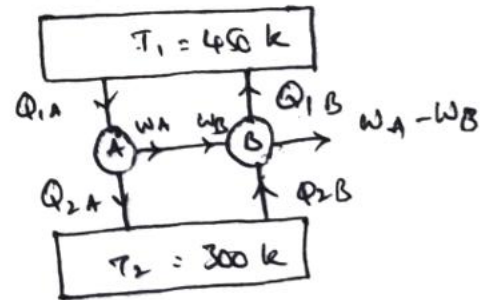
$$T_2 = 300 \text{ K}$$

$$Q_{2A} = 200 \text{ KJ}$$

$$Q_{2B} = 50 \text{ KJ}$$

$$(\text{COP})_B = 2.5$$

$$W_{\text{net}} = 20 \text{ KJ}$$



$$(\text{COP})_{\text{HP}, B} = \frac{Q_{1B}}{W_B} = 2.5$$

$$\Rightarrow Q_{1B} = W_B \times 2.5 \rightarrow \textcircled{1}$$

$$\text{Also } W_B = Q_{1B} - Q_{2B} \Rightarrow Q_{1B} = W_B + Q_{2B} \rightarrow \textcircled{2}$$

Equate ① & ②, we get $W_B \times 2.5 = W_B + Q_{2B}$

$$W_B \times 2.5 = W_B + 50$$

$$W_B = 33.3 \text{ KJ}$$

From data $W_A = W_B + W_{\text{output}}$

$$W_A = 33.3 + 20 = 53.3 \text{ kJ}$$

Applying 1st law to engine A,

$$Q_{1A} = Q_{2A} + W_A = 200 + 53.3 = 253.3 \text{ kJ}$$

$$\eta_A = \frac{W_A}{Q_{1A}} = \frac{53.3}{253.3} = 0.21$$

$$\text{but } \eta_{\text{Carnot}} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{400} = 0.33$$

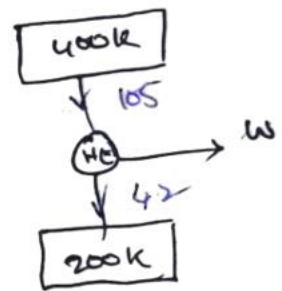
As $\eta_A < \eta_{\text{Carnot}}$, engine A is irreversible.

⑦ An inventor claims to have developed an engine that takes in 105 MJ at a temp of 400 K, rejects 42 MJ to a temp of 200 K. & delivers 15 kWh of mechanical work. Would you advise investing money to put this engine in the market.

Sol: Amt of heat given by source to the engine = $Q_1 = 105 \text{ MJ}$

$$Q_2 = 42 \text{ MJ}$$

$$\text{work delivered} = W_{\text{net}} = 15 \text{ kWh} = 54 \text{ MJ}$$



$$\frac{15 \times 3600}{1000} = 54 \text{ MJ}$$

$$\eta_{\text{HE}} = \frac{W_{\text{net}}}{Q_1} = \frac{W}{Q_1} = \frac{54}{105} = 0.5142$$

$$\eta_{\text{Carnot engine}} \quad \eta_c = 1 - \frac{T_2}{T_1} = 1 - \frac{200}{400} = 0.5$$

A/c to Carnot principle no engine has high η than Carnot engine

$$\text{Here } \eta_{\text{HE}} > \eta_{\text{Carnot}}$$

$\therefore$ It is not advisable to invest money to put this engine into market.

8. A heat engine receives half of its heat supply at 1000 K and half at 500 K while rejecting at a sink 300 K. What is the maximum thermal efficiency of the heat engine?

Given,

$$T_A = 1000 \text{ K}$$

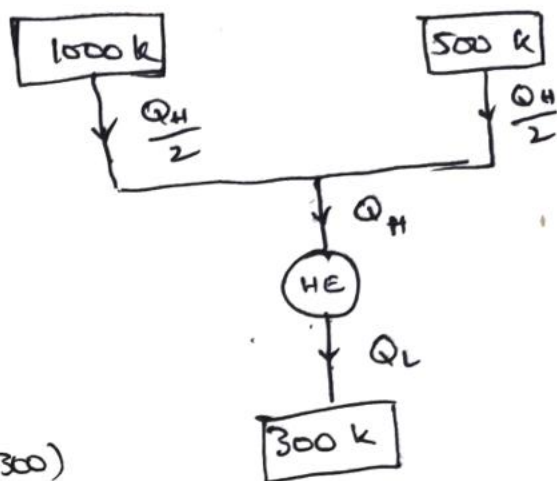
$$T_B = 500 \text{ K}$$

$$T_C = 300 \text{ K}$$

$$\begin{aligned} \text{Max. } \eta_{th} &= 1 - \left[\frac{T_A T_C + T_B T_C}{2 T_A T_B} \right] \\ &= 1 - \frac{(1000 \times 300) + (500 \times 300)}{2 \times 1000 \times 500} \end{aligned}$$

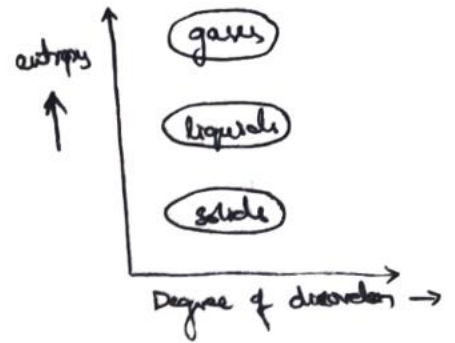
$$= 1 - 0.45 = 0.55$$

$$\therefore \eta_{th \text{ max}} = 55\%$$



Entropy is a measure of molecular disorder.

Consider a tank containing gas. The molecules of gas are in random motion & they move in different directions with different velocities colliding with each other.



Let us heat the gas, random motion of gas molecules further increases which leads to more irregularity (or) more disorderliness. This degree of disorderliness existing in a system is known as entropy.

The units of entropy $s = \frac{Q}{T} = \text{KJ/kgK}$

* I law of TD $\Rightarrow \boxed{Q = \Delta U + W}$

II law of TD $\Rightarrow \boxed{Q = T ds}$

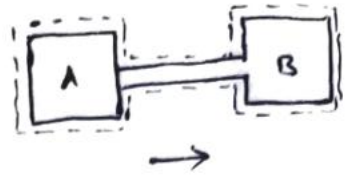
Uses of study of Entropy :-

- ① $\Rightarrow$ Since entropy is a property, it can be used to determine state of a substance.
- ② $\Rightarrow$ Entropy represents maximum amount of work obtainable per degree drop in temperature.
- ③ $\Rightarrow$ Entropy is generally assumed as zero at -273.15°C i.e. at 0K .
- ④ $\Rightarrow$ Since it depends on mass, entropy is an extensive property.
- ⑤ $\Rightarrow$ Entropy/unit mass = sp. entropy (intensive property)
- ⑥ $\Rightarrow$ A change in entropy shows that heat energy has been transferred to or from a system & the temp level at which that heat is transferred.
- ⑦ $\Rightarrow$ $\oint \frac{dQ}{T} \leq 0$ is called Clausius Inequality.

Applications :-

Heat transfer through finite temp difference :-

let A & B are two bodies
with temp's T_1 & T_2 resp.



change in entropy of body A $= \Delta S_A = \frac{-Q}{T_1}$

$$B = \Delta S_B = \frac{Q}{T_2}$$

For total isolated system $\Delta S = \Delta S_A + \Delta S_B$

$$\Delta S = -\frac{Q}{T_1} + \frac{Q}{T_2}$$

$$\Rightarrow \Delta S = Q \left[\frac{T_1 - T_2}{T_1 T_2} \right]$$

case(i): if $T_1 > T_2$ then $\Delta S > 0$ i.e., irreversible process

$T_1 < T_2$ then $\Delta S < 0$ i.e., impossible

$T_1 = T_2$ then $\Delta S = 0$ i.e., reversible process

- ⊕ I law of TD $\rightarrow$ Internal energy (extensive property)
- ⊕ II law of TD $\rightarrow$ Entropy (extensive property)

Entropy :- (S)

- It is a measure of molecular disorder
- ① → It is a thermodynamic property
- ② → It is a point function i.e., its value does not change unless the state of the system changes.
- ③ → Therefore entropy change of a system is zero, if state of system does not change during the process.

Ex:- Nozzle, compressor, turbines, pumps etc.

CLAUDE INEQUALITY :-

It is defined as,

"whenever a system executes a complete cycle, the cyclic integral of $\oint \frac{\delta Q}{T}$ is less than or in the limit equal to zero."

Consider an engine operating b/w two fixed temp reservoirs. Let dQ_1 units of heat be supplied at temp T_1 & dQ_2 units of heat be rejected at temp T_2 during a cycle. Then thermal efficiency of this engine,

$$\eta_E = \frac{dQ_1 - dQ_2}{dQ_1}$$

Thermal efficiency of a reversible engine operating b/w same temp

is given by $\eta_R = \frac{T_1 - T_2}{T_1}$

We know that $\eta_E \leq \eta_R$ or in limiting case $\eta_E = \eta_R$

Since no engine can be more efficient than a reversible engine (Carnot's theorem)

$$\therefore \frac{dQ_1 - dQ_2}{dQ_1} \leq \frac{T_1 - T_2}{T_1}$$

$$1 - \frac{dQ_2}{dQ_1} \leq 1 - \frac{T_2}{T_1}$$

$$\frac{dQ_2}{dQ_1} \leq \frac{T_2}{T_1}$$

$$\Rightarrow \frac{dQ_2}{T_2} - \frac{dQ_1}{T_1} \leq 0$$

Hence, in the limit

$$\oint \frac{dQ}{T} \leq 0$$

where the equality applies in case of reversible cycle & inequality applies in case of irreversible cycle.

Meaning & concept of Entropy:-

Entropy literally means 'transformation'. It is a thermodynamic state property of matter which is held constant if no heat is supplied in a reversible adiabatic process."

Entropy increases as heat is supplied to a system & decreases as heat is rejected from a system. But it remains constant if no heat is supplied nor rejected. That is why adiabatic process is called 'isentropic' process.

If dQ = a small change in heat transfer at T

T = absolute temp

$d\phi$ = change in entropy

$$\text{Then } d\phi = \frac{dQ}{T} \quad \text{or} \quad \phi = \int \frac{dQ}{T}$$

For a reversible cycle, $\oint \frac{dQ}{T} = 0$

Further we have $\eta_{\text{Carnot}} = \frac{\Delta T}{T} = \frac{W}{Q}$

If $\Delta T = 1$, then $W = \frac{Q}{T}$ (or) $dW = \frac{dQ}{T}$

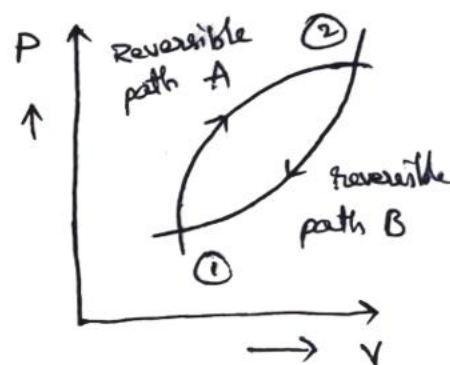
But $\frac{dQ}{T} = d\phi$

$\therefore d\phi = dW$

Hence entropy is regarded as a measure of rate of availability of heat for transformation into work.

Entropy - A Property :-

Let us consider two equilibrium states ① & ② connected by two reversible paths A & B as shown.



Now, if path B is reversed then the two paths make the cycle a reversible cycle.

Then $\oint \frac{dQ}{T} = 0$

we have $\oint \frac{dQ}{T} = \int_1^2 \left(\frac{dQ}{T}\right)_A + \int_2^1 \left(\frac{dQ}{T}\right)_B = 0$

$\int_1^2 \left(\frac{dQ}{T}\right)_A = - \int_2^1 \left(\frac{dQ}{T}\right)_B$

$\int_1^2 \left(\frac{dQ}{T}\right) = \int_2^1 \left(\frac{dQ}{T}\right)$

From the above it is clear that the expression $\int \frac{dQ}{T}$ is independent of the paths. It is a point function & hence represents a property of the system. This property is called entropy.

$$d\phi = \left(\frac{dQ}{T}\right)_{rev} \quad \& \quad \int_1^2 d\phi = \phi_2 - \phi_1$$

Temperature entropy ($T-\phi$) diagram:-

A reversible TD process may be plotted on Temp Vs Entropy diagrams.

Consider an elemental increase in entropy $d\phi$ due to a small amount of heat transfer dQ at an absolute temp T .

$$d\phi = \left(\frac{dQ}{T}\right)_{rev}$$

$$(dQ)_{rev} = T \times d\phi \quad (\text{or}) \quad \text{simply} \quad \boxed{dQ = T \times d\phi}$$

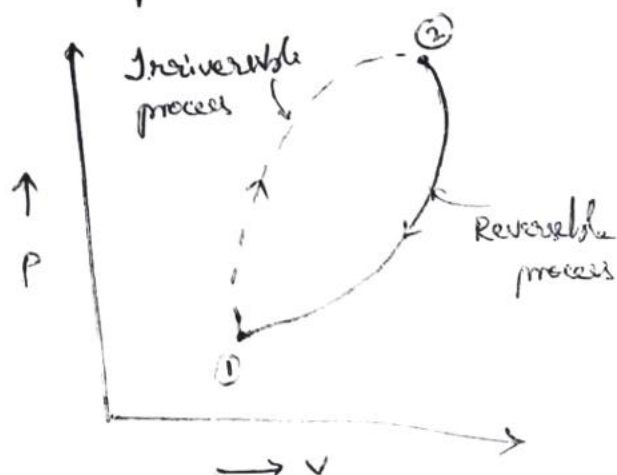
$$[dQ = T ds]$$

Thus area under a curve for a reversible process plotted on $T-\phi$ diagram, gives the quantity of heat transferred during the process.

Principle of Entropy Increase:-

The entropy of an isolated system increases in all real processes & is conserved in reversible process.

In order to prove this let us consider an irreversible process 1-2 along path A shown in fig.



Let the cycle be completed by a reversible process 2-1 along path B, so that process 1-A-2 & 2-B-1 together form an irreversible cycle. From the Clausius inequality, $\oint \frac{\delta Q}{T} < 0$ for irreversible process

$$(or) \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{irrev}} + \int_2^1 \left(\frac{\delta Q}{T} \right)_{\text{rev}} < 0$$

path-A path-B

$$\text{But } \int_2^1 \left(\frac{\delta Q}{T} \right)_{\text{rev}} = \int_2^1 ds$$

path-B path-B

But since path-B is reversible, the limits of the integral can be reversed & therefore

$$\int_2^1 \left(\frac{\delta Q}{T} \right)_{\text{rev}} = - \int_1^2 ds$$

$$\therefore \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{irrev}} - \int_1^2 ds < 0$$

path-A path-B

$$(or) \int_1^2 ds > \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{irrev}}$$

$$(or) ds > \left(\frac{\delta Q}{T} \right)_{\text{irrev}} \quad \text{But } ds = \left(\frac{\delta Q}{T} \right)_{\text{rev}}$$

Hence in general, we may write $ds \geq \frac{\delta Q}{T}$

$$or \Delta S = S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T}$$

where equality & inequality signs show the reversibility & irreversibility respectively.

For an adiabatic process whether reversible or irreversible $\delta Q = 0$

hence $\Delta S \geq 0$.

This means that entropy is zero in reversible adiabatic & greater than zero in irreversible adiabatic.

Although entropy change b/w two fixed states is the same for any path, reversible or irreversible, the heat transfer to the system is less (more, if heat is transferred from the system) for irreversible path than for the reversible one.

This is obvious from the fact that entropy increases in irreversible process & is conserved in reversible process.

① A qty of 400 kJ of heat is supplied per second to a heat engine at a fixed temp of 580 K. The heat rejection takes place at 280 K. The following results are obtained.

② 320 kJ/s are rejected ③ 193 kJ/s are rejected

④ 80 kJ/s are rejected

Classify which of results report a reversible cycle or irreversible cycle or impossible results.

Ans Applying Clausius Inequality to the cycle in each case, we have

$$\text{② } \oint \frac{dQ}{T} = \frac{400}{580} - \frac{320}{280} = -0.4532 < 0$$

Since $\oint \frac{dQ}{T} < 0$, cycle is irreversible.

$$\text{③ } \oint \frac{dQ}{T} = \frac{400}{580} - \frac{193}{280} = 0 \quad \therefore \oint \frac{dQ}{T} = 0, \text{ cycle is reversible}$$

$$\text{④ } \oint \frac{dQ}{T} = \frac{400}{580} - \frac{80}{280} = 0.4039 > 0 \quad \therefore \oint \frac{dQ}{T} > 0, \text{ cycle is impossible}$$

This shows Clausius inequality.

② One kg of air is heated reversibly from an initial state of 1 atm & 15°C to 1 atm & 75°C . For reversible heating of air at const. pressure, $\frac{dQ}{dT} = 1.2 \text{ kJ/K}$. Determine the change in entropy.

Sol. For a closed system undergoing a reversible process, change of entropy, by definition $ds = \frac{dQ}{T}$

$$\frac{dQ}{dT} = 1.2 \text{ kJ/K (given)}$$

$$dQ = 1.2 dT$$

$$\therefore ds = \frac{dQ}{T} = \frac{1.2 dT}{T}, \quad \text{Integrating}$$

$$\int_1^2 ds = 1.2 \int \frac{dT}{T}$$

$$(s)_1^2 = 1.2 [\log_e T]_{T_1}^{T_2}$$

$$s_2 - s_1 = ds = 1.2 [\log_e T_2 - \log_e T_1]$$

$$ds = 1.2 \log_e \frac{T_2}{T_1}$$

$$T_1 = 15 + 273 = 288 \text{ K}$$

$$T_2 = 75 + 273 = 348 \text{ K}$$

$$ds = 1.2 \log_e \frac{348}{288}$$

$$= 0.227 \text{ kJ/kg K. (0.23)}$$

AVAILABLE ENERGY:-

Heat energy derived from combustion of fossil fuels or from nuclear fusion is said to be low grade energy.

On the other hand, mechanical (shaft) work, electrical energy, water power, tidal power etc are rated as high grade energy.

The bulk of high grade energy is obtained from source of low grade energy. But the complete conversion of low grade energy i.e., 'heat' into high grade energy i.e., 'shaft work' is impossible a/c to II law of TD.

"That part of low grade energy which is available for conversion into high grade energy is called 'available energy'."

Available energy is also called as Energy.

Obviously the other part of energy, a/c to II law, must be rejected. It is known as Unavailable energy, which is also known as Unavailable Energy.

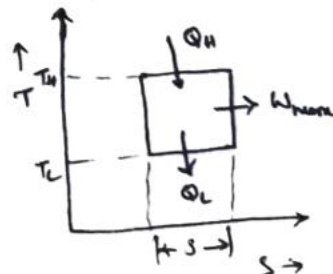
In a thermodynamically reversible cycle engine, max. work o/p obtainable from a certain heat input is called available energy (A.E.). The minimum energy that has to be rejected to the sink is called unavailable energy (U.E.). If Q_1 is heat input,

$$Q_1 = A.E. + U.E.$$

$$\text{Energy} = \text{Energy} + \text{Energy}$$

$$\begin{aligned} \text{(or)} \quad W_{\max} &= A.E. \\ &= Q_1 - U.E. \end{aligned}$$

If T_1 & T_2 are temp's of source & sink,



$$\eta_{\text{rev}} = 1 - \frac{T_2}{T_1}$$

For a given T_1 , the efficiency of reversible engine η_{rev} will increase with the decrease of T_2 . The lowest practicable value of temp of heat rejection is the temp of surroundings, T_0 .

Then, $\eta_{max} = 1 - \frac{T_0}{T_1} = \frac{W_{max}}{Q_1}$

$\therefore W_{max} = Q_1 \left[1 - \frac{T_0}{T_1} \right]$

$AE = Q \left[1 - \frac{T_0}{T_1} \right]$
 $= Q - T_0 \left(\frac{Q}{T} \right)$

$AE = Q - T_0 \cdot ds$

we know,

$W_{max} = AE$

$Q_1 = Q$ (say)

$T_1 = T$

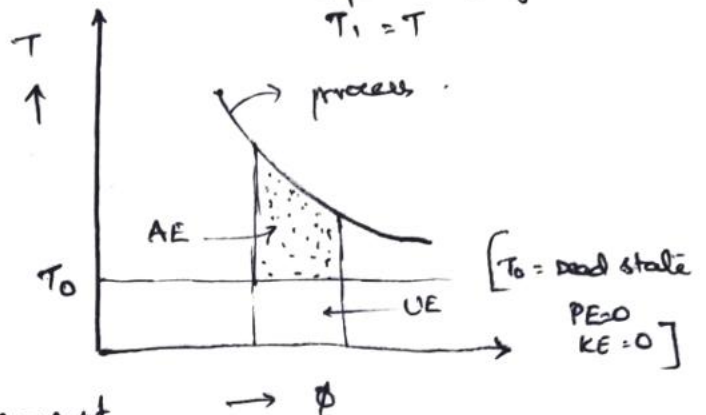


Figure shows the availability concept. $\rightarrow \phi$

T_0 is the dead state which is a level below which the system cannot by itself go down, like atm. temp, zero altitude ($PE=0$), zero velocity ($KE=0$).

- ① Calculate AE & UE of a system that absorbs 15000 kJ of heat from heat source at 500 K, while environment temp is 290 K.

$AE = Q_H - \frac{Q_H T_L}{T_H} = 15000 - \frac{15000 \times 290}{500} = 6300 \text{ kJ}$

$UE = 8700 \text{ kJ}$ $\because W_{max} = Q_H \left[1 - \frac{T_0}{T_1} \right]$

(or)
 $Q_H = T_H ds$

$ds = \frac{Q_H}{T_H} = \frac{15000}{500} = 30 \text{ kJ/K}$

$UE = T_L ds = 290 \times 30 = 8700 \text{ kJ}$

② $\eta = \frac{T_H - T_L}{T_H} = \frac{W_{max}}{Q_H}$

$W_{max} = \frac{(T_H - T_L) Q_H}{T_H}$
 $= 6300 \text{ kJ}$

- ③ In a Carnot cycle heat is supplied at 35°C & rejected at 27°C. Working fluid is water which absorbs heat and evaporates liquid at 35°C to steam at 35°C. The associated entropy change is 1.44 kJ/kg K.

☆ cycle operates with mass flow rate of 1 kg of water.

$T_H = 623 \text{ K}$ Calculate (1) Heat supplied, workdone

$T_L = 300 \text{ K}$

(2) cycle operates in steady flow

with power output of 20 kW. Determine mass flow rate of the system.

sol: $\eta = \frac{T_H - T_L}{T_H} = \frac{623 - 300}{623} = 52\%$

$ds = 1.44 \text{ KJ/kg K}$

$Q_H = T_H ds = 623 \times 1.44 = 897.12 \text{ KJ}$

$Q_L = T_L ds = 300 \times 1.44 = 432 \text{ KJ}$

$W = Q_H - Q_L = 465 \text{ KJ/kg} = AE$

Power = Mass ~~flow~~ rate $\times$ work

Mass flow rate = $\frac{\text{Power}}{\text{work}} = \frac{20}{465} = \underline{\underline{0.043 \text{ kg/sec}}}$

(3) Air at 1 bar & 30°C is heated in reversible manner at constant pressure until its temp reaches to 205°C .

(1) calculate amount of heat added per kg of air.

(2) Available energy if sink temp is 4°C .

sol: $T_1 = 30^\circ\text{C} = 303 \text{ K}$

$T_2 = 205^\circ\text{C} = 478 \text{ K}$

$T_0 = 4^\circ\text{C} = 277 \text{ K}$

$m = 1 \text{ kg}$

(P=c)

$\therefore Q = m C_p (T_2 - T_1)$
 $= 1 \times 1.005 (478 - 303) = 175.87 \text{ KJ/kg}$

Available Energy = $Q \left[1 - \frac{T_0}{T} \right]$
 $= 175.87 \left[1 - \frac{277}{478} \right] = 73.85 \text{ KJ/kg}$

4) 0.2 kg of air initially at 575 K temp receives 300 kJ of heat reversibly at const. pressure. Determine the available & unavailable energy of the heat added. C_p of air = 1.005 kJ/kgK
 Surrounding Temp = 300 K.

$$T_1 = 575 \text{ K}$$

$$Q = 300 \text{ kJ}$$

$$C_p = 1.005 \text{ kJ/kgK}$$

$$dS = \frac{dQ}{T}$$

$$= m C_p \ln \left(\frac{T_2}{T_1} \right)$$

$$= 0.2 \times 1.005 \ln \left(\frac{2067.5}{575} \right)$$

$$= 0.2572 \text{ kJ/kgK}$$

$$\text{unavailable energy} = T_L \times ds \Rightarrow UE = 300 \times 0.2572 = 77.16 \text{ kJ}$$

$$dQ = m C_p dT$$

$$300 = 0.2 \times 1.005 \times (T_2 - 575)$$

$$T_2 = 2067.5 \text{ K}$$

$$= 300 \left(1 - \frac{2067.5}{575} \right)$$

$$AE = 216.56 \text{ kJ} = 256.4 \text{ kJ}$$

$$= 147.89 \text{ kJ/kg}$$

$$UE = \frac{575}{2067.5} \times 0.2572$$

$$= 531.81 \text{ kJ/kg}$$

Entropy change during a Process:-

$$dQ = du + dw$$

$$= du + p dv$$

$$\text{entropy } ds = \frac{dQ}{T} \Rightarrow T \cdot ds = dQ$$

$$\boxed{T \cdot ds = du + p dv} \rightarrow (1)$$

we know

$$h = u + pv$$

$$dh = du + d(pv)$$

$$dh = du + p dv + v dp$$

$$dh - v dp = du + p dv$$

$$[T ds = du + p dv]$$

$$\boxed{T \cdot ds = dh - v dp} \rightarrow (2)$$

① Constant volume Process :-

$$dQ = m c_v dT$$

$$ds = \frac{dQ}{T} = m c_v \frac{dT}{T} \quad \text{Integrate on both sides,}$$

$$\int_1^2 ds = \int_1^2 m c_v \frac{dT}{T}$$

$$s_2 - s_1 = m c_v (\ln T)_1^2$$

$$\left| \frac{P_1}{P_2} = \frac{T_1}{T_2} \right.$$

$$\boxed{s_2 - s_1 = m c_v \ln\left(\frac{T_2}{T_1}\right) \text{ (or) } = m c_v \ln\left(\frac{P_2}{P_1}\right)}$$

② Constant Pressure process :-

$$dQ = m c_p dT$$

$$ds = \frac{dQ}{T} = m c_p \frac{dT}{T} \quad \text{Integrate,}$$

$$\int_1^2 ds = \int_1^2 m c_p \frac{dT}{T}$$

$$\left| \frac{T_2}{T_1} = \frac{V_2}{V_1} \right.$$

$$\boxed{s_2 - s_1 = m c_p \ln\left(\frac{T_2}{T_1}\right) \text{ or } m c_p \ln \frac{V_2}{V_1}}$$

③ Isothermal process :-

$$dQ = du + dw \\ = du + p dv$$

$$[\text{For isothermal } du = 0]$$

$$\left| \begin{aligned} P_1 V_1 &= P_2 V_2 \\ \frac{P_1}{P_2} &= \frac{V_2}{V_1} \end{aligned} \right.$$

$$dQ = p dv$$

$$PV = RT$$

$$ds = \frac{dQ}{T} = \frac{p dv}{T}$$

$$P = \frac{RT}{V} \quad [\text{where } T = C]$$

$$ds = \frac{R}{V} dv, \quad \text{Integrate}$$

$$\int_1^2 ds = \int_1^2 \frac{R}{V} dv \Rightarrow \boxed{ds = R \ln\left(\frac{V_2}{V_1}\right) \text{ (or) } R \ln\left(\frac{P_1}{P_2}\right)}$$

④ Reversible Adiabatic process :-

$$dQ = 0$$

$$ds = \frac{dQ}{T} = 0$$

$$\Rightarrow s_2 - s_1 = 0 \Rightarrow s_2 = s_1$$

Entropy is constant. This is also known as Isentropic Process.

⑤ Polytropic Process :- $[PV^n = C]$

Case (i) :- $ds = \frac{dq}{T}$ $dq = du + dw$
 $= cvdT + pdv$

$ds = cv \frac{dT}{T} + p \frac{dv}{T}$, Integrate

$\int_1^2 ds = \int_1^2 cv \frac{dT}{T} + \int_1^2 p \frac{dv}{T}$

$s_2 - s_1 = cv \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right) \rightarrow \textcircled{1}$

Now in terms of Temp. & volume.

$\therefore \frac{PV}{T} = \text{constant} \Rightarrow \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow \frac{T_2}{T_1} = \frac{P_2 V_2}{P_1 V_1}$

$\therefore$ Eq ① can be written as,

Case (ii) :- $s_2 - s_1 = cv \ln\left[\frac{P_2 V_2}{P_1 V_1}\right] + R \ln\left(\frac{v_2}{v_1}\right)$
 $= cv \ln\left(\frac{P_2}{P_1}\right) + cv \ln\left(\frac{v_2}{v_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$
 $= cv \ln\left(\frac{P_2}{P_1}\right) + (cv + R) \ln\left(\frac{v_2}{v_1}\right)$ $[\because C_p = C_v + R]$
 $s_2 - s_1 = cv \ln\left(\frac{P_2}{P_1}\right) + C_p \ln\left(\frac{v_2}{v_1}\right) \rightarrow \textcircled{2}$

Case (iii) :- $\frac{PV}{T} = \text{const} \therefore \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow \frac{v_2}{v_1} = \frac{P_1 T_2}{P_2 T_1}$

$\therefore$ Eq ①, can be written as,

$s_2 - s_1 = cv \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{P_1 T_2}{P_2 T_1}\right)$
 $= cv \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{P_1}{P_2}\right) + R \ln\left(\frac{T_2}{T_1}\right)$
 $= (cv + R) \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{P_1}{P_2}\right)$

$s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{P_1}{P_2}\right) \rightarrow \textcircled{3}$

Calc IV:- In terms of n & γ

$$dq = \left[\frac{\gamma - n}{\gamma - 1} \right] dw$$

$$P = \frac{RT}{V}$$

$$ds = \frac{dq}{T} = \left[\frac{\gamma - n}{\gamma - 1} \right] \frac{dw}{T} = \left[\frac{\gamma - n}{\gamma - 1} \right] P \frac{dV}{T}$$

$$ds = \left[\frac{\gamma - n}{\gamma - 1} \right] R \frac{dV}{V}, \quad \text{Integrate ...}$$

$$\int_1^2 ds = \int_1^2 \left[\frac{\gamma - n}{\gamma - 1} \right] R \frac{dV}{V}$$

$$\boxed{S_2 - S_1 = \frac{\gamma - n}{\gamma - 1} R \ln \left(\frac{V_2}{V_1} \right)} \rightarrow (4)$$

$$\text{But } \frac{R}{\gamma - 1} = C_V$$

$$\frac{C_P}{C_V} = \gamma$$

$$\downarrow C_V = C_P$$

$$\therefore S_2 - S_1 = (\gamma - n) C_V \ln \left(\frac{V_2}{V_1} \right)$$

$$= \gamma \cdot C_V - n C_V \ln \left(\frac{V_2}{V_1} \right)$$

$$\boxed{S_2 - S_1 = C_P - n \cdot C_V \ln \left(\frac{V_2}{V_1} \right)} \rightarrow (5) \quad [\text{in terms of vol}]$$

$$PV^n = \text{const.}$$

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2} \right)^{1/n}, \quad \text{then eq (5) becomes,}$$

$$S_2 - S_1 = (C_P - n \cdot C_V) \ln \left(\frac{P_1}{P_2} \right)^{1/n}$$

$$\boxed{S_2 - S_1 = (C_P - n \cdot C_V) \frac{1}{n} \ln \left(\frac{P_1}{P_2} \right)} \rightarrow (6) \quad [\text{in terms of pressure}]$$

we know,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{n-1/n}, \quad \text{take log on both sides,}$$

$$\ln \left(\frac{T_2}{T_1} \right) = \left(\frac{n-1}{n} \right) \ln \left(\frac{P_2}{P_1} \right)$$

$$\therefore \ln \left(\frac{P_2}{P_1} \right) = \left(\frac{n}{n-1} \right) \ln \left(\frac{T_2}{T_1} \right) \Rightarrow \ln \left(\frac{P_1}{P_2} \right) = - \left(\frac{n}{n-1} \right) \ln \left(\frac{T_2}{T_1} \right)$$

$\therefore$ Eq. (6) can be written as,

$$S_2 - S_1 = (C_p - n \cdot C_v) \cdot \frac{1}{\gamma} \cdot - \left(\frac{n}{n-1} \right) \ln \left(\frac{T_2}{T_1} \right)$$

$$= \frac{n C_v - C_p}{n-1} \ln \left(\frac{T_2}{T_1} \right)$$

$$\left| \frac{C_p}{C_v} = \gamma \right.$$

$$S_2 - S_1 = \frac{C_v \left(n - \frac{C_p}{C_v} \right)}{n-1} \ln \left(\frac{T_2}{T_1} \right)$$

$$\therefore S_2 - S_1 = \left(\frac{n-1}{n-1} \right) C_v \ln \left(\frac{T_2}{T_1} \right) \rightarrow (7)$$

- (i) Heat flows from a reservoir at 800 K to another reservoir at 250 K. If entropy change of the hot reservoir is -4 kJ/kg K . Determine the entropy change of cold reservoir.

Sol:

$$\frac{dQ}{T} = (ds)_H$$

$$\frac{dQ}{800} = -4 \Rightarrow dQ = -3200 \text{ kJ}$$



Same amount of heat is supplied to cold body.

For cold body,

$$\frac{dQ}{T} = (ds)_c \Rightarrow \frac{+3200}{250} = ds$$

$$(ds)_c = 12.8 \text{ kJ/kg K}$$

- (ii) Heat flows from a hot reservoir at 800 K to another reservoir at 250 K. If the entropy change of overall process is 4.25 kJ/kg K . Make calculations for heat flowing out of high temp reservoir.

change of entropy of hot & cold body = 4.25 kJ/kg K

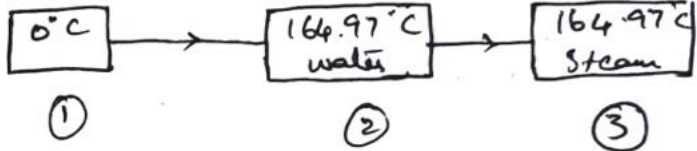
$$\left(\frac{dQ}{T}\right)_h + \left(\frac{dQ}{T}\right)_c = 4.25$$

$$-\frac{dQ}{800} + \frac{dQ}{250} = 4.25$$

$$Q = 1545.45 \text{ kJ}$$

- ③ Water is heated at constant pressure of 0.7 MPa , boiling point is 164.97°C . The initial temp of water is 0°C . Latent heat of evaporation is 2066.3 kJ/kg . Find the increase in entropy of water if final state is steam at 164.97°C . $C_p \text{ of water} = 4.2 \text{ kJ/kg K}$.

From (1-2) $(m=1)$



$$S_2 - S_1 = m C_p \ln \frac{T_2}{T_1}$$

$$= 4.2 \times 1 \times \ln \left(\frac{437.97}{273} \right)$$

$$= 1.985 \text{ kJ/kg K}$$

From (2-3)

$$S_3 - S_2 = \frac{Q}{T} = \frac{\text{latent heat}}{T} = \frac{2066.3}{437.97} = 4.717 \text{ kJ/kg K}$$

Total change in entropy

$$S_2 - S_1 + S_3 - S_2 = 1.985 + 4.717$$

$$= 6.7029 \text{ kJ/kg K}$$

- ④ 2 kg of water at 80°C is mixed adiabatically with 3 kg of water at 30°C at a constant pressure process of 1 atm . Find the increase in entropy of total mass of water due to mixing process.

20x: Heat lost by 2 kg of water at 80°C = Heat gained by 3 kg of water at 30°C .

$$\therefore 2 c_p (80 - t) = 3 c_p (t - 30) \quad [Q = mc_p \Delta T]$$

$$t = 50^\circ\text{C} = 323\text{ K} \quad [c_{\text{water}} = 4.2 \text{ kJ/kg K}]$$

$$(\Delta S)_1 = \int_{T_1}^{T_2} \frac{\delta Q}{T} = \int_{353}^{323} \frac{m c_p dT}{T}$$

$$= 2 \times 4.2 \ln \left(\frac{T_2}{T_1} \right)_{353}^{323} = 8.4 \ln \left(\frac{323}{353} \right)$$

$$= -0.746 \text{ kJ/kg K}$$

Entropy change for 3 kg of water,

$$(\Delta S)_2 = \int_{T_1}^{T_2} \frac{\delta Q}{T} = \int_{303}^{323} \frac{m c_p dT}{T}$$

$$= 3 \times 4.2 \times \ln \left(\frac{323}{303} \right) = 0.8054 \text{ kJ/kg K}$$

Entropy increase of total mass of water due to mixing,

$$= (\Delta S)_1 + (\Delta S)_2$$

$$= -0.746 + 0.8054$$

$$= 0.059 \text{ kJ/kg K}$$

Thermodynamic Potential:-

The energy when expressed in terms of parameters like pressure, vol, temp, U , H , S etc are called Thermodynamic potentials.

Gibbs Function:-

It is denoted by G . It is defined as the diff b/w enthalpy & product of temp & entropy.

$$G = H - TS$$

Helmholtz's Functions :-

It is defined as diff b/w I.E & product of temp & entropy. It is denoted by 'F'.

$$F = U - TS$$

Maxwell's Equations :-

Out of 8 properties (P, V, T, U, H, S, G & F), any one property can be expressed as a function of any other two properties. The relation b/w properties of pure substance are known as "Maxwell relations".

① Internal Energy :-

A/c to I law of TD,

$$Q = U + W$$

$$dQ = dU + dW$$

$$Tds = du + pdv$$

$$\left[ds = \frac{dq}{T} \Rightarrow Tds = dq \right]$$

$$du = Tds - pdv \rightarrow ①$$

② Enthalpy :-

$$H = U + PV$$

$$dH = dU + d(PV)$$

$$= Tds - pdv + pdv + vdp$$

$$[du = Tds - pdv]$$

$$dH = Tds + vdp \rightarrow ②$$

③ Gibb's Function :-

$$G = H - TS$$

$$dG = dH - d(TS)$$

$$= Tds + vdp - Tds - SdT$$

$$dG = vdp - SdT \rightarrow ③$$

④ Helmholtz Functions:-

$$F = U - TS \Rightarrow dF = dU - d(TS) \quad (dU = Tds - pdv)$$

$$= Tds - pdv - Tds - SdT$$

$$\boxed{dF = -pdv - SdT} \rightarrow \textcircled{4}$$

The above ④ eq's are known as Maxwell's Relations. These are in the form of $\boxed{dz = Mdx + Ndy}$

The differential eq. must satisfy condition of

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y \text{ for exactness Test.}$$

Applying exactness test for Maxwell's Relations:-

① $dU = Tds - pdv$

Comparing with standard differential eqn.

$$Z = U, M = T, x = S, N = -P, y = V$$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y \Rightarrow \boxed{\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V} \rightarrow \textcircled{5}$$

② $dH = Tds + vdp$, Apply std. diff. eqn.

$$Z = H, M = T, x = S, N = V, y = P$$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y \Rightarrow \boxed{\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P} \rightarrow \textcircled{6}$$

Sp. heats

$C_p = T\left(\frac{\partial S}{\partial T}\right)_P$
$C_v = T\left(\frac{\partial S}{\partial T}\right)_V$

③ $dG = vdp - SdT$

$$Z = G, M = V, N = -S, x = P, y = T$$

$$\boxed{\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T} \rightarrow \textcircled{7}$$

④ $dF = -pdv - SdT$

$$Z = F, M = -P, N = -S, x = V, y = T$$

$$\boxed{+\left(\frac{\partial P}{\partial T}\right)_V = +\left(\frac{\partial S}{\partial V}\right)_T} \rightarrow \textcircled{8}$$

- ① The HE receives heat at the rate of 500 kW, from a source temp of 1200 K. It rejects waste heat at 300 K. In this process it delivers a power o/p of 180 kW. Determine
 (i) η of engine (ii) Ideal power o/p (iii) Irreversibility.

Sol: ① $\eta = \frac{T_H - T_L}{T_H} = \frac{1200 - 300}{1200} = \underline{75\%}$

② ~~Ideal power o/p = heat i/p = 500 kW~~

③ $W_{\text{actual}} \quad \eta = \frac{Q_H - Q_L}{Q_H} = \frac{W}{Q_H}$

Ideal power $W = 0.75 \times 500 = \underline{375 \text{ kW}}$

③ Irreversibility = Ideal power - Actual power
 $= 375 - 180$
 $= \underline{195 \text{ kW}}$

② Using Maxwell Relations derive two Tds equations.

① Consider entropy to be a function of temp & vol.
 $S = f(T, V)$

Sol $ds = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$

Multiplying above eq. with T on both sides,

$$Tds = T \left(\frac{\partial S}{\partial T}\right)_V dT + T \left(\frac{\partial S}{\partial V}\right)_T dV \quad \left[\because C_V = T \left(\frac{\partial S}{\partial T}\right)_V \right]$$

$$\boxed{Tds = C_V dT + T \left(\frac{\partial P}{\partial T}\right)_V dV} \quad \left[\text{From MW relation } \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \right]$$

② Consider entropy as a function of temp & pressure, then
 $S = f(T, P)$

$$ds = \left(\frac{\partial s}{\partial T} \right)_p dT + \left(\frac{\partial s}{\partial p} \right)_T dp$$

Multiply with T ,

$$\left[c_p = T \left(\frac{\partial s}{\partial T} \right)_p \right]$$

$$T ds = T \left(\frac{\partial s}{\partial T} \right)_p dT + T \left(\frac{\partial s}{\partial p} \right)_T dp$$

From M.W.R $\left(\frac{\partial s}{\partial p} \right)_T = - \left(\frac{\partial v}{\partial T} \right)_p$, substitute

$$T ds = c_p dT - T \left(\frac{\partial v}{\partial T} \right)_p dp \rightarrow \textcircled{2}$$

- ③ Consider combustion of fuel-air mixture in an IC engine which generates hot gases at 2773 K in cylinder. The gas expands a/c to polytropic process $p v^{1.38} = c$. Volume ratio is 9. & surrounding air temp is 293 K. Determine ① work transfer ② change in I.E ③ Heat Transfer. Assume $R = 0.26 \text{ kJ/kgK}$
 $c_v = 0.82 \text{ kJ/kgK}$

$$T_H = 2773 \text{ K}$$

$$T_L = 293 \text{ K}$$

$$\textcircled{1} \text{ work transfer} = \frac{p_1 v_1 - p_2 v_2}{n-1} = \frac{m R T_1 - m R T_2}{1.38-1}$$

$$\text{Given } \frac{v_2}{v_1} = 9$$

In polytropic process

$$T_2 = 1203 \text{ K}$$

$$T_1 = 2773 \text{ K}$$

$$\left(\frac{v_2}{v_1} \right)^{n-1} = \frac{T_1}{T_2} = \left(\frac{2773}{1203} \right) = 2.3024$$

$$T_2 = \frac{2773}{2.3024} = 1203 \text{ K}$$

$$\text{work} = \frac{m R (T_1 - T_2)}{n-1}$$

$$= \frac{0.26 (2773 - 1203)}{1.38 - 1} = \underline{\underline{1074 \text{ kJ}}}$$

$$\textcircled{2} \Delta u = m c_v (T_2 - T_1)$$

$$= 1 \times 0.82 (1203 - 2773) = \underline{\underline{-1287 \text{ kJ}}}$$

$$\textcircled{3} \quad Q = \Delta u + w$$

$$= -1287 + 1074 = -213 \text{ kJ}$$

Heat transfer $Q = 213 \text{ kJ}$ (rejected).

$\textcircled{4}$ A cylinder contains 0.115 m^3 of gas at 1 bar & 90°C . The gas is compressed to a vol. of 0.0288 m^3 . The final pressure of gas is 5.67 bar. Calculate

(i) Mass of gas (ii) Value of index of compression

(iii) Increase in internal energy of the gas (Already Completed)

(iv) Heat transfer during compression.

Assume $\gamma = 1.4$, $R = 0.3 \text{ kJ/kg K}$, $c_v = 0.75 \text{ kJ/kg K}$.

Sol: $V_1 = 0.115 \text{ m}^3$

$$P_1 = 1 \text{ bar} = 100 \text{ kPa}$$

$$V_2 = 0.0288 \text{ m}^3$$

$$P_2 = 5.67 \times 10^2 \text{ kPa}$$

$$T_1 = 90^\circ\text{C} = 363 \text{ K}$$

(i) we have $P_1 V_1 = m R T_1$

$$m = \frac{P_1 V_1}{R T_1}$$

$$m = 0.1056 \text{ kg}$$

(ii) $P V^n = C$ i.e., $P_1 V_1^n = P_2 V_2^n$

$$100 \times (0.115)^n = 567 \times (0.0288)^n$$

$$n = 1.253$$

(iii) $\Delta u = m c_v (T_2 - T_1)$

$$= 0.1056 \times 0.75 (515.27 - 363)$$

$$\Delta u = 12.06 \text{ kJ}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{n-1}$$

$$\frac{363}{T_2} = \left[\frac{0.0288}{0.115} \right]^{1.253-1}$$

$$T_2 = 515.27 \text{ K}$$

(iv) $Q = \Delta u + w$

$$= 12.06 - 19.089 \text{ kJ}$$

$$w = \frac{P_1 V_1 - P_2 V_2}{n-1}$$

$$Q = -7.029 \text{ kJ}$$

$$= -19.089 \text{ kJ}$$

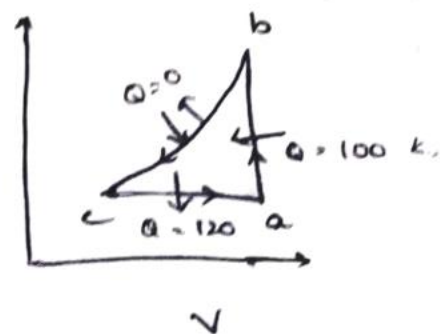
Heat transfer $Q = 7 \text{ kJ}$

⑤ Calculate internal energy & work transfer for the following p-v diagram.

$$U_a = 150 \text{ kJ}$$

$$\text{work done (b-c)} = -70 \text{ kJ} \quad (\text{on the gas during process})$$

$$U_b = ? , U_c = ? \quad W_{ca} = ?$$



sol: process (a-b),

$$Q = 100 \text{ kJ}$$

$$\Delta U = U_b - U_a = U_b - 150 \text{ kJ}$$

$$W = 0 \quad (\because \text{const. vol. process})$$

$$Q = \Delta U + W$$

$$100 = U_b - 150 - 0 \Rightarrow U_b = 250 \text{ kJ}$$

process (b-c), $Q = 0$

$$\Delta U = U_c - U_b = U_c - 250$$

$$Q = \Delta U + W$$

$$0 = U_c - 250 - 70 \Rightarrow U_c = 320 \text{ kJ}$$

process (c-a) :-

$$Q = -120 \text{ kJ}$$

$$\Delta U = U_a - U_c = 150 - 320 = -170$$

$$Q = \Delta U + W$$

$$W = Q - \Delta U = -120 + 170 = 50 \text{ kJ}$$

$$W = \underline{\underline{50 \text{ kJ}}}$$

Third Law of Thermodynamics:-

Third law of TD was formulated by W.H. Nernst & Max. Planck. It states that the entropy of a perfect crystalline substance is zero at the absolute zero temp & it represents the maximum degree of order.

"Entropy is zero at absolute temperature."

- 459.67 F

Available Energy:-

$$AE = Q \left[1 - \frac{T_0}{T_1} \right]$$

$$W_{max} = Q \left[1 - \frac{T_0}{T_1} \right]$$

$$dW_{max} = dQ_1 - \frac{T_0}{T_1} dQ_1$$

Integrate on both sides,

$$\int_1^2 dW_{max} = \int_1^2 dQ_1 - \int_1^2 T_0 \left(\frac{dQ_1}{T_1} \right)$$

$$W_{max} = \int_1^2 dQ_1 - T_0 \int_1^2 \left(\frac{dQ_1}{T_1} \right)$$

$$W_{max} = AE = Q_{1-2} - T_0 (S_2 - S_1)$$

Availability in non-flow system:-

$$AE = W_{max} = (u_1 + p_0 v_1 - T_0 s_1) - (u_0 + p_0 v_0 - T_0 s_0)$$

$$W_{max} = a_1 - a_0$$

Availability in steady flow process:-

$$AE = W_{max} = (h_1 - T_0 s_1) - (h_0 - T_0 s_0)$$

$$AE = (h_1 - h_0) - T_0 (s_1 - s_0)$$

Irreversibility:-

$$I = W_{max} - W$$

Irreversibility is also called as degradation (or) dissipation.

This constant value cannot be zero in any other parameter characterizing in closed system.

Such as pressure, volume, applied magnetic field. At Absolute zero the system must be in a state with minimum possible energy.

Entropy is related to the number of accessible microstates, and then it typically increases with increasing energy.

At such a case, the entropy at absolute zero will be exactly zero.

$$\begin{cases} W = h - T_0 s \\ AE = \dots \end{cases}$$

Gouy Stodola Theorem:-

It states that the rate of loss of AE in a process is ~~directly~~ proportional to the rate of entropy generation ($\dot{S}_G$)

$$\boxed{\dot{I} \propto \dot{S}_G}$$

$$\dot{I} = \dot{W}_{\text{lost}} = T_0 \dot{D}S_{\text{universe}} = T_0 \dot{S}_G$$

$$\therefore \dot{I} = T_0 \dot{S}_G \rightarrow \text{known as Gouy, Stodola eq.}$$

$\therefore$ A thermodynamically efficient process would involve minimum energy loss with minimum rate of entropy generation.

- ⑥ Calculate the decrease in AE when 25 kg of water at 95°C mix with 35 kg of water at 35°C . The pressure being taken as constant & the temp of surroundings being 15°C . $C_p = 4.2 \text{ kJ/kg}^\circ\text{C}$

Sol:- The AE of a system of mass m , C_p , & T is given by.

$$AE = m C_p \int_{T_0}^{T_1} \left(1 - \frac{T_0}{T}\right) dT$$

$$Q_{12} = T_0 \left(\frac{Q}{T}\right)$$

$$(AE)_{25} = 25 \times 4.2 \int_{273+15}^{273+95} \left(1 - \frac{288}{T}\right) dT$$

$$Q_1 \left(1 - \frac{T_0}{T_1}\right) + m C_p \int \left(1 - \frac{T_0}{T}\right) dT$$

$$= 105 \left[(368 - 288) - 288 \ln\left(\frac{368}{288}\right) \right] = 987.49 \text{ kJ}$$

$$(AE)_{35} = 35 \times 4.2 \int_{288}^{308} \left(1 - \frac{288}{T}\right) dT$$

$$= 97.59 \text{ kJ}$$

Total AE before mixing

$$(AE)_{\text{total}} = 987.49 + 97.59 = 1085.08 \text{ kJ}$$

After mixing if t_f is final temp, then

$$25 \times 4.2 (95 - t_f) = 35 \times 4.2 (t_f - 35)$$

$$t_f = 60^\circ\text{C}$$

$$\therefore \text{Total mass after mixing} = 35 + 25 = 60 \text{ kg}$$

$$(AE)_{60} = AE \text{ of } 60 \text{ kg of water at } 60^\circ\text{C}$$

$$= 60 \times 4.2 \int_{298}^{333} \left(1 - \frac{298}{T}\right) dT$$

$$= 803.27 \text{ kJ}$$

$\therefore$ Decrease in AE due to mixing,

$$= (AE)_{\text{before mixing}} - (AE)_{\text{after mixing}}$$

$$= 1085.08 - 803.27$$

$$= 281.81 \text{ kJ}$$

$$(AE) = 2016.8$$

$$AE = 1532.14$$

$$\text{Deem} = 484.85 \text{ kJ}$$

7. One kg of ice at -5°C is exposed to the atmosphere which is at 25°C . The ice melts & comes into thermal equilibrium. Find the entropy increase of the universe.

sol: Given $m = 1 \text{ kg}$.

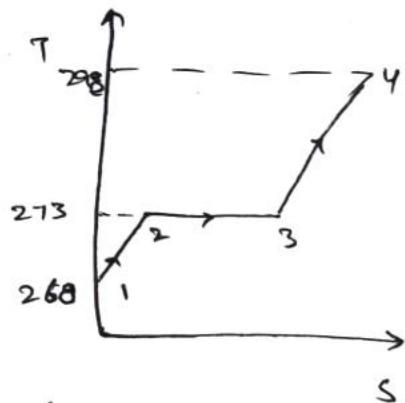
$$T_1 = -5^\circ\text{C} = 268 \text{ K}$$

$$T_u = 25^\circ\text{C} = 298 \text{ K}$$

$$\text{Take } c_{\text{ice}} = 2.093 \text{ kJ/kg K}$$

$$c_{\text{pw}} = 4.187 \text{ kJ/kg K}$$

$$\text{Latent heat of fusion} = 334.5 \text{ kJ/kg}$$



$$\Delta S_{\text{Total}} = \Delta S_{(1-2)} + \Delta S_{(2-3)} + \Delta S_{(3-4)}$$

$$= m c_{\text{ice}} \ln \frac{T_2}{T_1} + \frac{m h_{\text{fg}}}{T_2} + m c_{\text{pw}} \ln \frac{T_u}{T_3}$$

$$= 1 \times 2.093 \ln \frac{273}{268} + \frac{1 \times 334.5}{273} + 1 \times 4.187 \ln \frac{298}{273}$$

$$\Delta S_{\text{Total}} = 0.038689 + 1.2252 + 0.3668$$

$$\Delta S_{\text{Total}} = 1.63 \text{ kJ/kg K}$$

$$Q = m h_{\text{fg}}$$

$$\Delta S_{2-3} = \frac{Q}{T}$$

PURE SUBSTANCES

Pure Substances:-

A pure substance is either a single substance or a mixture of substances which have same chemical composition throughout its mass. The phases may be different but molecular structure should be same. If it doesn't undergo chemical change during heat transfer process.

Ex 1:- i. mixture of water & ice, mixture of water & steam are two examples of pure substances.

ii. Air and steam, Air & petrol are not examples for pure substances.

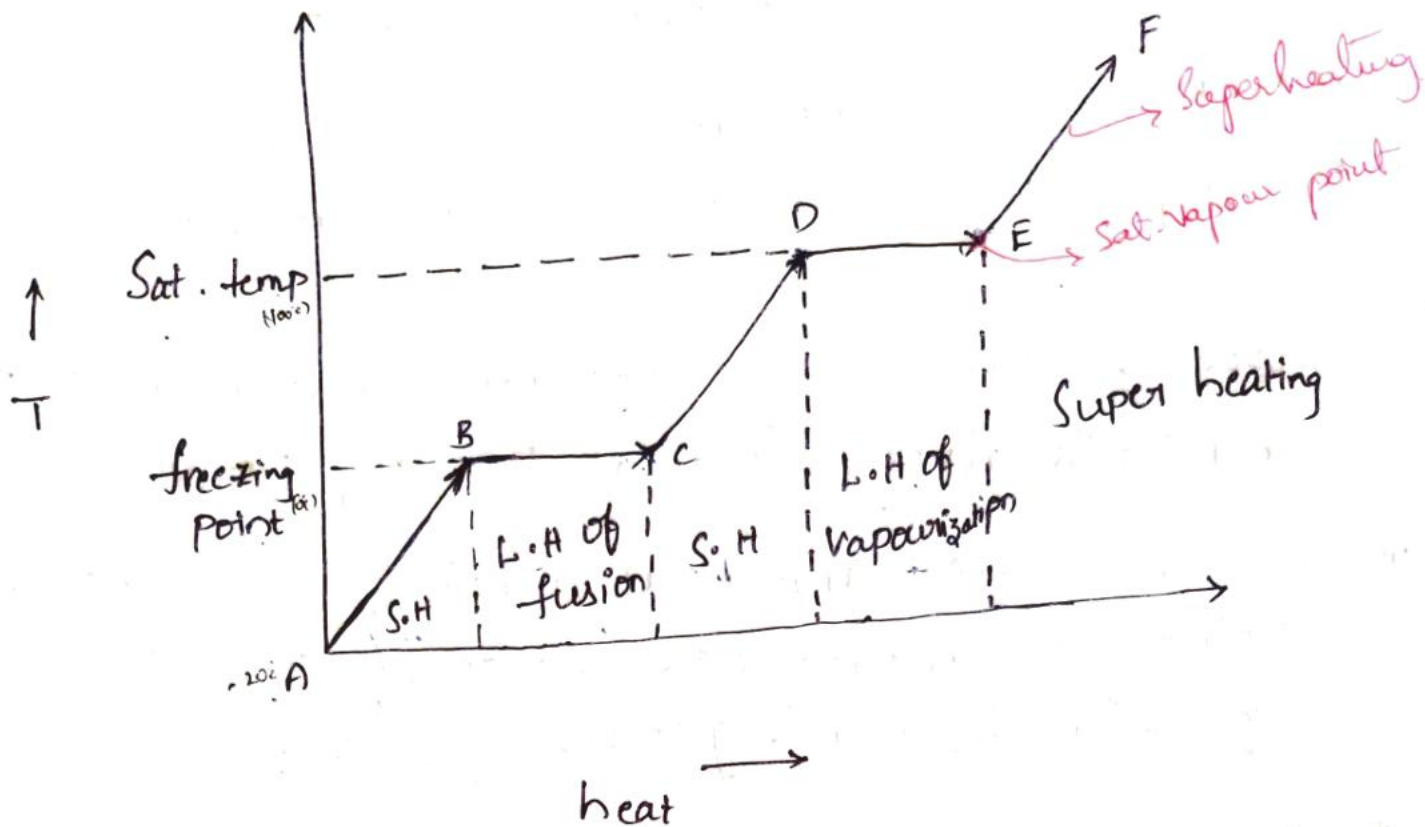
Phase Transformation:-

i. Consider 1 kg of ice at -20°C in a cylinder under a pressure of 1 atm. due to supply of heat the temp of ice rises and approaches zero (0°C) on further heating the temp remains constant and ice starts melting.

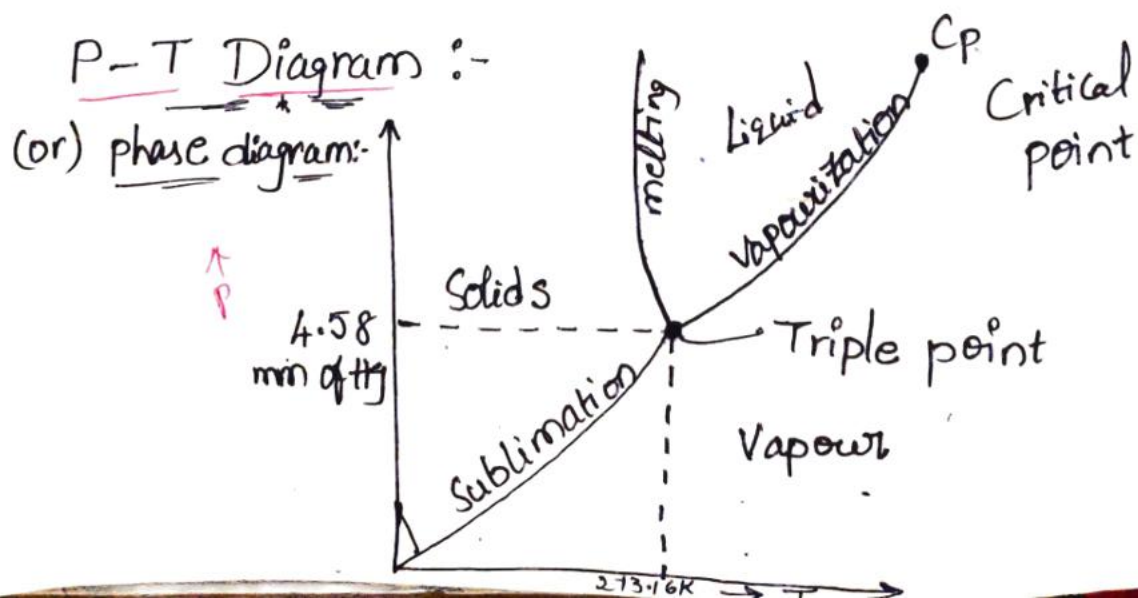
ii. A change of phase takes place from solid to liquid at same temp 0°C . The two phases ice and water exist in equilibrium. Until final flake of ice melts and converts into water there is no rise of temp. The quantity of heat required to convert solid into liquid at a const temp

is known as latent heat of fusion. (or) enthalpy of fusion

- iii. The temp at which the substance change its phase is known as freezing point of temp.
- iv. With the further ~~heating~~ heat addition the temp of water from 0°C to starts rising to 100°C .
- v. When more heat is added further water starts evaporating.
- vi. The temp at which evaporation is occurred is boiling point temp. which is also known as saturation temp.
- vii. The state of water at 100°C , 1 atm and it's called called saturated liquid state.
- viii. The process of ^{evaporation} continues without any rise in temp and heat added is known as latent heat of vaporization.
- ix. The temp will not rise until the final temp droplet of water converts into steam.
- x. This state at which we have complete steam at saturation temp is known as saturation vapour point.
- xi. With further heat addition the temp of steam rises. This process is called super heating of steam.

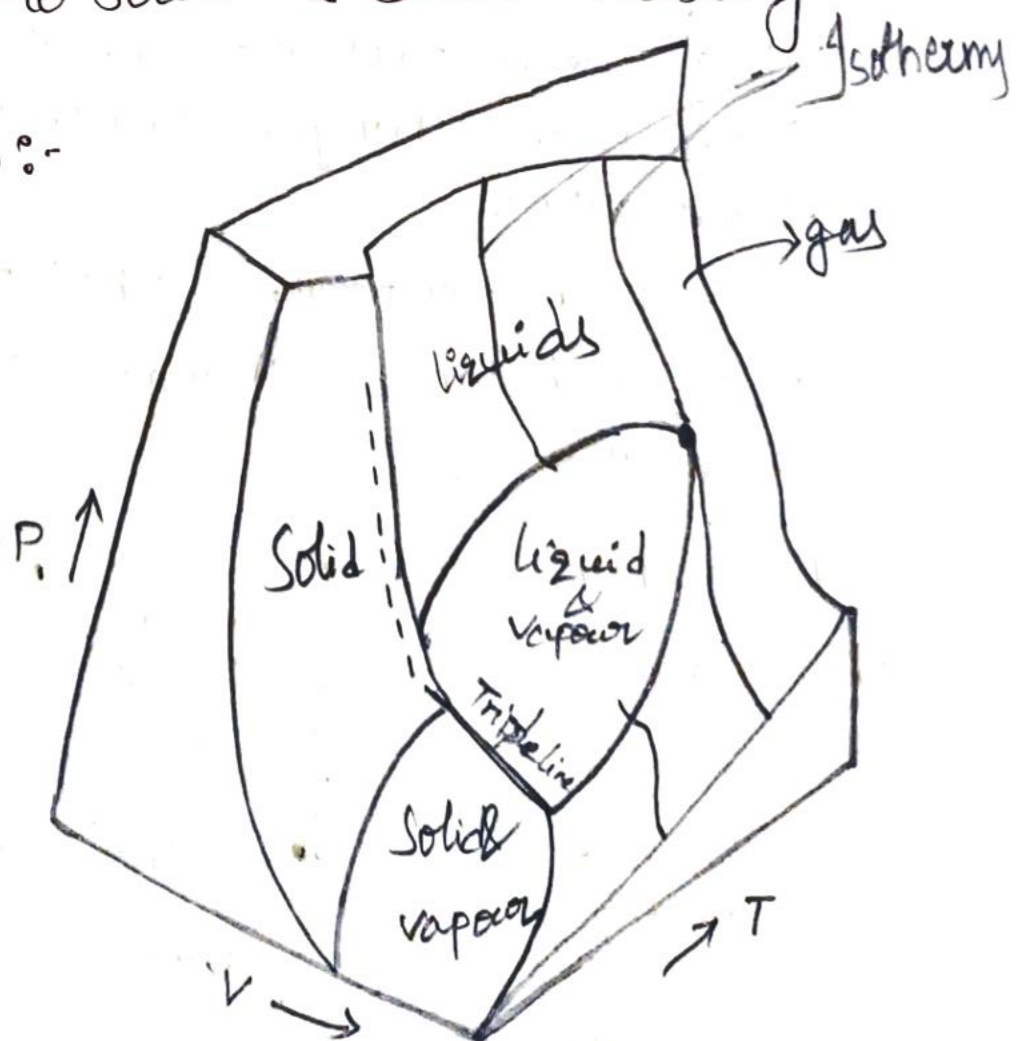


- xii) In any pure substance with increasing pressure the freezing point and boiling point of substance increases.
- xiii) At a given pressure the temp at which a pure substance changes its phase is called saturation temperature.
- xiv) At a given temperature the pressure at which pure substance changes its phase is called saturation pressure.



- * Sublimation Curve Separates Solid phase and vapour
- * melting Curve Separates Solids and liquids
- * melting Curve is also known as ^{point}freezing Curve and fusion Curve.
- * vaporization Curve Separates liquid and vapour and It is also called as Boiling point Curve.
- * All the Curves meet at a point called Triple point. It represents State of Equilibrium of solid, liquid & gaseous (vapour).
- * Triple point of water is at 273.16 K & 4.58 mm of Hg (0.162 kPa)
- * The reverse Sublimation i.e. vapour directly converted to solid is called frosting.

P-V-T diagram :-



Important terms related to Steam formation:-

1. Sensible heat (h_f):-

Quantity of heat absorbed by 1 kg of water when it is heated from 0°C to boiling point without any phase transformation.

2. Latent heat (h_{fg}):-

Amount of heat required to convert water into steam at const temp. (where phase transformation takes place).

3. Dryness fraction (x):-

Ans: The fraction of vapour present in 1 kg of mixture of liquid & vapour is called Dryness fraction.

$$x = \frac{\text{mass of vapour}}{\text{mass of vapour + liquid}}$$

4. Wet Steam ($x < 1$)

mixture of water & steam.

5. Dry Steam ($x = 1$)

There will not be any water droplets.

6. Superheated steam (.

When steam is heated after it has become dry and saturated it is called superheated steam.

7. Degree of Superheat:-

It is the difference b/w Superheated temp and saturation temp.

Properties of Steam:-

Let p = Absolute pressure in bar (or) kpa.

T_s = Saturation temperature in $^{\circ}\text{C}$

h_f = Enthalpy of Saturated liquid in kJ/kg

s_f = Entropy of Saturated liquid in kJ/kg K

v_f = Specific volume of Saturated liquid in m^3/kg

h_{fg} = Enthalpy (or) latent heat of vaporization

s_{fg} = Entropy of vaporization in kJ/kg K

v_{fg} = Specific volume of vaporization in m^3/kg

h_g = Enthalpy of Saturated vapour in kJ/kg

s_g = Entropy of Saturated vapour in kJ/kg K

v_g = Specific volume of Saturated vapour m^3/kg

Suffix

f $\rightarrow$ liquid state

g $\rightarrow$ gaseous state

$h_{fg} = h_g - h_f$ = Change of Enthalpy during Evaporation.

$s_{fg} = s_g - s_f$ = Change of Entropy during Evaporation.

$v_{fg} = v_g - v_f$ = Change of Specific volume during Evaporation.

* To Calculate Specific volume $v = x \times v_g$

* To Calculate Enthalpy $h = h_f + x h_{fg}$

$$h = h_f + x (h_g - h_f)$$

where x = dryness fraction

* To Calculate entropy $S = S_f + x S_{fg}$

$$S = S_f + x (S_g - S_f)$$

The following Equations can be used to Calculate Enthalpy Entropy and Specific volume of Super heated Stream

i. $h = h_g + C_{ps} [T_{sup} - T_s]$

$$C_{ps} = 2.1 \text{ KJ/kgK}$$

Specific heat at
Constant pressure

ii. $S = S_g + C_{ps} \ln \left[\frac{T_{sup}}{T_s} \right]$

iii. $v = v_g \left[\frac{T_{sup}}{T_s} \right]$

Problems:-

- * Steam pressure is 10 bar Calculate Enthalpy and Entropy of the Steam i. If the Steam is dry.
ii. If the Steam is 90% dry.

Sol:- Given $P = 10 \text{ bar}$

i. $x = 1$

ii. $x = 0.9$

From Steam tables

At 10 bar:- $h_f = 762.6 \text{ KJ/kg}$

$$h_{fg} = 2013.6$$

$$h_g = 2776.2$$

$$S_f = 2.138 \text{ KJ/kgK}$$

$$S_{fg} = 4.445$$

$$S_g = 6.583$$

At $x = 1$
i. $h = h_f + x h_{fg}$

$$= h_f + h_{fg} = 762.6 + 2013.6 = 2776.2 \text{ KJ/kg}$$

$$\begin{aligned}
 \text{Entropy } S &= S_f + x S_{fg} \\
 &= S_f + S_{fg} \\
 &= 2.138 + 4.445 = 6.583 \text{ kJ/kg K.}
 \end{aligned}$$

If $x = 0.9$.

$$\begin{aligned}
 \text{Enthalpy } h &= h_f + x h_{fg} \\
 &= 762.6 + 0.9(2013.6) \\
 &= 2574.84 \text{ kJ/kg}
 \end{aligned}$$

$$\begin{aligned}
 \text{Entropy } S &= S_f + x S_{fg} \\
 &= 2.138 + 0.9(4.445) \\
 &= 6.1385 \text{ kJ/kg K}
 \end{aligned}$$

2. In the above problem calculate enthalpy and entropy

- i. if it is 50% dry steam
- ii. pure liquid.

Given $P = 10 \text{ bar}$

i. $x = 0.5$

ii. $x = 0$

If $x = 0.5$

$$\begin{aligned}
 \text{Enthalpy } h &= h_f + x h_{fg} \\
 &= 762.6 + 0.5(2013.6) = 1769.4 \text{ kJ/kg}
 \end{aligned}$$

$$\begin{aligned}
 \text{Entropy } S &= S_f + x S_{fg} \\
 &= 2.138 + 0.5(4.445) \\
 &= 4.3605 \text{ kJ/kg K}
 \end{aligned}$$

If $x = 0$

$$\begin{aligned}
 \text{Enthalpy } h &= h_f + x h_{fg} \\
 &= h_f = 762.6 \text{ kJ/kg}
 \end{aligned}$$

$$\begin{aligned}
 \text{Entropy } S &= S_f + x S_{fg} \\
 &= S_f = 2.138 \text{ kJ/kg K}
 \end{aligned}$$

3. Steam pressure is 40 bar calculate enthalpy and Entropy i. If steam is dry saturated
ii. 70% dry.

Sol:- Given $p = 40 \text{ bar}$

$$\begin{aligned} h_f &= 1087.4 \text{ kJ/kg} & S_f &= 2.797 \text{ kJ/kg-K} \\ h_{fg} &= 1712.9 & S_{fg} &= 3.272 \\ h_g &= 2800.3 & S_g &= 6.069 \end{aligned}$$

If $x = 1$

$$\begin{aligned} \text{Entropy } S &= S_f + x S_{fg} \\ &= S_f + S_{fg} \\ &= 2.797 + 3.272 \\ &= 6.069 \text{ kJ/kg-K} \end{aligned}$$

$$\begin{aligned} \text{Enthalpy } h &= h_f + x h_{fg} \\ &= 1087.4 + 1712.9 \\ &= 2800.3 \text{ kJ/kg} \end{aligned}$$

If $x = 0.7$

$$\begin{aligned} \text{Enthalpy } h &= h_f + x h_{fg} \\ &= 1087.4 + 0.7(1712.9) \\ &= 2286.43 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \text{Entropy } S &= S_f + x S_{fg} \\ &= 2.797 + (0.7) 3.272 \\ &= 5.0874 \text{ kJ/kg-K} \end{aligned}$$

4. Calculate Enthalpy and Entropy and specific volume of steam at a pressure of 20 bar and 400°C .

Sol:- Given

$$\begin{aligned} \text{Spec}^s & - P = 20 \text{ bar} \\ & T = 400^\circ\text{C} \end{aligned}$$

At $P = 20 \text{ bar}$
 $T = 400^\circ\text{C}$

Specific volume $v = 0.1511 \text{ m}^3/\text{kg}$

Enthalpy $h = 3248.7 \text{ kJ/kg}$

Entropy $s = 7.130 \text{ kJ/kg K}$

5. Calculate Entropy, enthalpy and Specific volume of Steam at 12 bar and 258°C ?

Given $P = 12 \text{ bar}$ (Super heat steam)

$T_{\text{sup}} = 258^\circ\text{C}$
 $= 531 \text{ K}$

From Steam tables

$T_s = 188^\circ\text{C}$

$= 461 \text{ K}$

(It is not round up value so we can calculate saturated steam pressure table we can calculate in formula)

$h_g = 2782.7$, $s_g = 6.519$
 $v_g = 0.1632$

Enthalpy $h = h_g + C_{ps} (T_{\text{sup}} - T_s)$

$h = 2782.7 + 2.1 (531 - 461)$

$= 2929.7 \text{ kJ/kg}$

Entropy $s = s_g + C_{ps} \ln\left(\frac{T_{\text{sup}}}{T_s}\right)$

$= 6.519 + 2.1 \ln\left(\frac{531}{461}\right)$

$= 6.8159 \text{ kJ/kg K}$

Specific volume $v = \bar{v}_g \left(\frac{T_{sup}}{T_s} \right)$

$$= 0.16321 \left(\frac{531}{461} \right)$$

$$= 0.1880 \text{ m}^3/\text{kg}$$

6. Calculate Enthalpy, entropy of steam at 10 bar when it is i. saturated ii. 95% dry iii. Super heated to 260°C

Soln

$$P = 10 \text{ bar}$$

From steam table

$$t_s = 179.9^\circ\text{C} = 452.9 \text{ K}$$

$$h_f = 762.6$$

$$h_g = 2776.2$$

$$h_{fg} = 2013.6$$

$$s_f = 2.138$$

$$s_{fg} = 4.445$$

$$s_g = 6.583$$

i. Saturated Condition

$$x = 1$$

$$\begin{aligned} \text{Enthalpy } h &= h_f + x h_{fg} \\ &= 762.6 + (1)(2013.6) \\ &= 2776.2 \text{ kJ/kg} \end{aligned}$$

$$\text{Entropy } s = s_f + x(s_g - s_f)$$

or)

$$\begin{aligned} s &= s_f + x s_{fg} \\ &= 2.138 + 4.445 \\ &= 6.583 \text{ kJ/kgK} \end{aligned}$$

ii. 95% dry

$$x = 0.95$$

$$\begin{aligned} \text{Enthalpy } h &= h_f + x h_{fg} \\ &= 762.6 + 0.95(2013.6) \\ &= 2675.52 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \text{Entropy } s &= s_f + x s_{fg} \\ &= 2.138 + 0.95(4.445) \\ &= 6.36075 \text{ kJ/kgK} \end{aligned}$$

iii. Super heated

$$T_{sup} = 260^\circ\text{C} = 533 \text{ K}$$

$$\begin{aligned}\text{Enthalpy } h &= h_g + C_{ps} (T_{\text{sup}} - T_s) \\ &= 2776.2 + 2.1 (533 - 452.9) \\ &= 2947.41 \text{ kJ/kg}\end{aligned}$$

$$\begin{aligned}\text{Entropy } S &= S_g + C_{ps} \ln \left(\frac{T_{\text{sup}}}{T_s} \right) \\ &= 6.583 + 2.1 \ln \left(\frac{533}{452.9} \right) \\ &= 6.925 \text{ kJ/kg K}\end{aligned}$$

7. Calculate Entropy of 5 kg of Steam at 2 Mpa and 300°C

$$C_{ps} = 2.1 \text{ kJ/kg} \cdot \text{K}$$

$$20 \text{ bar} = 20 \times 100 \text{ kPa} = 2 \text{ Mpa}$$

Solⁿ

$$\text{Given } T_{\text{sup}} = 300^\circ\text{C}$$

$$P = 20 \text{ bar}$$

$$S = 6.770 \text{ kJ/kg K}$$

then for 5 kg of Steam

$$S = 33.85 \text{ kJ/kg K}$$

8. Steam enters an engine at 10 bar and 250°C and it is exhausted at 0.2 bar and 0.9 dry. Calculate

i. drop in Enthalpy

ii. Change in Entropy

Super heated

$$P_1 = 10 \text{ bar}$$

$$T_{\text{sup}} = 250^\circ\text{C}$$

$$h_1 = 2943 \text{ kJ/kg}$$

$$S_1 = 6.926 \text{ kJ/kg K}$$

$$P_2 = 0.2 \text{ bar}$$

$$x = 0.9$$

From steam table

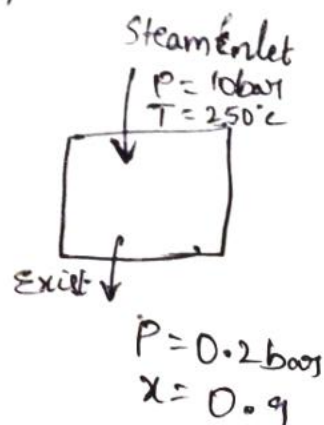
$$h_{f2} = 251.5$$

$$h_{fg2} = 2358.4$$

$$h_{g2} = 2609.9$$

$$S_{f2} = 0.832$$

$$S_{fg2} = 7.077$$



$$\begin{aligned}\text{Enthalpy } h_2 &= h_{f_2} + x h_{fg_2} \\ &= 251.5 + 0.9 (2358.4) \\ &= 2374.06 \text{ kJ/kg}\end{aligned}$$

$$\begin{aligned}\text{Entropy } s_2 &= s_{f_2} + x s_{fg_2} \\ &= 0.832 + 0.9 (7.077) \\ &= 7.2013 \text{ kJ/kg K}.\end{aligned}$$

$$\text{drop in Enthalpy} = h_1 - h_2 = 568.94 \text{ kJ/kg}$$

$$\text{Change in Entropy} = s_2 - s_1 = 0.2753 \text{ kJ/kg K}$$

9. Find the Enthalpy and Entropy of Steam when the pressure is 2 MPa and specific volume is $0.09 \text{ m}^3/\text{kg}$ and also calculate internal energy of a steam.

$$P = 2 \text{ MPa} = 20 \text{ bar}$$

$$\text{Specific volume} = 0.09 \text{ m}^3/\text{kg}$$

$$\text{At } P = 20 \text{ bar}$$

from steam tables

$$v_g = 0.099549 \text{ m}^3/\text{kg}$$

$$v = x \times v_g$$

$$0.09 = x \times (0.099549)$$

$$x = 0.9041$$

$$h_f = 908.6 \quad h_{fg} = 1888.7$$

$$s_f = 2.447 \quad s_{fg} = 3.890$$

$$\begin{aligned}\text{Enthalpy } h &= h_f + x h_{fg} \\ &= 908.6 + (0.9041)(1888.7) \\ &= 2616.17367 \text{ kJ/kg}\end{aligned}$$

$$\text{Entropy } S = S_f + x S_{fg}$$

$$= 2.447 + (0.9041)(3.890)$$

$$= 5.96394 \text{ kJ/kg K}$$

we know that

$$\text{Enthalpy } h = u + Pv$$

$$\text{Internal energy } u = h - Pv$$

$$u = 2616.17 - (2000)(0.09)$$

$$u = 2428 \text{ kJ/kg}$$

$$u = \cancel{2598.17} \text{ kJ/kg}$$

$$u = \cancel{816.17} \text{ kJ/kg}$$

10. If certain amount of heat is produced at a pressure of 8 bar and dryness fraction $x = 0.8$ calculate
- external work done
 - Internal energy of steam.

Sol:- Given $P = 8 \text{ bar}$
 $x = 0.8$

$$\text{we } w = Pv \quad v = x v_g$$

$$u =$$

$$h = u + Pv$$

$$u = h - Pv$$

At $P = 8 \text{ Bar}$ from steam tables.

$$v_g = 0.24026 \quad h_f = 720.9 \quad h_{fg} = 2046.5$$

$$\text{Specific volume } v = x v_g$$

$$= 0.8(0.24026)$$

$$= 0.192208 \text{ m}^3/\text{kg}$$

External work done $w = PV$

$$= (8 \times 100) \times 0.192208$$

$$= 153.7664 \text{ kJ/kg}$$

$P = (\text{KPa})^*$

$$= \text{KPa} \times \frac{\text{m}^3}{\text{kg}}$$

$$= \text{K} \frac{\text{N}}{\text{m}^2} \times \frac{\text{m}^3}{\text{kg}}$$

$$= \frac{\text{KN} \cdot \text{m}}{\text{kg}}$$

$$= \frac{\text{kJ}}{\text{kg}}$$

Enthalpy $h = h_f + x h_{fg}$

$$= 720.9 + 0.8 (2046.5)$$

$$= 2358.1 \text{ kJ/kg}$$

we know that

$$h = u + Pv$$

$$u = h - Pv$$

$$= 2358.1 - 153.7664$$

$$u = 2204.3336 \text{ kJ/kg}$$

11. Find the Enthalpy and internal energy of Steam
- at pressure of 7 bar and when it's quality is 0.8
 - when it is dry and saturated.
 - when it is Super heated ~~when~~ degree of Super heat is 65°C .

Sol: Given $P = 7 \text{ bar}$.

i. $x = 0.8$

$P = 7 \text{ bar}$ from steam tables

Enthalpy $h = h_f + x h_{fg}$

$$= 697.1 + 0.8 (2064.9)$$

$$= 2349.02 \text{ kJ/kg}$$

Specific volume $v = x v_g$

$$= 0.8 (0.2726)$$

$$= 0.218144$$

we know that $h = u + Pv$

$$u = h - Pv = 2349.02 - (700)(0.218144)$$

$$u = 2196.3192 \text{ kJ/kg}$$

ii. $P = 7$

$x = 1$

from steam tables

$h_f = 697.1$ $h_{fg} = 2064.9$

$v_g = 0.27268$ $h_g = 2762$

$h = h_f + x h_{fg}$

$h = 2762 \text{ kJ/kg}$

Specific volume

$v = x v_g$

$v = 0.27268$

we know that

$u = h - P v$

$= 2762 - (700)(0.27268)$

$u = 2571.124 \text{ kJ/kg}$

iii. $P = 7 \text{ bar}$

Degree of Superheat $= (T_{\text{sup}} - T_s) = 65^\circ\text{C}$

$(T_{\text{sup}} - T_s) = \cancel{328} 65$

Enthalpy $h = h_g + C_{ps} (T_{\text{sup}} - T_s)$

$= 2762 + 2.01 (\cancel{328}^{65})$

$= \cancel{3471.8} \text{ kJ/kg}$

$= 2898.5 \text{ kJ/kg}$

From steam tables

$T_s = 165^\circ\text{C}$

$(T_{\text{sup}} - T_s) = 65^\circ\text{C}$

$v = v_g \left(\frac{T_{\text{sup}}}{T_s} \right)$

$T_{\text{sup}} = 65 + T_s$

$= 230^\circ\text{C}$

$= 503 \text{ K}$

$v = 0.31314621 \text{ m}^3/\text{kg}$

$u = h - P v$

$= \cancel{3471.8}$

$= 2898.5 - (700)(0.31314621)$

$u = 2679.297653 \text{ kJ/kg}$

- 12 Find Saturation temp, Change in Specific volume
Change in entropy Latent heat of vaporization
of steam at 1 MPa.

At $p = 10 \text{ bar}$

From Steam tables

$$T_s = 179.9^\circ \text{C}$$

$$v_g = 0.19430$$

$$v_f = 0.001127$$

Change in Specific volume $v_{fg} = v_g - v_f$
 $= 0.193173 \text{ m}^3/\text{kg}$

Change in entropy $s_{fg} = 4.445 \text{ kJ/kg K}$

Latent heat of vaporization $h_{fg} = 2013.6 \text{ kJ/kg}$

13. 2 kg of steam initially at 12 bar and 250°C and expands polytropically to 1.2 bar. Calculate.

i. final volume

ii. final Condition of the steam

iii. Work done

iv. Change in entropy and assume index of

Expansion as 1.25.

Sol: Given $P_1 = 12 \text{ bar}$
 $T_1 = 250^\circ \text{C}$

$P_2 = 1.2 \text{ bar}$

Poly tropic process

$$Pv^n = C$$

From Steam table

$$P = 12 \text{ bar} \quad T_1 = 250^\circ \text{C}$$

Specific volume $v = 0.1924 \text{ m}^3/\text{kg}$

$$\Rightarrow v_2 = 0.1924 \left(\frac{12}{1.2} \right)^{\frac{1}{1.25}}$$

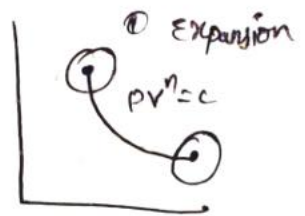
$$v_2 = 1.21396 = 1.214 \text{ m}^3/\text{kg}$$

$$P_1 v_1^n = P_2 v_2^n$$

$$\frac{v_2}{v_1} = \left(\frac{P_1}{P_2} \right)^{\frac{1}{n}}$$

$$v_2 = v_1 \left(\frac{P_1}{P_2} \right)^{\frac{1}{n}}$$

ii. By finding x_2 value we can find Condition of steam



w. know that $V_2 = x_2 V_{g2}$

At 1.2 bar from steam tables

$$V_{g2} = 1.4281$$

$$x_2 = \frac{V_2}{V_{g2}} = \frac{1.21396}{1.4281} = 0.85$$

iii. Work done = $\frac{P_1 V_1 - P_2 V_2}{n-1}$

$$= \frac{12 \times 100 \times 0.1924 - 120 \times 1.21396}{1.25-1}$$

$$= 340.8192 \text{ kJ/kg}$$

iv. Change in entropy

From steam tables

$P_1 = 12 \text{ bar}$ $T = 250^\circ\text{C}$

$$S_1 - S_2 = 6.831 - 6.40745$$

$$S_1 = 6.831 \text{ kJ/kg}$$

$$= 0.42355 \text{ kJ/kg-K}$$

$P_2 = 1.2 \text{ bar}$

$S_g = 1.361$ $S_{fg} = 5.937$

$$S_2 = S_f + x_2 S_{fg}$$

$$= 1.361 + 0.85(5.937)$$

$$S_2 = 6.40745$$

14. Two boilers one with Super heater and another without Super heater are delivering Equal quantity of steam into a Common main. The pressure in the boiler is 20 bar. The temp of steam from a boiler with a Super heater is 350°C and the temp of steam in the main is 250°C determine the quality of steam supplied by the other boiler. Take $C_{ps} = 2.25 \text{ kJ/kg}^\circ\text{C}$

ed:

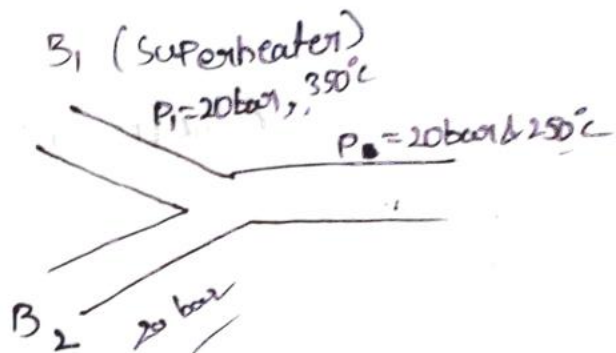
Boiler ①

$$P_1 = 20 \text{ bar}$$

$$T_1 = 350^\circ\text{C}$$

from steam-table

$$h_1 = 3138.6 \text{ kJ/kg}$$



Boiler ② :-

$$h_2 = h_{f2} + x_2 h_{fg2} = 908.6 + x_2 1888.7$$

from Steam-table (Saturated Condition)

$$\text{at } P = 20 \text{ bar} \quad h_{f2} = 908.6$$

$$h_{fg2} = 1888.7$$

main :

$$P = 20 \text{ bar} \rightarrow 250^\circ\text{C}$$

$$h_3 = 2902.4 \text{ kJ/kg} \rightarrow \text{dual quantity}$$

$$h_3 = 2 \times 2902.4 = 5804.8 \text{ kJ}$$

$$h_1 + h_2 = h_3$$

$$3138.6 + 908.6 + x_2 (1888.7) = 5804.8$$

$$4047.2 + x_2 (1888.7) = 5804.8$$

$$x_2 = 0.9305$$

15. Calculate Enthalpy & Entropy of Steam at a pressure of 30 bar
- When it dry Saturated
 - 90% dry
 - Superheated to 475°C
- Compare the values with mollier Chart.

i. $x = 1$

At $P = 30 \text{ bar}$

from steam tables

$$h_f = 1008.3$$

$$h_{fg} = 1794.0$$

$$S_f = 2.646$$

$$S_{fg} = 3.538$$

$$\begin{aligned}\text{Enthalpy } h &= h_f + x h_{fg} \\ &= 1008.3 + 1794 \\ &= 2802.3 \text{ kJ/kg}\end{aligned}$$

$$\begin{aligned}\text{Entropy } s &= s_f + x s_{fg} \\ &= 2.646 + 3.538 \\ &= 6.184 \text{ kJ/kg K}\end{aligned}$$

i. If $x = 0.9$

$P = 30 \text{ bar}$ from steam tables.

$$h_f = 1008.3 \quad h_{fg} = 1794$$

$$s_f = 2.646 \quad s_{fg} = 3.538$$

$$\begin{aligned}\text{Enthalpy } h &= h_f + x h_{fg} \\ &= 1008.3 + 0.9 (1794) \\ &= 2622.9 \text{ kJ/kg}\end{aligned}$$

$$\begin{aligned}\text{Entropy } s &= s_f + x s_{fg} \\ &= 2.646 + 3.538 \times 0.9 \\ &= 5.8302 \text{ kJ/kg K}\end{aligned}$$

iii. $T_{\text{sup}} = 475^\circ\text{C} = 748\text{K}$

For $p = 30 \text{ bar}$ From steam tables

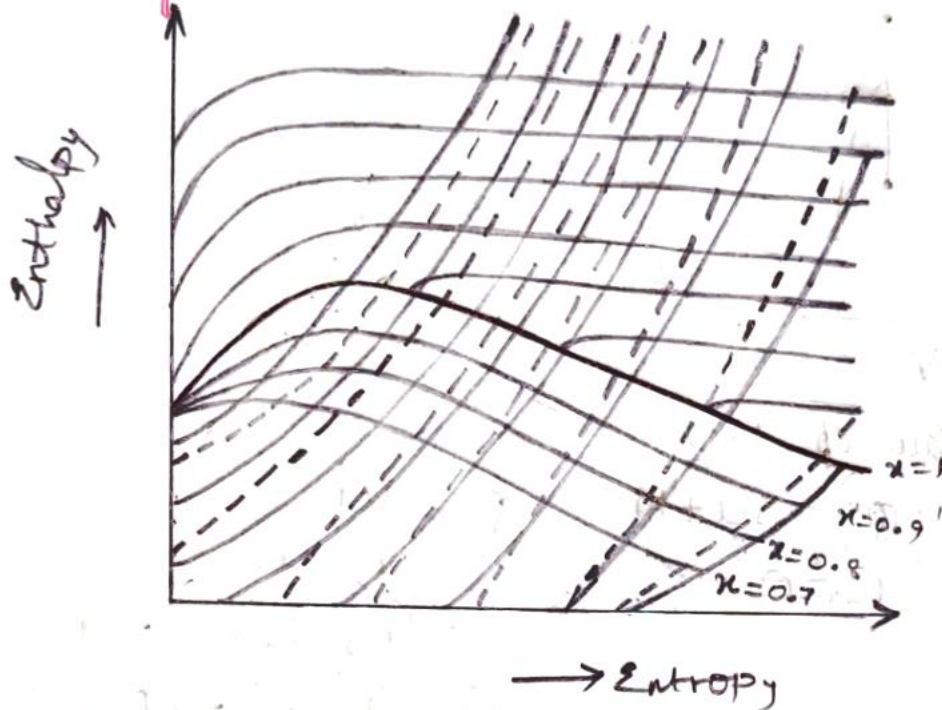
$$T_g = 233.8^\circ\text{C} = 506.8$$

$$h_g = 2802.3 \quad s_g = 6.184.$$

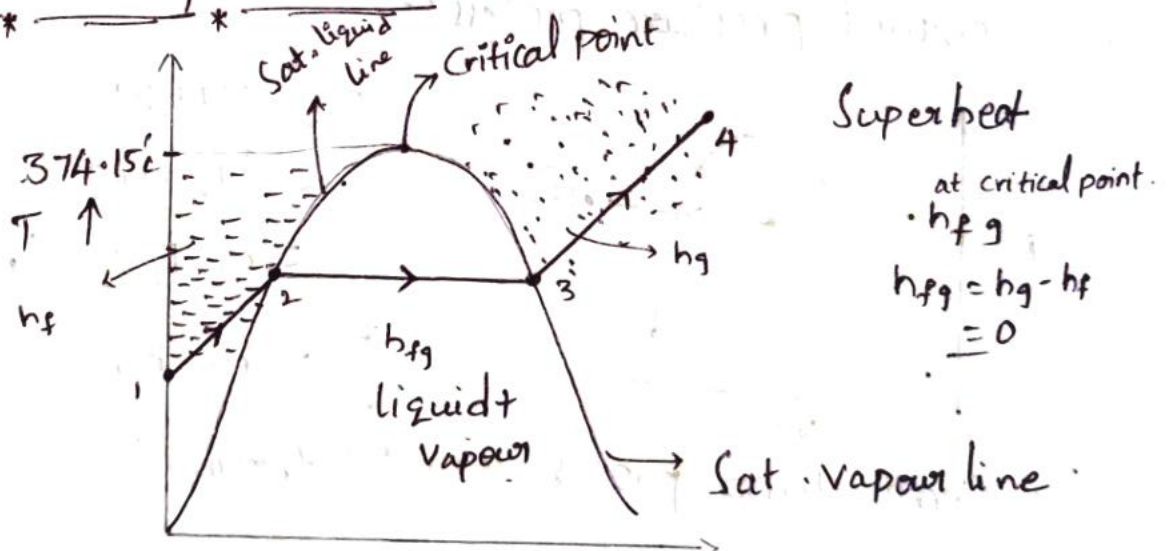
$$\begin{aligned}\text{Enthalpy } h &= h_g + C_{ps} (T_{\text{sup}} - T_g) \\ &= 2802.3 + 2.1 \times (748 - 506.8) \\ &= 3308.82 \text{ kJ/kg}\end{aligned}$$

$$\begin{aligned}
 \text{Entropy } S &= S_g + C_{ps} \ln \left(\frac{T_{\text{sup}}}{T_s} \right) \\
 &= 6.184 + 2.1 \ln \left(\frac{748}{506.8} \right) \\
 &= 7.0015 \text{ kJ/kgK}
 \end{aligned}$$

Mollier Diagram:



T-S diagram of pure substance



Critical point temp = 374.15°C

Critical pressure = 221.2 kPa

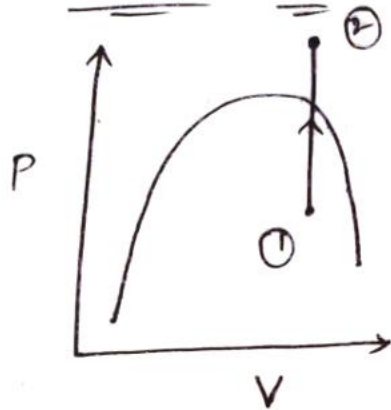
NOTE:-

At critical point Latent heat of vaporization is Zero, ~~vapour~~ po where the point contains liquid & vapour.

From that point is called critical point.

Thermodynamic * process of a pure substance :-

1. Constant Volume process :-



A/c to 1st law of T.D

$$Q = \Delta u + W$$

$$W = 0$$

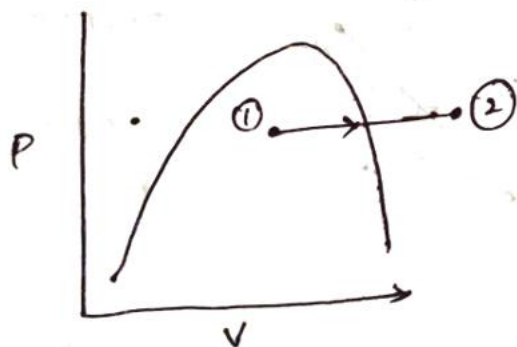
$$Q = \Delta u$$

$$= u_2 - u_1$$

$$= (h_2 - P_2 v_2) - (h_1 - P_1 v_1)$$

$$Q = (h_2 - h_1) - (P_2 v_2 - P_1 v_1)$$

2. Constant pressure process :-



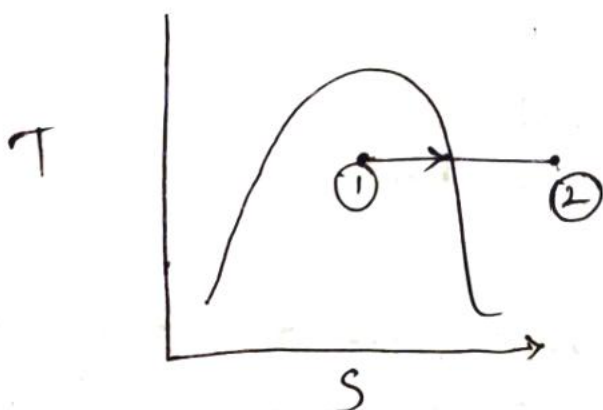
$$Q = \Delta u + W$$

$$= (u_2 - u_1) + P(v_2 - v_1)$$

$$= (h_2 - P_2 v_2) - (h_1 - P_1 v_1) + P_2 v_2 - P_1 v_1$$

$$\boxed{Q = h_2 - h_1}$$

3. Const temp process (or) Isenthalal :-

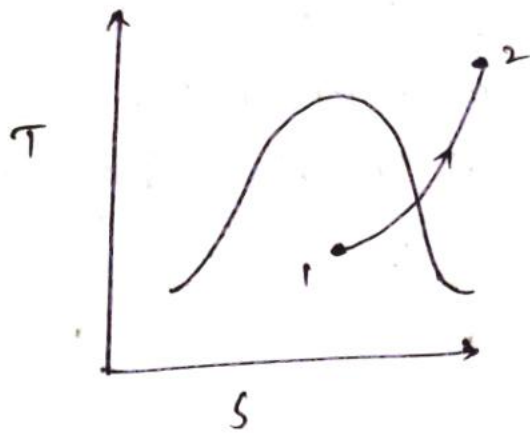


$$Q = \Delta u + W$$

$$= (h_2 - P_2 v_2) - (h_1 - P_1 v_1)$$

$$+ P_1 v_1 \ln\left(\frac{P_1}{P_2}\right)$$

④ Adiabatic process :-



$$Q = \Delta u + w$$

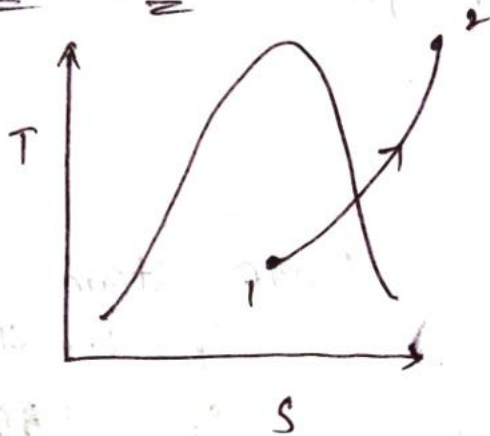
$$[Q = 0]$$

$$w = -\Delta u$$

$$= (h_1 - P_1 v_1) - (h_2 - P_2 v_2)$$

$$= (h_1 - h_2) - (P_1 v_1 - P_2 v_2)$$

⑤ Polytropic process :-

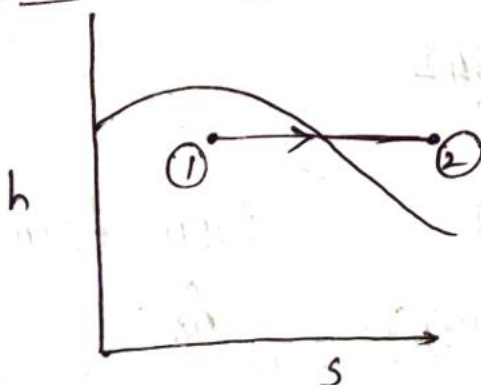


$$Q = \Delta u + w$$

$$= (h_2 - P_2 v_2) - (h_1 - P_1 v_1)$$

$$+ \frac{P_1 v_1 - P_2 v_2}{n-1}$$

⑥ Throttling process :-



For Throttling process heat transfer ($Q = 0$)

work transfer $w = 0$

It is also called as Constant Enthalpy process or

ISO Enthalpy process

$$\therefore h_1 = h_2$$

1. The steam at a pressure of 20 bar at 500°C expands isentropically to a pressure of 0.1 bar. Calculate work done per kg of steam & find the condition of steam. Represent the process on H-S diagram.

Sol:-

$$P_1 = 20 \text{ bar} \quad T_{\text{sup}} = 500^\circ\text{C}$$

From steam tables

$$h_1 = 3447.3 \text{ kJ/kg}$$

$$s_1 = 7.432 \text{ kJ/kgK}$$

From T-s diagram

$$s_1 = s_2$$

$$P_2 = 0.1 \text{ bar}$$

$$s_2 = s_{f2} + x_2 s_{fg2}$$

$$7.432 = 0.674 + x_2 7.444$$

$$6.758 = x_2 (7.444)$$

$$\underline{x_2 = 0.9041}$$

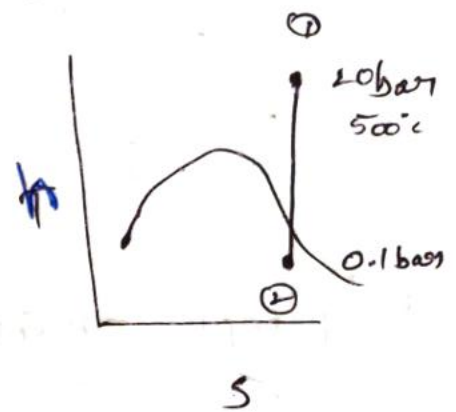
$$h_2 = h_f + x_2 h_{fg}$$

$$h_2 = 191.8 + (0.9041)(2392.9)$$

$$h_2 = 2372.219 \text{ kJ/kg}$$

$$\text{Work done} = h_1 - h_2$$

$$= 1075.08 \text{ kJ/kg}$$



From steam tables

$$s_{f2} = 0.674$$

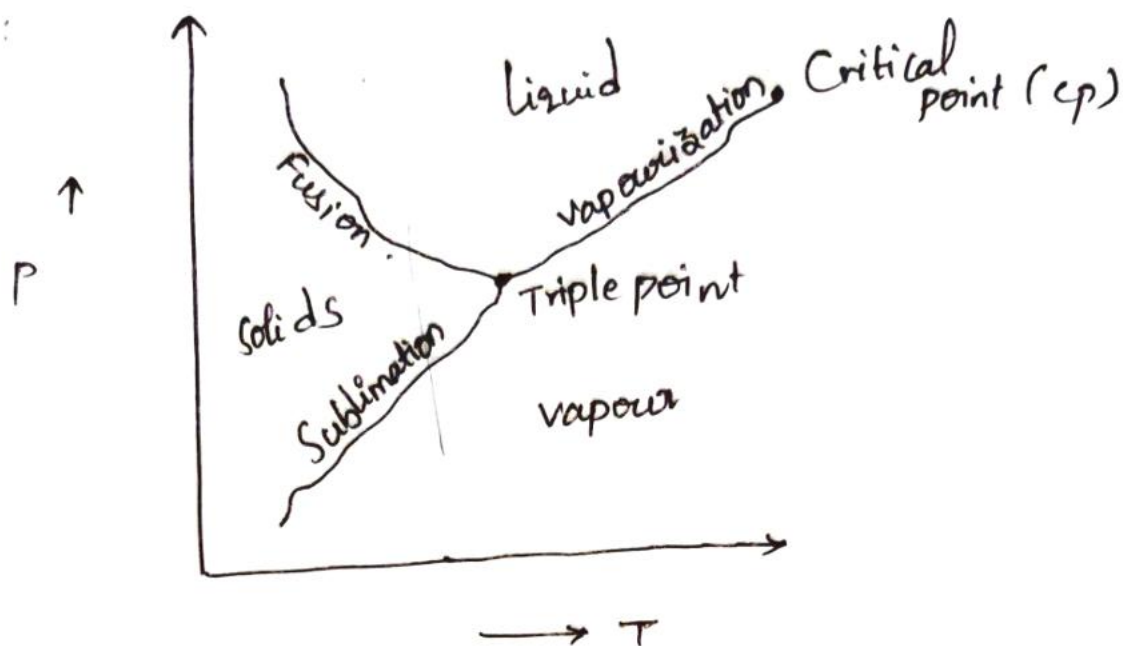
$$s_{fg2} = 7.502$$

From steam tables

$$h_f = 191.8$$

$$h_{fg} = 2392.9$$

Clausius Clapeyron Equation :-



Clausius Clapeyron Equation is a relationship b/w saturation pressure, Temperature, Enthalpy of evaporation and Specific volume of two phases involved.

This equation provides a basis for calculations of properties in a two phase region. It gives the slope of a curve separating the two phases in P-T diagram.

Consider one of the Maxwell relation for vaporization process.

$$\left(\frac{\partial P}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$$

Let us consider the change of state ^{from} Saturated liquid to Saturated vapour of a pure substance which takes place at const temp.

during the evaporation the pressure and temperatures are independent of volume.

$$\frac{dP}{dT} = \frac{ds}{dv}$$
$$\frac{dP}{dT} = \frac{s_g - s_f}{v_g - v_f} = \frac{s_{fg}}{v_{fg}}$$

$$\frac{dp}{dT} = \frac{h_{fg}}{T_s (V_{fg})} \rightarrow \text{①}$$

$$ds = \frac{dh}{T}$$

~~This Equation for~~ Clausius Clapeyron Equation for evaporation of liquid.

For fusion $\frac{ds}{dT} = \frac{L.H \text{ of fusion}}{T_s (V_f - V_g)}$

for vapourization $\frac{dp}{dT} = \frac{L.H \text{ of vapourization}}{T_s (V_g - V_f)}$

- * Thus if the substance expands on fusion $V_f > V_g$ and $\frac{dp}{dT}$ is positive.
- * If the substance contracts on melting then the fusion curve will have negative slope.

Ex:- Ice to water.

- * At temp below critical temp $V_g \gg V_f$

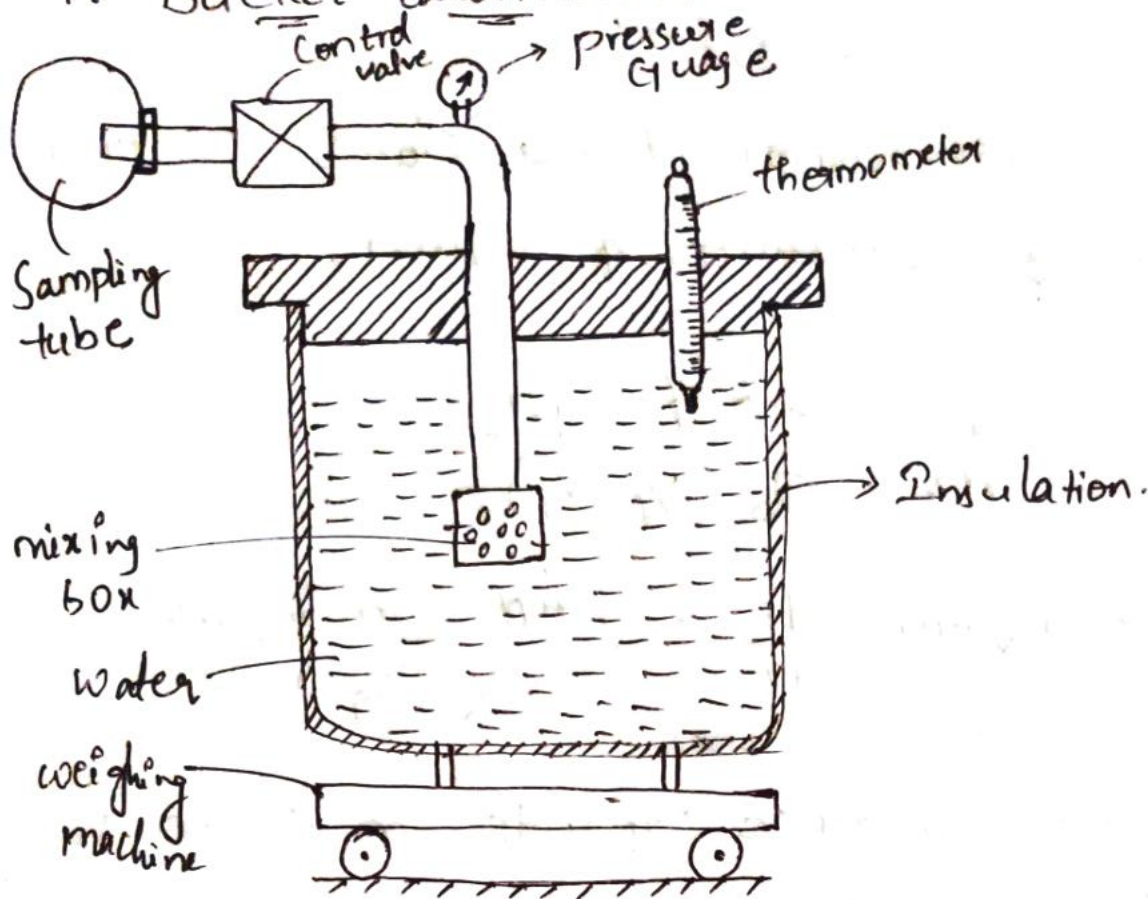
Methods of determining the dryness fraction :-

Basically four methods are available to find dryness fraction of steam experimentally and this process is known as Calorimetry.

Types of Calorimeter :-

1. Bucket Calorimeter
2. Separating calorimeter
3. Throttling Calorimeter
4. Separating and Throttling Calorimeter.

1. Bucket Calorimeter:-



- * Bucket Calorimeter Consists of Copper Container which has a measured quantity of water this container surrounded by an insulating material so as to prevent heat transfer.
- * A thermometer
- * The Container weight ~~first~~ and its temp is noted.
- * Now a sample of steam is passed into the container which discharges into cold water through fine exits holes.
- * The steam entering into the container gives out heat and condenses giving out entire latent heat & part of its sensible heat. This will raise the temp water.
- * The supply of steam is stopped & water temp

* Container weight is noted again.

* The increase in water, will give the weight of Steam Condensed and the increase in temp of water will give the amount of heat transfer.

Analysis:-

* According to the Energy balance heat lost by the Steam is Equal to heat Observed by the water.

$$M_s (h_s - h_w) = M \cdot C_{pw} (T_f - T_i)$$

where

M_s = mass of steam

h_s = Enthalpy of steam

h_w = Enthalpy of water.

C_{pw} = Specific heat of water

$$C_{pw} = 4.2 \text{ kJ/kgK}$$

$$M = M_s + M_w$$

T_f & T_i are the final & initial temperatures.

We know that $h_s = h_f + x h_{fg}$

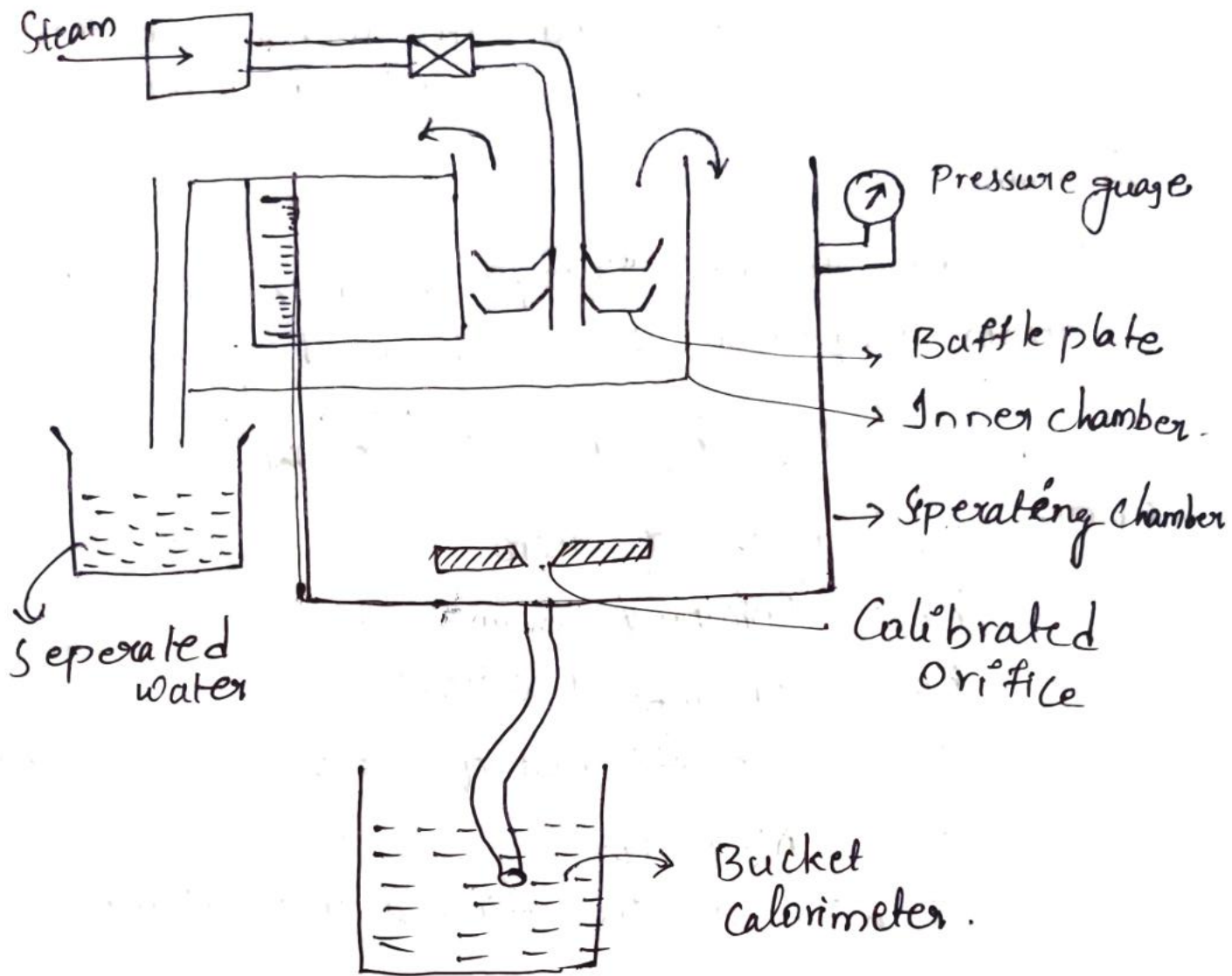
$$h_w = h_f$$

$$M_s (h_f + x h_{fg} - h_f) = M C_{pw} (T_f - T_i)$$

$$M_s (x h_{fg}) = M C_{pw} (T_f - T_i)$$

$$\boxed{\text{dryness fraction } x = \frac{M C_{pw} (T_f - T_i)}{M_s h_{fg}}}$$

2. Separating Calorimeter :-



* The wet steam enters the inner chamber of Separating Calorimeter through a tube in the inner chamber of calorimeter. Baffle plates are arranged in such a manner ~~arranged~~ to separate the moisture from steam.

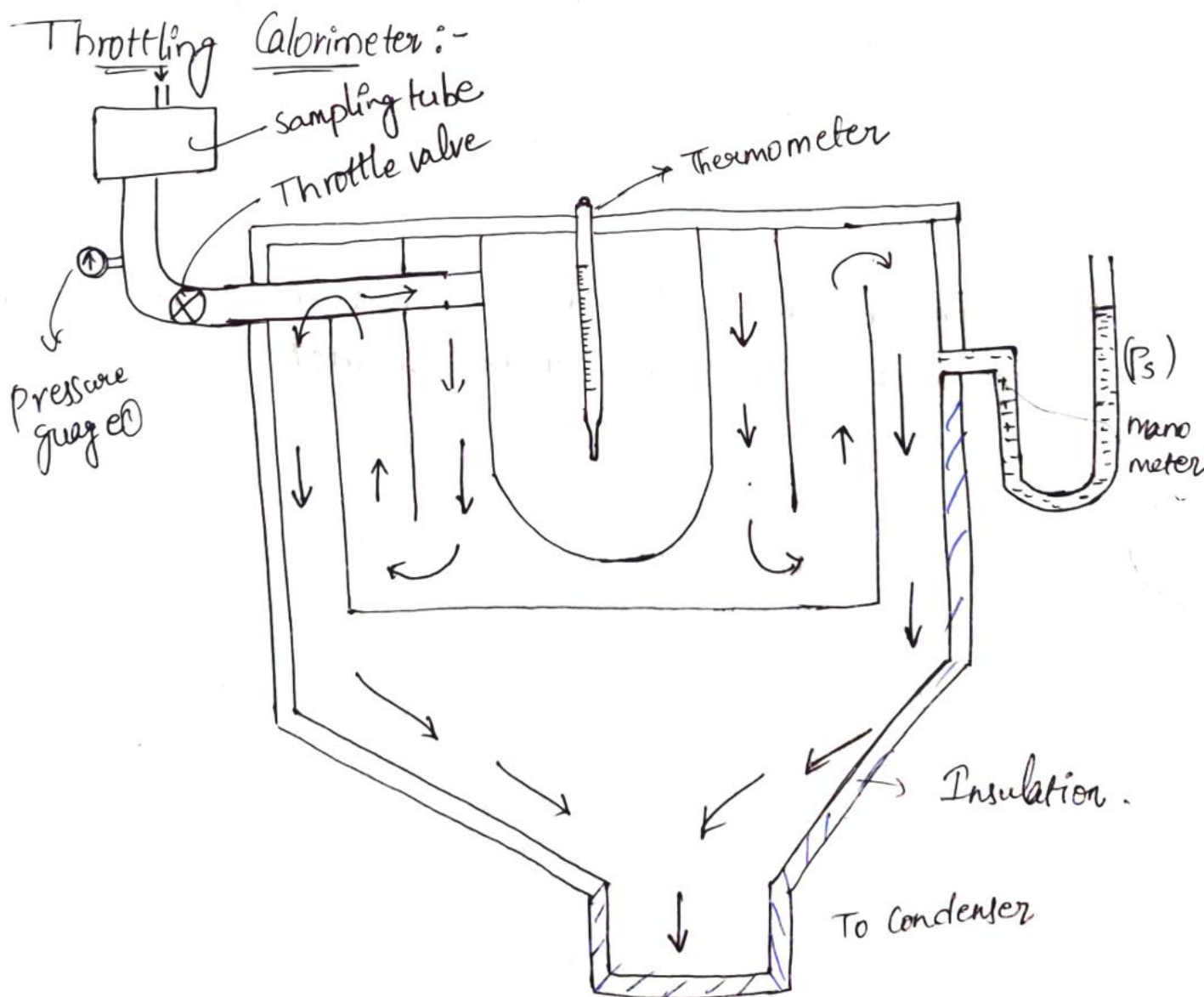
Hence the steam enters the separator & under goes sudden reverse flow direction when it strikes the plates.

This causes water particles which have greater inertia to separate from the wet steam.

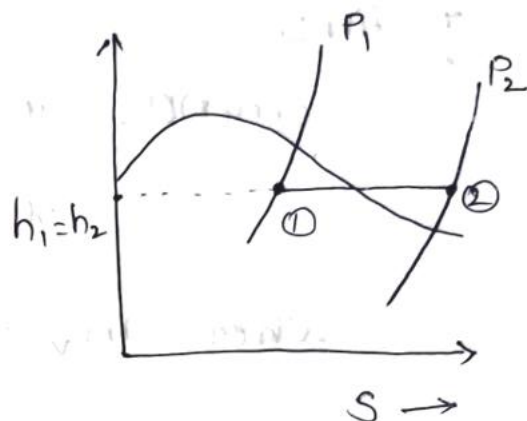
- * The water is collected in the bottom portion of inner chamber which is measured with the help of graduated scale showing the quantity of water separated.
 - * The dry steam moves upwards and enters the an empty space and goes down ward to bucket calorimeter. The amount of steam condensed in this can be weighted at different intervals of time.
- ∴ dryness can be calculated as

$$x = \frac{m_v}{m_v + m_w}$$

where m_v = mass of dry steam condensed in bucket calorimeter.



- * In this Calorimeter basic principle used is throttling process i.e. iso enthalpy process i.e. $h_1 = h_2$
- * The steam is supplied to the sampling tube when initial pressure of steam can be measured with the help of pressure gauge ①.
- * This steam is supplied to throttle valve in which enthalpy is constant.
- * With the help of thermometer and manometer final condition of steam i.e. P_2 & T_2 are measured



- * From the H-S diagram for the Calorimeter is drawn
 $\therefore h_1 = h_2$

$$h_{f,1} + x_1 h_{fg,1} = h_2$$

(at P_1 bar) (at P_2 bar & T_2)

- * With the help of above Equation initial condition of dryness fraction can be calculated

Problem:-

1. Throttling Calorimeter has steam entering pressure at 10 MPa and steam is coming out of Calorimeter at 0.05 MPa and 100°C determine the dryness fraction of the steam.

Sol:-

Given $P_1 = 10 \text{ MPa}$
 $= 10 \times 10^3 \text{ kPa}$
 $= 100 \text{ bar}$

$P_2 = 0.05 \text{ MPa}$
 $= 0.5 \text{ bar}$
 $T_2 = 100^\circ\text{C}$

From Steam tables

at $P_1 = 100 \text{ bar}$

$$h_{f,1} = 1408.1$$

$$h_{fg,1} = 1319.7$$

$$h_1 = h_{f,1} + x_1 h_{fg,1}$$

From Steam tables
at $P_2 = 0.5 \text{ bar}$ $T_2 = 100^\circ\text{C}$
 $h_2 = 2682.6 \text{ kJ/kg}$

we know that

$$h_1 = h_2$$

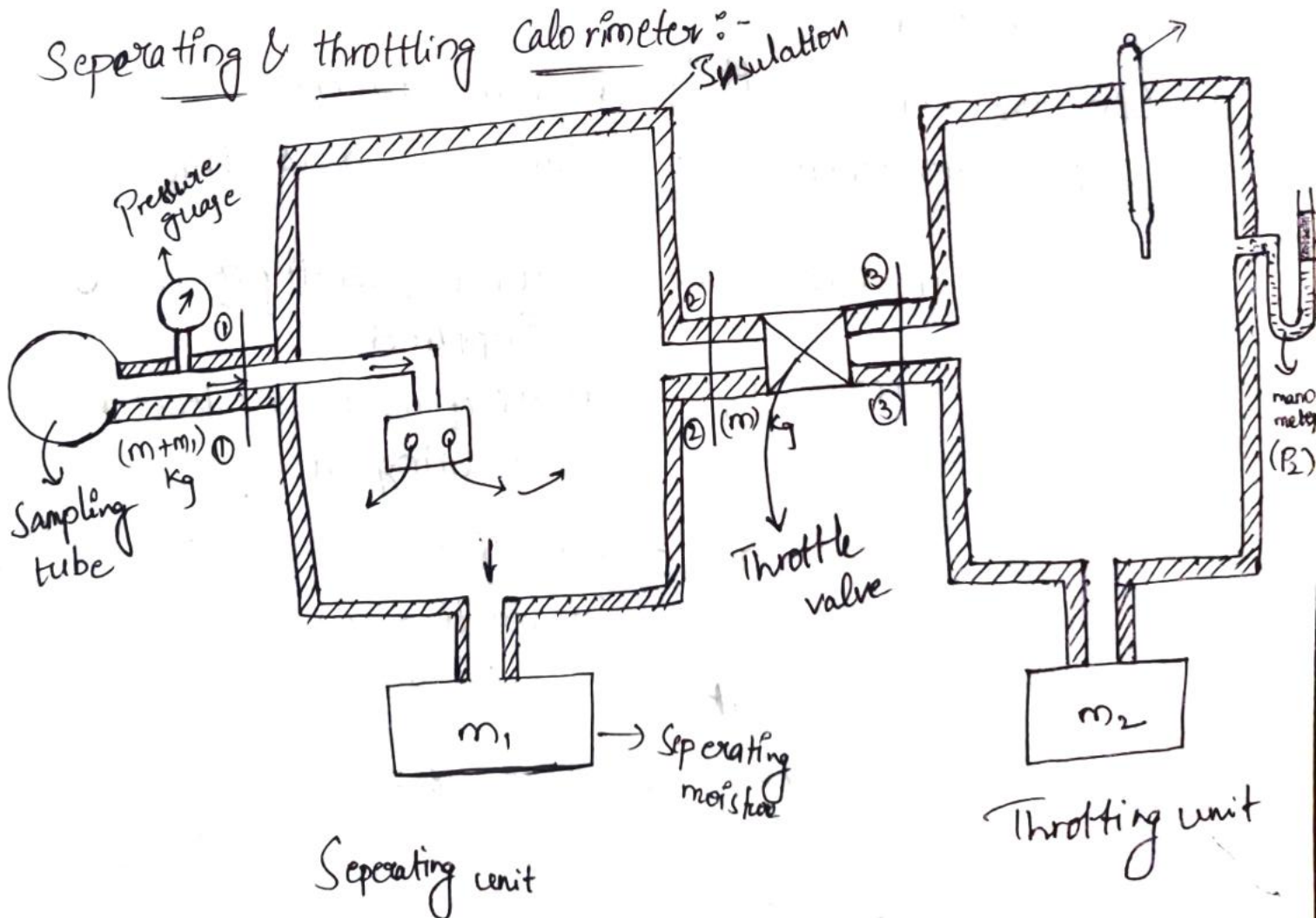
$$h_{f,1} + x_1 h_{fg,1} = 2682.6$$

$$1408.1 + x_1 (1319.7) = 2682.6$$

$$x_1 (1319.7) = 1274.5$$

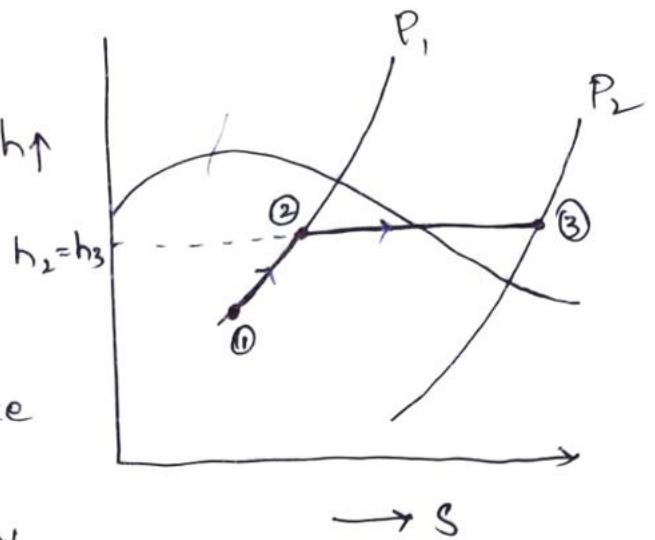
$$\text{dryness fraction } x_1 = \underline{0.9657}$$

Separating & throttling Calorimeter :-



Working:-

- * When the type of steam is very wet combined separating and throttling calorimeter is used.
- * It consists of two units
- * In separating unit moisture is separated by change in direction of flow ^{due to} gravity
- * In throttling unit steam passes through throttle valve at constant enthalpy
- * Quantities of water collected from both the units are measured.
- * Pressure P_1 and P_2 and also Temperature are also noted.
- * In the above $h-s$ diagram process 1-2 is moisture separation from wet steam & 2-3 is throttling process



Let $m + m_1$ kg - total quantity of wet steam supplied

m_1 kg - mass of water collected in separating unit.

m_2 kg - mass of water collected in the throttling unit.

We know that

$$\text{dryness fraction } x = \frac{\text{mass of steam}}{\text{Total quantity}}$$

In separating unit

$$\text{dryness fraction } x_1 = \frac{m}{m + m_1}$$

In throttling unit $x_2 = \frac{m - m_2}{m}$

For the Entire unit final

dryness fraction of Steam $\cdot x = \frac{m - m_2}{m + m_1}$

$$x = \frac{m}{m + m_1} - \frac{m_2}{m + m_1}$$

$$x = \frac{m}{m + m_1} \left[1 - \frac{m_2}{m + m_1} \times \frac{m + m_1}{m} \right]$$

$$= \frac{m}{m + m_1} \left[\frac{m(m + m_1) - m_2(m + m_1)}{m(m + m_1)} \right]$$

$$= \frac{m}{m + m_1} \left[\frac{(m - m_2)(m + m_1)}{m(m + m_1)} \right]$$

$$= \frac{m}{m + m_1} \left[\frac{m - m_2}{m} \right]$$

$$\boxed{x = x_1 \times x_2}$$

Problem :-

1. In a Separating throttling Calorimeter the pressure of steam before throttling is 10 bar the pressure and temp. of steam after throttling is 1.1 bar and 110°C. At the Separator 0.6 kg of water is ~~tapped~~ and 3.4 kg of Condensed water is collected from the Condenser. Determine the dryness fraction of Steam in the main pipe line Take Specific heat at Const pressure is 2.1 kJ/kgK

Sol.:-

Given In Separating unit
 $m = \text{3.4}$

$$m_1 = 0.6$$

$$m + m_1 = 3.4 + 0.6 = 4$$

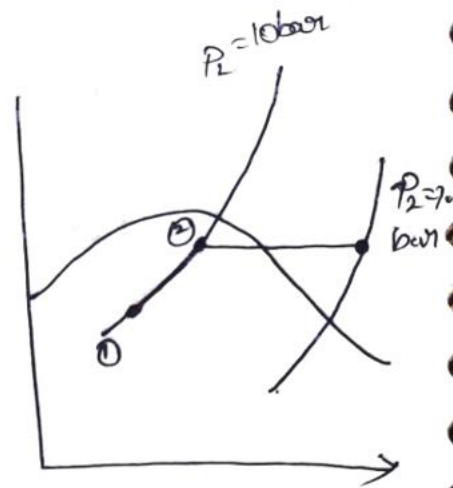
$$P_1 = 10 \text{ bar}$$

$$P_2 = 1.1 \text{ bar}$$

$$T_2 = 110^\circ\text{C}$$

In Separating unit

$$\begin{aligned} x_1 &= \frac{m}{m+m_1} \\ &= \frac{3.4}{3.4+0.6} \\ &= \frac{3.4}{4} = 0.85 \end{aligned}$$



$P_2 = 10 \text{ bar}$
From Steam tables

$$h_{f2} = 762.6$$

$$h_{fg2} = 2013.6$$

$$h_2 = h_{f2} + x_2 h_{fg2}$$

$$h_3 = 762.6 + x_2 (2013.6)$$

$$x_2 = \frac{2695.77 - 762.6}{2013.6}$$

$$x_2 = 0.96$$

$$0.85 \times 0.96 = 0.816$$

$$\begin{aligned} \text{Then } x &= x_1 \times x_2 = 0.85 \times 0.96 \\ &= 0.81 \end{aligned}$$

* The Capacity of pressure Cooker is 10 Liters, initially it Contains 1.5 liters of water and $8.5 \times 10^{-3} \text{ m}^3$ of steam In Equilibrium at 1 bar, heat is Supplied and the relief valve opens only when the pressure exceeds 5 bar. Calculate the heat to be Supplied to open the relief valve.

Sol:- Given volume of water = 1.5 lit = $1.5 \times 10^{-3} \text{ m}^3$

$$\text{Volume of steam} = 8.5 \times 10^{-3} \text{ m}^3$$

At 1 bar

$$v_f = 0.001043 \quad v_g = 1.6937 \text{ m}^3/\text{kg}$$

$$\text{mass of water } (m_w) = \frac{V_w}{v_f} = \frac{1.5 \times 10^{-3}}{0.001043} = 1.438 \text{ kg}$$

$$\text{mass of Steam } (m_s) = \frac{V_s}{v_g} = \frac{8.5 \times 10^{-3}}{1.6937} = 5.018 \times 10^{-3} \text{ kg}$$

$$\text{Specific volume of mixture} = \frac{\text{Total Vol}}{m_w + m_s} = \frac{10 \times 10^{-3}}{1.438 + 5.018 \times 10^{-3}}$$

At 1 bar

$$v_1 = 6.929 \times 10^{-3} \text{ m}^3/\text{kg}$$

initial internal Energy of steam and water in cooker

$$U_1 = m_w u_{w1} + m_s u_{s1}$$

$$= m_w (h_{f1}) + m_s (h_g - P v_g)$$

$$= 1.438 (417.5) + 5.018 \times 10^{-3} (2675.4 - (100(1.6937)))$$

$$U_1 = 612.94 \text{ kJ}$$

At 5 bar

In pressure cooker, volume is Constant

$$v_2 = v_1$$

$$v_{f2} + x_2 v_{fg2} = 6.929 \times 10^{-3}$$

$$0.001093 + x_2 (0.373567) = 6.929 \times 10^{-3}$$

$$x_2 = 0.0156$$

$$v_{fg} = v_g - v_f$$

from steam tables

$$v_{fg} = 0.37466 - 0.001093 = 0.373567$$

$$\text{mass of vapour } (m_{s2}) = \text{Total mass} \times x_2$$

$$= (1.438 + 5.018 \times 10^{-3}) \times 0.0156$$

$$= 0.0225 \text{ kg}$$

$$\text{mass of water } (m_{w_2}) = 1.438 - 0.0225 \\ = 1.415 \text{ kg}$$

At 5 bar

$$\begin{aligned} u_2 &= m_{w_2} u_{w_2} + m_{s_2} u_{s_2} \\ &= m_{w_2} h_{f_2} + m_{s_2} (h_{g_2} - P v_{g_2}) \\ &= 1.415 (640.1) + 0.0225 (2747.5 - 500 \times 0.37466) \\ &= 963.34 \end{aligned}$$

Heat Supplied to open relief valve

$$\begin{aligned} Q &= u_2 - u_1 \\ &= 963.65 - 612.74 \\ &= 350.405 \text{ kJ} \end{aligned}$$

* Following observations are recommended in a equipment with a Combined Separating throttling Calorimeter. Pressure in steam ^{main} ~~line~~ is 15 bar mass of water d from the Separation is 0.55 kg mass of steam. Condensed after passing through throttling valve is 4.2 kg. Pressure and temp at the throttling are 1 bar & 120°C Evaluate dryness fraction.

Given

$$P_2 = P_1 = 15 \text{ bar}$$

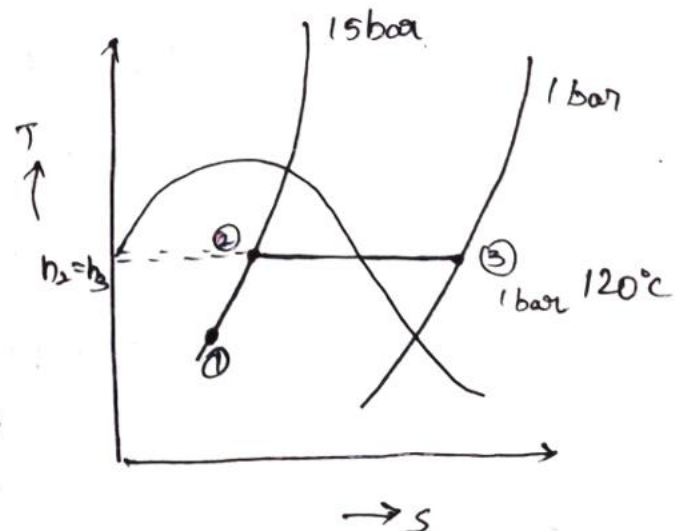
$$P_3 = 1 \text{ bar} \quad T_3 = 120^\circ \text{C}$$

At $P_2 = 15 \text{ bar}$

From steam tables.

$$h_{f_2} = 844.6 \quad h_{fg_2} = 1945.3$$

$$h_2 = h_{f_2} + x_2 h_{fg_2}$$



At 1 bar 120°C

From steam tables

$$P_3 = 1 \text{ bar} \quad T_p = T_3 = 120^\circ\text{C} = 120 + 273$$

$$h_g = 2675.4 \quad \text{kJ/kg} \quad = 393 \text{ K}$$

$$T_3 = 99.63^\circ\text{C} = 372.63 \text{ K}$$

$$h_3 = h_g + C_{ps} (T_{\text{sup}} - T_3)$$

$$= 2675.4 + 2.1 (393 - 372.63)$$

$$h_3 = 2718.177 \text{ kJ/kg}$$

From T-s diagram

$$h_2 = h_3$$

$$844.6 + x_2 (1945.3) = 2718.177$$

$$x_2 (1945.3) = 1873.577$$

$$x_2 = 0.963130$$

Given

$$m = 4.2 \text{ kg}$$

$$m_1 = 0.55 \text{ kg}$$

In separating unit

$$x_1 = \frac{m}{m+m_1} = \frac{4.2}{4.2+0.55} = 0.8842$$

dryness fraction $x = x_1(x_2)$

$$= 0.963130 (0.8842)$$

$$= 0.851599$$

Alternative method :-

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

A/k to interpolation method

$$y_3 = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x_3 - x_1)$$

Perfect Gas laws

Perfect gas (or) Ideal gas:-

Gases which don't change their phases during thermodynamic process and Obey a set of ^{common} * rules governing change of their properties are known as ideal gases.

* If the working fluid changes its phase during a thermodynamic process then it is called as vapour.

Equation of state of perfect gas:-

The relationship b/w three independent properties like pressure, volume and temp of a pure substance is known as Equation of state of perfect gas.

$$\text{i.e. } f(P, V, T) = 0$$

$$\text{or } P = f(V, T)$$

when ever there is change in pressure & Temperature there may be a change in volume. There exist a condition for all real gases at which they behave in similar manner.

$$PV = (RT)_{P \rightarrow 0} \quad \text{This gives the real gases}$$

(or)

$$P = \frac{RT}{V}$$

i.e. at very low pressures all the real gases behaves like an ideal gas and this behaviour is

Called ideal gas behaviour. This state of zero pressure is called ideal state.

For an ideal gas following conditions are applicable.

* $PV = RT$

* $U = f(T)$ alone

NOTE:-

There is no perfect gas. Some gases like H_2 , air, N_2 , CO_2 , O_2 which obeys the gas laws at normal pressure and temperatures. are considered as ideal gases.

Gas laws:-

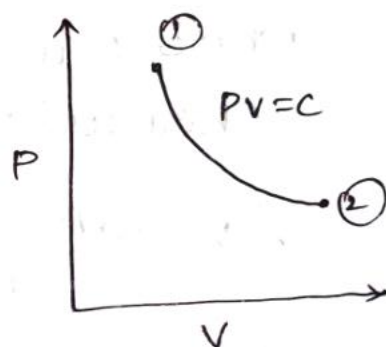
Boyle's law:-

It states that the volume of a given mass of perfect gas is inversely proportional to absolute pressure at constant temperature.

$$V \propto \frac{1}{P}$$

$$PV = C$$

$$\boxed{P_1 V_1 = P_2 V_2}$$



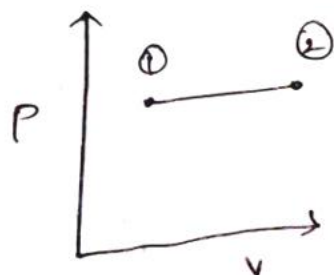
Charles's law:-

It states that the ~~gas which is~~ ^{volume} directly ~~proportional~~ with the absolute temperature if the ^{varies} pressure is constant.

$$V \propto T$$

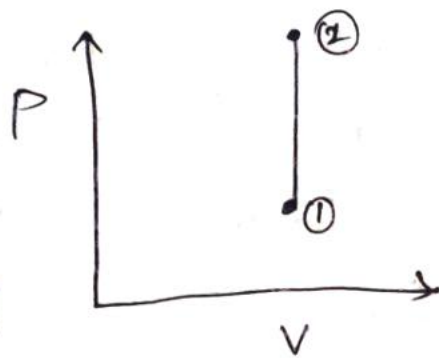
$$\frac{V}{T} = C$$

$$\boxed{\frac{V_1}{T_1} = \frac{V_2}{T_2}}$$



Process (1-2) $\rightarrow$ constant pressure heating process

Charles law state that
the pressure of given mass
of gas ~~where~~ varies directly
with ~~the~~ its absolute temp
if the volume is Const



$$P \propto T$$

$$\frac{P}{T} = C$$

$$\boxed{\frac{P_1}{T_1} = \frac{P_2}{T_2}}$$

Characteristic Equation of gas:-

By Combining Boyle's law and Charles law the relationship b/w pressure, temp & volume may be in the form of

$$\frac{PV}{T} = C = R$$

where R is the

Characteristic gas constant

$$\boxed{\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}}$$

Avagadro's law:-

It states that the volume of 1 kg^{mole} of all known ~~at~~ gases at normal pressure and temperature is same. ($V = 22.4 \text{ m}^3$)

$$\therefore R = \frac{\bar{R}}{M} \quad n = \frac{m}{M}$$

$$PV = mRT$$

(universal gas constant)

where

$n = \text{no. of molecules}$

$$PV = n\bar{R}T$$

is common for all the gases and it is constant value i.e. $\bar{R} = 8.314 \text{ kJ/kg K}$

$$P = 1.01325 \times 10^5 \text{ N/m}^2$$

$$\bar{R} = \frac{PV}{nT}$$

$$V = 22.4 \text{ m}^3$$

$$\bar{R} = 8.314 \text{ kJ/kg K}$$

$$T = 273 \text{ K}$$

$$n = 1$$

$M = \text{molecular weight.}$

For air $\bar{R} = 0.287 \text{ kJ/kg}\cdot\text{K}$

$\therefore$ universal gas constant $\bar{R} = \frac{\text{characteristic gas constant} \times \text{molecular weight}}{\text{molecular weight}}$

$$\bar{R} = R \times m.$$

Specific heat :- It is defined as the quantity of heat required to raise the temp of unit mass of gas by one degree.

Specific $C_{pw} = 4.2 \text{ kJ/kg}\cdot\text{K}$
heat of water

(steam) $C_{ps} = 2.1 \text{ kJ/kg}\cdot\text{K}$

$C_{pair} = 1.005 \text{ kJ/kg}\cdot\text{K}$

The ^{heat} observing capacity of water is much higher than air.

i.e. 1 kg of water requires 4.2 kJ of heat to raise its temperature by one degree.

and where as for air 1 kJ of heat is sufficient.

* heating is done in two ways.

i. Constant volume

ii. Constant pressure.

$$Q = \Delta U + W$$

$$m C_p \Delta T = m C_v \Delta T + P(V_2 - V_1)$$

$$m C_p (T_2 - T_1) = m C_v (T_2 - T_1) + m R (T_2 - T_1)$$

$$C_p = C_v + R$$

$$\boxed{C_p - C_v = R} \rightarrow (1)$$

divide with C_v

$$\frac{C_p}{C_v} - \frac{C_v}{C_v} = \frac{R}{C_v}$$

$$\frac{C_p}{C_v} - 1 = \frac{R}{C_v} \Rightarrow \frac{C_p}{C_v} = \frac{R}{C_v} + 1 \Rightarrow C_v = \frac{R}{\gamma - 1}$$

Substitute $E_2(2)$ in $E_2(1)$

$$C_p - \frac{R}{\gamma - 1} = R$$

$$C_p = R + \frac{R}{\gamma - 1}$$

$$= \frac{R(\gamma - 1) + R}{\gamma - 1}$$

$$= \frac{R\gamma - R + R}{\gamma - 1} \Rightarrow C_p = R \left(\frac{\gamma}{\gamma - 1} \right)$$

$$C_p - \frac{R}{\gamma - 1} = R \Rightarrow C_p = R \left(\frac{\gamma}{\gamma - 1} \right)$$

$$C_p = \gamma \left(\frac{R}{\gamma - 1} \right)$$

$$C_p = \gamma (C_v)$$

$$\boxed{\frac{C_p}{C_v} = \gamma}$$

Problems:-

- Calculate the gas Constant for oxygen, nitrogen, carbon dioxide, carbon monoxide and air where

$$\bar{R} = 8.314 \text{ kJ/kg-K}$$

Sol:- Given

$$\text{molecular wt } O_2 \rightarrow 32$$

$$N_2 \rightarrow 28$$

$$CO_2 \rightarrow 44$$

$$CO \rightarrow 28$$

$$\text{air} \rightarrow 28.9647$$

$$O_2 \rightarrow R = \frac{8.314}{32}$$

$$N_2 \rightarrow R = \frac{8.314}{28}$$

$$CO_2 \rightarrow R = \frac{8.314}{44}$$

$$CO \rightarrow R = \frac{8.314}{28}$$

$$\bar{R} = R \times m$$

$$R = \frac{\bar{R}}{m}$$

$$\text{air} \rightarrow R = \frac{8.314}{28.9647}$$

$$R = \frac{8.314}{28.9647}$$

$$R = 0.287 \text{ kJ/kg-K}$$

$$R_{\text{air}} = \frac{8.314}{28.9647}$$

- Find the molecular weight and gas Constant for the gas whose specific heat as follows.

$$C_p = 1.967 \text{ kJ/kg-K}$$

$$C_v = 1.507 \text{ kJ/kg-K}$$

Sol:-

Given

$$C_p = 1.967 \text{ kJ/kg-K}$$

$$C_v = 1.507 \text{ kJ/kg-K}$$

$$C_p - C_v = R$$

$$R = 0.46 \text{ kJ/kg-K}$$

$$R = \frac{\bar{R}}{m} \Rightarrow m = \frac{\bar{R}}{R} = \frac{8.314}{0.46}$$

$$= 18.07 \text{ kg}$$

3. One kg of ideal gas is heated from 18.3°C to 93.4°C
 assume $R = 287 \text{ J/kgK}$ and $\gamma = 1.18$ for the
 gas calculate i. Specific heats
 ii. Change in internal energy.

Sol: Given $R = 287 \text{ J/kgK} = 0.287 \text{ kJ/kgK}$
 $T_1 = 18.3^{\circ}\text{C} = 291.3 \text{ K}$
 $T_2 = 93.4^{\circ}\text{C} = 366.4 \text{ K}$

$$C_v = \frac{R}{\gamma - 1} = \frac{0.287}{1.18 - 1} = 1.5944 \text{ kJ/kgK}$$

$$C_p - C_v = R$$

$$C_p = R + C_v = 1.8814 \text{ kJ/kgK}$$

Change in internal energy $\Delta U = m C_v \Delta T$
 $= m C_v (T_2 - T_1)$
 $= 1.5944 (366.4 - 291.3)$

$$\Delta U = 119.73944 \text{ kJ}$$

4. Find the molecular weight and gas constant whose
 specific heats are as follows $C_p = 1.826 \text{ kJ/kgK}$
 $C_v = 1.23 \text{ kJ/kgK}$.

Given $C_p = 1.826 \text{ kJ/kgK}$ $\bar{R} = 8.314 \text{ kJ/kgK}$
 $C_v = 1.23 \text{ kJ/kgK}$

$$C_p - C_v = R$$

$$R = 0.596 \text{ kJ/kgK}$$

$$M = \frac{\bar{R}}{R} = 13.94966 \text{ kg} \quad \left| \quad R = \frac{\bar{R}}{M} \right.$$

5. A Spherical Shape balloon of 12 m diameter Contains Hydrogen at 30°C and 1.21 bar find the mass of hydrogen in the balloon.

$$V = \frac{4}{3} \pi r^3$$

Sol:-

Given

$$D = 12 \text{ m} \Rightarrow r = 6 \text{ m}$$

$$P = 1.21 \text{ bar} = 121 \text{ kPa}$$

$$T = 30^\circ\text{C} = 303 \text{ K}$$

$$V = \frac{4\pi}{3} r^3$$

$$= 904.7786 \text{ m}^3$$

$$R = \frac{\bar{R}}{M} = \frac{8.314}{2} = 4.157 \text{ kJ/kg K}$$

M = molecular weight
($M = 2$ for hydrogen)

$$PV = mRT$$

$$\text{mass } m = \frac{PV}{RT} = \frac{121 \times 904.7786}{4.157 \times 303}$$

$$\text{mass of hydrogen } m = 86.917 \text{ kg}$$

6. Determine the volume of 3 kg of air at a pressure of 2.95 bar and temp of 40°C assume $C_p = 1.005 \text{ kJ/kg K}$ and $C_v = 0.718 \text{ kJ/kg K}$.

Sol:-

Given

$$C_p = 1.005 \text{ kJ/kg K}$$

$$C_v = 0.718 \text{ kJ/kg K}$$

$$m = 3 \text{ kg}$$

$$P = 2.95 \text{ bar}$$

$$T = 313 \text{ K}$$

$$R = C_p - C_v$$

$$R = 0.287 \text{ kJ/kg K}$$

$$PV = mRT$$

$$V = \frac{mRT}{P} = \frac{3 \times 0.287 \times 313}{2.95}$$

$$V = 0.9135 \text{ m}^3$$

7. If the ratio's of Specific heat for Oxygen is 1.4 then Calculate C_p and C_v .

Sol:- Given $\frac{C_p}{C_v} = \gamma = 1.4$

For Oxygen $\bar{R} = 8.314 \text{ kJ/kg K}$

$$R = \frac{\bar{R}}{m}$$

$$C_v = \frac{R}{\gamma - 1} = \frac{8.314}{1.4 - 1} = 20.785 \text{ kJ/kg K}$$

$$= \frac{8.314}{3.2} = 0.2598125 \text{ kJ/kg K}$$

$$C_p = R \left(\frac{\gamma}{\gamma - 1} \right) = 0.90934 \text{ kJ/kg K}$$

8. 1.5 kg of air at 6 bar & volume of 0.2 m^3 . If air is expanded to a volume of 1.1 m^3 . Calculate the work done & heat transfer by the air.

i. If the process is isothermal

ii. If the process is adiabatic.

Sol:-

Given $m = 1.5 \text{ kg}$

$P_1 = 6 \text{ bar} = 600 \text{ kPa}$

$V_1 = 0.2 \text{ m}^3$

$V_2 = 1.1 \text{ m}^3$

i. If the process is Isothermal.

$$\text{work done (W.D)} = P_1 V_1 \ln \left(\frac{V_2}{V_1} \right)$$

$$= 600 \times 0.2 \ln \left(\frac{1.1}{0.2} \right)$$

$$= 204.56977 \text{ kJ}$$

Heat transfer $Q = \Delta u + W$ in isothermal

$$Q = W \quad \Delta u = 0$$

$Q = 204.56977 \text{ kJ}$

ii. If the process is adiabatic

$$W.D = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

$$= 148.3045 \text{ kJ}$$

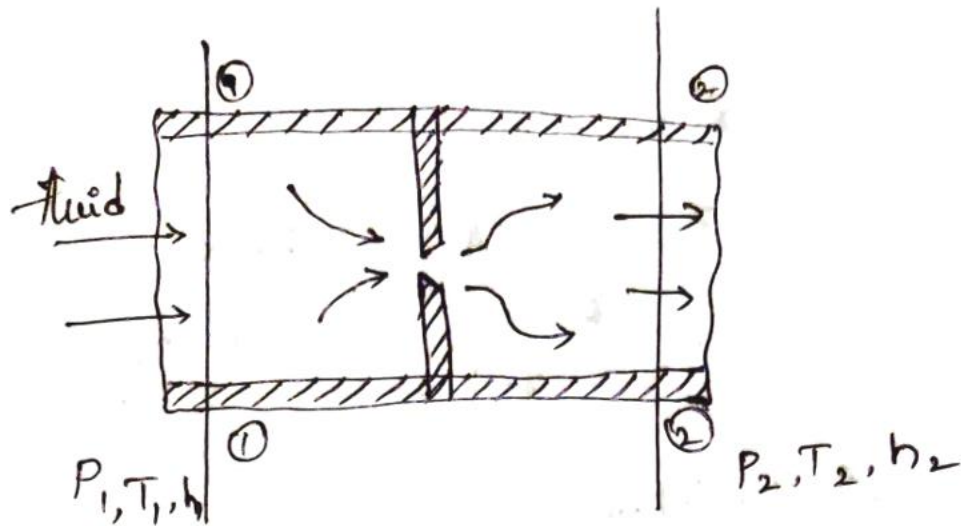
Heat transfer $Q = 0$ (adiabatic process)

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma$$

$$P_2 = 55.162 \text{ kPa}$$

Throttling process :-



It is an irreversible process in which the pressure of the fluid is reduced by keeping an obstruction in its flow path without any change in enthalpy under adiabatic conditions.

A fluid at P_1, T_1 is made to flow through a throttle valve which is thermally insulated and fluid comes out with a pressure $P_2 < P_1$.

By applying Steady Flow Energy Equation (SFEE)

$$m \left(h_1 + \frac{V_1^2}{2g} + Z_1 \right) + Q_1 = m \left(h_2 + \frac{V_2^2}{2g} + Z_2 \right) + W$$

$$h_1 = h_2$$

$Q = 0$ $\rightarrow$ since it is insulated

$W = 0$ $\rightarrow$ system doesn't exchange work from the surroundings

neglect the kinetic and potential energy

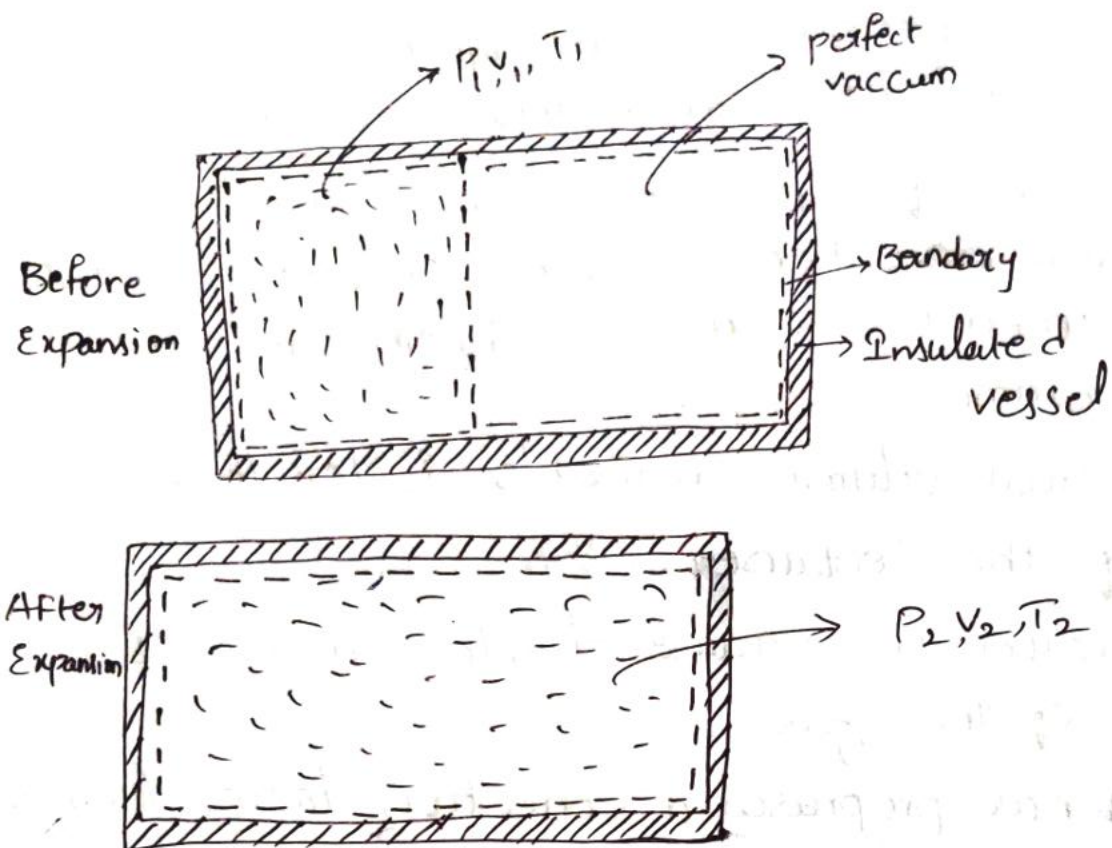
$$\therefore h_1 = h_2$$

Thus Enthalpy before throttling = Enthalpy after throttling.

NOTE:-

For an ideal gas temp remains const during throttling.
For a real gas temp is not const during throttling.

Free Expansion process:-



- * It is a non flow irreversible process and thermodynamic process.
- * It occurs when a fluid is allowed to expand suddenly into a vacuum chamber through an orifice of large dimension ^{area}.
- * when ~~the~~ gas is made to expand into vacuum which offers no restriction for its expansion is known as free expansion process. It is also known as Constant internal energy process.
- * Even after expansion there is no change in the boundary this is because of rigid vessel which means no work being done.
- * For free expansion process:- $Q=0$; $W=0$; pressure, temp decreases; volume increases; $\Delta u=0$.

Dutch physicist Johannes van der Waals (1837-1923)

* Vanderwaals's Equation of State (for real gas):-

- * The real gases has intermolecular attractive forces and hence the pressure exerted by molecules on the Container walls is less than the actual pressure.
- * Further if the no. of molecules in the Container occupies same volume and hence each molecule will not have a freedom to move through out the Container volume.
- * So The actual volume available is less than the volume of the Container.
- * This reduction in volume further increases the pressure of the gas.
- * Vanderwaals has proposed a Correction factor for both pressure and volume.

The Equation is stated as

$$\left[P + \frac{a}{V^2} \right] (V - b) = RT \rightarrow (1)$$

Where a, b are Constants which can determine Experimentally.

and V = Specific volume in $\text{m}^3/\text{kg mole}$

As the pressure approaches zero and Specific volume approaches infinity, $\frac{a}{V^2}$ and b are negligible and the Equation reduces to $PV = RT$

- * That is the reason why all real gases behave like ideal gas as pressure approaches to zero or volume is infinitely long such that the volume occupied by molecules is relatively negligible.
- * The vanderwall's constants a , and b can be completed in terms of its properties corresponding to the critical points by taking first and second

derivatives of pressure with respect to volume and equating them to zero.

Vanderwall's Equation at Critical State,

$$P_c = \frac{RT_c}{(V_c - b)} - \frac{a}{V_c^2} \rightarrow (2)$$

Where P_c , V_c & T_c are values at critical state.

$$\text{At critical state, } \left(\frac{\partial P}{\partial V}\right)_{T_c} = 0, \left(\frac{\partial^2 P}{\partial V^2}\right)_{T_c} = 0$$

Differentiate Eq (2)

$$-\frac{RT_c}{(V_c - b)^2} + \frac{2a}{V_c^3} = 0 \rightarrow (3)$$

$$-RT_c = -\frac{2a}{V_c^3} (V_c - b)^2$$

$$RT_c = \frac{2a}{V_c^3} (V_c - b)^2 \rightarrow (4)$$

Again differentiate Eq (3)

$$\frac{2RT_c}{(V_c - b)^3} - \frac{6a}{V_c^4} = 0 \rightarrow (5)$$

$$RT_c = \frac{6a}{V_c^4} \times \frac{(V_c - b)^3}{2} \rightarrow (6)$$

Equating Eq (4) & (6)

$$\frac{2a}{V_c^3} (V_c - b)^2 = \frac{6a}{V_c^4} \times \frac{(V_c - b)^3}{2}$$

$$1 = \frac{3(V_c - b)}{2V_c}$$

$$3(V_c - b) = 2V_c$$

$$\boxed{V_c = 3b} \rightarrow (7)$$

Substitute Eq (7) in Eq (6)

$$RT_c = \frac{6a}{(3b)^4} \times \frac{(3b - b)^3}{2} = \frac{8a}{27b}$$

$$\boxed{T_c = \frac{8a}{27Rb}}$$

$$T_c = \frac{8a}{27Rb} \rightarrow (8)$$

Now substitute ϵ_2 (7) & (8) in ϵ_2 (2)

$$\begin{aligned} P_c &= \frac{RT_c}{(V_c - b)} - \frac{a}{V_c^2} = \frac{R \left(\frac{8a}{27Rb} \right)}{(3b - b)} - \frac{a}{(3b)^2} \\ &= \frac{8a/27b}{2b} - \frac{a}{9b^2} \\ &= \frac{4a}{27b^2} - \frac{a}{9b^2} = \frac{4a - 3a}{27b^2} = \frac{a}{27b^2} \end{aligned}$$

$$P_c = \frac{a}{27b^2} \rightarrow (9)$$

divide ϵ_2 (8) with (9)

$$\frac{T_c}{P_c} = \frac{8a}{27Rb} \times \frac{27b^2}{a} = \frac{8b}{R}$$

$$\frac{T_c}{P_c} = \frac{8b}{R}$$

$$b = \frac{RT_c}{8P_c} \rightarrow (10)$$

Substitute ϵ_2 (10) in (8).

$$T_c = \frac{8a}{27R \left(\frac{RT_c}{8P_c} \right)} = \frac{64aP_c}{27T_c R}$$

$$a = \frac{27R^2T_c^2}{64P_c} \rightarrow (11)$$

Compressibility factor (chart) :-

It is the ratio of actual volume of the gas to the volume predicted by the ideal gas equation at same temp and pressure. It is denoted by 'Z'.

$$Z = \frac{V_{\text{actual}}}{V_{\text{ideal}}}$$

$$Z = \frac{PV}{RT} = \frac{V}{RT/P}$$

If $Z = 1$ for ideal gas.

If $Z \neq 1$ is real gas.

$\therefore$ Compressibility factor for real gases at critical point is

$$Z_c = \frac{P_c V_c}{RT_c}$$

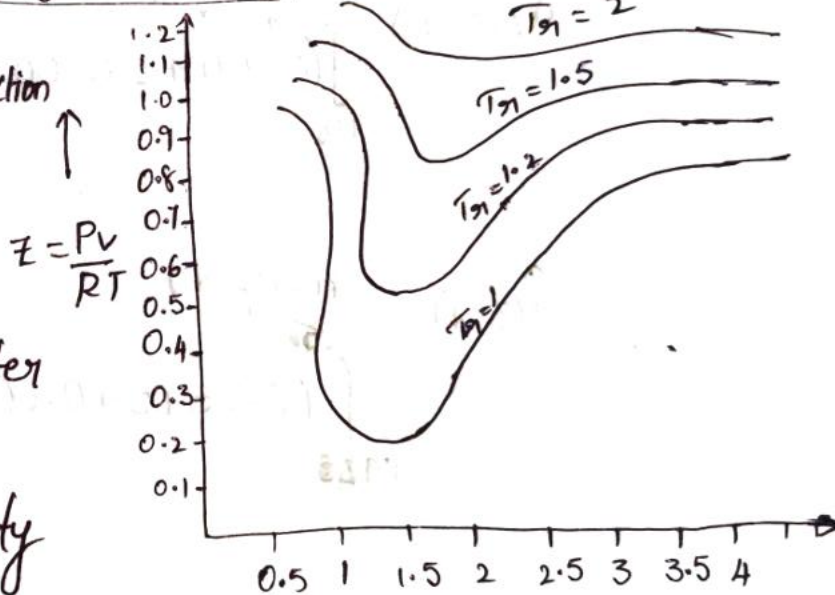
$$P_c = \frac{a}{27b^2}, \quad V_c = 3b, \quad T_c = \frac{8a}{27Rb}$$

$$\therefore Z_c = \frac{\frac{a}{27b^2} \times 3b}{R \frac{8a}{27Rb}} = \frac{3}{8}$$

$$Z_c = \frac{3}{8} = 0.375$$

(critical Compressibility factor)

* If Z is considered as function of $f(P_r, T_r)$ and Z is plotted against reduced pressure with reduced temperature as a parameter that chart is known as generalized Compressibility Chart.



Reduced pressure $P_r = \frac{P}{P_c} \Rightarrow P = P_r P_c$ Generalized Compressibility Chart

Reduced temp. $T_r = \frac{T}{T_c} \Rightarrow T = T_r T_c$

Reduced volume $V_r = \frac{V}{V_c} \Rightarrow V = V_r V_c$

$$Z = \frac{PV}{RT} = \frac{P_r P_c V_r V_c}{R T_r T_c} = \left(\frac{P_r V_r}{T_r} \right) \left(\frac{P_c V_c}{R T_c} \right)$$

reduced Compressibility factor $Z_r = \frac{3}{8} \frac{P_r V_r}{T_r}$

Problems:-

1. The temperature at the 5 Saliient points of a dual Combustion Cycle are 363k, 988k, 1294 k, 2135 k, 950k find thermal Efficiency of the Cycle. Assume $C_p = 0.2395 + 0.00003T$ and $C_v = 0.171 + 0.00003T$

Given

$$T_1 = 363 \text{ K}$$

$$T_2 = 988 \text{ K}$$

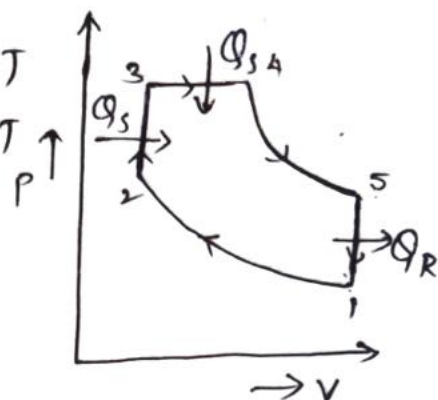
$$T_3 = 1294 \text{ K}$$

$$T_4 = 2135 \text{ K}$$

$$T_5 = 950 \text{ K}$$

$$C_p = 0.2395 + 0.00003T$$

$$C_v = 0.171 + 0.00003T$$



Assume $m = 1 \text{ kg}$

$$\text{Efficiency} = \frac{Q_s - Q_R}{Q_s}$$

$$Q_{s(2-3)} = m C_v dT$$

$$= \int_{988}^{1294} (0.171 + 0.00003T) dT = \left[0.171T + 0.00003 \frac{T^2}{2} \right]_{988}^{1294}$$
$$= 246.39054 - 183.59016$$
$$= 62.8 \text{ kJ}$$

$$Q_{s(3-4)} = m C_p dT$$

$$= \int_{1294}^{2135} (0.2395 + 0.00003T) dT = \left[0.2395T + 0.00003 \frac{T^2}{2} \right]_{1294}^{2135}$$
$$= 579.705875 - 335.02954$$
$$= 244.67633 \text{ kJ}$$

$$Q_R(5-1) = m C_v dT$$

$$= \int_{950}^{363} (0.171 + 0.00003T) dT$$

$$950$$

$$= \left[0.171T + 0.00003 \frac{T^2}{2} \right]_{950}^{363}$$

$$= 64.049535 - 175.9875$$

$$Q_R(5-1) = -111.93797 \text{ kJ}$$

$$\text{Efficiency} = \frac{Q_s - Q_R}{Q_s}$$

$$Q_5 = Q_5(2-3) + Q(3-4) = 62.8 + 244.67633 = 307.4$$

$$\eta = \frac{Q_5 - Q_R}{Q_5} >$$

2. The Specific volume of hydrogen at 90°C is $0.9 \text{ m}^3/\text{kg}$
- Determine the pressure Exerted by H_2 using vander wall's Equation
 - Compare the results obtained Considering hydrogen as an ideal gas Take $a = 25015 \text{ N-m}^4/(\text{kg mole})^2$
 $b = 0.0262 \text{ m}^3/\text{kg mole}$.

Sol! Given $a = 25015 \text{ N-m}^4/(\text{kg-mole})^2$ $a = 25.015 \text{ kN-m}^4/(\text{kg-mole})^2$
 $b = 0.0262 \text{ m}^3/\text{kg mole}$
 $V = 0.9 \text{ m}^3/\text{kg}$
 $T = 90^\circ\text{C} = 363 \text{ K}$

(i) Vanderwall Equation

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

$$R = \frac{\bar{R}}{M} = \frac{8.314}{2} = 4.157$$

$$P = \frac{RT}{V - b} - \frac{a}{V^2}$$

$$= \frac{4.157(363)}{(0.9) - 0.0262} - \frac{25.015}{(0.9)^2}$$

$$= 29155.7865$$

$$= 1696.0467 \text{ kPa}$$

$$= 1.69604 \text{ MPa}$$

(ii)

$$PV = RT$$

$$P = \frac{RT}{V}$$

$$= \frac{4.157(363)}{(0.9)}$$

$$= 1676.6567 \text{ kPa}$$

$$= 1.6766 \text{ MPa}$$

3. 1 kg of CO_2 has a volume of 0.003 m^3 and a pressure of 100 atm. Compute the temp. By perfect gas Equation (ii) By vanderwall Equation.

Sol! Given $m = 1 \text{ kg}$ $V = 0.003 \text{ m}^3$
 $P = 100 \text{ atm}$

$$\text{CO}_2$$

$$M = 44$$

$$R = \frac{\bar{R}}{M} = \frac{8.314}{44} = 0.18895$$

$$100 \text{ atm} = 101.325 \text{ bar}$$

$$= 10132.5 \text{ kPa}$$

$$1 \text{ atm} = 1.01325 \text{ bar}$$

By Perfect gas Equation

$$PV = RT$$

$$T = \frac{PV}{R} = \frac{10132.5 \times 0.003}{0.18895}$$

$$= 160.875 \text{ K}$$

By van der Waal's Equation

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT$$

$$b = \frac{RT_c}{8P_c} = \frac{0.18895(160.875)}{8(10132.5)} = 0.000375 \text{ m}^3/\text{kg.mole}$$

$$a = \frac{27R^2T_c^2}{64P_c} = \frac{27(0.18895)^2(160.875)^2}{64(10132.5)} = 0.0385 \text{ kNm}^4/\text{kg.mole}^2$$

$$T = \frac{\left(P + \frac{a}{v^2}\right)(v - b)}{R} = \frac{\left[10132.5 + \frac{0.0385}{(0.003)^2}\right](0.003 - 0.000375)}{0.18895}$$

$$= 200.19 \text{ K}$$

4. Determine the Compressibility factor for oxygen

i. At 100 bar and -70°C

ii. At 50 bar & 30°C use $T_c = -118.12^\circ\text{C}$ and

$$P_c = 50.6 \text{ bar}$$

Sol:

$$P = 100 \text{ bar}$$

$$T = -70^\circ\text{C}$$

$$P_{r1} = \frac{P}{P_c} = \frac{100}{50.6} = 1.97628$$

$$T_{r1} = \frac{-70}{-118.12} = 0.59267$$

$$P = 50 \text{ bar}$$

$$T = 30^\circ\text{C}$$

$$P_{r1} = \frac{50}{50.6}$$

$$T_{r1} = \frac{30}{-118.12}$$

From Compressibility Chart

$$Z = 0.98$$

$$Z = 0.97$$

5. 1 kg mole of nitrogen is contained in vessel of volume 3.5 m^3 at 200°C .
- Evaluate mass, pressure, Specific volume of gas.
 - If the ratios of specific heat is $\frac{C_p}{C_v} = 1.4$
 - Subsequently the gas cools to the atmospheric temp of 35°C . find C_p & C_v ,
 - Evaluate final pressure of gas.
 - Evaluate the increase in specific internal energy, specific enthalpy, entropy and magnitude & sign of heat transfer.

$$N_2 = 28$$

Sol:

Given

$$V_1 = 3.5 \text{ m}^3$$

$$T_1 = 200^\circ\text{C} = 473 \text{ K}$$

$$\frac{C_p}{C_v} = 1.4$$

$$T_2 = 35^\circ\text{C} = 308 \text{ K}$$

$$\text{mass } m = 28 \text{ kg mole}$$

$$R = \frac{\bar{R}}{M} = \frac{8.314}{28} = 0.2969$$

$$P_1 = 1123.57 \text{ kPa}$$

$$P_1 V_1 = m R T_1$$

$$V_1 = \frac{V}{m} = \frac{3.5}{28}$$

$$= 0.125 \text{ m}^3/\text{kg}$$

$$P_1 = \frac{m R T_1}{V_1} = 28 \times \frac{8.314}{28} \times \frac{473}{3.5}$$

$$P_1 = 1123.57 \text{ kPa}$$

(16).

$$C_p - C_v = R$$

dividing C_v on both sides

$$\frac{C_p}{C_v} - \frac{C_v}{C_v} = \frac{R}{C_v}$$

$$1.4 - 1 = \frac{0.2969}{C_v} \Rightarrow C_v = \frac{0.2969}{0.4} = 0.74225 \text{ kJ/kgK}$$

$$C_p = \gamma(C_v) = 1.03915 \text{ kJ/kgK}$$

The entire process is Constant volume process

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}$$

$$P_2 = \frac{P_1 T_2}{T_1} = \frac{1123.57 \times 308}{473} \\ = 731.6269 \text{ kPa}$$

iv. Internal Energy $\Delta u = m C_v (T_2 - T_1)$

$$= 28 (0.7422) (308 - 473) \\ = -3428.964 \text{ kJ}$$

Enthalpy $\Delta h = m C_p (T_2 - T_1)$

$$= -4800.18 \text{ kJ}$$

Entropy $\Delta s = m C_v \ln\left(\frac{T_2}{T_1}\right)$

$$= 28 (0.7422) \ln\left(\frac{308}{473}\right) \\ = -8.9152$$

$$Q = \Delta u + W$$

$$Q = -3428.964 \text{ kJ}$$

1. Show that $PV^\gamma = \text{Const}$ for reversible adiabatic process

Sol:-

$$PV = mRT$$

Differentiating on both sides

$$Pdv + vdp = mRdT$$

$$dT = \frac{Pdv + vdp}{mR} \rightarrow \textcircled{1}$$

A/c to first law

$$\delta Q = \delta u + \delta w$$

In adiabatic process heat transfer

$$\delta Q = 0$$

$$\delta w = -\delta u$$

$$\delta W = -m C_v dT$$

$$P dv = -m C_v dT$$

$$P dv + m C_v dT = 0 \rightarrow (2)$$

Sub Eq (1) in Eq (2)

$$P dv + m C_v \frac{P dv + v dP}{m R} = 0$$

$$P dv + C_v \frac{P dv + v dP}{R} = 0$$

$$\frac{P R dv + C_v (P dv + v dP)}{R} = 0$$

$$P R dv + C_v (P dv + v dP) = 0$$

$$C_v (P dv + v dP) + P dv (C_p - C_v) = 0$$

$$\boxed{R = C_p - C_v}$$

$$C_v P dv + C_v v dP + P C_p dv - C_v P dv = 0$$

$$C_v v dP + P dv C_p = 0$$

dividing on both sides by $C_v P v$

$$\frac{C_v v dP + P C_p dv}{C_v P v} = 0$$

$$\frac{dP}{P} + \frac{C_p}{C_v} \frac{dv}{v} = 0$$

$$\frac{dP}{P} + \gamma \frac{dv}{v} = 0$$

Integrated the above Eq

$$\log_e P + \gamma \log_e v = \log C$$

$$\log_e P + \log_e v^\gamma = \log C$$

$$\log_e P v^\gamma = \log C$$

$$\underline{P v^\gamma = C}$$

Deviations from Ideal Gas law behaviour:

The behaviour of real gases usually agree with the predictions of the ideal gas equation to within 5% at normal temp & pressures, ~~and~~ At low temp's or high pressures, real gases deviate significantly from ideal gas behaviour.

Cause of deviation from ideal behaviour:

The cause may be due to the following two assumptions of kinetic theory of gases.

→ The vol. occupied by gas molecules is negligibly small as compared to the vol. occupied by the gas.

→ The forces of attraction b/w gas molecules are negligible.

The first assumption is valid only at low pressure & high temp's, when the vol. occupied by the gas molecule is negligible as compared to the total volume. But at low temp & at high pressure, the molecules are no more negligible.

The second assumption is not valid when the pressure is high & temp is low. But at high pressure or low temp, when the total vol of gas is small, the forces attraction becomes appreciable & cannot be ignored.

The deviation from ideal behaviour is expressed by introducing a factor z known as compressibility factor in the ideal gas equation. z may be expressed as $z = \frac{PV}{nRT}$

$$\left[\begin{array}{l} z=1 \rightarrow \text{ideal gas} \\ z \neq 1 \rightarrow \text{real gas} \end{array} \right]$$

MIXTURE OF PERFECT GASES

Mixture of perfect gases:-

A mixture is defined as the combination of different gases distinguished from one another by virtue of a chemical structure.

Air is mixture of O_2 , CO_2 , H_2 , N_2 , CO etc. Similarly the products of Combustion in IC Engines is a mixture of various gases like CO_2 , CO , N_2 , SO_2 water vapour etc.

Mass fraction:-

If a mixture consists of no. of ideal gases A, B, C the total mass of mixture is equal to the sum of individual masses of component gases

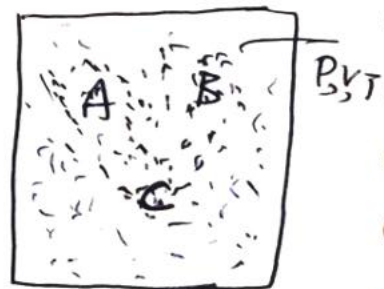
$$\therefore \text{Total mass of mixture } m_m = m_a + m_b + m_c + \dots$$

$\therefore$ mass fraction is defined as the ratio of mass of single component to the total mass of mixture. It is denoted by 'y'.

mass fraction of gas 'a' is $y_a = \frac{m_a}{m_m}$

mass fraction of gas 'b' is $y_b = \frac{m_b}{m_m}$

mass fraction of gas 'c' is $y_c = \frac{m_c}{m_m}$



Sum of all mass fractions

$$y = y_a + y_b + y_c + \dots$$

$$y = \frac{m_a}{m_m} + \frac{m_b}{m_m} + \frac{m_c}{m_m} + \dots$$

$$y = \frac{m_a + m_b + m_c}{m_m} = 1$$

$\therefore$ Sum of mass fraction of all the gases is equal to the one (1).

Mole fraction (x):-

It is the ratio of no. of moles of individual components to that of total no. of moles.

$$\therefore \text{mole fraction of gas a} = x_a = \frac{n_a}{n_m}$$

$$\text{mole fraction of gas b} = x_b = \frac{n_b}{n_m}$$

$$\text{mole fraction of gas c} = x_c = \frac{n_c}{n_m}$$

Sum of mole fraction in the mixture

$$x = x_a + x_b + x_c$$

$$= \frac{n_a}{n_m} + \frac{n_b}{n_m} + \frac{n_c}{n_m}$$

$$= 1$$

Dalton's Law of Additive pressures:-

It states that total pressure of a mixture of perfect gas is the sum of partial pressures of constituents each measured alone at the temp and volume of mixture.

$\frac{m_a P_a}{V T}$	+	$\frac{m_b P_b}{V T}$	+	$\frac{m_c P_c}{V T}$
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$$= \frac{m_m P_m}{V T}$$

$$P_m = P_a + P_b + P_c + \dots = \sum_{i=1}^{\infty} P_i$$

For gas a $\rightarrow P_a V = m_a R_a T$

$$P_a V = n_a \bar{R} T \rightarrow (1)$$

$$b \rightarrow P_b V = m_b R_b T = n_b \bar{R} T \rightarrow (2)$$

$$c \rightarrow P_c V = m_c R_c T = n_c \bar{R} T \rightarrow (3)$$

$$(P_m = P_a + P_b + P_c + \dots)$$

mixture $P_m V = n_m \bar{R} T \rightarrow (4)$

This is same as the equation of state of perfect gas $PV = nRT$

divide $E_2(1)$ with $E_2(4)$

$$\frac{P_a V}{P_m V} = \frac{n_a \bar{R} T}{n_m \bar{R} T}$$

$$\boxed{\frac{P_a}{P_m} = \frac{n_a}{n_m} = X_a}$$

$$\frac{P_a}{P_m} = X_a$$

$$P_a = X_a P_m$$

$\therefore$ Partial pressure of gas A is equal to product of total pressure of gas mixture and mole fraction of that gas.

Amagat's law of Additive volume:-

This law states that the total volume of mixture of gases is the sum of partial volume of each individual gases each measured alone at the pressure and temp of mixture.

$$P V_a = n_a \bar{R} T \rightarrow (1)$$

$$P V_b = n_b \bar{R} T \rightarrow (2)$$

$$P V_m = n_m \bar{R} T \rightarrow (3)$$

divide $E_2(1)$ with $E_2(3)$

$$\frac{P V_a}{P V_m} = \frac{n_a \bar{R} T}{n_m \bar{R} T}$$

$$\boxed{\frac{V_a}{V_m} = \frac{n_a}{n_m} = X_a}$$

$$V_a = X_a V_m$$

$\therefore$ mole fraction $\boxed{X_i = \frac{n_i}{n} = \frac{V_i}{V} = \frac{P_i}{P}}$

Equivalent

Equivalent gas Constant of mixture:-

For the mixture of gases $PV = mRT$

If the mixture contains three gases.

then

$$P_a V = m_a R_a T$$

$$P_b V = m_b R_b T$$

$$P_c V = m_c R_c T$$

By adding the above Equation we get

$$(P_a + P_b + P_c) V = (m_a R_a + m_b R_b + m_c R_c) T \rightarrow (1)$$

$$P_m V = m_m R_m T \rightarrow (2)$$

By Comparing two Equations

$$m_m R_m = m_a R_a + m_b R_b + m_c R_c$$

$$\left[\therefore y = \frac{m}{m_m} \right]$$

$$R_m = y_a R_a + y_b R_b + y_c R_c$$

Avogadro's law of additive volumes:-

It states that equal volume of all perfect gases at same temp and pressure has equal no. of moles.

Volume of 1 kg mole of all gases at normal pressure, temp gases (1.01325 bar, 0°C) is same and is equal to 22.4 m^3 .

$$\bar{R} = 8.314$$

$$\bar{R} = \frac{PV}{nT} = \frac{1.0132 \times 10^5 \times 22.4}{1 \times 273} = 8314 \text{ KJ/kgmole K}$$

$$R = \frac{\text{KJ}}{\text{kg} \cdot \text{K}}$$

Hence proved.

Mole:- A mole of substance has a mass numerically equal to the molecular weight of the substance.

ex:-

1 gram mole of $\text{N}_2 = 28 \text{ grams}$

1 kg mole of $\text{O}_2 = 32 \text{ kg}$

Law of additive volumes:-

$$n_{\text{total}} = n_a + n_b + n_c + \dots$$

$$V_{\text{total}} = V_a + V_b + V_c + \dots \text{ at same pressure \& temp}$$

No. of kg moles of gas $n = \frac{\text{mass of gas}}{\text{molecular weight}} = \frac{m}{M}$

Molar Volume :-

$$\bar{V} = \frac{V}{n} \quad \text{m}^3/\text{kg-mole}$$

molar volume is ratio of total volume of mixture and no. of moles.

Internal Energy, Entropy, Specific heat of gas mixtures :-

According to Gibbs's theorem the internal energy of gas mixture is equal to the sum of internal energy of individual components each taken at the temp & volume of gas mixture.

These also true for Remaining properties like enthalpy (h), entropy (s), specific volume (C_v) specific pressure (C_p), F, G

$$U = U_a + U_b + U_c + \dots$$

$$\text{Total mass} = m_m$$

$$m_m U = m_a U_a + m_b U_b + m_c U_c + \dots$$

$$U = \frac{m_a U_a}{m_m} + \frac{m_b U_b}{m_m} + \frac{m_c U_c}{m_m} + \dots$$

$$U = y_a U_a + y_b U_b + y_c U_c + \dots$$

$$s = y_a s_a + y_b s_b + y_c s_c + \dots$$

$$C_p = y_a C_{p_a} + y_b C_{p_b} + y_c C_{p_c} + \dots$$

$$C_v = y_a C_{v_a} + y_b C_{v_b} + y_c C_{v_c} + \dots$$

$$H = y_a H_a + y_b H_b + y_c H_c + \dots$$

Gravimetric Analysis :-

When the analysis of gas mixture is made on the basis of mass (or) weight is known as gravimetric analysis (or) mass analysis.

$$m_m = m_a + m_b + m_c + \dots = \sum_{i=1}^{\infty} m_i$$

$$\text{mass fraction} = \frac{m_a}{m_m} + \frac{m_b}{m_m} + \frac{m_c}{m_m} + \dots$$

$$y = y_a + y_b + y_c + \dots = 1$$

Volumetric analysis:-

When the analysis of gas mixture is made on the basis of no. of moles then it is known as volumetric analysis (or) molar analysis is

$$V_m = V_a + V_b + V_c + \dots = \sum_{i=1}^{\infty} V_i$$

$$\text{mole fraction } x = \frac{n_i}{n_{\text{total}}}$$

$$x = x_a + x_b + x_c + \dots = 1$$

1. A gas mixture consists of 0.4 kg of CO, 1.1 kg of CO₂ and 1.5 kg N₂ determine
- mass fraction of each component
 - mole fraction of each component
 - Average molar mass of mixture.
 - gas constant of mixture.

Sol:- Given $m_1 = 0.4 \text{ kg}$ $m_2 = 1.1 \text{ kg}$ $m_3 = 1.5 \text{ kg}$

$$\text{mass of mixture} = m_1 + m_2 + m_3$$

$$m_m = 0.4 + 1.1 + 1.5 = 3 \text{ kg}$$

$$\text{mass fraction } y_1 = \frac{m_1}{m_m} = \frac{0.4}{3} = 0.133$$

$$M_{\text{CO}} = 28$$

$$y_2 = \frac{m_2}{m_m} = \frac{1.1}{3} = 0.3667$$

$$M_{\text{CO}_2} = 44$$

$$y_3 = \frac{m_3}{m_m} = \frac{1.5}{3} = 0.5$$

$$M_{\text{N}_2} = 28$$

$$\text{No. of moles, } n_1 = \frac{m_1}{M_1} = \frac{0.4}{28} = 0.0143$$

$$n_2 = \frac{m_2}{M_2} = \frac{1.1}{44} = 0.025$$

$$n_3 = \frac{m_3}{M_3} = \frac{1.5}{28} = 0.0536$$

$$\text{Total no. of moles of mixture } n_m = n_1 + n_2 + n_3 = 0.0929$$

$$\text{mole fraction } x_1 = \frac{n_1}{n_m} = \frac{0.0143}{0.0929} = 0.154$$

$$x_2 = \frac{n_2}{n_m} = \frac{0.025}{0.0929} = 0.269$$

$$x_3 = \frac{n_3}{n_m} = \frac{0.0536}{0.0929} = 0.576$$

Average molar mass of mixture

$$M_m = \frac{m_m}{n_m} = \frac{3}{0.0929} = 32.293 \text{ mole}$$

$$\text{gas Constant of mixture } R_m = \frac{\bar{R}}{M_m} = \frac{8.314}{32.293} = 0.2574 \text{ kJ/kgK}$$

2. A vessel of 6 m^3 Capacity Contains 2 gases 'a' & 'b' in Proportion of 45% & 55% respectively at 30°C . If the value of R for the gases is 0.288 kJ/kgK and 0.295 kJ/kgK and if the total ~~rate~~ ^{weight} of mixture is 2 kg calculate
- partial pressure
 - mean value of ' R ' in the mixture

Sol:

Given

$$V = 6 \text{ m}^3$$

$$T = 30^\circ\text{C} = 303 \text{ K}$$

$$R_a = 0.288 \text{ kJ/kgK}$$

$$R_b = 0.295 \text{ kJ/kgK}$$

$$m_a = 0.45 \times 2 = 0.9 \text{ kg}$$

$$m_b = 0.55 \times 2 = 1.1 \text{ kg}$$

$$PV = mRT$$

$$P_a V = m_a R_a T$$

$$P_a = \frac{m_a R_a T}{V} = \frac{0.9 \times 0.288 \times 303}{6}$$

$$= 13.0896 \text{ kPa}$$

$$= 0.1308 \text{ bar}$$

$$P_b = \frac{m_b R_b T}{V} = \frac{1.1 \times 0.295 \times 303}{6}$$

$$= 16.3875 \text{ kPa}$$

$$= 0.1638 \text{ bar}$$

$$R_m = y_a R_a + y_b R_b = \frac{m_a}{m_m} R_a + \frac{m_b}{m_m} R_b = 0.2915 \text{ kJ/kg K}$$

mean value of 'R' in the mixture

$$R = \frac{R_a + R_b}{2} = 0.2915 \text{ kJ/kg K}$$

3. An engine ^{uses} mixture of fuel and air consisting of 0.08 kg of Octane and 1.2 kg of air. The pressure and temperature in the intake manifold is 300 k & 101.3 kPa make calculations for volume of the mixture and partial pressure of air in the mixture.

Sol. Given Octane (C_8H_{18}).

$$m_{\text{Oct}} = 0.08 \text{ kg}$$

$$P = 101.3 \text{ kPa}$$

$$m_{\text{air}} = 1.2 \text{ kg}$$

$$T = 300 \text{ K}$$

(molecular weight) $M_{\text{Oct}} = 114$

$$M_{\text{air}} = 29$$

$$n_{\text{Oct}} = \frac{m_{\text{Oct}}}{M_{\text{Oct}}} = \frac{0.08}{114} = 0.000701$$

$$n_{\text{air}} = \frac{m_{\text{air}}}{M_{\text{air}}} = \frac{1.2}{29} = 0.0414$$

$$n_m = n_{\text{Oct}} + n_{\text{air}}$$

$$PV = nRT$$

$$PV = n \bar{R} T$$

$$101.3 \times V = 0.0421 \times 8.314 \times 300$$

$$V = 1.0366 \text{ m}^3$$

$$\left[x_a = \frac{n_a}{n_m} = \frac{P_a}{P_m} \right]$$

$$\frac{n_{\text{Oct}}}{n_m} = \frac{P_{\text{Oct}}}{P_m} \Rightarrow P_{\text{Oct}} = \frac{n_{\text{Oct}}}{n_m} \times P_m$$

$$= \frac{0.000701}{0.0421} \times 101.3$$

$$P_{\text{OCT}} = 1.686 \text{ kPa}$$

$$\frac{n_{\text{air}}}{n_m} = \frac{P_{\text{air}}}{P_m} \Rightarrow P_{\text{air}} = \frac{n_{\text{air}}}{n_m} \times P_m$$

$$= \frac{0.0414}{0.0421} \times 101.3$$

$$= 99.61 \text{ kPa}$$

Conversion from volumetric to gravimetric and vice versa :-

1. The gas mixture have following volume analysis
 $O_2 = 30\%$ $CO_2 = 40\%$ $N_2 = 30\%$ determine analysis on mass basis

Sol:

Given

$$O_2 = 30\% \quad CO_2 = 40\% \quad N_2 = 30\%$$

Gas	Volume %	mole fraction $[x_a = \frac{V_i}{V}]$	mole wt (M_a)	mass $m_a = x_a M_a$	Gravimetric analysis $(\frac{m_a \times 100}{m_m})$
O_2	30	0.3	32	9.6	26.96%
CO_2	40	0.4	44	17.6	49.43%
N_2	30	0.3	28	8.4	23.59%
	100%			35.6	

Gas	Gravimetric (%)	mole wt (M_a)	$n_a (\text{mole} = \frac{\text{Gravimetric}}{\text{mole wt}(M)})$	Volumetric Analysis $(\frac{n_a \times 100}{n_m})$
O_2	26.96	32	$\frac{26.96}{32} = 0.842$	$\frac{0.842}{2.803} \times 100 = 30\%$
CO_2	49.4	44	$\frac{49.4}{44} = 1.122$	40%
N_2	23.59	28	$\frac{23.59}{28} = 0.849$	30%
			2.803	

2. A gas mixture consists of 60% N_2 and 40% CO_2 by mole basis. determine gravimetric analysis of mixture.

Sol: Given $N_2 = 60\%$ $CO_2 = 40\%$

Gas	Volume %	mole fraction $x_a = \frac{V_i}{V}$	mol wt	$m_a = x_a M_a$	Gravimetric Analysis $(\frac{m_a}{M_m} \times 100)$
N_2	60	0.6	28	16.8	$\frac{16.8}{34.4} \times 100 = 48.84$
CO_2	$\frac{40}{100}$	0.4	44	$\frac{17.6}{34.4}$	51.162

3. The volumetric analysis of dry ~~flue~~ ^{tray} gases in a boiler ~~tray~~ is given in % as 13% CO_2 , 1.5% CO , 3.5% O_2 and 82% N_2 determine the % in gravimetric analysis and also find the gas constant of mixture.

Sol:

Gas	Volume %	mole fraction $x_a = \frac{V_i}{V}$	mol wt (M)	$m_a = x_a M_a$	Gravimetric Analysis $(\frac{m_a}{M_m} \times 100)$
CO_2	13	0.13	44	5.72	$\frac{5.72}{30.22} \times 100 = 18.93$
CO	1.5	0.015	28	0.42	1.39
O_2	3.5	0.035	32	1.12	3.71
N_2	82	0.82	28	22.96	75.98
					$\frac{132}{30.22}$
Gas Constant $R = \frac{\bar{R}}{M_m} = \frac{8.314}{132} = 0.063 \text{ kJ/kg K}$					

$$R = Y_{O_2} R_{CO_2} + Y_{CO} R_{CO} + Y_{O_2} R_{O_2} + Y_{N_2} R_{N_2}$$

$$n_m = n_{CO_2} + n_{CO} + n_{O_2} + n_{N_2}$$

$$\left[n = \frac{m}{M} \right]$$

$$n_m = \frac{5.72}{44} + \frac{0.42}{28} + \frac{1.12}{32} + \frac{22.96}{28}$$

$$= 1$$

$$\text{mean molecular weight } M_m = \frac{m_m}{n_m}$$

$$M_m = m_m = 30.222$$

Gas Constant of mixture

$$R = \frac{\bar{R}}{M_m} \text{ (or) } \frac{\bar{R}}{m_m}$$

$$R = Y_a R_a + Y_b R_b + Y_c R_c + \dots$$

$$R = \frac{0.314}{30.22} = 0.275 \text{ kJ/kg K}$$

4. A gas mixture Contains 1kg of O_2 3.5 kg of N_2
 The pressure and temp of mixture are 1bar and $27^\circ C$ determine
- i. mass fraction
 - ii. mole fraction
 - iii. Average molecular weight of mixture
 - iv. Partial pressures of the Components
 - v. Specific gas Constants.
 - vi. mixture volume
 - vii. mixture density

Sol:

Given

$$m_{O_2} = 11 \text{ kg}$$

$$m_{N_2} = 3.5$$

$$M_{O_2} = 32$$

$$M_{N_2} = 28$$

$$T = 27^\circ\text{C} = 300 \text{ K}$$

$$P = 1 \text{ bar} = 100 \text{ kPa}$$

$$m_m = 1 + 3.5 = 4.5 \text{ kg}$$

$$(i) \quad y_{O_2} = \frac{m_{O_2}}{m_m} = \frac{1}{4.5} = 0.2222$$

$$y_{N_2} = \frac{m_{N_2}}{m_m} = \frac{3.5}{4.5} = 0.7778$$

$$y_m = y_{O_2} + y_{N_2} = 1$$

$$(ii) \quad n_{O_2} = \frac{m_{O_2}}{M_{O_2}} = \frac{1}{32} = 0.03125$$

$$n_{N_2} = \frac{3.5}{28} = 0.125$$

$$n_m = n_{O_2} + n_{N_2} = 0.15625$$

$$x_{O_2} = \frac{n_{O_2}}{n_m} = 0.2$$

$$x_{N_2} = \frac{n_{N_2}}{n_m} = 0.8$$

$$(iii) \quad \text{Average molecular weight } M_m = \frac{m_m}{n_m} = \frac{4.5}{0.15625}$$

$$M_m = 28.8 \text{ kg/mole}$$

$$(iv) \quad \text{mole fraction } x_i = \frac{P_i}{P} = \frac{n_i}{n}$$

$$\frac{n_{O_2}}{n_m} = \frac{P_{O_2}}{P_m}$$

$$P_{O_2} = \frac{n_{O_2}}{n_m} \times P_m$$

$$= \frac{0.03125}{0.15625} \times 100$$

$$= 20 \text{ kPa}$$

$$P_{O_2} = 0.2 \text{ bar}$$

$$(v) R_{O_2} = \frac{\bar{R}}{M_{O_2}} = \frac{8.314}{32}$$

$$= 0.2598$$

$$R_m = y_a R_a + y_b R_b$$

$$= (0.222)(0.2598) + (0.7778)(0.2969)$$

$$R_m = 0.28865$$

$$(vi) PV = mRT$$

$$V_m = \frac{m_m R_m T_m}{P_m}$$

$$= \frac{0.28865 \times 4.5 \times 300}{100}$$

$$V_m = 3.8968 \text{ m}^3$$

$$(vii) \text{ density } \rho = \frac{m_m}{V_m} = \frac{4.5}{3.8968}$$

$$= 1.155 \text{ kg/m}^3.$$

$$P_{N_2} = \frac{n_{N_2}}{n_m} \times P_m$$

$$= \frac{0.125}{0.15625} \times 100$$

$$= 80 \text{ kPa}$$

$$P_{N_2} = 0.8 \text{ bar}$$

$$R_{N_2} = \frac{\bar{R}}{M_{N_2}} = \frac{8.314}{28}$$

$$R_{N_2} = 0.2969$$

Gas Power Cycles

A T.D cycle consists of a series of processes in which the working fluid undergoes in some order before returning to its initial state.

Important terms related to I.C Engine:-

Bore

Top dead Center (TDC)

Bottom dead Center (BDC)

Inner dead Center (IDC)

Outer dead Center (ODC)

Swept volume (or) piston displacement

$$V_s = \frac{\pi}{4} d^2 l$$

Clearance volume (V_c)

Stroke length (L) = $2r$

where r = crank radius

$$\text{Compression ratio } \eta = \frac{V_s + V_c}{V_c} = \frac{V_1}{V_2}$$

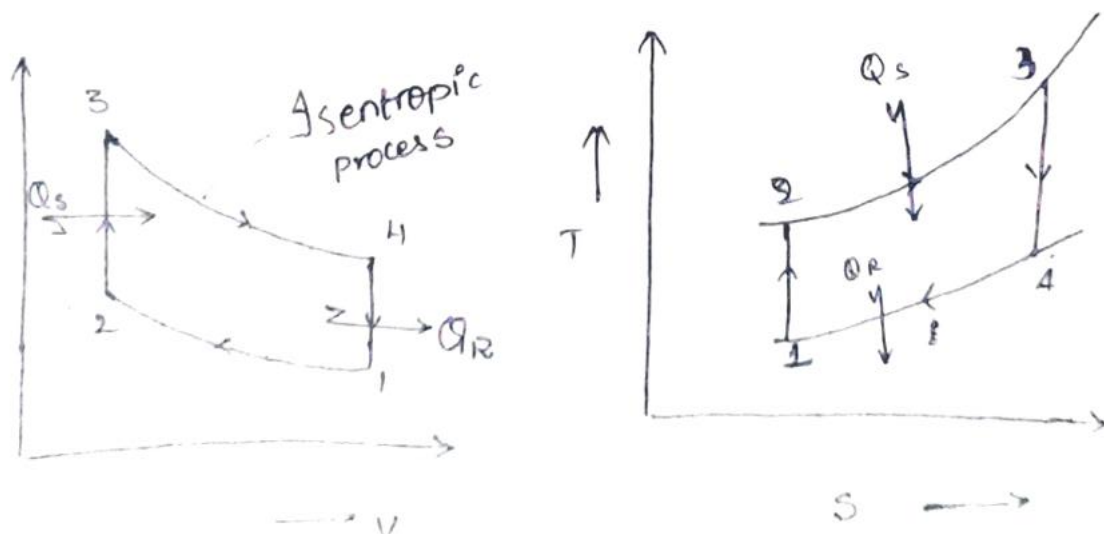
Assumptions made in air standard cycles :-

- * The working substance consists of fixed mass of air and behaves as an ideal gas, which obeys gas laws.
- * The working fluid is homogeneous at all times and known reaction takes place.
- * The cyclic process is reversible.
- * All the processes in the cycle are frictionless.

- * Compression and Expansion process are adiabatic i.e. no heat is gained or lost during the process and these operations take place without external friction in gases.
- * The System is Considered as Closed System so no intake or Exhaust process.
- * The Combustion process is replaced by an equivalent heat addition process from external source.
- * There is no Exhaust process This is replaced by an equivalent heat rejection process.
- * The specific heats C_p and C_v of air do not vary with temperature.

Air Standard Cycles :- (Repeat Carnot Cycle once again)

Otto Cycle or Petrol Cycle :- (Constant volume cycle)



This cycle was invented by Dr. Nicholas A. Otto (1832-1891). (German engineer)

Process (1-2) : Isentropic Compression

Process (2-3) : Constant volume heat addition

Process (3-4) : Isentropic Expansion

process (4-1) : Constant volume heat rejection.

* Heat is supplied at constant volume so this cycle is called Constant volume Cycle.

$$\text{Heat supplied } Q_S = Q_{(2-3)} = m C_v (T_3 - T_2)$$

$$\text{Heat rejection } Q_R = Q_{(4-1)} = m C_v (T_4 - T_1)$$

$$\begin{aligned} \text{Workdone} = W_D &= Q_S - Q_R \\ &= m C_v (T_3 - T_2) - m C_v (T_4 - T_1) \\ &= m C_v [(T_3 - T_2) - (T_4 - T_1)] \end{aligned}$$

$$\begin{aligned} \eta_{\text{otto}} &= \frac{\text{workdone}}{\text{Heat supplied}} = \frac{Q_S - Q_R}{Q_S} \\ &= \frac{m C_v [(T_3 - T_2) - (T_4 - T_1)]}{m C_v (T_3 - T_2)} \end{aligned}$$

$$\eta_{\text{otto}} = \left[1 - \frac{T_4 - T_1}{T_3 - T_2} \right] \Rightarrow \boxed{\eta_{\text{otto}} = 1 - \frac{Q_R}{Q_S}}$$

From the above P-V diagram we can write as follows.

$$\begin{aligned} P_1 V_1 &= m R T_1 \\ P_2 V_2 &= m R T_2 \end{aligned}$$

$$\frac{P_1 V_1}{P_2 V_2} = \frac{T_1}{T_2}$$

Process (1-2) : Isentropic process

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1} \right)^\gamma$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^\gamma \left(\frac{V_1}{V_2} \right)$$

$$\left[\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{\gamma-1} \right]$$

Similarly from p-v diagram

$$\boxed{\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}}$$

$$V_1 = V_4$$

$$V_3 = V_2$$

$$\frac{T_1}{T_2} = \frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \text{ (or) } \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

Also write as

$$\frac{T_3}{T_2} = \frac{T_4}{T_1}$$

$$\therefore \eta_{\text{otto}} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{T_1 \left(\frac{T_4}{T_1} - 1\right)}{T_2 \left(\frac{T_3}{T_2} - 1\right)} \quad \left(\because \frac{T_3}{T_2} = \frac{T_4}{T_1}\right)$$

$$\eta_{\text{otto}} = 1 - \frac{T_1}{T_2}$$

$$\eta = 1 - \frac{T_1}{T_2}$$

$$= 1 - \frac{1}{\left(\frac{T_2}{T_1}\right)} = 1 - \frac{1}{\left(\frac{V_1}{V_2}\right)^{\gamma-1}}$$

$$\boxed{\eta_{\text{otto}} = 1 - \frac{1}{(r)^{\gamma-1}}}$$

Mean Effective Pressure:-

It is defined as the ratio of net work done to the displacement volume of piston.

$$\text{W.D.} = m c_v [(T_3 - T_2) - (T_4 - T_1)]$$

$$\text{Displaced volume} = (V_1 - V_2)$$

$$= V_1 \left(1 - \frac{V_2}{V_1}\right)$$

$$= V_1 \left(1 - \frac{1}{\frac{V_1}{V_2}} \right)$$

$$= V_1 \left(1 - \frac{1}{r} \right)$$

$$= \frac{m R T_1}{P_1} \left(1 - \frac{1}{r} \right)$$

$$\left(\frac{V_1}{V_2} = r \right)$$

$$P_1 V_1 = m R T_1$$

$$V_1 = \frac{m R T_1}{P_1}$$

$$(\because R = C_v (\gamma - 1))$$

$$\text{Displacement} \cdot \text{Vol} = \frac{m C_v (\gamma - 1) T_1}{P_1} \left(1 - \frac{1}{r} \right)$$

$$\therefore m e p = \frac{\omega \cdot D}{D \cdot V}$$

$$= \frac{m C_v [(T_3 - T_2) - (T_4 - T_1)]}{\frac{m C_v (\gamma - 1) T_1}{P_1} \left(\frac{r - 1}{r} \right)}$$

$$m e p = \frac{P_1}{T_1} \frac{1}{\gamma - 1} \left(\frac{r}{r - 1} \right) [(T_3 - T_2) - (T_4 - T_1)] \quad \rightarrow (1)$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma - 1} = (r)^{\gamma - 1}$$

$$T_2 = T_1 (r)^{\gamma - 1} \rightarrow (2)$$

$$\frac{T_3}{T_2} = \frac{P_3}{P_2} \Rightarrow T_3 = T_2 \frac{P_3}{P_2}$$

$$(\because \frac{P_3}{P_2} = r_p)$$

$$T_3 = T_2 (r_p)$$

$$= T_1 (r)^{\gamma - 1} (r_p) \rightarrow (3)$$

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4} \right)^{\gamma - 1} = \left(\frac{1}{r} \right)^{\gamma - 1}$$

$$\left[\because \frac{V_4}{V_3} = \frac{V_1}{V_2} = r \right]$$

$$T_4 = T_3 \left(\frac{1}{r} \right)^{\gamma - 1}$$

$$= T_1 r_p (r)^{\gamma - 1} \left(\frac{1}{(r)^{\gamma - 1}} \right)$$

$$T_4 = T_1 r_p \rightarrow (4)$$

Substitute γ 2, 3, 4 in ϵ_2 ①

$$mep = \frac{P_1}{T_1} \frac{1}{\gamma-1} \left(\frac{\gamma}{\gamma-1} \right) \left[(T_1 \gamma)^{\gamma-1} \gamma_P - T_1 (\gamma)^{\gamma-1} - (T_1 \gamma_P - T_1) \right]$$

$$= \frac{P_1}{\gamma-1} \left(\frac{\gamma}{\gamma-1} \right) \left[((\gamma)^{\gamma-1} \gamma_P - (\gamma)^{\gamma-1}) - (\gamma_P - 1) \right]$$

$$= \frac{P_1}{\gamma-1} \left(\frac{\gamma}{\gamma-1} \right) \left[(\gamma)^{\gamma-1} (\gamma_P - 1) - (\gamma_P - 1) \right]$$

$$= \frac{P_1}{\gamma-1} \left(\frac{\gamma}{\gamma-1} \right) \left[(\gamma_P - 1) ((\gamma)^{\gamma-1} - 1) \right]$$

$$mep = \frac{P_1 \gamma (\gamma_P - 1) ((\gamma)^{\gamma-1} - 1)}{(\gamma-1) (\gamma-1)} \rightarrow \underline{\underline{\epsilon_2}}$$

NOTE:-

- * In case of petrol engine (or) Spark ignition Engines the Compression ratio is about 6 to 12.
- * In case of diesel Engine (or) Combustion ignition engine the Compression ratio is about 12 to 24.
- * Stroke volume is the Cubic Capacity of the engine

Problem:-

① An engine has 200mm bore and 250mm stroke and it has a clearance volume of 1570cc calculate Compression ratio and identify the engine. also calculate the air standard efficiency of cycle.

② Given bore $d = 200 \text{ mm} = 20 \text{ cm}$
 $l = 250 \text{ mm} = 25 \text{ cm}$

$$V_c = 1570 \text{ cm}^3$$

$$\underline{\text{req:}} \quad \textcircled{1} \quad \eta = \frac{V_1}{V_2} = \frac{V_s + V_c}{V_c}$$

$$\text{Stroke Volume } V_s = \frac{\pi}{4} d^2 l = \frac{\pi}{4} (20)^2 (25) \\ = 7853.982 \text{ cm}^3$$

$$\text{Compression ratio} = \frac{V_s + V_c}{V_c} \\ = \frac{7853.982 + 1570}{1570} \\ = 6.0025$$

The engine is S.I engine (or) Petrol engine.

$$\eta_{\text{otto}} = 1 - \frac{1}{(\eta)^{\gamma-1}} = 1 - \frac{1}{(6.0025)^{\gamma-1}} \\ = 0.5117 = 51.2 \%$$

2 An engine working on Otto Cycle has following conditions. pressure at the beginning of Compression is 1 bar and at the end of Compression is 12 bar. Calculate i. Compression ratio ii. Air Standard efficiency of engine.

Sol:- Given $P_1 = 1 \text{ bar}$
 $P_2 = 12 \text{ bar}$

$$\eta = \frac{V_1}{V_2} = \frac{1}{\frac{V_2}{V_1}} \\ = 5.899 \\ \approx 5.9$$

$$\eta_{\text{otto}} = 1 - \frac{1}{(\eta)^{\gamma-1}} \\ = 1 - \frac{1}{(5.9)^{\gamma-1}} \times 100 \\ = 50.83 \%$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma \\ \frac{P_1}{P_2} = \left(\frac{V_2}{V_1} \right)^\gamma \\ \frac{1}{12} = \left(\frac{V_2}{V_1} \right)^\gamma \\ \frac{V_2}{V_1} = \left(\frac{1}{12} \right)^{\frac{1}{\gamma}} \\ \frac{V_2}{V_1} = 0.16947$$

- (3). In a Car working on otto cycle initial pressure is 1 bar and pressure after Compression is 10 bar
find i. Compression ratio
ii. efficiency
iii. % of clearance.

Sol:

Given $P_1 = 1 \text{ bar}$
 $P_2 = 10 \text{ bar}$

$$\eta = \frac{V_1}{V_2} = 5.179$$

$$\eta = 1 - \frac{1}{(\eta)^{\gamma-1}}$$

$$= 1 - \frac{1}{(5.179)^{0.4}} \times 100$$

$$\eta = 48.20\%$$

$$\left[\% \text{ clearance} = \frac{V_c}{V_s} \times 100 \right] \Rightarrow \% \text{ of clearance}$$

$$\eta = \frac{V_s + V_c}{V_c}$$

$$\eta = \frac{V_s}{V_c} + 1$$

$$\frac{V_s}{V_c} = \eta - 1$$

$$\frac{V_c}{V_s} = \frac{1}{\eta - 1}$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^\gamma$$

$$\frac{V_1}{V_2} = \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}}$$

$$= \left(\frac{10}{1} \right)^{\frac{1}{1.4}}$$

$$= 5.179$$

$$= \frac{1}{\eta - 1} \times 100$$

$$= 23.929\%$$

- (4) In An air Standard otto Cycle Compression ratio is '8' and at starting of Compression process temperature is 26°C and pressure is 1 bar. The maximum Cycle temp is $T_3 = 1080^\circ\text{C}$. Calculate.
i. heat supplied, ii. heat rejected iii. Work done
iv. efficiency and also draw P-v and T-s diagrams.

Given data

$$\eta = 8$$

$$\frac{V_1}{V_2}$$

$$T_1 = 26^\circ\text{C}$$

$$P_1 = 1 \text{ bar}$$

$$T_3 = 1080^\circ\text{C} = 1353 \text{ K}$$

$$C_v = 0.117$$

$$\text{kJ/kg K}$$

$$m = 1 \text{ kg}$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\left(\frac{V_1}{V_2}\right)^\gamma = \frac{P_2}{P_1}$$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^\gamma$$

$$= 1 (8)^{1.4}$$

$$= 18.379 \text{ bar}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 299 (8)^{\gamma-1} = 299 (8)^{0.4}$$

$$T_2 = 686.92 \text{ K}$$

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$T_4 = T_3 \left(\frac{V_2}{V_1}\right)^{\gamma-1} = T_3 \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$T_4 = 1353 \left(\frac{1}{8}\right)^{\gamma-1} = 588.9274$$

Heat Supplied $Q_s = m C_v (T_3 - T_2)$

$$= 0.117 (1353 - 686.92)$$

$$= 477.579 \text{ kJ}$$

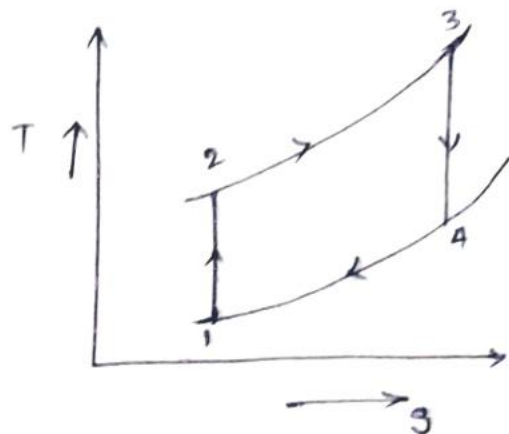
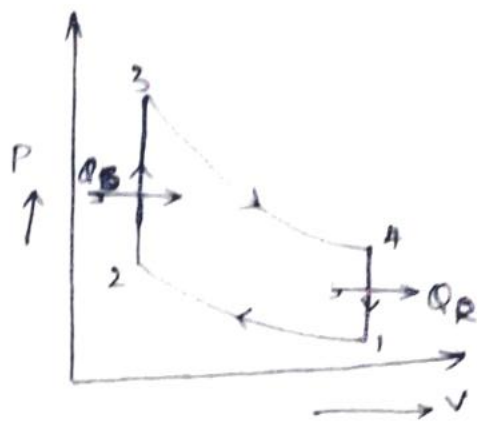
Heat rejected $Q_R = m C_v (T_4 - T_1)$

$$= 0.117 (588.9274 - 299)$$

$$= 207.8779 \text{ kJ}$$

Work done $W = Q_s - Q_R = 477.597 - 207.8779$

$$= 269.7191 \text{ kJ}$$



$$\eta_{\text{otto}} = 1 - \frac{1}{(r_1)^{\gamma-1}}$$

$$= 1 - \frac{1}{8^{0.4}} = 0.564 \times 100 = 56.41\%$$

4. An air standard Otto cycle has a Compression ratio of 8. At starting of Compression the pressure and temp are 1 bar and 27°C the maximum temp of the cycle is 727°C Calculate i. heat supplied, ii. heat rejected and iii. work done iv. efficiency.

Given data

$$r_1 = \frac{V_1}{V_2} = \frac{V_4}{V_3} = 8$$

$$P_1 = 1 \text{ bar}$$

$$T_1 = 300 \text{ K} = 27^\circ\text{C}$$

$$T_3 = 727^\circ\text{C} = 1000 \text{ K}$$

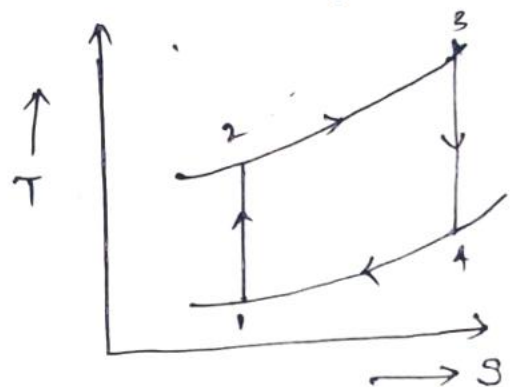
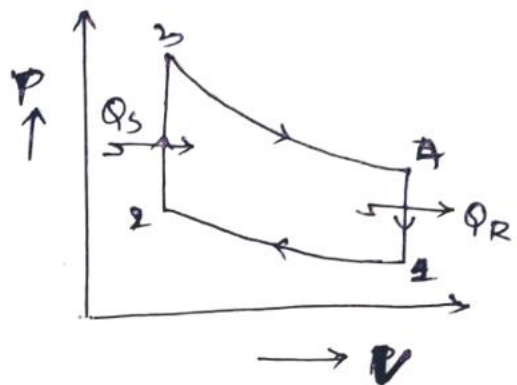
$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300 (8)^{\gamma-1} = 689.219 \text{ K}$$

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \Rightarrow T_4 = T_3 \left(\frac{1}{r_1}\right)^{\gamma-1} = 1000 \left(\frac{1}{8}\right)^{\gamma-1} = 435.275$$

$$\text{Heat Supplied } Q_S = m c_v (T_3 - T_2) = 0.717 (1000 - 689.219) = 222.8299 \text{ KJ}$$

$$\text{Heat rejected } Q_R = m c_v (T_4 - T_1) = 0.717 (435.275 - 300)$$



$$= 96.992 \text{ kJ}$$

$$\text{work done} = Q_s - Q_R$$

$$= 125.8377 \text{ kJ}$$

$$\text{Efficiency } \eta_{\text{otto}} = 1 - \frac{1}{(9)^{\gamma-1}} = 56.47 \%$$

6. Find efficiency of Otto-cycle to a Compression ratio of 6. using ideal gases having specific heat ratio's as 1.35, 1.43 and 1.62, ii. What is the air Standard cycles, why are the cycles consumed?

Given

$$\frac{C_p}{C_v} = \gamma$$

$$\gamma = 1.35$$

$$\gamma = 1.43$$

$$= 1.62$$

$$\eta = 1 - \frac{1}{(9)^{\gamma-1}} = 0.4658$$

$$= 46.58 \%$$

$$\eta = 1 - \frac{1}{(6)^{1.43-1}} = 53.71 \%$$

$$\eta = 1 - \frac{1}{(6)^{1.62-1}} = 67.073 \%$$

- iii. What are the advantages of helium as working substance in the cycle?

'He' is having high ' γ ' value than compare to other working substance and this leads to high efficiency.

7. The minimum pressure and temp of Otto-cycle are 100 kPa and 27°C the amount of heat added per cycle is 1500 kJ/kg

i. Determine the temp at all points.

ii. Determine the pressures at all the points.

iii. Calculate Specific workdone.

iv. What is the thermal efficiency of the cycle if Compression ratio is 8:1. Assume $C_p = 1.005$ and $C_v = 0.714 \text{ kJ/kg K}$ represent the process on P-v and T-s diagrams.

Given data

$$P_1 = 100 \text{ kPa}$$

$$T_1 = 27^\circ\text{C} = 300 \text{ K}$$

$$Q_s = 1500 \text{ kJ/kg}$$

$$r = 8 = \frac{V_1}{V_2} = \frac{V_A}{V_3}$$

$$C_p = 1.005 \text{ kJ/kg K}$$

$$C_v = 0.714 \text{ kJ/kg K}$$

$$Q_s = m C_v (T_3 - T_2)$$

$$1500 = 0.714 (T_3 - 689.219)$$

$$T_3 - 689.219 = 2100.84$$

$$T_3 = 2790.059 \text{ K}$$

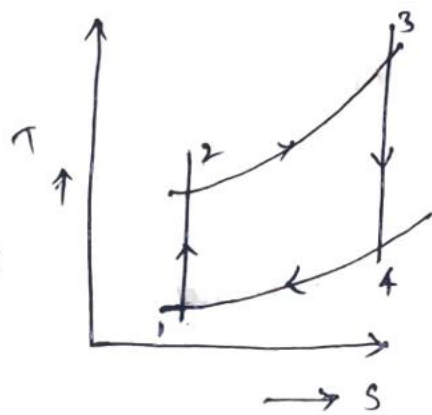
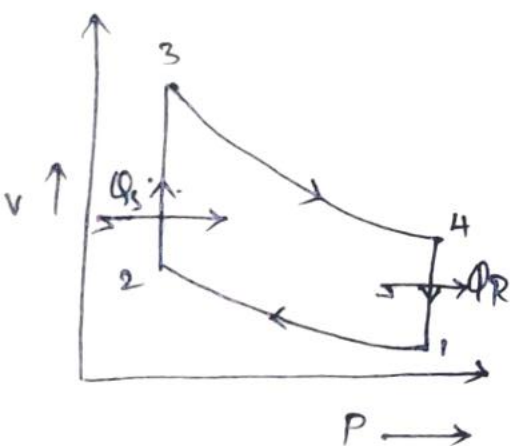
$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 300 (8)^{1.4-1}$$

$$= 689.219 \text{ K}$$

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \Rightarrow T_4 = T_3 \left(\frac{1}{r}\right)^{\gamma-1}$$

$$= 1214.439 \text{ K}$$



work done

adiabatic

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^\gamma$$

$$= 100 (8)^{1.4}$$

$$= 1837.917 \text{ kPa}$$

$$P_3 V_3^\gamma = P_4 V_4^\gamma$$

Process (2-3) (const volume)

$$P_2 V_2 = m R T_2$$

$$P_3 V_3 = m R T_3$$

$$\frac{P_2}{P_3} = \frac{T_2}{T_3}$$

$$P_3 = P_2 \left(\frac{T_3}{T_2}\right)$$

$$P_3 = 1440.1334 \text{ kPa}$$

(3-4) Process
adiabatic

$$P_3 V_3^\gamma = P_4 V_4^\gamma$$

$$P_4 = P_3 \left(\frac{V_3}{V_4}\right)^\gamma$$

$$= P_3 \left(\frac{1}{r}\right)^\gamma$$

$$= 404.8131 \text{ kPa}$$

Work done

$$W.D = Q_S - Q_R$$

$$= 1500 - 652.9092$$

$$= 847.090 \text{ KJ/kg}$$

$$\begin{aligned} \text{Heat reject } Q_R &= m C_v (T_A - T_1) \\ &= 0.714 (1214.439 - 300) \\ &= 652.9094 \text{ KJ/kg} \end{aligned}$$

$$\begin{aligned} \text{Efficiency } \eta_{\text{otto}} &= 1 - \frac{1}{(r)^{\gamma-1}} \\ &= 0.5647 = 56.47\% \end{aligned}$$

8. A gas engine operating on an ideal otto-cycle has Compression ratio of 6:1. The pressure and temp after commencement of ~~suction~~ ^{suction} process are 1 bar & 27°C. Heat supplied at Const volume. Combustion process is 1170 KJ. Determine.
- Pressure & temp at all the points.
 - workdone & efficiency Assume $\gamma = 1.4$
 $C_v = 0.717 \text{ KJ/kg K}$.

Given data

$$r = 6 = \frac{V_1}{V_2} = \frac{V_4}{V_3}$$

$$P_1 = 1 \text{ bar}$$

$$T_1 = 300 \text{ K}$$

$$Q_S = 1170 \text{ KJ}$$

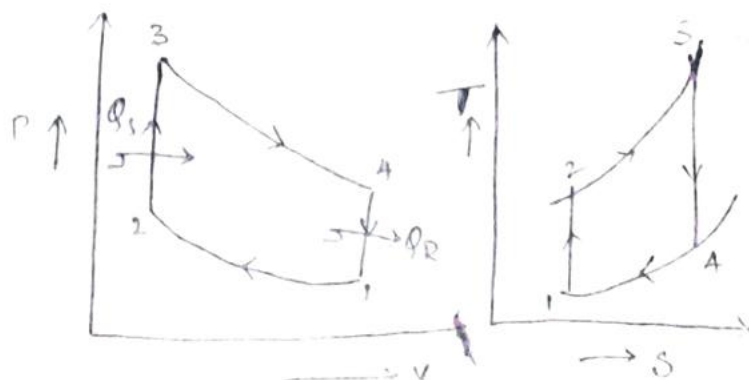
$$\begin{aligned} \frac{T_2}{T_1} &= \left(\frac{V_1}{V_2} \right)^{\gamma-1} \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} \\ &= 300 (6)^{1.4-1} \\ &= 614.3017 \text{ K} \end{aligned}$$

$$\begin{aligned} Q_S &= m C_v (T_3 - T_2) \\ 1170 &= 0.717 (T_3 - 614.3017) \end{aligned}$$

$$1631.799 = T_3 - 614.3017$$

$$T_3 = 2246.1008 \text{ K}$$

$$\begin{aligned} \frac{T_3}{T_4} &= \left(\frac{V_4}{V_1} \right)^{\gamma-1} \\ T_4 &= T_3 \left(\frac{1}{r} \right)^{\gamma-1} \\ T_A &= 977.6721 \text{ K} \\ &1096 \end{aligned}$$



Process (1-2)
adiabatic

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma$$

$$= 1 (6)^{1.4}$$

$$= 12.286 \text{ bar}$$

Process (2-3)
constant volume

$$P_2 V_2 = m R T_2$$

$$P_3 V_3 = m R T_3$$

$$\frac{P_2}{P_3} = \frac{T_2}{T_3}$$

$$P_3 = P_2 \left(\frac{T_3}{T_2} \right)$$

$$= 12.286 \left(\frac{2246.1008}{614.3017} \right)$$

$$= 44.922 \text{ bar}$$

Process (3-4)
adiabatic

$$P_3 V_3^\gamma = P_4 V_4^\gamma$$

$$P_4 = P_3 \left(\frac{V_3}{V_4} \right)^\gamma$$

$$= P_3 \left(\frac{1}{91} \right)^\gamma$$

$$= 3.656 \text{ bar}$$

heat rejection $Q_R = m C_v (T_4 - T_1)$

$$= 0.717 (977.6721 - 300)$$

$$= 485.891 \text{ kJ} \quad 517 \text{ kJ}$$

Work done $= Q_s - Q_R$

$$= 1170 - 485.891$$

$$= 684.11 \text{ kJ} \quad 598 \text{ kJ}$$

Efficiency $\eta_{\text{otto}} = 1 - \frac{1}{(91)^{\gamma-1}} = 1 - \frac{1}{(6)^{\gamma-1}} \times 100$

$$= 51.164 \%$$

9. In an air standard Otto Cycle Compression ratio is 7 and the Compression begins at 350°C & 0.1 MPa . The maximum temp of cycle is 1100°C determine
- The temp & pressure at all cardinal points of cycle
 - Heat Supplied
 - work done
 - Efficiency
 - displacement mean-effective pressure.

Given data:-

$$r = \frac{V_1}{V_2} = 7 = \frac{V_4}{V_3}$$

$$T_1 = 350^\circ\text{C} = 623 \text{ K}$$

$$P_1 = 0.1 \text{ MPa}$$

$$T_3 = 1100^\circ\text{C} = 1373 \text{ K}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 623(-1)^{0.4}$$

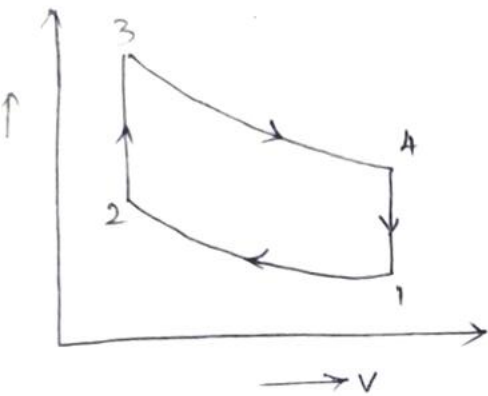
$$= 1356.835 \text{ K}$$

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$$

$$T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$= 1373 \left(\frac{1}{7}\right)^{0.4}$$

$$= 630.422 \text{ K}$$



Process (1-2)

adiabatic

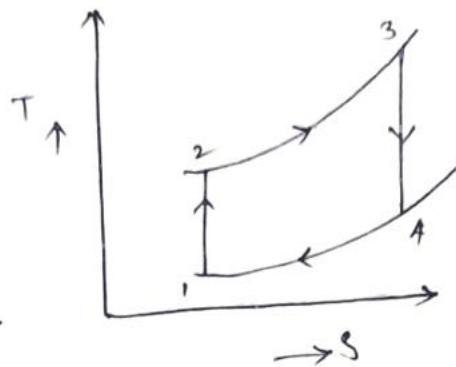
$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^\gamma$$

$$= 0.1 (7)^{1.4}$$

$$P_2 = 1.5245 \text{ MPa}$$

$$= 1.525 \text{ MPa}$$



Process (2-3)

Constant volume

$$\frac{P_2}{P_3} = \frac{T_2}{T_3}$$

$$P_3 = P_2 \left(\frac{T_3}{T_2}\right)$$

$$= 1.525 \left(\frac{1373}{1356.835}\right)$$

$$P_3 = 1.543 \text{ MPa}$$

Process (3-4)

adiabatic

$$P_3 V_3^\gamma = P_4 V_4^\gamma$$

$$P_4 = P_3 \left(\frac{V_3}{V_4}\right)^\gamma$$

$$= 1.543 \left(\frac{1}{7}\right)^{1.4}$$

$$P_4 = 0.1012 \text{ MPa}$$

Heat Supplied $Q_s = m C_v (T_3 - T_2)$

$$= 0.717 (1373 - 1356.835)$$

$$= 11.59 \text{ kJ}$$

Heat rejection:-

$$Q_R = m C_v (T_4 - T_1)$$

$$= 0.717 (630.422 - 623)$$

$$= 5.322 \text{ kJ}$$

$$\text{Workdone} = Q_s - Q_R$$

$$= 11.59 - 5.322$$

$$= 6.268 \text{ kJ}$$

$$\text{Efficiency } \eta_{\text{otto}} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(7)^{0.4}}$$

$$= 0.5408 \times 100$$

$$= 54.08 \%$$

$$\text{Mean effective pressure} = \frac{10.0}{\text{Displaced volume}}$$

$$\text{Displaced volume} = (V_1 - V_2)$$

$$= V_1 \left(1 - \frac{V_2}{V_1}\right)$$

$$= \frac{m R T_1}{P_1} \left(1 - \frac{1}{r}\right)$$

$$= \frac{m C_v (\gamma - 1) T_1}{P_1} \left(1 - \frac{1}{r}\right)$$

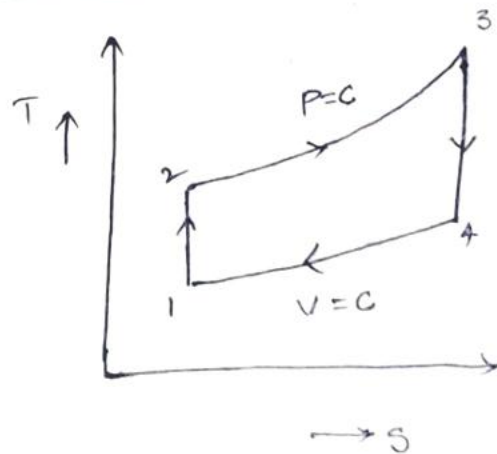
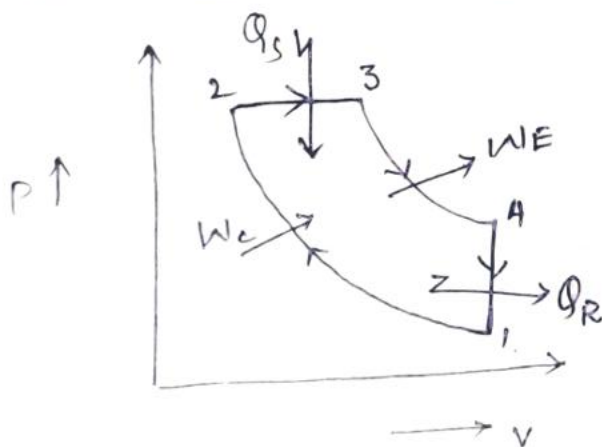
$$= \frac{0.717 (0.4) 623}{100} \left(1 - \frac{1}{7}\right)$$

$$= 1.532$$

$$\text{Mean effective pressure} = \frac{W.D}{\text{Displaced volume}}$$

$$= \frac{6.268}{1.532} = 4.091$$

Diesel Cycle :- RUDOLF DIESEL (1892) (Compression ignition)



Heat Supplied is carried out during Constant pressure process so this cycle also called as Constant pressure cycle.

Process (1-2) - Reversible adiabatic Compression

(2-3) - Constant pressure heat addition.

(3-4) - Reversible adiabatic Expansion

(4-1) - Constant volume heat rejection

Consider m kg of working ~~fuel~~ fluid. from the diagram.

Heat Supplied $Q_s = m C_p (T_3 - T_2)$

Heat Rejected $Q_R = m C_p (T_4 - T_1)$

Work done $= Q_s - Q_R$

$$\eta = 1 - \frac{Q_R}{Q_s}$$

$$\eta = 1 - \frac{m C_v (T_4 - T_1)}{m C_p (T_3 - T_2)}$$

$$= 1 - \frac{1}{\gamma} \left[\frac{T_4 - T_1}{T_3 - T_2} \right] \quad \left(\because \frac{C_p}{C_v} = \gamma \right)$$

Process (1-2) process
reversible adiabatic process
compression

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$\left(\because \gamma = \frac{V_1}{V_2} = \frac{V_4}{V_3} \right)$$

$$T_2 = T_1 (\gamma)^{\gamma-1} \longrightarrow \textcircled{1}$$

Process (2-3)

Cut off ratio :- $\boxed{\frac{T_3}{T_2} = \frac{V_3}{V_2} = \gamma_c = \rho}$ $\left| \begin{array}{l} P_2 V_2 = m R T_2 \\ P_3 V_3 = m R T_3 \\ \frac{V_2}{V_3} = \frac{T_2}{T_3} \end{array} \right.$

Cut off ratio:-

Hot body is withdrawn at point 3. implied

Cut off heat Supplied

$$T_3 = T_2 \gamma_c$$

$$T_3 = \gamma_c (\gamma)^{\gamma-1} T_1 \longrightarrow \textcircled{2}$$

Process (3-4)

adiabatic Expansion

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4} \right)^{\gamma-1}$$

$$= \left(\frac{V_3}{V_2} \cdot \frac{V_2}{V_4} \right)^{\gamma-1}$$

$$= \left(\eta_c \cdot \frac{1}{\eta} \right)^{\gamma-1}$$

$$\frac{T_4}{T_3} = \left(\frac{\eta_c}{\eta} \right)^{\gamma-1}$$

$$T_4 = T_3 \left(\frac{\eta_c}{\eta} \right)^{\gamma-1}$$

$$= \eta_c (\eta)^{\gamma-1} \left(\frac{\eta_c}{\eta} \right)^{\gamma-1} \cdot T_1$$

$$= \eta_c \cdot \eta_c^{\gamma-1} T_1$$

$$T_4 = \eta_c^{\gamma} T_1 \quad \longrightarrow \textcircled{3}$$

Substitute eq ①, eq ② & ③ in η

$$\eta_{th} = 1 - \frac{1}{\gamma} \left[\frac{T_4 - T_1}{T_3 - T_2} \right]$$

$$= 1 - \frac{1}{\gamma} \left[\frac{\eta_c^{\gamma} T_1 - T_1}{\eta_c (\eta)^{\gamma-1} T_1 - T_1 (\eta)^{\gamma-1}} \right]$$

$$= 1 - \frac{1}{\gamma} \left[\frac{\eta_c^{\gamma} - 1}{\eta_c (\eta)^{\gamma-1} - \eta^{\gamma-1}} \right]$$

$$= 1 - \frac{1}{\gamma} \left[\frac{\eta_c^{\gamma} - 1}{(\eta)^{\gamma-1} (\eta_c - 1)} \right]$$

$$\eta_{th} = 1 - \frac{1}{\gamma (\eta)^{\gamma-1}} \left[\frac{\eta_c^{\gamma} - 1}{\eta_c - 1} \right]$$

$$\eta_{th} = 1 - \frac{1}{\gamma (\eta)^{\gamma-1}} \left[\frac{e^{\gamma} - 1}{e - 1} \right]$$

Mean effective pressure :-

$$MEP = \frac{WD}{D.V} = \frac{m C_p (T_3 - T_2) - m C_v (T_4 - T_1)}{V_1 - V_2}$$

$$V_1 - V_2 = V_1 \left(1 - \frac{V_2}{V_1}\right)$$

$$= V_1 \left(1 - \frac{1}{r}\right)$$

$$= \frac{m R T_1}{P_1} \left(\frac{r-1}{r}\right)$$

$$V_1 - V_2 = \frac{m C_v (r-1) T_1}{P_1} \left(\frac{r-1}{r}\right)$$

$$m.e.p = \frac{m C_p (T_c (r)^{r-1} - T_1 (r)^{r-1}) - m C_v (T_c^* T_1 - T_1)}{m C_v (r-1) T_1 \left(\frac{r-1}{r}\right)}$$

$$\left[\frac{C_p}{C_v} = r \right]$$

$$= P_1 r \left[\frac{r ((T_c - 1) r^{r-1}) - (T_c^* - 1)}{(r-1)(r-1)} \right]$$

$$m.e.p = P_1 r \left[\frac{r (r^{r-1} (T_c - 1)) - (T_c^* - 1)}{(r-1)(r-1)} \right]$$

Problems:

1. Diesel Engine working on Diesel cycle operates with Compression ratio of 17 and Cutt off ratio of '2'. Calculate the air standard efficiency of Engine.

$$\eta_{th} = 1 - \frac{1}{r (r^{r-1})} \left[\frac{T_c^* - 1}{T_c - 1} \right] \text{ Given data :-}$$

$$r = 17$$

$$r_c = 2$$

$$= 1 - \frac{1}{r (r^{r-1})} \left[\frac{T_c^* - 1}{T_c - 1} \right]$$

$$= 1 - \frac{1}{1.4 (17^{0.4})} \left[\frac{2^{1.4} - 1}{2 - 1} \right] = 0.6231 \times 100$$

$$= 62.31\%$$

2. Diesel Engine operating on a air Standard cycle as a Compression ratio of 15. pressure and temp at the begining of ~~suction~~ ^{compression} are 1.04 bar and 15°C the maximum temp of cycle is 2330 K calculate.

i. Cut off ratio

ii. Efficiency of diesel cycle.

Sol: Given $P_1 = 1.04 \text{ bar} = 104 \text{ kPa}$

$$T_1 = 15^\circ\text{C} = 288 \text{ K}$$

$$T_3 = 2330 \text{ K}$$

$$r = \frac{V_1}{V_2} = \frac{V_4}{V_2} = 15$$

required:- $\rho = \frac{T_3}{T_2} = \frac{V_3}{V_2}$

$$2) \quad \eta = 1 - \frac{1}{r^{1/\gamma} (\gamma)^{\gamma-1}} \left[\frac{\rho^\gamma - 1}{\rho - 1} \right]$$

Cut off ratio $\rho = \frac{T_3}{T_2}$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (15)^{1.4-1} = 15^{0.4}$$

$$T_2 = (15)^{0.4} \times 288$$

$$= 850.8 \text{ K}$$

$$\rho = \frac{2330}{850.8} = 2.7386$$

$$\eta_{th} = 1 - \frac{1}{1.4 (15)^{1.4-1}} \left[\frac{(2.7386)^{1.4} - 1}{2.7386 - 1} \right] \times 100$$

$$\eta_{th} = 56.92 \%$$

3. A Diesel Cycle Compression ratio 15 and Compression begins at 0.1 MPa and at 40°C heat added to the cycle is 1.675 MJ/kg Calculate i. maximum temp (T_3)
 ii. Temperature at the end of Compression (T_2)
 iii. Temp at the end of Expansion (T_4)
 iv. Cut off ratio v. Work done
 vi. Efficiency of Cycle.

Sol:-

Given data:-

$$P_1 = 0.1 \text{ MPa}$$

$$r_1 = \frac{V_1}{V_2} = \frac{V_4}{V_2} = 15$$

$$T_1 = 40^\circ\text{C} = 313 \text{ K}$$

$$Q_s = 1.675 \text{ MJ/kg} = 1675 \text{ kJ/kg}$$

Process (1-2)

adiabatic compression:-

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma$$

$$= 0.1 (15)^{1.4}$$

$$P_2 = 4.43127 \text{ MPa}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$= 313 \left(\frac{15}{1} \right)^{1.4-1}$$

$$T_2 = 924.6573 \text{ K}$$

Cut off ratio

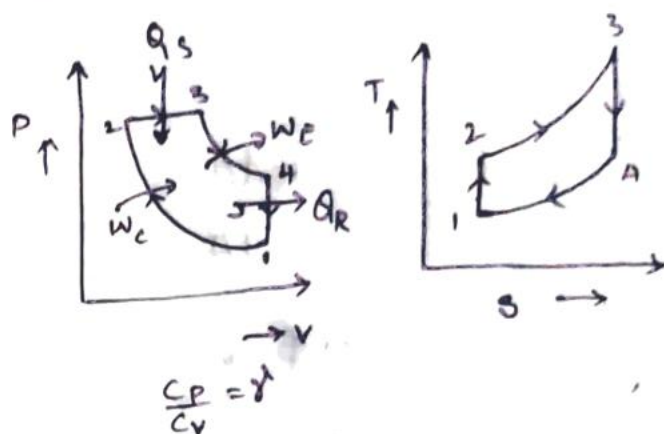
$$\frac{V_3}{V_2} = \frac{T_3}{T_2}$$

$$\frac{V_3}{V_2} = 2.81 = \rho$$

$$Q_R = m C_v (T_4 - T_1)$$

$$= 0.717 (1331.093 - 313)$$

$$= 729.9733 \text{ kJ/kg}$$



heat supplied $Q_s = m C_p (T_3 - T_2)$

$$= m \gamma C_v (T_3 - T_2)$$

$$1675 = 1.4 (0.714) (T_3 - 924.6573)$$

$$T_3 - 924.6573 = 1675.67$$

$$T_3 = 2600.32$$

Process (3-4)

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3} \right)^{\gamma-1}$$

$$= \left(\frac{V_4}{V_2} \times \frac{V_2}{V_3} \right)^{\gamma-1}$$

$$= \left(\frac{V_4}{V_2} \times \frac{V_2}{V_3} \right)^{\gamma-1}$$

$$\frac{T_3}{T_4} = \left(\frac{r_1}{r_c} \right)^{\gamma-1} = \left(\frac{15}{2.8122} \right)^{1.4-1}$$

$$T_4 = \frac{2600.32}{1.9535} = 1331.093$$

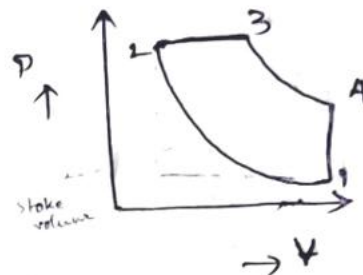
$$\text{work done} = Q_B - Q_R = 945.0267 \text{ kJ/kg}$$

$$\begin{aligned} \text{Efficiency } \eta_{\text{Dies}} &= 1 - \frac{1}{r^{(\gamma-1)}} \left(\frac{e^{\gamma} - 1}{e - 1} \right) \\ &= 1 - \frac{1}{1.4(15)^{0.4}} \left(\frac{(2.8122)^{1.4} - 1}{2.8122 - 1} \right) \times 100 \\ &= 56.60\% \end{aligned}$$

Q. A Diesel Engine has Compression ratio of 14 Cut off takes place at 6% of stroke volume calculate Cut off ratio and Air Standard Cycle Efficiency.

Given data:-

$$r_1 = 14 = \frac{V_1}{V_2}$$



Cut off volume = 6% stroke volume

$$V_3 - V_2 = \frac{6}{100} (V_1 - V_2)$$

$$\frac{V_3 - V_2}{V_1 - V_2} = 0.06$$

$$\frac{\frac{V_3}{V_2} - 1}{\frac{V_1}{V_2} - 1} = 0.06$$

$$\frac{V_3}{V_2} = (0.06)14 + 1 = 1.78 \quad \checkmark$$

$$e = \frac{V_3}{V_2} = 1.78$$

$$\eta_{\text{th}} = 1 - \frac{1}{r^{(\gamma-1)}} \left(\frac{e^{\gamma} - 1}{e - 1} \right) \times 100$$

$$= 1 - \frac{1}{1.4(14)^{0.4}} \left(\frac{1.78^{1.4} - 1}{1.78 - 1} \right) \times 100$$

$$\eta_{\text{th}} = 60.43\% \quad \checkmark$$

5. In an ideal diesel cycle the Compression ratio is 15:1 and the Expansion ratio is 7.5:1. The pressure and temp at the beginning of Compression are 98 kN/m^2 and 44°C pressure at the end of Expansion is 258 kN/m^2 determine.

- The maximum temp attained during the cycle.
- Thermal efficiency of the cycle Take $\gamma = 1.4$

Sol:-

Given

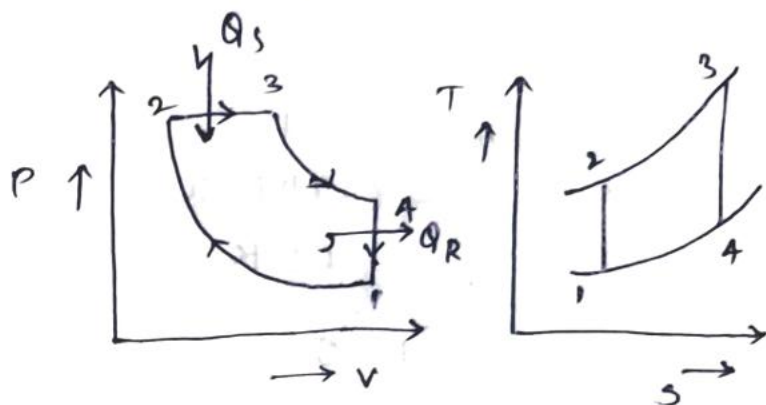
$$P_1 = 98 \text{ kN/m}^2$$

$$T_1 = 44^\circ\text{C} = 317 \text{ K}$$

$$P_4 = 258 \text{ kN/m}^2$$

$$\gamma_1 = \frac{V_1}{V_2} = \frac{V_4}{V_3} = 15$$

$$\gamma_e = \frac{V_4}{V_3} = \frac{V_1}{V_2} = 7.5$$



Process (1-2) :-

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 317 (15)^{0.4}$$

$$T_2 = 936.474 \text{ K}$$

Process (3-4)

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$$

$$T_3 = T_4 \left(\frac{V_4}{V_3}\right)^{\gamma-1}$$

$$T_3 = 1868.432 \text{ K}$$

Process (1-4)

Const volume

$$P_1 V_1 = m R T_1$$

$$P_4 V_4 = m R T_4$$

$$\frac{P_1}{P_4} = \frac{T_1}{T_4}$$

$$T_4 = T_1 \frac{P_4}{P_1}$$

$$= 317 \times \frac{258}{98}$$

$$T_4 = 834.551 \text{ K}$$

Thermal efficiency

$$\eta_{th} = 1 - \frac{1}{\gamma} \left(\frac{T_4 - T_1}{T_3 - T_2} \right)$$

$$= 1 - \frac{1}{1.4} \left(\frac{834.551 - 317}{1868.432 - 936.474} \right) \times 100$$

$$= 60.333 \%$$

6. Two Engines are operated on Otto and diesel Cycles with the following data. maximum temp 1400 K Exhaust temp $= 700\text{ K}$ State the air at the beginning of Compression of 0.1 MPa , 300 K determine the Compression ratio, maximum pressure, Efficiency and rate of work outputs (for 1 kg/min of air) of the respective Cycles.

Sol: Given:-

$$T_3 = 1400\text{ K}$$

$$T_A = 700\text{ K}$$

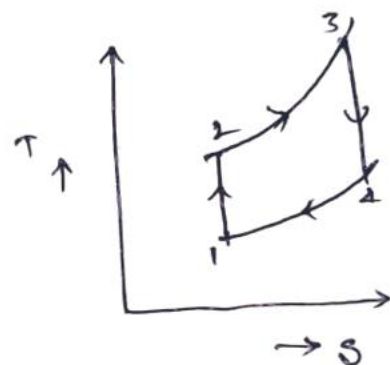
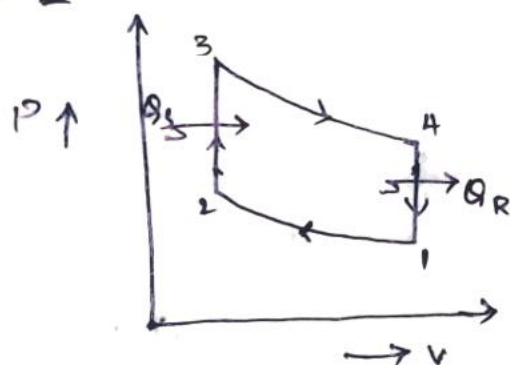
$$P_1 = 0.1\text{ MPa}$$

$$T_1 = 300\text{ K}$$

required:-

$$\eta = \frac{V_1}{V_2} = \frac{V_4}{V_3}$$

Otto Cycle:-



Process (1-2)
(adiabatic process):-

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$T_2 = (5.657)^{\gamma-1} 300$$

$$T_2 = 600\text{ K}$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$P_2 = 0.22\text{ MPa} \quad 1.1314\text{ MPa}$$

Process (3-4)

adiabatic process

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$\left(\frac{V_4}{V_3}\right)^{\gamma-1} = \left(\frac{T_3}{T_4}\right)$$

$$\frac{V_4}{V_3} = \left(\frac{T_3}{T_4}\right)^{\frac{1}{\gamma-1}}$$

$$\eta = \frac{V_4}{V_3} = \left(\frac{1400}{700}\right)^{\frac{1}{0.4}}$$

$$\eta = 5.657$$

Process (2-3)
Constant volume

$$\frac{P_2}{P_3} = \frac{T_2}{T_3}$$

$$P_3 = P_2 \left(\frac{T_3}{T_2} \right) \\ = 1.1314 \left(\frac{1400}{600} \right)$$

$$P_3 = 2.6399 \text{ MPa} \checkmark$$

heat supplied

$$Q_s = m C_v (T_3 - T_2) \\ = 1 \times 0.717 (1400 - 600)$$

$$Q_s = 573.6 \text{ kJ/min}$$

$$\text{Efficiency} = 1 - \left(\frac{T_4 - T_1}{T_3 - T_2} \right)$$

$$\eta_{\text{otto}} = 1 - \left(\frac{T_4 - T_1}{T_3 - T_2} \right) \\ = 1 - \left(\frac{700 - 300}{1400 - 600} \right)$$

$$\eta_{\text{otto}} = 50\%$$

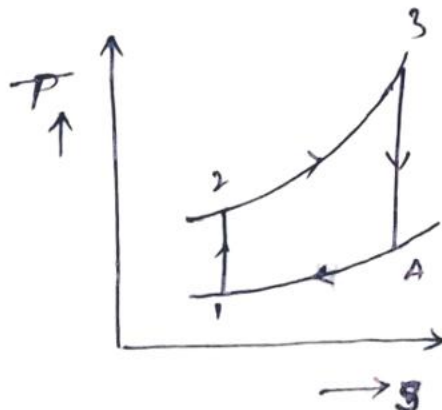
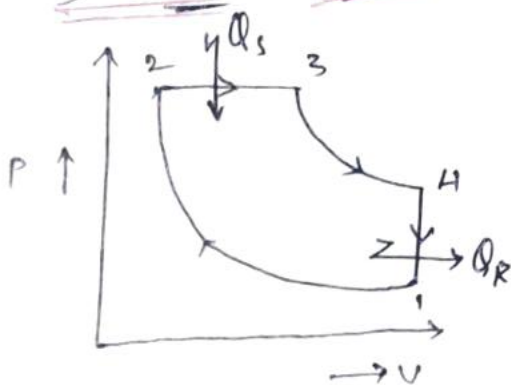
$$\eta_{\text{otto}} = \frac{W.D}{Q.s}$$

$$W.D = \eta_{\text{otto}} \times Q.s$$

$$= 0.5 \times 573.6$$

$$\checkmark W.D = 286.8 \text{ kJ/min}$$

Diesel Cycle:-



Process (3-4):-
adiabatic process

$$\frac{T_A}{T_B} = \left(\frac{V_3}{V_4} \right)^{\gamma-1}$$

$$\left(\frac{V_4}{V_3} \right)^{\gamma-1} = \frac{T_3}{T_A}$$

$$\frac{V_4}{V_3} = \left(\frac{T_3}{T_A} \right)^{\frac{1}{\gamma-1}} = 5.65685$$

$$P_3 V_3^{\gamma} = P_4 V_4^{\gamma}$$

$$P_3 = P_4 \left(\frac{V_4}{V_3} \right)^{\gamma}$$

$$= 0.233 (5.65685)^{1.4}$$

$$P_3 = 2.6398 \text{ MPa}$$

Process (A-1)

$$P_1 V_1 = m R T_1$$

$$P_A V_A = m R T_A$$

$$\frac{P_A}{P_1} = \frac{T_A}{T_1}$$

$$P_A = P_1 \left(\frac{T_A}{T_1} \right)$$

$$= 0.1 \left(\frac{700}{300} \right)$$

$$P_A = 0.2333 \text{ MPa}$$

Process (1-2)

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\left(\frac{V_1}{V_2}\right)^\gamma = \frac{P_2}{P_1}$$

$$\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} = \left(\frac{2.63986}{0.1}\right)^{\frac{1}{1.4}}$$

Compression ratio $r = \frac{V_1}{V_2} = 10.3613$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 300 (10.3613)^{0.4}$$

$$T_2 = 764.3411 \text{ K}$$

Process (2-3) :-

Constant pressure :-

$$P_2 V_2 = m R T_2$$

$$P_3 V_3 = m R T_3$$

$$r = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{1400}{764.3411} = 1.831$$

$$\text{Efficiency} = 1 - \frac{1}{r^{\gamma(\gamma-1)}} \left(\frac{r^\gamma - 1}{r - 1} \right)$$

$$= 1 - \frac{1}{1.4(10.3613)^{0.4}} \left(\frac{(1.831)^{1.4} - 1}{1.831 - 1} \right)$$

$$= 0.5505 \times 100$$

$$\eta_{th} = 55.05\%$$

Heat Supplied

$$Q_s = m C_p (T_3 - T_2)$$

$$= m \gamma C_v (T_3 - T_2)$$

$$= (1.4)(0.711)(1400 - 764.3411)$$

$$Q_s = 638.07438$$

$$\text{kJ/min}$$

$$\eta_{th} = \frac{W.D}{Q_s}$$

$$W.D = \eta_{th} \times Q_s = 0.5505 \times 638.07438$$

$$= 351.2 \text{ kJ/min}$$

Comparison

Otto Cycle

1. Homogeneous mixture of air and fuel formed in the carburetor and is supplied to engine cylinder
2. Ignition is initiated by means of an electrical Spark plug
3. Power output is controlled by varying the mass of air fuel mixture by means of throttle valve in the carburetor

Diesel Cycle

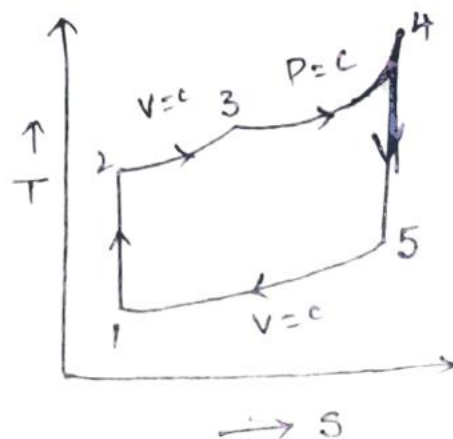
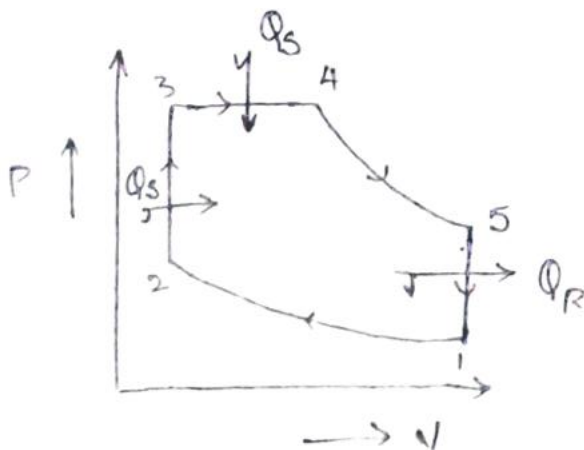
No carburetor is used for air & fuel mixture. They enter into engine cylinder ^{individually} ~~indually~~. Fuel, air mixture is heterogeneous.

No spark plug is used. Compression ratio is high and high temp of air ignites the fuel.

No throttle valve is used. Power output is controlled only by means of the mass of fuel injected by the injector.

DUAL Cycle:-

This is combination of Otto Cycle & Diesel Cycle. It's also called as Limited pressure cycle.



Process (1-2) - Isentropic Compression

Process (2-3) - Constant volume heat addition Process

Process (3-4) - Constant pressure heat addition

Process (4-5) - Isentropic Expansion

Process (5-1) - Constant volume heat rejection

Heat addition $Q_s = m C_v (T_3 - T_2) + m C_p (T_4 - T_3)$

Heat rejection $Q_R = m C_v (T_5 - T_1)$

$W.D = Q_s - Q_R$

Efficiency $= 1 - \frac{Q_R}{Q_s}$

$$\eta = 1 - \frac{m C_v (T_5 - T_1)}{m C_v (T_3 - T_2) + m C_p (T_4 - T_3)}$$

$$\eta = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma (T_4 - T_3)}$$

The Efficiency of the Cycle can be expressed in the terms of following ratio.

1. Compression ratio $r_1 = \frac{V_1}{V_2} = \frac{V_1}{V_3} = \frac{V_5}{V_2} = \frac{V_5}{V_3}$

2. Cutoff ratio $\rho = \frac{V_4}{V_3} = \frac{V_4}{V_2}$

3. Expansion ratio $r_e = \frac{V_5}{V_4} = \frac{V_1}{V_4}$

4. Constant volume pressure ratio $r_p = \frac{P_3}{P_2}$

Process (1-2):- (isentropic process)

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1 (r)^{\gamma-1} \rightarrow (1)$$

Process (2-3):- (constant volume)

$$\frac{T_3}{T_2} = \frac{P_3}{P_2} \Rightarrow T_3 = T_2 \left(\frac{P_3}{P_2}\right) = T_2 (r_p) = T_1 (r)^{\gamma-1} \cdot (r_p) \rightarrow (2)$$

Process (3-4):- (constant pressure)

$$\frac{T_4}{T_3} = \frac{V_4}{V_3} \Rightarrow T_4 = T_3 \left(\frac{V_4}{V_3}\right) = T_3 (r_c) = T_3 (e)$$

$$T_4 = T_1 (r)^{\gamma-1} (r_p) (e) \rightarrow (3)$$

Process (4-5):- (isentropic expansion):-

$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{\gamma-1}$$

$$= \left(\frac{V_4}{V_2} \times \frac{V_2}{V_5}\right)^{\gamma-1} = \left(e \times \frac{1}{r}\right)^{\gamma-1}$$

$$T_5 = T_4 \left(\frac{e}{r}\right)^{\gamma-1}$$

$$T_5 = T_1 (r)^{\gamma-1} \cdot r_p \cdot e \times \frac{e^{\gamma-1}}{r^{\gamma-1}}$$

$$\boxed{T_5 = T_1 r_p e^{\gamma}} \rightarrow (4)$$

$$\text{Efficiency } \eta_{th} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

$$= 1 - \frac{(T_1 r_p e^{\gamma} - T_1)}{(T_1 (r)^{\gamma-1} r_p - T_1 (r)^{\gamma-1}) + \gamma(T_1 (r)^{\gamma-1} r_p e - T_1 (r)^{\gamma-1} r_p)}$$

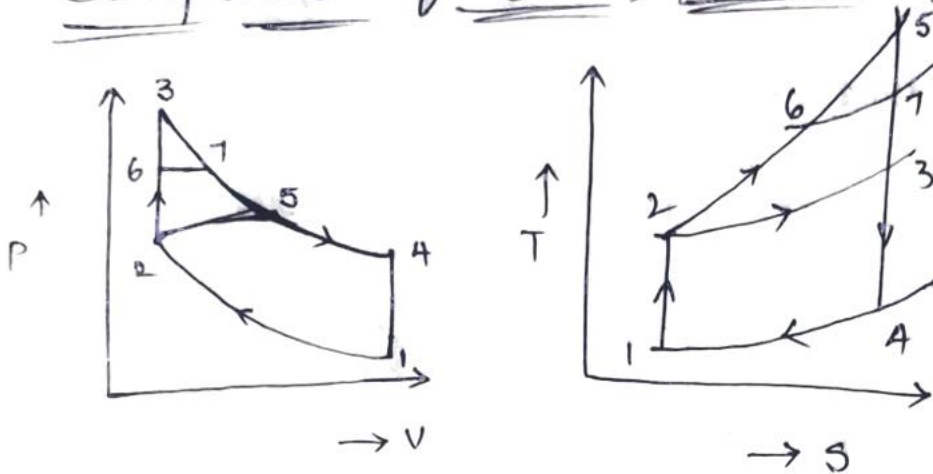
$$= 1 - \frac{1}{(r)^{\gamma-1}} \frac{(r_p e^{\gamma} - 1)}{((r_p - 1) + \gamma r_p (e - 1))}$$

Mean effective pressure

$$mep = \frac{WD}{D \times v_d} = \frac{WD}{v_1 - v_2}$$

$$mep = \frac{P_1 r}{(r-1)(\gamma-1)} \left[r^{\gamma-1} [(r_p-1) + \gamma r_p (e-1)] - (r_p e^{\gamma}-1) \right]$$

Comparison of Otto, Diesel & Dual Cycle:-



Otto Cycle :- 1-2-3-4-1

Diesel Cycle :- 1-2-5-4-1

Dual Cycle :- 1-2-6-7-4-1

The three cycles can be compared on the basis of either the same compression ratio (or) the same maximum pressure and temp.

Case 1:-

For the same compression ratio and same heat rejection.

* In the above diagram compression ratio $r = \frac{V_1}{V_2}$ for all the three cycles.

* Heat rejection is also same for all the three cycles
i.e. $Q_R = mc_v(T_4 - T_1)$

* Generally work done is represented as area occupied by P-v diagram i.e. more area of P-v diagram more will be the work done.

* From the P-v diagram it is evident that

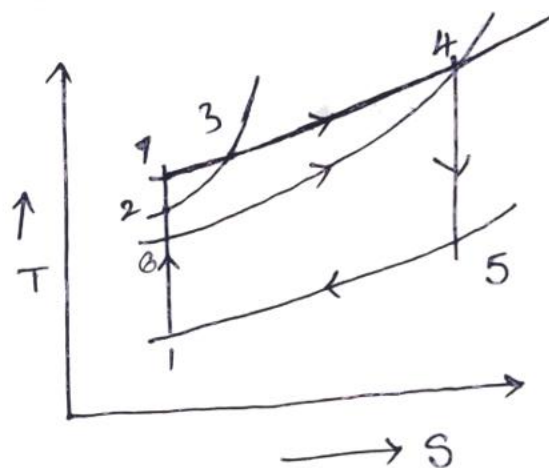
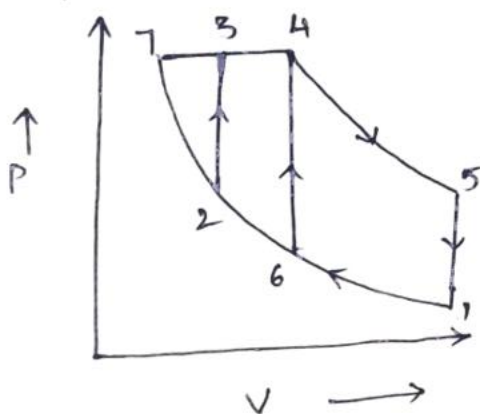
$$\text{Area of Otto} > \text{Area of dual} > \text{Area of Diesel}$$

$$W.D \text{ of Otto} > W.D \text{ of dual} > W.D \text{ of Diesel}$$

$$\eta_{\text{Otto}} > \eta_{\text{dual}} > \eta_{\text{Diesel}}$$

As work done increases efficiency also increases

Case 2:- For the same maximum pressure & temp & same heat rejection



Otto Cycle $\rightarrow (1-2-4-5-1) \rightarrow \eta = \frac{v_1}{v_2}$

Diesel Cycle $\rightarrow (1-7-4-5-1) \rightarrow \eta = \frac{v_1}{v_7}$

Dual Cycle $\rightarrow (1-2-3-4-5-1) \rightarrow \eta = \frac{v_1}{v_3}$

* maximum pressure $P_1 = P_3 = P_4$ for all 3 cycles

$$W.D \text{ Diesel} > W.D \text{ Dual} > W.D \text{ Otto}$$

$$\eta_{\text{Diesel}} > \eta_{\text{Dual}} > \eta_{\text{Otto}}$$

* maximum temp T_4 for all 3 cycles.

1. In a Compression ignition Engine working on dual Combustion cycle the pressure and temp at the starting of compression are 1 bar and 300 K. At the end of compression pressure reaches a value of 25 bar. 420 KJ of heat is supplied per kg of air during constant volume heating and pressure becomes 2.8 bar at the end of isentropic expansion. Estimate the ideal thermal efficiency take $C_p = 1.005 \text{ kJ/kg}\cdot\text{K}$ and $C_v = 0.712 \text{ kJ/kg}\cdot\text{K}$

Sol:-

Given data :-

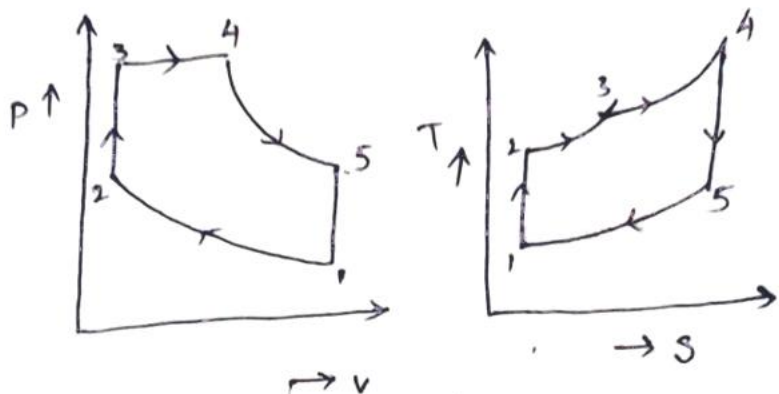
$$P_1 = 1 \text{ bar}$$

$$T_1 = 300 \text{ K}$$

$$P_2 = 25 \text{ bar}$$

$$Q_{s(2-3)} = 420 \text{ kJ}$$

$$P_5 = 2.8 \text{ bar}$$



Process (1-2) :-

(Isentropic Process)

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^\gamma$$

$$\frac{V_1}{V_2} = \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} = \left(\frac{25}{1} \right)^{\frac{1}{1.4}}$$

$$\eta = \frac{V_1}{V_2} = 9.966$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = 300 (9.966)^{0.4}$$

$$T_2 = 752.5453 \text{ K}$$

Heat supplied

$$Q_{s(2-3)} = m C_v (T_3 - T_2)$$

$$420 = 1.005 (T_3 - 752.5453)$$

$$489.986 = T_3 - 752.5453$$

$$T_3 = 1342.43 \text{ K}$$

$$\eta_p = \frac{P_3}{P_2} = \frac{T_3}{T_2} = \frac{1342.43}{752.5453}$$

$$\eta_p = 1.784$$

$$P_3 = 1.784 \times 25$$

$$= 44.596 \text{ bar}$$

Process (4-5)

$$\frac{P_4}{P_5} = \left(\frac{V_5}{V_4} \right)^{\gamma} \Rightarrow \frac{V_5}{V_4} = \left(\frac{P_4}{P_5} \right)^{\frac{1}{\gamma}}$$

$$P_4 = P_3$$

$$\frac{V_5}{V_4} = 7.222$$

$$\frac{V_4}{V_5} = 0.138$$

Cut off ratio $\rho = \frac{V_4}{V_3}$

$$\rho = \frac{V_4}{V_5} \times \frac{V_5}{V_3} = \frac{V_4}{V_5} \times \frac{V_1}{V_2}$$

$$= 0.138 \times 9.96$$

$$\rho = 1.379$$

$$\eta_{\text{Dual}} = 1 - \frac{1}{(\gamma)^{\gamma-1}} \left[\frac{\gamma_P \rho^{\gamma} - 1}{(\gamma_P - 1) + \gamma \gamma_P (\rho - 1)} \right]$$

$$= 1 - \frac{1}{(1.4)^{1.4-1}} \left[\frac{1.784 (1.379)^{1.4} - 1}{(1.784 - 1) + 1.4 (1.784) (1.379 - 1)} \right]$$

$$= 0.5858 \times 100$$

$$\eta_{\text{Dual}} = 58.58\%$$

- (2) An ideal dual cycle engine works with a stroke volume of 10 ltr of air with a compression ratio of 16. The pressure & temperature of air before isentropic compression is 1 bar & 300 K respectively. If heat is added at constant pressure of 70 bar & for 5% of the stroke, Determine (i) Pressure ratio

- ② Cut off ratio ③ Mass of air ④ Heat added per cycle
 ⑤ Heat rejected/cycle ⑥ WD ⑦ η ⑧ mep
 $C_p = 1 \text{ kJ/kg K}$; $C_v = 0.714 \text{ kJ/kg K}$

Given:-

$$V_3 = V_1 - V_2 = V_5 - V_3 = 10 \text{ lit}$$

$$V_1 - V_2 = \frac{10}{1000} = 0.01 \text{ m}^3$$

$$r_1 = \frac{V_1}{V_2} = 16 = \frac{V_5}{V_3}$$

$$P_1 = 1 \text{ bar}$$

$$T_1 = 300 \text{ K}$$

$$P_3 = P_4 = 70 \text{ bar}$$

Cut off $r = 5\%$ of stroke volume.

reg: $r_p = \frac{P_3}{P_2}$

$$2. \rho = \frac{V_4}{V_3}$$

$$3. P_1 V_1 = m R T_1$$

$$4. Q_s = m C_v (T_3 - T_2) + m C_p (T_4 - T_3)$$

$$5. Q_R = m C_v (T_5 - T_1)$$

$$6. \text{W.D} = Q_s - Q_R$$

$$7. \eta = \frac{\text{W.D}}{Q_s}$$

$$\text{m.e.p} = \frac{\text{W.D}}{\text{D.V}} = \frac{\text{W.D}}{V_s}$$

Process (1-2)

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^{1.4} = P_1 (16)^{1.4}$$

$$P_2 = 48.05 \text{ bar}$$

$$P_2 = 48.5 \text{ bar}$$

$$1. r_p = \frac{P_3}{P_2} = \frac{70}{48.5} = 1.44 \text{ --- ①}$$

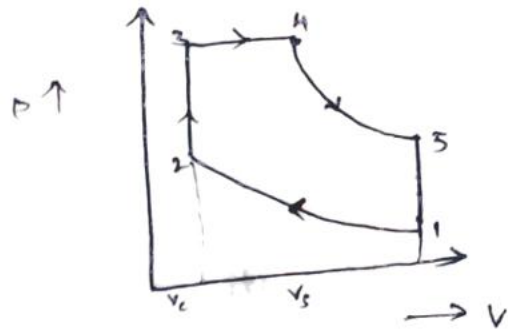
$$2. \rho = \frac{V_4}{V_3} = \frac{V_4}{V_2}$$

$$\frac{V_1}{V_2} = 16 \Rightarrow V_1 = 16 V_2$$

$$V_1 - V_2 = 0.01 \text{ (given)}$$

$$V_2 = 0.67 \times 10^{-3} \text{ m}^3$$

$$\left[\because V_3 = V_2 \right]$$



$$V_4 = V_3 + \frac{5}{100} \times 0.01$$

$$= (0.67 \times 10^{-3}) + (0.05 \times 0.01)$$

$$V_4 = 1.17 \times 10^{-3} \text{ m}^3$$

$$e = \frac{V_4}{V_2} = \frac{1.17 \times 10^{-3}}{0.67 \times 10^{-3}} = 1.746 \rightarrow \textcircled{2}$$

③ mass of air

$$P_1 V_1 = m R T_1$$

$$V_1 = 16 V_2 = 16(0.67 \times 10^{-3})$$

$$V_1 = 0.01072 \text{ m}^3$$

$$P_1 = 1 \text{ bar} = \text{kN}$$

$$R = 0.287 \text{ kJ}$$

$$T_1 = 300 \text{ K}$$

$$m = \frac{P_1 V_1}{R T_1} = 0.0124 \text{ kg} \rightarrow \textcircled{3}$$

④ process (1-2)

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 300 (16)^{0.4} = 909.429 \text{ K}$$

process (2-3)

const pressure volume

$$\frac{P_2}{P_3} = \frac{T_2}{T_3}$$

$$T_3 = \frac{P_3}{P_2} \times T_2 = \gamma_p T_2$$

$$= 1309.57 \text{ K}$$

Heat

process (3-4) (const pressure)

$$\frac{V_4}{V_3} = \frac{T_4}{T_3}$$

$$T_4 = T_3 \left(\frac{V_4}{V_3}\right) = T_3 \times e$$

$$= 2286.52 \text{ K}$$

$$Q_s = m C_v (T_3 - T_2) + m C_p (T_4 - T_3)$$

$$= 15.73 \text{ kJ/kg} \rightarrow \textcircled{4}$$

$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5} \right)^{\gamma-1}$$

$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_3} \times \frac{V_3}{V_5} \right)^{\gamma-1}$$

$$T_5 = \left(\rho \times \frac{1}{r_1} \right)^{\gamma-1} T_4$$

$$= 1.746 \times \frac{1}{16} \times 2286.52$$

$$= 942.637 \text{ K} \quad \checkmark$$

$$Q_R = m C_v (T_5 - T_1)$$

$$= 0.0124 \times 0.717 (942.637 - 300)$$

$$= 5.71 \text{ kJ/kg} \quad \checkmark$$

$$W.D = Q_S - Q_R$$

$$= 10.01 \text{ kJ/kg} \quad \checkmark$$

$$\eta = \frac{W.D}{Q_S} = 0.6363 \times 100$$

$$= 63.63\% \quad \checkmark$$

$$mep = \frac{W.D}{V_1 - V_2} = \frac{10.01}{0.01}$$

$$= 1001 \text{ kN/m}^2$$

$$\text{kN/m}^2 = \text{kPa}$$

$$\frac{\text{kJ/s}}{\text{m}^3/\text{s}} = \frac{\text{kN-m}}{\text{m}^3\text{-m}}$$

3. An air standard dual cycle has a Compression ratio of 16 and Compression begins at 1 bar and 50°C, the maximum pressure is 170 bar the heat transferred to air at Constant pressure is equal to that at Constant volume. Estimate
- The pressure and temperatures at all Coordinate points of the cycle

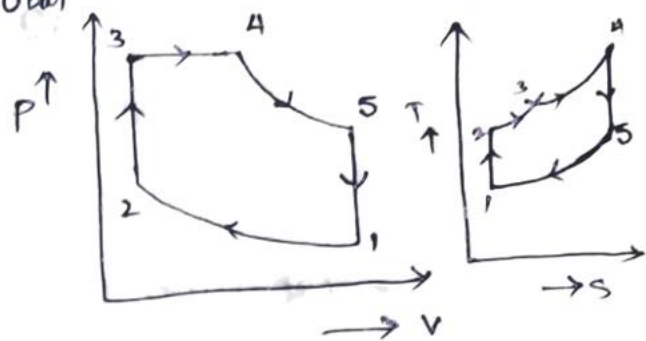
ii. Thermal Efficiency of dual cycle.

iii. mean effective pressure Tak $C_v = 0.718 \text{ kJ/kg K}$

$$C_p = 1.005 \text{ kJ/kg K}$$

$$P_3 = P_4 = 70 \text{ bar}$$

$$\begin{aligned} \text{Given } r &= 16 \\ &= \frac{V_1}{V_2} \\ &= \frac{V_5}{V_3} \end{aligned}$$



Process (1-2)

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^\gamma$$

$$\left(\frac{V_1}{V_2} \right) = \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}}$$

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma$$

$$P_2 = 1 (16)^{1.4} = 48.5 \text{ bar}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$\Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = 323 (16)^{0.4}$$

$$T_2 = 979.1529 \text{ K}$$

Process 2-3

$$\frac{P_3}{P_2} = \frac{T_3}{T_2}$$

$$T_3 = \frac{P_3}{P_2} T_2$$

$$= \frac{70}{48.5} \times 979.1529$$

$$T_3 = 1413.21 \text{ K}$$

Process (3-4)

Const pressure

$$\frac{T_3}{T_4} = \frac{V_3}{V_4} \Rightarrow \frac{V_3}{V_4} = \frac{1413.21}{1723.25}$$

$$\frac{V_4}{V_3} = \frac{1723.25}{1413.21} = 1.2196$$

Given heat supplied Const volume $m C_v (T_3 - T_2) = m C_p (T_4 - T_3)$

is Equal to Const pressure $m C_v (T_3 - T_2) = m \gamma C_v (T_4 - T_3)$

$$T_3 - T_2 = \gamma (T_4 - T_3)$$

$$310 = T_4 - 1413.21$$

$$T_4 = 1723.25 \text{ K}$$

Process (4-5)

$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5} \right)^{\gamma-1}$$

$$T_5 = T_4 \left(\frac{V_4}{V_3} \times \frac{V_3}{V_5} \right)^{\gamma-1}$$

$$\frac{P_5}{P_4} = \left(\frac{V_4}{V_5} \right)^\gamma$$

$$\frac{P_5}{P_4} = \left(\frac{V_4}{V_3} \times \frac{V_3}{V_5} \right)^\gamma$$

$$T_5 = 1723.25 \left(1.2196 \times \frac{1}{16} \right)^{0.4}$$

$$T_5 = 615.442 \text{ K}$$

$$T_5 = 615.442 \text{ K}$$

$$P_5 = P_4 \left(1.2196 \times \frac{1}{16} \right)^{1.4}$$

$$= 70 \left(\frac{1.2196}{16} \right)^{1.4}$$

$$P_5 = 1.9056 \text{ bar}$$

Heat Supplied $Q_s = m C_v (T_3 - T_2) + m C_p (T_4 - T_3)$

$$= 0.718(1413.21 - 979.1529) + 1.005(1723.25 - 1413.21)$$

$$Q_s = 623.2432 \text{ kJ/kg}$$

Heat Rejection $Q_R = m C_v (T_5 - T_1)$

$$= 0.718(615.4422 - 323)$$

$$= 209.9735 \text{ kJ/kg}$$

workdone $= Q_s - Q_R$

$$W.D = 413.2697 \text{ kJ/kg}$$

$$\eta = 1 - \frac{Q_R}{Q_s} = 0.6631$$

$$\eta = 66.31\%$$

mean effective pressure