Course: **Network Analysis and Synthesis**

Course Code: **24AEEE21T**

Branch(s): **EEE**

Prepared by: **Dr. M.Padma Lalitha**

Designation: **Professor**

Department: **Electrical and Electronics Engineering**

Title of the Course: Network Analysis and Synthesis
Category: PCC
Semester: III
Course Code: 24AEEE32T
Branch/es: EEE

Lecture Hours	Tutorial Hours	Practice Hours	Credits
2	1	0	3

Course Objectives:

1. To introduce the principles of three-phase circuits and power measurement techniques.
2. To explain the fundamentals and applications of Laplace transforms in circuit analysis.
3. To familiarize students with the transient behavior of electrical circuits under DC and AC excitations.
4. To provide a foundation in Fourier series and Fourier transforms for analyzing periodic and aperiodic signals.
5. To develop an understanding of network functions and the synthesis of RL, RC, and LC networks.

Course Outcomes:

At the end of the course, the student will be able to …

1. Analyze three-phase circuits and measure real and reactive power using two-wattmeter method.
2. Apply Laplace transform techniques to solve electrical circuits and interpret initial and final conditions of system responses.
3. Determine the transient response of RL, RC, and RLC circuits for both DC and AC excitations using time-domain analysis.
4. Compute the Fourier series and transforms of electrical signals and analyze their frequency-domain characteristics.
5. Evaluate network functions and synthesize RL, RC, and LC one-port networks satisfying positive real function criteria.

Unit 1 Analysis of three phase circuits **9**

Phase sequence, star and delta connection of sources and loads, relation between line and phase quantities, analysis of balanced three phase circuits, analysis of unbalanced delta, Analysis of unbalanced 3-wire star using millman's theorem, analysis of 4-wire star, measurement of active and reactive power using two-wattmeter method for measurement of three phase power.

Unit 2 Laplace Transforms **9**

Definition of Laplace transform – advantages - Laplace transform of important functions and signals – inverse Laplace transform - application of Laplace transform to electrical circuits – initial and final value theorem.

Unit 3 Transient Analysis (DC & AC) **9**

Introduction- Initial Conditions -DC Transient response of RL, RC and RLC series circuits -Response of RL, RC and RLC series circuits with AC excitation.

Unit 4 Fourier Series & Fourier Transform **9**

Introduction- trigonometric Fourier series - waveform symmetry, exponential form of Fourier series, effective value, application to electrical circuits. Fourier transforms & Properties, Fourier transform of important signals and periodic signals.

Unit 5 Network Functions and Synthesis **9**

Network functions- necessary conditions for driving point function-necessary conditions for transfer function –

Hurwitz polynomials – positive Real functions - definitions and properties - synthesis of single port networks (RL, RC and LC networks.

Prescribed Textbooks:

1. A. Chakrabarti. Circuit theory (Analysis and Synthesis) 7th Revised edition 2020.

Reference Books:

1. M.E. Van Valkenberg. Network Analysis. 3rd edition, Pearson Publications, New Delhi 2019.
2. A. Sudhakar, Shyammohan S Palli. Circuits and Networks. (Analysis and Synthesis), 5th edition, Tata McGrawHill Publishing company Ltd, 2017.
3. Umesh Sinha. Network Analysis and Synthesis. 5th edition, Satyaprakashan, New Delhi, 2018.
4. D. Roy Choudhury. Networks and Systems. 3rd edition, New Age international publishers 2023.

Web Resources:

1. https://www.youtube.com/watch?v=eUOFLraM0G8
2. https://www.youtube.com/watch?v=JIFwqRn0LQk
3. https://archive.nptel.ac.in/courses/108/105/108105159/
4. https://www.youtube.com/watch?v=I4Fr3qG0xVY
5. https://www.youtube.com/watch?v=CykSP552LUs

CO-PO-PSO Mapping:

Course Outcomes	Engineering Knowledge	Problem Analysis	Design/Development of solutions	Conduct investigations of complex problems	Engineering Tool Usage	The Engineer and The World	Ethics	Individual and Collaborative Team work	Communication	Project management and finance	Life-long learning	PSO1	PSO2
24AEEE32T.1	3	3	2	2	3	2	1	2	2	2	2	3	2
24AEEE32T.2	3	3	2	2	2	1	–	2	2	1	2	3	2
24AEEE32T.3	3	2	2	3	3	1	–	2	2	2	3	3	3
24AEEE32T.4	3	3	2	3	3	2	–	2	2	2	3	3	3
24AEEE32T.5	3	3	3	3	3	2	1	2	2	3	3	3	3

ANNAMACHARYA UNIVERSITY, RAJAMPET

(ESTD UNDER AP PRIVATE UNIVERSITIES (ESTABLISHMENT AND REGULATION) ACT, 2016
RAJAMPET, Annamayya District, AP, INDIA

Department of Electrical & Electronics Engineering

II B.Tech I Semester

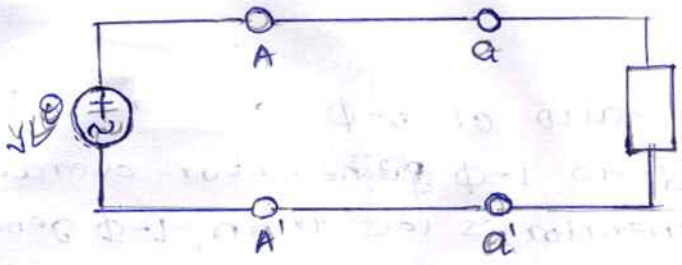
Network Analysis and Synthesis (24AEEE21T)
(Electrical and Electronics Engineering)

Unit-I (Analysis of Three Phase Circuits)

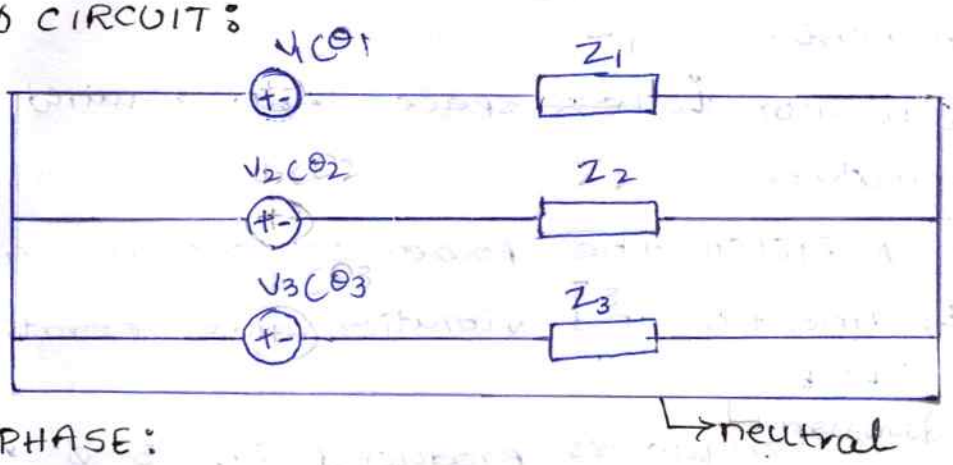
Syllabus:

Phase sequence, star and delta connection of sources and loads, relation between line and phase quantities, analysis of balanced three phase circuits, analysis of unbalanced delta, Analysis of unbalanced 3-wire star using millman's theorem, analysis of 4-wire star, measurement of active and reactive power using two-wattmeter method for measurement of three phase power.

1-ϕ circuit :



3-ϕ CIRCUIT :



$|V_1| = |V_2| = |V_3| = V$

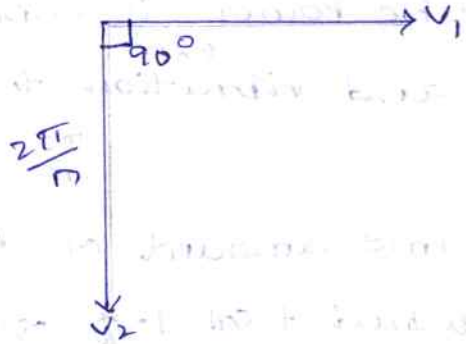
POLYPHASE :

Poly phase is a circuit consisting two(2) or more independent AC sources connected to the one impedance to each phase.

BALANCED POLY PHASE :

A system with 'n' Phases is said to be balanced when the voltage sources have equal magnitude with a progressive phase difference of "$\frac{2\pi}{n}$" where $n=3,4\ldots\infty$ and for 2-ϕ system the phase difference is "$\pi/2$"

$|V_1| = |V_2| = V$, $V_1 = V\angle 0^\circ$, $V_2 = V\angle -90^\circ$

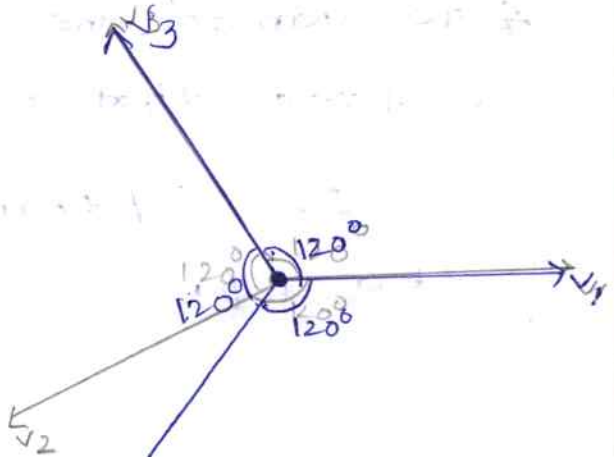


$|V_1| = |V_2| = |V_3| = V$

Phase difference $= \frac{2\pi}{3} = 120^\circ$

$V_1 = V\angle 0^\circ$, $V_2 = V\angle -120^\circ$

$V_3 = V\angle -240^\circ = V\angle 120^\circ$



ADVANTAGES:

Generator:

* Power to weight ratio of 3-ϕ generator is high when it compared to 1-ϕ generator overall transistor 3-ϕ generator is less than 1-ϕ generator.
* Due to reduce weight transformer of generator is convienent.
* 3-ϕ generator is less space required for 3-ϕ generator.
* In 3-ϕ system the power is constant which results smooth and vibration free operation of vessels.
* Uniform torque is produced in 3-ϕ motors where in 1-ϕ motors pulsating torque is Produced.

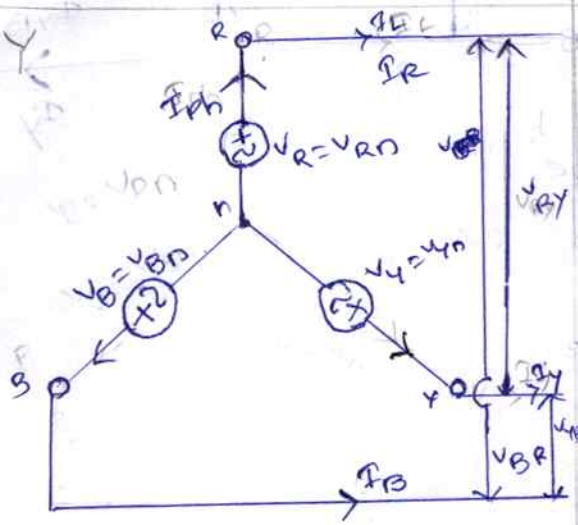
TRANSMISSION:

* Power to weight ratio of 3-ϕ transformer is high when it is compared to 1-ϕ transformer. overall ~~transistor~~ of 3-ϕ transformer is less than 1-ϕ transistor
* 3-ϕ transformer requires less space than 1-ϕ
* In 3-ϕ system the power is constant which results in smooth and vibration free operation of vessels.
* TO transtor the same amount of power over a fixed that required for 1-ϕ system.

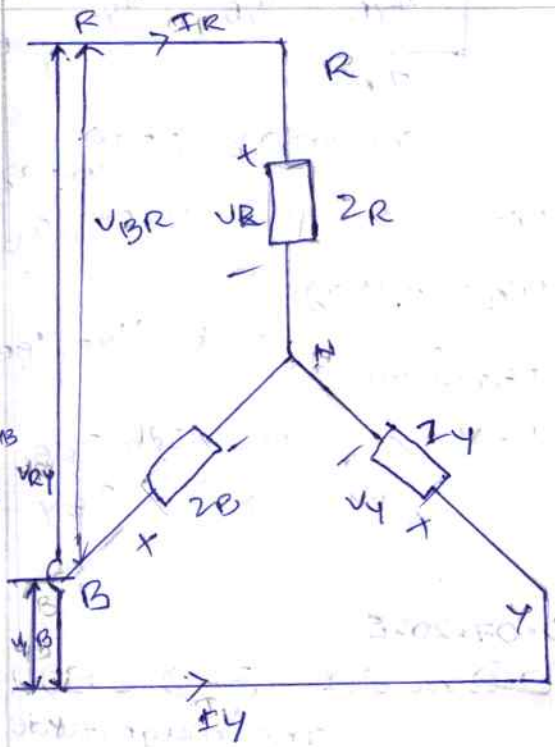
∴ 3-ϕ transmission is economical compared to 1-ϕ.

09-09-2025

3-ϕ sources



3-ϕ loads



The conductor which connects source and load is called Line.

* The current passing through the line is line current

Line current = I_R, I_Y, I_B

* Voltage blw too lines is line voltage.

* The each branch of the 3-ϕ source (or) line is known as phase

* The current through the phase is phase current.

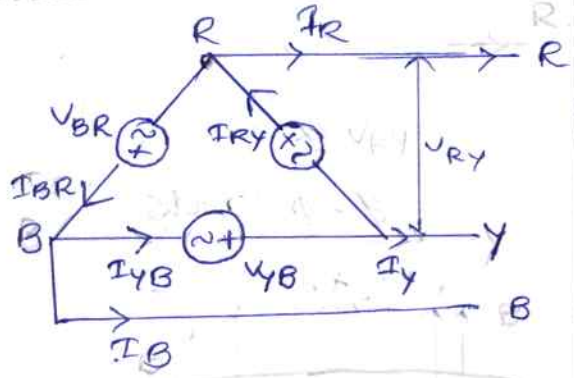
* The voltage across a phase is known as phase voltage.

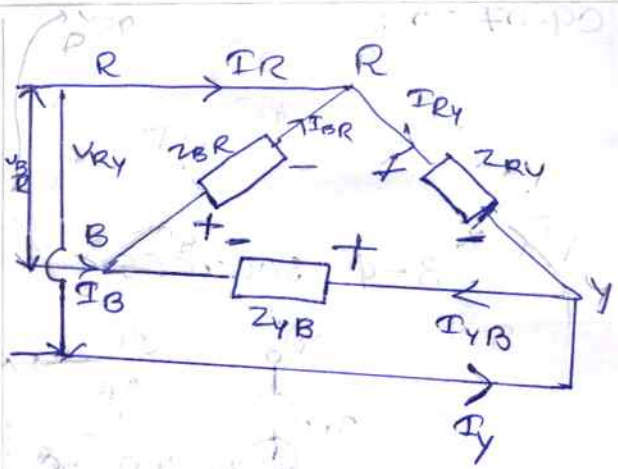
Line current = I_R, I_Y, I_B

Line voltage = V_{RY}, V_{YB}, V_{BR}

Phase voltage = V_R, V_Y, V_B

Delta





Line voltage = phase voltage

Line currents - I_R, I_Y, I_B

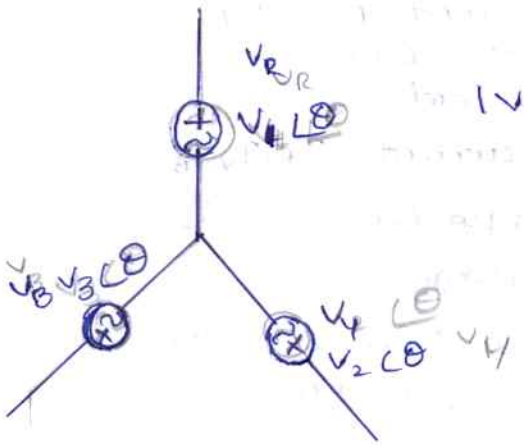
Line voltage & Phase voltage - V_{RY}, V_{YB}, V_{BR}

Phase currents - I_{RY}, I_{RB}, I_{BY}

10.07.2025

Balanced 3-Ø sources:

The magnitude of all voltage sources are same or equal and phase difference is 120° (they are displaced by 120°)



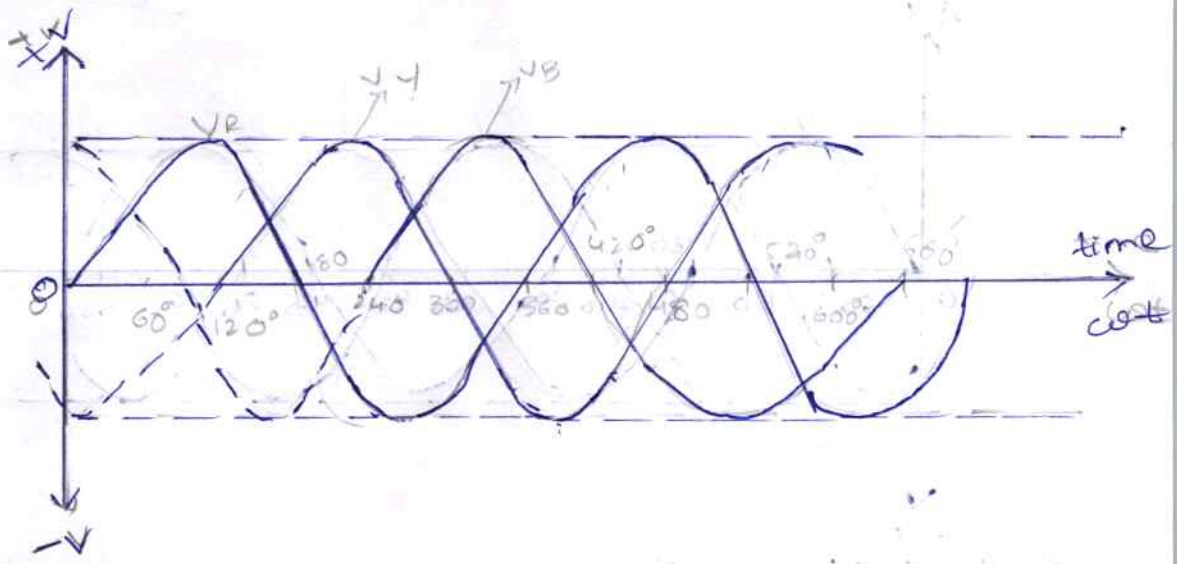
magnitude of V phase

$|V_R| = |V_Y| = |V_B| = V_{ph}$

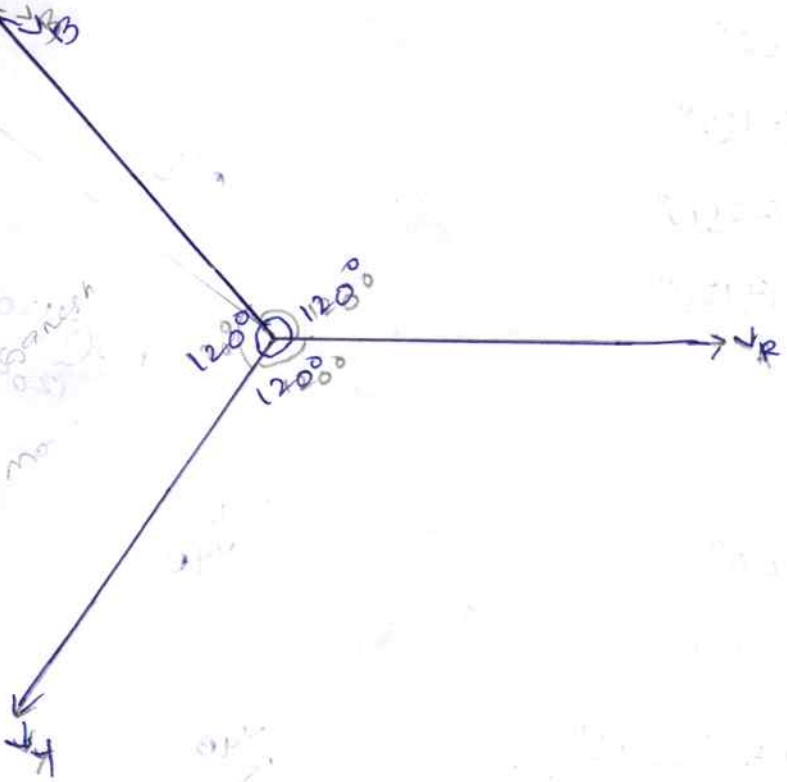
Phase sequence:

It is the order in which the phase voltages reaches it's peak values. For a 3-Ø system they are 2 possible sequences - are 1. +ve sequence (R, Y, B) (A B C), 2. -ve sequence (R B Y)

+ve (or) RYB sequence



Phasor:



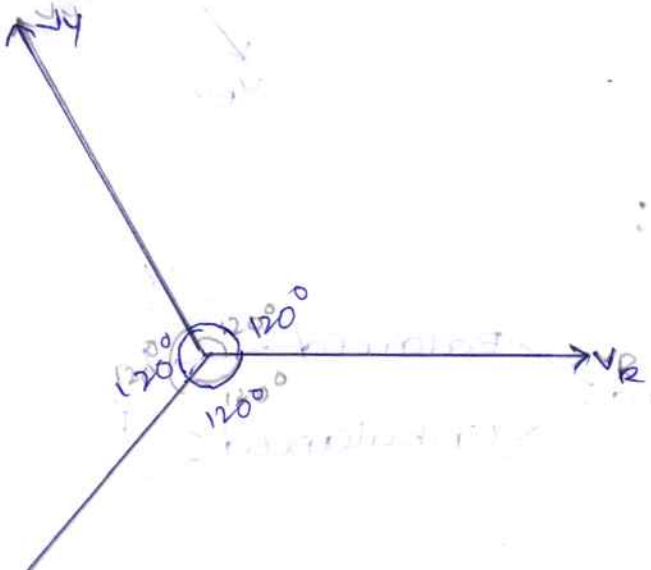
$V_R = V_{Ph} \angle 0^\circ$

$V_Y = V_{Ph} \angle -120^\circ$

$V_B = V_{Ph} \angle -240^\circ$

$= V_{Ph} \angle +120^\circ$

-ve (or) RBY sequence:

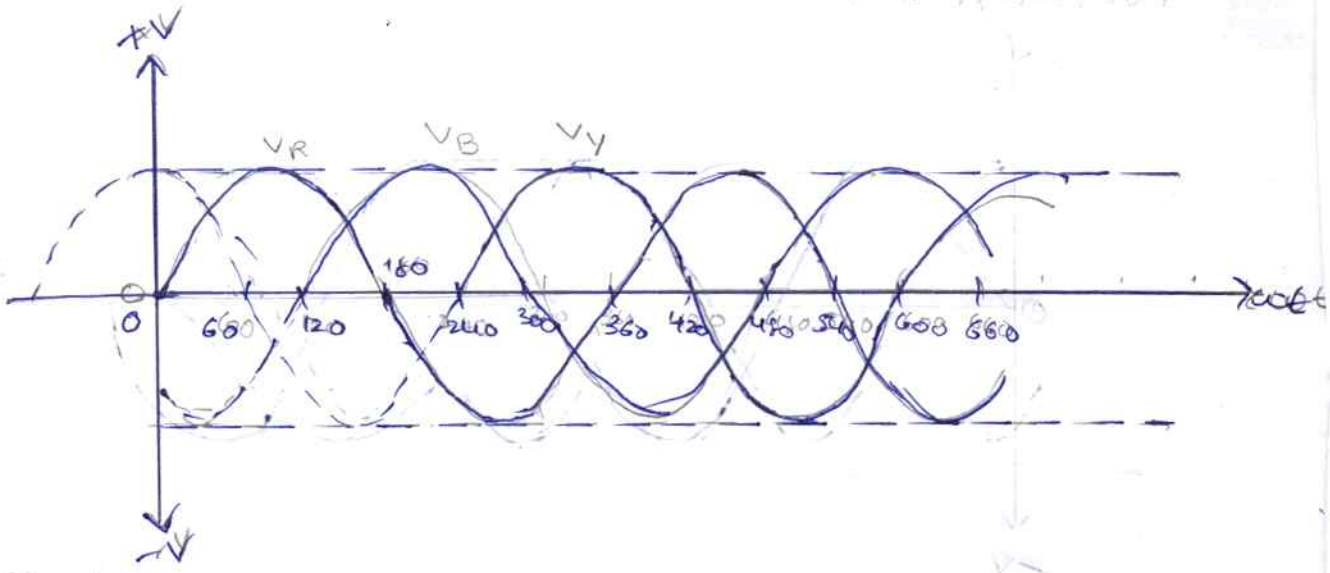


$V_R = V_{Ph} \angle 0^\circ$

$V_B = V_{Ph} \angle -120^\circ$

$V_Y = V_{Ph} \angle -240^\circ$

$= V_{Ph} \angle +120^\circ$



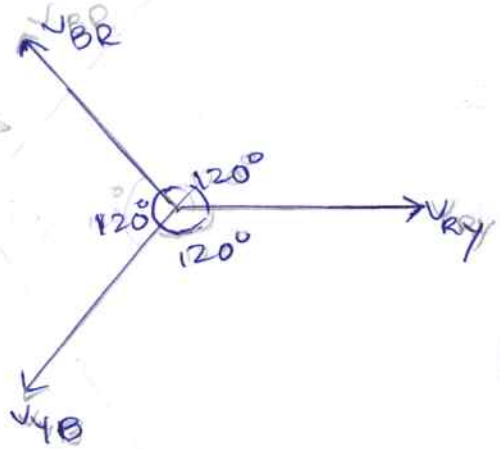
3-ϕ delta Connected :

$V_{RY} = V_{Ph} \angle 0°$

$V_{YB} = V_{Ph} \angle -120°$

$V_{BR} = V_{Ph} \angle -240°$

$= V_{Ph} \angle +120°$

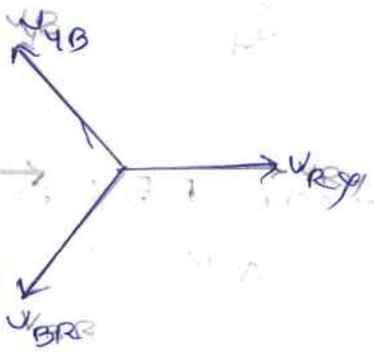


$V_{RY} = V_{Ph} \angle 0°$

$V_{BR} = V_{Ph} \angle -120°$

$V_{YB} = V_{Ph} \angle -240°$

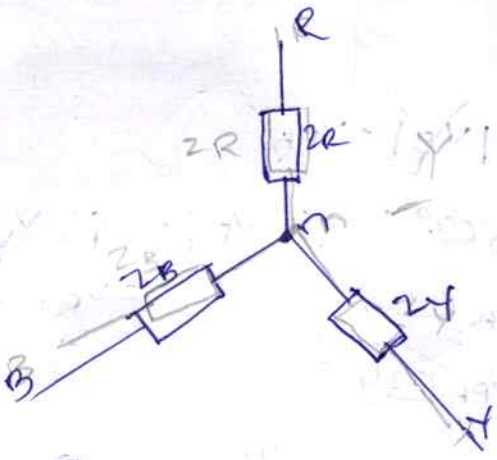
$= V_{Ph} \angle +120°$

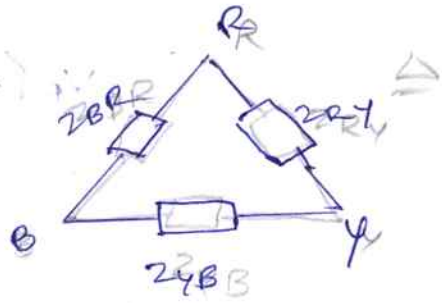


3-ϕ LOADS:



Balanced 3-ϕ Loads :





$Z_R = Z_Y = Z_B$ magnitude and angles must be same then it is balanced 3-ϕ load.

Examples:

$Z_{RY} = 10 \angle 30^\circ$
$Z_{YB} = 10 \angle 30^\circ$
$Z_{BR} = 1 \angle 30^\circ$

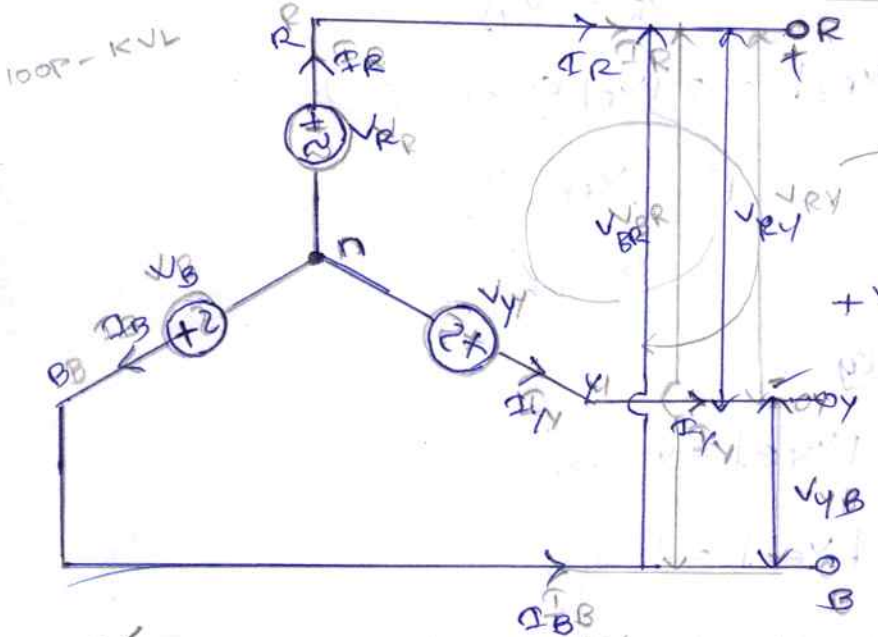
$Z_R = 10 \angle 0^\circ$
$Z_Y = 10 \angle -120^\circ$
$Z_B = 10 \angle +120^\circ$

unbalanced

$Z_R = 5 \angle 30^\circ$
$Z_Y = 5 \angle 30^\circ$
$Z_B = 5 \angle 30^\circ$
} Balanced

Both balanced & unbalanced

Voltages, Currents and Power in 'Y' system:



Line current = Phase current ✓

$+V_R - V_{RY} - V_Y = 0$

$V_{RY} = V_R - V_Y$

$V_{BR} = V_B - V_R$

$V_{YB} = V_Y - V_B$ ✓

V.V.IMP

Balanced star: (relation b/w line & phase vb of a balanced 'Y')

$\therefore V_L = \sqrt{3} V_{Ph}$

$|V_R| = |V_Y| = |V_B| = V_{Ph}$

$V_R = V_{Ph} \angle 0^\circ$

$V_Y = V_{Ph} \angle -120^\circ$

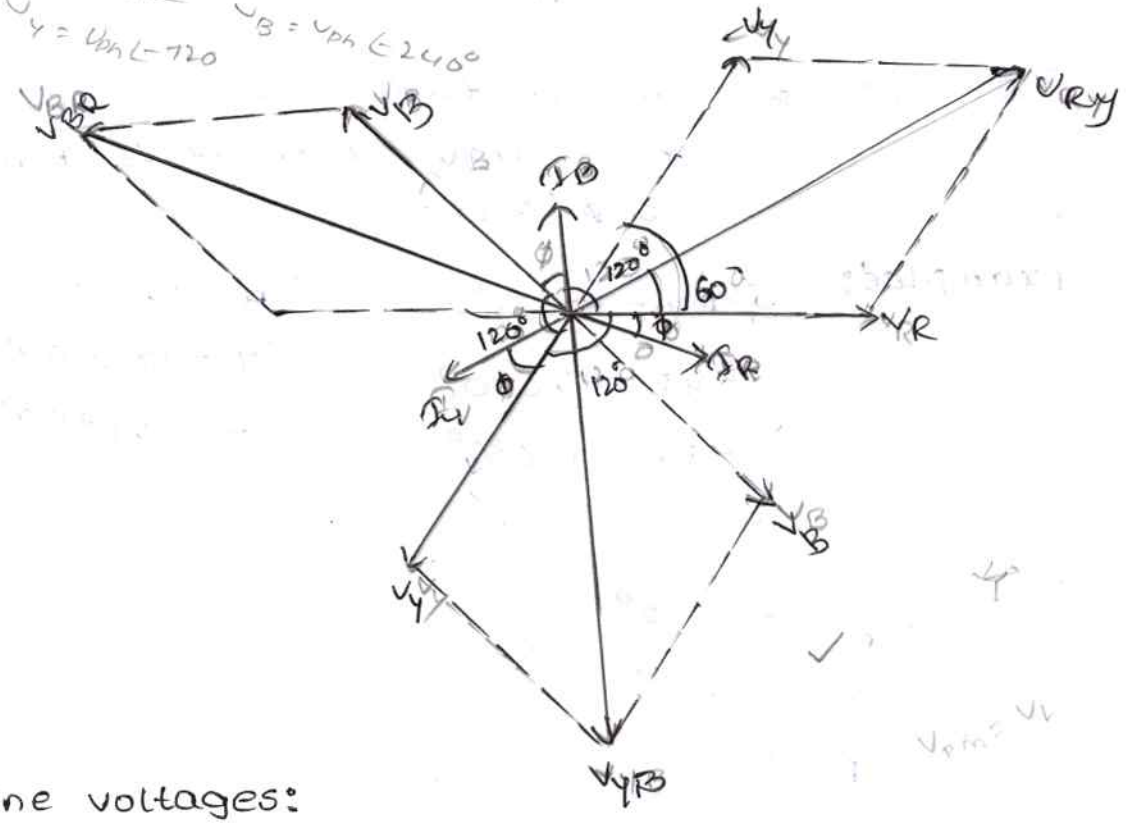
$V_B = V_{Ph} \angle -240^\circ$

$\left(V_{RY} = V_R - V_Y = V_R + (-V_Y) \right)$ ×

$V_R = V_Y = V_B = V_{Ph}$

$V_R = V_{Ph} \angle 0 \quad V_B = V_{Ph} \angle 240^\circ$

$V_Y = V_{Ph} \angle -120$



Line voltages:

$V_{RY} = V_R - V_Y$

$= V_R + (-V_Y)$

$|V_{RY}| = 2|V_R| \cos\left(\frac{60^\circ}{2}\right)$

$= 2 V_{Ph} \frac{\sqrt{3}}{2}$

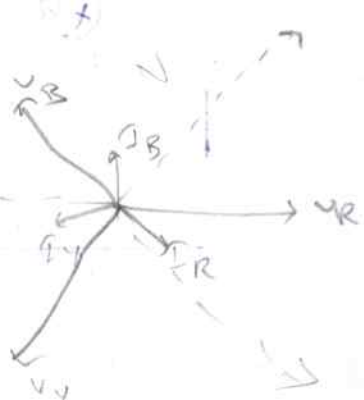
$= \sqrt{3} V_{Ph}$

similarly

$|V_{YB}| = \sqrt{3} V_{Ph}$

$|V_{BR}| = \sqrt{3} V_{Ph}$

$|V_{RY}| = |V_{YB}| = |V_{BR}| = V_L$



The line voltage leads the respective phase voltage by angle of 30°.

$$V_{RY} = \sqrt{3}\, V_{Ph} \angle 30^\circ$$

$$V_{YB} = \sqrt{3}\, V_{Ph} \angle -90^\circ$$

$$V_{BR} = \sqrt{3}\, V_{Ph} \angle 210^\circ$$

Line current

phase currents lags phase voltage by angle ϕ

Assumed inductive load

I_R, I_Y, I_B

Power:

$$P = |V||I| \cos\phi$$

Power in phase R $= P_R = |V_R||I_R| \cos\phi_R$

$$P_R = V_{Ph}\, I_{Ph} \cos\phi$$

$$P_Y = |V_Y||I_Y| \cos\phi_Y$$

$$= V_{Ph} \cdot I_{Ph} \cos\phi$$

$$P_B = |V_B||I_B| \cos\phi_B$$

$$= V_{Ph}\, I_{Ph} \cos\phi$$

Total 3-ϕ power

$$P = P_R + P_Y + P_B$$

$$P = 3\, V_{Ph}\, I_{Ph} \cos\phi$$

$$= 3\, \frac{V_L}{\sqrt{3}}\, I_L \cos\phi$$

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

$V_L = \sqrt{3}\, V_{Ph}$

$V_{Ph} = \frac{V_L}{\sqrt{3}}$

$I_{Ph} = I_L$

$Q = \sqrt{3} V_L I_L \sin\phi$

$S = \sqrt{3} V_L I_L$

14/07/2025

Summary for STAR connections:

Line current

$I_L = I_{Ph}$

Balanced Y

Balanced Y:

$$I_L = I_{ph}$$

$$V_L = \sqrt{3} V_{ph}$$

Line vlgs leads the respective phase vlgs by 30°

Line current lags with respective line voltage by $30^\circ + \phi$ for Inductive load.

Line current lags with respective line voltage by $30^\circ - \phi$ for capacitor load.

Phase power $P = 3 V_{ph} I_{ph} \cos\phi$

$$= \sqrt{3} V_L I_L \cos\phi$$

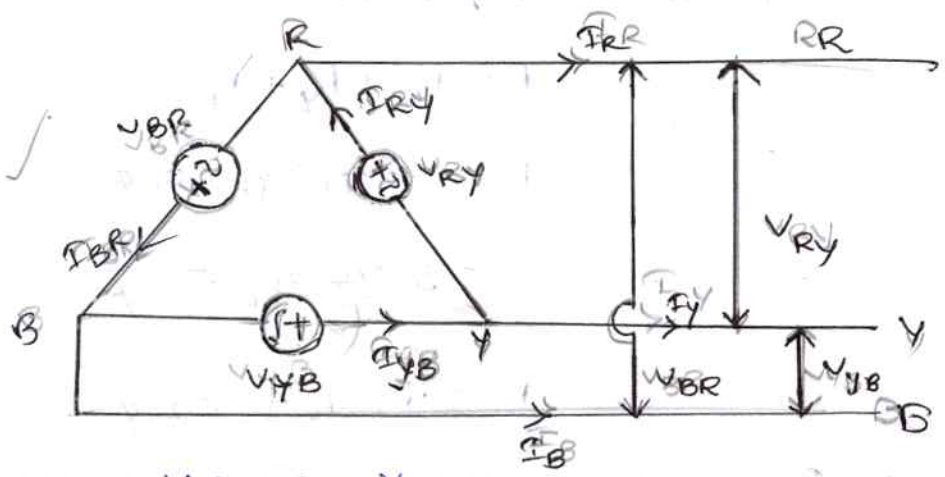
$$Q = 3 V_{ph} I_{ph} \sin\phi$$

$$= \sqrt{3} V_L I_L \cos\phi$$

Apparent power $S = 3 V_{ph} I_{ph}$

$$= \sqrt{3} V_L I_L$$

Voltages, Currents and Powers in Δ system:



Line voltages — V_{RY}, V_{YB}, V_{BR}

Phase voltages — V_{RY}, V_{YB}, V_{BR}

Line voltage = phase voltage.

KCL at node R:

$$I_{RY} = I_R + I_{BR}$$

$$\boxed{I_R = I_{RY} - I_{BR}}$$ ✓

similarly

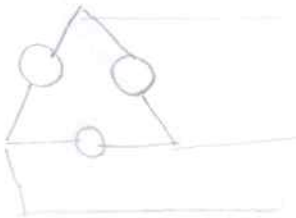
$$I_Y = I_{YB} - I_{RY}$$ ✓

$$I_B = I_{BR} - I_{YB}$$ ✓

Balanced Delta: (derive the relation b/w line & phase current in balanced Δ) (OL)

above figure:

P.T $I_L = \sqrt{3} I_{Ph}$ for balanced Δ connected load (or) source)



$$|V_{RY}| = |V_{YB}| = |V_{BR}| = V_{Ph} = V_L$$

Assume V_{RY} as reference

Phase current:

Assume inductive load

Phase currents lags Phase voltage by $\phi°$

$$|I_{RY}| = |I_{YB}| = |I_{BR}| = I_{Ph}$$

$$V_{RY} = V_L \angle 0°$$

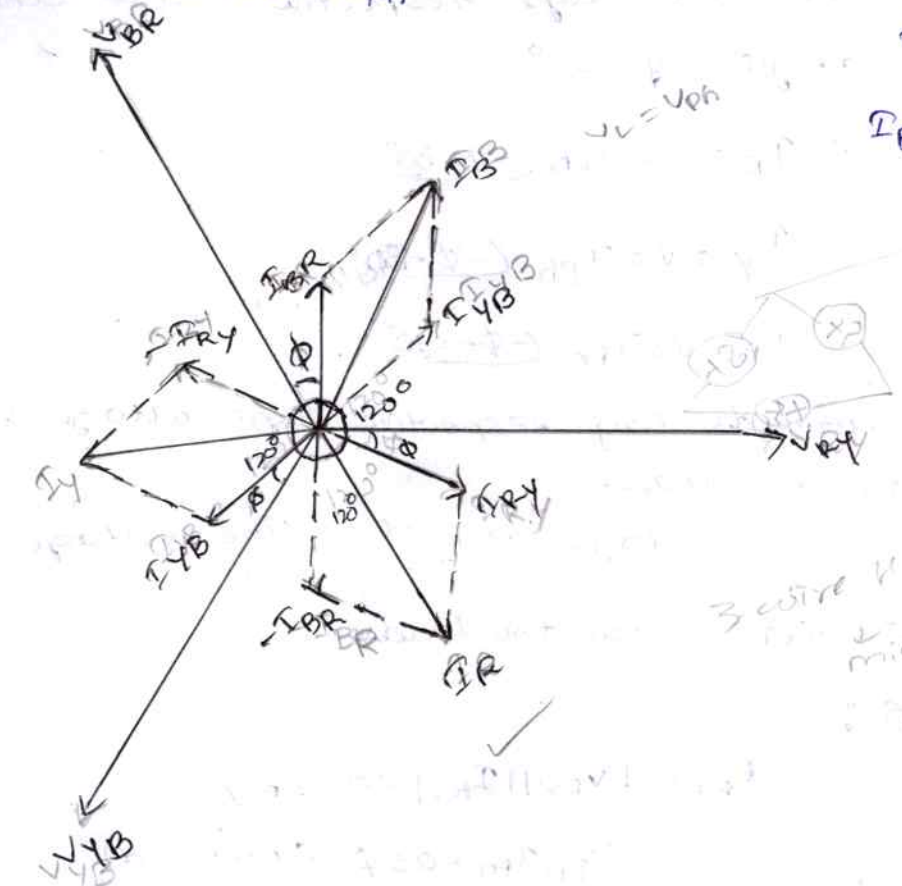
$$V_{YB} = V_L \angle -120°$$

$$V_{BR} = V_L \angle -240° = V_L \angle 120°$$

$$I_{RY} = I_{Ph} \angle -\phi°$$

$$I_{YB} = I_{Ph} \angle -\phi - 120°$$

$$I_{BR} = I_{Ph} \angle -\phi - 240°$$



Line currents

$$I_R = I_{RY} - I_{BR}$$

I_{RY} & I_{BY} are having equal magnitudes and displaced by 60°

$$\therefore |I_R| = 2\,|I_{RY}| \cos\left(\frac{60}{2}\right)$$

$$= 2\, I_{Ph} \frac{\sqrt{3}}{2}$$

similarly $= \sqrt{3}\, I_{Ph}$

$$I_Y = I_{YB} - I_{RY}$$

$$|I_Y| = \sqrt{3}\, I_{Ph}$$

$$I_B = I_{BR} - I_{YB}$$

$$|I_B| = \sqrt{3}\, I_{Ph}$$

$$|I_R| = |I_B| = |I_Y| = \sqrt{3}\, I_{Ph} = I_L$$

$$\boxed{I_L = \sqrt{3}\, I_{Ph}}$$

Line currents lags respective phase currents by an angle of 30°

$$\therefore I_R = \sqrt{3}\, I_{Ph} \angle -\phi - 30^\circ$$

$$I_Y = \sqrt{3}\, I_{Ph} \angle -\phi - 150^\circ$$

$$I_B = \sqrt{3}\, I_{Ph} \angle -\phi - 270^\circ$$

Line currents lag respective line voltage by $30^\circ + \phi^\circ$ for inductive load.

Line currents lags respective line voltage by $30 - \phi^\circ$ for capacitor loads.

POWERS:

$$P_{RY} = |V_{RY}||I_{RY}| \cos\phi_{RY}$$

$$= V_{Ph} I_{Ph} \cos\phi \quad (\text{balanced})$$

Similarly

$P_{BR} = |V_{BR}||I_{BR}| \cos\phi_{BR}$

$= V_{ph} I_{ph} \cos\phi$

$P = P_{RY} + P_{BR} + P_{YB}$

$\boxed{P = 3 V_{ph} I_{ph} \cos\phi}$ $\quad V_L = V_{ph}$

$P = 3 V_L \frac{I_L}{\sqrt{3}} \cos\phi$ $\quad I_L = \sqrt{3} I_{ph}$

$\boxed{P = \sqrt{3} V_L I_L \cos\phi}$

$Q = 3 V_{ph} I_{ph} \sin\phi$

$Q = \sqrt{3} V_L I_L \sin\phi$

$S = 3 V_{ph} I_{ph}$

$S = \sqrt{3} V_L I_L$

Summary of Δ connection:

Line voltages = Phase voltages

Line currents

$I_R = I_{RY} - I_{BR}$ Both balanced & unbalanced

$I_Y = I_{YB} - I_{RY}$

$I_B = I_{BR} - I_{YB}$

Balanced Δ:

$V_L = V_{ph}$

Line voltages = Phase voltages

$I_L = \sqrt{3} I_{ph}$

Line current lags Phase current by 30°

$I_R = \sqrt{3} I_{ph} \angle -\phi - 30^\circ$

$I_Y = \sqrt{3} I_{ph} \angle -\phi - 150^\circ$

$I_B = \sqrt{3} I_{ph} \angle -\phi - 270^\circ$

Line current lag line voltage by $30 + \phi^\circ$ for Inductive load.

Line current lag line voltage by $30 - \phi^\circ$ for

Real power

$P = 3V_{ph} I_{ph} \cos\phi$ (phase)

$P = \sqrt{3} V_L I_L \cos\phi$ (line)

Reactive power

$Q = 3V_{ph} I_{ph} \sin\phi$ (phase)

$Q = \sqrt{3} V_L I_L \sin\phi$ (line)

Apparent power

$S = 3 V_{ph} I_{ph}$ (phase)

$S = \sqrt{3} V_L I_L$ (line)

15/07/2025

Analysis of Δ connected load : (balanced & unbalanced)

Given data

Magnitude line voltage (V_L)

Phase impedance Z_{RY}, Z_{YB}, Z_{BR}

Phase sequence

Calculate - line current and power drawn by load.

$Z_{RY} = Z_{YB} = Z_{BR} = Z\angle\phi$ → Balanced

1. Phase voltage:

Phase voltage = Line voltage

$V_{ph} = V_L$

Phase voltages of load are balanced.

+ve (or) RYB (or) ABC

$V_{RY} = V_L \angle 0°$

$V_{YB} = V_L \angle -120°$

$V_{BR} = V_L \angle -240°$

$V_{RY} = V_L \angle 0°$

$V_{BR} = V_L \angle 120°$

$V_{YB} = V_L \angle -240°$

2. Phase currents:

$I_{ph} = \frac{V_{ph}}{Z_{ph}}$

$I_{RY} = \frac{V_{RY}}{Z_{RY}}$

$I_{YB} = \frac{V_{YB}}{Z_{YB}}$

$I_{BR} = \frac{V_{BR}}{Z_{BR}}$

3. Line currents:

$I_R = I_{RY} - I_{BR}$

If balanced

$I_L = \sqrt{3} I_{ph}$

Line current lags its phase

4. POWERS:

calculate the power using

Q1. A 3-ϕ Δ connected load of $4+j8\Omega$ is connected across a 400V 3-ϕ balanced supply. Determine phase currents and line currents. Assume R, Y, B sequence and also calculate power in each phase and total V_L and reactive power drawn by the load.

delta connected load $- V_L = V_{Ph}$

Balanced load

$- Z_{RY} = Z_{YB} = Z_{BR} = 4+8j\Omega$

$V_L = 400V \quad \phi = 63.43^\circ \quad = 8.944 \angle 63.43^\circ$

Phase sequence $-$ +ve sequence (or) ABC

Phase Voltages:

$I_L = \sqrt{3} I_{Ph}$

$V_{Ph} = V_L = 400V$

$V_{RY} = 400 \angle 0^\circ \ V$

$V_{YB} = 400 \angle -120^\circ V$

$V_{BR} = 400 \angle -240^\circ V$

Phase Currents:

$$I_{RY} = \frac{V_{RY}}{Z_{RY}} = \frac{400 \angle 0^\circ}{8.94 \angle 63.43^\circ} = 44.742 \angle -63.43^\circ$$

$$I_{YB} = \frac{V_{YB}}{Z_{YB}} = 44.742 \angle 176.57 \text{ (or) } 44.742 \angle -183.43^\circ$$

$$I_{BR} = \frac{V_{BR}}{Z_{BR}} = 44.742 \angle 56.57 \text{ (or) } 44.742 \angle -303.43^\circ$$

Line Currents:

$I_Y = I_{YB} - I_{RY}$ $\quad I_B = I_{BR} - I_{YB}$

Powers: Power in each phase $= V_{ph} I_{ph} \cos\phi$

Real power $P = 3 V_{ph} I_{ph} \cos\phi$
$= 3 \times 400 \times$ $= 400 \times 44.74 \times \cos(63.43)$
$= 8004.717\ W$

3-ϕ Real power $= 3 V_{ph} I_{ph} \cos\phi$
$= 3 \times 8004.717$
$= 24.014\ kW$ or $24014\ W$

3-ϕ Reactive power: $3 V_{ph} I_{ph} \sin\phi$
$= 3 \times 400 \times 44.74 \times \sin(63.43)$
$= 3 \times 16005.97$
$Q = 48017.93\ VAR$

3-ϕ Apparent power $S = 3 V_{ph} I_{ph}$
$= 53688\ VA$

Q2. A 3-ϕ balanced Y connected alternator (→ generator) supplies 110V to a 3-ϕ Δ load of $3.54 + j\,3.54\ \Omega$. Determine line currents for negative phase sequence and also calculate the total Real and Reactive power drawn by the load. and draw Phasor diagram.

Given data, Δ connected load

$V_L = 110V = V_{ph}$

Balanced load $Z_{RY} = Z_{YB} = Z_{BR} = 3.54 + j\,3.54\ \Omega$
$= 5.006 \angle 45^\circ\ \Omega$

$\phi = 45^\circ$ $I_{ph} = 5.006\ A$

Negative sequence – RBY → $V_{RY} = V_L \angle 0^\circ$
$V_{BR} = V_L \angle -120^\circ$
$V_{YB} = V_L \angle -240^\circ$

Phase voltages:

$V_{RY} = V_L \angle 0^\circ = 110 \angle 0^\circ$
$V_{YB} = V_L \angle -240^\circ$

Phase currents :

$$I_{Ph} = \frac{V_{Ph}}{Z_{Ph}}$$

$$I_{RY} = \frac{V_{RY}}{Z_{RY}} = 21.97 \angle -45^\circ A$$

$$I_{BR} = \frac{V_{BR}}{Z_{BR}} = 21.97 \angle -165^\circ A$$

$$I_{YB} = \frac{V_{YB}}{Z_{YB}} = 21.97 \angle -285^\circ A$$

(or)

$$21.97 \angle 75^\circ A$$

Line Currents :

$$I_R = I_{RY} - I_{BR}$$

$$= 38.053 \angle -15^\circ A$$

$$I_Y = I_{YB} - I_{RY}$$

$$= 38.053 \angle 105^\circ A$$

$$I_B = I_{BR} - I_{YB}$$

$$= 38.053 \angle -135^\circ A$$

POWERS:

Power in each phase $= V_{Ph} I_{Ph} \cos\phi$

$= 110 \times 21.97 \times \cos(45)$

$= 1708.86 W$

3-ϕ Real power $P = 3 \times V_{Ph} I_{Ph} \cos\phi$

$= 3 \times 1708.86$

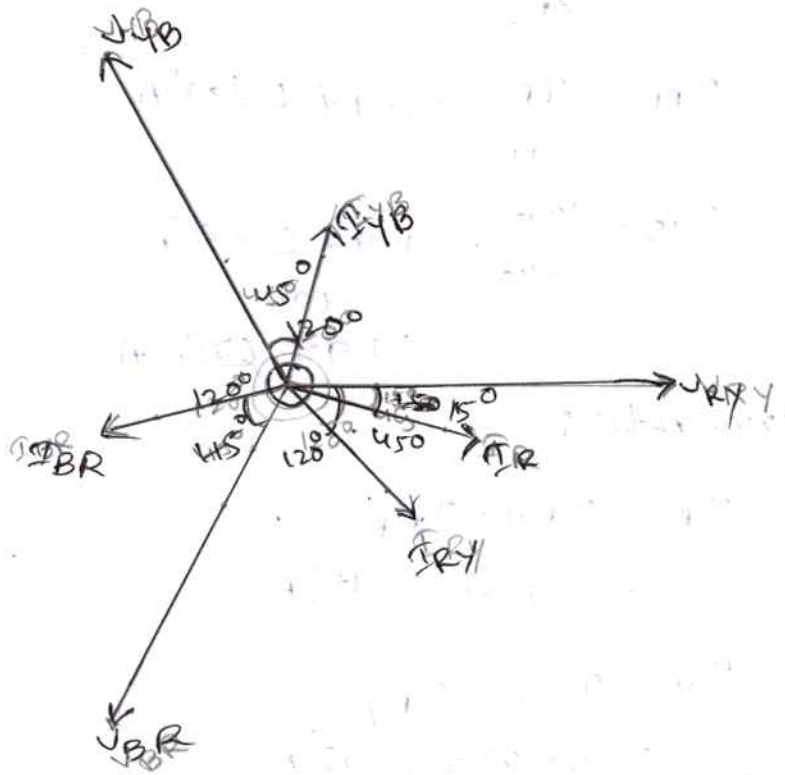
$= 5126.594 W$

3-ϕ reactive power $\phi = 3 \times V_{Ph} I_{Ph} \sin\phi$

$= 3 \times 110 \times 21.97 \times \sin(45^\circ)$

$= 5126.594 VAR$

Phasor Diagram:



3 impedances $Z_1 = 20\angle 30^\circ \Omega$, $Z_2 = 40\angle 60^\circ \Omega$, $Z_3 = 10\angle -90^\circ \Omega$ are delta connected across a 400V, 3-ϕ balanced supply. Determine phase currents, line currents and total power drawn by the load.

Given data

Δ connected load

$V_L = V_{ph}$

$V_L = 400V$

$Z_{RY} = 20\angle 30^\circ \Omega$, $Z_{YB} = 40\angle 60^\circ \Omega$, $Z_{BR} = 10\angle -90^\circ \Omega$

It is a unbalanced load.

positive sequence → RYB

Phase voltages:

$V_L = V_{ph} = 400V$

$V_{RY} = 400\angle 0^\circ V$

$\angle -120^\circ V$

Phase currents:

$$I_{RY} = \frac{V_{RY}}{Z_{RY}} = \frac{400\angle 0^\circ}{20\angle 30^\circ}$$

$$= 20\angle -30^\circ \text{ A}$$

$$I_{YB} = \frac{V_{YB}}{Z_{YB}} = \frac{400\angle -120^\circ}{40\angle 60^\circ}$$

$$= 10\angle 180^\circ \text{ A}$$

$$I_{BR} = \frac{V_{BR}}{Z_{BR}} = \frac{400\angle -240^\circ}{10\angle -90^\circ}$$

$$= 40\angle -150^\circ \text{ A}$$

Line currents:

$$I_R = I_{RY} - I_{BR}$$

$$= 52.91\angle 110.89^\circ \text{ A}$$

$$I_Y = I_{YB} - I_{RY}$$

$$= 29.093\angle 159.89^\circ \text{ A}$$

$$I_B = I_{BR} - I_{YB}$$

$$= 31.73\angle -140.93^\circ \text{ A}$$

Powers:

$$P_{RY} = |V_{RY}|\,|I_{RY}|\cos\phi_{RY}$$

$$= 400 \times 20 \times \cos(30^\circ)$$

$$= 6.93 \text{ kW}$$

$$P_{YB} = |V_{YB}|\,|I_{YB}|\cos\phi_{YB}$$

$$= 400 \times 40 \times \cos(60^\circ)$$

$$= 8 \text{ kW}$$

$$P_{BR} = |V_{BR}|\,|I_{BR}|\cos\phi_{BR}$$

$$= 400 \times 10 \times \cos(-90)$$

$$= 0 \text{ W}$$

HW Q4. A delta connected load has 10Ω between R & Y, 6.36mH between Y & B, 634μF between B & R. The supply voltage is 400V, 50Hz. determine the line current and total power drawn by the load for RBY sequency

Line voltage = phase voltage

$V_L = V_{ph} = 400V$

Δ connected load

$(\forall_{RY} = 400 \angle 0^\circ V)_x$

Phase voltages:

$V_{RY} = 400 \angle 0^\circ V$

$V_{BR} = 400 \angle -120^\circ V$

$V_{YB} = 400 \angle -240^\circ V$

Impedances:

$Z_{RY} = 10\Omega = 10 \angle 0^\circ \Omega$

$Z_{YB} = j\omega L$

$= j(2\pi \times 50 \times 6.36 \times 10^{-3})$

$= 2j\Omega = 2 \angle 90^\circ \Omega$

$Z_{BR} = \frac{1}{j\omega C}$

$= \frac{1}{j(2\pi \times 50 \times 634 \times 10^{-6})}$

$= -j(0.000106)\Omega = +j93.62\Omega$

$= 93.62 \angle -90^\circ$

phase current:

$I_{RY} = \frac{V_{RY}}{Z_{RY}} = \frac{400 \angle 0^\circ}{10 \angle 0^\circ} = 40A$

$I_{BR} = \frac{V_{BR}}{} = \frac{400 \angle -120^\circ}{} = 4.272 \angle -30$

$I_{YB} = \frac{V_{YB}}{Z_{YB}} = \frac{400 \angle -240^\circ}{2 \angle 90^\circ} = 200 \angle 30^\circ$

Line currents:

$I_R = I_{RY} - I_{BR}$

$= 40 - 4.272 \angle -30$

$= 36.363 \angle -3.367$

$I_Y = I_{YB} - I_{RY}$

$= 200 \angle 30 - 40$

$= 166.564 \angle 36.696$

$I_B = I_{BR} - I_{YB}$

$= 4.272 \angle -30 - 200 \angle 30$

$= 197.898 \angle -148.928$

Powers:

Real power $P = 3$

$P_{RY} = |V_{RY}||I_{RY}| \cos\phi_{RY}$

$= 400 \times 40 \times \cos(0)$

$= 16\,kW$

$P_{YB} = |V_{YB}||I_{YB}| \cos\phi_{YB}$

$= 400 \times 200 \times \cos(90)$

$= 0\,W$ VAR

$P_{BR} = |V_{BR}||I_{BR}| \cos\phi_{BR}$

$= 400 \times 4.272 \times \cos(-90)$

$= 0\,W$

Total power $P = P_{RY} + P_{YB} + P_{BR}$

$= 16\,kW + 0\,W + 0\,W$

$= 16\,kW$

16.07.2025

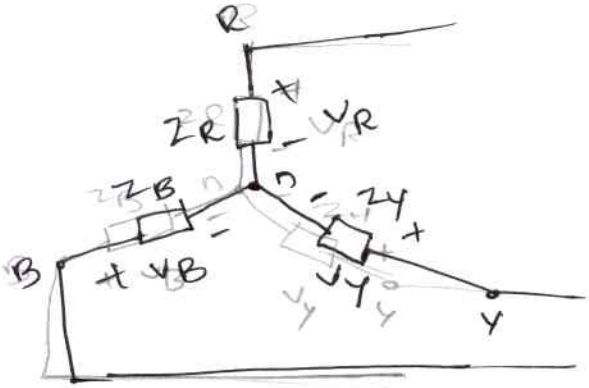
Analysis of star Y connected load:

Y connected

Balanced | Unbalanced

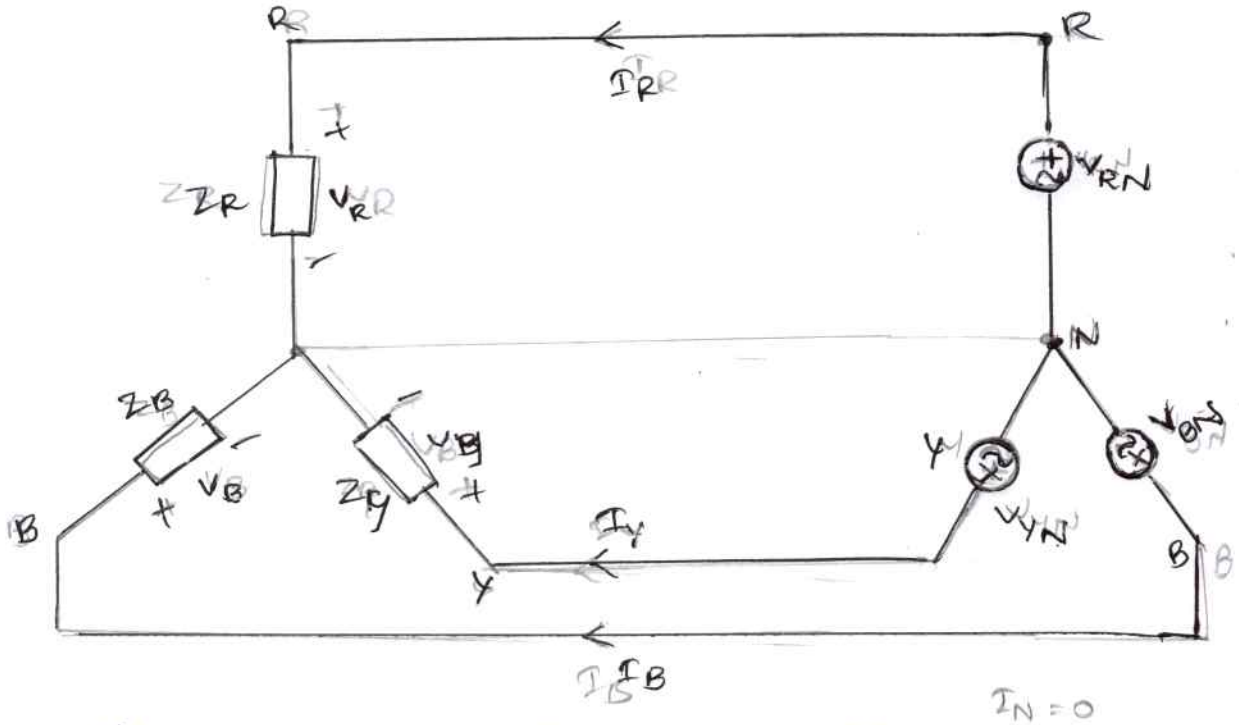
3-wire | 4wire

Balanced star:



$Z_R = Z_Y = Z_B$

Unbalanced 4-wire star connections:



$I_N = 0$

V_{RN}, V_{BN}, V_{YN}

$I_N = I_R + I_Y + I_B = 0$

V_R, V_B, V_Y – phase voltage of load

1. Phase voltage

$V_L = \sqrt{3}\, V_{Ph}$

$V_{Ph} = \frac{V_L}{\sqrt{3}}$

$V_R = V_{Ph} \angle 0^\circ$

$V_Y = V_{Ph} \angle -120^\circ$

$V_B = V_{Ph} \angle -240^\circ$

2. Phase currents

$I_{Ph} = \frac{V_{Ph}}{Z_{Ph}}$

$I_R = \frac{V_R}{Z_R}$

$I_Y = \frac{V_Y}{Z_Y}$

$I_B = \frac{V_B}{Z_B}$

3. Line Currents:

Line current = phase current

$I_L = I_{Ph}$

This procedure is for balanced and unbalanced 4-wire star.

Q5 A 3-ϕ Y connected load having an impedance $(15 + j20)\,\Omega$ for phase is connected to a 3-ϕ 440V, 50Hz. Find line currents and real, reactive power absorbed by the load?

Y connected load $I_L = I_{Ph}$

Ph impedances

Balanced

$Z_R = Z_Y = Z_B = 15 + 20i$

$= 25 \angle 53.130^\circ\ \Omega$

$V_L = 440V$

Phase sequence = Assume positive sequence

Phase Voltages:

$V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03V$

$V_R = V_{Ph} \angle 0^\circ = 254.03 \angle 0^\circ\ V$ $V_B = V_{Ph} \angle -240^\circ$

Phase Current:

$$I_R = \frac{254.03\angle 0^\circ}{25\angle 53.13} = 10.16\angle -53.13^\circ A$$

$$I_Y = \frac{254.03\angle -120^\circ}{25\angle 53.13} = 10.16\angle -173.13^\circ A$$

$$I_B = \frac{254.03\angle -240^\circ}{25\angle 53.13} = 10.16\angle 66.78^\circ A$$

Line currents:

I_n Δ Y connections

Line current = phase currents

$$I_R = 10.16\angle -53.13^\circ$$

$$I_Y = 10.16\angle -173.13^\circ$$

$$I_B = 10.16\angle 66.78^\circ A$$

Powers:

$$P_R = V_R I_R \cos\phi_R$$

$= 2450 \times$ xarong

3-ϕ real power $P = 3 V_{ph} I_{ph} \cos\phi$

$= 3 \times 254.03 \times 10.16 \times \cos(53.13)$

$= 4645.711 W$

3-ϕ reactive power

$Q = 3 V_{ph} I_{ph} \sin\phi$

$= 6194.20 VAR$

$= 6.194 KVAR$

Q6. H.W

A Y connected load has a parallel combination of resistance of 5Ω and capacitive reactance $-5j\Omega$ on each phase across a balanced 3-ϕ Δ connected alternator having a line voltage of 120V. Find line currents and power absorbed by the

Given that

Y connected, balanced load

$Z_R = Z_Y = Z_B$

$Z = 5 \angle -55 = 3.53 \angle -45$

$V_L = 120V$

$Z_R = Z_Y = Z_B = 3.53 \angle -45°$ +ve sequence

Phase voltages:

$$V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{120}{\sqrt{3}} = 69.28V$$

$V_R = 69.28 \angle 0° V$

$V_Y = 69.28 \angle -120° V$ $V_B = 69.28 \angle -240° V$

Phase Currents:

$$I_{Ph} = \frac{V_{Ph}}{Z_{Ph}}$$

$$I_R = \frac{69.28 \angle 0°}{3.53 \angle -45°} = 19.26 \angle 45° A$$

$$I_Y = \frac{69.28 \angle -120°}{3.53 \angle -45°} = 19.26 \angle -75° A$$

$$I_B = \frac{69.28 \angle -240°}{3.53 \angle -45} = 19.26 \angle -195° A$$

Line Currents:

In star connection

$I_L = I_{Ph}$

Powers:

$P_R = V_{Ph} I_{Ph} \cos\phi$

$= 69.28 \times 19.26 \times \cos(-45)$

$= 943.515 W$

$P_Y = 943.515 W$

$P_B = 943.515 W$

Total power $P = 3 V_{Ph} I_{Ph} \cos\phi$

$= 3 \times 943.515$

Q7. A 3-ϕ of 4-wire Y connected load $Z_1 = 4 + j8\,\Omega$ $Z_2 = 3 + j4\,\Omega$, $Z_3 = 15 + j20\,\Omega$ connected to a balanced 3-ϕ 400 V supply. Find line current and neutral currents and power absorbed by load.

Given data

4-wire star connected - unbalanced

$Z_R = 4 + j8\,\Omega$, $Z_y = 3 + j4\,\Omega$, $Z_B = 15 + j20\,\Omega$

$= 8.94 \angle 63.43^\circ\,\Omega$ $= 5 \angle 53.130^\circ\,\Omega$ $= 25 \angle 53.130^\circ\,\Omega$

Line voltage $V_L = 400V$

Phase voltages:

$$V_{Ph} = \frac{V_L}{\sqrt{3}}$$

$$= \frac{400}{\sqrt{3}} = 230.940$$

$V_R = 230.940 \angle 0^\circ$

$V_Y = 230.940 \angle -120^\circ$

$V_B = 230.940 \angle -240^\circ$

Phase Currents:

$$I_R = \frac{V_R}{Z_R} = 25.83 \angle -63.43\,A$$

$$I_Y = \frac{V_Y}{Z_Y} = 46.18 \angle -173.13\,A$$

$$I_B = \frac{V_B}{Z_B} = 9.23 \angle 66.87\,A$$

Line Currents

$$I_L = I_{Ph}$$

POWERS:

$$P_R = |V_R||I_R| \cos\phi_R$$

$$= 230.940 \times 25.83 \times \cos(63.43)$$

Powers

$P_R = |V_R||I_R| \cos\phi_R$ (angle of Z_R)

$P_Y = |V_Y||I_Y| \cos\phi_Y$

$P_B = |V_B||I_B| \cos\phi_B$

$P = P_R + P_Y + P_B$

$P_Y = |V_Y||I_Y| \cos\phi_Y$

$= 230.940 \times 46.18 \times \cos(53.13)$

$= 6.39\,kW$

$P_B = |V_B||I_B| \cos\phi_B$

$= 230.940 \times 9.23 \times \cos(53.15)$

$= 1.27\,kW$

Total power $P = P_R + P_Y + P_B$

$= 2.66 + 6.39 + 1.27$

$= 10.32\,kW$

Neutral current

$I_N = I_R + I_Y + I_B$

$= 25.83 \angle -63.43 + 9.23 \angle 66.87 + 46.18 \angle -173.13$

$I_N = 36.689 \angle -146.71 A$

17.07.2025

Unbalanced 3-wire Y connected load :

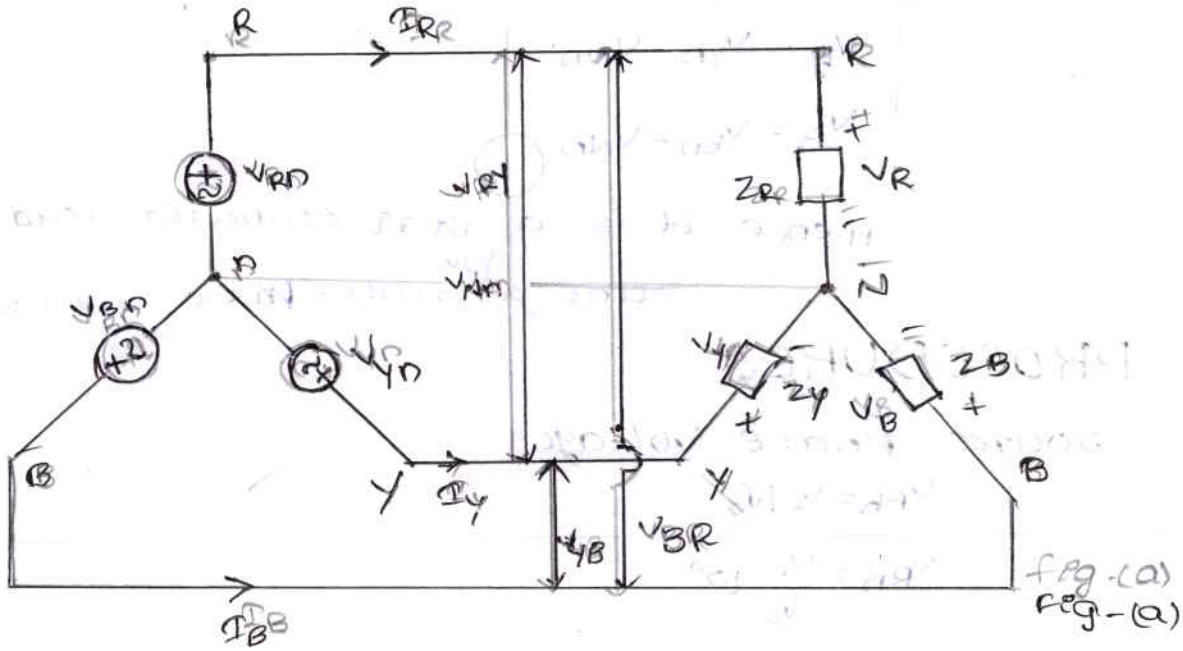


fig-(a).

$V_R \neq V_{RN}$

Phase voltages of load are unbalanced

They are 3 methods to solve

Solution by using Millman's Theorem:

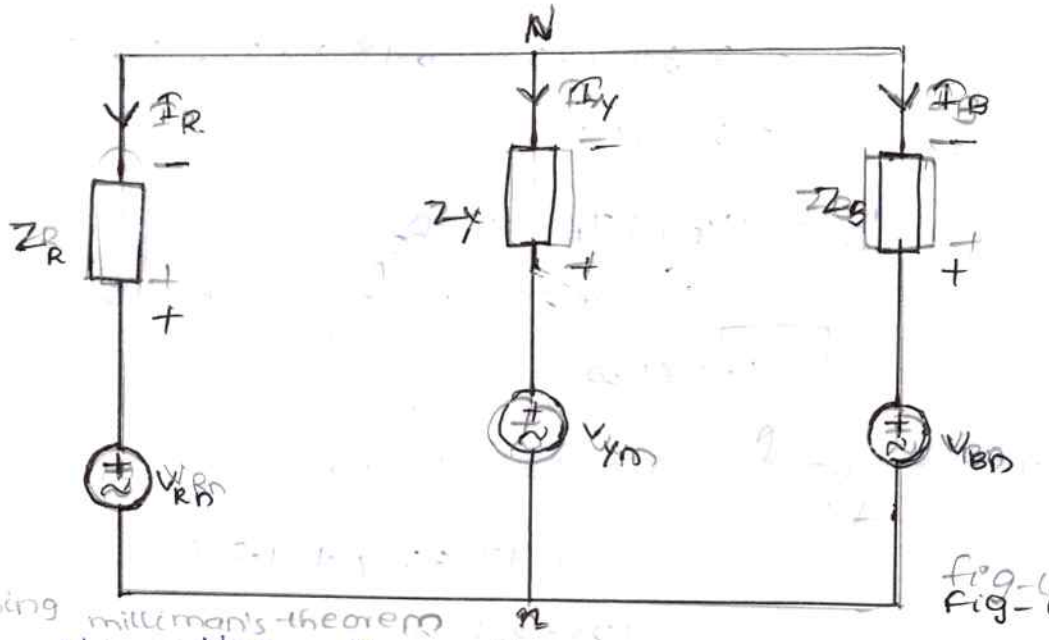


fig-(b)

using millman's theorem

fig-(b)

$$V_{NN} = \frac{\frac{V_{RN}}{Z_R} + \frac{V_{YN}}{Z_Y} + \frac{V_{BN}}{Z_B}}{\frac{1}{Z_R} + \frac{1}{Z_Y} + \frac{1}{Z_R}} = \frac{V_{RN}Y_R + V_{YN}Y_Y + V_{BN}Y_B}{Y_R + Y_Y + Y_B}$$

Applying KVL to fig (a)

$$V_{RN} - V_R - V_{NN} = 0$$

$$V_R = V_{RN} - V_{NN}$$

Similarly

$$V_Y = V_{YN} - V_{NN}$$

$$V_B = V_{BN} - V_{NN}$$

Hence it is a star connected load

Line currents = Phase currents

PROCEDURE:

Source phase voltage

$$V_{Ph} = V_L / \sqrt{3}$$

$$V_{RN} = \frac{V_L}{\sqrt{3}} \angle 0^\circ$$

$$V_{YN} = \frac{V_L}{\sqrt{3}} \angle -120^\circ$$

$$V_{BN} = \frac{V_L}{\sqrt{3}} \angle +120^\circ$$

V_n

$$V_{Nn} = \frac{V_{Rn}Y_R + V_{Yn}Y_Y + V_{Bn}Y_B}{Y_R + Y_Y + Y_B}$$

Load phase voltage:

$V_R =$

$V_B =$

$V_Y =$

Q8 A $\sqrt{3} \times 100$V 3-ϕ 3-wire balanced supply feeds a load consisting of 3 impedances $Z_1 = Z_R = 1+j1\,\Omega$, $Z_Y = 1+j2\,\Omega$, $Z_B = 3+j4\,\Omega$ in star connection. Compute voltages and currents in 3-phases of the load.

Given 3-ϕ, 3-wire Y connected load, unbalanced

$V_L = \sqrt{3}\,100$V

$Z_R = 1+1j\,\Omega = 1.414\angle 45^\circ = \sqrt{2}\angle 45^\circ\,\Omega$

$Z_Y = 1+2j\,\Omega = 2.236\angle 63.43 = \sqrt{5}\angle 63.43^\circ\,\Omega$

$Z_B = 3+4j = 5\angle 53.130^\circ\,\Omega$

$Y_R = \frac{1}{Z_R} = \frac{1}{1.414\angle 45^\circ} = 0.707\angle -45^\circ\,\mho$

$Y_Y = \frac{1}{Z_Y} = \frac{1}{2.236\angle 63.43} = 0.447\angle -63.43\,\mho$

Source phase voltage:

$$V_{Ph} = \frac{V_L}{\sqrt{3}} = 100$$

$$V_{Rn} = 100 \angle 0^\circ V$$

$$V_{Yn} = 100 \angle -120^\circ V$$

$$V_{Bn} = 100 \angle -240^\circ V$$

V_{NN}

$$V_{NN} = \frac{V_{Rn}Y_R + V_{Yn}Y_Y + V_{Bn}Y_B}{Y_R + Y_B + Y_Y}$$

$$= \frac{31.807 \angle -65.423}{1.0332 \angle -52.213}$$

$$= 23.879 \angle -13.21$$

$$V_R = V_{RN} - V_{NN}$$

$$= 76.910 \angle 4.05 V$$

18.07.2025

Q9 A balanced Y connected load is connected to a 3-ϕ 400V supply the phase impedance is $4+j3\Omega$ find power factor, real power, reactive power and apparent power drawn by the load.

Balanced Y connected load

$$I_L = I_{Ph}$$

$$V_L = \sqrt{3} V_{Ph} \Rightarrow \frac{400}{\sqrt{3}} = 230.94V$$

$$Z = 4+j3\Omega = 5\angle 36.86^\circ \Omega, Z_{Ph} = 5\Omega$$

$$V_L = 400V, \phi = 36.86^\circ$$

$$V_{Ph} = 230.94V \quad I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = 46.188A$$

3-ϕ real power $P = 3 V_{Ph} I_{Ph} \cos\phi = \sqrt{3} V_L I_L \cos\phi$

$= 25.603 kW$

3-ϕ reactive power $Q = 3 V_{Ph} I_{Ph} \sin\phi$

$= 19.195 KVAR$

3-ϕ apparent power $S = 3 V_{Ph} I_{Ph}$

$= 31.999 KVA$

Power factor $= \cos\phi$

$= 0.8$

Q10 A Balanced Δ connected load is connected to a 3-ϕ 400V supply. The phase current is 5A and is lagging behind phase voltage by 30°. find impedance for phase, real power, reactive power drawn by the load.

Balanced Δ $V_L = V_{Ph}$, $I_L = \sqrt{3} I_{Ph}$, $I_{Ph} = 5A$

$\phi = 30^\circ$ 8.66

$$V_L = V_{Ph} = 400V$$

$$Z_{Ph} = \frac{V_{Ph}}{I_{Ph}} = \frac{400}{5} = 80\Omega$$

$$Z = 80\angle 30^\circ \Omega$$

3-ϕ reactive power $Q = 3 V_{ph} I_{ph} \sin\phi$

$= 3$ KVAR

Q4 A balanced load draws 8kw at a lagging power factor of 0.8. If the line voltage is 440V find line current, reactive power and apparent power

$P = 8$kw $\quad$ P.F $= 0.8$

$V_L = 440$V $\quad$ $\cos\phi = 0.8$

$\phi = \cos^{-1}(0.8)$

$= 36.86^\circ$

$P = \sqrt{3} V_L I_L \cos\phi$

$$\frac{P}{\sqrt{3} V_L \cos\phi} = I_L$$

$$I_L = \frac{8000}{\sqrt{3} \times 440 \times 0.8}$$

$= 13.121$ A

3-ϕ reactive power $\phi = \sqrt{3} V_L I_L \sin\phi$

$= 5.998$ KVAR

3-ϕ apparent power $S = \sqrt{3} V_L I_L$

$= 9.999$ KVA

Q12 3 identical impedances are connected in Y to a 415V 3-ϕ supply and drawing a total power of 1.8kw Obtained resistance and inductance values of each phase impedance assume line current to be 10A

Balanced Y $\quad I_L = I_{ph}$

$V_L = \sqrt{3} V_{ph}$ $\quad V_L = 415$V

$P = 1.8$kw $= 1800\omega$

$I_L = 10A = I_{ph}$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.60 \text{ V}$$

$$\frac{P}{\quad} = \cos\phi = 0.25$$

$$\phi = \cos^{-1}(0.25)$$

$$= 75.52^\circ$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = 23.96\,\Omega$$

$$\therefore Z = 23.96 \angle 75.52^\circ\,\Omega$$

$$= 5.991 + 23.19i\ \Omega$$

$$= R + j\omega L \qquad R = 5.99\,\Omega$$

$$\omega L = 23.19$$

$$L = \frac{23.19}{\omega}$$

$$L = 0.073\,H$$

Q3 3-ϕ Load has a impedance of $3+j4\,\Omega$ in each phase is connected in 1. star 2. delta to a 400V 3-phase supply compare the power consumed in the two cases.

$$Z = 3 + j4\,\Omega = 5 \angle 53.13^\circ,\ Z_{ph} = 5$$

$$V_L = 400V$$

1. star:

$$I_L = I_{ph}$$

$$V_L = \sqrt{3}\, V_{ph}$$

$$V_{ph} = 230.94V$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = 46.188A$$

3-ϕ real power $P_Y = 3\, V_{ph} I_{ph} \cos\phi$

$$= 19.196kW$$

2. Delta:

$$V_L = V_{ph} = 400V$$

$$I_L = \sqrt{3} I_{ph}$$

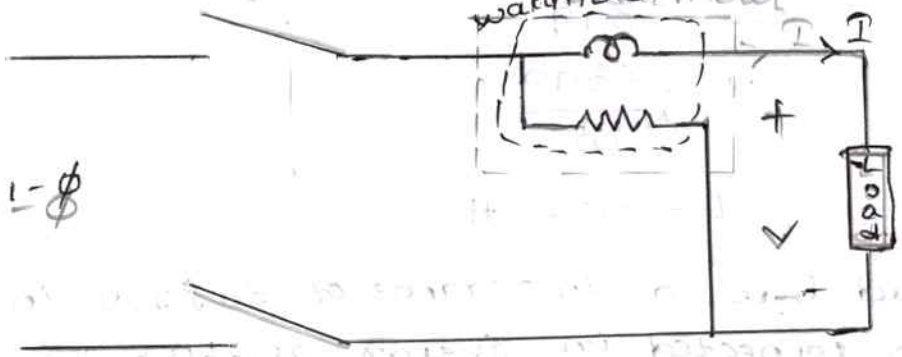
3-ϕ real power $P_\Delta = 3 V_{ph} I_{ph} \cos\phi$

$$\frac{P_\Delta}{P_Y} = 3 \Rightarrow \boxed{P_\Delta = 3P_Y}$$

Power consumed by the Δ connected is 3 times the power consumed by the Y connected for the same load and supply. So we will connect load only in Y connection in the power systems.

21.07.2025

Measurement of 3-ϕ Power:



$$W = |V||I| \cos\phi$$

where
ϕ = angle between voltage and current

Balanced:

$$P = 3 V_{Ph} I_{Ph} \cos\phi$$

out of syllabus $= \sqrt{3} V_L I_L \cos\phi$

Three-wattmeter method:

It is used is employed to measure power for 3-ϕ, 4-wire system.

Two-wattmeter methods:

Advantages:

It can be used for both balanced and unbalanced loads.

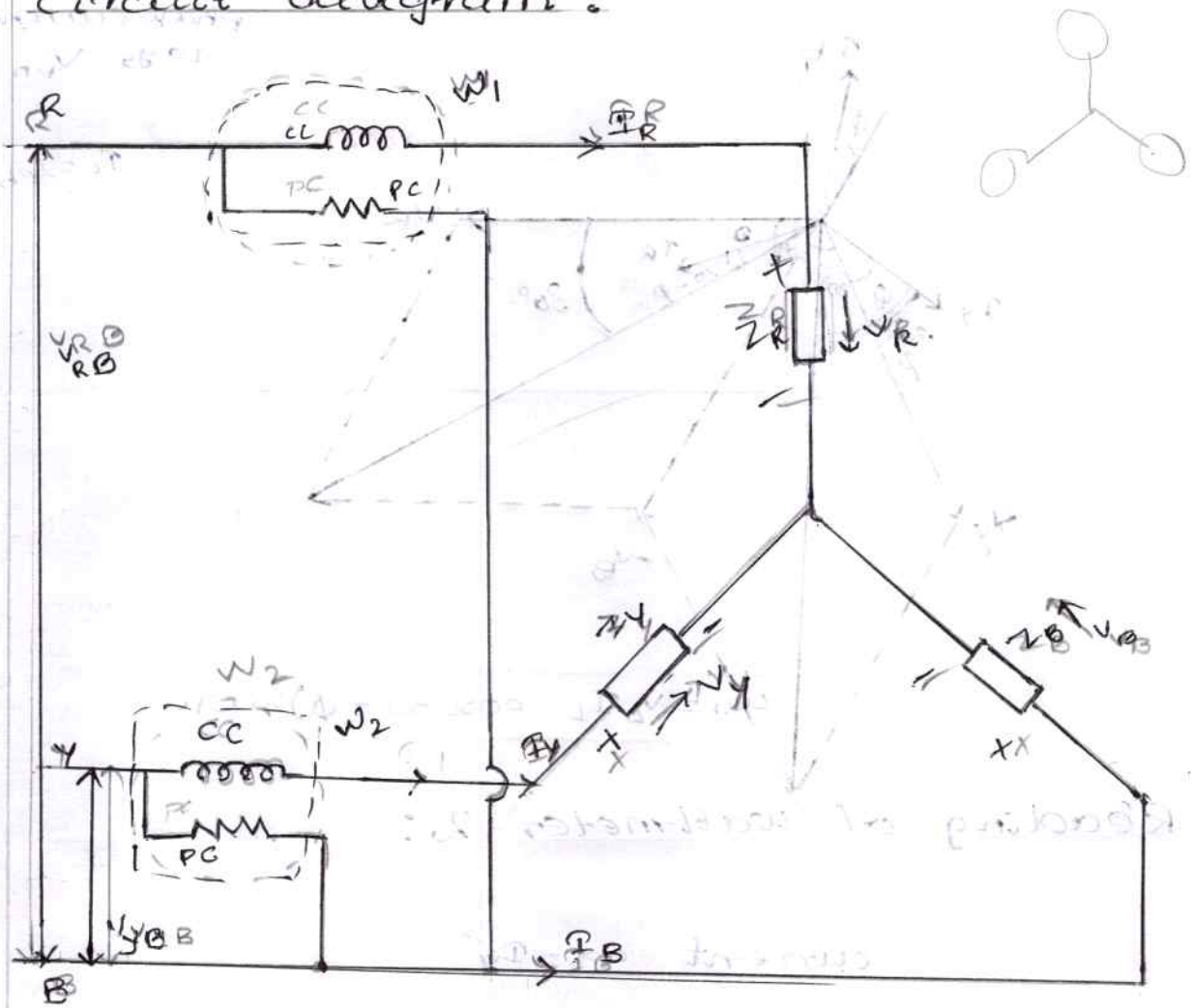
It can be used for both Y, Δ connected load

In balanced system in addition to 3-ϕ real power we can measure 3-ϕ reactive power and power

Limitations :

we cannot measure power in 4-wire system using this method.

Circuit diagram :



Reading of wattmeter-1 :

- current through CC of $W_1 = I_R$ I_R
- voltage across PC of $W_1 = V_{RB}$

(Readin)

- $W = |V_{RB}||I_R| \cos(\phi)$
- where ϕ = angle between V_{RB} and I_R

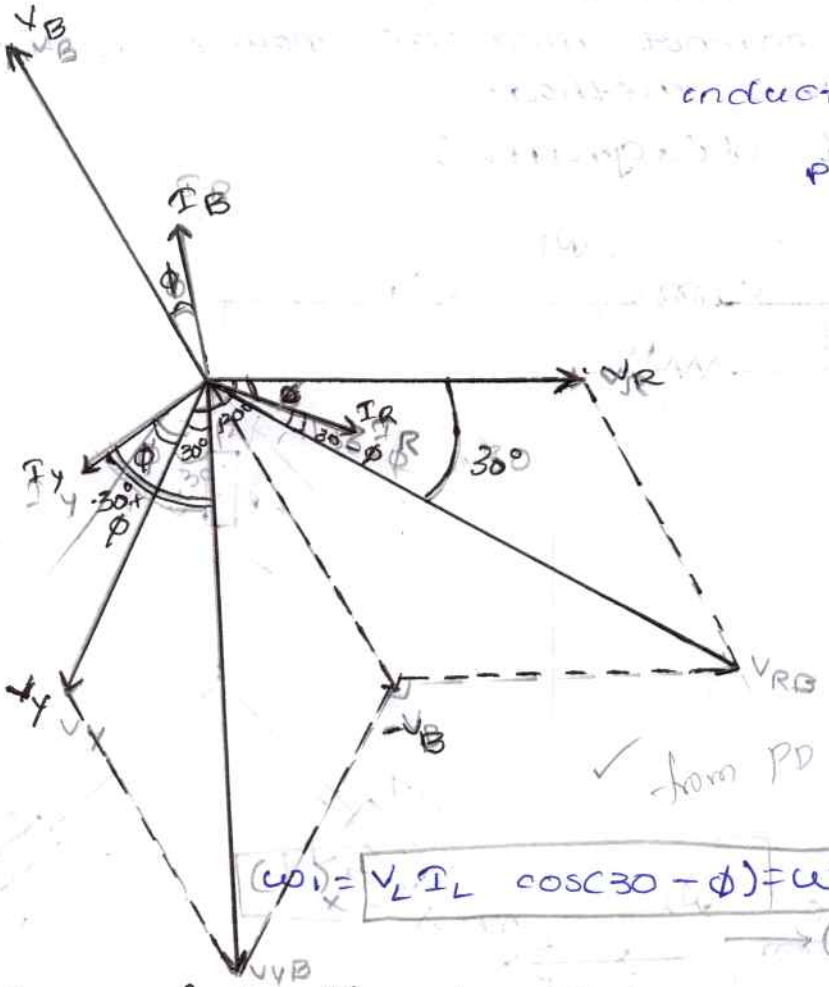
If it is balanced

$$|V_R| = |V_Y| = |V_B| = V_{ph}$$

$$V_R = V_{ph} \angle 0^\circ$$

$$V_Y = V_{ph} \angle -120^\circ$$

$$V_B = V_{ph} \angle -240^\circ$$



Assuming inductive load

phase current lags V_{ph} by ϕ

$I_L = I_{ph}$

from PD Angle b/t V_{RB} & I_R is $30-\phi$

$$W_1 = V_L I_L \cos(30-\phi) = W_1 \quad \checkmark$$

→ (1)

Reading of wattmeter-2:

current $W_1 = I_Y$

voltage at $W_2 = V_{YB}$

$$W_2 = |V_{YB}||I_Y| \cos(\phi)$$

where ϕ = Angle between V_{YB} and I_Y

the angle between V_{YB} and I_Y is $30+\phi$ from phasor diagram.

$$W_L = V_L I_L \cos(30+\phi)$$

$$W_2 = V_L I_L \cos(30+\phi)$$

→ (2)

22.07.25

Add eq (1), (2)

$$W_1 + W_2 = V_L I_L [\cos(30 - \phi) + \cos(30 + \phi)]$$

$$W_1 + W_2 = V_L I_L [\cos 30^\circ \cos\phi + \sin 30^\circ \sin\phi + \cos 30^\circ \cos\phi - \sin 30^\circ \sin\phi]$$

$$W_1 + W_2 = V_L I_L [2 \cos 30^\circ \cos\phi] = V_L I_L \left[2 \cdot \frac{\sqrt{3}}{2} \cdot \cos\phi\right]$$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi \rightarrow (3)$$

$\therefore$ 3-ϕ real power $P = W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$

$\rightarrow (4)$

Add substrate eqn (1), (2)

$$W_1 - W_2 = V_L I_L [\cos(30 - \phi) - \cos(30 + \phi)]$$

$$W_1 - W_2 = V_L I_L [\cos 30^\circ \cos\phi + \sin 30^\circ \sin\phi - \cos 30^\circ \cos\phi + \sin 30^\circ \sin\phi]$$

$$W_1 - W_2 = V_L I_L [2 \sin 30^\circ \sin\phi]$$

[$\because \sin 30^\circ = \frac{1}{2}$]

$$W_1 - W_2 = V_L I_L \left[2 \cdot \frac{1}{2} \cdot \sin\phi\right]$$

$$W_1 - W_2 = V_L I_L \sin\phi \rightarrow (5)$$

$\therefore$ 3-ϕ reactive power

$\rightarrow$ larger value

$$Q = \sqrt{3} (W_1 - W_2) = \sqrt{3} V_L I_L \sin\phi$$

$\rightarrow (6)$

divide eqn (5), (3)

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \frac{\tan\phi}{\sqrt{3}}$$

$$\tan\phi = \sqrt{3} \frac{W_1 - W_2}{W_1 + W_2}$$

$\rightarrow (7)$

$$\phi = \tan^{-1}\left[\sqrt{3} \frac{W_1 - W_2}{W_1 + W_2}\right]$$

Power factor $P.F = \cos\phi$

Effect of Power factor On Wattmeter Readings:

Power factor = $\cos\phi$ $\quad \phi = 0^\circ \text{ — } \pm 90^\circ$

max value = 1, min value = 0 (pure resistance; 2nd cap)

Case-1 : Unity power factor

$P.f = 1 = \cos\phi$

$\phi = 0^\circ$

$W_1 = V_L I_L \cos(30^\circ - \phi)$

$W_1 = V_L I_L \cos 30^\circ$

$$\boxed{W_1 = \frac{\sqrt{3} V_L I_L}{2}}$$

$W_2 = V_L I_L \cos(30^\circ + \phi)$

$W_2 = V_L I_L \cos 30^\circ$

$$\boxed{W_2 = \frac{\sqrt{3}}{2} V_L I_L}$$

$$W_1 = W_2 = \frac{P}{2}$$

wattmeter readings are equal then the reading is half of the real power.

Case-2 :

$P.f = 1 \text{ to } \frac{1}{2}$

$\phi = 0^\circ \text{ to } 60^\circ$

$W_1 = V_L I_L \cos(30 - \phi)$

W_1 increases

$W_2 = V_L I_L \cos(30 + \phi)$

W_2 decreases

$W_1 > W_2$ but both the values are '+ve'

case-3 :

Power factor P.F = 0.5

$\phi = 60^\circ$

$\omega_1 = V_L I_L \cos(30 - \phi)$

$= V_L I_L \cos(30^\circ - 60^\circ)$

$= \frac{\sqrt{3}}{2} V_L I_L$

ω_1 increases

$\omega_2 = V_L I_L \cos(30 + \phi)$

$= V_L I_L \cos(30 + 60)$

$= 0$

ω_2 becomes zero ($\omega_2 = 0$)

case-4 :

Power factor = 0.5 to 0

$\omega_1 = V_L I_L \cos(30 - \phi)$ $\Rightarrow \omega_1$ increase

$\omega_2 = V_L I_L \cos(30 + \phi)$

$\phi > 60^\circ$ becomes negative

To read the second wattmeter reading we need to reverse either current coil connection (or) pressure coil connection. Then consider that values as negative in power calculation.

case-5 : zero power factor

P.F = 0

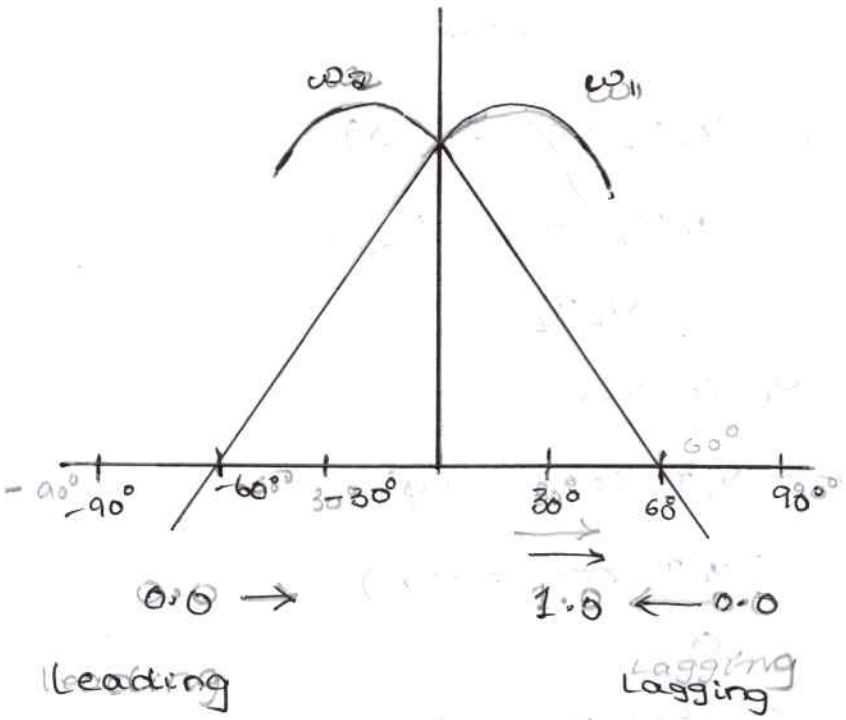
$\cos\phi = 0$

$\phi = \cos^{-1}(0)$

$\phi = 90^\circ$

$P = \sqrt{3} V_L I_L \cos\phi = 0 = \omega_1 + \omega_2$

The readings of the two wattmeter readings are equal and opposite.



Summary:

$$\omega_1 = V_L I_L \cos(30 - \phi)$$

$$\omega_2 = V_L I_L \cos(30 + \phi)$$

$$P = \omega_1 + \omega_2 \qquad P.f = \cos\phi$$

$$Q = \sqrt{3}(\omega_1 - \omega_2)$$

Q14 The readings of two wattmeters are 15 kw and 4kw for a 3-ϕ load if the supply voltage is 440V, find real power and reactive power drawn by the load and power factor of the load and Line current.

Given

$W_1 = 15kW$ $\quad P = ?$ $\quad P.f = ?$

$W_2 = 4kW$ $\quad \Phi = ?$ $\quad I_L = ?$

$V_L = 440V$

$W_1 + W_2 = P = \sqrt{3}$

$P = 19kW$

$P.f = \tan\phi$

$= \sqrt{3}\ \frac{W_1 - W_2}{W_1 + W_2}$

$= 1.002$

$\Phi = \sqrt{3}(W_1 - W_2)$

$\Phi = 19.052 kVAR$

$\phi = \tan^{-1}(1.002)$

$= 45.05°$

$P.F = \cos\phi$

$= \cos(45.05°)$

$= 0.706$

$P = \sqrt{3} V_L I_L \cos\phi$

$I_L = \frac{P}{\sqrt{3} V_L \cos\phi}$

$I_L = 35.313A$

Q15 In two wattmeter method the readings of the 2 wattmeters are $-4kW$ & $15kW$, If supply voltage is 400V, find real power, reactive power, power factor and Line current.

$W_1 = 15kW, W_2 = -4kW$

$V_L = 400V$

$P = W_1 + W_2$ $\quad \Phi = \sqrt{3}(W_1 - W_2)$

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

$$= 2.99$$

$$\phi = \tan^{-1}(2.99)$$

$$= 71.507^{0}$$

$$P.f = \cos\phi$$

$$= \cos(71.507)$$

$$= 0.317$$

$$P = \sqrt{3} V_L I_L \cos\phi$$

$$I_L = \frac{P}{\sqrt{3} V_L \cos\phi}$$

$$I_L = 50.085\,A$$

Q16 A 3-ϕ 400V has a power factor of 0.4 find the readings of two wattmeters if power is 10KW.

$V_L = 400V$,

$P.f = \cos\phi = 0.4$ $\quad \cos^{-1}(0.4) = 66.42\,\phi$

$P = 10KW$ $\quad \phi = 66.42$

$W_1 + W_2 = 10KW$ $\quad \tan(66.42) = 2.29$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = 2.29$$

$$W_1 - W_2 = \frac{2.29 \times (W_1 + W_2)}{\sqrt{3}}$$

$$W_1 - W_2 = 13.22 \times 10^{3} \quad \rightarrow (2)$$

$W_1 + W_2 = 10K$

$W_1 - W_2 = 13.22K$

$2W_1 = 23.22 \times 10^{3}$ $\quad W_1 = \frac{23.22 \times 10^{3}}{2}$

sub

$W_2 = 10 - W_1$

$= -1.61 \text{ kW}$

Q17 (HW) The readings of two wattmeter 800W & 400W the later reading is obtain after reversal of current coil. Calculate real power, reactive power and power factor.

$W_1 = 800W, W_2 = -400W$

real power $P = W_1 + W_2$

$= 800 - 400$

$P = 400W$

Reactive power $\phi = \sqrt{3}(W_1 - W_2)$

$= \sqrt{3}(800 - (-400))$

$\phi = 2078.46 \text{ VAR}$

$\tan\phi = \sqrt{3}\frac{(W_1 - W_2)}{W_1 + W_2}$

$= \frac{2078.46}{400}$

$= 5.196$

$\phi = \tan^{-1}(5.196)$

$= 79.106$

Power factor $P.f = \cos(79.106)$

$= 0.189$

23.07.2025

Q18 two wattmeters are connected to measure the power in a balanced 3-ϕ circuit the readings are 500W and 2000W. Find the power factor of the circuit and real powell (i) when both readings are positive (ii) when first one is obtained after reversing the current coil.

<u>case-1</u>: when both readings are '+ve'

$W_1 = 2000W$

$W_2 = 500W$

$P = W_1 + W_2$

$= 2500W$

$\tan\phi = \sqrt{3}\left[\frac{W_2 - W_1}{W_1 + W_2}\right]$

$= 1.039$

$\phi = \tan^{-1}(1.039)$

$= 46.095$

case-2: first reading taken after reversing cc

[so 500w is -500w]

$W_1 = 2000w$

$W_2 = -500w$

Real power $P = W_1 + W_2$

$= 2000 - 500$

$P = 1500w$

$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$

$= 2.886$

$\phi = \tan^{-1}(2.886)$

$= 70.88$

Power factor $P.f = \cos\phi$

$= \cos(70.88)$

$= 0.327$

Q19. Two wattmeters are used to measure power in a 3-ϕ balanced load. Determine total power, reactive power and power factor. if two wattmeters read i) 1000w each ii) 1000w each but one is opposite

case-1: 1000w each

$W_1 = 1000w$

$W_2 = 1000w$

real power $P = W_1 + W_2$

$= 1000 + 1000$

$P = 2000w$

$\tan\phi = \frac{\varphi}{P}$

$= \frac{0}{2000} = 0^\circ$

Reactive power $\varphi = \sqrt{3}(W_1 - W_2)$

$= \sqrt{3}(1000 - 1000)$

$\varphi = 0$

$\phi = \tan^{-1}(0)$

$\phi = 0$

Power factor $P.f = \cos\phi$

$= \cos(0)$

$= 1$

case-2: 1000w each but one is opposite

$W_1 = 1000w$

$W_2 = -1000w$

$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$

$= \frac{10}{3.464 \times 10^3}$

real power $P = W_1 + W_2$

Reactive Power $Q = \sqrt{3}(W_1 - W_2)$ $\quad \phi = \tan^{-1}(10000)$

$= 5.464$ KVAR $\quad = 90°$

Power factor $P.f = \cos\phi$

$= \cos(90)$

$= 0$

Q20 Two wattmeters are used to measure power in a balanced 3-ϕ circuit. what is the power factor of load when

i) both meters read equal

ii) Both meters read equal but one is negative

iii) one reading is twice the other reading

Case-1: Both the meters read equal

$W_1 = W_2$

$V_L I_L \cos(30 - \phi) = V_L I_L \cos(30 + \phi)$

$\cos(30 - \phi) = \cos(30 + \phi)$

$30 - \phi = 30 + \phi \quad [\because \phi = 0]$ ✗

$[30 = 30]$ ✗

$\phi = -\phi$ if $\phi = 0$

∴ Power factor is $\cos(0)$ is 1

Case-2: Both the meters reads equal but one is negative

$W_1 = +W$

$W_2 = -W_1$

∴ real power $P = W_1 + W_2$

$= W_1 - W_1$

$P = 0 = \sqrt{3} V_L I_L \cos\phi$

$\therefore \cos\phi = 0 = P.f$

Case-3: one reading is twice the other reading

$W_1 = W_2 \times 2$

$\sqrt{3} V_L I_L \cos(30 - \phi) = 2 V_L I_L \cos(30 + \phi)$

$$2\cos 30^\circ \cdot \cos\phi - \cos 30^\circ \cdot \cos\phi = \sin 30^\circ \cdot \sin\phi + 2\sin 30^\circ \cdot \sin\phi$$

$$\cos 30^\circ \cdot \cos\phi = 3 \sin 30^\circ \cdot \sin\phi$$

$$\frac{\sin\phi}{\cos\phi} = \frac{\cos 30^\circ}{3\sin 30^\circ}$$

$$\tan\phi = \frac{\sqrt{3}/2}{3 \times \frac{1}{2}} = \frac{1}{\sqrt{3}} = 0.577$$

$$\phi = \tan^{-1}(1/\sqrt{3}) = 30^\circ$$

Power factor $PF = \cos\phi$

$$= \cos(30)$$

$$= \frac{\sqrt{3}}{2}$$

Q4 Each phase of a balanced Y connected load consis-t of $R = 10\Omega$ and $C = 10\mu F$ in series. Calculate line current, Real power, reactive power when a symmetric 400V, 50Hz 3-ϕ supply is applied to it. If two wattmeters are used to measure the power what will be the readings of two wattmeters

Balanced Y

$$I_L = I_{ph}$$

$$V_L = \sqrt{3}\, V_{ph}$$

$$R = 10\Omega, \quad C = 10 \times 10^{-6} F$$

$$X_C = \frac{1}{\omega C} = 318.47\Omega$$

$$Z = R - jX_C$$

$$= 10 - j318.47\Omega$$

$$= 318.62 \angle -88.2^\circ$$

$$\phi = -88.2^\circ$$

$$V_L = 400V$$

$$V_L = \sqrt{3}\, V_{ph}$$

$$V_{ph} = \frac{400}{\sqrt{3}} = 230.94$$

$$W_1 = V_L I_L \cos(30 - \phi)$$

$$= 400 \times 0.724 \times \cos(30 + 88.2)$$

$$= -136.55W$$

$$W_2 = V_L I_L \cos(30 + \phi)$$

$$= 400 \times 0.724 \times \cos(30 - 88.2)$$

$$= 152.60W$$

$$P = W_1 + W_2 \qquad \phi = \sqrt{3}(W_1 - W_2)$$

$$= 15.75W \qquad = -501.34 \text{ VAR}$$

$$P.f = \cos\phi$$

$$= \cos(-88.2)$$

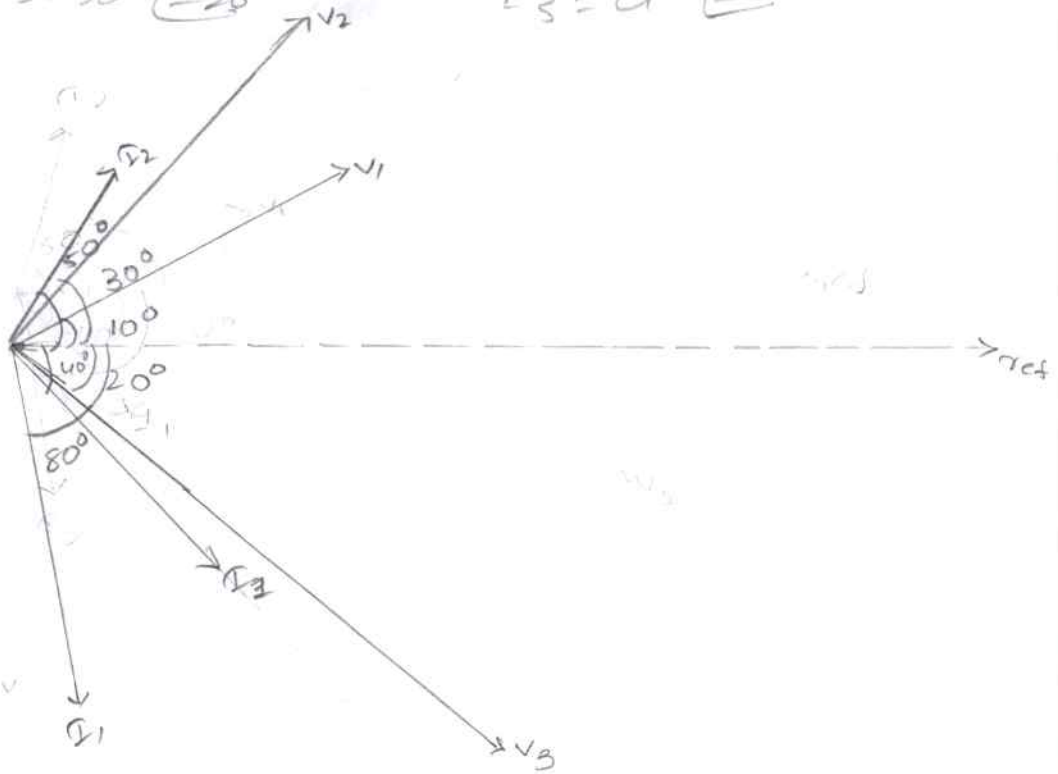
$$= 0.031 \text{ (leading)}$$

24.07.2025
Revision notes

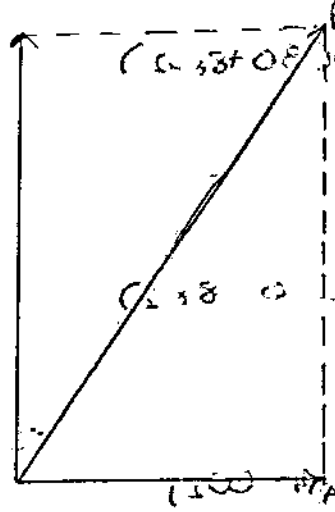
$V_1 = 10 \angle 10^\circ$ $\quad I_1 = 2 \angle -40^\circ$

$V_2 = 20 \angle 30^\circ$ $\quad I_2 = 5 \angle 50^\circ$

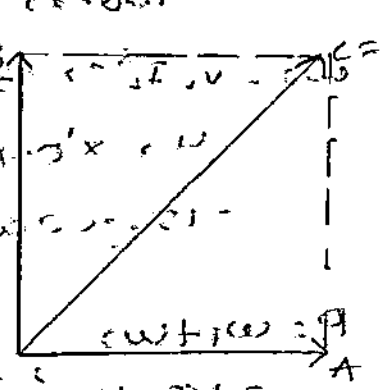
$V_3 = 30 \angle -20^\circ$ $\quad I_3 = 1 \angle 80$

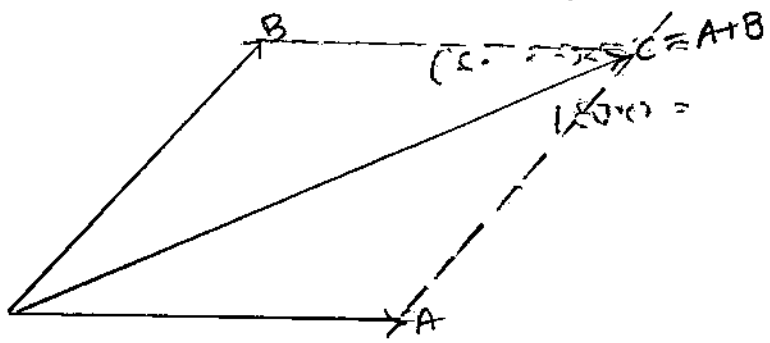


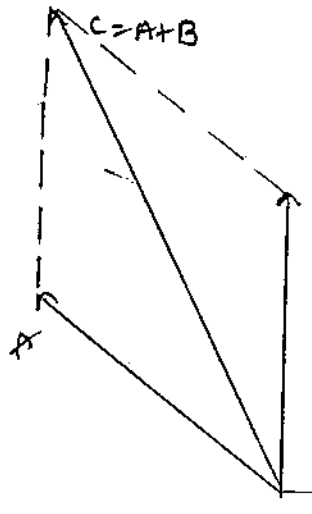
2 vector adding

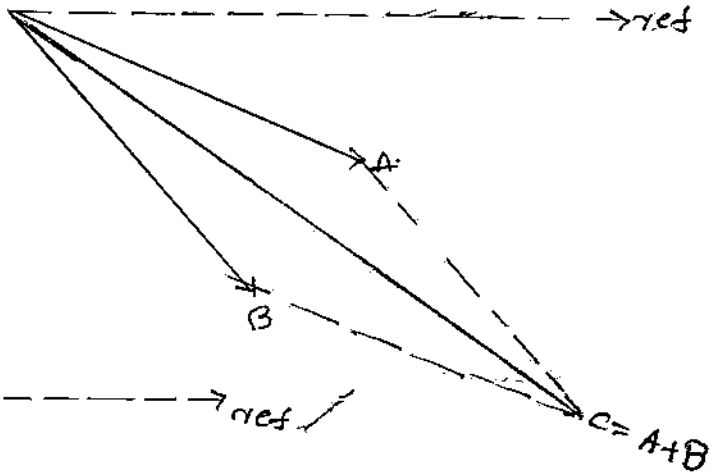


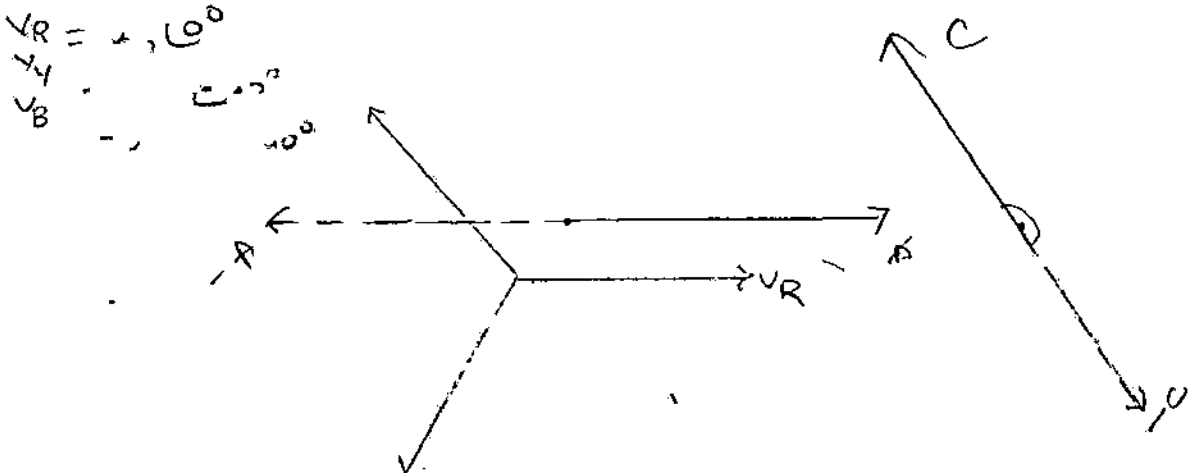
$|C| = 2|A|\cos(\frac{\alpha}{2})$









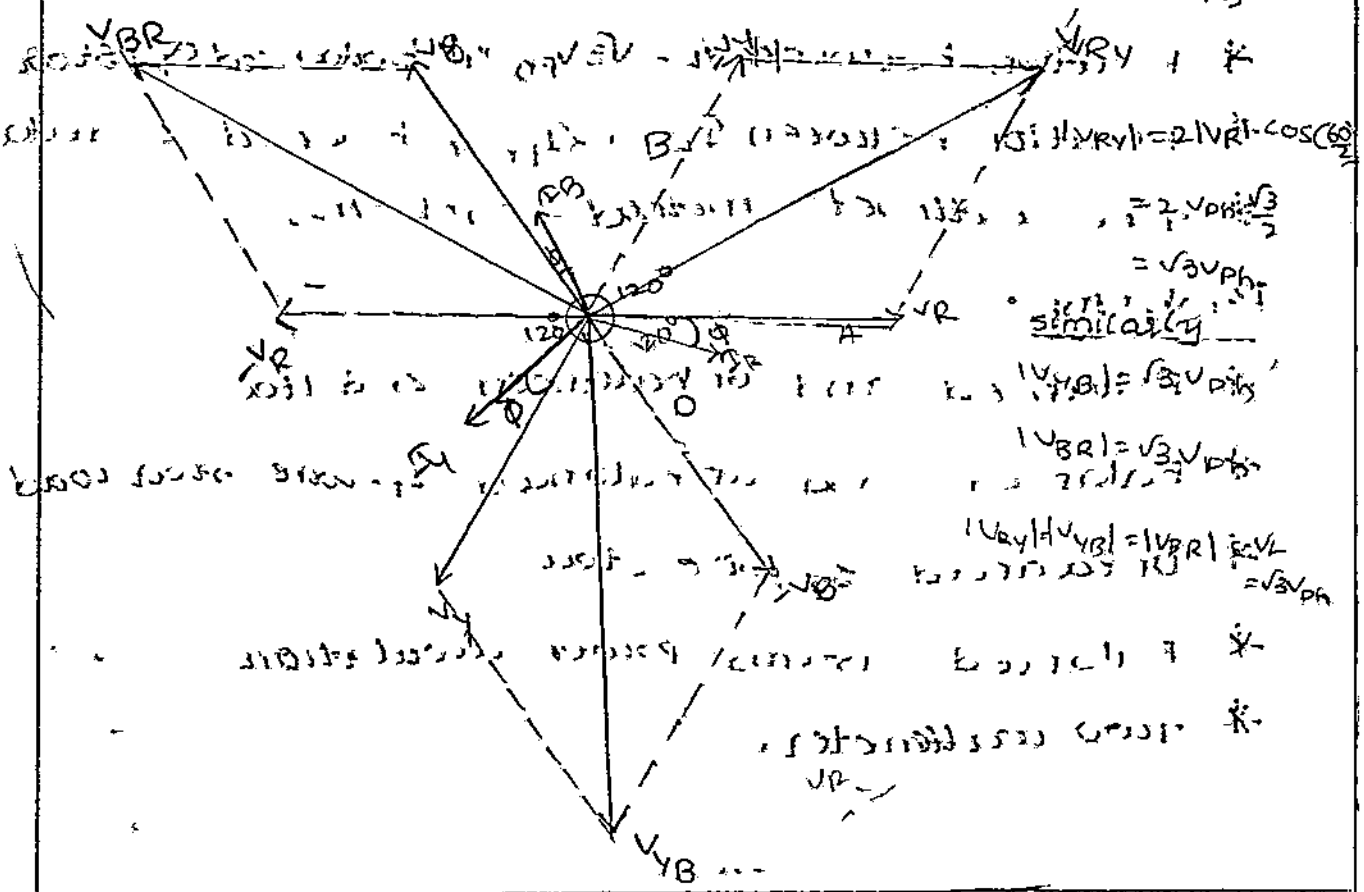


Inductive load

$I_L = I_{ph}$

$V_{RY} = V_R + (-V_Y)$

$V_{YB} = V_Y + (-V_B)$



$|V_{RY}| = 2|V_R| \cdot \cos(60)$

$= 2 V_{ph} \frac{\sqrt{3}}{2}$

$= \sqrt{3} V_{ph}$

similarly

$|V_{YB}| = \sqrt{3} V_{ph}$

$|V_{BR}| = \sqrt{3} V_{ph}$

$|V_{RY}| = |V_{YB}| = |V_{BR}| = V_L$

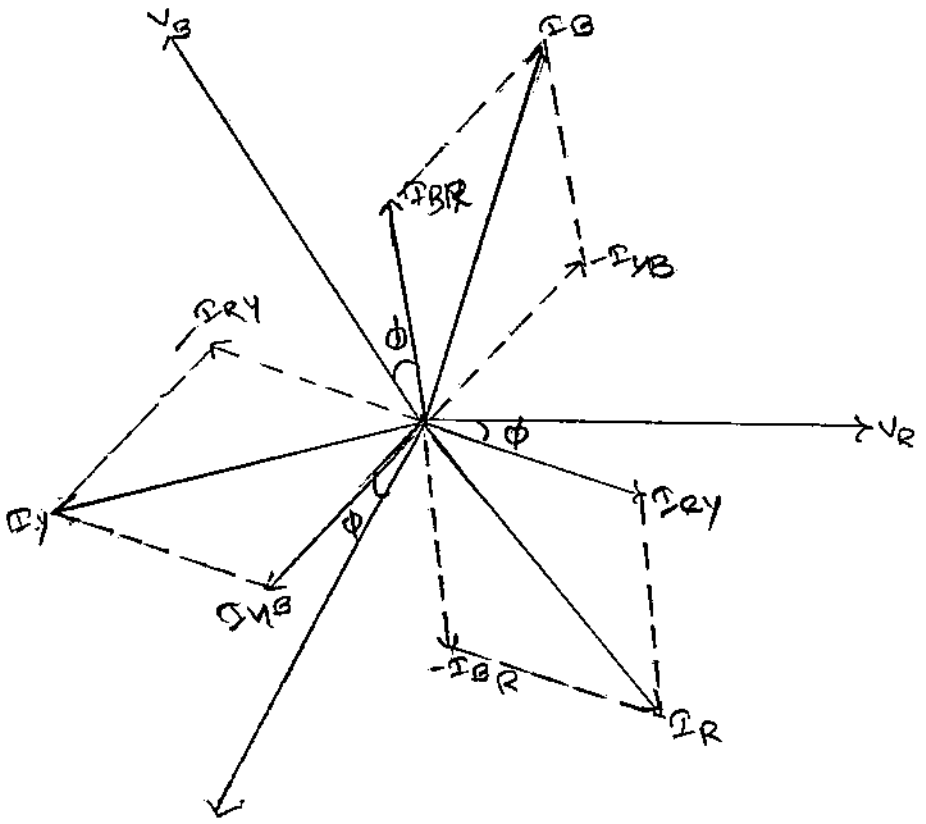
$= \sqrt{3} V_{ph}$

Phase Voltage = Line voltage

At Ecclal node -1

$I_{RY} = I_{BR} + I_R$

$I_R = I_{RY} - I_{BR}$



Unit-1 Theory Questions:

* Advantages of 3-ϕ system
* Relation between $V_L = \sqrt{3} V_{ph}$ in balanced (Y) star
* Relation between $I_L = \sqrt{3} I_{ph}$ in balanced (Δ) delta
* Two wattmeter method (- problems)

Problems:

* Balanced and unbalanced (Δ) delta
* Balanced and unbalanced 4-wire star load
* Unbalanced 3- wire star
* Balanced systems - power calculation
* Two-wattmeter.

laplace TT
↳ to solve electrical circuits where i/p time varying o/p

fourier T,
↳ when i/p is periodic

Z-trans
discrete time

ANNAMACHARYA UNIVERSITY, RAJAMPET

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RAJAMPET, Annamayya District, AP, INDIA

Department of Electrical & Electronics Engineering

II B.Tech I Semester

Network Analysis and Synthesis (24AEEE21T)
(Electrical and Electronics Engineering)

Unit-2 (Laplace Transforms)

Syllabus:

Definition of Laplace transform – advantages - Laplace transform of important functions and signals – inverse Laplace transform - application of Laplace transform to electrical circuits – initial and final value theorem.

UNIT-2

Laplace Transforms

28.07.2025

Laplace Transform:

A function $f(t)$ of laplace transform is

$$L\{f(t)\} = F(s) = \int_0^\infty f(t) e^{-st}\, dt$$

Laplace transform exist for $f(t)$ if and only if

$$\int_0^\infty |f(t) e^{-\sigma t}|\, dt < \infty$$

where s is complex frequency

$s = \sigma + j\omega$ → real part, → imaginary part

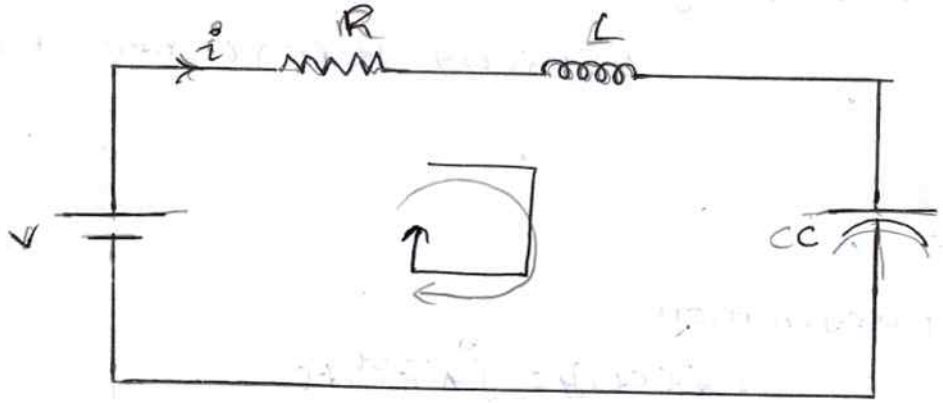
Inverse laplace transform:

The Inverse laplace transform of function $f(s)$ is given by

$$L^{-1}[F(s)] = f(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s) e^{st}\, ds$$

Advantages of L.T:

1. Integral and derivatives equations can be converted into algebraic equations by using laplace transform



$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\, dt$$

L.T

$$V(s)\left[\frac{dv}{dt}\right] = R\,I(s) + L \cdot s\, I(s) + \frac{1}{C} \cdot \frac{I(s)}{s}$$

~~V(s) =~~

$$I(s) = \frac{V(s)}{\left(R + Ls + \frac{1}{Cs}\right)}$$

2. This method gives the total solution in single operation (or) step.

Total solution $= C.F + P.I$

3. The initial conditions are incorporated in the equation itself rather than last step in solving differential equation.

4. The time involve in solving differential equation using laplace transform is less than that of classical method.

5.

Super Position Property :

If $L\{f_1(t)\} = F_1(s)$ and $L\{f_2(t)\} = F_2(s)$ then

$$L\{f_1(t) + f_2(t)\} = L\{f_1(t)\} + L\{f_2(t)\}$$
$$= F_1(s) + F_2(s)$$

Linear Property :

If $L\{f(t)\} = F(s)$ then

$L\{kf(t)\} = kF(s)$ (where k is constant

Laplace

$L\{f(t)\} = A$

By defination

$$L\{f(t)\} = \int_0^\infty A e^{-st} dt$$

$$= A \int_0^\infty e^{-st} dt$$

$$= A \left[\frac{e^{-st}}{-s}\right]_0^\infty = A \left[\frac{e^{-\infty}}{-\infty} - \frac{e^0}{-s}\right]$$

iii) $f(t) = t$

By defination

$$\mathcal{L}\{f(t)\} = \int_0^\infty t \cdot e^{-st}\, dt$$

$$= t\,\frac{e^{-st}}{-s} - 1\int \frac{e^{-st}}{-s}$$

$$= \left[t\,\frac{e^{-st}}{-s} - 1\,\frac{e^{-st}}{(-s)^2}\right]_0^\infty$$

$$= \left[t\,\frac{e^{-st}}{-s}\right]_0^\infty + \frac{1}{s}\left[\frac{e^{-st}}{-s}\right]_0^\infty$$

$$= 0 - 0 + \frac{1}{s}\left[\frac{e^{-\infty}}{-s} + \frac{e^{0}}{s}\right]$$

$$= \frac{1}{s}\left[\frac{1}{s}\right]$$

$$= \frac{1}{s^2}$$

iii) $f(t) = e^{at}$

By defination

$$f(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$= \int_0^\infty e^{-at} \cdot e^{-st}\, dt$$

$$= \int_0^\infty e^{(-at-st)}\, dt$$

$$= \int_0^\infty e^{-(s+a)t}\, dt$$

$$= \left[\frac{e^{-(s+a)t}}{-(s+a)}\right]_0^\infty$$

$$= \frac{e^{\infty}}{-(s+a)} + \frac{e^{0}}{s+a}$$

$$= \frac{1}{s+a}$$

Pvt $f(t) = e^{at}$

By definition

$$f(s) = \int_0^\infty f(t) \cdot e^{-st} dt$$

$$= \int_0^\infty e^{at} \cdot e^{-st} dt$$

$$= \int_0^\infty e^{(at-st)} dt$$

$$= \int_0^\infty e^{(a-s)t} dt$$

$$= \left[\frac{e^{(a-s)t}}{(a-s)} \right]_0^\infty$$

$$= \frac{e^\infty}{a-s} - \frac{e^0}{a-s}$$

$$= -\frac{1}{a-s}$$

$$= \frac{1}{s-a}$$

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Vb find Laplace transform of $f(t) = \sin at$

Using $e^{\pm j\theta}$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2}$$

$$f(t) = \sin at$$

$$= \frac{e^{jat} - e^{-jat}}{2j}$$

$$\sinh\theta = \frac{e^\theta - e^{-\theta}}{2}$$

$$\cosh\theta = \frac{e^\theta + e^{-\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$L\{\sin at\} = L\left\{ \frac{e^{jat} - e^{-jat}}{2j} \right\}$$

$$= \frac{1}{2j} L\{e^{jat} - e^{-jat}\}$$

$$= \frac{1}{2j} \{L\{e^{jat}\} - L\{e^{-jat}\}\}$$

$$= \frac{1}{2j} \left\{ \frac{1}{s-ja} - \frac{1}{s+ja} \right\}$$

$$= \frac{1}{2j}\left[\frac{2ja}{s^2+a^2}\right]$$

$$= \frac{a}{s^2+a^2}$$

vi) find Laplace transform of $f(t) = \cos at$

Using e

$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$$

$$f(t) = \cos at$$

$$L\{\cos at\} = L\left\{\frac{e^{j\theta}+e^{-j\theta}}{2}\right\}$$

$$= \frac{1}{2}\{L\{e^{j\theta}\} + L\{e^{-j\theta}\}\}$$

$$= \frac{1}{2}\left\{\frac{1}{s-a} + \frac{1}{s+a}\right\}$$

$$= \frac{1}{2}\left\{\frac{s+a+s-a}{s^2+a^2}\right\}$$

$$= \frac{1}{2}\left\{\frac{2s}{s^2+a^2}\right\}$$

$$= \frac{s}{s^2+a^2}$$

vii) find Laplace transform of $f(t) = \sinh at$

Using e

$$\sinh\theta = \frac{e^{\theta}-e^{-\theta}}{2}$$

$$f(t) = \sinh at$$

$$= \frac{e^{at}-e^{-at}}{2}$$

$$L\{\sinh at\} = L\left\{\frac{e^{at}-e^{-at}}{2}\right\}$$

$$= L\frac{1}{2}\{L\{e^{at}\} - L\{e^{-at}\}\}$$

$$= \frac{1}{2}\left\{\frac{1}{s-a} - \frac{1}{s+a}\right\}$$

$$= \frac{1}{2}\left\{\frac{s+a-s+a}{s^2-a^2}\right\}$$

$$= \frac{a}{s^2 - a^2}$$

viii) find laplace transform of $f(t) = \cosh at$

Using

$$\cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}$$

$$f(t) = \cosh at$$

$$= \frac{e^{at} + e^{-at}}{2}$$

$$L\{\cosh at\} = L\left\{\frac{e^{at} + e^{-at}}{2}\right\}$$

$$= \frac{1}{2}\{L\{e^{at}\} + L\{e^{-at}\}\}$$

$$= \frac{1}{2}\left\{\frac{1}{s-a} + \frac{1}{s+a}\right\}$$

$$= \frac{1}{2}\left\{\frac{s+a+s-a}{s^2-a^2}\right\}$$

$$= \frac{1}{2}\left\{\frac{2s}{s^2-a^2}\right\}$$

$$= \frac{s}{s^2-a^2}$$

ix) find Laplace transform of $f(t) = t^n$

$$L\{t^n\} = \int_0^{\infty} t^n e^{-st}\, dt$$

$$= t^n \frac{e^{-st}}{-s}\Big|_0^{\infty} - \int n t^{n-1} \cdot \frac{e^{-st}}{-s}\, dt$$

$$= [0-0] + \frac{n}{s}\int_0^{\infty} t^{n-1} e^{-st}\, dt$$

$$L\{t^n\} = \frac{n}{s} L\{t^{n-1}\}$$

$$L\{t^{n-1}\} = \frac{n-1}{s} L\{t^{n-2}\}$$

$$L\{t^{n-2}\} = \frac{n-2}{s} L\{t^{n-3}\}$$

$$\vdots \quad \vdots \quad \vdots$$

$$\therefore \mathcal{L}\{t^n\} = \frac{n}{s}\mathcal{L}\{t^{n-1}\}$$

$$= \frac{n}{s}\cdot\frac{n-1}{s}\mathcal{L}\{t^{n-2}\}$$

$$= \frac{n}{s}\cdot\frac{n-1}{s}\cdot\frac{n-2}{s}\mathcal{L}\{t^{n-3}\}\ldots$$

$$= \frac{n}{s}\cdot\frac{n-1}{s}\cdot\frac{n-2}{s}\ldots\ldots\frac{1}{s}\cdot\frac{1}{s}$$

$$= \frac{n!}{s^{n+1}}$$

Q2. find Laplace transforms of $f(t) = 4t^3 + t^2 - 6t + 7$

$$f(t) = 4t^3 + t^2 - 6t + 7$$

$$\mathcal{L}\{f(t)\} = 4\mathcal{L}\{t^3\} + \mathcal{L}\{t^2\} - 6\mathcal{L}\{t\} + 7\mathcal{L}\{1\}$$

$$= 4\cdot\frac{3!}{s^{3+1}} + \frac{2!}{s^{2+1}} - 6\cdot\frac{1}{s^2} + 7\cdot\frac{1}{s}$$

$$= 4\cdot\frac{3!}{s^4} + \frac{2!}{s^3} - \frac{6}{s^2} + \frac{7}{s}$$

$$= \frac{24}{s^4} + \frac{2}{s^3} - \frac{6}{s^2} + \frac{7}{s}$$

Q3. find Laplace transform of $f(t) = 3t^4 - 2t^3 + 4e^{-3t} - 2\sin 5t + 3\cos 2t$.

$$f(t) = 3t^4 - 2t^3 + 4e^{-3t} - 2\sin 5t + 3\cos 2t$$

$$\mathcal{L}\{f(t)\} = 3\cdot\mathcal{L}\{t^4\} - 2\mathcal{L}\{t^3\} + 4\cdot\mathcal{L}\{e^{-3t}\} - 2\mathcal{L}\{\sin 5t\} + 3\cdot\mathcal{L}\{\cos 2t\}$$

$$= 3\cdot\frac{4!}{s^{4+1}} - 2\cdot\frac{3!}{s^{3+1}} + 4\cdot\frac{1}{s+3} - 2\cdot\frac{5}{s^2+5^2} + 3\cdot\frac{s}{s^2+2^2}$$

$$= \frac{72}{s^5} - \frac{12}{s^4} + \frac{4}{s+3} - \frac{10}{s^2+5^2} + \frac{3s}{s^2+2^2}$$

Sol: Find Laplace transform of $f(t) = \cos^2 t$ $\quad \cos^2\theta = \frac{1+\cos\theta}{2}$

$$\cos^2 t = \frac{1+\cos 2t}{2}$$

$$L\{\cos^2 t\} = L\left\{\frac{1+\cos 2t}{2}\right\}$$

$$= \frac{1}{2} L\left\{\frac{1}{2}\right\} + L\left\{\frac{\cos 2t}{2}\right\}$$

$$= \frac{1}{2}\{L\{1\} + L\{\cos 2t\}\}$$

$$= \frac{1}{2} \cdot \left\{\frac{1}{s} + \frac{s}{s^2+2^2}\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{s} + \frac{s}{s^2+4}\right\}$$

$$= \frac{1}{2}\left\{\frac{s^2+4+s^2}{s(s^2+4)}\right\}$$

$$= \frac{1}{2}\left\{\frac{2(s^2+2)}{s(s^2+4)}\right\}$$

$$= \frac{s^2+2}{s(s^2+4)}$$

Some useful Laplace transform theorem:

Differentiation theorem:

$f(0) = f(t)$ at $t=0$

If laplace transform of $f(t)$ is $L\{f(t)\} = F(s)$

then $L\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0)$

$$L\left\{\frac{d^n}{dt} f(t)\right\} = s^n F(s)\, s^{n-1} f(0) - f'(0) \cdot s^{n-2} \ldots\ldots$$

$$L\left\{\frac{d^2}{dt} f(t)\right\} = s^2 F(s) - sf(0) - f'(0)$$

Integration theorem:

If laplace transform of $f(t)$ is $L\{f(t)\} = F(s)$

then $L\{\int f(t)\,dt\} = \frac{F(s)}{s}$

$\frac{f(t)}{t}$ — t-domain

$\frac{df(t)}{dt}$

$\int f(s)\,ds$ — s-domain

$sF(s)$

multiplication with t:

If laplace transform of $f(t)$ is $L\{f(t)\} = F(s)$

then $L\{t f(t)\} = -\frac{d}{ds} F(s)$

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$$

Division with t:

If Laplace transform of $f(t)$ is $L\{f(t)\} = F(s)$

then $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)\, ds$

Q5) Find Laplace transform of $f(t) = t^2$

$f(t) = t^2$

$f_1(t) = t$

$L\{f_1(t)\} = L\{t\}$

$= \frac{1}{s^2}$

$L\{f(t)\} = L\{t^2\}$

$= L\{t f(t)\}$

$= -\frac{d F_1(s)}{ds}$

$= -\frac{d}{ds}\{\frac{1}{s^2}\}$

$L\{t^2\} = \frac{2}{s^3}$

Q6) Find Laplace transform of $f(t) = 2t \sin 3t$

$f(t) = 2t \sin 3t$

$f_1(t) = 2 \sin 3t$

$F_1(s) = 2 \cdot \frac{3}{s^2+9} = \frac{6}{s^2+9}$

$L\{t\, 2\sin 3t\} = L\{t f_1(t)\}$

$= -\frac{d}{ds}\left[\frac{6}{s^2+9}\right]$

Q3) Find laplace transform of $f(t) = t e^{at}$

$$f(t) = t e^{at}$$

$$f_1(t) = e^{at}, \quad F_1(s) = \frac{1}{s-a}$$

$$L\{f(t)\} = L\{t f_1(t)\}$$

$$= \frac{-d}{ds}\left[\frac{1}{s-a}\right]$$

$$= -\left[-\frac{1}{(s-a)^2}\right]$$

$$= \frac{1}{(s-a)^2}$$

Q4) Find laplace transform of $f(t) = \frac{2-2e^{2t}}{t}$

$$f(t) = \frac{2-2e^{2t}}{t}$$

$$f_1(t) = 2 - 2e^{2t}$$

$$L\{f_1(t)\} = F_1(s) = \frac{2}{s} - 2\cdot\frac{1}{s+2}$$

$$L\{f(t)\} = L\left\{\frac{f_1(t)}{t}\right\}$$

$$= \int_s^\infty F_1(s)\,ds$$

$$= \int_s^\infty \left[\frac{2}{s} - \frac{2}{s+2}\right] ds$$

$$= 2\left[\int_s^\infty \frac{1}{s}\,ds - \int_s^\infty \frac{1}{s+2}\,ds\right]$$

$$= 2\left\{[\log s - \log(s+2)]_s^\infty\right\}$$

$$= 2\left\{\log\frac{s}{s+2}\right\}_s^\infty$$

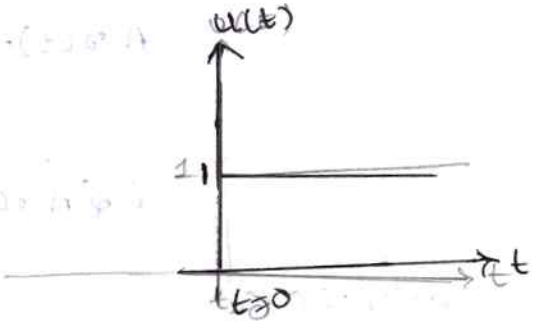
$$= 2\left\{\log\infty - \log\frac{s}{s+2}\right\} = 2$$

$$= 2\cdot\log\frac{s+2}{2}$$

$$= \log\left[\frac{s+2}{2}\right]^2$$

Laplace transforms of standard function:

a) step function: (unit)



unit:

$$u(t) = 0 \quad t < 0$$
$$= 1 \quad t > 0$$

$$\mathcal{L}\{u(t)\} = \int_0^\infty u(t) e^{-st} dt$$

$$= \int_0^\infty 1 \cdot e^{-st} dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_0^\infty$$

$$= \left[\frac{e^\infty}{\infty} + \frac{e^0}{s}\right]$$

$$= \left[0 + \frac{e^0}{s}\right]$$

$$= \frac{1}{s}$$

Normal:

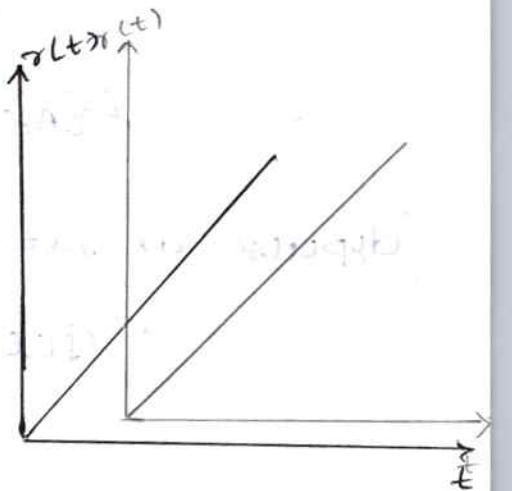
$$A\,u(t) = 0 \quad t < 0$$
$$= A \quad t > 0$$

$$\mathcal{L}[A u(t)] = \frac{A}{s}$$

b) Ramp function:

unit

$$r(t) = 0 \quad t < 0$$
$$= t \quad t \geq 0$$



$$\mathcal{L}[r(t)] = \int_0^\infty t\, e^{-st}\, dt$$

$$= t\,\frac{e^{-st}}{-s} - 1 \int \frac{e^{-st}}{-s}$$

$$= \left[t\,\frac{e^{-st}}{-s} + \frac{e^{-st}}{s^2}\right]_0^\infty$$

$$= \left[\frac{t e^{-st}}{-s}\right]_0^\infty + \frac{1}{s}\left[\frac{e^{-st}}{-s}\right]_0^\infty$$

$$= 0 + \frac{1}{s}\left[\frac{e^{-\infty}}{-\infty} - \frac{e^0}{-s}\right] = \frac{1}{s}\left[\frac{1}{s}\right]$$

$$\frac{d(\text{ramp})}{dt} = \text{step}$$

$$\frac{dr(t)}{dt} = u(t)$$

$$r(t) = \int u(t)$$

$$\mathcal{L}[r(t)] = \mathcal{L}\left[\int u(t)\right]$$

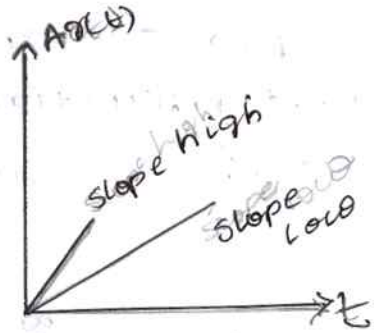
$$= \frac{U(s)}{s} = \frac{1}{s^2}$$

Normal:

$$A\,r(t) = 0 \quad t < 0$$

$$At \quad t \geq 0$$

$$L\{A\,r(t)\} = \frac{A}{s^2}$$



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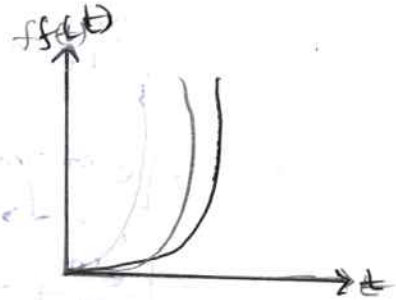
c) parabola function:

$$f(t) = 0 \quad t < 0$$

$$= \frac{t^2}{2} \quad t \geq 0$$



$$L\{f(t)\} = L\left\{\frac{t^2}{2}\right\}$$

$$= \frac{1}{2}\{L\{t^2\}\}$$

$$= \frac{1}{2} \cdot \frac{2!}{s^{2+1}}$$

$$= \frac{1}{s^3}$$

$\frac{d}{dt}(\text{Par}) = r$ ramp

$\text{Par} = \int r(t)$

$L[\text{Par}] = L[\int r(t)]$

$= \frac{R(s)}{s} = \frac{1/s}{s}$

$$A f(t) = 0 \quad t < 0$$

$$= \frac{A t^2}{2} \quad t \geq 0$$

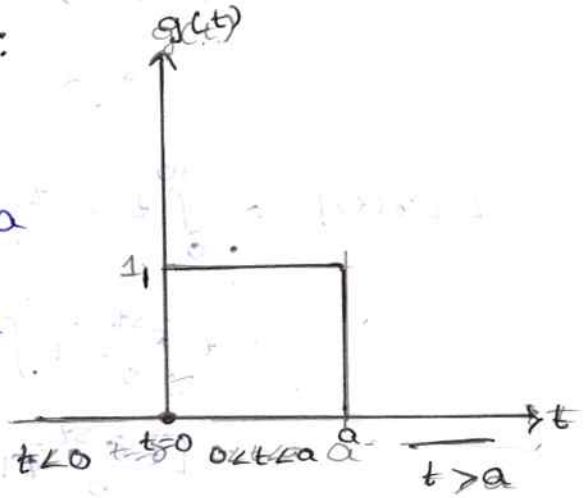
$$L\{A f(t)\} = \frac{A}{s^3}$$

d) pulse or Gate function:

$$g(t) = 0 \quad t < 0$$

$$= 1 \quad 0 < t < a$$

$$= 0 \quad t > a$$



$$L\{g(t)\} = \int_0^\infty g(t)\, e^{-st}\, dt$$

$$= \int_0^a g(t)\, e^{-st}\, dt + \int_a^\infty g(t)\, e^{-st}\, dt$$

$$= \int_0^a e^{-st}\,dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_0^a$$

$$= \frac{e^{-as}}{-s} + \frac{e^{0}}{s}$$

$$= \frac{1}{s} + \frac{e^{-as}}{-s}$$

$$= \frac{1}{s} - \frac{e^{-as}}{s}$$

$$= \frac{1}{s}\left[1 - e^{-as}\right]$$

differentiation: Impulse → Step → Ramp → Parabola

integration

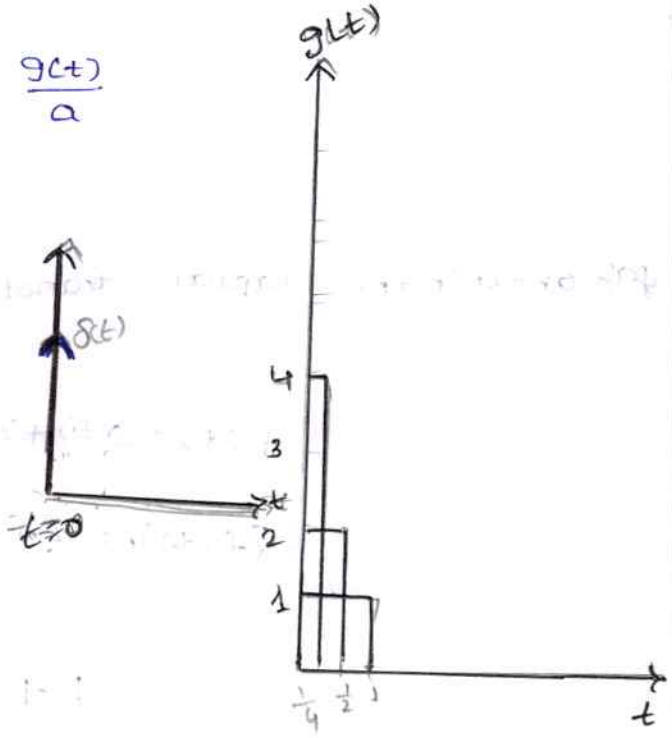
e) Impulse function:

$$\delta(t) = \lim_{a \to 0} \frac{g(t)}{a}$$

$$\delta(t) = 0 \quad t \neq 0$$

$$= \infty \quad t = 0$$

$$\int_0^\infty \delta(t)\,dt = 1$$



Parabola:

$$f(t) = 0 \quad t < 0$$

$$= \frac{t^2}{2} \quad t \geq 0$$

ramp derivative of parabola & ramp parabola is integral of ramp

$$r(t) = \frac{d f(t)}{dt} = 0 \quad t < 0$$

$$= t \quad t \geq 0$$

$$r(t) = \frac{df(t)}{dt}$$

$$f(t) = \int r(t)\,dt$$

$$u(t) = \frac{d r(t)}{dt} = 0 \quad t < 0$$

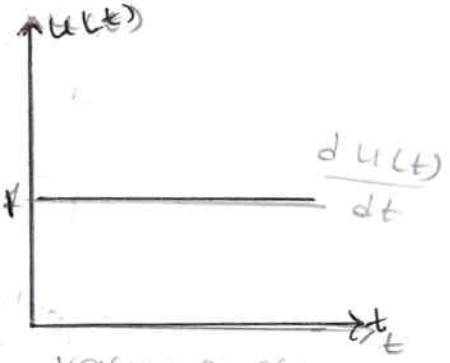
$$1 \quad t \geq 0$$

$$u(t) = \frac{dr(t)}{dt}$$

$$r(t) = \int u(t)\,dt$$

$$\frac{d\,u(t)}{dt} = \begin{cases} 0 & t < 0 \\ 0 & t > 0 \\ \infty & t = 0 \end{cases}$$

$\frac{d}{dt}$ ↓ para, ramp, step, impulse ↑ $\int$



value is changing from 0 to 1 at $t = 0$

$$\delta(t) = \frac{d\,u(t)}{dt}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{0} = \infty$$

$$\mathcal{L}\{\delta(t)\} = \mathcal{L}\left\{\frac{d\,u(t)}{dt}\right\}$$

$$= s\,U(s)$$

$$= s \times \frac{1}{s}$$

$$= 1$$

$$\mathcal{L}\{\delta(t)\} = 1$$

$$\mathcal{L}\{u(t)\} = \frac{1}{s}$$

$$\mathcal{L}\{r(t)\} = \frac{1}{s^2}$$

$$\mathcal{L}\{f(t)\} = \frac{1}{s^3}$$ parabola

Q) obtain the laplace transform of $f(t) = \delta(t) + 2u(t) + 3e^{2t}$

$$f(t) = \delta(t) + 2\,u(t) - 3\,e^{2t}$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{\delta(t) + 2u(t) - 3e^{2t}\}$$

$$= 1 + 2 \cdot \frac{1}{s} - 3\,\frac{1}{s+2}$$

$$= 1 + \frac{2}{s} - \frac{3}{s+2}$$

Shifting Theorems:

1) frequency shifting:

If $\mathcal{L}\{f(t)\} = F(s)$ then

$$\mathcal{L}\{e^{-at} f(t)\} = F(s+a)$$

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$$

6) Time shifting:

If $L\{f(t)\} = F(s)$ then

$L\{f(t-a)u(t-a)\} = e^{-as} F(s)$

$L\{f(t+a)u(t+a)\} = e^{as} F(s)$

Q08 Find laplace transform of $f(t) = e^{-at} \sin bt$.

$f(t) = e^{-at} \sin bt$

$f_1(t) = \sin bt$

$F_1(s) = L\{f_1(t)\}$

$= \frac{b}{s^2+b^2}$

$f(t) = e^{-at} f_1(t)$

$L\{f(t)\} = L\{e^{-at} f_1(t)\}$

$= F_1(s+a)$

$= \frac{b}{(s+a)^2+b^2}$

$= \frac{b}{s^2+a^2+2as+b^2}$

Q11 find laplace transform of $f(t) = e^{-at} \cos bt$

$f(t) = e^{-at} \cos bt$

$f_1(t) = \cos bt$

$F_1(s) = L\{f_1(t)\}$

$= \frac{s}{s^2+b^2}$

$f(t) = e^{-at} f_1(t)$

$L\{f(t)\} = L\{e^{-at} f_1(t)\}$

$= F_1(s+a)$

$= \frac{s+a}{(s+a)^2+b^2}$

Q12 find Laplace transform of $f(t) = t^2 e^{-t}$

$$f(t) = t^2 e^{-t}$$

$$f_1(t) = t^2$$

$$F_1(s) = L\{t^2\}$$

$$= \frac{2!}{s^{2+1}}$$

$$= \frac{2}{s^3}$$

$$L\{e^t f_1(t)\} = F(s+1)$$

$$= \frac{2}{(s+1)^3}$$

Q13 find laplace transform of $f(t) = (t+2)^2 e^t$

$$f(t) = (t+2)^2 e^t$$

$$f_1(t) = (t+2)^2$$

$$F_1(s) = L\{(t+2)^2\}$$

$$= L\{t^2 + 4 + 4t\}$$

$$= \frac{2}{s^3} + \frac{4}{s} + \frac{4}{s^2}$$

$$L\{e^t \cdot (t+2)^2\} = F_1(s-1)$$

$$= \frac{2}{(s-1)^3} + \frac{4}{(s-1)} + \frac{4}{(s-1)^2}$$

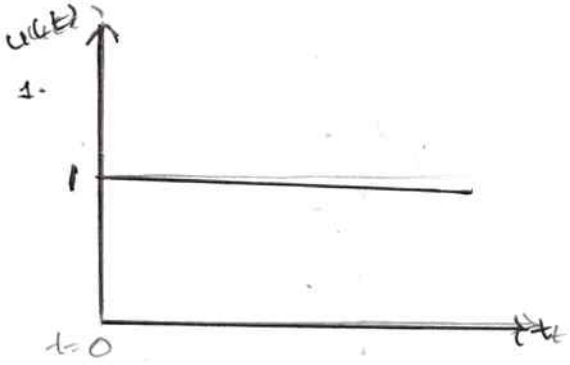
$f(t)$	$F(s)$
A	A/s
t	$1/s^2$
t^2	$2/s^3$
t^n	$\frac{n!}{s^{n+1}}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
e^{at}	$\frac{1}{s-a}$
e^{-at}	$\frac{1}{s+a}$
$u(t)$	$\frac{1}{s}$
$r(t)$	$\frac{1}{s^2}$
$\delta(t)$	1
Parabola $f(t) = \frac{t^2}{2}$	$\frac{1}{s^3}$
$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\frac{d}{dt} f(t)$	$sF(s)$
$\int f(t)\,dt$	$\frac{1}{s}F(s)$
te^{-at}	$\frac{1}{(s+a)^2}$
$\frac{t^2}{2}e^{at}$	$\frac{1}{(s+a)^3}$

31-07-2025

Shifted functions: (Delayed functions)

Unit step:

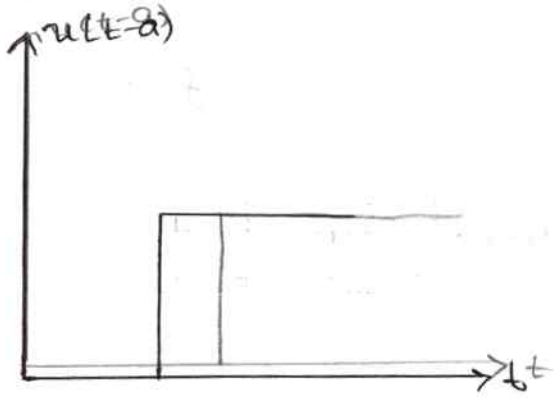
1.



$u(t) = 0 \quad t < 0$

$\quad\quad 1 \quad t > 0$

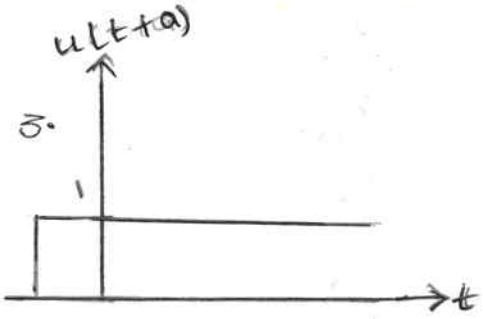
2.



$u(t-a) = 0 \quad t < a$

$\quad\quad = 1 \quad t \geq a$

3.



$u(t+a) = 0 \quad t < -a$

$\quad\quad 1 \quad t \geq -a$

1. $\mathcal{L}\{u(t)\} = \frac{1}{s}$

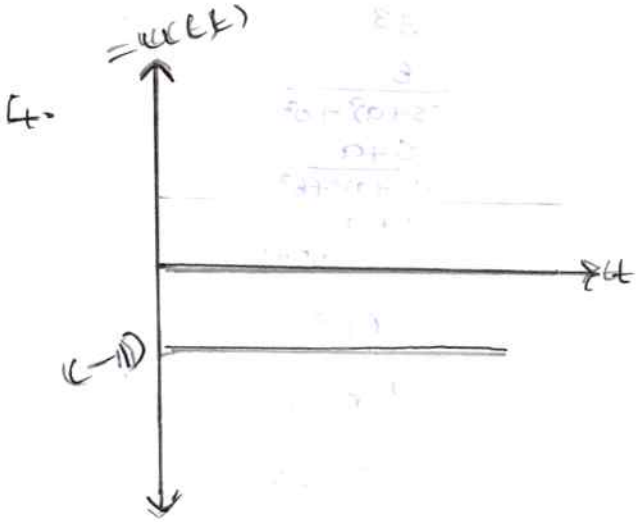
2. $\mathcal{L}\{u(t-a)\} = e^{as} \times \frac{1}{s}$

$= \frac{e^{as}}{s}$

3. $\mathcal{L}\{u(t+a)\} = e^{as} \times \frac{1}{s}$

$= \frac{e^{as}}{s}$

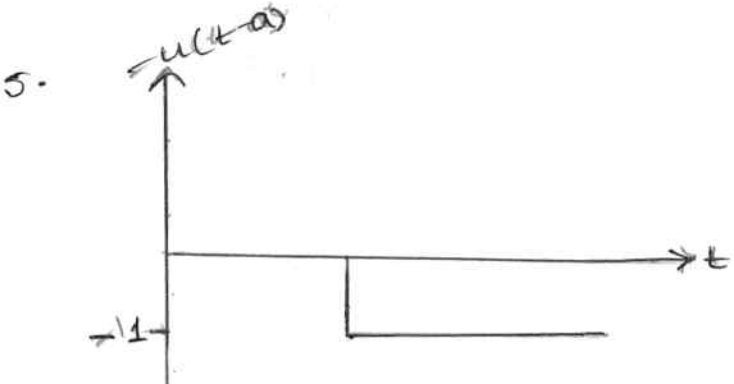
4.



$-u(t) = 0 \quad t < 0$

$\quad\quad -1 \quad t \geq 0$

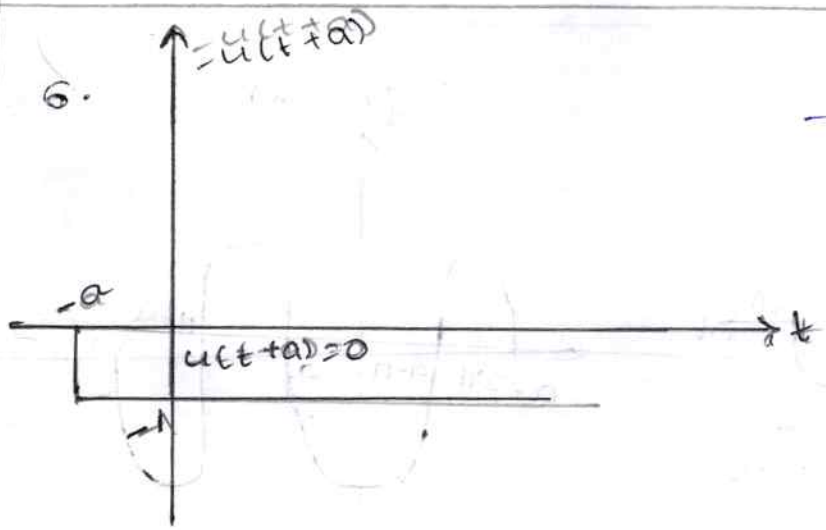
5.



$-u(t-a) = 0 \quad t < a$

$\quad\quad = -1 \quad t \geq a$

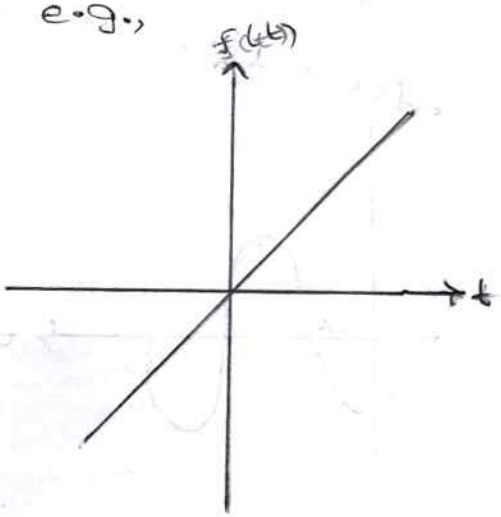
6. $-u(t+a)$



$-u(t+a) = 0 \quad t < -a$

$-1 \quad t > -a$

e.g.,



$u(t) = 0 \quad t < 0$

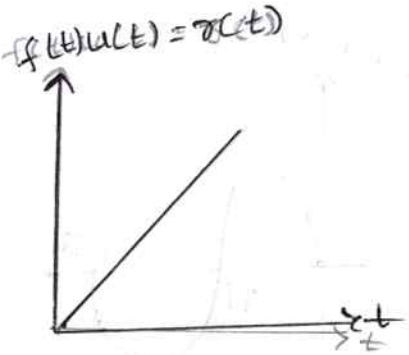
$= 1 \quad t \geq 0$

$u(t-a) = 0 \quad t < a$

$1 \quad t \geq a$

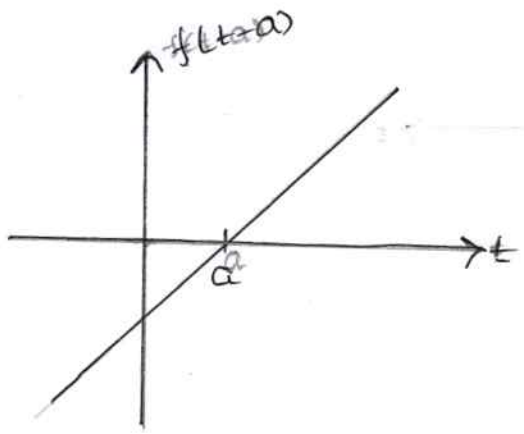
$f(t)u(t-a)$

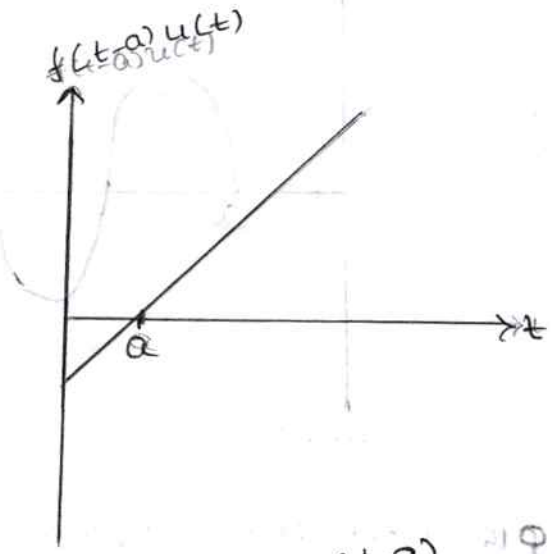
$f(t)u(t) = ?(t)$

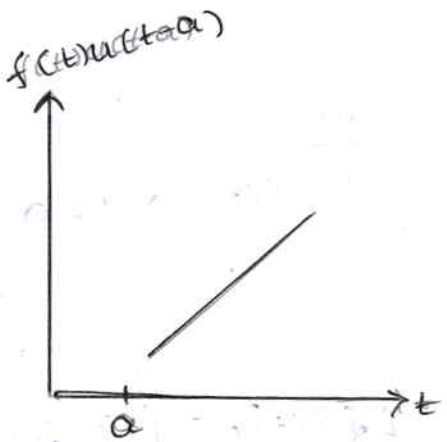


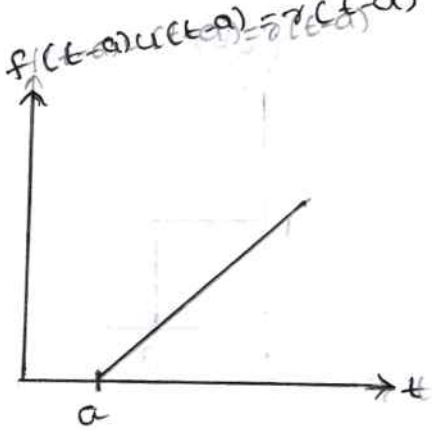
$f(t)u(t) = 0 \quad t < 0$

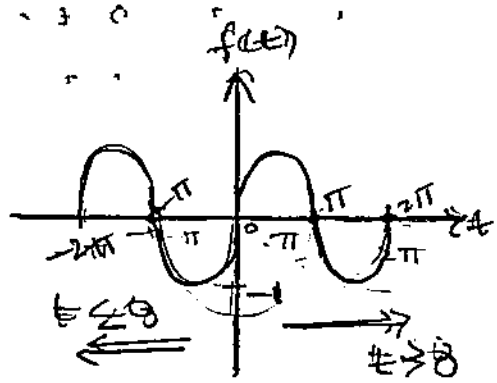
$t \geq 0$

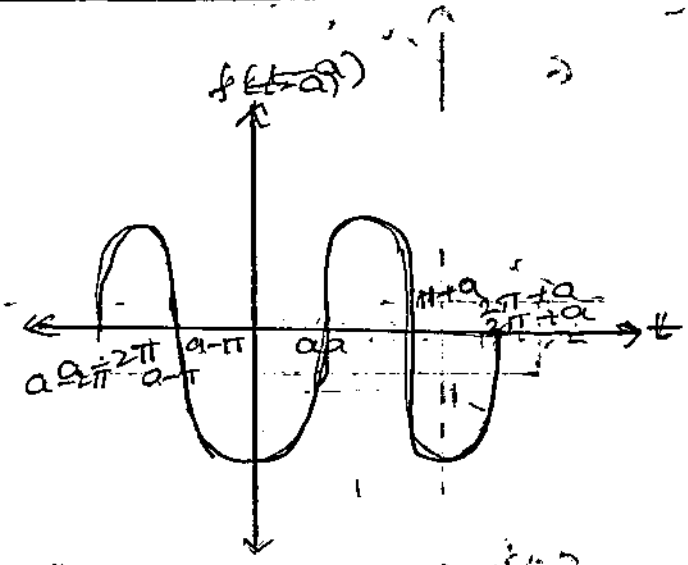


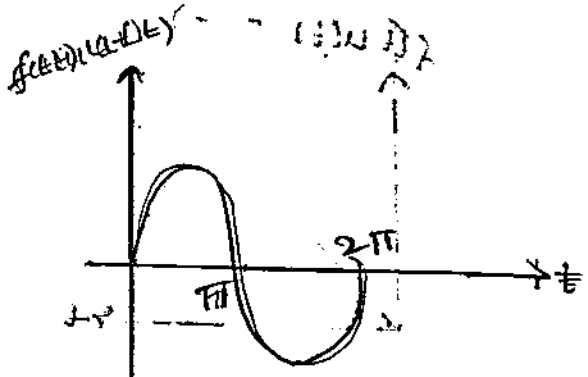


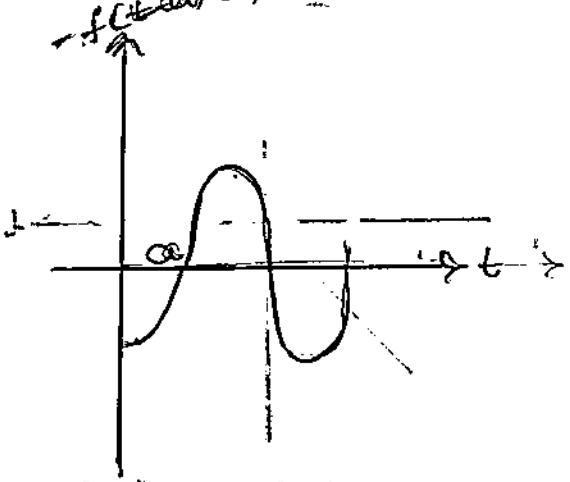


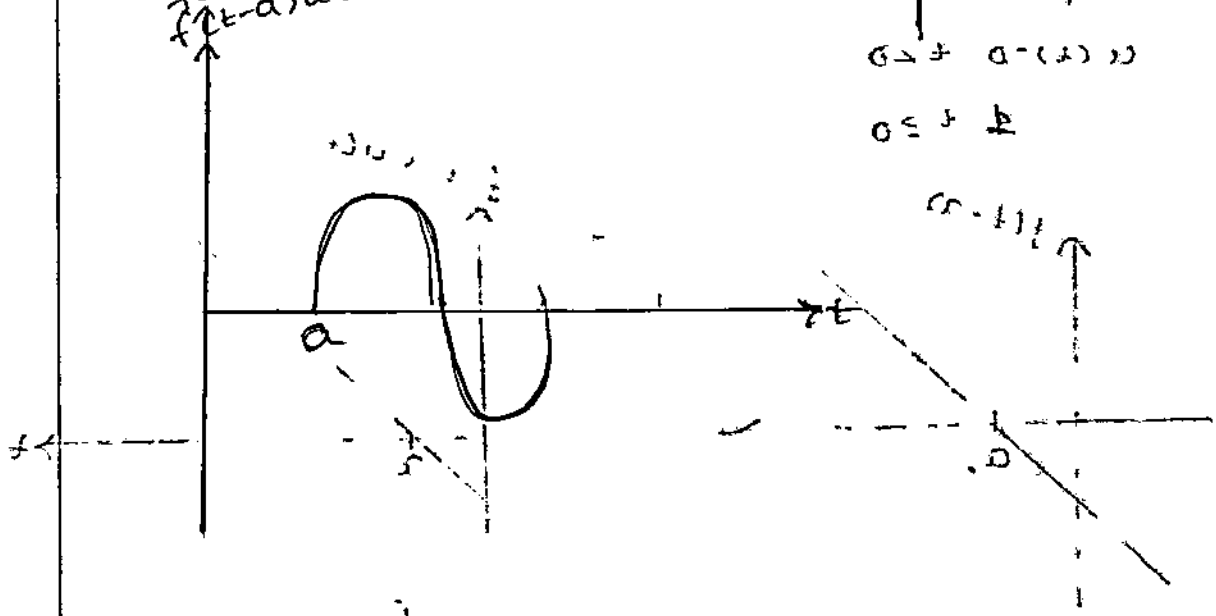




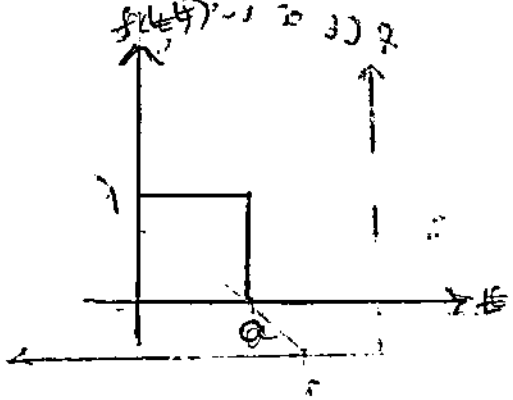








Q14 find the laplace transform of the given function

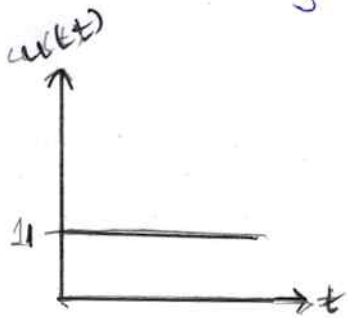


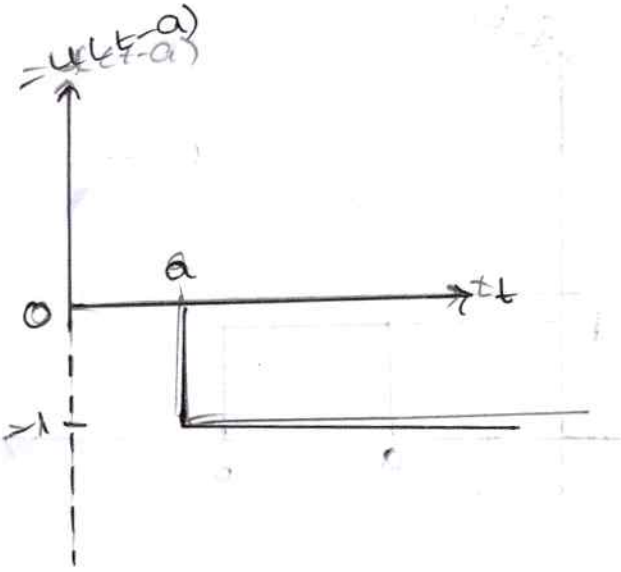
Sol

$$f(t) = 0 \quad t < 0$$
$$= 1 \quad 0 < t < a$$
$$= 0 \quad t \geq a$$

$$L\{f(t)\} = \int_0^a 1 \cdot e^{-st}\,dt + \int_a^\infty 0 \cdot e^{-st}\,dt$$

$$= \frac{(1-e^{-as})}{s}$$





$$f(t) = u(t) - u(t-a)$$

$$L\{f(t)\} = L\{u(t)\} - L\{u(t-a)\}$$

$$= \frac{1}{s} - \frac{e^{as}}{s}$$

$$= \frac{(1-e^{as})}{s}$$

01/08/2025

$$\boxed{g(t) = u(t) - u(t-a)}$$

Initial value	final value	change in value f-g	change is occured at
0	1	1	$t=0$
1	0	-1	$t=a$

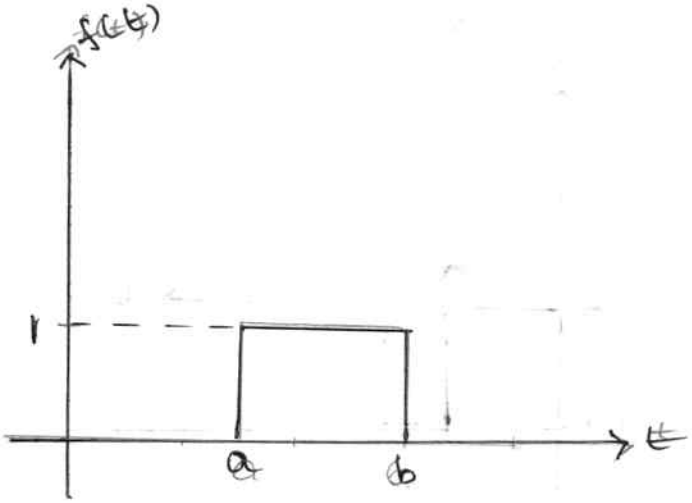
$$f(t) = 1\,u(t) - 1\,u(t-a)$$

$$L\{f(t)\} = L\{u(t)\} - L\{u(t-a)\}$$

$$= \frac{1}{s} - \frac{e^{as}}{s}$$

$$= \frac{1-e^{as}}{s}$$

Q5 Find the laplace transform of the given function



Initial value	Final value	change in value	change is occured at
0	1	1	$t=a$
1	0	-1	$t=b$

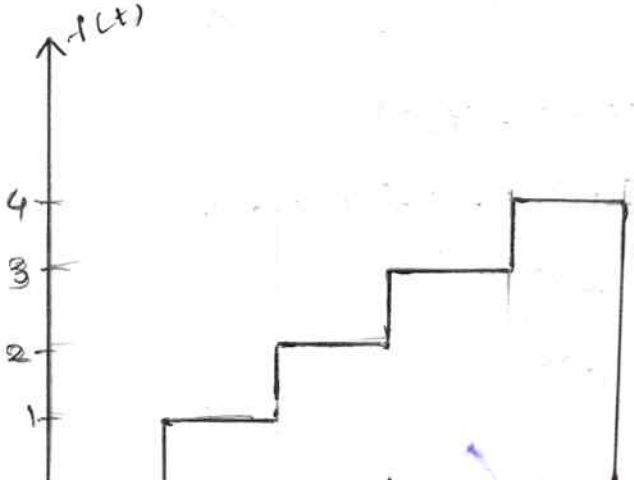
$$f(t) = 1\,u(t-a) - 1\,u(t-b)$$

$$L\{f(t)\} = L\{u(t-a)\} - L\{u(t-b)\}$$

$$= \frac{e^{-as}}{s} - \frac{e^{-bs}}{s}$$

$$= \frac{e^{-as} - e^{-bs}}{s}$$

Q6 Fend the laplace transform (function) of given function

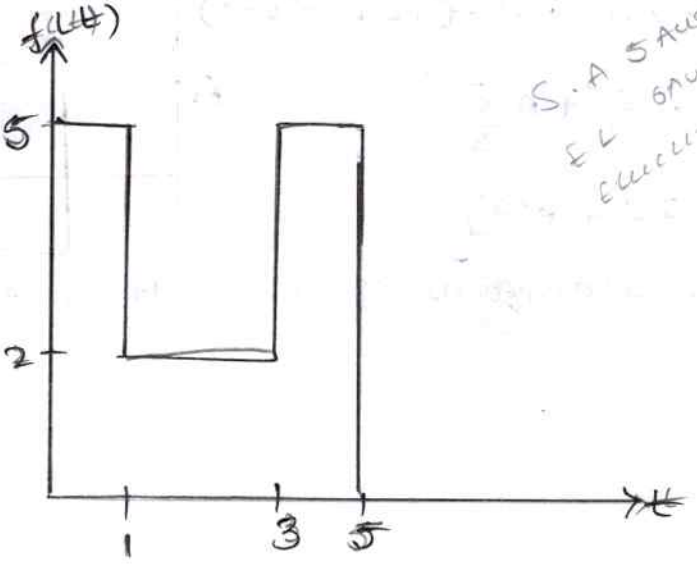


Initial value	final value	change in value	change is occured at
0	1	1	$t=2$
1	2	1	$t=4$
2	3	1	$t=6$
3	4	1	$t=8$
4	0	-4	$t=10$

$$f(t) = 1u(t-2) + 1u(t-4) + 1u(t-6) + 1u(t-8) - 4u(t-10) = 0$$

$$L\{f(t)\} = \frac{e^{-2s}}{s} + \frac{e^{-4s}}{s} + \frac{e^{-6s}}{s} + \frac{e^{-8s}}{s} - \frac{4e^{-10s}}{s}$$

Q1) Find the laplace transform of given function



Initial value	Final value	Change in value	change is occured at
0	5	5	$t=0$
5	2	-3	$t=1$
2	5	3	$t=3$

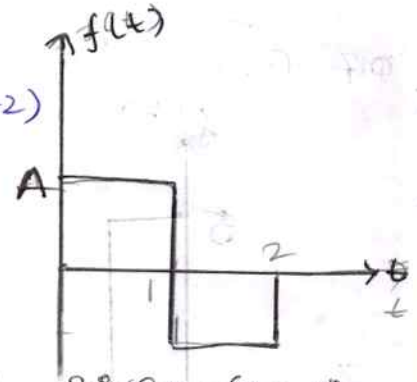
$$f(t) = 5u(t) - 3u(t-1) + 3u(t-3) - 5u(t-4)$$

$$L\{f(t)\} = \frac{5}{s} - 3\cdot\frac{e^{-s}}{s} + 3\cdot\frac{e^{-3s}}{s} - 5\frac{e^{-4s}}{s}$$

$$= \frac{1}{s}[5 - 3e^{-s} + 3e^{-3s} - 5e^{-4s}]$$

Q18 Find the Laplace transform for given function.

Initial value	Final value	Change in value	Change is occured at
0	A	A	$t=0$
A	-A	-2A	$t=1$
-A	0	A	$t=2$

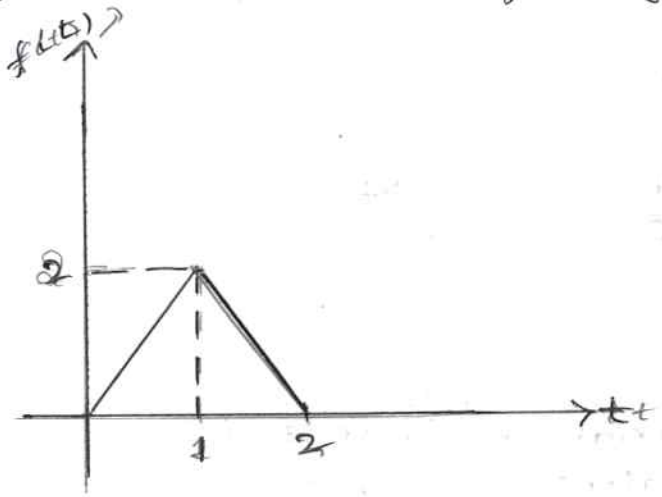


$$f(t) = A\,u(t) - 2A\,u(t-1) + A\,u(t-2)$$

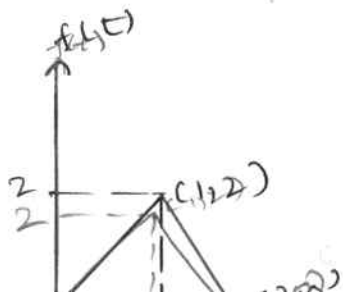
$$= \frac{A}{s} - 2A\frac{e^{-s}}{s} + A\frac{e^{-2s}}{s}$$

$$= \frac{A}{s}[1 - 2e^{-s} + e^{-2s}]$$

Q19 Find the laplace transform of the given function



A



$$L\{r(t)\} = \frac{1}{s^2}$$

$$L\{r(t-a)\}$$

Initial value	Final value	change in value	change occured at
00	2	2	$t=0$
22	-2	-4	$t=1$
-22	0	2	$t=2$

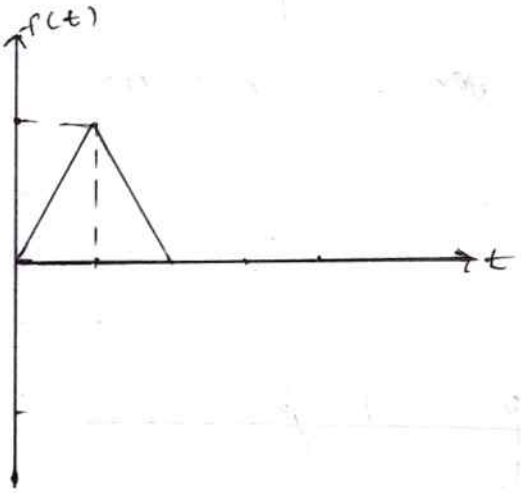
$$f(t) = 2\delta(t) - 4\delta(t-1) + 2\delta(t-2)$$

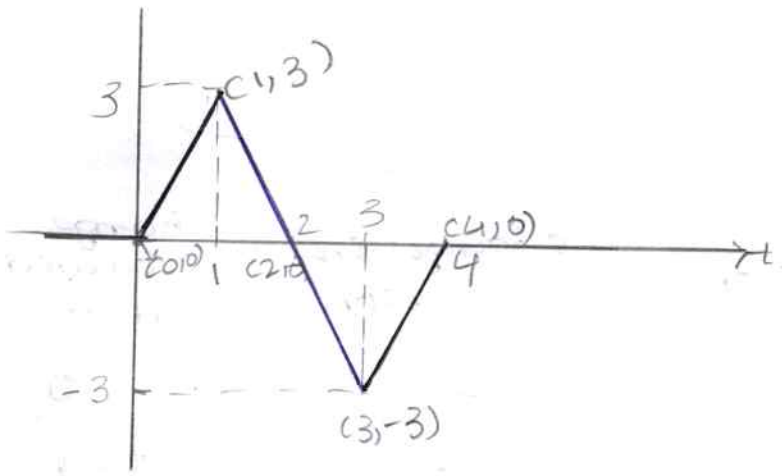
$$= 2 \cdot \frac{1}{s^2} - 4\frac{e^{-s}}{s^2} + 2\frac{e^{-2s}}{s^2}$$

$$= \frac{2}{s^2}\left[1 - 2e^{-s} + e^{2s}\right]$$

04/08/2025

Q Find the Laplace transform of given function





Initial value/ slope	final slope	change in slope	change in occurring at t
Ramp 0	3	3	$t=0$
3	-3	-6	$t=1$
-3	3	6	$t=3$
3	0	-3	$t=4$

Slope $= \frac{y_2-y_1}{x_2-x_1}$

$= 3$

$\frac{-3-3}{3-1} = \frac{-6}{2}$

$\frac{0-3}{2-1}$

$\frac{3-1}{-3-3} = \frac{2}{-6}$

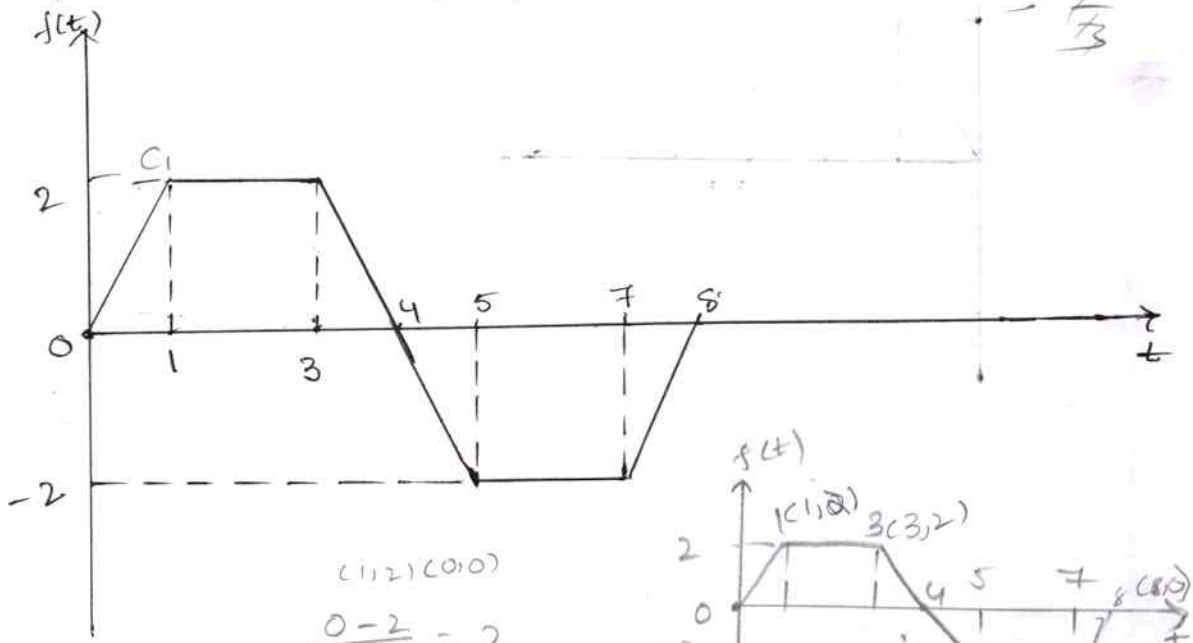
$= -\frac{1}{3}$

$$f(t) = 3r(t) - 6r(t-1) + 6r(t-3) - 3r(t-4)$$

$$\mathcal{L}\{f(t)\} = \frac{3}{s^2} - 6\frac{e^{-s}}{s^2} + 6\frac{e^{-3s}}{s^2} - 3\frac{e^{-4s}}{s^2}$$

$$= \frac{3}{s^2}\left[1 - 2e^{-s} + 2e^{-3s} - e^{-4s}\right]$$

Q Find laplace transform of given function.



(1,2) (0,0)

$\frac{0-2}{0-1} = 2$

Initial slope	Final slope	change in slope	change occured at t
0	2	2	$t=0$
2	0	-2	$t=1$
0	-2	-2	$t=3$
-2	0	2	$t=5$

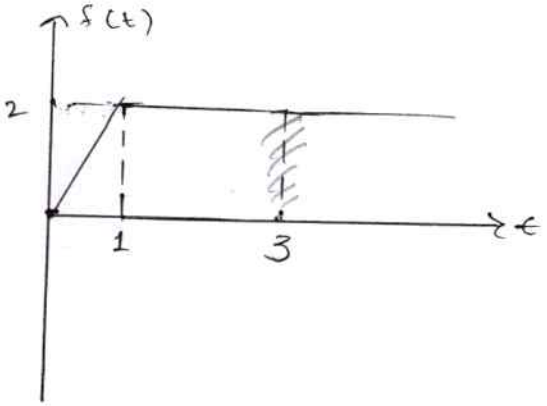
Initial slope	final slope	change in slope	change occuring at
2	0	−2	$t = 8$

$$f(t) = 2r(t) - 2r(t-1) - 2r(t-3) + 2r(t-5) + 2r(t-7) - 2r(t-8)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{2r(t) - 2r(t-1) - 2r(t-3) + 2r(t-5) + 2r(t-7) - 2r(t-8)\}$$

$$= \frac{2}{s^2} - \frac{2e^{-s}}{s^2} - \frac{2e^{-3s}}{s^2} + \frac{2e^{-5s}}{s^2} + \frac{2e^{-8s}}{s^2}$$

Q Find the laplace transform of given function



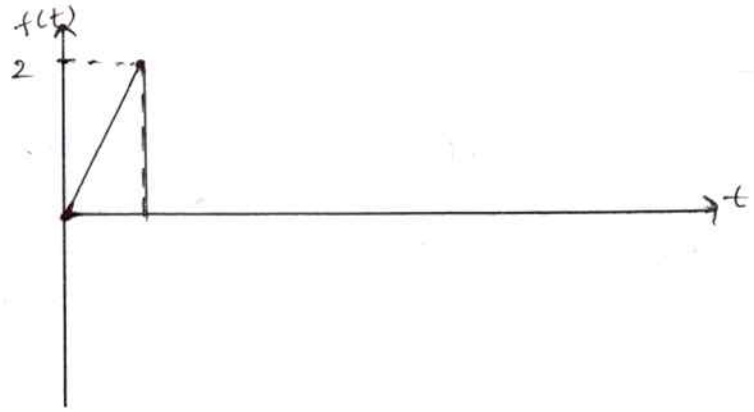
Initial value	Final value	change in value	change is occured at
0	2	2	$t=0$
2	0	−2	$t=1$

$$f(t) = 2r(t) - 2r(t-1)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{2r(t) - 2r(t-1)\}$$

$$= \frac{2}{s^2} - \frac{2e^{-s}}{s^2}$$

Q find the laplace transform of given function



Method-1:

$f(t) = 2t \quad 0 < t < 1$

$= 0 \quad t > 1$

$L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt$

$= \int_0^1 f(t)e^{-st}dt + \int_1^\infty f(t)e^{-st}dt$

$= \int_0^1 2te^{-st}dt + 0$

$=$

Method-2:

Ramp

Initial slope	Final slope	change in slope	change in occuring at
0	2	+2	$t=0$
2	0	−2	$t=1$

Steep

Initial slope ratio	Final slope	change in slope	change in occuring at

$$f(t) = 2\gamma(t) - 2\gamma(t-1) - 2u(t-1)$$

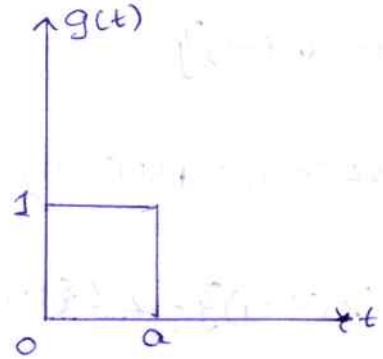
$$\mathcal{L}\{f(t)\} = \mathcal{L}\{2\gamma(t) - 2\gamma(t-1) - 2u(t-1)\}$$

$$= \frac{2}{s^2} - \frac{2e^{-s}}{s^2} - \frac{2e^{-s}}{s}$$

$$\left(= \frac{2}{s^2}[1 - e^{-s} - e^{-s}]\right)$$

Method - 3:

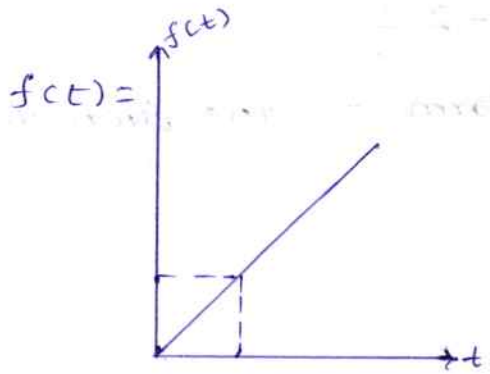
By using gate function



$f(t) \times g(t) = 0 \quad t < 0$

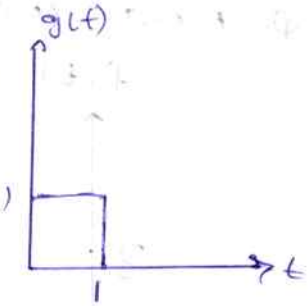
$= f(t) \quad 0 < t < a$

$= 0 \quad t > a$



$f(t) =$

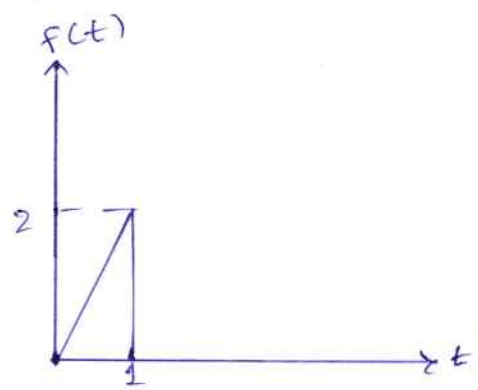
$\times \quad g(t) =$



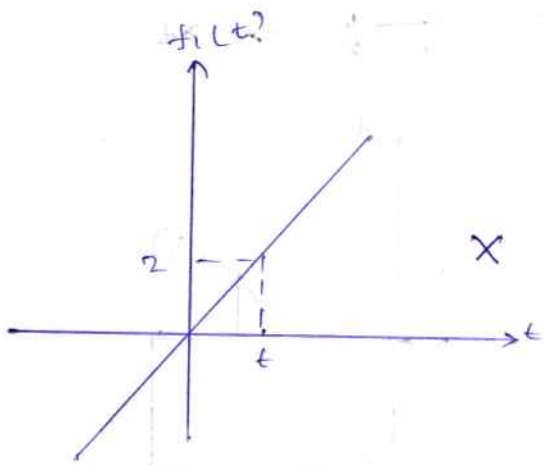
$f(t) = 2\gamma(t)$

$f(t) = f_1(t) \times g(t)$

05/08/25

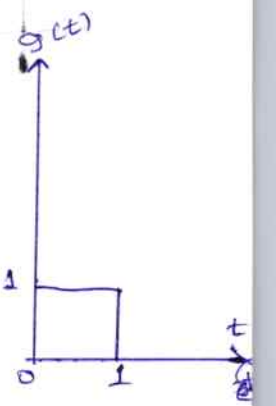


$f(t)$



$f_1(t) = 2t$

$\times$



$g(t) = 0 \quad t < 0$

$= 1 \quad 0 < t < 1$

$= 0 \quad t > 1$

$f(t) = f_1(t) \times g(t) = 0 \quad t < 0$

$$f(t) = 2t\,[u(t) - u(t-1)]$$

$$= 2t\,u(t) - 2t\,u(t-1)$$

$$\mathcal{L}\{f(t)\} = 2\mathcal{L}\{t\,u(t)\} - 2\mathcal{L}\{t\,u(t-1)\}$$

$\because \mathcal{L}\{e^{-at} f(t)\} = F(s+a)$

$\mathcal{L}\{f(t-a)\,u(t-a)\} = e^{-as} F(s)$

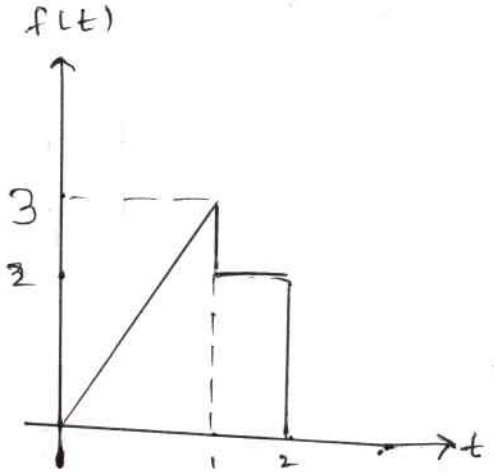
$$= \frac{2}{s^2} - 2\mathcal{L}\{(t-1+1)u(t-1)\}$$

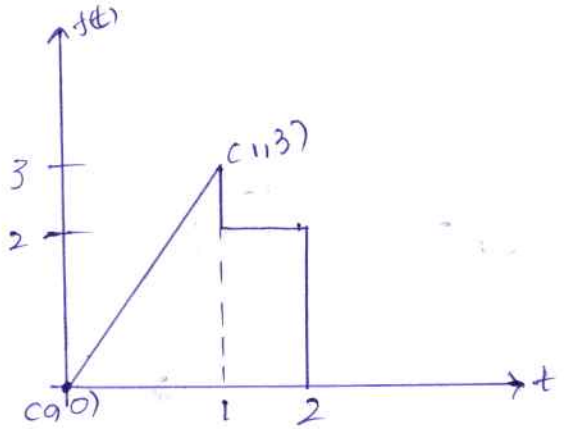
$$= \frac{2}{s^2} - 2\mathcal{L}\{(t-1)u(t-1) + 1\,u(t-1)\}$$

$$= \frac{2}{s^2} - 2\mathcal{L}\{(t-1)u(t-1)\} - 2\mathcal{L}\{u(t-1)\}$$

$$= \frac{2}{s^2} - 2\frac{e^{-s}}{s^2} - 2\frac{e^{-s}}{s}$$

Q Find the laplace transform of the given function.





Ramp

Initial slope	Final slope	change in slope	change is occured at
0	3	3	$t=0$
3	0	-3	$t=1$

step

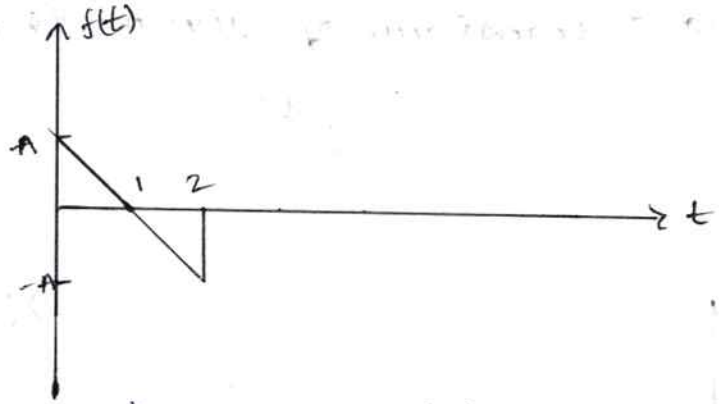
Initial slope	Final slope	change in slope	change is occured at
3	2	-1	$t=1$
2	0	-2	$t=2$

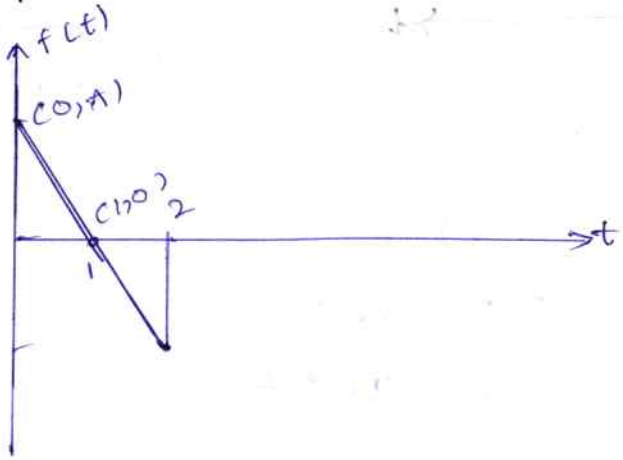
$$f(t) = 3r(t) - 3r(t-1) - u(t-1) - 2u(t-2)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{3r(t) - 3r(t-1) - u(t-1) - 2u(t-2)\}$$

$$= \frac{3}{s^2} - \frac{3e^{-s}}{s^2} - \frac{e^{-s}}{s} - \frac{2e^{-2s}}{s}$$

Q Find the laplace transform of the given function





step Initial value	Final value	change in value	change is occured at
0	A	A	$t=0$

Ramp Initial value	Final value	change in value	change is occured at
0	$-A$	$-A$	$t=0$
$-A$	0	A	$t=2$

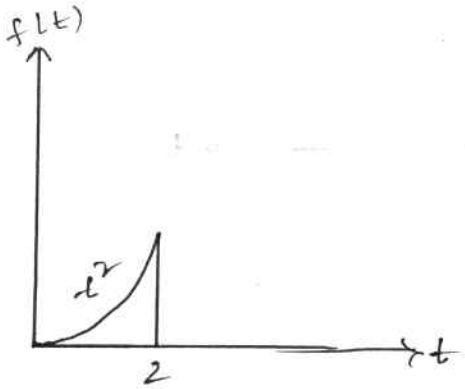
step Initial value	Final value	change in value	change is occured at
$-A$	0	A	$t=2$

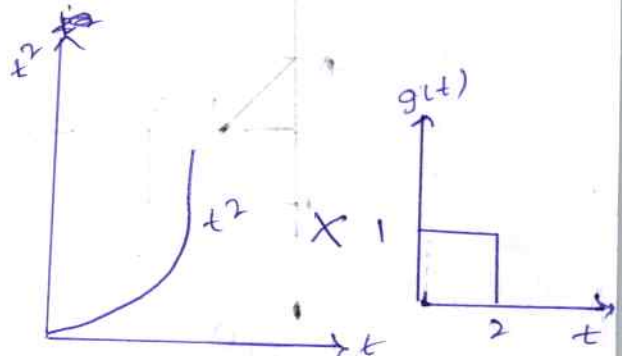
$$f(t) = A\,u(t) - A\,r(t) + A\,r(t-2) + A\,r\,u(t-2)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{A\,u(t) - A\,r(t) + A\cdot r(t-2) + A\cdot u(t-2)\}$$

$$= \frac{A}{s} - \frac{A}{s^2} + A\frac{e^{-2s}}{s^2} + \frac{Ae^{-2s}}{s}$$

Q Find the laplace transform of given function





$f_1(t) = t^2$

$g(t)$

$= 0 \quad t<0$

$= 1 \quad 0<t<2$

$= 0 \quad t>2$

$f(t) = f_1(t) \times g(t)$

$= 0 \quad t<0$

$= t^2 \quad 0<t<2$

$= 0 \quad t>2$

$= t^2 u(t) - t^2 u(t-2)$

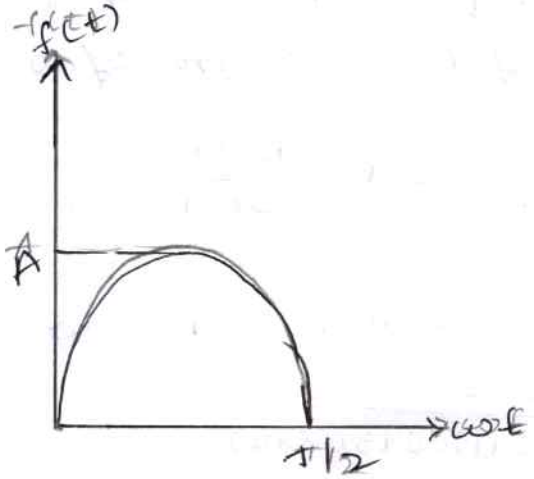
$= t^2 u(t) - (t-2+2)^2 u(t-2)$

$= t^2 u(t) - [(t-2)^2 + 4 + 4(t-2)] u(t-2)$

$f(t) = t^2 u(t) - (t-2)^2 u(t-2) + 4u(t-2) + 4(t-2)u(t-2)$

$\mathcal{L}\{f(t)\} = \frac{2}{s^3} - \frac{2e^{-2s}}{s^3} + \frac{4e^{2s}}{s} + \frac{4e^{2s}}{s^2}$

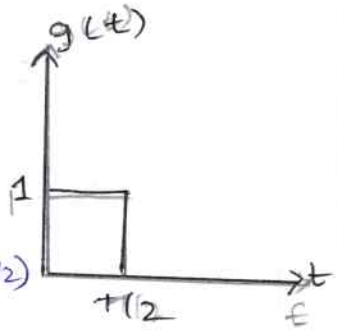
Q find the laplace transform of the given function



$\omega = 2\pi f = \frac{2\pi}{T}$

$f(t) = A \sin\left(\frac{2\pi}{T}\right) t \quad 0 < t < T/2$

$f(t) = A \sin\left(\frac{2\pi}{T}\right) t \times g(t)$



$f(t) = A \sin\left(\frac{2\pi}{T}\right) t\, u(t) - A \sin\left(\frac{2\pi}{T}\right) t\, u(t-T/2)$

$= A \sin\left(\frac{2\pi}{T}\right) t\, u(t) * A \sin\left(\frac{2\pi}{T}\right)\left(t - \frac{T}{2} + \frac{T}{2}\right) u(t-T/2)$

$= A \sin\left(\frac{2\pi}{T}\right) t \cdot u(t) - A\{\sin\frac{2\pi}{T}(t-\frac{T}{2}) \cos\frac{2\pi}{T}\times\frac{T}{2} + \cos\frac{2\pi}{T} \times$

$(t-T/2) \cdot \sin\frac{2\pi}{T}\cdot\frac{T}{2}\} u(t-\frac{T}{2})$

$= A\sin\left(\frac{2\pi}{T}\right)t + A\sin\frac{2\pi}{T}(t-T/2)\, u(t-T/2)$

$\mathcal{L}\{f(t)\} = \frac{A\frac{2\pi}{T}}{s^2 + \left(\frac{2\pi}{T}\right)^2} + \frac{A\frac{2\pi}{T} e^{-T/2 s}}{s^2 + \left(\frac{2\pi}{T}\right)^2}$

Inverse Laplace Transformation:

Using Partial Fraction transformation:

$$f(t) \longrightarrow F(s)$$

$$F(s) \longrightarrow f(t)$$

$$F(s) = \frac{P(s)}{Q(s)}$$

Degree of $P(s) <$ Degree of $Q(s)$

Degree of $P(s) =$ Degree of $Q(s)$

$$F(s) = K + \frac{R(s)}{Q(s)}$$

roots of $Q(s)$:

1. real and non repeated

e.g. $F(s) = \frac{s+2}{(s+1)(s+3)(s+4)}$

$s = -1, -3, -4$

2. real and repeated

e.g. $F(s) = \frac{s+2}{(s+1)^2(s+3)}$

$s = -1, -1, -3$

3. Complex conjugate

e.g. $F(s) = \frac{s+2}{(s+1)^2+2^2}$

$s^2+1+2s+4$

$s^2+2s+5=0$

$s = -1 \pm j2$

4. Pure imaginary

$a = 0$

$F(s) = \frac{s+1}{s^2+2^2}$

Case-1 : Real and Non Repeated Roots :

$$F(s) = \frac{P(s)}{Q(s)} = \frac{P(s)}{(s+a)(s+b)(s+c)\cdots}$$

$$f(s) = \frac{c_1}{s+a} + \frac{c_2}{s+b} + \frac{c_3}{s+c} + \cdots\cdots$$

$$c_1 = f(s)(s+a) = \frac{P(s)}{(s+b)(s+c)}\Big|_{s=-a}$$

$$c_2 = f(s)(s+b) = \frac{P(s)}{(s+a)(s+c)}\Big|_{s=-b}$$

$$c_3 = f(s)(s+c) = \frac{P(s)}{(s+a)(s+b)}\Big|_{s=-c}$$

Q1. Find the inverse Laplace transform of $\frac{2s+3}{(s+1)(s+3)}$

Given

$$F(s) = \frac{2s+3}{(s+1)(s+3)} = \frac{c_1}{s+1} + \frac{c_2}{s+3}$$

$$c_1 = f(s)(s+1) = \frac{2s+3}{(s+3)}\Big|_{s=-1}$$

$$= \frac{2(-1)+3}{(-1)+3}$$

$$= \frac{-2+3}{-1+3} = \frac{1}{2}$$

$$c_2 = F(s)(s+3) = \frac{2s+3}{s+1}\Big|_{s=-3}$$

$$= \frac{2(-3)+3}{(-3)+1}$$

$$= \frac{-6+3}{-3+1} = \frac{3}{2}$$

$$F(s) = \frac{1/2}{s+1} + \frac{3/2}{s+3}$$

$$f(t) = L^{-1}\left\{\frac{1/2}{s+1}\right\} + L^{-1}\left\{\frac{3/2}{s+3}\right\}$$

$$f(t) = \frac{1}{2}e^{-t} + \frac{3}{2}e^{-3t}$$

Q2 Find $f(t)$ for $F(s) = \dfrac{s^2+s+1}{s(s+3)(s+5)}$

Given

$$F(s) = \frac{s^2+s+1}{s(s+3)(s+5)}$$

roots $s = 0, -3, -5$

$$F(s) = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+5}$$

$$C_1 = F(s)\,s = \frac{P(s)}{(s+3)(s+5)}$$

$$= \frac{s^2+s+1}{(s+3)(s+5)}\Big|_{s=0}$$

$$= \frac{1}{15}$$

$$C_2 = F(s)(s+3) = \frac{s^2+s+1}{s(s+5)}\Big|_{s=-3}$$

$$= \frac{(-3)^2+(-3)+1}{(-3)(-3+5)}$$

$$= \frac{-7}{6}$$

$$C_3 = F(s)(s+5) = \frac{s^2+s+1}{s(s+3)}\Big|_{s=-5}$$

$$= \frac{(-5)^2+(-5)+1}{-5(-5+3)}$$

$$= \frac{21}{10}$$

$$F(s) = \frac{1/15}{s} - \frac{7/6}{s+3} + \frac{21/10}{s+5}$$

$$L^{-1}\{F(s)\} = L^{-1}\left\{\frac{1/15}{s}\right\} - L^{-1}\left\{\frac{7/6}{s+3}\right\} + L^{-1}\left\{\frac{21/10}{s+5}\right\}$$

$$= \frac{1}{15} - \frac{7}{6}e^{-3t} + \frac{21}{10}e^{-5t}$$

06.08.2025

Q3. Find $f(t)$ for $F(s) = \frac{2s^2+1}{s^2+4s+3}$

Given

$$F(s) = \frac{2s^2+1}{s^2+4s+3}$$

roots for s^2+4s+3 are $-1, -3$

$$= \frac{2s^2+1}{(s+1)(s+3)}$$

$$C_1 = F(s)(s+1) = \frac{2s^2+1}{s+3}\Big|_{s=-1}$$

$$= \frac{2(-1)^2+1}{3-1}$$

$$= \frac{3}{2}$$

$$C_2 = F(s)(s+3) = \frac{2s^2+1}{s+1}\Big|_{s=-3}$$

$$= \frac{2(-3)^2+1}{-3+1}$$

$$= \frac{-19}{2}$$

$$F(s) = \frac{\frac{3}{2}}{s+1} - \frac{19/2}{s+3}$$

$$\mathcal{L}^{-1}\{F(s)\} = \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - \frac{19}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}$$

$$= \frac{3}{2}e^{-t} - \frac{19}{2}e^{-3t}$$

$$s^2+4s+3 \;)\; 2s^2+1 \;(\; 2$$

$$\underline{2s^2+8s+6}$$

$$-8s-5$$

$$F_1(s) = 2 - \frac{8s+5}{s^2+4s+3}$$

$$C_1 = F(s)(s+1) = \frac{8s+5}{s+3}\Big|_{s=-1}$$

$$= \frac{8(-1)+5}{-1+3}$$

$$= \frac{-3}{2}$$

$$C_2 = F(s)(s+3) = \frac{8s+5}{s+1}\Big|_{s=-3}$$

$$= \frac{8(-3)+5}{-3+1}$$

$$= \frac{19}{2}$$

$$\mathcal{L}^{-1}\{F(s)\} = \frac{-\frac{3}{2}}{s+1} + \frac{\frac{19}{2}}{s+3}$$

$$\mathcal{L}^{-1}\{F(s)\} = -\frac{3}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \frac{19}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}$$

$$f(t) = -\frac{3}{2}e^{-t} + \frac{19}{2}e^{-3t}$$

$$= \delta(t) + \frac{3}{2}e^{-t} + \frac{19}{2}e^{-3t}$$

× wrong

$$F(s) = 2 + \frac{\frac{3}{2}}{s+1} + \frac{\frac{19}{2}}{s+3}$$

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\{2\} + \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \frac{19}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}$$

$$= 2\delta(t) + \frac{3}{2}e^{-t} + \frac{19}{2}e^{-3t}$$

Case – 2: roots are real and repeated.

$$F(s) = \frac{P(s)}{(s+a)^n}$$

E.g. $\dfrac{s+1}{(s+2)^3(s+3)(s+4)}$

roots of $(s+2)^3(s+3)(s+4)$ are

$$F(s) = \frac{A_1}{(s+a)} + \frac{A_2}{(s+a)^2} + \cdots + \frac{A_n}{(s+a)^n} + \frac{B}{s+b} + \frac{C}{s+c}$$

$$f(s) = \frac{A_1}{s+2} + \frac{A_2}{(s+2)^2} + \frac{A_3}{(s+3)^3} + \frac{B}{s+3} + \frac{C}{s+4}$$

$$B = F(s)(s+b)\big|_{s=-b}$$

$$C = F(s)(s+b)\big|_{s=-c}$$

$$A_n = F(s)(s+a)^n \big|_{\text{at } s=-a}$$

$$A_{n-1} = \frac{d}{ds}\left[F(s)(s+a)^n\right]\Big|_{\text{at } s=-a}$$

$$A_{n-2} = \frac{1}{2!}\frac{d^2}{ds}\left[F(s)(s+a)^n\right]\Big|_{\text{at } s=-a}$$

$$\vdots$$

Q1. Find inverse laplace of $F(s) = \frac{s-5}{s(s+2)^2}$

Given

$$F(s) = \frac{s-5}{s(s+2)^2}$$

$$= \frac{A}{s} + \frac{B_1}{s+2} + \frac{B_2}{(s+2)^2}$$

$$A = F(s)\cdot s = \frac{s-5}{(s+2)^2}\Big|_{s=0}$$

$$= \frac{0-5}{(0+2)^2}$$

$$= \frac{-5}{4}$$

$B_1 = F(s)\frac{d}{ds}\left[\ \right]$ wrong

$$B_2 = \frac{s-5}{s(s+1/2)^2} \cdot (s+2)^2 \Big|_{s=-2}$$

$$= \frac{-2-5}{-2} = \frac{-7}{-2}$$

$$= \frac{7}{2}$$

$$B_1 = \frac{d}{ds}\left[\frac{s-5}{s}\right]\Big|_{at\ s=-2} \quad \left[\because \frac{d}{ds}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}\right]$$

$$= \frac{s \times 1 - (s-5)\times 1}{s^2}$$

$$= \frac{s - s + 5}{s^2}$$

$$= \frac{+5}{s^2}$$

$$= \frac{5}{4}$$

$$f(s) = \frac{-5/4}{s} + \frac{7/2}{s+2} + \frac{5/4}{(s+2)^2}$$

$$L^{-1}\{f(s)\} = -\frac{5}{4}L^{-1}\left\{\frac{1}{s}\right\} + \frac{7}{2}L^{-1}\left\{\frac{1}{s+2}\right\} + \frac{5}{4}L^{-1}\left\{\frac{1}{(s+2)^2}\right\}$$

$$f(t) = -\frac{5}{4} + \frac{7}{2}e^{-2t} + \frac{5}{4}e^{-2t} \cdot t$$

HW

Q2 $$F(s) = \frac{2s+1}{s(s+1)^2(s+2)}$$

$$F(s) = \frac{2s+1}{s(s+1)^2(s+2)}$$

$$= \frac{A}{s} + \frac{B_1}{s+1} + \frac{B_2}{(s+1)^2} + \frac{C}{s+2}$$

$$A = sF(s)\Big|_{s=0} = \frac{2s+1}{(s+1)^2(s+2)} = \frac{1}{2}$$

$$B_2 = (s+1)^2 F(s)\Big| \qquad \frac{2s+1}{} = \frac{-1}{}$$

$$B_1 = \frac{d}{ds}\left[F(s)(s+1)^2\right]\Big|_{s=-1}$$

$$= \frac{d}{ds}\left[\frac{2s+1}{s(s+2)}\right]$$

$$= \frac{d}{ds}\left[\frac{2s+1}{s^2+2s}\right]$$

$$= \frac{s^2+2s(2) - (2s+1)(2s+2)}{(s^2+2s)^2}$$

$$= \frac{2s^2+4s-4s^2-4s-2s-2}{s^4+4s^3+4s^2}$$

$$= \frac{-2s^2-2s-2}{s^4+4s^2+4s^3}$$

$$= \frac{-2+i-2}{1+4-4}$$

$$= -2$$

$$C = F(s)(s+2)\Big|_{s=-2}$$

$$= \frac{2s+1}{s(s+1)^2}$$

$$= \frac{-4+1}{-2(-1)^2}$$

$$= \frac{-3}{-2} = \frac{3}{2}$$

$$F(s): \frac{1/2}{s} - \frac{2}{s+1} + \frac{1}{(s+1)^2} + \frac{3/2}{s+2}$$

Apply Inverse Laplace transform

$$\mathcal{L}^{-1}\{F(s)\} = \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - 2\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} + \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}$$

$$f(t) = \frac{1}{2} - 2e^{-t} + te^{-t} + \frac{3}{2}e^{-2t}$$

Case-3 : Complex conjugate.

a. when roots are pure imaginary

$s = \pm jb$ $\quad s^2+b^2$

If s^2+b^2 is a factor then we get above equation (roots)

b. when roots are complex conjugate

$s = -a \pm jb$ $\quad -a+jb$, $-a-jb$

Case-a : roots are pure imaginary :

$$F(s) = \frac{P(s)}{(s^2+b^2)} \qquad s^2+b^2 = 0$$

$$= \frac{Bs+C}{(s^2+b^2)} \qquad s^2 = -b^2$$

$$s = -j\sqrt{b^2}$$

$$s = \pm jb$$

Q1. Find the inverse Laplace of $F(s) = \frac{2}{s(s^2+2)}$

$$F(s) = \frac{2A}{s(s^2+2)}$$

$$F(s) = \frac{A}{s} + \frac{Bs+C}{s^2+2}$$

$$\frac{2}{s(s^2+2)} = \frac{A(s^2+2)+(Bs+C)s}{s(s^2+2)}$$

$$As^2 + 2A + Bs^2 + Cs = 2$$

Comparing s^2 coefficient

$$A + B = 0 \longrightarrow (1)$$

Comparing s coefficient

$$\boxed{C = 0}$$

Comparing constant

$$2A = 2$$

from eqn (1)

$$A + B = 0$$

$$A = -B$$

$$\boxed{B = -1}$$

$$F(s) = \frac{1}{s} + \frac{(-1)s + 0}{s^2 + 2}$$

$$= \frac{1}{s} - \frac{s}{s^2 + 2}$$

$$\frac{s}{s^2 + (\sqrt{2})^2} = \cos\sqrt{2}t$$

$$\because \mathcal{L}^{-1}\left\{\frac{s}{s^2 + a^2}\right\} = \cos at$$

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{s}{s^2 + 2}\right\}$$

$$f(t) = 1 - \cos\sqrt{2}t$$

Q2 Find $f(t)$ if $F(s) = \dfrac{2s^2 + s + 8}{s(s^2 + 4)}$

Given

$$F(s) = \frac{2s^2 + s + 8}{s(s^2 + 4)^2}$$

$$= \frac{A}{s} + \frac{Bs + C}{s^2 + 4}$$

$$\frac{2s^2 + s + 8}{s(s^2 + 4)} = \frac{A(s^2 + 4) + (Bs + C)s}{s(s^2 + 4)}$$

$$As^2 + 4A + Bs^2 + Cs = 2s^2 + s + 8$$

compare s^2 coefficient

$$A + B = 2 \longrightarrow (1)$$

compare s coefficient

$$\boxed{C = 1}$$

compare constant

$$4A = 8$$

$$\boxed{A = 2}$$

$B = 2 - 2$

$\boxed{B = 0}$

$$F(s) = \frac{2}{s} + \frac{1}{s^2+4}$$

$\frac{1}{b}\mathcal{L}\{\sin bt\} = \frac{1}{s^2+b^2}$

$$\mathcal{L}^{-1}\{F(s)\} = 2\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}$$

$$= 2 + \frac{1}{2}\sin 2t$$

Case-b: roots are complex conjugate

$$s = -\alpha \pm j\beta.$$

If factor is $s^2 + k_1 s + k_2 = 0$ then we get above roots

$$s^2 + k_1 s + k_2 = (s+\alpha)^2 + \beta^2$$

Q1. Find $f(t)$ for $F(s) = \frac{s+1}{s^2+2s+2}$

Given

$$F(s) = \frac{s+1}{s^2+2s+2}$$

The roots of s^2+2s+2 are $-1 \pm j1$

$$F(s) = \frac{s+1}{(s+1)^2+1}$$

$\mathcal{L}^{-1}\left\{\frac{s+a}{(s+a)^2+b}\right\} = e^{-at}\cos bt$

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+1}\right\}$$

$$f(t) = e^{-t}\cos t$$

Q2 Find $f(t)$ for $F(s) = \frac{s+2}{s^2+2s+2}$

Given

$$F(s) = \frac{s+2}{s^2+2s+2}.$$

The roots of $s^2+2s+2 = 0$ are $-1 \pm j1$

$$F(s) = \frac{s+2}{(s+1)^2+1}$$

$$= \frac{s+1+1}{(s+1)^2+1}$$

$$\mathcal{L}^{-1}\left\{\frac{s+a}{(s+a)^2+b^2}\right\} = e^{-at}\cos bt$$

$$\mathcal{L}^{-1}\left\{\frac{b}{(s+a)^2+b^2}\right\} = e^{-at}\sin bt$$

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\}$$

$$f(t) = e^{-t}\cos t + e^{-t}\sin t$$

$$= e^{-t}[\sin t + \cos t]$$

Q3 Find $f(t)$ if $F(s) = \frac{s+5}{s(s^2+2s+5)}$

Given

$$F(s) = \frac{s+5}{s(s^2+2s+5)}$$

The roots for s is zero(0) and $s^2+2s+5=0$ is $-1 \pm j2$

$$F(s) = \frac{A}{s} + \frac{Bs+C}{s^2+2s+5}$$

$$\frac{s+5}{s(s^2+2s+5)} = \frac{A(s^2+2s+5)+(Bs+C)(s)}{s(s^2+2s+5)}$$

$$As^2+2As+5A+Bs^2+Cs = s+5$$

compare s^2 coefficient

$$A+B=0 \quad \rightarrow (1)$$

compare s coefficient

$$2A+C=1 \quad \rightarrow (2)$$

compare constant

$$5A=5$$

$$\boxed{A=1}$$

from eq (1) $B=-A$

$$\boxed{B=-1}$$

from eq ②

$$2A + C = 1$$

$$C = 1 - 2A$$

$$C = 1 - 2(1)$$

$$\boxed{C = -1}$$

$$F(s) = \frac{1}{s} - \frac{s+1}{s^2+2s+5}$$

Apply inverse laplace transform

$$L^{-1}\{F(s)\} = L^{-1}\left\{\frac{1}{s}\right\} - L^{-1}\left\{\frac{s+1}{(s+1)^2+4}\right\}$$

$$f(t) = 1 - L^{-1}\left\{\frac{s+1}{(s+1)^2+2^2}\right\}$$

$$f(t) = 1 - e^{-t}\cos 2t$$

Q4 Find $f(t)$ if $F(s) = \frac{2s+1}{(s+1)(s^2+2s+5)}$

Given

$$F(s) = \frac{2s+1}{(s+1)(s^2+2s+5)}$$

The roots are $-1 \pm j2$

$$\frac{2s+1}{(s+1)(s^2+2s+5)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+2s+5}$$

$$\frac{2s+1}{(s+1)(s^2+2s+5)} = \frac{A(s^2+2s+5)+(Bs+C)(s+1)}{(s+1)(s^2+2s+1)}$$

$$2s+1 = As^2+2As+5A+Bs^2+Cs+Bs+C$$

Compare s^2 coefficient

$$A + B = 0 \longrightarrow ①$$

compare s coefficient

$$2A + B + C = 2 \quad \rightarrow (2)$$

compare constant

$$5A + C = 1 \quad \rightarrow (3)$$

(solving (1), (2))

from eqn (1)

$$A = -B \Rightarrow B = -A$$

Put A in eqn (2)

$$+2B + B + C = 2$$
$$-B + C = 2$$

Put B in eqn (2)

$$2A - A + C = 2$$
$$A + C = 2 \quad \rightarrow (4)$$

Solve eqn (3), (4)

$$5A + C = 1$$
$$-A + C = 2$$
$$4A = -1$$

$$\boxed{A = \frac{-1}{4}}$$

Put A in eqn (3)

$$5\left[\frac{-1}{4}\right] + C = 1$$

$$C = 1 + \frac{5}{4} \quad \therefore f(t) =$$

$$\boxed{C = \frac{9}{4}}$$

11.08.2025

Initial value and Final value theorem:

Any time function $f(t)$

so, initial value of $f(t) = f(0) = f(t)|_{t=0} = \lim_{t \to 0} f(t)$

E.g. i) $f(t) = t^2$ ii) $f(t) = 2t + 3$

$f(0) = 0$ $f(0) = 3$

The final value of $f(t) = f(\infty) = f(t)|_{t=\infty}$ $\lim_{t \to \infty} f(t)$

E.g. i) $f(t) = t^2$ ii) $f(t) = 2t + 3$ iii) $f(t) = e^{2t}$

$f(\infty) = \infty$ $f(\infty) =$ $f(\infty) = e^{2\infty}$

$= e^{\infty}$

INITIAL VALUE THEOREM:

If $f(t)$ and it's 1st derivative are laplace transformable. Then the initial value of $f(t)$ [$f(0)$] is given by $f(0) = \lim_{s\to\infty} sF(s)$

PROOF:

$$\text{Let } \mathcal{L}\{f(t)\} = F(s)$$

$$\mathcal{L}\left\{\frac{d}{dt} f(t)\right\} = sF(s) - f(0)$$

By defination

$$\mathcal{L}\left\{\frac{d}{dt} f(t)\right\} = \int_0^\infty \frac{d}{dt} f(t) e^{-st}\, dt = sF(s) - f(0)$$

$$f(0) = \lim_{s\to\infty} s \cdot F(s)$$ → to apply $\lim_{s\to\infty}$ by using this formula we don't remember

Apply $\lim_{s\to\infty}$ on both sides

$$\lim_{s\to\infty} \int_0^\infty \frac{d}{dt} f(t) e^{-st}\, dt = \lim_{s\to\infty} [sF(s) - f(0)]$$

$$\int_0^\infty \frac{d}{dt} f(t) \cdot \lim_{s\to\infty} (e^{-st})\, dt = \left[\lim_{s\to\infty} s \cdot F(s)\right] - f(0)$$

$$\int_0^\infty \frac{d}{dt} f(t)\,[0]\, dt = \left[\lim_{s\to\infty} s \cdot F(s)\right] - f(0)$$

$$0 = \lim_{s\to\infty} sF(s) - f(0)$$

$$f(0) = \lim_{s\to\infty} s \cdot F(s)$$

FINAL VALUE THEOREM:

If $f(t)$ and 1st derivative are laplace transformable; then the initial value of $f(t)$ [$f(\infty)$] is given by $f(\infty) = \lim_{s\to 0} s \cdot F(s)$

PROOF:

$$\text{Let } \mathcal{L}\{f(t)\} = F(s)$$

$$\mathcal{L}\left\{\frac{d}{dt} f(t)\right\} = sF(s) - f(0)$$

By defination

$$\mathcal{L}\left\{\frac{d}{dt} f(t)\right\} = \int_0^\infty \frac{d}{dt} f(t) e^{-st}\, dt = s \cdot F(s) - f(0)$$

Apply $\lim_{s\to 0}$ on both sides

$$\lim_{s\to 0} \int_0^\infty \frac{d}{dt} f(t) e^{-st}\, dt = \lim_{s\to 0} [sF(s) - f(0)]$$

$$\int_0^\infty \frac{d}{dt} f(t) \cdot \lim_{s\to 0} e^{st}\, dt = \left[\lim_{s\to 0} s\cdot F(s)\right] - f(0)$$

$$\int_0^\infty \frac{d}{dt} f(t)\, e^0 \cdot dt = \left[\lim_{s\to 0} s\cdot F(s)\right] - f(0)$$

$$\int_0^\infty \frac{d}{dt} f(t)\, dt = \lim_{s\to 0} s\cdot F(s) - f(0)$$

$$\left[f(t)\right]_0^\infty = \lim_{s\to 0} s\cdot f(s) - f(0)$$

$$f(\infty) - f(0) = \lim_{s\to 0} \cdot s\cdot F(s) - f(0)$$

$$\boxed{f(\infty) = \lim_{s\to 0} s\,F(s)}$$

Q1. Fenal initial and final values theorem for $f(t) = 2t+3$

$$f(t) = 2t + 3$$

Initial value

$$f(0) = \lim_{t\to 0} f(t)$$

$$= 2(0) + 3 = 3$$

final value

$$f(\infty) = \lim_{t\to\infty} f(t)$$

$$= 2(\infty) + 3 = \infty$$

By initial value theorem

$$f(0) = \lim_{s\to\infty} sF(s)$$

$$f(t) = 2t + 3$$

$$\mathcal{L}\{f(t)\} = \frac{2}{s^2} + \frac{3}{s}$$

$$= \frac{2+3s}{s^2}$$

$$f(0) = \lim_{s\to\infty} sF(s)$$

$$= \lim_{s\to\infty} s\cdot \frac{3s+2}{s^2}$$

$$= \lim_{s\to\infty} \cdot \frac{3s+2}{s}$$

$$= \lim_{s\to\infty} \cdot \frac{s\left[3+\frac{2}{s}\right]}{s}$$

$= 3 + \frac{2}{\infty} = 3 + 0$

$= 3$

Hence

Initial value theorem is proved

By final value theorem

$$f(\infty) = \lim_{s \to 0} s \cdot F(s)$$

$$= \lim_{s \to 0} s \cdot \frac{3s+2}{s^2}$$

$$= \lim_{s \to 0} \frac{3s+2}{s}$$

$$= \frac{3(0)+2}{0}$$

$$= \infty$$

Hence, Final value theorem is proved.

Q2. Verify initial and final value theorems for the following functions.

i, $5e^{-4t}$

given

$$f(t) = 5e^{-4t}$$

Initialvalue

$$f(t) = f(0) = \lim_{t \to 0} f(t)$$

$$= \lim_{t \to 0} 5e^{-4t}$$

$$= \left[\lim_{t \to 0}\right] 5e^{-4(0)} \quad [\because e^0 = 1]$$

$$= \left(\lim_{t \to 0}\right) 5(1)$$

$$= 5$$

Final value

$$f(t) = f(\infty) = \lim_{t \to \infty} f(t)$$

$$= \lim_{t \to \infty} 5e^{-4t}$$

$$= 5e^{-\infty} \quad [\because e^{-\infty} = 0]$$

$$= 0$$

By Initial value theorem

$$f(0) = \lim_{s\to\infty} s\cdot F(s)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{5e^{-4t}\}$$

$$= 5\cdot\frac{1}{s+4}$$

$$= \lim_{s\to\infty} s\cdot\frac{5}{s+4}$$

$$= \lim_{s\to\infty} s\cdot\frac{5}{s\left(1+\frac{4}{s}\right)}$$

$$= \lim_{s\to\infty} \frac{5}{1+\frac{4}{s}}$$

$$= \frac{5}{1+\frac{4}{\infty}}$$

$$= \frac{5}{1+0} = 5$$

By final value theorem

$$f(\infty) = \lim_{s\to 0} sF(s)$$

$$= \lim_{s\to 0} s\cdot\frac{5}{s+4}$$

$$= \lim_{s\to 0} s\cdot\frac{5}{4\left(\frac{s}{4}+1\right)}$$

$$= \lim_{s\to 0} 0\cdot\frac{5}{s+4}$$

$$= 0$$

iii) $2-e^{-5t}$

Given $f(t) = 2-e^{-5t}$

Initial value

$$f(t) = f(0) = \lim_{t\to 0} f(t)$$

$$= \lim_{t\to 0} [2-e^{-5t}] = 2-1 = 1$$

final value

$$f(t) = f(\infty) = \lim_{t\to\infty} f(t)$$

$$= \lim_{t\to\infty} [2-e^{-5t}]$$

$$= 2$$

By initial value theorem

$$f(0) = \lim_{s\to\infty} sF(s)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{2-e^{-5t}\}$$

$$= \frac{2}{s} - \frac{1}{s+5}$$

$$= \frac{2(s+5)-s}{s(s+5)} = \frac{2s+10-s}{s(s+5)}$$

$$= \frac{s+10}{s(s+5)}$$

$$= \lim_{s\to\infty} s\cdot\frac{s+10}{s(s+5)} = \lim_{s\to\infty}\frac{s\left(1+\frac{10}{s}\right)}{s\left(1+\frac{5}{s}\right)}$$

By final value theorem

$$f(\infty) = \lim_{s\to 0} s\cdot F(s)$$

$$= \lim_{s\to 0} s\cdot\frac{s+10}{s(s+5)}$$

$$= \lim_{s\to 0}\frac{s+10}{s+5}$$

$$= \frac{10}{5}$$

$$= 2$$

iii) $f(t) = 2 + e^{-3t}\cos 2t$

Initial value

$$f(t) = f(0) = \lim_{t\to 0} f(t)$$

$$= \lim_{t\to 0} (2 + e^{-3t}\cos 2t)$$

$$= 2 + 1 = 3$$

final value

$$f(t) = f(\infty) = \lim_{t\to\infty} f(t)$$

$$= \lim_{t\to\infty} (2 + e^{-3t}\cos 2t)$$

$$= 2$$

$f(t) = 2 + e^{-3t}\cos 2t \quad L\{f(t)\} = \frac{2}{s} + \frac{s+3}{(s+3)^2+4}$

By initial value theorem

$$f(0) = \lim_{s\to\infty} s\,F(s)$$

$$= \lim_{s\to\infty} s\left[\frac{2}{s} + \frac{s+3}{(s+3)^2+4}\right]$$

$$= \lim_{s\to\infty} s\left[\frac{2s^2+18+12s+8+s^2+3s}{s(s+3)^2+4s}\right]$$

$$= \lim_{s\to\infty} s\left[\frac{s^2(2+\frac{18}{s^2}+\frac{12}{s}+\frac{8}{s^2}+1+\frac{3}{s})}{1+\frac{13}{s^2}+\frac{6}{s}}\right]$$

$$= 2 + 1 = 3$$

By final value theorem

$$f(\infty) = \lim_{s\to 0} s\,F(s)$$

$$= \lim_{s\to 0} s\left[\frac{2}{s} + \frac{s+3}{(s+3)^2+4}\right]$$

$$= \lim_{s\to 0}\left[2 + \frac{s(s+3)}{(s+3)^2+4}\right]$$

$$= 2$$

iv) $f(t) = 6(1 - e^{-t})$

Initial value

$$f(t) = f(0) = \lim_{t \to 0} f(t)$$

$$= \lim_{t \to 0} 6(1 - e^{-t})$$

$$= 0$$

Final value

$$f(t) = f(\infty) = \lim_{t \to \infty} f(t)$$

$$= \lim_{t \to \infty} (6(1 - e^{-t}))$$

$$= 6$$

By initial value theorem

$$f(0) = \lim_{s \to \infty} sF(s)$$

$$L\{f(t)\} = L\{6(1 - e^{-t})\}$$

$$= \frac{6}{s} - \frac{6}{s+1}$$

$$= \lim_{s \to \infty} s\left[\frac{6}{s} - \frac{6}{s+1}\right]$$

$$= \lim_{s \to \infty} s\left[\frac{6}{s(s+1)}\right]$$

$$= \lim_{s \to \infty} s\left[\frac{6}{1 + \frac{1}{s}}\right]$$

$$= \infty\left[\frac{6}{1+0}\right] = 0$$

By final value theorem

$$f(\infty) = \lim_{s \to 0} sF(s)$$

$$= \lim_{s \to 0} s\left[\frac{6}{s} - \frac{6}{s+1}\right]$$

$$= \lim_{s \to 0} s\left[\frac{6}{s(s+1)}\right]$$

$$= 6$$

v) $f(t) = e^{t}(\sin 3t + \cos 5t)$

Initial value

$$f(t) = \lim_{t \to 0} f(t)$$

$$= 1(0+1)$$

$$= 1$$

final value

$$f(t) = \lim_{t \to \infty} f(t)$$

$$= 0$$

By initial value theorem

$$f(0) = \lim_{s \to \infty} s\left[\frac{3}{(s+1)^2+9} + \frac{5}{(s+1)^2+25}\right]$$

$$= \lim_{s \to \infty} s\left[\frac{3s^2+7s+65+s^3+2s^2+10s}{(s^2+2s+10)(s^2+2s+2)}\right]$$

$$= \lim_{s \to \infty} s\left[\frac{s^3\left[1+\frac{3}{s}+\frac{7s}{s^3}+\frac{6}{s^2}+\frac{2}{s}+\frac{10}{s^2}\right]}{s^4\left[1+\frac{2s}{s^2}+\frac{2}{s^2}+\frac{8}{s^3}-\frac{4}{s^2}+\frac{10}{s^2}+\frac{2s}{s^4}+\frac{2}{s^4}\right]}\right]$$

$$= \frac{1}{1} = 1$$

By final value theorem

$$f(\infty) = \lim_{s \to 0} sF(s)$$

$$= \lim_{s \to 0} s\left[\frac{3}{(s+1)^2+9} + \frac{5}{(s+1)^2+25}\right]$$

$$= 0$$

12.08.2025

Q3 find initial and final values of the function whose laplace transform is $\frac{20}{(s+3)(s^2+8s+25)}$

Given

$$F(s) = \frac{20}{(s+3)(s^2+8s+25)}$$

by initial value theorem

$$f(0) = \lim_{s\to\infty} s \cdot F(s)$$

$$= \lim_{s\to\infty} s \cdot \left[\frac{20}{(s+3)(s^2+8s+25)}\right]$$

$$= \lim_{s\to\infty} s \cdot \left[\frac{20}{s\left(1+\frac{3}{s}\right)s^2\left(1+\frac{8}{s}+\frac{25}{s^2}\right)}\right]$$

$$= \lim_{s\to\infty} s \cdot \left[\frac{20}{\left(1+\frac{3}{\infty}\right)\infty^2\left(1+\frac{8}{\infty}+\frac{25}{\infty}\right)}\right]$$

$$= 0$$

By final value theorem

$$f(\infty) = \lim_{s\to 0} s\, f(s)$$

$$= \lim_{s\to 0} s \cdot \left[\frac{20}{(s+3)(s^2+8s+25)}\right]$$

$$= 0 \cdot \left[\frac{20}{3\times 25}\right]$$

$$= 0$$

$$F(s) = \frac{3s^2+2s+10}{s(s+3)(s^2+4s+4)}$$

given that

$$F(s) = \frac{3s^2+2s+10}{s(s+3)(s^2+4s+4)}$$

By ~~final~~ initial value theorem

$$f(0) = \lim_{s\to\infty} s.F(s)$$

$$= \lim_{s\to\infty} s.\left[\frac{3s^2+2s+10}{s(s+3)(s^2+4s+4)}\right]$$

$$= \lim_{s\to\infty} s.\left[\frac{s^2\left(3+\frac{2}{s}+\frac{10}{s^2}\right)}{s(s)\left(1+\frac{3}{s}\right)s^2\left(1+\frac{4}{s}+\frac{4}{s^2}\right)}\right]$$

$$= \lim_{s\to\infty}\left[\frac{3+\frac{2}{s}+\frac{10}{s^2}}{s\left(1+\frac{3}{s}\right)\left(1+\frac{4}{s}+\frac{4}{s^2}\right)}\right]$$

$$= \left[\frac{3+\frac{2}{\infty}+\frac{10}{\infty^2}}{\infty\left(1+\frac{3}{\infty}\right)\left[1+\frac{4}{\infty}+\frac{4}{\infty^2}\right]}\right]$$

$$= \frac{3+0+0}{\infty[1][1]}$$

$$= \frac{3}{\infty} = 0$$

By final value theorem

$$f(\infty) = \lim_{s\to 0} s.F(s)$$

$$= \lim_{s\to 0} s.\left[\frac{3s^2+2s+10}{s(s+3)(s^2+4s+4)}\right]$$

$$= 0 \times \frac{3s^2+2s+10}{s(s+3)(s^2+4s+4)}$$

$$= 0\ \frac{10}{3\times 4}$$

$$= \frac{10}{12} = \frac{5}{6}$$

$$F(s) = \frac{3s+5}{s(s+1)}$$

By initial value theorem

$$f(0) = \lim_{s\to\infty} s \cdot F(s)$$

$$= \lim_{s\to\infty} s \cdot \left[\frac{3s+5}{s(s+1)}\right]$$

$$= \lim_{s\to\infty} \frac{s\left[3+\frac{5}{s}\right]}{s\left[1+\frac{1}{s}\right]}$$

$$= \lim_{s\to\infty} \frac{3+\frac{5}{s}}{1+\frac{1}{s}}$$

$$= \frac{3+\frac{5}{\infty}}{1+\frac{1}{\infty}}$$

$$= \frac{3+0}{1+0}$$

$$= 3$$

By final value theorem

$$f(\infty) = \lim_{s\to 0} s \cdot F(s)$$

$$= \lim_{s\to 0} s \cdot \frac{3s+5}{s(s+1)}$$

$$= \lim_{s\to 0} \frac{3s+5}{s+1}$$

$$= \frac{3(0)+5}{0+1}$$

$$= \frac{5}{1}$$

$$= 5$$

APPLICATIONS OF LAPLACE TRANSFORM TO ELECTRICAL CIRCUITS

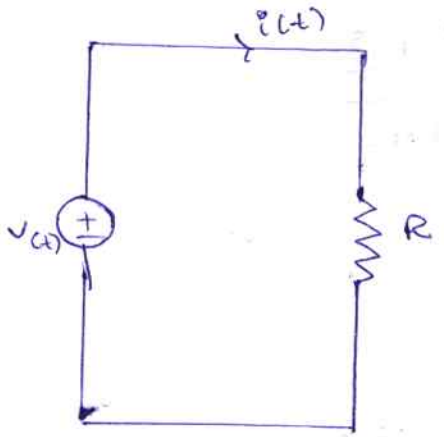
Transformed Impedance Z(s):

It is the ratio of laplace transform of voltage to laplace transform of the current under the zero initial condition.

Mathematically,

$$Z(s) = \frac{V(s)}{I(s)} \Big|_{zic}$$

Resistor:



$$v(t) = R\,i(t) \longrightarrow (1)$$

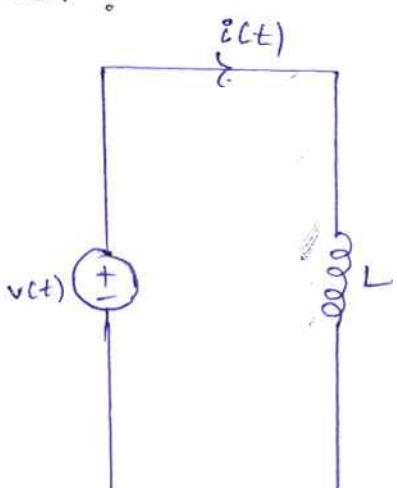
Apply laplace transform for eqv (1)

$$V(s) = R\,I(s)$$

$$\frac{V(s)}{I(s)} = R$$

$$R = Z(s)$$

Inductor:



$$v(t) = L\frac{di}{dt} \longrightarrow (1)$$

Apply laplace transform for eqv (1)

$$V(s) = L\left[sI(s) - i(0)\right]$$

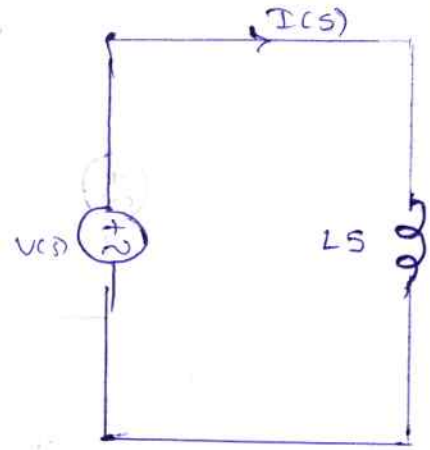
At zero initial condition

$i(0) = 0$

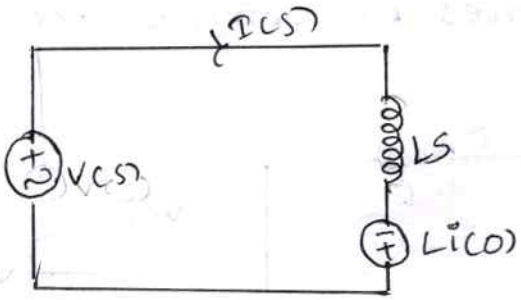
$V(s) = L\,s\,I(s)$

$L \cdot s = \frac{V(s)}{I(s)}$

$L \cdot s = Z(s)$

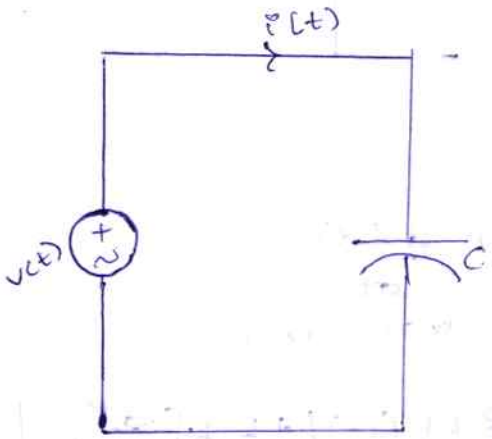


At initial condition



s- drawn in domain with gc

Capacitor:



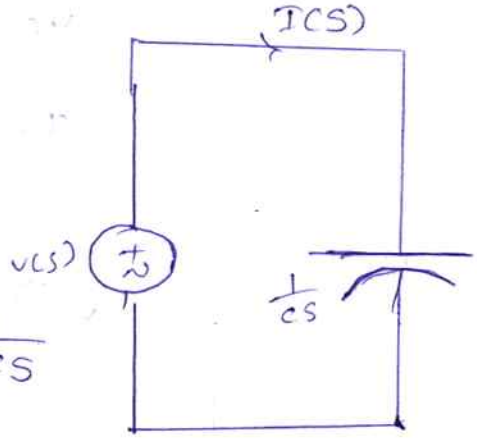
$$i(t) = C\,\frac{dv(t)}{dt} \rightarrow (1)$$

Apply laplace transform for above eqn (1)

$I(s) = C[sV(s) - V(0)]$

$= CsV(s) - CV(0)$

At zero initial condition:



$V(0) = 0$

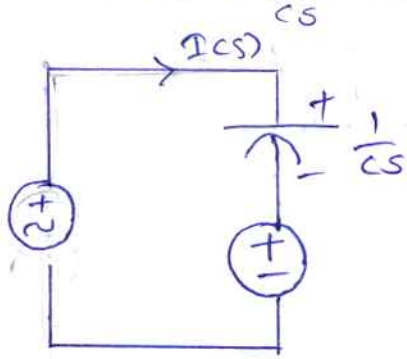
$I(s) = CsV(s)$

$Z_c(s) = \frac{V(s)}{I(s)} = \frac{1}{Cs}$

$I(s) = CsV(s) - CV(0)$

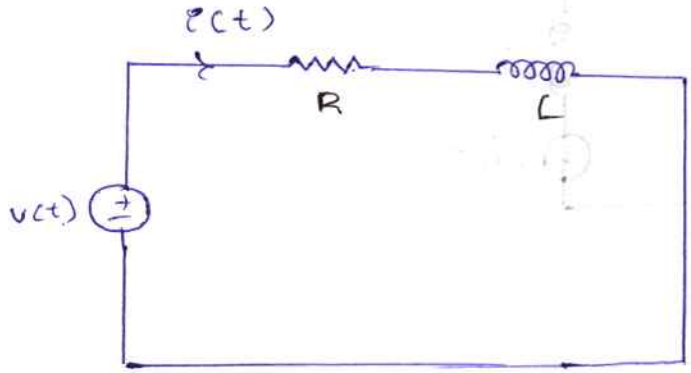
$CsV(s) = I(s) + CV(0)$

$$V(s) = \frac{1}{Cs} I(s) + \frac{v(0)}{s}$$



STEP RESPONSE OF RL SERIES CIRCUIT :

•Consider series RL circuit excited by



v(t) = step I/P
= V u(t)

method-1:

$$v(t) = R\,i(t) + L\,\frac{d\,i(t)}{dt}$$

Apply laplace transform

$$L[v(t)] = R\,L[i(t)] + L\,L\left[\frac{d\,i(t)}{dt}\right]$$

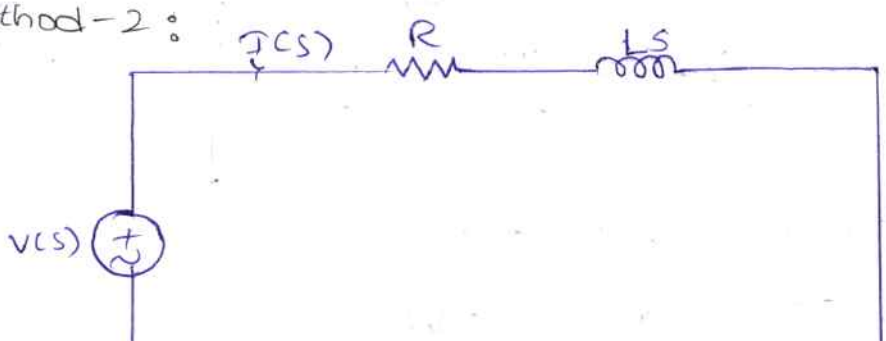
$$V(s) = R\,I(s) + LsI(s) - i(0) \quad (\text{But } i(0) = 0)$$

$$V(s) = R\,I(s) + LsI(s)$$

$$V(s) = (R + Ls)\,I(s)$$

$$I(s) = \frac{V(s)}{(R + Ls)}$$

method-2:



$$V(S) = I(S)(R+LS)$$

$$I(S) = \frac{V(S)}{R+LS}$$

$$V(t) = V\,u(t)$$

$$V(S) = L\{V(t)\} = \frac{V}{S}$$

$$I(S) = \frac{V(S)}{R+LS} = \frac{V}{S(R+LS)}$$

$$= \frac{V}{SL\left(\frac{R}{L}+S\right)}$$

$$= \frac{V/L}{S\left(S+\frac{R}{L}\right)}$$

$$= \frac{A}{S} + \frac{B}{S+\frac{R}{L}}$$

$$A = SI(S) = \frac{V/L}{S+R/L}\Big|_{\text{at } S=0}$$

$$= \frac{V/L}{R/L}$$

$$= \frac{V}{R}$$

$$B = I(S)\left(S+\frac{R}{L}\right) = \frac{V/L}{S}\Big|_{\text{at } S=-\frac{R}{L}}$$

$$= \frac{V/L}{-R/L}$$

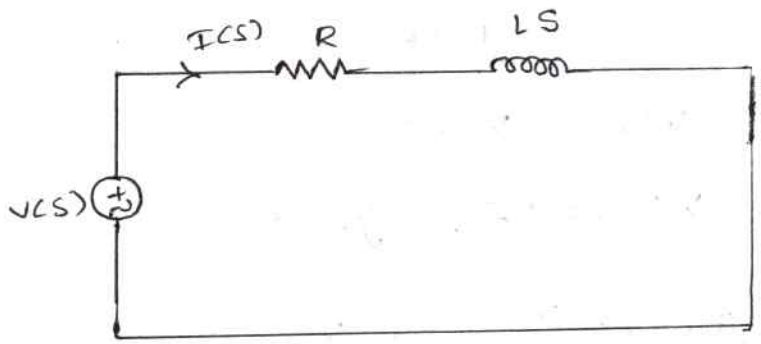
$$= \frac{-V}{R}$$

Apply Inverse laplace transform

$$i(t) = \frac{V}{R} - \frac{V}{R}e^{-\frac{R}{L}t}$$

$$i(t) = \frac{V}{R}\left[1 - e^{-\frac{R}{L}t}\right]$$

Impulse response of series RL circuit:



$v(t)$ = Impulse $I/P = \delta(t)$

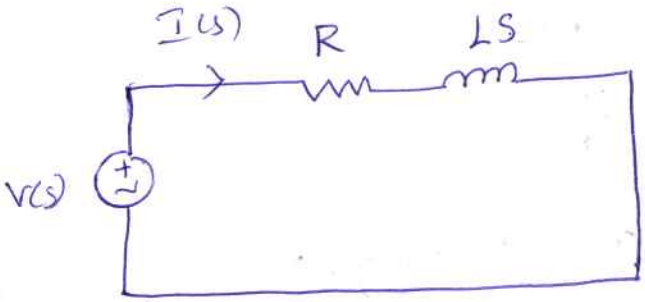
$$I(s) = \frac{V(s)}{R+LS}$$

$$V(s) = L\{\delta(t)\} = 1$$

$$I(s) = \frac{1}{R+LS} = \frac{1}{L(s+R/L)}$$

$$i(t) = \frac{1}{L} e^{-\frac{R}{L}t}$$

RAMP RESPONSE OF SERIES RL CIRCUIT:



$v(t)$ = ramp $I/P = v\,r(t)$; $V(s) = \frac{V}{s^2}$

$$I(s) = \frac{V(s)}{R+LS}$$

$$I(s) = \frac{V}{s^2(R+LS)}$$

$$= \frac{V/L}{s^2(s+R/L)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+R/L}$$

$$B = I(s)\,s^2 = \frac{V}{R+Ls}\Big|_{A+S=0} = \frac{V}{R}$$

$$A = \frac{d}{ds}\left[I(s)\,s^2\right] = \frac{d}{ds}\left[\frac{V}{R+Ls}\right]$$

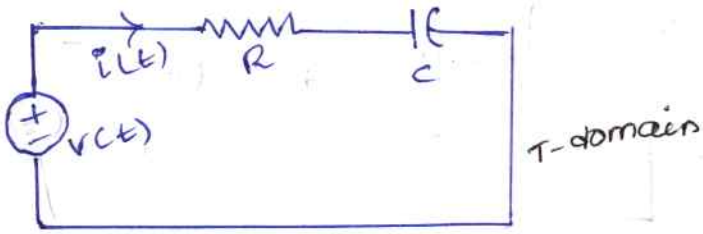
$$= -\frac{V \times L}{(R+Ls)^2}\Big|_{A+S=0} = -\frac{VL}{R}$$

$$C = \frac{V/L}{s^2}\Big|_{s=-R/L} = \frac{V/L}{R^2/L^2} = \frac{VL}{R}.$$

$$\therefore I(s) = -\frac{VL/R}{s} + \frac{V/R}{s^2} + \frac{VL/R}{s+R/L}$$

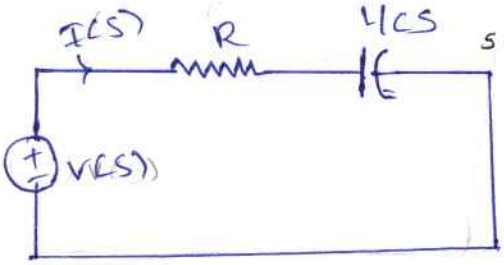
$$\therefore i(t) = -\frac{VL}{R} + t\,\frac{V}{R} + \frac{VL}{R}\,e^{-\frac{R}{L}t}.$$

STEP RESPONSE OF SERIES RC CIRCUIT:



$V(t) = V u(t)$ (∵ step ip)

$V(s) = \frac{V}{s}$



$V(s) = \left(R + \frac{1}{Cs}\right) I(s)$

$I(s) = \frac{V(s)}{\left(R + \frac{1}{CR}\right)}$

$= \frac{V(s)}{\frac{RCs+1}{CR}}$

$= \frac{CsV(s)}{RC\left[s + \frac{1}{CR}\right]}$

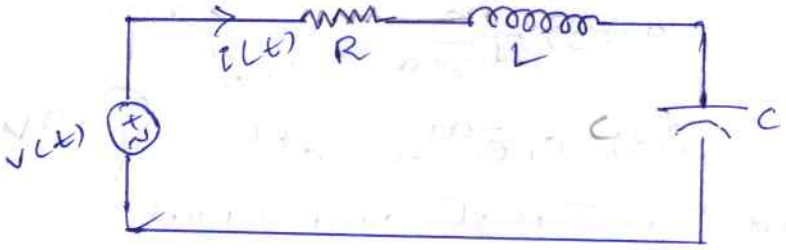
$= \frac{sV(s)}{R\left[s + \frac{1}{CR}\right]}$

$I(s) = \frac{s \cdot V/s}{R\left[s + \frac{1}{CR}\right]}$ $\left[\because V(s) = \frac{V}{s}\right]$

$I(s) = \frac{V}{R\left[s + \frac{1}{CR}\right]} = \frac{V/R}{\left[s + \frac{1}{CR}\right]}$

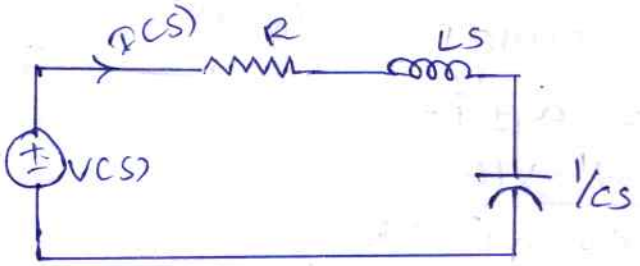
$i(t) = \frac{V}{R} e^{-\frac{1}{RC}t}$

STEP RESPONSE OF SERIES RLC CIRCUIT:



$V(t) = V\,u(t)$ (∵ step input)

$= \frac{V}{S}$



consider the RLC series circuit as shown

$$v(t) = (R + L + C)\, i(t)$$

$$v(t) = R\,i(t) + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Apply Laplace transform on both sides

$$V(s) = R\,I(s) + LS\,I(s) + \frac{1}{CS}\,I(s)$$

$$V(s) = \left(R + LS + \frac{1}{CS}\right) I(s)$$

$$I(s) = \frac{V(s)}{R + LS + \frac{1}{CS}}$$

$$I(s) = \frac{V/S}{\frac{RCS + LCS^2 + 1}{CS}}$$

$$I(s) = \frac{CSV/S}{LC\left[\frac{RCS}{LC} + \frac{LCS^2}{LC} + \frac{1}{LC}\right]}$$

$$\left[= \left[\frac{CSV/S}{\quad}\right]_{\text{wrong}}\right.$$

case-1: roots are real and unequal

roots = $-a, -b$

$$I(s) = \frac{c_1}{s+a} + \frac{c_2}{s+b}$$

$$i(t) = c_1 e^{-at} + c_2 e^{-bt}$$

case-2: roots are real and equal

roots = $-a, -a$

$$I(s) = \frac{V/L}{(s+a)^2}$$

$$i(t) = \frac{V}{L} t e^{-at}$$

case-3: roots are complex conjucate

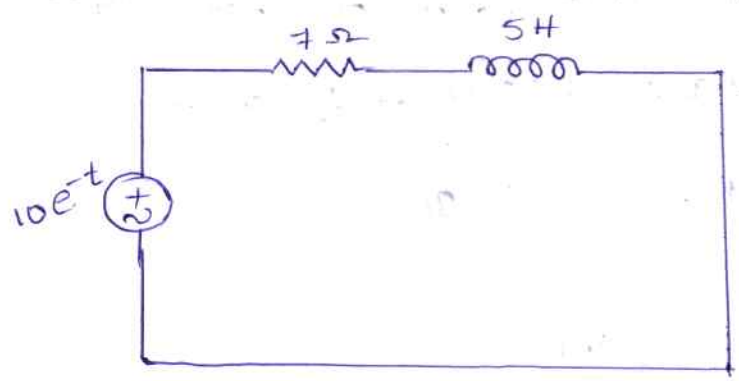
$$s_1, s_2 = -a \pm jb$$

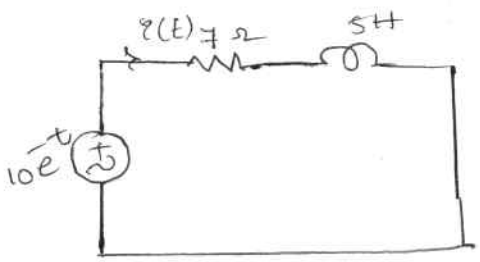
$$I(s) = \frac{V/L}{(s+a)^2 + b^2}$$

$$= \frac{V}{bL} e^{-at} \sin bt$$

13.08.2025

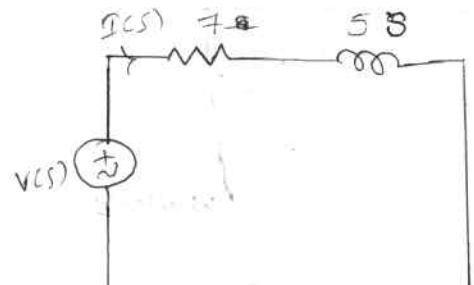
Q1 find $I(t)$ in the given circuit under zero initial conditions.





$$V(t) = 10e^{-t}$$

$$V(s) = \frac{10}{s+1}$$



$$V(s) = (7+5s)\, I(s)$$

$$I(s) = \frac{V(s)}{7+5s}$$

$$= \frac{10}{(s+1)5\left[5+\frac{7}{5}\right]}$$

$$T(s) = \frac{2}{(s+1)(s+1.4)}$$

$$= \frac{A}{s+1} + \frac{B}{s+1.4}$$

$$[A = F(s)(s+1) =]_{x}$$

$$\frac{2}{(s+1)(s+1.4)} = \frac{A(s+1.4)+B(s+1)}{(s+1)(s+1.4)}$$

$$2 = As + A(1.4) + Bs + B$$

compare s coefficient

$$A + B = 0 \rightarrow A = -B$$

compare constant

$$1.4A + B = 2$$

$$1.4(-B) + B = 2$$

$$B - 1.4B = 2$$

$$-0.4B = 2$$

$$B = \frac{-2}{0.4}$$

$$\boxed{B = -5}$$

from $A = -B$

$$A = -(-5)$$

$$\boxed{A = 5}$$

(OR)

$$A = I(s)(s+1) = \frac{2}{s+1.4}\Big|_{s=-1}$$

$$= \frac{2}{1.4-1}$$

$$= 5$$

$$B = I(s)(s+1.4) = \frac{2}{s+1}\Big|_{s=-1.4}$$

$$= \frac{2}{-1.4+1}$$

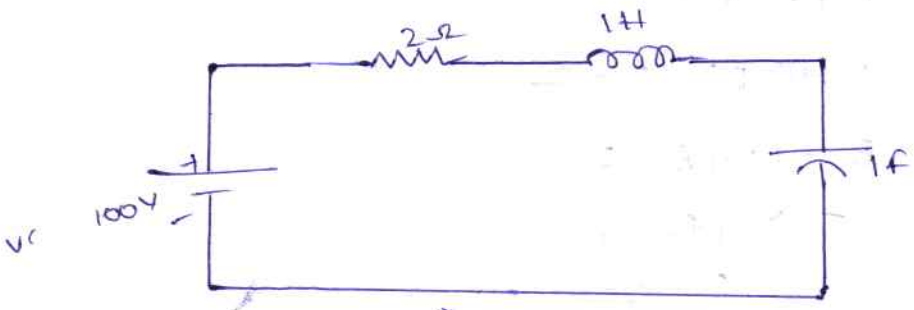
$$= -5$$

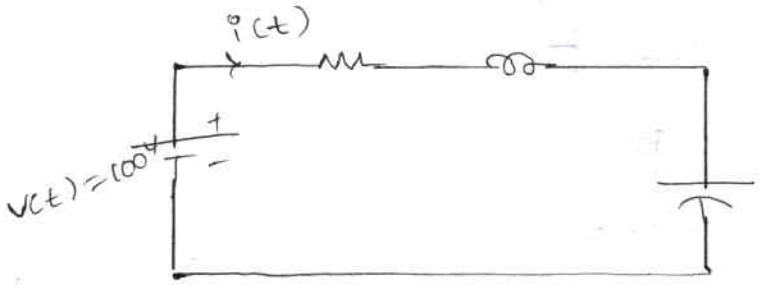
$$I(s) = \frac{5}{s+1} - \frac{5}{s+1.4}$$

Apply I.L.T

$$i(t) = 5e^{-t} - 5e^{-1.4t} \text{ A}$$

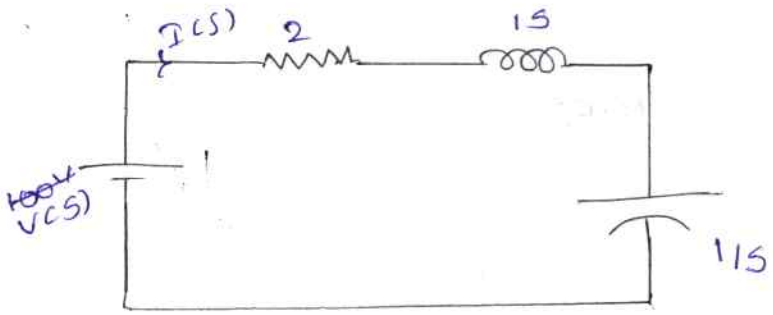
Q2. Find i(t) in the given circuit for zero initial condition





$$V(t) = 100$$

$$V(s) = \frac{100}{s}$$



s - domain

$$I(s) = \frac{V(s)}{2 + s + \frac{1}{s}}$$

$$= \frac{V(s)}{\frac{s^2+2s+1}{s}} = \frac{s\,V(s)}{s^2+2s+1}$$

$$= \frac{s \cdot \frac{100}{s}}{s^2+2s+1}$$

$$= \frac{100}{s^2+2s+1}$$

roots for s^2+2s+1 are $-1, -1$

$$I(s) = \frac{100}{(s+1)^2}$$

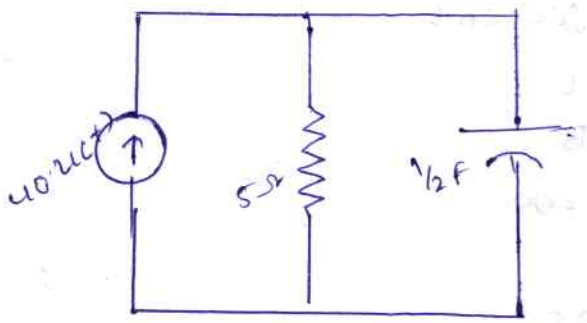
$\because \mathcal{L}^{-1}\left[\frac{1}{s^2}\right] = t$

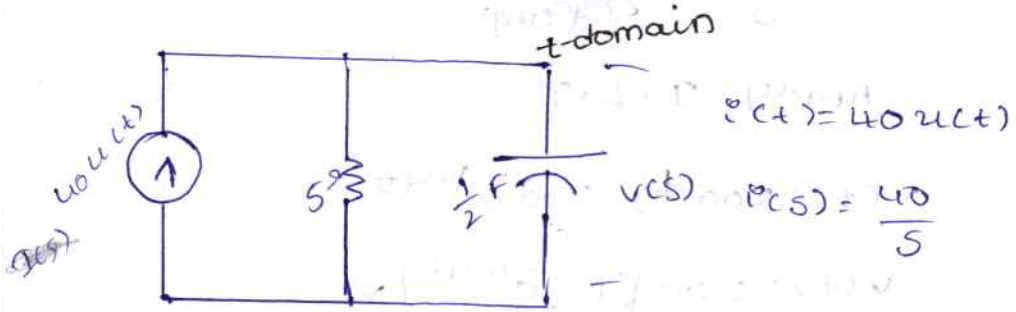
$\mathcal{L}^{-1}\left[\frac{1}{(s+1)^2}\right] = e^{-t} \cdot t$

Apply $\mathcal{I}^{-1}$

$$i(t) = 100\, e^{-t}\, t$$

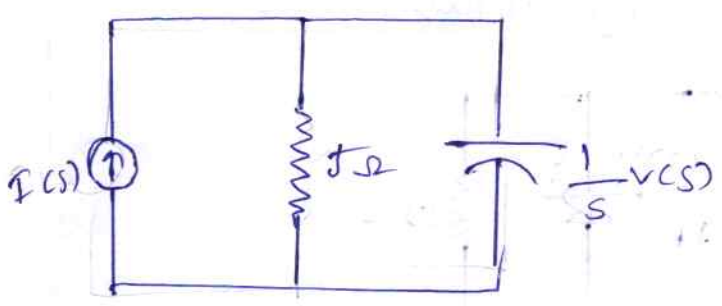
Q3 find $v(t)$





$i(t) = 40\,u(t)$

$I(s) = \frac{40}{s}$



$$I(s) = \frac{V(s)}{s} + \frac{V(s)}{\frac{2}{s}}$$

$$= V(s)\left[\frac{1}{s} + \frac{s}{2}\right]$$

$$V(s) = \frac{10\,I(s)}{2+5s}$$

$$= \frac{10\,I(s)}{5\left(\frac{2}{5}+s\right)}$$

$$= \frac{2\,I(s)}{s+0.4} =$$

$$= \frac{2 \times 40/s}{s+0.4}$$

$$V(s) = \frac{80}{s(s+0.4)}$$

$$-V(s) = \frac{A}{s} + \frac{B}{s+0.4}$$

$$80 = A(s) + A(0.4) + Bs$$

constant

$$A = 80$$

$$0.4A = 80$$

$$\boxed{A = 200}$$

compare s coefficient

$$A + B = 0$$

$$A = -B$$

$$\boxed{B = -200}$$

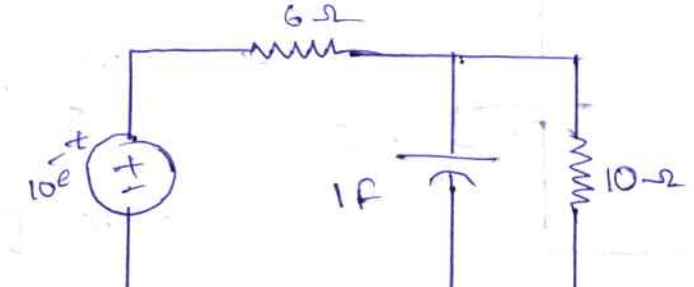
$$V(s) = \frac{200}{s} - \frac{200}{s+0.4}$$

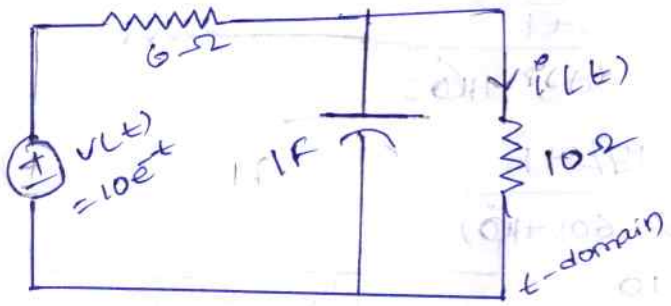
Apply I.L.T

$$v(t) = 200(1) - 200\,e^{-0.4t}$$

$$v(t) = 200\left[1 - e^{-0.4t}\right] V$$

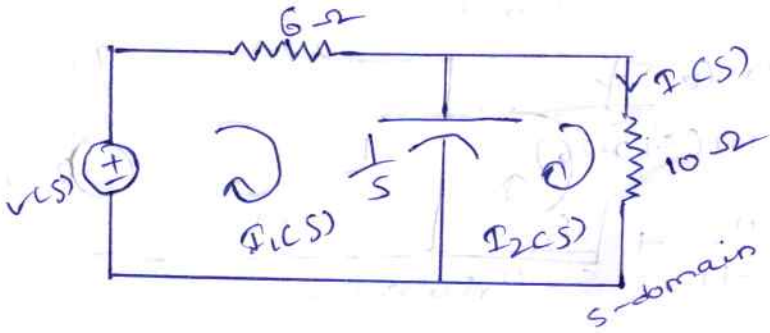
Q4 find current flowing through 10Ω





$V(t) = 10e^{-t}$

$V(s) = \frac{10}{s+1}$



loop-1

$$\frac{10}{s+1} = (6+\tfrac{1}{s})\,I_1(s) - \tfrac{1}{s}\,I_2(s) \longrightarrow (1)$$

loop-2

$$0 = -(\tfrac{1}{s})\,I_1(s) + (10+\tfrac{1}{s})\,I_2(s) \longrightarrow (2)$$

From eq (2)

$$I_1(s)\left[\tfrac{1}{s}\right] = \left[10+\tfrac{1}{s}\right] I_2(s)$$

$$I_1(s)\left[\tfrac{1}{s}\right] = \frac{10s+1}{s}\, I_2(s)$$

$$I_1(s) = (10s+1)\, I_2(s) \longrightarrow (3)$$

Sub eq (3) in eq (1)

$$\frac{10}{s+1} = (6+\tfrac{1}{s})(10s+1)\, I_2(s) - \tfrac{1}{s}\, I_2(s)$$

$$V(s) = \left[\frac{(6s+1)(10s+1)-1}{s}\right] I_2(s)$$

$$I_2(s) = \frac{s\,V(s)}{(6s+1)(10s+1)-1}$$

$$= \frac{s \cdot \frac{10}{s+1}}{(6s+1)(10s+1)-1}$$

$$= \frac{5 \cdot \frac{10}{s+1}}{60s^2 + 160s}$$

$$= \frac{5 \cdot 10/s+1}{5 \cdot (60s + 160)}$$

$$= \frac{10}{(s+1)60\left(s + \frac{16}{60}\right)}$$

$$I_2(s) = \frac{0.16}{(s+1)(s+0.26)}$$

$$\frac{0.16}{(s+1)(s+0.26)} = \frac{A}{s+1} + \frac{B}{s+0.26}$$

$$0.16 = A(s+0.26) + B(s+1)$$

$$0.16 = As + 0.26A + Bs + B$$

compare s coefficient

$$A + B = 0$$

$$B = -A$$

compare constant

$$0.26A + B = 0.16$$

$$0.26A - A = 0.16$$

$$-0.74A = 0.16$$

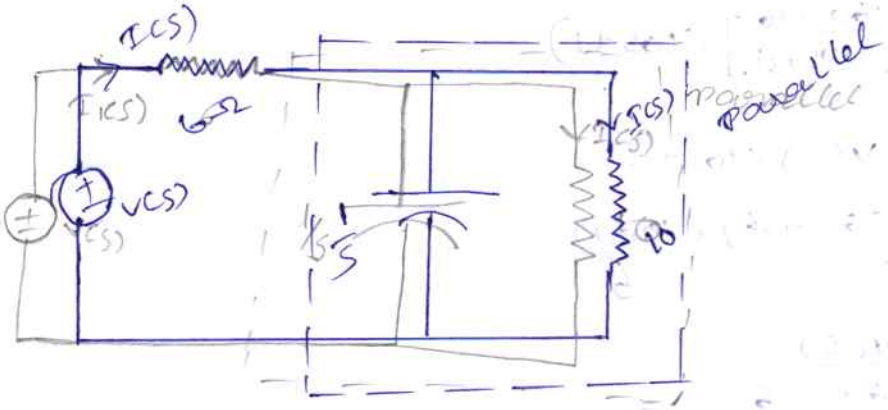
$$A = 0.216$$

$$B = -(-0.216)$$

$$B = 0.216$$

14.08.2025

method - 2 :



$$= \frac{\frac{1}{s} \times 10}{\frac{1}{s} + 10}$$

$$= \frac{10/s}{\frac{10s+1}{s}}$$

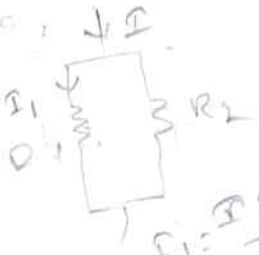
$$= \frac{10}{10s+1}$$

$$Z(s) = \frac{10}{10s+1} + 6$$

$$= \frac{10 + 6(10s+1)}{10s+1}$$

$$= \frac{60s + 6 + 10}{10s+1} = \frac{60s+16}{10s+1}$$

current division



$$I_1 = \frac{I R_2}{R_1 + R_2}$$

$$I_1(s) = \frac{V(s)}{Z(s)}$$

$$I(s) = \frac{I_1(s) \times 1/s}{10 + \frac{1}{s}}$$

$$= \frac{\frac{V(s)}{Z(s)} \times \frac{1}{s}}{10 + 1/s}$$

$$= \frac{V(s)}{}$$

$$= \frac{V(s)}{s\left[\frac{60s+16}{10s+1}\right](10s+1)}$$

$$= \frac{V(s)(10s+1)}{s\left[(60s+16)\frac{(10s+1)}{s}\right]}$$

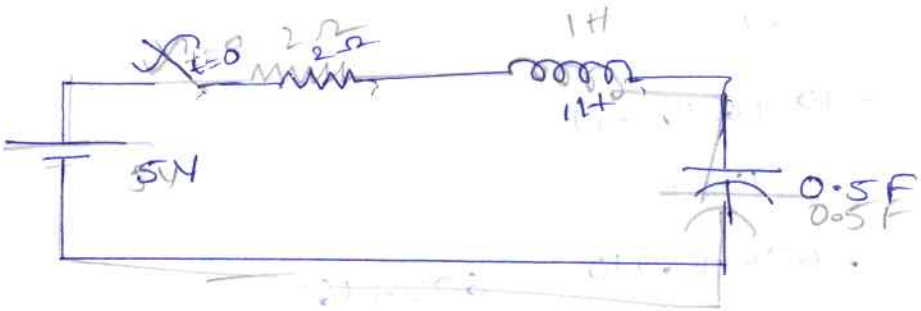
$$= \frac{V(s)}{60s+16}$$

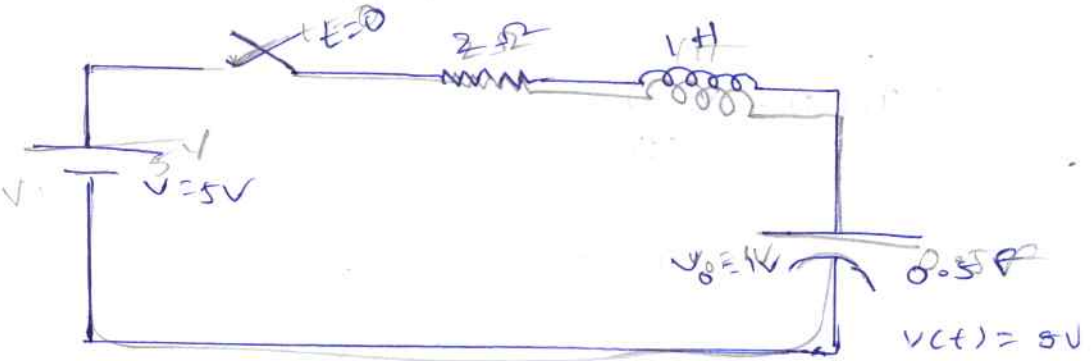
$$= \frac{10}{(s+1)(60s+16)}$$

$$= \frac{10}{(s+1)60\left(s+\frac{16}{60}\right)}$$

$$I(s) = \frac{0.16}{(s+1)(s+0.26)}$$

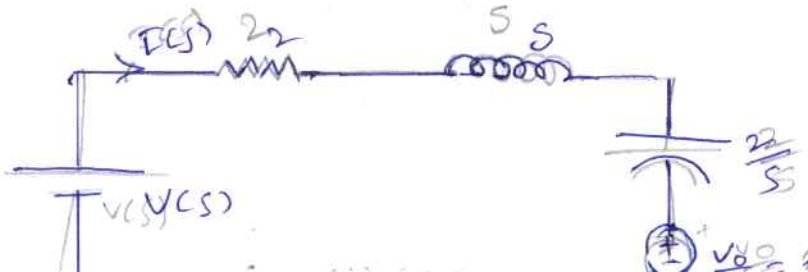
Q Find $I(t)$ after the switch is closed when the initial voltage across the capacitor is 1 V.





$v(t) = 5V$

$V(s) = \frac{5}{s}$



$$V(s) - \frac{1}{s} = \left(2 + s + \frac{2}{s}\right) I(s)$$

$$\frac{4}{s} = \left[\frac{2s + s^2 + 2}{s}\right] I(s)$$

$$I(s) = \frac{4}{s^2 + 2s + 2}$$

$s^2 + 2s + 2 = -1 \pm 1i$

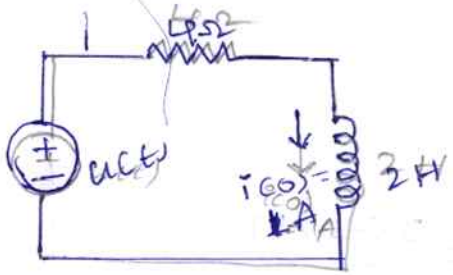
roots for $s^2 + 2s + 2$ is $-1 \pm i$ $\quad (s+a)^2 + b^2 = -a \pm ib$

$$I(s) = \frac{4}{(s+1)^2 + 1}$$

Apply I.L.T :

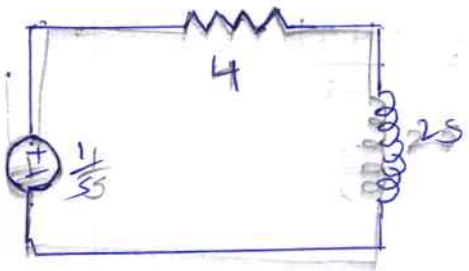
$$i(t) = 4 e^{-t} \sin t$$

Determine $i(t)$ for $t > 0$

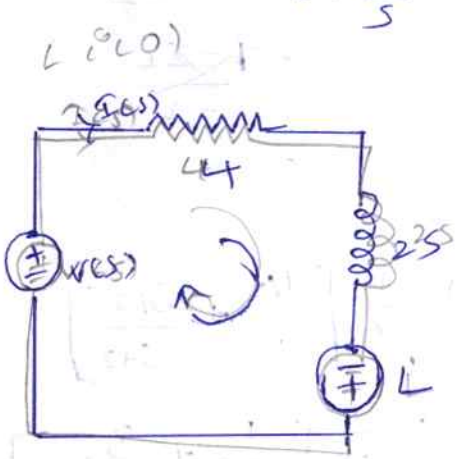


$V(t) = u(t)$

$V(s) = \frac{1}{s}$



s-domain



$$V(s) + L = (4 + 2s) I(s)$$

$$\frac{1}{s} + L = (4 + 2s) I(s)$$

$$\frac{Ls + 1}{s} = (4 + 2s) I(s)$$

$$I(s) = \frac{Ls + 1/s}{4 + 2s}$$

$$= \frac{2s+1}{s(4+2s)}$$

$$= \frac{2s+1}{s^2 2(s+2)}$$

$$= \frac{1}{2}\left[\frac{2s+1}{s(s+2)}\right]$$

$$I(s) = \left[\frac{A}{s} + \frac{B}{s+2}\right]\frac{1}{2}$$

$$A = sI(s) = \frac{2s+1}{s+2}\Big|_{s=0}$$

$$= \frac{2(0)+1}{0+2}$$

$$= \frac{1}{2}$$

$$B = (s+2)I(s)\Big|_{s=-2}$$

$$= \frac{2s+1}{s}$$

$$= \frac{2(-2)+1}{-2} \qquad = \frac{2(-2)+1}{-2}$$

$$= \frac{1-2L}{-2} \qquad = \frac{-4+1}{-2}$$

$$= \qquad = \frac{3}{2}$$

$$I(s) = \frac{1}{2}\left[\frac{1/2}{s} + \frac{3/2}{s+2}\right]$$

$$= \frac{1}{2}\left[\frac{1}{2s} + \frac{3}{2(s+2)}\right]$$

$$= \frac{1}{4}\left[\frac{1}{s} + \frac{3}{s+2}\right]$$

$$i(t) = \frac{1}{4}\left[L^{-1}\left[\frac{1}{s}\right] + 3L^{-1}\left[\frac{1}{s+2}\right]\right]$$

$$= \frac{1}{4}\left[1 + 3e^{-2t}\right]$$

ANNAMACHARYA UNIVERSITY, RAJAMPET

(ESTD UNDER AP PRIVATE UNIVERSITIES (ESTABLISHMENT AND REGULATION) ACT, 2016
RAJAMPET, Annamayya District, AP, INDIA

Department of Electrical & Electronics Engineering

II B.Tech I Semester

Network Analysis and Synthesis (24AEEE21T)
(Electrical and Electronics Engineering)

Unit-3 (Transient Analysis (DC & AC))

Syllabus:

Introduction- Initial Conditions -DC Transient response of RL, RC and RLC series circuits - Response of RL, RC and RLC series circuits with AC excitation.

Unit-3:

Transients

08.09.2025

First order Differential equation:

$$y' + P \times (y) = Q(y)$$

$$I(x) = \int_{@} P(x)\,dx$$

Second order Differential Equation:

$$y'' + y' + y = 0$$

$$f(D)y = 0$$

$$y = y_1 \| y_2$$

Resistor; Inductor and Capacitor:

Resistor, inductor and capacitor are linear passive elements.

R - Energy dissipating element [static element]

L, C - Energy storing element [Dynamic elements]

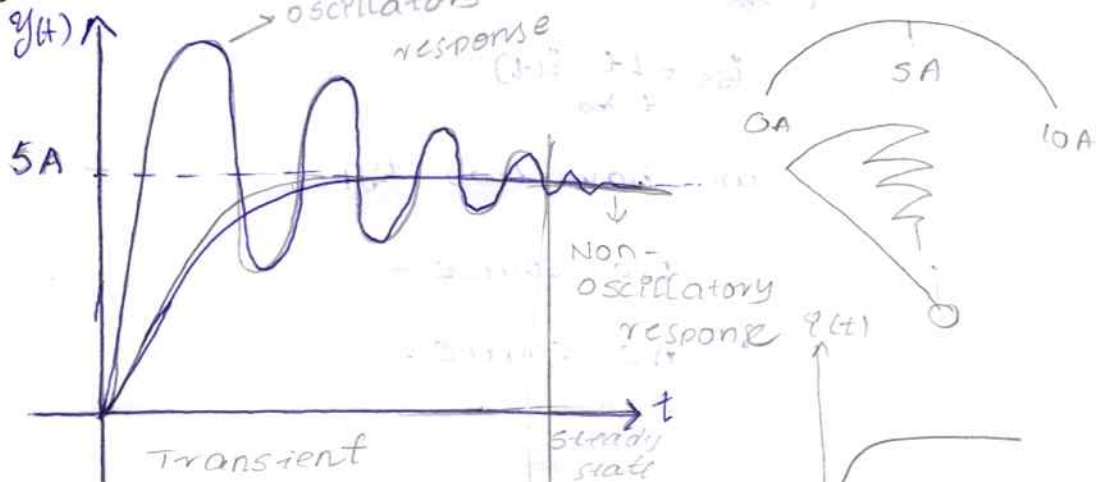
$R: V = IR$, $L: V = L\frac{di}{dt}$, $C: V = \frac{1}{C}\int i\,dt$

inductor has magnetic field

conductor has electric field

The combination both inductor, conductor is Integro differential

Steady state Analysis:



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DEFINATION OF STEADY STATE:

A network in which currents and voltages are not changing with respective time is said to be in steady state [FOR DC SOURCE]

AC SOURCE:

A circuit with AC sources is said to be in steady state when voltages and currents are having constant amplitude and frequency.

TRANSIENT STATE:

The network is changed from one steady state to another steady state when

* Source is connected to a network
* Source is disconnected to a network
* Any change in the elements of the network.

The time taken ~~for~~ by the network to reach the steady state is known as TRANSIENT STATE. So, the voltages and currents changes with respective time, in transient state

$$i(t) = i_t(t) + i_{ss}(t).$$

where

$i_t(t)$ = transient response

$i_{ss}(t)$ = steady state response

$\lim_{t \to \infty} i_t(t) = 0$ [∵ no transients in steady state]

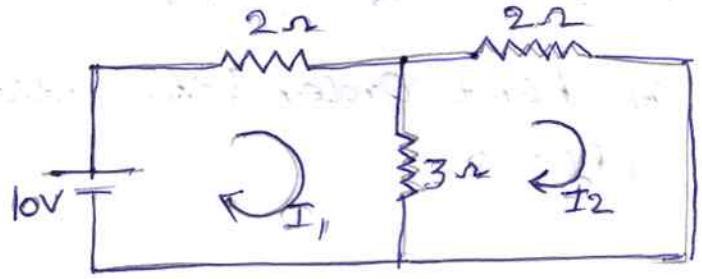
$$i_{ss} = \lim_{t \to \infty} i(t)$$

we have two types

DC SOURCE -

AC SOURCE -

Consider :



$$5I_1 - 3I_2 = 10$$

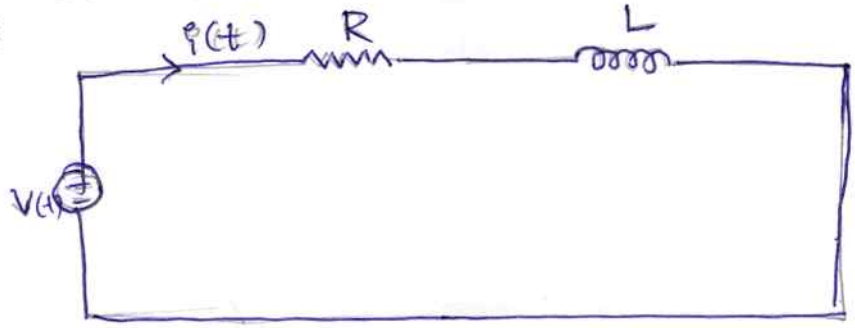
$$-3I_1 - 5I_2 = 0$$

It is a static system

No transients in resistive circuits

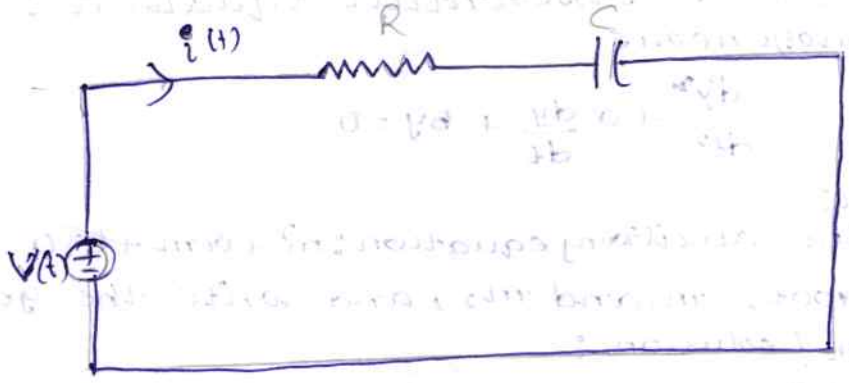
Consider Dynamic system :

RL :



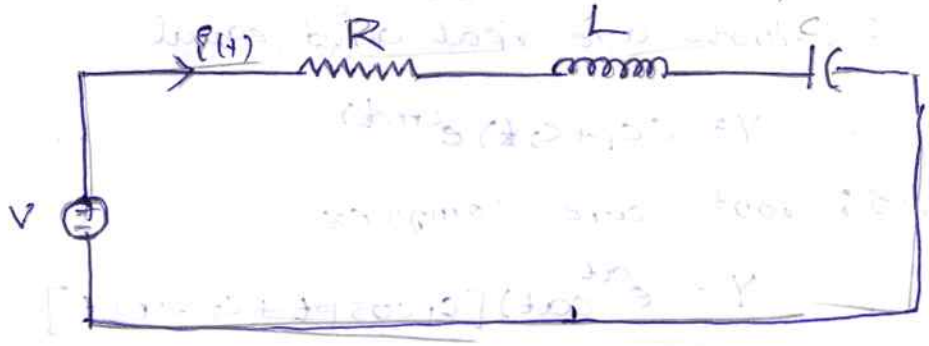
$$V = Ri + L\frac{di}{dt}$$

RC :



$$V = Ri + \frac{1}{C}\int i\,dt \qquad \frac{dV}{dt} = R\frac{di}{dt} + \frac{i}{C}$$

RLC :



$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

order = no. of energy storing elements

Solution of First-Order Differential Equation :

Form: $\frac{dy}{dt} + Py = Q$

Solution:

$$CF = K e^{-Pt}$$

$$PI = e^{-Pt} \int_0^x e^{Pt} Q \, dt$$

general solution = $K e^{-Pt} + e^{-Pt} \int_0^t e^{Pt} Q \, dt$ [$\because$ G.S = CF + P.I]

Second-Order Differential Equation :

FORM: Homogeneous

$$\frac{d^2y}{dt^2} + a\frac{dy}{dt} + by = 0$$

Solution:

1. solve the auxiliary equation: $m^2 + am + b = 0$
2. Find roots m_1 and m_2, and write the general equation / solution :

Case-1: roots are real and distinct

$$Y = c_1 e^{(m_1 t)} + c_2 e^{(m_2 t)}$$

Case-2: roots are real and equal

$$Y = (c_1 + c_2 t) e^{(mt)}$$

Case-3: roots are complex

$$Y = e^{at} (at) [c_1 \cos \beta t + c_2 \sin \beta t]$$

10.09.2025

DYNAMIC SYSTEM:

When energy storing elements like inductors and capacitors present in the circuit is called DYNAMIC SYSTEM.

The Dynamic system is represented by Integro - Differential equation.

Transients are occurs in Dynamic system.

Solution of differential Equations:

There are two methods to solve the Differential equations.

a) classical (or) Direct Method [t - Domain]:

1. Obtain General solution

$$\text{General solution } G.S = C.F + P.I$$

no. of arbitary constants = order of system

2. Calculate arbitary constants using initial condition

b) Using Laplace transforms [s - Domain]:

1. Calculate the initial condition
2. Draw the s - domain circuit
3. Obtain the expresion for I(s) or V(s)
4. Obtain the partial fraction expansion.
5. Obtain inverse laplace transform of I(s) or V(s) to get i(t) or v(t)

G.S $$a_1 \frac{dy}{dt} + a_0 y = b$$

$$\frac{dy}{dt} + \frac{a_0}{a_1} y = \frac{b}{a_1}$$

$$\frac{dy}{dt} + Py = Q$$

Solution:

C.F P.I

$$y = e^{-\int P dt} \int e^{\int P dt} Q \, dt + k e^{-\int P dt}$$

C.F:

Independent of I/P

[O I/P response]

Depends on nature of system

Solution for Second-order D.E:

$$\frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_2 y = b$$

auxillary equation

$$D^2 + a_1 D + a_2 = 0$$

Let the roots of auxillary equations are

$$m_1, m_2$$

Case-1: roots are real and unequal

$$G.S = c_1 e^{m_1 t} + c_2 e^{m_2 t}$$

case-2: roots are real and equal

$$G.S = (c_1 + c_2 t) e^{m_1 t}$$

case-3: roots are complex conjucate

$$m_1, m_2 = a \pm jb$$

$$G.S = e^{ax} [c_1 \cos bt + c_2 \sin bt]$$

case-3(a): roots are pure imaginary

$$m_1, m_2 = \pm jb$$

$$G.S = c_1 \cos bt + c_2 \sin bt$$

$$= k \sin(bt + \phi)$$

Initial Conditions:

Initial conditions are used to

Defination:

Initial conditions are those conditions that exist in the circuit immediately after switching operation. At $t=0$ switching operation occurs

There are two cases

Just before switching operation $t[0^-]$

Immediately after switching operation $t[0^+]$

Initial condition depends on past history

[v and i at $t=0$]

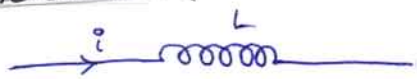
2. circuit at $t=0^+$

1. Resistor:



$v = Ri$

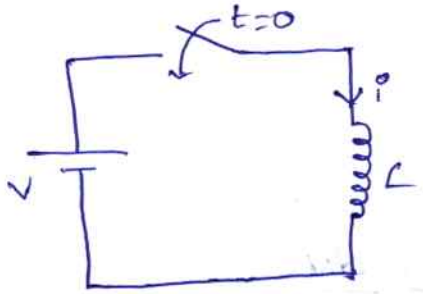
2. Inductor:



$$v = L\frac{di}{dt}$$

Inductor opposes change in current

∴ The current in inductor can't change instantaneously.

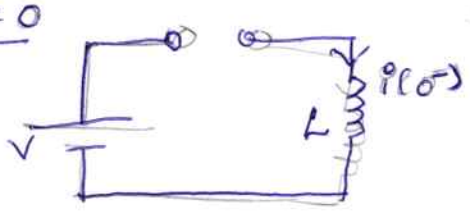


$i(0^-)$ = current through (⊖) Inductor just before closing the switch

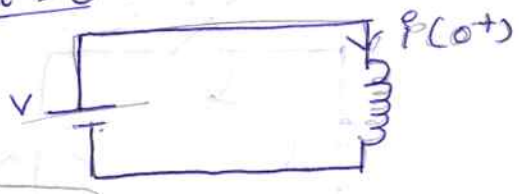
$i(0^+)$ = current passing through (⊖) Inductor immediately after closing the switch

$\frac{1}{2}CV^2$

At $t=0^-$



At $t=0^+$



$i_L(0^+) = i_L(0^-)$

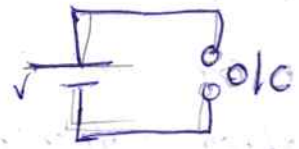
∴ Since current in Inductor

i. Zero Initial condition:

$$i(0^+) = 0$$

$$i(0^-) = 0$$

At $t = 0^+$



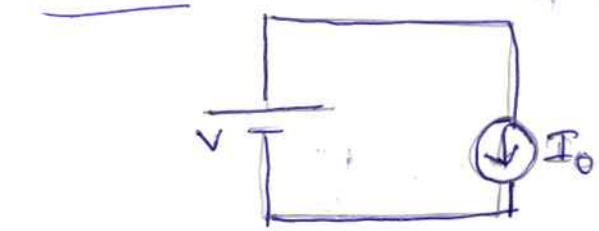
Inductor act as a open circuit at initial condition

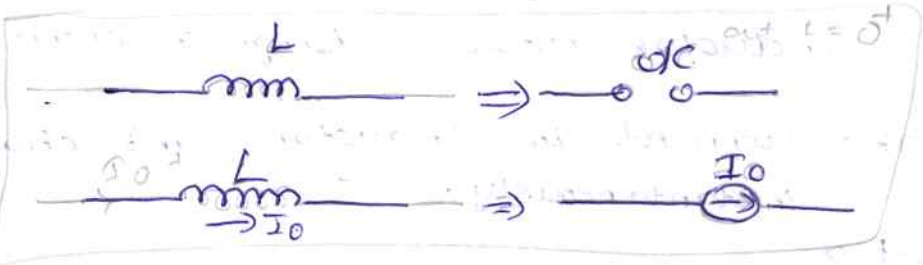
ii. Non-Zero Initial condition:

$$i(0^-) = I_0$$

$$i(0^+) = i(0^-) = I_0$$

At $t = 0^+$





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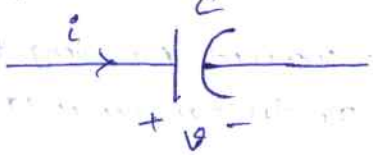
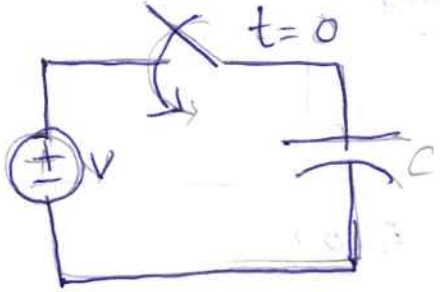
3. Capacitor:

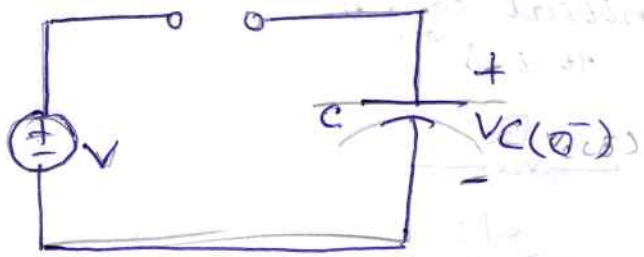


$$i = C \frac{dv}{dt}$$

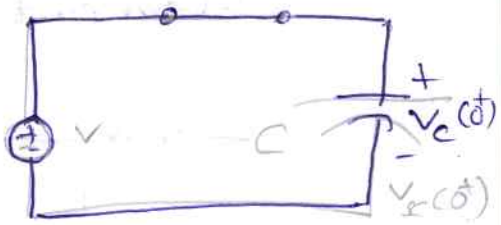
Capacitor opposes sudden change in voltage



At $t = 0^-$



At $t = 0^+$



Initial condition is $V_c(0^+) = V_c(0^-)$

(∵ Since voltage across a capacitor cannot change instantly.)

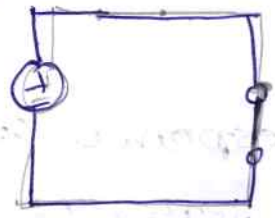
Case-1: zero initial condition

$$V_c(0^-) = 0$$

$$V_c(0^+) = 0$$

∴ At $t = 0$ capacitor acts as short circuit

At $t = 0^+$



S/C (∵ $V_c(0^+) = 0$)

$V_c(0^+) = 0$

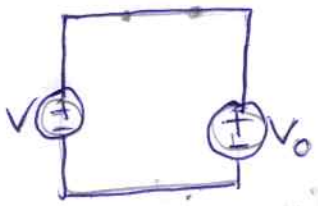
Case-2: Non-zero initial condition

$$V_c(0^-) = V_0$$

$$V_c(0^+) = V_0$$

$$V_c(0^-) = V_c(0^+) = V_0$$

At $t = 0^+$

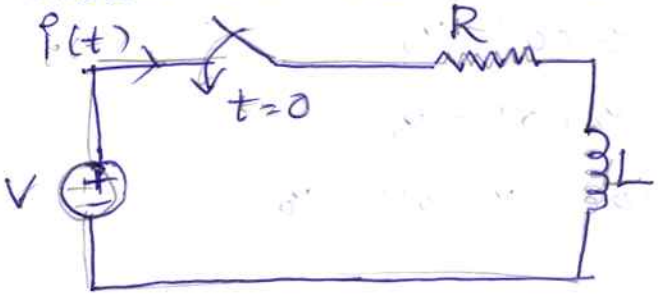


At $t = 0^+$ capacitor acts as voltage source V_0

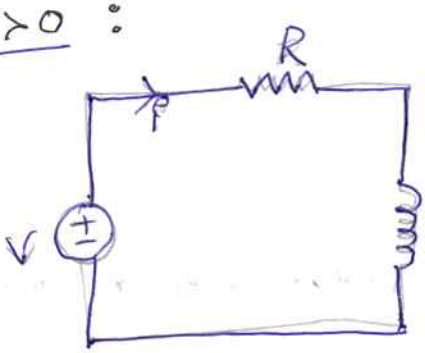
Element	Initial condition At $t=0^+$
R	R
L (Zero IC)	o/c
L ($\rightarrow I_0$ Non-zero)	I_0
C (Zero)	s/c
C ($+V_0-$ non-zero)	V_0

DC Transient Response of series RL circuit:

consider a series RL circuit connected to a DC voltage source through a switch as shown in below figure



$t > 0$:



$$V = Ri + L\frac{di}{dt}$$

$$L\frac{di}{dt} + Ri = V$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$$

For the differential equation $\frac{dy}{dt} + Py = Q$

The general equation is

$$y(t) = ke^{-Pt} + e^{-Pt} \int e^{Pt} Q \, dt$$

from eq ①

$$P = \frac{R}{L}, \quad Q = \frac{V}{L}$$

∴ General solution

$$i(t) = ke^{-\frac{R}{L}t} + e^{-\frac{R}{L}t} \int e^{\frac{R}{L}t} \frac{V}{L} dt$$

$$= ke^{-\frac{R}{L}t} + \frac{V}{L} e^{-\frac{R}{L}t} \int e^{R/L\, t} \, dt$$

$$= ke^{-\frac{R}{L}t} + \frac{V}{L} e^{-\frac{R}{L}t} \frac{e^{\frac{R}{L}t}}{R/L}$$

$$= ke^{-\frac{R}{L}t} + \frac{V}{R} e^{-\frac{R}{L}t + \frac{R}{L}t}$$

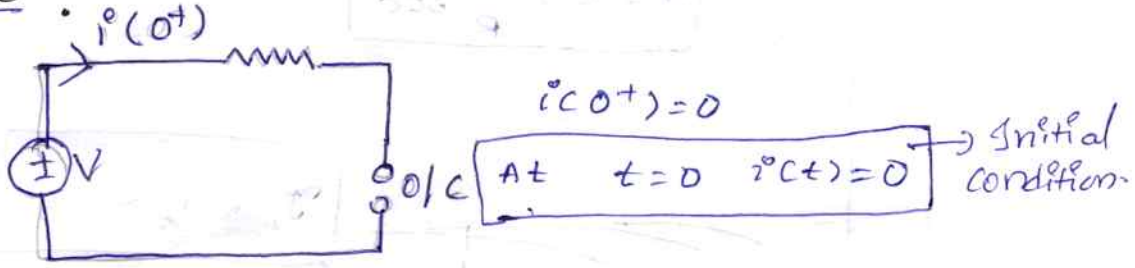
$$= ke^{-\frac{R}{L}t} + \frac{V}{R} e^{0}$$

$$= ke^{-\frac{R}{L}t} + \frac{V}{R}$$

$$\therefore \; i(t) = ke^{-\frac{R}{L}t} + \frac{V}{R} \longrightarrow ②$$

At zero initial condition :

At $t = 0^+$:



from eq ②

At $t = 0$

$$0 = Ke^{0} + \frac{V}{R}$$

$$0 = \frac{V}{R} + K$$

$$\boxed{K = \frac{-V}{R}}$$

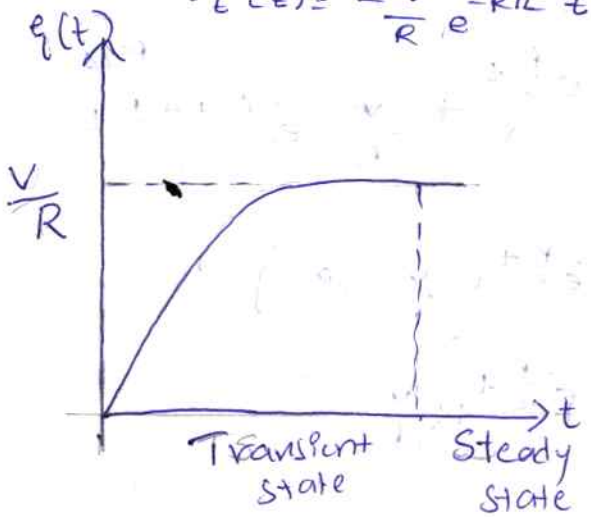
∴ The complete solution

$$i(t) = \frac{-V}{R} e^{-\frac{R}{L}t} + \frac{V}{R}$$

$$\boxed{i(t) = \frac{V}{R}\left[1 - e^{-\frac{R}{L}t}\right]}$$

$$i_{ss}(t) = \lim_{t \to \infty} i(t) = \frac{V}{R}$$

$$i_t(t) = -\frac{V}{R} e^{-R/L\, t}$$



At $t = 0$

$i(t) = \frac{V}{R}(1-1) = 0$

At $t = \infty$

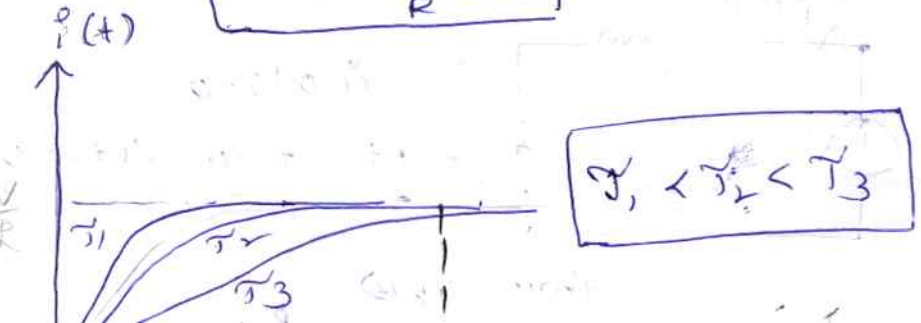
$i(t) = \frac{V}{R}$

Time Constant:

Time constant is the ~~constant~~ time at which the power of exponential term is unity (1)

$\frac{R}{L} t = 1$ Time constant of RL series circuit

$$\boxed{\tau = t = \frac{L}{R} \text{ sec}}$$



$$\boxed{\tau_1 < \tau_2 < \tau_3}$$

At $t = \tau$ $i(t) = \frac{V}{R}[1 - e^{-\frac{R}{L} \cdot \frac{L}{R}}]$

$= \frac{V}{R}[1 - e^{-1}]$

$= 0.632 \frac{V}{R}$

At $t = 2\tau$ $i(t) = \frac{V}{R}[1 - e^{-2\frac{R}{L} \cdot \frac{L}{R}}]$

$= \frac{V}{R}[1 - e^{-2}]$

$= 0.864 \frac{V}{R}$

At $t = 3\tau$ $i(t) = \frac{V}{R}[1 - e^{-3\frac{R}{L} \cdot \frac{L}{R}}]$

$= \frac{V}{R}[1 - e^{-3}]$

$= 0.950 \frac{V}{R}$

At $t = 4\tau$ $i(t) = \frac{V}{R}[1 - e^{-3+\frac{R}{L} \cdot \frac{L}{R}}]$

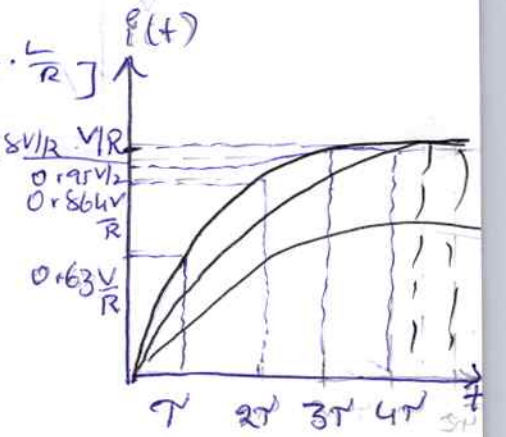
$= \frac{V}{R}[1 - e^{-4}]$

$= 0.98 \frac{V}{R}$

At $t = 5\tau$ $i(t) = \frac{V}{R}[1 - e^{-5\frac{R}{L} \cdot \frac{L}{R}}]$

$= \frac{V}{R}[1 - e^{-5}]$

$= 0.993 \frac{V}{R}$

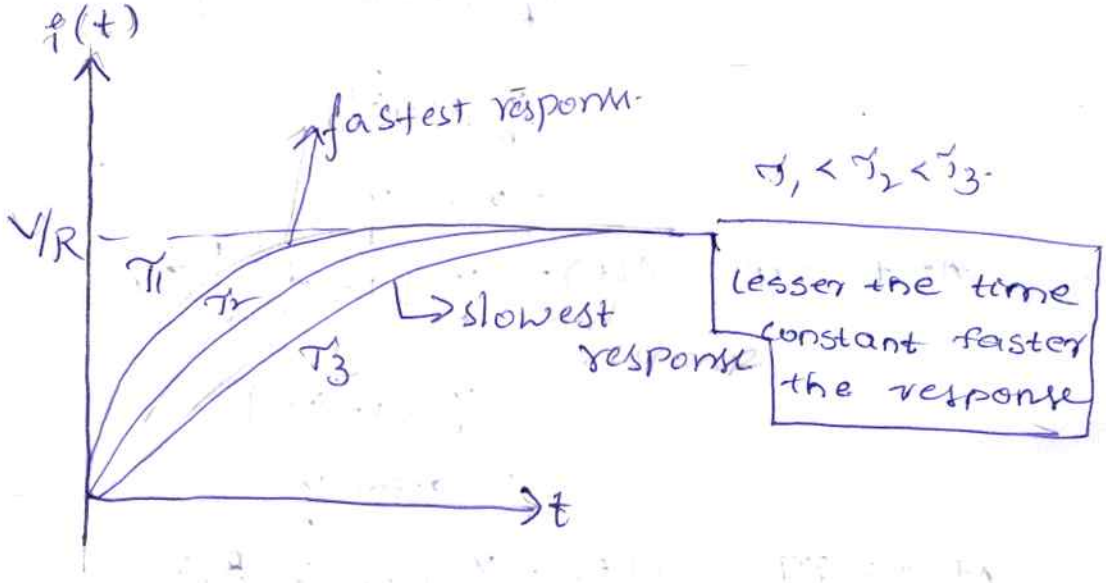


Time Constant:

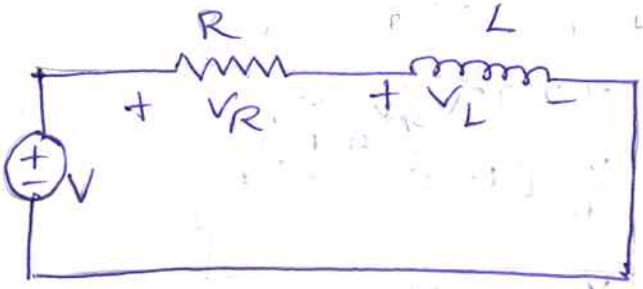
Time constant is defined as the time required for the response to reach 63.2% of it's final value for raising exponentials.

The time required for any system to reach the steady state is equal to 4τ

Speed of the response depends on time constant



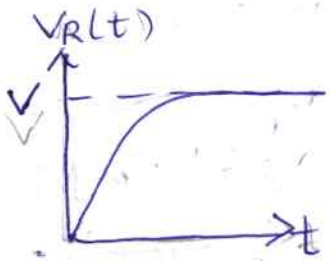
Voltage across Resistor and Inductor :



$V_R = Ri$

$V_R = R \cdot \frac{V}{R}\left[1 - e^{-\frac{R}{L}t}\right]$

$$V_R = V\left[1 - e^{-\frac{R}{L}t}\right] \rightarrow (4)$$



$V_L = L\frac{di}{dt}$

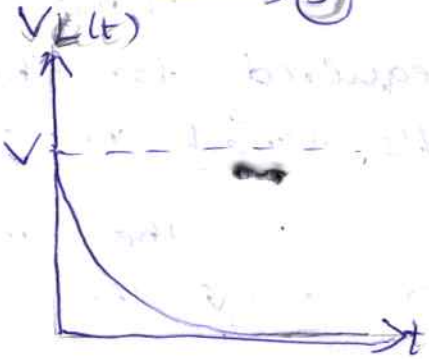
$= L\frac{V}{R}\frac{d}{dt}\left[1 - e^{-\frac{R}{L}t}\right]$

$= -\frac{VL}{R}\left[-\frac{R}{L}e^{-\frac{R}{L}t}\right]$

$$V_L = Ve^{-\frac{R}{L}t} \rightarrow (5)$$

At $t = 0$, $V_L = V$

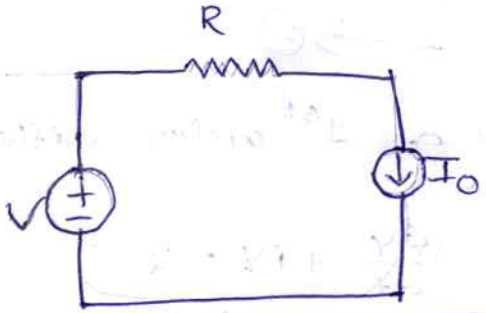
At $t = \infty$, $V_L = 0$



At Non-zero Initial condition :

$i_L(0^-) = I_0$

At $t = 0^+$



At $t = 0$, $i(t) = I_0$ → Initial Condition.

General solution

$$i(t) = t e^{-R/L\, t} + \frac{V}{R}$$

At $t = 0$

$$I_0 = k e^{0} + \frac{V}{R}$$

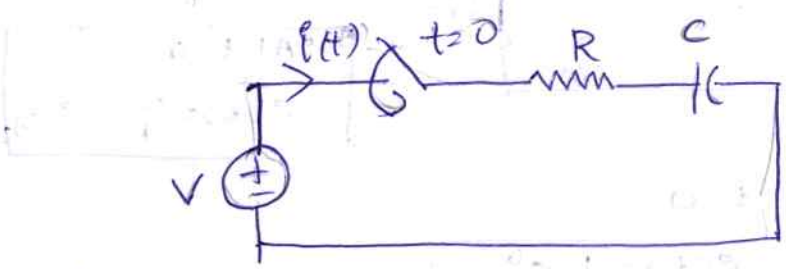
$$k = I_0 - \frac{V}{R}$$

$$\therefore i(t) = \left[I_0 - \frac{V}{R}\right] e^{-\frac{R}{L}t} + \frac{V}{R}$$

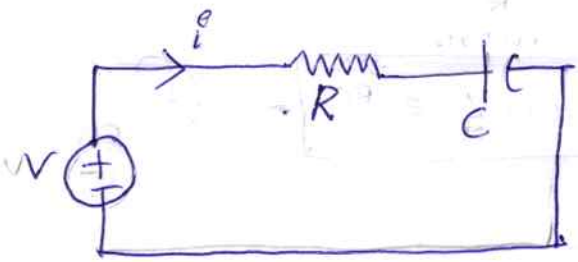
15.09.2025

DC TRANSIENT RESPONSE OF SERIES RC CIRCUIT :

Consider a series RC circuit connected to the voltage source, through a switch as shown in figure.



for $t > 0$:



$$V = Ri + \frac{1}{C}\int i\,dt \rightarrow (1)$$

Apply differentiation

$$\frac{d}{dt}V = \frac{d}{dt}\left[Ri + \frac{1}{C}\int i\,dt\right]$$

$$\frac{dV}{dt} = \frac{di}{dt}R + \frac{1}{C}i$$

$$0 = R\frac{di}{dt} + \frac{i}{C} \qquad [\because v \text{ is constant}]$$

$$\frac{di}{dt} + \frac{i}{RC} = 0 \longrightarrow (2)$$

standard eqn of 1st order differential eqn

$$\frac{dy}{dt} + Py = Q$$

The general solution is

$$y(t) = ke^{-Pt} + e^{-Pt}\int e^{Pt}\, Q\, dt$$

$$P = \frac{1}{RC}, \quad Q = 0$$

∴ general solution for eqn (2)

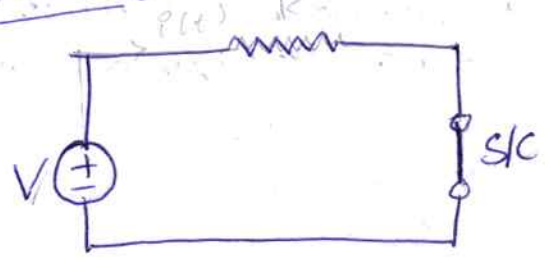
$$i(t) = ke^{-t/RC} + e^{-\frac{t}{RC}}\int e^{\frac{t}{RC}}.0\, dt$$

$$\boxed{i(t) = ke^{-t/RC}} \longrightarrow (3)$$

Initial condition :

case-1 : zero initial condition

At $t = 0^+$



$$i(t) = \frac{V}{R}$$

At $t = 0$

$$i(t) = \frac{V}{R} = i(0)$$

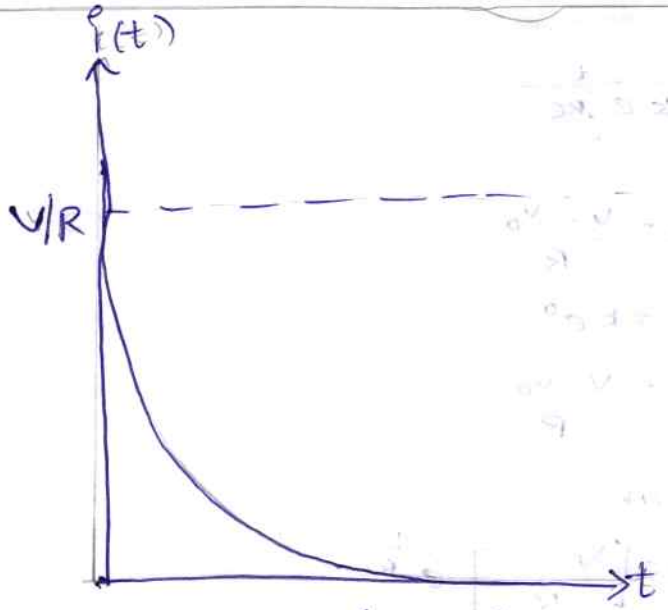
from eqn (3) At $t = 0$

$$i(t) = ke^0 = \frac{V}{R}$$

$$k = \frac{V}{R}$$

complete solution

$$\boxed{i(t) = \frac{V}{R}e^{-\frac{t}{RC}}} \longrightarrow (4)$$



At $t=0$, $i(t) = \frac{V}{R}$

At $t=\infty$, $i(t) = 0$

$\frac{t}{RC} = 1$

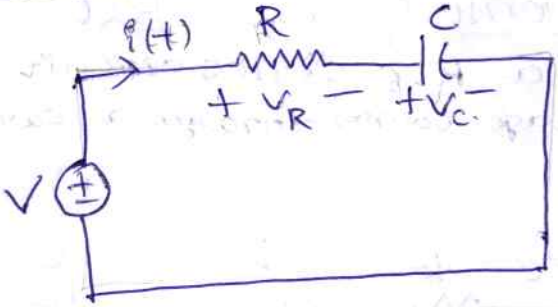
$\tau = RC$

$t = RC = \tau$

$i(t) = \frac{V}{R} e^{-t/\tau}$

Time constant of RC series circuit $\tau = RC$

Voltage ax R & C: (V_R & V_C)



$V_R = R i = R \cdot \frac{V}{R} e^{-t/RC}$

$V_R = V e^{-t/RC}$

$V_C = \frac{1}{C} \int_0^t i\, dt$ (or) $V_C = V - V_R$

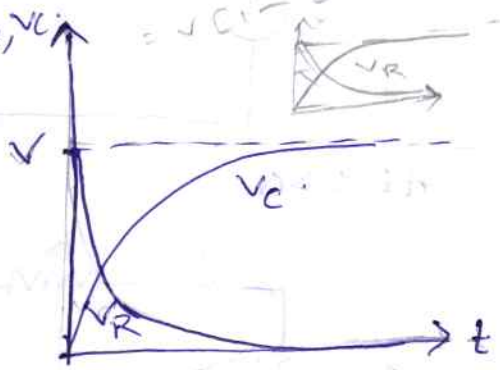
$= V - V e^{-t/RC}$

$= V(1 - e^{-t/RC})$

$= \frac{1}{C} \int_0^t \frac{V}{R} e^{-t/RC}$

$= \frac{V}{RC} \int_0^t e^{-t/RC}\, dt$

$= \frac{V}{RC} \left[\frac{e^{-t/RC}}{-\frac{1}{RC}} \right]_0^t$



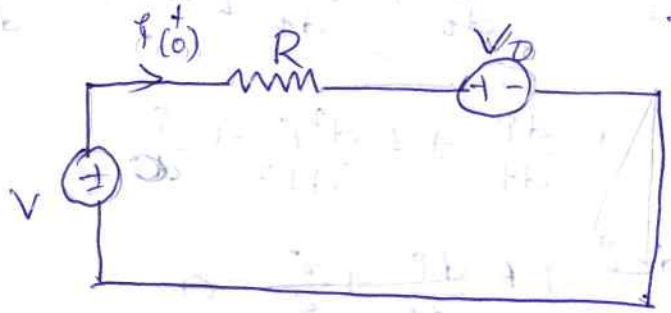
$V_C = -V e^{-t/RC} + V$

$V_C = V(1 - e^{-t/RC})$

case-2: Non-zero initial condition:

$V_C(0) = V_0$

At $t = 0^+$



At $t = 0$

$i(0) = \frac{V - V_0}{R}$

Final condition:

General solution

$$i(t) = k e^{-\frac{t}{RC}}$$

At $t=0$

$$i(t) = \frac{V - V_0}{R}$$

$$= k e^0$$

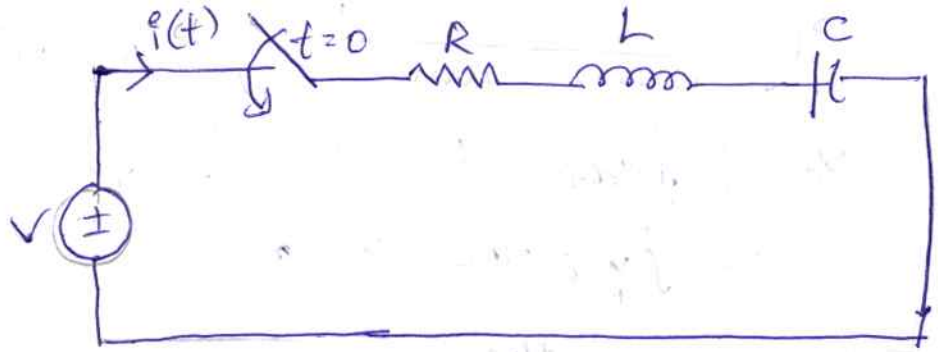
$$k = \frac{V - V_0}{R}$$

Final solution

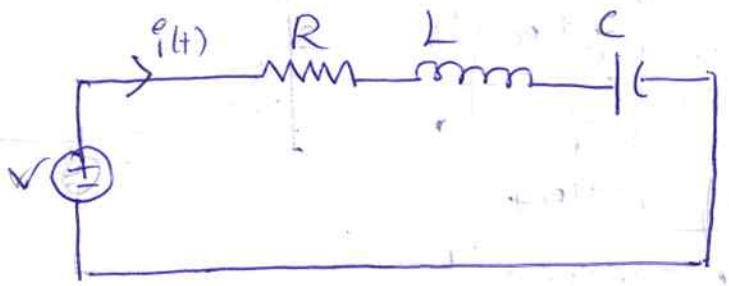
$$i(t) = \left[\frac{V - V_0}{R}\right] e^{-\frac{t}{RC}}$$

DC Transient Response of Series RLC circuit :-

consider a RLC series circuit connected to a dc voltage source through a switch as shown in figure



At $t > 0$



$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \quad \rightarrow (1)$$

$$\frac{dV}{dt} = \left[R\frac{di}{dt}\right] \frac{d}{dt}\left[Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt\right]$$

$$0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C}$$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0$$

standard second order differential eqn is

$$\frac{d^2y}{dt} + P\frac{dy}{dt} + Qy = b$$

$$\therefore \frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0 \longrightarrow (2)$$

Auxillary eqn

$$D^2 + \frac{R}{L}D + \frac{1}{LC} = 0 \longrightarrow (3)$$

Roots of Auxillary eqn m_1, m_2

$$= \frac{-\frac{R}{L} \pm \sqrt{\left[\frac{R}{L}\right]^2 - \frac{4}{LC}}}{2}$$

$$= \frac{-R}{2L} \pm \frac{1}{2}\sqrt{\left[\frac{R}{L}\right]^2 - \frac{4}{LC}}$$

$$m_1 m_2 = \frac{-R}{2L} \pm \sqrt{\left[\frac{R}{2L}\right]^2 - \frac{1}{LC}} \longrightarrow (4)$$

$$\left[\frac{R}{2L}\right]^2 - \frac{1}{LC} = 0$$

$$\left[\frac{R}{2L}\right]^2 = \frac{1}{LC}$$

$$\frac{R}{2L} = \sqrt{\frac{1}{LC}}$$

$$R = \frac{2L}{\sqrt{LC}}$$

$$R = 2\sqrt{\frac{L^2}{LC}}$$

Critical Resistance $\boxed{R_{CR} = 2\sqrt{\frac{L}{C}}}$ R_{CR} = critical resistance

Critical Resistance:

The value of resistance for which the imaginary part of the roots of auxillary equation is equal to zero.

$$R_{cr} = 2\sqrt{\frac{L}{C}}$$

Damping Ratio or Damping Factor (ζ):

Damping ratio is defined as the ratio of resistance of the circuit to the critical resistance.

$$\zeta = \frac{R}{R_{cr}}$$

Damping ratio is opposition to oscillations

Depending upon types of roots there will be 4 types of responses.

i) undamped ($\zeta = 0$) response:

when $R = 0$

$$m_1, m_2 = \pm\sqrt{-\frac{1}{LC}}$$

roots are pure imaginary $= \pm j\sqrt{\frac{1}{LC}}$

ii) under damped ($\zeta < 1$) response:

when $R < R_{cr}$

$$m_1, m_2 = -\frac{R}{2L} \pm \sqrt{\left[\frac{R}{2L}\right]^2 - \frac{1}{LC}}$$

roots are complex conjugate $-\frac{R}{2L} \pm j\sqrt{\frac{1}{LC} - \left[\frac{R}{2L}\right]^2}$

iii) critically damped response ($\zeta = 1$)

when $R = R_{cr}$

$$m_1, m_2 = -\frac{R}{2L}, -\frac{R}{2L}$$

roots are real and equal.

Over damped response ($\zeta > 1$):

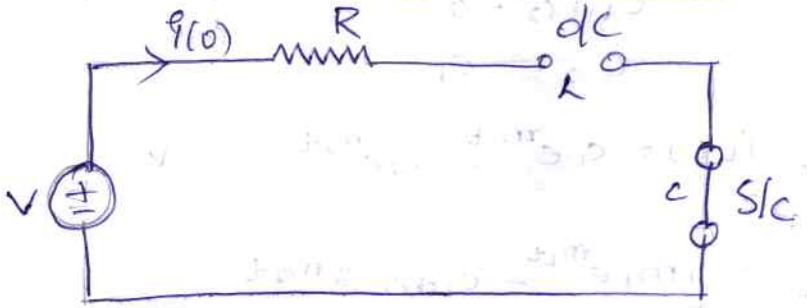
when $R > R_{cr}$

$$m_1, m_2 = \frac{-R}{2L} \pm \sqrt{\left[\frac{R}{2L}\right]^2 - \frac{1}{LC}}$$

roots are real and unequal

Initial condition:

At $t = 0^+$ Assume zero initial condition



$i(0) = 0$ [Open circuit]

At $t = 0$, $i(t) = 0$ ⟶ Initial condition 1. ⟶ (5)

$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

voltage across cap

$$V_c(0^+) = V_c(0^-) = 0 = \frac{1}{C}\int i\,dt$$

At $t = 0$

$$V = Ri + L\frac{di}{dt}$$

$$V = L\frac{di}{dt} \quad [\because i(0) = i = 0 \text{ at } t = 0]$$

At $t = 0$ $\quad \frac{di}{dt} = \frac{V}{L}$ ⟶ Initial condition 2. ⟶ (6)

Initial conditions

At $t = 0$ $\quad i(t) = 0$

$$\frac{di}{dt} = \frac{V}{L}$$

case-1: over damped response:

roots are real and unequal

$$m_1, m_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

General solution

$$i(t) = c_1 e^{m_1 t} + c_2 e^{m_2 t}$$

At $t=0$, $i(t) = 0 = c_1 e^0 + c_2 e^0$

$$c_1 + c_2 = 0$$

$$c_2 = -c_1$$

At $t=0$, $i(t) = c_1 e^{m_1 t} - c_1 e^{m_2 t}$

$$\frac{di}{dt} = c_1 m_1 e^{m_1 t} - c_1 m_2 e^{m_2 t}$$

$$= c_1 m_1 e^0 - c_1 m_2 e^0$$

$$= c_1 (m_1 - m_2) = \frac{V}{L}$$

$$c_1 = \frac{V}{L(m_1 - m_2)}, \quad c_2 = -\frac{V}{L(m_1 - m_2)}$$

∴ final solution

$$i(t) = \frac{V}{L(m_1 - m_2)} \left[e^{m_1 t} - e^{m_2 t} \right]$$

m_1, m_2 are roots of auxillary equation

16.09.2025

~~case-2: critically damped response:~~

$$\zeta = 1$$

roots are real and equal

square roots term in the roots becomes zero

$$m_1, m_2 = -\frac{R}{2L}, -\frac{R}{2L}$$

∴ general solution

$$i(t) = (c_1 + c_2 t) e^{m_1 t} \longrightarrow (1)$$

$$i(t) = 0 = (c_1 + c_2 \times 0)\, e^0$$

$$\boxed{0 = c_1}$$

$$\therefore i(t) = c_2 t e^{m_1 t}$$

At $t=0$ $\quad \frac{di}{dt} = \frac{V}{L}$

$$\frac{di}{dt} = c_2 1 e^{m_1 t} + c_2 t m_1 e^{m_1 t}$$

$$\frac{di}{dt} = \frac{V}{L} = c_2 e^0 + 0$$

$$\boxed{\frac{V}{L} = c_2}$$

$\therefore$ complete solution

$$i(t) = \frac{V}{L} t e^{m_1 t}$$

$$\boxed{i(t) = \frac{V}{L} t e^{-\frac{Rt}{2L}}}$$

Case-3: ~~Under damped~~ response:

$0 < \zeta < 1$

roots are complex conjucate

$$m_1, m_2 = -\frac{R}{2L} \pm j\sqrt{\frac{1}{LC} - \left[\frac{R}{2L}\right]^2}$$

$$= a \pm jb$$

General solution

$$i(t) = e^{at}\,[c_1 \cos bt + c_2 \sin bt]$$

At $t=0$ $\quad i(t) = 0 = e^0\,[c_1 \cos 0 + c_2 \sin 0]$

$$\boxed{0 = c_1}$$

$$\therefore i(t) = c_2 e^{at} \sin bt \quad [\because c_1 = 0]$$

$$\frac{di}{dt} = a t$$

At $t=0$ $\quad \frac{di}{dt} = \frac{V}{L}$

$$= c_2 a e^{0} \sin 0 + c_2 e^{0} b \cos 0$$

$$= 0 + c_2 b$$

$$c_2 = \frac{V}{bL}$$

$$\therefore i(t) = \frac{V}{bL} e^{at} \sin bt$$

where a, b are real part and imaginary part of the roots respectively

Case-4: Undamped Response:

$\zeta = 0$

Roots are pure imaginary [$\because$ real part $= 0$ i.e., $R = 0$]

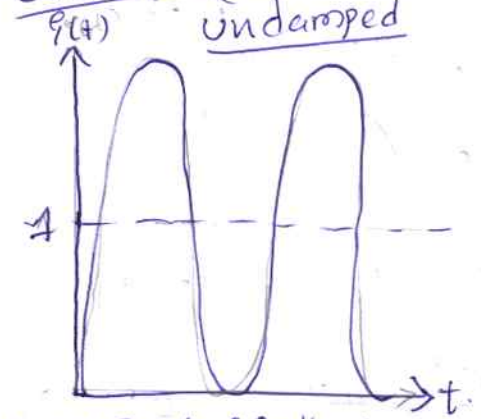
$$m_1, m_2 = \pm j \sqrt{\frac{1}{LC}}$$

$$= \pm jb$$

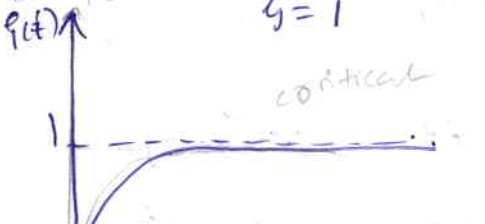
$$i(t) = \frac{V}{bL} e^{at} \sin bt$$

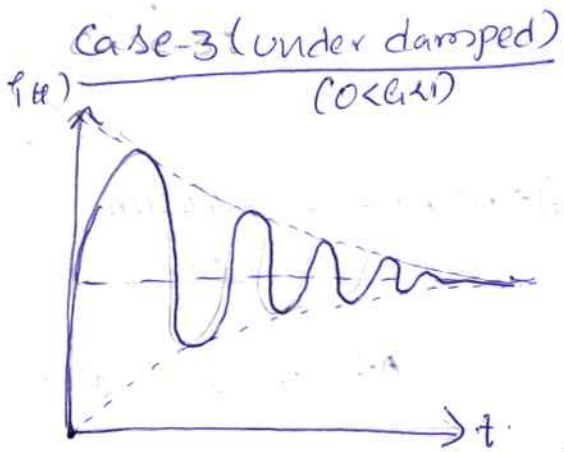
Substitute $a = 0$ in the above solution

Case-4: ($\zeta = 0$) $\quad i(t) = \frac{V}{bL} \sin bt$

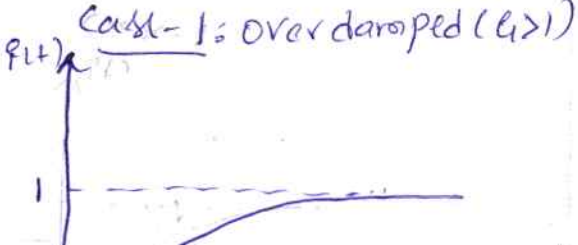


Case-2: (critically damped) $\zeta = 1$



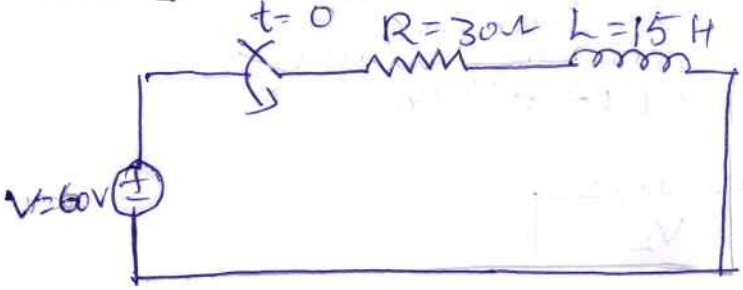


Case-1: over damped ($\zeta > 1$)

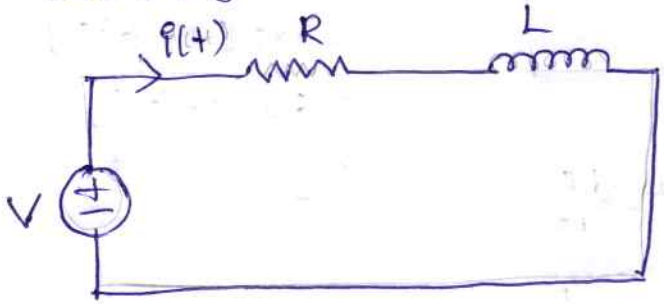


Problem-1:

Find the expression for the current flowing through the circuit, voltage across the resistor and voltage across inductor for $t > 0$



Sol) for $t = 0$



$$V = Ri + L\frac{di}{dt}$$

$$60 = 30i + 15\frac{di}{dt}$$

$$15\frac{di}{dt} + 30i = 60$$

$$\frac{di}{dt} + 2i = 4 \longrightarrow (1)$$

$$\frac{dy}{dt} + Py = Q \qquad P = 2,\ Q = 4$$

$$i(t) = ke^{-Pt} + e^{-Pt}\int e^{Pt} Q\, dt$$

$$= ke^{-2t} + e^{-2t}\int e^{2t} 4\, dt$$

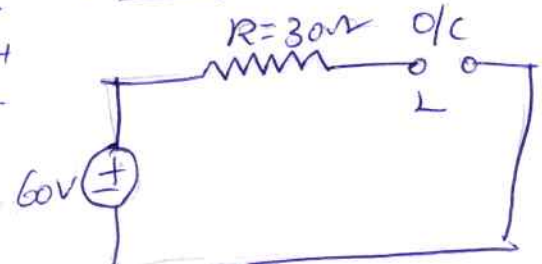
$$= ke^{-2t} + e^{-2t} 4 \cdot \frac{e^{2t}}{2}$$

$$= ke^{-2t} + e^{-2t}\, 2\, e^{2t}$$

$$i(t) = ke^{-2t} + 2$$

initial condition

At $t = 0^+$



At $t = 0$, $i(t) = 0$

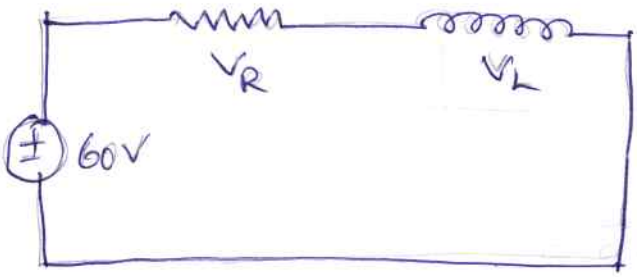
At $t = 0$ $\quad i(t) = 0 = Ke^{0} + 2$

$$K + 2 = 0$$

$$K = -2$$

$$\therefore \; i(t) = -2e^{-2t} + 2$$

$$= 2\left[1 - e^{-2t}\right] A$$



$$V_R = Ri$$

$$= 30 \times 2\left[1 - e^{-2t}\right]$$

$$= 60\left[1 - e^{-2t}\right]$$

$$= 60 - 60e^{-2t} \; V$$

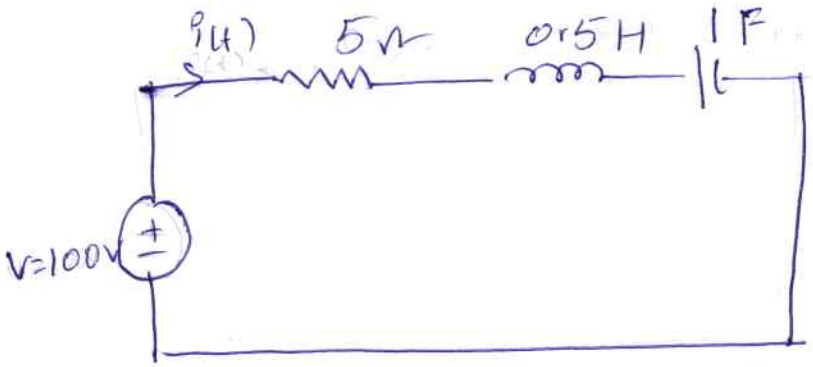
$$V_L = L\frac{di}{dt}$$

$$= V - V_R$$

$$= 60 - 60 + 60e^{-2t}$$

$$V_L = 60e^{-2t} \; V$$

Pb Find the expression for current $i(t)$ in the given circuit. (Assume Zero Initial Condition)



$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \quad \text{---(1)}$$

$$100 = 5i + 0.5\frac{di}{dt} + 1\int i\,dt$$

Apply differentiation on both sides

$$\frac{d}{dt}(100) = \frac{d}{dt}(5i) + 0.5\frac{d^2i}{dt^2} + \frac{di}{dt}\int i\,dt$$

$$0 = 5\frac{di}{dt} + 0.5\frac{d^2i}{dt^2} + i$$

$$\frac{d^2i}{dt^2} + 10\frac{di}{dt} + 2i = 0 \quad \text{---(1)}$$

Auxiliary eqn is

$$D^2 + 10D + 2 = 0$$

$$m_1, m_2 = -0.2041, -9.79$$

The roots are real and unequal

General solution is

$$i[t] = C_1 e^{-9.79t} + C_2 e^{-0.204t} \longrightarrow (2)$$

At initial condition:

At $t=0$, $i(t)=0$

At $t=0$ $\frac{di}{dt} = \frac{V}{L} = \frac{100}{0.5} = 200$

from eqn (2) At $t=0$

$$i(t) = C_1 e^0 + C_2 e^0$$
$$= C_1 + C_2$$

from IC $i(t)=0$ at $t=0$

$$C_1 + C_2 = 0 \longrightarrow (3)$$

differentiate eq (2)

$$\frac{di}{dt} = C_1(-9.79)\, e^{-9.79t} + C_2(-0.204)\, e^{-0.204t}$$

$$\frac{di}{dt} = -9.79 C_1 - 0.204 C_2 = 200 \longrightarrow (4)$$

[IC $\frac{di}{dt} = 200$]

Solve eq (3), (4)

$$C_1 + C_2 = 0$$
$$-9.79 C_1 - 0.204 C_2 = 200$$

$$C_1 = -20.86$$
$$C_2 = 20.86$$

∴ final solution is

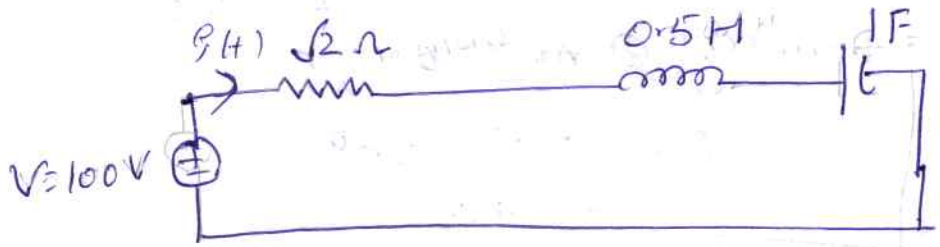
$$i(t) = -20.86 e^{-9.79t} + 20.86 e^{-0.204t}$$

critical resistance

$$R_{cr} = 2\sqrt{\frac{L}{C}}$$
$$R_{cr} = 1.414\ \Omega$$
$$R = 5\ \Omega$$

$R > R_{cr}$ overdamped

Pb Find the expression for current i(t).



$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \quad \text{——} (1)$$

$$100 = 1.414\,i + 0.5\frac{di}{dt} + 1\int i\,dt$$

Apply differentiation on both sides

$$\frac{d}{dt}(100) = 1.414\frac{di}{dt} + 0.5\frac{d^2i}{dt^2} + \frac{d}{dt}\int i\,dt$$

$$0 = 1.414\frac{di}{dt} + 0.5\frac{d^2i}{dt^2} + i$$

$$\frac{d^2i}{dt^2} + 2.828\frac{di}{dt} + 2i = 0$$

Auxillary eqn

$$D^2 + 2.828D + 2 = 0$$

$$m_1, m_2 = -1.414, -1.414$$

The roots are real and equal

∴ General solution is

$$i(t) = (C_1 + C_2 t)e^{m_1 t}$$

$$= (C_1 + C_2 t)e^{-1.414t} \quad \text{——} (2)$$

At initial conditions

at $t = 0$, $i(t) = 0$

from eq① $C_1 = 0$ $\quad i(t) = C_2 t e^{-1.414t}$

$$\frac{di}{dt} = C_2 \left[t(-1.414) e^{-1.414t} + e^{-1.414t}(1) \right]$$

$$\frac{di}{dt} = C_2 e^{-1.414t} \quad [+\,+1]$$

At $t = 0$ $\quad \frac{di}{dt} = \frac{V}{L}$

$$\frac{V}{L} = C_2 e^{0} [0 + 1]$$

$$C_2 = \frac{V}{L}$$

∴ final value solution

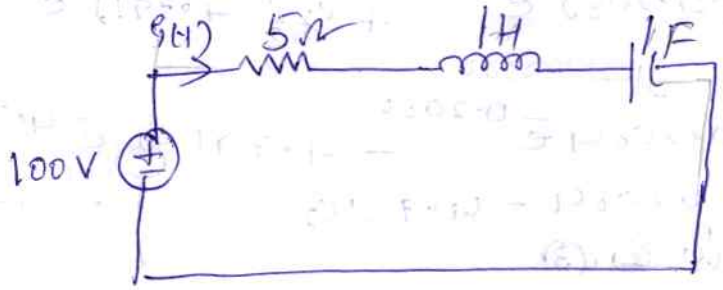
$$i(t) = \left[\frac{V}{L} t \right] e^{-1.414t}$$

$$i(t) = 200t\, e^{-1.414t} \quad \text{——} (3)$$

18.09.2025

Problem

$V = 100V$, $R = 5\Omega$, $L = 1H$ and $C = 1F$ Series RLC circuit find the expression for $i(t)$.



$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \quad \text{——} (1)$$

$$100 = 5i + \frac{di}{dt} + \int i\,dt$$

Apply differentiation

$$\frac{d}{dt}(100) = 5\frac{di}{dt} + \frac{di}{dt}\left[\frac{di}{dt}\right] + \frac{d}{dt}\int i\,dt$$

$$0 = 5\frac{di}{dt} + \frac{d^2i}{dt^2} + i$$

Auxillary equation

$$D^2 + 5D + 1 = 0$$

$$m_1 = -0.208, \quad m_2 = -4.791$$

The roots are real and unequal

∴ General solution

$$i(t) = c_1 e^{m_1 t} + c_2 e^{m_2 t}$$

$$= c_1 e^{-0.208t} + c_2 e^{-4.791t} \longrightarrow (1)$$

At initial condition:

At $t = 0$ $i(t) = 0$

At $t = 0$ $\frac{di}{dt} = \frac{V}{L} = \frac{100}{1} = 100$

$$i(t) = c_1 e^{-0.208t} + c_2 e^{-4.791t}$$

$$i(0) = c_1 e^0 + c_2 e^0$$

$$0 = c_1 + c_2 \longrightarrow (3)$$

$$\frac{di}{dt} = c_1(-0.208) e^{-0.208t} + c_2(-4.791) e^{-4.791t}$$

$$100 = -0.208 c_1 e^{-0.208t} - 4.791 c_2 e^{-4.791t}$$

$$100 = -0.208 c_1 - 4.791 c_2 \longrightarrow (4)$$

from eqn (4), (3)

$$c_1 + c_2 = 0$$

$$-0.208 c_1 - 4.791 c_2 = 100$$

$$c_1 = 21.819, \quad c_2 = -21.819$$

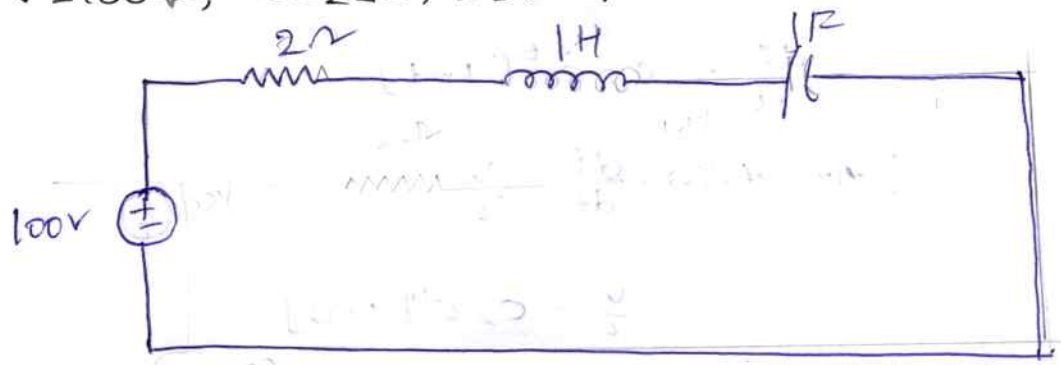
∴ Final solution

$$i(t) = 21.819 e^{-0.208t} + (-21.819) e^{-4.791t}$$

$$= 21.819 [e^{-0.208t} - e^{-4.791t}]$$

Problem

$V = 100V$, $R = 2\Omega$, $L = 1H$, $C = 1F$



$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \quad \text{——} (1)$$

$$100 = 2i + \frac{di}{dt} + \int i\,dt$$

Apply differentiation

$$\frac{d}{dt}100 = \frac{d}{dt}(2i) + \frac{d}{dt}\left(\frac{di}{dt}\right) + \frac{d}{dt}\int i\,dt$$

$$0 = 2\frac{di}{dt} + \frac{d^2i}{dt^2} + i$$

$$\frac{d^2i}{dt^2} + 2\frac{di}{dt} + i = 0$$

Auxillary eqn

$$D^2 + 2D + 1 = 0$$

$$m_1 = m_2 = -1$$

The roots are real and equal

∴ General solution

$$i(t) = (C_1 + C_2 t)e^{m_1 t}$$

$$= (C_1 + C_2 t)e^{-t} \quad \text{——} (2)$$

At initial conditions

At $t = 0$ $i(t) = 0$

At $t = 0$ $\frac{di}{dt} = \frac{V}{L} = \frac{100}{1} = 100$

$$\frac{di}{dt} = c_2[-t - 1e^{-t} + e^{-t}]$$

$$\frac{di}{dt} = c_2 e^{-t}[1-t]$$

At $t = 0$ $\frac{di}{dt} = \frac{V}{L}$

$$\frac{V}{L} = c_2 e^{0}[1-0]$$

$$\frac{V}{L} = c_2$$

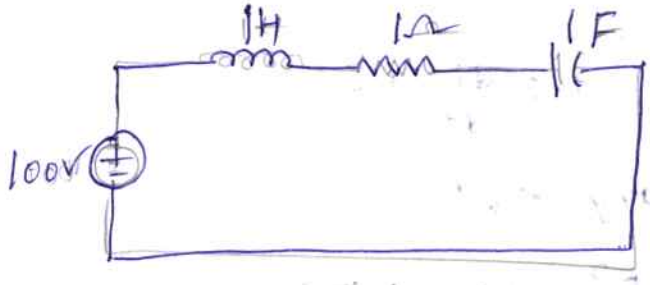
∴ final solution

$$i(t) = \frac{V}{L} t e^{-t}$$

$$\boxed{i(t) = 100 t e^{-t}}$$

Problem

$V = 100V$, $R = 1\Omega$, $L = 1H$, $C = 1F$



$$V = Ri + L\frac{di}{dt} + \frac{1}{c}\int i\,dt \quad \text{——} (1)$$

$$100 = i + \frac{di}{dt} + \int i\,dt$$

Apply differentiation

$$0 = \frac{di}{dt} + \frac{d^2i}{dt} + i$$

$$\frac{d^2i}{dt} + \frac{di}{dt} + i = 0$$

Auxillary eqn

$$D^2 + D + 1 = 0$$

$$m_1, m_2 = -0.5 + 0.866i$$

∴ roots are complex conjugate

∴ general solution

$$i(t) = e^{-0.5t}[C_1 \cos[0.866]t + C_2 \sin(0.866]t]$$

(2)

At initial conditions

At $t=0$, $i(t)=0$

At $t=0$, $\frac{di}{dt} = \frac{V}{L} = \frac{100}{1} = 100$

$$i(t) = e^{-0.5t}[C_1 \cos 0.866t + C_2 \sin 0.866t]$$

$$i(0) = e^{-0.5(0)}[C_1 \cos 0 + C_2 \sin 0]$$

$$0 = 1[C_1 + 0]$$

$$\boxed{C_1 = 0}$$

$$\frac{di(t)}{dt} = e^{-0.5t}[C_2 \sin(0.866]t]$$

$$= C_2[\sin 0.866t[-.05]e^{-0.5t} + e^{-0.5t}(0.866]\cos 0.866t]$$

$$= C_2 e^{-0.5t}[-0.5 \sin 0.866t + 0.866 \cos 0.866t]$$

At $t=0$ $\frac{di}{dt} = \frac{V}{L} = 100$

$$\frac{V}{L} = C_2 e^0[-0.5 \sin 0 + 0.866 \cos 0]$$

$$\frac{V}{L} = C_2 \cdot [0.866]$$

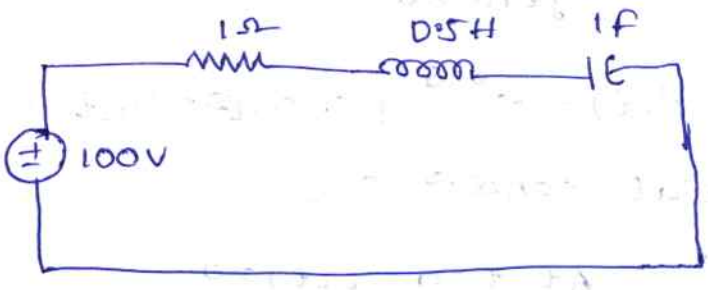
$$\frac{100}{0.866} = C_2 \quad f_2 = 115.473$$

∴ final solution

$$i(t) = e^{-0.5t}[115.473 \sin 0.866t]$$

19.09.2025

find the expression for the circuit in the series RLC circuit shown below.





$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

$$100 = i + 0.5\frac{di}{dt} + \int i\,dt \longrightarrow (1)$$

Differentiate

$$\frac{d}{dt}(100) = \frac{d}{dt}(i) + 0.5\frac{d}{dt}\left(\frac{di}{dt}\right) + \frac{d}{dt}\int i\,dt$$

$$0 = \frac{di}{dt} + 0.5\frac{d^2i}{dt^2} + i$$

$$\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 2i = 0$$

Auxillary eqn is

$$D^2 + 2D + 2 = 0$$

$$m_1, m_2 = -1 \pm j$$

The roots are complex conjugate

General solution

$$i(t) = e^{-t}[C_1\cos t + C_2 \sin t] \longrightarrow (2)$$

$\longrightarrow (2)$

At initial condition:

$$\text{At } t=0,\ i(t)=0$$

$$\text{At } t=0,\ \frac{di}{dt}=\frac{V}{L}=200$$

from eqn (2) At $t=0$

$$i(t)=e^{0}(C_1\cos 0+C_2\sin 0)$$

$$\therefore C_1=0$$

differentiate eq (2)

$$i(t)=e^{at}\,C_2\sin bt \quad [\because C_1=0]$$

$$\frac{di}{dt}=C_2\left[e^{at}(\cos bt)\,b+e^{at}(\sin bt)\,a\right]$$

$$\text{At } t=0 \quad \frac{di}{dt}=C_2[b]$$

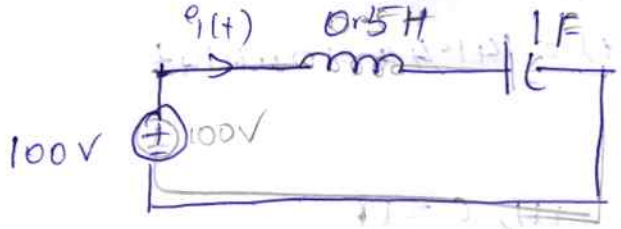
$$200=C_2\,b$$

$$\boxed{C_2=200}$$

$\therefore$ final solution

$$\boxed{i(t)=e^{-t}[200\sin t]}$$

Find the expression for current in given circuit



$$V=L\frac{di}{dt}+\frac{1}{C}\int i\,dt$$

$$100=0.5\frac{di}{dt}+\int i\,dt$$

$$\frac{d}{dt}(100)=0.5\,\frac{d}{dt}\left[\frac{di}{dt}\right]+\frac{d}{dt}\int i\,dt$$

$$\frac{d^2i}{dt^2} + 2i = 0$$

$$D^2 + 2 = 0$$

$$m_1, m_2 = \pm 1.414i$$

∴ The roots are pure imaginary

$$i(t) = c_1 \cos bt + c_2 \sin bt$$

$$= c_1 \cos 1.414t + c_2 \sin 1.414t$$

At initial condition;

At $t = 0$ $i(t) = 0 \Rightarrow c_1 = 0$

At $t = 0$ $\frac{di}{dt} = \frac{V}{L} = 200$

$$\frac{di}{dt} = c_2 [1.414 \cos 1.414t]$$

$$200 = c_2 [1.414] \quad [\because \cos 0 = 1]$$

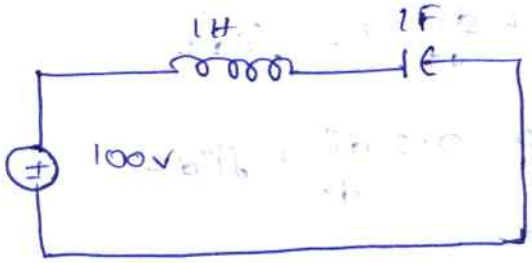
$$c_2 = \frac{200}{1.414}$$

$$c_2 = 141.44$$

∴ final solution

$$i(t) = 141.44 \sin 1.414t$$

$V = 100V$, $L = 1H$, $C = 1F$



$$V = L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Differentiate

$$\frac{d^2i}{dt^2} + i^0 = 0$$

Auxiliary eqn

$$D^2 + 1 = 0$$

$$m_1, m_2 = \pm i^0$$

∴ roots are pure imaginary

$$i^0(t) = c_1 \cos bt + c_2 \sin bt$$

$$i^0(t) = c_1 \cos t + c_2 \sin t$$

At initial condition:

$$At\ t=0 \quad i(t) = 0 \Rightarrow c_1 = 0$$

$$At\ t=0 \Rightarrow \frac{di^0}{dt} = \frac{V}{L} = 100$$

$$\frac{di^0}{dt} = c_2 \cos t$$

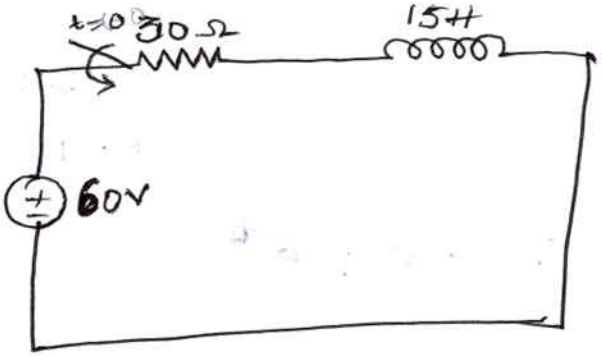
$$\frac{V}{L} = c_2 \cos(0)$$

$$c_2 = 100$$

∴ final solution

$$i(t) = 100 \sin t$$

find the expression for current i(t) for the given circuit for series RL circuit when initial current through inductor is 3A





$$V = Ri + L\frac{di}{dt}$$

$$60 = 30i + 15\frac{di}{dt}$$

$$\frac{di}{dt} + 2i = 4$$

General solution

$$i(t) = ke^{-Pt} + e^{-Pt}\int e^{Pt} \cdot Q\, dt$$

$$= ke^{-2t} + e^{-2t}\int e^{2t} 4\, dt$$

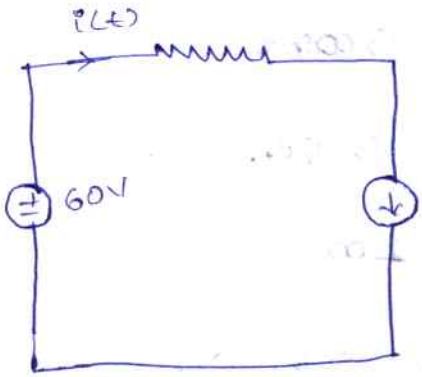
$$= ke^{-2t} + 4e^{-2t}\frac{e^{2t}}{2}$$

$$= ke^{-2t} + 2e^{0}$$

$$i(t) = ke^{-2t} + 2 \longrightarrow (1)$$

Initial condition

At $t = 0^+$



Zero initial condition

At $t = 0, i(t) = 0$

With initial current I_0

At $t = 0, i(t) = I_0$

$i(t) = 3A$

At, $i(t) = 3A$

$t = 0$

from eq (1)

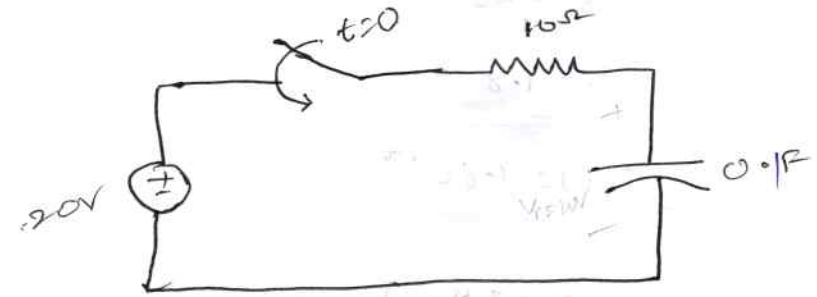
At $t = 0$ $\quad i(t) = 2 + ke^{0} = 3$

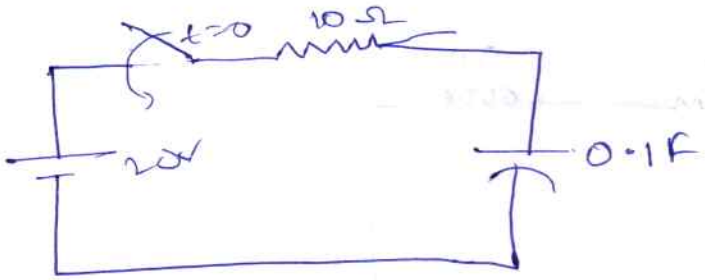
$k = 3 - 2$

$k = 1$

$\therefore i(t) = 2 + e^{-2t}$

Assume the initial voltage across capacitor is 2V and find the current after switch is closed in the given circuit.





$$V = Ri + \frac{1}{C}\int i\,dt$$

$$20 = 10i + \frac{1}{0.1}\int i\,dt$$

$$10i + 10\int i\,dt = 20$$

$$i + \int i\,dt = 2$$

Differentiate

$$\frac{di}{dt} + i = 0$$

General solution

$$i(t) = Ke^{Pt} + e^{-Pt}\int e^{Pt}\,\phi\,dt$$

$$i(t) = Ke^{-t} \longrightarrow (1)$$

Zero initial condition

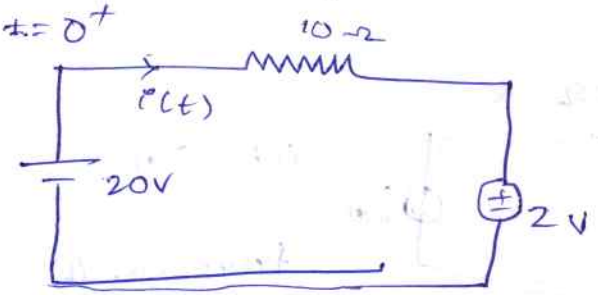
At $t=0$, $i(t) = \frac{V}{R}$

With initial voltage

At $t=0$, $i(t) = \frac{V - V_0}{R}$

Initial condition:

At $t = 0^+$



At $t=0$ $i(t) = \frac{V - V_0}{R}$

$= \frac{20-2}{10}$

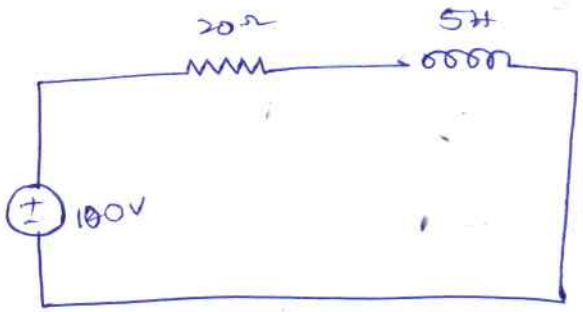
At $t=0$ from eqn (1)

$$i^o(t) = ke^0$$

$$k = 1.8$$

$$\therefore \quad i^o(t) = 1.8e^{-Pt}$$

$V = 100V \quad R = 20\Omega \quad L = 5H, \; I_0 = 2A$



$$V = Ri + L\frac{di}{dt}$$

$$100 = 20i + 5\frac{di}{dt}$$

$$\frac{di}{dt} + 4i = 20$$

General solution

$$i(t) = ke^{-Pt} + e^{-Pt}\int e^{Pt} Q\, dt$$

$$= ke^{-4t} + e^{-4t}\int e^{4t} 20\, dt$$

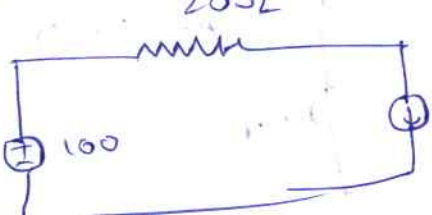
$$= ke^{-4t} + 20e^{-4t} \times \frac{e^{4t}}{4}$$

$$= ke^{-4t} + 5e^0$$

$$i(t) = ke^{-4t} + 5$$

initial condition

At $t = 0^+$



$i(t) = 2A$

from eqn (b)

~~K = 5~~

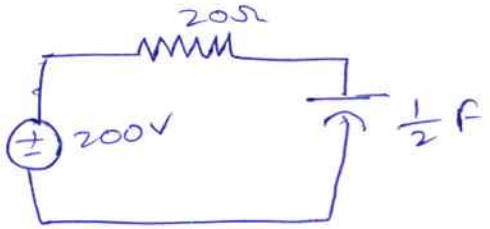
$K + 5 = 2$

$K = -3$

$$i(t) = -3e^{-4t} + 5$$

RC:

$V = 200V$, $R = 20\Omega$, $C = \frac{1}{2}F$, $V_0 = 10$



$$V = Ri + \frac{1}{C}\int i\,dt$$

$$20i + \frac{1}{0.5}\int i\,dt = 200$$

$$20i + 2\int i\,dt = 200$$

$$i + 0.1\int i\,dt = 10$$

Differentiate

$$\frac{di}{dt} + 0.1i = 0$$

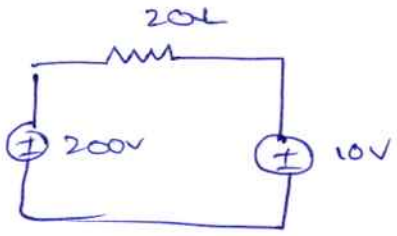
$$s = -0.1$$

$$i(t) = Ke^{-Pt} + e^{-Pt}\int e^{Pt}\,\phi\,dt$$

$$i(t) = Ke^{-0.1t}$$

Initial condition

$t = 0^+$



At $t = 0$ $i(t) = \frac{V - V_0}{R}$

$= \frac{200 - 10}{20}$

At $t=0$, from eq ①

$$v(t) = Ke^{0}$$

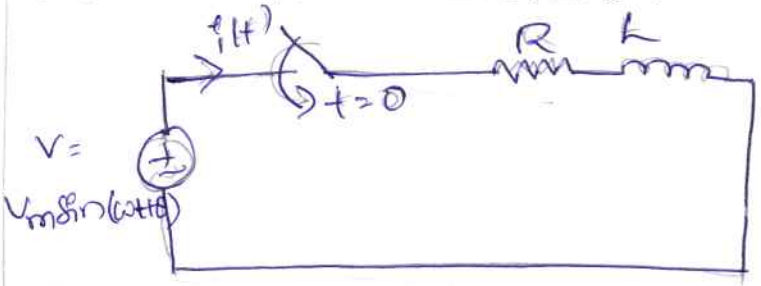
$$K = 9.5$$

$$v(t) = 9.5e^{-0.1t}$$

23.09.2025

AC Transients:

Series RL circuit:



$$V_m \sin(\omega t+\theta) = Ri + L\frac{di}{dt}$$

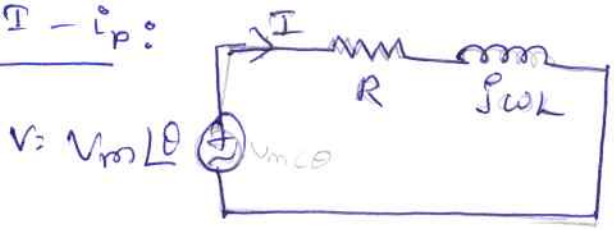
$$\frac{di}{dt} + \frac{Ri}{L} = \frac{V_m}{L}\sin(\omega t+\theta)$$

General solution $i(t) = CF + PI = i_c + i_p$ → (1)

$$CF = i_c = ke^{-Pt} = ke^{-\frac{R}{L}t} \rightarrow (2)$$

~~[illegible]~~ ×

P.I – i_p:



$$Z = R + j\omega L$$
$$= |Z| \angle\phi$$

$$I = \frac{V}{Z} = \frac{V_m \angle\theta}{|Z| \angle\phi}$$

$$I = \frac{V_m}{|Z|} \angle\theta - \phi$$

$$i_p = \frac{V_m}{|Z|}\sin(\omega t + \theta - \phi) \rightarrow (3)$$

General solution

$$i(t) = i_c + i_p$$

$$i(t) = ke^{-\frac{R}{L}t} + \frac{V_m}{|Z|}\sin(\omega t + \theta - \phi)$$

Source:

DC source
V
AC source

$V = V_m \sin(\omega t+\theta)$

$V_m \sin(\omega t+\theta) = V_m \angle\theta$

frequency → ω

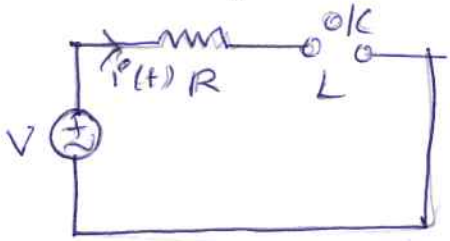
$Z_R = R$

$Z_L = j\omega L$

$Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$

Zero initial condition:

At $t = 0^+$



At $t=0$ from eq (4)

$$i(t) = ke^0 + \frac{V_m}{|Z|}\sin(0+\theta-\phi)$$

$$0 = K + \frac{V_m}{|Z|}\sin(\theta-\phi)$$

$$\therefore K = -\frac{V_m}{|Z|}\sin(\theta-\phi)$$

∴ final solution

$$i(t) = -\frac{V_m}{|Z|}\sin(\theta-\phi)\, e^{-\frac{R}{L}t} + \frac{V_m}{|Z|}\sin(\omega t + \theta - \phi)$$

$$i_{ts} = \frac{-V_m}{|Z|}\sin(\theta-\phi)\, e^{-\frac{R}{L}t}$$

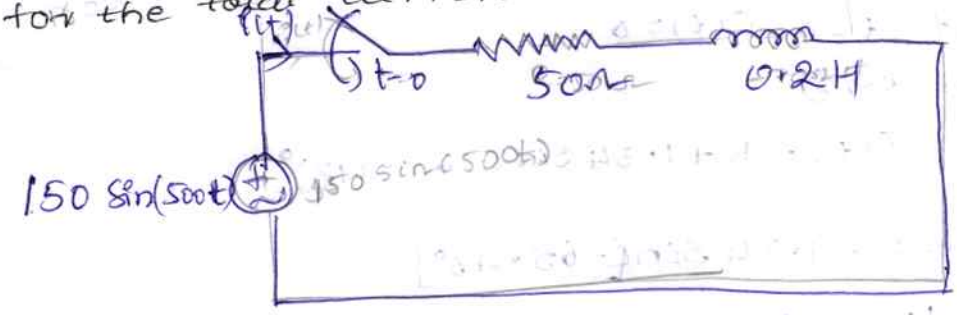
$$\sin(\theta-\phi) = 0$$

$$\theta - \phi = n\pi \quad n = 0, 1, 2, 3, 4 \ldots\ldots \infty$$

$$\boxed{\theta = \phi + n\pi}$$

when $\theta = \phi + n\pi$ then there is no transient in the AC circuits.

Q Find the expression for i(t) in the given circuit below. A series RL circuit with R = 50Ω and L = 0.2H has a sinusoidal voltage source $v = 150\sin[500t + \phi]$ applied at a time when $\phi = 0$. Find the expression for the total current. non-initial At $t=0$, $i(t) = I_0$



$\omega = 500$ rad/sec

$$V = Ri + L\frac{di}{dt}$$

$$150\sin(500t) = 50i + 0.2\frac{di}{dt}$$

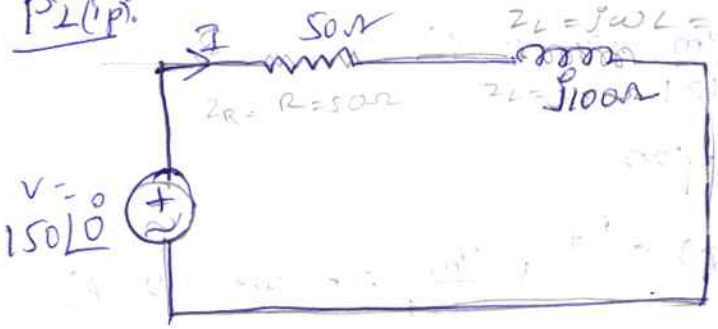
$$0.2\frac{di}{dt} + 50i = 150\sin(500t)$$

$$\frac{di}{dt} + 250i = 750\sin(500t]$$

$$\therefore C.F = i_c = Ke^{-250t}$$

→ (1)

PI(i_p):



$Z = Z_R + Z_L = (50 + j100)\Omega$

$150 \sin(500t) = 150\angle 0°$

$$I = \frac{V}{Z} = \frac{150\angle 0°}{50 + j100}$$

$$= 1.34 \angle -63.43°$$

$\therefore i_p = 1.34 \sin(500t - 63.43°)$

$\therefore$ General solution

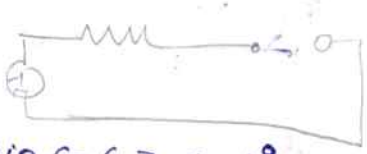
$i(t) = i_c + i_p$

$i(t) = K e^{-250t} + 1.34 \sin(500t - 63.43°)$

initial condition:

Assume zero initial condition

At $t = 0$ $i(t) = 0$



from eqn (2)

$i(t) = K + 1.34 \sin(-63.43)°$

$K = -1.34 \sin[-65.43°]$

$K = 1.197$

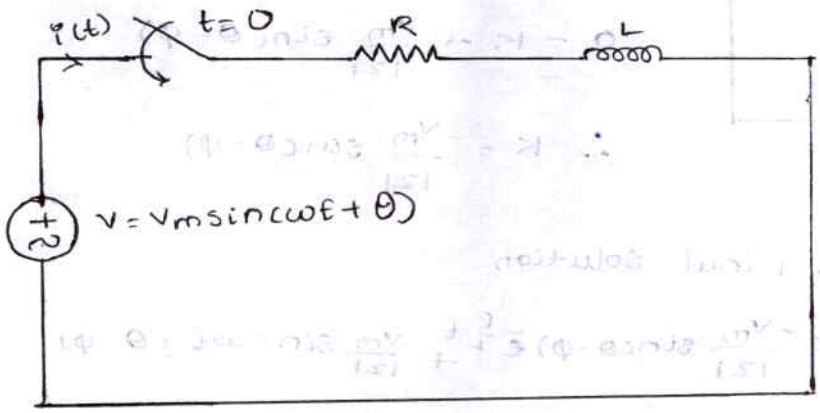
$\therefore$ Final solution

$i(t) = 1.197\, e^{-250t} + 1.34 \sin(500t - 63.43)$

23.09.2025

AC TRANSIENTS :

SERIES RL CIRCUIT FOR AC TRANSIENTS :



$$V = Ri + L\frac{di}{dt}$$

$$V_m \sin(\omega t + \theta) = Ri + L\frac{di}{dt}$$

$$L\frac{di}{dt} + Ri = V_m \sin(\omega t + \theta)$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V_m}{L}\sin(\omega t + \theta)$$

(P) (Q)

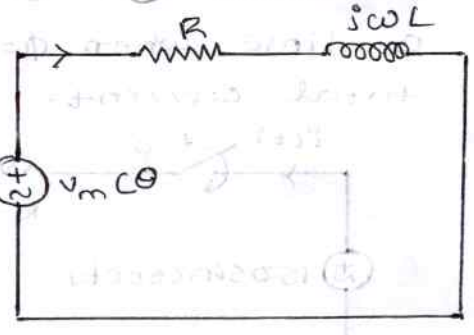
General solution

$$i(t) = C.F + P.I = i_c + i_p \longrightarrow (1)$$

$$C.F = i_c = ke^{-Pt}$$

$$i_c = ke^{-\frac{R}{L}t} \longrightarrow (2)$$

P.I



$$Z = R + j\omega L$$

$$= |Z| \angle \phi$$

$$I = \frac{V}{Z} = \frac{V_m \angle \theta}{|Z| \angle \phi}$$

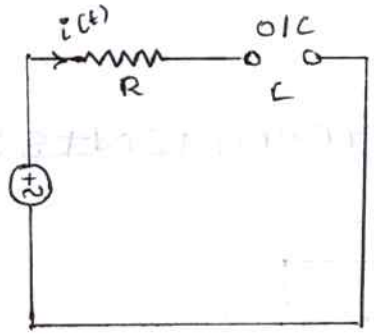
∴ General solution

$$i(t) = i_c + i_p$$

$$\boxed{i(t) = K e^{-\frac{R}{L}t} + \frac{V_m}{|Z|}\sin(\omega t + \theta - \phi)} \rightarrow (4)$$

Zero initial Condition:

At $t = 0^+$



At $t = 0$ from eqn (4)

$$i(t) = Ke^0 + \frac{V_m}{|Z|}\sin(0 + \theta - \phi)$$

$$0 = K + \frac{V_m}{|Z|}\sin(\theta - \phi)$$

$$\therefore K = -\frac{V_m}{|Z|}\sin(\theta - \phi)$$

∴ Final solution

$$i(t) = -\frac{V_m}{|Z|}\sin(\theta - \phi)\, e^{-\frac{R}{L}t} + \frac{V_m}{|Z|}\sin(\omega t + \theta - \phi)$$

$$i_{ts} = -\frac{V_m}{|Z|}\sin(\theta - \phi)\, e^{-\frac{R}{L}t}$$

$$\sin(\theta - \phi) = 0$$

$$\theta - \phi = n\pi, \quad n = 0, 1, 2, 3, 4 \ldots \infty$$

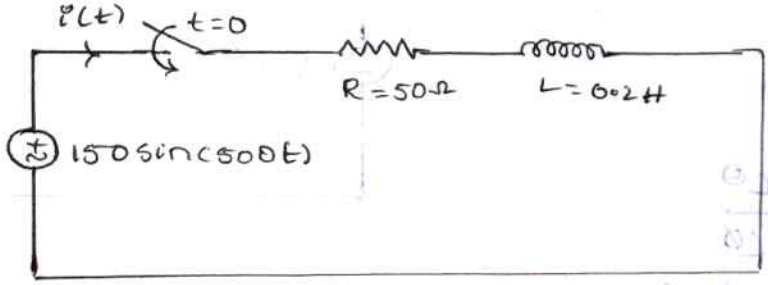
$$\theta = \phi + n\pi$$

when $\theta = \phi + n\pi$ then there is no transient in the AC circuits.

NOTE:

At non-initial condition $t = 0$, $i(t) = I_0$

Q A series RL circuit with $R = 50\Omega$ and $L = 0.2H$ has a sinusoidal voltage source of $v = 150\sin(500t + \phi)$ applied at a time when $\phi = 0$. Find the expression for the total current.



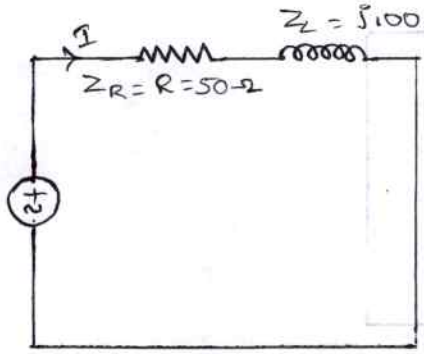
$\omega = 500$ rad/sec

$$V = Ri + L\frac{di}{dt}$$

complementary function

$$C.F = i_c = k e^{-250t} \quad \rightarrow (1)$$

Particular Integral



$$Z = Z_R + Z_L$$

$$= 50 + j100\Omega$$

$$150\sin(500t) = 150\angle 0^\circ$$

$$I = \frac{V}{Z}$$

$$I = \frac{150\angle 0^\circ}{50 + j100}$$

$$I = 1.34\angle -63.43^\circ$$

$$\therefore i_p = 1.34\sin(500t - 63.43)$$

$\therefore$ General solution

$$i(t) = i_c + i_p$$

$$i(t) = k e^{-250t} + 1.345\sin(500t - 63.43^\circ)$$

Initial condition:

Assume zero initial condition

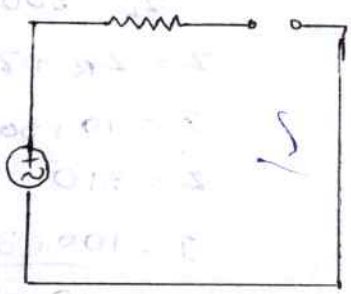
At $t = 0$, $i(t) = 0$

from eq ②

$$i(t) = K + 1.345\sin(-63.43)$$

$$K = -1.345\sin(-63.43)$$

$$K = 1.197$$

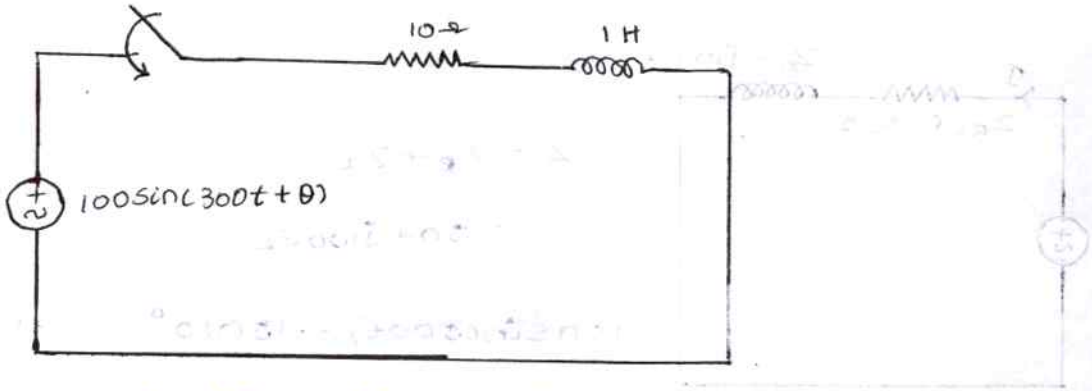


$\therefore$ final solution

$$i(t) = 1.197 e^{-250t} + 1.34\sin(500t - 63.43)$$

24.09.2025

In the given circuit the switch is closed at $\theta = 30°$. Find the expression for the current. Assume 0.1A current is following through the inductor before the switch closed.

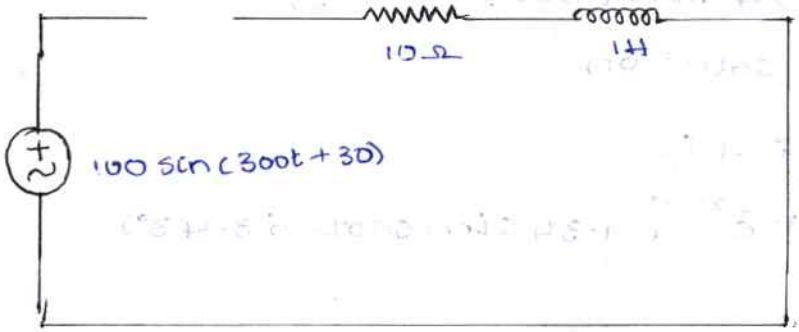


$$V = Ri + L\frac{di}{dt}$$

$$100\sin(300t + 30) = 10i + \frac{di}{dt}$$

$$\frac{di}{dt} + 10i = 100\sin(300t + 30°)$$

$$\therefore C.F = i_c = ke^{-10t}$$



$$RI = i_p$$

$$Z_R = 10$$

$$Z_L = 300$$

$$Z = Z_R + Z_L$$

$$Z = 10 + 300$$

$$Z = 310$$

$$I = \frac{100\angle 30°}{10 + j300}$$

$$I = 0.33\angle -58.04$$

$$i_p = 0.33\sin(300t - 58.04)$$

$\therefore$ General Solution

$$i(t) = i_c + i_p$$

Assume initial condition

$\therefore$ At $t=0, i(t)=0$

from eq ②

At $t=0$

$i(t) = K + 0.33 \sin(-58.09)$

$K = -0.33 \sin(-58.09) + 0.1$

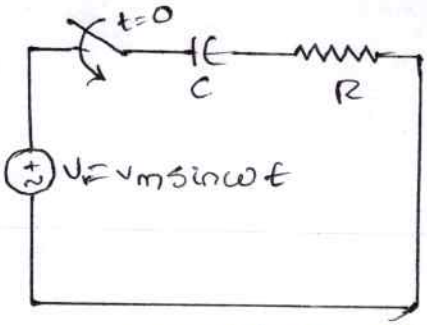
$K = 0.28 + 0.1$

$K = 0.38$

$\therefore$ final solution

$i(t) = 0.38e^{-10t} + 0.33 \sin(300t - 58.09)$

AC TRANSIENT RESPONSE OF SERIES RC CIRCUIT:



consider the series RC circuit connected to an ac voltage source with a switch

$$V = Ri + \frac{1}{C}\int i\,dt$$

$$V_m \sin\omega t = Ri + \frac{1}{C}\int i\,dt$$

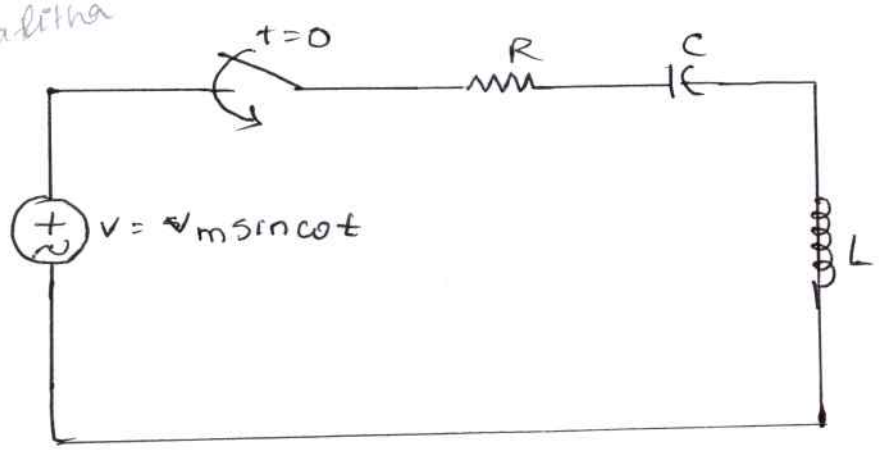
Differentiate on both side

$$V_m \omega \cos\omega t = R\frac{di}{dt} + \frac{i}{C}$$

R

AC TRANSIENT RESPONSE OF SERIES RCL CIRCUIT:

Padmavathi M



consider

$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

$$V_m \sin(\omega t+\theta) = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

$$\frac{d}{dt}\left(V_m \sin(\omega t+\theta)\right) = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C}$$

$$V_m \omega \sin(\omega t+\theta) = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C}$$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = V_m \omega \cos(\omega t+\theta)$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{V_m}{L}\omega\cos(\omega t+\theta)$$

$\therefore$ General solution $i = i_c + i_p$

C.F - i_c :

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

auxillary eqn

$$D^2 + \frac{R}{L}D + \frac{1}{LC} = 0$$

$$m_1 m_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{-\frac{R}{2L} \pm 2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Case-1: roots are real and unequal

$$i_c = C_1 e^{m_1 t} + C_2 e^{m_2 t}$$

Case-2: roots are real

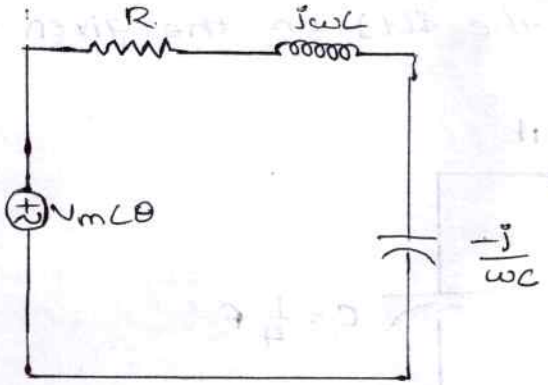
Case-3: roots are complex conjucate

$$i_c = e^{at}(C_1 \cos bt + C_2 \sin bt)$$

Case-4: roots are pure imaginary

26.09.2025.

Particular Integral P.I = i_p :



$$V = V_m \sin(\omega t + \theta)$$
$$= V_m \angle\theta$$

$$Z = R + j\omega L - \frac{j}{\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= |Z| \angle\phi$$

$$I = \frac{V}{Z} = \frac{V_m \angle\theta}{|Z| \angle\phi}$$

$$= \frac{V_m}{|Z|} \angle\theta - \phi$$

$$i_p = \frac{V_m}{|Z|} \sin(\omega t + \theta - \phi)$$

∴ General solution

$$i(t) = i_c + i_p$$

Initial conditions :

case-1 : zero initial condition

$$I_0 = 0, \quad V_0 = 0$$

At $t=0$, $i(t) = 0$

At $t=0$, $\frac{di}{dt} = \frac{V}{L}$.

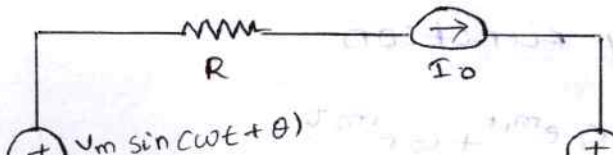
$$= \frac{V_m \sin(\omega t + \theta)}{L}$$

$$= \frac{V_m \sin\theta}{L}$$

case-2 : Non- initial conditions

At $t=0$, $i(t) = I_0$, $v(t) = V_0$

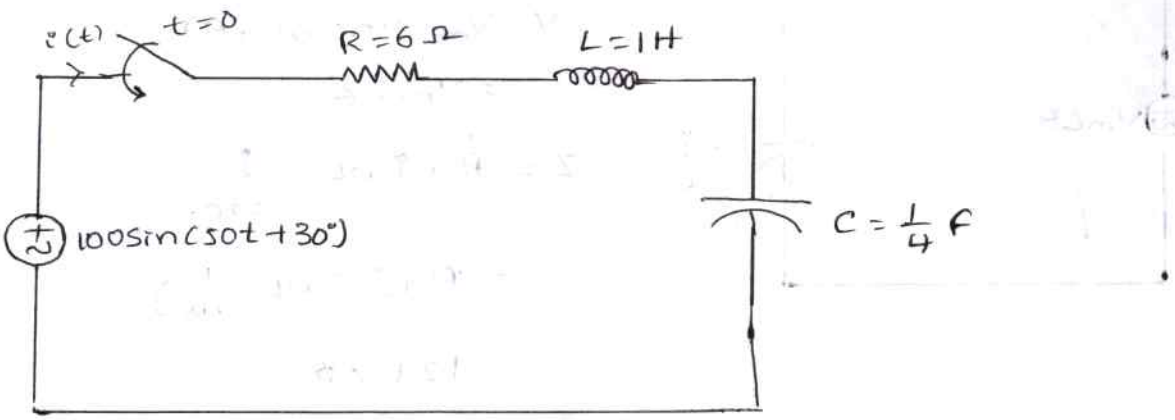
At $t=0$, $i(t) = I_0$



$$V_m \sin\theta = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Using the initial conditions: evaluate C_1, C_2 in the general solution to get the final solution

Q. Find the expression for the $i(t)$ in the given circuit



Given data

$R = 6\Omega,\ L = 1H,\ C = \frac{1}{4}F$

$V = 100\sin(50t + 30°)$

$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

$$100\sin(50t+30°) = 6i + \frac{di}{dt} + 4\int i\,dt$$

Differentiate on both sides

$$100\frac{d}{dt}(\sin(50t+30)] = 6\frac{di}{dt} + \frac{d}{dt}\left[\frac{di}{dt}\right] + 4\frac{d}{dt}\left[\int i\,dt\right]$$

$$\frac{d^2i}{dt^2} + 6\frac{di}{dt} + 4i = 100\cos(50t+30)\cdot 50$$

$$\frac{d2i}{dt^2} + 6\frac{di}{dt} + 4i = 5000\cos[50t+30]$$

The auxiliary equation is

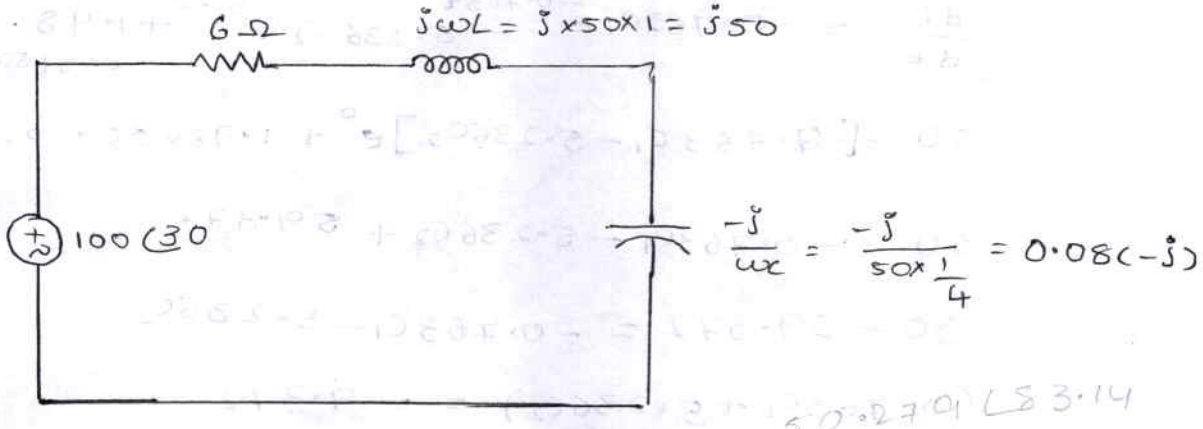
$$D^2 + 6D + 4 = 0$$

$m_1, m_2 = -0.763, -5.236$ [∵ roots are real, unequal]

Complimentary function

$$C.F = i_c = C_1 e^{m_1 t} + C_2 e^{m_2 t}$$

Particular integral P.F = i_p



$Z = Z_R + Z_L + Z_C$

$= 6 + j50 - j(0.08)$

$= 6 + j(50 - 0.08)$

$Z = 6 + j(49.92) = 50.279 \angle 83.14$

$I = \frac{V}{Z} = \frac{100 \angle 30}{50.279 \angle 83.14}$

$= 1.98 \angle -53.14$

$i_p = 1.98 \sin[50t - 53.14]$

∴ General solution

$i(t) = i_c + i_p$

$i(t) = c_1 e^{-0.763t} + c_2 e^{-5.236t} + 1.98 \sin[50t - 53.14]$ → ②

Initial condition:

case: zero initial condition.

At $t = 0$, $i(t) = 0$

At $t = 0$, $\frac{di}{dt} = \frac{V_m \sin\theta}{L} = \frac{100 \sin 30}{1} = 50$

from eq ②

$i(t) = c_1 e^0 + c_2 e^0 + 1.98 \sin(0 - 53.15)$

$0 = c_1 + c_2 + 1.98 \sin(-53.15)$

$c_1 + c_2 = -1.98 \sin(-53.15)$

$c_1 + c_2 = 1.58$

Differentiate eq ②

$$\frac{di}{dt} = -0.763C_1 e^{-0.763t} - 5.236C_2 e^{-5.236t} + 1.98 \cdot \cos[50t - 53.15]50$$

$$50 = [-0.763C_1 - 5.236C_2]e^{0} + 1.98 \times 50 \times \cos[53.15]$$

$$50 = -0.763C_1 - 5.236C_2 + 59.372$$

$$50 - 59.372 = -0.763C_1 - 5.236C_2$$

$$-(0.763C_1 + 5.236C_2) = -9.372$$

$$0.763C_1 + 5.236C_2 = 9.372$$

$$C_1 = -0.245e^{-0.763t} + 1.829e^{-5.236t}\sin(50t - 53.15^\circ)$$

9 Find the expression for the i(t) in the given circuit.

Given data

$R = 4\Omega$, $L = 1H$, $C = \frac{1}{4}F$

$V = 100\sin(50t + 30^\circ)$

$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

$$100\sin(50t + 30^\circ) = 4i + \frac{di}{dt} + 4\int i\,dt$$

Differentiate on both sides

$$\frac{d}{dt}[100\sin(50t + 30)] = \frac{d}{dt}[4i] + \frac{d}{dt}\left[\frac{di}{dt}\right] + \frac{d}{dt}\left[4\int i\,dt\right]$$

$$\frac{d^2i}{dt^2} + 4\frac{di}{dt} + 4i = 100 \times 50 \times \cos(50t + 30)$$

$$\frac{d^2i}{dt^2} + 4\frac{di}{dt} + 4i = 5000\cos(50t + 30)$$

Auxillary Equation

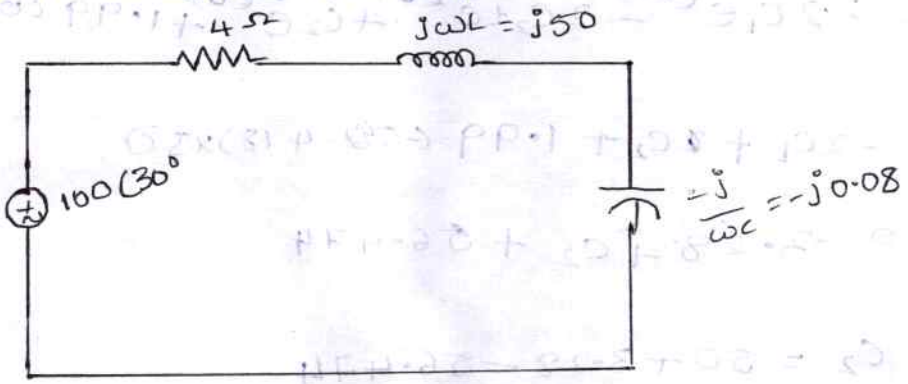
The roots are real and equal

complimentary function

$$c.f = i_c = (C_1 + C_2 t)\, e^{m_1 t}$$

$$\boxed{i_c = (C_1 + C_2 t)\, e^{-2t}}$$

Particular integral $p.f = i_p$ → (1)



$$Z = Z_R + Z_L + Z_C \qquad V = 100\angle 30^\circ$$

$$= 4 + j50 - j0.08$$

$$= 4 + j(50 - 0.08)$$

$$Z = 4 + 49.92j$$

$$Z = 50.08\angle 85.418^\circ$$

$$I = \frac{V}{Z}$$

$$= \frac{100\angle 30^\circ}{50.08\angle 85.418^\circ}$$

$$= 1.996\angle -55.418 \qquad i_p = 1.99 \sin(50t - 55.418)$$

General solution

$$i(t) = i_c + i_p$$

$$\boxed{i(t) = (C_1 + C_2 t)e^{-2t} + 1.99 \sin(50t - 55.418)}$$

Initial condition:

At $t = 0$, $i(t) = 0$

At $t = 0$, $\frac{di}{dt} = \frac{V_m \sin\theta}{L} = \frac{100 \sin 30}{1} = 50$

from eq (2)

Differentiate eqn ②

$$\frac{di}{dt} = -2(c_1 + c_2 t)e^{-2t} + 1.99\cos(50t - 55.418)\cdot 50$$

$$50 = (c_1 + c_2 t)(-2e^{-2t}) + c_2 e^{-2t} + 1.99\cos(50t - 55.418) \times 50$$

$$50 = -2c_1 e^{-(2\times 0)} - 2c_2 t e^{-(2\times 0)} + c_2 e^{-(2\times 0)} + 1.99\cos(50(0) - 55.418) \times 50$$

$$50 = -2c_1 + 2c_2 + [1.99(-55.418)\times 50]$$

$$50 = -3.28 + c_2 + 56.474$$

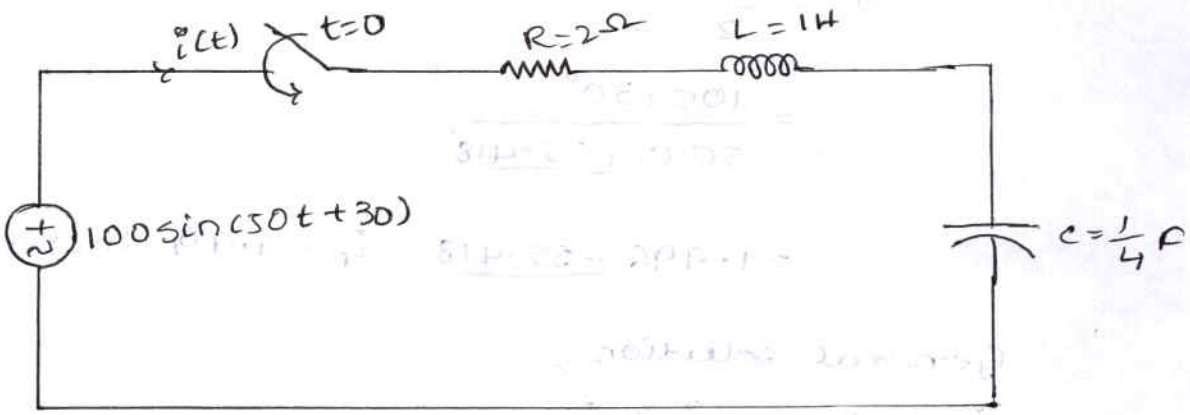
$$c_2 = 50 + 3.28 - 56.474$$

$$c_2 = -3.194$$

final solution

$$i(t) = (1.638 - 3.194t)e^{-2t} + 1.99\sin(50t - 55.418)]$$

Q. Find the expression for the $i(t)$ for the given circuit.



$$V = Ri + L\frac{di}{dt} + \frac{1}{c}\int i\,dt$$

$$100\sin(50t + 30) = 2i + \frac{di}{dt} + 4\int i\,dt$$

Differentiate on both sides

$$\frac{d}{dt}(2i) + \frac{d}{dt}\left[\frac{di}{dt}\right] + \frac{d}{dt}[4\int i\,dt] = \frac{d}{dt}[100\sin(50t+30)]$$

Auxillary equation

$$D^2 + 2D + 4 = 0$$

$$m_1, m_2 = -1 \pm 1.732j$$

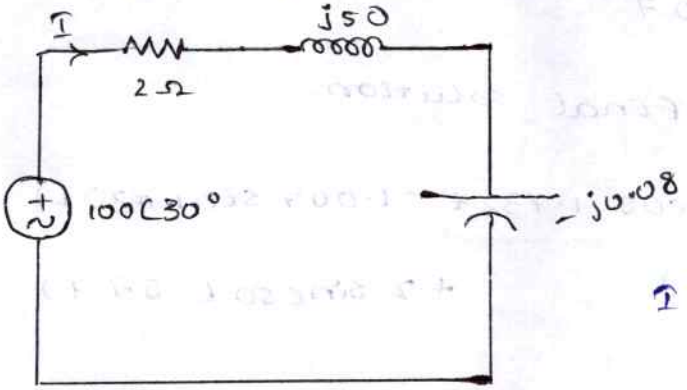
The roots are complex conjate

Complimentary function

$$C.F = i_c = e^{at}[C_1 \cos bt + C_2 \sin bt]$$

$$i_c = e^{-t}[C_1 \cos(1.732t) + C_2 \sin(1.732t)]$$

Particular integral $P.I = i_p$



$$\therefore Z = 2 + j50 - j0.08$$

$$= 2 + j49.92$$

$$I = \frac{V}{Z} = \frac{100\angle 30^\circ}{2 + j49.92}$$

$$= 2\angle -57.7 \text{ A}$$

$$\therefore i_p = 2 \sin(50t - 57.7)$$

∴ General solution

$$i(t) = i_c + i_p$$

$$i(t) = e^{-t}(C_1 \cos 1.732t + C_2 \sin 1.732t) + 2 \sin(50t - 57.7)$$

At initial conditions:

$$\text{At } t = 0,\ i(t) = 0$$

$$\text{At } t = 0,\ \frac{di}{dt} = \frac{V_m \sin\theta}{L} = 50$$

from eqn (2)

At $t = 0$

$$i(t) = e^0(C_1 \cos 0 + C_2 \sin 0) + 2 \sin(0 - 57.7)$$

$$C_1 - 1.67 = 0$$

$$C_1 = 1.67$$

$$\frac{di}{dt} = C_1[-\cos 1.732t\, e^{-t} - e^{-t}\sin(1.732)t\,(1.732)] + C_2[-\sin 1.732\, e^{-t} + 1.732\, e^{-t}\cos 1.732t] + 100\cos(50t - 57.7)$$

At $t = 0$

$$\frac{di}{dt} = 1.69\,[-e^{0} - 0] + C_2[0 + 1.732\, e^{0}] + 100\cos(0 - 57.7)$$

$$50 = -1.69 + 1.732\,C_2 + 53.435$$

$$1.732\,C_2 = 50 + 1.69 - 53.435$$

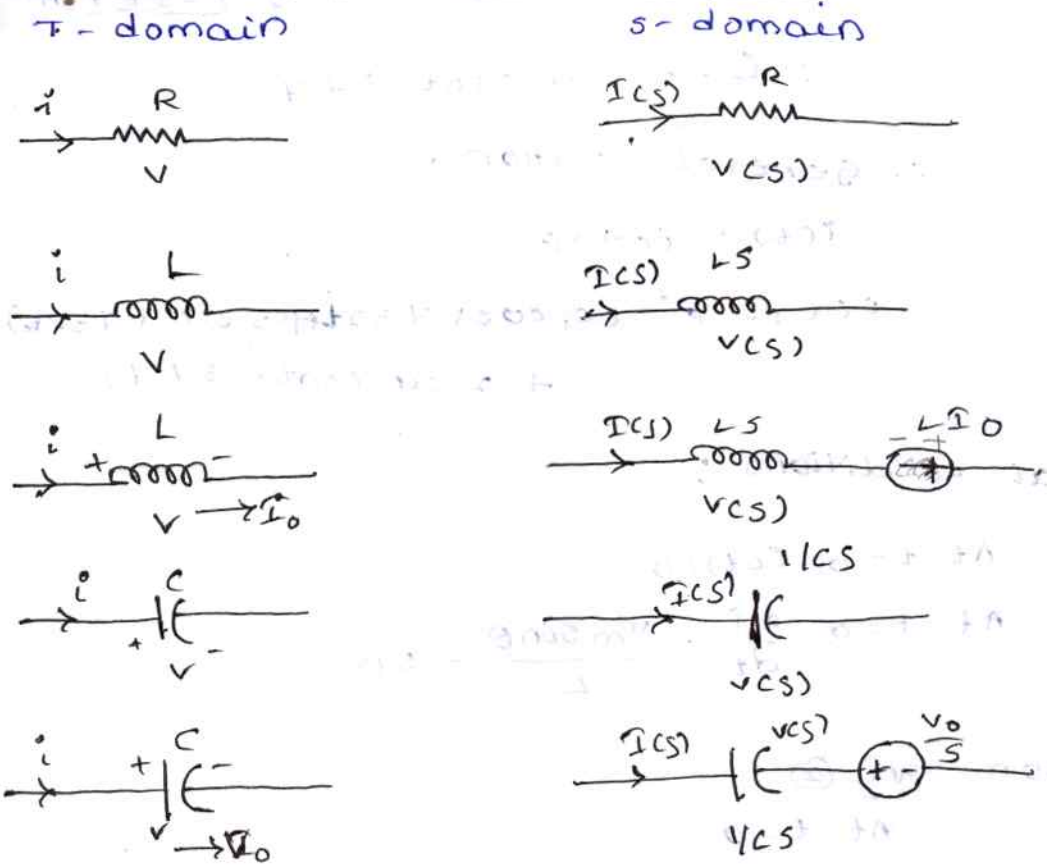
$$C_2 = -1.007$$

∴ Final solution

$$i(t) = e^{-t}(1.69\cos 1.732t - 1.007\sin 1.732t) + 2\sin(50t - 57.7)$$

04.10.2025

USING LAPLACE TRANSFORM:

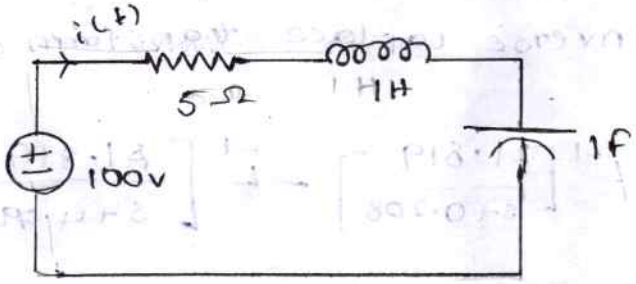


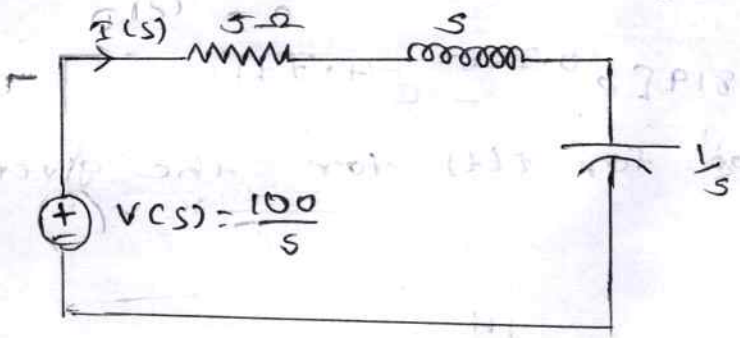
Procedure:

convert the circuit in to s-domain

Apply Inverse laplace transform to get $i(t)$

Q1. Find the expression for $i(t)$ for the given circuit





$$\frac{100}{s} = I(s)\left[5 + s + \frac{1}{s}\right]$$

$$\frac{100}{s} = I(s)\left[\frac{5s + s^2 + 1}{s}\right]$$

$$I(s) = \frac{100}{s^2 + 5s + 1}$$

$$s^2 + 5s + 1 = 0$$

$$\therefore s = -0.208, -4.791$$

$$I(s) = \frac{100}{(s + 0.208)(s + 4.791)} = F(s)$$

$$= \frac{A}{s + 0.208} + \frac{B}{s + 4.791}$$

$$A = I(s)(s + 0.208) = \frac{100}{s + 4.791}\Big|_{s = -0.208}$$

$$= 21.819$$

$$I(s) = \frac{21.819}{s+0.208} - \frac{21.819}{s+4.791}$$

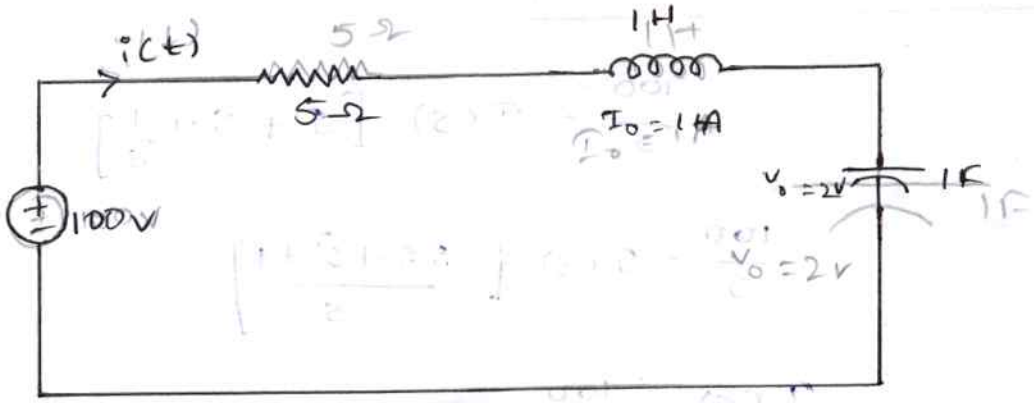
Apply Inverse laplace transform on both side

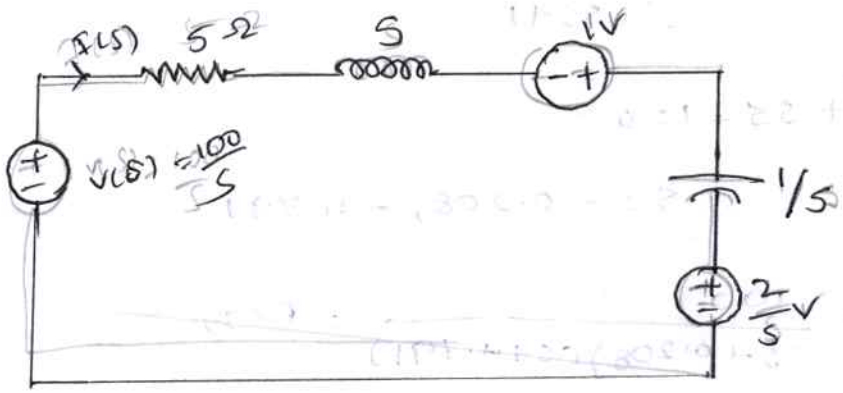
$$\mathcal{L}^{-1}[I(s)] = \mathcal{L}^{-1}\left[\frac{21.819}{s+0.208}\right] - \mathcal{L}^{-1}\left[\frac{21.819}{s+4.791}\right]$$

$$i(t) = 21.819\, e^{-0.208t} - 21.819\, e^{-4.791t}$$

$$\boxed{i(t) = 21.819\left[e^{-0.208t} - e^{-4.791t}\right]}$$

Q Find the expression for $i(t)$ for the given circuit





$$\frac{100}{s} + 1 - \frac{2}{s} = I(s)\left[5 + s + \frac{1}{s}\right]$$

$$\frac{100 + s - 2}{s} = I(s)\left[\frac{5s + s^2 + 1}{s}\right]$$

$$98 + s = I(s)\left[s^2 + 5s + 1\right]$$

$$\frac{98 + s}{}$$

roots for s^2+5s+1 are

$$s = -0.208, -4.791$$

$$I(s) = \frac{s+98}{(s+0.208)(s+4.791)}$$

$$= \frac{A}{s+0.208} + \frac{B}{s+4.791}$$

$$A = I(s)(s+0.208) = \frac{s+98}{s+4.791}\Big|_{s=-0.208}$$

$$= 21.33$$

$$B = I(s)(s+4.791) = \frac{s+98}{s+0.208}\Big|_{s=-4.791}$$

$$= -20.337$$

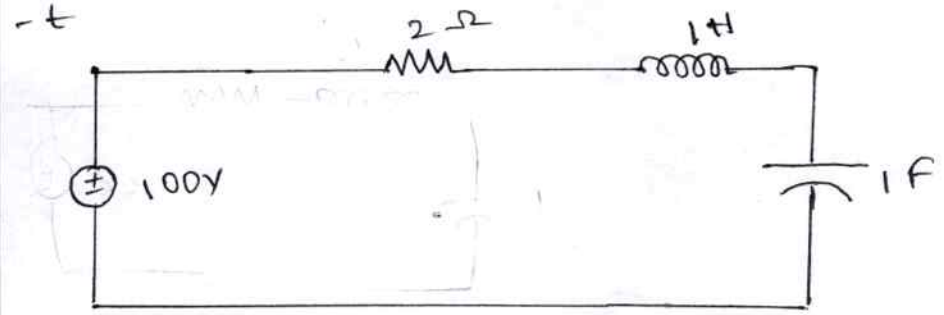
$$I(s) = \frac{21.33}{s+0.208} - \frac{20.337}{s+4.791}$$

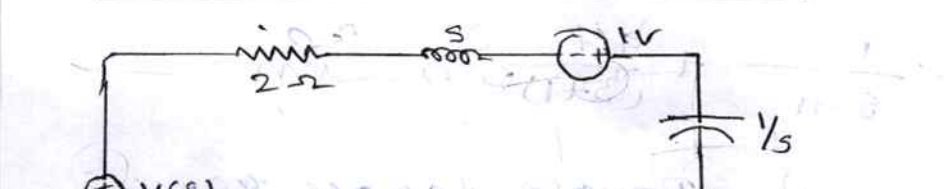
Apply Inverse laplace transform

$$L^{-1}\{I(s)\} = 21.33\{L^{-1}\frac{1}{s+0.208}\} - 20.337\, L^{-1}\{\frac{1}{s+4.791}\}$$

$$i(t) = 21.33\, e^{-0.208t} - 20.337\, e^{-4.791t}$$

Q. Find the expression for $i(t)$ for the given circuit





$$\frac{100}{s} + 1 - \frac{2}{s} = I(s)\left[2 + s + \frac{1}{s}\right]$$

$$\frac{100 + s - 2}{s} = I(s)\left[\frac{2s + s^2 + 1}{s}\right]$$

$$I(s)\left[2s + s^2 + 1\right] = 98 + s$$

$$I(s) = \frac{98 + s}{2s + s^2 + 1}$$

The roots for $2s + s^2 + 1$ are $s = -1, -1$

$$I(s) = \frac{s + 98}{(s+1)^2}$$

$$= \frac{A}{(s+1)} + \frac{B}{(s+1)^2}$$

$$\frac{s + 98}{(s+1)^2} = \frac{A(s+1)^2 + B}{(s+1)^2}$$

$$s + 98 = A(s^2 + 1 + 2s) + B$$

compare s terms

$$2A = 1$$

$$B = s + 98 \Big|_{s=-1}$$

$$= 97$$

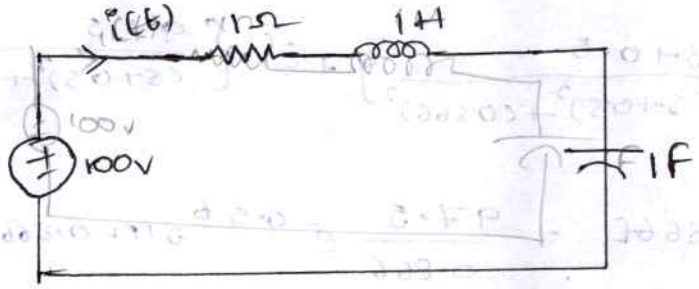
$$A = \frac{d}{ds}(s + 98)\Big|_{s=-1}$$

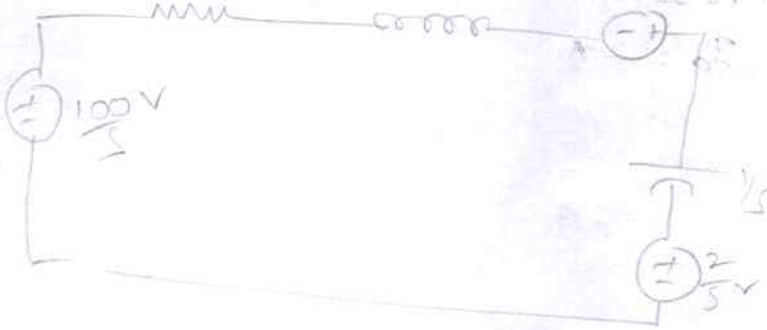
$$= 1$$

$$I(s) = \frac{1}{s+1} + \frac{97}{(s+1)^2}$$

$$i(t) = e^{-t} + 97 e^{-t} \cdot t$$

9 Find the expression for $i(t)$ for the given circuit.





$$\frac{100}{s} + 1 - \frac{2}{s} = \left(1 + s + \frac{1}{s}\right) I(s)$$

$$\frac{100 + s - 2}{s} = I(s) \left[\frac{s + s^2 + 1}{s}\right]$$

$$I(s)\left[s^2 + s + 1\right] = s + 98$$

$$I(s) = \frac{s + 98}{s^2 + s + 1}$$

The roots for $s^2 + s + 1$ are

$$s = -0.5 \pm j\,0.866 \Rightarrow a \pm jb$$

$$I(s) = \frac{s + 98}{(s + 0.5)^2 + (0.866)^2}$$

NOTE: $\mathcal{L}^{-1}\left\{\frac{s}{s^2 + a^2}\right\} = \cos bt$, $\mathcal{L}^{-1}\left\{\frac{s + a}{(s + a)^2 + b^2}\right\} = e^{at}\cos bt$

$\mathcal{L}^{-1}\left\{\frac{1}{\quad}\right\}$

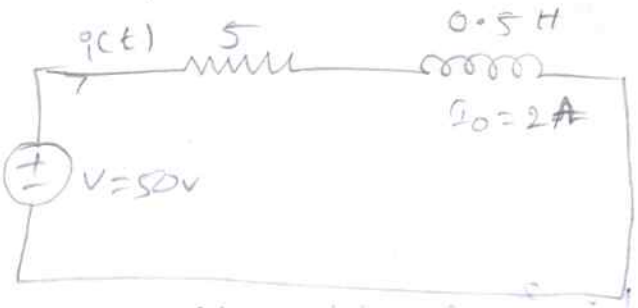
$$I(s) = \frac{s+0.5+97.5}{(s+0.5)^2+(0.866)^2}$$

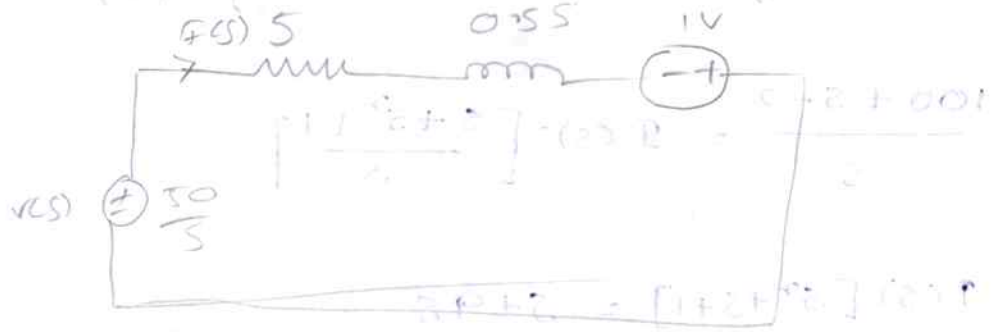
Apply Inverse Laplace transform

$$\mathcal{L}^{-1}\{I(s)\} = \mathcal{L}^{-1}\left\{\frac{s+0.5}{(s+0.5)^2+(0.866)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{97.5}{(s+0.5)^2+(0.866)^2}\right\}$$

$$i(t) = e^{-0.5t}\cos 0.866t + \frac{97.5}{0.866}e^{-0.5t}\sin 0.866t$$

Q Find the expression for $i(t)$ for the given circuit; where $I_0 = 2A$





$$\frac{50}{s} + 1 = [5 + 0.5s]I(s)$$

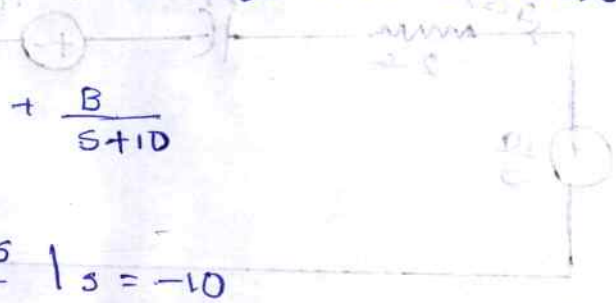
$$\frac{50+s}{s} = I(s)[5+0.5s]$$

$$I(s) = \frac{50+s}{s[5+0.5s]}$$

$$I(s) = \frac{50+s}{5s+0.5s^2}$$

$$= \frac{50+s}{0.5\left[s^2+\frac{5}{0.5}s\right]}$$

The roots for s^2+10s are $s=-10, 0$



$$I(s) = \frac{A}{s+10} + \frac{B}{s+10}$$

$$A = \frac{100+25}{s} \Big|_{s=-10}$$

$$A = \frac{0.55+25}{s+10} \qquad A = \frac{100-20}{-10}$$

$$A = -8$$

$$B = \frac{100+25}{s+10} \Big|_{s=0}$$

$$B = \frac{100}{10}$$

$$B = 10$$

$$I(s) = \frac{-8}{s+10} + \frac{10}{s}$$

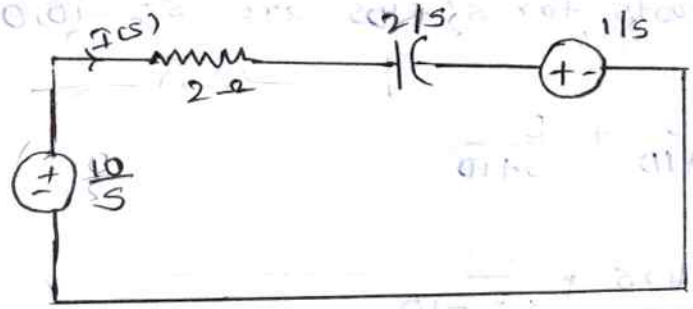
Apply Inverse laplace transform

$$L^{-1}\{I(s)\} = -8\, L^{-1}\left\{\frac{1}{s+10}\right\} + 10\, L^{-1}\left\{\frac{1}{s}\right\}$$

$$i(t) = -8\, e^{-10t} + 10s$$

$$i(t) = 10s - 8\, e^{-10t}$$

Q Find the expression for $i(t)$ for the given circuit



$$\frac{10}{s} - \frac{1}{s} = \left[2 + \frac{2}{s}\right] I(s)$$

$$\frac{9}{s} = \left[\frac{2s+2}{s}\right] I(s)$$

$$I(s)[2s+2] = 9$$

$$I(s) = \frac{9}{2s+2}$$

$$I(s) = \frac{9}{2[s+1]}$$

$$I(s) = \frac{4.5}{s+1}$$

Apply inverse laplace transform

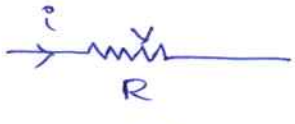
$$\mathcal{L}^{-1}\{I(s)\} = \mathcal{L}^{-1}\left\{\frac{4.5}{s+1}\right\}$$

$$= 4.5\,\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$$

$$i(t) = 4.5\,e^{-t}$$

09.10.2025

STEADY STATE CONDITION OR FINAL CONDITION OF DC CIRCUITS:

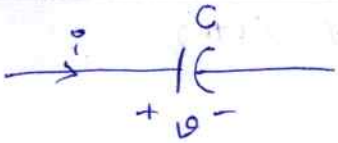


$V = iR$

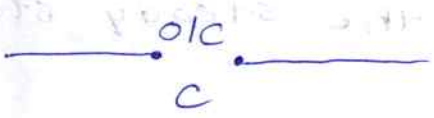
steady state







$i = C\frac{dv}{dt}$



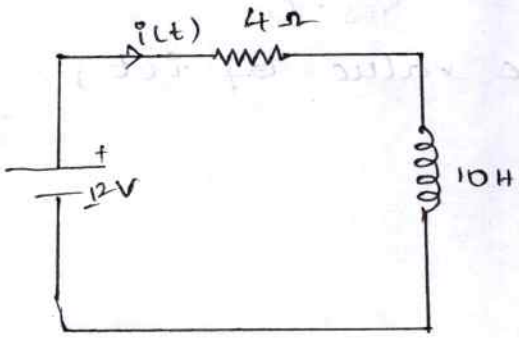
'C' behaves as o/c at DC

At SS v = constant

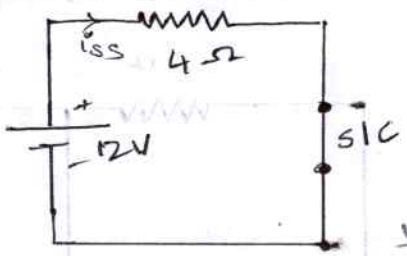
$\frac{dv}{dt} = 0$

$i = 0$

Find the steady state value of $i(t)$.



At steady state (SS)

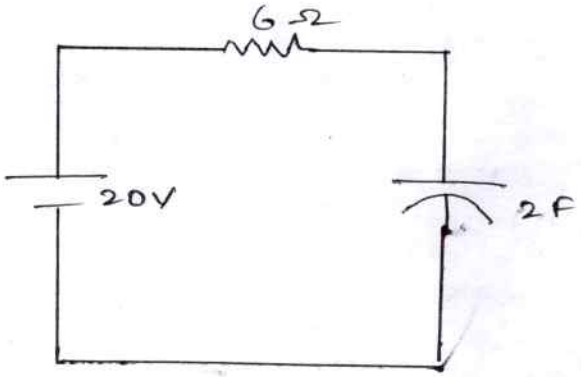


$\frac{V}{R}(1 - e^{-t/\tau})$

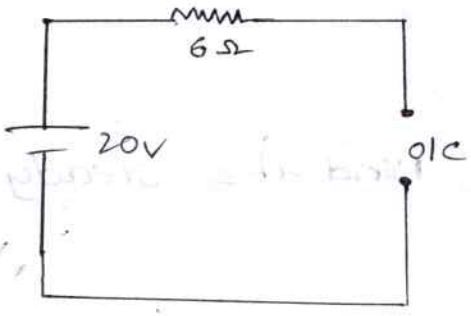
$i_{ss} = \frac{12}{4}$

$= 3A$

Find the steady state value of $i(t)$

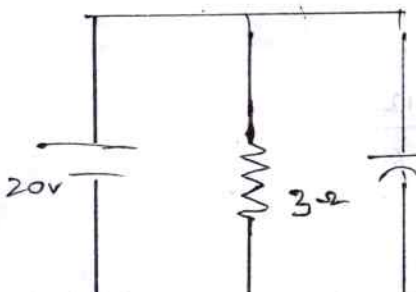


At steady state (SS)

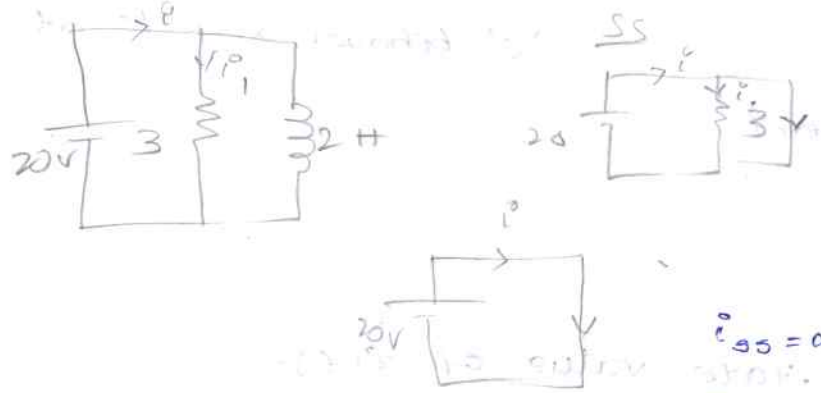


$i_{ss} = 0$

Find the steady state value of $i(t)$



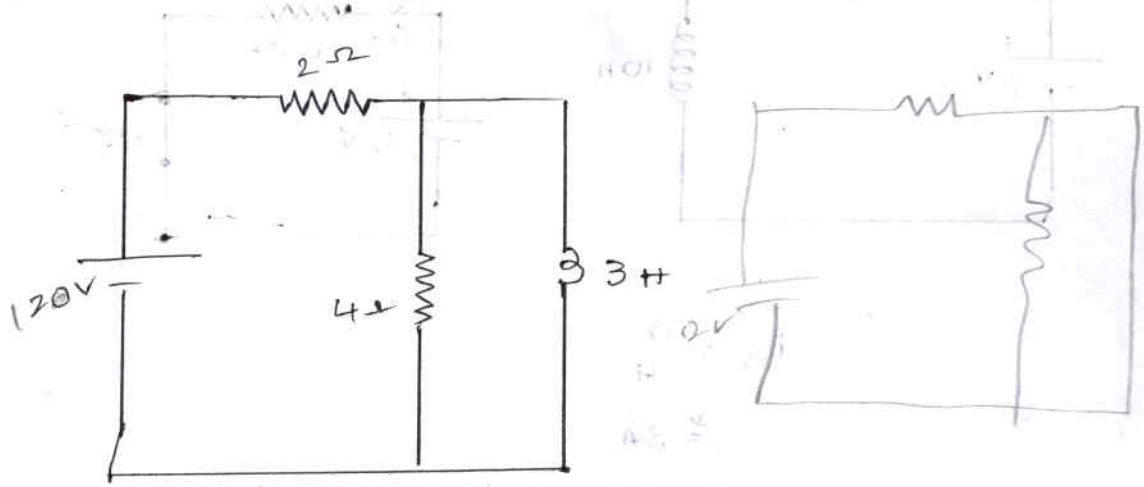
find the steady state value of $i(t)$

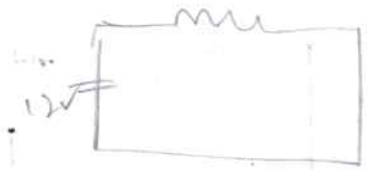


$i_{ss} = \infty$

$i_{ss} = 0$

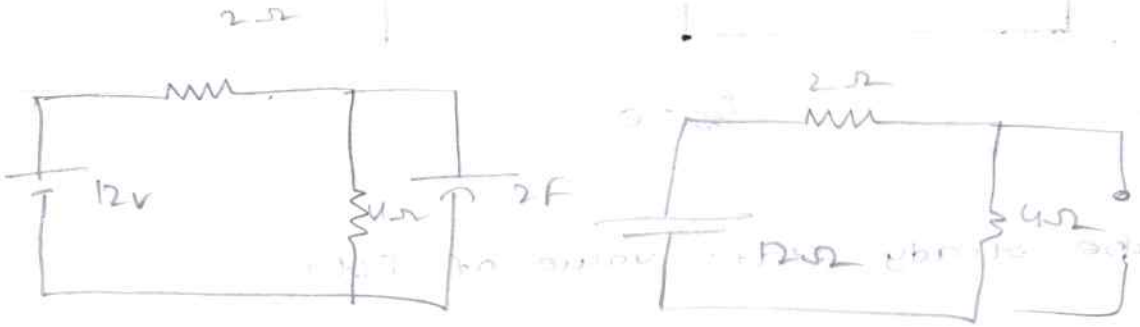
find the steady state value of $i(t)$





$i_{ss} = 6V$

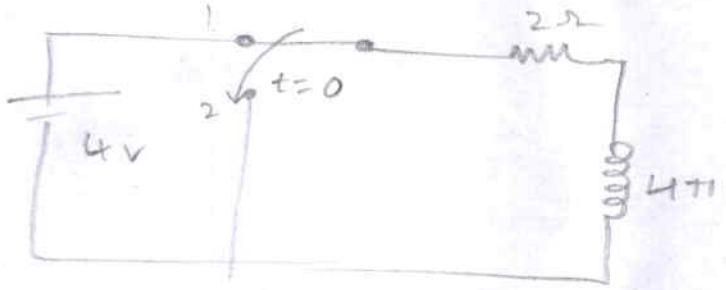
find the steady



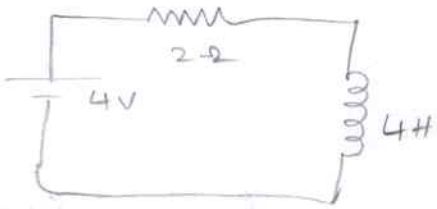
$$P_{ss} = \frac{12}{2+4} = \frac{12}{6}$$

$$= 2$$

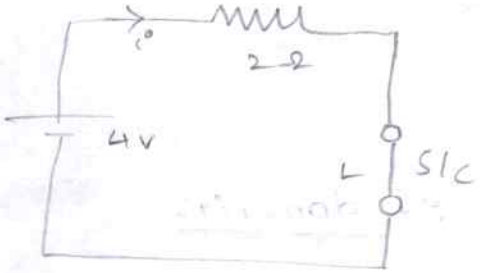
Find the initial value of the current in the given circuit when the switch is moved from 1 to 2 at $t=0$



At $t<0$



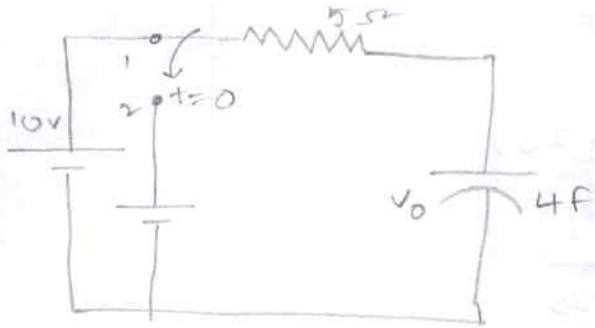
Before switching ss



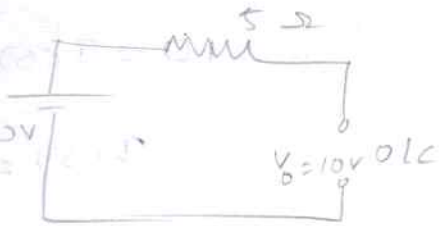
$i(0) = \frac{4}{2}$

$= 2A$

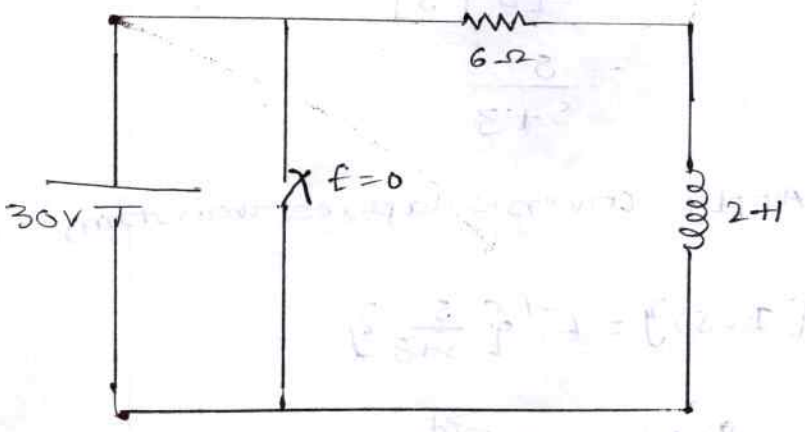
Find the initial value of the voltage in the given circuit when the switch is moved from 1 to 2 at $t=0$



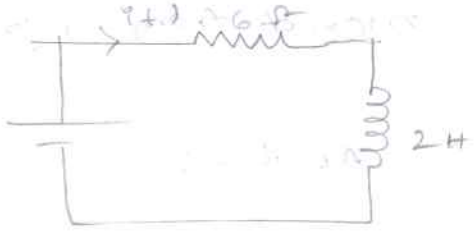
At $t<0$



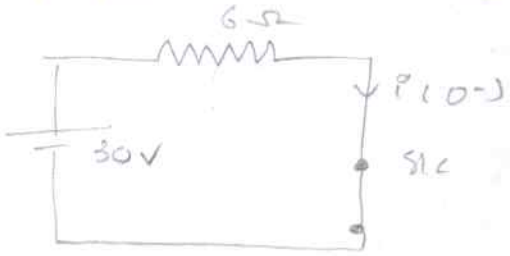
Find the expression for current for $t>0$



$t < 0$



The circuit is in steadystate

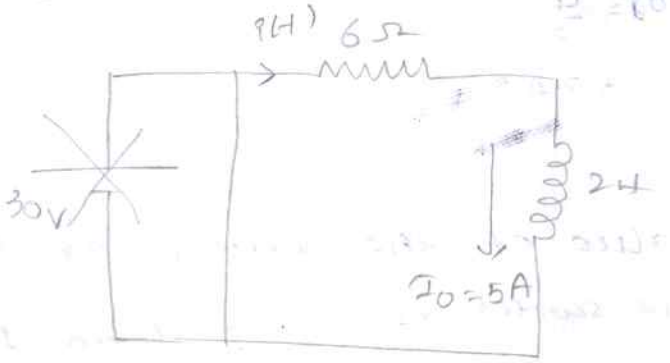


$$i_{ss}\ (or)\ i(0^-) = \frac{30}{6}$$

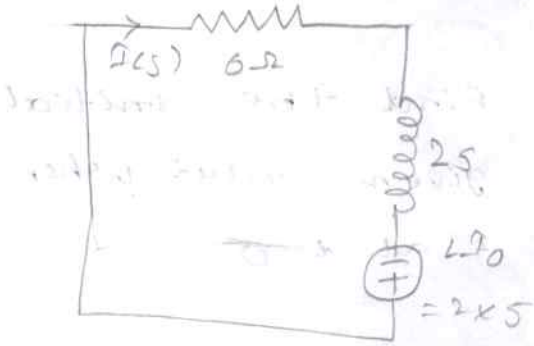
$$= 5A$$

$$\therefore I_0 = 5A$$

for $t > 0$



s - domain



$$10 = I(s)(6 + 2s)$$

$$I(s) = \frac{10}{6+2s}$$

$$= \frac{10}{2[s+3]}$$

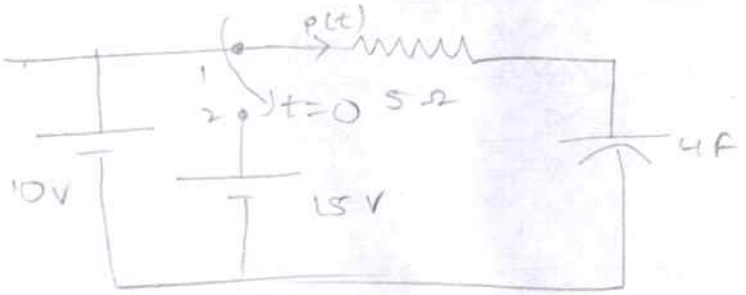
$$= \frac{5}{s+3}$$

Apply inverse laplace transform

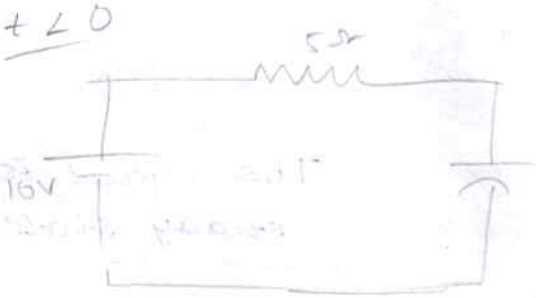
$$L^{-1}\{I(s)\} = L^{-1}\left\{\frac{5}{s+3}\right\}$$

$$i(t) = 5e^{-3t}$$

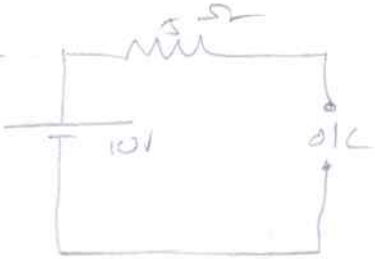
find the expression for current for $t > 0$, when the switch is move from position 1 to 2 at $t = 0$



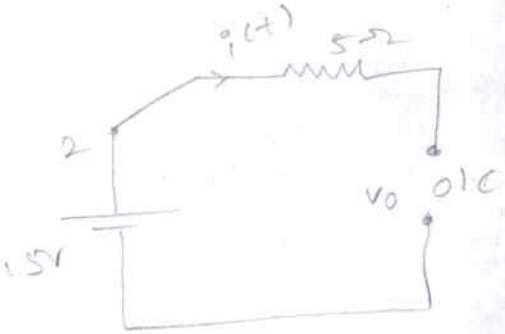
$t < 0$

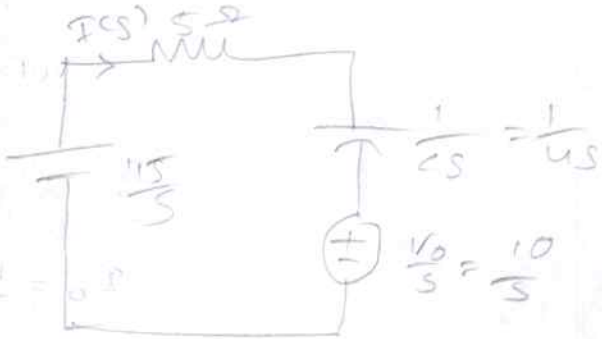


CKT is in SS



$V_0 = 10V$





$$\frac{15}{s} - \frac{10}{s} = I(s)\left[5 + \frac{1}{4s}\right]$$

$$\frac{5}{s} = I(s)\left[\frac{20s+1}{4s}\right]$$

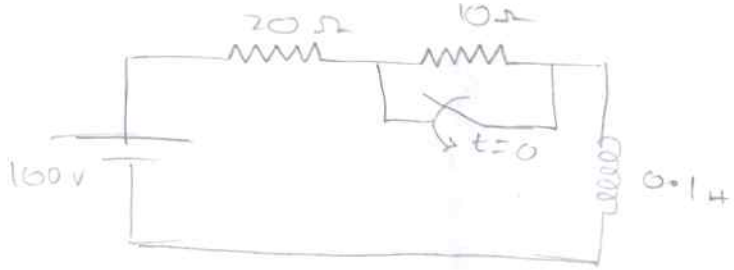
$$I(s) = \frac{20}{20s+1}$$

$$= \frac{20}{20\left[s + \frac{1}{20}\right]}$$

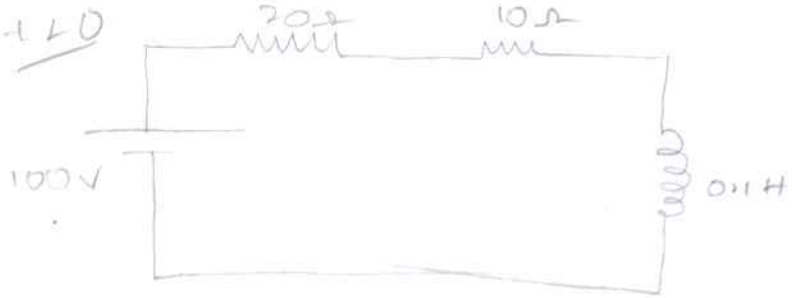
$$= \frac{1}{s + 0.05}$$

Apply inverse laplace

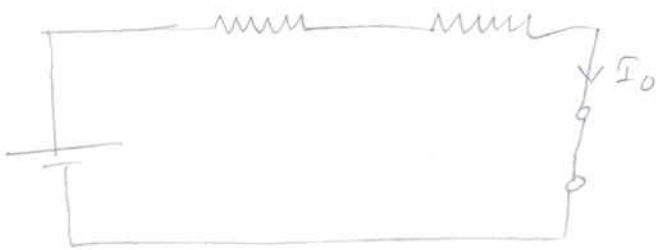
Find the expression for the current through the inductor for $t > 0$



Initial condition:



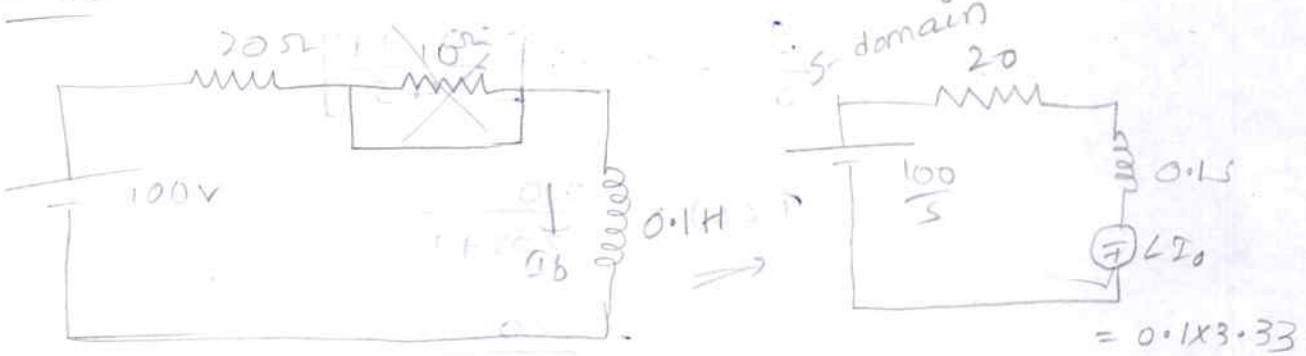
The circuit is in steady state



$$I_0 = \frac{100}{30}$$

$$= 3.33A$$

$t > 0$



$= 0.1 \times 3.33$

$= 0.333V$

$$\frac{100}{s} + 0.333 = I(s)[20 + 0.1s]$$

$$\frac{100 + 0.3335}{s} = I(s)[20 + 0.1s]$$

$$I(s) = \frac{100 + 0.333s}{s \times 0.1 (200+s)}$$

$$I(s) = \frac{1000 + 3.333s}{s(s+200)}$$

$$= \frac{A}{s} + \frac{B}{s+200}$$

$$A = \left.\frac{1000+3.333s}{s+200}\right|_{At\ s=0}$$

$$= \frac{1000}{200} = 5$$

$$B = \left.\frac{1000 + 3.333s}{s}\right|_{s=-200}$$

$$B = \frac{1000 - 666}{-200}$$

$$= -1.67$$

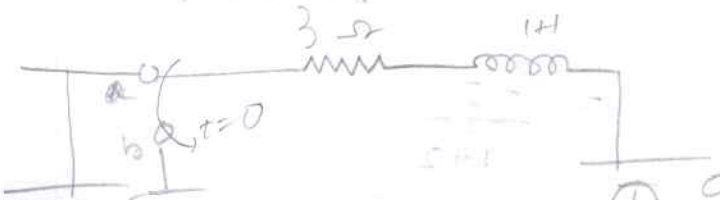
$$I(s) = \frac{5}{s} - \frac{1.67}{s+200}$$

Apply inverse laplace

$$L^{-1}\{I(s)\} = L^{-1}\left\{\frac{5}{s}\right\} - L^{-1}\left\{\frac{1.67}{s+200}\right\}$$

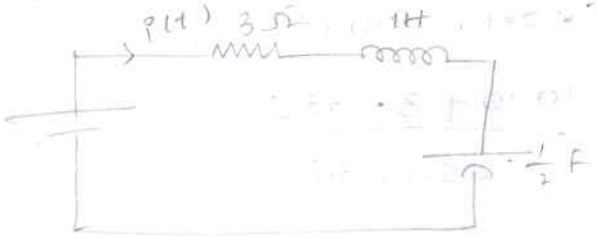
$$i(t) = 5u - 1.67e^{-200t}$$

In the below figure switch is thrown from position A to B at $t=0$. Find $i(t)$ after switching operation



Initial conditions:

At $t < 0$

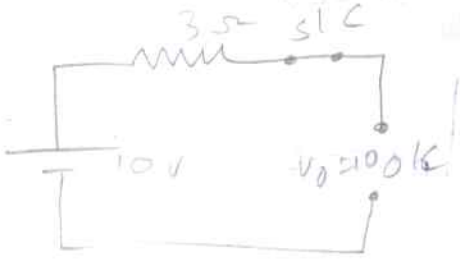


The circuit is in SS

under steady state

$L \rightarrow$ SC

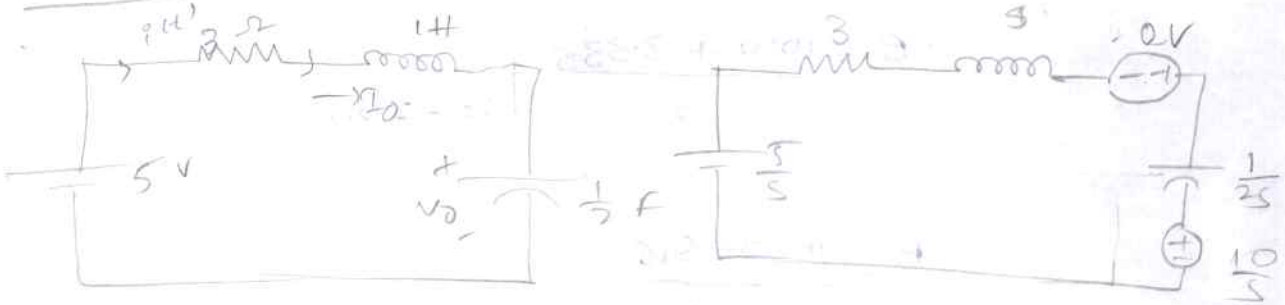
$C \rightarrow$ OC



$V_0 = 10V$

$I_0 = 0$

At $t > 0$



$$\frac{5}{s} - \frac{10}{s} = \left(3 + s + \frac{2}{s}\right) I(s)$$

$$\frac{-5}{s} = I(s)\left[\frac{3s + s^2 + 2}{s}\right]$$

$$I(s) = \frac{-5}{s^2 + 3s + 2}$$

the roots for $s^2 + 3s + 2$ are $s = -1, -2$

$$I(s) = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = I(s)(s+1) = \frac{-5}{s+2} \Big| \text{ At } s = -1$$

$$= \frac{-5}{-1+2}$$

$$B = I(s)(s+2) = \frac{-5}{s+1} \Big|_{\text{at } s=-2}$$

$$= \frac{-5}{-2+1}$$

$$= \frac{-5}{-1}$$

$$B = 5$$

2.82Ω → Real & unequal

1Ω → complex conjugate

$$I(s) = \frac{-5}{s+1} + \frac{5}{s+2}$$

Apply I.L.T

$$L^{-1}\{I(s)\} = L^{-1}\left\{\frac{-5}{s+1}\right\} + L^{-1}\left\{\frac{5}{s+2}\right\}$$

$$i(t) = -5e^{-t} + 5e^{-2t}$$

ANNAMACHARYA UNIVERSITY, RAJAMPET

(ESTD UNDER AP PRIVATE UNIVERSITIES (ESTABLISHMENT AND REGULATION) ACT, 2016
RAJAMPET, Annamayya District, AP, INDIA

Department of Electrical & Electronics Engineering

II B.Tech I Semester

Network Analysis and Synthesis (24AEEE21T)
(Electrical and Electronics Engineering)

Unit-4 (Fourier Series & Fourier Transform)

Syllabus:

Introduction- trigonometric Fourier series - waveform symmetry, exponential form of Fourier series, effective value, application to electrical circuits. Fourier transforms & Properties, Fourier transform of important signals and periodic signals.

UNIT - 4:

Fourier series & Fourier Transform

Laplace transform – g/p – Aperiodic

ex: DC, step, ramp, exponential

$s = \sigma + j\omega$ Li

FT, $s = j\omega$ $x(t) = e^{\sigma t}$

g/p – periodic – Fourier series

TRIGNOMETRIC FORM OF FOURIER SERIES:

Any periodic function $f(t)$ which satifies Dirichlet conditions. can be represented by an infinite series.

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

where

$\omega = \frac{2\pi}{T}$, T = time period of $f(t)$

a_0, a_n, b_n are fourier coeffients

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$ dc value or average value

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\,dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\,dt$$

$$f(t) = a_0 + (a_1\cos\omega t + b_1\sin\omega t) + (a_2\cos 2\omega t + b_2 \sin 2\omega t + \dots \infty$$

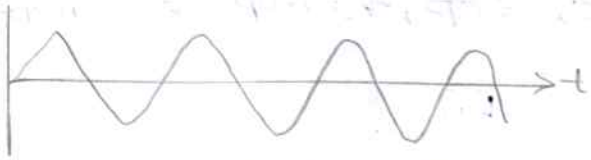
where

n = harmonic number

$n \neq 0$ → DC value

$n = 1$ → fundamental

1. In each period the function has finite no. of maximum and minimum value.



2. In each period the function has finite no. of discontinuities.

3. If $f(t)$ is integrable then

$$\int_0^T |f(t)|\,dt < \infty$$

FORMULAS:

1. $\int_0^T \sin n\omega t\,dt = 0 \quad n = 0, 1, 2, 3, \ldots \infty$

2. $\int_0^T \cos n\omega t\,dt = 0 \quad n = 1, 2, \ldots \infty \; [n \neq 0]$

for $n = 0$, $\int_0^T \cos n\omega t\,dt = T$

3. $\int_0^T \cos n\omega t \cdot \cos m\omega t\,dt = 0 \quad n \neq m$

if $n = m$ then $\int_0^T \cos^2 n\omega t\,dt = \frac{T}{2}$

4. $\int_0^T \sin n\omega t \cdot \sin m\omega t\,dt = 0 \quad n \neq m$

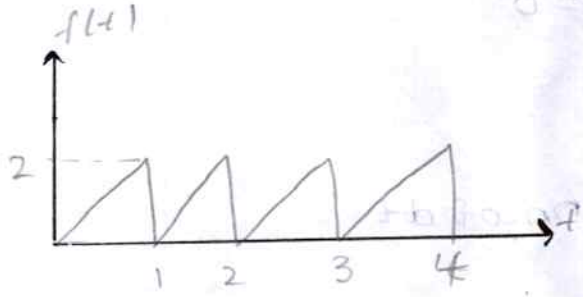
if $n = m$ then $\int_0^T \sin^2 n\omega t\,dt = T/2$

5. $\int_0^T \cos n\omega t \sin m\omega t\,dt = 0$

6. $\int t \cos at\,dt = \frac{1}{a^2}[\cos at + at \sin at]$

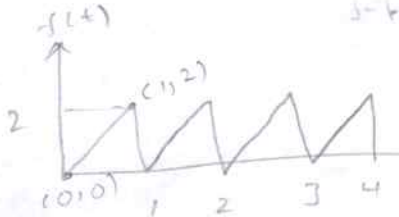
7. $\int t \sin at\,dt = \frac{1}{a^2}[\sin at - at \cos at]$

Find the fourier series for the given function



$Y = mx$

$f(t) = mt$



$T = 1$ second

$Y = mx$

$f(t) = mt$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{1-0} = 2$

$f(t) = 2t \quad 0 < t < 1$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$= \frac{1}{1}\int_0^1 2t\,dt$$

$$= \left[\frac{2t^2}{2}\right]_0^1$$

$$= [t^2]_0^1$$

$$= 1^2 - 0^2$$

$$\boxed{a_0 = 1}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\,dt \quad \left[\omega = \frac{2\pi}{T} = 2\pi\right]$$

$$= \frac{2}{1}\int_0^1 2t\cos 2\pi n t\,dt$$

$$= 4\int_0^1 t\cos 2\pi n t\,dt$$

$$\boxed{\int t\cos at\,dt = \frac{1}{a^2}[\cos at + at\sin at]}$$

$$= 4 \cdot \frac{1}{(2\pi n)^2}\left[\cos 2\pi n t + 2\pi n t \sin 2\pi n t\right]_0^1$$

$$= \frac{1}{\pi^2 n^2}[\cos 2\pi n + 2\pi n \sin 2\pi n - \cos 0 - 0]$$

$$= \frac{1}{\pi n^2}[1-1] = 0$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{2}{T}\int_0^1 f(t)\sin n\omega t\, dt$$

$$= 2\int_0^1 2t \sin 2\pi n t\, dt$$

$$= 4\int_0^1 t \sin 2\pi n t\, dt$$

$$\because \int t\sin at\, dt = \frac{1}{a^2}[\sin at - at\cos at]$$

$$= 4\left[\frac{1}{(2\pi n)^2}\left[\sin 2\pi n t - 2\pi n t\cos 2\pi n t\right]_0^1\right.$$

$$= \frac{1}{(\pi n)^2}\left[\sin 2\pi n - 2\pi n\cos 2\pi n - 0\right]$$

$$= \frac{1}{(\pi n)^2}\left[0 - 2\pi n\cdot 1\right]$$

$$= \frac{-2}{\pi n}$$

$$\boxed{b_n = \frac{-2}{\pi n}}$$

$$\therefore f(t) = a_0 + \sum_{n=1}^{\infty}[a_n\cos n\omega t + b_n\sin n\omega t]$$

$$\boxed{f(t) = 1 + \sum_{n=1}^{\infty}\left[\frac{-2}{n\pi}\sin n\omega t\right]}$$

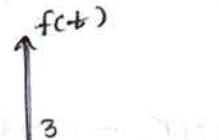
10.10.2025

WAVE FORM SYMMETRY:

a. EVEN FUNCTION:

A function is said to be even function, then, if

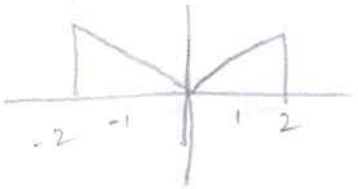
$$f(-t) = f(t)$$

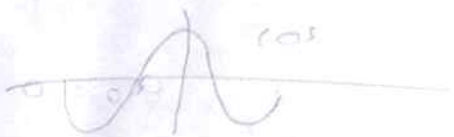


$$f(1) = 3$$

$$f(-1) = 3$$

> If a function is symmetrical above +ve Y-axis then the function is a even function.







If incase of even function

$$a_0 = \frac{1}{T/2} \int_0^{T/2} f(t)\,dt$$

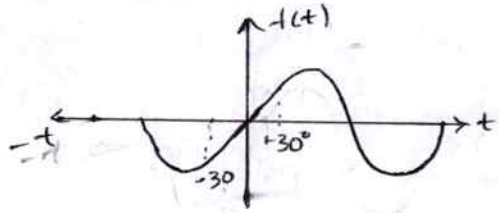
$$a_n = \frac{2}{T/2} \int_0^{T/2} f(t) \cos n\omega t\,dt$$

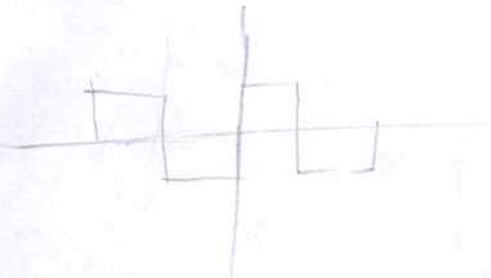
$$b_n = 0$$

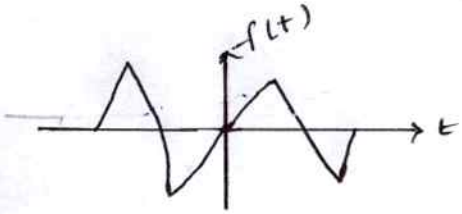
Incase of odd function

B. ODD FUNCTION:

A function is said to be odd function if $f(-t) = -f(t)$









$$a_0 = 0$$

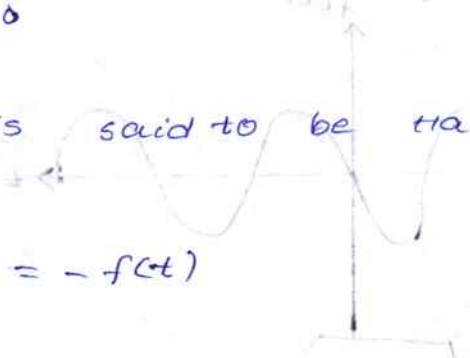
$$a_n = 0$$

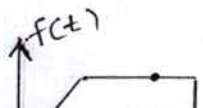
$$b_n = \frac{2}{\frac{T}{2}} \int_0^{T/2} f(t) \sin n\omega t\,dt$$

C. HALF WAVE SYMMETRY:

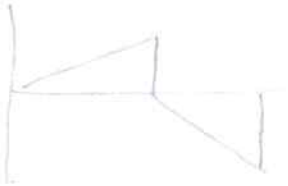
A function is said to be Half wave symmetry, if

$$f(t + T/2) = -f(t)$$





> The second half cycle is mirror image of the first half cycle.



$a_0 = 0$

d. QUATER WAVE SYMMETRY :

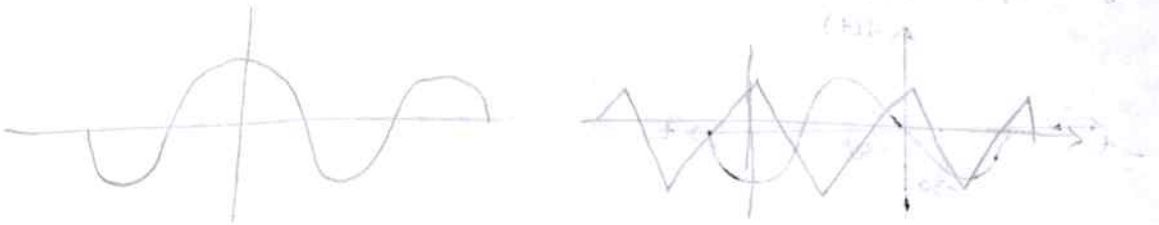
Two types

> Half wave + even function

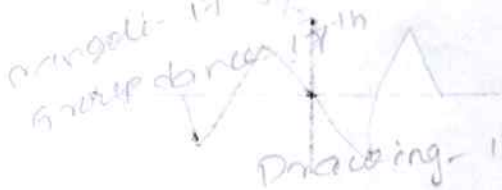
$a_0 = 0$

$b_n = 0$

$$a_{neven} = \frac{2}{T/4} \int_0^{T/4} f(t) \cos n\omega t \, dt$$







Drawing- 13

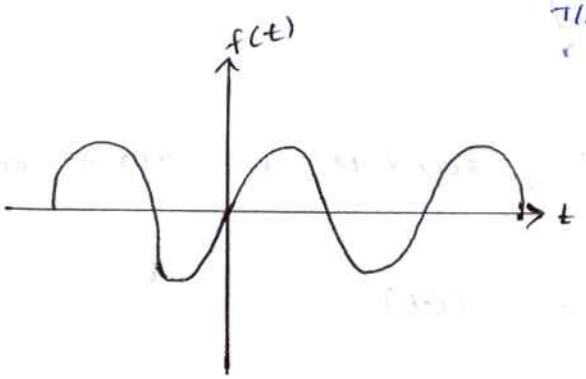
3:00

> Half wave + odd function

$a_0 = 0$ [$\because$ Half wave]

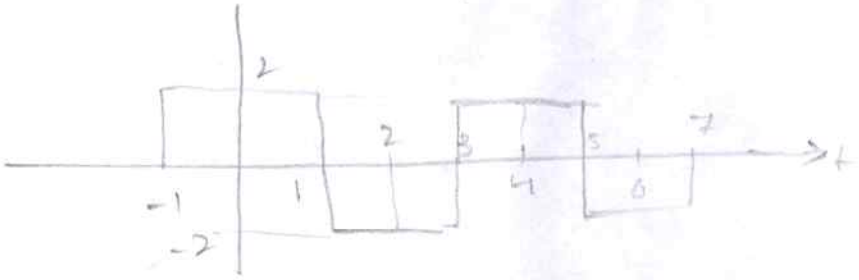
$a_n = 0$ [$\because$ odd]

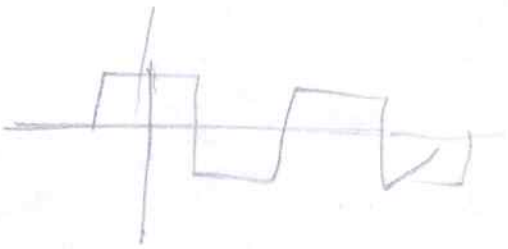
$$b_n = \frac{2}{T/4} \int_0^{T/4} f(t) \sin n\omega t \, dt$$





Find the fourier series for the given function





The waveform has half wave and even function. So, it is a Quater wave symmetry

$$a_0 = 0, \quad b_n = 0$$

$$a_n = \frac{2}{\frac{T}{4}} \int_0^{\frac{T}{4}} f(t) \cos n\omega_0 t \, dt$$

$T = 4$ sec, $\frac{T}{4} = 1$ sec

$$f(t) = 2 \quad 0 < t < 1$$

$$a_n = 2 \int_0^1 2 \cos n\omega_0 t \, dt$$

$$= 4 \int_0^1 \cos n\omega_0 t \, dt \qquad \left[\omega = \frac{2\pi}{T}, \quad \omega = \frac{\pi}{2}\right]$$

$$= 4 \int_0^1 \cos \frac{n\pi t}{2} \, dt$$

$$= 4 \left[\frac{\sin \frac{n\pi t}{2}}{\frac{n\pi}{2}}\right]_0^1$$

$$= \frac{8}{n\pi} \left[\sin \frac{n\pi}{2}\right]$$

$\sin \frac{n\pi}{2} = 0$ for n = even

$= \pm 1$ for n = odd → 1 for $n = 1, 5, 9, \ldots$; -1 for $n = 3, 7, 11, \ldots$

$$f(t) = \sum a_n \cos n\omega_0 t$$

HW



The waveform has half wave symmetry & odd function so it is a quarter wave symmetry

$$a_0 = 0,\ a_n = 0 \qquad T = 1\text{sec},\ T/4 = \frac{1}{4}\text{sec}$$

$$b_n = \frac{2}{\frac{T}{4}} \int_0^{T/4} f(t)\sin n\omega t\,dt \qquad \therefore f(t) = 1$$

$$b_n = \frac{2}{\frac{1}{4}} \int_0^{1/4} \sin n\omega t\,dt \qquad \omega = \frac{2\pi}{T} = 2\pi$$

$$= 8\left[\frac{-\cos n\omega t}{n\omega}\right]_0^{1/4}$$

$$= 8\left[\frac{-\cos 2n\pi\left(\frac{1}{4}\right)}{2n\pi} + \frac{1}{2n\pi}\right]$$

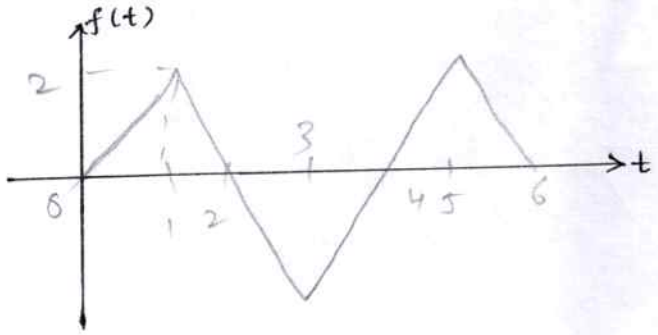
$$= 8\left[\frac{-\cos\frac{n\pi}{2}}{2n\pi} + \frac{1}{2n\pi}\right]$$

$$= \frac{8}{2\pi n}\left[1 - \cos\frac{n\pi}{2}\right]$$

$$b_n = \frac{4}{n\pi}\left[1 - \cos\frac{n\pi}{2}\right]$$

$$f(t) = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{n}\left[\left(1 - \cos n\frac{\pi}{2}\right)\sin n\omega t\right]$$

Find the fourier series for the given waveform.



The function is odd function and half wave symmetry. So, it is a quater wave symetry

$a_0 = 0, a_n = 0$

$T = 4$ seconds

$\frac{T}{4} = 1$ seconds

$$b_n = \frac{2}{\frac{T}{4}} \int_0^{T/4} f(t) \sin n\omega t \, dt$$

slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= 2$

$\therefore f(t) = 2t \quad 0 < t < 1$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$

$$= 2 \int_0^1 2t \sin n\omega t \, dt$$

$$= 4 \int_0^1 t \sin \frac{n\pi t}{2} dt$$

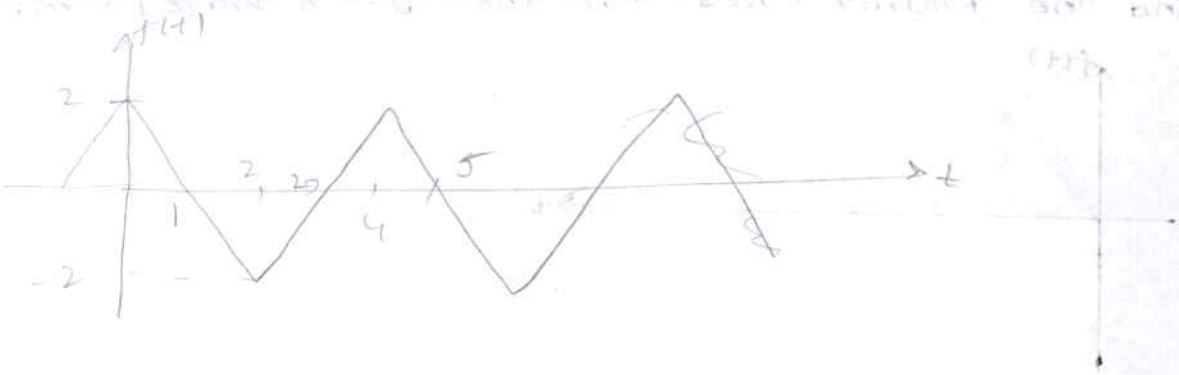
$$\because \int t \sin at \, dt = \frac{1}{a^2} [\sin at - at \cos at]$$

$$= 4 \cdot \frac{1}{\left(\frac{n\pi}{2}\right)^2} \left[\sin \frac{n\pi t}{2} - \frac{n\pi}{2} t \cos \frac{n\pi t}{2} \right]_0^1$$

$$= \frac{16}{(n\pi)^2} \left[\sin \frac{n\pi}{2} - \frac{n\pi}{2} \cos \frac{n\pi}{2} - 0 - 0 \right]$$

$$= \frac{16}{(n\pi)^2} \left[\sin \frac{n\pi}{2} - \frac{n\pi}{2} \cos \frac{n\pi}{2} \right]$$

H.W



The waveform is half wave and even function. So, the wave form is quarter wave symmetry

$$a_0 = 0, \quad b_n = 0$$

$$a_n = \frac{8}{T}\int_0^{T/4} f(t)\cos n\omega t\, dt$$

$T = 4\text{sec} \quad \frac{T}{4} = 1\text{sec}, \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$

$f(t) = 2t$

$$= 2\int_0^1 2t\cos n\frac{\pi}{2}t\, dt$$

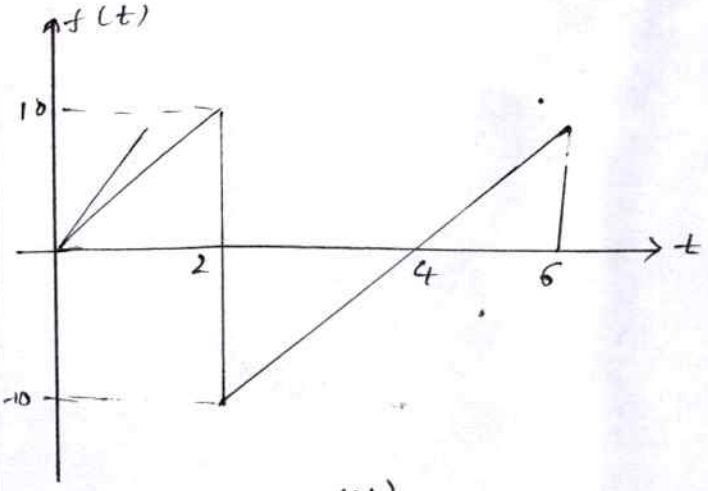
$$= 4\int_0^1 t\cos n\frac{\pi}{2}t\, dt$$

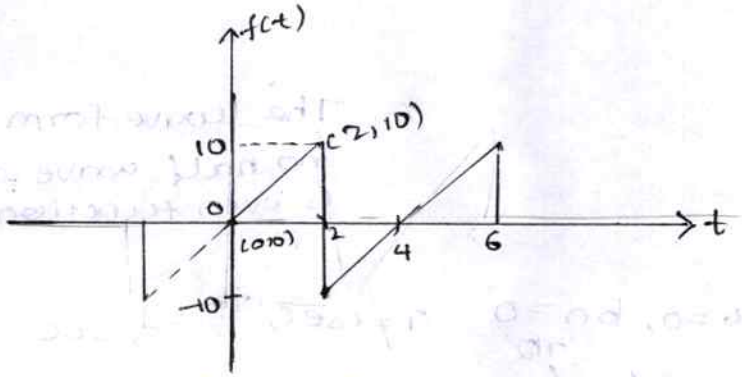
$$= \frac{4}{\left(\frac{n\pi}{4}\right)^2}\left[\cos n\frac{\pi}{2}t + n\frac{\pi}{2}\sin n\frac{\pi}{2}t\right]_0^1$$

$$= \frac{16}{(n\pi)^2}\left[\cos n\frac{\pi}{2} + n\frac{\pi}{2}\sin n\frac{\pi}{2} - 1\right]$$

$$f(t) = \sum_{n=1}^{\infty}\frac{16}{(n\pi)^2}\left[\cos n\frac{\pi}{2} + n\frac{\pi}{2}\sin n\frac{\pi}{2} - 1\right]\cos n\omega t$$

Find the fourier series for the given waveform





The waveform is a odd function

$y = mx$

$T = 2\,sec$

$\frac{T}{2} = 2\,sec$

$m = 5, \omega = \frac{2\pi}{T} = \frac{\pi}{2}$

$f(t) = 5t$

$a_0 = 0, a_n = 0$

$$b_n = \frac{2}{T/2}\int_0^{T/2} f(t)\sin n\omega t\,dt$$

$$= \frac{2}{2}\int_0^2 5t\sin n\frac{\pi t}{2}\,dt$$

$$= 5\int_0^2 t\sin\frac{n\pi t}{2}\,dt$$

$$\int_a t\sin at\,dt = \frac{1}{a^2}[\sin at - at\cos at]$$

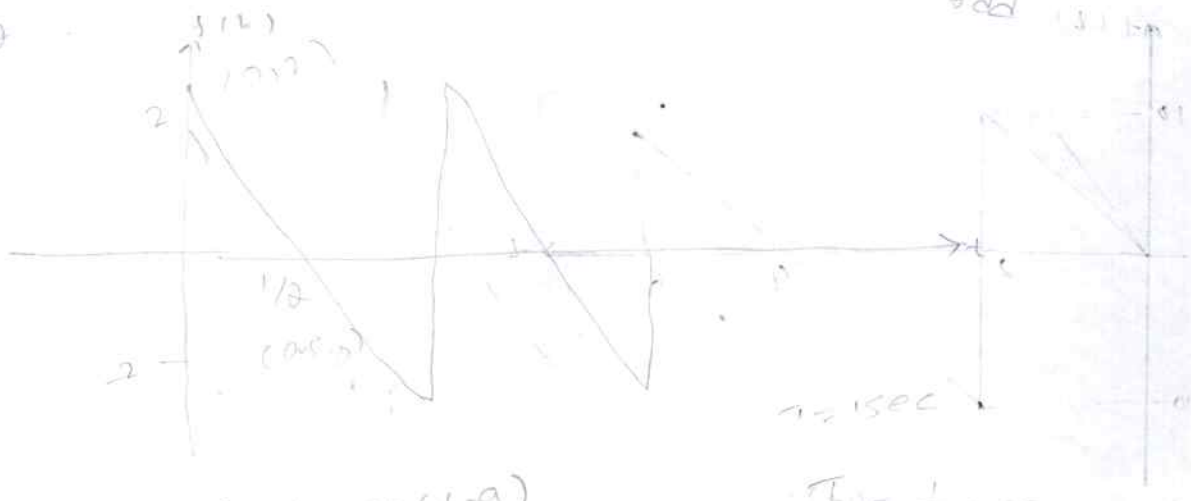
$$= 5\cdot\frac{1}{\left(\frac{n\pi}{2}\right)^2}\left[\sin\frac{n\pi t}{2} - \frac{n\pi t}{2}\cos\frac{n\pi t}{2}\right]_0^2$$

$$= \frac{5\times 4}{(n\pi)^2}[\sin n\pi - \cos n\pi\cdot n\pi]$$

$$= \frac{20}{n^2\pi^2}[\sin n\pi - n\pi\cdot\cos n\pi] = -\frac{20}{(n\pi)^2}[n\pi\cos n\pi]$$

$$\sum_{n=1}^{\infty}$$

H.W



The wave form has no half wave and it is even function.

$a_0 = 0,\ b_n = 0 \quad T = 1\text{sec},\ \frac{T}{2} = \frac{1}{2}\text{ sec}$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega t\, dt$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1} = 2\pi \qquad a = 0.5$$

$$m = \frac{0-2}{0.5-0} = \frac{-2}{0.5} = -4$$

$$f(t) = m(t-a)$$
$$= -4(t - 0.5)$$

$$a_n = \frac{2}{\frac{1}{2}}\int_0^{1/2} -4(t-0.5)\cos 2\pi n t\, dt$$

$$= 4\int_0^{1/2} -4(t-0.5)\cos n 2\pi t\, dt$$

$$= -16\int_0^{1/2} (t-0.5)\cos n 2\pi t\, dt$$

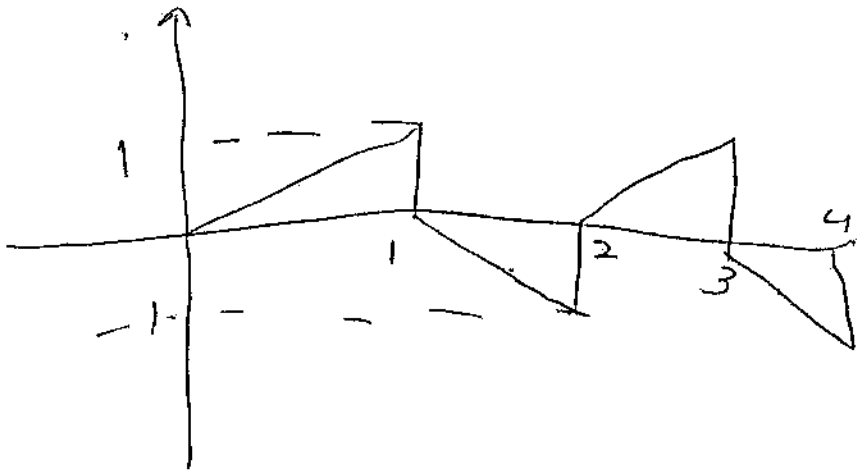
$$= -16\left[\frac{1}{(4\pi n)^2}\left[\cos 2n\pi t + 2n\pi t \sin 2n\pi t\right]_0^{1/2}\right] + 0.5\left[\frac{\sin n 2\pi t}{2\pi n}\right]_0^{1/2}$$

$$= -16\left[\frac{1}{4n^2\pi^2}\left[\cos n\pi + n\pi \sin n\pi - \cos 0\right]\right] - 16\left[0.5\left[\frac{\sin n\pi}{2n\pi}\right]\right]$$

$$= \frac{-4}{n^2\pi^2}\left[\cos n\pi - 1\right] - (5)$$

$$f(t) = \sum_{n=1}^{\infty} \frac{-4}{n^2\pi^2}\left[\cos n\pi - 1\right]\cos n\omega t$$

HW



11.10.2025

ALTERNATE FORMS OF FOURIER SERIES :

$$f(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega t + b_n \sin n\omega t]$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega t + \phi_n)$$

$$A_n = \sqrt{a_n^2 + b_n^2}, \quad \phi_n = \tan^{-1}\left[\frac{b_n}{a_n}\right]$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} A_n \sin(n\omega t + \psi_n)$$

$$A_n = \sqrt{a_n^2 + b_n^2}, \quad \psi_n = \tan^{-1}\left[\frac{a_n}{b_n}\right]$$

EXPONENTIAL FORM OF FOURIER SERIES :

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega t}$$

$$c_0 = \frac{1}{T}\int_0^T f(t)\,dt = a_0$$

$$c_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega t}\,dt$$

From exponential to Trignometry :

$$a_0 = c_0$$

$$[a_n = a_n - j b_n]_x$$

$$a_n = c_n + c_{n-}$$

$$b_n = j(c_{n-} - c_n)$$

From Trignometry to exponential :

$$c_0 = a_0$$

$$c_n = \frac{a_n - j b_n}{2}$$

$$c_{n-} = \frac{a_n + j b_n}{2}$$

FORMULAS:

1. $\int t\, e^{at}\, dt = t\frac{e^{at}}{a} - 1\frac{e^{at}}{a^2}$

$= \frac{e^{at}}{a^2}[at-1]$

2. $\int t\, e^{-at}\, dt = t\frac{e^{-at}}{a} - 1\frac{e^{-at}}{a^2}$

$= -\frac{e^{-at}}{a^2}[at+1]$

3. $\int e^{-at}\sin bt\, dt = -\frac{e^{-at}}{a^2+b^2}[a\sin bt + b\cos bt]$

4. $\int e^{-at}\cos bt\, dt = -\frac{e^{-at}}{a^2+b^2}[a\cos bt - b\sin bt]$

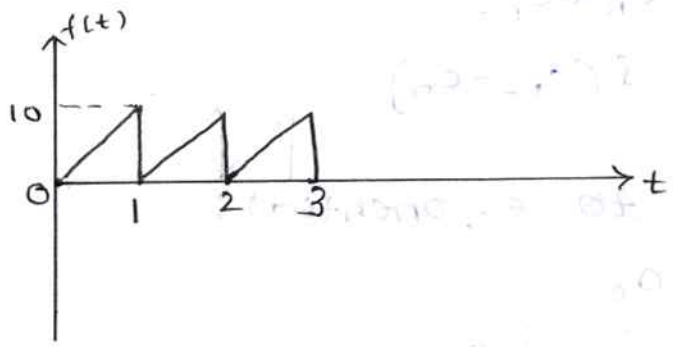
5. $e^{-j2\pi n} = 1$ for all n

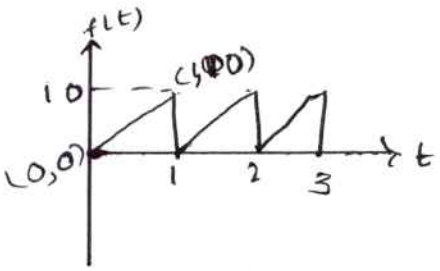
6. $e^{-j\pi n} = 1$ for even

$= -1$ for odd

$= (-1)^n$ fr

find the exponential form of fourier series for the given waveform.





$T = 1$ sec

$f(t) = 10t \quad 0 < t < 1$

$\omega = \frac{2\pi}{T} = 2\pi$

$C_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega t}\, dt$

$$= 10 \int_0^1 t e^{-2j\pi nt} dt$$

$$= 10 \left[t \frac{e^{-j2\pi nt}}{-j2\pi n} - \frac{e^{-j2\pi nt}}{(-j2\pi n)^2} \right]_0^1$$

$$= 10 \left[\frac{e^{-j2\pi n}}{-j2\pi n} - \frac{e^{-j2\pi n}}{(-j2\pi n)^2} - 0 + \frac{1}{(-j2\pi n)^2} \right]$$

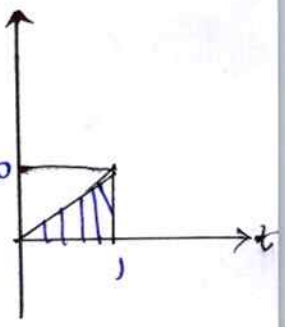
$$= 10 \left[\frac{1}{-j2\pi n} - \frac{1}{(-j2\pi n)^2} \right] + \frac{1}{(-j2\pi n)^2}$$

$$= \frac{j5}{\pi n}$$

This expression is not valid for C_0

$$\therefore C_0 = a_0 = \frac{1}{T} \int_0^T f(t)\, dt$$

$$= \frac{\text{Area under } f(t) \text{ in one T}}{T}$$



$$= \frac{\frac{1}{2} \times 10 \times 1}{1}$$

$$= \frac{10}{2} = 5$$

OR

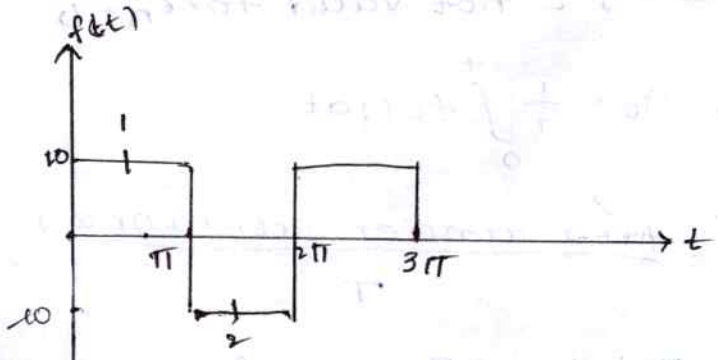
$$a_0 = \int_0^1 10t\, dt$$

$$= 10 \left[\frac{t^2}{2} \right]_0^1$$

$$= \frac{10}{2} = 5$$

$$f(t) = 5 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{j5}{n\pi} e^{jn\omega t}$$

find the exponential form of fourier series for the given wave form.



$T = 2\pi$

$f(t) = 10 \quad 0 < t < \pi$

$= -10 \quad \pi < t < 2\pi$

$$C_n = \frac{1}{2\pi} \int_0^{2\pi} f(t) e^{-jn\omega t} dt$$

$$C_n = \frac{1}{2\pi} \left[\int_0^{\pi} 10\, e^{-jn\omega t} dt + \int_{\pi}^{2\pi} (-10) e^{-jn\omega t} dt \right]$$

$$= \frac{5}{\pi} \left[\int_0^{\pi} e^{-jn\omega t} dt - \int_{\pi}^{2\pi} e^{-jn\omega t} dt \right]$$

$$= \frac{5}{\pi} \left[\int_0^{\pi} e^{-jnt} dt - \int_{\pi}^{2\pi} e^{-jnt} dt \right]$$

$$= \frac{5}{\pi} \left[\left[\frac{e^{-jnt}}{-jn} \right]_0^{\pi} - \left[\frac{e^{-jnt}}{-jn} \right]_{\pi}^{2\pi} \right]$$

$$= \frac{5}{\pi} \left[\frac{e^{-jn\pi}}{-jn} - \right.$$

$$= \frac{5}{\pi} \left[-\frac{(-1)^n}{jn} + \frac{1}{jn} + \frac{1}{jn} - \frac{(-1)^n}{jn} \right]$$

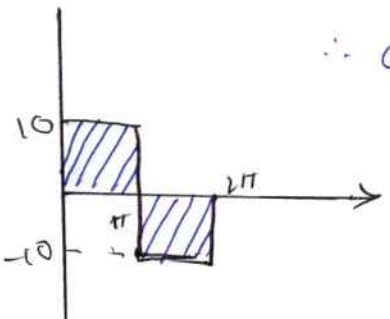
$$= \frac{5}{jn\pi} \left[-2(-1)^n + 2 \right]$$

$$= -\frac{10j}{n\pi} \left[-(-1)^n + 1 \right]$$

$$= \frac{10j}{n\pi} \left[(-1)^n - 1 \right]$$

This expression is not valid for $C_n = 0$

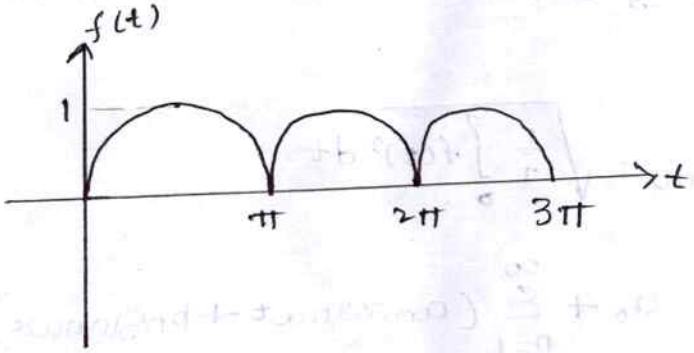
$$\therefore C_0 = a_0 = \frac{1}{T} \int_0^T f(t) dt$$



$= \frac{\text{Area under } f(t) \text{ in one } T}{T}$

$$\therefore f(t) = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{j10}{n\pi}\left((-1)^n - 1\right) e^{jnwt}.$$

find the exponential form of the fourier series for the given wave form.



$T = \pi \qquad \omega = \frac{2\pi}{T} = 2$

$f(t) = 1 \sin\omega t \quad 0 < t < \pi$

$= \sin 2t \quad 0 < t < \pi$

$$\therefore C_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega t}\, dt$$

$$C_n = \frac{1}{\pi}\int_0^{\pi} \sin 2t\, e^{-j2nt}\, dt$$

$$\therefore \int e^{at}\sin bt\, dt = \frac{e^{at}}{a^2+b^2}\left[a\sin bt + b\cos bt\right]$$

$$= \frac{1}{\pi}\left[\left[\frac{e^{-j2nt}}{(-j2n)^2+4}\right]\left[-j2n\sin 2t + 2\cos 2t\right]\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{e^{-j2\pi n}}{-4n^2+4}\left(-j2n\sin 2\pi + 2\cos 2\pi\right) + \frac{1}{-4n^2+4}\left[0 + 2\cos 0\right]\right]$$

$$= \frac{1}{4\pi(1-n^2)}\left[2+2\right]$$

$$= \frac{4}{4\pi(1-n^2)}$$

Here, we no need to calculate C_0

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{2}{\pi(1-n^2)(4)} e^{jn\omega t}$$

RMS VALUE OF A PERIODIC FUNCTION:

If $f(t)$ is a periodic function with time period T

$$f_{rms} = \sqrt{\frac{1}{T}\int_0^T f(t)^2\,dt}$$

if $f(t) = a_0 + \sum_{n=1}^{\infty}(a_n\cos n\omega t + b_n \sin n\omega t)$

$$f_{rms} = \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)}$$

$V(t) = 215\sin 377t + 140\cos 754t + 70\sin 1131t$. Find the rms value of $v(t)$.

Given

$V(t) = 215\sin 317t + 140\cos 754t + 70\sin 1131t$

fundamental frequency $\omega = 377$ rad/sec

$$754 = 2\omega$$

$$1131 = 3\omega$$

$$V(t) = a_0 + a_1\cos\omega t + b_1\sin\omega t + a_2\cos 2\omega t + b_2\sin 2\omega t + a_3\cos 3\omega t + b_3\sin 3\omega t$$

$$V(t) = 215\sin \underbrace{377}_{\omega} t + 140\cos 2\omega t + 70\sin 3\omega t$$

$$a_0 = 0,\ a_1 = 0,\ b_1 = 215$$

$$a_2 = 140,\ b_2 = 0,\ a_3 = 0$$

$$b_3 = 70$$

$$V_{rms} = \sqrt{a_0^2 + \frac{1}{2}(a_1^2 + b_1^2) + \frac{1}{2}(a_2^2 + b_2^2) + \frac{1}{2}(a_3^2 + b_3^2)}$$

H.W A circuit has a DC current of 10A and AC fundamental component of 10A. Determine rms value of current.

$$i(t) = 10 + 10\sin\omega t$$

$$a_0 = 10,\ a_1 = 0$$

$$b_1 = 10$$

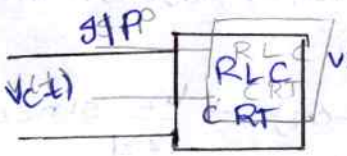
$$V_{rms} = \sqrt{10^2 + \frac{1}{2}(10^2)}$$

$$= 12.24\,V$$

13.Oct.2025

FOURIER SERIES APPLICATIONS TO THE ELECTRIC CIRCUIT:

Input to electrical circuits is a non-sinusiodal periodic function like square, triangular, sawtooth, trapizoidal.



$$v(t) = V_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$v(t) = V_0 + \sum_{n=1}^{\infty} V_n \sin(\omega t n + \phi_n)$$

$$V_n = \sqrt{a_n^2 + b_n^2}$$

$$\tan\phi_n = \frac{a_n}{b_n}$$

$$v(t) = V_0 + V_1 \sin(\omega t + \phi_1) + V_2 \sin(2\omega t + \phi_2) + \dots \infty$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Z is dependent on frequency (ω)

$$Z_n = R + j\left(n\omega L - \frac{1}{n\omega C}\right) \longrightarrow (2)$$

$$Z_0 = R$$

$$Z_1 = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z_2 = R + j\left(2\omega L - \frac{1}{2\omega C}\right)$$

when the input is $V_1 \sin(\omega t + \phi_1)$

$$\text{current} = \frac{V_1 \sin(\omega t + \phi_1)}{|Z_1| \angle \theta_1}$$

$$= \frac{V_1}{|Z_1|} \sin(\omega t + \phi_1 - \theta_1)$$

Similarly the input is $V_2 \sin(2\omega t + \phi_2)$

$$\text{current} = \frac{\angle V_2 \sin(2\omega t + \phi_2)}{|Z_2| \angle \theta_2}$$

$$= \frac{V_2}{|Z_2|} \sin(\omega t + \phi_2 - \theta_2)$$

. . . . . .

Total current By superposition theorem

$$i(t) = \frac{V_0}{R} + \frac{V_1}{|Z_1|} \sin(\omega t + \phi_1 - \theta_1) + \frac{V_2}{|Z_2|} \sin(\omega t + \phi_2 - \theta_2) \longrightarrow (3)$$

$$i(t) = I_0 + I_1 \sin(\omega t + \psi_1) + I_2 \sin(2\omega t + \psi_2) + \cdots \infty \longrightarrow (4)$$

$$P = I_{rms}^2 R$$

$$I_{rms}^2 = \sqrt{I a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} (I a_n^2 + I b_n^2)}$$

$$I_{rms} = \sqrt{I_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} I_n^2} \longrightarrow (5)$$

$P = I_{rms}^2 R$

$P = V_{rms} I_{rms} \cos\phi$

$P.f = \frac{P}{V_{rms} I_{rms}}$

similarly

$$V_{rms} = \sqrt{V_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} V_n^2} \longrightarrow (6)$$

$$P = V_0 I_0 + \frac{V_1 I_1}{2} \cos(\phi_1 - \psi_1) + \frac{V_2 I_2}{2} \cos(\phi_2 - \psi_1) + \cdots \infty$$

$$P = V_0 I_0 + \frac{1}{2} \sum_{n=1}^{\infty} V_n I_n \cos(\phi_n - \psi_n) \longrightarrow (7)$$

The voltage applied to a load is $v(t) = 50 + 50\sin 314t + 50\sin 942t$ and current flowing through the load is $i(t) = 10\sin(314t + 60^\circ) + 15\sin(942t + 45^\circ)$. Determine the power consumed by load and power factor of the load.

Given

$$v(t) = 50 + 50\sin 314t + 50\sin 942t$$

$$i(t) = 10\sin(314t + 60^\circ) + 15\sin(942t + 45^\circ)$$

$$\omega = 314 \text{ rad/sec}$$

$$942 = 3\omega$$

compare eqn (1) with $v(t)$

$$v(t) = V_0 + V_1\sin(\omega t + \phi_1) + V_2\sin(2\omega t + \phi_2) + V_3\sin(3\omega t + \phi_3)$$

$V_0 = 50$

$V_1 = 50$, $\phi_1 = 0^\circ$

$V_2 = 0$, $\phi_2 = 0^\circ$

$V_3 = 50$, $\phi_3 = 0^\circ$

$$\therefore V_{rms} = \sqrt{V_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} V_n^2}$$

$$= \sqrt{50^2 + \frac{1}{2}[V_1^2 + V_2^2 + V_3^2]}$$

$$= \sqrt{50^2 + \frac{1}{2}[50^2 + 0^2 + 50^2]}$$

$$V_{rms} = 70.71\text{V}$$

compare eqn (2) with $i(t)$

$$i(t) = I_0 + I_1\sin(\omega t + \psi_1) + I_2\sin(\omega t + \psi_2) + I_3\sin(\omega t + \psi_3)$$

$I_0 = 0$

$I_1 = 10$, $\psi_1 = 60^\circ$

$$I_{rms} = \sqrt{I_0^2 + \frac{1}{2}(I_1^2 + I_2^2 + I_3^2)}$$

$$= 12.74\,A$$

Real power consumed by circuit

$$P = V_0 I_0 + \frac{V_1 I_1}{2}\cos(\phi_1 - \psi_1) + \frac{V_2 I_2}{2}\cos(\phi_2 - \psi_2) + \frac{V_3 I_3}{2}\cos(\phi_3 - \psi_3)$$

$$= 50\times 0 + \frac{50\times 10}{2}\cos(0-60) + \frac{0\times 0}{2}\cos(0-0) + \frac{50\times 15}{2}\cos(0-45^\circ)$$

$$P = 390.165\ W$$

Power factor of the circuit

$$p.f = \frac{P}{V_{rms}\cdot I_{rms}}$$

$$= \frac{390.165}{70.71 \times 12.74}$$

$$= 0.433$$

If the current through 20 Ω resistor is given by $i(t) = 4 + 5\sin(20t) - 3\cos 3\omega t$. Determine the power consumed by the resistor.

Given

$R = 20\,\Omega$

$i(t) = 4 + 5\sin\omega t - 3\cos 3\omega t$

14.10.2025

Determine the expression for current in a circuit having $R = 10\Omega$, $L = 0.0318\,H$, with aquired emf $v(t) = 200\sin 314t + 40\sin(942t + 30°) + 10V$. Calculate RMS values of the voltage and current as well as power factor of the circuit.

Given

$R = 10\Omega$, $L = 0.0318\,H$

$v(t) = 200\sin(314t) + 40\sin(942t + 30°) + 10V$

$v(t) = 10 + 200\sin(314t) + 40\sin(942t + 30°)$ ——→ ①

$\omega = 314$ rad/s

$942 = 3\omega$

$Z = Z_R + Z_L$

$Z = R + j\omega L$, $Z_n = R + jn\omega L$

$Z_0 = R = 10\Omega$

$Z_1 = 10 + j\omega L$

$= 10 + j9.98$

$Z_1 = 14.12\angle 44.94°\ \Omega$

$Z_2 = 10 + j3\omega L$

$= 10 + j29.95$

$= 31.57\angle 71.53°\ \Omega$

$$i(t) = \frac{R}{Z_0} = \frac{10}{Z_0} + \frac{200\sin(314t)}{Z_1} + \frac{40\sin(942t+30°)}{Z_2}$$

$$= \frac{10}{10} + \frac{200\sin(314t)}{14.12\angle 44.94} + \frac{40\sin(942t+30°)}{31.57\angle 71.53}$$

$= 1 + 14.16\sin(314t - 44.94) + 1.26\sin(942t + 30° - 71.53)$ ——→ ②

compare eqn ① with $v(t)$

$v(t) = V_0 + V_1\sin(\omega t + \phi_1) + V_2\sin(2\omega t + \phi_2) + V_3\sin(3\omega t + \phi_3)$

$V_0 = 10V$ $\quad \phi_1 = 0°$

$V_1 = 200V$ $\quad \phi_2 = 0$

$V_2 = 0V$ $\quad \phi_3 = 30°$

$V_3 = 40V$

$$= \sqrt{10^2 + \frac{1}{2}(200^2 + 40^2)}$$

$$= 144.56\,V$$

Compare eqn ② with $i(t)$

$$i(t) = I_0 + I_1 \sin(\omega t + \psi_1) + I_2 \sin(2\omega t + \psi_2) + I_3 \sin(3\omega t + \psi_3)$$

$I_0 = 1$

$I_1 = 14.16 \quad \psi_1 = -44.94$

$I_2 = 0 \quad \psi_2 = 0$

$I_3 = 1.26 \quad \psi_3 = -41.53°$

$$\therefore I_{rms} = \sqrt{I_0^2 + \frac{1}{2}[I_1^2 + I_3^2]}$$

$$= \sqrt{1 + \frac{1}{2}[14.16^2 + 1.26^2]}$$

$$= 10.101\,A$$

Real power consumed by circuit

$$P = V_0 I_0 + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos(\phi_n - \psi_n)$$

$$= V_0 I_0 + \frac{V_1 I_1}{2}\cos(\phi_1 - \psi_1) + \frac{V_2 I_2}{2}\cos(\phi_2 - \psi_2) + \frac{V_3 I_3}{2}\cos(\phi_3 - \psi_3)$$

$$= 10 + \frac{200 \times 14.16}{2}\cos(0 + 44.94) + \frac{40 \times 1.26}{2}\cos(30 + 41.53)$$

$$P = 1020.29\,W$$

$$P.f = \frac{P}{V_{rms} I_{rms}}$$

$$= 0.698$$

A series RLC circuit with $R = 25\,\Omega$, $L = 1H$, $C = 10\,\mu F$ is excited by a source $v(t) = 15\sin(100t) + 20\sin(200t) + 5\sin(300t)$. Determine the power factor of circuit.

Given

$R = 25\Omega$, $L = 1H$

$C = 10\,\mu F = 10 \times 10^{-6} = 10^{-5}\,F$

$$Z = Z_R + Z_L + Z_C$$

$$= R + j\omega L - \frac{j}{\omega C}$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z_n = R + j\left(n\omega L - \frac{1}{n\omega C}\right)$$

$$Z_1 = 25 + j\left[100 - \frac{1}{100\times10^{-5}}\right] = 900.347 \angle -88.408\ \Omega$$

$$Z_2 = 25 + j\left[200 - \frac{1}{200\times10^{-5}}\right] = 301.039 \angle -85.236\ \Omega$$

$$Z_3 = 25 + j\left[300 - \frac{1}{300\times10^{-5}}\right] = 41.66 \angle -53.13\ \Omega$$

$$i(t) = \frac{15}{Z_1}\sin(100t) + \frac{20}{Z_2}\sin(200t) + \frac{5}{Z_3}\sin(300t)$$

$$i(t) = \frac{15}{900.347}\sin(100t + 88.408) + \frac{20}{301.039}\sin(200t + 85.236)$$

$$+ \frac{5}{41.66}\sin(300t + 53.13)$$

$$i(t) = 0.016\sin(100t + 88.408) + 0.066\sin(200t + 85.236)$$

$$+ 0.12\sin(300t + 53.13^\circ) \longrightarrow (2)$$

compare with eq^n (1) with $v(t)$

$$v(t) = v_0 + v_1\sin(\omega t + \phi_1) + v_2\sin(2\omega t + \phi_2) + v_3\sin(3\omega t + \phi_3)$$

$v_0 = 0$

$v_1 = 15$, $\phi_1 = 0$

$v_2 = 20$, $\phi_2 = 0^\circ$

$v_3 = 5$, $\phi_3 = 0^\circ$

$$\therefore V_{rms} = \sqrt{V_0^2 + \frac{1}{2}(V_1^2 + V_2^2 + V_3^2)}$$

$$= \sqrt{\frac{1}{2}[15^2 + 20^2 + 5^2]}$$

$$= 18.02\,V$$

compare eq^n (2) with $i(t)$

$I_0 = 0$

$I_1 = 0.016, \quad \psi_1 = 88.408$

$I_2 = 0.066 \quad \psi_2 = 85.236$

$I_3 = 0.12 \quad \psi_3 = 53.13$

$$\therefore I_{rms} = \sqrt{I_0^2 + \frac{1}{2}(I_1^2 + I_2^2 + I_3^2)}$$

$$= \sqrt{\frac{1}{2}[0.016^2 + 0.066^2 + 0.12^2)}$$

$$I_{rms} = 0.097 \text{ A}$$

Real power consumed by the circuit

$$P = V_0 I_0 + \frac{1}{2} \sum_{n=1}^{\infty} V_n I_n \cos(\phi_n - \psi_n)$$

$$= V_0 I_0 + \frac{V_1 I_1}{2} \cos(\phi_1 - \psi_1) + \frac{V_2 I_2}{2} \cos(\phi_2 - \psi_2) + \frac{V_3 I_3}{2} \cos(\phi_3 - \psi_3)$$

$$= \frac{0.24}{2} \cos(-88.408) + \frac{1.32}{2} \cos(-85.236) + \frac{0.6}{2} \cos(-53.13)$$

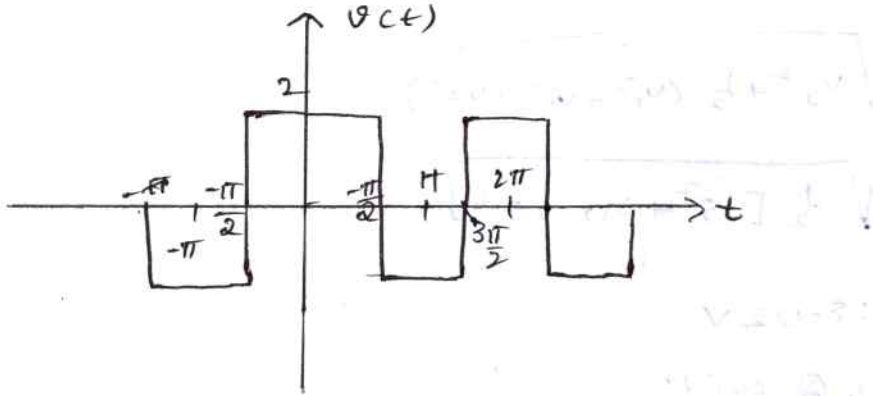
$$P = 0.238 \text{ W}$$

$$P.F = \frac{P}{V_{rms} I_{rms}}$$

$$= 0.136$$

15.09.2025

The voltage waveform of below figure is applied to RL series circuit $R = 1\Omega$, $L = 1H$. Find the expression for the circuit and find the power factor of the circuit



The voltage applied to a load $v(t) = 215\sin(377t) + 140\cos(754t) + 70\sin(1131t)$ and current through the load is $i(t) = 15\cos(377t + 30°) + 4\cos(754t - 60°) + 3\sin(1131t + 50°)$. Determine the real power and power factor of the circuit.

Given

$$v(t) = 215\sin(377t) + 140\cos(754t) + 70\sin(1131t)$$

$$i(t) = 15\sin(377t + 60) + 4\cos(754t - 60°) + 3\sin(1131t + 50)$$

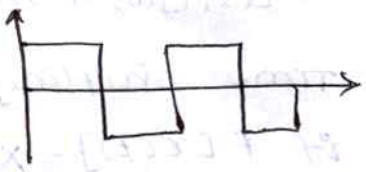
15.10.2025

FOURIER TRANSFORMS :

FS — periodic fn

as $T \longrightarrow \infty$

Periodic fn $\longrightarrow$ non periodic fn



↓

FS $\longrightarrow$ FT

$= \Sigma$ → distance

$= \int$ → sumation

The fourier transform of a non periodic function $f(t)$ is

$$F[f(t)] = F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

compare with $L\{f(t)\}$

$$L\{f(t)\} = F(s) = \int_{0}^{\infty} f(t) e^{-st} dt$$

Differences

1. Limits
2. $s = \sigma + j\omega$ FS $\longrightarrow j\omega$ ($\sigma = 0$)

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$$

PROPERTIES OF FOURIER TRANSFORMS:

1. Linearity property:

If $F(x_1(t)) = X_1(j\omega)$ and $F(x_2(t)) = X_2(j\omega)$ then

$$F[a x_1(t) + b x_2(t)] = a X_1(j\omega) + b X_2(j\omega)$$

PROOF:

$$F[a x_1(t) + b x_2(t)] = \int_{-\infty}^{\infty} [a x_1(t) + b x_2(t)] e^{-j\omega t} dt$$

$$= a\int_{-\infty}^{\infty} x_1(t)e^{-j\omega t}dt + b\int_{-\infty}^{\infty} x_2(t)e^{-j\omega t}dt$$

$$= aX_1(j\omega) + bX_2(j\omega)$$

2. Time shifting property:

if $F[x(t)] = X(j\omega)$ then

$$F[x(t-a)] = e^{-j\omega a}X(j\omega)$$

PROOF:

$$F[x(t-a)] = \int_{-\infty}^{\infty} x(t-a)e^{-j\omega t}dt$$

Let $p = t-a$

$t = p+a$

$dt = dp$

$$F(x(t-a)) = \int_{-\infty}^{\infty} x(p)\, e^{-j\omega(p+a)}\, dp$$

$$= \int_{-\infty}^{\infty} x(p)\, e^{-j\omega p}\cdot e^{-j\omega a}\, dp$$

$$= e^{-j\omega a}\int_{-\infty}^{\infty} x(p)e^{-j\omega p}\, dp$$

$$= e^{-j\omega a}X(j\omega)$$

3. Frequency shifting property:

If $F[x(t)]$ is $X(j\omega)$ then $[\mathcal{L}\{e^{at}f(t)\} = F(s+a)$

$$F[e^{jat}x(t)] = X(j(\omega-a))$$

PROOF:

$$F[e^{jat}x(t)] = \int_{-\infty}^{\infty} e^{jat}x(t)\, e^{-j\omega t}dt$$

$$\int_{-\infty}^{\infty} e^{-jt[\omega-a]}$$

16.10.2025

4. Time Reversal property:

If $F[x(t)] = X(j\omega)$ then $F[x(-t)] = X(-j\omega)$

PROOF:

$$F[x(-t)] = \int_{-\infty}^{\infty} x(-t)\, e^{-j\omega t}\, dt$$

$$p = -t, \quad t = -p \Rightarrow dt = dp$$

$$t = -\infty \Rightarrow p = \infty$$

$$t = \infty \Rightarrow p = -\infty$$

$$F[x(-t)] = \int_{\infty}^{-\infty} x(p)\, e^{j\omega p}(-dp)$$

$$= \int_{-\infty}^{\infty} x(p)\, e^{j\omega p}\, dp$$

$$= X(-j\omega)$$

5. Time differentiation property:

If $F[x(t)] = X(j\omega)$ then $F\left[\frac{d}{dt}x(t)\right] = j\omega X(j\omega)$

PROOF:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

Apply differentiation on both sides

$$\frac{d}{dt}x(t) = \frac{d}{dt}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega\right]$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\frac{d(e^{j\omega t})}{dt}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}(j\omega)\, d\omega$$

$$\frac{d(x(t))}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} j\omega X(j\omega)\, e^{j\omega t}\, d\omega$$

$$d(x(t)) = \frac{1}{2\pi}[\ldots]$$

6. Time integration property :

If $F[x(t)] = X(j\omega)$ then $F\left[\int x(t)\,dt\right] = X(j\omega)/j\omega$

PROOF :

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

Apply integration on both sides

$$\int x(t)\,dt = \int\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega\right] dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\left[\int (e^{j\omega t})\,dt\right] d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\,\frac{e^{j\omega t}}{j\omega}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{X(j\omega)}{j\omega}\right) e^{j\omega t}\, d\omega$$

$$= F^{-1}\left[\frac{X(j\omega)}{j\omega}\right]$$

$$F\left[\int x(t)\,dt\right] = \frac{X(j\omega)}{j\omega}$$

7. Frequency differentiation Property or multiplication with t.

If $F[x(t)] = X(j\omega)$ then $F[t\cdot x(t)] = j\,\frac{d}{d\omega} X(j\omega)$

PROOF:

$$F[x(t)] = X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\frac{d}{d\omega} X(j\omega) = \frac{d}{d\omega}\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} x(t)\,\frac{d}{d\omega}\, e^{-j\omega t}\, dt$$

$$\frac{d}{d\omega} X(j\omega) = -j \int_{-\infty}^{\infty} t\, x(t)\, e^{-j\omega t}\, dt$$

$$= -j\, F[t\,x(t)]$$

$$F[t\,x(t)] = j \frac{d}{d\omega} X(j\omega)$$

8. Time scaling property: $[[x(at)] = \frac{1}{a} X[\frac{s}{a}]$

if $F[x(t)] = X(j\omega)$ then $F[x(at)] = \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$

PROOF:

~~for a>0~~ for a>0

$$F[x(at)] = \int_{-\infty}^{\infty} x(at)\, e^{-j\omega t}\, dt$$

$p = at \quad t = \frac{p}{a} \Rightarrow dt = \frac{dp}{a}$

$$F[x(at)] = \int_{-\infty}^{\infty} x(p)\, e^{-j\omega \frac{p}{a}}\, \frac{dp}{a}$$

$$= \frac{1}{a} \int_{-\infty}^{\infty} x(p)\, e^{-\frac{j\omega}{a} p}\, dp$$

$$= \frac{1}{a} X\left[\frac{j\omega}{a}\right]$$

for a<0

$$F[x(at)] = +\frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$

∴ for both cases

$$F[X(at)] = \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$

17.10.2025

9. Time convolution property:

If $F[x_1(t)] = X_1(j\omega)$ and $F[x_2(t)] = X_2(j\omega)$

then $F[x_1(t) * x_2(t)] = X_1(j\omega) X_2(j\omega)$

PROOF:

$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(p) \cdot$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} x_1(\tau) \int_{-\infty}^{\infty} x_2(t-\tau)\, e^{-j\omega t}\, dt\, d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau) \left(\int_{-\infty}^{\infty} x_2(p)\, e^{-j\omega(p+\tau)}\, dp \right) d\tau$$

$p = t - \tau$
$t = p + \tau$
$dt = dp$

$$= \int_{-\infty}^{\infty} x_1(\tau) \int_{-\infty}^{\infty} x_2(p)\, e^{-j\omega p} e^{-j\omega\tau}\, dp\, d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\, e^{-j\omega\tau} \int_{-\infty}^{\infty} x_2(p)\, e^{-j\omega p}\, dp\, d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\, e^{-j\omega\tau}\, X_2(j\omega)\, d\tau$$

$= X(j\omega)$

$$= X_2(j\omega) \int_{-\infty}^{\infty} x_1(\tau) e^{-j\omega\tau}\, d\tau$$

$$= X_1(j\omega)\, X_2(j\omega)$$

10. Frequency convolution property :

If $F[x_1(t)] = X_1(j\omega)$ and $F[x_2(t)] = X_2(j\omega)$ then

$$F[x_1(t)\, x_2(t)] = \frac{1}{2\pi}[x_1(j\omega) * x_2(j\omega)]$$

11. Conjucate property :

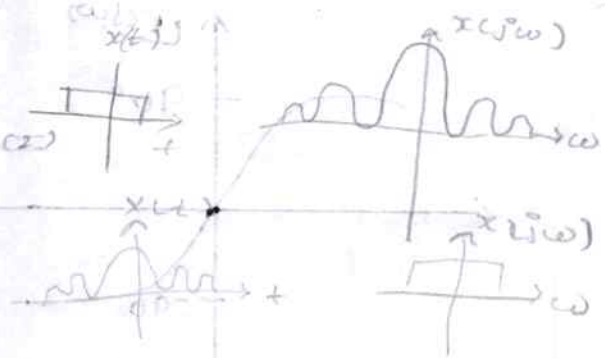
If $F[x(t)] = X(j\omega)$ then

$$F[x^*(t)] = X^*(-j\omega)$$

12. Symmetry property :

If $F[x(t)] = X(j\omega)$

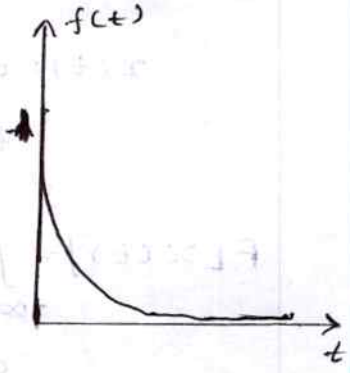
$$F[X(t)] = \frac{1}{2\pi} x(j\omega)$$



FOURIER TRANSFORM OF IMPORTANT SIGNALS :

1. Fourier transform single sided decaying exponential function :

$$f(t) = 0 \quad t < 0$$
$$= e^{at} \quad t \geq 0$$



$$F[f(t)] = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} f(t)\, e^{-j\omega t}\, dt + \int_{0}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} 0\, dt + \int_{0}^{\infty} e^{at} e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-t(a+j\omega)}\, dt = \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty}$$

$$= -\frac{1}{a+j\omega} [0 - 1]$$

$$= \frac{1}{a+j\omega}$$

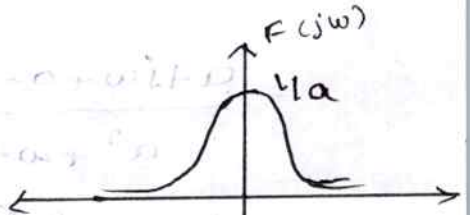
$\frac{1}{s+a} \hookrightarrow j\omega$

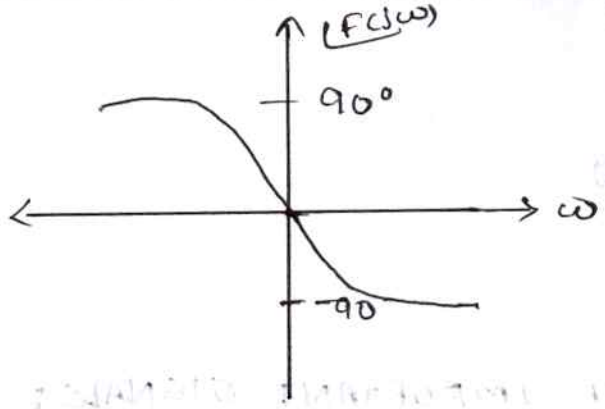
Frequency spectrum
- magnitude spectrum $|X(j\omega)|$ vs ω
- phase spectrum $(X(j\omega))$ vs ω

$$F[f(t)] = \frac{1}{a+j\omega}$$

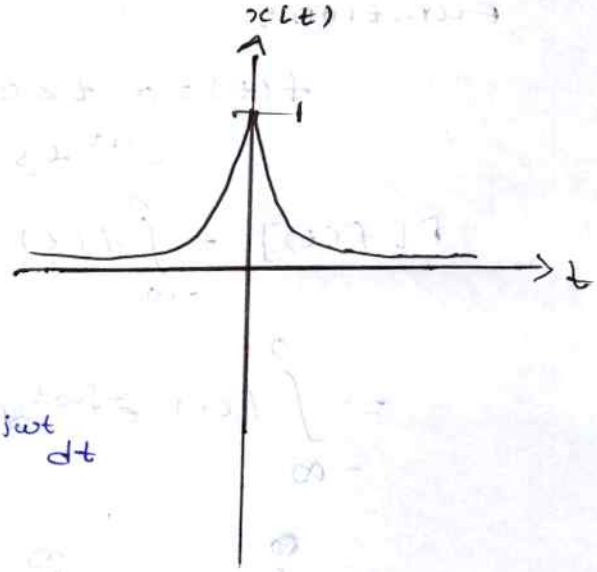
$a + jb$

$\tan^{-1}[b/a]$ $\quad |F(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$





2. Double sided exponential function:



$$x(t) = e^{at} \quad \text{for } t < 0$$
$$= e^{-at} \quad t \geq 0$$

$$F[x(t)] = \int_{-\infty}^{\infty} x[t]\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} x(t)\, e^{-j\omega t}\, dt + \int_{0}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

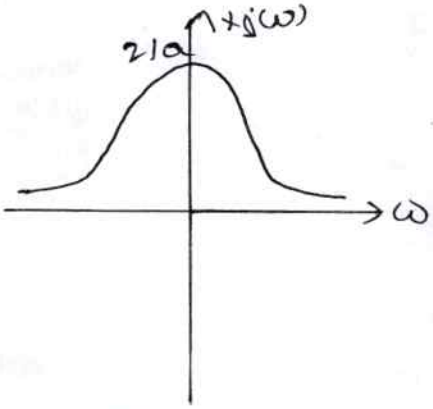
$$= \int_{-\infty}^{0} e^{t(a-j\omega)}\, dt + \int_{0}^{\infty} e^{-t[a+j\omega]}\, dt$$

$$= \left[\frac{e^{t(a-j\omega)}}{a-j\omega} \right]_{-\infty}^{0} + \left[\frac{e^{-t[a+j\omega]}}{a+j\omega} \right]_{0}^{\infty}$$

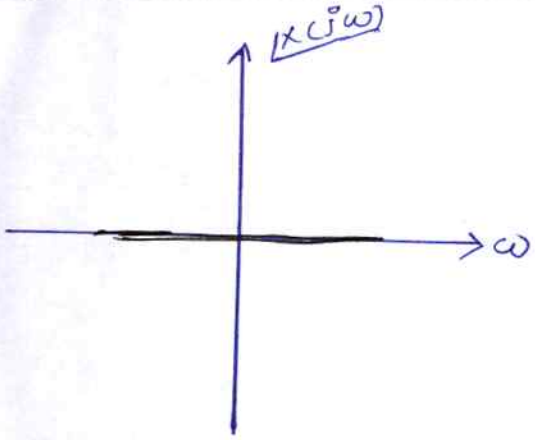
$$= \left[\frac{1}{a-j\omega} - 0 \right] + \left[\frac{1}{a+j\omega} \right]$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$= \frac{a+j\omega+a-j\omega}{a^2+\omega^2}$$



magnitude spectrum



Department of Electrical & Electronics Engineering

II B.Tech I Semester

Network Analysis and Synthesis (24AEEE21T)
(Electrical and Electronics Engineering)

Unit-5 (Network Functions and Synthesis)

Syllabus:

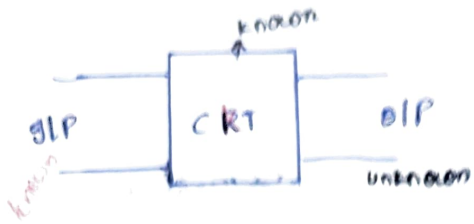
Network functions- necessary conditions for driving point function-necessary conditions for transfer function –Hurwitz polynomials – positive Real functions - definitions and properties - synthesis of single port networks (RL, RC and LC networks.

UNIT - 5

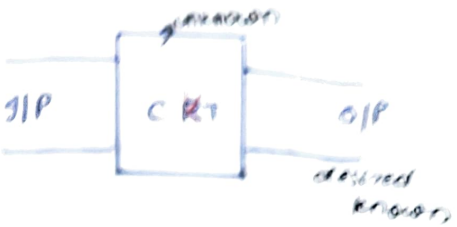
NETWORK SYNTHESIS

24-10-25

Analysis :



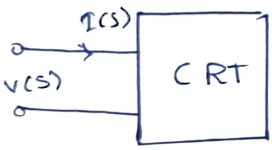
Synthesis :



NETWORK FUNCTIONS:

Network Function = $\frac{\text{Laplace transform of output}}{\text{Laplace transform of Input}}$

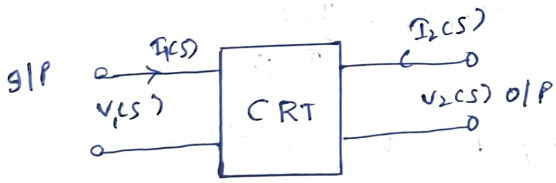
SINGLE PORT NETWORK:



$$Y(s) = \frac{I(s)}{V(s)} \qquad Z(s) = \frac{1}{Y(s)}$$

$$Z(s) = \frac{V(s)}{I(s)}$$

TWO PORT NETWORK:



N/w fn → driving point function
N/w fn → Tr function

Driving point function = $\frac{\text{O/P of same port}}{\text{I/P of same port}}$

1. Driving point function of impedance of I/P port

$$Z_{11} = \frac{V_1(s)}{I_1(s)}$$

2. Driving point function of admittance of I/P port

$$Y_{11} = \frac{I_1(s)}{V_1(s)}$$

3. Driving point function of impedance at o/p port

$$Z_{22} = \frac{V_2(s)}{I_2(s)}$$

4. Driving point function of admittance at o/p port

$$Y_{22} = \frac{I_2(s)}{V_2(s)}$$

Immitance = impedance (or) admittance

Transfer function = $\frac{\text{output of one port}}{\text{input of other port}}$

1. Tr. fn of impedance

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$$

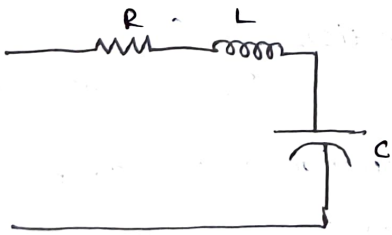
Tr fn of admittance

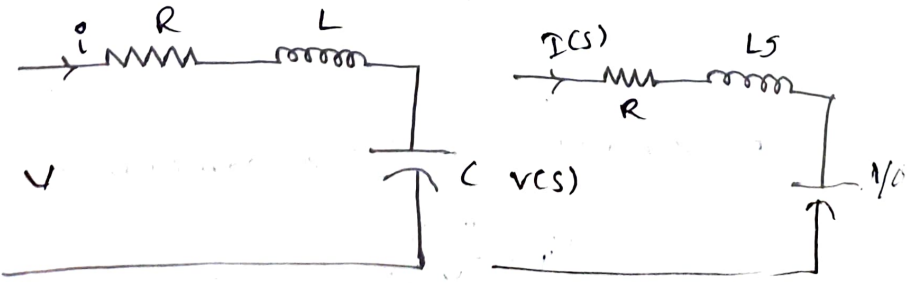
$$Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$$

Voltage ratio = $G_{21}(s) = \frac{V_2(s)}{V_1(s)}$

current ratio = $\alpha_{21}(s) = \frac{I_2(s)}{I_1(s)}$

find the driving point function of given function (or) network of impedance.





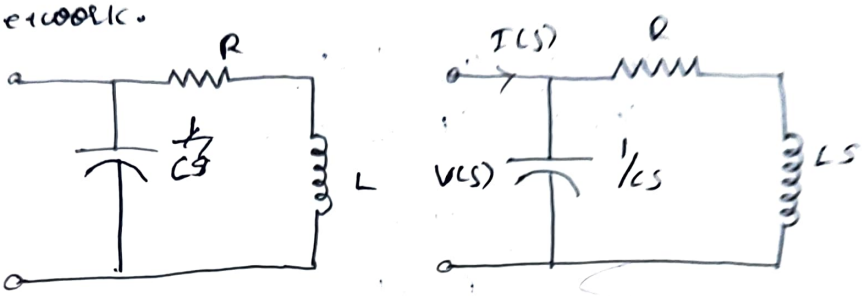
$$Z(s) = \frac{V(s)}{I(s)}$$

$$V(s) = \left(R + LS + \frac{1}{CS}\right) I(s)$$

$$= \frac{(R + LS + 1/CS)\, I(s)}{I(s)}$$

$$Z(s) = \frac{LCS^2 + SRC + 1}{SC}$$

Find the driving point function of given network.



$$Z_1 = \frac{1}{CS}, \quad Z_2 = R + LS$$

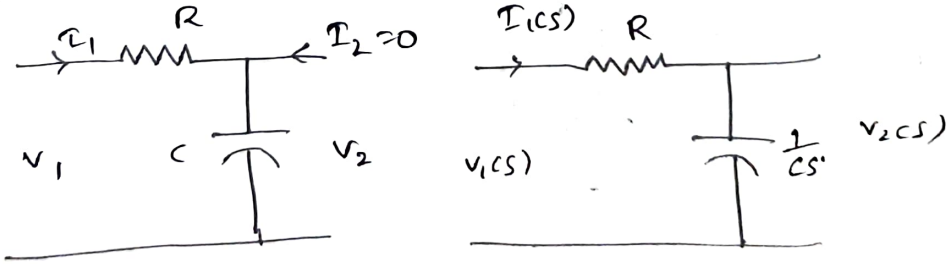
$$Z(s) = \frac{Z_1 * Z_2}{Z_1 + Z_2}$$

$$= \frac{R + \frac{LS}{CS}}{R + LS + \frac{1}{CS}}$$

$$= \frac{\frac{R + LS}{CS}}{\frac{1}{CS}\left[\frac{RCS}{CS} + \frac{CLS^2}{CS} + 1\right]}$$

$$Z(s) = \frac{LS + R}{LCS^2 + RCS + 1}$$

find the driving point voltage Transfer funtion of given function.



$$V_1(s) = \left(R + \frac{1}{CS}\right) I_1(s)$$

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$$

$$= R + \frac{1}{Cs}$$

$$= \frac{RCs+1}{Cs}$$

From eqn ① $I_1(s) = \frac{V_1(s)}{R + \frac{1}{Cs}}$

$$V_2(s) = I_1(s) \frac{1}{Cs} \quad \text{②}$$

divide eq ② with eq ①

$$\frac{V_2(s)}{V_1(s)} = \frac{I_1(s) \cdot 1/Cs}{(R + \frac{1}{Cs}) I_1(s)}$$

$$= \frac{1/Cs}{\frac{RCs+1}{Cs}}$$

$$= \frac{1}{RCs+1}$$

from eq ②

$$Z_{21} = \frac{V_2(s)}{I_1(s)} = \frac{1}{Cs}$$

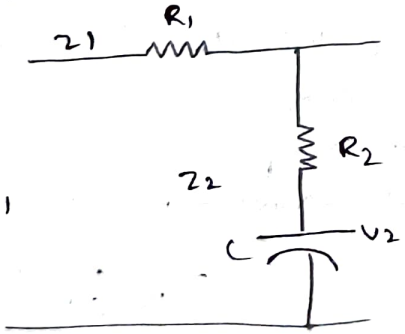
voltage division rule

$$V_{21}(s) = \frac{V_{Z_1}(s)}{Z_1(s) + Z_2(s)}$$

$$V_2(s) = \frac{V_1(s)\, Z_2}{Z_1 + Z_2}$$

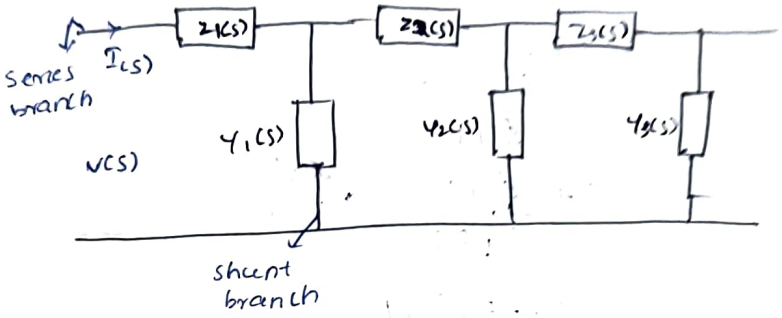
find the voltage transfer ratio of given circuit network.

1.



NOTE :

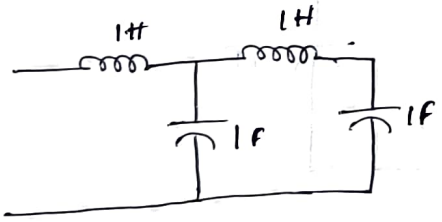
LADDER NETWORKS :

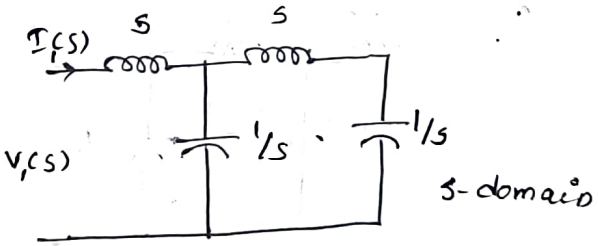


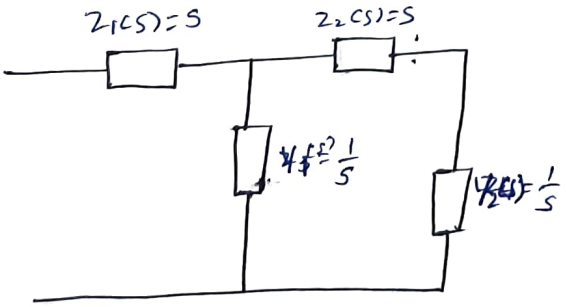
driving point impedance $= Z_1(s) + \cfrac{1}{Y_1(s) + \cfrac{1}{Z_2(s) + \cfrac{1}{Y_2(s)}}}$

find the driving point impedance of ladder function.

a.







$$\frac{V_1(s)}{I_1(s)} = Z_1(s) + \cfrac{1}{Y_1(s) + \cfrac{1}{Z_2(s) + \cfrac{1}{Y_2(s)}}}$$

$$= s + \cfrac{1}{s + \cfrac{1}{s + \frac{1}{s}}}$$

$$= s + \cfrac{1}{s + \cfrac{1}{\cfrac{s^2+1}{s}}}$$

$$= s + \cfrac{1}{s + \cfrac{s}{s^2+1}}$$

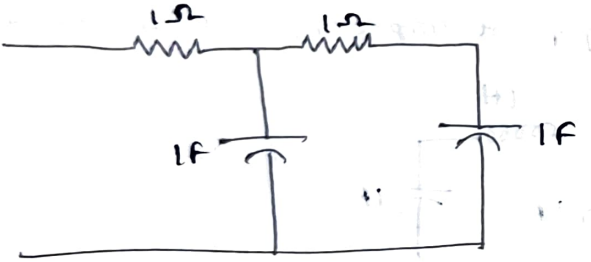
$$= s + \cfrac{1}{\cfrac{s^3+s+s}{s^2+1}}$$

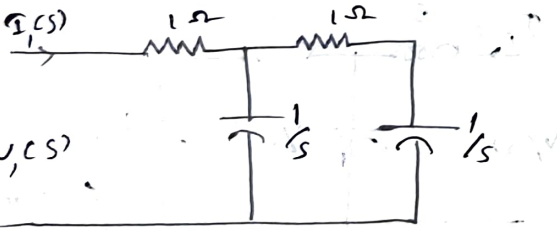
$$= s + \cfrac{s^2+1}{s^3+2s}$$

$$= \frac{s^4+2s^2+s^2+1}{s^3+2s}$$

$$= \frac{s^4+3s^2+1}{s^3+2s}$$

b.







PROPERTIES OF DRIVING POINT IMMITTANCE AND TRANSFER FUNCTIONS :

$$N(s) = \frac{P(s)}{\Phi(s)} \qquad N(s) = \frac{s^4 + 3s^2 + 1}{s^3 + 2s}$$

$P(s)$ & $\Phi(s)$ – polynomials of 's'

Zero's are the values of 's' for which $N(s) = 0$

example :

$$N(s) = \frac{(s+1)(s+4)}{s(s+2)(s+3)}$$

Zero's are roots of eqn numerator of $T \cdot fn = 0$

$$P(s) = 0$$

$$(s+1)(s+4) = 0$$

$s = -1, -4 \Rightarrow$ zero's

Pole's are the values of 's' for which $N(s) = \infty$

example :

$$N(s) = \frac{(s+1)(s+4)}{s(s+2)(s+3)}$$

Pole's are roots of eqn denominator of $T \cdot fn = 0$

$$\Phi(s) = 0$$

$$s(s+2)(s+3) = 0$$

$s = 0, -2, -4 \Rightarrow$ poles

example :

$$N(s) = \frac{s(s+1)}{(s+2)(s+3)(s+4)}$$

Zero's : $s(s+1) = 0$ $\quad$ Pole's : $(s+2)(s+3)(s+4) = 0$

$s = 0, -1$ $\quad$ $s = -2, -3, -4$

Poles – real

complex conjugate

Simple pole - not repeated
multiple pole - repeated

Degree of polynomial - highest power of 's'

$$N(s) = \frac{s(s+1)}{(s+2)(s+3)(s+4)}$$

$$= 3$$

PROPERTIES:

SNO	Driving - point Immittance function	Transfer function
1.	coefficients in polynomials $P(s)$ & $\phi(s)$ of $N(s) = \frac{P(s)}{\phi(s)}$ must be real and positive $N(s) = \frac{s^2+2s+1}{s^3+s^2+3s+4} = \frac{P(s)}{\phi(s)}$	coefficient in polynomials $P(s)$, $\phi(s)$ must be real; coefficient of $\phi(s)$ must be positive. $N(s) = \frac{s^2+2s+1}{s^3+s^2+3s+4}$
2.	complex poles and zeros must occurs in conjucate pair $N(s) = \frac{s^2+2s+1}{s^3+s^2+3s+4}$ Zero's $= s^2+2s+2=0$ $s = -1 \pm j1$ $= -1+j, -1-j$	complex poles and zeros must occurs in conjucate pair $N(s) = s^2+2$ Zero's $= s^2+2s+2=0$ $s = -1 \pm j$ $= -1+j, -1-j$
3.	Real parts of all poles and zeros ~~must~~ not be positive [0 or -ve] $N(s) = \frac{s^2+2s+2}{s(s+1)(s+2)}$ Poles $= 0, -1, -2$	Real parts of all poles must not be positive. must be -ve or 0.

SNO

4. If the real part of zero, the pole & zero must be simple [includes the origin]

If the real part is zero, the pole must be simple [includes origin].

$$N(s) = \frac{s^2+2s+2}{s^2(s+1)(s+2)^2}$$

Zero = $s = -1, \pm j$

Poles = $s = 0, 0, -1, -2, -2$

5. P(s) and Q(s) must not have missing term blw highest and lowest degree unless all even or all odd terms are missing

$$N(s) = \frac{s^4+3s^2+1}{s^3+2s}$$

Q(s) shouldn't have missing terms blw highest & lowest degree, unless all even or all odd are missing. P(s) may have missing terms, and some coefficients may be negative

6. Degree's of P(s) and Q(s) may differ by 0 ~~and~~ or 1

Degree of P(s) can be as small as zero [indepent of Q(s))

7. lowest-degree terms in P(s) and Q(s) may differ by 1 degree at most.

$$\frac{s^2+2s}{s^3+2s^2+s+1}$$ +1

For G_{n1} & α_{n1}: max degree of P(s) = degree of Q(s)

For Z_{21} and Y_{21}: the maximum degree of P(s) is the degree of Q(s) plus one.

$G_{21} = \frac{V_2}{V_1}$ $\alpha_{21} = \frac{I_2}{I_1}$

examples:

1. $\frac{4s^4+s^2-3s+1}{s^3+2s^2+2s+40}$

It doesnot satisfies the the properties of driving-point immittance

2. $\frac{15(s^3+2s^2+3s+2)}{s^4+6s^3+8s^2}$

It does not satisfies the the properties of driving point immittance

HURWITZ POLYNOMIAL:

A polynomial is said to be Hurwitz if the following conditions are satisfied:

P(s) is real when s is real.

The roots of P(s) have real parts which are zero or negative

NOTE: Here P(s) is any polynomial in s

$$P(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \cdots + a_{n-1} s + a_n$$

example:

x^2+2x+1

The only real solution is -1, as it factors to:

$(x+1)^2$

PROPERTIES:

1. All coefficients of 's' terms must be real and positive.

2. Both the odd and even parts of the Hurwitz polynomial have roots on the imaginary axis only

$P(s) = s^2+2s+1$

odd parts $M(s) = 2s$ roots $s=0$

'even' part $N(s) = s^4 + 1 = 0$

$s^2 = -1$

$s = \pm j1$

3. The continued fraction expansion of the ratio of the even to odd parts of Hurwitz polynomial gives all positive quotient terms.

example:

$P(s) = s^4 + 3s^3 + 4s^2 + 3s + 1$

odd part $M(s) = 3s^3 + 3s$

even part $N(s) = s^4 + 4s^2 + 1$

$$3s^3 + 3s \overline{)\, s^4 + 4s^2 + 1 \,(\, \frac{s}{3}}$$

$$s^4 + s^2$$

$$3s^2 + 1 \overline{)\, 3s^3 + 3s \,(\, s}$$

$$3s^3 + s$$

$$2s \overline{)\, 3s^2 + 1 \,(\, \frac{3}{2}s}$$

$$3s^2$$

$$1 \overline{)\, 2s \,(\, 2s}$$

$$2s$$

$$0$$

4. The polynomials satisfies the condition of Hurwitz, the polynomial then is Hurwitz to any even multiplicative factor $w(s)$, i.e., if $P_1(s) = w(s)\, P(s)$ then if $P(s)$ is Hurwitz, $P_1(s)$ must be Hurwitz.

5. If the ratio of polynomial P(s) and its derivative P'(s) gives a continued fraction expansion with all positive coefficients, then P(s) is Hurwitz. In case the polynomial is either only even or only odd, the method of continued fraction is not possible, and this method is beneficial.

check the polynomial $s^4+s^3+2s^2+3s+2$ is Hurwitz or not

$$P(s) = s^4+s^3+2s^2+3s+2$$

$$M(s) = s^4+2s^2+2$$

$$N(s) = s^3+3s$$

$$Z(s) = \frac{M(s)}{N(s)} = \frac{s^4+2s^2+2}{s^3+3s}$$

using continued fraction method, the function Z(s) is tested as following.

$$s^3+3s \;]\; s^4+2s^2+2 \;[\; s$$
$$s^4+3s^2$$
$$-s^2+2 \;]\; s^3+3s \;[\; -s$$
$$s^3-2s$$
$$5s$$

Since the continued fraction contain negative quotient. Hence the given polynomial is not Hurwitz

25.10.2025

Check whether the given polynomial is Hurwitz or not

$$P(s) = s^4 + 11s^3 + 39s^2 + 51s + 20$$

$$P(s) = s^4 + 11s^3 + 39s^2 + 51s + 20$$

$$M(s) = s^4 + 39s^2 + 20$$

$$N(s) = 11s^3 + 51s$$

$$Z(s) = \frac{M(s)}{N(s)} = \frac{s^4 + 39s^2 + 20}{11s^3 + 51s}$$

Using continued fraction method, the function $Z(s)$ is tested as following

$$11s^3 + 51s \overline{)\, s^4 + 39s^2 + 20 \,(\, \frac{s}{11}}$$

$$s^4 + \frac{51}{11}s^2$$

$$\frac{378}{11}s^2 + 20 \overline{)\, 11s^3 + 51s \,(\, \frac{121}{378}s}$$

$$11s^3 + \frac{1210}{189}s$$

$$\frac{8429}{189}s \overline{)\, \frac{378}{11}s^2 + 20 \,(\, \frac{378 \times 189}{11 \times 8429}s}$$

$$\frac{378}{11}s^2$$

$$20 \overline{)\, \frac{8429}{189}s \,(\, \frac{8429}{189 \times 20}s}$$

$$\frac{8429}{189}s$$

$$0$$

Hence, the polynomial is Hurwitz

$$\psi(s) = \frac{M(s)}{N(s)} = \frac{s}{11} + \frac{1}{0.325 + \cdots}$$

All the coefficient of continued fraction expansion are +ve. So it is Hurwitz

(b) $P(s) = s^4 + s^3 + 6s^2 + 3s + 6$

$M(s) = s^4 + 6s^2 + 6$

$N(s) = s^3 + 3s$

$s^3+3s)\,s^4+6s^2+6\,(\,s$

$s^4 + 3s^2$

$3s^2+6)\,s^3+3s\,(\,\frac{s}{3}$

$s^3 + 2s$

$s)\,3s^2+6\,(\,3s$

$3s^2$

$6)\,s\,(\,\frac{s}{6}$

s

0

$$\psi(s) = \frac{M(s)}{N(s)} = s + \cfrac{1}{\frac{s}{3} + \cfrac{1}{3s + \cfrac{1}{\frac{s}{6}}}}$$

All the coefficients of continued fraction expansion are positive. Hence it is positive.

(c) $P(s) = s^4 + 3s^2 + 2$

$P(s) = s^4 + 3s^2 + 2$

$\frac{d}{ds}(P(s)) = 4s^3 + 6s$

$\frac{P(s)}{P'(s)} = \frac{s^4 + 3s^2 + 2}{4s^3 + 6s}$

$4s^3+6s)\ s^4+3s^2+2\ (\frac{1}{4}s$

$s^4+\frac{3}{2}s^2$

$\frac{3}{2}s^2+2)\ 4s^3+6s\ (\frac{8}{3}s$

$4s^3+4s$

$2s)\ \frac{3}{2}s^2+2\ (\frac{3}{4}s$

$\frac{3}{2}s^2$

$2)\ 2s\ (s$

$2s$

0

All the coefficients of continued fraction expansion are positive, Hence the polynomial is Hurwitz.

d. $P(s) = s^4+s^3+3s^2+2s+2$.

$P(s) = s^4+s^3+3s^2+2s+2$

$M(s) = s^4+3s^2+2$

$N(s) = s^3+2s$

$s^3+2s)\ s^4+3s^2+2\ (s$

s^4+2s^2

$s^2+2)\ s^3+2s\ (s$

s^3+2s

0

$\psi(s) = \frac{M(s)}{N(s)} = s+\frac{1}{s}$

All coefficients of continued fraction expansion are positive, Hence the polynomial is Hurwitz

$P(s) =$

$= (P_1(s))(w(s))$

$w(s) = s^2+2$

$P_1(s) = \frac{s^4+s^3+3s^2+2s+2}{s^2+2}$

$s^2+2)\ s^4+s^3+3s^2+2s+2\ (s^2+s+1$

$s^4+\quad +2s^2$

s^3+s^2+2s+2

$s^3+\quad +2s$

s^2+2

s^2+2

0

$\therefore P(s) = (s^2+s+1)(s^2+2)$

If both (s^2+s+1) & (s^2+2) are Hurwitz

$P(s)$ is hurwitz

$P_1(s) = s^2+s+1$

$M(s) = s^2+1$

$N(s) = s$

$P_1(s)$ is Hurwitz

$s)\ s^2+1\ (s$

s^2

$1)\ s\ (s$

s

0

NOTE: as^2+bs+c

→ Hurwitz if a,b,c is +ve

METHOD-2: Routh Array

s^n	a_0	a_2	a_4
s^{n-1}	a_1	a_3	a_5
	b_1	b_2	b_3
	c_1	c_2	c_3
	⋮		
s^0	a_n		

$b_1 = \dfrac{a_1a_2 - a_0a_3}{a_1}$

$b_2 = \dfrac{a_1a_4 - a_0a_5}{a_1}$

$c_1 = \dfrac{b_1a_3 - a_1b_2}{b_1}$

All the first column elements are +ve then it is Hurwitz

$P(s) = s^4 + 11s^3 + 39s^2 + 31s + 20$

s^4	1	39	20
s^3	11	31	0
s^2	34.363	20	
s	44.59	0	
s^0	20		

$P(s) = s^4 + 3s^2 + 2$ $\quad p(s) = 4s^3 + 6s$

s^4	1	3	2
s^3	4	6	0
s^2	6/5	2	
s	0.66	0	
s^0	2		

$P(s) = s^4 + s^3 + 3s^2 + 2s + 2$

s^4	1	3	2
s^3	1	2	0
s^2	1	2	
s	0	0	
s^0	2		

$w(s) = s^2 + 2$

$w(s) = 2s$

27·10·2025

POSITIVE REAL FUNCTION:

Real function $F(s)$ whose real part is +ve for values of s with +ve real part is called a positive real function.

Any driving point impedance

$$F(s) = \frac{A(s)}{B(s)}$$

$$= \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

$F(s)$ is +ve real function if becomes

i) $F(s)$ is real for 's' real

ii) $A(s) \times B(s)$ is Hurwitz

iii) If $F(s)$ has poles on $j\omega$ axis the poles should be simple.

iv) Real part of $F(j\omega) \geq 0$ for all ω

Ex: $s = j\omega$

$$F(s) = \frac{s+1}{s+2} = \frac{j\omega+1}{j\omega+2}$$

$$= \frac{j\omega+1}{j\omega+2} \times \frac{j\omega-1}{j\omega-2}$$

$$= \frac{(j\omega)^2 - 2j\omega + j\omega - 2}{(j\omega)^2 - 2^2}$$

$$= \frac{-\omega^2 - j\omega + 2}{-\omega^2 - 4}$$

$$= \frac{\omega^2 + j\omega + 2}{\omega^2 + 4}$$

$$= \frac{\omega^2 + 2}{\omega^2 + 4} \geq 0 \quad \text{for all } \omega$$

PROPERTIES OF PR FUNCTION:

1. Both A(s) and B(s) polynomials are Hurwitz. They may have factors of the forms $(s^2+\omega^2)$ only simple poles with +ve real residues can exists on the $j\omega$ axis.
2. The highest and lowest power of A(s) and B(s) may differs by almost by unity. This condition, prohibits multiple poles and zero at $s=\infty$ and at $s=0$ respectively.

$$\frac{s+2}{s^2(s+1)}$$

3. If F(s) is a positive real function, the reciprocal of F(s) is also a PR function.
4. The sum of PR function is also a preal function

PROCEDURE FOR TESTING PR FUNCTION:

Necessary conditions:

1. All coefficient of A(s), B(s) polynomials must be +ve reals.
2. The highest and lowest power of A(s) and B(s) may differs atmost by unity. This condition prohibits multiple poles and zero at $s=\infty$ and at $s=0$ respectively.
3. There should be no missing term between higher and lower starters of A(s) and B(s) polynomials less they are even or odd polynomial.

Sufficient conditions:

1. Both A(s), B(s) must be Hurwitz
2. Real part of $F(j\omega) \geq 0$ for all ω

$$F(s) = \frac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)}$$

$M_1(s)$ = even part of $A(s)$

$N_1(s)$ = odd part of $A(s)$

$M_2(s)$ = even part of $B(s)$

$N_2(s)$ = odd part of $B(s)$

$$\frac{M_1(s) + N_1(s)}{M_2 + N_2} \times \frac{M_2 - N_2}{M_2 - N_2}$$

$$= \frac{(M_1 M_2 - N_1 N_2) - (M_1 N_2 + M_2 N_1)}{M_2^2 - N_2^2}$$

(even) (odd)

$$= \frac{M_1 M_2 - N_1 N_2}{M_2^2 - N_2^2} + \frac{M_2 N_1 - M_1 N_2}{M_2^2 - N_2^2}$$

$$\text{Real}[F(s)] = \frac{M_1 M_2 - N_1 N_2}{M_2^2 - N_2^2}$$

denominator is always positive

Test $F(s) = \frac{s+2}{s+1}$ is a PR function or not

Given

$$F(s) = \frac{s+2}{s+1}$$

coefficients of $A(s)$ and $B(s)$ are +ve

The difference b/w the highest and lowest degree is zero.

No terms missing.

sufficient condition:

Both $A(s)$ & $B(s)$ is Hurwitz

$s = -2, -1$

$F(s) = \frac{s+2}{s+1}$ $M_1 = 2,\ N_1 = 3$

$M_2 = 1,\ N_2 = 5$

$M_1M_2 - N_1N_2 = 2 - 5^2$

sub $s = j\omega$

$D(j\omega) = 2 + (j\omega)^2$

$\therefore 2 + \omega^2 \geq 0$ for all ω

$\therefore$ the given function is PR function

Test $F(s) = \frac{s+6}{s^2+4s+2}$ is a PR function or not

Given

$F(s) = \frac{s+6}{s^2+4s+2}$

coefficients of $A(s)$, $B(s)$ are +ve

The difference btw the highest and lowest power is one.

No missing terms.

Condition:

Both $A(s) + B(s)$ are Hurwitz

ax^2+bx+c is Hurwitz polynomial for +ve values of a, b, c

$s = -6,$

$s = -2 \pm \sqrt{2}$

$M_1 = 6 \quad N_1 = s$

$M_2 = s^2+2 \quad N_2 = 4s$

$M_1M_2 - N_1N_2 = 6(s^2+2) - s(4s)$

$= 6s^2 + 12 - 4s^2$

$= 2s^2 + 12$

$= 2[s^2+6]$

Sub $s = j\omega$

$D(j\omega) = 2[(j\omega)^2+6] = 2[-\omega^2+6] = 12 - 2\omega^2$

for some values of ω

$D(j\omega) \not\geq 0$

The given fn is not +ve real function

Test $F(s) = \frac{s^2+10s+4}{s+2}$

Necessary:

All coefficient of $A(s), B(s)$ are positive.

The difference b/w highest and lowest power is one

No missing terms

Sufficient condition:

Both $A(s)$ and $B(s)$ are Hurwitz

$$F(s) = \frac{s^2+10s+4}{s+2}$$

$M_1 = s^2+4 \quad N_1 = 10s$

$M_2 = 2 \quad N_2 = s$

$M_1M_2 - N_1N_2 = (s^2+4)2 - s(10s)$

$= 2s^2+8-10s^2$

$= -8s^2+8$

Sub $s = j\omega$

$D(s) = D(j\omega)$

$= -8(j^2\omega^2)+8$

$= -8(-1)\omega^2 = 8+8\omega^2 \geq 0$

~~for~~

∴ Given function is PR function.

Test: $F(s) = \frac{s^2+2s+36}{s^2+3s+49}$ is PR function or not

Sufficient:

Both $A(s)$ & $B(s)$ are Hurwitz

$$F(s) = \frac{s^3 + 2s + 36}{s^4 + 3s^2 + 49}$$

$M_1 = s^2 + 36 \qquad N_1 = 2s$

$M_2 = s^4 + 49 \qquad N_2 = 3s$

$M_1M_2 - N_1N_2 = (s^2+36)(s^4+49) - 3s(2s)$

$= s^6 + 49s^2 + 36s^4 + 1764 - 6s^2$

$= s^6 + 36s^4 + 43s^2 + 1764$

$D(s) = s^6 + 36s^4 + 43s^2 + 1764$

Put $s = j\omega$

$D(j\omega) = D(s)|_{s=j\omega}$

$= (j\omega)^6 + 36(j\omega)^4 + 43(j\omega)^2 + 1764$

$= -\omega^6 + 36\omega^4 - 43\omega^2 + 1764$

$= -\omega^6 + 36\omega^4 - 43\omega^2 + 1764$

$\therefore$ Given $F(s)$ is a PR function

Test $F(s) = \frac{s^2 + s + 6}{s^2 + s + 1}$ is a PR function or not

Necessary condition:

- All the coefficients of $A(s)$, $B(s)$ are positive for given $F(s)$
- The difference between highest and lowest power is one
- No missing terms.

Sufficient conditions:

Both $A(s)$, $B(s)$ are Hurwitz

$$F(s) = \frac{s^2+s+6}{s^2+s+1}$$

$M_1 = s^2 + 6 \quad N_1 = s$

$M_2 = s^2 + 1 \quad N_2 = s$

$$M_1M_2 - N_1N_2 = (s^2+6)(s^2+1) - (s)(s)$$

$$= s^4 + s^2 + 6s^2 + 6 - s^2$$

$$= s^4 + 6s^2 + 6$$

Put $s = j\omega$

$$D(s) = s^4 + 6s^2 + 6$$

$$= (j\omega)^4 + 6(j\omega)^2 + 6$$

$$= \omega^4 - 6\omega^2 + 6$$

$$= (\omega^2 - 3)^2 + 6 - 9$$

$$= (\omega^2 - 3)^2 - 3$$

∴ Given $F(s)$ is not a PR function

Test $F(s) = \frac{s^3+5s^2+9s+3}{s^3+4s^2+7s+9}$ is PR function or not

Sufficient condition:

Both A(s), B(s) are Hurwitz.

$$F(s) = \frac{s^3+5s^2+9s+3}{s^3+4s^2+7s+9}$$

$M_1 = 5s^2+3$, $N_1 = s^3+9s$

$M_2 = 4s^2+9$, $N_2 = s^3+7s$

$$M_1M_2 - N_1N_2 = (5s^2+3)(4s^2+9) - (s^3+9s)(s^3+7s)$$

$$= 20s^4+45s^2+12s^2+27 - [s^6+7s^4+9s^4+63s^2]$$

$$= 20s^4+45s^2+12s^2+27 - s^6-7s^4-9s^4-63s^2$$

$$= -s^6-16s^4+20s^4]_x$$

$$= -s^6+4s^4-6s^2+27$$

$s = j\omega$

$$= -((-1)+4(1)-6(-1)+27]_x$$

$$= -(j\omega)^6+4(j\omega)^4-6(j\omega)^2+27$$

$$= +\omega^6+4\omega^4+6\omega^2+27$$

∴ Given function is PR function.

test F(s) = $\frac{s^2+4}{s^3+3s^2+3s+1}$ is PR function or not

sufficient condition:

Both A(s) & B(s) are Hurwitz

$s = -jQ$

$s = -1, -1$

$M_1 = s^2+4$, $N_1 = 0$

$M_2 = 3s^2+1$, $N_2 = s^3+3s$

$M_1M_2 - N_1N_2 = (s^2+4)(3s^2+1) - 0$

$= 3s^4 + s^2 + 12s^2 + 4$

$= 3s^4 + 13s^2 + 4$

$s = j\omega$

$= 3(j\omega)^4 + 13(j\omega)^2 + 4$

$= 3\omega^4 - 13\omega^2 + 4$

$= (\omega^4 - \frac{13}{3}\omega^2 + \frac{4}{3})3$

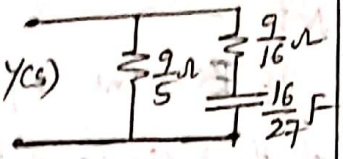
$= 3[\omega^2 - \frac{13}{6}]^2 + \frac{4}{3} - (\frac{13}{6})^2]$

$= 3[\omega^2 - 2.160]^2 - 3.36]$

for $\omega < 0$

∴ given function is not a +ve real function

$$Z_2(s) = \frac{9s+27}{16s} = \frac{9}{16} + \frac{27}{16s} = \frac{9}{16} + \frac{1}{s(16/27)}$$



The complete realization is shown in fig.

Synthesis of one-port L-C networks:-

It should be noted that the network synthesis does not provide unique solution for a given specifications. Here, any LC network can be realized by using two methods designated by the names of the inventors, Foster and Cauer.

① Foster form, which uses partial fraction of the given LC immittance function.

② Cauer form, which utilizes continued fraction expansion of the function. By virtue of the technique the network comes out of the ladder formation.

(*) While synthesizing such a network into each term in the partial fraction is associated with an element (or) a pair of elements and then these are to be connected in series.

An LC immittance function, being positive real, has poles and zeros on the $j\omega$-axis only. The partial fraction expansion of the function is expressed in general terms as

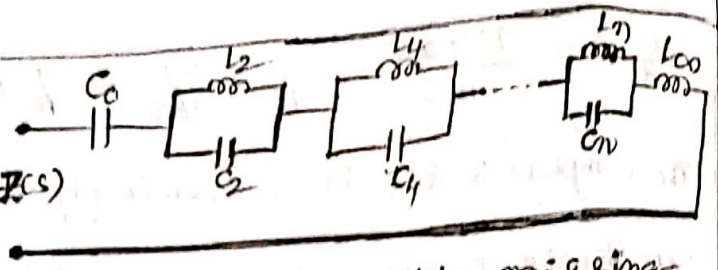
$$Z(s) = \frac{k_0}{s} + \frac{2k_2 s}{s^2+\omega_2^2} + \frac{2k_4 s}{s^2+\omega_4^2} + \cdots + \frac{2k_j s}{s^2+\omega_j^2} + \cdots + k_\infty s$$

The network in general consists of a series capacitor, a number of parallel LC networks and an inductor as shown.

The term $\frac{k_0}{s}$ represents capacitor of capacitance $C_0 = \frac{1}{k_0}$ F

The last term $k_\infty s$ corresponds to an inductor $L_\infty = k_\infty$ H.

The term $\frac{2k_j s}{s^2+\omega_j^2}$ corresponds to parallel LC tank circuit consists of a capacitor $= C_j = \frac{1}{2k_j}$ F, $L_j = \frac{2k_j}{\omega_j^2}$ H



Note :-

If Z(s) has no pole at the origin (and so has a zero), the first term in the partial fraction will be missing ie C_0 will not be present. Similarly if there is a zero rather than a pole at infinity then the inductor L_∞ will be missing

(*)(1) First Foster Form :- (Canonical form)

If the given LC-function F(s) is an impedance function Z(s) then it can be realized in the first foster form. The n/w consists of, in general, a series capacitor, a number of parallel LC networks and an inductor as shown below. In which C_0 is present if Z(s) has a pole at $s=0$ and L_∞ is present if Z(s) has a pole at infinity.

A positive Real function Z(s) of LC network being a ratio of given either even to odd (or) even to odd polynomials.

2016

(Q) Find the first Foster form of LC network for $Z(s) = \frac{s(s^2+2)}{(s^2+1)(s^2+3)}$

(A) It is clear that $P = 4$, $Z = 3$, $P - Z = 1$, Hence one zero is present at infinity, so L_∞ will not be present. one zero is located at origin instead of pole, hence C_0 will not be present.

$$Z(s) = \frac{s^3+2s}{(s^2+1)(s^2+3)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+3}, \quad A = 1/2, \; B = D = 0, \; C = 1/2$$

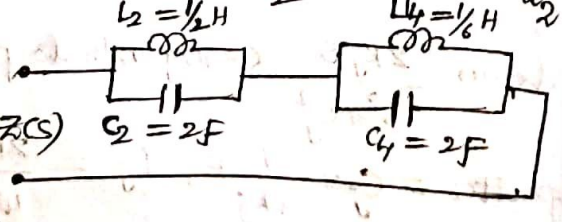
$$Z(s) = \frac{s(1/2)}{s^2+1} + \frac{s(1/2)}{s^2+3}, \quad 2K_2 = 1/2$$

$$C_2 = \frac{1}{2K_2} = 2F \qquad \omega_2^2 = 1, \quad L_2 = \frac{1}{\omega_2^2 C_2} = \frac{1}{2}H$$

$$2K_4 = 1/2$$

$$C_4 = \frac{1}{2K_4} = 2F$$

$$L_4 = \frac{1}{\omega_4^2 C_4} = \frac{1}{3 \times 2} = \frac{1}{6}H$$



(5) Synthesize the network in the first foster form for the LC imp-function

$$Z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

(A) It is clear that $P=3$, $Z=4 \Rightarrow Z-P = 4+3=1$, $\therefore$ one pole is present at infinity hence L_∞ is present, Ac. The pole is located at $s=0$ hence C_0 is present.

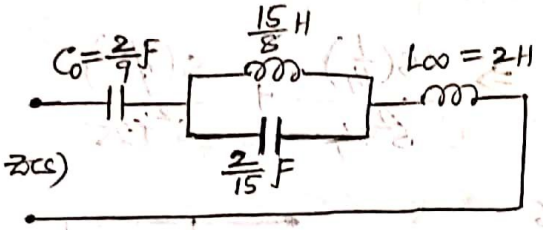
$$Z(s) = 2s + \frac{12s^2+18}{s(s^2+4)}$$

$$\frac{12s^2+18}{s(s^2+4)} = \frac{K_0}{s} + \frac{2K_2 s}{s^2+4} = \frac{(9/2)}{s} + \frac{s(15/2)}{s^2+4}$$

$$Z(s) = 2s + \frac{(9/2)}{s} + \frac{s(15/2)}{s^2+4}$$

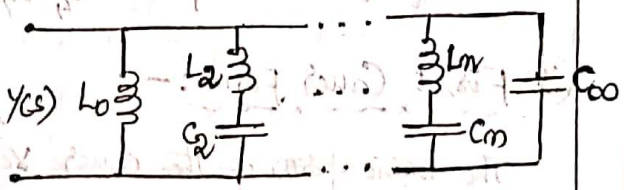
$C_0 = \frac{2}{9}F$, $L_\infty = 2H$, $2K_2 = 15/2$, $C_2 = \frac{1}{2K_2} = \frac{2}{15}F$

$$L_2 = \frac{1}{\omega_2^2 C_2} = \frac{1}{4 \times \frac{2}{15}} = \frac{15}{8}H.$$



(2) Second Foster form :-

If the given function $F(s)$ is an admittance $Y(s)$, then the partial fraction of $Y(s)$ gives us a network consisting of parallel branches are shown in fig.



L_0 is present if $Y(s)$ has a pole at $s=0$ and C_∞ is present if $Y(s)$ has a pole at ∞.

2016

(3) Let us synthesize the LC network function $Z(s) = \frac{s(s^2+2)}{(s^2+1)(s^2+3)}$ in second Foster form.

(A) Networks in the second foster form are realized for admittance LC functions.

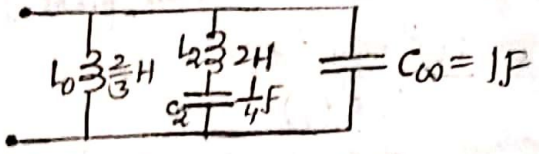
$$Y(s) = \frac{(s^2+1)(s^2+3)}{s(s^2+2)} = s + \frac{(3/2)}{s} + \frac{s(1/2)}{s^2+2}$$

$C_\infty = 1F$, $L_0 = \frac{2}{3}H$

$2k_2 = 1/2$

$L_2 = \frac{1}{2k_2} = 2H$

$C_2 = \frac{1}{\omega_2^2 L_2} = \frac{1}{2\times 2} = \frac{1}{4}F$



(9) The driving point impedance of a one port LC network is given by $Z(s) = \frac{8(s^2+4)(s^2+25)}{s(s^2+16)}$ obtain second Fosters form of network.

(A) In order to find the second Fosters form, we shall have to represent the given function into admittance function.

$$Y(s) = \frac{s(s^2+16)}{8(s^2+4)(s^2+25)}$$

$P = 4$, $Z = 3$, $P - Z = 1$, one zero at infinity $\therefore C_\infty$ is not present, one more zero at $s = 0$, $\therefore L_0$ is not present.

$$Y(s) = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+25} \Rightarrow \frac{s(1/14)}{s^2+4} + \frac{s(3/56)}{s^2+25}$$

$2k_2 = \frac{1}{14}$

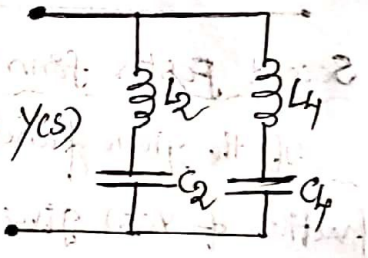
$L_2 = \frac{1}{2k_2} = 14H$

$C_2 = \frac{1}{\omega_2^2 L_2} = \frac{1}{56}F$

$2k_4 = 3/56$

$L_4 = \frac{1}{2k_4} = \frac{56}{3}$

$C_4 = \frac{1}{\omega_4^2 L_4} = \left(\frac{3}{1400}\right)F$



(3) First Cauer Form :-

The basic form of the Cauer realization is LC ladder type network. The given polynomial is arranged as numerator and denominator in the descending power of 's' and we eliminate during every step a pole at infinity.

An important property of LC - immittance function is that the numerator and denominator polynomials always differ in degree by one. Therefore there is always a pole(or) zero at infinity. By successive removal of pole at infinity, we obtain the ladder network structure.

Let us consider the case of an LC-impedance function $Z(s)$ whose numerator is of degree $2n$ and denominator of degree $(2n-1)$ giving a pole at infinity. We can remove this pole by removing sL_1 (an inductor) which is the quotient obtained by dividing the numerator by denominator. Therefore the remainder function $Z_2(s)$ is still a L-C function and is given by

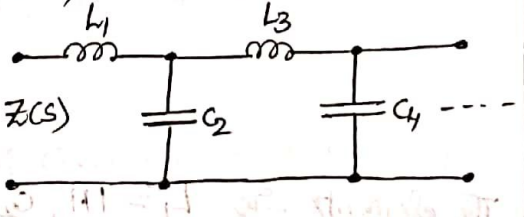
$$Z_2(s) = Z(s) - sL_1$$

Degree of denominator of $Z_2(s)$ is $(2n-1)$ where as degree of numerator becomes $(2n-2)$ because the numerator & denominator differ in degree by one. As a result $Z_2(s)$ has zero at $s = \infty$

If we invert $Z_2(s)$ to give $Y_2(s) = \frac{1}{Z_2(s)}$ will have a pole at ∞. The pole can be removed to give a capacitor sC_2 which will form shunt element of the two terminal network and a remainder term $Y_3(s)$

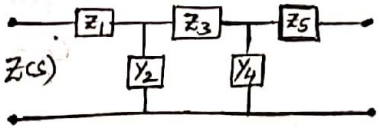
$$Y_3(s) = Y_2(s) - sC_2$$

This process of successively remove a pole by dividing and inverting continues till remainder is zero. Each time we remove a pole, we remove an inductor (or) capacitor depending upon the whether the function is impedance (or) admittance respectively. Hence final structure of ladder network is shown below, whose series arms are inductors and shunt arms are capacitors.



The synthesizing could be done by continued fraction expansion as

$$Z(s) = sL_1 + \cfrac{1}{sC_2 + \cfrac{1}{sL_3 + \cfrac{1}{sC_4 + \cfrac{1}{sL_5 + \cdots}}}}$$



In general form it can be represented as

$$Z(s) = Z_1(s) + \cfrac{1}{Y_2(s) + \cfrac{1}{Z_3(s) + \cfrac{1}{Y_4(s) + \cfrac{1}{Z_5(s) + \cdots}}}}$$

Since the expansion of Z(s) looks like a ladder, the name ladder network is arrived. The representation of the ladder network as shown above. The driving point impedance of the nature of first element of the network is fixed by nature of Z(s) at $s=\infty$. If Z(s) has a pole at $s=\infty$ first element is inductor, if it has zero at $s=\infty$, first element is capacitor.

(9) Synthesize the impedance function $Z(s)=\frac{s(s^2+3)(s^2+5)}{(s^2+2)(s^2+4)}$ using Fisrt form of Cauer network.

(A) Arrange the numerator and denominator polynomials in the descending power of s.

$$Z(s)=\frac{s^5+8s^3+15s}{s^4+6s^2+8}$$

$s^4+6s^2+8\overline{)\,s^5+8s^3+15s}\,(s \rightarrow Z_1$

s^5+6s^3+8s

$2s^3+7s\overline{)\,s^4+6s^2+8}\,(\frac{s}{2} \rightarrow Y_2$

$s^4+\frac{7}{2}s^2$

$\frac{5}{2}s^2+8\overline{)\,2s^3+7s}\,(\frac{4}{5}s \rightarrow Z_3$

$2s^3+\frac{32}{5}s$

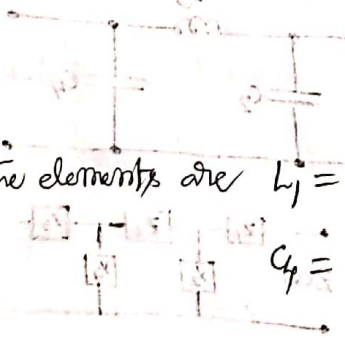
$\frac{3}{5}s\overline{)\,\frac{5}{2}s^2+8}\,(\frac{25}{6}s \rightarrow Y_4$

$\frac{5}{2}s^2$

$8\overline{)\,\frac{3}{5}s}\,(\frac{3}{40}s \rightarrow Z_5$

$\frac{3}{5}s$

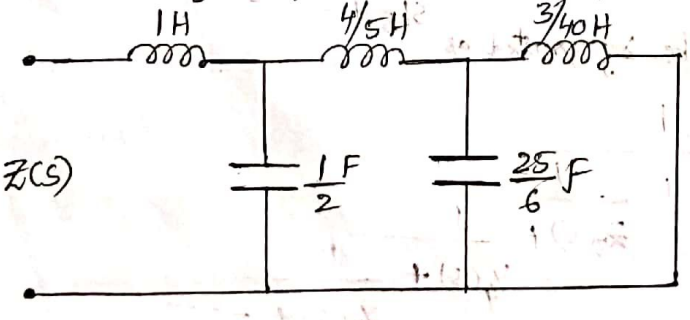
0



The elements are $L_1=1H$, $C_2=\frac{1}{2}F$, $L_3=\frac{4}{5}H$

$C_4=\frac{25}{6}F$, $L_5=\frac{3}{40}H$

The total realization shown in below fig.



(Q) Synthesize L-C immittance function $Z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$ into first Cauer form.

(A) $Z(s) = \frac{2s^4 + 20s^2 + 18}{s^3 + 4s}$

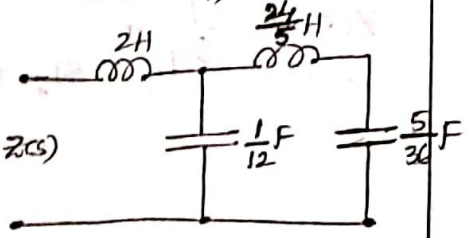
$s^3+4s) 2s^4 + 20s^2 + 18 (2s \rightarrow Z_1$

$\frac{2s^4 + 8s^2}{12s^2+18)\ s^3+4s (\frac{s}{12} \rightarrow Y_2}$

$\frac{s^3 + \frac{3}{2}s}{\frac{5}{2}s)\ 12s^2+18 (\frac{24}{5}s \rightarrow Z_3}$

$\frac{12s^2}{18)\ \frac{5}{2}s (\frac{5}{36}s \rightarrow Y_4}$

$\frac{\frac{5}{2}s}{0}$



The elements are

$L_1 = 2H,\ C_2 = \frac{1}{12}F,\ L_3 = \frac{24}{5}H,\ C_4 = \frac{5}{36}F$

Note :-

If given function is an impedance, then first Quotient must necessarily be impedance, if given function is admittance, then first Quotient must necessarily be admittance.

(4) Second Cauer Method :-

The second form of ladder network developed by Cauer consists of capacitances in series arms and inductances in shunt arms. We know that lowest degrees of numer and denominator polynomials of L-C immittance function must differ by unity. This indicates that there must be a zero (or) pole at origin.

Following similar procedure of First Cauer form and removing successively pole at $s=0$ it is possible to develop alternate ladder structure. This is called Second Cauer form.

For removing that pole at $s=0$, we follow the procedure, that first we arrange the given function polynomials in the ascending powers of 's'

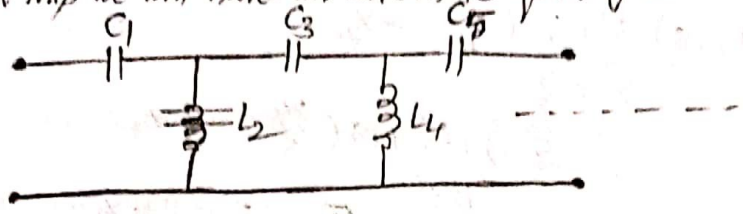
Therefore, $Z(s) = \frac{a_0 + a_1 s + \cdots + a_n s^n}{b_1 s + b_2 s^2 + \cdots + b_m s^m}$ where $n-m \neq \pm 1$

We now divide the lowest power of the numerator by the lowest power of the denominator and then we invert the reminder and divide again.

Thus the obtained continued-fraction expansion is

$$Z(s) = \frac{1}{sC_1} + \cfrac{1}{sL_2 + \cfrac{1}{sC_3 + \cfrac{1}{sL_4 + \cfrac{1}{sC_5 + \cdots}}}}$$

with this we will have an alternate form of ladder network as shown.



(9) Synthesize the admittance function $Y(s) = \frac{(s^2+2)(s^2+4)}{s(s^2+3)(s^2+5)}$ using second form of Cauer network.

(A) $Y(s) = \frac{s^4+6s^2+8}{s^5+8s^3+15s}$

Arrange $Y(s)$ in ascending power of s in the numerator and denominator

we have $Y(s) = \frac{8+6s^2+s^4}{15s+8s^3+s^5}$

$15s+8s^3+s^5 \overline{)\, 8+6s^2+s^4 \,(\, \frac{8}{15s} \rightarrow Y_1}$

$8+\frac{64}{15}s^2+\frac{8}{15}s^4$

$\frac{26}{15}s^2+\frac{7}{15}s^4 \overline{)\, 15s+8s^3+s^5 \,(\, \frac{225}{26s} \rightarrow Z_2}$

$15s+\frac{105}{26}s^3$

$\frac{103}{26}s^3+s^5 \overline{)\, \frac{26}{15}s^2+\frac{7}{15}s^4 \,(\, \frac{676}{1545s} \rightarrow Y_3}$

$\frac{26}{15}s^2+\frac{676}{1545}s^4$

$\frac{3}{103}s^4 \overline{)\, \frac{103}{26}s^3+s^5 \,(\, \frac{103\times103}{785s} \rightarrow Z_4}$

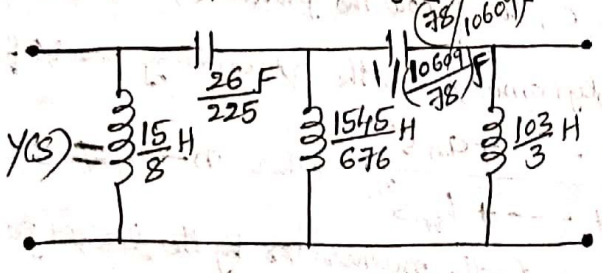
$\frac{103}{26}s^3$

$s^5 \overline{)\, \frac{3}{103}s^4 \,(\, \frac{3}{103s} \rightarrow Y_5}$

$\frac{3}{103}s^4$

0

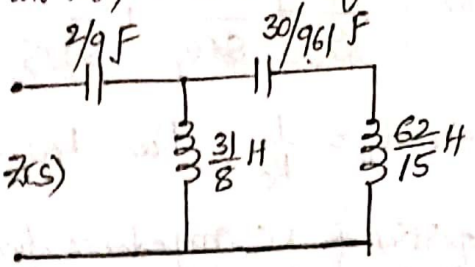
The network shown in fig.



(6) Synthesize L-C immittance function in the second Cauer form.

$$Z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

(A) $$Z(s) = \frac{18+20s^2+2s^4}{4s+s^3}$$



$$4s+s^3 \overline{)\,18+20s^2+2s^4\,(\tfrac{9}{2}s \rightarrow Z_1}$$

$$\frac{18+\frac{9}{2}s^2}{\frac{31}{2}s^2+2s^4}$$

$$\frac{31}{2}s^2+2s^4 \overline{)\,4s+s^3\,(\tfrac{8}{31}s \rightarrow Y_2}$$

$$\frac{4s+\frac{16}{31}s^3}{\frac{15}{31}s^3}$$

$$\frac{15}{31}s^3 \overline{)\,\tfrac{31}{2}s^2+2s^4\,(\tfrac{961}{30}s \rightarrow Z_3}$$

$$\frac{\frac{31}{2}s^2}{2s^4}$$

$$2s^4 \overline{)\,\tfrac{15}{31}s^3\,(\tfrac{15}{62}s \rightarrow Y_4}$$

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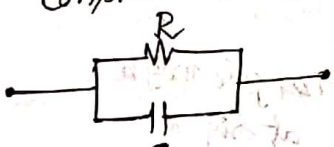
(1) RC impedance function :-

The study of RC network is of fundamental importance as they form building blocks of filters.

Consider a simple parallel RC and series RC network as shown below





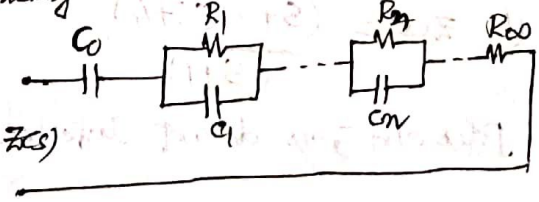
The impedance of parallel network $Z_{RC}(s) = \frac{R(\frac{1}{sC})}{R+\frac{1}{sC}} = \frac{1}{C}\left[\frac{1}{s+\frac{1}{RC}}\right]$

The admittance of series network $Y_{RC}(s) = \frac{1}{Z_{RC}(s)} = \frac{1}{(R+\frac{1}{sC})}$

$$Y_{RC}(s) = \frac{sC}{sRC+1} = \frac{1}{R}\left[\frac{s}{s+\frac{1}{RC}}\right]$$

By inspection we can say that both Z_{RC}, Y_{RC} have their poles on negative real axis in the S-plane. Foster realization of RC impedance function is obtained by simply replacing all inductances by resistances.

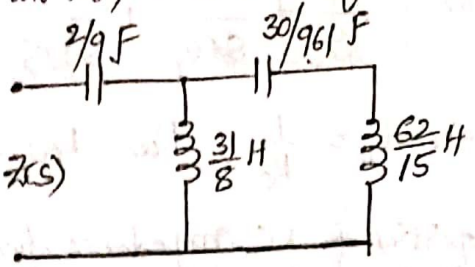
The general RC impedance circuit configuration network can be represented as shown.



(6) Synthesize L-C immittance function in the second Cauer form.

$$Z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

(A) $$Z(s) = \frac{18+20s^2+2s^4}{4s+s^3}$$



$$4s+s^3 \overline{)\,18+20s^2+2s^4\,(\tfrac{9}{2}s \rightarrow Z_1}$$

$$\frac{18+\frac{9}{2}s^2}{\frac{31}{2}s^2+2s^4}$$

$$\frac{31}{2}s^2+2s^4 \overline{)\,4s+s^3\,(\tfrac{8}{31}s \rightarrow Y_2}$$

$$\frac{4s+\frac{16}{31}s^3}{\frac{15}{31}s^3}$$

$$\frac{15}{31}s^3 \overline{)\,\tfrac{31}{2}s^2+2s^4\,(\tfrac{961}{30}s \rightarrow Z_3}$$

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(1) RC impedance function :-

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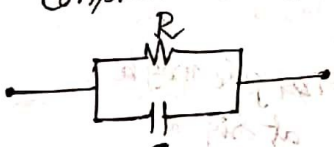



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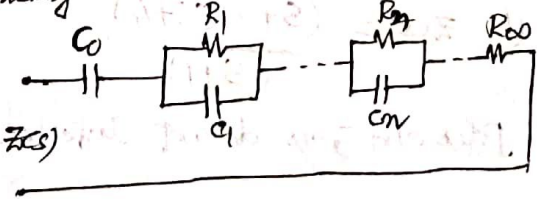


$$Z(s) = \frac{K_0}{s} + \frac{K_1}{s+\sigma_1} + \frac{K_2}{s+\sigma_2} + \cdots + \frac{K_n}{s+\sigma_n} + \cdots + K_\infty$$

where $C_0 = \frac{1}{K_0}$, $R_\infty = K_\infty$, $C_1 = \frac{1}{K_1}$, $R_1 = \frac{K_1}{\sigma_1}$, $C_i = \frac{1}{K_i}$, $R_i = \frac{K_i}{\sigma_i}$

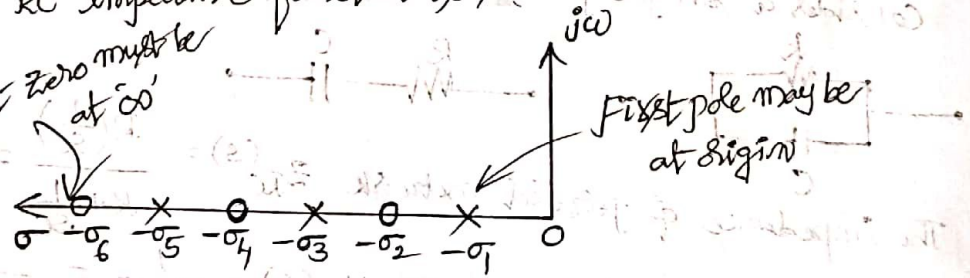
Properties of RC impedance function :- (RL admittance function)

(1) The poles of RC driving point impedance are on −ve real axis. Similarly poles of an RL admittance function are also on −ve real axis. We can thus conclude that the zeros of RC impedance are also on −ve real axis.

(2) The residues of the poles K_i are real & positive.

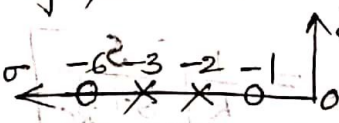
(3) All poles & zeros are simple, alternate and are located on negative real axis. (4) $\frac{dz}{d\sigma}$ (slope) is negative. (5) $Z(0) > Z(\infty)$

(6) The critical frequency nearest to the origin (0) must be a pole (Singularity nearest to origin is pole), whereas critical freq nearest to infinity (∞) must be a zero (Singularity nearest to infinity is zero). Since there can be no pole at infinity, the degree of the numerator cannot exceed that of the denominator. The typical pole-zero diagram for RC impedance function is shown.



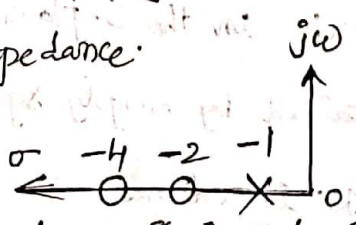
(7) Test if following function represent RC driving point imp function

(a) $Z(s) = \frac{(s+1)(s+6)}{(s+2)(s+3)}$



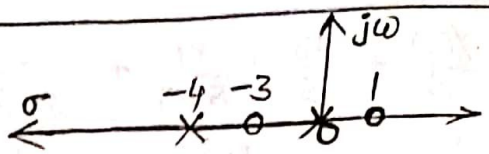
All poles & zeros are simple & located on negative real axis. poles & zeros do not interlace and singularity nearest to origin is zero. Hence it is not RC impedance.

(b) $Z(s) = \frac{(s+2)(s+4)}{(s+1)}$



poles and zeros do not interlace hence it is not RC − imp function

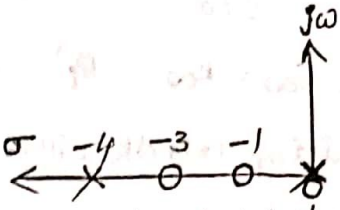
(c) $Z(s) = \frac{(s+1)(s+3)}{s(s+4)}$



Zero is lying on positive real axis.

Hence it is not RC imp. function.

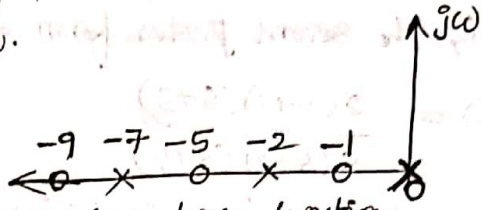
(d) $Z(s) = \frac{(s+1)(s+3)}{s(s+4)}$



~~Zero is lying on positive real axis.~~ poles & zeros do not interlace,

Hence it is not RC impedance function.

(e) $Z(s) = \frac{(s+1)(s+5)(s+9)}{s(s+2)(s+7)}$



As it satisfies all properties it is RC impedance function.

Synthesis of RC impedance function & RL admittance function:-

① First-Foster form of RC network:-

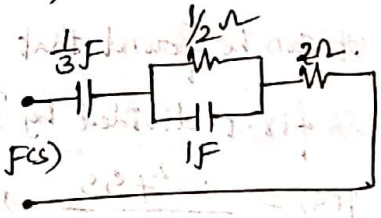
It is obtained by partial fraction of $Z_{RC}(s)$:

$$Z_{RC}(s) = \frac{K_0}{s} + \frac{K_1}{s+\sigma_1} + \cdots + \frac{K_i}{s+\sigma_i} + \cdots + K_\infty$$

where $K_0, K_1, \cdots K_i$ are residues of poles at origin, $-\sigma_1, -\sigma_2 \cdots -\sigma_i$. All these constants are real & positive. If there is no pole at origin $K_0 = 0$, if zero at ∞, $K_\infty = 0$

(g) Ex:- obtain first foster form realization of $F(s) = \frac{2(s+1)(s+3)}{s(s+2)}$

(A) The numerator and denominator of same degree, divide numerator by denominator $F(s) = 2 + \frac{4s+6}{s^2+2s}$



$$\frac{4s+6}{s^2+2s} = \frac{A}{s} + \frac{B}{s+2} \Rightarrow A = 3, B = 1$$

$$F(s) = 2 + \frac{3}{s} + \frac{1}{s+2} = \frac{1}{s(1/3)} + \frac{1}{s+2} + 2$$

$C_0 = 1/3\,F$, $R_\infty = 2\Omega$, $C_1 = \frac{1}{1} = 1F$, $R_1 = \frac{1}{2}\Omega$

② Second-Foster form of RC network:-

The second Foster form is obtained by expanding specified RC admittance by partial fraction of $Y(s) = \frac{1}{Z(s)}$

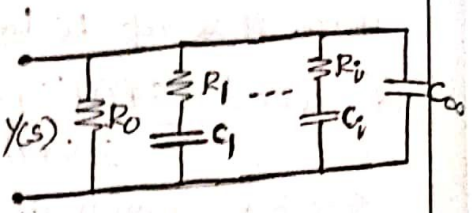
In case of RC networks, the partial fraction expansion of $Y(s)$ normally leads to negative residues at its poles. Therefore for RC networks, $\frac{Y(s)}{s}$ is expanded instead of $Y(s)$.

$$\frac{Y(s)}{s} = \frac{k_0}{s} + \frac{k_1}{s+\sigma_1} + \cdots + \frac{k_i}{s+\sigma_i} + \cdots + k_\infty$$

$$Y(s) = k_0 + \frac{k_1 s}{s+\sigma_1} + \cdots + \frac{k_i s}{s+\sigma_i} + \cdots + k_\infty s$$

$R_0 = 1/k_0$, $C_\infty = k_\infty$, $R_i = \frac{1}{k_i}$, $C_i = \frac{k_i}{\sigma_i}$

The resulting network will be shown in fig.



⑤ Realize the second Foster form of the driving point imp. function given by

$$Z(s) = \frac{2(s+1)(s+3)}{s(s+2)(s+4)}$$

(A) The second Foster form is obtained by expanding admittance function by partial fraction

$$Y(s) = \frac{1}{Z(s)} = \frac{s(s+2)(s+4)}{2(s+1)(s+3)} = \frac{s^3+6s^2+8s}{2s^2+8s+6}$$

The degree of the numerator is one greater than the degree of denominator. Hence, we shall divide the numerator by denominator.

$$2s^2+8s+6 \;\overline{)\; s^3+6s^2+8s \;(\; \frac{s}{2}}$$

$$\frac{s^3+4s^2+3s}{2s^2+5s}$$

$$Y(s) = \frac{s}{2} + \frac{2s^2+5s}{2s^2+8s+6} = \frac{s}{2} + P(s)$$

It can be found that one of the residues would be negative. Hence it is first divided by 's' and then the residues are evaluated.

$$P(s) = \frac{2s^2+5s}{2s^2+8s+6} \Rightarrow \frac{P(s)}{s} = \frac{2s+5}{2s^2+8s+6} = \frac{(2s+5)}{2(s+1)(s+3)}$$

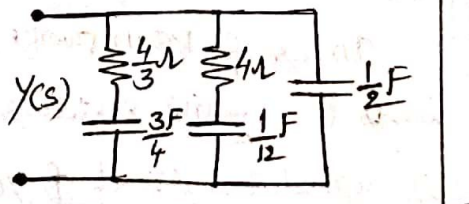
$$\frac{P(s)}{s} = \frac{1}{2}\left[\frac{A}{s+1} + \frac{B}{s+3}\right] \Rightarrow A = \frac{3}{2},\ B = \frac{1}{2}$$

$$\frac{P(s)}{s} = \frac{3}{2(s+1)} + \frac{1}{2(s+3)} \Rightarrow P(s) = \left[\frac{3s}{2(s+1)} + \frac{s}{2(s+3)}\right]\frac{1}{2}$$

$$Y(s) = \frac{s}{2} + \left[\frac{3s}{2(s+1)} + \frac{s}{2(s+3)}\right]\frac{1}{2} = \frac{s(3/4)}{(s+1)} + \frac{s(1/4)}{(s+3)} + s\left(\frac{1}{2}\right)$$

$R_1 = \frac{1}{k_1} = \frac{4}{3}\Omega$ | $R_2 = \frac{1}{k_2} = 4\Omega$ | $C_\infty = \frac{1}{2}F$

$C_1 = \frac{k_1}{\sigma_1} = \frac{(3/4)}{1} = \frac{3}{4}F$ | $C_2 = \frac{k_2}{\sigma_2} = \frac{1}{12}F$

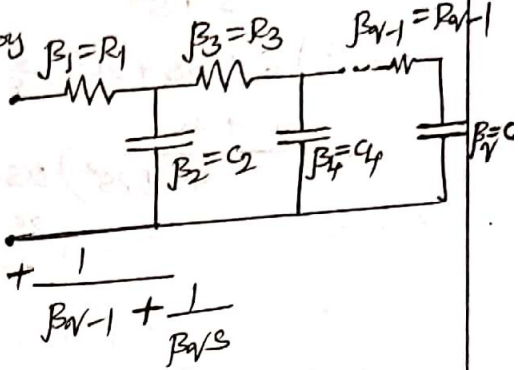


(3) First Cauer form of RC network :-

The whole synthesis process developed by continued fraction expansion. The quotients represent elements of ladder network. The given polynomial is arranged as numerator and denominator in the descending power of 's'. In which the resistors are connected as series arms and capacitors are connected as shunt arms.

The continued fraction expansion is given by

$$F(s) = \beta_1 + \cfrac{1}{\beta_2 s + \cfrac{1}{\beta_3 + \cfrac{1}{\beta_4 s + \cfrac{1}{\beta_5 + \cdots + \cfrac{1}{\beta_{n-1} + \cfrac{1}{\beta_n s}}}}}}$$



2016

(Q) obtain first Cauer form realization of $F(s) = \frac{2(s+1)(s+3)}{s(s+2)}$

(A)

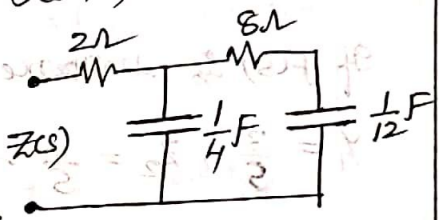
$s^2+2s \;)\; 2s^2+8s+6 \;(2 \rightarrow Z_1$

$\frac{2s^2+4s}{4s+6}) \; s^2+2s \;(\frac{s}{4} \rightarrow Y_2$

$s^2+\frac{3}{2}s \;)\; 4s+6 \;(8 \rightarrow Z_3$

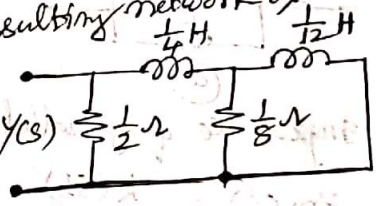
$\frac{4s}{6}) \; \frac{1}{2}s \;(\frac{1}{12}s \rightarrow Y_4$

$\frac{\frac{1}{2}s}{0}$



If $F(s)$ denotes admittance function then resulting network is

$Y_1 = 2\,\upsilon$, $Z_2 = \frac{1}{4}H$, $Y_3 = 8\,\upsilon$, $Z_4 = \frac{1}{12}H$

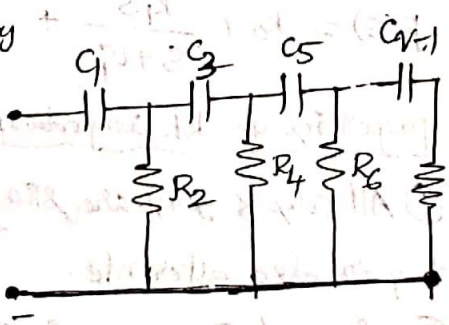


(4) Second Cauer form of RC network :-

The second Cauer form is obtained by expanding RC impedance function in continued fraction expansion. We first arrange the both numerator and denominator of RC impedance function in ascending order of 's'.

The continued fraction expansion is given by

$$F(s) = \frac{1}{sC_1} + \cfrac{1}{\frac{1}{R_2} + \cfrac{1}{\frac{1}{sC_3} + \cfrac{1}{\frac{1}{R_4} + \cfrac{1}{\frac{1}{sC_5} + \cdots}}}}$$



(9) Synthesize the given function in second Cauer form

2016

$$F(s) = \frac{2(s+1)(s+3)}{s(s+2)}$$

(A) Arrange the numerator & denominator in Ascending power of 's'.

$$F(s) = \frac{6+8s+2s^2}{2s+s^2}$$

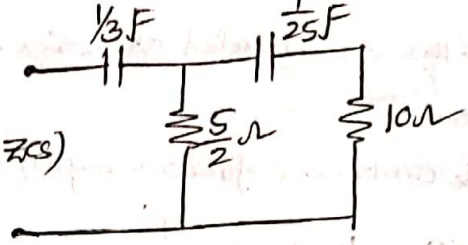
$2s+s^2 \overline{)\, 6+8s+2s^2 \,(\frac{3}{s} \rightarrow Z_1}$

$\frac{6+3s}{5s+2s^2} \overline{)\, 2s+s^2 \,(\frac{2}{5} \rightarrow Y_2}$

$\frac{2s+\frac{4}{5}s^2}{\frac{1}{5}s^2} \overline{)\, 5s+2s^2 \,(\frac{25}{5} \rightarrow Z_3}$

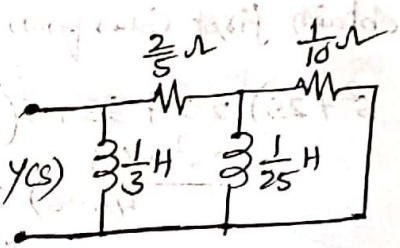
$\frac{5s}{2s^2} \overline{)\, \frac{1}{5}s^2 \,(\frac{1}{10} \rightarrow Y_4}$

$\frac{\frac{1}{5}s^2}{0}$



If $F(s)$ is admittance function

$Y_1 = \frac{3}{s}$, $Z_2 = \frac{2}{5}\Omega$, $Y_3 = \frac{25}{5}$, $Z_4 = \frac{1}{10}\Omega$

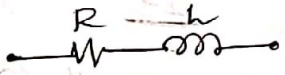


(2) RL impedance functions :-

Consider parallel RL, Series RL networks



(a)



(b)

Impedance of parallel R-L network is

$$Z_{RL}(s) = \frac{R(sL)}{R+sL} = \frac{RsL}{L[s+\frac{R}{L}]} = \frac{1}{L}\left[\frac{sR}{s+R/L}\right]$$

Admittance of Series R-L network is

$$Y_{RL}(s) = \frac{1}{Z_{RL}(s)} = \frac{R+sL}{RsL} = \frac{1}{R+sL} = \frac{1}{L(s+\frac{R}{L})}$$

$$F(s) = k_0 + \frac{k_1 s}{s+\sigma_1} + \frac{k_2 s}{s+\sigma_2} + \cdots + \frac{k_i s}{s+\sigma_i} + \cdots + k_\infty s$$

Properties of RL impedance functions :- (RC admittance function)

(1) All poles & zeros are simple and are located on negative real axis and they are also alternate.

(2) poles and zeros are interlace.

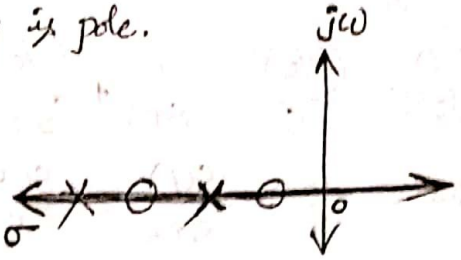
③ The singularity nearest to origin is zero.

④ The singularity nearest to infinity is pole.

⑤ The residues are real & positive

⑥ Slope $\frac{d\,Z(\sigma)}{d\sigma}$ is positive

⑦ $Z(\infty) > Z(0)$



pole-zero diagram of RL imp (or) RC adm is shown above.

Synthesis of RL impedance (or) RC admittance function:-

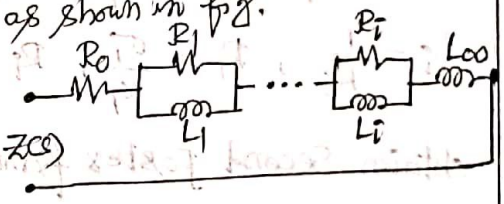
Residues of poles of $Z_{RL}(s)$ must be real & negative. Because of this property a partial fraction of $Z_{RL}(s)$ contain negative residues, hence this does not represent RL impedance at all. To obtain foster form of RL imp we expand $\frac{Z_{RL}(s)}{s}$ into partial fractions, which yields positive residues

$$\frac{Z_{RL}(s)}{s} = \frac{k_0}{s} + \frac{k_1}{s+\sigma_1} + \cdots + \frac{k_i}{s+\sigma_i} + \cdots + k_\infty, \text{ multiply by 's'}$$

$$Z_{RL}(s) = k_0 + \frac{k_1 s}{s+\sigma_1} + \cdots + \frac{k_i s}{s+\sigma_i} + \cdots + k_\infty s$$

① Foster First form of RL impedance:-

The first foster form is synthesized as shown in fig.



where $R_0 = k_0$, $L_\infty = k_\infty$

$$R_1 = k_1, \quad L_1 = \frac{R_1}{\sigma_1} = \frac{k_1}{\sigma_1}$$

$$R_i = k_i, \quad L_i = \frac{k_i}{\sigma_i}$$

⑥ Realize the first foster form of the RL network function $Z(s) = \frac{3(s+1)(s+4)}{(s+3)(s+5)}$

Ⓐ $$Z(s) = \frac{3s^2+15s+12}{s^2+8s+15} = 3 + \frac{-(9s)+(33)}{s^2+8s+15}$$

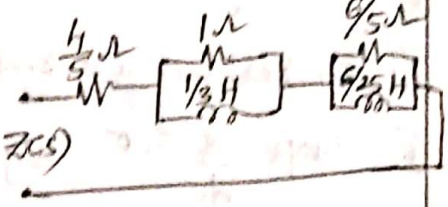
It is evident that, because of the negative sign of the second term, we should consider $\frac{Z(s)}{s}$ and obtain its partial fraction expansion.

$$\frac{Z(s)}{s} = \frac{3(s+1)(s+4)}{s(s+3)(s+5)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+5}$$

$A = \frac{4}{5}$, $B = 1$, $C = \frac{6}{5}$

$$\frac{Z(s)}{s} = \frac{(4/5)}{s} + \frac{1}{s+3} + \frac{(6/5)}{s+5}$$

$$Z(s) = \frac{4}{5} + \frac{s(1)}{s+3} + \frac{s(6/5)}{s+5}$$



$R_0 = \frac{4}{5}\Omega$, $K_1 = 1$ | $K_2 = 6/5$

$L_\infty = 0\,H$ | $R_1 = K_1 = 1\Omega$ | $R_2 = K_2 = 6/5\,\Omega$

$\therefore L_1 = \frac{K_1}{\sigma_1} = \frac{1}{3}H$ | $L_2 = \frac{K_2}{\sigma_2} = \frac{6}{5\times 5} = \frac{6}{25}H$

② Second Foster form of RL network :-

It is possible to realize to second Foster form of a RL network by considering driving point admittance function $Y_{RL}(s)$

Let $Y(s) = \frac{a_0 s^n + a_1 s^{n-1} + \cdots + a_n}{b_0 s^m + b_1 s^{m-1} + \cdots + b_m}$ $(m > n)$

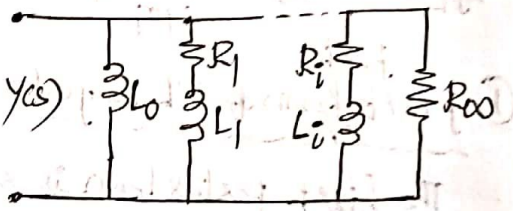
By partial fraction we can express $Y(s)$ as

$$Y(s) = \frac{K_0}{s} + \frac{K_1}{s+\sigma_1} + \cdots + \frac{K_i}{s+\sigma_i} + \cdots + K_\infty$$

$L_0 = \frac{1}{K_0}$, $R_\infty = \frac{1}{K_\infty}$

$L_1 = \frac{1}{K_1}$, $\frac{1}{R_1} = \frac{1}{\sigma_1 L_1} = \frac{1}{(K_1/\sigma_1)}$

$L_i = \frac{1}{K_i}$, $R_1 = \frac{\sigma_1}{K_1}$; $R_i = \frac{\sigma_i}{K_i}$

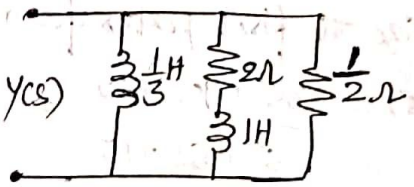


Q) obtain second Foster form of $F(s) = \frac{2(s+1)(s+3)}{s(s+2)}$

A) Second Foster form realized for $Y(s) = F(s) = \frac{2(s+1)(s+3)}{s(s+2)}$

$$Y(s) = 2 + \frac{4s+6}{s^2+2s}$$

$$\frac{4s+6}{s^2+2s} = \frac{A}{s} + \frac{B}{s+2} = \frac{3}{s} + \frac{1}{s+2}$$



$$Y(s) = \frac{3}{s} + \frac{1}{s+2} + 2$$

$L_0 = 1/3\,H$ | $K_1 = 1$ | $\sigma_1 = 2$

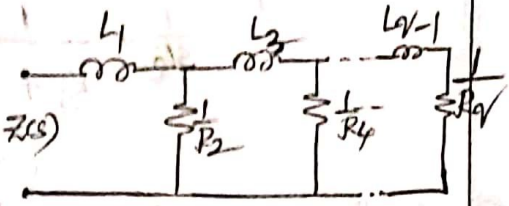
$R_\infty = (1/2)\Omega$ | $L_1 = \frac{1}{K_1} = 1H$ | $R_1 = \frac{\sigma_1}{K_1} = \frac{2}{1} = 2\Omega$

(3) Cauer form - I realization of RL impedance :-

In which the numerators & denominators polynomials are arranged in descending power of 's' and realize the function using continued fraction expansion.

If the network function is Z(s), then

$$Z(s) = sL_1 + \cfrac{1}{\cfrac{1}{R_2} + \cfrac{1}{sL_3 + \cfrac{1}{\cfrac{1}{R_4} + \cdots}}}$$



(4) Realize the first cauer form of the impedance function

$$Z(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}$$

(A) $$Z(s) = \frac{s^2+4s+3}{s^2+6s+8}$$

It is seen that in the continued fraction expansion, some of the quotient terms are negative. Hence we shall consider the admittance function $Y(s) = \frac{s^2+6s+8}{s^2+4s+3}$

$$s^2+4s+3 \overline{)\, s^2+6s+8 \,(1} \rightarrow Y_1$$

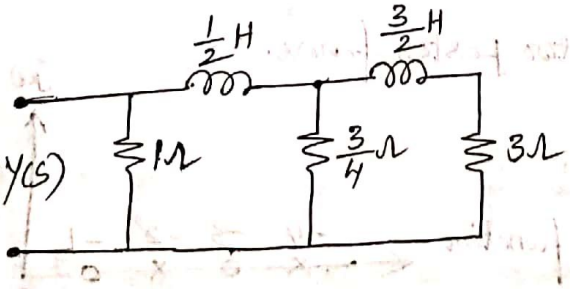
$$\frac{s^2+4s+3}{2s+5} \overline{)\, s^2+4s+3 \,(\frac{s}{2}} \rightarrow Z_2$$

$$\frac{s^2+\frac{5}{2}s}{\frac{3}{2}s+3} \overline{)\, 2s+5 \,(\frac{4}{3}} \rightarrow Y_3$$

$$\frac{2s+4}{1} \overline{)\, \frac{3}{2}s+3 \,(\frac{3}{2}s} \rightarrow Z_4$$

$$\frac{\frac{3}{2}s}{3} \overline{)\, 1 \,(\frac{1}{3}} \rightarrow Y_5$$

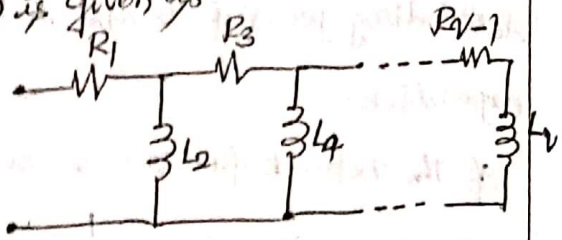
$$\frac{1}{0}$$



(4) Second Cauer form of RL network :- Arrange the polynomials in ascending power of 's'

If the network function is Z(s) is synthesized in the second Cauer form as the ladder network then Z(s) is given as

$$Z(s) = R_1 + \cfrac{1}{\frac{1}{sL_2} + \cfrac{1}{R_3 + \cfrac{1}{\frac{1}{sL_4} + \cdots}}}$$



(5) Synthesize F(s) in second Cauer forms for $F(s) = \frac{2(s+1)(s+4)}{(s+2)(s+6)}$

(A) Arrange numerators & denominators polynomials in ascending order of 's'

$12+8s+s^2)\ 8+10s+2s^2\ (\frac{2}{3} \rightarrow Z_1, Y_1$

$\frac{8+\frac{16}{3}s+\frac{2}{3}s^2}{\frac{14}{3}s+\frac{4}{3}s^2)\ 12+8s+s^2\ (\frac{18}{7s} \rightarrow Y_2, Z_2}$

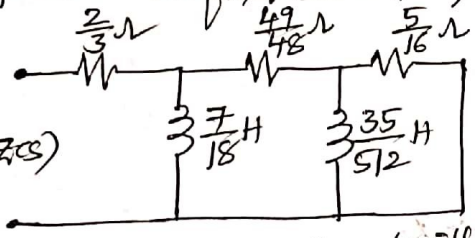
$\frac{12+\frac{24}{7}s}{\frac{32}{7}s+s^2)\ \frac{14}{3}s+\frac{4}{3}s^2\ (\frac{49}{48} \rightarrow Z_3, Y_3}$

$\frac{\frac{14}{3}s+\frac{49}{48}s^2}{\frac{5}{16}s^2)\ \frac{32}{7}s+s^2\ (\frac{512}{355} \rightarrow Y_4, Z_4}$

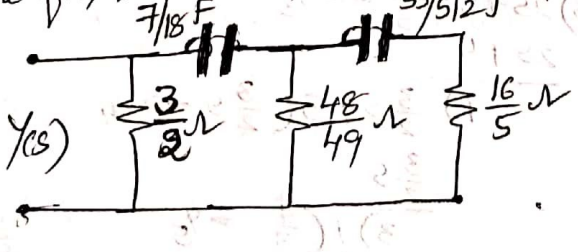
$\frac{\frac{32}{7}s}{s^2)\ \frac{5}{16}s^2\ (\frac{5}{16} \rightarrow Z_5, Y_5}$

$\frac{\frac{5}{16}s^2}{0}$

If F(s) denotes the impedance function the first element is the imp..



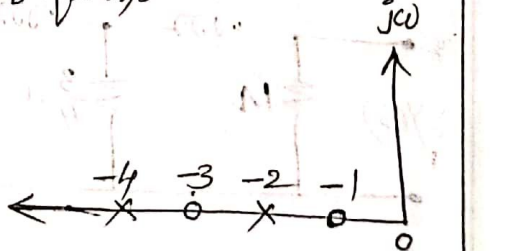
If F(s) denotes the admittance function, the first element is the admittance



(6) Synthesize following F(s) in two Fosters forms.

$$F(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}$$

(A) The pole-zero diagram for the function F(s) as shown.



Above represents RL impedance function (or) RC admittance function

① poles & zeros are simple and lying on negative real axis.

② poles & zeros interlace

③ Singularity nearest to origin is zero

④ Singularity nearest to infinity is pole

⑤ $Z(\infty) > Z(0)$

$$F(s) = \frac{s^2+4s+3}{s^2+6s+8} = 1 + \frac{\left(\frac{-1}{2}\right)}{s+2} + \frac{\left(\frac{-3}{2}\right)}{s+4}$$

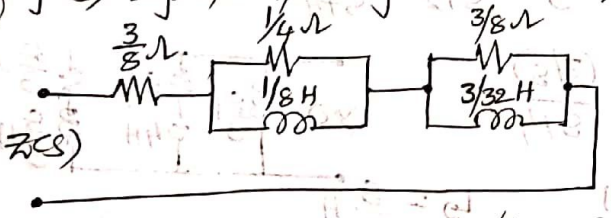
As residues are negative

$$\frac{F(s)}{s} = \frac{(s+1)(s+3)}{s(s+2)(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$\frac{F(s)}{s} = \frac{\left(\frac{3}{8}\right)}{s} + \frac{\left(\frac{1}{4}\right)}{s+2} + \frac{\left(\frac{3}{8}\right)}{s+4}$$

$$F(s) = \frac{3}{8} + \frac{s\left(\frac{1}{4}\right)}{s+2} + \frac{s\left(\frac{3}{8}\right)}{s+4} = Z_{RL}(s) = Y_{RC}(s)$$

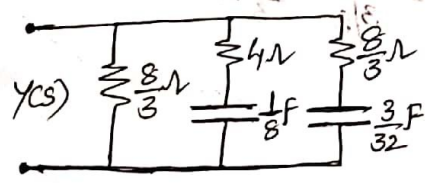
If $F(s)$ represents impedance it is synthesized in FF-①



$R_i = K_i$ $\quad R_1 = \frac{1}{4}\Omega, L_1 = \frac{1}{8}H$

$L_i = \frac{K_i}{\sigma_i}$ $\quad R_2 = \frac{3}{8}\Omega, L_2 = \frac{3}{32}H$

If $F(s)$ represents admittance it is synthesized in FF-②

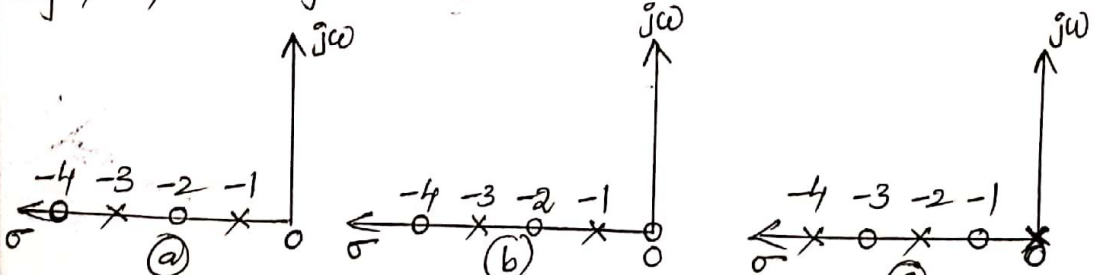


$R_i = \frac{1}{K_i}, \quad C_i = \frac{K_i}{\sigma_i}$

$R_1 = 4\Omega \quad R_2 = \frac{8}{3}\Omega$

$C_1 = \frac{1}{8}F \quad C_2 = \frac{3}{32}F$

⑨ Three pole-zero diagrams shown in figs. pick up the diagram that represents RL impedance function and synthesize in two Foster forms.



(a) (b) (c)

(A) fig (b) represents pole-zero diagram of RL impedance function as the singularity nearest to origin is zero and nearest to infinity is pole.

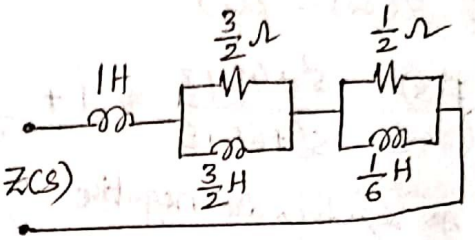
First Fosters form:-

$$Z_{RL}(s) = \frac{s(s+2)(s+4)}{(s+1)(s+3)}$$

$$\frac{Z(s)}{s} = \frac{(s+2)(s+4)}{(s+1)(s+3)} = 1 + \frac{2s+5}{(s+1)(s+3)}$$

$$\frac{Z(s)}{s} = 1 + \frac{\left(\frac{3}{2}\right)}{s+1} + \frac{\left(\frac{1}{2}\right)}{s+3}$$

$$Z(s) = s + \frac{s\left(\frac{3}{2}\right)}{s+1} + \frac{s\left(\frac{1}{2}\right)}{s+3}$$



$L_0 = 1H \quad K_1 = \frac{3}{2} \quad K_2 = \frac{1}{2}$

$R_\infty = 0\Omega \quad R_1 = K_1 = \frac{3}{2}\Omega \quad R_2 = K_2 = \frac{1}{2}\Omega$

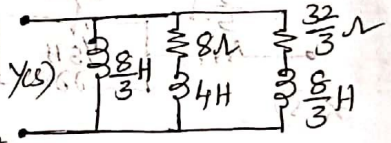
$L_1 = \frac{K_1}{\sigma_1} = \frac{3}{2}H \quad L_2 = \frac{K_2}{\sigma_2} = \frac{1}{6}H$

Second Fosters form:-

Second Fosters form realized for $Y_{RL}(s)$

$$Y_{RL}(s) = \frac{(s+1)(s+3)}{s(s+2)(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$Y_{RL}(s) = \frac{(3/8)}{s} + \frac{(1/4)}{s+2} + \frac{(3/8)}{s+4}$$



$L_0 = \frac{8}{3}H, \quad L_1 = \frac{1}{K_1} = 4H \quad L_2 = \frac{8}{3}H$

$R_1 = \frac{\sigma_1}{K_1} = \frac{2}{1/4} = 8\Omega \quad R_2 = \frac{4}{3/8} = \frac{32}{3}\Omega$